Liquidity Traps and Monetary Policy: Managing a Credit Crunch

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Abstract

We study a model with heterogeneous producers that face collateral and cash in advance constraints. These two frictions give rise to a non-trivial financial market in a monetary economy. A tightening of the collateral constraint results in a credit-crunch generated recession. The model reproduces several features of the recent financial crisis, like the persistent negative real interest rates, the prolonged period at the zero bound for the nominal interest rate, the collapse in investment and low inflation, in spite of the very large increases of liquidity adopted by the government. The model can suitable be used to study the effects on the main macroeconomic variables - and on welfare of each individual - of alternative monetary - and fiscal - policies following the credit crunch. The policy implications are in sharp contrast with the prevalent view in most Central Banks, based on the New Keynesian explanation of the liquidity trap.

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1 Introduction

In this paper, we study the effect of monetary and debt policy following a negative shock to the efficiency of the financial sector. We build a model that combines the financial frictions literature, like in Kiyotaki (1998), Moll (2015) and Buera and Moll (2012) with the monetary literature, like Lucas (1982), Lucas and Stokey (1987) and Svensson (1985). The first branch of the literature gives rise to a non-trivial financial market by imposing collateral constraints on debt contracts. The second gives rise to a money market by imposing cash-in-advance constraint in purchases. We show that the model qualitatively reproduces several aspects of the recent great recession. More importantly, we also show that a calibrated version can quantitatively match many salient features of the US experience since 2007. Finally, we use the model to learn about the effects of alternative policies.

The year 2008 will long be remembered in the macroeconomics literature. This is so not only because of the massive shock that hit global financial markets, particularly the bankruptcy of Lehman Brothers and the collapse of the interbank market that immediately followed, but also because of the unusual and extraordinary policy response that followed. The Federal Reserve doubled its balance sheet in just three months—from $900 billion on September 1 to $2.1 trillion by December 1, and it reached around $3 trillion by the end of 2012. At the same time, large fiscal deficits implied an increase in the supply of government bonds, net of the holdings by the Fed, of roughly 30% of total output in just a few years, a change never seen during peace time in the United States. Similar measures were taken in other developed economies.

Somewhat paradoxically, however, the prominent models used for policy evaluation at the time of the crisis ignored the financial sector in one hand, and the role of changes in outside liquidity.\footnote{These models have been further adapted to try to address these issues (see Curdia and Woodford (2010), Christiano, Eichenbaum and Rebelo (2011) or Werning (2011)). As we will discuss, our model captures different effects and the policy implications differ substantially.} There were good historical reasons for both: big financial shocks have seemed to belong exclusively to emerging economies since the turbulent 1930s. In addition, monetary economics developed during the last two decades around the central bank rhetoric of exclusively emphasizing the short-term nominal interest rate, while measures of liquidity or money were completely ignored as a stance of monetary policy.\footnote{A likely reason is that the empirical relationship between monetary aggregates, interest rates, and prices, which remained stable for most of the 20th century, broke down in the midst of the banking deregulation that started in the 1980s. For a detailed discussion, as well as a reinterpretation of the evidence that strongly favors the view of a stable “money demand” relationship, see Lucas and Nicolini (2013).} But 2008 seriously challenged those generally accepted views. Consequently, we need general equilibrium models that can be used for policy evaluation during times of financial distress. The purpose of this paper is to provide one such model and to analyze the macroeconomic effects of alternative policies.

An essential role of financial markets is to reallocate capital from wealthy individu-
als with no profitable investment project to individuals with profitable projects and no wealth. The efficiency of these markets determines the equilibrium allocation of physical capital across projects and therefore equilibrium intermediation and total output. The financial frictions literature, from which we build, studies models of intermediation with these properties, the key friction being an exogenous collateral constraint on investors.\textsuperscript{3} The equilibrium allocation critically depends on the nature of the collateral constraints: The tighter the constraints, the less efficient the allocation of capital and the lower are total factor productivity and output, so a tightening of the collateral constraint creates disintermediation and a recession. We interpret this reduction in the ability of financial markets to properly allocate capital across projects as the negative shock that hit the US economy at the end of 2007.\textsuperscript{4}

We modify this basic model by imposing a cash-in-advance constraint on households. Monetary policy has real effects, not only because of the usual well-understood distortionary effects of inflation in a cash-credit world, but, more importantly, because of the zero bound constraint on nominal interest rates that naturally arises in monetary models. Conditional on policy, the bound on the nominal interest rate may become a bound on the real interest rate. For example, imagine a policy successfully that targets a constant price level: if inflation is zero, the Fisher equation implies a zero lower bound on the real interest rate. The way in which negative real interest rates interact with the zero bound on nominal interest rates, given a target for inflation, is at the heart of the mechanism by which policy affects outcomes in the model discussed in the paper.

The exogenous nature of both frictions raises legitimate doubts regarding the policy invariant nature of them. Precise microfoundations for the collateral or the cash-in-advance constraints have been hard to develop in macro models that remain tractable and general enough to provide insights on the effects of the policies we study in this paper. We thus view this as a first exploration into the role of the above mentioned monetary and debt policies, hoping that the answers we provide, both positive and normative, can shed light into the questions we pose, as in Robert E. Lucas (2000), and Alvarez and Lippi (2009) for models with cash-in-advance constraints, and Kiyotaki (1998) or Moll (2015) for models with collateral constraints.

As we show in a simplified version of the model that can be solved analytically, if the shock to the collateral constraint that causes the recession is sufficiently large, the equilibrium real interest rate becomes negative and persistent as long as the shock is persistent. We find this property of the model particularly attractive, since a very special feature of the last years is a substantial and persistent gap between real output and its trend, together with a substantial and persistent negative real rate of interest. This feature is specific to the credit crunch: If the recession is driven by an equivalent

\textsuperscript{3}We closely follow the work of Buera and Moll (2012), who apply to the study of business cycles the model originally developed by Moll (forthcoming) to analyze the role of credit markets in economic development. See Kiyotaki (1998) for an earlier version of a related framework.

\textsuperscript{4}As we explain in detail in Section 4.1, the behavior of the real interest rate, the variable we use to identify the shock, dates the beginning of the recession in the third quarter of 2007.
but exogenous negative productivity shock, the real interest rate remains positive.

The reason for the drop in real interest rates is that savings must be reallocated
to lower productivity entrepreneurs, but they will only be willing to do it for a lower
interest rate. To put it differently, the “demand” for loans falls, which in turn pushes
down the real interest rate. Several other properties of the recession generated by a
tightening of the collateral constraint in the model are in line with the events that have
unfolded since 2008, such as the persistent negative real interest rate, the sustained
periods with an effective zero bound on nominal interest rates, and the substantial
drops in investment, total factor productivity and output. In addition, the model
implies very large increases in liquidity while the zero bound binds.

A calibrated version of the full model can quantitatively account for the behavior
of the real interest rate, output, capital, total factor productivity and, with somewhat
less success, measures of leverage, since 2007. The one variable the model misses is
labor input, that dropped substantially in the US following 2007 and is constant in the
model. We find this quantitative performance remarkable, given the small number of
parameters. We also find it reassuring in using the model to perform policy analysis.

The paper proceeds as follows. In Section 2 we present the model and characterize
the individual problems. In Section 3, we define an equilibrium and characterize its
properties for a particular case that, by shuting down the endogenous evolution of the
wealth distribution, can be soved analytically. In Section 4 we calibrate the full model
and show that, once the monetary and debt policies implemented by the US authorities
are taken into account, it can explain the evolution of all relevant macro-variables, the
exception being labor input as we already mentioned. In solving the model, we study
the deterministic equilibrium path following the shock to intermediation. In looking
at the data, we focus at the medium frequency evolution of the data, which is the
frequency the model implies we should focus on, as we explain in detail. In particular,
we ignore the high frequency business cycle fluctuations that are the focus of the RBC
literature.

In Section 5, we perform several policy counterfactuals that help put the policies
undertaken in the United States starting in 2008 into perspective. First, we solve for
the equilibrium in the absence of a policy reaction. Specifically, we assume that there
is no injection of liquidity on impact and that there is no further increase in the stock
of government bonds (safe assets). The model implies that the nominal interest rate
will be at its zero lower bound for a finite number of periods and that there will be
an initial deflation, followed by an inflation rate that is higher that the steady state.
These effects are the natural response of the no arbitrage condition between money and
bonds. In the case that private debt contracts are indexed to the price level, the real
effects are minor. On the contrary, in the more realistic case in which debt obligations
are in nominal terms, the deflation strongly accentuates the recession well beyond the
one generated by the credit crunch, due to a debt deflation problem. A similar result
obtains with sticky wages, commonly assumed in modern monetary models. We then
study active inflation-targeting policies for low values of the inflation target. In these
cases, the deflation with its associated real effects can be avoided by a sufficiently large
increase in the supply of government liabilities that must accommodate the credit crunch. This exercise is reminiscent of the discussion in Friedman and Schwartz, who argued that the Fed should have substantially increased its balance sheet in order to avoid the deflation during the Great Depression.\footnote{In 2002, Bernanke, then a Federal Reserve Board governor, said in a speech in a conference celebrating Friedman’s 90th birthday, “I would like to say to Milton and Anna: Regarding the Great Depression. You’re right, we did it. We’re very sorry. But thanks to you, we won’t do it again” (speech published in Milton Friedman and Anna Jacobson Schwartz, The Great Contraction, 1929-1933) (Princeton, NJ: Princeton University Press, 2008), 227.} Was the different monetary policy recently adopted the reason why the Great Contraction was much less severe than the Great Depression? Our model suggests this may well be the case.\footnote{Claiming that he “prevented an economic catastrophe,” Time magazine named then-Chairman Bernanke Person of the Year on December 2009.}

In studying inflation targeting policies, we show that the number of periods that the economy will be at the zero bound and the amount of liquidity that must be injected depend on the target for the rate of inflation. The evolution of output critically depends on the increase in liquidity. The target for inflation and the zero bound constraint on nominal rates imply a floor on how low the real interest rate can be. But for this to be an equilibrium, private savings must end up somewhere else: this is the role of the increase in government liabilities. In this heterogeneous credit-constrained agents model, outside liquidity affects equilibrium interest rates even if taxes are lump sum: Ricardian equivalence does not hold if agents discount future flows at different rates, as is the case when collateral constraints bind. As a consequence, the issuance of government liabilities (money or bonds, which are perfect substitutes at the zero bound) crowds out private investment and slows down capital accumulation. But increases in liquidity have an additional effect. In the model, a credit crunch generates a recession because total factor productivity falls. The reason, as we mentioned above, is that capital needs to be reallocated from high productivity entrepreneurs for which the collateral constraint binds to low productivity entrepreneurs for which the collateral constraint does not bind. An increase in liquidity prevents the real interest rate from falling too much, and ameliorates the drop in productivity.

In summary, a target for inflation, if low enough, ameliorates the drop in productivity (i.e., there will be less reallocation of capital to low productivity workers). But it requires a larger increase in total outside liquidity and makes the recession more prolonged (i.e., capital accumulation falls because of the crowding-out effect). The model therefore challenges the interpretation of the events following 2009 provided by a branch of the literature that, using New Keynesian models, places a strong emphasis on the interaction between the zero bound constraint on nominal interest rates and price rigidities.\footnote{See Krugman (1998), Eggertsson and Woodford (2003), Christiano et al. (2011), Correia et al. (2013), and Werning (2011).} This is also the dominant view of monetary policy at major central banks, including the Fed. According to this view, a shock—often associated with a shock to the efficiency of intermediation—drove the natural real interest rate to negative terri-

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\footnote{See Krugman (1998), Eggertsson and Woodford (2003), Christiano et al. (2011), Correia et al. (2013), and Werning (2011).}

\footnote{See, for example, Curdia and Eggertsson (2009), Drautzburg and Uhlig (2011), and Galí et al.}
tory. The optimal monetary policy in those models is to set the nominal interest rate equal to the natural real interest rate. However, because of the zero bound, that is not possible. But it is optimal, unambiguously, to keep the nominal interest rate at the zero bound, as the Fed has been doing for over seven years now. Furthermore, these models imply that it is unambiguously optimal to maintain the nominal interest rate at zero even after the negative shock reverts. This policy implication, called “forward guidance,” has dominated the policy decisions in the United States since 2008 and remains the conceptual framework that justifies the “exit strategy”.

On the contrary, the model we study stresses a different and novel trade-off between ameliorating the initial recession and delaying the recovery. When the central bank chooses a lower inflation target, it must inject more liquidity. As a result, the liquidity trap lasts longer and the real interest rate is constrained to be higher, ameliorating the drop in productivity. The counterpart of the milder drop in TFP is a drop in investment due to the crowding out, leading to a substantial and persistent decline in the stock of capital and a slower recovery.

Which is the optimal policy? In this heterogenous agents model, answering that question requires taking a stand on Pareto weights. We do not pursue this line, but in Section 6, we compute the distribution of welfare changes across all agents. A final Section concludes.

**Related Literature**  We consider a monetary version of the model in Buera and Moll (2012), who apply to the study of business cycles the framework originally developed by Moll (forthcoming) to analyze the role of credit markets in economic development. Kiyotaki (1998) is an earlier example that focuses on a two-point distribution of shocks to entrepreneurial productivity. This framework is related to a long tradition that studies the role of firms’ balance sheet in business cycles and during financial crises, including Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Bernanke et al. (1999), Cooley et al. (2004), Jermann and Quadrini (2012). 9

Kiyotaki and Moore (2012) study a monetary economy where entrepreneurs face stochastic investment opportunities and friction to issue and resell equity on real assets. They also consider the aggregate effects of a shock to the ability to resell equity. In their environment, money is valuable provided that frictions to issue and resell equity are tight enough. They use their model to study the effect of open market operations that consist of the exchange of money for equity. Brunnermeier and Sannikov (2013) also study a monetary economy with financial frictions, emphasizing the endogenous determination of aggregate risk and the role of macroprudential policy. As in our model, a negative aggregate financial shock results in a deflation, although both of these papers consider environments where, for the relevant cases, the zero lower bound on the nominal interest rate is binding in every period.

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9See Buera and Moll (2012) for a detailed discussion of the connection of the real version of our framework with related approaches in the literature.
Guerrieri and Lorenzoni (2011) also study a model where workers face idiosyncratic labor shocks. In their model, a credit crunch leads to an increase in the demand of bonds and therefore results in negative real rates. Although our model also generates a large drop in the real interest rate, the forces underlying this result are different. In our framework, the drop in the real interest rate is the consequence of a collapse in the ability of productive entrepreneurs to supply bonds (i.e., to borrow from the unproductive entrepreneurs and workers), as opposed to an increase in the demand for bonds by these agents. In our model, a credit crunch has an opposite, negative effect on investment.

2 The Model

In this section we describe the model, which closely follows the framework in Moll (forthcoming), modified by imposing a cash-in-advance constraint on the consumer’s decision problem. As mentioned, the analysis will be restricted to a perfect foresight economy in which starting at the steady state, all agents learn at time zero that, starting next period, the collateral constraint will be tightened for several periods.

2.1 Households

All agents have identical preferences, given by

$$\sum_{t=0}^{\infty} \beta^t \left[ \nu \log c_{1t}^j + (1 - \nu) \log c_{2t}^j \right],$$

(1)

where $c_{1t}^j$ and $c_{2t}^j$ are consumption of the cash good and of the credit good, for agent $j$ at time $t$, and $\beta < 1$. Each agent also faces a cash-in-advance constraint,

$$c_{1t}^j \leq \frac{m_{1t}^j}{p_t},$$

(2)

where $m_{1t}^j$ is the beginning of period money holdings and $p_t$ is the money price of consumption at time $t$.

The economy is inhabited by two classes of agents, a mass $L$ of workers and a mass $1$ of entrepreneurs, which we now describe.

Entrepreneurs Entrepreneurs are heterogeneous with respect to their productivity (which is exogenous) and their wealth (which is endogenous). We assume that productivity of each entrepreneur, $z \in Z \subset \mathbb{R}^+$, is constant through their lifetime. We let $\Psi(z)$ be the measure of entrepreneurs of type $z$. Every period, each entrepreneur must choose whether to be active in the following period (to operate a firm as a manager) or to be passive and offer her wealth in the credit market. Thus, there are four state
variables for each entrepreneur: her financial wealth (capital plus bonds), money holdings, the occupational choice (active or passive) made last period and productivity. She must decide the labor demand if active, how much to consume of each good, whether to be active in the following period, and if so, how much capital to invest in her own firm. An entrepreneur’s investment is constrained by her financial wealth at the end of the period $a$ and the amount of bonds she can sell $-b$, $k \leq -b + a$, where we assume that the amount of bonds that can be sold are limited by a simple collateral constraint of the form

$$-b^j \leq \theta k^j$$

for some exogenously given $\theta \in [0, 1]$.

If the entrepreneur decides not to be active (i.e., to allocate zero capital to her own firm), then she invests all her nonmonetary wealth to purchase bonds.

We assume that the technology available to entrepreneurs of type $z$ is a function of capital and labor

$$y = (zk)^\alpha l^{1-\alpha}.$$ 

This technology implies that revenues of an entrepreneur net of labor payments is a linear function of the capital stock, $gzk$, where $g = \alpha ((1 - \alpha)/w)^{(1-\alpha)/\alpha}$ is the return to the effective units of capital $zk$, and $w$ denotes the real wage. Thus, the end of period investment and leverage choice of entrepreneurs with ability $z$ and wealth $a$ solves the following linear problem:

$$\max_{k,d} gzk + (1 - \delta)k + (1 + r)b$$

$$k \leq a - b,$$

$$-b \leq \theta k,$$

where $r$ is the real interest rate. It is straightforward to show that the optimal capital and leverage choices are given by the following policy rules, with a simple threshold property\(^{10}\)

$$k(z, a) = \begin{cases} a/(1 - \theta), & z \geq \hat{z}, \\ 0, & z < \hat{z} \end{cases},$$

$$b(z, a) = \begin{cases} -(1/(1 - \theta) - 1)a, & z \geq \hat{z} \\ a, & z < \hat{z} \end{cases},$$

where $\hat{z}$ solves

$$g\hat{z} = r + \delta.$$  

\(^{10}\)See Buera and Moll (2012) for the details of these derivations.
Given entrepreneurs’ optimal investment and leverage decisions, they would face a linear return to their nonmonetary wealth that is a simple function of their productivity

\[ R(z) = \begin{cases} 
1 + r, & z < \hat{z} \\
\frac{(\varrho z - r - \delta)}{(1-\vartheta)} + 1 + r, & z \geq \hat{z}.
\end{cases} \quad (5) \]

Given these definitions, the budget constraint of entrepreneur \( j \), with net worth \( a_j^t \) and productivity \( z_j \), will be given by

\[ c_{1t}^j + c_{2t}^j + a_{t+1}^j + \frac{m_{t+1}^j}{p_t} = R_t(z_j^j)a_t^j + \frac{m_t^j}{p_t} - T_t^e, \quad (6) \]

where we assume, for simplicity, that lump-sum taxes (transfers if negative) do not depend on the productivity of entrepreneurs.\(^{11}\)

These budget constraints imply that agents choose, at \( t \), money balances \( m_{t+1}^j \) for next period, as the cash-in-advance constraints (2) make clear. Thus, we are adopting the timing convention of Svensson (1985), in which agents buy cash goods at time \( t \) with the money holdings they acquired at the end of period \( t-1 \).\(^{12}\) An advantage of this timing for our purposes is that it treats all asset accumulation decisions symmetrically, using the standard timing from capital theory, where production by entrepreneurs at time \( t \) is done with capital goods accumulated at the end of period \( t-1 \).

**Workers** There is a mass \( L \) of identical workers, endowed with a unit of time each period that they inelastically supply to the labor market. Thus, their budget constraints are given by

\[ c_{1t}^W + c_{2t}^W + a_{t+1}^W + \frac{m_{t+1}^W}{p_t} = (1 + r_t)a_t^W + \frac{w_t^W}{p_t} - T_t^W, \quad (7) \]

where \( a_{t+1}^W \) and \( m_{t+1}^W \) are real financial assets and nominal money holdings chosen at time \( t \), and \( T_t^W \) are lump-sum taxes paid to the government. If \( T_t^W < 0 \), these represent transfers from the government to workers. We impose on workers a nonborrowing constraint, so \( a_t^W \geq 0 \) for all \( t \).\(^{13}\)

\(^{11}\)Note that efficiency implies transferring resources from low productivity entrepreneurs to high productivity ones. However, given the exogenous nature of the collateral constraints, we do not find those exercises illuminating.

\(^{12}\)This assumption implies that unexpected changes in the price level have welfare effects, since agents cannot replenish cash balances until the end of the period.

\(^{13}\)This is a natural constraint to impose. It is equivalent to impose on workers the same collateral constraints entrepreneurs face, since workers will never decide to hold capital in equilibrium.
2.2 Demographics

As we will show below, active entrepreneurs will increase their wealth permanently, while inactive entrepreneurs will deplete asymptotically. In order for the model to have a nondegenerate asymptotic distribution of wealth across productivity types, we assume that a fraction \( 1 - \gamma \) of entrepreneurs depart for Nirvana every period, and are replaced by an equal number of new entrepreneurs. The productivity \( z \) of the new entrepreneurs is drawn from the same distribution \( \Psi(z) \), i.i.d. across entrepreneurs and over time. We assume that there are no annuity markets and that each new entrepreneur inherits the assets of a randomly drawn departed entrepreneur. Agents do not care about future generations, so if we let \( \tilde{\beta} \) be the pure discounting factor, they discount the future with the compound factor \( \beta = \tilde{\beta} \gamma \), which is the one we used above.

2.3 The Government

In every period the government chooses the money supply \( M_{t+1} \), issues one-period bonds \( B_{t+1} \), and uses type-specific lump-sum taxes (subsidies) \( T^e_t \) and \( T^W_t \). Government policies are constrained by a sequence of period-by-period budget constraints:

\[
B_{t+1} - (1 + r_t)B_t + \frac{M_{t+1}}{p_t} - \frac{M_t}{p_t} + T^e_t + LT^W_t = 0, \quad t \geq 0.
\]

We denote by \( T_t \) the total tax receipts of the government:

\[
T_t = T^e_t + LT^W_t.
\]

Ricardian equivalence does not hold in this model for two related reasons. First, agents face different rates of return to their wealth. Thus, the present value of a given sequence of taxes and transfers differs across agents. Second, lump-sum taxes and transfers will redistribute wealth in general, and these redistributions do affect aggregate allocations, due to the presence of the collateral constraints. In the numerical sections, we will be explicit regarding the type of taxes and transfers we consider.

2.4 Optimality Conditions

The optimal problem of agents is to maximize (1) subject to (2) and (6) for entrepreneurs or (7) for workers. Note that the only difference between the two budget constraints is that entrepreneurs have no labor income. For workers, as for inactive entrepreneurs, the real return to their nonmonetary wealth equals \( 1 + r_t \). In what follows, to save on notation, we drop the index for individual entrepreneurs \( j \) unless strictly necessary. Since this is a key aspect of the model, we first briefly explain the zero bound equilibrium restriction on the nominal interest rate that arises from the
agent’s optimization problem.\textsuperscript{14} Then, we discuss the other first-order conditions.

In this economy, gross savings (demand for bonds) come from inactive entrepreneurs and, potentially, from workers. Note that the return on holding financial assets for these agents is $R_t(z) = (1 + r_t)$, while the return on holding money—ignoring the liquidity services—is given by $p_t/p_{t+1}$. Thus, if there is intermediation in equilibrium, the return on holding money cannot be higher than the return on holding financial assets. If we define the nominal return as $(1 + r_t) p_t/p_{t-1}$, then for intermediation to be nonzero in equilibrium, the zero bound constraint

$$
(1 + r_t) \frac{p_t}{p_{t-1}} - 1 \geq 0
$$

must hold for all $t$.

The first-order conditions of the household’s problem imply the standard Euler equation and intratemporal optimality condition between cash and credit goods:

$$
\frac{1}{\beta} \frac{c_{2t+1}(z)}{c_2(z)} = R_{t+1}(z), \quad t \geq 0,
$$

$$
\frac{\nu}{1 - \nu} \frac{c_{2t+1}(z)}{c_{1t+1}(z)} = R_{t+1}(z) \frac{p_{t+1}}{p_t}, \quad t \geq 1.
$$

Solving forward the period budget constraint (6), using the optimal conditions (10) and (11) for all periods, and assuming that the cash-in-advance constraint is binding at the beginning of period $t = 0$, we obtain the solutions for consumption of the credit good and financial assets for agents that face a strictly positive opportunity cost of money in period $t + 1$,\textsuperscript{15}

$$
c_{2t}(z) = \frac{(1 - \nu) (1 - \beta)}{1 - \nu (1 - \beta)} \left[ R_t(z) a_t - \sum_{j=0}^{\infty} \frac{T^e_{t+j}}{\prod_{s=1}^{j} R_{t+s}(z)} \right]
$$

$$
a_{t+1}(z) = \beta \left[ R_t(z) a_t - \sum_{j=0}^{\infty} \frac{T^e_{t+j}}{\prod_{s=1}^{j} R_{t+s}(z)} \right] + \sum_{j=1}^{\infty} \frac{T^e_{t+j}}{\prod_{s=1}^{j} R_{t+s}(z)}.
$$

These equations always characterize the solution for active entrepreneurs even when nominal interest rates are zero. The reason is that for them, the opportunity cost of holding money is given by $R_t(z)p_{t+1}/p_t > (1 + r_t) p_{t+1}/p_t \geq 1$, where the last inequality follows form (9). The solution also characterizes the optimal behavior of inactive entrepreneurs, as long as $(1 + r_t) p_{t+1}/p_t - 1 > 0$.

The solution for inactive entrepreneurs in periods in which the nominal interest

\textsuperscript{14}Formal details are available from the authors upon request.

\textsuperscript{15}Note that it could be possible that initial money holdings are so large for an active entrepreneur that the cash-in-advance constraint will not be binding in the first period. This case will not be relevant provided initial real cash balances are not too big.
rate is zero, \((1 + r_t) p_{t+1}/p_t - 1 = 0\), is

\[
a_{t+1}(z) + \frac{m_{t+1}(z)}{p_t} - \frac{m^T_{t+1}(z)}{p_t} = \beta \left[ R_t(z) a_t - \sum_{j=0}^{\infty} \frac{T_{t+j}}{\prod_{s=1}^{j} R_{t+s}(z)} \right] + \sum_{j=1}^{\infty} \frac{T^c_{t+j}}{\prod_{s=1}^{j} R_{t+s}(z)},
\]

where

\[
\frac{m^T_{t+1}(z)}{p_t} = \frac{\nu(1 - \beta)\beta}{1 - \nu(1 - \beta)} \left[ R_t(z) a_t - \sum_{j=0}^{\infty} \frac{T^c_{t+j}}{\prod_{s=1}^{j} R_{t+s}(z)} \right]
\]

are the real money balances that will be used for transaction purposes in period \(t + 1\). Thus, \(m_{t+1}/p_t - m^T_{t+1}/p_t \geq 0\) are the excess real money balances, hoarded from period \(t\) to \(t + 1\).

The optimal plan for workers is slightly more involved, since their income is non-homogeneous in their net worth and they will tend to face binding borrowing constraints in finite time. In particular, as long as the \((1 + r_\infty)\beta < 1\), as will be the case in the equilibria we will discuss, where \(r_\infty\) is the real interest rate in the steady state, workers drive their wealth to zero in finite time and are effectively hand-to-mouth consumers in the long run. That is, for sufficiently large \(t\),

\[
c^W_{2,t} = \frac{(1 - \nu)(w_t - T^W_t)}{1 - \nu(1 - \beta)} \quad \text{and} \quad c^W_{1,t+1} = \frac{m^W_{t+1}}{p_{t+1}} = \frac{\nu(w_t - T^W_t) \beta p_t}{1 - \nu(1 - \beta) p_{t+1}}.
\]

Along a transition, workers may accumulate assets for a finite number of periods. This would typically be the case if they expect a future drop in their wages—as in the credit crunch we consider—or they receive a temporarily large transfer, \(T^W_t < 0\).

### 3 Equilibrium

Given policies \(\{M_t, B_t, T^W_t\}_{t=0}^{\infty}\) and collateral constraints \(\{\theta_t\}_{t=0}^{\infty}\) an equilibrium is given by prices \(\{r_t, w_t, p_t\}_{t=0}^{\infty}\) and corresponding quantities such that:

- Entrepreneurs and workers maximize, taking as given prices and policies,
- The government budget constraint is satisfied, and
- Bond, labor, and money markets clear:

\[
\int_z b_{t+1}(z)dz + Lb^W_t + B_{t+1} = 0, \quad \int_z l_t(z)dz = L, \quad \int_z m_t(z)dz + Lm^W_t = M_t, \quad \text{for all } t.
\]

To illustrate the mechanics of the model, we first provide a partial characterization of the equilibrium dynamics of the economy for the case in which the zero lower bound is never binding, \(1 + r_{t+1} > p_t/p_{t+1}\) for all \(t\), workers are hand-to-mouth, \(a^W_t = 0\) for
all \( t \), and the share of cash goods is arbitrarily small, \( \nu \approx 0 \). Second, we discuss some properties of the model when the zero bound constraint binds. Finally, we study a very special case for which we can obtain closed-form solutions.

### 3.1 Equilibrium Away from the Zero Bound

Let \( \Phi_t(z) \) be the measure of wealth held by entrepreneurs of productivity \( z \) at time \( t \). Integrating the production function of all active entrepreneurs, equilibrium output is given by a Cobb-Douglas function of aggregate capital \( K_t \), aggregate labor \( L \), and aggregate productivity \( Z_t \),

\[
Y_t = Z_t K_t^\alpha L^{1-\alpha},
\]

where aggregate productivity is given by the wealth-weighted average of the productivity of active entrepreneurs, \( z \geq \hat{z}_t \),

\[
Z_t = \left( \frac{\int_{\hat{z}_t}^{\infty} z \Phi_t(dz)}{\int_{\hat{z}_t}^{\infty} \Phi_t(dz)} \right)^\alpha.
\]

Note that \( Z_t \) is an increasing function of the cutoff \( \hat{z}_t \) and a function of the wealth measure \( \Phi_t(z) \). In turn, given the capital stock at \( t+1 \), which we discuss below, the evolution of the wealth measure is given by

\[
\Phi_{t+1}(z) = \gamma \left[ \beta \left( R_t(z) \Phi_t(z) - \sum_{j=0}^{\infty} \frac{T_{t+j}^e \Psi(z)}{\prod_{s=1}^{j} R_{t+s}(z)} \right) + \sum_{j=1}^{\infty} \frac{T_{t+j}^e \Psi(z)}{\prod_{s=1}^{j} R_{t+s}(z)} \right] + (1 - \gamma) \Psi(z) (K_{t+1} + B_{t+1}),
\]

where the first term on the right-hand side reflects the decision rules of the \( \gamma \) fraction of entrepreneurs that remain alive, and the second reflects the exogenous allocation of assets of departed entrepreneurs among the new generation.

Then, given the (exogenous) value for \( \theta_{t+1} \) and the wealth measure \( \Phi_{t+1}(z) \), the cutoff for next period is determined by the bond market clearing condition

\[
\int_{0}^{\hat{z}_{t+1}} \Phi_{t+1}(dz) = \frac{\theta_{t+1}}{1 - \theta_{t+1}} \int_{\hat{z}_{t+1}}^{\infty} \Phi_{t+1}(dz) + B_{t+1}.
\]

To obtain the evolution of aggregate capital, we integrate over the individual decisions and use the market clearing conditions. It results in a linear function of aggregate output, the initial capital stock, and the aggregate of the (individual-specific) present
value of taxes,

\[ K_{t+1} + B_{t+1} = \beta [\alpha Y_t + (1 - \delta)K_t + (1 + r_t)B_t] - \beta \int_0^\infty \sum_{j=0}^\infty \frac{T_{t+j}^e}{\prod_{s=1}^j R_{t+s}(z)} \Psi(dz) + \int_0^\infty \sum_{j=1}^\infty \frac{T_{t+j}^e}{\prod_{s=1}^j R_{t+s}(z)}(dz). \]  

(17)

Solving forward the government budget constraint (8), using that \( \nu \approx 0 \), and substituting into (17),

\[ K_{t+1} = \beta [\alpha Y_t + (1 - \delta)K_t] + (1 - \beta) \int_0^\infty \sum_{j=1}^\infty \frac{T_{t+j}^e}{\prod_{s=1}^j R_{t+s}(z)} \Psi(dz) - (1 - \beta) \int_0^\infty \sum_{j=1}^\infty \frac{T_{t+j}^e}{\prod_{s=1}^j (1 + r_{t+s})} \Psi(dz) + \beta LT_t^W - (1 - \beta)L \sum_{j=1}^\infty \frac{T_{t+j}^W}{\prod_{s=1}^j (1 + r_{t+s})}. \]  

(18)

The first term gives the evolution of aggregate capital in an economy without taxes. In this case, aggregate capital in period \( t + 1 \) is a linear function of aggregate output and the initial level of aggregate capital. The evolution of aggregate capital in this case is equal to the accumulation decision of a representative entrepreneur (Moll, forthcoming; Buera and Moll, 2012). The second term captures the effect of alternative paths for taxes, discounted using the type-specific return to their nonmonetary wealth, while the last term is the present value of taxes from the perspective of the government. For instance, consider the case in which the government increases lump-sum transfers to entrepreneurs in period \( t \), financing them with an increase in government debt and, therefore, with an increase in the present value of future lump-sum taxes. In this case, future taxes will be discounted more heavily by active entrepreneurs, implying that the last term is bigger than the second. Thus, this policy crowds out private investment and results in lower aggregate capital next period.

Finally, we describe the determination of the price level. In the previous derivations, in particular, to obtain (18), we have used that \( \nu \approx 0 \), and therefore, the money market clearing condition is not necessarily well defined.\(^{16}\) More generally, given monetary and

\(^{16}\)To determine the price level in the cashless limit, we need to assume that as \( M_{t+1}, \nu \to 0 \), \( M_{t+1}/\nu \to \tilde{M}_{t+1} > 0 \). See details in Section 3.3.3.
fiscal policy, the price level is given by the equilibrium condition in the money market,
\[
\frac{M_{t+1}}{p_t} = \frac{\nu(1 - \beta)^2}{1 - \nu(1 - \beta)} \left[ \alpha Y_t + (1 - \delta)K_t + (1 + r_t)B_t + \int_0^\infty \sum_{j=0}^\infty \frac{T^{e\ell}_{t+j}}{\prod_{s=1}^j R_{t+s}(z)} \Psi(dz) \right].
\]
(19)
The nominal interest rate is obtained from the intertemporal condition of inactive entrepreneurs,
\[
\frac{1}{\beta} \frac{c_{2t+1}}{c_{2t}} = \frac{1 + i_{t+1}}{\frac{p_{t+1}}{p_t}} = 1 + r_{t+1}.
\]
(20)

Note that, except for the well-known Sargent-Wallace initial price level indeterminacy result, we can think of monetary policy as sequences of money supplies, \( \{M_t\}_{t=0}^\infty \), or sequences of nominal interest rates, \( \{i_t\}_{t=0}^\infty \). We will think of policy as determining exogenously one of the two sequences, abstracting from the implementability problem.\(^{17}\)

There are two important margins in this economy. The first is the allocation of capital across entrepreneurs, which is dictated by the collateral constraints and which determines measured TFP (see (14)). The second is the evolution of aggregate capital over time, which, in the absence of taxes, behaves as in Solow’s model (see (18) and set \( T_{t+j}^e = 0 \)). Clearly, fiscal policy has aggregate implications: the net supply of bonds affects (16) and taxes affect (18). However, monetary policy does not, since none of those equations depend on nominal variables. Monetary policy does have effects, since it distorts the margin between cash and credit goods, but in a fashion that resembles the effects of monetary policy in a representative agent economy. This is the case only if, as assumed above, the zero bound does not bind.

### 3.2 Equilibrium at the Zero Bound

In periods in which the zero bound binds, successful inflation-targeting policies will affect equilibrium quantities if the target for inflation is tight enough. The reason is that, by successfully controlling inflation, monetary policy, together with the zero bound on nominal interest rates, can impose a bound on real interest rates. To see this, use (20) and the zero bound to write
\[
1 + i_t = \frac{p_t}{p_{t-1}} (1 + r_t) \geq 1, \text{ so } r_t \geq \frac{p_{t-1} - p_t}{p_t} = -\left( \frac{\pi_t}{1 + \pi_t} \right),
\]
(21)
where \( \pi_t \) is the inflation rate.\(^{17}\)

\(^{17}\)Because we use log utility, there is a unique solution for prices, given the sequence \( \{M_t\}_{t=0}^\infty \).
Imagine now an economy with zero net supply of bonds that enters a credit crunch, generated by a drop in maximum leverage, \( \theta_t \). Equation (16) implies that the threshold \( \hat{z}_{t+1} \) has to go down, to reduce the left-hand side and increase the right-hand side so as to restore the equilibrium. This drop in the gross supply of private bonds will reduce the real interest rate, so the marginal entrepreneurs that were lending capital now start borrowing until market equilibrium is restored (i.e., the net supply of bonds is zero). If the credit crunch is large enough, the equilibrium real interest rate may become negative. If inflation is not high enough, the bound (21) may be binding. Imagine, for instance, the case of inflation targeting with a target equal to zero. Then, the real interest rate cannot become negative.

What will an equilibrium look like? In order to support the zero inflation policy, the government needs to inject enough liquidity so that the net supply of money (or bonds, since they are perfect substitutes at the zero bound) goes up to the point where conditions (16) and (21) are jointly satisfied. This policy will have implications for the equilibrium cutoff \( \hat{z}_{t+1} \). In addition, as can be seen in (17), the injection of liquidity (increases in \( B_{t+1} \)) affects capital accumulation. Thus, at the zero bound, the level of inflation chosen by the central bank, if low enough, can affect the two relevant margins in the economy. To further explore these implications in this general model, we need to solve it numerically. But before doing that, we now present a particular (very special) case that can be analytically solved and analyzed, which we find very useful in isolating and understanding some of the mechanisms of the model.

### 3.3 A Simple Case with a Closed-Form Solution

An interesting feature of the model is the interaction between the credit constraints and the endogenous savings decisions. This interaction generates dynamics that imply long-run effects that are very different from the ones obtained on impact, precisely through the endogenous decisions agents make over time to save away from those constraints. A complication is that the endogenous wealth distribution becomes a relevant state variable, and it becomes impossible to obtain analytical results.

It is possible, however, to obtain closed-form solutions if we shut down that endogenous evolution of the wealth distribution. Some of the effects that the simulations of the general model exhibit are also present in this simplified version, where they are easier to understand. We now proceed to discuss that example.

Consider the case in which \( \gamma \to 0 \), but \( \beta \to \infty \), such that \( \beta = \hat{\beta} \gamma \) is kept constant. In this limit, agents live for only a period but the saving decisions are not modified so equation (15) becomes

\[
\Phi_{t+1} (z) = \Psi (z) (K_{t+1} + B_{t+1}).
\]

We also let \( z \) be uniform in \([0, 1]\). Then, the equilibrium condition for the credit
market (16) becomes

$$\dot{z}_{t+1} = \theta_{t+1} + (1 - \theta_{t+1}) b_{t+1},$$

where $b_t = \frac{B_t}{K_t + b_t}$, and the value of TFP in equation (14) becomes

$$Z_t = \left(1 + \frac{\theta_t + (1 - \theta_t) b_t}{2}\right)^{\alpha}. \quad (22)$$

Finally, we normalize $L = 1$ so the law of motion for capital in this special case is given by

$$K_{t+1} = \beta \left[\alpha \left(1 + \frac{\theta_t + (1 - \theta_t) b_t}{2}\right)^{\alpha} K_t^{\alpha} + (1 - \delta)K_t\right] + (1 - \beta) \left[\int_0^\infty \sum_{j=1}^\infty T_{t+j}^e \frac{R_{t+j}(z)}{\prod_{s=1}^j \frac{\delta}{\theta_t + (1 - \theta_t) b_t}} dz - B_{t+1}\right]. \quad (23)$$

Note that given sequences of policies and collateral constraints, this equation fully describes the dynamics of capital.

The economy behaves as it does in Solow’s growth model, except that the collateral constraint as well as the fiscal policy both matter. The collateral constraint matters because it affects aggregate TFP. Policy matters because the model, as we show in detail below, does not exhibit Ricardian equivalence. Finally, the real interest rate is given by

$$\delta + r_{t+1} = \alpha \frac{\theta_{t+1} + (1 - \theta_{t+1}) b_{t+1}}{Z_{t+1}^{\frac{1-\alpha}{\alpha}} K_{t+1}^{\frac{1-\alpha}{\alpha}}}. \quad (24)$$

In addition, the constraint (21) must be satisfied.

In order to gain understanding of some of the effects on equilibrium outcomes of changes in the collateral constraint and on the effects of monetary and fiscal policy, we now solve several simple exercises. In the first two exercises, we solve for the real economy, where $\nu = 0$ and focus the discussion on the evolution of real variables. We first set all transfers to zero and study the effect of a credit crunch: an anticipated drop in $\theta_t$ that lasts several periods and then goes back to its steady state value. We show that total factor productivity, output, and capital accumulation drop, so the effect of output is persistent. We also show that if the credit crunch is large enough, the real interest rate becomes negative. We then keep $\theta_t$ constant and study the effect of debt-financed transfers to show the effect of an increase in the outside supply of bonds in the equilibrium. We show that debt issuance crowds out private investment but increases total factor productivity, so the effect on output is ambiguous. In addition, debt issuance increases the real interest rate. Finally, we consider the cashless limit and
study the behavior of the price level following a credit crunch where the real interest rate becomes negative and the zero bound on the nominal interest rate becomes binding. We show that if the central bank does not change the nominal quantity of money, a deflation follows.

### 3.3.1 The Effect of a Credit Crunch

To isolate the effect of a credit crunch, we set $b_t = 0$ and $T^e_t = T^w_t = 0$. In this case,

$$\hat{z}_t = \theta_t \text{ and } Z_t = \left(\frac{1 + \theta_t}{2}\right)^\alpha.$$  

Given the level of capital, the interest rate is given by

$$\delta + r_t = \frac{\theta_t}{(1 + \theta_t)^{1-\alpha} K_t^{1-\alpha}},$$  

which implies that the real interest rate falls with $\theta_t$. This drop in the real interest rate is what provides the incentives to the less efficient entrepreneurs to enter until the credit market clears, thereby reducing TFP. The drop in the real interest rate as the collateral constraint falls enough is a general feature of the model, which does not depend on the particular simplifying assumptions used in this example. In general, $r_t = \hat{z}_t / E[z | z \geq \hat{z}_t]^{1-\alpha} K_t^{\alpha-1} - \delta$, which tends to $-\delta$ as $\theta_t$, and therefore $\hat{z}_t$, converges to zero. We find this feature of the model particularly attractive in studying the great recession: A single shock can explain both the gap between output and trend and the negative real interest rates. We will further discuss this issue in the calibration section.

The law of motion for capital is given by

$$K_{t+1} = \beta \left[ \alpha \left(\frac{1 + \theta_t}{2}\right)^\alpha K_t^\alpha + (1 - \delta)K_t \right].$$  

We model a temporary credit crunch as

$$\theta_0 = \theta^{ss},$$

$$\theta_t = \theta_t < \theta^{ss} \text{ for } t = 1, 2, ..., T,$$

$$\theta_t = \theta^{ss} \text{ for } t > T$$

and assume that all agents have perfect foresight. The effect on capital is identical to a temporary drop in TFP in Solow’s model: capital does not change on impact, but starts going down until $T$. Then, it starts going up to the steady state. The interest rate drops on impact, since the ratio $\frac{\theta_{t+1}}{(1+\theta_{t+1})^{1-\alpha}}$ is increasing on $\theta_{t+1}$. Note that

$$\delta + r_1 = \frac{\theta_t}{(1 + \theta_t)^{1-\alpha} K_t^{1-\alpha}},$$

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so the real interest rate will be negative if $\theta_t$ is low enough. The evolution of the interest rate depends on the evolution of capital and on the collateral constraint parameter.

### 3.3.2 The Effect of Policy

We consider the case in which there are no taxes or transfers to workers (so $T_t^w = 0$). First, note that (22) trivially implies that total factor productivity is increasing on the ratio of debt to total assets. We show now that it also crowds out investment.

Assume that the economy starts at the steady state and consider the policy

$$B_0 = 0, \quad B_1 = -T_0 = B > 0 \quad \text{and} \quad (1 + r_1) T_0 + T_1 = 0,$$

so $B_t = 0$ for $t \geq 2$. Thus, at time 0 there is a deficit (a transfer to all entrepreneurs) financed by issuing debt. Then, at time 1, there is a surplus (tax to all entrepreneurs) that is enough to payoff all bonds issued at time zero.

Given this policy, the law of motion for capital (23) becomes

$$K_1 = K_{ss} - (1 - \beta)B \left[ 1 - \int_0^1 \frac{(1 + r_1)}{R_1(z)} dz \right].$$

When $\theta = 1$, $R_1(z) = (1 + r_1)$ for all $z$ (and $\hat{z} = 1$), then

$$(1 + r_1) \int_0^1 \frac{1}{R_1(z)} dz = 1$$

and Ricardian equivalence holds. But when $\theta \in (0, 1)$, from (5) it follows that $R_1(z) > (1 + r_1)$ for $z > \hat{z}$ (and $\hat{z} \in (0, 1)$), then

$$0 < \int_0^1 \frac{(1 + r_1)}{R_1(z)} dz < 1.$$ 

Thus, the level of capital is lower than the steady state for any positive level of debt. The opposite is true if the government lends such that $B < 0$.

Finally, note that the interest rate on period 1 is given by

$$\delta + r_1 = \frac{\theta + b_1(1 - \theta)}{(1 + \theta + b_1(1 - \theta))^{1-\alpha}} \frac{\alpha^{2-\alpha}}{K_1^{1-\alpha}},$$

\[\text{We solve the case in which workers are also taxed in Appendix B, where, to keep analytical tractability, we also assume that workers are hand to mouth. In our simulations we solve for the general case in which workers can hold bonds.}\]

\[\text{The opposite is true if the government lends such that } B < 0.\]

\[\text{The relationship between government debt and aggregate capital is nonmonotonic. In particular, one can show that as } B \to \infty, \text{ aggregate capital converges to the steady state value in an economy with } \theta = 1.\]
where the first term is increasing on \(b_1\), so the interest rate will be higher than in the steady state.

To summarize, a credit crunch and an increase in debt have opposite effects on total factor productivity and on the real interest rate: while the credit crunch reduces both, the increases in debt increase both. On the contrary, both the credit crunch and the debt increase reinforce each other in that they reduce capital accumulation. As increases in outside liquidity (bonds plus money at the zero bound) dampen the drop in the real interest rate, they will be effective at achieving a target for inflation when the zero bound binds. Doing so also implies a lower drop in TFP (a higher threshold) but a larger drop in capital. The net effect on output is in general ambiguous, but it can be shown to be positive in the neighborhood of \(B = 0\).

This trade-off will be present in our simulations of the general model that allows for rich dynamics of the wealth distribution and uses alternative functional forms for the distribution of \(z\).

### 3.3.3 Deflation Follows Passive Policy

We want to discuss the behavior of the price level following a credit crunch that drives the real interest rate into negative territory and such that the zero bound constraint binds during at least one period. The characterization is simpler in the case in which the zero bound is binding for only one period. As it turns out, if the parameters satisfy certain properties, this will indeed be the case. Thus, as we explain in the example, we will make two assumptions that parameters must satisfy for the equilibrium to be such that the zero bound binds only in one period. Under these conditions, we then explain why deflation will be the result of a credit crunch if policy does not respond.

**The cashless limit** We consider the limiting case of the cashless economy (i.e., \(\nu \to 0\)). In this case, the distortions associated with a positive nominal interest rate do not affect the real allocation.\(^{22}\) In taking the limit, though, we also let nominal money balances shrink at the same rate so we can still meaningfully determine the equilibrium price level. The details follow.

When the cash-in-advance constraint is binding, the first-order condition is

\[
p_t c_t^1 = \frac{\nu}{1 - \nu} c_t^2 p_{t-1} \frac{1}{R_t(z)}.
\]

We define \(\overline{m}_t = m_t / \nu\), so \(p_t c_t^1 = m_t = \overline{m}_t \nu\). Replacing above and taking the limit when

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\(^{21}\)In the case in which workers are the only individuals being taxed and subsidized, the net effect on output is negative. The analysis of the net effect on output is presented in Appendix B.

\(^{22}\)In the general case, nonnegligible money balances crowd out capital and ameliorate the drop in the real interest rate and in total factor productivity.
\( \nu \to 0 \), we obtain
\[
\overline{m}_t = c_t^2 p_{t-1} \frac{1}{R_t(z)}.
\]

Finally, using the optimal rule for the credit good specialized for the limiting case
\[ c_t^2 = (1 - \beta) R_t(z) k_t \]
and aggregating over all agents, we obtain
\[
\overline{M}_t = (1 - \beta) K_t p_{t-1}.
\] (27)

Because of the cashless limit and since debt and transfers are all zero, the real variables follow the solution described in (25) and (24), irrespectively of the evolution of the price level. However, the price level does depend on the behavior of real variables, so it is useful to obtain some explicit solutions.

If we let \( \beta \equiv (1 + \rho)^{-1}, \rho > 0 \), the steady state is given by
\[
K_{ss} = \left( \frac{\alpha}{\rho + \delta} \right)^{\frac{1}{1 - \alpha}} \left( \frac{1 + \theta_{ss}}{2} \right)^{\frac{\alpha}{1 - \alpha}}
\] (28)

and
\[
\left( \frac{2\theta_{ss}}{1 + \theta_{ss}} \right) (\rho + \delta) = r_{ss} + \delta.
\] (29)

We assume that
\[
\frac{2\theta_{ss}}{(1 + \theta_{ss})} > \beta,
\] (30)

which implies that the real interest rate is positive in the steady state.\(^{23}\) During the credit crunch, at \( t = 1 \) the real interest rate is
\[
\delta + r_1 = (\rho + \delta) \frac{2\theta_t}{(1 + \theta_t)} \left( \frac{1 + \theta_t}{1 + \theta_{ss}} \right)^{\alpha},
\]
which is negative as long as
\[
\frac{2\theta_t}{(1 + \theta_t)} \left( \frac{1 + \theta_t}{1 + \theta_{ss}} \right)^{\alpha} < \frac{\delta}{(\rho + \delta)}. \tag{31}
\]

\(^{23}\)The necessary condition for positive interest rates in the steady state, \( \frac{2\theta_{ss}}{(1 + \theta_{ss})} > \frac{\delta}{\rho + \delta} \), is weaker. The stronger condition that we assume will also imply that the zero bound on nominal interest rates binds one period at most and simplifies the example.
Clearly, there exists a value for $\theta_l \in (0, \theta_{ss})$ such that this constraint is satisfied.

**The conditions that determine the price level**  As we assume policy is passive, we let $\bar{M}_{t+1} = \bar{M}$ and assume that in equilibrium $i_t > 0$ for $t \geq 2$. Then, using (27) we obtain that for all $t \geq 2$

$$1 + i_t = (1 + r_t) \frac{p_t}{p_{t-1}} = (1 + r_t) \frac{K_t}{K_{t+1}}.$$  

Note that the real interest rate is positive, but there is deflation. It is possible to show, however, that under assumption (30) the deflation is not enough to make the nominal interest rate negative from time 2 on.

**Lemma 1:** Given assumption (30), $i_t > 0$ for $t \geq 2$.

**Proof:** See Appendix A.

Since all cash-in-advance constraints are binding from $t \geq 2$,

$$p_t = \frac{\bar{M}}{(1 - \beta)K_{t+1}} \text{ for } t \geq 1.$$  

We now show that under certain conditions on the parameters, the zero bound is binding at $t = 1$.

**Lemma 2:** If $\delta > \rho$, then there exists a $\tilde{\theta}_l > 0$ such that $i_1 = 0$ for all $\theta_l \in (0, \tilde{\theta}_l]$.

**Proof:** See Appendix A.

Finally, we show that if the economy starts at the steady state, at time zero, when agents learn there is a credit crunch, the equilibrium price level must be strictly below its steady state value.

**Lemma 3:** Under the assumptions of Lemmas 1 and 2, $p_0 < p_{ss}$ for all $\theta_l \in (0, \tilde{\theta}_l)$.

**Proof:** See Appendix A.

The credit crunch drives the real interest rate below zero to the point at which the zero bound is reached. At this point, there is an excess demand for money as a “store of value.” That excess demand is, of course, real rather than nominal. As the nominal quantity is fixed by policy, the demand pressure results in deflation. The excess demand for money as a store of value will be positive until future inflation is high enough such that the return on money is as negative as the return on bonds. The
initial deflation allows for future inflation along the path, required for the arbitrage condition to hold, with a zero “long-run” inflation. This zero long run inflation is the natural consequence of a constant nominal money supply.

4 Calibration and Evaluation of the model

In this Section we first calibrate the model using data for the United States and discuss how to take the model to the data. We discuss in detail the frequency we want to focus on and the way we de-trend the data. Second, we numerically solve the model and compare it to the data. Finally, we perform several policy experiments to illustrate the role of policy during the Great Recession, and to consider the impact of alternative policy responses.

4.1 Calibration

The model has a very small number of parameters. We first set the the share of capital in output to be one third and the (annual) depreciation of capital to be seven percent, which are standard values. The preference parameter for the cash good only determines the total level of money to output in the steady state. For simplicity we consider the cashless limit in which \( \nu \to 0 \) and we also let the initial stock of money go to zero at the same rate, so the price level can still be determined and the zero bound constraint in interest are still must hold. We set the distribution of abilities to be log-normal, \( z \sim \ln \mathcal{N}(0, \sigma_z) \), truncated from above at \( z = 10 \), and choose the standard deviation \( \sigma_z \) so that the log dispersion of productivity among entrepreneurs in the model matches that among manufacturing establishments in the United States, as reported by Hsieh and Klenow (2009).\(^{24}\) We choose the rate at which entrepreneurs die \( 1 - \gamma = 0.10 \) to match to the average exit rate of US establishments from the Business Dynamic Statistics (BDS). The initial parameter of the collateral constraint, \( \theta_0 = 0.65 \), is chosen to match the ratio of liabilities to capital for the US non-corporate sector in the second quarter of 2007.\(^{25}\) We assume a value of debt to GDP equal to 62%, the value of total public federal debt in the US, in the second quarter of 2007. Finally, we set the discount factor in preferences, \( \beta = 0.985 \), to match, in the steady state, a real interest rate of two percent. Table 4.1 summarizes the parameter values we use.\(^{26}\)

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\(^{24}\) If there are diminishing returns to scale all entrepreneurs will be active in equilibrium. Therefore, by matching the log dispersion of productivity among all entrepreneurs, active and inactive, the calibration is robust to the inclusion of arbitrary small diminishing returns to scale.

\(^{25}\) We measure the ratio of liabilities to capital by dividing the ratio of liabilities to gross value added of the US non-corporate sector from the flow of funds by the aggregate capital to output ratio.

\(^{26}\) We also need to specify the relative number of workers and entrepreneurs in the economy. We assume that workers are 25% of the population, \( L/(1 + L) = 1/4 \). We choose a low share of workers, who in our model choose to be against their borrowing constraint in a steady state, to limit the non-Ricardian elements in the model.
We then assume that, starting at that steady state, all agents learn that the collateral constraint will tighten for several periods and that the Fed and the central government will substantially increase their liabilities, i.e., their supply of liquid assets. Specifically, we chose a sequence $\{\theta_t\}_{t=0}^{\infty}$ so that the model broadly matches the evolution of the real interest rate, given the path for government liabilities, which is a direct input in our calibration. The imputed evolution of government liabilities is associated with the path of taxes and transfers. To be conservative, we assume that transfers are given solely to entrepreneurial types that are active in the initial steady state, while taxes are lump-sum. In Section 4.3.2 we consider the case with lump-sum subsidies.

We chose to focus on the behavior of the real interest rate in the United States following the financial crisis to calibrate that sequence for a couple of reasons. First, as discussed in detail in the last section, it is the reduction in $\theta_t$ that drives down the real interest rate. The direct theoretical relationship between the unobservable shock and the real rate makes it an attractive target for the calibration.

Second, the very long period of negative real interest rates, depicted in the left panel of Figure 1, is one of the most remarkable features of the events of the last decade and speaks to the persistence of the shock that drove the real rates substantially below zero. The series labeled “computed” is just the difference between the short term nominal interest rate between quarter $t$, and quarter $t+1$ and the observed inflation rate, using the consumer price index, also between $t$ and $t+1$. The series labeled ”ABC 2014” is obtained from Ajello et al. (2012), which compute it using a no-arbitrage model that jointly explains the dynamics of consumer prices as well as the nominal and real term structure of risk-free rates.

We use the behavior of this real rate to identify the timing, the severity and the persistency of the shock. As it can be seen in the Figure, the drop in the rate is in the third quarter of 2007, so we choose this quarter as the onset of the crisis. We then chose the sequence $\theta_t$ such that the simulated series for the real interest rate matches the solid line in the left Panel of Figure 1, which is a simple, piece-wise linear smooth version of the data. The simulation starts at the steady state - taken to be the third...

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Targets</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 1/3, \ 1 - (1 - \delta)^4 = 0.07$</td>
<td>Standard Values</td>
</tr>
<tr>
<td>$z \sim \ln N(0, 3.36)$</td>
<td>Log dispersion of estab., US manuf.</td>
</tr>
<tr>
<td>$1 - \gamma^4 = 0.10$</td>
<td>avg. exit rate of US establishments</td>
</tr>
<tr>
<td>$B_0/(4Y_0) = 0.62$</td>
<td>Total Public, Federal Debt in the US, 2007:Q2</td>
</tr>
<tr>
<td>$\beta = 0.985$</td>
<td>2% real interest rate</td>
</tr>
<tr>
<td>$\theta_0 = 0.65$</td>
<td>Liabilities to capital, US corporate sector 2007:Q2</td>
</tr>
</tbody>
</table>

Table 1: Calibration, Initial Steady State
As the model makes clear, the equilibrium does depend on the reaction of monetary and debt policy following the shock - the injection of total liquidity. Thus, in the simulation we also assume that agents have perfect foresight with respect to the policy followed and assumed the total supply of outside liquidity to be the one observed in the data starting in 2007.

The second panel of Figure 1 shows the evolution of the supply of liquid assets by the consolidated federal government, which is a direct input into our simulations. This series is the sum of the total public federal debt and the balance sheet of the Federal Reserve Banks net of their holdings of Treasury bonds. An important part of the policy response to the crisis by the Fed was to increase the supply of liquidity in term of bank reserves in exchange for Mortgage Back Securities. In addition, there were large transfer and tax programs that resulted in an unprecedented increase in the level of the public debt. Furthermore, we assume a path for the money supply that is consistent with an inflation of 2% per year in periods in which the nominal rate is strictly positive. When the nominal rate is at the zero lower bound, the inflation rate equals minus the real interest rate, i.e., the observed path of inflation. As we will argue in the following sections, according to the model, these policies are responsible for avoiding a large initial deflation, and for “implementing” relatively low inflation rates.

4.2 Evaluation of the model

We now compare the simulations of the model with the US data since the third quarter of 2007 - the date identified as the beginning of the crisis by the real interest rate - till the first quarter of 2015, the end of our sample. In looking at variables such as output, capital, labor or productivity, we face a difficulty that does not arise when looking

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Figure 1: Real Interest Rate, Credit Growth and Public Liquidity, Data and Model Calibrated Paths
at real interest rates: the trend in the data. Our model is stationary, but it can be modified to incorporate exogenous productivity growth, the same way it is done in the Solow model. To the extent that the exogenous component of productivity grows at a constant rate, allowing for this exogenous component is equivalent to removing a linear trend to the natural logarithm of the data. That is the strategy we pursue in comparing the data to the model.

In the next Table we show the quarterly growth rate of the linear trend for the natural logarithm of output, capital, hours and productivity using quarterly data. We report results for three different starting periods, 1947, 1960 and 1980. In all cases, the last period was the third quarter of 2007, the period where the crisis started according to our calibration.

<table>
<thead>
<tr>
<th>Initial Period</th>
<th>1947:Q3</th>
<th>1960:Q1</th>
<th>1980:Q1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>0.0088</td>
<td>0.0081</td>
<td>0.0080</td>
</tr>
<tr>
<td>Hours</td>
<td>0.0033</td>
<td>0.0042</td>
<td>0.0039</td>
</tr>
<tr>
<td>Capital</td>
<td>0.0092</td>
<td>0.0086</td>
<td>0.0077</td>
</tr>
<tr>
<td>Productivity</td>
<td>0.0027</td>
<td>0.0023</td>
<td>0.0028</td>
</tr>
</tbody>
</table>

Table 2: Linear Trends of (log) GDP, Hours, Capital and TFP

The results are surprisingly robust to the initial date used for output and hours. It is less so for capital. However, a detailed inspection of the data shows that a linear trend does not adjust well for the first two samples: The data cuts the trend only twice for the first two periods, suggesting a slowdown of the trend over time. However, for the sample that starts in 1980, the trend cuts the data several times. Thus, we do choose the value for the sample that starts in 1980. The trend for productivity seems also relatively stable, but that hides the fact that it grew very rapidly from 1947 to 1973 (a slope of 0.0043) and then it remained essentially constant till 1983. It then grew at 0.0028 the rate reported for the sample that starts in 1980.

Given this discussion, we chose the values for the trends to be the ones that result from the last period, the ones reported in the last column of Table 2.

To show the effect of de-trending in a way that makes clear the long lasting effect of the crisis that started in 2007, in Figure 2 we depict the difference between the natural logarithm of the data and its trend (computed from 1980 to 2007) for output, hours, capital and productivity. As it can be seen, there are fluctuations around trend that, till 2007 never go beyond 5% in absolute value for any of the series. However, after 2007, the deviations are all negative and way larger than anything that has been seen before.

\footnote{The linear trend has been computed by ordinary least squares regressions of the log of the corresponding variable on time.}
We now argue that, except for hours, these events can be explained by the single shock we model, and that we calibrated to match the evolution of the real interest rate.

The deviations from trend exhibit in Figure 2 are the ones we compare to the simulation of our model. To begin with, we should notice we assumed the labor supply to be constant, so the model will be unable to replicate the very large drop in hours since 2007. This would not be different if we had leisure in the utility function: a shock to the collateral constraint, as we model it, does not have a direct effect on the labor market.\textsuperscript{28} If the reader was hoping to learn something meaningful regarding the relationship between credit constraints and the labor market, she should stop reading now.

In Figure 3 we compare the de-trended data for output, productivity, capital, and hours, with the simulation of the model. The solid (red) line is the simulation of the model. The slashed (blue) line is the data. The two panels in the left show that the model captures the direction and the persistency of the drops in capital and output - relative to trend - but misses the magnitudes: it only explains around one third of the output drop and about half of the drop in capital. The first panel in the left shows that the model does a very good job at tracking the behavior of productivity, beyond missing the high frequency movements in 2009. It is important to highlight that no parameter has been chose to fit this curve - or any of the other pictures in this figure! The lower panel in the left shows that the model, with constant labor, misses the behavior of hours. One could conjecture, therefore, that part of the reason why the model misses the magnitudes in explaining output and capital is related to the failure of explaining labor. One way to evaluate this conjecture, given that in the model labor supply is exogenous, is to simply impose in the simulation the behavior of hours that we saw in the data. The result is depicted in the Figure with the dotted (green) line.

\textsuperscript{28}As we show below, adding sticky wages a la Calvo cannot explain it either for parameter values used in the literature. The effects in the model lasts less than three years.
Figure 3: Great Recession in the Benchmark Model and Model with Exogenous Hours

Once we feed the model with the observed value for hours, the match of the model is remarkably good.

Figure 4: Credit Growth in the Benchmark Model vs. The Non-Corporate Sector

The solid line in the second panel of Figure 4 shows the path for the growth rate of credit in the calibrated model. We also show the growth rate of credit to the non-financial, non-corporate sector, normalized to be zero at the beginning of the crisis (dashed line). While there is substantial correlation between the data and the model, the simulation over predicts the speed with which credit drops in the data, and correspondingly, under predicts the persistence of the contraction.
A natural caveat regarding the behavior of credit growth in the model is that debt contracts have one period maturity, so the speed at which firms are force to deleverage is very high. In the data, debt contracts last for many periods, and they take a few periods to get processed, approved and executed. Thus, the comparison between the model that abstracts from all these complications and the data is trickier than what it appears at first sight. All in all, it can be seen that indeed, a big change occurs in 2007 Q3, the quarter the real interest rate identifies as the beginning of the crisis. In addition, the growth rate almost fully recovers by the end of our sample. Would this indicate that maybe the crisis is reverting (in the model credit to capital starts growing when the collateral parameter starts growing) and the main variables will now begin converging to its trend, putting an end to the "secular stagnation"? This model certainly implies so, with the exception of labor input, with its corresponding impact on output and capital.

Taken all together, these exercises provide evidence that the mechanism discussed in the model capture many of the relevant features of the post 2007 events, the big exception is the behavior of hours. Something else ought to explain why hours behaved as they did.

The interpretation provided by the model then, is that the credit crunch lasted at least 8 years, with some weak indication that some reversal may be taking place: The real interest rate seems to be trending upwards and credit growth, while very small, became positive at the end of 2014. With this acceptable background as a model to explain the recent events, we now use it to study the effect of policy.

4.3 Alternative Policy Responses

Was the policy respond to the financial crisis, i.e., the large increase in the supply of liquidity, instrumental in avoiding an even larger, Great Depression like recession? What would have been the consequence of an even more aggressive response, e.g., a path of policy implementing a larger inflation target? To explore these question we our calibrated model to analysis alternative policy responses.

For all the experiments we consider we proceed as before: we start the economy at the steady state and assume that in the first period, agents learn that there will be a deterministic credit crunch as calibrated in the previous section. All remaining parameters are also kept at the calibrated values. We then consider different scenarios for monetary and debt policy.

First, we illustrate the predicted evolution for the economy as a result purely of the credit crunch, in the absence of any policy response. For this case, we also consider extensions with nominal debt and sticky wages. Second, we assume that monetary and fiscal policies are such that inflation is kept low and constant at alternative targeted values that are consistent with the typical mandates of central banks. To achieve the desired target, monetary and fiscal policy must be active, and the equilibrium outcome will depend on the accompanying debt, tax, and transfer policies. Thus, we consider alternative lump-sum tax and subsidy schemes. The dynamics of the benchmark model
discussed in the previous section is a particular case of these examples, in which we interpret the observe policy response as implementing the actual path of inflation.

4.3.1 Nonresponsive Policy

We now assume that the quantity of money and bonds remain fixed all time. Note that although we focus on the case of money rules, in an equilibrium, given a money rule, we obtain a unique sequence of interest rates. One could therefore think of policies as setting those same interest rates. Since there is no change in policy, we do not need to change transfers either.

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29If one were to think of policy as setting a sequence of interest rates, the issue of price level determination should be addressed. The literature has adopted two alternative routes: the Taylor principle or the fiscal theory of the price level. We abstract from those implementation issues in this paper.

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Figure 5: Nonresponsive Policy, Alternative Nominal Frictions

In Figure 5 we present results for the benchmark model with indexed debt that features flexible wages, and for extensions of the model with nominal debt and/or sticky wages. The results for the case with indexed debt and flexible wages (blue, dotted line) are consistent with the theoretical results of the special case model discussed in Section 3.3. A credit crunch results in a large decline in the return of real assets, which is up
to 100 basis points larger than the drop in the benchmark model with the observed policy response (right panel of Figure 1). At the zero bound there is an excess demand for “store of value,” leading in equilibrium to the hoarding of real money balances by inactive entrepreneurs, in excess of the ones needed for transaction purposes. Since the supply of money is held fixed in this exercise, the price level must drop initially so that, in equilibrium, the supply of real balances meets the excess demand of real balances of inactive entrepreneurs and the return on money is the same as the return on bonds.

As discussed above and illustrated in the middle right panel, there is a large deflation on impact and positive inflation afterward, guaranteeing that agents are indifferent between holding real balances and bonds paying a low return.

In the context of the benchmark model, the deflation has little effect. But the results suggest that a potential problem may arise if debt instruments are nominal obligations or if there is downward wage rigidity as the New Keynesian models assume. We now discuss these two extensions.

**Nominal Bonds** We next consider the case in which entrepreneurs issue nominal bonds only. As before, the real value of bond issuance is restricted by the collateral constraint in (3). The results, which are substantially different, are given by the red-dashed line in Figure fig: Fixed M. The recession is deeper and more persistent, driven mainly by a sharper decline in TFP (middle, left panel). The intuition for the large negative effect of the debt deflation is simple: the initial deflation implies a large redistribution from high productivity, leveraged entrepreneurs toward bondholders, who are inactive, unproductive entrepreneurs. The ability of productive entrepreneurs to invest is now hampered by both the tightening of collateral constraints and the decline of their net worth. As a consequence, there needs to be a larger decline in the real interest rate so that in equilibrium more capital is reallocated from productive to unproductive entrepreneurs (bottom left panel), which results in a larger deflation and a nominal interest rate that remains at zero for longer (middle, right panel).

This example shows that the initial deflation can be very costly in terms of output and could provide motivation for policy interventions to stabilize the price level and output. An alternative motivation is given by the existence of nominal price rigidities.

**Sticky Wages** We consider now the model with nominal wage rigidities, which can be easily implemented following the New Keynesian tradition. In particular, we consider workers that are grouped into households with a continuum of members supplying differentiated labor inputs. Each member of the household is monopolistically competitive and gets to revise the wage in any given period with a constant probability as in Calvo (1983). A detailed and totally standard description of this extension is provided

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30 This “debt deflation” problem has been mentioned as one of the possible costs of deflations before, particularly in reference to the Great Depression (Fisher, 1933).
in Appendix C. The parametrization and calibration of this version is totally standard in the New Keynesian literature.

The solid, green line in Figure 5 shows the evolution of the economy for the case with rigid wages. As in the first example, we assume that private bonds are indexed to the price level and the supply of money and bonds remain constant. With rigid wages, the initial deflation causes an increase in the real wage (lower right panel) and a sharp decline in the labor input (top right panel). This results in a substantially more severe recession. As in the previous examples in this figure, the real interest rate becomes negative and the nominal interest rate is at the zero lower bound for various periods.\(^{31}\)

Is there an interesting interaction between these two nominal frictions, nominal debt contracts and sticky nominal wages? The dramatic dynamics illustrated by the solid lines with circle markers, which corresponds to that of an economy with nominal debt contracts and sticky wages, provides a loud answer to this question. As with discussed, in an economy with nominal debt contracts the credit crunch results in a large redistribution of wealth from debtors (productive entrepreneurs) towards creditors (unproductive entrepreneurs). This implies a lower real interest rate, and a larger initial deflation. With sticky wages, initially the real wage is larger, and therefore, there is a substantially larger drop in employment. This feedback into a lower profitability of productive entrepreneurs, and lower TFP.

The discussion in last three examples suggests that the initial deflation can be very costly in terms of output. An obvious question, then, is what can monetary and fiscal policy do, if anything, to stabilize the price level and output? As we hinted when discussing the calibration of the benchmark model, the unprecedented policy response were important to rationalize the moderate and stable inflation observed during the crisis. We next consider cases where the government implements alternative inflation targets to clarify the role of the observed policy response, and the merits of alternative policies.

### 4.3.2 Inflation Targeting

We now consider the cases in which the government implements alternative inflation targets, which for simplicity we assume constant, \(\pi^*\). We compare these cases with the benchmark economy which we calibrate to the US experience during the financial crisis. This is a particular example of the policies described in this section, where the inflation target is the path of inflation in the data.

As long as the zero bound does not bind, so \(r_t > -\frac{\pi^*}{1+\pi^*}\), inflation is determined by standard monetary policy. However, if given the target, the natural interest rate is inconsistent with the zero bound, the government needs to increase real money balances \(M_{t+1}/p_t\) and/or government bonds \(B_{t+1}\) in order to satisfy the excess demand for real assets. In order to do so, it will also need to implement a particular scheme

\(^{31}\)Note that for this parametrization, which follows the literature, the effect of sticky wages on the evolution of total hours last about two years and a half. Thus, while the size of the drop in labor is almost as in the data, it reverts way to fast in the model.
of taxes and transfers. Because of the collateral constraints, redistributions may have significant effects on the economy, so the way in which the taxes and transfers are executed matters a lot.

It is important to emphasize that once the economy is at the liquidity trap, real money and bonds are perfect substitutes and the only thing that matters is the sum of the two, so there is an indeterminacy in the composition of total outside liquidity. We will further discuss the subtle distinction between monetary and debt policy at the zero bound, but in order to focus on the effects of policy (monetary and/or fiscal) and without loss of generality, we assume that the government sets the quantity of money to be equal to the money required by individuals to finance their purchases of cash goods in every period, $m^{T}_{t+1}$, given by equation (12). Then, public debt accommodates the excess demand for bonds in periods during which the real interest rate equals the constant return of money $r_{t+1} = -\pi/(1 + \pi)$,

$$B_{t+1} = \begin{cases} 
B_t & \text{if } r_{t+1} > -\frac{\pi}{1+\pi} \\
\int_0^{z_{t+1}} \Phi_{t+1}(dz) - \frac{a_{t+1}}{1-r_{t+1}} \int_{z_{t+1}}^{\infty} \Phi_{t+1}(dz) & \text{if } r_{t+1} = -\frac{\pi}{1+\pi}.
\end{cases}$$

Obviously, lump-sum taxes (subsidies) must be adjusted accordingly to satisfy the government budget constraint in (8).

These conditions fully determine the evolution of the money supply, government bonds, and the aggregate level of taxes (transfers), but they leave unspecified how taxes (transfers) are distributed across entrepreneurs and workers. We consider two simple cases: first, as in the benchmark calibration, we consider the case in which taxes are purely lump sum for all periods. But in periods when the government increases the supply of bonds, we assume that the proceeds from the sale of bonds, net of interest payments and the adjustment of the supply of real balances, are transferred only to the entrepreneurs so $T^{W}_t = 0$ whenever $T^e_t < 0$. This case captures a scenario in which the government responds to a credit crunch by bailing out productive entrepreneurs and bondholders. We refer to this as the “bailout” case.\textsuperscript{32} The second case that we consider is one in which taxes (transfers) are purely lump sum, that is, $T^{W}_t = T^e_t$ for all $t, z$. We refer to this as the “lump-sum” case.

We now address the question, can the government mitigate the consequences of the credit crunch by choosing alternative inflation targets? In particular, is it desirable that the government chooses a sufficiently high inflation target in order to avoid the zero lower bound? We explore these questions in Figure 6. There we present the evolution of three economies differing in the level of the inflation target. The solid line correspond to the benchmark economy, where policy implements the observed path of inflation, closed to 2 percent per year. The other two curves show the evolution of two counterfactual economies where policy implements a lower and a higher inflation target,\textsuperscript{33}

\textsuperscript{32}The transfer to bondholders is consistent with the evidence presented by Veronesi and Zingales (2010) for the bailout of the financial sector in 2008.
\( \pi = 0 \) (dashed line) and \( \pi = 0.03 \) (dotted line). In all these cases, we assume that the government rebates the proceeds from the increase in the debt solely to entrepreneurs (bailout case).

To avoid the deflation induced by the excess demand of mediums to serve as a “store of value” the government must increase the supply of government bonds plus or money (lower right panel).\(^{33}\) The required increase in the supply of liquidity is a decreasing function of the desired inflation target. For instance, to implement the case with zero inflation, the government needs to engineer a path with zero real interest rates during the credit crunch. This requires an increase in the supply of liquidity that is even larger than in the benchmark case. On the contrary, a path with sufficiently high inflation, e.g., \( \pi = 0.03 \) in our calibration, can be implemented with an arbitrarily small increase in the supply of liquidity. The key difference is that when the inflation target is large enough, the economy does not enter a liquidity trap and, therefore, inflation is determined by standard monetary policy and not by the behavior of the real interest rate.

As the top left panel of Figure 6 shows, with a lower inflation target the government accomplishes a slightly less pronounced recession at the cost of significantly more pro-

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\(^{33}\)The required increase in the supply of bonds is larger than the initial excess demand for bonds, as the increase in the supply of government bonds induces a further increase in the demand of these bonds by unconstrained entrepreneurs, since these agents save in anticipation of the higher taxes that will be raised to pay the interest on this debt.
tracted recovery. The milder recession is explained by the smaller drop in TFP (top, right panel). To attain a low inflation, the real interest rate needs to be higher (less negative), and therefore there is less reallocation of capital toward less productive, and previously inactive, entrepreneurs. The counterpart of the milder drop in TFP is a collapse in investment, leading to a substantial and persistent decline in the stock of capital (bottom, left panel).

In our framework increases in government debt crowd out private investment. Constrained entrepreneurs discount future taxes at a rate higher than the interest rate (the return to their project), and therefore, they consume part of the transfers they receive. In addition, even inactive entrepreneurs increase their consumption, as the transfers they receive outweighs the present value of taxes, since workers are also taxed and not subsidized. The consumption of workers do not respond to the future taxes, as they are against their liquidity constraint after the first two periods. These effects explain why Ricardian equivalence do not hold in the model.

![Graphs of GDP, TFP, Capital Stock, and Government Debt over time](image)

**Figure 7: Alternative Inflation Targets**

The crowding out of private investment is true for both ways of designing the tax and transfer scheme, as shown in Figure 7. Note, however, that the magnitude of the crowding out is much higher when the government uses pure lump-sum taxes (solid line). In this case, part of the transfers go to workers, who in equilibrium have a large marginal propensity to consume, since they will be against their borrowing constraint in finite time.\(^{34}\) In comparison to the lump-sum case, the recovery is faster when the

\(^{34}\)In a steady state, the interest rate is strictly lower than the rate of time preferences, \((1+r_\infty)\beta < 1.\)
government rebates the proceeds from the increase in the debt solely to entrepreneurs (dashed line).\textsuperscript{35}

The case of a government implementing a low inflation rate seems an attractive approach to interpreting the Great Recession in the United States. Following the 2008 crisis, the economy has been at the zero bound for several quarters, while the Fed has substantially increased its balance sheet. The Fed policy has been directed explicitly to provide the US economy with safe zero nominal interest rate moneylike assets, while inflation has been under total control. In the same direction, the federal government has substantially increased its indebtedness (supply of real assets). All of these features are reproduced by this example. Moreover, the presumption is that these policies avoided a more severe recession, although the recovery is seen as unusually slow—again, a feature of the aggregate economy in this example.

4.3.3 Monetary or Fiscal Policy?

At the zero bound, real money and bonds are perfect substitutes. Thus, standard open market operations in which the central bank exchanges money for short-term bonds have no impact on the economy. What is needed is an effective increase in the supply of government liabilities, which at the zero bound can be money or bonds. How can these policies be executed? Clearly, one way to do it is through bonds, taxes, and transfers. But another way is through a process described long ago: helicopter drops, whereby increases of money are directly transferred to agents. Sure enough, to satisfy the government budget constraint, these helicopter drops need to be compensated with future “vacuums” (negative helicopter drops).

Although the distinction between a central bank or the Treasury making direct transfers to agents may be of varying relevance in different countries because of alternative legal constraints, there is little conceptual difference in the theory. To fully control inflation during a severe credit crunch, the sum of real money plus bonds must go up at the zero bound. Otherwise, there will be an initial deflation, followed by an inflation rate that will be determined by the negative of the real interest rate. If these policies are understood as being outside the realm of central banks, then central banks should not be given tight inflation target mandates: inflation is out of their control during a liquidity trap.

Therefore, workers, who earn a flow of labor income each period, will choose to be against their borrowing constraint in finite time.

\textsuperscript{35}See Appendix B for analytical comparative static results of the effect of changes in government debt in the neighborhood of $B_1 = 0$ for the simple example introduced in Section 3.3. There we consider the case in which all taxes and transfers are made to entrepreneurs, and the polar case in which workers are the only agents that are taxed and receive transfers.
5 Distribution of Welfare Impacts

In the previous section, we focused on the impact of policies on aggregate outcomes and factor prices. The aggregate figures suggest a relatively simple trade-off at the aggregate level. These dynamics, though, hide very disparate effects of a credit crunch and alternative policies among different agents. Although workers are hurt by the drop in wages, the profitability of active entrepreneurs and their welfare can increase as a result of lower factor prices. Similarly, unproductive entrepreneurs are bondholders in equilibrium, and therefore their welfare depends on the behavior of the real interest rate.

Figure 8: Distribution of Welfare Gains

Figure 7 presents the impact of a credit crunch on the welfare of workers and on entrepreneurs of different ability under alternative inflation targets for the bailout case. We measure the welfare impact of a credit crunch in terms of the fraction of consumption that an individual is willing to permanently forgo in order to experience a credit crunch. If positive (negative), we refer to this measure as the welfare gains (losses) from a credit crunch and alternative policy responses.

The dotted line shows the welfare gains for entrepreneurs from a policy that implements a 3% inflation rate as a function of the percentile of their ability distribution. This case is very similar to the real benchmark, since the zero bound is binding only in a couple of quarters. Unproductive entrepreneurs are clearly hurt by a credit crunch, since the return on the bonds they hold becomes negative for over 10 quarters and only gradually returns to the original steady state. Their losses amount to over 5% of permanent consumption. On the contrary, entrepreneurs who become active as the credit crunch lowers factor prices, and who increase their profitability, benefit the most. The same effect increases welfare for previously active entrepreneurs, but they are hurt by

For entrepreneurs, we consider the welfare of individuals that at the time of the shock have wealth equal to the average wealth of its type. For workers, their welfare is calculated assuming, as is true in the steady state of the model, that they own no wealth when the credit crunch is announced.
the tightening of collateral constraints, which limit their ability to leverage their high productivity. The welfare losses for workers are shown by the legend of each curve. Clearly, workers are hurt by experiencing a credit crunch, since the wages drop for a number of periods. The credit crunch amounts to a permanent drop of over half a percentage point in their consumption, $w^W_g = -0.005$.

The other two curves show the welfare consequences of lower inflation targets. The lower the inflation target, the higher the real interest rate, both during the credit crunch and in the new steady state. Unproductive entrepreneurs benefit from the highest interest rate. Similarly, productive entrepreneurs benefit from the lowest wages associated with the lowest capital during the transition and in the new steady state. Although individual entrepreneurs do not internalize it, collectively they benefit from the lower wages associated with a lower aggregate stock of capital. The lower the inflation target, the lower the capital stock and the lower the wages, so the welfare of workers goes down when the target goes down.

6 Conclusions

A contraction in credit due to a tightening of collateral constraints leads to a recession and a drop in the return of safe assets. In a monetary economy, the nominal return on safe assets cannot be negative, so the negative of the rate of inflation is a lower bound on its real return. We showed that if the contraction in credit is large enough, then this constraint becomes binding and the economy enters a liquidity trap. In this case, a deflation occurs if policy is passive. This deflation may interact with collateral constraints, creating debt deflation and worsening the recession if debt obligations are in nominal terms. In addition, it creates a large drop in employment if wages are sticky.

We characterize a policy that avoids that costly deflation. That policy resembles the one followed by the Federal Reserve as a reaction to the 2008 crisis and is in line with Friedman and Schwartz’s explanation of the severity of the Great Depression.

The policies that avoid the deflation involve a large increase in money or bonds, which are perfect substitutes at the zero bound. These policies do stabilize prices and output. There is a side effect of these policies, though: they generate slow recovery. We argue that many of the features of the model capture the characteristics of the last

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37 Given the debt policy equation (??), the government debt in the new steady state will be higher the lower the inflation target is. In the model, a higher level of government debt implies a lower level of capital in the new steady state.

38 The nonmonotonic nature of the welfare effects is related to the heterogeneous impact due to the changing nature of the occupational choice of agents during the transition. For example, the entrepreneur that benefits the most is the most productive inactive entrepreneur in the steady state. As the real rate goes down, that agent becomes an entrepreneur and starts borrowing to profit from the difference between his productivity and the now low interest rate and also from the lower equilibrium wage. On the other hand, the most productive entrepreneur also benefits from the low input prices, but is hurt by the reduction in ability to borrow. Thus, although she gets a higher margin per unit of capital, she can only manage a lower amount of capital.
financial crisis that hit the United States starting in 2008 and the one that hit Japan in the early 1990s.

The interpretation of the crisis provided by the model in this paper is in contrast to the dominant view in most central banks and is supported by a literature that emphasizes price frictions. According to that literature, it is unambiguously optimal to maintain the economy at the zero bound even after the shock that drove real interest rates to negative values reverts. The model of this paper implies that avoiding the zero bound or not implies nontrivial trade-offs: ameliorating the drop in output at the cost of a slower recovery. The policy trade-offs are even more subtle when the heterogeneous effects across agents are taken into account.

Our model rationalizes the notion that the inflation determination mechanisms differ substantially when the policy authority decides to be at the zero bound. Away from the zero bound, it depends on standard monetary mechanisms. But at the zero bound, it is total outside liabilities that matter: inflation can be controlled only by managing the real interest rate so it does not become too negative.

A Additional Proofs

Proof of Lemma 1: Use the solutions for the real interest rate and capital (25) and (24) to write

\[ 1 + i_t = \left[ \alpha \theta_{ss} \left( \frac{1 + \theta_{ss}}{2} K_t \right)^{\alpha-1} + (1 - \delta) \right] \frac{K_t}{K_{t+1}} = \frac{\left[ \alpha \theta_{ss} \left( \frac{1 + \theta_{ss}}{2} \right)^{\alpha-1} K_t^\alpha + (1 - \delta) K_t \right]}{\beta \left[ \alpha \left( \frac{1 + \theta_{ss}}{2} \right)^\alpha K_t^\alpha + (1 - \delta) K_t \right]} \].

Assume now, toward a contradiction, that

\[ 1 + i_t = \left[ \alpha \theta_{ss} \left( \frac{1 + \theta_{ss}}{2} \right)^{\alpha-1} K_t^\alpha + (1 - \delta) K_t \right] \frac{\alpha \theta_{ss} \left( \frac{1 + \theta_{ss}}{2} \right)^{\alpha-1} K_t^\alpha + (1 - \delta) K_t}{\beta \left[ \alpha \left( \frac{1 + \theta_{ss}}{2} \right)^\alpha K_t^\alpha + (1 - \delta) K_t \right]} < 1. \]

Then,

\[ \alpha \theta_{ss} \left( \frac{1 + \theta_{ss}}{2} \right)^{\alpha-1} K_t^\alpha + (1 - \delta) K_t < \beta \left[ \alpha \left( \frac{1 + \theta_{ss}}{2} \right)^\alpha K_t^\alpha + (1 - \delta) K_t \right], \]

which can be written as

\[ \alpha \left( \frac{1 + \theta_{ss}}{2} \right)^\alpha K_t^\alpha \left( \frac{2 \theta_{ss}}{1 + \theta_{ss}} - \beta \right) + (1 - \delta) K_t (1 - \beta) < 0. \]

Assumption (30) implies that the first term on the left-hand side is positive. As \( \delta \) and \( \beta \in (0, 1) \), this is a contradiction. \( \square \)
Proof of Lemma 2: Assume, toward a contradiction, that \( i_1 > 0 \). Then

\[
\bar{M} = (1 - \beta) K_1 p_0
\]

so

\[
\frac{p_1}{p_0} = \frac{K_1}{K_2},
\]

and the solution for the nominal interest rate is given by

\[
1 + i_1 = \frac{(1 + r_1) p_1}{p_0} = \left[ \frac{2 \theta t}{(1 + \theta t)} \left( \frac{1 + \theta t}{1 + \theta ss} \right)^\alpha (\rho + \delta) + (1 - \delta) \right] \frac{K_1}{K_2},
\]

but

\[
\frac{K_1}{K_2} = \frac{\alpha \left( \frac{1 + \theta ss}{2} \right)^\alpha K_{ss}^\alpha + (1 - \delta) K_{ss}}{\alpha \left( \frac{1 + \theta ss}{2} \right)^\alpha K_{ss}^\alpha + (1 - \delta) K_{ss}^{1 - \alpha}} = \frac{\alpha \left( \frac{1 + \theta ss}{2} \right)^\alpha + (1 - \delta) K_{ss}^{1 - \alpha}}{\alpha \left( \frac{1 + \theta ss}{2} \right)^\alpha + (1 - \delta) K_{ss}^{1 - \alpha}}.
\]

Replacing the solution for \( K_{ss} \), we obtain

\[
\frac{K_1}{K_2} = \frac{\alpha \left( \frac{1 + \theta ss}{2} \right)^\alpha + (1 - \delta) \frac{\alpha}{\beta - 1 + \delta} \left( \frac{1 + \theta ss}{2} \right)^\alpha}{\alpha \left( \frac{1 + \theta ss}{2} \right)^\alpha + (1 - \delta) \frac{\alpha}{\beta - 1 + \delta} \left( \frac{1 + \theta ss}{2} \right)^\alpha} = \frac{1}{\beta} \frac{\left( \frac{1 + \theta ss}{1 + \theta ss} \right)^\alpha (\rho + \delta) + (1 - \delta)}{\left( \frac{1 + \theta ss}{1 + \theta ss} \right)^\alpha (\rho + \delta) + (1 - \delta)}.
\]

Then

\[
1 + i_1 = \frac{1}{\beta} \frac{2 \theta t}{\left( \frac{1 + \theta ss}{1 + \theta ss} \right)^\alpha (\rho + \delta) + (1 - \delta)}.
\]

Thus, for the lower bound on the nominal interest rate to be binding, we need

\[
\frac{1}{\beta} \frac{2 \theta t}{\left( \frac{1 + \theta ss}{1 + \theta ss} \right)^\alpha (\rho + \delta) + (1 - \delta)} < 1,
\]

which implies that

\[
\frac{2 \theta t}{(1 + \theta t)} \left( \frac{1 + \theta ss}{1 + \theta ss} \right)^\alpha (\rho + \delta) + (1 - \delta) < \beta \left( \frac{1 + \theta ss}{1 + \theta ss} \right)^\alpha (\rho + \delta) + \beta (1 - \delta)
\]

or

\[
(1 - \delta)(1 - \beta) < \left( \frac{1 + \theta ss}{1 + \theta ss} \right)^\alpha (\rho + \delta) \left[ \beta - \frac{2 \theta t}{(1 + \theta t)} \right].
\]
We now briefly characterize the function

\[ f(\theta_l) = \left( \frac{1 + \theta_l}{1 + \theta_{ss}} \right)^\alpha (\rho + \delta) \left[ \beta - \frac{2\theta_l}{(1 + \theta_l)} \right]. \]

Equation (30) implies that \( f(\theta_{ss}) = 0 \). On the other hand,

\[ f(0) = \left( \frac{1}{1 + \theta_{ss}} \right)^\alpha \frac{(\rho + \delta)}{1 + \rho}. \]

As \( \delta > \rho \),

\[ \frac{\delta}{1 - \delta} > (1 - \beta) = \frac{\rho}{1 + \rho} \]

so

\[ \theta_{ss} < 1 < \frac{\delta}{1 - \delta} \frac{1 + \rho}{\rho} \]

and

\[ 1 + \theta_{ss} < \frac{\delta + \rho}{(1 - \delta) \rho}. \]

Thus,

\[ (1 + \theta_{ss})^\alpha < 1 + \theta_{ss} < \frac{\delta + \rho}{(1 - \delta) \rho} \]

and

\[ (1 + \theta_{ss})^\alpha (1 - \delta) \rho < \delta + \rho \]

or

\[ \frac{\rho}{1 + \rho} (1 - \delta) = (1 - \beta) (1 - \delta) < \left( \frac{1}{1 + \theta_{ss}} \right)^\alpha \frac{\delta + \rho}{1 + \rho}. \]

Thus, by the intermediate value theorem, there exists a \( \bar{\theta}_l \in (0, \theta_{ss}) \) such that

\[ (1 - \delta)(1 - \beta) = \left( \frac{1 + \bar{\theta}_l}{1 + \theta_{ss}} \right)^\alpha (\rho + \delta) \left[ \beta - \frac{2\bar{\theta}_l}{(1 + \bar{\theta}_l)} \right]. \]

Since \( f(\theta_l) \) is decreasing, the zero bound will bind for all \( \theta_l \in (0, \bar{\theta}_l] \). □
Proof of Lemma 3: The ratio of the price level at \( t = 0 \) to the price level in the steady state \( p_{ss} \) is given by

\[
\frac{p_0}{p_{ss}} = (1 + r_1) \frac{p_1}{p_{ss}} = \frac{1}{\beta} \left( \frac{1 + \theta_1}{1 + \theta_{ss}} \right)^\alpha \left( \rho + \delta \right) + (1 - \delta)
\]

which is equal to the right-hand side of (32) and, therefore, it is strictly less than one provided \( \theta_l \in (0, \tilde{\theta}_l) \). □

B The Effect of Public Debt Around \( B = 0 \)

In this appendix we characterize the effect of public debt on GDP for two limiting cases. First, we consider the example presented in Section 3.3.2, where only entrepreneurs pay taxes and receive subsidies associated with the temporary one-period increase in government debt. For this case, we show that GDP tends to be an increasing function of the level of public debt in the neighborhood of \( B = 0 \). Second, we consider the polar case in which only workers pay taxes and receive subsidies associated with the temporary one-period increase in government debt. In this case, we show that GDP is a decreasing function of the level of public debt in the neighborhood of \( B = 0 \). These examples illustrate that the net effect of government debt on aggregate output depends on the particular implementation of the debt policy and on the relative size of workers and entrepreneurs in the population.

B.1 Taxing/Subsidizing Only Entrepreneurs

Differentiating (26) around \( B_1 = 0 \),

\[
\frac{\partial K_1}{\partial B_1} \bigg|_{B_1 = 0} = -(1 - \beta) \left[ 1 - \int \frac{(1 + r_{ss})}{R_{ss}(z)} dz \right]. \tag{33}
\]

Similarly, differentiating (22) around \( B_1 = 0 \),

\[
\frac{\partial Z_1}{\partial B_1} \bigg|_{B_1 = 0} = \alpha Z_{ss} K_{ss}^{-1} \frac{1 - \theta}{1 + \theta}. \tag{34}
\]
Thus, the net effect on GDP around \( B_1 = 0 \) is as follows:

\[
\left. \frac{\partial Y_1}{\partial B_1} \right|_{B_1=0} = \alpha Z_{ss} K_{ss}^{\alpha - 1} \left[ \frac{1 - \theta}{1 + \theta} - \alpha Z_{ss} K_{ss}^{\alpha - 1} (1 - \beta) \left[ 1 - \int \frac{(1 + r_{ss})}{R_{ss}(z)} \, dz \right] \right].
\]

Finally, using the expressions for \( R_1(z) \) and solving the integral, we have

\[
\left. \frac{\partial Y_1}{\partial B_1} \right|_{B_1=0} = \alpha Z_{ss} K_{ss}^{\alpha - 1} \left[ \frac{1 - \theta}{1 + \theta} - (1 - \beta) \left[ 1 - \int \frac{(1 + r_{ss})}{R_{ss}(z)} \, dz \right] \right],
\]

where around \( B_1 = 0 \) the real interest rate \( r_{ss} = (\rho + \delta)2\theta/(1 + \theta) - \delta \). It is straightforward to show that this expression is positive for \( \beta \) close to 1 or \( \theta \) close to 0.

### B.2 Taxing/Subsidizing Only Workers

In this case,

\[
\left. \frac{\partial K_1}{\partial B_1} \right|_{B_1=0} = -1,
\]

and the effect on TFP is also given by (34). Thus,

\[
\left. \frac{\partial Y_1}{\partial B_1} \right|_{B_1=0} = -\alpha Z_{ss} K_{ss}^{\alpha - 1} \frac{2\theta}{1 + \theta} < 0.
\]

### C Environment with Sticky Wages

In this appendix we describe the extension with rigid wages that is solved in Section 4.3.1. In order to allow for sticky wages, we now consider the case in which workers are grouped into households with a continuum of members indexed by \( h \in [0, 1] \), each supplying a differentiated labor input \( l_{ht} \). Each member is endowed with a unit of time. Preferences of the household are described by

\[
\sum_{t=0}^{\infty} \beta^t \left[ \zeta \nu \log c_{1t}^W + \zeta (1 - \nu) \log c_{2t}^W + (1 - \zeta) \log (N_t) \right],
\]

where leisure is

\[
N_t = 1 - \int_0^1 l_{ht} \, dh.
\]
The differentiated labor varieties aggregate up to the labor input $L_t$, used in production by individual entrepreneurs, according to the Dixit-Stiglitz aggregator

$$L_t = \left[ \int_0^1 l_{ht} \frac{w_{ht}}{w_t} \, dh \right]^{\frac{\eta}{\eta - 1}}, \eta > 1. \tag{37}$$

Each member of the household, which supplies a differentiated labor variety, behaves under monopolistic competition. They set wages as in Calvo (1983), with the probability of being able to revise the wage $1 - \alpha^w$. This lottery is also i.i.d. across workers and over time. The workers that are not able to set wages in period 0 all share the same wage $w_{-1}$. Other prices are taken as given.

There is a representative firm that produces homogeneous labor to be used in production by the entrepreneurs using the production function (37). The representative firm minimizes $\int_0^1 w_{ht} l_{ht} \, dh$, where $w_{ht}$ is the wage of the $h$-labor, for a given aggregate $L_t$, subject to (37). The demand for $n_{ht}$ is

$$l_{ht} = \left( \frac{w_{ht}}{w_t} \right)^{-\eta} L_t, \tag{38}$$

where $W_t$ is the aggregate wage level, given by

$$w_t = \left[ \int_0^1 w_{ht}^{1-\eta} \, dh \right]^{\frac{1}{1-\eta}}. \tag{39}$$

It follows that $\int_0^1 w_{ht} n_{ht} \, dh = w_t L_t$. In order to simplify the analysis, we also assume that workers are hand to mouth. In this case, the representative worker maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \zeta \nu \log c_{1t}^W + \zeta (1 - \nu) \log c_{2t}^W + (1 - \zeta) \log (N_t) \right]$$

subject to

$$c_{1t}^W + c_{2t}^W + \frac{m_{t+1}^W}{p_t} = \frac{1}{p_t} \int_0^1 w_{ht} l_{ht} \, dh + \frac{m_t^W}{p_t} - T_t^W,$$

$$l_{ht} = \left( \frac{w_{ht}}{w_t} \right)^{-\eta} L_t,$$

and

$$c_{1t}^W \leq \frac{m_t^W}{p_t}.$$ 

Note that although consumption and total labor will not be stochastic, each particular
$w_{ht}$ will be a random variable. From the first-order conditions of representative workers, we obtain

$$w_{ht} = \tilde{w}_t = \frac{\eta}{\eta - 1} \sum_{j=0}^{\infty} \xi_{t+j} \frac{1 - \zeta}{\zeta(1 - \nu)} \frac{p_{t+j}CW_{t+j}}{N_{t+j}},$$

where

$$\xi_{t+j} = \frac{(\beta w)^j \frac{\zeta(1-\nu)}{\zeta} \frac{1}{p_{t+j}} w_{t+j}^\eta L_{t+j}}{\sum_{j=0}^{\infty} (\alpha w \beta)^j \frac{\zeta(1-\nu)}{\zeta} \frac{1}{p_{t+j}} w_{t+j}^\eta L_{t+j}}$$

and

$$\sum_{j=0}^{\infty} \xi_{t+j} = 1.$$

The evolution of the cost of a composite unit of labor is

$$w_t = \left( (1 - \alpha w) \tilde{w}_t^{1-\theta_w} + \alpha w w_{t-1}^{1-\theta_w} \right)^{\frac{1}{1-\theta_w}},$$

and

$$L_t = \left[ \alpha w \left( \frac{w_{t-1}}{w_t} \right)^{-\theta_w} + (1 - \alpha w) \left( \frac{\tilde{w}_t}{w_t} \right)^{-\theta_w} \right]^{-1} (1 - N_t)$$

solves for the aggregate composite labor input given aggregate leisure.

To implement this extension, we follow Correia et al. (2013) and calibrate $\zeta = 0.3$, $\eta = 3$, and $\alpha_w = 0.85$. To simplify the calculations, we consider the cashless limit. The other parameter values are set as in the other numerical examples.
References


