Debt and the U.S. Economy*

Kaiji Chen† Ayşe İmrohoroğlu‡‡

This Version: April 2012

Abstract

Publicly held debt to GDP ratio in the U.S. has reached 68% in 2011 and is expected to continue rising. Many proposals regarding the ways to curb the government deficit and the resulting debt are being discussed. In this paper, we use the standard neoclassical growth model to examine the implications of some of these policy proposals on the future path of the budget deficit and debt in the U.S. economy. The experiments we conduct reveal how difficult it will be for the U.S. debt to GNP ratio to return back to its average level of 40%. In fact, unless there is a significant increase in the growth rate of GNP, debt to GNP ratios above 100% are more likely to continue into the future.

†Department of Economics, Emory University, Atlanta, GA 30322-2240
‡‡Department of Finance and Business Economics, Marshall School of Business, University of Southern California, Los Angeles, CA 90089-1427

*We would like to thank the seminar participants at the Federal Reserve Bank of Cleveland, Ohio State University, University of Quebec at Montreal, and Utah State University for helpful comments.
1 Introduction

Publicly held debt to GDP ratio in the U.S. has risen from 36% in 2007 to 68% in 2011.\footnote{Historical Tables, Office of Management and Budget, the White House. Table 7.1: Federal Debt at the End of Year: 1940-2017.} Figure 1 summarizes how the current levels of debt to GDP ratio are significantly higher than what the U.S. has experienced since World War II. In addition, projections by the Congressional Budget Office (CBO) indicate significant increases in this ratio, ranging between 80% to 187% in 2035 under their two different scenarios. This fiscal outlook is generating significant academic, public, and political debate in the U.S.

Currently, many proposals regarding how to curb the government deficit and the resulting debt are being discussed. In this paper, we use the standard growth model to study the effects of some of these proposals on the future path of the U.S. economy. The setup is an infinite horizon, complete markets framework that has been successfully used to address a variety of economic issues.\footnote{Our model is similar to the ones used in Hayashi and Prescott (2002), and Chen, Imrohoroglu, and Imrohoroglu (2006 and 2009).} We calibrate the economy to the U.S. data for the 1960-2010 period and show that it is able to capture the long run movements in many of the key variables we are interested in. In particular, deficit and debt ratios relative to GNP generated by the model mimic the data during this period reasonably well. We use this model to understand the impact of several fiscal policy actions on the future path of employment, output, deficits, and debt and compare our results with the projections provided by the CBO.

Our benchmark results indicate a debt to GNP ratio of 131% by 2030. Many things
affect this ratio and are easily investigated in this framework. Most of the experiments we conduct, however, reveal how difficult it will be for the U.S. debt to GNP ratio to stabilize around 80% by 2030 as forecasted in CBO’s extended baseline scenario let alone go back to its historical average level of 40%. In fact, unless there is a significant increase in the growth rate of GNP, debt to GNP ratios above 100% are more likely to continue into the future. Our results also highlight the importance of incorporating behavioral responses into calculations regarding deficits and debt especially when large changes in tax rates are being evaluated as in the CBO calculations. We find that debt to GDP ratios in 2030 can easily be 17% to 29% higher than what is projected by the CBO when labor supply responses are taken into account.

The main reason for the recent increases in the debt to GDP ratio was the 2007-2009 recession. Real GDP declined by 0.3% in 2008 and 3.5% in 2009. To combat the great recession, the U.S. enacted an $800 billion stimulus package in 2009. The increase in expenditures, together with the decline in revenues, resulted in a significant increase in the government deficit that reached 9% of GDP in 2010 and 8.7% of GDP in 2011. These deficits, coupled with projected increases in future health care spending, are resulting in large projected debt levels in the U.S. Intense negotiations took place recently as the U.S. debt reached its ceiling in May 2011. These negotiations involved various options to cut expenditures and raise taxes.

In this paper, we examine the implications of different expenditure projections on the future debt to GDP ratio in the U.S. First set of expenditure projections are provided by the CBO. We use their extended baseline scenario as a starting point for our analysis. Next, we examine the proposal by the National Commission on Fiscal Responsibility and Reform (Bowles-Simpson in short) that involves cuts in expenditures beyond what the CBO projections consider. There are vast differences across the projected debt to GDP ratios between the alternative projections provided by the CBO as well as the projections provided by the Bowles-Simpson commission. Under the extended baseline scenario of the CBO, publicly held debt to GDP ratio stabilizes around 80% while under their alternative scenario it reaches 187% by 2035. A major reason for this difference is the assumption about tax revenues. In the extended baseline scenario, marginal tax rates on labor income are assumed to increase from 29% in 2010 to 35% in 2025 and to 38% in 2035, leading to significant increases in tax revenues. In the second scenario tax revenues as a percentage of GDP remain close to their historical levels. The Bowles-Simpson Commission proposes larger cuts in government spending than the CBO projections. They also propose elimination of many tax deductions coupled with lower marginal tax rates. Their calculations result in a debt to

---

3The National Commission on Fiscal Responsibility and Reform was created by President Obama in 2010 to identify policies to improve the fiscal situation in the U.S. Alan Simpson and Erskine Bowles were the co-chairs of the commission.
GDP ratio of 65.5% in 2020.

None of the above projections take into account the potential distortions or benefits that may stem from changes in tax rates. In this paper, we examine the consequences of these policies, especially the one under the extended baseline scenario in detail in a fully calibrated general equilibrium model. This framework allows us to model the reactions of labor and capital due to changes in policy that impact the projected debt to GDP ratios. Our findings indicate that depending on the elasticity of labor supply, debt to GDP ratio can be 17% to 29% higher than what is projected by the extended baseline scenario of the CBO.

There are many important issues regarding deficits and debt that we leave for future work. We do not have a theory of debt and do not model the rate of return dominance. We take the interest rate on government debt exogenously and can not address potential changes in interest rates following extended deficits.\footnote{See for example Evans, Kotlikoff, and Phillips (2012) and Reinhart and Rogoff (2009).}

In Section 2, we summarize the model economy. Calibration is presented in Section 3 and the results in Section 4. Section 5 concludes.

2 The Standard Neoclassical Growth Model

In this model, a representative household makes consumption and saving decisions taking the factor prices and government policy as given. A stand-in firm maximizes its profits, setting factor prices equal to their marginal productivities. There is a government that finances exogenously given government purchases and transfer payments by taxing factor incomes and consumption, or by issuing new one-period bonds at an exogenously given interest rate. The engine of growth in the model is exogenously growing TFP. Agents in this perfect foresight environment maximize their objective functions, taking into account future policy and prices.\footnote{Chen, İmrohoğlu, and İmrohoğlu (2007) and Braun, Ikeda, and Joines (2009) develop overlapping generations models with incomplete markets to study the Japanese economy. By construction, these models deliver richer implications by disaggregating the economy into cohorts and different income and wealth groups. However, their aggregate predictions on the main macro variables seem to be consistent with those from the standard model with infinite horizon and complete markets.}

Below, we present our model in detail.

2.1 Household’s Problem

Time is discrete, starting from period 0. There is a representative household with \( N_t \) working-age members at date \( t \), facing the following problem in a complete markets environment:

\[
\max_{\mathbf{c_t}} \sum_{t=0}^{\infty} \beta^t N_t [\log c_t + \alpha \log (1 - h_t)]
\]
subject to

\[ C_t + K_{t+1} \leq [1 + (1 - \tau_{k,t})(r_t - \delta_t)] K_t + (1 - \tau_{h,t}) w_t H_t + TR_t + N_t \pi^p_t, \]

where \( c_t = C_t / N_t \) is consumption per household member, \( h_t = H_t / N_t \) is the fraction of hours worked per member of the household, \( \beta \) is the subjective discount factor, \( \alpha \) is a parameter that indicates the relative weight of leisure in the utility functions. \( H_t \) is total hours worked by all working-age members of the household, \( \tau_{h,t} \) and \( \tau_{k,t} \) are tax rates on labor and capital income, respectively, \( w_t \) is the real wage, \( TR_t \) is aggregate government transfers, \( \pi^p_t \) is the per-member primary balance of the government, \( r_t \) is the rental rate of capital, and \( \delta_t \) is the time-\( t \) depreciation rate.\(^7\) Beginning of period \( t \) assets are denoted by \( K_t \). Population growth is given by the change in the size of the household, which evolves over time exogenously at the rate \( n_t = N_t / N_{t-1} \). We assume that the representative household receives the interest earnings on the government debt \( I_k \).

It is well known that the neoclassical growth model has difficulty accounting for the hours boom in the 1990s. To accommodate the model to generate more reasonable results, we feed in a labor wedge as in Chari, Kehoe, and McGrattan (2007) or Ohanian, Raffo, and Rogerson (2008).

The labor wedge, \( \Delta^H_t \), is defined to satisfy the following equation when consumption, \( c_t \), hours worked, \( h_t \), and output, \( y_t \), are all from the data:

\[
\frac{-u_{h_t}(c_t, 1 - h_t)}{u_{c_t}(c_t, 1 - h_t)} = \Delta^H_t (1 - \tau_{h,t}) MPL_t
\]

where \( u_x (x = c_t \text{ or } h_t) \) is the partial derivative of period utility to \( x \), and \( MPL_t \) denotes the marginal product of labor. After computing the labor wedge, we replace \( (1 - \tau_{h,t}) \) in the first order condition for the consumption-leisure trade-off with \( (1 - \tau_{h,t}) \Delta^H_t \). Since we already have taxes in this model, the labor wedge is interpreted as a proxy for changes in labor distortions other than taxes. Other interpretations of the changes in the labor wedge may be related to factors that caused increases in the labor force participation of women or the intangible capital explanation advanced by McGrattan and Prescott (2010) or the change in wage markups argued by Smets and Wouters (2007).

### 2.2 Firm’s Problem

There is a stand-in firm with access to a constant returns to scale Cobb-Douglas production function given by:

\[ Y_t = A_t K_t^\theta H_t^{1-\theta}, \]

\(^6\)Lower case letters refer to per-capita items and upper case letters denote economy-wide aggregate quantities.
where $\theta$ is the income share of capital and $A_t$ is total factor productivity, which grows exogenously at the rate $\gamma_t = A_t/A_{t-1}$. Aggregate capital stock follows the law of motion

$$K_{t+1} = (1 - \delta_t)K_t + X_t,$$

where $X_t$ is gross investment at period $t$.

The representative firm maximizes its profits by choosing capital and labor, taking factor prices as given. This produces the usual equilibrium conditions that equate factor prices to their marginal productivities.

### 2.3 Government Budget

The government faces exogenously given streams of government purchases $G_t$, transfer payments $TR_t$, and interest payments to holders of its debt $I_t$. These can be financed by taxing consumption, income from labor, and capital, or by raising new debt. In this paper, we do not explicitly model government debt. Modeling debt requires a way of introducing rate-of-return dominance of private capital over government debt as we observe in the data which is beyond the scope of this paper. Instead, we focus on the additions to existing debt by carefully modeling the government’s flow budget constraint. Denoting the (per-capita) budget balance by $\pi_t^b$ and the primary balance by $\pi_t^p$, we specify the government budgets as follows:

$$G_t + TR_t + I_t = \tau_{h,t}w_tH_t + \tau_{k,t}(r_t - \delta_t)K_t - N_t\pi_t^b,$$

$$G_t + TR_t = \tau_{h,t}w_tH_t + \tau_{k,t}(r_t - \delta_t)K_t - N_t\pi_t^p.$$  

#### 2.3.1 Constructing Debt-Output Ratio in the Model

Government debt is constructed to be consistent with the NIPA accounts. In nominal terms, the evolution of debt follows:

$$\tilde{B}_t^{\theta} = \tilde{B}_t^{\theta} + GB_t,$$

where $GB_t$ is net borrowing. We describe the construction of $GB_t$ in Section 3.

All variables in our model are real and detrended, so we need to deflate and detrend the above law of motion for government debt. In real terms, equation 4 can be written as:

$$B_t^{\theta} = (\tilde{B}_t^{\theta} + GB_t) P_t/P_{t+1}$$

where $P_t/P_{t+1}$ is the inverse of the inflation rate. For every nominal variable $\tilde{x}_t$ ($\tilde{x}_t = \tilde{B}_t^{\theta}$ or $\tilde{GB}_t$), we let $x_t$ denote the corresponding real variable, which is given as $\tilde{x}_t/P_t$. 


Accordingly, law of motion of the debt to output ratio follows:

$$\frac{B_{t+1}^p}{Y_{t+1}} = \frac{(B_{t}^p + G_{t})}{Y_t} \frac{P_t}{P_{t+1}} \frac{Y_t}{Y_{t+1}}. \quad (5)$$

The interest rate on government debt, denoted as $i_t$, is computed as $I_t = B_t^p$. In this paper, we take the interest rate on government debt exogenously. As equation 5 implies, the relevant interest rate to compute debt is the nominal interest rate, once we explicitly take the inflation into account.

It should be emphasized that we do not have a theory as to household’s holding of government debt. There is no consensus in the literature on the optimal size of government debt primarily because there is no agreement on a theory of debt. For this reason, we concentrate on the effects of its financing on the economy as well as the effects of fiscal policy on the size of debt. In this sense, debt is endogenous in our model as we determine its level by accumulating budget deficits that are endogenously determined by the interaction of demographics, policy, and private sector behavior. Note that the projected increases in $G_t$ and $R_t$ will proxy for the impact of the demographic transition in U.S. Government’s fiscal policy will be represented with the assumed paths of the expenditure items and the tax rates. Finally, the private sector will optimally respond to changes in this environment by adjusting its consumption-saving behavior, and the general equilibrium effects will show up as the wage rate and rate of return to capital adjust accordingly.

### 2.4 Competitive Equilibrium

For a government fiscal policy $\{G_t, TR_t, i_t, \tau_h, \tau_k,t\}_{t=0}^{\infty}$, a competitive equilibrium consists of an allocation $\{C_t, X_t, H_t, K_{t+1}, Y_t\}_{t=0}^{\infty}$, a budget balance $\pi_t$, a primary balance $\pi^p_t$, and factor prices $\{w_t, r_t\}_{t=0}^{\infty}$ such that:

- the allocation solves household’s problem,
- the allocation solves the firm’s profit maximization problem with factor prices given by: $w_t = (1 - \theta)A_tK_t^\theta H_t^{-\theta}$ and $r_t = \theta A_t K_t^{\theta-1} H_t^{1-\theta}$,
- the government budget is satisfied,
- the goods market clears: $C_t + X_t + G_t = Y_t$. 


2.5 Equilibrium Conditions

We can combine the equilibrium conditions of the model in three equations below:

\[ \frac{\alpha C_t/N_t}{1-H_t/N_t} = (1 - \tau_{h,t}) (1 - \theta) A_t K_t^\theta H_t^{1-\theta} \]  

\[ \frac{C_{t+1}}{N_{t+1}} = \frac{C_t}{N_t} \beta \left\{ 1 + \frac{1}{1-t_{k,t+1}} \right\} \left[ \theta A_{t+1} K_t^\theta H_{t+1}^{1-\theta} - \delta_{t+1} \right] \]  

\[ K_{t+1} = (1 - \delta_t) K_t + A_t K_t^\theta H_t^{1-\theta} - C_t - G_t. \]

Our approach is to start from given initial conditions and then compute an equilibrium transition path towards a balanced growth path at which per capita aggregate variables grow at the rate \( g_t = \gamma_t / (1-\theta) \). For a variable \( z_t \), detrending is done by applying \( \tilde{z}_t = z_t / \left[ A_t^{1-\theta} N_t \right] \). Using this change of variables to equations (6), (7), and (8), we obtain equations:

\[ \frac{\alpha \tilde{c}_t}{1-h_t} = (1 - \tau_{h,t}) (1 - \theta) x_t^\theta \]

\[ \tilde{c}_{t+1} = \frac{\tilde{c}_t}{g_{t+1}} \beta \left\{ 1 + (1 - \tau_{k,t+1}) \left[ \theta x_{t+1}^\theta - \delta_{t+1} \right] \right\} \]

\[ \tilde{k}_{t+1} = \frac{1}{g_{t+1} n_{t+1}} \left[ (1 - \delta_t) + (1 - \psi_t) x_t^\theta \right] \tilde{k}_t - \tilde{c}_t, \]

where \( \psi_t \) is the ratio of government purchases to output, \( G_t/Y_t \), and \( x_t \) is detrended capital-labor ratio, \((K_t/H_t)/A_t^{1-\theta}\).

The steady-state conditions are obtained by setting \( \tilde{z}_t = \tilde{z} \) for all \( t \):

\[ \frac{\alpha \tilde{c}}{1-h} = (1 - \tau_h) (1 - \theta) x^\theta \]

\[ 1 = \frac{1}{g} \beta \left\{ 1 + (1 - \tau_k) \left[ \theta x^\theta - \delta \right] \right\} \]

\[ \tilde{k} = \frac{1}{g n} \left[ (1 - \delta) + (1 - \psi) x^\theta \right] \tilde{k} - \tilde{c}. \]

These two equations deliver the steady-state values of detrended capital and consumption where \( \tilde{\delta}, \tilde{\tau}_h, \) and \( \tilde{\tau}_k \) are the steady-state depreciation rate, labor income tax rate and capital income tax rate, respectively.

In our model, we can compute the labor wedge as:

\[ \Delta^H_t = \frac{\alpha c_t h_t}{(1-h_t)(1-\tau_{h,t})(1-\theta) y_t}. \]
Finally, the detrended law of motion for debt-output ratio is:

\[ \frac{\tilde{B}_{t+1}^g}{\tilde{Y}_{t+1}} = \left( \frac{\tilde{D}_t^g + \tilde{G}B_t}{\tilde{Y}_t} \right) \frac{\tilde{Y}_t A_t^{\frac{\delta}{\beta}} N_t}{\tilde{Y}_{t+1} A_{t+1}^{\frac{\delta}{\beta}} N_{t+1}} P_t. \]

3 Measurement and Calibration

In order for us to make predictions about the fiscal position of the U.S. in the near future, we want our model economy to generate aggregate behavior and fiscal outcomes that resemble their counterparts in the U.S. economy. First, we make adjustments to observed macroeconomic aggregates so that data accounts are in line with our model accounts. Second, we make adjustments to government accounts and bring them closer to what a government does in the standard growth model. Third, we calibrate our model economy to generate certain targets from the U.S. economy. Below, we will describe these calibration issues.

3.1 Adjustment to National Account

We use data from the 2011 revision of National Income and Product Accounts (NIPA) and Fixed Asset Tables (FAT) of Bureau of Economic Analysis (BEA) for the years 1960-2010. Our adjustments to measured macroeconomic aggregates follow Cooley and Prescott (1995). We define capital \( K \) as the sum of the fixed assets, stock of consumer durables, inventory stock, and net foreign assets. Output \( Y \) corresponds to \( GNP \) plus the service flows from government capital and consumer durable, and capital depreciation is the sum of consumption of fixed capital and depreciation of consumer durables. Thus for the rest of the analysis, we use \( GNP \) as our measure of output.

3.2 Adjustment to Government Debt

This subsection describes how the General Government Accounts are arranged so that the government accounts in the data are in line with those in the model. The aim is to have primary and budget balances in the data and model to align conceptually. The ultimate goal of the paper is to quantify how close the standard growth theory comes in generating observed budget balance and debt figures and to use the model to deliver short run predictions on both the government accounts and national accounts.

The budget balance, \( \tau_{h,t} w_t H_t + \tau_{k,t} (r_t - \delta_t) K_t - (G_t + TR_t + I_t) \), corresponds to net government saving in the NIPA data. The government tax revenue, \( \tau_{h,t} w_t H_t + \tau_{k,t} (r_t - \delta_t) K_t \), is measured as current receipt in the data.\(^7\) \( G_t \) is measured as government consumption.

\(^7\)Since our measured \( K_t \) includes consumer durable and government fixed capital, when we compute the capital income tax revenue, we tease out the share of consumer durable and government fixed capital, which
Transfer payments, $TR_t$, are calculated as

\[
\text{Current Transfer Payment - Current Transfer Receipts}
\]

Interest payment on government debt, $I_t$, is calculated as

\[
\text{Interest Payment - Income Receipts on Asset}
\]

Therefore, the net borrowing, $GB_t$, is calculated in the data as

\[
\begin{pmatrix}
\text{Current Receipts} \\
\text{− Transfer Payment} \\
\text{− Interest Payment} \\
\text{− Government Consumption} \\
\text{− Gross Government Investment} \\
\text{+ Consumption of Fixed Capital}
\end{pmatrix}
\]

Note that $GB_t$ includes both the budget deficit $N_t \pi^b_t$ and net government investment (denoted as $GI_t$), the latter of which is not explicitly modeled in this paper. To assess the model’s prediction on net borrowing, we construct $GB_t$ in the model as the sum of $-N_t \pi^b_t$ and $GI_t$.

After we compute the net borrowing in the data, we construct government debt in each period between 1961 and 2010 following equation 4, taking the 1959 value of the federal government debt held by the public as the initial debt level for 1960.

### 3.3 Calibration

The starting year for the analysis is 1960 and the last period for which we have data for all of the variables is 2010. The model takes the observations for the exogenous inputs as given for the 1960-2010 period, and makes certain assumptions about their values for 2011 and beyond. A steady state is assumed to be reached far into the future so that we have a two-point boundary problem, starting with given initial conditions in 1960, and ending at a steady state. Following Hayashi and Prescott (2002), we use a shooting algorithm to calculate an equilibrium transition path that connects these two boundary points. We make sure that our assumptions about that steady state have minimal effects on the immediate future along the transition path.

The following two subsections present the calibration choices in detail summarizing the parameters that are constant throughout the analysis, the exogenous inputs for which we have direct observations, and the assumptions made for the values of these exogenous inputs are not taxed in reality.
for 2011 and beyond.

While there are many calibration issues, it may be useful to provide an overall summary for the growth rate of some of the variables in the benchmark economy. Equation 5 shows that there are two growth rates that are critical for the evolution of the debt to output ratio. These are the growth rate of output and the inflation rate. Our assumption for the inflation rate after 2010 is 2%. Growth rate of output which is determined endogenously turns out to be 2.75% at the steady state. CBO assumptions for the inflation rate and the growth rate of output are 2% and 2.2% respectively.

3.3.1 Constant Parameters and Steady State Calibration

There are three parameters that are time invariant throughout our analysis. The capital share parameter, $\theta$, is set to 0.4. The subjective discount factor, $\beta$, is set to 0.969 so that the capital-output ratio is 3.2 at the final steady state. The share of leisure in the utility function, $\alpha$, is set to 1.45 to match an average workweek of 35 hours in the U.S. The log utility function we use implies a Frisch elasticity of labor equal to 1. We set the steady state depreciation rate, $\delta$, at 0.055, its average value between 1990 and 2010. We use a TFP growth rate of 1.1% and population growth rate of 1% in the steady state. We set $\tau_k = 0.392$ and $\tau_h = 0.273$, the average of corresponding tax rates between 1960 and 2008. Government transfer to output ratio, $TR/Y$ and consumption to output ratio, $\psi$, are set equal to 0.17 and 0.05 at the steady state to be consistent with the projections for these variables under the extended baseline scenario of the CBO. Since the steady state is assumed to be reached far into the future, the steady-state values do not affect our short-term predictions. These choices are summarized below.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>Capital Income Share</td>
<td>0.40</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Utility Parameter for Leisure</td>
<td>1.45</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Subjective Discount Factor</td>
<td>0.969</td>
</tr>
<tr>
<td>$g-1$</td>
<td>TFP Growth Rate</td>
<td>1.1%</td>
</tr>
<tr>
<td>$n-1$</td>
<td>Population Growth Rate</td>
<td>1.0%</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Government Consumption to Output Ratio</td>
<td>0.05</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation Rate</td>
<td>0.055</td>
</tr>
<tr>
<td>$TR/Y$</td>
<td>Transfers to Output Ratio</td>
<td>0.17</td>
</tr>
<tr>
<td>$\tau_k$</td>
<td>Capital Income Tax Rate</td>
<td>0.392</td>
</tr>
<tr>
<td>$\tau_n$</td>
<td>Labor Income Tax Rate</td>
<td>0.273</td>
</tr>
</tbody>
</table>
3.3.2 Inputs for 1960-2010 and Beyond

Calibration of the Initial Conditions: We use the initial capital-output ratio for the U.S. in 1960, 3.2, to pin down the initial capital stock. In addition, the initial debt level in 1960 is set to target an initial debt to output ratio of 39.8%, which is the ratio of federal government debt held by the public to output at the beginning of 1960.

Calibration of the 1960-2010 period: In our benchmark simulation, we use the actual time series data for the U.S. between 1960-2010 for TFP growth rates, $g_t - 1$, population growth rates, $n_t - 1$, shares of government purchases in measured GNP, $\psi_t$, share of transfer payment in measured GNP, $TR_t/Y_t$, the depreciation rate, $\delta_t$, and capital and labor income tax rates, $\tau_{k,t}, \tau_{h,t}$. The inflation rate, $P_{t+1}/P_t$, is taken as the growth rate of the GNP deflator. We take the nominal interest rate on government bonds as exogenously given. There are several possible choices to use for this interest rate. Given that the U.S. government bonds are in many different maturities, we chose to compute the implicit interest rate as the ratio of interest payments by the government to the stock of gross federal debt from the previous period. Figure 2 displays this imputed interest rate against the 3-month T-bill rate and the 10-year Treasury note rate between 1960 and 2010 as well as the projections used by the CBO for the T-bill and T-note rates until 2020.

We use data from Statistics of Income (SOI), Individual Income Tax Returns (1960-2010), Social Security Bulletin and National Incomes and Product Accounts (1960-2010), and the
Figure 3: Data and Assumptions for the Future


**Calibration of 2011 and Beyond:** We assume that U.S. starts from a given set of initial conditions in 1960 and eventually converge to a steady-state in 2070. We make certain assumptions about the future path of the exogenous variables and check the sensitivity of our results to these assumptions. For our benchmark, we assume that the capital and labor income tax rates continue at their long run averages (steady state values given in Table 1) after 2010. We also consider an alternative case where tax rates increase gradually, as assumed by the CBO’s alternative scenario, from 25% in 2010 to 38% in 2035. TFP growth rate is also assumed to continue at its historical average for the benchmark simulations. We do examine cases with higher growth rates after 2010 as well. We set the population growth rate equal to 1%, which is consistent with the projections of the Census Bureau, and the depreciation rate equal to 5%. We assume the inflation rate after 2010 to be 2% and the nominal interest rates on government bonds to be 3.3% both consistent with the CBO assumptions.

Figure 3 summarizes the data and the assumptions for the future path of the capital and labor income tax rates, population growth rate, and the growth rate of the TFP factor.

For the projections on government expenditures and transfer payments, we use data
from two sources: one from the Congressional Budget Office (CBO) and the other from the Bowles-Simpson Commission. Both provide projections on different categories of government spending for the federal government. In the CBO projection, $TR_t$ is computed as the sum of Social Security, Medicare, Medicaid, CHIP, and Exchange Subsidies, while $G_t$, together with net government investment, constitutes the other non-interest spending. For the Bowles-Simpson projections, we combine their projections on Social Security and health care into $TR_t$ and "other mandatory spending" and "discretionary spending" into $G_t$. Figure 4 summarizes the differences between the CBO and Bowles-Simpson projections used in our experiments.

Since $Y_t$ is endogenous in our model, we make an adjustment to the projections provided by the CBO and the Bowles-Simpson commission. We use the projected growth rates of the level of $G_t$ and $TR_t$ from this data to compute their corresponding shares in total output. To accomplish that, we first compute the model generated $G_t$ and $TR_t$ for 2010, as the product of $Y_t$ and the corresponding spending shares in year 2010. Next, we project the level of $G_t$ and $TR_t$ starting from 2011 using the corresponding projected growth rate of the federal government spending and the initial levels at year 2010, which are computed in the first step. Lastly, we compute $G_t/Y_t$, and $TR_t/Y_t$ using the model generated $Y_t$. It is important to mention that the projections by the CBO and the Bowles-Simpson plan are for the federal government expenditures and revenues only. In our benchmark exercises we assume that the state and local government expenditures will decline at the same rate as the federal expenditures as projected by the CBO. Later, we examine the sensitivity of our results to this assumption taking into account the fact that federal government constitutes about 40% of government consumption and 33% of government investment.
4 Results

In the first part of this section, we evaluate the impact of the future path of expenditures assumed in the CBO baseline versus the Bowles-Simpson plan on the future debt to GNP ratio in the U.S. using our model. The calibration of the economy remains constant between these two cases except for the differences in the future path of expenditures. Consequently, differences in the resulting debt to GNP ratio by 2030 only reflect the differences in the future path of expenditures that are fed into the model and the endogenous response of the model economy to them. In the second section we conduct several experiments to assess the likely consequences of additional scenarios such as an increase in the future inflation rate or the growth rate of TFP on the debt to GNP ratio by 2030.

4.1 Benchmark Calculations

We start this section by examining if the model generated economy can serve as a useful laboratory for studying the effects of different policies on the future debt to GNP ratios. In this benchmark, we use the CBO projections for the future path of the expenditures. The first two panels in Figure 5 summarize the GNP per person and hours per capita generated by the model as well as their data counterparts. The lower two panels summarize the results on the capital output ratio and the consumption output ratio. Notice that the model is not able to capture the continued increase in the consumption to output ratio that took place throughout the 1990s and 2000s. This is similar to the neoclassical model’s inability to capture the hours boom in the 1990s. In addition, model generated GNP per person and hours per capita are higher than their counterparts in the 1990s.

Figure 6 displays the overall government budget balance and debt to GNP ratios generated by the model economy as well as their counterparts in the data. The simulated series, that are generated through a mix of exogenously fed series such as government purchases and transfer payments and endogenous variables such as labor and capital which lead to GNP and tax revenues, captures the general increase in the government deficit and the debt to GNP ratio reasonably well. Model generated debt to GNP ratio in the 1990s is lower than the data due to lower deficits generated by the model in the 1990s. By 2010, however, both the budget balance and the debt ratio are close to their data counterparts.

While the model economy may not be able to capture all aspects of the U.S. economy precisely, it may be able to help us evaluate the implications of different expenditure projections on the future debt to GNP ratios. In the first experiment, we use the expenditure projections given by the CBO under the extended baseline scenario. In the second one, we

\[9\] See McGrattan and Prescott (2010) for a discussion of this issue. Since we only feed in the labor wedge and not the other wedges, the model generated hours per capita is not identical to the data.
Figure 5: Benchmark Economy

Figure 6: Government’s Budget
use the proposed expenditure path by the Bowles-Simpson commission. We keep the tax rates beyond 2010 set at their steady state values as described in the calibration section.

4.1.1 Benchmark Projections

In the first panel of Figure 7, we present the model generated budget balance up to 2030 and its data counterpart until 2010. The projections on expenditures are taken from the \textit{extended baseline} scenario of the CBO. Tax rates are the ones that are consistent with the historical averages. Our results indicate an initial improvement in the budget deficit that declines from about 9\% in 2010 to around 7\% until 2020 and deteriorates back to 9\% by 2030. The series labeled "data" in the second panel of Figure 7 displays the actual debt to GNP ratio until 2010 and the CBO projections for it after 2010. CBO projections are the ones obtained under the \textit{extended baseline} scenario where debt to GNP ratio reaches 80\% by 2030\textsuperscript{10}. The model generated debt to GNP ratio is 131\% in 2030.

The main reason behind the differences in debt to output ratios obtained in the model versus the CBO is due to the differences in the projected revenues between the two. Figure 8 presents the government revenue to GNP ratio in the data, until 2010, and the model

---

\textsuperscript{10}In their second scenario CBO assumes that tax rates remain at their historical averages. This assumption together with a slightly different path on expenditures results in a projected debt to GDP ratio of 187\% in 2035.
Figure 8: Revenues

until 2030. For the values after 2010 in the data, we take the CBO projections (under the extended baseline scenario) for revenues which are provided for the federal government only and assume that the growth rate of this series reflects the growth rate of total revenues.\footnote{We multiply the government revenue to GNP ratio in 2010 by the growth rate of the ratio of federal government to GDP implied by the CBO projections.}

Projected increases in revenues in the CBO calculations arise due to two assumptions. They assume that the tax cuts that were enacted since 2001 are allowed to expire as scheduled in 2012 and the exemption amounts for the individual alternative minimum tax (AMT) revert to their 2001 levels in 2012. Both of these assumptions result in marginal tax rates on labor income to increase from about 25% in 2011 to 38% in 2035. CBO projections do not model the potential decline in labor supply due to changes in tax rates. Consequently, tax revenues rise significantly after 2010 which result in the projected debt to GNP ratio of 80% in 2030. In our benchmark calculations we assumed that tax rates will continue at their historical levels which gives rise to tax revenues that are labeled "model" in Figure 8. In the next section we examine the response of the model economy to increases in tax rates as assumed by the CBO projections.

\textbf{Higher Taxes} CBO projections ignore the distortionary impact of higher taxes on labor supply and tend to overestimate the impact of higher tax rates on curbing the debt-to-output ratio. While higher tax rates tend to reduce the debt to output ratio through their positive impact on tax revenues, their negative impact on output mitigates some of the gains in tax

\hspace{1cm}
revenues.

Equation 2 shows that an increase in $\tau_{h,t}$ has a positive impact on tax revenues through its impact on revenues from labor income given by $\tau_{h,t} (1 - \theta)$.

$$\frac{N_t \pi^b_t}{Y_t} = \frac{\tau_{h,t} w_t H_t + \tau_{k,t}(r_t - \delta_t) K_t}{Y_t} - \frac{(G_t + TR_t + I_t)}{Y_t}$$

$$= \tau_{h,t} (1 - \theta) + \tau_{k,t} \theta - \tau_{k,t} \frac{\delta_t K_t}{Y_t} - \frac{(G_t + TR_t + I_t)}{Y_t} \quad (9)$$

However, a higher $\tau_{h,t}$ reduces $Y_t$ via its distortionary impact on $H_t$. The reduction in $Y_t$ reduces the revenues that can be obtained through the tax increase. The magnitude of the change in $Y_t$ depends on the Frisch elasticity of labor supply. There is significant discussion about the right elasticity of labor supply in the profession. The log utility function we have used so far implies a Frisch elasticity of one. In the next two experiments we examine the impact of the changes in the tax rates on the debt to GNP ratio under the benchmark (with a Frisch elasticity of one) and an alternative case with a Frisch elasticity of 0.5, as well as an exogenous labor case.

**Benchmark-Log Utility** In this experiment, we increase the tax rate on labor income from about 25% in 2010 gradually to 38% in 2035, as assumed in the CBO projections. Rest of the calibration is exactly same as the benchmark calibration. Figure 9 summarizes the resulting budget balance and debt to GNP ratios for this case. Notice that government budget balance improves in this case compared to the results presented in Figure 7. In particular, the budget deficit remains around 6% until 2030 as opposed to reaching 9% if labor tax rates were to remain at 25%. The resulting debt to GNP ratio of 115% in 2030, is also slightly better than the benchmark case. However, 115% is significantly higher than the CBO projection of 80%.

The impact of higher taxes on tax revenues can been observed in Figure 10 as well. Tax revenues rise from about 22% in 2010 to above 26% in 2030. However, these tax revenues do not come close to matching what is projected by the CBO in order for the debt to GNP ratio to stabilize around 80%. In addition, higher tax rates result in a 2.8% lower GNP per capita by 2030 due to the distortions on labor supply.

**Alternative Utility Function** In this specification we use the utility function given in equation 10 where we set $\gamma$, the Frisch elasticity of labor supply, to 0.5.\(^{12}\)

$$U(c, l) = \log c - \frac{\chi l^{1+\frac{1}{\gamma}}}{1 + \frac{1}{\gamma}} \quad (10)$$

\(^{12}\)\(\chi\) is set to 16.85 so that workers on average spend one-third of their time on market work.
Figure 9: Budget with Higher Taxes

Figure 10: Revenue with Higher Taxes
The result for this case generates a debt to GDP ratio of 122% in 2030 under historical taxes and 104% with higher taxes. That is, with a smaller labor supply response, debt to GNP ratio declines by 15% in response to a tax increase. In our benchmark case, the decline in the debt to GNP ratio was 12%, from 131% to 115%.

**Exogenous Labor Supply** As mentioned earlier, the CBO projections do not take into account the labor supply response to higher taxes, practically assuming a zero Frisch elasticity. In the next experiment, we keep the labor supply exogenous and repeat the exercise where the tax rate on labor income is increased from 25% in 2010 gradually to 38% in 2035. The debt to GNP ratio implied by the model in this case is 89% in 2030. Debt to GNP ratio with historical tax rates under the exogenous labor case is equal to 112%. Thus, with exogenous labor, increases in taxes result in an 26% decline in the debt to GNP ratio.

4.1.2 Bowles-Simpson Proposal for Expenditures:

The National Commission on Fiscal Responsibility and Reform headed by Erskine Bowles and Alan Simpson has a proposal that involves larger cuts in both discretionary and mandatory spending categories such as Social Security and Medicare until 2020. They also make some recommendations on changes in the tax code.

The main differences between the CBO projections and the Bowles-Simpson proposal can be summarized as follows. Total discretionary spending, which is the spending category $G_t$ in the model economy, between 2012 and 2020 is set at $9,724 billion in the Bowles-Simpson plan as opposed to $11,666 billion in the CBO baseline. Expenditures on transfer payments (Social Security and health care) is also lower under the Bowles-Simpson plan. These differences are evident from Figure 4, which shows that the projected ratio of government expenditure to GDP under the Bowles-Simpson plan declines at a faster rate than its counterpart under the CBO projection. In this experiment, we implement the path of expenditures in $G_t$ and $TR_t$ according to their proposal and examine the resulting debt to GNP ratio after 2010.

Using the growth rate of the level of expenditures proposed by the Bowles-Simpson Commission results in government budget balance and debt to $GNP$ ratios that are displayed in Figure 11 labeled "model". Data until 2010 and the projections provided by the Bowles-Simpson plan until 2020 are labeled "data". Compared with the results of the CBO baseline presented in Figure 7, Bowles-Simpson proposal results in smaller budget deficits as well as a smaller debt to $GNP$ ratio by 2030 (100% versus 131%). The model is not rich enough to evaluate the changes in the tax code that are discussed in the Bowles-Simpson plan that focuses on increasing the tax base. The resulting debt to GDP in this plan is 62% in 2020. A simple increase in the marginal tax rates as proposed in the CBO baseline, however, has a relatively small effect on the debt to $GNP$ ratio in 2030.
4.2 Additional Experiments

In this section we conduct several additional experiments by making changes in the benchmark economy one at a time.

4.2.1 High Employment

There are many uncertainties about the future that make it difficult to provide forecasts. For example, in the benchmark results, we assumed that the labor wedge will stay at its 2010 level, which causes the average hours worked to be much lower than its peak in the 1990s. Consequently, \( GNP \) per person generated for the period after 2010 is also lower than its peak in the 1990s. These features of the simulated data contribute to the high debt to \( GNP \) ratios generated for the benchmark. In the first counterfactual experiment, we assume that the labor wedge after 2010 will go back to its average level in the 1990s. Rest of the calibration is same as in the benchmark calibration where future projections on expenditures come from CBO’s extended baseline scenario. Such an increase in labor supply results in a significantly higher \( GNP \) per person in 2011 as displayed in Figure 12. This jump implies a high growth rate of GNP per capita between 2010 and 2011. Under this fairly rosy scenario, debt to \( GNP \) ratio, displayed in Figure 13, reaches a relatively lower level of about 80% in 2030. This is due to the fact that the budget deficit shrinks immediately. A better fiscal outcome early on results in lower interest payments later on. If hours worked increase more
gradually instead of jumping to a higher level immediately, the resulting growth rate in \( GNP \) per capita in 2011 is lower. In that case, debt to \( GNP \) ratio in 2030 reaches 90%.

### 4.2.2 Federal Versus State and Local Expenditures

In our benchmark results, we assumed that both federal and state and local government purchases will decline after 2011 as projected by the CBO. However, CBO projections are primarily for the federal government, which constitutes about 40% of government consumption and 33% of government investment.\(^\text{13}\) In the next experiment, we assume that state and local government expenditures as a percent of \( GNP \) stay constant after 2011 and only the federal expenditures decline as a percent of \( GNP \) as projected by the CBO. In this case, debt to output ratio, displayed in Figure 14, reach 173% in 2030.

### 4.2.3 Inflation

In the benchmark simulations, we assumed the inflation rate in 2011 and beyond to continue at 2%, its level in 2011. In addition, the nominal interest rate on government debt is assumed to be 3.3% into the future. In the next set of counterfactual experiments, we increase the inflation rate to 4% and 6% without making any changes in the assumed nominal interest rate. Consequently, real interest rates are -0.7%, and -2.7%. We find that by 2030, debt to \( GNP \) ratio declines from 131% in the benchmark case to 102%, and 82% for these three cases.

\(^{13}\)In 2010, consumption expenditures of the federal government were $1,054 billion while the total government consumption was $2,497 billion. Gross government investment by the federal government was $168 billion. The same year state and local governments spent $336 billion on investment expenditures.
Figure 13: Debt with Higher Growth

Figure 14: Higher Expenditures
4.2.4 Summary

Table 2 summarizes the findings of various experiments conducted above. A useful starting point is the comparison between the extended benchmark scenario of the CBO, with a 80% debt to GDP ratio projected for 2030, and the model generated one. Important assumptions by the CBO for this case include tax rates increasing from 25% in 2010 to about 38% in 2035 and no labor supply responses. Our calibration that is most similar to the CBO benchmark is the case with higher tax rates and exogenous labor. This case results in a debt to GNP ratio of 89%. Incorporating labor supply responses in a utility function with a Frisch elasticity of one increases this ratio to 115%. An alternative utility function with a Frisch elasticity of 0.5, results in a debt to GNP ratio of 104%. Our benchmark case, which we view as the more likely scenario, where tax rates are assumed to continue at their historical averages results in a debt to GNP ratio of 131%. Larger cuts proposed by the Bowles-Simpson plan for the benchmark case result in a debt to GNP ratio of 100%. All these experiments assume that the projected cuts in these plans apply to both the federal and the state and local governments. In an event that this assumption fails to hold and state and local expenditures continue at their historical levels, projected debt to GNP ratio climbs to 173%.

Table 2: Summary Table

<table>
<thead>
<tr>
<th>Model</th>
<th>Debt to GNP in 2030</th>
</tr>
</thead>
<tbody>
<tr>
<td>CBO benchmark projections</td>
<td>80%</td>
</tr>
<tr>
<td>CBO expenditures; high tax; exogenous labor</td>
<td>89%</td>
</tr>
<tr>
<td>CBO expenditures; higher tax; endogenous labor; Frisch elasticity 0.5</td>
<td>104%</td>
</tr>
<tr>
<td>CBO expenditures; higher tax; endogenous labor; Frisch elasticity 1.0</td>
<td>115%</td>
</tr>
<tr>
<td>CBO expenditures; historical tax; endogenous labor (benchmark)</td>
<td>131%</td>
</tr>
<tr>
<td>CBO expenditures for federal gov only; historical tax; endogenous labor</td>
<td>173%</td>
</tr>
<tr>
<td>CBO expenditures; historical tax; endogenous labor, 4% inflation</td>
<td>102%</td>
</tr>
<tr>
<td>Bowles Simpson expenditures; historical tax; endogenous labor</td>
<td>100%</td>
</tr>
</tbody>
</table>

5 Conclusions

In this paper, we evaluate the likely consequences of the paths of government expenditures foreseen by the CBO baseline and the Bowles-Simpson proposal in a fully calibrated general equilibrium model. While the framework is relatively simple, it incorporates the general equilibrium effects of policy that are often missing from the CBO projections. We find that debt to GDP ratios in 2030 can easily be 17% to 29% higher when labor supply responses are taken into account. Our results also suggest that it is fairly difficult for the U.S. economy to reach debt to \textit{GNP} ratios around 80% by 2030 as forecasted in the baseline CBO calculations.
Unless there is a significant increase in the growth rate of $GNP$, debt to $GNP$ ratios above 100% are more likely to continue into the future.

We leave many important and interesting issues for future work. Impact of the projected debt levels on different generations and the sustainability of the U.S. fiscal policy are among them.
References


6 Appendix

6.1 Calibration of the Benchmark Economy

In this section, we provide the details of our calibration for the benchmark economy. We use data from the 2010 revision of National Income and Product Accounts (NIPA) and Fixed Asset Tables (FAT) of Bureau of Economic Analysis (BEA) for the years 1960-2010. Our adjustments to measured macroeconomic aggregates follow Cooley and Prescott (1995).

Denote measured GNP as follows:

\[(cs + cnd + icd) + g + i + nx + nfp = GNP = dep + NNP\] (A-1)

where \(cs, cnd, icd\) denote service flow of consumer durables, consumption of nondurable and expenditure on consumer durable. \(g\) denotes the sum of government consumption, denoted as \(gc\), and gross government investment, denoted as \(gi\). \(i\) denotes gross private investment. \(nx\) denotes net export and \(nfp\) denotes net factor payments on foreign assets. \(dep\) denotes consumption of fixed capital.

First, we include government capital in the definition of the capital stock. Once we include the service flow from government capital, \(sg\), A-1 becomes:

\[(cs + cnd + icd + sg) + gc + (i + gi) + nx + nfp = GNP + sg = dep + (NNP + sg)\] (A-2)

where \(dgi\) denotes depreciation of government fixed assets and \(dep - dgi\) is depreciation of private fixed asset.

Second, we treat the stock of consumer durable as part of capital stock. Then A-2 becomes:

\[(cs + cnd + csd + sg) + gc + (i + nicd + dcd + gi) + nx + nfp = GNP + sg + csd = (dep + dcd) + (NNP + sg + csd - dcd)\] (A-3)
where \( csd \) is service flow from consumer durable and \( dcd \) denote depreciation of consumer durable. Therefore, total private consumption becomes \((cs + cnd + csd + sg)\) and total investment becomes \((i + icd + gi)\) or \((i + nicd + dcd + gi)\), where \( nicd \) is referred to as net investment in consumer durable and \( dcd \) denotes depreciation of consumer durable. Total depreciation becomes \((dep + dcd)\).

Third, we treat net foreign asset as part of capital stock. A-3 then becomes:

\[
\begin{align*}
(cs + cnd + csd + sg) + gc + (i + nicd + dcd + gi + nx + nfp) &= GNP + sg + csd \\ &= (dep + dcd) + (NNP + csd + sg - dcd)
\end{align*}
\]

Now total investment becomes \((i + nicd + dcd + gi + nx + nfp)\).

In summary, we define capital \( K \) as the sum of the fixed assets, stock of consumer durables, inventory stock land, and net foreign assets. Output \( Y \) corresponds to \( GNP + sg + csd \) and total depreciation corresponds to \( dep + dcd \).

Following McGrattan and Prescott (2000), we assume that the rate of returns for consumer durable and government fixed assets are equal to the rate of return for non-corporate capital stock. Specifically, we have

\[
i = \frac{\text{(Accounting Returns + Imputed Returns)}}{\text{(Non-corporate capital +land+inventory+Capital of Foreign Subsidiary)}}
\]

\[
= \frac{(0.0603 + 1.6803i)}{(2.976 + 0.0095/i)}
\]

where 0.0603 is non-corporate profit plus net interest less intermediate financial services, 1.6803 is the sum of the net stock of government capital, consumer durable, land and inventory; 2.976 is the sum of net stock of non-corporate business, government capital, consumer durable, land and inventory. 0.0095 is the net profit from foreign subsidiaries.

The above equation gives a value of \( i \) at 3.93% over the period between 1960 and 2000.

\( Y_{sd} \) and \( Y_{sg} \) denote the service flows from consumer durables and government capital, respectively, which are computed following Cooley and Prescott (1995).

\[
Y_{sd} = csd = (i + \delta_d) K_D \\
Y_{sg} = (i + \delta_g) K_G
\]

Then the capital share in the output function \( \alpha \) is computed as:

\[
\alpha = \frac{Y_{kp} + Y_{sd} + Y_{sg}}{GNP + Y_{sd} + Y_{sg}},
\]
where $Y_{kp}$ is the income from private fixed assets

$$
Y_{kp} = \text{Unambiguous capital income} + \theta_p \times (\text{proprietors’ income} + \text{indirect business tax}) + \text{depreciation} = \theta_p \times GNP
$$

This gives a value 0.32 for $\theta_p$ and a value of 0.41 for $\alpha$. 