Why Might the Old Want to Honor Sovereign Debt?

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Abstract

Survey evidence indicates that, during periods of high default risk, older individuals tend to favor sovereign debt repayment. I model optimal risk-sharing between a country and foreign lenders to reflect these life-cycle preferences. Efficient allocations under political constraints deter default by promising higher future consumption to the young. Thus, although the elderly face lower direct costs from financial exclusion, they may be most affected by default. The model is consistent with the observed weak correlation between output or debt and default. Calibration shows that even with low risk aversion and no output losses, the threat of financial autarky can sustain debt of about 42% of GDP.

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> "It is a partnership not only between those who are living, but between those who are living, those who are dead, and those who are to be born."
>
> — Edmund Burke, *Reflections on the Revolution in France*

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1 Introduction

Survey evidence from episodes of sovereign debt distress points to a recurring generational pattern: older individuals are more inclined than younger individuals to support repayment of external public debt. The evidence comes from different countries, institutional settings, and moments in the default cycle. Tomz (2004) documents a positive relationship between age and support for repayment in Argentina six months after the 2001 default. Survey evidence around Iceland's 2011 Icesave referendum displays the same age gradient in a setting centered on sovereign debt resettlement (Curtis et al. (2014)). Greece's 2015 referendum, held under immediate default and euro-exit risk, shows a stark divide between young and old voters (The Economist (2015)). In a survey I commissioned in Argentina in 2019, when debt sustainability was central to the presidential campaign, older respondents were again more likely to oppose default. In summary, age-based repayment preferences appear as a stylized fact of sovereign-debt distress.

The Greek referendum is a useful illustration. On July 5, 2015, voters were asked whether to accept a reform and fiscal-adjustment program proposed by the European Commission, the European Central Bank (ECB), and the International Monetary Fund (IMF). Greece had already missed an IMF payment on June 30, and a vote against the proposal was widely viewed as raising the likelihood of exit from the eurozone and default on obligations to European governments and the ECB.[1] Public Issue, an opinion-polling company in Greece, documented a large age gradient in voting intentions: only 15% of voters aged 18 to 24 supported the Yes vote, whereas 55.1% of those aged 65 and older did so, as shown in Figure 1. Financial circumstances also mattered, but the age profile was especially pronounced: older voters were the only age group in which a majority supported accepting the program.

This pattern is puzzling from the perspective of standard sovereign-debt theory. In reputational models, repayment is sustained because default triggers exclusion from international capital markets. If the cost of default is mainly the loss of future market access, then older voters should be the least willing to repay: they have the shortest horizon over which to benefit from preserving access. Representative-agent models abstract from this conflict altogether. Some political-economy models do introduce generational conflict, but they tend to sharpen rather than resolve the puzzle. For example, Alichi (2008) studies sovereign debt in democracies and emphasizes that an old generation with little concern for future market access may force default when it is politically decisive. The survey evidence therefore raises a question that is not answered by the standard mechanism: why might short-lived older voters want to honor sovereign debt?

This paper answers that question by embedding an overlapping-generations structure in a limited-commitment model of sovereign borrowing. The economy receives stochastic endowment, or output, shocks and trades state-contingent claims with risk-neutral

[1] Total outstanding foreign liabilities were approximately €340 billion.

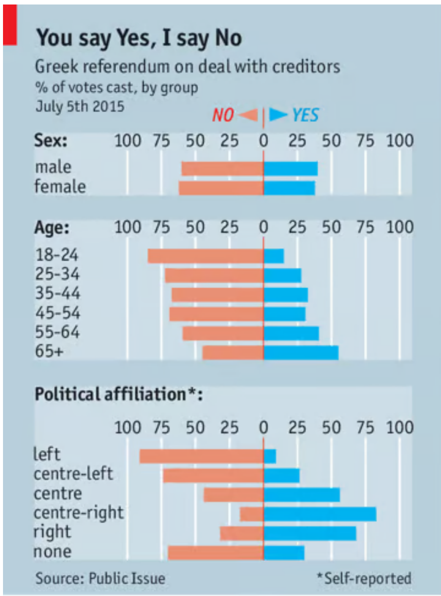


Figure 1: Greece's 2015 referendum. The Economist (2015)

foreign lenders. Individuals live for two periods and have no altruism toward unborn generations. A planner designs efficient risk-sharing arrangements with foreign lenders, but these arrangements must be politically credible. At each history, a period government representing the young and the old who are currently alive can repudiate the contract and choose default. The contract is feasible only if the politically weighted utility of the living generations under repayment is at least as high as under default and financial autarky. I refer to this requirement as the political credibility constraint. The weights in this constraint are derived from probabilistic voting, so that the period government's objective reflects the aggregation of voter preferences in the political process.

The mechanism can be summarized in four steps. First, as in standard reputational models, risk-sharing is sustained by the threat of exclusion from international capital markets after default. Second, as in dynamic contracting models, promises of future utility summarize the relevant history of the lending relationship. Third, unlike in a representative-agent model, promises are made to generations with finite lives. When a high output shock makes default attractive, the contract can preserve political support by increasing current consumption for both cohorts and by promising higher old-age consumption to the young. Fourth, those promises become political claims in the next period: a cohort that was compensated when young may support repayment when old, even though it has little direct interest in future market access.

This logic clarifies what happens when the political credibility constraint binds. The constraint is a weighted average of the contractual surplus of the old and the young relative to default. If both generations strictly preferred repayment, the constraint would be slack; if both strictly preferred default, the contract would not be credible. Thus, except for knife-edge cases in which one group is exactly indifferent, a binding constraint implies a generational conflict: one generation favors repayment and the other favors default. The model predicts that the old are more likely to favor repayment when past binding constraints have given them high current consumption relative to what they would receive in autarky. Conversely, if the old have strong political power or weak promised entitlements, they may favor default. The model therefore does not assume that old voters are always pro-repayment; it identifies when they become the repayment coalition.

The same mechanism generates history dependence in default incentives and debt dynamics. With complete markets and an infinitely lived representative agent, promised value can often be summarized by debt, and default incentives are tightly linked to current output and outstanding liabilities. In the OLG economy, the relevant state includes promises made to cohorts. A positive output shock today may raise future old-age consumption for the currently young, making tomorrow's old more willing to repay. As a result, the temptation to default depends not only on current output or debt, but also on the past output shocks that shaped the distribution of promises across generations. This breaks the tight relation between default, output, and debt, and it allows default incentives to arise even when the country is a net recipient of capital inflows.

Numerical solutions are obtained using two complementary approaches. The first solves the recursive contracting problem directly by value function iteration, approximating the lenders' value function over promised utility. The second solves the expectational difference equation implied by the first-order conditions for the Lagrange multiplier on the political credibility constraint. This multiplier recursion summarizes the dynamics of the contract because, conditional on the promised value to the planner, current allocations can be represented in terms of the current output shock and the lagged multiplier. Using both methods is important for two reasons. First, it provides an independent check on the quantitative results: when the state space admits an analytical characterization of the lower bound for promised utility, the value functions, policy functions, and debt levels obtained from the two procedures are essentially identical. Second, the multiplier-based method is computationally useful beyond this special case, since it can be implemented even when the promised-utility state is not analytically characterized.

In numerical simulations I calibrate the endowment process using GDP measured at ten-year intervals for a set of large emerging markets. The OLG structure implies a significantly higher borrowing capacity than a standard representative-agent reputational model with low risk aversion and no output losses from default. With CRRA preferences and risk aversion equal to 2, financial autarky alone can sustain debt of about 42% of annual GDP when the country is ex ante indifferent between the contract and autarky. The reason is a limited immiseration result. Because political credibility requires support from the living electorate on average, rather than an individual participation constraint for every cohort, the efficient contract can assign some young cohorts expected lifetime utility below their autarky value while preserving the political support needed to avoid default. This expands the resources that can be pledged to foreign lenders and increases debt capacity.

The quantitative exercise also rationalizes two empirical regularities that are difficult for simple reputational models to match. First, default is only weakly correlated with output: high output creates incentives to default, but past high output also raises promised consumption to cohorts that later become old, mitigating current default incentives. Second, debt is an imperfect sufficient statistic for default incentives, because the same level of state-contingent liabilities can be associated with different distributions of promised utility across generations.

Literature review

Much of the sovereign-debt literature studies the penalties that sustain repayment when a sovereign lacks commitment. Reputational models emphasize the restriction of a defaulting country's access to international capital markets.[2] If legal or reputational mechanisms fully excluded a country from capital markets after default, they could in principle support debt or insurance contracts, as in Eaton and Gersovitz (1981). Quantitatively, however, Aguiar and Gopinath (2006) and Arellano (2008) show that this mechanism alone has difficulty generating the large debt levels observed in the data. In their models, as in Lucas (1987), the welfare cost of consumption fluctuations is relatively small. The present paper shows that, once political credibility is imposed across overlapping generations, the efficient contract can sustain substantially higher debt even without output losses from default.

Empirically, sovereign defaults are not tightly tied to the states in which reputational models would most strongly predict default. Using a historical dataset of default episodes since the end of the Napoleonic Wars, Tomz and Wright (2007) show that defaults are more common in downturns but that the relationship between defaults and output is much weaker than the simplest models imply: countries often repay in bad times and sometimes default when output is above trend. The history dependence generated by cohort-specific promises offers a mechanism for weakening the link between current output and default incentives.

A related literature studies debt renegotiation after default. Most defaults end with a return to international capital markets and a reduction in the present value of outstanding claims. Sturzenegger and Zettelmeyer (2008) estimate investor losses using yields on newly issued bonds, and Cruces and Trebesch (2013) estimate an average haircut of 37% across 180 sovereign restructurings from 1978 to 2010. Bi (2008) and Benjamin and Wright (2008) model post-default renegotiation, emphasizing that with incomplete markets and non-contingent debt there may be incentives to delay settlement until repayment is more

[2] See Bulow and Rogoff (1989), Worrall (1990), and Kletzer and Wright (2000).

credible. The extension in Section 6 introduces temporary exclusion and renegotiation into the present complete-markets environment.

Domestic politics has become increasingly important in sovereign-debt research. Guembel and Sussman (2009) show how domestic creditors can support repayment even without direct default penalties. Empirical work by Tomz (2004), Curtis et al. (2014), and Nelson and SteinBerg (20181) studies public attitudes toward repayment and debt disputes. The closest antecedent on age and sovereign default is Alichi (2008), which develops a model in which generations disagree because the old care less about future access to capital markets. The contribution here is different: old voters may support repayment precisely because past incentive provision has transformed earlier promises into current old-age entitlements.

The characterization of constrained-efficient allocations using promised utility links the paper to the dynamic-contracting literature on risk sharing under limited commitment and incentive constraints. In this class of models, the planner's problem is made recursive by using promised utilities as state variables; see Golosov et al. (2016) for an overview. Spear and Srivastava (1987), Thomas and Worrall (1988, 1994), and Kocherlakota (1996) are early contributions that operationalize this promised-utility approach. More recently, Lancia et al. (2024) applies related methods in an overlapping-generations economy with intergenerational insurance in a closed economy, showing how promised utility to the old can serve as a sufficient state variable.

The closest work in terms of methods is Dovis et al. (2023), who use a dynamic optimal-contract framework with an OLG structure to study cycles of debt crises and inequality. Their mechanism centers on conflict between current and future governments over debt and inequality. Here the focus is on how a single political credibility constraint aggregating the preferences of currently living generations changes the enforcement of external sovereign debt and generates age-based repayment coalitions.

The rest of the paper is organized as follows. Section 2 presents survey evidence on age-based preferences for sovereign debt repayment. Section 3 introduces the endowment-economy framework, and section 4 characterizes constrained-efficient allocations and describes the numerical methods. Section 5 examines the model's quantitative implications. Section 6 concludes. The appendix contains proofs, auxiliary calculations, numerical details, and additional material.

2 Evidence on preferences for repayment by age

2.1 Argentina 2019

In October 2019, I commissioned a survey with close to 2,000 respondents in Argentina, providing valuable insights into public preferences regarding sovereign debt repayment. The data reveals a clear positive correlation between age and the propensity to support debt repayment. One of the unique aspects of this survey is that it was conducted between Argentina's primary elections on August 11 and the first round of presidential elections on October 27. Leading up to the primaries, polls indicated a narrow lead for the left-wing populist opposition over the moderate incumbent seeking re-election. Although the populists were not advocating for outright default on sovereign debt, they had criticized the previous year's agreement with the IMF, which had failed to prevent the country's growing loss of creditor confidence. On August 12, following the surprise populists' victory margin of over 15 percentage points, the Argentine peso devalued by approximately 25%, and Argentine stocks traded in U.S. exchanges lost nearly 50% of their value. Given Argentina's historical struggles with sovereign debt and the salience of this issue in the political discourse, this survey provides an insightful snapshot of public opinion on debt repayment during an economically turbulent time.

The survey was conducted online by Seido, a local consulting firm with an established database of respondents and a history of conducting regular online surveys via its Facebook page. Participants were incentivized with the chance to win one of twenty $500 Argentine peso prizes (roughly $8.50 USD). A total of 1,984 valid responses were collected. The full details of the questionnaire and its ethics approval process can be found in Appendix 7.1. The key question regarding preferences for debt repayment was "Given the country's economic problems, should Argentina stop paying its public debt?" (in Spanish: "En vista de los problemas económicos que tiene el país, Argentina debería dejar de pagar su deuda pública?"). The possible responses ranged from "Strongly agree" to "Strongly disagree", with an option for "N/A." The variable "default" is a binary variable that classifies respondents who agreed or were neutral about Argentina stopping its public debt payments as supporting default. Results were consistent whether the default preference was measured with this binary variable or as a categorical variable with values from 1 to 5. Since default would be on public debt and some respondents might be debt holders this might explain why the old might be more in favor of repayment, as they are more likely to have public debt on their portfolios.[3] While this might be at least in part controlled by income to validate the result I use a second default dummy on attitudes towards repayment of debt with the IMF.

The main regressions control for variables such as age, gender, income (proxied by savings behavior), education, self-identified political orientation (ranging from nationalism to liberalism on a scale of 1 to 10), a measure of trust, a dummy for being a public employee or receiving a public subsidy (excluding pensions), self-evaluation of the change in economic conditions in the past year, and an evaluation of the incumbent president. Table 1 presents the regression results using weights to account for stratification along age, gender, and education. Results in columns (1) for public debt and (3) for debt with

[3]In 2016 Argentina had a tax amnesty that offered lower tax rates for those participants who opted to hold newly issued public bonds. I expected that asking directly about participation in this tax amnesty might have led to attrition and bias among respondents.

the IMF use age as a control, while results in columns (2) and (4) use a dummy for being age 60 or above.[4] All regressions show that age is negatively correlated with support for default. Specifically, the estimated coefficient implies that being age 60 or above reduces the likelihood of supporting default by approximately 10%. The main explanatory variable in all regressions is evaluation of the incumbent: Respondents that have a favorable view of the president are more strongly against default.[5]

Table 1: Age, default preferences and expectations in Argentina 2019

	(1) default public debt	(2) default public debt	(3) default IMF debt	(4) default IMF debt	(5) fear future	(6) fear future
age	-0.002791**		-0.00348***		-0.00378***	
	(0.00139)		(0.00130)		(0.00106)	
old		-0.10795***		-0.08176**		-0.12452***
		(0.04062)		(0.03941)		(0.03491)
gender	0.05795	0.05457	0.03001	0.02979	0.03968	0.03764
	(0.04060)	(0.04063)	(0.03889)	(0.03946)	(0.03178)	(0.03176)
trust	-0.08622**	-0.07088*	-0.02969	-0.00399	-0.07941***	-0.05982**
	(0.04067)	(0.04015)	(0.03971)	(0.03928)	(0.02987)	(0.02993)
government	-0.16664***	-0.19333***	-0.27271***	-0.28471***	- 0.11420***	-0.11832***
	(0.04848)	(0.04261)	(0.04106)	(0.04168)	(003911)	(.04040)
No. of Obs.	1738	1738	1762	1762	1846	1846
R-Squared	0.1156	0.1121	0.1903	0.1880	0.1231	0.1180

Robust standard errors in parenthesis. Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

It is important to note, however, that due to the cross-sectional nature of the data, age and birth year are perfectly correlated. This means the observed relationship between age and default preferences could reflect the fact that older cohorts tend to have more pro-repayment views—perhaps due to the specific economic, social and political environment in which they were raised—rather than an effect of aging on individual preferences.[6]

In contrast, there is recent evidence from cross country surveys that, across different cultures and settings, individual preferences vary systematically with gender, cognitive

[4] The reason for the main econometric specification to be OLS is that it is well known that when the dependent variable is binary, linear regression is about as good as non-linear models like Probit. See Angrist and Pischke (2009).

[5] This is consistent with results in Nelson and Steinberg (2018) that study attitudes towards repayment to holdouts and find that presidential approval is significant and aligned with government policy. Importantly they have survey data for two governments one that resisted repayment the other that favored it.

[6] Cohort effects have been shown to be a significant determinant of preferences in other contexts. For instance, Giuliano and Spilimbergo (2014) find that preferences for redistribution in the United States are influenced by macroeconomic shocks experienced during individuals' formative years (ages 18-25).

ability, and age, see Falk et al. (2018). In particular older individuals tend to be less patient and more risk averse. Impatience cannot explain the regression results as being more impatient would, if anything, make respondents more inclined to favor default.[7] To check whether repayment preferences might reflect risk aversion I run a third set of regressions, now with the answer to a question about fear that the economic situation might worsen in the following year as the dependent variable. Results, are reported in columns (5) and (6) again with age or a dummy for being aged 60 or above as the main controls of interest. Results strongly reject that the old might be against default because they are more risk averse.

3 Model

3.1 Structure and preferences

Consider a small open endowment economy with an infinite sequence of dates, $t = 0, 1, \ldots$. The economy is subject to exogenous endowment shocks which follow a finite-state i.i.d. process.[8] Let $y_t = y(s_t)$ denote the period t realization of the per capita endowment, and $\pi(s_t)$ its probability. Denote by $s^t \in S^t$ a history of the economy observed up to time t, where S is the set of possible states. The economy is inhabited by two-period lived overlapping generations, a sequence of governments indexed by $t = 0, 1, \ldots,$ and external lenders. Population growth at rate ν and the young know that they will be alive when old only with probability $0 < p \leq 1$. I abstract from any shocks to the distribution of the endowment across generations and assume that it is evenly split among young and old.[9]

Preferences of individuals born in period t and history s^t are described by the utility function

$$u_t(s^t) = u(c^y(s^t)) + \delta p E_t[u(c^o(s^{t+1}))],$$

where $u(\cdot)$ is a twice differentiable, increasing, strictly concave function, with $\lim_{c\to 0} u'(c) = \infty$; $\delta \in (0, 1)$ is the discount factor; expectations are conditional on observed history;[10] and $(c^y(s^t), c^o(s^{t+1}))$ denotes a consumption path over the life cycle.

We are interested in characterizing efficient allocations, and therefore need a social welfare function to rank outcomes. It is assumed that cohorts are given weights proportional to their size, and that time is discounted at rate δ, i.e. the social planner has

[7] Falk et al. (2018) also find that cognitive skills are correlated with patience. In the regressions those with incomplete primary education were more likely to favor default.

[8] I make this assumption for simplicity and because it is validated by GDP data at ten year intervals; see section 5. It also highlights the history dependence of optimal allocations. Most results directly generalize to a Markov chain from some initial state y_0.

[9] For a thorough analysis of constrained efficient risk sharing in a closed economy with shocks to aggregate endowment and its distribution see Lancia et al. (2024).

[10] Although shocks are i.i.d., the potential history-dependence of allocations requires the use of conditional expectations.

the same discount factor as individuals. I assume that $\delta\nu < 1$. The utilitarian welfare function would then be

$$U(s_0) = \frac{p}{\nu + p} E_0 \left[u(c^o(s_0)) \right] + \sum_{t=0}^{\infty} (\delta\nu)^t E_0 \left[\frac{\nu}{\nu + p} u(c^y(s^t)) + \frac{\nu}{\nu + p} \delta p u(c^o(s^{t+1})) \right], \quad (1)$$

where the welfare is normalized such that the first period population has mass one and the aggregate endowment evolves as $\nu^t y_t$. Competitive infinitely-lived external lenders are risk neutral, discount the future at factor δ', and have the ability to commit to their obligations. Lending takes place using a full set of state-contingent assets with prices $q^*(s^t)$. Denoting by x_t their consumption in period t, their preferences are thus given by

$$\sum_{t=0}^{\infty} \delta'^t E_0[x_t].$$

3.2 Government and credibility constraints

The government centralizes decisions in the economy. This implies that given the observed state, s^t, individual consumption is determined, $c^y(s^t), c^o(s^t)$, payments are made to lenders $b(s^t)$, and borrowing takes place using a portfolio of state-contingent liabilities, $b(s^t, s_{t+1})$, (raising resources $\sum_{s^{t+1}} \frac{q^*(s^{t+1})}{q*(s^t)} b(s^{t+1})$). Since markets are complete the resource constraint of the small open economy can be written as

$$b_0 \leq \sum_{t \geq 0, s^t} q^*(s^t) \nu^t \left(y(s^t) - \frac{\nu}{\nu + p} c^y(s^t) - \frac{p}{\nu + p} c^o(s^t) \right), \quad (2)$$

where $b_0 \equiv b(s_0)$ is the initial net foreign liability position. The right-hand side is the present expected discounted value of net exports. Balance of payments accounting requires that this be equal to initial net foreign liabilities. Alternatively b_0 can be interpreted as the equivalent initial debt that solves bargaining between borrowers and lenders when they start a long-term lending relationship. If the sovereign has no initial assets it must satisfy $0 \leq b_0 \leq \underline{b}$ where the maximum debt level, $\underline{b}$, corresponds to maximized utility according to the planner's preferences (1) equal to that of financial autarky, v^{AUT}, see appendix 7.4.

The first-best outcome would have lenders absorbing all endowment uncertainty, and implementing a consumption profile that reflected the relative impatience of the country vis a vis the rest of the world.[11] Instead, the small open economy has limited commitment such that at each point in time its government can decide to renege on its obligations. More specifically, policy decisions are made by a sequence of governments indexed by $t = 0, 1, \ldots$. Government t cares only about agents that are currently alive and evaluates

[11]For example, if $\delta > \delta'$ then consumption would be increasing over time.

welfare according to the utilitarian criterion,

$$\omega \frac{p}{\nu + p} u(c^o(s^t)) + (1-\omega) \frac{\nu}{\nu + p} \left(u(c^y(s^t)) + \delta p E_t[u(c^o(s^{t+1}))] \right), \tag{3}$$

with relative weights $0 < \omega < 1$ and $1 - \omega$ on the old and young, respectively.[12] In Appendix 7.3 I show how this utilitarian criterion can be justified if policy is decided sequentially and decisions are aggregated by probabilistic voting.

With lack of commitment, and under the assumption that deviation implies permanent exclusion from international capital markets, efficient allocations must satisfy the constraint that each period government chooses to adhere to the arrangement instead of defaulting, i.e. we must consider self-enforcing allocations that satisfy

$$\frac{\omega p}{\nu + p} u(c^o(s^t)) + \frac{(1-\omega)\nu}{\nu + p} \left(u(c^y(s^t)) + \delta p E_t[u(c^o(s^{t+1}))] \right) \geq \underline{V}(y_t). \tag{4}$$

where optimal consumption under a deviation and the autarky payoff $\underline{V}(y_t) = \frac{\omega p}{\nu+p} u(c^{oD}(y_t)) + \frac{(1-\omega)\nu}{\nu+p} \left(u(c^{yD}(y_t)) + \delta p E[u(c^{oD}(y_{t+1}))] \right)$, are characterized in appendix 7.4.

I will interpret constraint (4) following the contract-theoretic literature: It restricts the contracts, and thus the risk sharing available to the sovereign, that can be written between the sovereign and foreign lenders. An alternative interpretation is game theoretic and considers that a repeated game takes place between the sovereign and foreign lenders. This would require to enrich the history of the game to h^t by including the history of payments made between the sovereign and lenders.[13] Any self-enforcing allocation can be implemented as a subgame perfect equilibrium of the repeated game between the sovereign and foreign lenders, and with complete information there are no deviations in equilibrium. To emphasize the equivalence I will henceforth refer to the history as h^t even though only the history of endowment shocks is relevant.

3.3 First best

Foreign lenders' risk neutrality implies that the period expected (risk-free) gross return is $R = \frac{1}{\delta'}$. The equilibrium international price of a unit of consumption in state h^t in units of period zero consumption is given by $q^*(h^t) = \frac{\pi(h^t)}{R^t}$, with $\pi(h^t) = \prod_{i=0}^{t} \pi(y_i)$. Since

[12] See Fahri et al. (2012) for an application of this utilitarian criterion in a different context.

[13] Each possible history of the game up to any date t defines a subgame of the repeated game beginning at that date. Given a history h^t, the strategies determine policy and debt contracts in period t, generating an updated history h^{t+1}. Starting from $t = 0$ the corresponding consumption allocations are thus determined (c_t^o, c_t^y) for any strategy profile. Our interest is in characterizing the best subgame perfect equilibrium (SPE) from the perspective of the social welfare function (1). Following Chari and Kehoe (1990) and Abreu (1988), this SPE can be found using the threat of reversion to a punishment SPE with the worst continuation value in the face of a deviation. This corresponds to a "self-enforcing" equilibrium, or in the terminology of Chari and Kehoe (1990), a "sustainable plan". With permanent exclusion from capital markets the harshest punishment is given by $\underline{V}(y_t)$.

households are risk averse and foreign lenders are risk neutral, the first best would have the latter absorbing all uncertainty. This would happen if credibility constraints where always slack. In this case it is straightforward to show that optimal allocations satisfy

$$\begin{aligned} u'(c^y(h^t)) &= u'(c^o(h^t)), \\ u'(c^y(h^{t+1})) &= \delta R u'(c^y(h^t)). \end{aligned} \tag{5}$$

Thus, consumption is equalized between the young and the old, and the intertemporal profile reflects the difference between foreign and social discount factors. In particular, when $\delta = \delta'$, $c^y(h^t) = c^o(h^t) = \bar{c}$, and this constant level of consumption satisfies the resource constraint (2).

Assumption 1. Let $\bar{c}(b_0)$ satisfy (2) when initial debt is b_0 and $\delta R = 1$, i.e. $\bar{c}(b_0)$ is the first best consumption for the young and old. Then

$$\frac{\omega p}{\nu + p} u(\bar{c}(0)) + \frac{(1-\omega)\nu}{\nu + p} \left(u(\bar{c}(0)) + \delta p u(\bar{c}(0))\right) < \underline{\mathrm{V}}(y_S).$$

Under assumption 1 the first best allocation violates the participation constraints in at least the state with the highest output realization even when there is no initial debt.[14] Henceforth we assume this assumption holds and note when it implies a binding constraint on the economy's parameters.

4 Constrained efficient allocations

With the above characterization of the small open economy and international capital markets, an optimal allocation under limited commitment is a sequence of functions $c^y(h^t)$, $c^o(h^t)$, and $b(h^t)$ such that (1) is maximized, with constraint (2) holding, and period governments at all times and histories prefer the continuation allocation from the contract to deviating to autarky, i.e. (4) holds. Denote by v the maximized value of (1). I will characterize constrained efficient allocations recursively using the promised utility approach.

As is standard in the literature on dynamic contracts with limited commitment (see, among others, Aguiar, Amador and Gopinath, 2009), I work with the dual formulation of the optimization problem. Rather than directly maximizing the planner's objective subject to feasibility and incentive constraints, I characterize the constrained-efficient frontier in terms of promised utilities and reformulate the problem as one of maximizing the present discounted value of lenders' profits subject to promise-keeping and incentive constraints. The social planner must receive at least per capita utility v, defined as the

[14] It is trivial to see that $\frac{d\bar{c}(b_0)}{db_0} < 0$, thus if assumption 1 holds for $b_0 = 0$ it holds for $b_0 > 0$.

maximal per capita utility attainable to the planner in a constrained optimal allocation.[15] And in all dates and histories period governments do not want to deviate. Given that period governments represent a weighted average of the utilities of young and old, then an additional promise keeping constraint will be that the current surviving old receive at least the utility they were promised in the past when they were young.

With the problem thus rewritten, histories are summarized by the promised expected per capita utilities to the social planner and to the old, state variables v and η. Thus, we can rewrite (4) as

$$\frac{\omega p}{\nu + p} u(c^o(h^t)) + \frac{(1-\omega)\nu}{\nu + p} \left(u(c^y(h^t)) + \delta p \theta(h^t) \right) \geq \underline{\mathrm{V}}(y_t), \tag{6}$$

where $\theta(h^t)$ is the promised utility to the current young for next period, contingent on being alive. Define $B(v, \eta)$ as the maximal present discounted value of lenders' profits, normalized by borrower's population, among feasible allocations that deliver the promised pair (v, η) and satisfy incentive constraints (6) in every period and history.[16]

In order to apply standard results from stochastic dynamic programming (as in Chapter 9 of Stokey et al. (1989)), I consider a compact set for feasible (v, η) values. For simplicity I assume that $\delta' = \delta$ or $\delta R = 1$.[17] To set a lower bound for v I assume the planner would never enter a contract that at any contingency gave it per capita utility below $\underline{v} = v^{AUT} \equiv E[\frac{\frac{p}{\nu+p}u(c^{oD}(y_t)) + \frac{\nu}{\nu+p}u(c^{yD}(y_t))}{1-\delta\nu}]$.[18] For the upper bound for v, I consider the first best per capita value when there is no debt, $\bar{v} = v^*(0) \equiv E[\frac{u(\bar{c}(0))}{1-\delta\nu}]$.[19] Thus, $v \in [\underline{v}, \bar{v}]$. For η first note that risk sharing and the i.i.d. nature of shocks would imply that the country consumes less than its endowment for the highest output realization. Thus, a natural upper bound for c^y and c^o is y_S. This implies an upper bound for promised utility to the old, $\bar{\eta} = u(y_S)$. We can use (6) and the upper bounds for consumption to find a lower bound $\underline{\eta}$.[20] Denote the set of possible values for the endogenous states by $X = [\underline{v}, \bar{v}] \times [\underline{\eta}, \bar{\eta}]$.

The constrained optimal allocation solves the following Bellman equation with choice

[15] More precisely, with initial population normalized to one v is the value of (1) at an optimum given that debt b is outstanding and subject to budget and incentive constraints.

[16] Denoting by $\mathcal{B}(v, \eta; t)$ the maximal present discounted value of lenders' *aggregate* profits, the Bellman equation in proposition 1 would be equivalently satisfied for $\mathcal{B}(v, \eta; t)$ by noting that $\mathcal{B}(v, \eta; t) \equiv \nu^t B(v, \eta)$ and for aggregate profits the Bellman equation would be $\mathcal{B}(v, \eta; t) = \max_{(c^y(\cdot), c^o(\cdot), w(\cdot), \theta(\cdot)) \in Y} E_t \left[\nu^t \left(y(h^t) - \frac{\nu}{\nu+p} c^y(h^t) - \frac{p}{\nu+p} c^o(h^t) \right) + \delta \mathcal{B}(w(h^t), \theta(h^t); t+1) \right]$.

[17] The general case would feature a force that tilts consumption intertemporally. While it would still be possible to define a compact set for state variables, the case $\delta R = 1$ is particularly relevant as it shows that impatience is not needed to sustain positive levels of sovereign debt in equilibrium.

[18] As will become clear this can be relaxed such that the planner would never enter a contract that gave it *initial* per capita utility below v^{AUT}.

[19] Incentives to default, and thus $\underline{\mathrm{V}}(y_t)$, would be different if the country had assets, so I restrict attention to cases in which the economy has no initial assets, i.e. $b_0 \geq 0$.

[20] Formally $\underline{\eta}$ is determined by $\frac{\omega p}{\nu+p} u(y_S) + \frac{(1-\omega)\nu}{\nu+p} (u(y_S) + \delta p \underline{\eta}) = \frac{\omega p}{\nu+p} u(c^{oD}(y_1)) + \frac{(1-\omega)\nu}{\nu+p} (u(c^{yD}(y_1)) + \delta p E[u(c^{oD})]$.

variables $c^y(h^t)$, $c^o(h^t)$, $w(h^t)$, and $\theta(h^t)$, where $(w,\theta) \in X$ are promised (per capita) utilities for the following period and $(c^y(h^t), c^o(h^t)) \in [c^{min}, y_S]^2$, with $c^{min} > 0$ are (per capita) consumptions for young and old.[21] Denote by $Y = [c^{min}, y_S]^{2S} \times X^S$ the action space. The constraints on the choice set are described by correspondence $\Gamma : X \to Y$, i.e. $\Gamma(v,\eta)$ is the set of all measurable contingent plans for consumption and promised utilities that satisfy the promise keeping and incentive constraints.

Proposition 1. $B(v,\eta)$ is the unique solution to the following recursive problem:

$$\begin{aligned}
B(v,\eta) &= \max_{(c^y(\cdot),c^o(\cdot),w(\cdot),\theta(\cdot))\in Y} E_t\left[y(h^t) - \frac{\nu}{\nu+p}c^y(h^t) - \frac{p}{\nu+p}c^o(h^t) + \delta\nu B(w(h^t),\theta(h^t))\right] \\
\text{s.t.} \quad v &\leq E_t\left[\frac{p}{\nu+p}u(c^o(h^t)) + \frac{\nu}{\nu+p}u(c^y(h^t)) + \delta\nu w(h^t)\right] \\
\eta &\leq E_t\left[u(c^o(h^t))\right] \\
\underline{V}(y_t) &\leq \frac{\omega p}{\nu+p}u(c^o(h^t)) + \frac{(1-\omega)\nu}{\nu+p}\left(u(c^y(h^t)) + \delta p\theta(h^t)\right) \quad \forall h^t
\end{aligned}$$

Moreover the correspondence $G : X \to X$ defined by

$$\begin{aligned}
G = \{(c^y(\cdot), c^o(\cdot), w(\cdot), \theta(\cdot)) \in \Gamma(v,\eta) : B(v,\eta) = E_t\Bigg[y(h^t) - \frac{\nu}{\nu+p}c^y(h^t) \\
- \frac{p}{\nu+p}c^o(h^t) + \delta\nu B(w(h^t),\theta(h^t))\Bigg]
\end{aligned}$$

is non-empty, compact-valued and upper hemi-continuous.

By restricting $v \in [\underline{v}, \bar{v}]$ and having $\underline{v} = v^{AUT}$ the promise keeping constraint to the social planner will never be slack. But the promise keeping constraint to the old might be slack for some η. This will render the value function B flat over some subset of its domain, preventing further characterization of the unique value function B and its policy correspondence G. I will conjecture that for all $v \in [\underline{v}, \bar{v}]$ there exists $\underline{\eta}(v) > \underline{\eta}$ such that denoting $\tilde{X} = [\underline{v}, \bar{v}] \times [\underline{\eta}(v), \bar{\eta}]$, both promise keeping constraints bind for $x \in \tilde{X}$, B is decreasing and strictly concave in $\tilde{X}$ and differentiable in $(\underline{v}, \bar{v}) \times [\underline{\eta}(v), \bar{\eta})$. This implies that the correspondence G is a continuous single-valued function.

[21] The following equations characterize lower bounds for consumption of young and old:

$$\begin{aligned}
\frac{\omega p}{\nu+p}u(c^{min,o}) + \frac{(1-\omega)\nu}{\nu+p}(u(y_S) + \delta p\underline{\eta}) &= \underline{V}(y_1), \\
\frac{\omega p}{\nu+p}u(y_S) + \frac{(1-\omega)\nu}{\nu+p}(u(c^{min,y}) + \delta p\underline{\eta}) &= \underline{V}(y_1)
\end{aligned}$$

Then simply take $c^{min} = \min(c^{min,y}, c^{min,o})$. It is straightforward that, with $0 < \omega < 1$, $c^{min} > 0$.

4.1 Constrained allocations and limit consumption behavior

Letting $\lambda(h^t)\pi(h^t)$ denote the multipliers on credibility constraints (6), $\mu_t\pi(h^t)$ on the promise keeping constraint to the social planner, and $\nu_t\pi(h^t)$ on the promise keeping to the old, under the conjecture we get the following first order conditions for $(v,\eta) \in \tilde{X}$[22]

$$-1 + \left(\mu_t + (1-\omega)\lambda(h^t)\right) u'(c^y(h^t)) = 0, \tag{7}$$

$$-1 + \left(\mu_t + \omega\lambda(h^t) + \frac{\nu + p}{\nu}\nu_t\right) u'(c^o(h^t)) = 0, \tag{8}$$

$$B'_v(w(h^t), \theta(h^t)) + \mu_t = 0, \tag{9}$$

$$B'_\eta(w(h^t), \theta(h^t)) + (1-\omega)\lambda(h^t)\frac{p}{\nu + p} = 0. \tag{10}$$

We also have the following envelope conditions

$$B'_v(v, \eta) = -\mu_t, \tag{11}$$

$$B'_\eta(v, \eta) = -\nu_t. \tag{12}$$

From (9) and (11) it follows that $B'_v(v,\eta) = B'_v(w(h^t), \theta(h^t))$ for all h^t, implying that $\mu_t = \mu$. The constancy of this promise keeping multiplier is a direct consequence of markets being complete. Since under the conjecture B is strictly concave this implies that $w(h^t) = v$ for all h^t. Thus, while keeping in mind that the optimal contract depends on v, η_t is the only relevant endogenous state variable. From the promise keeping constraint to the social planer holding with equality, $w(h^t) = v$, implies:

$$\frac{p}{\nu + p}\eta + \frac{\nu}{\nu + p}E_t\left[u(c^y(h^t))\right] = (1-\delta\nu)v \tag{13}$$

Thus, an increase in the promised utility to the old must reduce the expected utility of the young. When the promise keeping constraint to the old is not binding the planner's preferences over consumption imply that at an optimum $u'(c^o(h^t)) = u'(c^y(h^t))$ and thus that $c^o(h^t) = c^y(h^t)$. This relation and (13) implicitly determine $\underline{\eta}(v)$, the promised utility to the old at which the corresponding promise keeping constraint starts to bind, $\underline{\eta}(v) = (1-\delta\nu)v$. In particular, $\underline{\eta}(v^{AUT}) = E[\frac{\nu}{\nu+p}u(c^{yD}) + \frac{p}{\nu+p}u(c^{oD})]$ regardless of the functional form of preferences. Given that we characterized $\underline{\eta}(v)$ let's prove the rest of the conjecture.

Proposition 2. The unique solution $B(v,\eta)$ of the Bellman equation is decreasing and strictly concave in $\tilde{X}$ and the correspondence G is a continuous single-valued function. $B(v,\eta)$ is differentiable for all $(v,\eta) \in (\underline{v}, \bar{v}) \times [\underline{\eta}(v), \bar{\eta})$.

[22]Formally there is also one complementary slackness condition for (6), and under the conjecture there is no need to consider the complementary slackness conditions of the promise keeping constraints.

Combining (10) and (12) it follows that $-B'_\eta(v,\eta) = \nu_t = (1-\omega)\frac{p}{\nu+p}\lambda(h^{t-1})$. Thus, given strict concavity of B, there is a one-to-one positive relation between $\lambda(h^{t-1})$ and $-B'_\eta(v,\eta)$, i.e. a steeper slope of the value function in the η-direction in period t corresponds to a higher value of the multiplier on the incentive constraint in period $t-1$. Finally, the optimality conditions are given by:

$$\begin{aligned} \left(\mu + (1-\omega)\lambda(h^t)\right) u'(c^y(h^t)) &= 1, \\ \left(\mu + \omega\lambda(h^t) + \frac{p}{\nu}(1-\omega)\lambda(h^{t-1})\right) u'(c^o(h^t)) &= 1. \end{aligned} \tag{14}$$

When the incentive constraint binds, consumptions for both the young and old will increase. The multiplier μ is associated with the promised per capita utility to the social planner, v, and to the level of *minimum* consumption that the young are guaranteed, $\underline{c}(v)$, characterized by $u'(\underline{c}(v)) = \frac{1}{\mu}$. The strict concavity of B implies that an increase in v leads to a higher μ and thus higher minimum consumption (given that the optimal contract features a constant v, I henceforth drop v as an argument for $\underline{c}$, μ or $\underline{\eta}$).[23] Whenever the incentive constraint is not binding we have $c^y(h^t) = \underline{c}$ for the young and a constant consumption for the old (see characterization below, (16)), i.e. consumption is smoothed over states for which the incentive constraint is slack. In these states we also have that $\lambda_t = 0$ implying $B'_\eta(w(h^t), \theta(h^t)) = 0$ and thus that $\theta(h^t) = \underline{\eta}$. But consumption for the young and old will be higher if the incentive constraint is binding in the current period. And (14) shows that consumption smoothing implies that for the young this translates into higher consumption for them in the following period when old. This is achieved through a higher promised utility, $\theta(h^t) > \underline{\eta}$.

To further characterize the optimum let's rewrite the optimality conditions as

$$u'(c^y(h^t)) = \frac{1}{\frac{1}{u'(\underline{c})} + (1-\omega)\lambda(h^t)}, \tag{15}$$

$$u'(c^o(h^t)) = \frac{1}{\frac{1}{u'(\underline{c})} + \omega\lambda(h^t) + \frac{p}{\nu}(1-\omega)\lambda(h^{t-1})} = \frac{1}{\frac{p}{\nu u'(c^y(h^{t-1}))} + \omega\lambda(h^t) + \frac{\nu-p}{\nu u'(\underline{c})}}, \tag{16}$$

where the second equality in (16) uses (15) in period $t-1$. If in period t the credibility constraint is not binding, $\lambda(h^t) = 0$, and

$$c^y(h^t) = \underline{c}, \tag{17}$$

$$u'(c^o(h^t)) = \frac{1}{\frac{p}{\nu u'(c^y(h^{t-1}))} + \frac{\nu-p}{\nu u'(\underline{c})}}, \tag{18}$$

thus the young receive $\underline{c}$ while the old receive a consumption level that depends on the

[23]Note the difference with the case with a representative agent with an infinite horizon with $\delta R = 1$ in which the multiplier of the resource constraint is associated with the constant level of consumption once the highest endowment is realized for the first time. See Worrall (1990) and Aguiar and Amador (2014).

consumption they were allocated when young in the previous period. Therefore, when the credibility constraint is binding and the young are given a consumption above $\underline{c}$, they know that this determines the floor on their old age consumption as well.

4.2 History dependence and capital flows

To study the effect of history, as summarized by state variable η_t, on the correlation between current output and incentives to deviate we start by defining $\bar{y}_t(\eta_t)$ as the per capita endowment level at which the credibility constraint starts to bind. Since $\underline{V}(y_t)$ is monotonous this threshold is well defined, and it must be the case that $\lambda_t = 0$ (and $\theta(h^t) = \underline{\eta}$) when $y_t \leq \bar{y}_t(\eta_t)$ and $\lambda_t > 0$ (and $\theta(h^t) > \underline{\eta}$) when $y_t > \bar{y}_t(\eta_t)$. Since the higher is $\bar{y}_t(\eta_t)$ the more likely it is that consumption will be smoothed, increases in this threshold can be seen as improvements in the economy's ability to share risk with the rest of the world.

$$\frac{\omega p}{\nu + p} u(c^y(h^{t-1})) + \frac{(1-\omega)\nu}{\nu + p} \left(u(\underline{c}) + \delta p \underline{\eta} \right) = \underline{V}(\bar{y}_t(\eta_t)). \tag{19}$$

Given that an increase in η_t does not affect $c^y(h^t) = \underline{c}$ nor $\underline{\eta}$ but it increases $c^y(h^{t-1})$, then $\bar{y}_t(\eta_t)$ increases with η_t. Thus the incentives to deviate in a given period are not only a function of current, but also of past output: When in the past output was high, consumption increased to avoid default. But for those who were young in the past, this increase was spread over their lifetime. Therefore today the old have high consumption and the overall incentive to deviate is muted.

Turning now to the case $\lambda_t > 0$, or $\theta(h^t) > \underline{\eta}$, and a given output state y_t

$$\frac{\omega p}{\nu + p} u(c^o(h^t)) + \frac{(1-\omega)\nu}{\nu + p} \left(u(c^y(h^t)) + \delta p \theta(h^t) \right) = \underline{V}(y_t). \tag{20}$$

From (16) it can be seen that an increase in η_t, (and thus λ_{t-1}) implies an increase in $c^o(h^t)$ and since the right-hand side of (20) does not depend on η_t, $u(c^y(h^t)) + \delta p \theta(h^t)$ must decrease. Since from (15) $c^y(h^t)$ and $\theta(h^t)$ are positively related, this implies that $c^y(h^t)$, $\theta(h^t)$, and λ_t are reduced. Thus, the policy functions are monotonic in η_t.

Whether average consumption, $\frac{\nu}{\nu+p} c^y(h^t) + \frac{p}{\nu+p} c^o(h^t)$, increases or decreases with η_t when the credibility constraint is binding depends on parameters. Since an increase in average consumption is interpreted as a reduction in state contingent debt, this implies that in general there is no monotonous relation between debt and state variables. Therefore debt cannot replace promised utility η as a state variable as can be done in standard models, and incentives to default are not tightly related to outstanding (state-contingent) debt.[24]

[24] Debt is always a function of state variables, $b(h^t) = b(y_t, \eta_t)$, but in general it is not always true that $\frac{db(y_t,\eta_t)}{d\eta_t} < 0$.

Since average consumption is constant, at level $\frac{\nu}{\nu+p}\underline{c}+\frac{p}{\nu+p}c^y(h^{t-1})$, for low endowment realizations, the country is a net recipient of capital flows for bad shocks. These capital inflows are financed by capital outflows in other output states. Thus, at least one threshold per capita income level, $\hat{y}_t(\eta_t)$, exists such that when output is (locally) above it the country experiences net capital outflows, and conversely when output is (locally) below this threshold.

In models with an infinitely lived representative agent $\hat{y}_t(\eta_t) \leq \bar{y}_t(\eta_t)$, i.e. the credibility constraint only binds in states for which the country is expected to make a positive payment to its lenders (see Aguiar and Amador (2014)). The next proposition shows that, with commitment problems arising from the aggregation of default preferences among different generations, there might be incentives to default even when the country is a net recipient of resources from the rest of the world, i.e. $\bar{y}_t(\eta_t) < \hat{y}_t(\eta_t)$. It also presents parameter restrictions such that average consumption is always increasing (and thus debt is decreasing) in η_t.

Proposition 3. When $\omega \leq \omega^S$ (with $\omega^S \geq \frac{1}{2}$) there is a monotone relation between debt and η. There exists a unique threshold income level, $\hat{y}_t(\eta_t)$, such that whenever $y_t < \hat{y}_t(\eta_t)$, the country is a net recipient of capital flows. For some parameters it is the case that $\bar{y}_t(\eta_t) < \hat{y}_t(\eta_t)$.

That there is an unique threshold income level, $\hat{y}_t(\eta_t)$ was expected since insurance implies that average consumption is higher than per capita endowment for bad shocks, and lower for good shocks. Thus, for given η_t, average consumption increases at a lower rate than the per capita endowment.

4.3 Political power and preferences for default by age

We turn to the motivating evidence presented in section 2 and appendix 7.2 that in survey data at times of distress the elderly are more likely to favor repayment of sovereign debt. To this effect it is illustrative to rewrite the credibility constraint (6) as

$$\frac{\omega p}{\nu+p}\left(u(c^o(h^t)) - u(c^{oD}(y_t))\right) \quad + \quad \frac{(1-\omega)\nu}{\nu+p}\Big(u(c^y(h^t)) + \delta p\theta(h^t) \\ -(u(c^{yD}(y_t)) + \delta p E[u(c^{oD}(y_{t+1}))])\Big) \geq 0. \quad (21)$$

The left hand side is the weighted average of the surplus that young and old get from the contract. When the constraint is binding this average is zero, implying that one type of voters favors default while the other is against it. In particular, whenever $c^o(h^t) > c^{oD}(y_t)$, the old will favor repayment.

To see what determines the old's preferences for default, it is illustrative to consider first the case of equal generational per capita weights in the political process, $\omega = \frac{1}{2}$, and equal shares in the population, $p = \nu$. In this case $c^o(h^t) \geq c^y(h^t)$ with equality holding whenever $\lambda_{t-1} = 0$ (i.e. $\eta_t = \underline{\eta}$).

When $p = \nu$, $\omega = \frac{1}{2}$, $y_t > \bar{y}_t(\eta_t)$, and $\eta_t = \underline{\eta}$ such that $c^o(h^t) = c^y(h^t)$ and $c^{oD}(h^t) = c^{yD}(h^t) = \frac{y_t}{2}$, regrouping terms in (20) as done above in (21) results in

$$2\left[u(c^y(h^t)) - u(\frac{y_t}{2})\right] = \delta p\left(E_t[u(\frac{y_{t+1}}{2})] - \theta(h^t)\right). \tag{22}$$

That $y_t > \bar{y}_t(\eta_t)$ (i.e. $\lambda_t > 0$) implies $\theta(h^t) > \underline{\eta} \geq E_t[u(\frac{y_{t+1}}{2})]$. Thus, the RHS of (22) is negative, which implies that $c^y(h^t) < \frac{y_t}{2}$. This then implies that $\frac{\nu}{\nu+p}c^y(h^t) + \frac{p}{\nu+p}c^o(h^t) < y_t$, i.e. whenever the constraint is binding in this period, but not in the previous one, net exports are positive, i.e. $\hat{y}_t(\underline{\eta}) \leq \bar{y}_t(\underline{\eta})$. As mentioned before, this is a standard result from a representative agent economy. It holds in this special case because with equal weighting and $\eta_t = \underline{\eta}$ the optimal allocation when $\nu = p$ gives equal consumption to young and old, and under default consumptions are also equalized. Another implication of (22) is that $c^o(h^t) < c^{oD}(y_t) = \frac{y_t}{2}$. Thus whenever the credibility constraint is binding, but was not binding in the past, the old are in favor of default when $\nu = p$ and $\omega = \frac{1}{2}$.[25]

The following proposition shows how the old's preferences for repayment are affected by past outcomes.

Proposition 4. When $y_t > \bar{y}_t(\eta_t)$ and $\omega \geq \omega^D(\eta_t)$, with $\frac{d\omega^D(\eta_t)}{d\eta_t} \geq 0$, the old prefer a default and the young oppose it. Conversely, when $\omega < \omega^D(\eta_t)$ the old oppose a default while the young are in favor of one.

Not surprisingly, the old favor a default when the political process is biased towards them such that they would be able to extract a larger share of resources in autarky. But this preference for defaulting is mitigated if they are entitled to high current consumption as a result of past promises. Thus, higher past endowment realizations increase the probability that the elderly oppose default.

The optimal contract reflects two types of pressures. First, the overall willingness to default when the country experiences good endowment shocks. Second, the tension between desired generational equity and the distribution of political power: When the credibility constraint is not binding the desired social allocation has consumption for young and old that satisfy (18). But when the credibility constraint is binding the allocation gets distorted towards the autarkic one with marginal utilities reflecting the relative political power of the old, $\frac{\omega}{(1-\omega)}$.

A change in ω affects the autarky allocation, whether the incentive constraint binds or not for a given state, and the value of the multiplier for the states in which the constraint is binding. Thus, it is not possible to determine the effects of changes in political power

[25]Note that $y_t = \bar{y}_t(\eta_t)$ and $v = v^{AUT}$ was the case considered in the proof of proposition 3 to show that we can have $\bar{y}_t(\eta_t) < \hat{y}_t(\eta_t)$. In this, knife edge, case we have that $\hat{y}_t(\underline{\eta}) = \bar{y}_t(\underline{\eta})$, as can be seen from (22), since in this case its RHS is equal to 0.

on the efficient allocation. The exception being that when $\omega = 1$ no risk sharing contract is feasible and $c_t^o = c_t^{oD} = y_t$.

4.4 Dynamics, convergence, and numerical methods

In reputational models of sovereign debt with an infinitely lived representative agent it is well know that when $\delta R = 1$ the optimal constrained efficient allocation features per capita consumption growing over time with the economy eventually reaching constant per capita consumption.[26] This is done through the accumulation of foreign assets that relax the incentive constraint and thus serve as collateral to sustain perfect risk sharing. I will now show that with finite lives, and under mild assumptions, the promised utility to the old, η, and thus debt, converge to a non-degenerate ergodic distribution.

The policy function G specifies a consumption allocation and it induces a first order Markov process for η. This can be summarized in the transition function P on measurable space $(\tilde{X}, \tilde{\mathcal{X}})$, where $\tilde{\mathcal{X}}$ is the σ-algebra of $\tilde{X}$[27]

$$P(\eta, A) = Prob\{\eta_{t+1} \in A | \eta_t = \eta\} = \sum_{s=1}^{S} \pi_s \mathbf{1}_{\theta(h^t) \in A},$$

where $\mathbf{1}_{\theta(h^t)\in A} = 1$ if $\theta(h^t) \in A$ and 0 otherwise. Transition function P has an associated operator T^* that maps the sets of probability measures on $(\tilde{X}, \tilde{\mathcal{X}})$, $\mathcal{P}(\tilde{X}, \tilde{\mathcal{X}})$, into itself. Thus, if $f_t \in \mathcal{P}(\tilde{X}, \tilde{\mathcal{X}})$ is the probability measure on the state η_t in period t then the probability measure on the state in $t+1$, f_{t+1} is given by $T^* f_t$.

Assumption 2.

$$\min_{\eta} \left[\frac{\omega p}{\nu + p}\eta + \frac{(1-\omega)\nu}{\nu + p} \left(\frac{(1-\delta\nu)v(\nu + p) - p\eta}{\nu} + \delta p \underline{\eta}(v) \right) \right] > E[\underline{\mathrm{V}}(y)].$$

Given that the LHS in this relation is linear in η, we need only evaluate it at $\underline{\eta}(v)$ and $\bar{\eta}$. Importantly both sides of the condition depend on primitives and v. Furthermore if assumption 2 is satisfied for v^{AUT} it will be satisfied for all v. The next proposition implies that the time averages obtained from simulated trajectories correspond to state (ensemble) averages.

Proposition 5. Under assumption 2 there exists a unique probability measure $f^* \in \mathcal{P}(\tilde{X}, \tilde{\mathcal{X}})$ such that $f^* = T^* f^*$ and $T^{*n} f_0$ converges to f^* for all $f_0 \in \mathcal{P}(\tilde{X}, \tilde{\mathcal{X}})$.

Optimality conditions (15) and (16) show that the allocation following h^t can be summarized by the values of multipliers, μ, λ_{t-1}, and λ_t. Given that the participation

[26] See for example Worrall (1990), Thomas and Worrall (1994) or Aguiar et al. (2009).

[27] For simplicity I keep notation $\tilde{X}$ despite the fact that I showed that for the constrained efficient allocation the promise to the social planner is constant.

constraint shows that λ_t is determined by y_t and λ_{t-1}, and that μ is determined by the initial condition b_0 (as this determines v), we can study the evolution of the contract over time using λ_{t-1} as a state variable instead of η.[28] Propositions 1 and 2 imply that there exists well defined policy functions $c^y(v, \eta)$ and $c^o(v, \eta)$ (and $\theta(v, \eta)$). They also imply that $B'(\underline{\eta}) = 0$ and $\lim_{\eta \to 0} B'(\eta) = -\infty$ for preferences for which $\lim_{c \to \infty} u(c) = 0$ as will be the case for the baseline model.

When $v = v^{AUT}$ there exists an analytical characterization of $\underline{\eta}$ and estimation via value function iteration is straightforward. In particular, I use a projection method to approximate the value function that incorporates the theoretical properties $B'(\eta)$ described above. More precisely, I approximate B as the sum of a singular basis function

$$\phi(\eta; \alpha) \; = \; (-\eta)^\alpha - (-\underline{\eta})^\alpha - \alpha(-\underline{\eta})^{\alpha-1}(\eta - \underline{\eta}),$$

which captures, for $0 < \alpha < 1$, the theoretically required divergence $B'(\eta) \to -\infty$ as $\eta \to 0^-$, and a smooth remainder represented by a truncated Chebyshev expansion of degree N_{cheb}. The zero-slope condition $B'(\underline{\eta}) = 0$, is imposed analytically by eliminating one Chebyshev coefficient. The remaining free parameters are updated at each iteration by projecting the Bellman operator output onto the parametric family via nonlinear least squares, with damping to ensure convergence. An outer bisection loop over the resource floor $\bar{c}$ pins down the equilibrium by requiring the simulated average utility to equal the autarky value. The solution is robust to the choice of singularity exponent $\alpha \in \{0.05, 0.5, 0.95\}$, confirming that α is not identified from the fixed-point condition on the interior of the state space; I fix $\alpha = 0.5$ throughout. Full details of the algorithm and numerical parameters are reported in appendix 7.10.1.

An alternative numerical approach, and one that can be directly implemented when one has no analytical characterization of $\underline{\eta}$, is to solve the expectational difference equation given by the law of motion of λ_t. When the participation constraint is not binding we know $\lambda_t = 0$, when it is binding it must satisfy

$$\frac{\omega p}{\nu + p} u(c^o(y_t, \lambda_{t-1})) + \frac{(1-\omega)\nu}{\nu + p} \left(u(c^y(y_t, \lambda_{t-1}) + \delta p E_t[u(c^o(y_{t+1}, \lambda_t))] \right) = \underline{\mathrm{V}}(y_t). \quad (23)$$

Its solution, that in numerical applications is found by using projection methods to approximate the expectation term $E_t[u(c^o(y_{t+1}, \lambda_t))]$, is a function of λ_{t-1} and y_t, see appendix 7.10.2.[29] Thus there is no loss of generality in writing its solution as $\lambda(y_t, \lambda_{t-1})$, and since there is a monotonic relation between λ_{t-1} and η_t the former will inherit the ergodic properties of the latter such that numerical simulations are informative of the

[28] Recall that the strict concavity of B implies a one-to-one positive relation between λ_{t-1} and $-B'_\eta(v, \eta)$.

[29] It might also be a function of future expectations of y. But these are constant in the i.i.d. case and would all be determined by y_t if the endowment followed a Markov process.

long run properties of the economy. Simulations allow the construction of pseudo policy functions for c^y, c^o, and θ as functions of λ_{t-1} and b_0 (or v). From these it is direct to recover policy and value functions as functions of η and v.

As done in the value function iteration before, the algorithm to solve the expectational difference equation takes the minimum consumption of the young, $\underline{c}$, as a parameter and solves for the optimal contract. It then verifies if this satisfies that the contract either delivers utility v to the social planner or satisfies the resource constraint, (2), with initial debt level b_0. If not the parameter $\underline{c}$ is adjusted until convergence. To assess the goodness of fit of the solution, I exploit an independent characterization of $\underline{\eta}(v)$ that is not imposed in the algorithm.

Anticipating results from the next section, when there is an analytical characterization of $\underline{\eta}$, the planner's value function $B(\eta)$, and the initial levels of debt, obtained numerically using value function iteration and solving the expectational difference equation are essentially identical.

5 Quantitative implications

To evaluate the quantitative implications of the model, I begin by setting one period equal to ten years. This choice is consistent with Tomz and Wright (2007), who document that, in a sample of 176 countries over almost two centuries, the average duration of default episodes is 9.9 years. For the endowment process, I use GDP data from the World Development Indicators for a set of large emerging economies: Argentina, Bangladesh, Brazil, Chile, Colombia, Egypt, India, Indonesia, Malaysia, Mexico, Nigeria, Pakistan, the Philippines, South Africa, Thailand, and Turkey. The sample covers the period 1960-2020. For each country, I compute decade-level log deviations from a log-linear trend. To assess persistence, I estimate AR(1) regressions with country fixed effects. In only one case (Pakistan) is the lag coefficient statistically significant, and it is negative (Bangladesh is borderline significant with a positive coefficient). These results support the assumption that the process can be treated as i.i.d.

Pooling the data, both Kolmogorov-Smirnov and Shapiro-Wilk tests indicate that the distribution of shocks is well approximated by a Normal. I therefore discretize the state space from the fitted Normal distribution using Gauss-Hermite quadrature nodes and probabilities. In the baseline, I employ six nodes, which implies a probability of close to 0.5 percent for the most extreme realizations, and I simulate sequences of one million shocks.[30]

Preferences are assumed to be CRRA with risk aversion parameter σ. To solve the expectational difference equation for λ_t, (23), I use standard projection method to ap-

[30] The sample mean is –0.02982 and the standard deviation is 0.14192. In the simulations I set the mean to zero such that average consumption is normalized to one in the first best.

Table 2: Model simulations

	$\sigma = 2$		$\sigma = 1$	
	max debt	zero debt	max debt	zero debt
$\frac{b_0}{E[y]\frac{1-r^y}{1-1/R}}$	0.418	0	0.200	0
$\text{std}(c_t^y)$	0.109	0.096	0.106	0.101
$\text{std}(c_t^o)$	0.110	0.105	0.116	0.113
$\text{corr}(c_t^y, c_t^o)$	0.606	0.630	0.606	0.616
$\text{Prob}(\lambda_t > 0)$	0.806	0.707	0.769	0.713
$\text{Prob}(c_t^o < \frac{y_t}{2} \mid \lambda_t > 0)$	0.492	0.604	0.517	0.570
$\frac{CEV}{E[y]/2}$	0.973	0.985	0.984	0.989
$\frac{CEV^{AUT}}{E[y]/2}$	0.978	0.978	0.989	0.989

proximate for $E_t[u(c^o(y_{t+1}, \lambda_t))]$ using Chebyshev polynomials of degree four on both state variables, λ_{t-1} and y_t, see details in appendix 7.10.2. In the baseline calibration, I set $\sigma = 2$, $\omega = \frac{1}{2}$, $\nu = p = 1$, and $\delta = \frac{1}{1.02^{10}}$ which corresponds to a yearly interest rates in international capital markets of $r^y = 0.02$ under the maintained assumption that $\delta R = 1$. Results are reported in table 2. Even without output costs of default, the model generates a maximum debt-to-GDP ratio of about 4.6% over a decade, equivalent to roughly 41.8% of annual GDP. This result is obtained by calibrating $\underline{c}$ such that utility under the contract equals that in autarky. In this extreme scenario, the participation constraint binds with probability 80.6% and in 49.2% of these cases it is the old that want to default.

The ex-ante expected utility of any generation falls *below* its autarky level, corresponding to a consumption-equivalent loss of about 0.5 percent. The intuition is that, under the efficient contract, default is avoided as long as the *average* utility of generations alive exceeds the autarky benchmark (proposition 4). Incentives are delivered by promising higher consumption later in life, which generates an increasing consumption profile across the life cycle. While the planner values this profile based on aggregate utilities without discounting future consumption, individuals *discount* future utility when making their own comparisons. As a result, individual welfare is on average lower than in autarky, even though this need not hold from the planner's perspective. This "immiseration" property of the optimal contracts is what allows the model to sustain high levels of borrowing even in the absence of output costs.[31]

When the minimum consumption floor is raised by about 5.4 percent, debt falls to zero. In this case, the average generation experiences a welfare gain from risk sharing, equivalent to roughly 0.7 percent of consumption. With no initial debt, the probability that the participation constraint binds declines to 70.7%, and conditional on binding, the

[31] Simulations of a modified model in which there are two incentive constraints, one for the young and one for the old, show that the maximum debt is significantly lower, at around 20% of GDP.

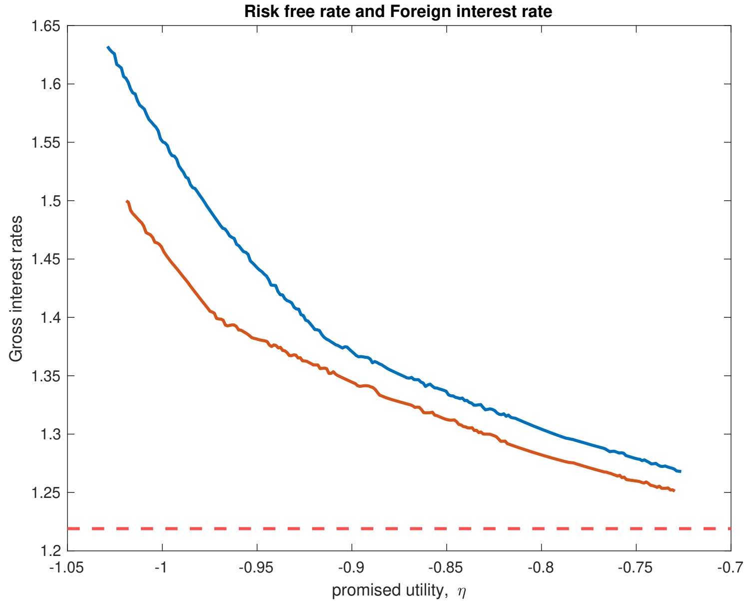


Figure 2: Interest rates - high (zero) debt in blue (orange)

probability that the old favor default rises to 60.4%. In both the maximum debt and zero-debt scenarios, the volatility of consumption is about two thirds that of output. The correlation between the consumption of young and old declines to 60.6-63%, compared with perfect correlation in autarky. Solving the model with logarithmic preferences yields a maximum debt level of about 20% of annual GDP, with the incentive constraint binding at similar frequencies as in the baseline case. Similar magnitude of welfare losses for high debt, but negligible welfare gains for zero debt.

When debt is high, and $\eta_t < -0.932$ ($\lambda_{t-1} < 0.712$), which happens with probability 51%, the participation constraint binds when $y_t > y_2$, which happens with probability 89.5%. And when $\eta_t \geq -0.932$ the participation constraint only binds when $y_t > y_3$, which happens with probability 50%. Thus, as stated in section 4.2 the incentives to default depend not only on current output, but on past output through the effect this had on contractual promises. Similar results hold when there is no initial debt. In this case the threshold value of η_t is -0.979 (0.287 for λ_{t-1}) and the probability that $\eta_t < -0.979$ is 69%. With lower initial debt the contract specifies a higher level of minimum consumption making it less likely that the incentive constraint is binding.

Figure 2 reports the domestic risk-free interest rate implied by the model, which from (16) is given by $\left(\delta E_t\left[\frac{u'(c^o_{t+1})}{u'(c^y_t)}\right]\right)^{-1}$. When η_{t+1} (equivalently λ_t), and thus c^y_t, is low (corresponding to "increases" in debt), the domestic rate is high; the same pattern emerges in the zero-debt case. Thus, given that η_{t+1} (λ_t) is positively correlated with y_t, domestic interest rates are countercyclical, as in the data. These interest rate movements are not driven by changes in default premia, as could be the case if markets were incomplete,

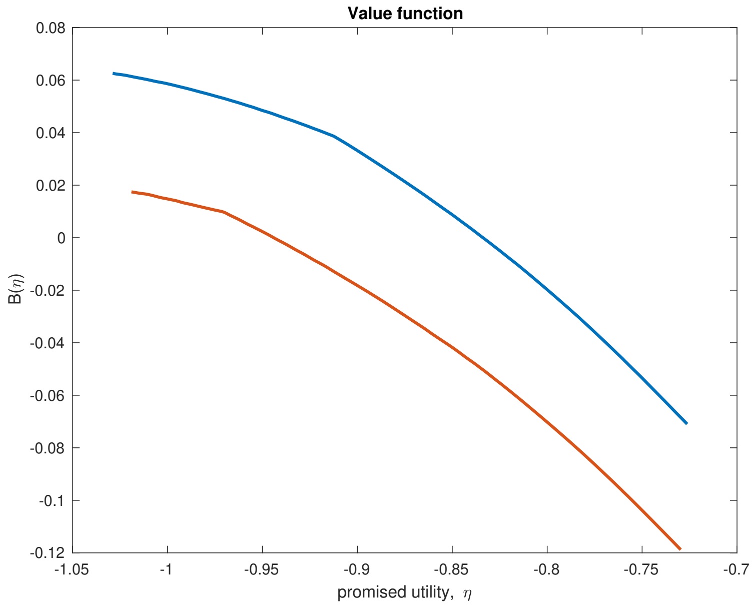
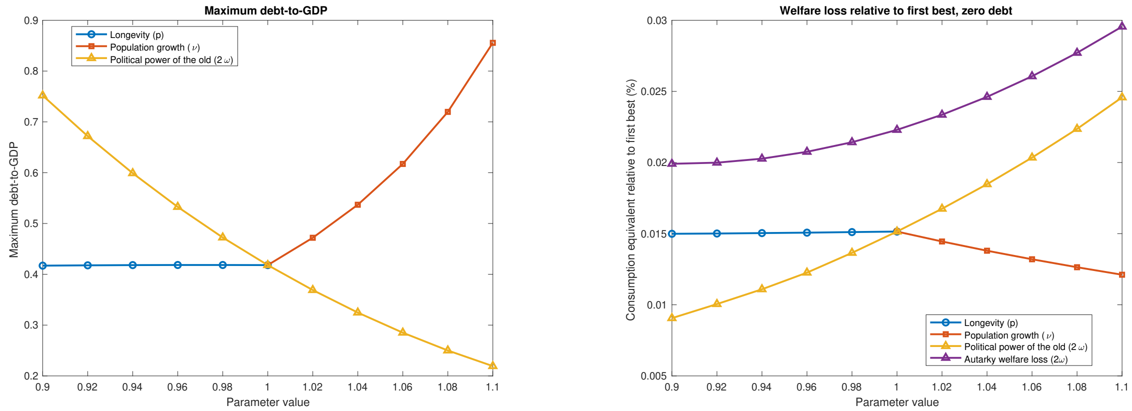


Figure 3: Lenders' value function - high (zero) debt in blue (orange)

see e.g. Arellano (2008). The reason for this correlation is that when η_{t+1} (λ_t) is low, the young expect to receive higher consumption when old. As η_{t+1} (λ_t) increases, raising consumption for the young, expected consumption growth declines.

Figure 3 shows the value functions for maximum and zero debt as a function of η_t. As expected, foreign lenders' profits are higher when they had initially lent resources to the country. The fact that B is negative when η_t is high (and thus debt was "reduced" in the previous period) reflects that, when default incentives were strong, the young had been promised a high consumption floor in old age, leading lenders to, on average, transfer resources to the country in the current period.

Regarding the validity of the numerical simulations I compare the value of $\underline{\eta}(v)$ (which only depends on primitives for $v = v^{AUT}$) with the minimum value of $\eta = E_t[\underline{u}(c^o(h^{t+1}))]$ in the numerical simulations. For both maximum and zero debt the difference is less than 2%. Next I consider the effect of changes in population growth, longevity and political power of the old on maximum debt and the welfare gains when there is no debt, see figures 4a and 4b. I find that longevity has negligible effects for borrowing or welfare gains. As expected, population growth increases borrowing capacity as the country can pledge larger amounts of future resources. Population growth also increases the welfare gains (the figure shows welfare loses relative to first best, population growth reduces them). In the figure ω increases from 0.45 to 0.55. Maximum debt and welfare gains are steeply decreasing in ω. Welfare loses relative to first best are also shown for the autarky allocation, showing that the contract welfare gain relative to autarky decreases with ω.



(a) Maximum debt-to-GDP

(b) Welfare cost relative to first best

Figure 4: Comparative statics with respect to longevity $p \in [0.9, 1]$, population growth $\nu \in [1, 1.1]$, and political power of the old $2\omega \in [0.9, 1.1]$.

6 Conclusions

Survey evidence and recent empirical studies challenge the predictions of representative agent models of sovereign risk. Surveys conducted during sovereign debt distress indicate that older individuals are more likely to favor repayment than younger cohorts—a pattern confirmed by a survey I commissioned in Argentina during a critical recent presidential election. Meanwhile, empirical studies show a weaker correlation between default and either output or debt than standard models would predict. This raises important questions about how repayment preferences are distributed across a country's residents and how they ultimately shape policy.

To address these gaps, this paper incorporates an overlapping-generations structure into the standard limited-commitment framework. I characterized the efficient consumption allocations from the perspective of a social planner, that faces the constraint that policy can be challenged by period governments representing the interest of generations alive.

Results show that, as in representative agent models, risk-sharing remains constrained because the economy has incentives to deviate when output is high. Like these models consumption is used to provide incentives in the past, but the fact that this only works for one period for each generation prevents the country from accumulating assets to achieve the first best. Thus, default incentives fluctuate over time—even when shocks are i.i.d.—since high past output reduces the temptation to deviate in the present. Notably, this relationship does not always stem from lower outstanding debt, as there is no monotone relation between state-contingent debt payments and the intensity of default incentives.

Unlike infinite-horizon representative agent models, this framework allows for default even when a country is a net recipient of capital flows. This helps explain the weak empirical relationship between default and output and the fact that some countries default at relatively low debt-to-GDP ratios. Furthermore, the model provides insight into the mentioned survey evidence on generational preferences: The elderly are more likely to support repayment when they are politically weak, when past policy has promised them a high level of current consumption, or when the country has a higher level of initial debt.

When calibrated to match GDP per capita fluctuations over a ten-year horizon, the model predicts higher debt capacity than standard representative agent models, even when both generations hold veto power. Debt capacity rises to around 42% of GDP in large part because the planner is willing to enter into a contract that reduces generational welfare. Importantly, these results emerge for low levels of risk aversion and under the assumption that the only threat that sustains sovereign debt is the exclusion from international capital markets.

Future research should extend this model to a production economy to establish the growth costs of sovereign risk and debt overhang. Additionally, an important avenue for exploration is how risk-sharing changes if markets are incomplete, restricting sovereigns to non-contingent debt contracts with foreign lenders.

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7 Appendix

7.1 Argentina 2019 survey

I hired an Argentina external consultant, Seido, that at the time used a series of online tools (email campaigns, and social media) to run a short survey a few weeks prior to the Argentine presidential election of October 2019. Respondents were encouraged to participate through a monetary reward using a lottery among participants. Participants were made aware of the academic nature of this survey, and told that its objective was to learn about the relationship between politics and economics in the Argentine context. Thus, while they were not fully aware that we wanted to elicit their default preferences there was no deception in the survey design. Any person over 18 years of age residing in Argentina could participate in the online poll. They only had to be a fan of Seido's Facebook page. About a month after the survey was finished a lottery was held among legitimate participants (the consultant eliminated those answers deemed to come from either bots or from accounts that seemed to be created with the sole purpose of participating in this survey), and a monetary prize was awarded to some respondents to be paid by bank transfer (only winners had to provide the consultant with this information). The total amount to be distributed among the winners was 10.000 Argentine pesos, and individual winners received around eight and a half U.S. dollars

Data was collected using a questionnaire with 26 questions that seeked to elicit preferences for repayment or default of public debt, their age, and other determinants such as political affiliation, education, employment status that might be correlated with this preference. Given that previous studies have shown that political affiliation is correlated with preferences for default (e.g. with left-leaning voters favoring default more than right-leaning voters), I need to control for their political preferences. Thus, the survey asked them how much they care about politics, what was their political conviction, which party they intended to vote in the presidential election, and whether they viewed themselves as "nationalists" or "liberals". Other studies have suggested that income is correlated with preferences for default, but I was told by the consultant that respondents did not like to be asked about their income, so information is elicited by asking them about their savings behavior (if they have saved in the past year, and if they bought foreign currency), whether they have debts, and how their financial condition changed in the last months.

Use of personal data in research was approved by the Faculty of Social Sciences, University of Copenhagen - 1879090/4242.[32] A risk assessment was carried out and impact assessment deemed not relevant according to the criteria that would prompt such an

[32] At the time the survey was conducted I was working at the University of Copenhagen. At the time the data collection was conducted, there was no institutional review board (IRB) at the Department of Economics. In 2020 a Research Ethics Committee at the level of the Department of Economics was created and evaluated that the project activities were in accordance with the relevant International and Danish ethical guidelines and regulations. Approval was granted May 31, 2021.

assessment.

The start and end time of responses was also recorded, and 1984 answers were received by the end of the survey.

Survey questionnaire

Survey on Politics and Economics

You are invited to participate in a survey conducted for academic purposes by a researcher from the University of Copenhagen, Denmark. The aim of this survey is to understand the relationship between politics and economics in the current Argentine context. Participation is voluntary, and you are free to leave the survey at any time. The survey should take no more than 10 minutes to complete.

The survey will be available from October 4 to October 18. The draw will take place on October 21 among the participants. The winners will receive an email and will have one week to claim their prize. If the winner(s) do not respond within the specified timeframe after being notified, the prize will be awarded to the next person. Participation in the draw is free. Any natural person over the age of 18 residing in the Republic of Argentina and who is a fan of the "Encuestas Online" Facebook page may participate. Only one valid response per person. The individual information collected in the survey is confidential and will not be sold or shared.

Survey link:

1. Gender (1=Male, 2=Female, 3=Prefer not to answer).

2. Age.

3. Which category best describes your level of education? (1=Incomplete primary, 2=Incomplete secondary, 3=Complete secondary, 4=Incomplete university or technical studies, 5=Complete university or technical studies, 6=Postgraduate studies).

4. What is your employment status? (1=Full-time employee, 2=Part-time employee, 3=Self-employed, 4=Unemployed and looking for work, 5=Student and not working, 6=Retired or full-time parent).

5. Where were you born? (1=Argentina, 2=Another Latin American country, 3=Another country).

6. In which province do you live?

7. Which best describes your place of residence? (1=Lives in a city, 2=Lives in an urban suburb, 3=Lives in a rural area).

8. Are you a public sector employee? (1=Yes, 2=No).

9. Are you a beneficiary of a government social program (e.g., AUH, PNC, complementary social salary, Hacemos Futuro)? (1=Yes, 2=No).

10. Would you say that most people can be trusted, or that one must be very careful when dealing with others? (0=No trust, to 10).

11. How interested are you in politics? (1=None, 2=Little, 3=Quite a bit, 4=A lot).

12. Which political party do you identify with the most? (1=Frente de Todos, 2=Juntos por el Cambio, 3=Consenso Federal, 4=Frente de Izquierda Unida, 5=Frente NOS, 6=Unite por la Libertad y la Dignidad, 7=NS/NC).

13. In politics, people often talk about "left" and "right." Where would you place yourself on a scale from 0 to 10, where '0' represents the extreme left and '10' the extreme right?

14. Politics also classifies preferences between "nationalism," understood as the prioritization of national interests, and "liberalism," understood as the prioritization of freedom and equality before the law. Where would you place yourself on a scale from 0 to 10, where '0' represents extreme nationalism and 10' extreme liberalism?

15. Do you agree that Mauricio Macri is doing a reasonable job as president of the country? (1=Strongly agree, 2=Somewhat agree, 3=Neither agree nor disagree, 4=Somewhat disagree, 5=Strongly disagree, 6=NS/NC).

16. Do you have savings in a bank or investments abroad? (1=Yes, 2=Yes but not in a bank, 3=No but make ends meet, 4=No and has cut back on spending, 5=NS/NC).

17. In the past 12 months, have you bought U.S. dollars or another foreign currency for savings? (1=Yes, 2=No).

18. Do you have debt with a bank and/or on your credit card? (1=Yes, 2=No).

19. In the past 12 months, has your economic situation... (1=Worsened a lot, 2=Worsened somewhat, 3=Stayed the same, 4=Improved somewhat, 5=Improved a lot, 6=NS/NC).

20. It is important that Argentina retains its ability to decide its own policies. Do you agree? (1=Strongly agree, 2=Somewhat agree, 3=Neither agree nor disagree, 4=Somewhat disagree, 5=Strongly disagree, 6=NS/NC).

21. Some say that globalization, understood as a general opening of countries to world markets, is good. Do you agree? (1=Strongly agree, 2=Somewhat agree, 3=Neither agree nor disagree, 4=Somewhat disagree, 5=Strongly disagree, 6=NS/NC).

22. Given the country's economic problems, Argentina should stop paying its public debt. Do you agree? (1=Strongly agree, 2=Somewhat agree, 3=Neither agree nor disagree, 4=Somewhat disagree, 5=Strongly disagree, 6=NS/NC).

23. Given the country's economic problems, Argentina should stop paying the International Monetary Fund. Do you agree? (1=Strongly agree, 2=Somewhat agree, 3=Neither agree nor disagree, 4=Somewhat disagree, 5=Strongly disagree, 6=NS/NC).

24. Regarding the future, how afraid are you that the economic situation will worsen next year? (1=Very afraid, 2=Somewhat afraid, 3=A little afraid, 4=Not afraid at all, 5=NS/NC).

25. A preliminary agreement between the European Union and Mercosur was recently announced. Do you think this will be good for the country? (1=Strongly agree, 2=Somewhat agree, 3=Neither agree nor disagree, 4=Somewhat disagree, 5=Strongly disagree, 6=NS/NC).

26. Given the country's economic problems, Argentina should reduce pensions or raise the retirement age. Do you agree? (1=Strongly agree, 2=Somewhat agree, 3=Neither agree nor disagree, 4=Somewhat disagree, 5=Strongly disagree, 6=NS/NC).

The data shows unbalanced responses by age, see figure 5. The consultant provided me with a weight which stratifies the responses by age, education and gender. This is used in the weighted regressions reported in the paper.

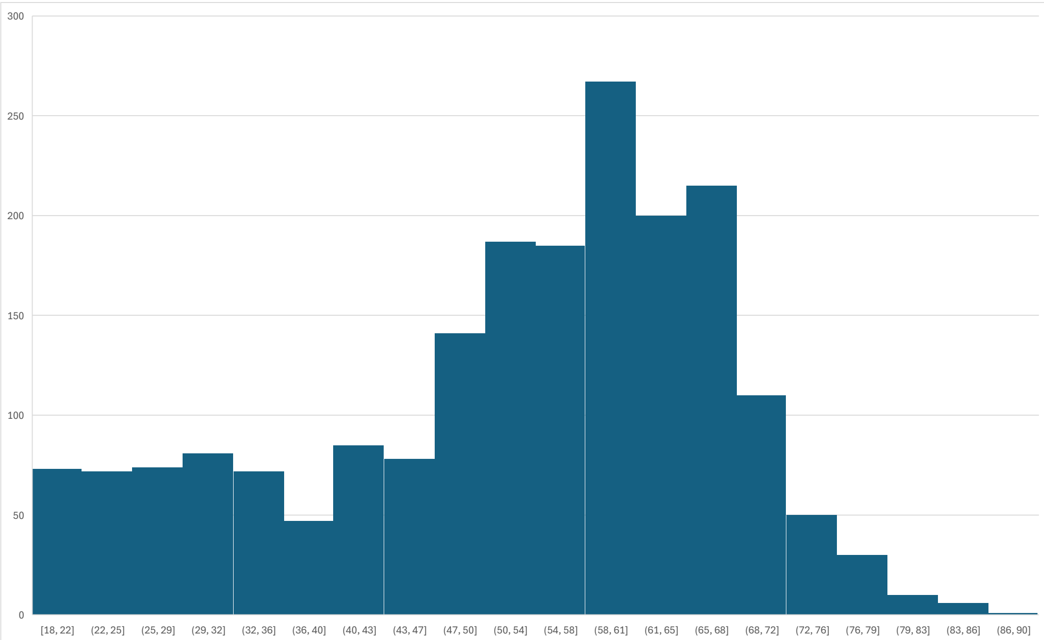

Figure 5: Histogram of Respondents' Age Distribution

7.2 Further survey evidence on repayment preferences by age

7.2.1 Iceland 2011

Iceland was severely affected by the Great Recession, with 85% of its banking sector collapsing. The government nationalized the largest banks, including Landsbanki, which had been funding operations through internet-based savings accounts called "Icesave" from European customers. When the bank could not repay depositors, the British and Dutch governments stepped in to provide compensation, later demanding reimbursement from Iceland. While Iceland accepted this in principle, it held two referendums on whether to accept the repayment terms. Only the second referendum, held in 2011, is informative, as a better offer was available during the first referendum in 2010.

Curtis et al. (2014) conducted a survey shortly after the second referendum on April 9, 2011. Although voters were not directly deciding whether to repay the sovereign debt but rather the timing and terms, their responses on the repayment plan revealed preferences

Table 3: Preference for repayment in Iceland's 2011 referendum

	(1)	(2)	(3)
age	0.0024**	0.003**	
	(0.0012)	(0.0012)	
old dummy			0.098***
			(0.0508)
R^2	0.269	0.262	0.259
N	628	630	630

Source: Curtis et al. (2014)

for repayment. The authors found that self-interest, along with partisan, ideological, educational, and sociotropic effects, influenced voting behavior. Their data shows that 39% of voters aged 18–24 supported repayment, while nearly 50% of those aged 65 and above did.

Curtis et al. (2014) do not control for age in their regressions.[33] I perform four regressions, with results reported in Table 3. Column (1) reports the results of the regression model in Curtis et al. (2014) including age.[34] The age coefficient is positive and statistically significant. To ensure robustness, I apply a Lasso-type procedure, see Belloni et al. (2014), which confirms a positive and significant age effect (Column 2).[35] In Column (3), I replace age with a dummy for older individuals.

As with Argentina, disentangling age and cohort effects is challenging. To check for cohort effects, I regress attitudes towards foreigners and national identity on age.[36] The elderly are more likely to identify as Icelandic and express colder attitudes toward foreign countries, which aligns with European data showing older individuals are less cosmopolitan. Thus, for preferences for repayment to be driven by cohort effects, these would have to be beliefs that make older Icelanders to be more insular but at the same time more adamant on breaking an obligation with foreign governments.

[33] In footnote 47 the authors claim age to be highly collinear with both education and investment behavior. Using the publicly available data I found these correlations to be -0.16 and 0.18 respectively.

[34] Specifically all the controls of the first specification plus age and "prime minister approval" which is used as an alternative measure of domestic political cues. Similar results hold if the latter variable is omitted.

[35] Not surprisingly the explanatory variables that the procedure select are similar to those employed in the regression in Curtis et al. (2014). The fact that "prime minister approval" was selected *alongside* the other measures of domestic political cues was what motivated me to use it in the regression reported in column (1).

[36] The first variable captures respondents' ratings toward the UK, Netherlands and India and the latter is a variable based on whether respondents proclaim themselves European or Icelandic.

7.2.2 Argentina 2002

Between March 1991 and January 2002, Argentina operated under a currency board, which limited its ability to use monetary policy as a stabilization tool, making it vulnerable to supply shocks. In the late 1990s, a series of negative shocks worsened the situation: the US dollar, the currency of Argentina's peg, was very strong, Brazil, its main trading partner, devalued in January 1999, and US interest rates remained high until the dot-com bubble burst in 2001. These factors led to three years of declining production and employment, with citizens blaming the weak government coalition of Fernando De La Rúa. In October 2000, the breakdown of the governing coalition caused a sharp rise in country risk measures, and Argentina faced a near-total shutdown of capital markets.

The IMF offered a rescue package, but it proved inadequate given the government's fragile political position. After violent street riots in December 2001, De La Rúa resigned, and, after two weeks of turmoil, Congress appointed Eduardo Duhalde as interim president. Duhalde devalued the currency and suspended nearly USD 100 billion in foreign debt payments, resulting in the largest sovereign default in history.

To study how adjustment costs and reputational benefits influenced repayment preferences in Argentina in 2002, Michael Tomz surveyed 442 Buenos Aires voters in July 2002, six months after the default, see Tomz (2004). A limitation of his sample is its concentration in Buenos Aires, which has higher income and education levels than other parts of the country. Tomz (2004) controlled for various factors, including income and education proxies, employment status, and nationalism.[37] He also included age as a control, and given the other variables, it is likely that age's coefficient captures the pure effect of interest. In all regressions of repayment preferences, age was positively correlated and statistically significant.

7.3 Probabilistic voting

The proposed utilitarian functions postulated for period governments, (3), can be rationalized if policy preferences are aggregated via probabilistic voting. Assume that the young and old alive in period t, in shares $\frac{\nu}{\nu+p}$ and $\frac{p}{\nu+p}$ respectively, vote on candidates representing competing platforms. Voters support a candidate not only for the private utility derived from her policy platform, but also for other characteristics like "ideology" that are orthogonal to the economic policy dimensions of interest. These characteristics are permanent and cannot be credibly altered in the course of electoral competition; their valuation differs across voters (even if voters agree about the preferred policy platform) and is subject to random aggregate shocks, realized after candidates have chosen their platforms. This setup thus renders the probability of winning a voter's support a

[37] More specifically, wealth is proxied by the number and kind of cars, goods and services in the households. Although there is no variable for education attainment respondents were asked a number of questions to determine their degree of "economic knowledge". See Tomz (2004).

continuous function of the competing policy platforms.

In a Nash equilibrium with two candidates maximizing their expected vote share, both candidates propose the same policy platform.[38] This platform maximizes a convex combination of the objective functions of young and old voters, where the weights reflect the groups' sensitivity of voting behavior to policy changes. Groups that care a lot about policy platforms relative to the candidates' other characteristics have more political influence as they are more likely to shift their support from one candidate to the other in response to small changes in the proposed platform. In equilibrium, these groups of "swing voters" thus tilt policy in their own favor. Letting ω and $1-\omega$ denote the per-capita political influence of old and young, respectively, the objective function of candidates for period t government is precisely (3).

7.4 Autarky allocation

When a country defaults and is permanently excluded from capital markets the autarky allocation is given by the solution to the following equations:[39]

$$\begin{aligned} \omega u'(c^{oD}(y_t)) &= (1-\omega)u'(c^{yD}(y_t)), \qquad (24)\\ \frac{\nu}{\nu+p}c^{yD}(y_t) + \frac{p}{\nu+p}c^{oD}(y_t) &= y_t. \end{aligned}$$

In the absence of distribution shocks, the planner cannot improve on $v^{AUT} = \frac{E[\nu u(c^{yD}(y_t))+pu(c^{oD}(y_t))]}{(\nu+p)(1-\delta\nu)}$. (Lancia et al., 2024). I use notation $\underline{V}(y_t) \equiv \frac{\omega p}{\nu+p}u(c^{oD}(y_t)) + \frac{(1-\omega)\nu}{\nu+p}\left(u(c^{yD}(y_t)) + \delta p E[u(c^{oD}(y_t))]\right)$ as the autarky political value.

7.5 Proof of proposition 1

Denote by $Y = [\underline{c}, y_S]^{2S} \times X^S$ the action space, and $a = (c^y(\cdot), c^o(\cdot), w(\cdot), \theta(\cdot)) \in Y$ a possible choice. Rewriting the Bellman equation

$$\begin{aligned} B(v,\eta) &= \max_{(c^y(\cdot),c^o(\cdot),w(\cdot),\theta(\cdot))\in Y} E_t\left[y(h^t) - \frac{\nu}{\nu+p}c^y(h^t) - \frac{p}{\nu+p}c^o(h^t) + \delta\nu B(w(h^t),\theta(h^t))\right]\\ \text{s.t.}\quad v &\leq E_t\left[\frac{p}{\nu+p}u(c^o(h^t)) + \frac{\nu}{\nu+p}u(c^y(h^t)) + \delta\nu w(h^t)\right] \equiv \psi_1(a) \qquad (25)\\ \eta &\leq E_t\left[u(c^o(h^t))\right] \equiv \psi_2(a) \qquad (26)\\ \underline{V}(y_s) &\leq \frac{\omega p}{\nu+p}u(c^o(h^t)) + \frac{(1-\omega)\nu}{\nu+p}\left(u(c^y(h^t)) + \delta p\theta(h^t)\right) \equiv \psi_3(a;y_s)\ \forall h^t = (v,\eta,y_s) \qquad (27) \end{aligned}$$

[38] See Persson and Tabellini (2000) for a textbook discussion on probabilistic voting.

[39] Absent interactions between period governments, these are bound to choose the static autarky optimal consumption allocation. Given that shocks are i.i.d. expectations are unconditional.

The correspondence $\Gamma : X \to Y$ is then given by

$$\Gamma(v, \eta) = \{a \in Y : \psi_1(a) \geq v; \psi_2(a) \geq \eta; \psi_{3,s}(a) \geq \underline{V}(y_s) \ \forall y_s\},$$

and its graph A is given by the set

$$A = \{((v, \eta), a) : a \in \Gamma(v, \eta)\}.$$

Define the Bellman operator T as

$$(TB)(v, \eta) = \max_{a \in \Gamma(v,\eta)} E_t \left[y(h^t) - \frac{\nu}{\nu + p} c^y(h^t) - \frac{p}{\nu + p} c^o(h^t) + \delta \nu B(w(h^t), \theta(h^t)) \right]$$

The proof relies on theorem 9.6 of Stokey et al. (1989). For this we verify assumptions 9.4-9.7. First, by assumption X is compact and the product of convex sets, thus it is a closed and convex subset of $\mathbb{R}^2$ with its Borel subsets. Second, the set of exogenous states is countable (since shocks are i.i.d. and enter only through expectations, the value function does not depend on the current shock; it is the action a that is a vector contingent on the shock realization). Third, the correspondence $\Gamma : X \to Y$ is clearly nonempty as for each (v, η) there exists at least one allocation/promise plan satisfying the constraints.[40] Since Y is compact, for Γ to be compact it remains to be shown that $\Gamma(v, \eta)$ is closed in Y for each $(v, \eta) \in X$. Let $a_n \in \Gamma(v, \eta)$ and $a_n \to a$. Because $u(\cdot)$ is continuous and the constraints are finite sums of continuous functions in a, the right-hand sides of the promise-keeping constraints (25) and (26), and of the incentive constraint (27) are continuous in a. Therefore each inequality remains true in the limit a. Thus $a \in \Gamma(v, \eta)$ and hence $\Gamma(v, \eta)$ is closed in Y.

To show continuity I prove first that Γ is upper hemi-continuous using theorem 3.4 of Stokey et al. (1989). Given what was proved above this only requires to show that A is sequentially closed (as the action space Y is compact and thus images of bounded sets are bounded). Take any sequence $\{((v_n, \eta_n), a_n)\}_{n \geq 1} \subset A$ such that $(v_n, \eta_n) \to (v, \eta) \in X$ and $a_n \to a \in Y$. Since $((v_n, \eta_n), a_n) \in A$ for all n, we have

$$v_n \leq \Psi_1(a_n), \qquad \eta_n \leq \Psi_2(a_n), \qquad \underline{V}(y_s) \leq \Psi_{3,s}(a_n) \quad \forall s \in S.$$

Because the shock space S is finite and the functions $u(\cdot)$ and $\underline{V}(\cdot)$ are continuous on the relevant bounded domains, the functions Ψ_1, Ψ_2, and $\Psi_{3,s}$ are continuous in a. Therefore,

$$\Psi_1(a_n) \to \Psi_1(a), \qquad \Psi_2(a_n) \to \Psi_2(a), \qquad \Psi_{3,s}(a_n) \to \Psi_{3,s}(a) \quad \forall s \in S.$$

[40] Note that $\bar{a}$ given by $c^y = c^o = y_S$, $\theta = \bar{\eta}$ and $w = \bar{v}$ satisfies $\bar{a} \in \Gamma(v, \eta)$ for all $(v, \eta) \in X$.

Taking limits in the inequalities above yields

$$v \leq \Psi_1(a), \qquad \eta \leq \Psi_2(a), \qquad \underline{V}(y_s) \leq \Psi_{3,s}(a) \quad \forall s \in S.$$

Since Y is closed and $a_n \in Y$ for all n, it follows that $a \in Y$. Thus $a \in \Gamma(v, \eta)$, and hence $((v, \eta), a) \in A$. This proves that A is closed.

To prove that Γ is lower hemi-continuous I use theorem 3.5 in Stokey et al. (1989). For this we first show that A is convex. Take arbitrary points $((v_1, \eta_1), a_1) \in A$ and $((v_2, \eta_2), a_2) \in A$ and let $\lambda \in [0, 1]$. Define $(v_\lambda, \eta_\lambda) := \lambda(v_1, \eta_1) + (1 - \lambda)(v_2, \eta_2)$, $a_\lambda := \lambda a_1 + (1-\lambda)a_2$. We must show $((v_\lambda, \eta_\lambda), a_\lambda) \in A$, i.e. $a_\lambda \in \Gamma(v_\lambda, \eta_\lambda)$. Because Y is convex and $a_1, a_2 \in Y$, we have $a_\lambda \in Y$. Regarding Γ given that $u(\cdot)$ is concave and expectation is linear, all three functions $\psi_1(a)$, $\psi_2(a)$, $\psi_{3,s}(a)$ are concave (as they are finite sums of terms that are concave or linear in a). Thus, for $i = 1, 2$, $\psi_i(a_\lambda) \geq \lambda\psi_i(a_1) + (1-\lambda)\psi_i(a_2)$. Since $a_i \in \Gamma(v_i, \eta_i)$ this implies $\psi_1(a_\lambda) \geq v_\lambda$ and $\psi_2(a_\lambda) \geq \eta_\lambda$. For $\psi_{3,s}(s)$ we have that its superlevel sets $\{a \in Y : \ \Psi_{3,s}(a) \geq c\}$ are convex for any constant c. As $\underline{V}(y_s)$ is a fixed constant for each s, and since a_i satisfy $\underline{V}(y_s) \leq \Psi_{3,s}(a_i)$ for $i = 1, 2$, we obtain by concavity $\Psi_{3,s}(a_\lambda) \geq \lambda\Psi_{3,s}(a_1) + (1 - \lambda)\Psi_{3,s}(a_2) \geq V(y_s)$, so $\underline{V}(y_s) \leq \Psi_{3,s}(a_\lambda)$ for every $s \in S$. Thus, $a_\lambda \in \Gamma(v_\lambda, \eta_\lambda)$, hence $((v_\lambda, \eta_\lambda), a_\lambda) \in A$. Because the two points and $\lambda \in [0, 1]$ were arbitrary, the graph A is convex.

The remaining conditions of theorem 3.5 in Stokey et al. (1989) follow since both X and Y are compact and thus bounded. Take any $x \in X$, since Γ is non empty and $\Gamma(x) \subseteq Y$ then $\Gamma(x) \cap Y = \Gamma(x) \neq \emptyset$ for all $x \in X$. Take $\epsilon > 0$ and define $Bc(x, \epsilon) = \{x' \in \mathbb{R}^2 : \|x' - x\| \leq \epsilon\}$. Since X is closed and convex its intersection with closed ball Bc is closed and convex. Since Γ is both upper and lower hemicontinuous it is continuous.

Finally it is assumed that $0 < \delta\nu < 1$ and $E_t\left[y(h^t) - \frac{\nu}{\nu+p}c^y(h^t) - \frac{p}{\nu+p}c^o(h^t)\right]$ as a mapping from $X \times Y$ to $\mathbb{R}$ is bounded and continuous since Y is bounded and S finite. Thus, all assumptions for theorem 9.6 of Stokey et al. (1989) are satisfied. And the Bellman operator T that maps bounded functions into bounded functions satisfies Blackwell's sufficient conditions (monotonicity and discounting) for a contraction.[41] This completes the proof of proposition 1.

7.6 Proof of proposition 2

To prove that the value function is decreasing in $(v, \eta) \in \tilde{X}$ I first show that there is monotonicity of feasible sets. If $(v_1, \eta_1) \geq (v_2, \eta_2)$ then it is straightforward that $\Gamma(v_1, \eta_1) \subseteq \Gamma(v_2, \eta_2)$ as an increase in promised values reduces the set of feasible actions given that the promise keeping constraints are binding. Given that the objec-

[41] Clearly if B_1 and B_2 are bounded functions with $B_1(v, \eta) \geq B_2(v, \eta)$ then $TB_1(v, \eta) \geq TB_2(v, \eta)$ for all $(v, \eta) \in X$. and $T(B + a)(v, \eta) \leq TB(v, \eta) + \delta\nu a$ for all bounded B, $(v, \eta) \in X$, and $a \geq 0$.

tive $E_t\left[y(h^t) - \frac{\nu}{\nu+p}c^y(h^t) - \frac{p}{\nu+p}c^o(h^t)\right]$ does not depend on (v,η) then we conclude that $B(v,\eta)$ is (weakly) decreasing in it arguments.

In the proof of proposition 1 it was established that A is convex. But $E_t\left[y(h^t) - \frac{\nu}{\nu+p}c^y(h^t) - \frac{p}{\nu+p}c^o(h^t)\right]$ does not depend on (v,η) such that theorem 9.8 of Stokey et al. (1989) cannot be directly applied to establish strict concavity of B in $\tilde{X}$. For this I follow Aguiar et al. (2009) and Golosov et al. (2016) and restate the optimization as if utilities for young and old, $u^y(h^t)$ and $u^o(h^t)$, are chosen instead of consumptions. Since there was a positive lower bound for consumptions there will be a lower bound for utilities and the choice set is compact. With this change of variables the constraint set is linear and the objective now reads $E_t\left[y(h^t) - \frac{\nu}{\nu+p}C(u^y(h^t)) - \frac{p}{\nu+p}C(u^o(h^t))\right]$ where $C(\cdot)$ is a cost function given by $C(u) = u^{-1}(u)$. Given that utility is increasing and concave, $C(\cdot)$ is increasing, differentiable and strictly convex.

Since the objective is concave and the constraint set is convex the value function is weakly concave and thus continuous in the interior of $\tilde{X}$. To show that it is strictly concave I consider first a given v and two η_1 and η_2 with $\eta_i \in [\underline{\eta}(v), \bar{\eta}]$ with $\eta_1 \neq \eta_2$. Let a_1^* and a_2^* the corresponding optimal policy choices. From the promise keeping constraint to the old it is immediate that $u^{o*}(h_1^t) \neq u^{o*}(h_2^t)$ and from the promise keeping to the social planner $u^{y*}(h_1^t) \neq u^{y*}(h_2^t)$.[42] Let $\alpha \in [0,1]$ and $\eta_\alpha = \alpha\eta_1 + (1-\alpha)\eta_2$ with corresponding optimal policy choice a_α^*. Since with the change of variable the constraint set is linear we have that

$$(\alpha u^{y*}(h_1^t) + (1-\alpha)u^{*y}(h_2^t), \alpha u^{o*}(h_1^t) + (1-\alpha)u^{o*}(h_2^t), \alpha w^*(h_1^t) + (1-\alpha)w^*(h_2^t),$$
$$\alpha\theta^*(h_1^t) + (1-\alpha)\theta^*(h_2^t)) \in \Gamma(v,\eta_\alpha).$$

Thus, the value function satisfies

$$\begin{aligned}
B(v,\eta_\alpha) &= E_t[y(h^t) - \frac{\nu}{\nu+p}C(u^{y*}(h_\alpha^t)) - \frac{p}{\nu+p}C(u^{o*}(h_\alpha^t)) + \delta\nu B(w^*(h_\alpha^t), \theta^*(h_\alpha^t))] \\
&\geq E_t[y(h^t) - \frac{\nu}{\nu+p}C(\alpha u^{y*}(h_1^t) + (1-\alpha)u^{y*}(h_2^t)) - \frac{p}{\nu+p}C(\alpha u^{o*}(h_1^t) + (1-\alpha)u^{o*}(h_2^t)) \\
&+ \delta\nu B(\alpha w^*(h_1^t) + (1-\alpha)w^*(h_2^t), \alpha\theta^*(h_1^t) + (1-\alpha)\theta^*(h_2^t))].
\end{aligned}$$

The strict convexity of $C(\cdot)$ and the weak concavity of $B(\cdot)$ then imply that

$$\begin{aligned}
B(v,\eta_\alpha) &> \alpha E_t[y(h^t) - \frac{\nu}{\nu+p}C(u^{y*}(h_1^t)) - \frac{p}{\nu+p}C(u^{o*}(h_1^t)) + \delta\nu B(w^*(h_1^t), \theta^*(h_1^t))] \\
&\quad +(1-\alpha)E_t[y(h^t) - \frac{\nu}{\nu+p}C(u^{y*}(h_2^t)) - \frac{p}{\nu+p}C(u^{o*}(h_2^t)) + \delta\nu B(w^*(h_2^t), \theta^*(h_2^t))].
\end{aligned}$$

A similar argument can be applied for given η and $v_1 \neq v_2$ as long as $\eta \geq \underline{\eta}(v_i)$. We thus

[42] I employ notation h_i^t to refer to the history summarized by (v, η_i, y_s). The same applies later for h_α^t. In constrast, since shocks are i.i.d. and thus do not depend on η, for output I keep notation $y(h^t)$.

conclude that B is strictly concave in $\tilde{X}$. Strict concavity then implies that B is strictly decreasing. It also implies that the policy correspondence G is single valued and since it is upper hemi continuous (from proposition 1) then it is continuous.

Now I will establish differentiability of B in the interior of $\tilde{X}$. For this take fixed v and $\underline{\eta}(v) < \eta_0 < \bar{\eta}$ and define the test function

$$W(v,\eta) = E_t[y(h^t) - \frac{\nu}{\nu+p}C(u^{y*}(h^t_{\eta_0}) - (\eta-\eta_0)) - \frac{p}{\nu+p}C(u^{o*}(h^t_{\eta_0}) + (\eta-\eta_0)) + \delta\nu B(w^*(h^t_{\eta_0}), \theta^*(h^t_{\eta_0})).$$

The policy choices $u^{o*}(h^t_{\eta_0}) + (\eta - \eta_0), u^{y*}(h^t_{\eta_0}) - (\eta - \eta_0))$ satisfy both promise keeping constraints. Therefore for all η in the interior of $\tilde{X}$ it must be that $W(v,\eta) \le B(v,\eta)$. Since $W(\cdot)$ is concave and differentiable then by application of the Benveniste-Scheinkman theorem (see Stokey et al. (1989), theorem 4.10) $\frac{\partial B(v,\eta)}{\partial \eta}|_{\eta=\eta_0}$ exists and

$$\frac{\partial B(v,\eta)}{\partial \eta}|_{\eta=\eta_0} = \frac{\partial W(v,\eta)}{\partial \eta}|_{\eta=\eta_0} = E_t[\frac{p}{\nu+p}C'(u^{y*}(h^t_{\eta_0})) - \frac{p}{\nu+p}C'(u^{o*}(h^t_{\eta_0}))]. \quad (28)$$

Similar reasoning implies that $\frac{\partial B(v,\eta)}{\partial v}|_{v=v_0} = -E_t[C'(u^{y*}(h^t_{\eta_0}))]$. It remains to be proved that the value function is also differentiable at $\eta = \underline{\eta}(v)$. But this follows since $\lim_{\eta_0 \to \underline{\eta}(v)} \frac{\partial B(v,\eta)}{\partial \eta}|_{\eta=\eta_0} = 0$.[43] But for $\eta \le \underline{\eta}(v)$ the value function is flat, i.e. $B(\eta, v) = B(\underline{\eta}(v))$, thus the left partial derivative of B is also zero at $\underline{\eta}(v)$ implying B is differentiable at $\eta = \underline{\eta}(v)$.

7.7 Proof of proposition 3

From (16) it can be seen that an increase in η_t (i.e. λ_{t-1}) implies an increase in $c^o(h^t)$ and since the right-hand side of (20) does not depend on η_t, both $c^y(h^t)$ and $\theta(h^t)$ must decrease. Thus, $\frac{dc^o(h^t)}{d\eta_t} > 0$ and $\frac{dc^y(h^t)}{d\eta_t} < 0$, and write $\delta p \frac{d\theta(h^t)}{d\eta_t} \equiv \psi u'(c^y(h^t))\frac{dc^y(h^t)}{d\eta_t}$, with $\psi \ge 0$. Then from (20) we have that $\frac{\omega p}{\nu+p}u'(c^o(h^t))\frac{dc^o(h^t)}{d\eta_t} = -\frac{(1-\omega)\nu}{\nu+p}(1+\psi)u'(c^y(h^t))\frac{dc^y(h^t)}{d\eta_t}$. Using this relation to see the marginal effect of η_t on average consumption we get

$$\begin{aligned}
\frac{d\left(\frac{p}{\nu+p}c^o(h^t) + \frac{\nu}{\nu+p}c^y(h^t)\right)}{d\eta_t} &= \frac{dc^o(h^t)}{d\eta_t}\left[\frac{p}{\nu+p} - \frac{\nu}{\nu+p}\frac{\omega p u'(c^o(h^t))}{(1+\psi)(1-\omega)\nu u'(c^y(h^t))}\right] \\
&\frac{p}{\nu+p}\frac{dc^o(h^t)}{d\eta_t}\left[1 - \frac{\omega(\frac{1}{u'(\underline{c})} + (1-\omega)\lambda_t)}{(1+\psi)(1-\omega)(\frac{1}{u'(\underline{c})} + \omega\lambda_t + p/\nu(1-\omega)\lambda_{t-1})}\right].
\end{aligned}$$

Where for the last relation we used (16). Thus, at least for some values of λ_{t-1} and λ_t, for ω high enough the term in brackets will be negative and average consumption falls with an increase in η_t.

[43] When promise keeping to the old is not binding the optimal allocation must satisfy $u'(c^y(h^t)) = u'(c^o(h^t))$. Since $C'(u(c)) = \frac{1}{u'(c)}$ implies $\frac{\partial B(v,\eta)}{\partial \eta}|_{\eta=\eta_0} = \frac{p}{\nu+p}E_t[\frac{1}{u'(c^y(h^t))} - \frac{1}{u'(c^o(h^t))}]$ the result follows.

From this last equation we can see that if $\omega \leq \omega^S$, with $\omega^S \geq \frac{1}{2}$ ($\omega^S = \frac{1}{2}$ if $\psi = 0$), then $\frac{d\left(\frac{p}{\nu+p}c^o(h^t)+\frac{\nu}{\nu+p}c^y(h^t)\right)}{d\eta_t} > 0$.

To show that $\hat{y}_t(\eta_t)$ is unique, we start noting that we cannot have an even number of states for which average consumption equals aggregate income as this would imply that capital flows have the same sign for extreme positive and negative shocks. Next note that $\theta(h^t) - E_t[u(c^{oD}(y_{t+1})]$ is weakly increasing in the current output realization and this difference must be positive when $y_t = \hat{y}_t(\eta_t) > \bar{y}_t(\eta_t)$.[44] Finally, for states with higher current output in (20), the ratio of the marginal utilities of the consumption pair $c^o(h^t)$ and $c^y(h^t)$ moves closer to its value under autarky (see (15), (16), and (24)). And in the limit as output, and thus λ_t, increases the ratio of marginal utilities converges to

$$\lim_{\lambda_t \to \infty} \frac{u'(c^o(h^t))}{u'(c^y(h^t))} = \frac{(1-\omega)\nu}{\omega p} = \frac{u'(c^{oD}(y_t))}{u'(c^{yD}(y_t))}.$$

Since consumption is stabilized when $y_t \leq \bar{y}_t(\eta_t)$, then at most one of the odd number of thresholds might lie at the left of $\bar{y}_t(\eta_t)$. I will now show that a contradiction arises if two thresholds exist to the right of $\bar{y}_t(\eta_t)$. Let's call them $\hat{y}_t^1(\eta_t)$ and $\hat{y}_t^2(\eta_t) > \hat{y}_t^1(\eta_t) > \bar{y}_t(\eta_t)$. At both thresholds (20) holds and $\frac{p}{\nu+p}c^{o1}(h^t) + \frac{\nu}{\nu+p}c^{y1}(h^t) = \hat{y}_t^1(\eta_t)$, $\frac{p}{\nu+p}c^{o2}(h^t) + \frac{\nu}{\nu+p}c^{y2}(h^t) = \hat{y}_t^2(\eta_t)$. And since $\theta(h^t) - E_t[u(c^{oD}(y_{t+1})]$ is weakly increasing in the current output, and positive for $y_t \geq \hat{y}_t^1(\eta_t)$, it must be the case that

$$\frac{\omega p}{\nu+p}u(c^{o2}(h^t)) + \frac{(1-\omega)\nu}{\nu p}u(c^{y2}(h^t)) - \left(\frac{\omega p}{\nu+p}u(c^{oD}(\hat{y}_t^2(\eta_t))) + \frac{(1-\omega)\nu}{\nu p}u(c^{yD}(\hat{y}_t^2(\eta_t)))\right) <$$
$$\frac{\omega p}{\nu+p}u(c^{o1}(h^t)) + \frac{(1-\omega)\nu}{\nu p}u(c^{y1}(h^t)) - \left(\frac{\omega p}{\nu+p}u(c^{oD}(\hat{y}_t^1(\eta_t))) + \frac{(1-\omega)\nu}{\nu p}u(c^{yD}(\hat{y}_t^1(\eta_t)))\right) < 0. \quad (29)$$

But for $\hat{y}_t^2(\eta_t)$ the optimal contract's ratio of marginal utilities must be closer to its autarky value than for $\hat{y}_t^1(\eta_t)$. And since the autarky allocation, by definition, maximizes $\frac{\omega p}{\nu+p}u(c^o) + \frac{(1-\omega)\nu}{\nu p}u(\frac{(\nu+p)y-pc^o}{\nu})$, then (29) implies a contradiction. This proves that there has to be a unique $\hat{y}_t(\eta_t)$ if $\hat{y}_t(\eta_t) \geq \bar{y}_t(\eta_t)$. But since we must have an odd number of thresholds and there can be at most one to the left or right of $\bar{y}_t(\eta_t)$ this proves $\hat{y}_t(\eta_t)$ is unique.

To show that it might be the case that $\hat{y}_t(\eta_t) \leq \bar{y}_t(\eta_t)$, assume that $v = v^{AUT}$, $\omega = \frac{1}{2}$ and $y_t = \bar{y}_t(\eta_t)$ such that $\theta(h^t) = \underline{\eta} = E_t[u(c^{oD}(y_{t+1})]$. Constraint (20) then implies

$$\frac{p}{2(\nu+p)}u(c^o(h^t)) + \frac{\nu}{2(\nu+p)}u(\underline{c}) = \frac{p}{2(\nu+p)}u(c^{oD}(y_t)) + \frac{\nu}{2(\nu+p)}u(\frac{(\nu+p)y_t - pc^{oD}(y_t)}{\nu}).$$

[44]Note that this is straightforward since shocks are i.i.d. The second term is constant and the former increases with y_t as this increases $c^y(h^t)$ and $\theta(h^t)$. That the difference must be positive at $\hat{y}_t(\lambda_{t-1}) > \bar{y}_t(\eta_t)$ follows from considering (20) and noting that $\frac{p}{\nu+p}c^o(h^t)+\frac{\nu}{\nu+p}c^y(h^t) = y_t$ and, under the assumption on the planner, the autarky allocation maximizes $\omega p u(c^o) + (1-\omega)\nu u(\frac{(\nu+p)y-pc^o}{\nu})$.

Under the assumption on the planner the RHS maximizes the weighted average of utilities for $y_t = \bar{y}_t(\eta_t)$. This implies that for the equality to hold it must be the case that $\frac{p}{\nu+p}c^o(h^t) + \frac{\nu}{\nu+p}\underline{c} \geq y_t$, implying $\hat{y}_t(\eta_t) \leq \bar{y}_t(\eta_t)$.

7.8 Proof of proposition 4

Consider the case that only the welfare of the old enters the utilitarian welfare function. In this case, $\omega = 1$ and the credibility constraint when binding, (20), reduces to

$$u(c^o(h^t)) = u(y_t),$$

since in this case the optimal consumption under a deviation gives all the endowment to the old. Thus, when the constraint is binding net exports are zero. Therefore the economy is unable to receive resources from the rest of the world in bad states and the optimal allocation corresponds to $c^o(h^t) = y_t$ in every period and state.

When the incentive constraint (6) is binding we have that

$$\frac{\omega p}{\nu+p}\left(u(c^o(h^t)) - u(c^{oD}(y_t))\right) + \frac{(1-\omega)\nu}{\nu+p}\left(u(c^y(h^t)) + \delta p\theta(h^t) - u(c^{yD}(y_t)) - \delta pE[u(c^{oD}(y_{t+1}))]\right) = 0.$$

Thus the weighted average of the surplus from the contract for young and old is zero. This implies that in general one group will be in favor of default while the other will be against. Whenever $c^o(h^t) > c^{oD}(y_t)$ the old will be against a default.

When the credibility constraint starts to bind, i.e. $y_t = \bar{y}_t(\eta_t)$, from (18) $c^o(h^t)$ is determined by $c^y(h^{t-1})$. At the same time, the autarky consumption of the old is a weakly increasing function of ω as seen in (24). Thus there exists $\omega^D(\eta_t)$ such that $c^{oD}(\bar{y}_t(\eta_t)) = c^y(h^{t-1})$, and when $\omega > \omega^D(\eta_t)$ the old favor a default. Since $c^y(h^{t-1})$ and η_t are positively related we have $\frac{d\omega^D(\eta_t)}{d\eta_t} > 0$.

7.9 Proof of proposition 5

Integrating the incentive constraint (6) we get[45]

$$\begin{aligned}
\frac{\omega p}{\nu+p}E_t[u(c^o(h^t))] + \frac{(1-\omega)\nu}{\nu+p}\left(E_t[u(c^y(h^t))] + \delta pE_t[\theta(h^t)]\right) &\geq E[\underline{V}(y)] \\
\frac{\omega p}{\nu+p}\eta_t + \frac{(1-\omega)\nu}{\nu+p}\left(\frac{(1-\delta\nu)v(\nu+p) - p\eta_t}{\nu} + \delta pE_t[\theta(h^t)]\right) &\geq E[\underline{V}(y)].
\end{aligned}$$

The second inequality uses the promise keeping constraint of the old and the social planner to replace for the expected utility from consumption for the young and old. Since the

[45]Note that while in the LHS we have conditional expectations, given that shocks are i.i.d. the RHS features unconditional expectations of the political payoff under autarky.

LHS is linear in η_t and $E_t[\theta(h^t)] \geq \underline{\eta}(v)$ assumption 2 guarantees that at least for one state the incentive constraint is not binding. And given that $\underline{V}(y)$ is monotone then we know for sure that the incentive constraint will not bind for y_1.

Assumption 2 thus guarantees that $Prob(\lambda_t = 0) \geq \pi(y_1) > 0$, i.e. that the promised utility to the old gets reset to its lowest value $\underline{\eta}$ with probability $\pi(y_1)$ (or higher if the incentive constraints does not bind for more than one state). Denoting $\epsilon = \pi(y_1)$ we have that

$$P(\eta, \underline{\eta}) \geq \epsilon \quad \forall \eta \in \tilde{X}.$$

From exercises 11.4 c) and 11.5 a) of Stokey et al. (1989) this implies that condition M is satisfied such that theorem 11.12 can be applied to show that for all $f_0 \in \mathcal{P}(\tilde{X}, \tilde{\mathcal{X}})$ there is geometric (uniform) convergence to a unique $f^* \in \mathcal{P}(\tilde{X}, \tilde{\mathcal{X}})$.

7.10 Numerical solution

7.10.1 Value Function Iteration

This appendix describes the algorithm used to solve for the planner's value function $B(\eta)$, where η denotes the promised continuation utility of the old generation and the state space is the compact interval $[\underline{\eta}, \bar{\eta}] \subset \mathbb{R}_{<0}$.

Projection Methods and Chebyshev Polynomials

The planner's value function satisfies a functional equation of the form $B = \mathcal{T}(B)$, where $\mathcal{T}$ is the Bellman operator defined in proposition 1. Because the state space is one-dimensional and continuous, we solve this equation using a *projection method*: the unknown function B is approximated by a finite-dimensional object whose coefficients are updated at each iteration of the Bellman operator until convergence.

Specifically, we represent the smooth component of B as a linear combination of Chebyshev polynomials of the first kind. The Chebyshev polynomials $\{T_k\}_{k=0}^{\infty}$, defined on $[-1, 1]$ by the recurrence

$$T_0(x) = 1, \qquad T_1(x) = x, \qquad T_k(x) = 2x\, T_{k-1}(x) - T_{k-2}(x), \quad k \geq 2,$$

constitute an orthogonal basis for the space of continuous functions on $[-1, 1]$ with respect to the weight $(1 - x^2)^{-1/2}$. The grid point $\eta \in [\underline{\eta}, \bar{\eta}]$ is mapped to $x(\eta) \in [-1, 1]$ via the affine transformation

$$x(\eta) \;=\; \frac{2(\eta - \underline{\eta})}{\bar{\eta} - \underline{\eta}} - 1.$$

A truncated Chebyshev expansion of degree N then approximates B on the grid as

$$R(\eta) \;=\; \sum_{k=0}^{N-1} a_k\, T_k\big(x(\eta)\big).$$

Chebyshev expansions are preferred over monomial or evenly-spaced polynomial bases because (i) they achieve near-minimax approximation error, meaning the maximum pointwise error is spread roughly uniformly across the domain rather than concentrated near the boundaries; (ii) the coefficients a_k decay rapidly for smooth functions, so low-degree expansions are highly accurate; and (iii) the recurrence structure makes evaluation of both the function and its derivative numerically stable and inexpensive. The derivative of the expansion with respect to η is obtained by the chain rule,

$$R'(\eta) \;=\; \frac{2}{\bar{\eta} - \underline{\eta}} \sum_{k=0}^{N-1} a_k \, T_k'\big(x(\eta)\big),$$

where $T_k'(x) = \mathrm{d}T_k/\mathrm{d}x$ is computed by the standard recurrence for Chebyshev derivative polynomials. This closed-form derivative is exploited directly in the computation of optimal policies, which depend on $B'(\eta)$ through the planner's first-order conditions.

At each inner iteration, the updated grid values $\{B_{\text{new}}(\eta_i)\}$ produced by the Bellman operator are fitted to the parametric family by nonlinear least squares, yielding a new coefficient vector $\mathbf{a}$. Damping of the form $\mathbf{a}^{(t+1)} = \lambda\,\mathbf{a}^{(t)} + (1-\lambda)\,\mathbf{a}_{\text{fit}}$ with $\lambda \in (0,1)$ is applied to stabilize convergence.

Singular Basis Augmentation

The planner's value function is expected to satisfy $B'(\eta) \to -\infty$ as $\eta \to 0^-$, reflecting the fact that with $\sigma > 1$ $\eta = 0$ is the *natural upper bound* of the state space corresponding to $c_s^o \to \infty$ for all $s \in S$. Standard Chebyshev polynomials are globally smooth and cannot represent this singularity, regardless of the degree of the expansion. Attempting to approximate a singular function with a smooth basis introduces Gibbs-type oscillations and poor accuracy near the boundary.

To address this, we augment the Chebyshev basis with a *singular basis function* that is designed to capture the correct asymptotic behaviour. Define

$$\phi(\eta;\,\alpha) \;=\; (-\eta)^{\alpha} - (-\underline{\eta})^{\alpha} - \alpha(-\underline{\eta})^{\alpha-1}(\eta - \underline{\eta}), \qquad \alpha \in (0,1),$$

recall $\underline{\eta} \le \eta \le \bar{\eta} < 0$. This function is constructed to satisfy three properties simultaneously:

i. $\phi(\underline{\eta}) = 0$: no level shift at the grid lower bound.

ii. $\phi'(\underline{\eta}) = 0$: the slope of ϕ vanishes at $\underline{\eta}$, so the singular component does not introduce a kink there.

iii. $\phi'(\eta) \to -\infty$ as $\eta \to 0^-$: the desired singularity is present at the natural lower bound.

The first two properties follow by direct computation:

$$\phi'(\eta;\,\alpha) \;=\; -\alpha(-\eta)^{\alpha-1} + \alpha(-\underline{\eta})^{\alpha-1},$$

which equals zero at $\eta = \underline{\eta}$ and diverges to $-\infty$ as $\eta \to 0^-$ because $\alpha - 1 < 0$.

The full approximation to the value function is therefore

$$B(\eta) \;=\; c\,\phi(\eta;\,\alpha) \;+\; R(\eta),$$

where $c \geq 0$ is a free scalar coefficient estimated jointly with the Chebyshev coefficients $\{a_k\}$.

Zero-Slope Constraint and Parameter Reduction

The theory imposes that $B'(\underline{\eta}) = 0$ at the grid lower bound: because the constraint $\eta \geq \underline{\eta}$ is slack with positive probability, the envelope condition requires the derivative of the value function to vanish there. Since $\phi'(\underline{\eta}) = 0$ by construction, this constraint reduces to requiring $R'(\underline{\eta}) = 0$, i.e. the smooth Chebyshev part must also have zero slope at $x = -1$.

The derivative of T_k at $x = -1$ is $T_k'(-1) = (-1)^{k+1}k^2$, so the constraint reads

$$R'(\underline{\eta}) = 0 \iff \frac{2}{\bar{\eta} - \underline{\eta}} \sum_{k=0}^{N-1} a_k\,(-1)^{k+1}k^2 = 0 \iff \sum_{k=0}^{N-1} a_k\,(-1)^{k+1}k^2 = 0.$$

This is a single linear restriction on the N Chebyshev coefficients. We exploit it to eliminate a_{N-1} analytically:

$$a_{N-1} \;=\; -\frac{1}{(-1)^N(N-1)^2} \sum_{k=0}^{N-2} a_k\,(-1)^{k+1}k^2,$$

so that the constraint is satisfied identically for any choice of the remaining $N - 1$ free coefficients $\{a_0, \ldots, a_{N-2}\}$.

Full Parameter Vector and Algorithm Summary

Collecting the free parameters, the full parameter vector is

$$\mathbf{a} \;=\; \big(c,\ a_0,\ a_1,\ \ldots,\ a_{N-2}\big)^{\top} \;\in\; \mathbb{R}^N,$$

with the constraint $c \geq 0$ enforced as a bound in the nonlinear least-squares step. The total number of free parameters equals N_{cheb}, the user-specified Chebyshev degree, which encompasses one singular coefficient and $N_{\text{cheb}} - 1$ free Chebyshev coefficients (with the last Chebyshev coefficient pinned by the zero-slope constraint).

The algorithm proceeds as follows.

Outer loop. A bisection procedure searches over the floor consumption parameter $\bar{c}$

(the resource constraint at the debt limit) to find the value consistent with the planner's autarky participation constraint, $\hat{V} = V^{\text{aut}}$, where $\hat{V}$ is the average utility generated by the simulated contract and V^{aut} is the autarky value.

Inner loop. For a fixed $\bar{c}$, the inner loop iterates on the coefficient vector $\mathbf{a}$:

i. Evaluate the Bellman operator at each grid point $\{\eta_i\}_{i=1}^{M}$ to obtain updated values $B_{\text{new}}(\eta_i)$. For each η_i, the optimal continuation utility θ, young consumption c^y, and old consumption c^o are recovered from the planner's first-order conditions, exploiting the closed-form expression for $B'(\eta)$.

ii. Project the updated grid values onto the parametric family by solving the bounded nonlinear least-squares problem

$$\min_{\mathbf{a}:\, c \geq 0} \sum_{i=1}^{M} \big[B(\eta_i;\, \mathbf{a}) - B_{\text{new}}(\eta_i)\big]^2.$$

iii. Update $\mathbf{a}$ with damping and check convergence in the sup-norm, $\sup_i |B(\eta_i;\, \mathbf{a}^{\text{new}}) - B(\eta_i;\, \mathbf{a}^{\text{old}})| < \varepsilon_{\text{inner}}$.

Simulation. Once the inner loop converges, the economy is simulated for T_{sim} periods using a pre-computed shock sequence. Optimal policies are pre-tabulated on a fine grid and interpolated at each simulation step. The simulated average utility $\hat{V}$ is compared to V^{aut} to update $\bar{c}$ in the outer loop.

Identification of the Singularity Exponent

The singularity exponent α controls the rate at which $B'(\eta) \to -\infty$ as $\eta \to 0^-$, but the equilibrium grid $[\underline{\eta}, \bar{\eta}]$ does not include the point $\eta = 0$. As a consequence, the Chebyshev coefficients can partially absorb changes in α on the interior of the grid, and α is not identified from the fixed-point condition alone. We verify this empirically: the shape of B, the initial debt-to-output ratio, and the simulated default frequency are all insensitive to the choice of $\alpha \in \{0.05, 0.5, 0.95\}$. We therefore fix $\alpha = 0.5$ throughout, which matches the closed-form two-parameter specification used as the baseline.

Calibration of Numerical Parameters

Table 4 summarises the numerical parameters used in the baseline solution.

7.10.2 Expectational difference equation

This appendix describes the Matlab routine that solves the expectational difference equation, (23), for the Lagrange multiplier, λ_t, associated with the incentive constraint. The state variable is the promised utility to the old, η. The code uses a projection method based on tensor-product Chebyshev polynomials to approximate the conditional expectation term $E_t[u^o(y_{t+1}, \lambda_t)]$. It then simulates the economy, constructs policy functions as

Table 4: Numerical Parameters

Parameter	Symbol	Value
Chebyshev degree (free params)	N_{cheb}	6
Singularity exponent	α	0.5
Eta grid points	M	100
Grid curvature exponent	—	1.5
Fine grid multiplier (simulation)	—	10
Inner loop tolerance	$\varepsilon_{\text{inner}}$	10^{-8}
Outer loop tolerance	$\varepsilon_{\text{outer}}$	5×10^{-7}
Damping factor	λ	0.85
Maximum inner iterations	—	500
Maximum outer iterations	—	150
Simulation length	T_{sim}	(shock sequence length)
Burn-in periods	—	1000

functions of the state, computes an implied domestic risk-free rate schedule, and finally computes a value function over promised utility and produces plots.

Environment, shocks, and preferences

A sequence of a million shocks Normally distributed is loaded and discretized into a Markov chain with the user being able to choose the number of nodes (six in the reported simulations) using a hybrid discretization routine. In particular, the routine `discretizeHybrid` combines Gauss–Hermite nodes (to place grid points) with empirical frequencies from the simulated series (to construct stationary probabilities). In the baseline the political weight of the old is $\omega = 0.5$, in other routines this parameter is allowed to change between 0.45 and 0.55. Preferences are CRRA, with the user choosing the coefficient of risk aversion. The baseline results use $\sigma = 2$ and some robustness is provided for logarithmic utility. The code also computes benchmark autarky objects used in the incentive constraint and welfare comparisons: (i) autarky lifetime utility for the representative agent and period government, (ii) a consumption-equivalent measure.

Multiplier recursion with an expectational term

The expectational difference equation followed by the optimal λ_t is of the form

$$F(\lambda_t, \lambda_{t-1}, y_t;\ \Psi(y_t, \lambda_{t-1})) = 0, \tag{30}$$

where the key nonlinearity is an expectation term $\Psi(\cdot)$ that depends on current output and the lagged multiplier (or, more generally, the endogenous state). In the code, this term is represented by the function

$$\Psi(y_t, \lambda_{t-1}) \equiv \mathbb{E}[\,\psi(\lambda_{t-1}, y_t) \,|\, y_t]\,,$$

and is approximated via Chebyshev projection. Given Ψ, the current multiplier λ_t is computed by solving (30) with a scalar root finder (Matlab's `fzero`), imposing the Kuhn–Tucker condition $\lambda_t \geq 0$.

Projection step: tensor-product Chebyshev approximation

The expectation object is approximated as a bivariate polynomial in $(\log(1+\lambda_{t-1}), \log y_t)$ on a rectangle domain. Let

$$x_\lambda \equiv \log(1+\lambda_{t-1}), \qquad x_y \equiv \log y_t.$$

These are linearly mapped to $[-1,1]$ using an evolving domain $\mathcal{D} = [x_\lambda^{\min}, x_\lambda^{\max}] \times [x_y^{\min}, x_y^{\max}]$:

$$\tilde{x}_\lambda = 2\frac{x_\lambda - x_\lambda^{\min}}{x_\lambda^{\max} - x_\lambda^{\min}} - 1, \qquad \tilde{x}_y = 2\frac{x_y - x_y^{\min}}{x_y^{\max} - x_y^{\min}} - 1,$$

with clamping to $[-1,1]$ to prevent extreme extrapolation. The code uses degrees 4 polynomials and approximates

$$\Psi(y_t, \lambda_{t-1}) \approx \sum_{i=0}^{n_\lambda} \sum_{j=0}^{n_y} a_{ij}\, T_i(\tilde{x}_\lambda)\, T_j(\tilde{x}_y), \tag{31}$$

where $T_k(\cdot)$ is the kth Chebyshev polynomial of the first kind, computed via the standard recurrence. Using the sequence of shocks a simulated sample of pairs (λ_{t-1}, y_t) and an implied target for the conditional expectation, `wl`, the code evaluates every basis function at every observation and stack the results into a matrix and estimates coefficients by least squares:

$$\hat{a} = \arg\min_a \|\Phi a - \texttt{wl}\|^2.$$

The domain $\mathcal{D}$ is updated each iteration to match the realized support of (x_λ, x_y).

Fixed point iteration

The solution is obtained by iterating on the Chebyshev coefficients until convergence:

i. Initialize a guess for minimum consumption $\underline{c}$.

ii. Given current coefficients, generate a time series $\{\lambda_t\}$ along the exogenous output sequence $\{y_t\}$ by solving (30) period-by-period.

iii. Using the simulated series, construct the regression targets `wl`. Intuitively, `wl` is the transformed object whose conditional expectation is approximated by (31).

iv. Refit the Chebyshev coefficients via least squares.

v. Check convergence using the sup norm of coefficient changes, $\max|\Delta a|$, and iterate until below tolerance (in the baseline 10^{-9}). After convergence, the code reports an in-sample R^2 comparing `wl` to the Chebyshev-predicted values, which is around 30% in the baseline.

vi. An outer loop adjusts $\underline{c}$ to hit one of two targets: social planner utility under the contract equal to that of autarky (maximum debt), or zero debt.

Simulation and construction of policy functions

With the converged $\{\lambda_t\}$ and $\underline{c}$ the code simulates allocations for young and old consumption:

$$c_t^y = \left(\underline{c}^\sigma + (1-\omega)\lambda_t\right)^{1/\sigma}, \qquad c_t^o = \left(\underline{c}^\sigma + \omega\lambda_t + (1-\omega)\lambda_{t-1}\right)^{1/\sigma},$$

It also computes:

- accumulated debt dynamics via the resource constraint,
- default indicators as the frequency with which the multiplier binds ($\lambda > 0$), and the ratio of times the old want to default in these states ($c_t^o > c_t^{oD}$),
- average domestic gross interest rates using an Euler-equation object:

$$R_t^d \propto \frac{(c_t^o)^\sigma}{\beta (c_{t-1}^y)^\sigma}.$$

To express policies as functions of the endogenous state, the code constructs a grid for λ (in the baseline of size 250) and, for each output node $y(i)$ and grid point $\lambda(j)$, averages simulated outcomes conditional on $(y = i, \lambda_{t-1} \approx \lambda(j))$. Missing cells are filled by linear interpolation in λ. This yields policy objects for consumption for young and old, and the implied multiplier value (a transition for λ_t).

Mapping to promised utility and computing value function

The promised utility to the old, η, is constructed on the λ grid by integrating (expectation under p) the old's period utility evaluated at the policy function for c^o:

$$\eta(\lambda_j) \equiv \sum_{i=1}^{S} p(i)\, u(c^o(y(i), \lambda_j)).$$

The mapping $\lambda \mapsto \eta$ is then sorted and inverted numerically to produce an evenly spaced grid for η and to interpolate other objects onto it.

The code computes two key schedules as functions of η:

i. **Domestic risk-free rate schedule.** Using the interpolated policies, it forms expected marginal utility of the old and computes a gross rate

$$R^d(\lambda_j) = \frac{u'\!\left(\underline{c} + \lambda_j/2\right)}{\beta\, \mathbb{E}[u'(c_{t+1}^o) \mid \lambda_j]}.$$

This is then interpolated to the η grid.

ii. **Value function over promised utility.** The code computes a fixed point for a value function $B(\lambda)$ satisfying a Bellman equation with flow payoff equal to net resources,

$$B(\lambda_j) = \sum_{i=1}^{S} p(i)\Big(y(i) - c^y(i,j) - c^o(i,j) + \delta\, B(\lambda_{t+1})\Big),$$

where λ_{t+1} is given by the corresponding policy function and $B(\lambda_{t+1})$ is obtained by interpolation. The converged B is finally interpolated to $B(\eta)$ on the grid on η.

Finally, for the two calibrated cases (maximum or zero debt), the code saves and plots: (i) the domestic rate schedule $R^d(\eta)$ together with the foreign gross rate $R^* = 1/\delta$, and (ii) the value function $B(\eta)$.