CREDIT SUPPLY AND THE HOUSING BOOM

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Abstract. The housing boom that preceded the Great Recession was due to a progressive loosening of lending constraints in the residential mortgage market. This view is consistent with a number of empirical observations, such as the rapid increase in house prices and household debt, the stability of debt relative to collateral values, and the fall in mortgage rates. These empirical facts are difficult to reconcile with the popular view that attributes the housing boom to a loosening of borrowing constraints associated with lower collateral requirements.

1. Introduction

The U.S. economy has recently experienced a very severe financial crisis, which precipitated the most damaging recession since the Great Depression. The behavior of the housing and mortgage markets in the first half of the 2000s has been identified by many as the crucial factor behind these events. Four key empirical facts characterize this behavior in the period leading up to the collapse in house prices and the ensuing financial turmoil.

Fact 1: House prices rose dramatically. Between 2000 and 2006 home prices increased anywhere between 40 and 70 percent, as shown in Figure 1.1. This boom is unprecedented in U.S. history, and was followed by an equally spectacular bust after 2006.

Fact 2: Households borrowed against the rising value of their real estate, expanding mortgage debt relative to income by a substantial amount. This fact is illustrated figure 1.2, which plots the ratio of households’ mortgages to GDP (panel a), and of mortgages to income for the fraction of households with little financial assets (panel b). Both ratios were stable in the 1990s, but increased by about 30 and 60 percentage points between 2000 and 2007, before beginning to fall during the financial crisis.

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Figure 1.1. Real house prices. The two measures of house prices are the FHFA (formerly OFHEO) all-transactions house price index and the CoreLogic repeated-sales index. Both indexes are deflated by the consumer price index, and normalized to 100 in 2000:Q1.

Fact 3: The increase in mortgage debt kept pace with the rapid appreciation of house prices, leaving the ratio of mortgages to the value of real estate roughly unchanged. This often under-appreciated fact is shown in figure 1.3, where we can also see that this measure of household leverage actually spiked when home values collapsed immediately before the recession.

Fact 4: Mortgage rates declined substantially. Figure 1.4 plots the 30-year conventional mortgage rate minus various measures of inflation expectations from the Survey of Professional Forecasters. Real mortgage rates were stable around 5% during the 1990s, but declined substantially afterward, with a fall of about 2.5 percentage points between 2000 and 2005.

We argue that the key factor behind these four phenomena was the relaxation of lending constraints since the late 1990s, which led to a significant expansion in the supply of mortgage loans. We also show that this simple mechanism is much more successful in explaining the facts than the relaxation of borrowing constraints, even though these constraints are the ones on which most of the literature has focused so far. In fact, in our framework,
Figure 1.2. (a): Mortgages-to-GDP ratio (Flow of Funds). Mortgages are defined as home mortgages from the balance sheet of U.S. households and nonprofit organizations in the Flow of Funds. (b): Mortgages-to-income ratio (SCF). Ratio between mortgage debt and income for the households with little financial assets in the Survey of Consumer Finances (see section 4.1 for details).

changes in the borrowers’ collateral constraints on their own have several counterfactual implications, although their interaction with lending constraints might well be needed to account for the entire boom and bust cycle in debt and house prices.

We develop this argument in a very simple model, to make the distinction between credit supply and loan demand as transparent as possible. In the model, patient households supply credit to impatient ones, who borrow in order to tilt their consumption profile towards the present. The ability to borrow is bounded by a collateral constraint, which limits mortgages to a certain fraction of the value of the real estate backing them, as in most of the literature.
following Kiyotaki and Moore (1997). What sets our model apart from other studies in this area is that we also assume a constraint on \textit{lending} in the residential mortgage market. This constraint captures, in reduced form, technological, regulatory and other factors hampering the free flow of funds from the savers to the mortgage market.

But if lending constraints are central in the model, what is the empirical evidence pointing to the presence of such constraints, and to their loosening during the credit and housing
boom? To answer this question, we would point to most of the same evidence on financial liberalization that has been so far mostly mentioned in the literature as being consistent with a slackening of borrowing constraints as the main driver of the boom. Our model’s microeconomic premises, as well as its macroeconomic implications, on the contrary, suggest that this evidence is better interpreted through the lens of a lending, rather than a borrowing constraint. In our interpretation, the well documented emergence of the shadow banking sector, intertwined with rapid growth in securitization, off-balance sheet financing and regulatory arbitrage, contributed to removing existing obstacles to the ability of the financial sector to direct funds towards mortgage financing.

For example, the rising importance of market-based financial intermediation—a key component of the so-called shadow banking—relative to traditional forms of financing through commercial banks, together with the lower cost opportunities for credit and maturity transformation provided by this evolution, have attracted a great deal of attention (Pozsar, Adrian, Ashcraft, and Boesky (2013)). The rise of securitization was a central aspect of this transformation of intermediation. The pooling and tranching of mortgages, with the
resulting creation of safe, highly-rated assets, contributed to channel into mortgage products a large pool of savings previously directed towards government debt, tapping a source of funds that was previously unavailable. In particular, securitization enabled certain institutional investors—money-market funds, pension funds and insurance companies, which are restricted to holding only the safest securities by explicit and implicit regulations—to finance mortgages (Brunnermeier (2009)). Furthermore, the standardization of mortgage products enhanced mortgage liquidity and reduced the dependence of loan supply from the availability of bank finance (Loutskina and Strahan (2009)). Finally, securitization has been shown to be the primary force behind the increased availability and access to credit of subprime borrowers (Mian and Sufi (2009), Keys, Mukherjee, Seru, and Vig (2010)).

On the regulatory side, capital requirements imposed on commercial banks imply lower charges for agency mortgage-backed-securities (and the senior tranches of private-label ones) than for the mortgages themselves. Combined with the rise of highly levered off-balance-sheet special purpose vehicles, which allowed their sponsors to expand their overall leverage at unchanged levels of regulatory capital, these forms of regulatory arbitrage drastically increased banks’ ability to make loans (Acharya and Richardson (2009), Acharya, Schnabl, and Suarez (2013)). While these developments date back at least to the 1980s, when the Government Sponsored Enterprises created the first mortgage-backed-securities, they really took off in the late 1990s and early 2000s, with the expansion of private-label securitizations beyond conforming mortgages and ultimately into subprime products.

We interpret all these developments in financial intermediation as sources of more relaxed lending constraints, and use our model to analyze their impact on the rest of the economy, both qualitatively and quantitatively. For the quantitative part of the analysis, we calibrate the model to match some key properties of the balance sheet of U.S. households in the 1990s, using the Survey of Consumer Finances and the Flow of Funds.

A key assumption underlying this quantitative exercise is that the US economy in the 1990s was constrained by a limited supply of funds to the mortgage market, rather than by a scarcity of housing collateral. Starting from this situation, we show that a progressive loosening of the lending constraint in the residential mortgage market increases household debt (fact 2). If the resulting shift in the supply of funds is sufficiently large, the availability of collateral also becomes a binding constraint. As a consequence, the price of houses
increase (fact 1), since their collateral value increases, while the interest rate falls (fact 4). The resulting relaxation of the collateral constraint drives a parallel increase in debt and home values, thus also reproducing fact 3. One interpretation of this mechanism is that the expansion of credit supply transformed houses into ATMs—a phrase often used to describe the impact of financial liberalization on housing—with the opportunity to borrow against home equity being reflected in real estate values.

The main objective of this paper is to point to barriers to the flow of funds to the mortgage market, and to their progressive lifting over time, as the fundamental driver of the housing and credit boom of the early 2000s. To make this shift of focus away from factors pertaining to the borrowers’ ability to obtain loans against collateral and towards the availability of funds for those loans, the model is kept deliberately simple. In particular, we do not include microeconomic foundations for the intricate developments in mortgage markets, and in the ensuing availability of credit for households that we just described. Instead, we model the lending limit as a simple parametric restriction on the savers’ ability to make loans, in the same spirit as the borrowing limit featured in Eggertsson and Krugman (2012) and in the literature spawned by Aiyagari (1994). Our contribution therefore is to highlight the importance of outward shifts in the supply of credit for determining the short-side of the market in an environment in which equilibrium prices and quantities are determined by constraints on both lenders or borrowers. As such, we expand the focus beyond borrowing constraints in collateral markets, which have been extensively analyzed in the literature.

This paper is mostly related to two strands of the recent macroeconomic literature. First, as Eggertsson and Krugman (2012), Guerrieri and Lorenzoni (2012), Hall (2011), Midrigan and Philippon (2011), Favilukis, Ludvigson, and Nieuwerburgh (2013), Bianchi, Boz, and Mendoza (2012), Boz and Mendoza (2014) and Garriga, Manuelli, and Peralta-Alva (2012), we use a model of borrowing and lending among households to analyze the drivers of the boom and bust in credit and house prices that precipitated the Great Recession. Similar to some of these papers, we assume that borrowing is limited by a collateral constraint, as in the pioneering contribution of Kiyotaki and Moore (1997), especially as declined by Iacoviello (2005) and Campbell and Hercowitz (2009b). Differently from these papers, however, which focus on borrowers’ limits, we bring attention to constraints on lending, and argue that their relaxation is crucial to understand the upswing in debt and real estate
values in the first half of the 2000s. Moreover, unlike most of those papers, we focus almost exclusively on the boom part of the credit cycle, rather than on its demise. This is where lending constraints appear most relevant, even though the interaction between lending and borrowing constraints in our model generates rich patterns of debt and collateral values that might account for the bust as well.

Second, the paper makes contact with the macro-finance literature that has analyzed the frictions on the balance sheets of financial intermediaries, and has thus concentrated on factors shifting the supply of credit (Geanakoplos and Fostel (2008), Adrian and Boyarchenko (2012, 2013), Brunnermeier and Sannikov (2014), He and Krishnamurthy (2013), Gertler and Kiyotaki (2010), Gertler, Kiyotaki, and Queralto (2012), Dewachter and Wouters (2012)). Unlike this line of work, our model focuses on the balance sheets of households, abstracting from the distinction between lenders and financial intermediaries. More important, another difference from these papers is that we explicitly link the greater supply of credit to mortgage debt and house prices. However, we abstract entirely from risk, which is instead naturally at the center of much of this work in macro-finance. Fully developing the microfoundations for our stark balance sheet constraints would most likely require bringing risk back into the picture, but we leave this challenging task to future research.

The rest of the paper is organized as follows. Section 2 presents the simple model of lending and borrowing with houses as collateral. Section 3 analyzes the properties of this model and characterizes its equilibrium. In section 4, we conduct a number of quantitative experiments to study the impact of looser lending and collateral constraints. Section 5 concludes.

2. The model

This section presents a simple model with heterogeneous households that borrow from each other, using houses as collateral. We use the model to establish that the crucial factor behind the boom in house prices and mortgage debt of the early 2000s was an outward shift in the supply of funds to borrowers, rather than an increase in the demand for funds driven by lower collateral requirements, as mostly assumed by the literature so far. We illustrate this point in the simplest possible endowment economy, abstracting from the complications arising from production and capital accumulation. However, none of these simplifications are crucial for the results, as we show in the Appendix (TBW).
2.1. **Objectives and constraints.** The economy is populated by two types of households, with different discount rates, as in Kiyotaki and Moore (1997), Iacoviello (2005), Campbell and Hercowitz (2009b) and our own previous work (Justiniano, Primiceri, and Tambalotti 2014b,a). Patient households are denoted by \( l \), since in equilibrium they save and lend. Their discount factor is \( \beta_l > \beta_b \), where \( \beta_b \) is the discount factor of the impatient households, who borrow in equilibrium.

At time 0, representative household \( j = \{ b, l \} \) maximizes utility

\[
E_0 \sum_{t=0}^{\infty} \beta^t_j [u(c_{j,t}) + v_j(h_{j,t})],
\]

where \( c_{j,t} \) denotes consumption of non-durable goods, and \( v_j(h_{j,t}) \) is the utility of the service flow derived from a stock of houses \( h_{j,t} \) owned at the beginning of the period. The function \( v(\cdot) \) is indexed by \( j \) for reasons that will become clear in section 2.3. Utility maximization is subject to the flow budget constraint

\[
c_{j,t} + p_t [h_{j,t+1} - (1 - \delta) h_{j,t}] + R_{t-1} D_{j,t-1} \leq y_{j,t} + D_{j,t},
\]

where \( p_t \) is the price of houses in terms of the consumption good, \( \delta \) is their depreciation rate, and \( y_{j,t} \) is an exogenous endowment. \( D_{j,t} \) is the amount of one period debt accumulated by the end of period \( t \), and carried into period \( t + 1 \), with gross interest rate \( R_t \). In equilibrium, debt is positive for the impatient borrowers and it is negative for the patient lenders, representing loans that the latter extend to the former. Therefore, borrowers can use their endowment, together with loans, to buy non-durable consumption goods and new houses, and to repay old loans with interest.

Households’ decisions are subject to two more constraints. First, on the liability side of their balance sheet, a collateral constraint limits debt to a fraction \( \theta \) of the value of the borrowers’ housing stock, along the lines of Kiyotaki and Moore (1997). This constraint takes the form

\[
(2.1) \quad D_{j,t} \leq \theta p_t h_{j,t+1},
\]
where $\theta$ is the maximum admissible loan-to-value (LTV) ratio.\footnote{This type of constraint is often stated as a requirement that contracted debt repayments (i.e. principal plus interest) do not exceed the future expected value of the collateral. The choice to focus on a contemporaneous constraint is done for simplicity and it is inconsequential for the results, since most of the results pertain to steady state equilibria.} Higher values of $\theta$ capture looser collateral requirements, such as those brought about by mortgages with higher initial LTVs, multiple mortgages on the same property (so-called piggy back loans), and home equity lines of credit. Together, they contribute to enhance households’ ability to borrow against a given value of their property. An emerging consensus in the literature identifies these looser credit conditions as the fundamental force behind the explosion of debt and house prices in the period leading up to the financial crisis. Prominent papers based on this hypothesis include Fovilukis, Ludvigson, and Nieuwerburgh (2013), Bianchi, Boz, and Mendoza (2012), Boz and Mendoza (2014), Garriga, Manuelli, and Peralta-Alva (2012), Lambertini, Mendicino, and Punzi (2013) and Korinek and Simsek (2014), while Mian, Rao, and Sufi (2011) informally support it as consistent with their influential empirical evidence on the macroeconomic effects of the mortgage boom and bust.

The second constraint on households’ decisions applies to the asset side of their balance sheet. This constraint consists in an upper bound on the total amount of mortgage lending they can engage in, as in

\begin{equation}
-D_{j,t} \leq \bar{L}.
\end{equation}

This very stark constraint is designed to create the cleanest possible contrast with the more familiar collateral constraint imposed on the borrowers. From a macroeconomic perspective, this lending limit produces an upward sloping supply of funds in the mortgage market, which mirrors the downward sloping demand for credit generated by the borrowing constraint, as illustrated in section 3.

In a more realistic setting with multiple assets, results would be similar if we imposed a constraint on mortgage lending as a fraction of the savers’ total assets, or of their share held in safe securities. In turn, in a stochastic environment, such a limit to households’ exposure to the mortgage market could result from the risk management practices of the financial intermediaries that manage households’ wealth, whether this practices are internal or imposed by regulators. For example, certain large institutional investors, such as money-market funds, pension funds and insurance companies, are restricted to holding only the...
safest securities, and cannot directly access mortgage lending (e.g. Brunnermeier (2009)). Alternatively, one could interpret (2.2) as an approximation of the regulatory capital-ratio constraints of financial institutions. If raising capital is costly—as typically assumed in the literature, e.g. Jermann and Quadrini (2012)—such a constraint would translate into a limit on overall mortgage lending, albeit one not as stark as the one we have assumed in (2.2) for simplicity.

To sum up, expression (2.2) is meant to capture all implicit or explicit regulatory and technological constraints on the economy’s ability to channel funds towards the mortgage market, which produces an upward sloping supply of such funds. Modeling the source of this constraint more explicitly would certainly be useful and interesting, but for now we focus on drawing its macroeconomic implications. This task is taken on in the next section, which characterizes the equilibrium of the model. In section 4, we will use this equilibrium, and its observable implications, to argue that the boom in credit and house prices of the early 2000s is best understood as the consequence of the relaxation of constraints on lending, rather than on borrowing, as most of the literature has been assuming (i.e. as an increase in \( \bar{L} \), rather than in \( \theta \)).

### 2.2. Equilibrium conditions

Given their lower propensity to save, impatient households borrow from the patient in equilibrium, and the lending constraint (2.2) does not influence their decisions. As a consequence, their optimality conditions are

\[
(1 - \mu_t) u'(c_{b,t}) = \beta_b R_t E_t u'(c_{b,t+1})
\]

\[
(1 - \mu_t \theta) u'(c_{b,t}) p_t = \beta_b v'_b(h_{b,t+1}) + \beta_b (1 - \delta) E_t [u'(c_{b,t+1}) p_{t+1}]
\]

\[
c_{b,t} + p_t [h_{b,t+1} - (1 - \delta) h_{b,t}] + R_{t-1} D_{b,t-1} = y_{b,t} + D_{b,t}
\]

\[\text{For instance, Leippold, Trojani, and Vanini (2006) show that a VaR constraint imposed on a bank that can invest in a risky and a riskless asset results in upper and lower bounds on the share of wealth invested in the former. Of course, these bounds are not parametric in their theory, but depend on the expected return and volatility of the risky security, and on the risk-free rate, a complication that we ignore in our deterministic setting. However, their result suggests an intuitive reason why the bound on mortgage lending \( \bar{L} \) might have slackened over the boom, if mortgages came to be seen as less risky over this period.}\]

\[\text{In our extremely stylized economy, this constraint results into a limit on households’ overall ability to save. But this is simply an artifact of the simplifying assumption that mortgages are the only financial assets in the economy, and is not important for the results.}\]
(2.6) \[ \mu_t (D_{b,t} - \theta_p h_{b,t+1}) = 0, \quad \mu_t \geq 0, \quad D_{b,t} \leq \theta_p h_{b,t+1}, \]

where \( u'(c_{b,t}) \cdot \mu_t \) is the Lagrange multiplier on the collateral constraint.

Equation (2.3) is a standard Euler equation weighting the marginal benefit of higher consumption today against the marginal cost of lower consumption tomorrow. Relative to the case of an unconstrained consumer, the benefit of higher current consumption is reduced by the cost of a tighter borrowing constraint. Equation (2.4) characterizes the borrowers’ housing demand: the cost of the forgone consumption used to purchase an additional unit of housing today must be equal to the benefit of enjoying this house tomorrow, and then selling it (after depreciation) in exchange for goods. The term \((1 - \mu_t \theta)\) on the left-hand side of (2.4) reduces the cost of foregone consumption, because the newly purchased unit of housing slackens the borrowing constraint when posted as collateral. Equation (2.4) makes it clear that the value of a house to a borrower is higher when the borrowing constraint is tighter, and when the ability to borrow against it (i.e. the maximum loan-to-value ratio) is higher. Finally, equation (2.5) is the flow budget constraint of the borrowers, and expressions (2.6) summarize the collateral constraint and the associated complementary slackness conditions.

Since the patient households lend in equilibrium, the lending constraint is the relevant one in their decisions. Their equilibrium conditions are

(2.7) \[ (1 + \xi_t) u'(c_{l,t}) = \beta_t R_t E_t u'(c_{l,t+1}) \]

(2.8) \[ u'(c_{l,t}) p_t = \beta_t u'_t (h_{t+1}) + \beta_t (1 - \delta) E_t [u'(c_{l,t+1}) p_{t+1}] \]

(2.9) \[ c_{l,t} + p_t [h_{t,t+1} - (1 - \delta) h_{l,t}] + R_{t-1} D_{l,t-1} = y_{l,t} + D_{l,t} \]

(2.10) \[ \xi_t (-D_{l,t} - \bar{L}) = 0, \quad \xi_t \geq 0, \quad -D_{l,t} \leq \bar{L}, \]

where \( u'(c_{l,t}) \cdot \xi_t \) is the Lagrange multiplier on the lending constraint. When this constraint is binding, the lenders would like to save more at the prevailing interest rate, but they cannot. To make it optimal for them to consume instead, the multiplier \( \xi_t \) boosts the marginal benefit of current consumption in their Euler equation (2.7), or equivalently reduces their perceived rate of return from postponing it. Contrary to what happens with the borrowers,
who must be enticed to consume less today not to violate their constraint, the lenders must
be driven to tilt their consumption profile towards the present when their constraint is
binding. Unlike the collateral constraint, though, the lending constraint does not affect the
demand for houses, since it does not depend on their value. Otherwise, equations (2.3)-(2.6)
have a similar interpretation to (2.7)-(2.10).

The model is closed by imposing that borrowing is equal to lending

\[ D_{b,t} + D_{l,t} = 0, \]

and that the housing market clears

\[ h_{b,t} + h_{l,t} = \bar{h}, \]

where \( \bar{h} \) represents a fixed supply of houses.

2.3. Functional forms. To characterize the equilibrium of the model, we make two con-
venient functional form assumptions. First, we assume that the lenders’ utility function
implies a rigid demand for houses at the level \( \bar{h}_l \). As a consequence, we replace equation
(2.8) with

\[ h_{l,t} = \bar{h}_l. \]

This assumption implies that houses are priced by the borrowers, which amplifies the poten-
tial effects of collateral constraints on house prices, since these agents face a fixed supply
equal to \( \bar{h}_b \equiv \bar{h} - \bar{h}_l \), and they are leveraged. This assumption is appealing for several
reasons. First, housing markets are highly segmented in practice (e.g. Landvoigt, Piazzesi,
and Schneider (2013)) and the reallocation of houses between rich and poor, lenders and
borrowers, is minimal. By assuming a rigid demand by the lenders, we shut down all reallo-
cation of houses between the two groups of agents, thus approximating reality. In addition,
this simple modeling device captures the idea that houses are priced by the most leveraged
individuals, as in Geanakoplos (2010).

The second simplifying assumption is that utility is linear in consumption. As a con-
sequence, the marginal rate of substitution between houses and non-durables does not depend
on the latter, and the level and distribution of income do not matter for the equilibrium in
the housing and debt markets. Therefore, the determination of house prices becomes very
simple and transparent, as we can see by re-writing equation (2.4) as

\[
(2.12) \quad p_t = \frac{\beta}{(1 - \mu t)} \left[ mrs + (1 - \delta) E_t p_{t+1} \right],
\]

where \( mrs = v' \left( \bar{h} - \bar{h}_t \right) \), with the constant marginal utility of consumption normalized to one.

According to this expression, house prices are the discounted sum of two components: the marginal rate of substitution between houses and consumption, which represents the “dividend” from living in the house, and the expected selling price of the undepreciated portion of the house. The discount factor depends on the maximum LTV ratio \( \theta \)—prices are higher if a larger fraction of the house can be used as collateral—and on the tightness of the borrowing constraint \( \mu t \). The tighter the constraint, the more valuable is every unit of collateral.

Although extreme, the assumption of linear utility has the virtue of simplifying the mathematical structure of the model, as well as its economics. With a constant marginal rate of substitution, the only source of variation in house prices is the tightness of the borrowing constraint, as captured by the multiplier \( \mu t \). To the extent that, in the data, part of the movement in prices is driven by changes in the marginal utility of housing services relative to other consumption, this simplification stacks the deck against our goal of explaining the behavior of the housing market in the 2000s. In any case, the appendix (TBW) shows that this simplification is essentially inconsequential for our quantitative results.

3. Characterization of the Equilibrium

The model we presented in the previous section features two balance sheet constraints, both limiting the equilibrium level of debt in the economy. The collateral constraint on the liability side of the borrowers’ balance sheet—a standard tool to introduce financial frictions in the literature—limits directly the amount of borrowing by impatient households to a fraction of the value of their houses \( D_{b,t} \leq \theta p_t \bar{h}_b \). The lending constraint, instead, puts an upper bound on the ability of patient households to extend mortgage credit. But in our closed economy, where borrowing must equal lending, this lending limit also turns into
a constraint on borrowing \((D_{b,t} \leq \bar{L})\).\(^4\) Which of the two constraints binds at any given point in time depends on the parameters \(\theta\) and \(\bar{L}\), but also on house prices, and is therefore endogenous. Moreover, both constraints bind simultaneously when \(\theta \bar{p} \bar{h}_b = \bar{L}\), a restriction that is not as knife-edge as one might think, as illustrated below.

To illustrate the interaction between the two balance sheet constraints, start from the standard case with only a borrowing limit, which is depicted in figure 3.1. The supply of funds is perfectly elastic at the interest rate represented by the (inverse of the) lenders’ discount factor. The demand of funds is also flat, at a higher interest rate determined by the borrowers’ discount factor, but only up to the borrowing limit, where it becomes vertical. The equilibrium is at the (gross) interest rate \(1/\beta_l\), where demand meets supply and the borrowing constraint is binding. This implies a positive value of the multiplier of the collateral constraint \((\mu)\) and a house price determined by equation (2.12), which in turn pins down the location of the kink in the demand of funds in figure (3.1).

Consider now the case of a model with a lending constraint as well, depicted in figure 3.2. With a lending constraint, the supply of funds also has a kink, at the value \(\bar{L}\). Whether this constraint binds in equilibrium depends on the relative magnitude of \(\bar{L}\) and \(\theta \bar{p} \bar{h}_b\). If

\(^4\)In an open economy model with borrowing from abroad, such as Justiniano, Primiceri, and Tambalotti (2014a), this constraint would become \(D_{b,t} \leq \bar{L} + L_{f,t}\), where \(L_{f,t}\) denotes the amount of foreign borrowing. Therefore, in such a model, \(L_{f,t}\) plays a similar role to \(\bar{L}\) in relaxing or tightening the constraint.
If $\bar{L} > \theta p\bar{h}_b$, as in the figure, the equilibrium is the same as in figure 3.1.\textsuperscript{5} In particular, small variations of the lending limit $\bar{L}$ do not affect equilibrium interest rates and house prices.

If instead $\bar{L} < \theta p\bar{h}_b$, the model behaves differently, as shown in figure 3.3. In this case, it is the lending limit to be binding and the interest rate settles at the level $1/\beta_b$, higher than before. Borrowers are limited in their ability to anticipate consumption not by the value of their collateral, but by the scarcity of funds that the savers are allowed to lend in the mortgage market. At the going rate of return, savers would be happy to expand their mortgage lending, but they cannot. As a result, the economy experiences a dearth of lending and a high interest rate. House prices are again determined by equation (2.12), but with $\mu_t = 0$, which puts them below those in the scenario of figures 3.1 and 3.2.

Qualitatively, the transition from a steady state with a low $\bar{L}$ (figure 3.2) to one with a higher $\bar{L}$ (figure 3.3), in which the lending constraint no longer binds, matches well what happened in the U.S. in the 2000s, with interest rates falling and household debt and house prices rising. Section 4 shows that this parallelism also works quantitatively, and that a slackening of the constraint on mortgage lending is also consistent with other patterns in the data.

In contrast, a slackening of the borrowing constraint caused by an increase in the LTV parameter $\theta$ would have opposite effects, making it an unlikely source of the boom observed.

\textsuperscript{5}For this to be an equilibrium, the resulting house price must be such that $\bar{L} > \theta p\bar{h}_b$. 

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure32.png}
\caption{Demand and supply of funds in a model with collateral and lending constraints, with the latter not binding.}
\end{figure}
in the U.S. in the 2000s. Assuming that the borrowing constraint binds initially, as in figure 3.2, an increase in \( \theta \) that is sufficiently large leads to an increase in interest rates from \( 1/\beta_l \) to \( 1/\beta_b \), as the vertical “arm” of the demand for funds crosses over the lending limit \( \bar{L} \), causing the latter to become binding. Moreover, as the borrowing constraint ceases to bind, the multiplier \( \mu_t \) falls to zero, putting downward pressure on house prices.\(^6\)

Intuitively, changes in \( \theta \) affect the demand for credit by its ultimate users, so that an increase in this parameter increases credit demand, driving its price (the interest rate) higher. And with higher mortgage rates, house prices fall. On the contrary, the lending limit \( \bar{L} \) controls the supply of funds from lenders, so that an increase in the limit drives the prices of mortgage credit down and house prices up, also leading to more household debt, at roughly unchanged levels of leverage.

Before moving on, it is useful to consider the case in which \( \bar{L} = \theta p_t \bar{h}_b \), when the vertical arms of the supply and demand for funds exactly overlap. This situation is not an unimportant knife-edge case, as the equality might suggest, due to the endogeneity of home prices. In fact, there is a large region of the parameter space in which both constraints bind, so that \( p_t = \bar{L} / \bar{h}_b \). Given \( p_t \), equation (2.12) pins down the value of the multiplier \( \mu_t \), which, in

\(^6\)Starting instead from a situation in which the lending constraint is binding, as in figure 3.3, an increase in \( \theta \) would leave the equilibrium unchanged.
turn, determines a unique interest rate
\[ R_t = \frac{1 - \mu_t}{\beta_b} \]
via equation (2.3). This is an equilibrium as long as the implied value of \( \mu_t \) is positive, and the interest rate lies in the interval \([1/\beta_t, 1/\beta_b]\).

We formalize these intuitive arguments through the following proposition.

**Proposition 1.** In the model of section 2 there exist two threshold house prices, \( p \equiv \frac{\beta_b \cdot mrs}{1 - \beta_b(1 - \delta)} \) and \( \bar{p}(\theta) \equiv \frac{\beta(\theta) \cdot mrs}{1 - \beta(\theta)(1 - \delta)} \), such that:

(i) if \( \bar{L} < \theta \bar{p} \tilde{h}_b \), the lending constraint is binding and
\[ p_t = p, \quad D_{b,t} = \bar{L} \quad \text{and} \quad R_t = \frac{1}{\beta_b}; \]

(ii) if \( \bar{L} > \theta \bar{p}(\theta) \tilde{h}_b \), the borrowing constraint is binding and
\[ p_t = \bar{p}(\theta), \quad D_{b,t} = \theta \bar{p}(\theta) \tilde{h}_b \quad \text{and} \quad R_t = \frac{1}{\beta_t}; \]

(iii) if \( \theta \bar{p} \tilde{h}_b \leq \bar{L} \leq \theta \bar{p}(\theta) \tilde{h}_b \), both constraints are binding and
\[ p_t = \frac{\bar{L}}{\theta \tilde{h}_b}, \quad D_{b,t} = \bar{L} \quad \text{and} \quad R_t = \frac{1}{\beta_b} \left[ 1 - \frac{1 - \beta_b(1 - \delta) - mrs \cdot \beta_b \theta \tilde{h}_b / \bar{L}}{\theta} \right]; \]

where \( mrs \equiv v'(\tilde{h} - \tilde{h}_t), \tilde{\beta}(\theta) \equiv \frac{\beta_b \beta_t}{\beta_b + (1 - \theta) \beta_t} \) and \( \bar{p}(\theta) \geq p \) for every \( 0 \leq \theta \leq 1 \).

**Proof.** See appendix A. \(\square\)

To further illustrate Proposition 1, figure 3.4 plots the equilibrium value of house prices, debt and interest rates, as a function of the lending limit \( \bar{L} \), for a given value of the loan-to-value ratio \( \theta \). On the left of the three panels, tight lending constraints are associated with high interest rates, low house prices and low levels of indebtedness. As \( \bar{L} \) rises and lending constraints become looser, interest rates fall, boosting house prices and allowing people to borrow more against the increased value of their home. However, the relation between lending limits and house prices is not strictly monotonic. When lending constraints become very relaxed, the model is akin a standard model with collateral constraints, and lending limits become irrelevant for the equilibrium.

The qualitative consequences of a transition towards looser lending constraints—a reduction in interest rates, an increase in house prices and debt, with a stable debt-to-collateral

ratio—square very well with the stylized fact of the period preceding the Great Recession that we have outlined in the introduction. In the next section we will use a calibrated version of our model to analyze its performance from a quantitative standpoint.

4. Quantitative Analysis

This section provides a quantitative perspective on the simple model illustrated above. The model is parametrized so that its steady state matches key statistics for the period of relative stability of the 1990s. We interpret this steady state as associated with a binding lending constraint, as in figure 3.3 above. This assumption seems appropriate for a period in which mortgage finance was still relatively unsophisticated, securitization was a nascent phenomenon, and as a result savers faced relatively high barriers to accessing mortgage loans.

Figure 3.4. Equilibrium house price, debt and interest rates as a function of $\bar{L}$, for a given $\theta$. 
We then analyze the extent to which a lowering of these barriers, in the form of a progressive increase in the lending limit $\bar{L}$, generates a surge in debt and house prices and a fall in interest rates comparable to those observed in the early 2000s. We attribute this slackening of the lending constraint to the diffusion of securitization, initially by the Government Sponsored Enterprises and subsequently also by private labels. Securitization turned an increasing share of mortgages into standardized, essentially safe assets (at least until they weren’t...) traded on financial markets, making them more easily accessible to savers. The main conclusion we draw from this experiment is that looser lending constraints are key to understanding the dynamics of debt, house prices and interest rates in the period leading up to the financial crisis. In contrast, a slackening of borrowing constraints, which we also consider as an alternative source of those dynamics, has entirely counterfactual implications.

4.1. **Parameter values.** Table 1 summarizes the model’s calibration, which is based on U.S. macro and micro data.

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$\beta_b$</th>
<th>$\beta_l$</th>
<th>$\theta$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.003</td>
<td>0.9879</td>
<td>0.9938</td>
<td>0.80</td>
<td>0.0056</td>
</tr>
</tbody>
</table>

Table 1. Calibration of the model.

Time is in quarters. We set the depreciation rate of houses ($\delta$) equal to 0.003, based on the Fixed Asset Tables. Real mortgage rates are computed by subtracting 10-year-ahead inflation expectations from the Survey of Professional Forecasters from the 30-year nominal conventional mortgage rate published by the Federal Reserve Board. The resulting series is plotted in figure 1.4. The average real rate in the 1990s is slightly less than 5% (4.63%) and it falls by about 2.5% between 2000 and 2005. Therefore, we set the discount factor of the borrowers to match a 5% real rate in the initial steady state, and we calibrate the lenders’ discount factor to generate a fall in interest rates of 2.5 percentage points following the relaxation of the lending constraint. The heterogeneity in discount factors consistent with this decline, $\beta_b = 0.9879$ and $\beta_l = 0.9938$, is in line with that chosen by Krusell and Smith (1998) or Carroll, Slacalek, and Tokuoka (2013) to match the distribution of household wealth in the US.
For the calibration of the remaining parameter—the maximum allowed LTV ratio ($\theta$)—we face two main challenges, due to some aspects of the theoretical model that are stark simplifications of reality. First, the model assumes a collateral constraint with a constant loan-to-value ratio over the entire life of a loan. This simple specification, which is by far the most popular in the literature, works well to provide intuition about the working of the model. However, calibrating $\theta$ to the initial loan-to-value ratio of the typical mortgage, say around 0.8, would overstate the aggregate debt-to-real estate ratio in the economy. This is because, in reality, mortgage contracts require a gradual repayment of the principal, leading to the accumulation of equity in the house and therefore to average loan-to-value ratios that are lower than the initial one.

To capture this feature of reality in our quantitative exercises, we generalize the model by replacing the collateral constraint (2.1) with

\begin{equation}
D_{b,t} \leq \theta p_t H_{b,t+1}
\end{equation}

\begin{equation}
H_{b,t+1} = \sum_{j=0}^{\infty} (1 - \rho)^j [h_{t+1-j} - (1 - \delta) h_{t-j}],
\end{equation}

where the last expression can be written recursively as

\begin{equation}
H_{b,t+1} = (1 - \rho) H_{b,t} + [h_{b,t+1} - (1 - \delta) h_{b,t}].
\end{equation}

The variable $H_{b,t+1}$ denotes the share of the housing stock that can be used as collateral, which does not necessarily coincide with the physical stock of houses $H_{b,t+1}$. Equation (4.2) describes the evolution and composition of $H_{b,t+1}$. The housing stock put in place in the latest period $(h_{t+1} - (1 - \delta) h_t)$ can all be used as collateral, and hence it can “sustain” an amount of borrowing equal to a fraction $\theta$ of its market value. However, only a fraction $(1 - \rho)^j$ of the houses purchased in $t - j$ is collateralizable, with the remaining share representing the amortization of the loan and the associated accumulation of equity by the borrowers. If $\rho = \delta$, amortization and depreciation of the housing stock proceed in parallel, so that the entire housing stock can always be used as collateral. As a result, the new collateral constraint is identical to (2.1). If $\rho > \delta$, however, amortization is faster than depreciation, reducing the borrowing potential of the housing stock and the average

\footnote{In this case, the effects of looser lending constraints would be even larger than in the baseline calibration.}
debt-to-real estate ratio in the economy, for any given value of the initial LTV $\theta$. Appendix B characterizes the solution of the model with this generalized version of the collateral constraint.

To calibrate $\theta$, we should look at the mortgages of households that resemble the borrowers in the model. One option would be identify as borrowers all households with mortgage debt in the micro data. However, in the data many borrowers also own a substantial amount of financial assets (Campbell and Hercowitz (2009a), Iacoviello and Pavan (2013), Kaplan and Violante (2014)). Our model is not equipped to capture this feature of the data, since in it agents only borrow because they are impatient, and as a result they do not own any asset, aside from their house. In the data, instead, people might choose to borrow against their houses (instead of running down their financial assets) for a variety of other reasons that are not explicitly considered in our model. To avoid including in the group of borrowers also many individuals possibly with the characteristics of a lender, an alternative, more prudent choice is to identify as borrowers all households with little financial assets.

More specifically, we separate the borrowers from the lenders using the Survey of Consumer Finances (SCF), which is a triennial cross-sectional survey of the assets and liabilities of U.S. households. We identify the borrowers as the households that appear to be liquidity constrained, namely those with liquid assets whose value is less than two months of their total income. Following Kaplan and Violante, we compute the value of liquid assets as the sum of money market, checking, savings and call accounts, directly held mutual funds, stocks, bonds, and T-Bills, net of credit card debt. We apply this procedure to the 1992, 1995 and 1998 SCF.

To calibrate the initial loan-to-value ratio $\theta$, we follow Campbell and Hercowitz (2009b) and take an average of the loan-to-value ratios of the mortgages of all households who purchased their home (or refinanced their mortgage) in the year immediately before the surveys. An average of these ratios computed over the three surveys of 1992, 1995 and 1998 yields a value for $\theta$ of 0.8, a very common initial LTV for typical mortgages, which is also broadly in line with the cumulative loan-to-value ratio of first time home buyers estimated by Duca, Muellbauer, and Murphy (2011) for the 1990s. As for $\rho$, the parameter that governs the amortization speed on the loans, we pick a value of 0.0056, so as to match

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8 Among these households, we restrict attention to the ones who borrow at least half of their home value, since those with lower initial LTVs are probably not very informative on the credit conditions experienced by the marginal buyers.
the average ratio of debt to real estate for the borrowers, which is 0.43 in the 1990s SCFs. Finally, the lending limit \( \bar{L} \) is chosen in the context of the experiments described in the next subsection.

4.2. An expansion in credit supply. In this subsection we study the quantitative effects of a relaxation of lending constraints in the mortgage market. The premise for this exercise is that, at the end of the 1990s, the U.S. economy was constrained by a limited supply of credit, as in figure 3.3 above. Starting in 2000, the lending constraint is gradually lifted, following the linear path depicted in figure 4.1. The experiment is timed so that the lending constraint is no longer binding in 2006. The dotted part of the line in figure 4.1 corresponds to the time periods in which the lending constraint becomes irrelevant for the equilibrium.

In the barebones model presented so far, an increase in \( \bar{L} \) affects the equilibrium house prices and interest rates only in the region in which both the lending and borrowing constraints are binding, as shown in the proposition 1. This is the region in which our numerical experiments put the economy between 2000 and 2006, because the dynamics associated with a relaxation of \( \bar{L} \) in this region are very consistent with the empirical facts we highlighted in the introduction. However, we think that it is plausible that the relaxation of lending constraints was in fact an ongoing phenomenon, that probably started in the 1990s, and possibly earlier. However, the model suggests that this process would have had relatively modest effects as long as the maximum volume of lending was far enough below the borrowing limit, which is why we ignore this earlier period in the simulations.

What real world phenomena are behind the relaxation of the lending limit in the model? Broadly speaking, we think of an increase in \( \bar{L} \) as capturing technological and regulatory developments that made it easier for savings to flow towards the mortgage market. Securitization, for instance, turned mortgages into Mortgage-Backed Securities (MBS). These standardized products could easily be traded on financial markets, and their higher-rated tranches could be acquired by institutional and other investors whose portfolios are restricted by regulation and their own statutes to include only safe assets. This innovation, in turn, opened up the possibility for a large pool of savings previously directed towards Government debt to get exposed to mortgage products, tapping a source of funds that was previously unavailable. Similarly, the capital requirements imposed by regulation on commercial banks imply lower charges for agency MBS (and the senior tranches of private
label ones) than for the mortgages themselves. This creates an interaction between financial innovation and regulation that, again, results into an increase in the amount of capital available for mortgage finance that evolves in parallel with the diffusion of securitization. This interaction was further amplified by the rise of highly levered off-balance-sheet special purpose vehicles, which allowed banks to bypass capital regulations altogether, further increasing the amount of funds directly intermediated towards mortgage lending.

These developments date back at least to the 1980s, when the Government Sponsored Enterprises created the first MBS, but they really took off in the late 1990s and early 2000s, with the expansion of private label securitizations beyond conforming mortgages and ultimately into subprime products. From the perspective of the model, this expansion in credit starts to have effects on the economy only at the point in which the amount of funds channeled to the mortgage market by the loosening of the lending limit approaches the borrowing limit. At least circumstantially, it seems plausible to date this approach of the two limits to the early 2000s, when enough funds had become available to mortgage finance that borrowers’ collateral (and creditworthiness) started to represent a material impediment to further credit expansion. The fact that these borrower-side factors, more than the ability of lenders to direct their savings towards the mortgage market, started to
Figure 4.2 plots the response of the key variables in the model to the loosening of the lending constraints described above. The expansion in credit supply lowers mortgage rates by 2.5 percentage points. This decline reflects the gradual transition from a credit-supply-constrained economy, where the interest rate equals $\frac{1}{\beta_b}$, to a demand-constrained economy, with interest rate $\frac{1}{\beta_l}$. This permanent fall in mortgage rates is a distinctive feature of our environment with lending constraints, and it would be difficult to obtain in the context of more standard models with only collateral constraints, in which the steady state interest rate is always pinned down by the discount factor of the lenders. The magnitude of the decline is in line with the evidence presented in the introduction, but this is just a function of our calibration of the discount factors of the two sets of households.

When the supply of credit is tight, the borrowers are unconstrained, and the interest rate is equal to the inverse of their discount factor. However, when lending constraints become
looser and mortgage rates fall below $\frac{1}{\beta_b}$, the impatient households increase their demand for credit up to the limit allowed by their collateral constraint, which becomes binding. The lower the mortgage rate, the more desirable is borrowing, and the higher is the shadow value $\mu_t$ of the collateral constraint. According to equation (2.12), a rise in $\mu_t$ increases the value of houses to the borrowers, who are the agents pricing them, because their collateral services become more valuable. In our calibration, house prices increase by almost 40 percent in real terms following a lending liberalization, a magnitude that is comparable to the US experience depicted in figure 1.1.

The substantial increase in house prices then relaxes the collateral constraint of the impatient households, allowing them to borrow more against the higher value of their homes. In the model, mortgage debt raises by approximately 30 percentage points of GDP. However, the debt-to-real estate ratio remains unchanged, with debt and home values rising in parallel. This is precisely what happened in the data through 2006, as shown in figure 1.3.

4.3. **Looser collateral requirements.** In this section, we contrast the implications of the loosening of lending constraints described above to what happens in response to a slackening of the borrowing constraint, either through a higher initial LTV, or through slower amortization. This comparison is important, because most of the literature focuses on an increase in borrowing limits as the trigger of the boom in household debt and, to a certain extent, in house prices. Our results suggest however that this view is difficult to reconcile with some of the key stylized facts discussed in the introduction.

We start with an experiment in which the collateral constraint is slackened within an economy without lending limits, which is the baseline case typically considered in the literature. The model is parameterized to match the same targets used in section 4.1, which produces the same values for most parameters, except for $\beta_l$ and $\beta_b$. In particular, $\beta_l$ is set equal to 0.9879 to match the 5 percent average real mortgage rate in the 1990s, since the equilibrium interest rate in this model corresponds to $\frac{1}{\beta_l}$. As for $\beta_b$, we choose the value 0.9820 to maintain the same gap from the discount factor of the lenders as in the previous calibration.

Given this parametrization, we study the effects of a gradual increase in the maximum LTV from 0.8, the baseline value of $\theta$, to 1.02, which corresponds to a situation in which borrowers can borrow up to the entire amount of the house (panel a in figure 4.3). This
increase in \( \theta \) generates the same increase in household debt as in the previous experiment, making the two simulations easily comparable.

Figure 4.4 plots the behavior of debt, interest rates and house prices in response to the change in \( \theta \), contrasting them to the responses of these variables to the relaxation in the lending constraint described above. Even though the model is extremely stylized, the contrast between the continuous and dashed lines highlights the remarkable success of the first experiment in reproducing the stylized facts we are focusing on. In comparison, not much happens when \( \theta \) increases.

First, interest rates remain unchanged as a result of the looser collateral requirements, since lenders are unconstrained and their discount factor pins down the interest rate. In a model with short-run dynamics, interest rates would even increase, in order to convince the patient households to lend additional funds to the now less constrained borrowers (Justíñano, Primiceri, and Tambalotti (2014a)). Second, house prices move very little in response to a rise in the maximum LTV. As a result, the increase in household debt stems from a combination of slightly higher house prices and a higher debt-to-collateral ratio, as shown in the lower-right panel in the figure. Once again, this is counterfactual because the debt-to-real estate ratio was essentially flat over this period.

We obtain very similar results if we drive an increase in household debt through a reduction in the speed of amortization \( \rho \), rather than a rise in \( \theta \). This experiment is depicted by the green dashed-dotted line in figure 4.4. The change in \( \rho \) is calibrated to generate the same dynamics of household debt as in the other two experiments, which requires gradually decreasing \( \rho \) from the initial value of 0.0056 to a value of 0.0041, as shown in panel b of
The response to a loosening of lending constraints and collateral constraints is similar to those generated by the previous experiment, and they are equally counterfactual. House prices increase little and, as a result, the increase in debt is achieved through an increase in leverage, rather than through an increase in the value of collateral that leaves leverage unchanged, as was the case in response to looser lending constraints.

We can think of the previous two experiments as having being conducted in a model in which the lending constraint is in fact present, but is slack enough compared to the borrowing constraint to be irrelevant for the equilibrium. Suppose instead now that $\bar{L}$ is not tight enough to bind when $\theta = 0.8$, but does become binding when $\theta$ reaches a certain threshold during its expansion from 0.8 to 1.02, say when $\theta$ equals 0.9. When this happens, both the collateral and the lending constraints will be binding simultaneously, putting us in case (iii) of proposition 1. A further increase in $\theta$ would then induce a fall in prices, as shown in figure 4.3, suggesting a potential mechanism to account for the reasons why house prices began to fall in 2006, even though credit liberalization was in full swing.
Figure 4.5. The response to a loosening of collateral constraints (an increase in the maximum loan-to-value ratio $\theta$). This figure illustrates what happens when lending constraints are slack given the initial collateral constraints, but become binding when collateral constraints are relaxed enough.

5. Concluding Remarks

An unprecedented boom and bust in house prices and household debt have been among the defining features of the U.S. macroeconomic landscape since the turn of the millennium. Most accounts of this cycle in credit and collateral values—and of the Great Recession and slow recovery that accompanied it—have pointed to changes in the tightness of borrowing constraints, and the consequent change in the demand for credit, as its key drivers. In this paper, we argued that the focus of this discussion should shift from constraints on borrowing to impediments to lending, in particular when it comes to understanding the boom phase of the cycle.

We make this point in a stylized model of borrowing and lending among households, which features both a collateral constraint on the borrowing side, and a constraint on households’ ability to lend in the mortgage market. A progressive loosening of this lending constraint is consistent with four key empirical facts characterizing the boom—the large increase in house prices and in mortgage debt, the stability of the ratio between mortgages and the value of the real estate collateralizing it, and the fall in mortgage interest rates.
The empirical success of the model depends on the interaction between the borrowing and lending constraints, but it cannot be reproduced with either of the two constraints in isolation. In fact, the interaction of the two constraints produces rich dynamics of interest rates, debt and house prices, which might account for both the boom and bust phases of the cycle. A fuller analysis of these dynamics, in a model in which the tightening and loosening of the two constraints is intertwined, is on our research agenda.

Appendix A. Proof of proposition 1

To prove proposition 1, suppose that the lending constraint is binding and the collateral constraint is not, so that \( D_{b,t} = \bar{L} < \theta p_t \bar{h}_b, \, \xi_t > 0 \) and \( \mu_t = 0 \). Under our assumption of linear utility in consumption, \( R_t = 1/\beta_b \) follows from equation (2.3), and equation (2.4) implies \( p_t = \frac{\beta_b}{1-\beta_b(1-\delta)} \cdot mrs \equiv \bar{p} \). For this to be an equilibrium, we have to verify that the collateral constraint is actually not binding, as assumed above. This requires that \( \bar{L} > \theta \bar{p} \bar{h}_b \).

These steps prove part (i) of the proposition.

Suppose now to be in the opposite situation in which the collateral constraint is binding, while the lending constraint is not. It follows that \( D_{b,t} = \theta p_t \bar{h}_b < \bar{L}, \, \xi_t = 0 \) and \( \mu_t > 0 \). We can now derive \( R_t = 1/\beta_l \) from equation (2.7), while equation (2.3) implies \( \mu_t = \beta_b/\beta_l - 1 \). Substituting the expression for \( \mu_t \) into equation (2.4) yields \( p_t = \frac{\tilde{\beta}(\theta)}{1-\theta(1-\delta)} \cdot mrs \equiv \bar{p}(\theta) \), where \( \tilde{\beta}(\theta) \equiv \frac{\beta_b \beta_l}{\beta_b(1-\theta(1-\delta))}. \) This is an equilibrium, provided that \( \bar{L} > \theta \bar{p}(\theta) \bar{h}_b \), which proves part (ii) of proposition 1.

To conclude, we must find the equilibrium of the model in the region of the parameter space in which \( \theta p_t \bar{h}_b \leq \bar{L} \leq \theta \bar{p}(\theta) \bar{h}_b \). The combination of equation (2.3) and (2.7) imply that at least one of the two constraints must be binding, and the results above show that the value of the parameters in this region is inconsistent with one of them being binding and the other not. It follows that both constraints must be simultaneously binding in this region of the parameter space, implying \( D_{b,t} = \tilde{L} = \theta p_t \bar{h}_b \) and \( \mu_t = \frac{L}{\partial \bar{h}_b} \). Substituting the expression for \( p_t \) into equation (2.4), we can compute the value of \( \mu_t = \frac{1-\theta(1-\delta)-mrs \beta_b \theta \bar{h}_b / L}{\theta - \beta_b (1-\delta) - mrs \beta_b \theta \bar{h}_b / L} \) and, using (2.3), for \( R_t = \frac{1}{\beta_b} \left[ 1 - \frac{1-\theta(1-\delta)-mrs \beta_b \theta \bar{h}_b / L}{\theta - \beta_b (1-\delta) - mrs \beta_b \theta \bar{h}_b / L} \right] \). Finally, notice that \( \mu_t \) satisfies \( \mu_t \geq 0 \) as long as \( \theta p_t \bar{h}_b \leq \bar{L} \leq \theta \bar{p}(\theta) \bar{h}_b \), which concludes the proof.
Appendix B. Solution of the model with home equity accumulation

The model used in section 4 to generate the quantitative results differs from the baseline because the collateral constraint allows for the gradual repayment of the principal of the mortgage. This generalization involves replacing expression (2.1) with (4.1) and (4.3). The optimality conditions of the problem of the borrowers become

\[(1 - \mu_t) u'(c_{b,t}) = \beta_b R_t E_t u'(c_{b,t+1})\]

\[(1 - \zeta_t) u'(c_{b,t}) p_t = \beta_b v'_b (h_{b,t+1}) + \beta_b (1 - \delta) E_t [(1 - \zeta_{t+1}) u'(c_{b,t+1}) p_{t+1}]\]

\[(\zeta_t - \theta \mu_t) u'(c_{b,t}) p_t = \beta_b E_t [(1 - \rho) \zeta_{t+1} u'(c_{b,t+1}) p_{t+1}]\]

\[c_{b,t} + p_t [h_{b,t+1} - (1 - \delta) h_{b,t}] + R_{t-1} D_{b,t-1} = y_{b,t} + D_{b,t}\]

\[\mu_t (D_{b,t} - \theta p_t H_{b,t+1}) = 0, \quad \mu_t \geq 0, \quad D_{b,t} \leq \theta p_t H_{b,t+1},\]

\[H_{b,t+1} = (1 - \rho) H_{b,t} + [h_{b,t+1} - (1 - \delta) h_{b,t}]\]

where \(u'(c_{b,t}) \cdot \mu_t\) and \(u'(c_{b,t}) \cdot p_t \cdot \zeta_t\) are the Lagrange multipliers on the constraint \(D_{b,t} \leq \theta p_t H_{b,t+1}\) and the evolution of \(H_{b,t+1}\) respectively. The optimality conditions of the problem of the lenders and the market clearing conditions are unchanged relative to the baseline model.

To solve the model, first notice that that

\[H_{b,t+1} = \frac{\delta}{\rho} \tilde{h}_b.\]

Suppose now that the lending constraint is binding and the collateral constraint is not, so that \(D_{b,t} = \tilde{L} < \theta p_t \frac{\delta}{\rho} \tilde{h}_b, \zeta_t > 0\) and \(\mu_t = 0\). Under our assumption of linear utility in consumption, \(R_t = 1/\beta_b\) follows from equation (B.1), and equations (B.2) and (B.3) imply \(p_t = \frac{\beta_b m_{r,s}}{1 - \beta_b (1 - \delta)} \equiv \tilde{p}\). For this to be an equilibrium, we have to verify that the collateral constraint is actually not binding, as assumed above. This requires that \(\tilde{L} < \theta \tilde{p} \frac{\delta}{\rho} \tilde{h}_b\).

Suppose now to be in the opposite situation in which the collateral constraint is binding, while the lending constraint is not. It follows that \(D_{b,t} = \theta p_t \frac{\delta}{\rho} \tilde{h}_b < \tilde{L}, \zeta_t = 0\) and \(\mu_t > 0\). We
can now derive $R_t = 1/\beta_t$ from equation (2.7), while equation (B.1) implies $\mu_t = \beta_b/\beta_t - 1$. Substituting the expression for $\mu_t$ into equation (B.3) and combining it with (B.2) yields

$$R_t = \frac{\beta_t \cdot mrs}{1 - \beta_t (1 - \delta)} \cdot \frac{1 - \beta_b (1 - \rho)}{1 - \beta_b (1 - \rho) - \theta (1 - \beta_b/\beta_t)} \equiv \tilde{p}(\theta, \rho) > p.$$ 

This is an equilibrium, provided that $\bar{L} > \theta \tilde{p}(\theta) \frac{\delta h_b}{\rho}$. 

Finally, we must find the equilibrium of the model in the region of the parameter space in which $\theta \bar{L} \rho \delta h_b \leq \bar{L} \leq \theta \tilde{p}(\theta) \frac{\delta h_b}{\rho}$. The combination of equation (B.1) and (2.7) imply that at least one of the two constraints must be binding, and the results above show that the value of the parameters in this region is inconsistent with only one of them being binding. It follows that both constraints must bind at the same time in this region of the parameter space, implying $D_{b,t} = \bar{L} = \theta p_t \frac{\delta h_b}{\rho}$ and $p_t = \frac{\rho}{\delta \rho h_b} \bar{L} = \beta_t$. Substituting the expression for $p_t$ into equations (B.2) and (B.3), we can compute the equilibrium value of $\mu_t = \frac{1 - \beta_b (1 - \rho) \cdot \delta \rho h_b / (\rho \bar{L})}{1 - \beta_b (1 - \rho)}$, and verify that it is positive if $\theta \bar{L} \rho \delta h_b \leq \bar{L} \leq \theta \tilde{p}(\theta) \frac{\delta h_b}{\rho}$. We can then obtain

$$R_t = \frac{1}{\beta_t} \left[ 1 - \frac{1 - \beta_b (1 - \delta) \cdot mrs \cdot \beta_b \delta \rho h_b / (\rho \bar{L})}{\theta} \cdot \frac{1 - \beta_b (1 - \rho)}{1 - \beta_b (1 - \delta)} \right]$$

These results can be summarized with the following proposition.

**Proposition 2.** In the model of section 4 there exist two threshold house prices, $p = \frac{\beta_t \cdot mrs}{1 - \beta_t (1 - \delta)}$ and $\tilde{p}(\theta, \rho) = \frac{\beta_t \cdot mrs}{1 - \beta_t (1 - \delta)} \cdot \frac{1 - \beta_b (1 - \rho)}{1 - \beta_b (1 - \rho) - \theta (1 - \beta_b/\beta_t)}$, such that:

(i) if $\bar{L} < \theta \tilde{p}(\theta) \frac{\delta h_b}{\rho}$, the lending constraint is binding and

$$p_t = p, \quad D_{b,t} = \bar{L} \quad \text{and} \quad R_t = \frac{1}{\beta_t};$$

(ii) if $\bar{L} > \theta \bar{L} \rho \delta h_b$, the borrowing constraint is binding and

$$p_t = \tilde{p}(\theta, \rho), \quad D_{b,t} = \theta \bar{L} \rho \delta h_b \frac{\delta h_b}{\rho} \bar{L} \quad \text{and} \quad R_t = \frac{1}{\beta_t};$$

(iii) if $\theta \rho \delta h_b \leq \bar{L} \leq \theta \tilde{p}(\theta) \frac{\delta h_b}{\rho}$, both constraints are binding and

$$p_t = \frac{\rho}{\delta \rho h_b} \bar{L}, \quad D_{b,t} = \bar{L} \quad \text{and} \quad R_t = \frac{1}{\beta_t} \left[ 1 - \frac{1 - \beta_b (1 - \delta) \cdot mrs \cdot \beta_b \delta \rho h_b / (\rho \bar{L})}{\theta} \cdot \frac{1 - \beta_b (1 - \rho)}{1 - \beta_b (1 - \delta)} \right];$$

where $mrs \equiv v'(\hat{h} - \hat{t})$ and $\tilde{p}(\theta) \geq p$ for every $0 \leq \theta \leq 1$. 

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