Human Capital Ladders, Cyclical Sorting, and Hysteresis *

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Abstract

Using administrative data, we document that workers acquire more human capital at more productive firms. Recessions distort workers-firm sorting, flatten the job ladder and impact human capital accumulation, as workers match on average to worse firms. To quantify the aggregate relevance of these effects, we build a directed search model with aggregate risk and worker-firm heterogeneity, in which human capital accumulation depends on firm quality. We estimate the model and show that recessions have persistent negative effects on the productivity of worker-firm matches, with distortions in sorting and human capital accumulation accounting for approximately 30% of cumulative output losses.

Keywords: Human Capital Accumulation; Hysteresis; Sorting; Scarring;
JEL Codes: J24; E32; E24; J63

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1 Introduction

Extensive research demonstrates that sorting – the degree to which more productive workers are matched with more productive firms – is highly cyclical. We introduce novel empirical evidence illustrating how changes in sorting can impact output dynamics across the business cycle. First, using matched employer-employee data from Italy, we establish a novel empirical finding: workers employed in more productive firms accumulate more human capital. Specifically, we show that earnings' returns to experience are larger for individuals employed in more productive firms, and persist long after individuals have moved on from those firms. Second, we show in the data that sorting deteriorates in recessions: workers who start a new job are on average going to do so in a relatively less productive firm, and will be less able to move towards higher rungs of the firm productivity ladder. Cyclical changes in sorting can then have long-term effects on the stock of human capital in the economy.

The implications of these empirical findings for the allocative effects of recessions are not straightforward. While recessions affect workers by forcing displacements and impacting their lifetime human capital accumulation, they also lead to labor reallocation across firms. This reallocation can potentially result in efficiency gains for the overall economy if workers are induced to move towards better jobs by the cleansing of less productive firms. It is then natural to ask whether recessions induce hysteresis, meaning persistent negative impacts on the productivity of workers and firms, or whether they promote long-term, efficiency-enhancing cleansing effects.

Evaluating the relative importance of hysteretic versus cleansing effects requires tracking the heterogeneity of workers in skill and career progression, their matching with firms of varying productivity, and how these distributions jointly evolve with business cycles. To this end, we develop a directed search model with heterogeneity among firms and workers, an overlapping generations structure, on-the-job search, and aggregate risk. The influence of firms on workers' earnings profiles is captured by match-dependent human capital accumulation, consistent with our empirical evidence. We thus introduce a new channel that links on-the-job learning with labor market sorting and human capital accumulation.

We estimate the model and find that that the persistent negative effects of recessions on the productivity of worker-firm matches outweigh any cleansing effects. Our findings are consistent with empirical evidence of initial cleansing effects, followed by persistent losses in productivity (Haltiwanger, Hyatt, McEntarfer and Staiger, 2022). We find that recessions impair the sorting process and distort the paths of human capital accumulation: nearly one-third of the cumulative post-recession output losses are due to the disruption in the sorting of workers to firms, a "flattening" of the job ladder, and the consequent erosion of aggregate human capital.

The heterogeneous and persistent costs of job displacement were previously not captured in other models of sorting over the business cycle (Lise and Robin 2017,Baley, Figueiredo and Ulbricht 2022). Conversely, models of the costs of job loss (Jarosch 2023, Huckfeldt 2022) abstracted from aggregate sorting implications. Considering these two aspects together yields a deeper understanding of the labor market dynamics around recessions.

A key feature our theoretical framework hinges on is that the production of human capital is linked to the match quality between workers and firms. In our model workers employed by more productive firms experience faster human capital accumulation.[1] This modeling choice is supported by our novel findings regarding differential experience returns across firms of varying productivity. We establish that, controlling for worker quality, the wage increase attributed to one year of experience at a top productivity firm is over twice that at a firm at the lower end of the productivity spectrum. Furthermore, we find that the experience returns are persistent, maintaining their value through periods of unemployment or displacement, and that this effect is robust to various specifications and definitions of firm quality.[2]

The evolution of human capital is not the only driver of workers' pay. Profit-maximizing firms strategically offer contracts to attract and retain the most productive workers. Given that workers cannot credibly commit to refraining from job searching while employed, the optimal contract design will take into account the evolution of search incentives as the match evolves, and the fact that workers accumulate human capital. Furthermore, considering workers' risk aversion, there is a demand for insurance against fluctuations in the value of the match. The state-contingent contract incorporates front-loading of firms' profits and back-loading of workers' compensation to optimize firm retention while offering partial insurance to workers against both idiosyncratic and aggregate shocks. Younger workers, who are in general less productive but highly mobile, receive significant wage increments as firms try to boost retention. Incentives to move onto the next job increase as workers accumulate human capital, and can target better vacancies. Job-to-job transitions thus emerge as a crucial driver of wage progression (Topel and Ward, 1992), with workers moving from less to more productive firms that offer higher-paying contracts. Life cycle earnings dynamics will be driven by human capital accumulation, tenure effects, and to

[1] An expanding body of literature highlights the significant role of firm characteristics in shaping workers' human capital accumulation and skill development (see Herkenhoff, Lise, Menzio and Phillips 2018; Jarosch, Oberfield and Rossi-Hansberg 2021; Mion, Opromolla and Ottaviano (2022); Arellano-Bover and Saltiel 2022; Gregory 2023).

[2] Changes in the firm class of employment imply earnings losses compatible with high but incomplete human capital portability across firms, or a limited role of firm-specific human capital (Kambourov and Manovskii, 2009), a feature that we reproduce in the model.

the climbing of the job ladder. The simulated model accurately reproduces observed patterns of wage growth both within and across firms.

In order to accurately capture the role and heterogeneity of cyclical displacements and endogenous layoffs, we need to impose further assumptions. The provision of insurance from firms to workers ensures that the wage schedule outlined in contracts remains non-decreasing as long as the match retains positive value, implying a floor for wage growth. We also posit that wages are downward rigid, meaning that adverse shocks could lead to separations that are inefficient despite both parties being fully informed about the contract's terms in advance.[3] With these assumptions, the model successfully matches the empirical patterns of separations due to firm layoffs, considering variations across age, worker, and firm productivity levels. Displacements occur more frequently in less productive firms, typically employing younger and less skilled workers, and these firms are also more likely to enact layoffs during recessions. The simulated output and unemployment volatilities in the model match their empirical counterparts, a challenging result for standard search and matching models.

Two main features ensure the tractability of our framework. First, we prove that workers can map the productivity of each firm to the value of the posted vacancy. This ensures that the optimal search strategy is unique, and that target firm productivity is monotonously increasing in workers' human capital level, reducing the number of states of the problem and excluding the presence of multiple optima in search strategies. Second, the choice of modeling search as directed and the assumption of free entry allow us to prove that the model features a block recursive equilibrium (Menzio and Shi, 2010). The model can then be solved without needing to track the distribution of agents as state variables in the optimization problem.

Incorporating firm-dependent human capital accumulation alters the interpretation of cleansing effects during the initial stages of recessions. While the exit of less productive firms is conventionally seen as cleansing, it predominantly affects younger workers in less productive matches, leading to displacements that have enduring effects (see Huckfeldt 2022, Jarosch 2023). Workers who are unable to climb the career ladder due to initial displacements largely experience job changes through movements in and out of employment, thus matching to less productive vacancies. Consequently, cleansing in the short term may actually hinder long-term growth, as displacements and lower quality employment-to-employment transitions impede human capital accumulation, trapping certain workers at lower levels of the job ladder.

[3] A significant body of research highlights strong resistance to cuts in nominal or real wages, suggesting that layoffs are a more probable reaction to negative productivity shocks (Altonji and Devereux 1999; Agell and Lundborg 2003; Grigsby, Hurst and Yildirmaz 2021; Bertheau, Kudlyak, Larsen and Bennedsen 2023*b*). Our analysis further shows that a model without this feature cannot accurately reproduce the observed joint dynamics of output and unemployment or the cyclical nature of firm layoffs.

Following a recessionary shock, the scarring effects on workers' careers stem from a decline in their outside options and an endogenous reduction in the economy's overall productive capacity. Multiple channels contribute to this outcome. First, many workers are displaced throughout a recession, and they need to restart climbing the job ladder from the bottom. Second, wage growth decreases for all surviving matches, as workers' outside options deteriorate. Third, the likelihood of workers moving across jobs if employed, or towards any job if unemployed, diminishes. The driving force behind the contraction in job mobility is the flattening of the job ladder for *all* workers, given a decrease in job openings.[4] Notably, this effect is stronger at higher rungs of the ladder, where vacancies are more expensive to create and exhibit greater sensitivity to economic cycles. A flatter job ladder impairs workers' productivity growth and their upward mobility. Workers match up the ladder with firms of lower quality, form less productive matches and thus accumulate less human capital. Therefore, changes in worker-firm sorting amplify and prolong the impact of recessions on earnings and aggregate output, leading to hysteresis.

The relative importance of distortions in sorting and human capital accumulation can be recovered by decomposing the cumulative output loss of recessions into four channels. Three years after a negative productivity shock, 30% of losses stems from a worsened sorting between workers and firms: 13% is caused by the decrease in firm quality in matches formed throughout the recessionary and recovery periods (*firm quality* sorting channel), and 17% arises from the decrease in human capital accumulation (*human capital* channel). In addition, distortions in workers' job finding probabilities with respect to normal times (*search* channel) account for approximately 35%. The remaining portion is ascribed to direct displacement effects (the *displacement* channel). This exercise implies that recessions have a sullying effect (Barlevy, 2002): distortions in sorting and human capital accumulation lead to a persistent deterioration of match quality, generating a loss in output that the economy never fully reabsorbs.

We further show that the severity of recession and length of recoveries are state-dependent and positively related to the average and skewness of the firm labor share distribution. State-dependent responses at the micro level, both in layoffs and in the search behavior, drive this aggregate feature, consistently with the idea of firm level fragility building up as labor share and tenure within matches grows.[5]

Finally, we estimate the welfare cost of business cycles. The incidence of these costs varies markedly by income and age. We find them to be greater for prime-age workers, and roughly U-shaped in income, with the greatest costs at the extremes of the income

[4] The pro-cyclicality of high-quality jobs has been documented, among others, by Moscarini and Postel-Vinay (2016).

[5] The role of labor share as a proxy of operating leverage within firms has been observed in corporate finance and labor studies (Favilukis, Lin and Zhao 2020, Acabbi, Panetti and Sforza 2023). Similar dynamics are also present in Ai and Bhandari (2021) and Dupraz, Nakamura and Steinsson (2022).

distribution. For low–skill workers, which tend to have lower wages, costs are driven by displacement and lower job-finding probabilities. For higher–skill workers, on the other hand, the distortions in sorting and human capital accumulation matter more than the immediate displacement effect.[6]

Related literature. Our work relates to three strands of empirical research.

First, we contribute to the existing body of empirical evidence on the importance of worker-firm matching in explaining earnings dispersion. Our novel contribution is to show that workers accumulate more human capital when matched in more productive firms. Our empirical exercise is close in spirit to Arellano-Bover and Saltiel (2022), who provide empirical methods to classify firms based on estimated firm-dependent human capital accumulation profiles. Other studies leverage specific firms characteristic determining differential human capital accumulation within firms: co-workers quality (Herkenhoff et al. 2018, Jarosch et al. 2021), international outreach (Mion et al., 2022), location within cities (De La Roca and Puga, 2017), or specific returns to formal education (Engbom and Moser 2017, Deming 2023). Our paper provides extensive evidence of heterogeneous firm dependent human capital accumulation, increasing in firm productivity, and estimates persistent firm-specific returns to experience on the universe of Italian private employment over 25 years of data and various business cycle conditions. We show that despite this finding, and consistently with what Gregory (2023) finds for Germany, in our model most of the human capital accumulation of workers takes place in relatively low productivity firms. This finding emerges endogenously as an equilibrium outcome of the model, absent any ad-hoc restriction on age-dependent learning ability.

Second, we relate to the empirical literature analyzing the impact of business cycles on workers' earnings, exploring how recessions impact workers both via worse career prospects and by forcing displacements (Kahn 2010; Oreopoulos, Von Wachter and Heisz 2012; Arellano-Bover 2022; Schmieder, Von Wachter and Heining 2023).[7] Our model accurately characterizes the so-called "scarring effects" of recession for workers, and provides an evaluation of their aggregate effects for output and productivity.

Third, we relate to the literature analyzing the effects of recessions on productivity, debating whether recessions tend to have long-lasting negative effects on the productivity distribution (Barlevy 2002; Haltiwanger et al. 2022; Crane, Hyatt and Murray 2023) or, by freeing up resources for more efficient uses they generate cleansing effects (Davis, Haltiwanger and Schuh 1996). Our findings align with empirical evidence showing initial

[6]These results for welfare echo and extend the results for earnings found by Heathcote, Perri and Violante (2020) and Doniger (2023), who find that recessions amplify income inequality and tend to have immediate stronger effects for lower income workers.

[7]Our results also reproduce, as untargeted moments, earning and employment dynamics of workers following displacement events (Jacobson, LaLonde and Sullivan, 1993, Schwandt and von Wachter, 2019, Schmieder et al., 2023, Bertheau, Acabbi, Barcelo, Gulyas, Lombardi and Saggio, 2023*a*).

cleansing effects, overshadowed by a sustained loss in productivity in the medium run (Haltiwanger et al. 2022). We use the insights from this empirical literature to calibrate our model and benchmark our untargeted results in the estimation, and then use the model to analyze the productivity long run dynamics after a recessionary period.

Our model of the labor market builds on the directed search models developed in Menzio and Shi (2010), Menzio, Telyukova and Visschers (2016). We adopt a standard wage setting protocol from the literature on dynamic contracts as in Thomas and Worral (1988) and, more recently, Balke and Lamadon (2022). In both cases, we extend their framework to explicitly account for two-sided heterogeneity and on-the-job human capital accumulation to study how business cycles interact with workers' decisions, firms' optimal retention policies, and influence labor market sorting.

The centrality of worker-firm sorting and on-the-job learning in busyness cycle dynamics is closely related to Lise and Robin (2017), Baley et al. (2022), Herkenhoff, Phillips and Cohen-Cole (2023), Gulyas (2023), Adda and Dustmann (2023), Carrillo-Tudela and Visschers (2023), Carrillo-Tudela, Visschers and Wiczer (2023), Blanco, Drenik, Moser and Zaratiegui (2023). By linking human capital accumulation to firm quality we extend previous analyses to quantify the feedback channels between labor market sorting and fluctuations in aggregate output.[8] In Carrillo-Tudela and Visschers (2023) and Carrillo-Tudela et al. (2023), workers accumulate occupation-specific human capital and the cyclical behavior of occupational mobility interacts with unemployment duration and workers' careers. The collapse of the job ladder following a recession makes the sullying effects of recessions long-lasting by dampening the returns to occupational mobility. Similarly, Baley et al. (2022) focuses on cyclical mismatches in tasks. In their setting, workers are uncertain about their ability. Displaced workers might find it optimal to switch occupations during recessions, when the cost of switching is comparatively lower. Recessions in their framework are thus cleansing. We abstract from modeling occupational mobility in our framework, and focus on workers' general human capital and life-cycle and cyclical employment along the productivity ladder.

Jarosch (2023) uses a search model in which workers progressively climb a "slippery" job ladder to gain job security, but accumulate human capital when employed. Persistent effects derive from the fact that jobs exogenously differ by productivity and layoff risk. In our framework,unemployment risk over the life cycle is a by-product of the endogenous sorting between workers and firms of different qualities.[9] A model

[8]For a discussion on the importance of human capital accumulation as a determinant of the dispersion in workers' career earnings see also Bagger and Lentz (2019), Lise and Postel-Vinay (2020), Taber and Vejlin (2020), Griffy (2021).

[9]Given the model's treatment of pay dynamics, our research is also related to a strand of literature in labor and finance analyzing the firms' management of liquidity and labor compensation dynamics (Xiaolan 2014; Favilukis et al. 2020; Acabbi and Alati 2022; Acabbi et al. 2023).

ignoring our novel empirical findings would fail to accurately characterize workers' scarring effects and output or unemployment dynamics. Absent match-dependent human capital accumulation, our model would fail to match output and unemployment volatilities, as well as the pattern of separations in the data. Our findings suggest that the evolution of pay dynamics and labor share within an employment spell, driven by firm retention incentives and human capital accumulation, are important ingredients to accurately characterize firm layoffs and workers' mobility.

2 Firms matter for human capital accumulation

We leverage unique longitudinal matched employer-employee data from Italy to shed new light on the role played by firms in shaping workers' human capital accumulation and career trajectories. We proxy human capital accumulation by looking at persistent earnings' returns to experience. We first highlight that experience returns are greater for individuals employed in highly productive firms, and their effect persists after individuals have moved away from those firms, even after displacement from the previous job. Second, we show that past firm quality matters for re-employment wages. Workers finding a job after an unemployment spell earn more the greater the productivity of their previous firm, even after controlling for their past wage. Lastly, we show that in the data workers tend to match with firms with lower productivity in recession periods.

We interpret these results as evidence that workers accumulate human capital at different rates based on the productivity of the firms they work for, and this effect varies over the cycle. As a consequence, in the rest of the paper we incorporate firm-dependent human capital accumulation in a labor market search model with heterogeneous workers and firms and aggregate risk, to evaluate the aggregate effect of distortions in sorting and human capital production along the business cycle.

2.1 Firm dependent returns to experience

Measuring human capital accumulation is challenging. In this section, we proxy human capital accumulation by looking at potentially different earnings' returns to experience across firms (Mion et al., 2022, Arellano-Bover, 2022). The key intuition is that, if workers accumulate more human capital in a certain firm type with respect to the other, this should translate into persistent changes in their lifetime earnings trajectory, regardless of unemployment events and job mobility.

We use administrative data on the universe of Italian workers and firms' employment, as well as firms' balance sheet data, spanning 1994 to 2019.[10] We assign each firm, f, to a

[10]We refer the reader to **Appendix A** for a detailed description of data characteristics and cleaning.

class according to its productivity. Firm classes are defined as quintiles of value added per employee. Then, for each worker i we construct a measure of experience in each firm-class and we estimate the following wage equation,

$$\log(w_{i,t}) = \alpha_i + \alpha_{j(i,t)} + \sum_{c=1}^{5} \beta_c e_{i,t}^c \times \mathbb{I}\{f(i,t) \in c\} + \sum_{c=1}^{5} \gamma_c e_{i,t}^c \times \mathbb{I}\{f(i,t) \notin c\} + \mathbf{X}_{i,t}'\theta + \varepsilon_{i,t}, \quad (1)$$

where $w_{i,t}$ is a monthly wage[11], $e_{i,t}^c$ are the number of years worker i has worked in firms belonging to productivity class c up to year t, $\mathbb{I}\{\cdot\}$ are indicator functions that take value one when the worker's current employer, $f(i,t)$, belongs to quintile c, and $\mathbf{X}_{i,t}$ are a set of controls that include age, sector-year and contract type (temporary versus open-ended, full-time versus part-time) fixed effects. In the spirit of the AKM literature (Abowd, Kramarz and Margolis, 1999), all regressions control for worker and firm fixed effects α_i and $\alpha_{j(i,t)}$. The firm fixed effect is estimated on one j of 10 latent firm types obtained by applying the K-means measure reduction method on the data as in Bonhomme and Manresa (2015), using the wage decile distribution for clustering similarity.

The main coefficients of interest are $\{\beta_c, \gamma_c\}_{c=1}^5$. They capture the returns to experience when working in a firm in the c^{th} quintile of the labor productivity distribution. Importantly, β_c measures the returns to experience when the worker is *currently* employed in firms belonging to a certain class, while γ_c captures the returns of having worked in a given firm class at some point in the worker's career. Distinguishing between the two cases is useful to assess how portable is the experience gained in any firm-class.[12] If one year of experience is more valuable when accrued in a more productive firm, we would expect $\{\beta_c, \gamma_c\}_{c=1}^5$ to be positive and increasing with firms' productivity. A positive relationship between firm productivity and wages could also be consistent with employers being forced to match outside offers, as in sequential bargaining models à la Postel-Vinay and Robin (2002), while workers are not becoming more productive over time. In order to rule out this alternative explanation, we estimate **Equation** (1) also on a sample of displaced workers, only for the years following their displacement event.[13] By virtue of the displacement, the bargaining position of workers is equalized to their unemployment value and the returns to experience should better

[11] Throughout the section we use monthly wages as the variable of reference for labor earnings. Results are qualitatively analogous when using annual earnings.

[12] In **Appendix B** we also report the estimate from the same class of regressions without distinguishing between current and past firm classes' experience returns, and allowing for quadratic returns to experience. The latter results highlight, consistently with our model, that firm-specific marginal experience returns are diminishing (Gregory, 2023).

[13] The definition of "displacement" follows the logic of Jacobson et al. (1993) and the criteria of Bertheau et al. (2023*a*). We identify displacement events as a decrease in employment beyond 30% for establishment with more than 49 employees. Displaced workers are separating workers younger than 50 years old, with at least 3 years of tenure within the firm. Displaced workers cannot be recalled, and we perform additional checks to control for false establishment closures (mergers, acquisitions, reorganizations within the same firm/group).

capture the premia associated with an increase in skills.

Results. **Figure 1** plots the returns to experience for the full sample of workers, Panel (a), and for the subsample of displaced workers, Panel (b).[14] Given the sample size, coefficient are estimated with an extreme degree of precision. In both cases, the returns to experience are increasing in firm classes. In particular, one year of experience in a firm belonging to the top 20% of the productivity distribution is associated with a wage increase more than twice as large than the one implied by working an extra year for a firm in the bottom 20%, 1.6% versus 0.6%. While the level of returns is lower once we remove returns to bargaining positions, the positive gradient in the wage experience profiles is preserved also in the sample of displaced workers, reinforcing the interpretation that experience in firms belonging to higher productivity classes benefits workers' earnings beyond the potential availability of better outside offers.[15] The results are robust to using quintiles of firm-cluster fixed effects estimated using a rolling-AKM algorithm (Lachowska, Mas, Saggio and Woodbury, 2023), in which we address the limited mobility bias by clustering firms ex-ante on their labor earnings distribution as in Bonhomme and Manresa (2015), Bonhomme, Lamadon and Manresa (2019). The estimated β and γ coefficients are of similar magnitude across both samples, with returns to experience outside of the firm class of reference (γ) amounting to between 70% and 90% of those within firm class (β). In sum, evidence suggests that experience returns are highly portable across firms, which is entirely consistent with the persistent accumulation of human capital on the job for workers.[16]

We take the results as evidence that workers' returns to being employed in different firms are highly heterogeneous and depend on firms' productivity. Firm characteristics and workers' employment histories therefore can have significant effects for the long-term dynamics of their careers.

2.2 Persistent effects of firm quality on workers' earnings

The results presented in **Figure 1** highlight how experience in differently productive firms persistently influences labor earnings. We now provide complementary evidence that this effect is present even when workers move to new employers from unemployment. While this context abstracts from the role of bargaining, it is still exposed to separate issues.

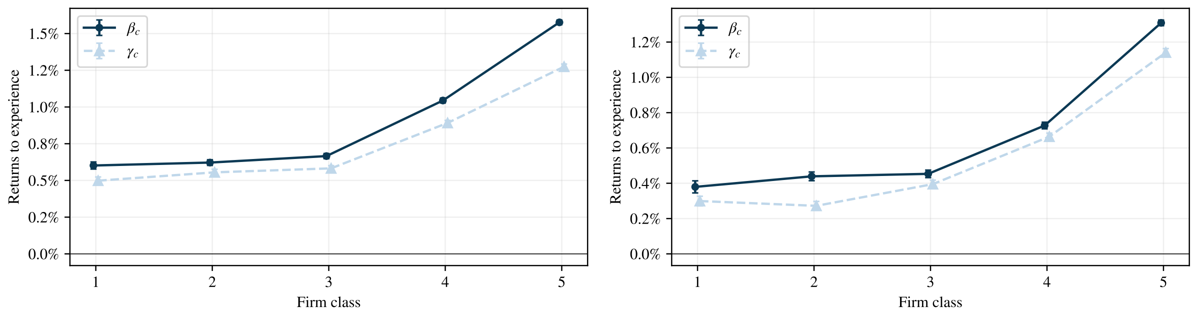
[14] In **Appendix B** we report robustness results using firm cluster AKM fixed effects quintiles as measures of firm quality, and the tables of coefficient for all regressions.

[15] Mion et al. (2022) shows on Portuguese administrative data that the experience wage premia for workers are higher in internationally active firms. Our results extend and complement theirs, by showing that firm productivity is monotonically correlated with experience premia for workers.

[16] Remarkably, our model delivers very close estimates of the share of portable human capital across jobs or into unemployment spells, without ever explicitly targeting the empirical evidence of this section in the calibration exercise.

Figure 1: Experience profiles

(a) Full sample **(b)** Displaced workers



Note: The figure reports the returns to experience estimated from Equation (1). The full sample includes all workers employed in private firms in Italy and includes approximately 98M observations (total worker-year). Displaced workers are identified from mass layoffs and the sample includes approximately 5M observations. Error bars are 99.9% confidence intervals with standard errors clustered at the worker level. When estimating the returns for the sample of displaced workers we include the worker and firm-class fixed effects estimated on the full sample as regressors. We report more details on the sample construction in Appendix A.

Conditioning on unemployment spell duration might induce selection on workers' quality out of unemployment. Not controlling for it, on the other hand, might confound the effect with duration dependence dynamics. Workers with lower skills tend to experience greater unemployment duration and to work in lower quality firms. As their human capital might deteriorate in unemployment, we might spuriously measure lower wage effects for low productive firms only because the workers themselves have lost human capital.[17]

We thus use two different subsamples of the data to address selection and negative duration dependence (Kroft, Lange and Notowidigdo, 2013, Schmieder and Von Wachter, 2016). First, we use the subsample of workers undergoing a displacement event, which is a standard way in the labor literature to identify plausibly exogenous separations. Second, we identify a subsample of workers undergoing an unemployment spell between two periods of *stable* employment at *different* firms: an employment-unemployment-employment (E-U-E) transition.[18]

We estimate how the origin firm's quality, before an unemployment spell, predicts labor earnings after the worker is re-employed:

$$\log(w_{i,t+1+\tau}) = \alpha \log(w_{i,t}) + \sum_{c=1}^{5} \beta_c \mathbb{I}\{f(i,t) \in c\} + \mathbf{X}'_{i,t}\theta + \varepsilon_{i,t}, \tag{2}$$

where $w_{i,t+\tau}$ is the wage earned by worker i, τ-periods after the E-U-E transition, $w_{i,t}$ is

[17]Recent evidence on labor displacement from Bertheau et al. (2023*a*) shows that for the Italian labor market the loss in earnings is mostly driven by lower employment probabilities.

[18]We follow Herkenhoff et al. (2018) for the definition of E-U-E events. Workers within this subsample must be employed the whole year in years t and $t+2$ in this small panel, and undergo at least one quarter of unemployment in year $t+1$. We exclude quits when identifying unemployment spells.

the wage at the time of separation and $\mathbb{I}\{f(i,t) \in c\}$ is an indicator function that takes value one if the employer at the time of separation belonged to productivity quintile c. $\mathbf{X}_{i,t}$ are a set of controls that include sex, contract type, occupation and sector-time fixed effects. The main coefficients of interest are $\{\beta_c\}_{c=2}^{5}$, $c = 1$ being the excluded category. They indicate how the quality of past employers influences future earnings at a specific horizon τ after unemployment spells. Controlling for past wages allows us to indirectly control for the possibility that workers in unemployment still retain different bargaining positions by virtue of potentially different generosity of unemployment benefits, a feature absent in the previous specification.

Results. Figure 2 reports the main coefficients of interest for two horizons τ: one and five years after a displacement event (left panel), or unemployment spell through stable jobs as defined in the previous section, (right panel).[19]

For both samples, two features are worth highlighting. First, the coefficients are monotonically increasing. This indicates that workers previously employed in relatively better firms, *conditional* on their past wages, obtain higher wages after an unemployment spell. Second, even five years after unemployment, past firms have a significant effect on workers' earnings. The impact of past firm quality is very persistent across all firm types, and similar across subsamples. The stability and significance of our estimated results over time and across subsamples provides reassuring evidence that selection and duration-dependence cannot explain our effects.

Overall, we conclude that the quality of past employers bears a long-lasting effect on workers' earnings even when workers undergo involuntary unemployment spells, consistently with persistent on-the-job human capital accumulation.

2.3 Worker-Firm Sorting and the Business Cycle

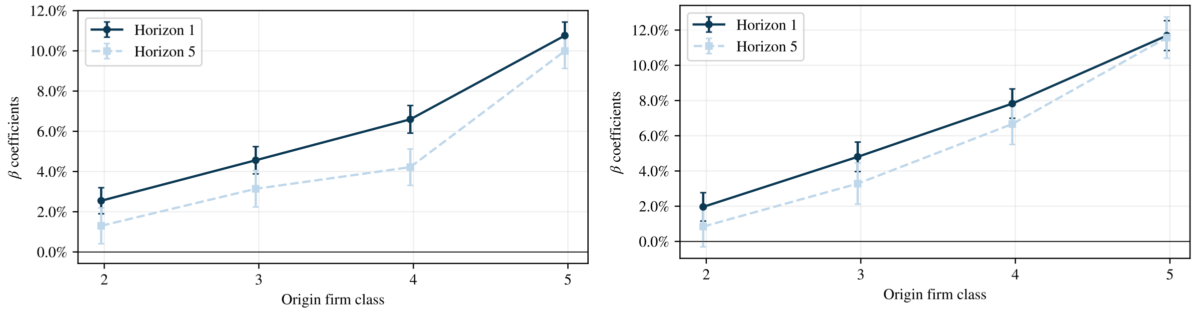
Workers accumulate more human capital when employed in more productive firms. This creates a job ladder in firm quality, which workers would want to climb to improve their human capital. **Figure 3a** shows that such ladder exists in the data. Each quarter, workers are grouped into quintiles of rolling-windows AKM person effects, while firm quality is again measured using value-added per employee. We then calculate the quarterly job-to-job transitions. The figure shows the average firm class destination for every class of worker across our sample: more than 25% of workers in the top quintile move towards firms in the top quintile, while less than 10% of workers in the median quintile do. Conversely, more than 25% of workers in the bottom quintile find jobs in the bottom quintile of firms.

While in general better workers will tend to move towards better firms during their

[19] In **Appendix Section B** we report robustness results using firm cluster AKM fixed effects quintiles as measures of firm quality, and the tables of cooefficient for all regressions.

Figure 2: Persistent effect of past firms for workers

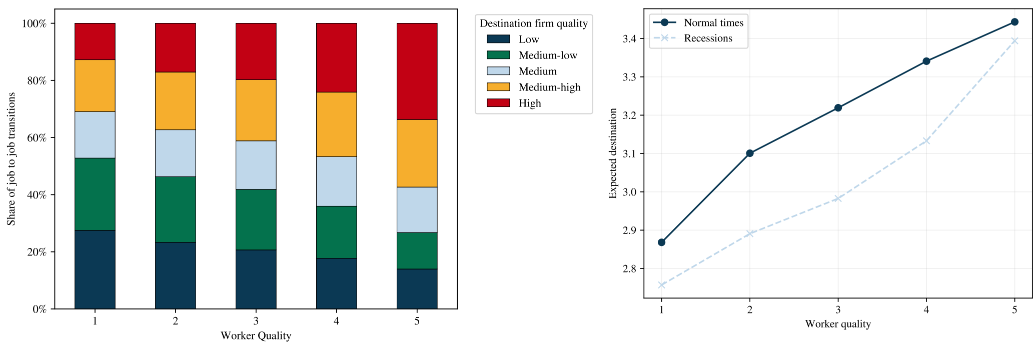
(a) Displaced workers **(b)** Short unemployment spells



Note: The figure reports the main coefficient of interest for the regression in **Equation 2**. The displaced workers sample includes workers that make E-U-E transitions following mass layoffs, while the short unemployment spell sample identifies all E-U-E transitions in which workers are employed for at least 4 quarters after an unemployment spell of less than one year. Firm quality is defined as (yearly) quintiles of firm value added per employee. N = 577,174 (left), 371,593 (right).

Figure 3: Worker and firm characteristics for new matches and over the cycle

(a) Sorting for new matches **(b)** Sorting over the cycle



Note: The figure presents how job-to-job flows are directed towards different quality firms in the data (Panel a) and how workers expected firm quality of destination changes over the cycle (Panel b). Workers are divided into (quarterly) quintiles of own AKM fixed effect, while the measure of firm quality is (quarterly) quintile of value added per worker. In Panel (a): each stacked bar reports the shares of job-to-job transitions ending into each firm quality bin. In Panel (b): the average firm quality of destination by worker quality, distinguishing between normal times and recession periods. Flows into unemployment are considered as zeros in the calculation. The figure uses quarterly rates from all pooled years, distinguishing quarters of recession from normal times, in the matched employer-employee dataset.

career, the job ladder for each worker might be relatively flatter in recessions. Because of the role of firms as learning environments, recessions would thus impact not only the allocations of workers to firms, but also the production of human capital in the aggregate.

Figure 3b shows that this is indeed what we see in the data: workers have a harder time matching with good firms in recessions. In the figure we consider all workers who are moving out of their current match - because of a job to job transition, a voluntary separation, a firing, or a dismissal. The expected firm class the worker is going to be employed at in their next employment spell is then displayed for different moments in the business cycle. By forcing workers who are moving to new jobs to move to comparatively worse firms (Hershbein and Kahn, 2018), recessions impact human capital accumulation. From the workers' perspective, a recession means getting a job of the same quality as in good times is comparatively harder.

Combining the results of this section, we point to long-run costs of business cycles that operate via a collapse of the job ladder that impacts human capital accumulation, and in turn persistently induces a lower worker-firm sorting. The next section is going to introduce a theoretical framework to account for the relative importance of these effects.

3 Model

We start by presenting the environment and the preference structure of workers. We discuss the features of the frictional labor market with directed search and finally we characterize the worker problem, the optimal contract and the equilibrium.

3.1 Environment

Time is discrete, runs forever and is indexed by $t \in \mathbb{N}$. We denote future values in recursive expressions by adding a $'$ to them, or index elements by t in non-recursive ones.

The economy is populated by a unit mass of $T \geq 2$ overlapping generations of finitely lived, hand-to-mouth, risk-averse workers and a continuum of risk-neutral entrepreneurs. All agents in the economy share the same discount factor $\beta \in (0, 1)$. Workers live for T periods, with age $\tau \in \mathcal{T} \equiv \{1, 2, 3, \ldots, T\}$, and maximize lifetime flow-utility from non-durable consumption:

$$\mathbb{E}_{t_0} \left(\sum_{\tau=1}^{T} \beta^{\tau} u(c_{\tau,t_0+\tau}) \right),$$

where t_0 characterizes the cohort year of entry into the labor market, and τ characterizes the age of the agent. Consequently, $c_{\tau,t_0+\tau}$ refers to the consumption of workers of age τ

in time $t_0 + \tau$.

Workers are characterized by heterogeneous human capital levels h, with $h \in \mathcal{H} \equiv [\underline{h}, \overline{h}]$, and are heterogeneous also with respect to their formal education level $\iota \in \mathcal{I} \equiv \{\text{g,s}\}$, which indicates college and high school education, respectively. Both types enter the labor market with a baseline level of human capital drawn from type-specific exogenous continuous distributions. Upon entry in the labor market, $\mathbb{E}[h|\text{g}] > \mathbb{E}[h|\text{s}]$. To account for the different number of education years in the data, graduate workers entry to the labor market is delayed accordingly. Workers exit the labor force when their employment prospects deteriorate below a certain utility threshold, effectively distinguishing long-term unemployment from non-participation.

Firms are characterized by different levels of quality $y \in \mathcal{Y} \equiv [\underline{y}, \overline{y}]$. We model the human capital accumulation process assuming that it depends on the quality of the firm workers are matched with, and on their own initial level of human capital, h. Workers accumulate human capital only while employed and according to a law of motion that is match-specific: $h' = \phi(h, y, \iota, \psi) = g_\iota(h, y) + \psi$, $\phi : \mathcal{H} \times \mathcal{Y} \times \mathcal{I} \times \Psi \rightarrow \mathcal{H}$, where g_ι is the deterministic component of the human capital accumulation dynamics, and $\psi \in \Psi$ constitutes a stochastic component. The function g_ι is concave in both its arguments.[20]

Human capital accumulation is risky: at any period any employed worker is subject to the idiosyncratic human capital shock ψ, which enters additively with respect to the deterministic component.[21] The shock affects workers' ability and can amplify, shrink, or even reverse human capital accumulation. We further allow for the possibility that human capital deteriorates while workers are unemployed, according to an arbitrary process g_u.[22] Lastly, we allow a share τ_{eu} of human capital to depreciate upon job loss. Similarly, in order to take into account the presence of firm-specific human capital, we allow the share of human capital to be retained after a job transition, τ_{ee}, to be below 1. Both τ_{eu} and τ_{ee} are estimated from data on wage dynamics upon separations and employment-to-employment transitions.

Firms are modeled as one worker–one job matches. Each job is characterized by a promised utility to the worker $V \in \mathcal{V} \equiv [\underline{v}, \overline{v}]$. We group worker-specific characteristics in a tuple $\chi \in \mathcal{X} \equiv \{\mathcal{H} \times \mathcal{T} \times \mathcal{I}\}$. The aggregate state of the economy Ω is characterized by the productivity level $a \in \mathcal{A} \subseteq \mathbf{R}_0^+$ and by the distribution of agents across states $\mu \in \mathcal{M}$

[20]The deterministic component of human capital accumulation is akin to a "catching-up" of the firm's quality, up to a point when the worker will not be able to learn any more from the match. The only difference across education levels is the speed of the "catching-up", with college-educated workers catching up faster. Workers who match with a low-quality firm will see their ability deteriorating with the same g_ι function.

[21]The additive nature of the shock keeps the properties of monotonicity and uniqueness of workers' search strategies unaltered, which is essential for tractability.

[22]This process might be without loss of generality deterministic or stochastic, and might or might not depend on current human capital h.

$: \{W, U\} \times \mathcal{Y} \times \mathcal{X} \times \mathcal{V} \rightarrow [0,1]$. Let $\Omega = (a, \mu) \in \mathcal{A} \times \mathcal{M}$ represent the aggregate state of the economy and let $\mathcal{M}$ represent the set of distributions μ over the states of the economy. Then $\mu' = \Phi(\Omega, a')$ is the law of motion of the distribution. Aggregate productivity is a stationary monotone increasing Markov process, namely $a' \sim F(a'|a) : \mathcal{A} \rightarrow \mathcal{A}$, with the Feller property.

3.2 Labor markets

Search is directed. Each labor market is organized as a continuum of submarkets indexed by the expected lifetime utility offered by firms of type y, $v_y \in \mathcal{V}$. Starting a firm amounts to posting a vacancy at a quality-specific cost $c(y)$, and will be described in **Section 3.5**.

The matching function $M(u, \nu)$ for each submarket has constant return to scale and is twice continuously differentiable. The tightness of each submarket in $\mathcal{X} \times \mathcal{V}$ is defined as $\theta = \nu/u$, with $\theta(\cdot) : \mathcal{X} \times \mathcal{V} \times \mathcal{A} \times \mathcal{M} \rightarrow \mathbf{R}_0^+$. Job finding rates are defined as $p(\theta(\cdot)) = M(u, \nu)/u$, where $p(\cdot) : \mathbf{R}_0^+ \rightarrow [0,1]$ is a twice continuously differentiable, strictly increasing, and strictly concave function with $p(0) = 0$, $\lim_{\theta \rightarrow +\infty} p(\theta) = 1$ and $p'(0) < \infty$. The vacancy-filling probability is defined as $q(\theta(\cdot)) = M(u, \nu)/\nu$, where $q(\cdot) : \mathbf{R}_0^+ \rightarrow [0,1]$ is twice continuously differentiable, strictly decreasing, and strictly convex, with $q(0) = 1$, $\lim_{\theta \rightarrow +\infty} q(\theta) = 0$ and $q'(0) < 0$, such that $q(\theta) = p(\theta)/\theta$, and $p(q^{-1}(\cdot))$ is concave.

Upon matching, workers produce according to the twice-continuous increasing and concave production function $f(h, y; a) + x(a) : \mathcal{A} \times \mathcal{H} \times \mathcal{Y} \rightarrow \mathbf{R}_0^+$. The $x(a)$ component of the production function is a cost which can depend on the aggregate productivity realization.[23] Workers' compensation is determined by dynamic contracts through which firms deliver a promised lifetime utility, as described in **Section 3.6**.

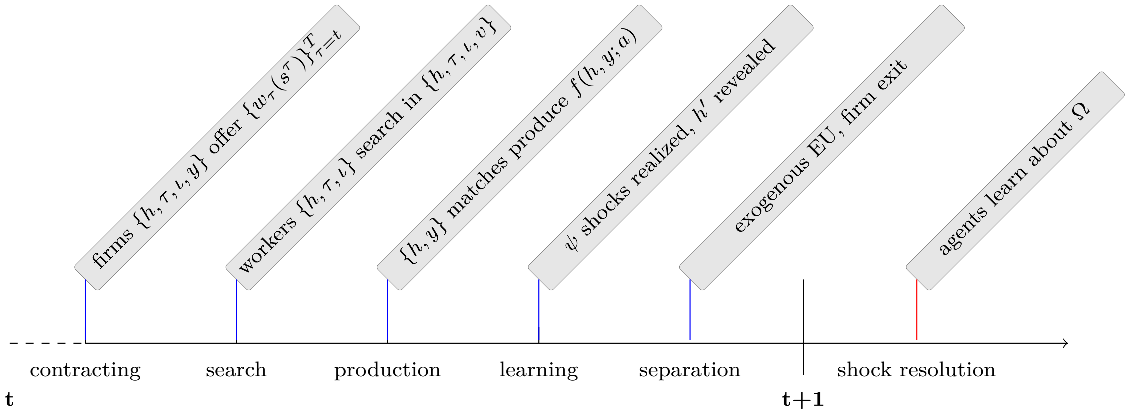
Workers search on the job with probability λ_e. Matches are destroyed each period at an exogenous rate, possibly varying by age, λ_τ. Matches separate also if the worker moves to another firm (poachings), or if the firm unilaterally walks away from the match (firings). Lastly, unemployed workers whose expected value of re-employment falls below a threshold $\underline{p}$ are assumed to permanently exit the labor force.[24]

Timing is represented in **Figure 4**. At the beginning of each period an aggregate productivity shock is drawn; entrepreneurs open vacancies across submarkets and post their offers; workers search from unemployment or on-the-job, and move to a new job if the search is successful; production takes place; workers accumulate human capital

[23] This is a reduced-form way of incorporating financial frictions in the model, which make fixed costs loom larger over flow-production in downturns.

[24] Workers could also voluntarily decide to quit to unemployment. We experimented with allowing workers quits to unemployment and found that in the model, given plausible parameterizations and in absence of additional shocks, quits to unemployment are never optimal. For this reason and for simplicity we remove the possibility from the model presented in this paper.

Figure 4: Timeline of Worker–Firm Match



depending on their employment status and idiosyncratic shock realization; an exogenous share of matches breaks down, while some firms endogenously exit.

3.3 Informational and contractual structure

Firms post fully state-contingent contracts. Each contract prescribes an action for each realization of the history of the worker–firm match. The tuple defining productivity and worker characteristics in a match with any firm y at time t is defined by $s_t = (h_t, \tau_t, \iota, a^t, \mu^t) \in \mathcal{S}^t = \mathcal{X} \times \Omega^t = \mathcal{H} \times \mathcal{T} \times I \times \Omega^t$, that is the worker skill, age, formal education, the history of aggregate productivity shocks, and workers' distributions across their employment history. A given history of realizations between t and k periods ahead is thus $s^{t+k} = (s_t, s_{t+1} \ldots., s_{t+k})$. The contract defines a transfer of utility from the risk-neutral firm to the risk-averse worker within the match for all future possible histories of shocks. We define τ_{t_0} as the age at which the worker is hired and T is the retirement age. The history of realizations between t_0, the time of hiring of the worker, and $t_0 + (T - \tau_{t_0})$, the time of maximum duration of the match with the worker before retirement, is thus $s^{t_0+(T-\tau_{t_0})}$.

Histories of workers and productivity shocks are common knowledge, and the future realizations of shocks are fully contractible. While the contract is state-contingent, workers' actions are private knowledge in the search stage, so firms are unable to counter outside offers. The contracts offered by firms are then defined as:

$$\mathcal{C}^{\tau_{t_0}} := (\mathbf{w}, \zeta) \text{ with } \mathbf{w} := \{w_t(s^{\tau_t - \tau_{t_0} + t_0})\}_{t=t_0}^{t_0+(T-\tau_{t_0})}, \text{ and } \zeta := \{v_t(s^{\tau_t - \tau_{t_0} + t_0})\}_{t=t_0}^{t_0+(T-\tau_{t_0})}. \tag{3}$$

Firms promise a series of state-contingent wages defined by the series of utility values v_t sought at each node of the history.[25] ζ is the action suggested by the contract, which

[25] Similarly to Menzio and Shi (2010), Tsuyuhara (2016), and Balke and Lamadon (2022), to guarantee that the problem is well behaved and the firm profit function is concave, the contract will require a two-

is bound to be incentive compatible for the worker. The resulting relationship between workers and firms is characterized by a contract with forward-looking constraints. The state space of the worker problem can be expressed in terms of their current lifetime utility, as in Spear and Srivastava (1987), so as to avoid having to keep track of all past histories s^t at each period. The relevant state space is then $\mathcal{X} \times \mathcal{V} \times \Omega$.

3.4 Worker problem

Given current lifetime utility V, job seekers with characteristics χ have to decide in which submarket to direct their search. Submarkets are indexed by worker type χ and by offered utility v associated to firms' posted vacancies. As discussed in **Section 3.5**, the choice over v will also indirectly determine which firm y the worker matches with, and thus the implied human capital accumulation path. For now, let us assume this (conditional) mapping exists. This amounts to assuming that the function $v(y; \chi, V)$ is a bijective function $f_v : \mathcal{X} \times \mathcal{V} \times \mathcal{Y} \times \mathcal{A} \times \mathcal{M} \to \mathcal{V}$. Upon observing a job offer with utility v, a worker χ with current utility V will be able to infer which firm type y is posting the offer.

A worker of type (χ, V) that enters the search stage has lifetime utility $V + \lambda_i \max\{0, R(\chi, V; \Omega\}$, where the second component of the expression embeds the option value of the search, with R being the search value function, and $\lambda_i = 1$ if the worker is unemployed or $\lambda_i = \lambda_e$ if they are employed. R is defined as:

$$R(\chi, V; \Omega) = \sup_{v} \Big[p(\theta(\chi, v; \Omega)) \big[v - V \big] \Big]. \tag{4}$$

We denote the solution of the search problem as $v^* = v^*(\chi, V; \Omega)$, and $p^*(\chi, v^*; \Omega) = p(\theta(\chi, v^*; \Omega))$ as the associated optimal job-finding probability. The lifetime utility of an unemployed worker at the beginning of the production stage can be defined as

$$\begin{aligned} U(h, \tau, \iota; \Omega) = \; & u(b(h, \tau)) + \beta \mathbb{E}_{\Omega, \psi} \Big(U(h', \tau + 1, \iota; \Omega') \\ & + \max\{0, R(h', \tau + 1, \iota, U(h', \tau + 1, \iota; \Omega'); \Omega')\} \Big), \end{aligned} \tag{5}$$

where $b(h, \tau)$ is a skill and age dependent unemployment benefit. Given finite workers' lives, $U(h, \tau, \iota; \Omega) = 0 \; \forall (\chi; \Omega) \in \mathcal{X} \times \mathcal{A} \times \mathcal{M}$ whenever $\tau > T$. The corresponding lifetime utility of a worker employed at firm y, with human capital h, age τ, education ι and promised utility V at the beginning of production stage can be expressed as:

point lottery, which specifies probabilities over the actions prescribed. We omit it here for conciseness.

$$V(h,\tau,\iota;\Omega) = u(w) + \beta\mathbb{E}_{\Omega,\psi}\Big(\lambda U(h',\tau+1,\iota;\Omega') + (1-\lambda)\Big[V(h',\tau+1,\iota;\Omega') + \lambda_e \max\{0, R(h',\tau+1,\iota,V(h',\tau+1,\iota;\Omega');\Omega')\}\Big]\Big), \tag{6}$$

where w is the promised wage, λ is the separation probability, and $V(h',\tau+1,\iota;\Omega')$ is next period's state-contingent promised utility of remaining in the current firm, which becomes the outside option in the search problem.Again, $V(h,\tau,\iota;\Omega) = 0\ \forall(\chi;\Omega) \in \mathcal{X} \times \mathcal{A} \times \mathcal{M}$ whenever $\tau > T$. Firms internalize incentives embedded in workers' strategies and post wages and utility offers to maximize profits by optimizing retention. This way, future promised utilities incorporate both future wages or option values of search. The policy functions are uniquely defined and identify target y as long as a bijective mapping between the offered utility v and y given χ exists.[26]

Definition 3.1 (Optimal retention probability and utility return)**.** *The solution to the worker's problem defines a retention function* $\widetilde{p} : \mathcal{X} \times \mathcal{V} \times \Omega \to [(1-\lambda)(1-\lambda_e), 1-\lambda]$ *and a utility return* $\widetilde{r} : \mathcal{X} \times \mathcal{V} \times \Omega \to \mathcal{V}$*:*

$$\widetilde{p}(\chi, V;\Omega) \equiv (1-\lambda)(1-\lambda_e p^*(\chi, v^*;\Omega)) \tag{7}$$

$$\widetilde{r}(\chi, V;\Omega) \equiv \lambda U(\chi;\Omega) + (1-\lambda)\Big[V + \lambda_e \max\{0, R(\chi, V;\Omega)\}\Big] \tag{8}$$

3.5 Vacancy creation and free entry

The economy is populated by a continuum of risk-neutral entrepreneurs. Each entrepreneur can invest to reach the desired level of firm quality y. The start-up costs of the firm are priced in terms of the consumption good and they coincide with vacancy posting costs in the frictional labor market. The cost of each vacancy is positively related to the quality of the firm being created through the cost function $c(y)$, which is increasing and strictly convex.[27]

At a generic time t each entrepreneur chooses in which submarket to post the vacancy by offering utility $W \in \mathcal{V}$. Each submarket is characterized by worker characteristics and current utility $(\chi, V) \in \mathcal{X} \times \mathcal{V}$, and we prove later in **Section 3.6** that the firm choice over W uniquely maps upon vacancy posting into firms' qualities $y \in \mathcal{Y}$ conditional on the submarket's characteristics of choice.

We define $J(h,\tau,\iota,W,y;\Omega) \in \mathcal{X} \times \mathcal{V} \times \mathcal{Y} \times \Omega$ as the value function of a firm, which

[26] Proofs of the uniqueness of policy functions are provided in Appendix Section C.1.

[27] We assume that entrepreneurs can borrow from risk-neutral, deep-pocketed financiers to finance the vacancy. As in Herkenhoff (2019) this assumption implies that the cost of credit for entrepreneurs coincides with the risk-free rate.

capitalizes all future profits from the match. As entrepreneurs choose the submarkets in which to open a vacancy, they face the following problem:

$$\Pi(h,\tau,\iota,W,y;\Omega) = \sup_{h,\tau,\iota,W} \ -c(y) + q(\theta(h,\tau,\iota,W;\Omega))[J(h,\tau,\iota,W,y;\Omega)] \tag{9}$$

Given perfect competition, free entry and the possibility for all entrepreneurs to choose *any* possible firm kind y, the expected profits from creating a vacancy are driven down to 0 in submarkets that actually open.This translates into a free entry condition:

$$\Pi(h,\tau,\iota,W,y;\Omega) \leq 0 \ \text{ for } \forall\{h,\tau,\iota,W,y;\Omega\} \in \{\mathcal{X}\times\mathcal{V}\times\mathcal{Y}\times\Omega\} \tag{10}$$

The equilibrium tightness in each open submarket is:

$$\theta(h,\tau,\iota,W;\Omega) = q^{-1}\left(\frac{c(y)}{J(h,\tau,\iota,W,y;\Omega)}\right). \tag{11}$$

3.6 Firm problem

The firm contract design plays out as a sequential equilibrium game with leader-follower dynamics, in which firms play as the principal/leader and workers are the agents/followers. Workers' limited commitment implies they will search for new jobs whenever they have the possibility to do so. Firms cannot observe poaching offers and thus cannot counter them. The sequence of past histories s^t is common knowledge, and while the firm cannot observe any of actions of its workers, it has enough information to internalize their optimal search policy decisions. The firm will decide whether to continue, in each period. For this purpose, it is useful to introduce the firm exit policy:

Definition 3.2 (Exit policy)**.** *The following indicator takes a value of one if the firm decides to exit :*

$$\eta(h,\tau,\iota,W,y;\Omega) = \begin{cases} 0 & \textit{if firm continues} \\ 1 & \textit{if firm exits} \end{cases}$$

Define also $\widetilde{p}(\chi,V;\Omega)$ as the optimal retention function and $\widetilde{r}(\chi,V;\Omega)$ as the optimal continuation utility for workers from workers' solution to the on-the-job search problem.[28] The firm chooses the wage(s) to be offered in the current period w_i, the utility promises W_i' and the probability π_i in a two-point lottery. The value function of an incumbent firm y in state $(h,\tau,\iota,W;\Omega)$ can be written recursively using the promised utilities as

[28]These expressions are defined in **Section 3.4**.

additional state variables, so that:

$$J(h,\tau,\iota,W,y;\Omega) = \sup_{\pi_i,\{\eta_i',w_i,W_i'\}} \sum_{i=1,2} \pi_i \Bigg(f(y,h;a) - w_i + \beta \mathbb{E}_\psi \Bigg[(1-\eta_i') \cdot \max \Bigg\{ 0, \mathbb{E}_\Omega \Bigg[\widetilde{p}(h',\tau+1,\iota,W_i';\Omega') J(h',\tau+1,\iota,W_i',y;\Omega') \Bigg] \Bigg\} \Bigg] \Bigg) \quad (12)$$

$$s.t.\ W = \sum_{i=1,2} \pi_i \Bigg(u(w_i) + \beta \mathbb{E}_{\Omega,\psi}((1-\eta_i')\widetilde{r}(h',\tau+1,\iota,W_i';\Omega') + \eta_i' U(h',\tau+1,\iota;\Omega')) \Bigg), \quad (13)$$

$$J(h',\tau+1,\iota,W_i',y;\Omega') < 0 \implies \eta' = 1, \text{ otherwise } \eta' = 0, \quad (14)$$

$$(w - w')(1-\eta') \leq 0 \quad (15)$$

$$\sum_{i=1,2} \pi_i = 1, \quad (16)$$

where **Equation** (13) is the promise keeping constraint ensuring that the current value of the contract is based on the current wage and future utility promises. **Equation** (14) captures the firm side of the limited commitment friction: at any point the entrepreneur can walk away and open a new vacancy, so the outside option against which the continuation of the match is compared is the *ex ante* value of opening a new vacancy, which is 0 because of free entry. Finally, **Equation** (15) limits wage adjustments by requiring wage growth to be (weakly) positive. In absence of the last constraint, the wage would be fully state-contingent, with workers facing wage decreases instead of displacements. We will discuss the empirical relevance of this assumption in **Section 4**.

Firms commit to the delivery of a utility value to workers, but exit when the present value of future profits becomes negative, i.e. when the constraint (14) binds - then $\eta' = 1$. Incumbent firms make their exit decisions before the realization of aggregate productivity but after the next period realization of idiosyncratic human capital shocks.[29] At the beginning of a period both firms and workers know the contingencies under which the firm will shut down. Exit is therefore state-dependent (see Appendix for details).

[29] This amounts to having the firm committing to a state-contingent exit decision in advance of the idiosyncratic shock's realization as in Gomes (2001) and Xiaolan (2014).

3.7 Equilibrium definition

Recursive Equilibrium. Let $\Theta = \mathcal{A} \times \mathcal{M} \times \mathcal{H} \times \mathcal{T} \times \mathcal{I}$. A recursive equilibrium in this economy consists of a market tightness $\theta : \Theta \times \mathcal{V} \to \mathbb{R}_+$, a search value function $R : \Theta \times \mathcal{V} \to \mathbb{R}$, a search policy function $v^* : \Theta \times \mathcal{V} \to \mathcal{V}$, an unemployment value function $U : \Theta \to \mathbb{R}$, a firm value function, $J : \Theta \times \mathcal{V} \times \mathcal{Y} \to \mathbb{R}$, a series of contract policy functions $\{c_\tau\}_{\tau=1}^{T} : \mathcal{S}^\tau \times \mathcal{Y} \to \mathcal{C}^\tau$, a bijective mapping between firm qualities and promised utilities at hiring $f_v : \Theta \times \mathcal{V} \times \mathcal{Y} \to \mathcal{V}$, an exit threshold for aggregate productivity $a^* : \Theta \times \mathcal{V} \times \mathcal{Y} \to \mathcal{A}$, a human capital accumulation process $\phi(h, y, \iota, \psi)$, $\mathcal{H} \times \mathcal{Y} \times \mathcal{I} \times \Psi \to \mathcal{H}$, and a law of motion for the aggregate state of the economy $\Phi_{\Omega,a} : \mathcal{A} \times \mathcal{M} \to \mathcal{A} \times \mathcal{M}$ such that:

1. Given the mapping f_v, market tightness satisfies **Equation** (11).
2. The unemployment value function solves **Equation** (5).
3. Search value functions solve the search problem in **Equation** (4) and v^* is the associated policy function.
4. Firm value functions and associated contract and exit policy functions solve **Equation** (12) for each $\tau \leq T$.
5. The law of motion for the aggregate state of the economy respects the search and contract policy functions and the exogenous process of aggregate productivity.

Definition 3.3 (Block Recursive Equilibrium)**.** *A Block Recursive Equilibrium (BRE) is a recursive equilibrium such that the value and policy functions depend on the aggregate state only through aggregate productivity, $a \in \mathcal{A}$ and not through the distribution of agents across states $\mu \in \mathcal{M}$.*

Property 3.1 (Existence of a BRE)**.** *A block recursive equilibrium exists for the model.*

Proof. See **Appendix Section D**. □

3.8 Model Properties: Discussion

The objective of our model of dynamic sorting is to understand the properties of jobs creation and worker search in a setting with two-sided heterogeneity. The following properties guarantee a high degree of tractability.[30]

[30] We refer the reader to Appendices C.1 and C.2 for a more in-depth discussion of the theoretical properties of the model and the proofs of this section.

Property 3.2 (Unique Bijective Mapping)**.** *Upon matching, firm quality y and utility promises in vacancy postings v are related by a bijective mapping conditional on the aggregate state of the economy, Ω, and workers characteristics (χ, V).*

The previous proposition establishes that workers' directed search toward promised values is equivalent to directed search toward firms' types. We then focus on the properties of the search strategy to get a complete view of how sorting works in equilibrium.

Property 3.3 (Search Monotonicity and Uniqueness)**.** *The optimal search strategy when unemployed, conditional on age τ, formal education ι and the aggregate state Ω, is unique and weakly increasing in workers' human capital h. The optimal search strategy when employed, conditional on age τ, formal education ι and the aggregate state Ω, is unique and weakly increasing in workers' human capital h and current level of lifetime utility V.*

Property 3.3 guarantees that, abstracting from idiosyncratic as well as aggregate shocks, workers sort positively with respect to their human capital. **Property 3.2**, in turn, guarantees that workers with same observable characteristics agree on firms' relative ranking. Firms are thus vertically differentiated, and there is a separating equilibrium whereby workers with different characteristics optimally search in distinct firms.

Because we are interested in how aggregate fluctuations shape the distribution of matches, we now turn to considering how they affect search strategies.

Property 3.4 (Search in Good and Bad Times)**.** *The optimal search strategy is increasing in the aggregate productivity level, a.*

Property 3.4 points to a key mechanism in the model: aggregate fluctuations modify sorting in the labor market. The value of vacancies posted by each firm in equilibrium changes with the business cycle, as submarkets become less productive and less tight in bad times. This also takes place due to the presence of increasing and convex vacancy costs, which do not vary with the cycle. Faced with a lower probability of successfully matching with the firm they would aim to match with in good times, risk-averse workers adjust their search downward. In turn, firms will adjust downwards their utility offers given the lower expected values of matches across the board.

Firms' offers will optimally respond to workers' incentives for on-the-job search.

Property 3.5 (Optimal Retention)**.** *Retention probabilities, $\widetilde{p}(h, \tau, \iota, W; \Omega)$ are:*

(i) increasing in the value of promised utilities, W.

(ii) decreasing in aggregate productivity, a.

Despite continuation values within the match being procyclical and workers searching more ambitiously in good times, matches will separate more often in expansions as workers transition to new jobs. This is consistent with employment-to-employment transitions being strongly pro-cyclical in the data.

Property 3.5 highlights another aspect of the incentives that shape the contract designed by firms: retention grows in continuation values W. To close the model, we need a rule for surplus sharing between firms and workers, that is, a wage protocol for firms to deliver lifetime utility promises to workers. We focus here on the wage protocol for a firm for which $\eta' = 0$, so that there is a wage to be paid in the future period.

Property 3.6 (Wage Protocol)**.** *The optimal contract for the continuing firm delivers a wage growth rule that satisfies:*

$$\frac{\partial \log \widetilde{p}(\chi', W_i'; \Omega')}{\partial W_i'} J(\chi', W_i', y; \Omega') = \frac{1}{u'(w_i')} - \frac{1}{u'(w_i)}, \tag{17}$$

with $\chi' \equiv (\phi(h, y, \iota, \psi), \tau + 1, \iota)$ *being the definition of individual characteristics and* w_i' *being the wage paid in the future state, conditional on realizations of idiosyncratic risk* ψ *and aggregate risk* a'*.*

This result extends the wage equation in Balke and Lamadon (2022) to an environment with two-sided heterogeneity. Wage growth is proportional to the residual continuation value of the match, J and the semi-elasticity of the worker's retention probability to future value promised. Limited liability provides the rationale for inefficient separations. At the same time, it also gives rise to wage rigidity, as it ensures that both elements in Equation 17 are weakly positive if the firm does not close down.[31]

Property 3.7 (Countercyclical Separations)**.** *Conditional on the existing contract and on worker and firm types, there exists an aggregate state* a^* *below which firms will not continue to operate. The threshold* a^* *is, all things being equal, increasing in the value promised to workers, and decreasing in worker and firm types.*

A clear implication of **Property 3.7** is that, at the onset of recessions, firms are significantly more likely to lay off workers. In addition, lower-skilled workers and low-productivity firms are more likely to separate in recessions. The counter-cyclicality of separations is a common feature in the data, together with the lower job security enjoyed by workers who are younger, less productive, or provided by firms that are less productive.[32]

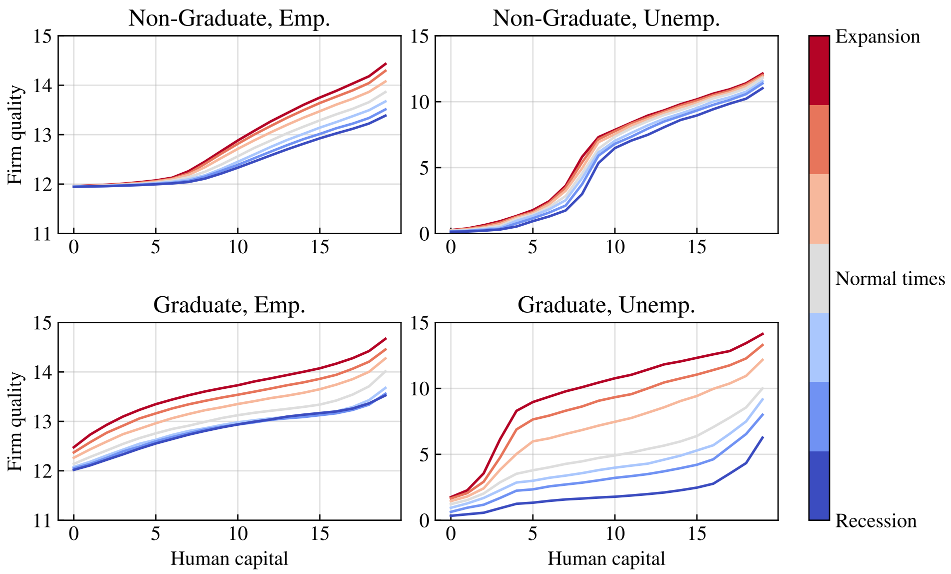
[31] Notice that, in the presence of risky human capital accumulation, J will fluctuate together with the human capital levels of the worker even in the absence of aggregate fluctuations. However, because the contract provides insurance to workers, changes in their human capital will have asymmetric effects on wage growth, thus weakly increasing the labor share over time (Ai and Bhandari, 2021).

[32] These relationships are observed in the data and explicitly modeled in Jarosch (2023), but emerge endogenously in our framework, without the need of specifying job security as a contract characteristic. Abstracting from labor market institutions is not limiting the ability of our model to partly capture the observed duality of the labor market.

3.9 Sorting in equilibrium

The theory discussed in this section predicts that workers' search is monotonic in individual characteristics and in the aggregate state (see **Proposition 3.3**).

Figure 5: Search Policies



Note: Search policy function by human capital level and aggregate state, averaged across labor market experience and wage promises.

In **Figure 5** we plot the equilibrium mapping between workers' human capital and search behavior resulting from our estimation, for different realizations of the aggregate state. Search is strongly monotonic in both dimensions. Search strategies of college-educated workers are more sensitive to shifts in aggregate conditions. This is due to the fact that expected human capital accumulation is more important for college-educated workers in the model, as it has greater long term effects for their careers. This implies that these workers will have ceteris paribus a greater tendency to search for highly productive matches, with more volatile vacancy creation, in good times. It also implies that they will moderate their search more in bad times, especially if unemployed, in order to minimize their unemployment risk and the consequent human capital deterioration. Because of these dynamics, vacancy creation tends to be of lower quality in recessions, which in turn increases misallocation and leads to a deterioration in sorting throughout the recovery. Our model thus endogenously rationalizes the empirical findings in Haltiwanger et al. (2022), who observe that recessions apparently feature an initial "cleansing" phase followed by a more prolonged "sullying" dynamics as the labor market re-builds its job ladder.

4 Bringing the model to the data

The model features internally and externally calibrated parameters. To estimate the first group of parameters, we target moments from Italian administrative data, provided by the Italian Social Security Administration (INPS), for all years between 1994 and 2019.[33] To obtain model moments, we simulate a population of overlapping generations working for 45 years (180 quarters, from 18 to 63 years old, the legal retirement age for most years in our period of analysis). We then use a simulated method of moments (SMM) approach. This section will first present the quantitative setup of the model, then calibration choices for the parameters that are set externally, and finally our estimation results.

4.1 Calibration and estimation

Quantitative Setup. **Table 1** collects all the functional form choices. We assume a Cobb-Douglas production function and allow for potentially cyclical maintenance costs, captured by the parameter x. We follow Schaal (2017) and Menzio and Shi (2010) in picking a CES function in market tightness. Vacancy creation imposes increasing costs in firm's quality y, according to the convex function $c(y)$. Workers are risk-averse with constant-relative-risk-aversion (CRRA) utility. The human capital production technology is concave in the firm quality, y, which is scaled by a parameter ξ, and in the existing stock of human capital, h. Future human capital is also subject to additive i.i.d. shocks, $\psi \sim \mathcal{N}(0, \sigma_\psi)$. Home production is increasing in the stock of human capital according to the parameter ξ_b. Finally, we allow the exogenous separation rate to be age dependent to capture age-specific aspects of worker quality that are unrelated to business cycles but still empirically relevant. [34] The model is then characterized by seven externally calibrated parameters and by 18 jointly estimated parameters.[35]

Table 1: Functional Forms

Functions	
Production function	$f(y,h) = Ay^\alpha h^{1-\alpha} - x(A-1)$
Job-finding probability	$p(\theta) = \theta(1+\theta^\gamma)^{-\frac{1}{\gamma}}$
Vacancy creation cost	$c(y) = \frac{y^\kappa}{\kappa}$
Utility function	$U(c) = \frac{c^{1-\nu}}{1-\nu}$
Human capital accumulation	$g_\iota(h,y) = (\xi y)^{\phi_\iota} h^{1-\phi_\iota} + \psi$
Home production	$b(h,\tau) = b + \xi_b h$
Exogenous exit rate	$\lambda_\tau = \frac{\lambda_b}{\lfloor \tau/4 \rceil}$

[33]Details of data sources and construction are discussed in Appendix Section A.
[34]The actual separation probability from the workers' perspective will thus be $\lambda = \max\{\lambda_\tau, \eta(\cdot)\}$
[35]Appendix Section E provides more details on the model solution and estimation procedure.

Calibration. Preference parameters (discount factor β, and agents' risk aversion ν), and the annualized risk-free rate r_f are set in line with the literature. We calibrate the persistence and volatility of the aggregate shock, (ρ_a, σ_a) by estimating an AR(1) on the detrended series of Italian real total factor productivity (TFP). In addition, workers draw their innate ability and human capital upon entry into the market from an initial distribution. We set the initial distribution of human capital for high school–educated workers as a $Beta(\mu_L, \sigma_L)$. College-educated workers draw their human capital from the same distribution plus a constant spread, ϑ. We set shape and scale of the beta distribution and internally estimate the scaling factor to match the ratio of average initial incomes between the two groups of workers. To properly account for the empirical age distribution, we weigh simulated data according to the distribution of the Italian working-age population.[36]

Table 2: Parameter Values

Parameter	**Description**	**Value**
	Externally Calibrated	
ν	Risk aversion	2.000
β	Discounting	0.990
r_f	Real interest rate	0.011
(μ_L, σ_L)	Shape and scale of initial human capital dist.	(2.50,10.00)
(ρ_A, σ_A)	Mean and std of TFP process	(0.95, 0.009)
	Jointly Estimated	
α	Production function elasticity to firm quality	0.552
γ	Matching function	1.090
ϕ	Human capital adjustment rate, High School	0.037
ϕ_g	Human capital adjustment rate, College	0.282
b	Unemployment benefit	1.103
λ_b	Exogenous separation prob., initial	0.116
κ	Vacancy cost	2.432
λ_e	On-the-job-search prob.	0.454
ξ	Scaling factor in human capital accumulation	0.640
ξ_b	UB dependence on human capital	0.063
l	Linear loss of humanc capital while unemployed	0.167
τ_{ee}	Human capital retention after EE	0.902
τ_{eu}	Human capital loss after EU	0.771
x	Cyclical component of cost function	-1.729
$\underline{p}$	Out of labor force threshold	0.037
σ_ψ	Std of idiosyncratic human capital shock	0.684
ϑ	Initial scaling in human capital distribution	0.350
$\underline{y}$	Lowest bound of firm distribution	2.844

Estimation and Identification. We estimate the remaining 18 parameters via SMM, targeting a set of standard labor market moments. **Table 2** reports estimated parameter values, **Table 3** reports simulated and empirical moments.

To examine the role of each moment in identifying our parameter values, we use the series generated by our global solver, and project simulated moments on the parameter

[36]Age weights are constructed following the age distribution of the 2010 census from the website of the Italian National Institute of Statistics (ISTAT).

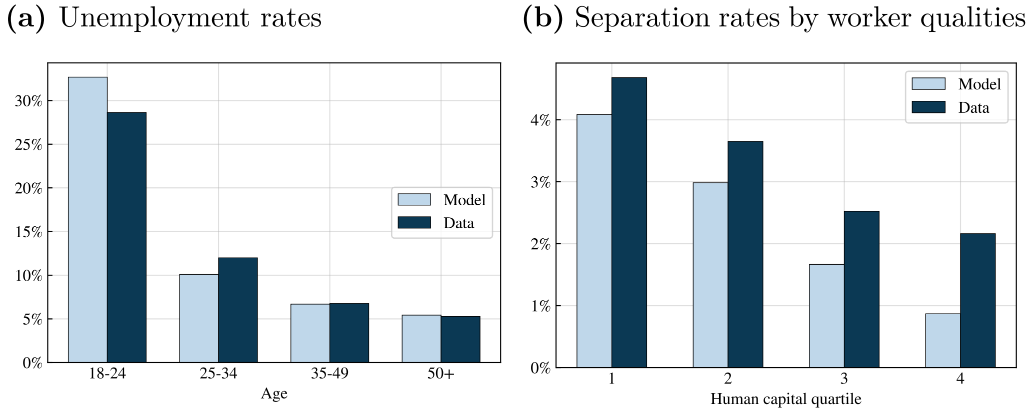
Table 3: Target Moments

Moments	Mean	
	Data	Model
A. Labor Market Flows		
Employment-to-Employment Transition Rate*	1.3%	0.7%
Employment-to-Unemployment Transition Rate*	4.1%	2.8%
Employment-to-Employment Correlation w/GDP	0.68	0.48
Employment-to-Unemployment Correlation w/GDP	-0.19	-0.14
B. Earnings		
Earnings Growth (College)*	129.7%	135.7%
Earnings Growth (High-School)*	60.7%	66.1%
Entry Salary Ratio: College to High School	1.46	1.02
C. Other Statistics		
Unemployment Rate	9.7%	7.2%
Labor Market Sorting	0.40	0.57
Inactivity Rate†	21.9%	25.7%
Firm Value Added Distribution**	1.18	1.38

Note: (*): We match the life-cycle profiles with nine age bins for each profile. The table reports average values. (**): We match the ratio between the third and fourth to the first quintiles. (†) The average inactivity rate is taken from the inactivity rates by age groups from 1994 to 2019 from ISTAT. We define sorting as the average over time of the correlation between the firm and worker fixed effects from an AKM model yearly estimated on the Italian administrative data. In the model, sorting is the correlation between firms' and workers' qualities.

space, then compare the impact of each parameter on moments, conditional on the others. The model fits EE flows by age, which provide information on the matching function elasticity, γ, and the human capital loss after a job-to-job transition, τ_{ee}. These parameters affect both the investment side of vacancy posting and the workers' willingness to climb the job ladder. The correlation of EE flows with the cycle, together with the end points of the earnings profiles of non-college workers, mostly identify the human capital accumulation parameter for non-college workers: higher ϕ implies a moderation in the depreciation of workers before retirement. Early EU flows, instead, provide information for estimating the rate of exogenous separations, λ_b. Late EU flows, which proxy the fragility of existing match, help identify a crucial fluidity parameter, λ_e: as workers struggle to move towards more suitable matches, their position in the firm becomes riskier. The correlation between EU transitions and the cycle, then, uniquely pins down the riskiness of human capital accumulation, σ_ψ: interactions between the two sources of risk allow us to match the increase of separations in recessions. We exploit early curvature in the earnings profiles of non-college workers to identify the cyclical fixed cost for firms, x, and the state dependence of unemployment benefits, ξ_b: these parameters drive respectively the expected tenure of employment and unemployment for young workers, important for those whose human capital accumulation on the job is less strong. The earnings profiles of college-educated workers inform over their human capital accumulation parameter, ϕ_g, as well as the ratio in

Figure 6: Cross-Sectional Features, Model and Data



Note: Panel (**a**) plots the unemployment rate by age groups in model simulations and in the data. *Sources:* unemployment rates are taken from the Italian National Statistical Agency (ISTAT). Panel (**b**) reports the average separation rates by worker quality. In the data worker quality is measured by the worker-specific AKM fixed effect. In the model simulations, worker quality is workers' human capital.

human capital levels at entry, ϑ.

Cross-sectional moments, like the distribution of value-added and the level of worker-firm sorting, are crucial to pin down the equilibrium firm distribution, and so mostly identify the vacancy cost parameter, κ, the elasticity to firm quality α, and the relative scale of workers' human capital and firm quality, which in the model depend on ξ and $\underline{y}$. Finally, the inactivity rate, together with the unemployment level, inform on the threshold that determines the exit from the labor force, $\underline{p}$, as well as the total loss of human capital during unemployment, l.

4.2 Model Properties

To assess the quality our estimation we check how the model performs in matching a series of untargeted moments both in the cross section and over time as well as both at the macro and micro level.

Cross-sectional properties. While in the estimation we target the average unemployment rate, the model exhibits a very good fit also for the age profile of the unemployment rate (see **Figure 6a**). In addition, **Figure 6b** compares the separation rates by worker types in the model and in the data, highlighting how the model is able to well capture the higher fragility of workers with low human capital, despite not directly targeting these moments in the estimation.

Our baseline model is also able to qualitatively reproduce the distribution of earnings. **Appendix Figure F.2** displays the cross-sectional distribution of earnings in the data and in the model. The empirical wage distribution is centered at slightly below €$2,000$ and skewed to the left, with most observations below €$4,000$. What the model fails to generate is the long right tail of wages in the data, which corresponds mainly to managerial figures whose earnings command premia that our mechanism is not meant to capture.

Exploiting the rich features of the model, in **Table 4** and **Appendix Figure F.3** we decompose the average growth in wages *within* jobs, that is within the same job spell, and *between* jobs, that is after a job-to-job transition. Consistent with the data, about a quarter of wage growth comes from tenure within the match, and this share decreases over life cycle. We also observe that average *within*-wage growth is declining in age and firm quality. The model generates a slightly steeper decline in the within share over the life cycle, potentially because automatic seniority-linked wage increases are present in the data but absent from our contract formulation - hence, tenure effects will persist later in the life cycle than the model would predict. Also, wage profiles for higher-productivity firms are flatter. In part, this is due to weaker retention elasticities: poaching a worker from a highly productive firm is harder. On top of this, highest skill workers tend to be older, with shorter expected residual match duration.

Table 4: Wage Growth of Movers vs. Stayers: Data and Model

Within/Between	Age				
	18-27	27-36	36-45	45-54	54-62
Data	0.239	0.250	0.243	0.207	0.136
Model	0.382	0.208	0.157	0.144	0.156

Note: The table reports the ratios between the median yearly wage growth of stayers (within wage growth) and movers (between wage growth). Data: INPS, for all years between 1994 and 2019.

Where does human capital accumulation take place? A key prediction of our model is that workers accumulate more human capital when employed at more productive firms. The business cycle properties of the model, however, depend on which firms account for a larger share of the aggregate creation of human capital. Because of the interactions between sorting and life cycle dynamics, learning might still take place in the least productive firms. We indeed find that, despite supermodularity in production and increasing returns to experience at better firms, more than half of the total human capital accumulation takes place in firms whose productivity level is around or below the median, which are the prime employers of younger, low-human capital workers.Firms at the top of the productivity distribution are not only less likely to lay off workers while paying them higher wages, but are also more likely to employ more senior workers, whose human capital accumulation accounts for a relatively lower share in the aggregate. This highlights the importance of less productive firms, which are more volatile and subject to aggregate fluctuations, for output dynamics in the medium term, and is consistent with evidence provided by Gregory (2023), who documents a negative overall correlation between firms' quality of learning environment and productivity in German administrative data.

Gregory's (2023) identification of this correlation stems from assumptions imposed on the learning ability of workers over the life cycle, specifically the assumption that senior workers are unable to learn. This assumption would be justified in our model: considering a firm with a certain level of productivity, and conditional on their human capital level, the probability that workers manage to match with it decreases with age, as they become shorter-term investments for firms. Older workers will then be less "ambitious" than younger ones in trying to upgrade the productivity of their matches: they will in equilibrium match with firms that tend to be higher up the productivity ladder, but are also unlikely to be able to provide strong human capital growth given their starting human capital. The fact that, ceteris paribus, high-productivity firms offer stronger human capital growth, will thus not imply a positive unconditional correlation of productivity and learning.

Time series properties. Replicating aggregate time-series properties of the data provides additional validation of the channels in the model. We project the detrended quarterly series of Italian TFP on a discrete grid to simulate a series of discretized aggregate shocks. The simulated model tracks the empirical series of GDP, capturing peaks and troughs as well as the overall behavior of the empirical series (see **Appendix Figure F.4**). Besides matching the volatility of output (the standard deviation of detrended log-output is approximately 3% both in the model and in the data) the model is able to generate also a realistic volatility for the unemployment rate, which is 1.6% in the model versus 1.4% in the data.[37]

Further validation of our framework is given by the ability to replicate the long-run effects of business cycles on workers' career outcomes. In particular, we adapt the reduced-form models proposed in the literature on the effects of recessions on labor market entrants (Kahn 2010, Schwandt and von Wachter 2019) and we run it on both the Italian administrative data and on a model-simulated panel.[38] Consistently with the literature, entering the labor market in a downturn is associated with persistent losses in earnings. As shown in **Appendix Figure F.5**, our baseline model is able to generate scarring effects that, on average, are approximately 25-30% of those observed in the data. The model matches reasonably well both the magnitude and the dynamics of the scarring effects of business cycles.

[37]To compute the volatility of unemployment we replicate the exercise in Shimer (2005). Specifically, we remove a slow moving trend from the time-series of unemployment, by filtering the quarterly series with an Hodrick-Prescott filter with smoothing equal to 10^5.

[38]In these empirical specifications we control for age, period, and cohort effects, proxied by the cyclical realization of GDP at cohort entry. We report the empirical estimates in Appendix F.

4.3 Alternative specifications

Our theoretical framework features two modeling choices which depart from previous models in the literature. Firstly, we characterize endogenous separations when a match is no longer profitable. Secondly, we depart from the standard human capital production function assumptions, by making it dependent on firm quality. To assess the significance of these assumptions, we compare our baseline model with two alternative specifications.

The first alternative, which we refer to as the "linear human capital accumulation" specification, assumes that human capital accumulation is independent of firm quality, and takes place as long as the worker remains employed. In this version, each worker type accumulates human capital according to a random walk with a linear trend.[39]

In the second alternative specification, we re-state the firm problem without the non-negative wage growth constraint, effectively allowing firms to adjust compensation downwards when their participation constraint is binding. This specification, called the "flexible wage" model, is the standard formulation of state-contingent wage contracts, and predicts separations when no wage promise can be made that will both satisfy the promise-keeping constraint and the participation constraint of the firm.

In the third alternative specification we eliminate the possibility of separations initiated by firms, by having firms continuing to operate even with negative values. In this "no (endogenous) separations" specification, matches between firms and workers continue even if the expected value of remaining in the match turns negative, effectively eliminating endogenous layoffs in firm-worker relationships.

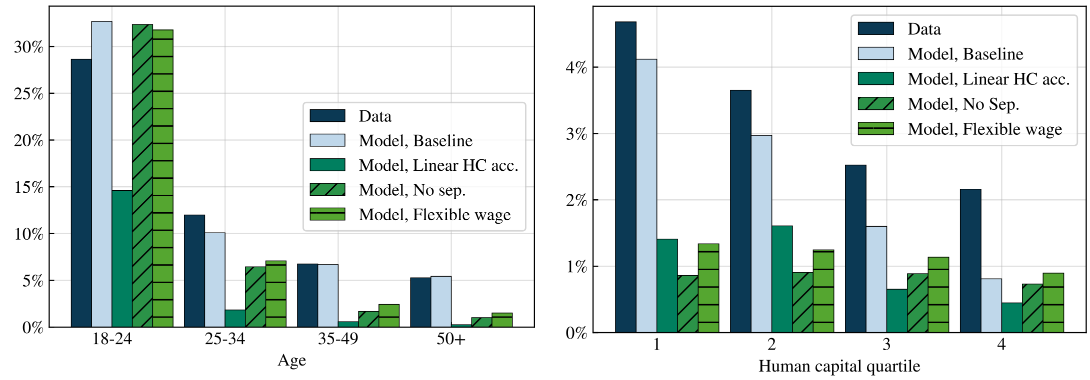
For all alternative specifications, we follow the same procedure outlined in the previous paragraphs and re-estimate the models using the same set of moments presented in **Table 3**. By comparing the results of these alternative specifications to our baseline model, we can gain insights into the importance of the assumptions regarding separations and firm-dependent human capital accumulation. **Panels 7a** and **7b** report the unemployment rate by age and the separation patterns for the alternative models. It is clear from the figure that, although the models are re-estimated to achieve the best possible fit on the same moments used in the estimation of the our baseline model, detaching human capital accumulation from sorting to firms makes the model unable to capture the important cross-sectional patterns for both unemployment and separations.

Time Series Properties and the Shimer Puzzle. The correlations between the simulated GDP and unemployment series in the baseline model and the two alternative models are presented in **Table 5**. When human capital accumulation is independent of firm quality, the model fails to replicate the co-movements of unemployment and the

[39]This is a common way of introducing human capital accumulation in search models (Herkenhoff et al., 2023, Jarosch, 2023).

Figure 7: Cross-Sectional Features, Alternative Specifications

(a) Unemployment rates **(b)** Separation rates by worker qualities



Note: Panel (**a**) plots the unemployment rate by age groups for the data and for alternative models: one in which the human capital accumulation is independent from firm quality, and one where there are no endogenous separations initiated by firms. Panel (**b**) reports the average separation rates by worker quality for model and data. Data is as in **Figure 6**.

cyclical component of GDP observed in Italian data. When we eliminate endogenous separations initiated by firms, instead, the model is still able to reproduce output dynamics and the correlation between unemployment and cyclical GDP. However, the model is unable to reproduce unemployment dynamics from the data.

Both our key modeling ingredients are necessary to achieve a good fit with the data. The baseline model exhibits a unique capability to capture the volatilities of output and unemployment. This is a feature that search and matching models typically struggle to generate (see Shimer 2005). By comparing the baseline model with alternative configurations, we delve deeper into the underlying causes of this emergent feature.[40]

When human capital accumulation does not depend on firm quality, the inherent value of being employed *per se* grows compared to the value of aiming at better matches. Search strategies adapt accordingly, shortening the average spell of unemployment. Workers accumulate human capital with tenure regardless of firm quality, so the match value rarely becomes negative during recessions even for low quality firms. Separations between workers and firms become less frequent in this setup, hence matching the unemployment level becomes harder. As the model struggles to characterize endogenous separations along the life cycle, the estimation procedure leads to trying to match unemployment levels by means of a higher likelihood of exogenous separations (λ_b). These exogenous separations are acyclical, so the model loses its ability to match at the same time unemployment volatility and level. This reasoning underscores why a human capital production function independent of firm quality would miss critical labor flows dynamics within our framework. It also emphasizes, more

[40] Schaal (2017) provides a notable exception where a model with idiosyncratic, time-varying volatility can also generate empirically consistent volatilities of output and unemployment.

Table 5: Co-movements at business cycle frequency

(a) Correlations between model generated and data time series

Corr. Model/Data	Baseline	Linear Human Capital Acc.	No Separations	Flexible Wages
Aggregate output	0.83	0.85	0.83	0.86
Unemployment	0.34	-0.22	0.07	0.41

(b) Correlation with aggregate output

	Data	Model			
		Baseline	Linear Human Capital Acc.	No Separations	Flexible Wage
Unemployment	-0.66	-0.85	-0.03	-0.32	-0.49

(c) Output and unemployment

	Data	Model			
		Baseline	Linear Human Capital Acc.	No Separations	Flexible Wage
Aggregate Output SD	2.3%	3.1%	1.7%	2.1%	2.2%
Unemployment Mean	9.4%	8.4%	2.4%	4.9%	5.4%
Unemployment SD	1.4%	1.6%	0.2%	0.3%	0.5%

Note: We report the correlation between model simulated series and their data counterparts (Panel 5a), the correlations with aggregate output in the model and in the data (Panel 5b) and the standard deviations of output and unemployment (Panel 5c). Simulations are obtained by feeding the model with a TFP series that matches the Italian TFP from 2000Q1 to 2019Q4. All series have been detrended. Panel 5a and 5b: Hamilton filter (4 lags and 8 leads); Panel 5c: Hodrick-Prescott filter (smoothing equal to 10^5, as in Shimer (2005)). We report the correlations for our baseline model and for three alternative ones. One in which human capital accumulation does not depend on firm quality and is given by $h_{\iota,t+1} = \phi_\iota + h_{\iota,t} + \epsilon_{\iota,t}$, a second in which wages can decrease in order to satisfy the firm participation constraint, and a third one in which firms fully commit to keeping matches open even if that induces present value losses. All the alternative models have been re-estimated as described in **Section 4.1**.

in general, the importance of accounting for heterogeneous patterns of human capital accumulation to adequately capture the volatility of unemployment.

A model incorporating our baseline human capital accumulation dynamics but not allowing firm-initiated endogenous separations also fails in matching unemployment volatility. Absent a cyclical increase in separations due to firm closures in recessions, the probability of losing a job will be almost identical across skill and age groups, failing to match unemployment volatility while contemporaneously matching its average level.

5 Anatomy of recessions

The model developed in **Section 3** can be used to analyze 1) how aggregate shocks propagate through the economy through changes in firm and worker sorting, and 2) what channels are responsible for their persistent effects on aggregate output.

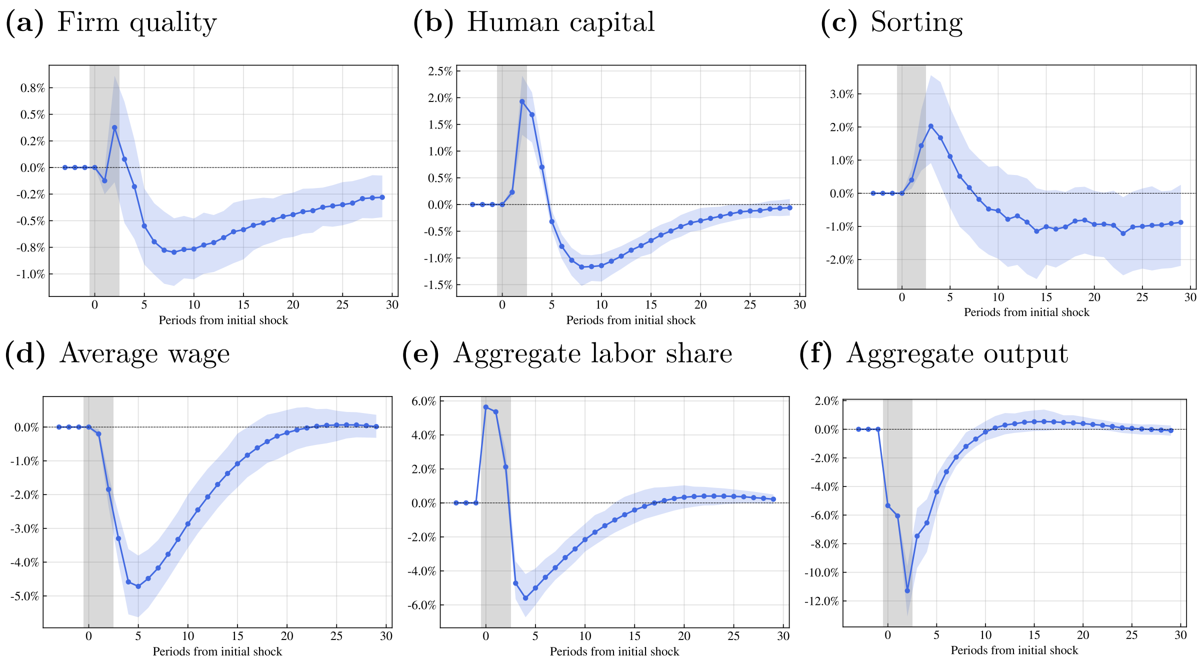
To illustrate these points we provide a series of decompositions of the economy's cumulative impulse responses to a TFP shock. We do this by first computing generalized impulse response functions. We compare a series of simulations of the model with aggregate shocks to otherwise identical counterparts, for which the only difference is that, at some exogenous time chosen to be the same for all simulations, the economy experiences three consecutive negative realizations of the TFP process.[41] We then focus on labor market outcomes of affected cohorts and the response of aggregate GDP throughout these specific recessions. We illustrate how the shock propagates in **Figure 8** and we then provide a decomposition of the response of aggregate output into sorting, human capital and displacement channels in **Figure 9**.

Shock propagation. In **Figure 8a** and **Figure 8b** we show the generalized impulse response functions of average firm quality and human capital. The onset of the recession is characterized by a sort of "Schumpeterian" dynamics, as implied by the initial marginal increase in the levels human capital and firm quality. **Figure 8c** shows our measure of sorting, the correlation of firms and workers' quality, relative to the baseline simulation. After a short-lived improvement, firm-worker sorting is persistently dampened by the recession. The prolonged drop in the quality of the factors of production increases the persistence of the initial shock on output beyond the original duration of the recession. **Figure 8f** shows that two years from the start of the recession, aggregate output its still approximately 1.5% below its counterfactual level and 0.5% below the no-recession scenario at a five years horizon.

The recession has a strong effect on both the average wage and the average labor share in the economy, as shown in **Figure 8d** and **8e** respectively. Due to the downward

[41] The cumulative drop in TFP is approximately 15% (equally split in each quarter).

Figure 8: Recession Experiment



Note: The panels in the figure plot the ratios of the aggregate variables between an economy in which we impose a three-quarter negative TFP realization and an economy without aggregate shocks, that serves as a benchmark. The gray shaded area are the quarters in which TFP is below trend, while the blue shaded area are the 90-10 quantile ranges across one hundred model simulations.

wage rigidty embedded in the contracts, average wages drop less than the average firm output. This allows for a brief spike in the labor share that quickly declines and remains below the counterfactual economy for a long time. The decline occurs for two reasons. First, the matches that form in the recovery period are subject to the sullying in firm and human capital qualities, lowering the average levels of starting wages. Second, due to the back-loading of compensation embedded in contracts, new contracts feature initially lower labor shares and greater profit shares for firms.

The quality of jobs created in recessions is different. In **Table 6**, we calculate the average tenure and wages for jobs created at different levels of the aggregate shock. Jobs created in recessions pay less on average than in "normal" times, and job stability is also affected, with jobs created in recessions lasting less than when the economy is running at full potential, as shown in Moscarini and Postel-Vinay (2016). This empirical pattern is replicated in the model. In addition, when we isolate periods within "normal" times that are immediately after a recession, we observe how it affects workers' careers even after the shock has been absorbed: jobs created after a recession have lower expected tenure and lower expected wages. Effects of the recessionary shocks will thus affect the trajectory of the labor market for years, potentially changing the transmission of additional shocks. To study this phenomenon in detail, the next subsection will study state-dependence through the lenses of our model.

Table 6: Job characteristics by aggregate state at their inception

	Bad Times	Recovery	Normal Times
Data			
Tenure	0.91	0.85	1.0
Wage	0.95	0.97	1.0
Model			
Tenure	0.95	0.95	1.0
Wage	0.95	0.98	1.0

Note: In the data, we take Hamilton filtered GDP, and define "*Bad Times*" as those periods in which GDP is below trend. "*Recovery*" periods are obtained by taking, for each GDP spell below trend, data from the first year in which GDP is below trend.

State-dependence. We can exploit the variation across the simulations used in **Figure 8** to check if specific states of the economy are correlated with the severity of the recessions, both in terms of cumulative losses and their persistence. We run a series of regressions in which we include the main moments of the *pre-recession* distributions of four variables of the model: firm quality, human capital, the average wage and labor share, plus a measure of labor market sorting. For each distribution, we examine their correlation with three model outcomes: i) short-term losses, which represent the cumulative output response one year after the shock; ii) medium-term losses, which capture the cumulative output response three years after the shock; and iii) the persistence of recessions, measured by the number of quarters from the onset of the shock until the output impulse response function (IRF) returns to zero. In total, we include all seventeen states in the regression analysis, and for ease of presentation, we present the results for the largest coefficients in **Table 7**.[42]

Output losses resulting from negative productivity shocks are primarily influenced by the first moments of the distributions. Specifically, if a recession hits an economy with firm quality one standard deviation higher than the average, the short-term output loss would be significantly stronger, reaching nearly seven standard deviations, and the medium-term loss would be around six standard deviations. This implies that when average firm quality is 1pp higher, the economy experiences larger cumulative output losses of approximately 1.3 and 1.8 percentage points, respectively. Similarly, a 1pp higher level of human capital increases the short-term output loss, resulting in a larger decline in output of 1.5 percentage points. This higher level of human capital extends the recovery period by approximately 1.2 quarters, although this coefficient is only marginally close to significant.

An average labor share 1pp higher instead, would imply a increase of output losses of 2.3pp in the short-term and of 2.8pp in the medium-term as well as an increase in the

[42] We report the full regression tables in Appendix F, Table F.1.

Table 7: State Dependence: Largest Coefficients

	Output Response		Persistence
	Short-term	Medium-term	
Firm Quality	-1.30***	-1.79**	-0.45
	(0.44)	(0.74)	(0.53)
Human Capital	1.54**	0.94	1.19
	(0.71)	(1.19)	(0.84)
Wage	0.39	1.01*	0.18
	(0.32)	(0.53)	(0.38)
Labor Share	-2.26***	-2.80**	1.38
	(0.76)	(1.27)	(0.90)
Sorting	1.07	1.89	-0.17
	(0.79)	(1.32)	(0.94)
R^2	0.57	0.52	0.16
N	100	100	100

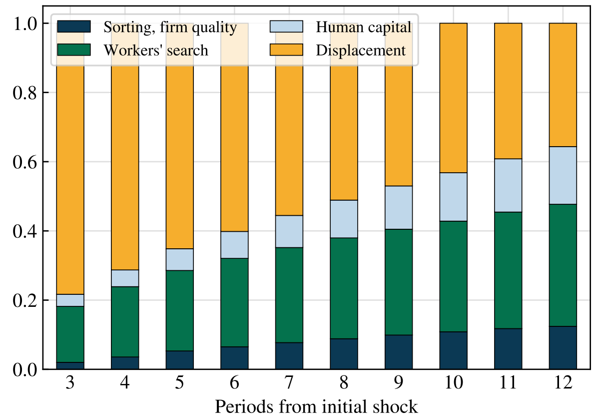
Note: Standard-errors in parenthesis. p-value: * < 0.1, ** < 0.05, *** < 0.01. The table reports the largest coefficients of regressing model outcomes after the shock on moments of relevant endogenous variables before the shock (one year average). The results are scaled to report the effect of changing the regressors by 1pp. Sorting is computed every quarter as the correlation between firm and worker quality and reported in the same table of the means even if technically is not a moment of a distribution. Model outcomes are: i) short-term cumulative output response (1 year after the shock); ii) the medium-term cumulative output response (3 years after the shock); and iii) the persistence of the shock (number of quarters before the output IRF is back at zero). For ease of exposition, the results are grouped by moments but the coefficients are computed including all moments in the same regression, **Table F.1** reports all the coefficients. The simulations are the same underlying the exercise in **Figure 8**.

length of the recovery by almost 1.4 quarters (although the coefficient on persistence is not statistically significant). It is important to highlight the significance of this result on the labor share. In fact, despite already accounting for the moments of the wage distribution, the labor share plays a crucial role in determining the severity of losses following the TFP shock.The labor share incorporates information about two crucial features for our model economy: the tenure distribution and the resulting quality of active job matches. In our contract formulation, highly tenured workers command higher wages and are situated in the flatter part of their human capital accumulation profiles within the match, being more vulnerable to separations. This result connects our model to previous findings in the literature (Engbom, 2021, Ai and Bhandari, 2021), suggesting that more fluid and dynamic labor markets, characterized by lower average tenures, might prove more resilient to recessions on an aggregate level.

5.1 Decomposing recessions

What explains the amplification and persistence of recessionary shocks? Different competing channels are at play. The first, which we call the human capital channel, captures the human capital accumulation that does not take place because of the recessionary event. The second, the sorting channel, amounts to the different joint

Figure 9: Decomposition of cumulative impulse response function of aggregate output



Note: The figure shows the relative importance of each transmission channel compared to the baseline recession for the cumulative response of GDP in the three years after the onset of the recession.

worker-firm distributions that emerge in the periods following the shock, because of different match formation along the cycle. The third, the search channel, is the impact of different search probabilities due to changes in aggregate and individual states. Finally, the standard displacement channel captures the job destruction that takes place because of the negative shock and its spillovers. We decompose the effects of recessions in the model economy by shutting down each channel at a time and then comparing the resulting dynamics to the one of a baseline recession.[43]

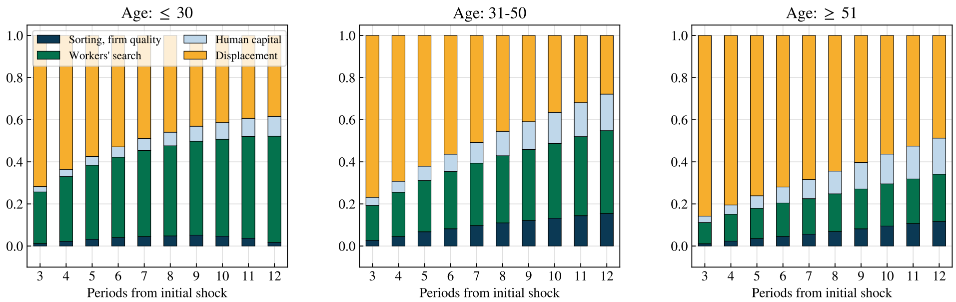
Figure 9 decomposes contributions to output after a shock into our four channels. The displacement channel is the main driver of the recession on impact, explaining the majority of output losses in the first year after the shock. During the recovery, the loss in output can be explained via a combination of a lower search quality from workers as well as a relatively lower firm quality for those that re-match during the transition. Recovery from displacement, while not immediate as search is frictional, is relatively fast, in part because unemployed workers have lower reservation wages.

Taken together, the sorting and human capital channels explain about 30% of cumulative output losses three years after the shock. While negligible in the short-run, the importance of lower firm quality builds up over time, amounting to approximately 13% after three years from the onset of the negative TFP shock. Human capital losses, instead, account for approximately 5% in the short-run and up to 17% at the three-years horizon. Worsened workers' search, finally, accounts for approximately 20%

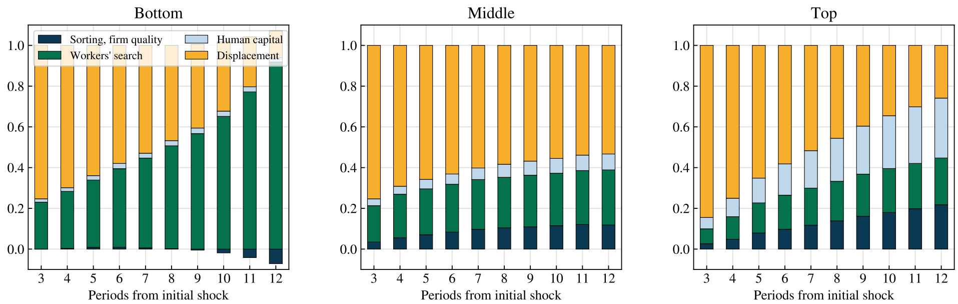
[43]Specifically, for the search component of the sorting channel we run model simulations in which the post-recession job-finding probabilities are those associated with search in the baseline simulation. Similarly, for the firm quality component of the sorting channel we keep employed workers in the same firm quality they have in their baseline simulation. For the human capital channel we erase human capital losses by forcing workers' human capital in our counterfactual simulation to be the same as in the baseline one. The displacement channel is obtained residually.

Figure 10: Decomposing aggregate output's cumulative response across the age and human capital distributions

(a) Age groups



(b) Human capital terciles



Note: For each age group, Panel (**a**) shows the relative importance of each transmission channel compared to the baseline recession for the cumulative impulse response of GDP across different age groups in the two years after the onset of the shock. Panel (**b**) plots the same decomposition across human capital terciles.

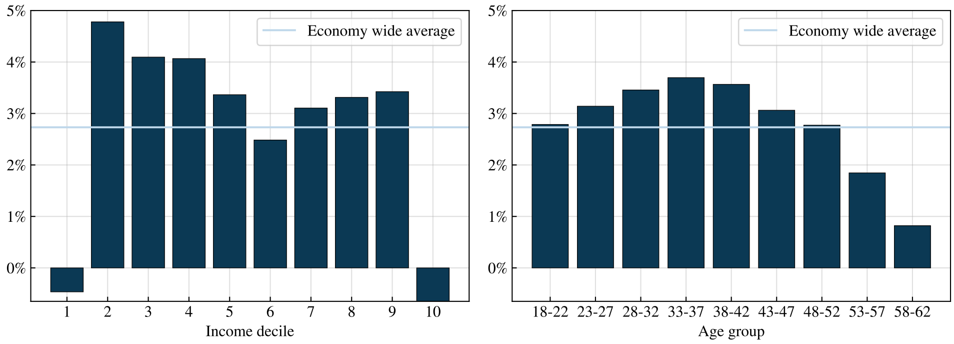
of output losses in the first year of the recovery and up to 35% three years after the start of the recession.

Average wage dynamics are instead dominated by the displacement effect, as shown in **Appendix Figure F.6**. The human capital channel increases over time but remains of second order importance. Interestingly, the effect of the sorting channel linked to workers' search is strongly negative at the beginning of the recovery (i.e. shutting down this channel would exacerbate the drop in average wage). This reflects the fact that when workers search facing the same job-finding probability of the baseline simulation a larger fraction of workers with relatively low human capital are pulled back into the labor market sooner than in the baseline, decreasing the average wage.

The four channels vary in importance across the workers' human capital or age cross-sectional distribution (see **Figure 10**). The displacement channel matters the most for the left tails of both distributions, as older and low-skilled workers are more likely to be separated. The sorting and human capital channels are instead more relevant for workers that are either young or in the right tail of the human capital distribution. For these workers, as a matter of fact, allocative efficiency and long-run human capital accumulation are particularly important. This intuition is confirmed also by the progressive reduction

Figure 11: Business Cycle Costs

(**a**) By income deciles (**b**) By age groups



Note: Panel (**a**) and (**b**) plot the reduction in consumption-equivalent utility due to aggregate fluctuations by income deciles and age groups.

in importance of the search component of the sorting channel as workers' age increases. This has important implications for the fragility of the economy: as the human capital distribution shifts to the right, recessions become less severe on impact, but might become more persistent and have greater long-run effects thought human capital scarring and firm quality match dynamics. As shown in **Table 6**, jobs that begin in expansions guarantee both higher wages and higher tenure and human capital accumulation within the spell.

5.2 The costs of recessions

Costs of Business Cycles. Lucas (1987) argues that welfare gains from reducing business cycle volatility are negligible, and quantifies them in less than 0.05 percentage points of aggregate consumption. By doing an analogous calculation, we estimate the cost of business cycles to be, on average, close to 3 percentage points, quantitatively close to the one in Barlevy (2004), who first theorized the potentially sullying effects of recessions. The rich heterogeneity in our model allows us to estimate how the welfare costs of business cycles vary along the income distribution, and analyze how they interact with income inequality: **Figure 11a** plots the cost of business cycles for different income deciles.[44] Lowest and highest deciles bear virtually no costs from business cycle fluctuations. This is due to the fact that the bottom decile mostly comprises very low skill unemployed workers, with very low probability of finding a stable employment regardless of the cycle. The very top of the distribution, on the other hand, comprises the highest skill workers, who have no cyclical unemployment risk and reached the maximum human capital attainable given firm opening costs and search frictions. The welfare costs of business cycles are, however, much larger for all other deciles, and have an interesting U-shape. Up until the sixth decile, the costs of aggregate fluctuations decrease with income: lower incomes are associated

[44]In Appendix Section F we also show, consistently with Heathcote et al. (2020), that our model reproduces the increase in income inequality throughout recessionary episodes.

with more-fragile jobs and to a higher likelihood of unemployment in recessions. For the remaining three deciles, costs of business cycles are instead associated with long-run impacts on careers. For workers closer to the top of the income distribution (but not quite there yet), recessions can have strong negative effects, as the deterioration in sorting and the ensuing lower human capital accumulation prevent them from realizing to their best possible earnings and human capital potential.

In **Figure 11b**, we report welfare costs by age groups. We calculate for each cohort the welfare amount agents would be willing to sacrifice out of the net present utility value of residual average lifetime earnings to avoid experiencing aggregate fluctuations. We observe that prime-age workers tend to incur in the greater welfare costs, due to two features. On the one hand, prime-age workers feature more highly educated workers than younger cohorts. For these workers distortions to sorting and human capital accumulation can be particularly detrimental. On the other hand, prime-age workers also feature older workers with lower education. For these workers displacement is riskier, as they have progressively lower re-employment probabilities and at lower productivity firms.

6 Conclusions

We develop a novel framework of on-the-job search and human capital accumulation with two-sided heterogeneity to study the role of sorting along the business cycle. In the model, search frictions and aggregate uncertainty prevent an efficient allocation of workers to firms, and the accumulation of human capital depends on the quality of worker-firm matches. Alterations to sorting induced by recessions are slow to reverse and contribute not only to slow recoveries but also to long-run changes in the distribution of firms and workers in the labor market. We show that these distortions account for 30% of the cumulative loss in output three years from a recessionary event.

The model is rich and tractable, and would naturally lend itself to policy analysis. Our results suggest that scarring effects and welfare effects of recessions are heterogeneous by education, skill and age. Different policies might thus be optimal for different kinds of workers, depending on whether unemployment risk or human capital deterioration are more relevant to them. The model might thus be deployed to analyze the impact and aggregate optimality of policies such as countercyclical unemployment benefits, short-term work, or retention schemes. We leave these studies for future research.

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Appendix for
"Human Capital Ladders, Cyclical Sorting, and Hysteresis"

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April 2024

Contents

A Data construction and sample selection

We rely on two main data sources provided by INPS through the VisitINPS Program: i) data on employment relationships in Italy from 1996 to 2018 (*Uniemens* dataset) and ii) balance sheet data for incorporated Italian firms from 1994 to 2019 (*Cerved* dataset).

Starting from data on virtually the universe of Italian private employment relationships at the contract level, we calculated for each worker their yearly gross real earnings and for the workers with multiple contracts we selected the information associated to the highest paid contract in the year and we find the associated annualized income using information on the number of actual weeks worked. For workers that experience a job flow or are promoted after 2009 we obtain the education level from the *Comunicazioni Obbligatorie* dataset, provided to INPS by the Ministry of Labor, and identify graduates and non-graduates. We label as graduates in the data both workers getting undergraduate education or above, or workers getting specialized diplomas after high-school.

We restrict our focus on workers employed under either full-time or part-time working schemes between 16 and 65 years old. Moreover, in order to compute AKM fixed effects and maintain the estimation computationally feasible, we analyze the connected set of firms and workers across firms with more than 15 employees. As in Bonhomme et al. (2019) we first cluster firms by means of a weighted K-means clustering based on deciles of their yearly wage distribution. The weight for the clustering is the yearly number of employees.

From *Cerved*, we have access to firm balance sheet data. From this dataset we obtain information of value added, overall labor costs and calculate quantiles of value added per employee. All monetary values (wages, value added, total compensation) are trimmed at the 1% level and deflated by the consumer price index for Italy.

B Additional Figures and Tables, Empirics

B.1 Returns to Experience

We discussed the role of firms for human capital accumulation in **Section 2**. Here, we present additional evidence that link firms to the on-the-job learning. What we want to test in this section is whether firms that are overall more productive are also better learning environments. To do so, we construct firm classes in two alternative ways: by quintile of firm-level value-added per employee, and by quintile of AKM fixed effects. Then, we construct a measure of experience in each firm-class for each worker i.

Define $\mathbb{I}_c \equiv \mathbb{I}\{f(i,t) \in c\}$ as the indicator function that take value one when the worker's current employer, $f(i,t)$, belongs to class c. We can then estimate the following wage equation,

$$\log(w_{i,t}) = \alpha_i + \alpha_{c(i,t)} + \sum_{c=1}^{5} \beta_c e^c_{i,t} + \mathbf{X}'_{i,t}\theta + \varepsilon_{i,t}, \tag{B.1}$$

where $w_{i,t}$ is a monthly wage, α_i and $\alpha_{c(i,t)}$ are worker and firm-class fixed effects, $e^c_{i,t}$ are the number of years worker i has worked in firms belonging to class c up to year t, and $\mathbf{X}_{i,t}$ are a set of controls that include age, sector-year and contract type (temporary versus open-ended, full-time versus part-time) fixed effects. A challenge to the identification of experience effects is the presence of firm-specific skills, outside offers and bargaining, match effects, firm productivity shocks, or seniority based pay schemes. However, since

Table B.1: Wage experience profiles by firm type

	Full (1)	Displaced (2)	Full (3)	Displaced (4)
Exp. in quantile 1	0.006	0.003	0.006	0.004
	(0.000)	(0.000)	(0.000)	(0.000)
Exp. in quantile 2	0.006	0.004	0.006	0.003
	(0.000)	(0.000)	(0.000)	(0.000)
Exp. in quantile 3	0.006	0.004	0.009	0.006
	(0.000)	(0.000)	(0.000)	(0.000)
Exp. in quantile 4	0.010	0.007	0.014	0.012
	(0.000)	(0.000)	(0.000)	(0.000)
Exp. in quantile 5	0.015	0.013	0.019	0.016
	(0.000)	(0.000)	(0.000)	(0.000)
N	98,571,505	4,799,423	98,571,505	4,799,423
R^2	0.86	0.85	0.86	0.85
Worker FEs	✓	✓	✓	✓
Contract FEs	✓	✓	✓	✓
Sector-Year FEs	✓	✓	✓	✓
Quant. type	VA	VA	AKM FE	AKM FE

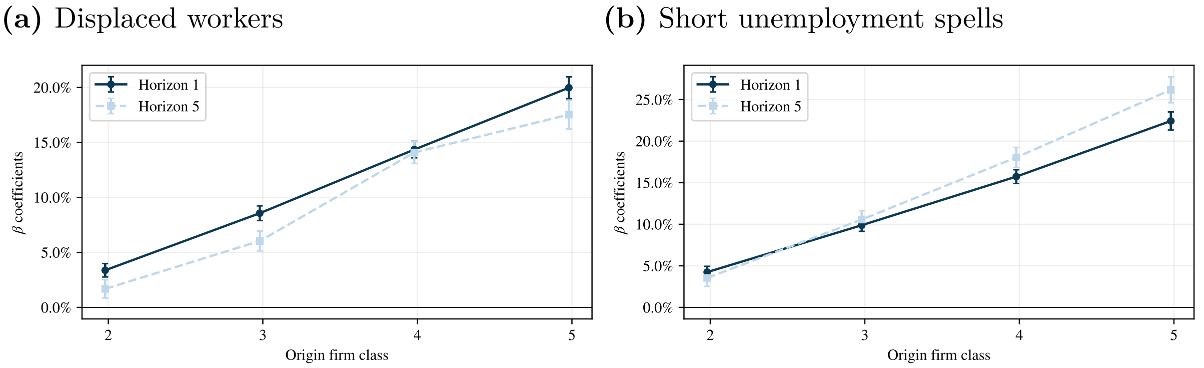
Note: The table reports the returns the returns to experience estimated from Equation (B.1). The full sample includes all workers employed in private firms in Italy. Displaced workers are identified from mass layoffs. When estimating the returns for the sample of displaced workers we include the worker and firm-class fixed effects estimated on the full sample as regressors. Contract FEs include controls for temporary versus open-ended, full-time versus part-time, and wage clusters fixed effects.

these should not impact post-displacement wages, we leverage the population-level coverage of both datasets and extend our results by presenting the same specification restricted to the sample of workers affected by firm closure and mass layoff events.

As described in **Section 2**, we identify displacement events as in Bertheau et al. (2023) as a decrease in employment beyond 30% for establishment with more than 49 employees. Displaced workers are separating workers younger than 50 years old, with at least 3 years of tenure within the firm. **Table B.1** presents returns to heterogeneous experiences resulting from the estimation of **Equation B.1** for both the full sample and the sample of displaced workers' first post-displacement observation.

We find that more productive firms increasingly contribute to the accumulation of skills, resulting in higher ex-post wages. Results are consistent across specifications, consistent with heterogeneous returns being driven by accumulation of portable skills. An implication of our modelling approach, however, is that the accumulation of skills at a certain firm cannot go on indefinitely: as tenure in a firm of a certain productivity increases, the skill gains progressively decrease. A simple way to test this hypothesis is to

Figure B.1: Persistent effect of past firms for workers



Note: The table figure reports the main coefficient of interest for the regression in **Equation 2**. The displaced workers sample includes workers that make E-U-E transitions following mass layoffs, while the short unemployment spell sample identifies all E-U-E transitions in which workers are employed for at least 4 quarters after an unemployment spell of less than one year. Firm quality is defined as (yearly) quintiles of firm cluster AKM fixed effect. N = 577,174 (left), 371,593 (right).

introduce a quadratic experience term, to verify that the class-specific experience profile is concave, that is running:

$$\log(w_{i,t}) = \alpha_i + \alpha_{c(i,t)} + \sum_{c=1}^{5} \beta_c e_{i,t}^c + \sum_{c=1}^{5} \gamma_c \left(e_{i,t}^c\right)^2 + \mathbf{X}'_{i,t}\theta + \varepsilon_{i,t}, \tag{B.2}$$

Results for **Equation B.2** are presented in **Table B.2**. The squared term is consistently negative, while the linear term remains positive and increasing in firm class. This confirms the initial hypothesis of decreasing returns to experience from spending time in a given firm class, and is consistent with the model intuition that, for workers to increase earnings over the life cycle, they need to climb a 'firm ladder' between jobs of different quality.

B.2 Persistent effects of firm quality on workers' earnings

In this section we report the coefficients for the specification in **Equation 2** and **Figure 2a** in **Table B.3**. In **Figure B.1** and **Table B.4** we report an analogous robustness specification, in which the firm class is a quintile of esttimated AKM fixed effects on firm cluster types as in Bonhomme and Manresa (2015).

B.3 Scarring Effects

In order to estimate the effect of entering the labor market in a recession we use an age-cohort-period model in which we break the collinearity among the three set of fixed effects by proxying the cohort fixed effects with the cyclical component of real GDP (Hamilton

Table B.2: Quadratic wage experience profiles by firm type

	Full	Displaced	Full	Displaced
	(1)	(2)	(3)	(4)
Exp. in quantile 1	0.006	0.005	0.006	0.004
	(0.000)	(0.000)	(0.000)	(0.000)
Exp^2 in quantile 1	-3.21e-5	-8.27e-5	6.47e-5	1.78e-5
	(8.14e-6)	(1.37e-5)	(3.97e-6)	(7.22e-6)
Exp. in quantile 2	0.005	0.002	0.004	0.001
	(0.000)	(0.000)	(0.000)	(0.000)
Exp^2 in quantile 2	-7.03e-5	0.0002	0.0001	0.0002
	(5.34e-6)	(1.06e-5)	(2.9e-6)	(6.09e-6)
Exp. in quantile 3	0.007	0.004	0.009	0.005
	(0.000)	(0.000)	(0.000)	(0.000)
Exp^2 in quantile 3	-5.2e-5	8.02e-5	-5.75e-6	6.47e-5
	(4.54e-6)	(9.53e-6)	(2.5e-6)	(4.75e-6)
Exp. in quantile 4	0.011	0.008	0.020	0.016
	(0.000)	(0.000)	(0.000)	(0.000)
Exp^2 in quantile 4	-0.0001	-9.66e-5	-0.0003	-0.0002
	(3.7e-6)	(7.33e-6)	(2.76e-6)	(4.77e-6)
Exp. in quantile 5	0.020	0.016	0.035	0.028
	(0.000)	(0.000)	(0.000)	(0.000)
Exp^2 in quantile 5	-0.0003	-0.0002	-0.0008	-0.0006
	(2.38e-6)	(4.7e-6)	(4.59e-6)	(7.29e-6)
N	98,571,505	4,799,423	98,571,505	4,799,423
R^2	0.86	0.85	0.86	0.85
Worker FEs	✓	✓	✓	✓
Contract FEs	✓	✓	✓	✓
Sector-Year FEs	✓	✓	✓	✓
Quant. type	VA	VA	AKM FE	AKM FE

Note: The table reports the returns the returns to experience estimated from Equation (B.1). The full sample includes all workers employed in private firms in Italy. Displaced workers are identified from mass layoffs. When estimating the returns for the sample of displaced workers we include the worker and firm-class fixed effects estimated on the full sample as regressors. Contract FEs include controls for temporary versus open-ended, full-time versus part-time, and wage clusters fixed effects.

Table B.3: Importance of firm quality for workers' earnings after E-U-E transitions

	Displaced workers		Short unemployment spells	
	Log-Wage$_t$	Log-Wage$_{t+5}$	Log-Wage$_t$	Log-Wage$_{t+5}$
	(1)	(2)	(3)	(4)
Firm Prod. Q2	0.025	0.013	0.020	0.008
	(0.002)	(0.003)	(0.002)	(0.004)
Firm Prod. Q3	0.046	0.031	0.048	0.033
	(0.002)	(0.003)	(0.003)	(0.004)
Firm Prod. Q4	0.066	0.042	0.078	0.067
	(0.002)	(0.003)	(0.003)	(0.004)
Firm Prod. Q5	0.108	0.100	0.117	0.116
	(0.002)	(0.003)	(0.003)	(0.004)
Controls	Yes	Yes	Yes	Yes
R^2	0.67	0.74	0.71	0.65
N	577,174	371,593	284,285	167,289

Note: Standard errors clustered at the worker level in parenthesis. The table reports the main coefficient of interest for the regression in **Equation 2**. The displaced workers sample includes workers that make E-U-E transitions following mass layoffs, while the short unemployment spell sample identifies all E-U-E transitions in which workers are employed for at least 4 quarters after an unemployment spell of less than one year. Firm quality is defined as (yearly) quintiles of firm value added per employee.

Table B.4: E-U-E wage regressions by past firm AKM firm cluster FE

	Displaced		Short unemployment spells	
	Log-wage$_{t+1}$	Log-wage$_{t+5}$	Log-wage$_{t+1}$	Log-wage$_{t+5}$
	(1)	(2)	(3)	(4)
Exp. in quantile 2	0.034	0.017	0.042	0.035
	(0.002)	(0.003)	(0.002)	(0.003)
Exp. in quantile 3	0.086	0.060	0.099	0.106
	(0.002)	(0.003)	(0.002)	(0.003)
Exp. in quantile 4	0.144	0.141	0.157	0.180
	(0.002)	(0.003)	(0.003)	(0.004)
Exp. in quantile 5	0.200	0.175	0.224	0.262
	(0.003)	(0.004)	(0.003)	(0.005)
Controls	Yes	Yes	Yes	Yes
R^2	0.68	0.66	0.71	0.66
N	577,174	371,655	284,314	167,310

Note: Standard errors clustered at the worker level in parenthesis. The table reports the main coefficient of interest for the regression in **Equation 2**. The displaced workers sample includes workers that make E-U-E transitions following mass layoffs, while the short unemployment spell sample identifies all E-U-E transitions in which workers are employed for at least 4 quarters after an unemployment spell of less than one year. Firm quality is defined as (yearly) quintiles of firm cluster AKM fixed effects.

filtered). In particular, we estimate the following yearly model:

$$\log(w)_{t,c,e} = \Phi_t + \Phi_e + \beta_e \widetilde{Y}_c \times \Phi_e + u_{t,c,e}, \tag{B.3}$$

where Φ_t, Φ_e, are dummies for calendar years and labor market experience, and $\widetilde{Y}_c$ is the cyclical realization of real GDP for cohort c at time of their labor market entry. The set of coefficients β_e, therefore, estimate the effect of aggregate conditions on real wages at each year of labor market experience.

Table B.5: Effects of initial aggregate conditions along the experience profile and experience growth profile

Dep.Variable: Log-Wage	Experience × Cycle	Experience
Experience Dummy		
0	1.968	
	(0.452)	
1	1.463	0.150
	(0.238)	(0.011)
2	1.473	0.246
	(0.283)	(0.014)
3	1.239	0.304
	(0.343)	(0.014)
4	1.250	0.342
	(0.349)	(0.015)
5	1.239	0.375
	(0.349)	(0.015)
6	1.301	0.399
	(0.287)	(0.015)
R^2	0.89	0.89
N	254,000,000	254,000,000
Age FE	✓	✓
Year FE	✓	✓
Sex FE	✓	✓
LLM FE	✓	✓

Note: The table reports regression coefficients for the empirical estimates in the data used to construct the profiles in Figure F.5.

C Discussion and Proofs

In this section we discuss the properties of the equilibrium of the model economy developed in the previous sections. All propositions and corresponding proofs related to the properties discussed in **Section 3.8** are reported in **Appendix C.1** and **C.2**.

C.1 Workers optimal behavior

For compactness of notation, we omit the dependence on education level, ι, which is a fixed characteristic, and the idiosyncratic human capital shock, which is additive, from the proofs in Appendices. The logic of the proofs follows without loss of generality.

The following propositions characterize the properties of workers' optimal search strategies that solve the search problem in (4), restated here for convenience:

$$R(h, \tau, V; \Omega) = \sup_{v} \Big[p(\theta(h, \tau, v; \Omega)) \big[v - V \big] \Big]. \tag{C.1}$$

Lemma C.1. *The composite function $p(\theta(h, \tau, v; \Omega))$ is strictly decreasing and strictly concave in v.*

Proof. For this proof we follow closely Menzio and Shi (2010), Lemma 4.1 (ii). From the properties of the matching function we know that $p(\theta)$ is increasing and concave in θ, while $q(\theta)$ is decreasing and convex. Consider that the equilibrium definition of $\theta(\cdot)$ is

$$\theta(h, \tau, v; \Omega) = q^{-1} \left(\frac{c(y)}{J(h, \tau, v, y; \Omega)} \right),$$

and that the first order condition for the wage and the envelope condition on V of the optimal contract problem in (12) implies

$$\frac{\partial J(h, \tau, v, y; \Omega)}{\partial v} = -\frac{1}{u'(w)}.$$

so that as $u'(\cdot) > 0$, $J(\cdot)$ is decreasing in v.

From the equilibrium definition of $\theta(\cdot)$ and noting that $q^{-1}(\cdot)$ is also decreasing due to the properties of the matching function we have that

$$\frac{\partial \theta(h, \tau, v; \Omega)}{\partial v} = \left. \frac{\partial q^{-1}(\xi)}{\partial \xi} \right|_{\xi = \frac{c(y)}{J(h,\tau,v,y;\Omega)}} \cdot \left(-\frac{\partial J(h, \tau, v, y; \Omega)}{\partial v} \right) \cdot \frac{c(y)}{(J(h, \tau, v, y; \Omega))^2} < 0,$$

which, in turn, implies that

$$\frac{\partial p(\theta(h,\tau,v;\Omega))}{\partial v} = \frac{\partial p(\theta)}{\partial \theta}\bigg|_{\theta=\theta(h,\tau,v;\Omega)} \cdot \frac{\partial \theta(h,\tau,v;\Omega)}{\partial v} < 0.$$

Suppressing dependence on the states (h, τ, y, Ω) for readability, to prove that $p(\theta(v))$ is concave, consider that $J(v)$ is concave[1] and a generic function $\frac{c}{v}$ is strictly convex in v. This implies that with $\alpha \in [0, 1]$ and $v_1, v_2 \in \mathcal{V}$, $v_1 \neq v_2$:

$$\frac{c}{J(\alpha v_1 + (1-\alpha)v_2)} \leq \frac{c}{\alpha J(v_1) + (1-\alpha)J(v_2)} < \alpha\frac{c}{J(v_1)} + (1-\alpha)\frac{c}{J(v_2)}.$$

As $p(q^{-1}(\cdot))$ is strictly decreasing the inequality implies that

$$\begin{aligned} p\left(q^{-1}\left(\tfrac{c}{J(\alpha v_1+(1-\alpha)v_2)}\right)\right) &\geq p\left(q^{-1}\left(\tfrac{c}{\alpha J(v_1)+(1-\alpha)J(v_2)}\right)\right) \\ &> \alpha p\left(q^{-1}\left(\tfrac{c}{J(v_1)}\right)\right) + (1-\alpha)p\left(q^{-1}\left(\tfrac{c}{J(v_2)}\right)\right), \end{aligned}$$

and as $\theta(v) = q^{-1}(\frac{c}{J(v)})$:

$$p(\theta(\alpha v_1 + (1-\alpha)v_2)) > \alpha p(\theta(v_1)) + (1-\alpha)p(\theta(v_2))$$

so that $p(\theta(v))$ is strictly concave in v. □

In the following proposition we summarize the main results regarding the behavior of the workers and their objective functions.

Proposition C.1. *Given the worker search problem, the following properties hold:*

(i) The returns to search, $p(\theta(h,\tau,v;\Omega))\big[v - V\big]$, are strictly concave with respect to promised utility, v.

(ii) The optimal search strategy

$$v^*(h,\tau,V;\Omega) \in \arg\max_v \left\{p(\theta(h,\tau,v;\Omega))\big[v - V\big]\right\}$$

is weakly increasing in V, Liptschitz continuous and unique.

(iii) For all promised utilities, the search gain $R(h,\tau,V;\Omega)$ is positive, weakly decreasing in V.

(iv) The survival probability of the match, given the optimal choice of the worker, is increasing in the value of current and future promised utilities, so $\widetilde{p}_t(h,\tau,v;\Omega)$ is increasing in v and V.

[1] $J(\cdot)$ concave give the two-point lottery in the structure of the contract. See Menzio and Shi (2010) Lemma F.1.

(v) *Search strategies are increasing in workers' human capital, h.*

Proof. The proofs follow closely Shi (2009), Lemma 3.1 and Menzio and Shi (2010), Corollary 4.4. More formally, for each triplet (h, τ, Ω) given at each search stage, we can re-define the search objective function as $K(v, V) = p(\theta(v))(v - V)$ and $v^*(V) \in \arg\max_v K(v, V)$ as the function that maximises the search returns (i.e. the optimal search strategy of the worker) and prove the following

(i) To show that $K(v, V)$ is strictly concave in v consider two values for v, $v_1, v_2 \in \mathcal{V}$ such that $v_2 > v_1$ and define $v_\alpha = \alpha v_1 + (1-\alpha)v_2$ for $\alpha \in [0,1]$.

Then by definition:

$$\begin{aligned}
K(v_\alpha, V) &= p(\theta(v_\alpha))(v_\alpha - V) \\
&\geq [\alpha p(\theta(v_1)) + (1-\alpha)p(\theta(v_2))][\alpha(v_1 - V) + (1-\alpha)(v_2 - V)] \\
&= \alpha K(v_1, V) + (1-\alpha)K(v_2, V) + \alpha(1-\alpha)[(p(\theta(v_1)) - p(\theta(v_2))](v_2 - v_1) \\
&> \alpha K(v_1, V) + (1-\alpha)K(v_2, V)
\end{aligned}$$

where the first inequality follows from the concavity of $p(\theta(\cdot))$ (this is true if $J(\cdot)$ concave with respect to V) and the second inequality stems from the fact that $p(\theta(\cdot))$ is strictly decreasing hence $\alpha(1-\alpha)[(p(\theta(v_1)) - p(\theta(v_2))](v_2 - v_1) > 0$.

(ii) **Weakly Increasing.** Consider a worker employed in a job that gives lifetime utility V. Given that $v \in [\underline{v}, \overline{v}]$, and that submarkets are going to open depending on realizations of the aggregate productivity, a, there is only one region in the set of promised utilities where the search gain is positive. This set is $[V, v(a)]$ with $v(a)$ being the highest possible offer that a firm makes in the submarket for the worker (h, τ). Any submarket that promises higher than $v(a)$ is going to have zero tightness. Therefore, the optimal search strategy for $V \geq v(a)$ is $v^*(V) = V$, as $K(V, v(a)) = K(V, V) = K(\overline{v}, V) = 0$ (the search gain is null given the current lifetime utility V). For $V \in [V, v(a)]$, instead, as $K(v, V)$ is bounded and continuous, the solution $v^*(V)$ has to be interior and therefore respect the following first order condition

$$V = v^*(V) + \frac{p(\theta(v^*(V)))}{p'(\theta(v^*(V)) \cdot \theta'(v^*(V))}. \tag{C.2}$$

Now consider two arbitrary values V_1 and V_2, $V_1 < V_2 < \overline{v}$ and their associated solutions $W_i = v^*(V_i)$ for $i = 1, 2$. Then, V_1 and V_2 have to generate two different values for the right-hand side of (C.2). Hence, $v^*(V_1) \cap v^*(V_2) = \emptyset$ when $V_1 \neq V_2$.

This also implies that the search gain evaluated at the optimal search strategy is higher than the gain at any other arbitrary strategy so that $K(W_i, V_i) > K(W_j, V_i)$

for $i \neq j$. This implies that

$$\begin{aligned}0 \geq & [K(W_2, V_1) - K(W_1, V_1)] + [K(W_1, V_2) - K(W_2, V_2)] \\ & = (p(\theta(W_2)) - p(\theta(W_1)))(V_2 - V_1),\end{aligned}$$

thus, $p(\theta(W_2)) \leq p(\theta(W_1))$. As $p(\theta(\cdot))$ is strictly decreasing (see Lemma C.1), then $v^*(V_1) \leq v^*(V_2)$.

Lipschitz continuous. We follow Menzio and Shi (2010) and show that, with generic points $V_1 \leq V_2$, $v^*(V_2) - v^*(V_1) \leq V_2 - V_1$. Given that in this case we know that $v^*(V_1) \leq v^*(V_2)$, then $v^*(V_2) - v^*(V_1) \geq 0$. If $v^*(V_2) - v^*(V_1) = 0$ then the claim is trivially satisfied. If $v^*(V_2) - v^*(V_1) > 0$, then let's consider a generic real number $\delta \in (0, (v^*(V_2) - v^*(V_1))/2)$. Given the definition of $v^*(\dot{})$, we know that $K(v^*(V_i), V_i) \geq K(v^*(V_i) + \delta, V_i)$. From this inequality and the definition for $K(\cdot, \cdot)$ it follows that $p(\theta(v^*(V_1)))(v^*(V_1) - V_1) \geq p(\theta(v^*(V_1) + \delta))(v^*(V_1) + \delta - V_1)$ and similarly $p(\theta(v^*(V_2)))(v^*(V_2) - V_2) \geq p(\theta(v^*(V_2) - \delta))(v^*(V_2)\delta - V_2)$. Then

$$\begin{aligned}v^*(V_1) - V_1 &\geq \frac{p(\theta(v^*(V_1) + \delta))\delta}{p(\theta(v^*(V_1))) - p(\theta(v^*(V_1) + \delta))} \\ v^*(V_2) - V_2 &\leq \frac{p(\theta(v^*(V_2) - \delta))\delta}{p(\theta(v^*(V_2) - \delta)) - p(\theta(v^*(V_2)))}.\end{aligned}$$

Knowing that $p(\theta(\cdot))$ is a decreasing function, and that $v^*(V_1) + \delta \leq v^*(V_2) - \delta$ given the domain of δ, then $p(\theta(v^*(V_1) + \delta) \geq p(\theta(v^*(V_2) - \delta)$. Also, given that $v^*(V_2) - v^*(V_1) > 0$ then $p(\theta(v^*(V_1))) - p(\theta(v^*(V_1) + \delta)) \leq p(\theta(v^*(V_2) - \delta)) - p(\theta(v^*(V_2)))$. This implies that we can rewrite the inequalities above as

$$\begin{aligned}v^*(V_2) - V_2 &\leq \frac{p(\theta(v^*(V_2) - \delta))\delta}{p(\theta(v^*(V_2) - \delta)) - p(\theta(v^*(V_2)))} \\ &\leq \frac{p(\theta(v^*(V_1) + \delta)}{p(\theta(v^*(V_2) - \delta)) - p(\theta(v^*(V_2)))} \\ &\leq \frac{p(\theta(v^*(V_1) + \delta)}{p(\theta(v^*(V_1))) - p(\theta(v^*(V_1) + \delta))} \leq v^*(V_1) - V_1\end{aligned}$$

which implies $v^*(V_2) - v^*(V_1) \leq V_2 - V_1$.

Unique. Uniqueness follows directly from strict concavity shown in (i).

(iii) The Bellman equation for the search problem is:

$$R(h, \tau, V; \Omega) = \sup_v \Big[p(\theta(h, \tau, v; \Omega)) \big[v - V \big] \Big]$$

hence a simple envelope argument shows that

$$\frac{\partial R(h,\tau,V;\Omega)}{\partial V} = -p(\theta(h,\tau,v;\Omega)) \leq 0,$$

as the job finding probability is weakly positive for all utility promises.

As $p(\theta(\cdot)) \geq 0$, $v^*(\cdot) \in [\underline{v}, \overline{v}]$ then $R(\cdot) \geq 0$.

(iv) Given the optimal search strategy, $v^*(h,\tau,V;\Omega)$, we can define the survival probability of the match as in (7):

$$\widetilde{p}(h,\tau,V;\Omega) \equiv (1-\lambda)(1-\lambda_e p(\theta(h,\tau,v^*;\Omega))).$$

Then, given (h,τ,Ω)

$$\frac{\partial \widetilde{p}(V)}{\partial V} = -\beta(1-\lambda)\lambda_e \left.\frac{\partial p(\theta)}{\partial \theta}\right|_{\theta=\theta(v^*)} \left.\frac{\partial \theta(v)}{\partial v}\right|_{v=v^*(V)} \frac{\partial v^*(V)}{\partial V} > 0,$$

because $p(\cdot)$ and $v^*(\cdot)$ are both increasing functions while $\theta(\cdot)$ is a decreasing function in promised utilities.

(v) We want to show that the optimal search strategy v^*, is increasing in human capital, so that $\frac{\partial v^*(h)}{\partial h} > 0$.

Knowing that the optimal search strategy has to satisfy the following first order condition:

$$\frac{\partial p}{\partial v}(v^* - V) = 0,$$

we can use the implicit function theorem to get

$$\frac{\partial v^*}{\partial h} = -\frac{(v^*-V)\frac{\partial^2 p}{\partial v \partial h} + \frac{\partial p}{\partial h}}{(v^*-V)\frac{\partial^2 p}{\partial v^2} + 2\frac{\partial p}{\partial v}}. \tag{C.3}$$

As $(v^* - V) > 0$, and $p(\cdot)$ decreasing and concave in W so that $\frac{\partial^2 p}{\partial W^2}, \frac{\partial p}{\partial W} < 0$, the denominator of **Equation C.3** is negative. Therefore for **Equation C.3** to be negative, it has to be that $(v^* - V)\frac{\partial^2 p}{\partial W \partial h} + \frac{\partial p}{\partial h} > 0$. This, in turn, implies that

$$\frac{\partial^2 p}{\partial W \partial h} > \frac{\partial p}{\partial W}\frac{\partial p}{\partial h}\frac{1}{p},$$

where the left hand side is negative as $p(\cdot)$ is increasing in h but decreasing in W.

□

The first statement implies that the marginal returns of searching towards better firms are decreasing. The intuition is that as workers search for work at firms granting better values, their job-finding probability decreases as better employment prospects are also subject to higher competition.

As a consequence of the strict concavity established in the first statement, workers' optimal search strategy is unique. The search strategy is also (weakly) increasing in the value of lifetime utilities granted by the current contract, which is the outside option for the worker.

The third statement follows from the fact that marginal returns to search are decreasing and the set of feasible utility promises is compact. The intuition is that employees at firms with higher utility promises have a relatively fewer chances of improving their position. Given a high outside option, the utility gain from moving is relatively lower, whereas the probability of matching with any firm does not depend on the *current* utility promise per se, but on the future promise offered by the vacancy.

The fourth statement finally follows from considering the implication of the previous ones. Given that the optimal search strategy is increasing in V workers' probability of leaving the firm at any time ends up depending negatively on V. This guarantees a longer expected duration of the match at higher current promised utility V, thus retention probabilities that are increasing in promised utilities v.

As human capital accumulation is tightly linked to the quality of the employer, workers that are able to start their working careers in good times have a greater chance of finding themselves on an higher path of human capital growth. As worker careers are limited and human capital accumulation follows a slow-moving process, business cycle effects on human capital quality fade only slowly and the quality of initial matches, both in terms of lifetime utility and firm quality, bears a long-standing effect on workers' careers.

C.2 Characteristics of the optimal contract

The optimization in the contracting problem balances a trade-off between insurance provision and profit maximization for firms. The contract implicitly takes into account workers' search incentives and their inability to commit to stay. The following propositions characterizes workers' incentives along the business cycle from the firms' standpoint.

Lemma C.2. *The Pareto frontier $J(h, \tau, W, y; \Omega)$ is concave in W.*

Proof. This is a direct consequence of using a two-point lottery for $\{w_i, W_i'\}$ as shown by Menzio and Shi (2010), Lemma F.1. □

Lemma C.3. *The Pareto frontier $J(h, \tau, W, y; \Omega)$ is increasing in y.*

Proof. The intuition for this proof follows the fact that a higher y firm, once the match exists, can always deliver a certain promise V and have resources left over. Within a dynamic contract, future retention is already optimized as the match is formed. This means that the promise V can be delivered by the greater capacity on the part of producing with respect to a close y firm. In presence of human capital accumulation, the worker is compensated through greater option values in the future, which again means that, even with lower retention, the firm cashes in more profits while decreasing wages (and respecting the V promise).

One can get to the same conclusion by starting from time T, noticing that the function J is trivially increasing in y in the last period, and the stepping back. At $T-1$, given V, any higher y function can make greater profits with the same delivery of value V, given the contract's optimal promise, which is a fortiori true with human capital accumulation (the option value is greater, so the firm can decrease w as a response). A more formal argument goes as follows: start from the optimal policies, as per Eq. (12) of a firm that has $y = \bar{y}$ and assume that one could exogenously increase its installed capital to $\bar{y} + \varepsilon$. We want to know whether, keeping policies constant, this would increase the flow of profits while keeping the worker indifferent. If this is true, then *a fortiori* it will be true that the firm value function J will be increasing in y. With a slight abuse of notation and for conciseness, we refer to the future J in Eq. (12) as J'.

This amounts to calculating:[2]

$$\left.\frac{d\bar{J}}{dy}\right|_{W,\{\pi_i,w_i,W_i'\}} = \frac{\partial f(\cdot)}{\partial y} + \beta\mathbb{E}_\Omega\left[\left.\frac{\partial\tilde{p}(\cdot)}{\partial y}\right|_{W,\{\pi_i,w_i,W_i'\}}\bar{J}'\right]$$

This first order condition presents the trade-off discussed in words above, namely that an increase in y will be instantaneously beneficial to production, but might also potentially have a longer term adverse effect on profits through decreased retention. Finding the sign of the derivative on the LHS hinges on understanding the sign of the derivative of the second element on the RHS, since $\frac{\partial f(\cdot)}{\partial y} > 0$ given the properties of the production function $f(\cdot)$. The change in y would affect search objective v_y through the variation in h due to the human capital accumulation dynamics, even taking the current firm policies as given.

Notice that:

$$\frac{\partial\tilde{p}(\cdot)}{\partial y} = -\lambda\lambda_E\frac{\partial p(\cdot)}{\partial\theta}\frac{\partial\theta(\cdot)}{\partial y}$$

Remember that $\frac{\partial p(\cdot)}{\partial\theta} > 0$ due to the properties of the job-finding probability function $p(\cdot)$. Assume first the case in which $\frac{\partial\theta(\cdot)}{\partial y} \leq 0$. In this case $\frac{\partial\tilde{p}(\cdot)}{\partial y} > 0$, which in turn implies,

[2]Without loss of generality, we assume that J' is constant with respect to y. Alternatively, one may start proving the result for contracts offered to workers just one period before retirement (for which $\frac{\partial J'}{\partial y}$ is trivially positive), and generalize the result with backward induction.

as argued, that J has to be increasing in y.

Now consider the second case, namely $\frac{\partial\theta(\cdot)}{\partial y} > 0$. By the free entry condition, we obtain:

$$\frac{\partial\theta(\cdot)}{\partial y} = \frac{\partial q^{-1}(c(y)/J(y))}{\partial y} = \frac{1}{q'\left(q^{-1}\left(c(y)/J(y)\right)\right)} \frac{c'(y)J(y) - \frac{\partial J(y)}{\partial y}c(y)}{J(y)^2}$$

The first term in the result is negative, given the properties of function $q(\cdot)$. Given our assumption on $\frac{\partial\theta(\cdot)}{\partial y}$ the second term in the result has to be negative as well. This requires: $c'(y)J(y) - \frac{\partial J(y)}{\partial y}c(y) < 0$, or $\frac{\partial J(y)}{\partial y} > \frac{c'(y)J(y)}{c(y)} > 0$. □

Proposition C.2. *The Pareto frontier $J(h, \tau, W, y; a, \mu)$ is increasing in the aggregate productivity shock a, while retention probabilities, $\widetilde{p}(h, \tau, W; a, \mu)$ decrease in aggregate productivity.*

Proof. We proceed by backward induction.[3] Following the logic of the proof of **Lemma C.3**, the proposition is trivially true for workers T periods old. Given that the firm increases its production while keeping the worker at least indifferent, J is at least weakly increasing in a. However, the firm can also feasibly increase the worker's wage by ε, with $\varepsilon < \frac{\partial f(\cdot)}{\partial a}$. J is thus strictly increasing in y.

Consider now a worker who is $T-1$ periods old. A firm matched to a worker in submarket $\{h, T-1, y, W\}$ will face the following Pareto frontier

$$\begin{aligned} J(h, T-1, y, W_y; a, \mu) = \sup_{w, W'} \Big(& f(h, y; a) - w \\ & + \mathbb{E}_\Omega \left[\widetilde{p}(h', T, W'; a', \mu')(f(h', y; a') - w')\right] \Big) \end{aligned}$$

Analogously to the proof in **Lemma C.3**, assume that aggregate productivity increases from $\bar{a}$ to $\bar{a} + \varepsilon$. Assume that the firm keeps its policies constant once again. We aim at proving that, even in such a case, firm value increases while keeping the worker at least indifferent. If this is the case, it is *a fortiori* true that J increases in a after reoptimizing firms' policies.

We are now interested in the sign of:[4]

$$\left.\frac{\partial \bar{J}}{\partial a}\right|_{W,\pi,w,\{W'\}} = \frac{\partial f(\cdot)}{\partial a} + \beta \mathbb{E}_\Omega \left[\left.\frac{\partial \tilde{p}(\cdot)}{\partial a}\right|_{W,\pi_i,w,\{W'\}} \bar{J}' \right]$$

[3]For compactness of notation, we omit without loss of generality the two-point lottery in the equations in the proof.

[4]We assume that $J' = f(h', y; a') - w'$ is constant with respect to a. It is possible to prove, by backward induction, that this assumption is without loss of generality for the sake of the proof.

Now notice that, in equilibrium,

$$\frac{\partial \widetilde{p}(\theta)}{\partial a} \propto -\frac{\partial p(\theta)}{\partial a}, \quad \text{with} \quad \frac{\partial p(\theta)}{\partial a} = \underbrace{\frac{\partial p(\theta)}{\partial \theta}}_{>0} \cdot \underbrace{\frac{\partial \theta}{\partial J(\cdot)}}_{>0} \cdot \frac{\partial J(\cdot)}{\partial a}$$

where the sign of the second derivative on the right hand side comes from the free entry condition and the properties of vacancy filling probability function $q(\cdot)$. Given this, it has to be that $\frac{\partial p(\theta)}{\partial a}$ and $\frac{\partial J(\cdot)}{\partial a}$ have the same sign in equilibrium. Now, if both are strictly positive, both statements of our proposition are immediately true. Let's now assume they are both negative or zero. If this is the case, then $\frac{\partial \tilde{p}(\cdot)}{\partial a} \geq 0$. But this implies $\frac{\partial \bar{J}}{\partial a} > 0$, which is a contradiction. □

The intuition behind this proposition relies on the observation that higher productivity realization are associated not only with better outcomes on impact but also to better future prospects, given that the productivity process is an increasing Markov chain.

A key property of the model is that it allows to characterize the workers' optimal behaviour along the business cycle. The following proposition summarizes how the search strategy changes depending on the aggregate productivity realization.

Proposition C.2 and Corollary C.3 have an important implication regarding firms' vacancy posting and workers' search decisions. The fact that at the posting stage profits J are increasing in aggregate productivity implies that more entry will take place in good times, and ceteris paribus more entrepreneurs will open up vacancies across the whole firms' distribution.[5] The resulting higher tightness impacts workers' optimal search behaviour as the job finding probability increases in all submarkets. As a consequence, workers respond optimally to the productivity increase searching in submarkets that guarantee higher lifetime utility promises.

Firms utility promises depend on the structure of the optimal contract. The contract provides insurance to workers through wage paths that are downward rigid, and at the same time allows firms to profit as wages only partially adjust to productivity realizations.

The following propositions provide a clear picture of the growth path prescribed by the optimal contract for a continuing firm. First, let us define the productivity threshold that determines whether a worker-firm match does not survive.

Corollary C.1. *There exists a productivity threshold $a^*(h, \tau, W, y)$ below which firms will not continue the operate.*

Proof. The proof follows immediately from **Proposition C.2** and the timing of the shock.

[5] In our model a better firm is a more productive firm. We do not specifically model the determinant of quality heterogeneity but we take the existence of profound differences in firm quality as a fact (Arellano-Bover, 2020, Arellano-Bover and Saltiel, 2022).

Given the timing of the shock, exit is fully determined by the current productivity shock and incumbent firms know in advance whether they are willing to produce in the next period.

Therefore, as the Pareto frontier is strictly increasing in a, firms are willing to continue the contract if $\mathbb{E}_\Omega[J(h', \tau+1, W', y; a', \mu')|h, \tau, W, y, a, \mu] \geq 0$, so that the threshold that determines exit is

$$a^*(h, \tau, W, y) : \mathbb{E}_\Omega[J(h', \tau+1, W', y; a', \mu')|h, \tau, W, y, a, \mu] = 0.$$

□

The intuition of why this has to be the case is linked to the fact that the Pareto frontier is strictly increasing in a and decreasing in the level of promised utilities to the worker. Hence once the aggregate state realizes a firm is able to perfectly predict whether next period it will exit the market or stay in (given the timing, the decision is based on expected profits, and is thus *not* state-contingent to next period's productivity). The choice is taken *before* new realizations of productivity, so it is possible that a firm makes negative profits for at most one period.

Corollary C.2. *The productivity threshold $a^*(h, \tau, y, W)$ below which firm y in match with worker (h, τ) and promised utility W exits the market in the aggregate state Ω is increasing in y.*

Proof. Consider two firms characterized by y_1, y_2 with $y_1 < y_2$. Consider the threshold for firm y_1, $a_1^* = a^*(h, \tau, , W_{y_1}, y_1)$. Firm y_1 makes 0 profits if state a_1^* materializes next period. Consider firm y_2 trying to mimic the current contract offered by y_1 to (h, τ). We know that J is increasing in y from **Lemma C.3**, which implies that the firm is making a profit at a_1^*. This completes the proof. □

Lemma C.4. *The Pareto frontier $J(h, \tau, W, y; \Omega)$ is strictly concave in y.*

Proof. **No human capital accumulation.** The proof without human capital accumulation is straightforward. Assuming there is no dependency of h' on y, one can write:

$$\frac{dJ}{dy} = f_y + \widetilde{p}\frac{dJ'}{dy}$$

$$\frac{d^2J}{dy^2} = f_{yy} + \widetilde{p}\frac{d^2J'}{dy^2}$$

where we take J' to represent next period's firm value $J(\cdot)$, and the dependence of all controls on y is ignored by virtue of the envelope condition. One can readily observe by induction that, given the concavity of $f(\cdot)$ in y, $\frac{d^2J}{dy^2} < 0$.

Human capital accumulation. With human capital accumulation the concavity of the function J on y depends on further assumptions on the concavity of the human capital accumulation function $g(\cdot)$. As these assumptions involve equilibrium objects, we state them here and verify ex-post numerically that they are always respected in our setting.

$$\frac{dJ}{dy} = f_y + g_y \frac{\partial \widetilde{p} J'}{\partial h} + \widetilde{p} \frac{dJ'}{dy}$$

$$\frac{d^2 J}{dy^2} = f_{yy} + \underbrace{g_{yy}}_{<0} \frac{\partial \widetilde{p} J'}{\partial h'} + g_y^2 \frac{\partial^2 \widetilde{p} J'}{\partial h'^2} + \underbrace{2 g_y}_{>0} \left(\frac{\partial \widetilde{p}}{\partial h'} f_y + \widetilde{p} f_{hy} \right) + \widetilde{p} \frac{d^2 J'}{dy^2}$$

Sufficient conditions for the previous expression to be true at $T-1$ would jointly be:

$$\tilde{p}_h (f - w) + f_h \tilde{p} \geq 0 \tag{C.4}$$

$$\tilde{p}_h f_y + \tilde{p} f_{h,y} \leq 0 \tag{C.5}$$

$$\tilde{p}_{h,h} (f - w) + \tilde{p} f_{h,h} + 2 \tilde{p}_h f_h \leq 0 \tag{C.6}$$

Rearranging and combining the first two would lead to:

$$(f - w) \leq \frac{f_y f_h}{f_{h,y}} \Rightarrow f \leq \frac{f_y f_h}{f_{h,y}} + w \tag{C.7}$$

Since $w > 0$, a more stringent condition would be:

$$\frac{f f_{h,y}}{f_y f_h} \leq 1 \rightarrow ES \leq 1 \tag{C.8}$$

This condition is not generally too restrictive, and holds with equality in the Cobb-Douglas case.

Going back to the third condition, and combining it with the first one, one gets

$$\frac{\tilde{p}}{\tilde{p}_h} \left(f_{h,h} - \frac{\tilde{p}_{h,h}}{\tilde{p}_h} f_h \right) + 2 f_h \geq 0 \tag{C.9}$$

A sufficient condition for that would be

$$h \frac{f_{h,h}}{f_h} \leq h \frac{\tilde{p}_{h,h}}{\tilde{p}_h} \tag{C.10}$$

In the Cobb-Douglas case this condition amounts to

$$\alpha \geq -h \frac{\tilde{p}_{h,h}}{\tilde{p}_h} \tag{C.11}$$

which bounds the returns on human capital. The economic interpretation is simple. For J to be concave even with endogenously increasing human capital, it has to be the case that the marginal product of human capital is not too big in the production function. For this to be the case, the effect of the relative flattening of the production function and decrease in marginal product of human capital has to be dominated by the relative increase in the marginal effect of the decrease in retention.

□

Proposition C.3. *For each state in which the firm is willing to continue the contract, the solution to the firm problem delivers the wage Euler equation:*

$$\frac{\partial \widetilde{p}(\Theta)}{\partial W_i'} \frac{J'(\Theta)}{\widetilde{p}(\Theta)} = \frac{1}{u'(w_i)} - \frac{1}{u'(w)} \tag{C.12}$$

with $\Theta \equiv (\phi(h,y), \tau+1, W_i'; \Omega')$ *being the definition of the relevant state and* w_i *is the wage paid in the future state.*

Proof. Consider the firm problem in Equation (12), restated here for convenience

$$J(h,\tau,\iota,W,y;\Omega) =$$

$$\sup_{\pi_i,\{\eta_i',w_i,W_i'\}} \sum_{i=1,2} \pi_i \Bigg(f(y,h;a) - w_i$$

$$+\beta \mathbb{E}_\psi \left[(1-\eta_i') \cdot \max \left\{ 0, \mathbb{E}_\Omega \left[\widetilde{p}(h',\tau+1,\iota,W_i';\Omega') J(h',\tau+1,\iota,W_i',y;\Omega') \right] \right\} \right] \Bigg)$$

$$s.t.\ [\mu] : W = \sum_{i=1,2} \pi_i \Bigg(u(w_i) + \beta \mathbb{E}_{\Omega,\psi}((1-\eta_i')\widetilde{r}(h',\tau+1,\iota,W_i';\Omega')$$

$$+ \eta_i' U(h',\tau+1,\iota;\Omega')) \Bigg),$$

$$[\theta] : J(h',\tau+1,\iota,W_i',y;\Omega') < 0 \implies \eta' = 1, \text{ otherwise } \eta' = 0,$$

$$[\psi] : (w - w')(1-\eta') \leq 0$$

$$\sum_{i=1,2} \pi_i = 1,$$

For $i = 1,2$, the first order conditions with respect to the wage and the promised utilities are:

$$[w_i]: \; -1 + \mu u'(w) - \tilde{\psi} = 0 \tag{C.13}$$

$$[W_i]: \; \tilde{p}(\cdot) \cdot \frac{\partial J(W')}{\partial W'} + J(W')\frac{\partial \tilde{p}}{\partial W'} + \mu \frac{\partial \tilde{r}}{\partial W'} + \theta \frac{\partial J(W')}{\partial W'} = 0. \tag{C.14}$$

where $\tilde{\psi} = \psi(1 - \eta')$. Note that by definition,

$$\widetilde{r}(h, \tau, V; \Omega) \equiv \lambda U(h, \tau; \Omega) + (1 - \lambda)\Big[V + \lambda_e \max\{0, R(h, \tau, V; \Omega)\}\Big]$$

therefore we can use the envelope theorem as in Benveniste and Scheinkman (1979), Theorem 1 and the definition in Equation (7) to derive an expression for the derivative of the employment value in $t+1$ as the period ahead of the following:

$$\frac{\partial \widetilde{r}(h, \tau, W; \Omega)}{\partial W} = \widetilde{p}(h, \tau, W; \Omega).$$

Similarly, using the envelope condition on the firm problem and the first order condition for the wage, we can establish that

$$\frac{\partial J(h, \tau, y, W; \Omega))}{\partial W} = -\mu \;\therefore\; \frac{\partial J(h, \tau, W, y; \Omega))}{\partial W} = -\frac{1 + \tilde{\psi}}{u'(w)}. \tag{C.15}$$

Moving these two expressions one period ahead, substituting them in (C.14), focusing on $\widetilde{p(\cdot)} > 0$ and $\pi_i > 0$ and rearranging we have that:

$$(1 + \tilde{\psi}')\frac{1 + \theta}{u'(w_i')} - \frac{1 + \tilde{\psi}}{u'(w)} = \frac{\partial \widetilde{p}(\Theta)}{\partial W_i} \frac{J(\Theta)}{\widetilde{p}(\Theta)},$$

with $\Theta \equiv (h', \tau + 1, W; \Omega')$ and where w' is the wage next period in state Ω'. We now turn to the behavior of the multipliers. To do so, let's focus on a contract at age $T - 1$, for which retirement is certain at age T and hence $\eta'' = 1 \implies \tilde{\psi}' = 0$. If $\eta' = 0$, can the firm participation constraint be binding? When this is the case, $J(\Theta) = 0$ and $\theta > 0$. But this implies the wage constraint will be binding, so $\psi > 0$. It is then easy to show that, for values of $h', \tau + 1, W; \Omega'$ for which the firm participation constraint is binding, $\eta' = 1$ and the firm exits. The argument can then be applied backwards for any value $\tau < T$ to show that if the firm state is such that the firm continues this period, then:

$$\frac{1}{u'(w_i')} - \frac{1}{u'(w)} = \frac{\partial \widetilde{p}(\Theta)}{\partial W_i} \frac{J(\Theta)}{\widetilde{p}(\Theta)},$$

□

The optimal contract links the wage growth to the realization of firms profits. The right hand side of Equation C.12 shows that, in providing insurance to the worker, the firm links wage growth to profits and to the incentive to maximize retention, incorporated in $\frac{\partial \log \widetilde{p}}{\partial W}$, the semi-elasticity of the retention probability to the utility offer. As the production stage takes place *after* exit choices are taken by the incumbent firms, the wage growth related to the continuation value of the contract is bound to be (weakly) positive, hence workers enjoy a non-decreasing wage profile under the optimal contract.[6]

A feature that the optimal contract derived in our model shares with the literature on long-term contracts with lack of commitment on the worker side is thus the backloading of wages.[7] Workers in our model make search decisions that affect the survival probability of the match. They do not however appropriate the full future value of the current match while making these search decisions (unless the firm makes zero profits). This makes it optimal for the firm to front-load profits and back-load wages. The reason is that the firm provides insurance and income smoothing to the worker, but given its risk neutrality it prefers to front-load its profits while providing an increasing compensation path to maximize retention. The contract thus optimally balances the consumption smoothing motives (i.e. the insurance provision of the contract) with the commitment problem of the worker.

Special case with log-utility. The wage Euler equation discussed in **Proposition C.3** can be simplified to a more intuitive interpretation in the log-utility case. In case of log-utility, in fact, $u'(w_{i,\Omega}) = \frac{1}{w_{i,\Omega}}$. Multiplying and dividing by wage levels and rearranging, we can express the elasticity of retention probability to offered utility as

$$\varepsilon_{\widetilde{p},W_y} = \underbrace{\frac{(w_i - w)}{w}}_{\text{Wage growth}} \quad \underbrace{\frac{w}{J(\Theta)}}_{\substack{\text{Ratio of wage}\\ \text{to match value}}} . \tag{C.16}$$

with $\varepsilon_{\widetilde{p},W} \equiv \frac{\partial \widetilde{p}(\Theta)}{\partial W_i \widetilde{p}(\Theta)}$.

The interpretation of this result is of interest to analyses that relate labor market dynamism to wage dynamics, like Engbom (2021). This is because $\varepsilon_{\widetilde{p},W_y}$, being a function of structural parameters of the matching technology, γ, search frictions λ_e, and measures of labor market tightness θ, provides us with a good proxy of labor market fluidity. The right hand side of (C.16), is composed entirely of observable quantities, as the ratio of wages to match value is a function of factor shares in value added. The quantity can then be used to compare the dynamism of different local, regional or national labor markets. The next proposition, instead, confirms our initial conjecture that in equilibrium firm

[6] As the exit decision takes place by considering *expected* profits next period, a firm operating at low but positive expected profits might end up, at most for a period, to have a negative continuation value. This would imply that wage growth *can* be negative before a firm's closure, which is actually a common finding in empirical studies (firstly observed in Ashenfelter (1978)).

[7] See for instance, Thomas and Worral (1988), Tsuyuhara (2016) and Balke and Lamadon (2022).

qualities and utility promises are related to a one-to-one mapping.

Lemma C.5. $\Pi(y, h, \tau, W; \Omega)$ *is concave in* W *and optimal utility promises are unique and increasing in* y *given worker characteristics* (h, τ) *and aggregate state* Ω*.*

Proof. Assuming the same (h, τ, y), the entrepreneur then chooses the optimal value $W^*(y) := \arg\max_W \Pi(W; \Omega)$ to deliver in the contract. For the rest of the proof we consider as given the dependence of the functions on (h, τ, Ω) and consider directly the composite function $q(\theta(W))$ as $q(W)$.

The optimization requires that W^* satisfies the following first order condition:

$$q_W(W^*)J(y, W^*) + q(W^*)J_W(y, W^*) = 0. \tag{C.17}$$

For W^* to be a unique maximum, $\Pi(\cdot)$ has to be concave hence the second-order condition must be negative. Taking into account explicitly of the composite function $q(\theta(W))$ implies the following second-order condition:

$$q_{\theta,\theta}(\theta_W)^2 J + q_\theta \theta_{W,W} J + 2q_\theta \theta_W J_W + qJ_{W,W} < 0 \tag{C.18}$$

$$(q_{\theta,\theta}\theta_W + q_\theta \frac{\theta_{W,W}}{\theta_W})\theta_W J + 2q_\theta \theta_W J_W + qJ_{W,W} < 0 \tag{C.19}$$

$$(q_{\theta,\theta}\theta_W + q_\theta \frac{\theta_{W,W}}{\theta_W}) \underbrace{\theta_W J}_{<0} + \underbrace{2q_\theta \theta_W J_W + qJ_{W,W}}_{<0} < 0 \tag{C.20}$$

where the inequalities in **Equation C.20** follow from knowing that $J(\cdot)$ is concave and decreasing in W, $J_W, J_{W,W} \leq 0$, $q(\cdot)$ is increasing in W, and $\theta(\cdot)$ is decreasing in promises as discussed in **Lemma C.1**. Note that if $q_{\theta,\theta}\theta_W + q_\theta \frac{\theta_{W,W}}{\theta_W} > 0$ then the inequality in C.20 would be respected and $\Pi(\cdot)$ would be concave. For this term to be positive it has to be that:

$$\frac{q_{\theta,\theta}}{q_\theta}\theta_W > -\frac{\theta_{W,W}}{\theta_W}. \tag{C.21}$$

As $q(\cdot)$ is decreasing and convex, $q_{\theta,\theta} > 0, q_\theta < 0$ and $\theta_W < 0$, the left-hand side of C.21 is positive. If $\theta_{W,W} < 0$ the right-hand side would be negative and the inequality would always be respected. We know from **Lemma C.1** that $\theta_W = -q_\xi^{-1} J_W \frac{c}{J^2}$, so that we can write $\theta_{W,W} = \frac{c(cJ_W^2 q_{\xi,\xi}^{-1} - J^2 J_{W,W} q_\xi^{-1} + 2JJ_W^2 q_\xi^{-1})}{J^4}$. As the cost function and the value of the contract are positive, $\theta_{W,W} < 0$ if the numerator is negative, so if $cJ_W^2 q_{\xi,\xi}^{-1} - J^2 J_{W,W} q_\xi^{-1} + 2JJ_W^2 q_\xi^{-1} < 0$.

This is true if

$$\frac{q_{\xi,\xi}^{-1}}{q_\xi^{-1}} > \left(\frac{J_{W,W}}{J_W}\frac{J}{J_W} - 2\right)\frac{J}{c}.$$

As the right-hand side of the condition above is negative given the properties of J and $q_\xi^{-1} < 0$, the inequality would always be respected if $q_{\xi,\xi}^{-1} < 0$. We verify that the conditions holds for our matching function and parametrization.

The uniqueness of the optimum for the firm problem at object implies, given the properties of $\mathcal{V}$ and the theorem of the maximum, $W^*(y)$ is also continuous.

To show that $W_y^*(y) > 0$ note that, by the implicit function theorem, the derivative of **Equation C.17** with respect to y is:

$$(q_{W,W}J + 2q_W J_W + qJ_{W,W})W_y + q_W J_y + qJ_{W,y} = 0. \tag{C.22}$$

The first term in parenthesis is negative, as per the second order condition in **Equation C.20**. The second term is positive, given that J_y is positive (**Lemma C.3**) and q_W is positive as well. From **Equation C.15** it is possible to show that $J_{W,y} = \Gamma W_y$, with $\Gamma = \frac{u''}{(u')^2}\frac{\partial w}{\partial W} < 0$ as $\frac{\partial w}{\partial W} \geq 0$ and $u'' < 0$. Therefore,

$$\underbrace{(q_{W,W}J + 2q_W J_W + qJ_{W,W} + q\Gamma)}_{<0} W_y + q_W J_y = 0.$$

This means that, in order for the equality to be respected, $W_y^* > 0$.

□

Proposition C.4. *The mapping defined by the function $W : \mathcal{Y} \to \mathcal{V}$ is bijective for each worker characteristic (h, τ) and aggregate state Ω.*

Proof. As shown in **Lemma C.5**, the optimal promise for each entrepreneur with quality y is unique, continuous and monotonically increasing. Free entry implies that $\Pi(W^*(y)) = c(y)$, pinning down y in equilibrium. As $W^*(y)$ is unique and increasing, for each y there can be only one $W^*(y)$ so that the mapping between firm qualities and optimal promises is bijective. □

Corollary C.3. *The optimal search strategy of the workers and vacancy posting of the firms is increasing in aggregate productivity.*

Proof. We start from the free entry condition, ignoring without loss of generality τ, μ from the notation, and ignoring the indirect dependence of $q(\cdot)$ on $\theta(\cdot)$:

$$q(h, W; a)J(h, W, y; a) = c(y)$$

We know from **Lemma C.5** and **Proposition C.4** that the optimal W^* choice is a unique, monotonically increasing function $W(y)$ given workers characteristics (h, τ) and aggregate state Ω. Given **Proposition C.1**, as the worker search policy is continuous, unique and increasing in h, we can also conclude that firms and workers qualities feature positive assortative matching for any aggregate state, according to a continuous function $y = \xi(h; a)$. We thus want to find conditions for which $\xi_a > 0$, which would imply that in equilibrium the "assignment" (search) firm quality y for worker quality h is higher when the state of the economy a is better. We write ξ_a simply as $\frac{\partial y}{\partial a}$ in the proof.

We obtain the partial derivative of the free entry condition by y:

$$c'(y) = q\frac{\partial J}{\partial y} \tag{C.23}$$

We then write the total derivative of of this equation by a, ignoring terms featuring W by envelope condition:

$$q\left(\frac{\partial^2 J}{\partial y \partial a} + \frac{\partial^2 J}{\partial^2 y}\frac{\partial y}{\partial a}\right) + \frac{\partial J}{\partial y}\frac{\partial q}{\partial a} = c''(y)\frac{\partial y}{\partial a} \tag{C.24}$$

which, in turn, leads to:

$$\frac{\partial y}{\partial a} = \frac{q\frac{\partial^2 J}{\partial y \partial a} + \frac{\partial J}{\partial y}\frac{\partial q}{\partial a}}{c''(y) - \frac{\partial^2 J}{\partial^2 y}} \tag{C.25}$$

The sign of the denominator is positive given the properties of the function $c(\dot{})$ and **Lemma C.4**, hence it is sufficient to sign the numerator, and $\frac{\partial y}{\partial a} > 0$ amounts to

$$q\frac{\partial^2 J}{\partial y \partial a} > -\frac{\partial J}{\partial y}\frac{\partial q}{\partial a}$$

We check for this condition in the solution and find it to be respected in the state space given our calibration.

□

Firm value $J(\cdot)$ is increasing in a, while retention is decreasing. As firm make more profits, they can marginally increase offered utilities to workers to maximize retention. The last inequality implies that for the workers to improve their target firm quality in better times it must be true that the first marginal effect, the increase in firm profits from better matches, must dominate the second, the relative decrease in retention, when aggregate productivity changes.

D Existence of a Block Recursive Equilibrium

In order to show that a Block Recursive Equilibrium (BRE) exists in our model we need to show that the equilibrium contracts, the workers' and the entrepreneurs value and policy functions do not depend on the distribution of employed and unemployed workers. This implies that the only element of the aggregate state that matters for a firm when making an hiring decision is the state of aggregate productivity but not the distribution of worker types (e.g. employed vs unemployed).

Proposition D.1. *A Block Recursive Equilibrium as defined in Definition 3.3 exists.*

Proof. We follow the approach in Menzio et al. (2016), Herkenhoff et al. (2023) and prove the existence of a BRE using backward induction.

Consider the lifetime values of an unemployed and an employed worker before the production stage in the last period of households lives with $\tau = T$:

$$U(h, T, \iota; \Omega) = u(b(h, T)) \tag{D.1}$$

$$V(h, T, \iota; \Omega) = u(w(a)), \tag{D.2}$$

their values trivially do not depend on the distribution of types as both valuations are 0 from $T + 1$ onward. Hence, $U(h, T, \iota; \Omega) = U(h, T, \iota; a)$ and $V(h, T, \iota; \Omega) = V(h, T, \iota; a)$.

The optimal contract for agents aged $\tau = T$, instead, solves the following problem

$$J(h, T, W, y; \Omega) = \sup_{w} [f(y, h; a) - w] \quad s.t.\ W = u(w),$$

that clearly does not depend on the distribution of worker types due to the directed search protocol and where the aggregate state only affects the promised utility and the optimal wage through realization of the aggregate productivity processes. Therefore, $J(h, T, \iota, W, y; \Omega) = J(h, T, \iota, W, y; a)$.

This also implies that the equilibrium market tightness

$$\theta(h, T, \iota, W; \Omega) = q^{-1} \left(\frac{c(y)}{J(h, T, \iota, W, y; a)} \right)$$

is independent from the distribution of worker types and it is only affected by realization of aggregate productivity, so $\theta(h, T, W; a)$.

This in turn implies that the search problem workers face at the beginning of the last period of their lives depends on the aggregate state only through aggregate productivity a:

$$R(h, T, \iota, V; a) = \sup_{v} \Big[p(\theta(h, T, \iota, v; a)) \big[v - V \big] \Big],$$

does not depend on the distribution of worker types.

Stepping back at $\tau = T - 1$, the value functions for the unemployed and the employed agents are solutions to the following dynamic programs

$$\sup_v u(b(h,T-1)) + \beta \mathbb{E}_{\Omega,\psi}\Big(U(h',T,\iota;a') + p(\theta(h,T,\iota,v;a'))\big[v - U(h',T,\iota,;a')\big]\Big)$$

$$u(w) + \beta \mathbb{E}_{\Omega,\psi}\left(\begin{array}{c} \lambda U(h',T,\iota;a') + \beta(1-\lambda)W + \\ +\beta(1-\lambda)\lambda_e \max(0, R(h',T,\iota,,W);a')]]\Big] \end{array}\right),$$

where both do not depend on the distribution of worker types.

The optimal contract at this step is a solution to

$$\begin{aligned} J_t(h,T-1,\iota,V,y;a) = \sup_{\{\pi_i,w_i,W_i\}} \sum_{i=1,2} \pi_i \Big(& f(y,h;a) - w_i \\ & + \mathbb{E}_{\Omega,\psi}\left[\widetilde{p}(h',T,W_{i,\Omega'};a')(J(h',T,y,W_i;a')\right] \Big) \end{aligned}$$

$$\begin{aligned} s.t.\ V = \sum_{i=1,2} \pi_i \left(u(w_i) + \mathbb{E}_{\Omega,\psi}\widetilde{r}(h',T,W_i;a')\right),\ h' = \phi(h,y,\iota,\psi) \\ \mathbb{E}_{\Omega,\psi}\sum_{i=1,2}\pi_i\left(\mathbb{E}_{\Omega,\psi}J(h',T,\iota,W_i,y;a')\right) \geq 0 \text{ and } t \leq T \end{aligned}$$

which does not depend on types distribution.

Therefore, also the equilibrium tightness and the search gain at $T-1$ are independent from types' distributions, as

$$\begin{aligned} \theta(h,T-1,\iota,W;a) &= q^{-1}\left(\frac{c(y)}{J(h,T-1,\iota,W,y;a)}\right) \\ R(h,T-1,\iota,V;a) &= \sup_v \Big[p(\theta(h,T-1,\iota,v;a))\big[v-V\big]\Big]. \end{aligned}$$

Stepping back from $\tau = T-1,...,1$ and repeating the arguments above completes the proof. □

E Model solution and estimation

Model solution. For each education level, we solve the model by backward induction on a regular grid of human capital, wage multiplier levels, firm quality and aggregate shocks. In particular, starting from the last period of workers lives we compute the terminal wage in each state, then solve the search problems for both the employed and the unemployed, which allows us to compute the market tightness in each submarket as well as the value of the contract and of unemployment. With these objects in hand we proceed backwards until workers labor market entry.

In the model simulations, we populate each cohort of agents with 42 individuals, 30 non-graduates and 12 graduates, consistent with the share of graduates in the Italian economy. Graduates enter the labor market in a staggered manner every quarter for the first three years of their life. We then simulate the model for 600 periods (we take 180 periods as burn in) and construct a panel with 419 periods and $7,560$ individuals per period. In the model, we consider one period as one quarter in the data.[8]

Estimation. To pin down the 18 internal parameters in the model, $\boldsymbol{\theta}$, we minimize the absolute error between model simulated moments, $m(\boldsymbol{\theta})$, and their empirical counterparts, $\mathbf{d}$. More formally, we pick the vector of parameters

$$\boldsymbol{\theta}^* = \arg\min_{\boldsymbol{\theta}} \left\{ (|m(\boldsymbol{\theta}) - \mathbf{d}|)' W (|m(\boldsymbol{\theta}) - \mathbf{d}|) \right\}.$$

In $\mathbf{d}$ and $m(\boldsymbol{\theta})$ we stack: the profiles of E-E and E-U transitions by age (9 age groups); wage growth relative to the initial wage by education level and experience (9 experience groups); the unemployment rate; the correlations of labor market flows with the cyclical component of aggregate output; the average inactivity rate; average sorting, measured as the average correlation between AKM fixed effects in the data and as the correlation of worker and firm qualities in the model; the ratio of initial wages between graduates and non-graduates, and the ratios of the second and third quintile of value added per employee to the first.

Jointly, these ten moments deliver 44 restrictions for 18 parameters. We construct each moment in the model weighting by age from Italian population weights. In the SMM we weight moments so that each profile, as a whole, has the same weight of the other moments and, for labor market flows, we give more weight to age groups that have a higher demographic share in the Italian population.[9] We collect this weighting scheme in the matrix W.

We solve for $\boldsymbol{\theta}^*$ numerically adopting two complementary strategies. In the first, we use a two step procedure and minimize the distance between model-generated and data-generated moments using a global solver on the largest feasible domain. For this, we rely on the particle swarm algorithm contained in the library developed by Blank and Deb (2020). In the second, we solve and simulate the model over a sparse grid of the parameter space.[10] In both cases, we refine the estimation using the best combination of

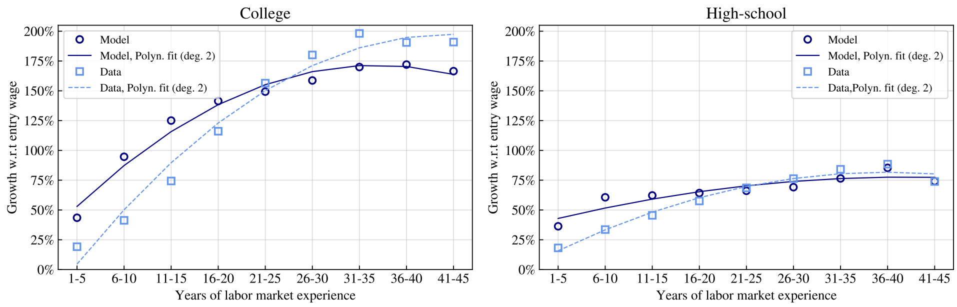
[8] We use a grid of $20 \times 15 \times 25 \times 7$ for each education level and we consider 180 quarters of workers life. Solving and simulating the model takes approximately 25 minutes with Python 3.11/Fortran95 on a Linux HPC cluster with 52 cores and 120G of RAM.

[9] For example, the weight for the EE rate of 18-22 years old is equal to $1/10 \times 0.091$, the share of 18-22 years old in the Italian population, while the weight on the unemployment rate instead is $1/10$, so that the entire EE profile, as a whole, weights as much as the unemployment rate.

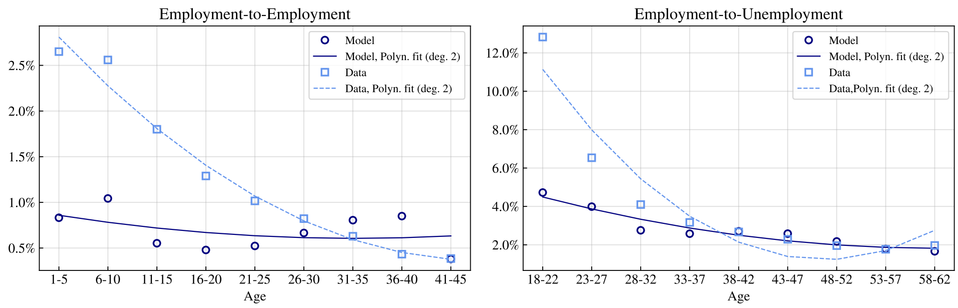
[10] The advantage of selecting the parameters with this procedure is that the grid is built so that the function domain is optimally covered with the least amount of possible points compared to other forms of approximation (e.g. equi-spaced or random grids). The sparse grid is built using the"Tasmanian"

Figure E.1: Target profiles

(a) Wage Profiles



(b) Transition rates



parameters as the starting point of a local minimization routine, for this we rely on the Nelder-Mead algorithm in the SciPy library. **Table 3** in the paper reports the averages for the profiles and the other targets, **Figure E.1** plots the whole profiles.

We eventually use the series generated by the particle swarm to examine the role of each moment in identifying our parameter values. We project simulated moments on the parameter space, and then compare the impact of each parameter on moments, conditional on the others.

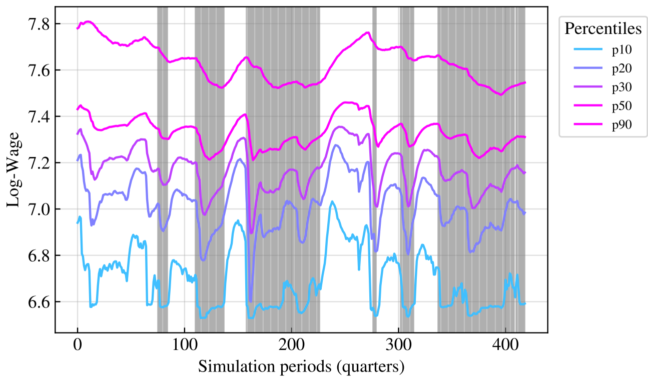
F Additional Figures and Tables, Model

Inequality dynamics. Heathcote et al. (2020) show that recessions have a persistent effect on inequality, affecting the earnings of workers in the left tail of the income distribution the most. **Figure F.1** illustrates the dynamics of inequality around business cycles by displaying the pattern of losses across the earnings distribution. Recessions hit the poorest workers the hardest, and worsening job prospects push some of them out of the labor force.

A prediction of the model that departs from existing literature is that the persistence of earnings losses varies across the distribution: while workers with less human capital

libraries (Stoyanov, 2015).

Figure F.1: Inequality dynamics



Note:The figure reports the dynamics of income for different percentiles of the income distribution.

display more volatility in earnings (mostly due to displacement, see **Figure F.6**), the impact on workers with high human capital is dampened but quite persistent. The presence of limited commitment to job matches by firms creates a subset of workers who regularly undergo shorter spells of employment. Their layoffs exhibit strong counter-cyclical behavior (a pattern highlighted in Jarosch 2023).

Due to match-specific human capital accumulation, the opportunity for workers to recover from low levels of human capital depends on climbing the job ladder. However, falling off the lowest rungs of the job ladder is more likely than from the highest rungs. Consequently, the model endogenously generates a dual economy with workers whose earnings cyclicality depends on the extensive margin of employment (the lowest deciles of income distribution), and higher income workers whose earnings cyclicality depend on the quality of their employment over time (see Doniger, 2023).

State dependence. We check for state dependence by running the following regression across different model simulations

$$Y_{\text{Post}} = \alpha + \textstyle\sum_{j=1}^{4} \beta_j \underbrace{\mathbb{E}[FQ^j_{\text{Pre}}]}_{\text{Firm Quality}} + \gamma_j \overbrace{\mathbb{E}[HC^j_{\text{Pre}}]}^{\text{Human Capital}} + \delta_j \underbrace{\mathbb{E}[W^j_{\text{Pre}}]}_{\text{Wages}} + \eta_j \overbrace{\mathbb{E}[LS^j_{\text{Pre}}]}^{\text{Labor Share}} + \theta \underbrace{\mathbb{C}(FQ_{\text{Pre}}, HC_{\text{Pre}})}_{\text{Sorting}} + \varepsilon,$$

in which $\mathbb{E}[X^j_{\text{Pre}}]$, denotes the j^{th} moments of an endogenous, cross-sectional distribution before the shock, and Y_{Post} are model outcomes after the shock.

Table F.1: State Dependence

(a) Mean

	Output Response		Persistence
	Short-term	Medium-term	
Firm Quality	-1.30***	-1.79**	-0.45
	(0.44)	(0.74)	(0.53)
Human Capital	1.54**	0.94	1.19
	(0.71)	(1.19)	(0.84)
Wage	0.39	1.01*	0.18
	(0.32)	(0.53)	(0.38)
Labor Share	-2.26***	-2.80**	1.38
	(0.76)	(1.27)	(0.90)
Sorting	1.07	1.89	-0.17
	(0.79)	(1.32)	(0.94)
R^2	0.6	0.5	0.1
N	100	100	100

(b) Variance

	Output Response		Persistence
	Short-term	Medium-term	
Firm Quality	0.22***	0.22*	0.13
	(0.08)	(0.13)	(0.09)
Human Capital	-0.79**	-0.54	-0.79*
	(0.34)	(0.56)	(0.40)
Wage	0.01	-0.27	0.52*
	(0.24)	(0.40)	(0.28)
Labor Share	-8.14	-17.23	-11.24
	(9.62)	(16.13)	(11.40)
R^2	0.6	0.5	0.1
N	100	100	100

(c) Skewness

	Output Response		Persistence
	Short-term	Medium-term	
Firm Quality	-0.17	-0.22	0.13
	(0.18)	(0.31)	(0.22)
Human Capital	0.53	0.68	0.02
	(0.80)	(1.34)	(0.94)
Wage	-0.23	-0.31	0.05
	(0.18)	(0.31)	(0.22)
Labor Share	0.40	0.77	0.20
	(0.32)	(0.53)	(0.38)
R^2	0.6	0.5	0.1
N	100	100	100

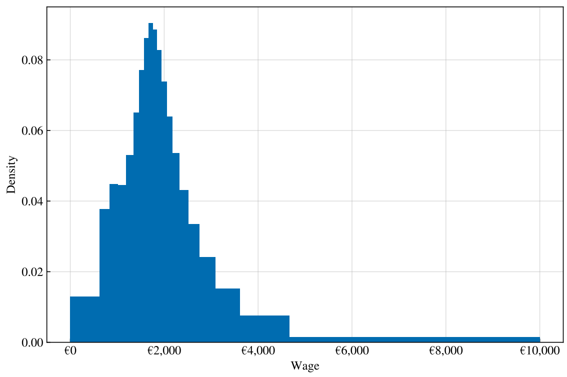
(d) Kurtosis

	Output Response		Persistence
	Short-term	Medium-term	
Firm Quality	0.02	0.03	-0.00
	(0.02)	(0.03)	(0.02)
Human Capital	-0.08	-0.19	0.03
	(0.11)	(0.18)	(0.13)
Wage	0.03	0.05	-0.02
	(0.03)	(0.05)	(0.04)
Labor Share	0.14	-0.08	-0.03
	(0.22)	(0.37)	(0.26)
R^2	0.6	0.5	0.1
N	100	100	100

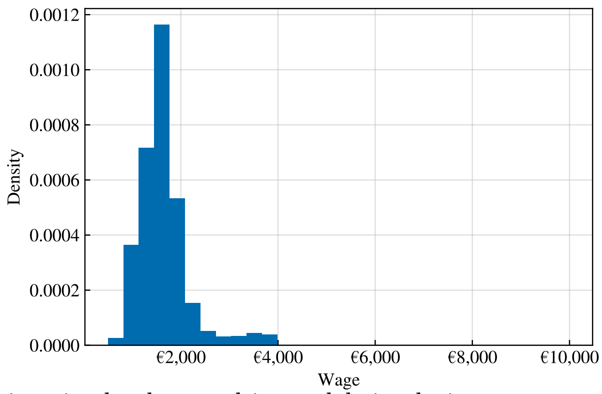
Note: Standard-errors in parenthesis. p-value: $^* < 0.1$, $^{**} < 0.05$, $^{***} < 0.01$. The table reports the coefficients of regressing model outcomes after the shock on moments of relevant endogenous variables before the shock (one year average). The results are scaled to report the effect of changing the regressors by 1pp. Sorting is computed every quarter as the correlation between firm and worker quality and reported in the same table of the means even if technically is not a moment of a distribution. Model outcomes are: i) short-term cumulative output response (1 year after the shock); ii) the medium-term cumulative output response (3 years after the shock); and iii) the persistence of the shock (number of quarters before the output IRF is back at zero). For ease of exposition, the results are grouped by moments but the coefficients are computed including all moments in the same regression. The simulations are those underlying **Figure 8**.

Figure F.2: Wage distributions

(a) Data

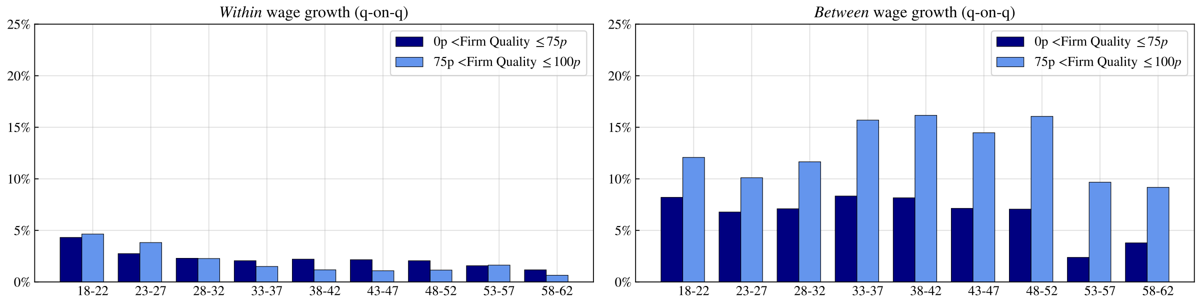


(b) Model



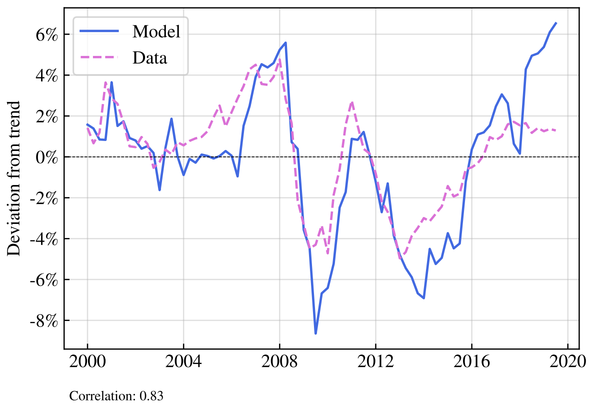
Note: The figure plots the wage distributions in the data and in model simulations.

Figure F.3: Within vs. Between Wage Growth by Age and Firm Quality in the Model



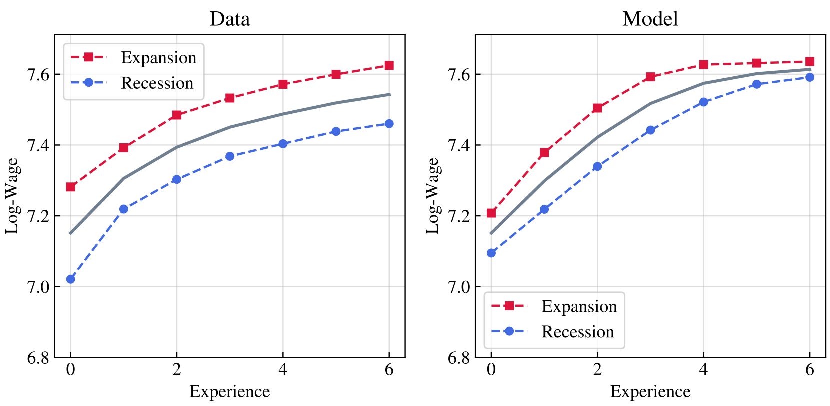
Note: The figure plots the average wage growth, by age and firm quality, *within* employment spells and after employment-to-employment transitions (*between*). For the *betweeen* component, the firm quality quartiles are computed on the distribution of origin firms.

Figure F.4: GDP: Model and Data



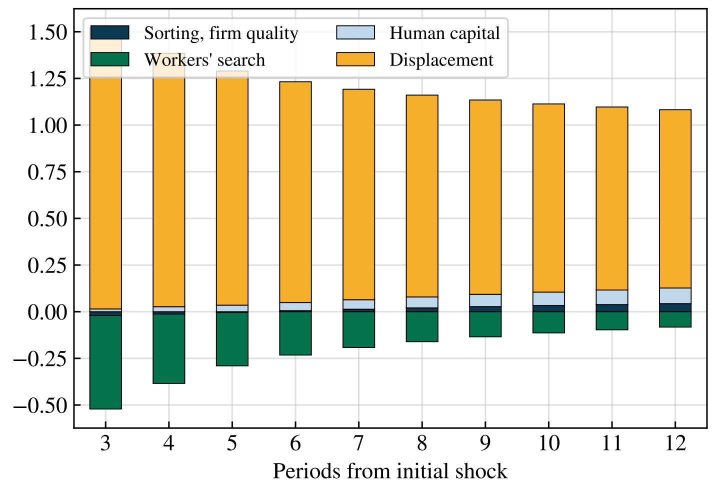
Note: The figure plots the cyclical components of real GDP for Italy and for a model simulation in which the TFP process is matched to the Italian TFP realizations from 2000 to 2019, both series are detrended using an Hamilton filter (4 lags and 8 leads) and their correlation is robust to the choice of the filter.

Figure F.5: Scarring Effect of Recessions



Note: The figure plots the wage profiles estimated on the data and on model simulations for cohorts of workers entering the labor market. The counterfactual profiles for expansions (recessions) are obtained considering a positive (negative) two standard deviation realization of cyclical GDP.

Figure F.6: Decomposition of cumulative impulse response function, average wage



Note: The figure shows the relative importance of each transmission channel compared to the baseline recession for the cumulative response of the average wage in the three years after the onset of the recession.

Table F.2: E-U-E transitions in the model

(a) Model

Dep. Variable: Log-wage after E-U-E transition	(1)
Firm quality bracket	
2 of 3	0.047
	(0.031)
3 of 3	0.068
	(0.030)
Log-wage at origin	0.112
	(0.009)
Controls	✓
Observations	16,598
R^2	0.11

Note: Standard errors in parentheses. The tables report a specification on datasets based on workers that experience experience an Employment to Unemployment to Employment transitions (E-U-E). *Controls* include the pre-transition human capital and a polynomial in labor market experience.

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