Abstract

This paper studies the effects of changes in the macroeconomic volatility on the sovereign risk premium and default probability. I first document a high positive correlation between risk premium and volatility for a set of European economies and high persistence of these variables. Then, I propose a model of endogenous sovereign risk premium for a small open economy as in Arellano (2008). Domestic output is exogenously determined and follows a first order autoregressive process with stochastic volatility. Using Gibbs Sampling I estimate the output process for Spain for the period 1980 to 2011. I feed the estimated parameters to the model and calibrate the remaining parameters to standard values. I find that changes in the macroeconomic volatility have a large impact on the saving and default decisions as well as on the sovereign risk premium, the price of debt and default probability. I find that an increase in volatility might induce an increase in savings. However, when there is a substantial default probability and a low cost of default, an increase in volatility might generate a decrease in savings. An increase in volatility is then associated to an increase in default probability and hence, induce a lower price of debt and a higher risk premium. This mechanism rationalizes the positive correlation between risk premium and volatility observed in the data.

Keywords: Default, Sovereign Risk Premium, Stochastic Volatility.

JEL Classification: E21, E32, F34, F41.
1 Introduction

This paper studies the effects of changes in the macroeconomic volatility on the sovereign risk premium and default probability of small open economies. In the last thirty years, the world experienced a variety of macroeconomic crisis affecting a broad set of economies, developed and emerging, with the common denominator of sharp increases in sovereign risk premiums (or reductions in credit ratings) and debt positions. These features raised the attention of policymakers and economists in academia towards the behavior of sovereign risk premium and the external borrowing conditions.

The current crisis experienced by West European countries such as Portugal, Spain, Italy, Greece or Ireland, is signed by weak fiscal fundamentals, depressed economic performance, high and volatile sovereign risk premiums and high macroeconomic uncertainty. Due to the poor macroeconomic performance, the fiscal conditions worsen dramatically while the burden of debt and the sovereign risk premium increases and the external financing conditions deteriorate while imposing increasing limitations for business cycle smoothing. This scenario is often associated to increasing macroeconomic volatility.

Even though the recent literature on open economies studied the impact of risk premium shocks and risk premium volatility shocks on the macroeconomic performance of small open economies, to my best knowledge, there have been no attempts to understand the way increasing macroeconomic volatility affects the sovereign risk premium itself, nor how it might affect the savings and default decisions.

The objective of this paper is to fill this gap in the literature. In this paper, I introduce a stochastic volatility shocks to the income process of a small open economy to study the way volatility shocks affect endogenous risk premium and domestic savings and indebtedness position. In this paper, the endogenous sovereign risk premium is observed as a consequence of debt repudiation, i.e. the government cannot commit to repay its debt.

In the first place, I study the data for a variety of European economies, in particular Spain, Italy, Portugal and Greece, during the debt crisis that started in 2008. I use Gibbs Sampling to estimate an autoregressive stochastic process for output with stochastic volatility for these economies. I compute the smoothed estimates for the time-varying volatility and I am able to establish the following stylized facts: first, I find that output is negatively correlated with the sovereign spread; second, I find that volatility of output is positively correlated with the sovereign risk premium; and third, I find that both sovereign risk premiums and output volatility are significantly persistent.

Then, the paper studies the way volatility changes affect the sovereign risk premium and default probabilities. To do so, I develop a dynamic stochastic general equilibrium model that generates an endogenous risk premium, as in Eaton and Gersovitz (1981) and Arellano (2008). The risk premium appears because the government cannot commit to honor its debts at any period. Hence, default can occur in equilibrium and in this way, the risk premium works as a price of the default probability. I model the output process following a stochastic volatility process with jumps.

\footnote{These features are documented in several papers, for instance Afonso et al. (2011), Broner et al. (2007), Mueller et al. (2011), Reinhart et al. (2003), Reinhart (2002a) and Reinhart (2002b).}

stochastic volatility model and I fit the income process to the Spanish estimates obtained in the empirical section.\footnote{I consider Spain to be a good application for this model and question. The main reason for using Spanish data is that it is available starting in 1970 and the series run up to these days and hence it is very convenient for the estimation of the stochastic volatility model. Second, it is one of the more compromised economies in the current crises. The use of a default model to analyze Spanish risk premium is due to the fact that these models provide us a way of endogenizing risk premium without removing the rational expectations assumption or imposing fiscal policy rules, as opposed to other papers such as Schabert (2010). Additionally, some authors have recently claimed that default episodes, even occurring long time in the past, might have long lasting consequences, see for instance Christophe Chamley’s article on 1575 Spanish default episode, in Bloomberg, July 31, 2011.} I feed the estimates into the general equilibrium model and calibrate the remaining parameters to standard values in the literature.

I find that an increase in the volatility of output has asymmetric impact on endogenous variables, depending on the level of debt and output. In a high output and high asset accumulation situation, that \textit{caeteris paribus} is associated to a nil default probability, an increase in volatility triggers precautionary savings behavior. On the other hand, increasing the volatility of output when the economy experiences a large default probability can actually diminish savings. In this case, default option becomes more attractive given a certain level of consumption conditional on default that bounds the value function from below.

Then, I inquire whether changes in volatility can generate substantial variation in the sovereign risk premium. I find that income level shocks alone can generate less than 1/3 of the risk premium variability. I find that for the benchmark calibration, the model with level and volatility shocks can generate a standard deviation of the sovereign risk premium of about 3.7%, similar to the one observed in the data. However, when I only consider income level shocks, the model can generate a standard deviation of the sovereign risk premium of about 1.1%. This suggests that the remaining 2/3 of the risk premium variability is explained directly by volatility shocks and by the interaction between volatility and level shocks. Additionally, the simple model is able to account for the observed output and consumption volatilities and, importantly, main stylized facts described in the empirical section, i.e. the fact that increases in volatility are associated to increases in the sovereign risk premium and that output is negatively correlated with sovereign risk premium. To my best knowledge, this paper is the first one to document the the fact that increases in volatility are associated to increases in the sovereign risk premium.

A natural question follows, why should we care about the impact of volatility shocks in the risk premium? There are two main reasons to study the impact of volatility shocks on risk premium, one from the asset pricing literature, and also from a policymaker perspective. First, from an asset pricing perspective, if the macroeconomic volatility affects the sovereign risk premium, the pricing of sovereign bonds in the absence of this factor might be significantly off. Moreover, not taking into account macroeconomic volatility as a relevant factor might induce researchers to overestimate the importance of other factors, such as the output growth. On the other hand, from a policymaker perspective, not addressing the impact of volatility shocks in the risk premium might lead to the erroneous conclusion that stabilizing output is the only prudential policy to control the risk premium dynamics. We show in the next sections that even if output is constant, there can be substantial variability in the risk premium due to changes in volatility and also the impact of income shocks is substantially
amplified by volatility shocks.

This paper is related to the literature on sovereign risk premium and default that started with the seminal paper by Eaton and Gersovitz (1981). In that paper, the authors model strategic default, a situation where agents take a default decision as part of an optimal plan. They show that in this framework, default occurs during bad times. In the same line, Arellano (2008) and Aguiar and Gopinath (2006) discussed and apply a variety of models in the same spirit to understand the macroeconomic behavior of Argentina during 2002 default. Lizarazo (2012) extends this framework to account for risk aversion of international lenders. Yue (2010), on the other hand, studies the default decisions under debt renegotiation. An optimal policy approach is taken by Cuadra and Sapriza (2008) who model the default decision for a government that has access to distortionary taxation. More recently, Mendoza and Yue (2012) study the default decision with endogenous output in order to reconcile the stylized facts of default episodes and the business cycle. This literature allows to model risk premium endogenously in fully rational perfect information environments. They do not, however, study the impact of volatility shocks in the sovereign risk premium.

This paper is also related to the literature on uncertainty and volatility shocks, such as Justiniano and Primiceri (2008), Bloom (2009), Bloom et al. (2007), Személy (2012), that uses general and partial equilibrium models to study the effect of changes in the volatility of technology shocks in general equilibrium models for closed economies. These papers, however, do not study the role of volatility shocks in accounting for the variability of an endogenous sovereign risk premium.

Additionally, my paper is closely related to Fernandez-Villaverde et al. (2011) and Gruss and Mertens (2009) that study the effect of changes in the volatility of risk premium shocks in macroeconomic dynamics. The authors, however, take the risk premium as exogenously determined and study the way changes in the volatility of risk premium shocks affect the macroeconomic performance of small open economies. In contrast, my paper studies the effect of changes in the volatility of the income process on the endogenous sovereign risk premium. Hence, I consider this paper is a complement to theirs.

Even though there is an extensive literature that studies risk premium behavior, there is no agreement on the factors driving this variable. One strand of the literature finds evidence suggesting that the sovereign risk premium is exogenously determined, or at least a large part of it does not depend on domestic conditions. In this line we can find Neumeyer and Perri (2005), Uribe and Yue (2006), Fernandez-Villaverde et al. (2011), Gruss and Mertens (2009) and Longstaff et al. (2011). On the other hand, Cline (2004) , Cantor and Packer (1996), Eichengreen and Mody (2000), among many other references, find that a large share of risk premium variability depends on domestic factors. In this paper, I follow the literature on default that models risk premium endogenously.

The remainder of the paper proceeds as follows. Section 2, presents and estimates a stochastic volatility model for the exogenous output process using Gibbs Sampling. Section 3 presents the model for the endogenous sovereign risk premium in the spirit of Eaton and Gersovitz (1981) and Arellano (2008), but allowing the income process to follow a stochastic volatility model. Section 4, discusses the benchmark calibration of the model. Section 5 discusses the main findings of the paper including policy functions and study of the dynamics conditional on no defaulting. Section 6 considers alternative parameterizations for sensitivity analyses. Finally, section 7 concludes. Additionally, Appendices A, B and C
provide a detailed explanation of the estimation algorithm, the solution algorithms and the data management, respectively.

2 A model for output with stochastic volatility

The main objective of this paper is to study the impact of volatility shocks in the external financing conditions of small open economies and on the sovereign risk premium. Hence, it is of first order importance to define a measure of aggregate macroeconomic volatility. In this paper, I proxy the unobserved series of aggregate macroeconomic volatility by the time-varying volatility of income.

I estimate the unobserved volatility process by assuming an exogenous stochastic process for output with stochastic volatility, as implied by the following model,

\[ y_t = \rho y_{t-1} + \exp(\sigma_t)\epsilon_t, \]  
\[ \sigma_t = (1 - \rho^\sigma)\bar{\sigma} + \rho^\sigma \sigma_{t-1} + \eta\epsilon_t^\sigma. \]

Here, \( y_t \) denotes the logarithm of linearly detrended and demeaned output; \( \sigma_t \) denotes the logarithm of standard deviation that evolves accordingly to equation 2. I assume \( \bar{\sigma} \) is the steady state of the logarithm of the standard deviation, \( \eta \) is a positive constant and \( |\rho^\sigma| < 1 \). These three parameters determine the features of the stochastic process of the log of standard deviation of \( \sigma_t \).

The model for output process is characterized by equations 1 and 2. As can be seen, output is affected by two shocks, \( \epsilon_t \) and \( \epsilon_t^\sigma \). These innovations affect the level of output and the volatility, respectively. I assume that these innovations follow standard normal distributions.

Given this setup, an innovation to output of a certain magnitude, \( \hat{\epsilon}_t \), can have different impact conditional on the realization of the volatility shock, \( \epsilon_t^\sigma \). In particular, a high \( \epsilon_t^\sigma \) will amplify \( \hat{\epsilon}_t \) that will, contemporaneously, have a larger impact on output.

The stochastic volatility model gives a convenient measure of time-varying volatility, given that output data is collected at quarterly frequencies, and because of its parsimonious specification (see a discussion about the stochastic volatility model in Fernandez-Villaverde et al. (2011)). Moreover, stochastic volatility model is a better approach for the interest of this paper given that, as opposed to the Generalized Autoregressive conditional heteroskedasticity model (GARCH), we can isolate the impact of two distinct shocks, one to the level of output and one to the volatility of output whereas in the GARCH model we only have a shock to the level of output. The interpretation of the volatility shock is straightforward but not unique. Here, I interpret higher volatility as higher risk regarding the realization of the income shock.

The key object of this section is to estimate the output process and the law of motion of volatilities and look for empirical regularities using data for Spain, Italy, Portugal and Greece. In the following sections I use Spanish estimates to calibrate an endogenous risk

\[ \text{as usually models such as “realized volatility” models are used with intraday data} \]
premium model. Using this model will allows us to provide a microfundated explanation for the facts presented in this section.

The Stochastic Volatility model is highly non-linear and its estimation is challenging. I follow Kim et al. (1998) strategy and implement a Gibbs Sampling procedure, together with a Bayesian approach. Table 1 presents the prior distributions used for estimation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Hyper-parameter 1</th>
<th>Hyper-parameter 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spain</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>Normal</td>
<td>0.9396</td>
<td>0.05438</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Normal</td>
<td>-4.868</td>
<td>0.01659</td>
</tr>
<tr>
<td>$\rho^\sigma$</td>
<td>Normal</td>
<td>0.9762</td>
<td>0.003389</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Inverse Gamma</td>
<td>151</td>
<td>6.112</td>
</tr>
<tr>
<td>Italy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>Normal</td>
<td>0.9518</td>
<td>0.227</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Normal</td>
<td>-4.938</td>
<td>0.0122</td>
</tr>
<tr>
<td>$\rho^\sigma$</td>
<td>Normal</td>
<td>0.9823</td>
<td>0.002481</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Inverse Gamma</td>
<td>154</td>
<td>3.437</td>
</tr>
<tr>
<td>Portugal</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>Normal</td>
<td>0.8924</td>
<td>0.3072</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Normal</td>
<td>-4.619</td>
<td>0.01171</td>
</tr>
<tr>
<td>$\rho^\sigma$</td>
<td>Normal</td>
<td>0.9651</td>
<td>0.002592</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Inverse Gamma</td>
<td>154</td>
<td>3.167</td>
</tr>
<tr>
<td>Greece</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>Normal</td>
<td>0.8714</td>
<td>0.5164</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Normal</td>
<td>-4.568</td>
<td>0.01033</td>
</tr>
<tr>
<td>$\rho^\sigma$</td>
<td>Normal</td>
<td>0.9936</td>
<td>0.00277</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Inverse Gamma</td>
<td>148</td>
<td>2.275</td>
</tr>
</tbody>
</table>

Note: For the Normal distribution “Hyper-parameter 1” stands for the mean while “Hyper-parameter 2” stands for the standard error. For the Inverse Gamma distribution they are degrees of freedom and scale matrix, respectively.

I need to assume conjugate priors to use Gibbs Sampling to draw from known probability distributions. This is the reason for assuming Normal priors for $\rho$, $\sigma$ and $\rho^\sigma$; and Inverse Gamma prior for $\eta$. To determine the prior hyper-parameters: means, standard errors, degrees of freedom and scale parameters, I proceed as follows. First, I use data from 1970Q1 to 1979Q4 as my training sample. I use it to estimate an AR(1) process without stochastic volatility using ordinary least squares. I use the point estimates of this model to set the
mean for $\rho$ and $\bar{\sigma}$ and I use their standard errors to determine the standard errors of the prior distribution (in particular, the standard errors of the priors distributions are 10 times the ones obtained by ordinary least squares). Then, using the residuals from the previous estimation, I compute a rolling window standard deviation and use its autocorrelation and standard error to determine the remaining hyper-parameters. I repeat this procedure for each country.

To estimate the stochastic volatility models, the Gibbs Sampling uses data from 1980Q1 to the latest available. Table 2 presents the posterior estimates and confidence sets.

Table 2: Posterior estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Posterior Median</th>
<th>16th Percentile</th>
<th>84th Percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spain</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{\sigma}$</td>
<td>-4.877</td>
<td>-5.086</td>
<td>-4.649</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.9957</td>
<td>0.9897</td>
<td>0.9988</td>
</tr>
<tr>
<td>$\rho^\sigma$</td>
<td>0.9385</td>
<td>0.9082</td>
<td>0.9669</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.2043</td>
<td>0.1952</td>
<td>0.2153</td>
</tr>
<tr>
<td>Italy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{\sigma}$</td>
<td>-4.918</td>
<td>-5.108</td>
<td>-4.712</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.9964</td>
<td>0.9913</td>
<td>0.9991</td>
</tr>
<tr>
<td>$\rho^\sigma$</td>
<td>0.9465</td>
<td>0.9206</td>
<td>0.9713</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.1514</td>
<td>0.1443</td>
<td>0.1602</td>
</tr>
<tr>
<td>Portugal</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{\sigma}$</td>
<td>-4.516</td>
<td>-4.666</td>
<td>-4.369</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.9952</td>
<td>0.9875</td>
<td>0.9988</td>
</tr>
<tr>
<td>$\rho^\sigma$</td>
<td>0.9344</td>
<td>0.9028</td>
<td>0.963</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.143</td>
<td>0.1369</td>
<td>0.1505</td>
</tr>
<tr>
<td>Greece</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{\sigma}$</td>
<td>-3.886</td>
<td>-4.286</td>
<td>-3.629</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.9714</td>
<td>0.9446</td>
<td>0.9896</td>
</tr>
<tr>
<td>$\rho^\sigma$</td>
<td>0.9713</td>
<td>0.9516</td>
<td>0.9874</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.1386</td>
<td>0.1278</td>
<td>0.1582</td>
</tr>
</tbody>
</table>

Note: Computed using Gibbs Sampling. Details on the procedure in Appendix A.

The previous table presents the main features of the posterior estimates. First, output process is highly autocorrelated in all the cases but specially for Spain with a point estimate of 0.996 and narrow confidence band around this number. On the other hand, volatility is

\footnote{For Greece my sample ends in 2011Q1, for Spain in 2011Q4 while for Italy and Portugal in 2012Q3.}
highly autocorrelated too, but slightly less than output in all the cases.

Additionally, note that Spain exhibits the larger standard deviation of the volatility, with $\eta = 0.20$. This means that, starting from steady state, a shock of magnitude 1 to the volatility process, will generate an increase $\exp((-4.87) + 0.20)$ to the volatility of output, from the average volatility of output innovation, 0.0077, to 0.0094; or in other words, 20%.

Using the point estimates in Table 2 and the realizations of the volatility innovations, it is possible to recover the smoothed estimates of the volatility. I plot these time series in the green-dashed line together with output, in solid blue line, in Figure 1 starting in 1980Q1 for each economy.

![Figure 1: Output and stochastic volatility](image)

Note: This figure plots the series for output (in dashed lines measured in the right axis) and the smoothed estimates of the stochastic volatility implied by our model (in solid lines measured in the left axis). Details on the computation of the time-varying volatility can be found in Appendix A.

As seen in the picture, for the case of Spain there are large changes in volatility with peaks in 1992, 1998 and 2009. From Figure 1 we see that these peaks are associated both to positive and negative deviations of output with respect to a linear trend. There seems to
be, hence, no clear correlation between the series of output and the smoothed volatilities. For instance, for the period 1980-1995, it seems that increases in volatility are associated to increases in output. On the other hand, it seems that after 1997, the correlation changed and increases in volatility seem to happen together with output drops, i.e. in the last 15 years, the correlation is remarkably negative. A similar feature is observed for Greece, Portugal and Italy. Although the evidence is supported for shorter samples, the recent crisis episode shows a larger increase in volatility together with a dramatic drop in output.

In summary, the figure shows that output exhibits substantial and persistent deviations from its linear trend while volatility shows also large variability. An important question is whether these changes in volatility are statistically significant. Figure 2, plots the stochastic volatility jointly with the 60% confidence bands.

Figure 2: Stochastic volatility and confidence bands

Note: This figure plots the smoothed estimates of the stochastic volatilities implied by our model (in solid lines) together with confidence bands corresponding to 1 standard deviation (in dotted lines). Details on the computation of the time-varying volatility and confidence bands can be found in Appendix A.

As seen in the figure, although there is substantial uncertainty on the actual volatility level, changes in the volatility of output are significant for all economies, specially during the
recent crisis. It is important to say that, even in this case, we cannot rule out that volatility has remained constant for several periods, such as the period between 1980-1990 or mid-1990s for Greece, Italy and Portugal. However, we can identify substantial contraction in volatility during during the 1990-2000 period, and a large increase in volatility starting in 2005.

For the objective of this paper, it is important to inquire how does the stochastic volatility of output correlate with the sovereign risk premium. Recall that in this framework the volatility of output is a measure of macroeconomic uncertainty and the main question of this paper is to study whether aggregate volatility can account for the variation of sovereign risk premium. Figure 3 plots the volatility of output and the risk premium for each of our four economies.\(^6\)

![Figure 3: Stochastic volatility and the sovereign risk premium](image)

Note: This figure plots the series for the sovereign risk premium (in dashed line measured in the right axis) and the smoothed estimates of the stochastic volatility implied by our model (in solid lines measured in the left axis). Details on the computation of the time-varying volatility can be found in Appendix A.

\(^6\)The measure of risk premium is the difference between the interest rate on long term sovereign bonds and the return of German long term bonds.
As seen in Figure 3, overall there seems to be a strong positive correlation between the volatility of output and the risk premium over the period 1980-2012 for Spain, as well as in shorter samples for Greece, Italy and Portugal. This is particularly the case for the two last major macroeconomic crisis, the 1992 European currency crisis and the current European debt crisis.

Table 3 presents related evidence, the point estimates and confidence bands of output and sovereign risk premium, and volatility and sovereign risk premium.

Table 3: Stylized facts

<table>
<thead>
<tr>
<th>Moments</th>
<th>Spain</th>
<th>Italy</th>
<th>Greece</th>
<th>Portugal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{\text{spread}<em>t,\text{spread}</em>{t-1}}$</td>
<td>97.74</td>
<td>96.98</td>
<td>94.14</td>
<td>97.25</td>
</tr>
<tr>
<td></td>
<td>(8.666)</td>
<td>(10.52)</td>
<td>(12.81)</td>
<td>(11.15)</td>
</tr>
<tr>
<td>$\rho_{\sigma_t,\sigma_{t-1}}$</td>
<td>98.49</td>
<td>98.46</td>
<td>98.73</td>
<td>99.37</td>
</tr>
<tr>
<td></td>
<td>(7.451)</td>
<td>(10.04)</td>
<td>(12.62)</td>
<td>(10.86)</td>
</tr>
<tr>
<td>$\rho_{y_t,\text{spread}_t}$</td>
<td>-57.04</td>
<td>12.81</td>
<td>-58.18</td>
<td>-63.93</td>
</tr>
<tr>
<td></td>
<td>(5.036)</td>
<td>(1.38)</td>
<td>(7.835)</td>
<td>(7.269)</td>
</tr>
<tr>
<td>$\rho_{\sigma_t,\text{spread}_t}$</td>
<td>34.46</td>
<td>23.29</td>
<td>55.39</td>
<td>33.2</td>
</tr>
<tr>
<td></td>
<td>(3.012)</td>
<td>(2.499)</td>
<td>(7.429)</td>
<td>(3.763)</td>
</tr>
</tbody>
</table>

Note: In this table, $\rho_{i,j}$ denotes the correlation between variable $i$ and $j$, $y_t$ denotes output, $\sigma_t$ denotes the log volatility and $\text{spread}_t$ denotes the sovereign risk premium. Standard errors are in parenthesis. All moments are in percentage terms.

As can be seen in the previous table, output and sovereign spreads are negatively correlated in these economies, except for the case of Italy where the correlation is slightly positive, while the volatility of output and the sovereign spread are positively correlated. An important issue is the persistence of both volatilities and sovereign risk premiums observed in these economies. As can be seen in the table these measures are highly volatile in all the cases, suggesting that changes in volatilities and spreads are likely to exhibit a long lasting effect.

This evidence suggest that a model that accounts for the risk premium behavior should also account for these features of the data. We next present the model for endogenous risk premium and use it to study the impact of volatility shocks in sovereign risk premium and default probabilities.

The model in the following section will be helpful to understand the dynamics behind the positive correlation between sovereign risk premium and volatility. In principle, note that if the standard precautionary savings motive holds, an increase in volatility will induce a decrease in the leverage of the economy, or an increase in savings. If this is the case, the risk premium would decrease after a volatility shock given that it is positively related to the level of debt. Hence, a standard model with precautionary savings behavior might not be able to capture this fact.
3 Endogenous sovereign risk premium

This paper models the endogenous sovereign risk premium following the literature on strategic default as Arellano (2008), Eaton and Gersovitz (1981) and Aguiar and Gopinath (2006). This literature develops models for a small open economy that trades an homogeneous good with the rest of the world and have access to an incomplete international asset market with one side commitment. The government of the small open economy cannot commit to honor its debt and, consequently, she is able repudiate it at any point in time. If the government defaults, it is assumed to loose access to international asset markets and experiences an output loss as long as it is in default. It is assumed, however, it can randomly return to international asset markets. It is also assumed that output is exogenously given, i.e. endowment economy, and that there is only a shock to the output process. In this paper, I depart from this last assumption by allowing the income process to follow an autoregressive process with stochastic volatility as the one estimated in the previous section. Hence, as shown in section 2, the output process is affected by a shock to the level and a shock to the volatility.

The remainder of this section specifies the model in detail, the recursive representation and defines the object of interest, the recursive competitive equilibrium.

3.1 Model’s Setup

The model is populated by a domestic private sector, the domestic government and the international financial investors. This section describes each agent in detail.

3.1.1 The domestic economy

Assume a small open economy populated by a large number of identical households that maximize the expected value of discounted utility given by,

$$
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t).
$$

Here, $u(c_t)$ denotes the instantaneous utility function. It is assumed this function has a positive first derivative and a negative second derivative. Intuitively, utility increases with consumption but at a decreasing rate. $\beta$ denotes the intertemporal discount factor, that can take values between 0 and 1, and $c_t$ denotes the consumption level of an homogeneous, perishable consumption good in period $t$.

It is assumed that households receive an exogenous stream of output that evolves accordingly to the process specified in Section 2, that for convenience is repeated here,

$$
y_t = \rho y_{t-1} + \exp\{\sigma_t\} \epsilon_t,
$$

This is a standard assumption in default models. For this paper it is convenient because of two reasons. First, the output and volatility processes can be directly estimated from available data and second, given that we do not consider capital accumulation, the set of states in simplified. The reader is invited to review Mendoza and Yue (2012) for a model with production, where output is produced using labor capital and imported inputs.
\[ \sigma_t = (1 - \rho^\sigma)\bar{\sigma} + \rho^\sigma \sigma_{t-1} + \eta \epsilon_t^\sigma. \]

As described earlier, \( y_t \) is the output level in period \( t \), \( \sigma_t \) is the logarithm of output innovation standard error, our measure of volatility or aggregate risk, \( \bar{\sigma} \) is the average level of the log volatility and \( \epsilon_t^\sigma \) is the innovation to the volatility process.

Assume there exists a benevolent government that maximizes households’ utility. The government is the only domestic agent that has access to international lending and borrowing markets and can buy international bonds \( b' \). It is assumed that \( b' > 0 \) means that the rest of the world is a debtor to the small open economy whereas \( b' < 0 \) means that the domestic economy has a liability with the rest of the world. Given that the government is benevolent and the households have concave utility functions, the government will use bond issuing to smooth consumption.

Assume the bonds are sold at discount at a price \( q(b', y, \sigma) \). The price of bonds is a function of the current realization of output, the current realization of volatility and the level of bonds chosen by the government in the current period. I assume the government internalizes the effect of government debt on the price of debt. This means that the government knows that its behavior in the asset market will affect the equilibrium price at which the bonds are traded.

The key assumption is that the government cannot commit on the issued debt. The government can default at any time. If the government choose at any period \( t \) to repay the outstanding debt, the resource constraint of the economy is given by,

\[ c = y + b - q(b', y, \sigma) b'. \]

Note that international lending and borrowing is the only way the government helps households to smooth consumption. Any returns from the international asset market operations is transfered to households using lump sum transfers or taxes.

If the government defaults, the budget constraint reduces to,

\[ c = y^{def}. \]

Here, \( y^{def} = h(y) \leq y \). With \( h(\bullet) \) being increasing in \( y \). We can think about this function as a penalty for defaulting that is increasing in the income realization. Another implication of the previous equation is that when the government defaults, the country is unable to access international asset markets and hence has to consume all its endowment given that it cannot be stored. We will refer to this situation as “Autarky”. As previous literature, I assume the country can return to asset markets with a probability \( \theta \) any period.

### 3.1.2 The foreign investors

The rest of the world is populated by a large number of international investors that are risk neutral and have access to a risk free asset and also trade the risky asset with the domestic economy. The risk free asset pays the risk free interest rate. Given that foreign investors are risk neutral, they allocate resources across different assets as long as they break even in expectation. Additionally, given the small open economy assumption any level of fund the domestic economy offers of demands will be traded at the appropriate equilibrium price.
Hence, foreign investors’ problem is to maximize profits from operating in the international asset markets. Using \( \Pi \) to denote the investors profits, we can formalize their problem as follows,

\[
\max \quad \Pi = q b' - \frac{1 - \delta}{R} b'.
\]

Here \( R \) is the risk free rate and \( \delta \) is the default probability. Taking first order conditions, the equilibrium requires \( q = \frac{1 - \delta}{R} \). Note that given that \((b, y, \sigma)\) are the states of the economy, and the timing is such that first the government chooses to default or not and then if no default is observed the government chooses how much debt take the current period, the price of debt has to satisfy,

\[
q(b'(b, y, \sigma), y, \sigma) = \frac{1 - \delta(b'(b, y, \sigma), y, \sigma)}{R},
\]

That is, the price of debt is equal to the inverse of the risk free rate times the probability of repayment of the debt contract.

### 3.2 Recursive formulation

The model is solved using value function iteration.\(^8\) For this reason, it is useful to write the recursive formulation of the problem. The states of the problem are the outstanding level of debt, as long as the government did not default the previous period; the output level; and the realization of the output’s volatility; that is \( s = (b, y, \sigma) \). Using the previous notation, under no default, the value function for the government is given by,

\[
v^c(b, y, \sigma) = \max_{c, b'} \left\{ u(y + b - q(b', y, \sigma)b') + \beta \int_{\sigma'} \int_{y'} v^o(b', y', \sigma') f(y, y') dy' g(\sigma, \sigma') d\sigma' \right\}.
\]

Here, we replaced out consumption using the budget constraint under no default and \( v^c(b, y, \sigma) \) stands for the value function under commitment. Also, \( f(y, y') \) denotes the transition function for output levels, \( g(\sigma, \sigma') \) denotes the transition function for volatility levels, and \( v^o(b', y', \sigma') \) denotes the optimal continuation value, given by,

\[
v^o(b, y, \sigma) = \max_{c, b'} \left\{ v^c(b, y, \sigma), v^d(y, \sigma) \right\},
\]

and \( v^d(y, \sigma) \) denotes the value of default, given by,

\[
v^d(y, \sigma) = u(y^{def}) + \beta \int_{\sigma'} \int_{y'} \left[ \theta v^o(0, y', \sigma') + (1 - \theta) v^d(y', \sigma') \right] f(y, y') dy' g(\sigma, \sigma') d\sigma'.
\]

Here, \( \theta \) is the probability of returning to international asset markets after default. In this way, the recursive formulation indicates that, the government’s objective is to maximize the

\(^8\)I provide a detailed description of the method and the pseudo-algorithm in Appendix A.
value function of the economy given by \( v^o (b', y', \sigma') \) using debt issuance, the default option and consumption.

In this framework, we can define a recursive competitive equilibrium as follows.

**Definition Recursive Equilibrium.** A recursive competitive equilibrium for this small open economy is a set of value functions \( v^o (b, y, \sigma) \), \( v^c (b, y, \sigma) \) and \( v^d (y, \sigma) \), a set of policy functions for the government asset holdings \( b'(s) \), a household’s consumption allocation \( c(s) \), a default decision \( d(s) \), and a pricing function for bonds \( q(b', y, \sigma) \), such that:

1. Given \( y, \sigma \) and the price of debt \( q \), the government policy function \( b' \) and default decisions, satisfies the government optimization problem.
2. Given \( y, \sigma \), the price of debt \( q \), the government policy function \( b' \) and default decisions \( c(s) \) satisfies the resource constraint.
3. The debt price function \( q \) is consistent with a zero expected profits for international lenders.

To solve for the equilibrium and policy functions we proceed by value function iteration. A detailed description of the algorithm can be found in Appendix A. The next section describes the calibration and steps implemented towards the discretization of the endogenous and exogenous states.

### 4 Functional forms, calibration and discretization

This section contains some of the implementation details regarding the solution of the model and the discretization of the state space. The model is solved using a variant of value function iteration with multigrid,\(^9\) on the grid of asset and linear interpolation. I discretize exogenous states as described below.

#### 4.1 Functional forms

I assume the following functional forms: the instantaneous utility function is given by

\[
u(c) = \frac{c^{1-\kappa}}{1-\kappa},\]

where \( \kappa \) is the coefficient of risk aversion. Additionally, the output penalty function under default is

\[ y^{def} = \gamma y, \]

if \( y \leq \gamma \bar{y} \) and \( y^{def} = y \) if \( y < \gamma \bar{y} \).

---

\(^9\) I first solve the model for a grid of 100 asset points, then 200 and finally 500.
4.2 Calibration

The benchmark calibration uses the estimates in section 2 together with standard parameter values in the related literature. I will later implement sensitivity analysis.

The parameterization for the income process is the one shown in Table 2 in Section 2. For the remaining parameters of the benchmark calibration I follow standard calibration in Arellano (2008) and related literature. The remaining parameters are shown in Table 4.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.9808</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>3</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.28</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.969</td>
</tr>
<tr>
<td>$R$</td>
<td>1.017</td>
</tr>
<tr>
<td>$\bar{y}$</td>
<td>10</td>
</tr>
</tbody>
</table>

As can be seen in the table, the benchmark calibration is similar to other papers in the default literature. The relative risk aversion coefficient equals 3, which is in the order of magnitude of several references in this literature. Arellano (2008) sets it to 2, however a slightly higher coefficient helps capturing the fact that the volatility of output is larger than the volatility of consumption in developed economies. Additionally, I set the risk free rate is set to 1.017, in line with Arellano (2008). Aguiar and Gopinath (2006) also uses a calibration of a similar order of magnitude, 1%. The output loss of default is about 4% and $\bar{y}$ is set to 10, both following Arellano (2008).

Given $\theta$, there is about 1/3 chances of going back to the international asset markets after a default episode. $\beta$ is set such that there is a 0.25% average default probability. In the sensitivity section we repeat the exercise considering $\beta = 0.98$ that implies an almost 0 average default probability. Other sensitivity analysis reported in that section includes increasing the cost of default and I also compare the results to the calibration in Arellano (2008) that assumes a much larger default probability.

4.3 Discretization

In addition to the calibration, we need to discretize the exogenous states, to do so I follow Tauchen (1986). This model has two exogenous states, the output and the volatility of output. For the volatility of output I use a 11 nodes grid assuming the mean, autocorrelation and standard deviations in Table 2 and I set the grid to cover 3 standard deviations around the mean. To build the grid for the income process I have to take into account the stochastic volatility that now is represented by the volatility grid and has, hence, 11 possible realizations. Given this, I apply Tauchen’s Method again. I specify a mean of zero and a correlation as the one in Table 2. However, to specify the standard deviation of the innovation, I first specify the exponent of the first element of the volatility grid, that generates a grid for output conditional on the lowest volatility level, then I construct another grid for
output using the second element of the grid for volatility, and so on. For each grid I use 21
nodes and a I set the grid to cover 3 standard deviations around the mean. Ultimately the
grid for income processes will be one of 11 times 21, each of them associated to a transition
probability matrix, as well as one transition probability matrix for the volatility grid.

5 Results

The main objective of the paper is to understand the effect of volatility shocks in macroeco-
nomic dynamics when the government cannot commit to honor its debt. We are particularly
interested in the effect of volatility shocks on the savings decision and on the sovereign risk
premium, or equivalently on the effect of macroeconomic volatility on the price of debt. To
address these questions I start by studying the features of the policy functions, prices and
interest rates. Then, I implement simulation exercises to study the dynamics of the model, in
particular I inquire how important are volatility shocks to capture the variability of the risk
premium. Finally, I study the dynamic behavior of savings and the sovereign risk premium
under a variety of sensitivity exercises.

5.1 Policy functions

To understand the effect of volatility changes in endogenous variables, such as sovereign risk
premiums and savings, I start by studying the policy functions. Given that we deal with
three state variables, this section presents the policy functions as function of output and
volatilities while keeping the asset positions fixed.

In this section, I study savings decision, price of debt and risk premium dynamics for
the benchmark parameterization. Section 6 will present the results of sensitivity analysis
to address how these findings depend on the risk aversion, the discount factor and default
penalty.

Figure 4 plots the policy function for savings in the space of income and volatility, as-
suming the debt level is fixed at middle debt grid-point.

As seen in the figure, when the economy does not default, the larger the income realiza-
tion, the larger the savings. Only when income realizations are low, the economy defaults
on its debt. On the other hand, the figure also indicates that the higher the volatility level,
the larger the savings for most of the state space. However, when the income level is close
to the default region, an increase in volatility might not generate an increase in savings.
As will be clear in the sensitivity analysis, the intuition on this behavior goes as follows.
In the benchmark parameterization, under default output is 4% smaller than output under
commitment. If the probability of default is large enough, the government might prefer
increasing consumption rather than decreasing it after a volatility increase because in the
event of a default, the households debt is set to zero and the economy can still experience
autarky consumption level.

The main difference of this model with standard model of incomplete markets as Bew-
ley models is that here, the gains associated to a high income realization are -net of debt
repayment- fully consumed by the households, whereas the losses associated to a bad in-
come realization are bounded for the households. In the the worst case, consumption will be
bounded by the autarky level. For this reason there is no precautionary savings when the economy can default on its debt.

Figure 5 contains two figures. The bottom figure plots several transversal cuts of the previous function for different levels of income shock: low income, medium income and high income shock levels. The low income level assumes the income realization is 1/3 of the highest possible realization, the medium income assumes income is fixed at the average or ergodic mean, this is the income level in the absence of any shock, finally, the high income level assumes the income realization is 2/3 of the highest possible realization. On the other hand, the top figure plots the savings function conditional on income level being fixed at the average, for different levels of initial asset position (low level of assets, medium level of assets and high level of assets).

As seen from the picture, when the economy has a high level of debt, and conditional on the ergodic income realization, the government does not increase savings when higher volatility levels are observed, this can be seen in the dotted line at the top of the top figure. In this case, the savings are fixed to zero for all volatility levels, this is the case because for this state combination, the government defaults for every level of volatility. This feature of the model partially vanishes as the asset position improves, as seen in the dashed and dotted lines. In the first case it is observed that the government exhibits precautionary savings, in terms of a contraction of its debt exposure, as volatility increases for low volatility levels only. For higher asset levels (the dotted full line), there is an increase in savings even when volatility approaches to 0.015, but is followed by a mild decrease after it. For higher levels of asset accumulation (not shown in the picture), an increase in savings is observed for any increase in volatility in the grid range.
The bottom figure, instead, shows that when the income shock is high and persistent, the economy is rich and behaves as an economy without default exhibiting high precautionary savings motive. To a lower degree it is also observed for the case of mid-income shocks. However, when the income shock is too small, the marginal cost of repayment is too large and hence, the government defaults and its savings are fixed to zero for any level of volatility.

To complete the picture, Figure 6 presents the price of sovereign debt \( q(b', y, \sigma) \) as a function of volatility and income levels, assuming that debt is fixed at middle debt grid-point.

As can be seen in the figure, for a wide region of the state space, the price of debt is fixed to the inverse of the risk free rate level, that is, the sovereign debt pays the same as the risk free rate asset given that default probability is nil. For very low income levels, the price of debt is zero, meaning that default probability is one and hence, the implied interest rate is infinite. There is a region around the ergodic level of income for which the price exhibits deviations from the extreme values, in particular this picture suggests that when volatility increases to higher levels, the price of debt drops, i.e. interest rate increases), however in principle, the picture suggests that the price level will not be too responsive to changes in the level of volatility. We turn now to study transversal cuts of this picture to clarify this point. Figures 7 and 8 present the price of debt and its inverse, the interest rate for different asset positions and income shocks, respectively.

As seen in the figure, increases in the volatility level affects the price of debt (and consequently the interest rate) at low volatility levels. However, this is not the only way volatility affects the price of debt. Note that these figures plot the direct impact of volatility changes in the price of debt conditional on a given savings decision, \( b' \). However, we have shown in
Figure 6: The price of sovereign debt

Note: This picture plots the small open economy price of sovereign debt as a function of volatility and income computed assuming the level of assets is fixed to its ergodic level.

Figure 7: Interest rate and the price of sovereign debt

Note: This picture plots transversal cuts of the small open economy price of sovereign debt, and its inverse (R), as a function of volatility and output, for high (dotted-full line), low (dotted line) and ergodic (dashed line) asset positions.

figures 4 and 5 that changes in volatility affects savings decision and through this channel an increase in volatility will affect the sovereign spread indirectly.

Note that, as discussed in this section, increases in volatility can have two opposite effects
on savings and on the price of debt. If the economy is relatively rich and default probability is small, an increase in volatility is likely to increase asset accumulations, in line with standard precautionary savings motive. This increase in savings (or deleveraging) contributes to a drop in default probability and subsequently an increase in the price of debt. On the other hand, if the economy is relatively poor, an increase in volatility can induce an increase in debt accumulation contributing to a higher default probability and to a lower price of debt. Hence, for the case of this model, generating a positive correlation between the volatility and the sovereign spread is a quantitative question. In the following section I study variance decomposition and simulation exercises and discuss the quantitative features of the model.

5.2 Dynamics

The objective of this section is to study the dynamic properties of the model. To this end I study how important are changes in volatility of output to explain the variability of the sovereign risk premium observed in the data and what is the effect of volatility innovations. Table 5 presents the volatility of output, consumption, trade balance to output ratio and the spread on sovereign debt computed using the full model and only each single shock.

Table 5 compares the moments implied by the full model before a default episode to the moments implied by the model that only allows for income or for volatility shocks. The way moments are computed matters. In this paper I run 1000 simulations of 100000 periods each. I identify the default episodes and discard the periods that the economy was in default. Then, for each default episode, denote the period of default by \( T^d \), I consider data in the sample period that goes from \( T^d - 125 \) to \( T^d - 4 \). That is I do not consider the data contained in the 4 quarters before a default episode. I do this because we don’t observe an in sample
default episode for Spain and, hence, we do not have information about the behavior of data right before default. Other arrangements have been tested and the correlations and relative volatilities are not different from the baseline model, although absolute volatilities are larger the closer we allow the sample to get to the default episode.\footnote{Arellano (2008) follows the same procedure but sets the sample from $T^d - 75$ to $T^d - 1$ because the data she uses covers a similar sample size and she observes a default episode.}

Note that the model is solved using non-linear methods. In this case the interpretation of a variance decomposition exercise is more complicated than the one for linear models, or linearly approximated models. In the linear case, the variance of the endogenous variables computes using 1 shock at a time will add up to the total variability explained by the model. This is not the case for non-linear models as the shocks might have non-linear interactions. This is one of the reasons that explain that the following tables in this section only presents the variability of the full model and the one with only level shocks.\footnote{The other reason is that an additional problem arises at this point, when level shocks to the income are fixed to zero, the model does not experience a default for the baseline calibration. In this case, I could divide the sample in subsamples of 125 periods and compute the moments using these subsamples and then averaging them. However, this strategy is not a good one, as it will underestimate the impact of volatility shocks because in this model much of the variability occur around a default episode and hence, the different experiments would not be implemented on the same ground.} For this reason, I stress the fact that the variability that is not explained by levels shocks only is variability that will be explained by a combination of volatility and income shocks.

Table 5: Variance Decomposition

<table>
<thead>
<tr>
<th></th>
<th>$\sigma^y$</th>
<th>$\sigma^c$</th>
<th>$\sigma^{by}$</th>
<th>$\sigma^{spread}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Model</td>
<td>4.68</td>
<td>4.538</td>
<td>0.7702</td>
<td>3.703</td>
</tr>
<tr>
<td>Only Level Shocks</td>
<td>2.184</td>
<td>2.451</td>
<td>0.5621</td>
<td>1.121</td>
</tr>
</tbody>
</table>

Note: The volatilities in this table are in percentage terms. The volatilities in the row named “Full Model” are computed by simulating the model considering shocks in level of output and volatilities. The ones in the row named “Only Level Shocks” are computed only using shocks to the income process but keeping volatility fixed in its average value. The number of Monte-Carlo simulations is $N = 1000$.

As seen in the table, the model generates a similar volatility for output and consumption of about 5% and the volatility for the spread of about 4%. All moments are in line with the ones in the data, moreover, given the larger $\kappa$, the model is able to generate a larger volatility for output than consumption. The second row depicts the moments computed only using income shocks while fixing the volatility level to the average. It can be seen that income shocks alone can generate up to $1/2$ of the total volatility of consumption and output and about 80% of the trade balance to output ratio. Instead, for the case of the sovereign spread, we can see that income shocks alone generates $1/3$ of the volatility compared to the one generated by the full model.

As can be seen, the level shocks left unexplained over $2/3$ of the sovereign risk premium variability explained by the model. This unexplained variability is driven by the fact that...
there are changes in volatility. Given that volatility changes affect the level of debt and interest rates autonomously, a share of the variability will be attributed directly to volatility shocks. There will be another share of the variability that would be explained by the interaction of volatility and income shocks. However, given the problems describe previously to disentangle both shares, I decide to provide only the information contained in the previous table.

The following table presents additional dynamics in terms of correlations for the full model.

Table 6: Correlations implied by the model.

<table>
<thead>
<tr>
<th></th>
<th>$\rho(y, spread)$</th>
<th>$\rho(y, \sigma)$</th>
<th>$\rho(spread, \sigma)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Model</td>
<td>-19.46</td>
<td>-4.634</td>
<td>21.86</td>
</tr>
<tr>
<td>Only Level Shocks</td>
<td>-17.02</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: The volatilities in this table are in percentage terms. The volatilities in the row named “Full Model” are computed by simulating the model considering shocks in level of output and volatilities. The ones in the row named “Only Level Shocks” are computed only using shocks to the income process but keeping volatility fixed in its average value. The number of Monte-Carlo simulations is $N = 1000$.

As can be seen in the table, the model, simple as it is, is able to capture the qualitative dynamics observed in the data. As can be seen the model captures a negative correlation between output and spread and the positive correlation observed between aggregate volatility and spread.

Importantly note that the interaction between level and volatility shocks also plays an important role in the dynamics of the model. In term of volatilities, we already discussed that the model with interaction of shocks generate a 70% of the total variability of sovereign risk premium and a large share of output and consumption variability. Hence, this suggests that this interaction play a big role in the savings decision of the households, increasing the response of savings to the macroeconomic environment. On the other hand, in terms of correlations, the interaction of these shocks has a substantial impact on the correlation between sovereign risk premium and volatility.

6 Sensitivity analysis

This section discusses the previous results under different parameterizations to inquire on the sources of the findings. I consider 2 sensitivity exercises: (1) an increase in the default penalty; and (2) the baseline calibration in Arellano (2008).

These sensitivity exercises are important to provide information on the role of volatility shocks in the risk premium dynamics under different default incentives. A higher default penalty is likely to induce a drop in default rates, however, it is not obvious how volatility and risk premiums should interact. It is likely that, by increasing the state space for which there will be no default, an increase in default penalty will lead to an increase the the variability.
of risk premium and in the importance of volatility shocks in explaining these movements. On the other hand, the comparison with the calibration in Arellano (2008) is presented to inquire about the impact of volatility shocks when default incentives are strong.

### 6.1 High default penalty

In this section, we compare the savings function and default decision for different volatility and income level shocks assuming $\gamma = 0.90$. Recall that $\gamma$ determines the penalty experienced by the small open economy in terms of output, as long as the economy is in bad standing with the international lenders. $\gamma = 0.90$ implies that while the economy is in a default scenario, it will suffer an output loss of about 10% each period. The functions for the baseline calibration and the high penalty calibration are presented in Figure 9.

![Figure 9: Savings function with $\gamma = 0.90$.](image)

Note: This picture plots the small open economy price of sovereign debt as a function of volatility and income computed assuming the level of assets is fixed to its ergodic level, for the baseline calibration and for the calibration with $\gamma = 0.90$.

The figure plots the savings function conditional on asset level fixed at the middle debt grid-point, as a function of volatility levels and output levels for both the baseline calibration, in red surface, and the alternative calibration with a higher penalty during default. As seen in the figure, both functions look similar. Specifically, note that in both cases, when output and volatility is high, the economy is lending to the rest of the world and a precautionary savings motive is observed. During bad income realizations in both cases there is a similar
default region, slightly larger for the baseline case rather than the high penalty case. The minimum and maximum savings levels are similar for both calibrations, suggesting that, while default penalty might have an impact on the default decision, it does not seem to affect the asset position decision. Additionally, savings decision became different for the two rules when income realizations are relatively lower. In particular, it can be seen that when income is low and volatility is high, the economy is prone to contract higher levels of debt, suggesting that incentives against precautionary savings motive described in the previous sections are stronger with low penalty than with high penalty. In other words, the larger the default penalty, the larger the precautionary savings motive.

Does the default penalty affect the moments implied by the model? Table 7 presents the volatilities in this case.

<table>
<thead>
<tr>
<th></th>
<th>$\sigma^y$</th>
<th>$\sigma^c$</th>
<th>$\sigma^{tby}$</th>
<th>$\sigma^{spread}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Model</td>
<td>4.981</td>
<td>4.885</td>
<td>0.8983</td>
<td>2.195</td>
</tr>
<tr>
<td>Only Level Shocks</td>
<td>2.185</td>
<td>2.365</td>
<td>0.3793</td>
<td>0.1517</td>
</tr>
</tbody>
</table>

Note: The volatilities in this table are in percentage terms. The volatilities in the row named “Full Model” are computed by simulating the model considering shocks in level of output and volatilities. The ones in the row named “Only Level Shocks” are computed only using shocks to the income process but keeping volatility fixed in its average value. The number of Monte-Carlo simulations is $N = 1000$.

Note that the higher default penalty does not have much impact on the moments implied by the model for output, consumption and the trade balance. However, note that the variability of the sovereign risk premium is smaller now and the share of this variability explained by level shocks is also smaller. The reason for this finding is in figure 9. As described in the previous paragraph, the model with low penalty exhibits more variability in debt for high values of volatility than the model with high penalty. This extra variability contributes to a larger volatility in the risk premium.

6.2 Calibration for Argentina

The final sensitivity exercise considered in this paper compares the baseline policy functions to the ones implied by the calibration in Arellano (2008), designed to fit the pre-default data in Argentina 2001. This calibration assumes the same calibration as I did in this paper except for $\kappa = 2$ and $\beta = 0.9528$. Figure 10 presents the policy functions under both calibrations.

As seen in the figure, the alternative calibration shows some important differences compared to the baseline calibration, but they are more of a quantitative rather than a qualitative issue. First, note that the alternative calibration shows a similar default region, although slightly larger for the Argentinean calibration, and a similar behavior during good times. Note however that default incentives are much larger in the case of 10 and hence debt accumulation is larger too, given that individuals in the baseline calibration are more patient than under the alternative calibration.
Figure 10: Savings function with $\beta = 0.95$, $\gamma = 0.969$ and $\sigma = 2$

Note: This picture plots the small open economy price of sovereign debt as a function of volatility and income computed assuming the level of assets is fixed to its ergodic level, for the baseline calibration and for the calibration with $\beta = 0.9528$, $\gamma = 0.969$ and $\sigma = 2$.

The intuition for these findings is that all the differences in calibration of the baseline parameterization compared to the one in Arellano (2008) go in the same direction of a lower penalty to default and a higher risk bearing environment, suggesting that in order to match the observed facts, the economy defaults more often in equilibrium and exhibits a lower precautionary savings behavior. The moments implied by this calibration under stochastic volatility are presented in Table 8.

As seen in this table, the economy is slightly more volatile. Notice that although income process is exogenous, the fact that we select the sample based on default episodes imply that the sample volatility of output can be larger than the one in the baseline calibration. Additionally, this calibration implies a much larger spread variability, which is a consequence of the larger debt accumulation during bad times.

Note that this table computes also the variability explained by volatility shocks. In this case volatility shocks alone can generate default episodes. We use these default episodes to select the samples we use to calculate the standard errors. As can be seen, with the alternative calibration the volatility shocks alone generate a very large variability of the spread. Actually, in this case, the interaction between level and volatility shocks is such that the full model generates a smaller variability than the one with either shocks shutted down.
Table 8: Variance Decomposition for the calibration in Arellano (2008)

<table>
<thead>
<tr>
<th></th>
<th>$\sigma^y$</th>
<th>$\sigma^c$</th>
<th>$\sigma^{by}$</th>
<th>$\sigma^{spread}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Model</td>
<td>5.722</td>
<td>6.037</td>
<td>1.114</td>
<td>6.835</td>
</tr>
<tr>
<td>Only Level Shocks</td>
<td>2.154</td>
<td>2.587</td>
<td>0.9289</td>
<td>7.825</td>
</tr>
<tr>
<td>Only Volatility Shocks</td>
<td>5.69e-14</td>
<td>0.8493</td>
<td>0.851</td>
<td>9.291</td>
</tr>
</tbody>
</table>

Note: The volatilities in this table are in percentage terms. The volatilities in the row named “Full Model” are computed by simulating the model considering shocks in level of output and volatilities. The ones in the row named “Only Level Shocks” are computed only using shocks to the income process but keeping volatility fixed in its average value. The number of Monte-Carlo simulations is $N = 1000$.

7 Conclusion

This paper studies the effects of macroeconomic volatility on the sovereign debt risk premium, default probability and savings decision of small open economies. The main question this paper answers is whether volatility shocks can contribute in a sizable manner to the variability of the sovereign risk premium or, on the other hand, the risk premium variability is a consequence of only shocks to output level. This question complements recent studies that try to analyze the impact of changes in risk premium in highly volatile environments.

References that study risk premium and stochastic volatility models assume exogenous risk premiums and hence, are unable to inquire whether the risk premium itself responds to volatility changes.

First, this paper establishes some stylized facts observed in European economies data. I estimate stochastic volatility models for Spain, Italy, Greece and Portugal using Gibbs Sampling and compute smoothed estimates for the volatilities of output. In particular, I find that output exhibits volatility changes, and positive changes in volatility are associated to increases in the risk premium and output drops. Also, I find that risk and volatility changes are persistent. Suggesting that they will have a long lasting impact on the macroeconomic dynamics.

Notice that explaining these facts is not trivial. In particular, if we were to work with a standard model with precautionary savings, we would find that an increase in volatility induce agents to save more and this will generate a lower debt level, that will induce a drop in the risk premium, which would give a counterfactual result.

To address the main question and explain he stylized facts, I develop a dynamic general equilibrium model for a small open economy in which the domestic economy cannot commit to honor its debt. In contrast to the related literature, as Eaton and Gersovitz (1981), Arellano (2008) or Aguiar and Gopinath (2006), I allow the income process to follow a stochastic volatility model. That is, income process is subject to level shocks, and shocks to the standard deviation of the innovations.

Using this model I find that the volatility of output has a differential impact on savings and risk premiums depending on whether the economy has relatively a large amount of
assets or a high income shock or, on the other hand is relatively poor and exposed to a high
debt level. When the economy is relatively rich, an increase in volatility induces increases
in savings, in line with the standard precautionary savings motive. This increase in savings
induce a drop in the risk premium. On the other hand, when the economy is relatively poor,
an increase in volatility can actually induce an increase in the debt level of the economy. The
increase in debt level translates in a higher exposure to risk and a higher risk premium. The
reason is that, when the economy is relatively poor and volatility increases, the government
understands that is a large good shock hits the economy, the returns from this shock are
going to imply a large consumption level and will be able to repay its debt, but if a large
negative shock happens, the economy can default and move to autarky.

The second finding of this paper is that risk shocks, i.e. shocks to the stochastic volatility
of the income process, are able to generate a substantial amount of sovereign spread variabil-
ity. The economy with level and volatility shocks can generate, for the baseline calibration,
a sovereign risk premium volatility of about 3.5% whereas the shocks to the level of output
alone can only generate 1/3 of this variability. This suggests that volatility shocks and its
interaction with level shocks account for the remaining 2/3.

These findings suggest that risk premium variability is not only due to actual income
shocks, but also to the fact that the economy becomes riskier. This finding has not been
taken into account by previous literature. In particular, previous literature assumes risk
premium is exogenous or a function of the level of some endogenous variables.

This finding is key, in particular for policymakers and asset pricing. For the former ones,
it suggests that stabilizing output alone will not necessarily lead to a stable risk premium,
reducing the risk associated to income shocks also matters.

Additionally, in terms of asset pricing, this paper implies that considering the impact of
risk shocks is key to correctly price sovereign bonds. If this factor is not included, the pricing
might end up concluding that changes in the level of output and debt to output ratios are
responsible for changes in prices that might have been induced only by changes in risk.

A third finding derived from the sensitivity analysis suggests that increases in macroeco-
omic volatility might generate an increase in consumption if the default penalty is small or
other default incentives are high. In this case, it is shown that if the incentives to default are
large because of different reasons such as low default penalty or low intertemporal discount
factor, combined with a high debt level, the government might increase consumption after
an increase in volatility, instead of decreasing it. In this case, the level of debt will increase
increasing the default probability. In other words, the more likely the economy is to default,
that is the higher default incentives, the lower degree of precautionary savings motive is
observed.

Hence, an increase in the volatility of income, can drive an increase in default probability
and hence, it might lead to a lower price of debt and a higher risk premium. This mechanism
rationalizes the positive correlation between risk premium and volatility observed in the data.
References


Reinhart, C., 2002b. Sovereign credit ratings before and after financial crises.


A Appendix: Solution algorithm

To solve the model I use a variant of value function iteration. The first step is to determine a grid for volatility, output and assets. First, I use the estimates from Section 2 and construct a 21 nodes grid for volatility using the method in Tauchen (1986). The second step is to, for each of the volatility nodes, construct a grid for output. Each grid for output has 21 nodes and is computed following Tauchen (1986). Finally, regarding the assets grid we follow a multi-grid approach. I start using a grid for assets of 200 nodes.

After fixing the grids, we propose a guess for the price of debt (start by setting \( q(b, y, \sigma) = 1/R \)). Given this price solve for the value function with linear interpolation and compute policy functions and the price of debt that clears asset markets. Compare the price of debt with the guess and repeat until convergence.

We use to value function from this step to build one with 500 grid points for asset position. To this end I use a linear interpolation of the value function and the price function and repeat the exercise until convergence of prices and value functions. I interpolate the price and value function two more times, using a grid for asset of 1000 and 2000 respectively.

B Appendix: Estimating SV using Gibbs Sampling

The empirical section of the paper estimates a stochastic volatility model given by,

\[
y_t = \rho y_{t-1} + \exp\{\sigma_t\} \epsilon_t, \tag{3}
\]

\[
\sigma_t = (1 - \rho^2) \bar{\sigma} + \rho^2 \sigma_{t-1} + \eta \epsilon_t, \tag{4}
\]

using output data for Spain, Italy, Greece and Portugal.

The procedure is an implementation of the standard Gibbs Sampling, a MCMC simulation method, as in Kim et al. (1998) and Garcia Cicco et al. (2012). The intuition of the Gibbs Sampler is extremely simple and a detailed explanation can be found in Kim and Nelson (1999). Suppose we have a model with 2 random variables, \( x_1 \) and \( x_2 \) and a joint density \( f(x_1, x_2, x_3, ..., x_n) \). Suppose we want to characterize the marginal density \( f(x_k) = \int ... \int f(x_1, x_2, ..., x_n) dx_1 dx_3 ... dx_{k-1} dx_{k+1} ... dx_n \). When integration is complicated, we can approximate this last function by MCMC as follows.

1. Set arbitrary starting values \( x_1^0, x_2^0, ..., x_n^0 \).
2. Draw \( x_2^1 \) from \( f(x_2|x_1^0, x_3^0, ..., x_n^0) \).
3. Draw \( x_3^1 \) from \( f(x_3|x_1^1, x_2^0, ..., x_n^0) \).
4. In the same fashion, draw all \( x_k^1 \) for all n variables.

These steps can be repeated a large number of times (in this paper we repeat this procedure 50,000 times). The distributions associated to the draws converge to the ones of \( x_1, x_2, x_3, ..., x_n \) when the number of repetitions goes to infinity.
B.1 The Gibbs Sampler for stochastic volatility

Given \( (i)' \)th draw for \( \sigma^i, \beta^i, \rho^i, \eta^i, \bar{\sigma}^i \) and \( s^i \), iteration \( i + 1 \) of the Gibbs Sampler goes as follows:

1. Draw \( \sigma^{i+1} \) from \( p(\sigma|y, \beta^i, \bar{\sigma}^i, \rho^i, \eta^i, s^i) \)
2. Draw \( s^{i+1} \) from \( p(s|y, \beta^i, \bar{\sigma}^i, \rho^i, \eta^i, \sigma^{i+1}) \)
3. Draw \( \beta^{i+1} \) from \( p(\beta|y, \sigma^{i+1}, \bar{\sigma}^i, \rho^i, \eta^i, s^{i+1}) \)
4. Draw \( \bar{\sigma}^{i+1} \) from \( p(\bar{\sigma}|y, \beta^{i+1}, \sigma^{i+1}, \rho^i, \eta^i, s^{i+1}) \)
5. Draw \( \eta^{i+1} \) from \( p(\eta|y, \beta^{i+1}, \bar{\sigma}^{i+1}, \rho^{i+1}, \sigma^{i+1}, s^{i+1}) \)

C Appendix: Data management

This paper uses data for Spain, Italy, Greece and Portugal. Spanish GDP is from IHS deflated using GDP deflator and linearly detrended for the period 1970Q1 to 2010Q4. In the cases of Italy (1980Q1 to 2011Q1), Greece (1980Q1 to 2011Q1) and Portugal (1980Q1 to 2012Q1), I use OECD data VPVOBARSA. These series are volume estimates computed in fixed PPP millions of US dollars at annual levels and seasonally adjusted. To these series and applied logs and linearly detrending and demeaned.

Sovereign risk premium series are computed using long term (10yr sovereign bonds) sovereign rates for Spain Italy, Greece and Portugal. Data is from OECD, Monthly Monetary and Financial Statistics (MEI). The risk free rate is the one for Germany, from the same dataset.

The sovereign risk premiums are the differences between the real rates and the real risk free rate.

The spread series are computed using available data. For the case of Spain the series run from 1980:Q1 to 2010Q4. For Portugal 1993Q1 to 2012Q1. For Italy 1991Q2 to 2011Q1. For Greece 1997Q3 to 2011Q1.