NBER WORKING PAPER SERIES

TECHNIFYING VENTURES

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Working Paper 33993
http://www.nber.org/papers/w33993

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
July 2025

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NBER Working Paper No. 33993
July 2025
JEL No. E13, G24, O30, O40

ABSTRACT

How do advanced technology adoption and venture capital (VC) funding impact employment and growth? An analysis of data from the US Census Bureau suggests that while both advanced technology use and VC funding matter on their own for firm outcomes, their joint presence is most strongly correlated with higher employment levels. VC presence is linked with a high increase in employment, though primarily among a limited subset of firms. In contrast, technology adoption is associated with a smaller rise in employment, yet it influences a considerably larger number of firms. A model of startups is created, focusing on decisions to use advanced technology and seek VC funding. The model is compared with firm-level data on employment, advanced technology use, and VC investment. Several thought experiments are conducted using the model. Some experiments assess the importance of advanced technology and VC in the economy. Others examine the reallocation effects across firms with different technology choices and funding sources in response to shifts in taxes and subsidies.

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1 Opening

Technological advancement plays a crucial role in driving economic growth, with innovative startups often serving as key contributors to the development and diffusion of new technologies. These firms are vital not only for generating employment and fostering economic dynamism but also for promoting the adoption of cutting-edge innovations. Understanding the factors that shape a startup's decision to adopt advanced technology—and how this choice influences employment and revenue across a firm's life cycle—is essential. The sources of startup funding, such as venture capital and more traditional options, may also be closely linked to technology adoption and firm performance. The interaction between funding types and technology use may also matter, influencing outcomes differently depending on the firm's strategic choices. At a broader level, the presence of venture capital and advanced technology also has potential implications for the economy as a whole, especially as subsidies and taxes affect firms' incentives for technology adoption and choices of financing type.

This study investigates the startup process, focusing specifically on how a firm's employment and revenue outcomes are linked to its adoption of advanced technologies and its source of funding, particularly venture capital.[1] The study is divided into two sections. The first part uses data from the U.S. Census Bureau to examine the connection between venture capital, firm outcomes, and the adoption of advanced technologies by startups. Until recently, analysis of the use of advanced technologies by U.S. firms has been challenging due to the lack of representative data. To address this gap, the Census Bureau has introduced new questions about advanced technology adoption in its recent surveys, particularly in the Annual Business Survey (ABS) for the years 2018-2023. These surveys include firm-level data on the adoption and use of advanced technologies like AI, robotics, and many others. The analysis compares the performance of high-tech startups and those backed by VC in terms of employment and revenue to startups that either do not adopt advanced technologies or rely on other sources of funding, such as banks. The findings reveal that startups with advanced technologies and VC backing tend to perform better in terms of both employment and revenue. Moreover, the results from difference-in-differences analyses, using matched control samples, suggest that adopting advanced technologies and having VC backing both have positive effects on firm employment. Interestingly, VC backing is more strongly connected with the performance of high-tech startups than with that of the startups without advanced technologies.

Based on assembled empirical evidence, the second part of the study builds a model to examine the relationship between the adoption of advanced technologies by startups and their choice of funding source—either banks or venture capitalists. Startups are born from entrepreneurial ideas, some with more potential than others. Additionally, some ideas are better suited to incorporating advanced technologies, which can influence the expected return on investment. Entrepreneurs seek funding from either banks, which offer loans with fixed interest rates, or venture capitalists, who provide funding in exchange for a share of the business's profits. Unlike banks, VCs also offer guidance and advice, which requires more involvement, especially for more ambitious projects. VCs are compensated based on their share of the startup's success, which is negotiated with the entrepreneur. Successful startups typically exit via an IPO or through a merger/acquisition. Entrepreneurs select the technology that maximizes their expected payoff, factoring in how this will influence the type of funding they will receive, which also has an impact on expected profits. The model developed here reflects how entrepreneurs balance these considerations and highlights the complementarities between venture capital and advanced technology.

The model is calibrated to match key empirical facts about technology adoption and startup financing in the United States. It is then used to conduct a series of experiments oriented to understanding

[1] Venture capital is a form of private equity financing that invests in *startups* with a high potential for rapid growth. The venture capitalists take shares in the startup in return for their advice and money. The exit strategy is to sell the startup on the stock market or to another business. Traditional private equity purchases *established* companies hoping to increase profits by restructuring the company's operations. Often this involves taking public traded companies private. The restructuring may involve bond financing via a leveraged buy out. Venture funded startups do not use bond financing.

the roles of advanced technology and venture capital in the economy. One experiment examines the consequences of shutting down VC presence in the economy, while another investigates the effects of eliminating access to advanced technologies. Both factors are found to play significant roles in the economy. The effects of changes in taxes and subsidies are also studied to explore reallocation effects across firms using different combinations of technology and sources of funding.

This research makes several contributions to existing knowledge. First, it provides some of the earliest systematic evidence linking advanced technology adoption to firm performance, using representative U.S. data that includes detailed information on adoption timing. Second, it offers new insights into how the effects of VC vary by firm technology type–previous work examined these factors separately. Notably, the positive relationship between technology adoption and firm outcomes is amplified for VC-funded startups. Third, the research develops a unified theoretical framework that jointly models firms' decisions concerning technology adoption and financing, enabling a quantitative analysis of their aggregate economic impact. The presence of advanced technology and VC in the economy has meaningful macroeconomic implications. Additionally, changes in taxes and subsidies can generate significant reallocation effects across firms with different mixes of technology and funding sources.

Some limitations are acknowledged. While the difference-in-differences analysis for exploring the relationships among technology adoption, VC, and firm outcomes uses well-matched control groups, it's still possible that firm-level unobserved factors could influence the decision to adopt technology or seek VC backing. In addition, the model assumes that the technology adoption decision is made at the outset by the entrepreneur, anticipating future VC backing. However, it does not allow VC to have a direct involvement in technology choice. Despite these limitations, this paper underscores the significant macroeconomic effects that technology adoption and VC funding can have.

2 The Advanced Technologies Studied

The technologies chosen for the analysis are detailed in Appendix A. The selection process was shaped by several factors. First, it was limited to technologies covered by Census surveys, which focus on those considered both advanced and impactful, excluding technologies that haven't yet gained significant traction– nevertheless, a wide variety of nascent technologies, such as nanotechnology, renewable energy technologies, advanced nuclear technologies, and human-machine interfaces, are included in the analysis. Second, the data for the chosen technologies were more uniformly and consistently collected in various ABS instruments between 2018 and 2023, providing a sufficient sample size for analysis. Additionally, data on the timing of adoption for AI and robotics (along with a few other technologies) are available in the 2023 ABS survey, but not in prior years or for other technologies. Third, the inclusion of more technologies, such as specialized equipment and software, yields similar results. So, while the findings don't apply to all advanced technologies, they offer valuable insights into key innovations that have been central in recent research, particularly AI and robotics. Next, five advanced technologies are highlighted as examples of the technologies studied in the empirical analysis; namely, additive manufacturing, artificial intelligence (AI), distributed ledgers, radio-frequency identification (RFID), and robotics.

2.1 RFID (Radio Frequency Identification)

RFID is a technology where a tag attached to an object allows it to be identified and tracked. The tag, once activated by an electromagnetic field, transmits information to a radio receiver. Mario Cardullo received the first patent for an RFID device in 1973, with an initial vision for applications in credit cards and toll systems. Today, RFID technology is used in a wide array of applications including access control, biometric data retrieval from passports and driver's licenses, contactless credit card payments, inventory management, and tracking of livestock, pets, and shipments.

2.2 Robotics

The history of robotics dates back to 1949, when William Grey Walter developed battery-powered robots with three wheels that could navigate around obstacles. These robots, which resembled tortoises, were capable of phototaxis, or moving toward light, and could return to their charging station when their battery was low. The fundamental elements of these early robots—sensor technologies, feedback loops, and decision-making—remain integral to modern robots. A decade later, General Motors introduced the first industrial robotic arm, designed to lift and stack hot metal parts. In 1972, engineers at the Stanford Research Institute created "Shakey," the first robot that incorporated AI. Shakey could break down commands into actionable steps, such as navigating to a specified room and moving an object to a desired location. Today, robots are widely used in manufacturing, healthcare (e.g., surgical assistance and food delivery in hospitals), law enforcement, military operations, and space exploration.

2.3 Artificial Intelligence (AI)

The concept of AI was first contemplated by Alan Turing in 1950, but it wasn't until 1956 that a computer program called The Logic Theorist was developed by Allen Newell, Cliff Shaw, and Herbert A. Simon, proving 38 theorems from *Principia Mathematica* by Alfred North Whitehead and Bertrand Russell. Despite its significance, the program was not well received at the 1956 Dartmouth Summer Research Project on Artificial Intelligence. An important milestone in AI came in 1967 when Joseph Weizenbaum created ELIZA, an early natural language processing program that could communicate with people using pattern matching. AI development was initially limited by computing power, but as computer technology advanced, breakthroughs occurred, such as IBM's Deep Blue defeating world chess champion Garry Kasparov in 1997. AI is now applied in various fields, from personalized advertising and autonomous vehicles to fraud detection, healthcare diagnostics, facial recognition, and voice assistants like Alexa.

2.4 Additive Manufacturing (3D Printing)

Additive manufacturing, or 3D printing, traces its origins to Johannes F. Gottwald's 1971 patent for the Liquid Metal Recorder, a modified inkjet printer that used liquid metal to create shapes. This technology revolutionized manufacturing by enabling rapid prototyping and direct production from design files. Additive manufacturing allows for complex shapes and custom designs, reduces material waste (since it doesn't require cutting or milling), and cuts down on assembly steps. Additionally, products can be manufactured on demand, which is particularly beneficial for specialized or low-volume items.

2.5 Distributed Ledgers

A distributed ledger is a decentralized system that records and verifies transactions across multiple locations and institutions through peer-to-peer networks, using cryptography for security. Every node in the network stores an identical copy of the ledger, and updates to the ledger require verification by a majority of nodes before they are recorded across the entire network. This system offers increased security, transparency, and eliminates the need for a central authority to oversee transactions. The concept of distributed ledgers was introduced by Satoshi Nakamoto in 2008, primarily for use in cryptocurrencies like Bitcoin.

These technologies represent a core group of innovations that have had a profound impact on various industries. While their applications continue to evolve, they are already reshaping fields such as manufacturing, healthcare, transportation, and security.

3 Literature Review

Using the technology module in the Census Bureau's 2018 ABS, Zolas et al. (2020) offer the first comprehensive estimates of U.S. firms' adoption of AI and Robotics in recent years. McElheran et al. (2024), leveraging the same 2018 ABS data, specifically explore the connection between early-stage firm characteristics and AI adoption, as well as how startup growth correlates with AI use. Acemoglu et al. (2022) document, using a different technology module from the 2019 ABS, the prevalence of five key technologies (AI, Robotics, Cloud Computing, Specialized Equipment, and Specialized Software) among US firms, as well as their impacts on the workforce, based on self-reports from firms. Their study also investigates the connection between firm characteristics and technology adoption, and the effects of technology use on firm outcomes. Bonney et al. (2024a,b) utilize data from the Business Trends and Outlook Survey to offer the latest estimates of current and expected near-term AI use rate by firms. Dinlersoz, Dogan, and Zolas (2024) draw on administrative microdata from the Census Bureau's Business Formation Statistics to analyze business applications focused on developing or using AI technologies. They compare the performance of AI-related business applications with those of other business applications, and study the performance of businesses originating from these applications in a number of outcomes, including employment, revenue, and failure rate. Dinlersoz and Wolf (2023) provide plant-level evidence on the connection between total factor productivity, the degree of automation, and labor share by using an earlier Census Bureau survey – the Survey of Manufacturing Technology.

A few stylized facts have emerged from this empirical literature based on Census Bureau surveys and administrative data. First, the adoption rates of advanced technologies, such as AI and Robotics, are relatively small–for instance, AI is currently used by around 9% of firms in producing goods or services (as of Spring 2025), up from about 3.2% for the period 2016-2018.[2] Second, the adoption of advanced technologies is concentrated in large firms, and in young firms, controlling for size. Third, advanced technology users generally exhibit better overall performance in terms of employment, revenue, and labor productivity. Furthermore, early characteristics of firms, including owner motivations and funding type in initial stages, are related to AI adoption. In particular, AI adoption is positively associated with early growth in startups. At the same time, users of advanced technologies overwhelmingly report overall upskilling of their workforce, together with an increase in STEM skills. However, there is little evidence that technology use necessarily results in instances of employment decline. For most technologies, a large fraction of firms report experiencing no change in employment. For those firms that experience a change in employment due to technology, an increase in employment seems to be more common than a decrease–though the relative incidences of the two outcomes depend on the technology. Despite the positive association between firm performance and technology presence, the analysis has so far not established a more causal link between firm outcomes and advanced technology use. The empirical analysis here takes steps in that direction by using a difference-in-differences approach with matched controls for firms adopting advanced technologies, with the caveat that selection into VC backing and technology adoption may remain due to unobserved factors correlated with the outcome measures.

Several studies use European data and a difference-in-differences framework to estimate the effect of technology adoption on employment. For instance, Koch et al. (2021) find that robot adoption results in net job creation, while Bonfiglioli et al. (2024) and Bessen et al. (2025) show a negative effect of automation technology on workers. Most other recent studies on US firms do not utilize information on the timing of adoption. For instance, Acemoglu et al. (2023) use the 2019 ABS data and the general adoption episodes for robotics (late 1990s to early 2000s) and AI (after 2010) in the United States to explore whether adopters grew more rapidly than non-adopters. The absence of adoption timing in the 2019 ABS, however, limits the ability to evaluate precisely selection and treatment effects. By incorporating firm-level adoption timing and using matching techniques to control for selection bias, the estimates presented here suggest a significant positive association between advanced technology

[2] See Business Trends and Outlook Survey data at https://www.census.gov/hfp/btos/data_downloads.

and employment in adopting US firms.

Macroeconomic models of venture capital financing are rare. Akcigit et al. (2022), Ando (2023), Ates (2018), and Greenwood, Han, and Sanchez (2022) are four examples. Akcigit et al. (2022) use US Census data to demonstrate empirically that startups funded by venture capital tend to outperform those that are not. They build a quantitative model that highlights the synergistic relationship between an entrepreneur and a venture capitalist, which is absent when entrepreneurs borrow from bankers in their framework. Ando (2024) adds angel investors into the mix. He finds that VC-backed firms perform better than those funded by angel investors, who in turn outperform firms financed by banks. A firm dynamics model of this process is then calibrated and matched with the data. A Schumpeterian growth model incorporating venture capital is advanced by Ates (2022). Firms choose how far to launch their productivities relative to incumbents, a feature also in Akcigit et al. (2022). Ates's model is not matched with data on firm startups. A dynamic contracting model of VC financing, embedded into an endogenous growth model, is developed in Greenwood, Han, and Sanchez (2022). Their model is matched up with data on the VC funding round process. They do not use firm-level US Census data. None of these studies, however, investigate the role venture capital plays in encouraging the adoption of advanced technologies. Additionally, they do not explore whether venture capital's contribution differs between high-tech and non-high-tech startups.

Finally, King and Levine (1993) provide some of the earliest evidence linking financial development to economic growth. Since then, a large body of research—both with and without borrowing constraints—has sought to explain this relationship [see, for example, Cavalcanti et al. (2023), Greenwood, Han, and Sanchez (2022), and Quadrini (2000)]. Empirically, however, it remains challenging to separate the pure financing effect of venture capital from its potential synergy effects. Akcigit et al. (2022, Table 2) do find evidence of a synergy effect even after controlling for initial funding amounts. The analysis presented here abstracts from borrowing constraints.

4 Data

The empirical analysis employs three data sets. The Annual Business Survey (ABS), conducted from 2018 to 2023 and designed jointly by the US Census Bureau and the National Center for Science and Engineering Statistics, provides information on advanced technology use among nonfarm employer businesses in the United States. Each survey collects data for the reference year prior to the survey year (e.g. the 2018 ABS collects data for the year 2017). The 2018 and 2023 ABS samples included 850,000 businesses, corresponding to the Economic Census years 2017 and 2022, while around 300,000 businesses were sampled annually between 2019 and 2021. The ABS is detailed in Appendix A. The 2018 and 2023 ABS samples included 850,000 businesses, corresponding to the Economic Census years 2017 and 2022, while around 300,000 businesses were sampled annually between 2019 and 2021. Examples of the business technologies in the survey include artificial intelligence, robotics, cloud computing, RFID, specialized software, and specialized equipment. The ABS provides information on whether a firm used any given advanced technology during the survey reference period. The 2023 ABS also records the timing of advanced technological adoption, measured in 5-year intervals.[3] The ABS also provides information on initial characteristics of startups and their owners.

The presence of VC financing at the founding of a business is recorded in the 2018 ABS. This information on VC financing is supplemented with data from Pitchbook and data on initial public offerings (IPOs).[4] As a result, VC funding is identified for all firms in the ABS for the from 2017 to 2023. Details such as deal size and equity stakes acquired by investors are sourced from Pitchbook–again, see Appendix A. This dataset is merged with the Census Bureau's Business Register/Standard Statistical Establishment List by name and address matching, and then linked to the ABS using firm identifiers in Census Bureau data sets.

[3]See the ABS website (https://www.census.gov/programs-surveys/abs.html) for additional information.

[4]The SDC New Issues and the IPO dataset combined and released by Jay Ritter are used.

Table 1: Descriptive Statistics

	Fraction (%)	Employment Share (%)	Revenue Share (%)
Tech Firms	11.63%	38.76%	44.11%
VC-backed Firms	0.64%	13.26%	14.98%

Note: The sample consists of firms in the ABS (2018–2022). Tech firms are defined as firms that have adopted advanced technologies. VC-backed firms are firms that have raised VC financing, according to the ABS (2018) or Pitchbook. Employment share is the share of employment contributed by tech firms or VC-backed firms out of total employment in the economy. Employment and revenue are obtained from the LBD (2017-2021). For firms whose revenue is missing in the LBD, sales in the ABS are used instead. All statistics are computed using the LBD weights.

Finally, some key firm outcomes and characteristics, including employment, revenue, firm age, industry, and location, are drawn from the Longitudinal Business Database (LBD). The LBD is a longitudinally linked dataset covering near all nonfarm employer businesses in the United States from 1976 to 2022 (the latest year available at the time of the analysis). The combined data set allows for the evolution of firm size and other outcomes to be examined separately for firms that use or do not use advanced technologies, and for firms that raise or do not raise VC financing.

5 Empirical Evidence

5.1 Descriptive Analysis

Turn now to some stylized facts on advanced technology adoption and VC backing in a startup and ensuing firm outcomes. This subsection documents that VC-backed firms have a significantly higher advanced technology adoption rate than non-VC-backed firms. In addition, firms that adopt advanced technologies and receive VC backing outperform firms that do not. The relationships are robust after controlling for detailed firm characteristics. Furthermore, it is shown that VC-backed firms with high-tech are the largest in employment and revenue, followed by VC-backed non-high-tech firms, and non-VC-backed high-tech firms. The non-VC-backed, non-high-tech firms have the lowest average size overall.

Table 1 provides descriptive statistics, based on the main sample of firms from the Annual Business Survey (ABS) 2018-2022.[5] The advanced technology adoption rate in the sample is 11.63%–sample weights are used so as to make the statistics representative of the US firm population. Despite the relatively small fraction of high-tech firms (i.e., firms that have adopted advanced technologies), they account for 38.76% of total employment and 44.11% of total revenue in the economy–consistent with prior findings from the ABS [see, e.g., Acemoglu et al. (2022)]. The fraction of VC-backed firms is even smaller, 0.64%. Their employment and revenue share in the economy is 13.26% and 14.98%, however, demonstrating that VC-backed firms are significantly larger than non-VC-backed firms and that they constitute an economically significant segment of firms [see also Dinlersoz et al. (2022)].

Table 2 documents the firm-level relationship between VC presence and technology adoption. It also shows the relationship between VC presence and technology adoption, on the one hand, and employment and revenue, on the other. An OLS regression is employed on a pooled cross-section for the measurement years 2017-2021. Fixed effects are included for year-industry (4-digit NAICS) interactions, state, and firm age.[6] Column (1) indicates that VC-backed firms have a significantly higher technology adoption rate than non-VC-backed firms. The technology adoption rate of VC-backed firms is 38.61%, 27 percentage points higher than the overall technology adoption rate in the sample (11.63%).

[5]This was the broadest sample available at the time of our initial analysis. The adoption timing information from the ABS (2023) is used in the later part of the analysis.

[6]The year $\times$ 4-digit NAICS fixed effects absorb industry-level price differences across years, which are relevant in the regression with revenue. For multi-unit firms, the industry and state refer to the industry and state with the largest employment share.

Table 2: Baseline Regression

	(1) adoption	(2) ln(emp)	(3) ln(rev)
VC-financed	0.170***		
	(0.00677)		
VC-tech		1.660***	1.479***
		(0.0358)	(0.0553)
VC-nontech		1.051***	1.048***
		(0.0298)	(0.0404)
NonVC-tech		0.483***	0.626***
		(0.00562)	(0.00706)
+ Fixed effects			
R-squared	0.0734	0.2403	0.2151
N (rounded)	1,050,000	1,050,000	1,050,000
Adoption rate (all firms)	11.63%		
Adoption rate (VC firms)	38.61%		

Notes: The fixed effects are: year×NAICS (4 digits), state, and firm age. The dependent variable in column (1) is firms that have adopted advanced technologies. The dependent variables in columns (2) and (3) are firm-level ln(employment) and ln(revenue), respectively. The independent variables are binary variables that are equal to one if firms belong to the category. The regressions use LBD weights. Standard errors are shown in parentheses and are clustered by firmid. * p<0.05 ** p<0.01 *** p<0.001. N is rounded to protect confidentiality.

Columns (2) and (3) in Table 2 compare firm-level employment and revenue, respectively, for each of the four mutually-exclusive groups of firms. Each group is defined by a pair of indicators that identify a distinct combination of VC presence and technology adoption status. VC-tech firms (i.e., firms that have VC backing and that have adopted advanced technologies) have significantly higher employment and revenue than the omitted group of firms that have no VC backing and no advanced technology. The size gap is approximately 425% [$= 100 \times (\exp(1.660) - 1)$] and 338% [$= 100 \times (\exp(1.479) - 1)$], respectively, for employment and revenue, based on the estimated coefficients. VC-backed, non-tech firms also exhibit significantly higher employment and revenue than the omitted group, but less so than VC-tech firms. Non-VC-backed, tech firms rank third in outcomes relative to the omitted group.

Regression analysis in Table 3 controls for a large number of business owner characteristics and firm business strategies/motivations related to innovation and growth. The regression examines, for robustness, the extent to which the relationships found in Table 2 are explained away by these intrinsic characteristics of the businesses that may be correlated with both VC backing and firm outcomes. The large set of control variables are drawn from the 2018 ABS and defined as follows: "Adv degree" is whether the business owner has a master's or higher degree; "Prior business" is whether the business owner has prior business ownership; "Age 35–54" is whether the business owner's age is between 35 and 54; "Age 55+" is whether the business owner's age is above 55; "Lifestyle" is whether the proprietor owns a business for a lifestyle reason (i.e., flexibility or work-life balance); "Process innov" is whether the business had process innovation (i.e., introduction or improvement in (i) methods of manufacturing, (ii) logistics, delivery or distribution methods, or (iii) supporting activities for processes); "Product innov" is whether the business had product innovation (the introduction or improvement of goods or services); "Patents" is whether the business owns patents or has pending patents; "IP important" is whether intellectual property is important for the business; and "Growth" is whether the business has a strategy to grow.[7, 8]

[7] Similar control variables are also used in McElheran et al. (2024) in their analysis of AI adoption. Here, the definition of technology includes a broad set of advanced technologies beyond AI. Moreover, firm outcomes are examined as a function of the interaction between VC backing and technology adoption.

[8] A business owner is defined as the person who owns the largest percentage in the business. If several owners have the same percentage in the business, "Owner 1" in the ABS form is chosen as the owner.

Table 3: Regression with detailed control variables using ABS (2018)

	(1) adoption	(2) adoption	(3) adoption	(4) ln(emp)	(5) ln(emp)	(6) ln(emp)	(7) ln(rev)	(8) ln(rev)	(9) ln(rev)
VC-financed	0.106***	0.102***	0.0606***						
	(0.00837)	(0.00836)	(0.00819)						
VC-tech				0.999***	0.919***	0.638***	0.821***	0.730***	0.362***
				(0.0485)	(0.0486)	(0.048)	(0.0761)	(0.0768)	(0.0774)
VC-nontech				0.661***	0.604***	0.459***	0.686***	0.618***	0.428***
				(0.0287)	(0.0286)	(0.0284)	(0.0384)	(0.0386)	(0.0386)
nonVC-tech				0.397***	0.384***	0.276***	0.484***	0.469***	0.327***
				(0.00681)	(0.00677)	(0.00678)	(0.00895)	(0.00889)	(0.00894)
Adv degree		0.0232***	0.0153***		0.131***	0.109***		0.174***	0.146***
		(0.00183)	(0.0018)		(0.00629)	(0.00619)		(0.00878)	(0.00864)
Prior business		0.0194***	0.0131***		0.181***	0.162***		0.184***	0.159***
		(0.00121)	(0.00119)		(0.00423)	(0.00419)		(0.0059)	(0.00583)
Age 35–54		0.0013	0.00908***		0.011	0.0316***		0.0218	0.0496***
		(0.00258)	(0.00256)		(0.00907)	(0.00902)		(0.0128)	(0.0128)
Age 55+		-0.00402	0.00857**		-0.172***	-0.134***		-0.291***	-0.240***
		(0.00269)	(0.00266)		(0.00948)	(0.00943)		(0.0134)	(0.0133)
Lifestyle		-0.000642	-0.00774***		-0.156***	-0.207***		-0.163***	-0.232***
		(0.00113)	(0.00113)		(0.00417)	(0.00414)		(0.00569)	(0.00565)
Process innov			0.0905***			0.155***			0.215***
			(0.00176)			(0.00521)			(0.00714)
Product innov			0.0176***			-0.00345			0.000273
			(0.00108)			(0.00401)			(0.00552)
Patents			0.0372***			0.261***			0.349***
			(0.0052)			(0.0152)			(0.0211)
IP important			0.0872***			0.273***			0.328***
			(0.00192)			(0.00574)			(0.00777)
Growth			0.0222***			0.301***			0.411***
			(0.0011)			(0.00415)			(0.00574)
+ Fixed fx									
R-squared	0.0363	0.0377	0.0719	0.2086	0.2203	0.2472	0.1745	0.1860	0.2132
N (rounded)	346,000	346,000	346,000	346,000	346,000	346,000	346,000	346,000	346,000
Adopt rate	10.03%								
VC-adopt rate	22.98%								

Notes: Included fixed effects are: year×NAICS (4 digits), state, and firm age. All independent variables are binary variables. The regressions are weighted by the LBD weights. Standard errors are in parentheses and are clustered by firmid. * $p<0.05$ ** $p<0.01$ *** $p<0.001$. N is rounded to protect confidentiality.

VC-backed firms have a higher technology adoption rate than non-VC-backed firms even after controlling for detailed business characteristics, as Table 3 shows. The coefficient on the VC-financed dummy declines from 0.106 in column (1) to 0.061 in column (3), where all control variables are employed. Firms' innovation activities are controlled for in column (3). In particular, process innovation is strongly associated with technology adoption. Columns (4)-(9) explore the relationships between the VC financing/technology adoption status and firm outcomes. The difference in firm size between VC-tech firms and the omitted group (i.e., non-VC, non-tech firms) attenuates as more business characteristics are controlled for, but remains significant (statistically and economically) in the regression with all control variables. A similar pattern is observed for the comparison between VC-backed, non-tech firms, and the omitted group, and for the comparison between non-VC, tech firms and the omitted group.

The difference in firm-size distributions between VC-backed and non-VC-backed firms, and between tech and non-tech firms, is examined separately in Table 4. Firm size is measured in terms of employment and revenue. The Kolmogorov-Smirnov test is employed to test whether one distribution stochastically dominates (in a first order sense) the other based on the difference between two cumulative distribution functions. The table shows that the VC-backed firm-size distribution stochastically dominates that of non-VC-backed firms (0 vs 1), because statistically speaking the cumulative distribution function of the latter lies above the former, while the opposite does not hold (1 vs 0). Likewise, the tech firm-size distribution stochastically dominates that of non-tech firms, but again no dominance in the opposite direction is detected.

Table 4: Kolmogorov-Smirnov test

Variable	**Context**	**D(0 vs 1)**	**D(1 vs 0)**	**p(0 vs 1)**	**p(1 vs 0)**
ln(emp)	VC	0.3331	0.0000	0.0000	1.0000
ln(emp)	Tech	0.2217	0.0000	0.0000	1.0000
ln(rev)	VC	0.3159	-0.0087	0.0000	0.1141
ln(rev)	Tech	0.2316	0.0000	0.0000	1.0000

Notes: Depending on the context, 0 and 1 refer to the empirical cumulative distribution functions for either ~VC and VC funded, respectively, or ~high-tech and high tech. D(0 vs 1) is the test statistic that measures the distance between the cumulative distribution functions 0 and 1. D(1 vs 0) is the test statistic that measures the distance between the cumulative distributions functions 1 and 0. p(0 vs 1) and p(1 vs 0) give the significance level (probability) of the tests. The sample consists of firms in the ABS (2018–2022).

Table 5: Regression with Timing of AI Adoption (Two-Way Fixed-Effect Model)

	(1)	**(2)**
	ln(**emp**)	ln(**rev**)
Adoption (during)	0.136***	0.174***
	(0.0116)	(0.0146)
Adoption (post)	0.168***	0.260***
	(0.0291)	(0.0379)
VC (during)	0.266***	0.118
	(0.0453)	(0.103)
VC (post)	0.844***	1.103***
	(0.0516)	(0.102)
+ Fixed effects		
R-squared	0.871	0.894
N (rounded)	6,590,000	4,800,000

Notes: The included fixed effects are: firm, year×NAICS (4 digits), and firm age. The sample consists of firms in ABS (2023) linked to LBD (1978-2021). Revenue is obtained from the LBD (1997-2021). The regressions are weighted by sampling weights [tabulation weights in ABS (2023) in the first column and by tabulation weights × probability weights associated with LBD revenue in the second column]. Standard errors are shown in parentheses and are clustered by firmid. * p<0.05 ** p<0.01 *** p<0.001. N is rounded to protect confidentiality.

Finally, the regression analysis in Table 5 exploits information on the timing of technology adoption. The ABS 2023 asked firms about the timing of AI adoption. The response options were five-year intervals (prior to 1990, 1991–1995, ..., 2016–2020, 2021–present). This data is linked to the Longitudinal Business Database (LBD) using firm identifiers. The dummy variable, Adoption (during), indicates that the year for the dependent variable, either employment or revenue, was within the five-year interval during which AI adoption occurred, and the dummy variable, Adoption (post), indicates that the year was after the adoption interval. These two dummy variables take on a value of zero for the years before the adoption interval and in all years for firms that have never adopted AI. Similarly, the dummy variable, VC (during), indicates that the firm raised its first VC financing in the year recorded for either employment or revenue, and the dummy variable, VC (post), means that the firm had raised VC financing before the year. Similar to the adoption timing case, the omitted category here is all years before VC financing and all years for firms that had no VC funding. The linear regression absorbs fixed effects of the firm, year-industry interactions, and firm age. Therefore, the coefficient on Adoption (during) examines how firm size changes during the years of AI adoption relative to years before AI adoption within the firm, controlling for year-industry effects and firm age effects. The table shows that firm size, measured as ln(employment) and ln(revenue), increases during AI adoption and even further after AI adoption. The size of VC-backed firms also increases after the VC funding year. The magnitudes of the estimated effects are larger for VC financing compared to AI adoption.

The findings are in line with a growing literature that documents that the adoption of automation technologies is associated with an increase in employment at the firm level–this literature is surveyed in Restrepo (2023). This literature is further extended here by documenting the increase in employment and revenue after technology adoption in a representative sample of US firms.

The empirical evidence thus far demonstrates that VC-backed firms are larger in size than non-VC-backed firms and that firms with advanced technologies are larger than firms using more conventional technologies. These results remain robust after controlling for detailed firm characteristics or firm fixed effects. Furthermore, firms with advanced technologies that receive VC funding stand out from others in terms of outcomes.

A natural question to ask is whether firms expand their size as a result of VC backing and technology adoption, or alternatively whether firms that receive VC backing and adopt advanced technologies are intrinsically different and perform better than other firms even in the absence of VC backing and advanced technologies. To partially address this question, difference-in-differences (DiD) analyses are employed to estimate separately the treatment effects of VC presence and technology adoption. Furthermore, potential heterogeneity in the treatment effect of VC presence for high-tech and non-high-tech firms is examined.

Difference-in-differences (DiD) analyses are conducted using matched control samples to estimate the average treatment effect on firm employment. The DiD estimator is valid under the parallel trends assumption and the assumption that there is no anticipation of treatment. The matched sample is constructed to improve similarity between treated and control firms, particularly with respect to pre-treatment trends and estimated probabilities of treatment. While matching can enhance the credibility of the DiD design, the results should be interpreted as suggestive of causal effects. This is because unobserved factors that simultaneously influence selection into VC backing or technology adoption and firm outcomes cannot be fully ruled out.

The sample used for estimating the VC treatment effects contains treated firms and their controls that appear in the ABS for the years 2018 to 2023–the broadest sample where both technology adoption *and* VC presence in firms are observed. The year of first VC funding for VC-treated firms is obtained from Pitchbook. Firm employment is drawn from the LBD (1978–2022). The matched sample used for estimating the treatment effects of advanced technology contains treated firms and their controls that appear in ABS for 2023, since the timing of technology adoption is available only for 2023. The latter sample is larger than the former because only a relatively small number of firms receive VC backing. For each treated firm, non-treated firms are selected in the base sample that share the same firm age, 3-digit NAICS industry, and employment-size bin as the treated firm in the year of treatment.

In addition, VC-treated firms are matched with non-VC-control firms that share the same technology adoption status. Likewise, tech-treated firms are matched with non-tech control firms that share the same VC status. This allows estimating the treatment effect of VC conditional on technology adoption status, and vice versa. Among non-treated firms matched with each treated firm, a control firm that is closest in the probability of treatment and ln(employment) before treatment is selected for the analysis.[9]

The ABS (2023) coarsely measures the timing of advanced technology adoption using 5-year intervals (e.g., whether adoption occurs in the windows 2001-2005, 2006-2010, etc.). The first year in a given 5-year interval is regarded as the year of technology adoption in the DiD analysis.[10] This yields a conservative estimate of treatment effects, as some treated firms under this definition are further treated after the first year of the interval. Figure 1 documents the fraction of firms with advanced technologies (i.e., AI and robotics) that adopt the technologies within a given firm age range (shown in the x-axis). Nearly half of the firms in the process of adopting advanced technologies are between 0 and 4 years old (and nearly 70% are less than 10 years old). While some firms adopt technology later in their life cycle, the high fraction of young firms (0-4 years of age) among technology adopters suggests that the technology adoption decision is often made at the beginning of a firm's life.[11]

The treatment effect on firm employment is estimated using three methodologies. The first approach is a DiD analysis using two-way fixed effects (TWFE) regression:

$$Y_{it} = \alpha_i + \phi_t + \lambda_{\tau_{it}} + \sum_{j=-3, j\neq 0}^{10} \gamma^j \cdot D_i \cdot \mathbf{1}\{\tau_{it} = j\} + \epsilon_{it},$$

where the subscript i refers to a firm and t to the time period. The variable $\tau_{it} \in \{-3, -2, -1, 0, 1, 2, \cdots, 10\}$ is time relative to (potential) treatment, where $\tau_{it} = 1$ indicates the first treated period.[12] There are three fixed effects: firm fixed effects, α_i, calendar year fixed effects, ϕ_t, and time relative to treatment fixed effects, $\lambda_{\tau_{it}}$. This specification is used to estimate the dynamic treatment effects γ^j in each post-treatment period ($j = 1, ..., 10$) relative to $\tau_{it} = 0$. The estimated coefficients γ^j for pre-treatment periods ($j = -3, ..., -1$) verify the absence of pre-treatment trends. In addition, a static TWFE model with only one dummy variable $\mathbf{1}\{\tau_{it} \geq 1\}$ is used to estimate the average effect from $\tau = 1$ to $\tau = 10$. Finally, a dynamic TWFE regression with interaction terms between τ_{it} for VC treatment and technology adoption status, $\sum_{j\neq 0} \gamma^{j,\text{het}} \cdot D_i \cdot \mathbf{1}\{\tau_{it} = j\} \cdot \mathbf{1}\{\text{tech}_i = 1\}$, is also used to estimate the heterogeneous treatment effect of VC backing between high-tech and non-high-tech firms.

While commonly used and easy to implement, the TWFE approach has shortcomings.[13] Therefore, two other methods are used that are designed to remedy these shortcomings. The first alternative

[9]For each treated-control pair, the distance is computed as $[\text{P(treat)}_i - \text{P(treat)}_{i'}]^2 + \sum_{\tau=-4}^{0}[\ln(\text{emp})_{i,\tau} - \ln(\text{emp})_{i',\tau}]^2$, where the subscript $\tau = 0, -1, ..., -4$ is time relative to is time relative to (potential) treatment and the subscripts (i, i') refer to treated-control pairs. The probability of treatment is estimated by a Probit model, where the right-hand-side variables include the detailed control variables from the ABS used in the previous regression analysis (Table 3), NAICS 4-digit industry code, ABS vintage, year, and tech-adoption status in the case of estimating the probability of receiving VC (or VC status in the case of estimating the probability of tech adoption). A similar approach to constructing a matched sample is adopted by Dinlersoz et al. (2023). See Ham and Mitraix (2024) for an analysis of the benefits and costs associated with matching on pre-treatment outcomes.

[10]The ABS (2023) reports the adoption of five technologies, specifically artificial intelligence, robotics, cloud-based computing systems, specialized software, and specialized equipment. AI and robotics are the advanced technologies used here. If a firm has both technologies, then the earliest year for the adoption of one of the technologies is used for the treatment event.

[11]The exact timing of adoption in the model may not be critical, as the nature of the business idea formed in the early stages of a firm may dictate the compatibility of advanced technology with the business, even if actual adoption occurs at a later stage.

[12]Denote the treatment period for treated firms by t_i^*. Then, $\tau_{it} = t - t_i^* + 1$. The treatment period for control group firms is defined by that of their matched treated firms [i.e., $t_i^* = t_{i'}^*$ for a treated-control pair (i, i')].

[13]The TWFE methodology does not correctly aggregate treatment effects across cohorts, since treatment timing varies across firms. Instead, it produces a weighted average of different treatment effects, which can be biased due to negative weighting, particularly if treatment effects evolve over time. In addition, if treatment effects are heterogeneous across cohorts, the TWFE estimate may not represent a meaningful average treatment effect on treated (ATT) firms.

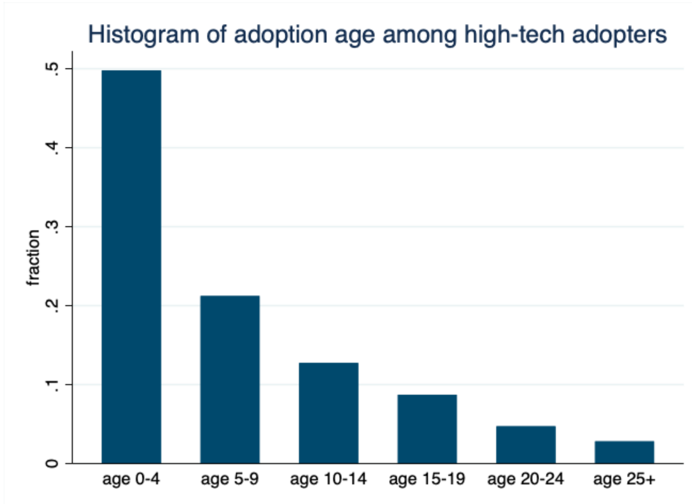

Figure 1: The timing of advanced technology adoption. The ABS (2023) reports the timing of technology adoption in 5-year intervals. The age of a firm reporting the adoption of high technology within one of these intervals is known. Thus, firms may appear in more than one age bin. The sample consists of firms born in or after 1991 that have adopted advanced technologies. Observations are weighted by the LBD weights.

method is a DiD estimation developed by De Chaisemartin and d'Haultfoeuille (2024), referred to as the "CD method" hereafter.[14] Finally, the treatment effect is also estimated using the "DiD by hand" method, following the approach of Callaway and Sant'Anna (2021).[15] In this method, the DiD is computed for each treated-control pair (i, i') and then aggregated by taking the sample average.[16]

Table 6 reports estimates of the treatment effects. Here, the average effects for the post-treatment periods from $\tau = 1$ to $\tau = 10$ are shown. The first row reports the treatment effect of VC, where the employment of VC-backed firms increases on average over the 10 year horizon by 0.35–0.40 ln points (or by 42-49%) after receiving VC treatment relative to control firms. The second row shows that the average VC treatment effect is larger for high-tech firms than for non-high-tech firms by 0.12-0.18 ln points (or by 13-20%).[17] The third row reports the estimated treatment effect of high-tech adoption. The employment of treated firms increases on average by 0.038–0.065 log points (or by 4-7%) following

[14] The CD method is robust to potential bias when the treatment effect is heterogeneous across groups or periods, and provides a DiD estimator that corrects for negative weighting issues. It also offers a robust estimator that recovers a weighted average of correct ATT values across cohorts. The CD method is implemented by using the *did_multiplegt_dyn* Stata package. The outcome variable is ln(employment), the group variable is firm ID, the time period variable is τ, and the treatment variable is equal to 1 if $\tau_{it} \geq 1$ for treated firms (and 0 otherwise). Only never-switchers are used as control units. Observations are weighted by the LBD weights.

[15] The advantage of this method is that it explicitly compares firms within each treated-control pair and properly accounts for treatment effect heterogeneity, while being robust to staggered treatment.

[16] The DiD for each pair is defined by $\text{DiD}_{i\tau} = (Y_{i,t_i^*-1+\tau} - Y_{i,t_i^*-1}) - (Y_{i',t_i^*-1+\tau} - Y_{i',t_i^*-1})$, where again t_i^* is the treatment period for treated firms. The treatment effect is estimated by the sample average of $\text{DiD}_{i\tau}$, where observations are weighted by the LBD weights to render results representative of all US employer firms. Standard errors are clustered by firms in the average treatment effect from $\tau = 1$ to $\tau = 10$.

[17] With a TWFE regression, the heterogeneity of the VC treatment effect is estimated by γ^{het} in the following specification:

$$Y_{it} = \alpha_i + \phi_t + \lambda_{\tau_{it}} + \gamma \cdot D_i \cdot \mathbf{1}\{\tau_{it} \geq 1\} + \gamma^{\text{het}} \cdot D_i \cdot \mathbf{1}\{\tau_{it} \geq 1\} \cdot \mathbf{1}\{\text{tech}_i = 1\} + \epsilon_{it}.$$

With CD estimation and DiD by hand, the treatment effect is separately estimated in the sample of high-tech firms and in the sample of non-high-tech firms. The difference between the two estimators is then taken. The standard errors are defined as $\text{se}^{\text{het}} = \sqrt{(\text{se}^{\text{tech}})^2 + (\text{se}^{\text{non-tech}})^2}$, as in the standard two sample unequal variance t-test.

Table 6: Difference-in-Differences Analyses (Average Effect from $\tau = 1$ to $\tau = 10$)

	Methodologies			
	TWFE	**CD**	**DiD by hand**	**Observations**
VC Treatment Effect	0.355	0.403	0.360	35,000
	(0.036)	(0.037)	(0.029)	
VC Heterogeneity (tech - non-tech)	0.183	0.156	0.123	
	(0.054)	(0.071)	(0.057)	
High-Tech Treatment Effect	0.044	0.065	0.038	214,000
	(0.007)	(0.009)	(0.007)	

Notes: The outcome variable is ln(employment). Standard errors are in parentheses. The DiD analysis is conducted three ways: namely, with (i) two-way fixed effects (TWFE) regressions, (ii) estimators developed by De Chaisemartin and d'Haultfoeuille (CD), and (iii) DiD by hand based on the sample average of the difference-in-differences between treated-control pairs. The sample used for estimating the VC treatment effects contains treated firms and their controls that appear in ABS (2018–2023)–the broadest sample where both advanced technology adoption *and* VC presence in firms are observed. The sample used for estimating the treatment effects of advanced technology contains only the treated firms and their controls in ABS (2023), since the timing of technology adoption is available only for 2023. The latter sample is larger than the former since a small number of firms receive VC backing. Observations are weighted by LBD weights. The number of observations is rounded to protect confidentiality.

Table 7: Difference-in-Differences Analyses (Treatment Effect at $\tau = 10$)

	Methodologies		
	TWFE	**CD**	**DiD by hand**
VC Treatment Effect at $\tau = 10$	0.705	0.712	0.548
	(0.073)	(0.093)	(0.067)
VC Heterogeneity at $\tau = 10$	0.495	0.372	0.285
	(0.120)	(0.187)	(0.141)
High-Tech Treatment Effect at $\tau = 10$	0.135	0.134	0.067
	(0.024)	(0.031)	(0.020)

Notes: The outcome variable is ln(employment). Standard errors are in parentheses. The DiD analysis is conducted three ways: namely, with (i) two-way fixed effects (TWFE) regressions, (ii) estimators developed by De Chaisemartin and d'Haultfoeuille (CD), and (iii) DiD by hand based on the sample average of the difference-in-differences between treated-control pairs. Observations are weighted by the LBD weights.

technology adoption, relative to the control firms without high-tech adoption. While the magnitude of the estimated treatment effects varies to some extent across estimation methods, overall the findings consistently suggest that both VC backing and technology adoption have a positive effect on firm employment. Moreover, the VC treatment effect is larger for firms that adopt advanced technology than for those that do not.

Table 7 reports estimates of the treatment effect in the 10th year after the treatment events. The treatment effect in the 10th year is larger than the average effect over the post-treatment periods from $\tau = 1$ to $\tau = 10$. Figures 2 and 3 illustrate the dynamic treatment effects from $\tau = -3$ to $\tau = 10$. In both figures, the pre-treatment differences between treated and control firms are mostly statistically insignificant, as a result of using matched control samples. The VC treatment effect in Figure 2 increases over time, whereas the high-tech treatment effect peaks around $\tau = 7$. These plots present the results of TWFE analyses, while the dynamic effects estimated using the CD method and DiD by hand are provided in Appendix B. All methods yield qualitatively similar results.

To conclude, estimates of the treatment effects of VC backing and technology adoption have been obtained. The analysis also provides new estimates of the VC effects on high-tech versus non-high-tech firms. While the previous literature estimates general effects of VC on firm employment [e.g., Puri and Zarutskie (2012) and Akcigit et al. (2022)], it has not quantified the heterogeneous effects of VC by firm technology type. The analysis also provides some of the very first estimates of the treatment

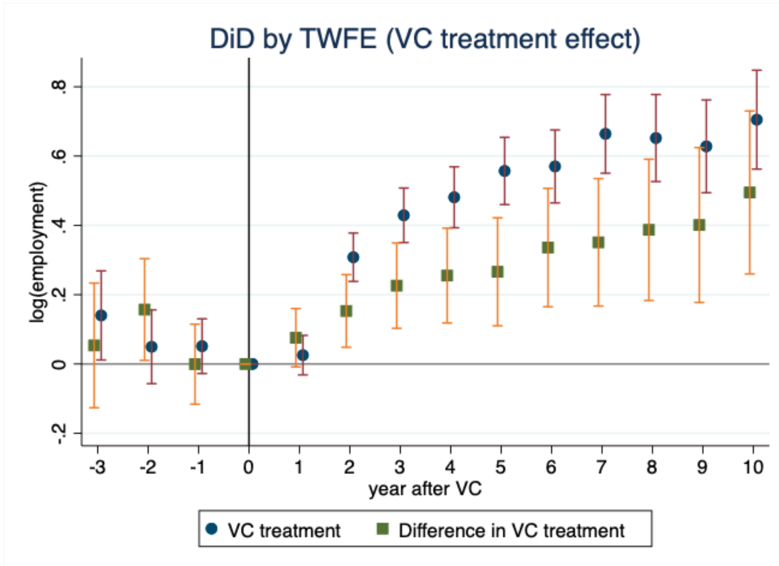

Figure 2: Dynamic Treatment Effect (TWFE) of venture capital backing. Difference in VC treatment refers to the difference in the impact of VC treatment on high-tech adopters versus non-high-tech ones.

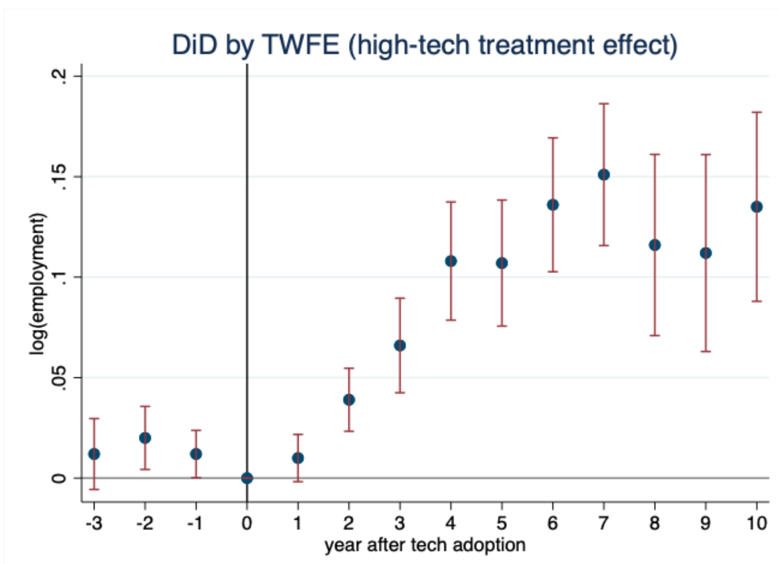

Figure 3: Dynamic Treatment Effect (TWFE) of high-tech adoption

effects of advanced technology adoption on firm employment in the United States. A lack of data on the timing of adoption for advanced technologies at the firm level in the United States has prevented such estimates to date. The current analysis does not identify the mechanisms driving the positive relationship between a startup's performance and venture capital or advanced technology—an issue to be explored in future research. Furthermore, the analysis so far does not necessarily speak to the general equilibrium effects associated with the presence of venture capital and advanced technology in the US economy. This task is tackled by the model below. Overall, the current findings motivate key premises of the model: advanced technology adoption tends to occur very early in the life-cycle of a firm, both VC and technology presence appear to matter for firm outcomes, and VC tends to matter more for firms with advanced technology.

6 The Model

The analysis develops a model with three phases of a firm's life cycle that focuses on the startup process. In the first phase of life, new entrepreneurs have an idea but no funding. Adolescent startups are funded by either a bank or a venture capitalist in the second phase. In the third phase, adult firms hire capital and labor on frictionless spot markets. The setup for the model is described first. Then, the analysis proceeds backwards in time starting with production by adult firms. Moving reversely, it then examines adolescent startups. Two types of funding contracts are considered: the partnership agreement between entrepreneurs and venture capitalists and the terms of a loan from bankers to entrepreneurs. After this, the source of funding for an adolescent startup is determined. Subsequently, production by a newborn startup in the first phase of life is formulated. Then, the choice of technology by a newborn entrepreneur is analyzed. Finally, an equilibrium for the model is specified.

6.1 Setting the Stage

In the first phase a unit mass of entrepreneurs are blessed with ideas. Newly born startups employ one of two production technologies: an advanced production technology (dubbed high tech) and a conventional one (dubbed non-high tech). An entrepreneur's production technology has two attributes: its current level of productivity and its potential for the future. It's possible for a startup's current productivity, and hence output, to be low while its future potential looks promising. The joint distributions over initial productivities and potentials differ across the advanced and conventional production technologies. Some ideas are better suited for using the advanced technology than others (military drones vs restaurants). An entrepreneur decides in the first phase which production technology to use based on its expected profits, which depends on odds of receiving bank or VC financing and the technology that will be adopted. Figure 1 shows that the modal firm in United States using advanced technologies is very young, less that 4 years old, so modeling this decision at an early phase seems appropriate. They then approach financiers, either banks or venture capitalists, for funding of their startups. The funding is used to acquire the capital needed for production in the second phase. Adolescent startups grow into adult firms in the third phase of the life cycle. Adolescent startups are sold off at the end of the second phase in anticipation of this. The amount received depends on the success of the startup. Labor is hired on a spot market in each period using current revenue. There is one unit of labor available in the economy. Behind the scene, banks and venture capitalists borrow funds from a representative consumer/worker at a fixed interest rate.

Projects are funded by venture capital based on their potential. The success of a startup is realized in the second phase. While potential is positively correlated with future output, it is an imperfect signal of that output. At the time of funding in the first phase, the entrepreneur and the venture capitalist decide on a sharing rule for phase-two profits. The sharing rule is determined by Nash bargaining. In the case of bank financing, the entrepreneur borrows funds from the bank in the first phase at a fixed interest rate. The loan is payed back in the second phase. Funding by a venture capitalist increases the likelihood that a startup will be successful due to a synergistic effect. That is,

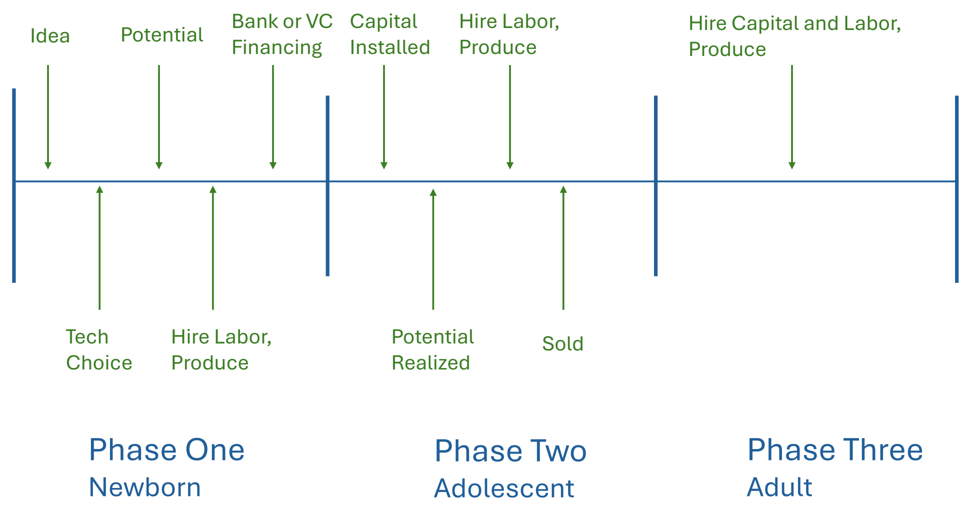

Figure 4: Timing of Events.

unlike a bank, a venture capitalist plays an instrumental role in starting up a venture. This requires effort by the venture capitalist. The contract between an entrepreneur and a venture capitalist rewards the latter for their value added to the project as well as for the effort they expend. The discussion below works backwards in time from phase-three production to the choice of technology in the first phase. Figure 4 summarizes the timing of events in a startup's life cycle.

6.2 Production by Adult Firms

By the third phase adult firms have been sold off by the founders at the end of the second phase either through a merger and acquisition or through an initial public offering. There are four generic types of adult firms operating in this period. Some adult firms became high-tech in the startup phase and were financed by either a bank or venture capitalists. Others chose non-high-tech production and utilized one of the two sources of financing. In the the third phase adult firms hire capital, $k^3_{\tau f}$, and labor, $l^3_{\tau f}$, on spot markets and produce output, $o^3_{\tau f}$, according to

$$o^3_{\tau f} = (z^3_{\tau f})^{\zeta}(k^3_{\tau f})^{\kappa}(l^3_{\tau f})^{\lambda}, \text{ with } \zeta + \kappa + \lambda = 1,$$

where the subscript $\tau = h, n$ denotes the high-tech, h, and the non-high-tech, n, production and the subscript $f = b, v$ represents the (initial) source of finance, either a bank, b, or a venture capitalist, v. Total factor productivity is given by $z^3_{\tau f}$. Total factor productivity in the third phase, $z^3_{\tau f}$, is a function of the level of total factor productivity realized in the second phase, $z^2_{\tau f}$. Specifically,

$$z^3_{\tau f} = \chi_{\tau f} z^2_{\tau f},$$

where the growth factor, $\chi_{\tau f}$, depends both on the type of technology and the source of finance. Labor is hired in the third phase at the wage rate w and capital can be raised at cost r.

The maximization problem for an adult firm is entirely standard and given by

$$\tilde{\pi}^3_{\tau f} = \max_{l^3_{\tau f}, k^3_{\tau f}} \{(z^3_{\tau f})^{\zeta}(k^3_{\tau f})^{\kappa}(l^3_{\tau f})^{\lambda} - wl^3_{\tau f} - rk^3_{\tau f}\},\ \text{for } \tau = h, n \text{ and } f = b, v, \tag{1}$$

where $\tilde{\pi}^3_{\tau f}$ is the per period profits for an adult firm. This above problem yields the familiar first-order conditions

$$\lambda(z^3_{\tau f})^{\zeta}(k^3_{\tau f})^{\kappa}(l^3_{\tau f})^{\lambda-1} = w \tag{2}$$

and

$$\kappa(z^3_{\tau f})^{\zeta}(k^3_{\tau f})^{\kappa-1}(l^3_{\tau f})^{\lambda} = r. \tag{3}$$

The discounted stream of profits from an adult firm at the beginning of phase 3 is

$$\pi^3_{\tau f} = \frac{1}{1-\delta^3}\tilde{\pi}^3_{\tau f} = \frac{1}{1-\delta^3} z^3_{\tau f}(1-\kappa-\lambda)[(\frac{\kappa}{r})^{\kappa}(\frac{\lambda}{w})^{\lambda}]^{1/\zeta},$$

where δ^3 is the survival-adjusted discount factor.

6.3 Production by Adolescent Startups

Output in the second phase is governed by

$$o^2_{\tau f} = (z^2_{\tau f})^{\zeta}(k^2_{\varsigma f})^{\kappa}(l^2_{\tau f})^{\lambda},\ \text{for } \tau = h, n \text{ and } f = b, v.$$

As can be seen, there are four generic types of adolescent firms. Total factor productivity, $z^2_{\tau f}$, is drawn in the second phase according to

$$\ln z^2_{\tau f} = \ln p_{\tau} + \ln \varepsilon_{\tau f},$$

and depends on two factors p_τ and $\varepsilon_{\tau f}$. The first factor is the project's potential, p_τ, which is known in the first phase. This depends on the technology adopted, τ. The second factor is a random shock, $\varepsilon_{\tau f}$, which is drawn in the second phase. The distribution function for this shock depends both on the source of finance, either a bank, b, or a venture capitalist, v, and on the technology used, τ. In particular,

$$\ln \varepsilon_{\tau f} \sim N(\gamma_{\tau f}, \sigma^2_{\epsilon_\tau}),\ \text{for } \tau = h, n \text{ and} f = b, v, \tag{4}$$

where $\gamma_{\tau v} > \gamma_{\tau b} = 0$. Therefore, a high-tech project funded by a venture capitalist draws the phase-two shock from a distribution function with a higher mean than a similar bank-funded venture. This captures the synergy effect from venture capital. The input of capital is decided in the first phase before $\varepsilon_{\tau f}$ is known. Labor is hired in the second phase after the shock $\varepsilon_{\tau f}$ is realized.

6.4 Venture Capital Financing

Banks and venture capitalists are risk neutral; there is no aggregate uncertainty so by lending to a large number of startups risk can be diversified. The profits from firms are distributed to a representative consumer/worker so they too are diversified. Consequently, entrepreneurs are also assumed to be risk neutral. The classical economist Frank Hyneman Knight felt entrepreneurs had high levels of risk tolerance and, indeed, there is empirical evidence suggesting that this may be the case—see Kerr, Kerr, and Xu (2018). Entrepreneurs are free to approach any venture capitalist. Since the VC industry is competitive the same deal will be offered by all VC funds. Alternatively, entrepreneurs can borrow from banks. Entrepreneurs will choose what is best for them.

The partnership agreement between a newborn entrepreneur and a venture capitalist takes place in the first phase before the phase-two technology shock is known. At this time the technology employed for the project, either h or n, and their associated potentials, p_h and p_n, are known. For a given technology, $\tau = h, n$, some ideas are better than others. Differences in potential could also reflect differences in entrepreneurial ability. The two parties agree on three things that are contingent on the technology employed: the initial investment in capital, $k^2_{\tau v}$, the amount of labor to be hired in the second phase contingent on the shock, $l^2_{\tau v}(z^2_{\tau v})$, and the venture capitalist's share of realized revenue, s_τ. The venture capitalist must expend effort, e_τ, overseeing the project according to

$$e_\tau = \alpha_\tau + \xi p_\tau, \text{ for } \tau = h, n,$$

which is increasing in a venture's potential, p_τ.

The Nash bargaining problem appears as

$$\max_{k^2_{\tau v}, l^2_{\tau v}(z^2_{\tau v}), s_\tau} \Big\{ E\Big[(1-s_\tau)[(z^2_{\tau v})^\zeta (k^2_{\tau v})^\kappa l_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2 \pi^3_{\tau v}] - \mathfrak{b}_\tau | p_\tau\Big]^\eta$$
$$\times E\Big[s_\tau[(z^2_{\tau v})^\zeta (k^2_{\tau v})^\kappa l_\tau(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] - \alpha - \xi p_\tau | p_\tau\Big]^{1-\eta}\Big\}, \text{ for } \tau = h, n, \quad (5)$$

where $\mathfrak{b}_\tau$ is the threat point of the entrepreneur or what they could receive from bank funding. The term on the first line is the entrepreneur's expected share of profits (after their threat point). This includes expected payoff from selling the startup just before phase-3 production starts, which is discounted at rate δ^2. This term is weighted by the entrepreneur's bargaining power, η. The term on the second line is the venture capitalist's expected share of profits net of their exertion on effort. Expected profits are conditioned both on an enterprise's potential, p_τ, and its source of finance, v.

Lemma 1. *(Nash Bargaining) The upshot of the Nash bargaining problem (5) is the following set of efficiency conditions (for $\tau = h, n$):*

$$\kappa E[(z^2_{\tau v})^\zeta (k^2_{\tau v})^{\kappa-1} l^2_{\tau v}(z^2_{\tau v})^\lambda | p_\tau] = r, \quad (6)$$

$$\lambda (z^2_{\tau v})^\zeta (k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^{\lambda-1} = w, \quad (7)$$

and the sharing rule

$$s_\tau = 1 - \eta + \frac{\eta(\alpha + \xi p_\tau) - (1-\eta)\mathfrak{b}_\tau}{E[(z^2_{\tau v})^\zeta (k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v} | p_\tau]}. \quad (8)$$

Proof. See Appendix C. □

The first condition sets the expected marginal product of capital equal to the rental rate. Labor is hired to the point where the realized marginal product of labor equals the wage rate, as the second condition specifies. Thus, capital and labor are hired in an efficient manner. The last condition gives the venture capitalist's share of profits. It states that the venture capitalist is entitled to the fraction $1-\eta$ of profits, plus the fraction η of the venture capitalist's effort and less the fraction $1-\eta$ of the entrepreneur's value elsewhere, both expressed as shares of profits. Denote the entrepreneur's expected profits from the Nash bargaining problem by

$$E[\pi^2_{\tau v}|p_\tau] = E\Big[(1-s_\tau)[(z^2_{\tau v})^\zeta (k^2_{\tau v})^\kappa l_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] | p_\tau\Big], \quad (9)$$

which includes the discounted expected profits from selling off a successful startup at the end of phase 2.

6.5 Bank Financing

Entrepreneurs can also approach banks for financing. Once again capital is installed in place before the technology shock is realized in the second phase. Labor is hired after the shock is known. Banking is a competitive industry. The banker and entrepreneur sign a loan contract. The contract specifies that for a loan of size $k^2_{\tau b}$ the entrepreneur will have to pay back the amount $\hat{r}_\tau(k^2_{\tau b}; z^2_{\tau b}, p_\tau)$. The loan payment, $\hat{r}_\tau(k^2_{\tau b}; z^2_{\tau b}, p_\tau)$, has two mutually exclusive parts: a fixed interest part, $\tilde{r}_\tau(k^2_{\tau b}; p_\tau)$, when the venture can cover its loan payment, and a default payment, $i_\tau(z^2_{\tau b}, k^2_{\tau b})$, when it can't. To understand the default payment, note that in some states the entrepreneur will not be able currently to repay all of the fixed loan payment, $\tilde{r}_\tau(k^2_{\tau b}; p_\tau)$, because

$$(z^2_{\tau b})^\zeta (k^2_{\tau b})^\kappa l^2_{\tau b}(z^2_{\tau b})^\lambda - w l^2_{\tau v}(z^2_{\tau v}) + \delta^2 \pi^3_{\tau b} < \tilde{r}_\tau(k^2_{\tau b}; p_\tau).$$

If this is the case, the bank seizes the startup, takes the current profits, $(z^2_{\tau b})^\zeta (k^2_{\tau b})^\kappa l^2_{\tau b}(z^2_{\tau b})^\lambda - w l^2_{\tau b}(z^2_{\tau b})$, and subsequently sells the enterprise for $\delta^2 \pi^3_{\tau b}$. Thus, upon a default the bank receives

$$i_\tau(z^2_{\tau b}, k^2_{\tau b}) = (z^2_{\tau b})^\zeta (k^2_{\tau b})^\kappa l^2_{\tau b}(z^2_{\tau b})^\lambda - w l^2_{\tau b}(z^2_{\tau b}) + \delta^2 \pi^3_{\tau b}. \tag{10}$$

Define z^{2*}_τ to be the value of $z^2_{\tau b}$ at which the entrepreneur can just make his fixed interest payment. Thus, z^{2*}_τ solves

$$i_\tau(z^{2*}_\tau, k_{\tau b}) = (z^{2*}_\tau)^\zeta k^\kappa_{\tau b} l_{\tau b}(z'^*_\tau)^\lambda - w l_{\tau b}(z^{2*}_\tau) + \delta^2 \pi^3_{\tau b} = \tilde{r}_\tau(k^2_{\tau b}; p_\tau). \tag{11}$$

This threshold is a function of $k^2_{\tau b}$, which in turn is a function of the enterprise's potential, p_τ. It also is a function of the fixed interest payment $\tilde{r}_\tau(k^2_{\tau b}; p_\tau)$. Therefore the loan payment takes the following form:

$$\hat{r}_\tau(k^2_{\tau b}; z^2_{\tau b}, p_\tau) = \begin{cases} i_\tau(z^2_{\tau b}, k^2_{\tau b}), & \text{if } z^2_{\tau b} < z^{2*}_\tau; \\ \tilde{r}_\tau(k^2_{\tau b}; p_\tau), & \text{if } z^2_{\tau b} \geq z^{2*}_\tau. \end{cases}$$

The bank's zero-profit condition reads

$$r k^2_{\tau b} = \Pr(z^2_{\tau b} \geq z^{2*}_\tau | p_\tau) \tilde{r}_\tau(k^2_{\tau b}; p_\tau) + [1 - \Pr(z^2_{\tau b} \geq z^{2*}_\tau | p_\tau)] E[i_\tau(z^2_{\tau b}, k^2_{\tau b}) | z^2_{\tau b} < z^{2*}_\tau, p_\tau],$$

where r is the interest rate paid to savers. The first term on the righthand side is the fixed interest payment, $\tilde{r}_\tau(k^2_{\tau b}; p_\tau)$, that is received with probability $\Pr(z^2_{\tau b} \geq z'^*_\tau | p_\tau)$. The second term is the expected value of the default payment, $E[i_\tau(z^2_{\tau b}, k^2_{\tau b}) | z^2_{\tau b} < z^{2*}_\tau, p_\tau]$, an event that occurs with the odds $1 - \Pr(z^2_{\tau b} \geq z^{2*}_\tau | p_\tau)$. This zero-profit condition implies that the fixed interest component, $\tilde{r}_\tau(k^2_{\tau b}; p_\tau)$, can be expressed as

$$\tilde{r}_\tau(k^2_{\tau b}; p_\tau) = \frac{r k^2_{\tau b} - [1 - \Pr(z^2_{\tau b} \geq z^{2*}_\tau | p_\tau)] E[i_\tau(z^2_{\tau b}, k^2_{\tau b}) | z^2_{\tau b} < z^{2*}_\tau, p_\tau]}{\Pr(z^2_{\tau b} \geq z^{2*}_\tau | p_\tau)}. \tag{12}$$

The entrepreneur's choice of capital and labor (for $\tau = h, n$) is given by

$$E[\pi^2_{\tau b} | p_\tau] = \max_{k^2_{\tau b}, l^2_{\tau b}(z^2_{\tau b})} \Big\{ \Pr(z^2_{\tau b} \geq z^{2*}_\tau | p_\tau) E[(z^2_{\tau b})^\zeta (k^2_{\tau b})^\kappa l^2_{\tau b}(z^2_{\tau b})^\lambda - \tilde{r}_\tau(k^2_{\tau b}; p_\tau) - w l^2_{\tau b}(z^2_{\tau b}) + \delta^2 \pi^3_{\tau b} | z^2_{\tau b} \geq z^{2*}_\tau, p_\tau] \Big\}. \tag{13}$$

The entrepreneur's expected profits, $E[\pi^2_{\tau b} | p_\tau]$, incorporates the discounted expectation of selling off the startup just before phase-3 production starts. Note that $\pi^2_{\tau b}$ is the entrepreneur's threat point in the Nash Bargaining problem (5) so that $E[\mathfrak{b}_\tau] = E[\pi^2_{\tau b} | p_\tau]$.

Lemma 2. *(Bank Financing) The solution to the bank financing problem (13) is the following set of efficiency conditions for capital and labor:*

$$\kappa E[(z^2_{\tau b})^\zeta (k^2_{\tau b})^{\kappa - 1} l^2_{\tau b}(z^2_{\tau b})^\lambda | p_\tau] = r \tag{14}$$

and

$$\lambda (z^2_{\tau b})^\zeta (k^2_{\tau b})^\kappa l^2_{\tau b}(z^2_{\tau b})^{\lambda - 1} = w. \tag{15}$$

Proof. Once again see Appendix C, where equation (28) gives the entrepreneur's expected profits, $E[\pi^2_{\tau b}|p_\tau]$. □

The lemma implies that capital accumulation and the hiring of labor are done efficiently. This transpires because: (i) banking is a competitive industry, (ii) both bankers and entrepreneurs are risk neutral, and (iii) upon a default bankers can seize the full value of a startup. The entrepreneur's expected profits are same as if they could finance the capital themselves, at an opportunity of cost of r per unit of lost savings–this can be seen from the form of the objective function (28) in Appendix C. The benefit from using venture capital is the synergy effect. The form of the contract resembles the costly state verification models of Townsend (1979) and Williamson (1986), but here there is no private information problem.

6.6 Determination of Financing

The entrepreneur selects the type of financing that yields the highest expected payoff. The payoff depends on the type of technology used and the idea's potential. The financing decision is given by (for $\tau = h, n$)

$$\begin{array}{ll} \texttt{VC}, & \text{if } E[\pi^2_{\tau v}|p_\tau] \geq E[\pi^2_{\tau b}|p_\tau]; \\ \texttt{Bank}, & \text{if } E[\pi^2_{\tau v}|p_\tau] < E[\pi^2_{\tau b}|p_\tau]. \end{array} \tag{16}$$

This can be rewritten in terms of a threshold rule. Specifically,

$$\begin{array}{ll} \texttt{VC}, & \text{if } p_\tau \geq p^*_\tau; \\ \texttt{Bank}, & \text{if } p_\tau < p^*_\tau, \end{array}$$

where the threshold p^*_τ solves the indifference condition

$$E[\pi^2_{\tau v}|p^*_\tau] = E[\pi^2_{\tau b}|p^*_\tau]. \tag{17}$$

The fraction of type-τ projects funded by venture capitalists is given by $\Pr[p_\tau \geq p^*_\tau]$, while the fraction receiving loans from banks is $1 - \Pr[p_\tau \geq p^*_\tau]$. The determination of financing is portrayed by Figure 5. Therefore, the model has both VC synergy and selection effects at work, as specified by equations (4) and (17). Since venture capitalists must be compensated for their effort expended advising and monitoring the startup [as specified by the sharing rule (8)], not all startups will choose this funding source.

6.7 Production by Newborn Startups

Output in the first phase is produced before the newly born entrepreneur approaches a bank or venture capitalists for funding but after the entrepreneur has made a technology choice. Phase-one output is governed by the production function (for $\tau = h, n$)

$$o^1_\tau = (z^1_\tau)^\zeta m^\kappa (l^1_\tau)^\lambda,$$

where m is a fixed amount of intangible entrepreneurial capital in the startup. So, there are two generic types of newborn startups. The technology shocks z^1_τ is not perfectly correlated with the potential p_τ. Specifically, the two follow the bi-variate normal distribution

$$\ln z^1_\tau, \ln p_\tau \sim N(\mu_{z^1_\tau}, \mu_{p_\tau}, \sigma^2_{z^1_\tau}, \sigma^2_{p_\tau}, \sigma_{z^1_\tau, p_\tau}).$$

A startup's potential depends upon which technology is used with some ideas being better than others. The draw for potential can also be thought of as reflecting an entrepreneur's talent. The bivariate

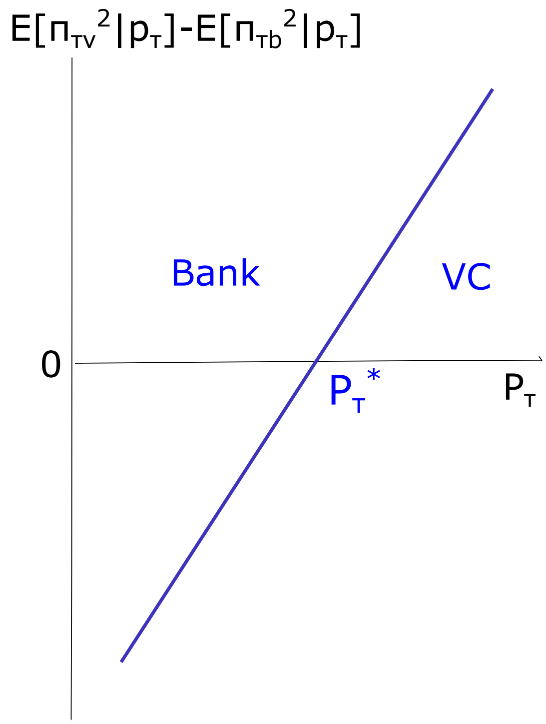

Figure 5: The determination of financing. At low levels of potential, $p_\tau < p_\tau^*$, the entrepreneur prefers using a bank. This transpires because of the fixed cost, α_τ, associated with a venture capitalist's effort. As the firm's potential rises the benefit of VC financing in terms of a *likely* higher level of productivity, $z_{\tau v}^2 > z_{\tau b}^2$, overcomes the presence of the fixed cost.

normal distribution implies that the initial employment and output of a very young firm will only imperfectly predict whether a startup will get VC funding. An infant startup using technology $\tau = h, n$ chooses labor to maximize its profits in line with

$$\pi_\tau^1 = \max_{l_\tau^1}\{(z_\tau^1)^\zeta m^\kappa (l_\tau^1)^\lambda - w l_\tau^1\}, \tag{18}$$

which yields the standard looking first-order condition

$$\lambda (z_\tau^1)^\zeta m^\kappa (l_\tau^1)^{\lambda-1} = w. \tag{19}$$

The entrepreneur lives off of all of the return from his entrepreneurial capital and the profits generated by the newborn startup.

Choice of Technology

At the beginning of phase one a newly born entrepreneur is endowed with an idea. They can implement the idea using either the high-tech or non-high-tech technology. They make this choice before knowing the potential of the respective technologies. Therefore, they do not know the type of finance that they will procure. Implementing the advanced technology involves a fixed implementation cost, ϕ, which is distributed according to a Gumbel distribution:

$$\phi \sim G(\mathfrak{g}, \mathfrak{l} = 1),$$

where $\mathfrak{g}$ and $\mathfrak{l}$ are the shape and location parameters. This feature captures the notion that some ideas are better suited for high tech (military drones) than others (restaurants). The entrepreneur picks the technology that yields the highest discounted expected profits, after factoring in its implementation cost. The presence of venture capital in the economy affects this choice. The unconditional expected payoffs from using the advanced and conventional technologies, $E[\pi^1|h]$ and $E[\pi^1|n]$, can be written as

$$E[\pi^1|h] = \pi_h^1 + \Pr[p_h \geq p_h^*]\delta^1 E[\pi_{hv}^2|p_h \geq p_h^*] + \Big[1 - \Pr[p_h \geq p_h^*]\Big]\delta^1 E[\pi_{hb}^2|p_h < p_h^*]$$

and

$$E[\pi^1|n] = \pi_n^1 + \Pr[p_n \geq p_n^*]\delta^1 E[\pi_{nv}^2|p_n \geq p_n^*] + (1 - \Pr[p_n \geq p_n^*])\delta^1 E[\pi_{nb}^2|p_n < p_n^*].$$

To understand these formulas take the first one. The probability of obtaining VC funding using the high-tech technology is $\Pr[p_h \geq p_h^*]$ while the expected discounted phase-2 payoff with VC funding is $\delta^1 E[\pi_{hv}^2|p_h \geq p_h^*]$, where δ^1 is the discount factor applied to phase two profits. With the complementary probability $1-\Pr[p_h \geq p_h^*]$ the entrepreneur obtains a bank loan from which phase-2 discounted profits are expected to be $\delta^1 E[\pi_{nb}^2|p_n < p_n^*]$.

The choice of technology is then summarized by

$$\begin{array}{ll} \texttt{High}-\texttt{Tech}, & \text{if } E[\pi^1|h] - \phi \geq E[\pi^1|n]; \\ \texttt{Non}-\texttt{High}-\texttt{Tech}, & \text{if } E[\pi^1|h] - \phi < E[\pi^1|n]. \end{array} \tag{20}$$

Condition (20) makes clear that the implementation cost can be equivalently interpreted as indicating that some ideas are more profitable to pursue using high-tech methods than others. That is, alternatively ϕ could be encapsulated into the $E[\pi^1|h]$ term. For example, $\ln p_\tau$, could be drawn from the bivariate normal distribution $N(\mu_{z_\tau^1}, \mu_{p_\tau} - \ln\phi, \sigma_{z_\tau^1}^2, \sigma_{p_\tau}^2, \sigma_{z_\tau^1, p_\tau})$, where ϕ is drawn from $G(\mathfrak{g}, \mathfrak{l} = 1)$ and is known at the time of the technology adoption decision. The fraction of high-tech startups reads

$$\Pr\Big[E[\pi^1|h] - \phi \geq E[\pi^1|n]\Big].$$

If the VC synergy effect is stronger for high-tech startups, this will tilt the adoption decision toward using advanced technologies.

6.8 Equilibrium

In the background, think about a representative consumer/worker living in a stationary equilibrium–for more detail see Appendix D. This consumer/worker supplies labor to firms and savings to banks and venture capitalists. The person earns income from their labor and savings and reaps the profits from firms and venture capital operation. Banks and venture capitalists can borrow funds at the fixed interest rate, ι, which represents the consumer/worker's rate of time preference. If the depreciation rate of physical capital is $\mathfrak{d}$, then the rental rate for capital, r, is

$$r = \iota + \mathfrak{d}. \tag{21}$$

Each period a unit mass of newborn startups flow into the economy. Newborn startups either survive to become adolescent startups with probability $\mathfrak{s}_n$ or suffer an infant death. Likewise, adolescent startups become adult ones with survival odds $\mathfrak{s}_a$. Similarly, adult firms survive a period with probability $\mathfrak{s}_e$. So, there will be 1 unit of newborn startups, $\mathfrak{s}_n$ units of adolescent startups, and $\mathfrak{s}_n\mathfrak{s}_a/(1-\mathfrak{s}_e)$ units of adult firms. The survival adjusted discount factors, δ^1 and δ^2, are accordingly given by

$$\delta^1 = \frac{\mathfrak{s}_n}{1+\iota} \text{ and } \delta^2 = \frac{\mathfrak{s}_a}{1+\iota}. \tag{22}$$

For an equilibrium to obtain the labor market must clear. There are 10 types of generic firms: newborn startups using either high-tech or non-high-tech production technology, adolescent high-tech and non-high-tech adopting startups that are financed by either banks or venture capitalists, and adult high-tech and non-high firms that are initially financed by one of the two sources. Thus, labor-market-clearing condition reads

$$\begin{aligned}
&\Pr\Big[E[\pi^1|h]-\phi \geq E[\pi^1|n]\Big]E[l_h^1(z_h^1)] + \Pr\Big[E[\pi^1|h]-\phi < E[\pi^1|n]\Big]E[l_n^1(z_n^1)] \\
&+\mathfrak{s}_n\Big\{\Pr\Big[E[\pi|h]-\phi \geq E[\pi|n]\Big]\{\Pr[p_h \geq p_h^*]E[l_{hv}^2(z_{\tau v}^2)|p_h \geq p_h^*] + \Pr[p_h < p_h^*]E[l_{hb}^2(z_{hb}^2)|p_h < p_h^*]\}\Big\} \\
&+\mathfrak{s}_n\Big\{\Pr\Big[E[\pi|h]-\phi < E[\pi|n]\Big]\{\Pr[p_n \geq p_n^*]E[l_{nv}^2(z_{nv}^2)|p_n \geq p_n^*] + \Pr[p_n < p_n^*]E[l_{nb}^2(z_{nb}^2)|p_n < p_n^*]\}\Big\} \\
&+\mathfrak{s}_n\mathfrak{s}_a\frac{1}{1-\mathfrak{s}_e}\Big\{\Pr\Big[E[\pi|h]-\phi \geq E[\pi|n]\Big]\{\Pr[p_h \geq p_h^*]\frac{\mathfrak{s}_a}{1-\mathfrak{s}_e}E[l_{hv}^3(z_{hv}^3)|p_h \geq p_h^*] + \Pr[p_h < p_h^*]E[l_{hb}^2(z_{hb}^3)|p_h < p_h^*]\}\Big\} \\
&+\mathfrak{s}_n\mathfrak{s}_a\frac{1}{1-\mathfrak{s}_e}\Big\{\Pr\Big[E[\pi|h]-\phi < E[\pi|n]\Big]\{\Pr[p_n \geq p_n^*]\frac{\mathfrak{s}_a}{1-\mathfrak{s}_e}E[l_{nv}^3(z_{nv}^3)|p_n \geq p_n^*] + \Pr[p_n < p_n^*]E[l_{nb}^3(z_{nb}^3)|p_n < p_n^*]\}\Big\} \\
&= 1.
\end{aligned} \tag{23}$$

The first line gives labor demand from newborn startups using high-tech and non-high-tech tech. The second line give labor demand from adolescent VC- and bank-funded firms using high-tech tech. The third line replicates the second line for the conventional technology. The fourth and fifth lines repeat the second and third lines for adult firms. The last line is labor supply.

Definition. An equilibrium consists of a solution for: (i) the fractions of newborn startups using the advanced and conventional technologies, $\Pr\Big[E[\pi^1|h]-\phi \geq E[\pi^1|n]\Big]$ and $\Pr\Big[E[\pi^1|h]-\phi < E[\pi^1|n]\Big]$; (ii) the labor hired by newborn startups using the high tech and conventional technologies, $l_h^1(z_h^1)$ and $l_n^1(z_n^1)$; (iii) the fractions of high-tech adopting adolescent startups funded by venture capital and banks, $\Pr[p_h \geq p_h^*]$ and $\Pr[p_h < p_h^*]$; (iv) the capital and labor hired by such adolescent startups, $k_{hv}^2(p_h \geq p_h^*)$, $k_{hb}^2(p_h < p_h^*)$, $l_{hv}^2(z_{hv}^2)$, and $l_{hb}^2(z_{hb}^2)$; (v) the fractions of conventional technology adopting adolescent startups funded by venture capital and banks, $\Pr[p_n \geq p_n^*]$ and $\Pr[p_n < p_n^*]$; (vi) the capital and labor hired by such adolescent startups, $k_{nv}^2(p_n \geq p_n^*), k_{nb}^2(p_n < p_n^*)$, $l_{nv}^2(z_{nv}^2)$, and $l_{nb}^2(z_{nb}^2)$; (vii) the threshold rules for venture capital funding of advanced and conventional technology projects, p_h^* and p_n^*; (viii) the amounts capital, $k_{hv}^3(z_{hv}^3), k_{hb}^3(z_{hb}^3), k_{nv}^3(z_{nv}^3)$, and $k_{nb}^3(z_{nb}^3)$, and labor hired by adult firms,

$l^3_{hv}(z^3_{hv}), l^3_{hb}(z^3_{hb}), l^3_{nv}(z^3_{nv})$, and $l^3_{nb}(z^3_{nb})$; and (ix) the wage rate, w. These allocations are determined such that:

1. The fractions of newborn startups using the advanced and conventional technologies, $\Pr\Big[E[\pi^1|h]-\phi \geq E[\pi^1|n]\Big]$ and $\Pr\Big[E[\pi^1|h] - \phi < E[\pi^1|n]\Big]$, are determined by the technology choice decision (20).

2. Newborn high-tech and conventional startups hire labor, $l^1_h(z^1_h)$ and $l^1_n(z^1_n)$, to maximize the entrepreneur's profits in accordance with (19).

3. The fractions of advanced technology adolescent startups funded by venture capitalists and banks, $\Pr[p_h \geq p^*_h]$ and $\Pr[p_h < p^*_h]$, are governed by the financing decision (16).

4. Adolescent high-tech and conventional startups that are funded by venture capitalists hire labor, $l^2_{hv}(z^2_{\tau v})$ and $l^2_{nv}(z^2_{nv})$, in keeping with the solution to the Nash Bargaining problems as specified by (6).

5. The fractions of conventional technology adolescent startups funded by venture capitalists and banks, $\Pr[p_n \geq p^*_n]$ and $\Pr[p_n < p^*_m]$, are ruled by the financing decision (16).

6. Adolescent startups that are funded by banks hire labor, $l^2_{hb}(z^2_{hb})$ and $l^2_{nb}(z^2_{nb})$, to maximize the entrepreneur's profits as stated by (15).

7. The threshold rules for VC funding for advanced and conventional technologies, p^*_h and p^*_n, are governed by (17).

8. Capital is hired by VC-funded adolescent startups, $k^2_{hv}(p_h \geq p^*_h)$ and $k^2_{nv}(p_n \geq p^*_n)$, in line with the outcome of the Nash Bargaining problem or with equation (7).

9. Capital is hired by bank-funded adolescent startups that adopt high tech, $k^2_{hb}(p_h < p^*_h)$ and $k^2_{nb}(p_n < p^*_n)$, as dictated by (14), to maximize the entrepreneur's profits.

10. Adult firms hire capital, $k^3_{hv}(z^3_{hv}), k^3_{hn}(z^3_{hn}), k^3_{nb}(z^3_{nb})$, and $k^3_{nb}(z^3_{nb})$, and labor, $l^3_{hv}(z^3_{hv})$, $l^3_{hn}(z^3_{hn})$, $l^3_{nb}(z^3_{nb})$, and $l^3_{nb}(z^3_{nb})$, to maximize profits as specified by (2) and (3).

11. The labor market clears in accordance with (23). This determines wages, w.

12. The rental rate on capital, r, and the survival adjusted discount factors, δ^1 and δ^2, are pinned down by (21) and (22).

7 Quantitative Analysis

The big picture for the quantitative analysis is this. Startups are small in terms of employment. The modal startup in the United States remains small over its lifetime, in line with the findings in Hurst and Pugsley (2011)–see also the recent work on Danish firms by Akcigit et al. (2025). By contrast, high-tech, VC-financed firms grow to become very large. They furnish a lot of workers to employment even though they represent a tiny fraction of firms. VC-backed firms that use conventional technologies also become large, but not as big as the high-tech ones. They contribute non-negligibly to employment, yet are still a fraction of firms. Last, most firms start off as bank financed—for quantitative analysis all non-VC-financed firms are dubbed as bank-financed firms. They are numerous and account for the majority of employment. Bank-financed, high-tech firms are bigger than the ones using conventional technologies, but are still much smaller than VC-financed firms. The modal firm in the economy is bank-financed and uses conventional technologies. These are very small in size, but given their large numbers contribute substantially to employment. VC-funded firms have a higher proclivity to use high-tech relative to bank-financed ones. The question is whether or not the developed model can match such facts.

To answer this question, the model is calibrated to replicate a rich set of data targets detailed shortly. The vast majority of data targets are grouped into three categories of firms: newborn startups, adolescent startups, and adult firms. A period in the model is taken to be a year. A startup is classified as newborn if it hasn't reached its fourth birthday. Adolescent startups are those that have had their third birthday but not their eleventh. Adults firms are 11+ years old.

To make the computational analysis more interesting, labor supply is endogenized. Again, think about a representative consumer/worker living in a stationary equilibrium–Appendix D presents more detail. Endow this person with a momentary utility function of the following form:

$$u = \ln(c - \upsilon \frac{l^{1+\theta}}{1+\theta}),$$

where c is consumption and l is labor effort. This utility function has the simple solution for labor supply

$$l = w^{1/\theta}, \tag{24}$$

so it is easy to append an endogenous labor supply onto the framework. So, now the labor-market-clearing condition (23) will have l instead of 1 on the righthand side.

Since the three phases are of different lengths some adjustments have to be made to theory in order to match the data. Recall that each period a unit mass of new startups is born. If the annual survival rate for newborns is $\mathfrak{s}_n$, then $\mathfrak{s}_n^3$ of these will survive into the adolescent phase. By summing across the 3 periods in the newborn phase, the mass of newborn startups is $(1-\mathfrak{s}_n^3)/(1-\mathfrak{s}_n)$. Suppose that the annual survival rate for an adolescent startup is $\mathfrak{s}_a$. Then, the mass of startups surviving into adulthood is $\mathfrak{s}_n^3\mathfrak{s}_a^8$. Taking into account that an adolescent startups potentially have eight periods of life implies that there will be $\mathfrak{s}_n^3(1-\mathfrak{s}_a^8)/(1-\mathfrak{s}_a)$ of them. Last, there will be $\mathfrak{s}_n^3\mathfrak{s}_a^8/(1-\mathfrak{s}_e)$ adult firms, where $\mathfrak{s}_e$ is the annual survival rate. When taking the theory to data, the survival-adjusted discounted profits for each phase need to be computed and the labor-market-clearing condition has to be modified to take into account the mass of firms in each phase–see Appendix E for the details.

The annual real interest rate is given a standard value of 4% implying that $\iota = 0.04$. From the US Census Bureau's Business Dynamics Statistics (BDS), the annual survival rates for newborns, adolescents, and adults are $\mathfrak{s}_n = 0.859$, $\mathfrak{s}_a = 0.920$,and $\mathfrak{s}_e = 0.949$. Hence, the annual survival-adjusted discount factors within the three phases are $0.859/(1.04) = 0.826$, $0.920/(1.04) = 0.885$ and $0.949/(1.04) = 0.913$. The annual depreciation rate of capital is taken to have a standard value of 8%, so that the rental rate on capital is $r = 0.04 + 0.08 = 0.12$. Appendices F to H derive the key formulas used for computing the model and matching it with the data.

7.1 Adolescent Startups, age 4 to 10 yrs

Now, start with data targets for adolescent startups. The facts for them are:

1. Average employment for adolescent high-tech adopting startups, computed separately for (a) bank and (b) VC financing.

2. The standard deviation of log employment for adolescent high-tech adopting startups, computed separately for (a) bank and (b) VC financing.

3. Average employment for adolescent non-high-tech adopting startups, computed separately for (a) bank and (b) VC financing.

4. The standard deviation of log employment for adolescent non-high-tech adopting startups, computed separately for (a) bank and (b) VC financing.

Table 8: Calibration Targets, Adolescent Startups, 4 to 10yrs

Target		**US Data**	**Model**
Employment Shares (% of all employment)			
T1	High-Tech, VC financed	0.28 (57.13)*	0.28
T2	High-Tech, Bank financed	2.398 (16.39)	0.29
T3	Non-High-Tech, VC financed	0.29 (39.36)	2.398
T4	Non-High-Tech, Bank financed	10.62 (9.446)	10.62
Standard Deviation of ln *Employment*			
T5	High-Tech, VC financed	1.528	1.302
T6	High-Tech, Bank financed	1.302	1.302
T7	Non-High-Tech, VC financed	1.47	1.15
T8	Non-High-Tech, Bank financed	1.15	1.15
Share of Startups (% of all firms)			
T9	High-Tech, VC financed	0.1147	0.1109
T10	High-Tech, Bank financed	3.398	3.285
T11	Non-High-Tech, VC financed	0.1712	0.1736
T12	Non-High-Tech, Bank financed	26.12	26.48
Equity Share (%)			
T13	All VC financed firms	55	66

* Numbers in parenthesis refer to absolute average employment size.

5. The fractions of VC funded adolescent firms that are (a) high-tech- and (b) non-high-tech adopting.

6. The share of high-tech adolescent startups in all firms.

7. The equity share of venture capitalists in adolescent startups.

The above facts are presented in Table 8. As can be seen from the table, for a given method of financing, adolescent startups using advanced technologies are much larger in terms of employment than those using conventional technologies. VC-backed startups are bigger than bank-financed ones, regardless of the technology used. The standard deviation of employment is large for all types of adolescent startups. For a given level of financing, the variance of employment is larger for high-tech startups. Holding the type of technology fixed, the variance of employment for VC-funded startups is bigger than for bank-financed ones. Not surprisingly, high-tech adolescent startups do not account for a large percentage of firms ($< 4\%$). VC-funded startups comprise a very small fraction of firms. The annual exit rate for adolescent startups is $100 \times (1 - \mathfrak{s}_a) = 8.0\%$. Last, to compute the average equity share earned by venture capitalist the sample of firms that had a first VC deal before 2010 was analyzed. The average equity share for a firm's last deal before age 10 was 55%. Some intuition about how these data targets help to identify some of the model's parameters is provided now.

High-tech startups. Start with high-tech adolescent startups. Data targets 1(a), 1(b), 2(a), 2(b), and 5(a) are useful for pinning down the mean and variance of high-tech startup's potentials, μ_{p_h} and $\sigma^2_{p_h}$, the variance of the second-period random productivity shock for high-tech firms, $\sigma^2_{\epsilon v}$, and for computing the VC threshold for high-tech startups, p^*_h. Intuitively, data target 5(a) is instrumental for determining the threshold for VC funding, p^*_h. Given this threshold, targets 1(a), 2(a), and 2(b) provide information on employment that can be used to back out the properties of the productivity distributions that determine the distributions for employment.

Non-high-tech startups. Likewise, for non-high-tech adolescent startups, data targets 3(a), 3(b), 4(a), 4(b), and 5(b) provide information for calibrating $\mu_{p_n}, \sigma^2_{p_n}, \sigma^2_{\epsilon b}$ and p^*_n.

Costs and benefits of VC. The information provided by data targets 5(a) and 5(b) help to pin down the venture capitalist's overseeing costs parameters, α_h,α_n,ξ, and the benefit from using VC, γ_{hv} and

Table 9: Calibration Targets, Newborn Startups, $\leq$ 3yrs

Target		US Data	Model
Employment Shares (% of all employment)			
T14	High-Tech	0.73 (8.167)*	0.73
T15	Non-High-Tech	4.23 (6.026)	4.23
Standard Deviation of ln *Employment*			
T16	High-Tech	1.141	1.141
T17	Non-High-Tech	1.05	1.05
Correlation of Newborn with Adult ln *Employment*			
T18	High-Tech, VC financed	0.6349	-
T19	High-Tech, Bank financed	0.6239	-
Share of Startups (% of all firms)			
T20	High-Tech	2.078	2.287
T21	Non-High-Tech	16.31	17.95

* Numbers in parenthesis refer to absolute average employment size.

γ_{nv}.[18] Heuristically speaking, a venture capitalist's share of an adolescent venture reflects the benefits and costs of using VC.

High-tech adoption cost. Last, data target 6 ties down the shape parameter, $\mathfrak{s}$, for the Gumbel distribution, which governs the fixed cost for a high-tech startup.

Adolescent survival rate. This is calculated using data from the BDS.

VC Equity Share. This number is useful for calibrating the venture capitalist's bargaining power, $1-\eta$.

7.2 Newborn Startups, age ≤ 3 yrs

A set of stylized facts is also collected for newborn startups–see Table 9. The facts concerning employment are now just broken down by technology. Newborn startups are much smaller than adolescent ones. They haven't had time to grow. Still the ones using advanced technologies are bigger than those using conventional ones. The standard deviation of employment is also large for newborn startups. It is larger for the high-tech newborn startups relative to the non-high-tech ones. Additionally, the correlation of a newborn's employment with that obtaining in its adult phase is also computed, where these are broken down both by the source of funding and the type of technology adopted. For VC-financed firms the correlation between their initial employment and the employment 11 plus years later (conditional on survival) is quite high. Unsurprisingly, newborn startups have a high exit rate of $100 \times (1-\mathfrak{s}_n) = 14.1\%$. These facts are useful for pinning down the mean and variance, $\mu_{z_\tau^1}$ and $\sigma^2_{z_\tau^1}$ for $\tau = h, n$, of a newborn startup's productivity as well as the covariance, $\sigma_{z_\tau^1 p_\tau}$, between a newborn's productivity and its potential.

7.3 Adult Firms, age 11+ yrs

Last, a limited set of facts is provided for adults firms–Table 10. High-tech adult firms are much larger than non-high-tech ones, holding fixed the method of finance. High-tech, VC-financed firms average almost 3,000 employees. VC-backed firms are much larger than bank-financed ones, regardless of the technology used. By comparison average firm size in the United States is only about 23 employees. VC-backed firms (using either advanced or conventional technologies) are only a small fraction of all firms ($< 0.2\%$). Yet, they still account for roughly 12.56% of employment. High-tech, VC-backed

[18] The indifference condition (17) between using bank and VC funding for each technology implies that two of these variables can be solved for in terms of the others.

Table 10: Calibration Targets, Adult Firms, 11+yrs

Target		**US Data**	**Model**
Employment Shares (% of all employment)			
T22	High-Tech, VC financed	8.63 (2,944)*	8.63
T23	High-Tech, Bank financed	26.72 (104)	26.72
T24	Non-High-Tech, VC financed	3.96 (736.8)	3.96
T25	Non-High-Tech, Bank financed	42.13 (21.43)	42.13
Share of Adult Firms (% of all firms)			
T26	High-Tech, VC financed	0.06807	0.183
T27	High-Tech, Bank financed	5.967	5.434
T28	Non-High-Tech, VC financed	0.1247	0.287
T29	Non-High-Tech, Bank financed	45.66	43.8

* Numbers in parenthesis refer to absolute average employment size.

firms makeup only 0.07% of firms in the economy, but they still comprise 8.6% of employment. Bank-financed adult firms using conventional technologies comprise by far the largest slice of firms in the economy and these firms tend to be small; actually, smaller than the average firm size in the economy. Bank-financed, high-tech firms are bigger than the ones using conventional technologies, but they are still much smaller than VC-funded firms regardless of the technology used by the latter. VC-funded firms are 4 times more likely to adopt high-tech than bank-financed ones.

The lefthand side panel of Figure 6 shows the model generated firm-size distributions for each of the four types of adult firms. As can be clearly seen, there is a greater mass of VC-funded firms in the right tail of the firm-size distributions for employment relative to bank-funded firms, regardless of the technologies that firms use. This is due to both selection and synergy effects. For a given type of finance, the firm-size distributions for high-tech firms dominate those using conventional technologies.

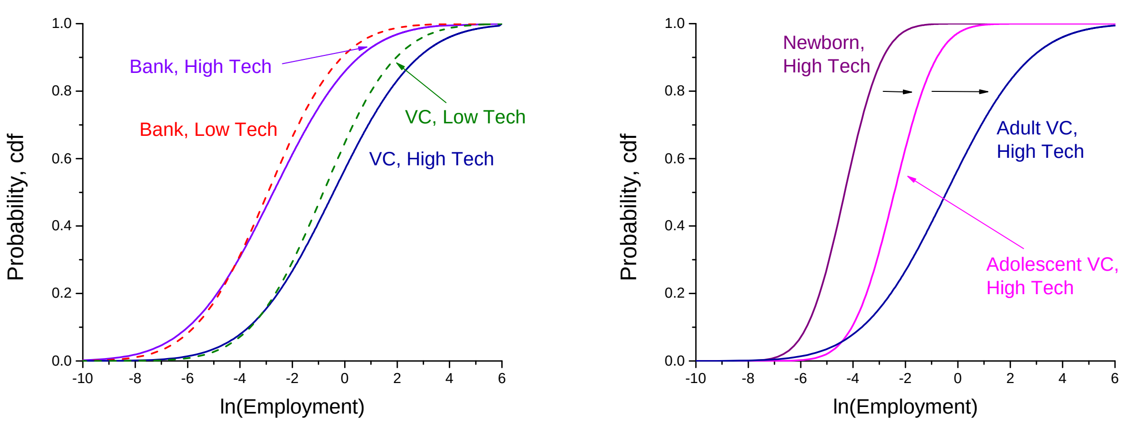

Figure 6: Firm-size distributions in the model measured by ln employment. The plots are in cumulative distribution function form. Left panel, the firm-size distributions are shown for the four types of adult firms. Right panel, the growth in the firm-size distributions from a newborn, high-tech startup to an adult, VC-financed, high-tech firm.

The fact that VC-funded adult firms are punching well above their weight is evident from Table 11, which displays the odds ratios for the various types of firms. The odds ratios give each type of firm's share of total employment relative to their share in the total number of firms. The odds ratios for VC-funded adult firms are huge, especially for firms using advanced technologies. Additionally, bank-

Table 11: Odds Ratios for Employment

Phase	Odds Ratios (%)			
Adolescent Startups	High-Tech, VC	High-Tech, Bank	Non-High-Tech, VC	Non-High-Tech, Bank
	246.00	70.58	169.49	40.68
Newborn Startups	High-Tech		Non-High-Tech	
	35.17		24.94	
Adult Firms	High-Tech, VC	High-Tech, Bank	Non-High-Tech, VC	Non-High-Tech, Bank
	12,677.04	447.83	3,172.71	92.28

financed, high-tech adult firms also contribute proportionally more to employment than their numbers warrant. Bank-financed firms using conventional technologies underperform in terms of employment; i.e, their odds ratio is less than 100%. Last, both high- and low-tech newborn firms are underachievers in terms of generating employment just because their employment is very low when starting out.

The shares of employment for the various types of adult firms are useful for calibrating the growth factors, $\chi_{\tau f}$ for $\tau = h, n$ and $f = b, v$. From Tables 8 and 10 it can be calculated that VC-funded, high-tech firms grow a phenomenal 52 times between the adolescent and adult phases. This is followed by VC-funded, non-high-tech firms that increase by a factor of 19 times. Bank-financed, high-tech firms have a growth factor of 6 times. Bank-financed, non-high-tech firms are relative laggards, only roughly doubling in size. The evolution, or rightward shifts, of the firm-size distributions for VC-financed, high-tech startups is shown in the righthand side panel of Figure 6. The annual exit rate for adult firms is $100 \times (1 - \mathfrak{s}_e) = 5.1\%$. Exit rates decline with age; i.e., $5.1 < 8.20 < 14.1\%$. The evolution, or rightward shifts, in the employment distributions starting from a newborn, high-tech startup to an adult, high-tech, VC-financed firm is shown in the righthand side panel of Figure 6. The parameter values obtained from the calibration are displayed in Table 12.

8 Thought Experiments

Next, a series of thought experiments are conducted. Some focus on examining the roles that advanced technology and venture capital play within the model. Others are designed to highlight the significant reallocation effects that can occur across different types of firms in response to changes in taxation and subsidies. Before proceeding, the model's measures of welfare, reallocation effects, and total factor productivity (TFP) are introduced.

To measure changes in welfare, consider two scenarios in the model, A and B, which differ by the perturbation of some exogenous parameters. The equivalent variation, ε, associated with a shift from A to B is given by

$$\varepsilon = 100\% \times \frac{\exp[c^B - \upsilon(l^B)^{1+\theta}/(1+\theta)] - \exp[c^A - \upsilon(l^A)^{1+\theta}/(1+\theta)]}{c^A},$$

where c^j and l^j, for $j = A, B$, are the consumptions and labor supplies in the two scenarios. This measures the amount, in terms of a percentage of regime A's consumption, that either a person would be willing to pay or have to compensated to move from A to B.

To study the reallocation effect, the change in aggregate employment (or in fact income and the capital stock) can be decomposed into the difference in each type of firm's average employment and the shift in the share of each type of firm in the economy. The formula for doing this is

$$l^B - l^A = \sum_{j=1}^{10} [(\frac{\#_j^B + \#_j^A}{2})(l_j^B - l_j^A) + (\frac{(l_j^B + l_j^A)}{2})(\#_j^B - \#_j^A)],$$

Table 12: Calibrated Parameter Values

Parameter Value	**Description**	**Identification**
Production function–all firms		
$\lambda = 0.6$	Labor share	Standard
$\kappa = 0.2$	Capital share	Standard
$\zeta = 0.2$	Profit share	Standard
$m = 0.1$	Entrepreneurial capital	Normalization
Production shocks, adolescent startups		
$\mu_{p_h} = -3.831, \sigma_{p_h} = 0.34, \sigma_{\epsilon_h} = 2.527$	Potential means, variances–high-tech	T1, T2, T5, T6
$\mu_{p_n} = -2.953, \sigma_{p_n} = 0.391, \sigma_{\epsilon_n} = 2.169$	Potential means, variances–non-high tech	T3, T4, T7, T8
Production shocks, newborn startups		
$\mu_{z_h^1} = -3.831, \mu_{z_n^1} = -4.239$	Productivity means, newborns	T14, T15
$\sigma_{z_h^1} = 2.282, \sigma_{z_n^1} = 2.100$	Potential variances, newborns	T16, T17
$\sigma_{z_h^1, p_h} = 0, \sigma_{z_n^1 p_n} = 0$	Covariance, newborn prod and potential	To do
Discount factors, rental rates, survival rates–annual		
$r = 0.04 + 0.08 = 0.12$	Capital rental rate, $r = \iota + \mathfrak{d}$	Standard
$\mathfrak{s}^1 = 0.859, \mathfrak{s}^2 = 0.920, \mathfrak{s}^3 = 0.949$	Survival rates	BDS
$\delta^1 = 0.826, \delta^2 = 0.885, \delta^3 = 0.913$	Discount factors, $\mathfrak{s}^i/(1.04)$	Surv Adj
Venture capital		
$\alpha_h = 3.619$	Oversight cost, constant, high tech	T9, T10
$\alpha_n = 1.087$	Oversight cost, constant, non-high tech	T11, T12
$\xi = 0.5$	Oversight cost, intercept	Imposed
$\gamma_{hv} = 0.502, \gamma_{nv} = 0.386$	Synergy effect	T1, T3
$\eta = 0.75$	Entrepreneur's bargaining power	T13
High-tech adoption cost		
$\mathfrak{g} = 0.33$	Gumbel distribution, shape	T20, T21
Adult firm growth		
$\chi_{hv} = 3.748, \chi_{hb} = 1.366$	Growth factors, high-tech	T22, T24
$\chi_{nv} = 2.542, \chi_{nb} = 0.739$	Growth factors, non-high-tech	T23, T25

where l_j^A and l_j^B are the average employments for a type-j firm in the two situations and $\#_j^A$ and $\#_j^B$ are the numbers for each type of firm. Reallocation effects discussed below relate to the distortion-induced misallocation effects emphasized by Guner, Ventura, and Xi (2008), Hsieh and Klenow (2009), and Restuccia and Rogerson (2008).

Aggregate TFP, as conventionally measured, is taken to be $\boldsymbol{z} = \boldsymbol{o}/(\boldsymbol{k}^{0.3}\boldsymbol{l}^{0.7})$, where $\boldsymbol{z}$ is TFP, $\boldsymbol{o}$ is GDP, $\boldsymbol{k}$ is the aggregate capital stock, and $\boldsymbol{l}$ is aggregate labor supply.

8.1 Tax rates

An interesting question is how business taxation affects startups Startups can be setup either as C corporations or a partnerships.[19] C corporations are the organizational form favored by venture capitalists, with 60% of VC-funded startups being set up this way. Partnerships are favored by other types of startups (80%). When operating as a C corporation an adult firm will pay corporate income tax at the rate τ_c. Traditional public finance theory suggests that the corporate income tax is nondistorting. As a profit tax, corporate income taxation does not affect the choice of capital and labor in the adult firm problem (1), because it just multiplies the maximand by $1 - \tau_c$. It influences the type of financing and technologies used by startups, however. When a startup is sold, the sellers are taxed at the capital gains tax rate, τ_g. The presence of the corporate income tax reduces the sale value of a startup by a factor of $1 - \tau_c$. This, combined with the capital gains tax, implies that the proceeds from the sale of a startup are effectively lessened by the factor $(1 - \tau_c)(1 - \tau_g)$. Alternatively, when operating as a partnership profits are taxed at the personal income tax rate, τ_l. Qualified business income can be deducted at the rate d. So, which business organization form is preferred for tax purposes depends on whether $(1 - \tau_c)(1 - \tau_g) \gtrless 1 - \tau_l(1 - d)$.

Currently, the corporate income tax is 21%. Capital gains are taxed at 20%. Suppose, as a thought experiment, both the corporate income and capital gains tax rates are raised to a much higher rate of 28%. Partnerships are taxed at personal income tax rate of 30% with a deduction for qualified business income of 20%. Tax receipts are rebated back to consumers via lump-sum transfer payments. Assume that all VC-funded startups are setup as C corporations and that all bank-funded ones are partnerships. The model is recalibrated assuming these tax rates and organizational structure.

The aggregate implications of the change in the tax rates is displayed in Figure 7. As can be seen, there is a significant drop in GDP and employment, 3.4 and 2.2%. Aggregate TFP drops by 1.4%. The high-tech adoption rate falls by 3.2%. There is a significant loss in welfare connected to the rise in taxes. The equivalent variation (EV) is -2.4% of aggregate consumption. The excess burden of the tax change is 2.7, implying that for every dollar raised in revenue there is a welfare loss of 2.7 dollars.

The aggregate results mask substantial reallocation effects. The reallocation effects can be gleaned from Table 13. The movement away from VC-financed to bank-financed firms is strong. As can be seen from column 3, VC-funded adult firms are large contributors to the fall in employment. The large drop in the fraction of adult VC-funded firms is a major factor in the fall of employment, as reflected in column 2. VC-funded adult firms are highly productive enterprises hiring a lot of labor. While it is true that the number of adolescent VC-funded startups also falls, at such a young stage they don't hire much labor. Still, the decline in adolescent VC-funded firms plays an important role because they grow up to be adult firms. Surprisingly, the average size of VC-funded firms actually increases because the threshold for funding, p_τ^*, rises. This works to increase their employment–column 1. The rise in average employment by adult bank-financed firms offsets significantly the decline in aggregate employment. This occurs for two reasons. First, there is a reallocation of some high productivity firms away from VC to bank financing. Second, the wage rate has fallen. Both of these effects operate to stimulate employment by adult bank-financed firms.

[19] Abraham et al. (2023) analyze the choice of a firm's organizational form.

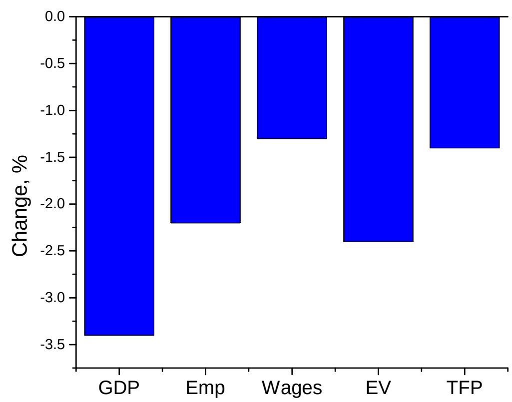

Figure 7: Tax Experiment. The bar chart shows the impact in the model of a change in taxes on some key macroeconomic variables.

Table 13: Reallocation Effect: Tax Experiment

Decomposition of Δ Employment, %	**Δ Empl**	**Δ Firms**	**Δ Total**
	1	***2***	***3***
Newborn Startups			
High-Tech	-1.1	-1.1	-2.3
Non-High-Tech	-6.5	0.8	-5.7
Adolescent Startups			
High-Tech, VC	-2.3	12.2	9.9
High-Tech, Bank	-9.4	-7.0	-16.4
Non-High-Tech, VC	-2.3	12.7	10.4
Non-High-Tech, Bank	-31.2	-0.7	-31.9
Adult Firms			
High-Tech, VC	-69.1	372.7	303.6
High-Tech, Bank	-104.9	-78.1	-183.0
Non-High-Tech, VC	-31.2	172.9	141.7
Non-High-Tech, Bank	-123.8	-2.6	-126.4

Since the change in aggregate employment is *negative*, for the newborn, adolescent, and adult categories, a *negative* number means that the entry *increased* employment, while a *positive* one implies it *decreased* it.

8.2 Subsidies

Two thought experiments regarding subsidies are considered. In the first experiment all adolescent startups are subsidized, whereas the second only subsidizes high-tech startups. The first experiment is operationalized by providing a subsidy equal to 10% of the cost of capital for an adolescent startup. In the second experiment the same 10% subsidy is targeted exclusively at high-tech adolescent startups. The two experiments amount to a reduction in the rental rate, r, for adolescent startups.

Subsidizing all startups

The first experiment results in a moderate increase in GDP and employment (0.8 and 0.5%), as the left panel of Figure 8 shows. There is a slight increase in the high-tech adoption rate of 0.5%. Despite the increases in GDP and employment, welfare decreases by 0.3%. Total factor productivity falls negligibly by 0.04%. This transpires because at the margin the subsidy is redirecting resources to adolescent startups with a relatively low level of productivity.

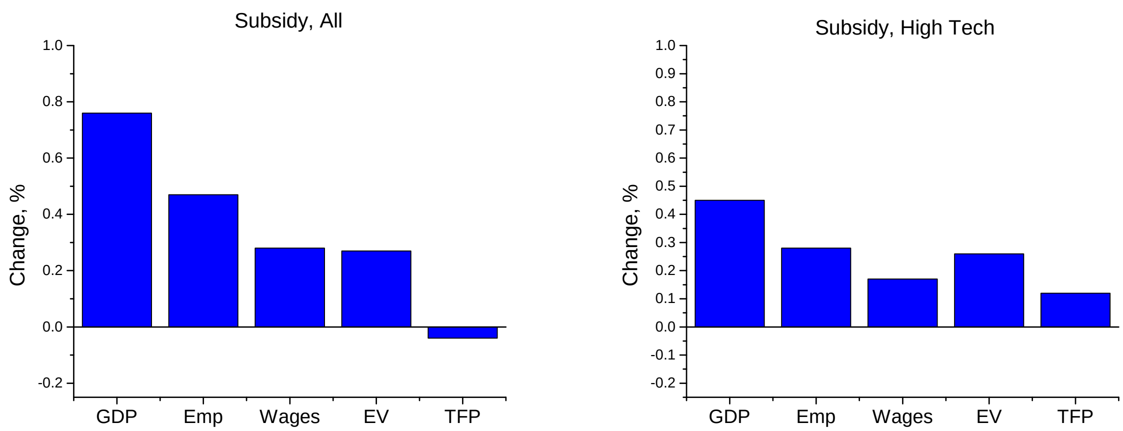

Figure 8: Subsidy Experiments. Left panel, the bar chart shows the impact in the model of providing subsidies to all adolescent startups on some key macroeconomic variables. Right panel, subsidies are provided to just high-tech adolescent startups.

From Table 14 it is clear that reallocation effects are important. The rise in aggregate employment is primarily driven by the increase in average employment by adolescent startups, as displayed in column 1. In particular, non-high tech, bank-funded adolescent firms increase their employment significantly and this accounts for most of the increase in aggregate employment, as can be deduced from columns 1 and 3. These startups on average have the lowest levels of productivity, however. When using bank funding an entrepreneur reaps all of the benefit from the subsidy. By contrast, the entrepreneur must share part of the benefit with the venture capitalist when using VC funding. There is a reduction in employment for all types of adult firms–again see column 1. This transpires for two reasons. First, the subsidy drives up the wage rate, and second, adult firms do not receive the subsidy. As column 2 illustrates, the drop in the fraction of both adolescent and adult high-tech, VC-funded firms works to decrease aggregate employment. Also note that the number of non-high-tech, bank-funded firms declines–again see column 2–as there is a shift toward high-tech, bank-financed ones and non-high-tech, VC-funded ones.

Table 14: Reallocation Effect: Subsidy, All

Decomposition of Δ Employment, %			
	Δ Empl	**Δ Firms**	**Δ Total**
	1	***2***	***3***
Newborn Startups			
High-Tech	-1.1	0.7	-0.4
Non-High-Tech	-6.3	-0.5	-6.8
Adolescent Startups			
High-Tech, VC	6.0	-0.7	5.3
High-Tech, Bank	50.4	2.7	53.1
Non-High-Tech, VC	6.0	0.8	6.8
Non-High-Tech, Bank	221.0	-1.6	219.4
Adult Firms			
High-Tech, VC	-16.6	-21.0	-37.6
High-Tech, Bank	-61.3	29.0	-32.3
Non-High-Tech, VC	-10.9	10.8	-0.1
Non-High-Tech, Bank	-101.4	-6.0	-107.4

Subsidizing just high-tech startups

Turn now to the second experiment where just high-tech adolescent startups are subsidized. Once again, aggregate GDP and output increase moderately (0.5 and 0.3%)–see Figure 8, right panel. The impact is smaller though, since the subsidy applies to a smaller fraction of startups. The high-tech adoption rate moves up by 230%. But, the rise in total factor productivity is fairly small (0.1%), because at the margin the productivity of high-tech firms has dropped. Again, welfare decreases by 0.3%.

Once again reallocation effects play a major role. The subsidy stimulates both a rise in employment for high-tech adolescent startups and their numbers, as Table 15, column 1 and 2 show. These extra startups grow into adult firms. The rising share of adult high-tech firms contribute significantly to the increase in aggregate employment, as can be gleaned from columns 2 and 3. Average employment for these types of firms, however, actually decreases–column 1. At the margin, less productive startups are enticed to adopt advanced technology and labor is now more expensive. The net increase in employment–column 3–occurs because adult high-tech firms are large on average. This comes at the expense of other types of firms. Non-high-tech, bank-financed adult firms provide the biggest offset to the rise in employment, because both of the decline in their average employment and in their share in the total number of firms—see columns1, 2, and 3.

8.3 No Venture Capital

Venture capital can be shut down by making it prohibitively expensive. Figure 9, left panel, shows the results. The impact on the aggregate economy is substantial. GDP drops by 4.4%. The loss in welfare is high, with the equivalent variation being -3.0%. This occurs because the synergy effect from VC has been forfeited. Surprisingly, the adoption of high tech increases by 4.5%. This transpires because non-high-tech, VC-funded startups switch into high-tech, bank-funded ones in order to maintain profitability. As a result, TFP falls because now high-tech firms are less productive on average due both to the absence of the VC synergy effect and the fact that these firms are less suitable for high technology.

Table 15: Reallocation Effect: Subsidy, High Tech

Decomposition of Δ Employment, %			
	Δ Empl	**Δ Firms**	**Δ Total**
	1	***2***	***3***
Newborn Startups			
High-Tech	-1.1	6.0	4.9
Non-High-Tech	-6.3	-4.4	-10.7
Adolescent Startups			
High-Tech, VC	10.4	3.2	13.5
High-Tech, Bank	88.6	20.5	109.1
Non-High-Tech, VC	-0.2	-4.3	-4.5
Non-High-Tech, Bank	-23.9	-10.1	-34.0
Adult Firms			
High-Tech, VC	-23.6	91.4	67.7
High-Tech, Bank	-66.4	216.1	149.8
Non-High-Tech, VC	-2.5	-58.3	-60.8
Non-High-Tech, Bank	-94.9	-40.1	-135.0

8.4 No High Tech

Similarly, the adoption of advanced technologies can be turned off by making it prohibitively expensive. The upshot for the aggregate economy is shown in Figure 9, right panel. GDP drops by 12.9% and employment by 8.2%. Measured TFP is lower by 5.5%. The loss in welfare is huge. The equivalent variation is -9.0%. It may seem surprising that switching off high-tech has a bigger effect than eliminating VC, because VC-funded firms have the highest levels of employment. Note that startups using VC funding are a much smaller fraction of firms than startups that use high tech.

9 Closing

Not all startups are created equal. Some entrepreneurs pursue bold, high-impact ideas, while others are more modest in scope. Some startups adopt advanced technologies (or high-tech), while others rely on more conventional technologies. Some are backed by venture capitalists, while others secure funding from alternative sources. This study explores how advanced technology adoption and VC financing interact within the startup ecosystem, offering new empirical evidence and theoretical insights into their combined impact.

Using newly available data from the U.S. Census Bureau, the empirical analysis finds that both advanced technology and venture capital are linked to stronger firm performance. Notably, the positive effects of venture capital are more pronounced among high-tech firms, suggesting complementarities between VC support and technology adoption.

A theoretical framework is developed to model how entrepreneurs jointly decide on financing and technology adoption. By calibrating the model to match key data patterns, the study reveals the significant economic role of both advanced technology and venture capital. Thought experiments suggest that both advanced technology and venture capital play significant roles in economic activity, with their elimination leading to substantial reductions in GDP and employment. The substantial reallocation effects demonstrated in the tax and subsidy experiments highlight how these instruments can have differential impacts across firm types. Understanding these heterogeneous effects is crucial for supporting innovation and entrepreneurship while considering their broader economic consequences.

Several avenues for future research emerge from this work. The potential for unobserved heterogeneity in technology adoption and VC selection decisions suggests that further refinement of identification

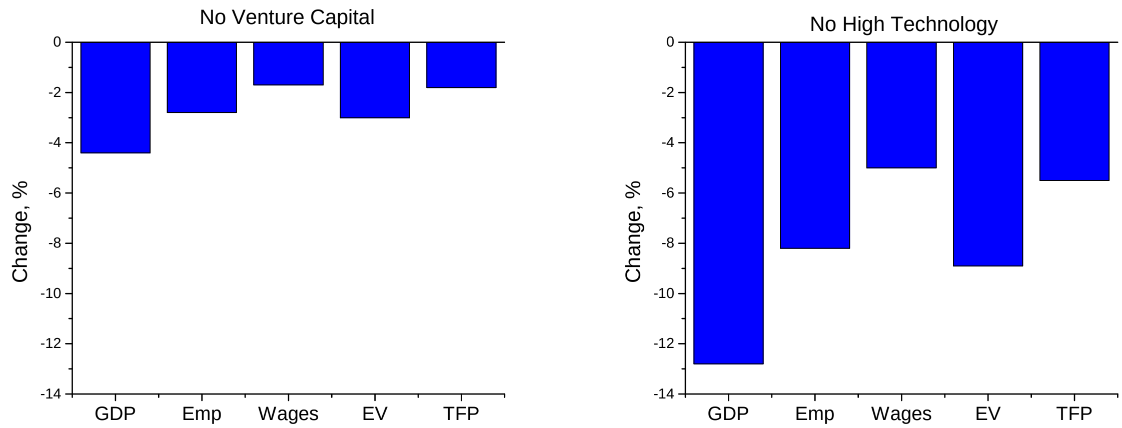

Figure 9: No Venture Capital and No High Tech Experiments. Left panel, the bar chart shows the impact in the model of eliminating venture capital on some key macroeconomic variables. Right panel, the impact in the model of eliminating high technology on some key macroeconomic variables.

strategies could strengthen inference. While the current model treats technology choice as a one-time event—consistent with data on early adoption—it could be extended to include ongoing upgrades. Additionally, future work might better capture how VCs influence technology choices beyond their current advisory roles.

As technology continues to advance and venture capital markets evolve, the interplay between finance and technology adoption will remain central to economic growth and innovation. This research lays the groundwork for further exploration into how they shape firm performance and impact the broader economy.

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A Data

The empirical analysis uses data from the Annual Business Survey (ABS) and Pitchbook. The technological variables are obtained from the ABS. For the most part, information on venture capital funding is harvested from Pitchbook with a limited amount of facts taken from the ABS.

A.1 The Use of Technology by Firms

The ABS collects data from firms regarding their use of various technology for the period 2017-2022, with this information obtained from ABS surveys conducted between 2018-2023. The specific questions asked about the use of technology vary by survey year. Only the technologies examined in the current analysis are listed for each question.

- **ABS (yr1 - 2018) Business Technologies**. The question asked was: *In 2017, to what extent did this business use the following technologies in producing goods or services?*
  Technologies included in the analysis: Augmented reality, Automated guided vehicles (AGV) or AGV systems, Automated storage and retrieval systems, Machine learning, Machine vision software, Natural language processing, Radio-frequency identification (RFID) inventory system, Robotics, and Voice recognition software. The extent of adoption was gauged by the choice selected in the following list: *No use; Testing but not using in production or service; In use for less than 5% of production or service; In use for between 5%–25% of production or service; In use for more than 25% of production or service; Don't know.* In the analysis the technology is counted as in use if the firm indicated in "use for less than 5%" or higher.
  Technologies excluded from the analysis: The following technology is excluded in the

analysis because it is too common: Touchscreens/ kiosks for customer interface (Examples: self-checkout, self-check-in, touchscreen ordering).

- **ABS (yr2 - 2019) Production Technology for Goods and Services**: The question asked was: *During the three years 2016 to 2018, to what extent did this business use the following technologies in production processes for goods or services?*
Technologies included in the analysis: Artificial Intelligence, Robotics. Extent was gauged by the choice selected in the following list: *Did not use; Tested, but did not use in production or service; Low use; Moderate use; High use; Don't know.* In the analysis the technology is counted as in use if the firm indicated "low use" or higher.
Technologies excluded from the analysis: Cloud-Based Computing Systems and Applications, Specialized Software, and Specialized Equipment are excluded. In general, the year-2 survey sampled a different set of firms in 2018 than the year-1 survey.

- **ABS (yr3 - 2020) Use of Digital Technologies**: The question asked was: *During the three years 2017 to 2019, to what extent does this business use the following digital technologies for innovation activities?*
Technologies included in the analysis: Artificial Intelligence, Digital technologies for distributed ledgers (blockchain). Extent was gauged by the choice selected in the following list: *A great extent; To some extent; To a small extent; Not at all.* A technology was counted as in use if the firm indicated to "some extent" or higher.
Technologies excluded from the analysis: Computer infrastructure (server technologies), cloud computing, automation, Internet-connected devices, mobile communication technologies, the use of digital technologies for collaboration, communication (i.e., through social media), and digital technologies for planning and management (i.e., enterprise resource planning, customer relationship management).

- **ABS (yr4 - 2021) Business Technologies**: The question asked was: *In 2020, did this business produce goods or provide services by using or applying any of the following technologies?* The technology module on this survey had a similar structure to that in ABS 2018.
Technologies included in the analysis: Augmented reality, Automated guided vehicles (AGV) or AGV systems, Machine learning, Machine vision, Natural language processing, Radio-frequency identification (RFID) system, Robotics, Voice recognition software, Additive manufacturing (3D printing), including prototyping. As in the ABS 2018, Artificial intelligence was defined as the collection of Machine learning, Machine vision software, Natural language processing, Augmented reality, AGVs, and Voice recognition software. The extent of adoption was gauged by the respondent's choice selected from the following list: *In use; In testing, but not in use; Not in use nor testing; Don't know.* The technology is counted as "In use" if the firm indicated "In use."

- **ABS (yr5 - 2022) Use of Technologies:** The question asked was: *During 2021, to what extent did this business use the following technologies?*
Technologies included in the analysis: Advanced sensing (e.g., machine vision, voice recognition, networked sensors and sensing, millimeter-wave radar, LIDAR, RFID, biointegrated sensors, electric grid measurement), Artificial intelligence (e.g., machine learning, planning, reasoning, and decision making), Autonomous systems and robotics (e.g., industrial robotics, automated guided vehicles (surface, aerospace, maritime)), Additive manufacturing (e.g., additive manufacturing (3-D Printing), smart manufacturing), Biotechnology (e.g., genetic engineering, DNA synthesis, genetic sequencing, tissue engineering, biomanufacturing, bioinformatics), Human-machine interfaces (e.g., augmented reality, virtual reality, brain-computer interfaces, human-machine teaming), Communication and networking technologies (e.g., radio frequency and mixed signal circuits, antennas and components, spectrum management technologies, communications and network security, mesh networks /infrastructure independent communication

technologies), Advanced financial technologies (e.g., digital technologies for distributed ledgers, blockchain), Advanced semiconductors and microelectronics (e.g., beyond CMOS electronics (including next generation semiconductor materials), design and electronic design automation (EDA) tools, manufacturing technologies and tooling, advanced lithography), Advanced engineering materials (e.g., materials by design, metamaterials, nanomaterials, smart materials, AM alloys, biomimetic, flexible electronics, material property characterization additive manufactured parts), Renewable energy generation and storage (e.g., wind, solar, and bio-based generation, electric and hybrid engines, batteries and grid backup/storage), Advanced gas turbine engine technologies [e.g., aero, marine, industrial (power generation), enabling component technologies], and Advanced nuclear energy technologies [e.g., aero, marine, industrial (power generation), enabling component technologies]. Extent was gauged by the choice selected in the following list: *A lot; Somewhat; A little; Not at all.* The technology is counted as in use if the firm indicated "a little" or higher.
TECHNOLOGIES EXCLUDED FROM THE ANALYSIS: Advanced computing (e.g., supercomputing, edge computing, cloud computing, data storage, advanced computing architectures).

- **ABS (yr6 - 2023) Production Technology for Goods and Services**: The question asked was: *During the three years 2020 to 2022, did this business adopt/use the following technologies?* The technology module on this survey had a similar structure to that in ABS 2019.
TECHNOLOGIES INCLUDED IN THE ANALYSIS: Artificial Intelligence and Robotics. For each technology, the survey asks about the timing of adoption. For example,
Timing of Adoption for Artificial Intelligence Technology–Processes and Methods: Approximately what year did this business first adopt or use Artificial Intelligence in processes and methods?
Prior to 1990, 1991–1995, 1996–2000, 2001–2005, 2006–2010, 2011–2015, 2016–2020, 2021–Present, Don't know.
TECHNOLOGIES EXCLUDED FROM THE ANALYSIS: Cloud-Based Computing Systems and Applications, Specialized Software, and Specialized Equipment are excluded, mainly because they are more commonplace as shown in Acemoglu et al (2024).

A.2 Venture Capital Funding for Firms

Data on venture capital funding is primarily sourced from Pitchbook, which gathers collects information on venture capital financing through public sources as well as its extensive network. As a leading provider of VC data, Pitchbook's datasets are also used by the National Venture Capital Association in its annual reports. This data is then merged with the Census Bureau's Business Register using name and address matching, utilizing company details such as name, state, city, zip code, and street address. According to Pitchbook's glossary, venture capital is defined as "a type of private equity investing that targets startups and early-stage companies with high growth potential." For the year 2018, Pitchbook's data is supplemented by responses to a specific question from the ABS.

- **ABS (yr1 - 2018) Capital Funding**: The question asked was: *What was the source(s) of capital used to start or initially acquire this business*?:
Personal/family savings of owner(s), Personal/family assets other than savings of owner(s), Personal/family home equity loan, Personal credit card(s) carrying balances, Business credit card(s) carrying balances, Government-guaranteed business loan from a bank or financial / institutions, including SBA-guaranteed loans, Business loan from a bank or financial institution, Business loan from a federal, state, or local government, Business loan/investment from family/friend(s), Investment by venture capitalist(s), Grants, Other source(s) of capital, Don't know, None needed.

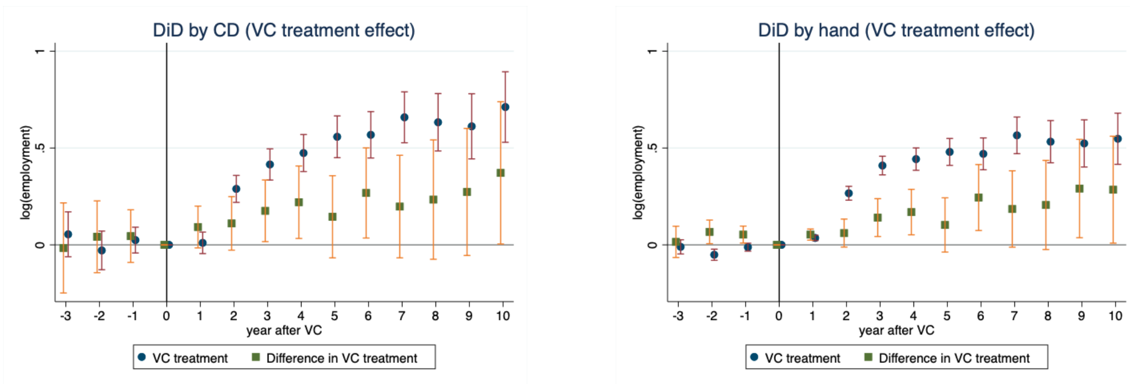

Figure 10: VC treatment effects are estimated using the DiD method developed by De Chaisemartin and d'Haultfoeuille (CD) and DiD by hand based on the sample average of the difference-in-differences between treated-control pairs. Difference in VC treatment refers to the difference in the impact of VC treatment on high-tech adopters versus non-high-tech ones.

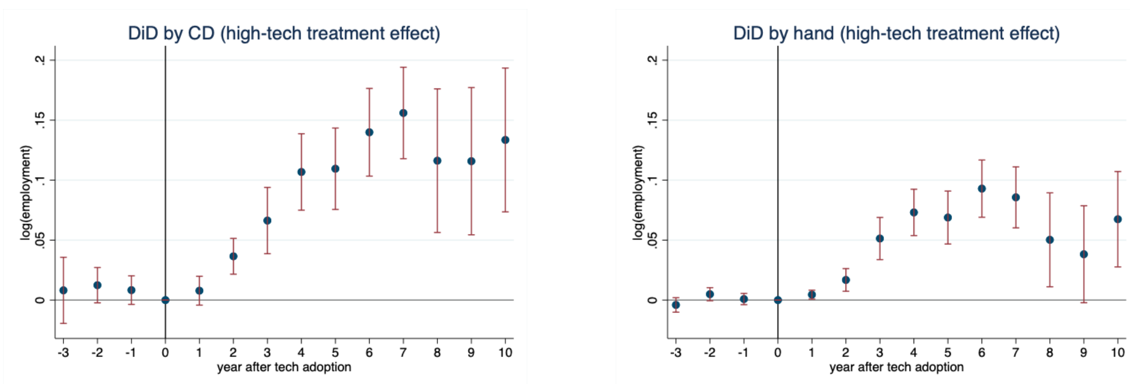

Figure 11: High-tech treatment effects are estimated using the DiD method developed by De Chaisemartin and d'Haultfoeuille (CD) and DiD by hand based on the sample average of the difference-in-differences between treated-control pairs.

B Additional Empirical Results

Figure 10 and Figure 11 display the dynamic treatment effects of VC backing and technology adoption, respectively. The methodologies are discussed in the main text. The figures confirm the absence of pre-trends and illustrate the positive effects of both treatments.

C Theory

Nash Bargaining–Lemma 1, Proof

The first-order conditions for $k^2_{\tau v}$,$l^2_{\tau v}(z^2_{\tau v})$, and s_τ associated with the Nash bargaining problem (5) between an entrepreneur and a venture capitalist are (for $\tau = h, n$):

$$\begin{aligned}
&\eta E\Big[(1-s_\tau)[(z^2_{\tau v})^\zeta(k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] - \mathfrak{b}_\tau | p_\tau\Big]^{\eta-1}(1-s_\tau)E[\kappa(z^2_{\tau v})^\zeta(k^2_{\tau v})^{\kappa-1}l^2_{\tau v}(z^2_{\tau v})^\lambda - r|p_\tau] \\
&\quad\times E\Big[s_\tau[(z^2_{\tau v})^\zeta(k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] - \alpha - \xi p_\tau | p_\tau\Big]^{1-\eta} \\
&\quad = E\Big[(1-s_\tau)[(z^2_{\tau v})^\zeta(k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] - \mathfrak{b}_\tau | p_\tau\Big]^{\eta} \\
&\quad\times (1-\eta)E\Big[s_\tau[(z^2_{\tau v})^\zeta(k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] - \alpha - \xi p_\tau | p_\tau\Big]^{-\eta} \\
&\qquad\qquad \times s_\tau E[\kappa(z^2_{\tau v})^\zeta(k^2_{\tau v})^{\kappa-1}l^2_{\tau v}(z^2_{\tau v})^\lambda - r|p_\tau], \quad (25)
\end{aligned}$$

$$\begin{aligned}
&\eta E\Big[(1-s_\tau)[(z^2_{\tau v})^\zeta(k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] - \mathfrak{b}_\tau | p_\tau\Big]^{\eta-1}(1-s_\tau)[\lambda(z^2_{\tau v})^\zeta(k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^{\lambda-1} - w] \\
&\quad\times E\Big[s_\tau[(z^2_{\tau v})^\zeta(k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] - \alpha - \xi p_\tau | p_\tau\Big]^{1-\eta} \\
&\quad = E\Big[[(1-s_\tau)(z^2_{\tau v})^\zeta(k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] - \mathfrak{b}_\tau | p_\tau\Big]^{\eta} \\
&\quad\times (1-\eta)E\Big[s_\tau[(z^2_{\tau v})^\zeta(k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] - \alpha - \xi p_\tau | p_\tau\Big]^{-\eta} \\
&\qquad\qquad \times s_\tau[\lambda(z^2_{\tau v})^\zeta(k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^{\lambda-1} - w], \quad (26)
\end{aligned}$$

and

$$\begin{aligned}
&\eta E\Big[(1-s_\tau)[(z^2_{\tau v})^\zeta(k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] - \mathfrak{b}_\tau | p_\tau\Big]^{\eta-1} \\
&\qquad \times E[(z^2_{\tau v})^\zeta(k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}|p_\tau] \\
&\quad\times E\Big[s_\tau[(z^2_{\tau v})^\zeta(k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] - \alpha - \xi p_\tau | p_\tau\Big]^{1-\eta} \\
&\quad = E\Big[(1-s_\tau)[(z^2_{\tau v})^\zeta(k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] - \mathfrak{b}_\tau | p_\tau\Big]^{\eta} \\
&\quad\times (1-\eta)E\Big[s_\tau[(z^2_{\tau v})^\zeta(k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] - \alpha - \xi p_\tau | p_\tau\Big]^{-\eta} \\
&\qquad\qquad \times E[(z^2_{\tau v})^\zeta(k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}|p_\tau]. \quad (27)
\end{aligned}$$

Start with the first order condition (25). It is automatically satisfied when (6) holds. Likewise, (7) will guarantee that (26) is fullfilled. To derive the solution for s_τ note that the term $E[(z^2_{\tau v})^\zeta(k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}|p_\tau]$ cancels out on both sides of (27). Then, divide both sides of equation (27) by $E\Big[(1-s_\tau)[(z^2_{\tau v})^\zeta(k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] - \mathfrak{b}_\tau | p_\tau\Big]^{\eta-1}$ and $E\Big[s_\tau[(z^2_{\tau v})^\zeta(k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] - \alpha - \xi p_\tau | p_\tau\Big]^{-\eta}$, and then solve for s_τ to get equation (8).

Bank Financing–Lemma 2, Proof

In the bank financing problem (13) the entrepreneur will make the interest payment $\tilde{r}_\tau(k^2_{\tau b}; p_\tau)$ with probability $\Pr(z^2_{\tau b} \geq z^{2*}_\tau | p_\tau)$. Solving out for $\tilde{r}_\tau(k^2_{\tau b}; p_\tau)$ in (13) by using the bank's zero-profit condition (12) gives

$$E[\pi^2_{\tau b}|p_\tau] = \max_{k^2_{\tau b}, l^2_{\tau b}(z^2_{\tau b})} \Big\{ \Pr(z^2_{\tau b} \geq z^{2*}_\tau|p_\tau) E[(z^2_{\tau b})^\zeta (k^2_{\tau b})^\kappa l^2_{\tau b}(z^2_{\tau b})^\lambda - w l^2_{\tau b}(z^2_{\tau b}) + \delta^2 \pi^3_{\tau b}|z^2_{\tau b} \geq z^{2*}_\tau, p_\tau]$$
$$- r k^2_{\tau b} + [1 - \Pr(z^2_{\tau b} \geq z^{2*}_\tau|p_\tau)] E[i_\tau(z^2_{\tau b}, k^2_{\tau b})|z^2_{\tau b} < z^{2*}_\tau, p_\tau] \Big\}.$$

Next, use (10) to substitute out for $i_\tau(z^2_{\tau b}, k^2_{\tau b})$ in the above maximization problem, which yields

$$E[\pi^2_{\tau b}|p_\tau] = \max_{k^2_{\tau b}, l^2_{\tau b}(z^2_{\tau b})} \Big\{ \Pr(z^2_{\tau b} \geq z^{2*}_\tau|p_\tau) E[(z^2_{\tau b})^\zeta (k^2_{\tau b})^\kappa l^2_{\tau b}(z^2_{\tau b})^\lambda - w l^2_{\tau b}(z^2_{\tau b}) + \delta^2 \pi^3_{\tau b}|z^2_{\tau b} \geq z^{2*}_\tau, p_\tau]$$
$$- r k^2_{\tau b} + [1 - \Pr(z^2_{\tau b} \geq z^{2*}_\tau|p_\tau)] E[(z^2_{\tau b})^\zeta (k^2_{\tau b})^\kappa l^2_{\tau b}(z^2_{\tau b})^\lambda - w l^2_{\tau b}(z^2_{\tau b}) + \delta^2 \pi^3_{\tau b}|z^2_{\tau b} < z^{2*}_\tau, p_\tau] \Big\}.$$

Finally, this gives the maximization problem shown below which has the associated first-order conditions (14) and (15).

$$E[\pi^2_{\tau b}|p_\tau] = \max_{k^2_{\tau b}, l^2_{\tau b}(z^2_{\tau b})} \Big\{ E[(z^2_{\tau b})^\zeta (k^2_{\tau b})^\kappa l^2_{\tau b}(z^2_{\tau b})^\lambda - r k^2_{\tau b} - w l^2_{\tau b}(z^2_{\tau b}) + \delta^2 \pi^3_{\tau b}|p_\tau] \Big\}. \tag{28}$$

D Supplemental Material, Households

Behind the scenes is a representative household residing in a stationary equilibrium. The household solves the following intertemporal maximization problem:

$$\max_{c_t, l_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t \ln(c_t - \upsilon \frac{l^{1+\theta}}{1+\theta}),$$

subject to

$$c_t + k_{t+1} = w_t l_t + r_t k_t + \pi_t + (1 - \mathfrak{d}) k_t,$$

where c_t, l_t, k_t, and π_t are period-t aggregate consumption, labor supply, capital, and profits. The solution to this problem gives the consumption Euler equation and the consumption/labor efficiency condition

$$\frac{1}{c_t} = \beta \frac{1}{c_{t+1}} (r_t + 1 - \mathfrak{d})$$

and

$$\upsilon l^\theta_t = w_t, \text{ cf. equation (24).}$$

D.1 Stationary Equilibrium

It is trivial to deduce equation from the consumption Euler equation that in a stationary equilibrium,

$$r = 1/\beta - 1 + \mathfrak{d} = \iota + \mathfrak{d}, \text{ cf. equation (21),}$$

where the subjective rate of time preference is $\iota \equiv 1/\beta - 1$. The consumer's budget constraint can be written as

$$c = wl + rk + \pi - \mathfrak{d}k.$$

Now, the national income identity implies that aggregate output, o, is given by,[20]

$$o = wl + rk + \pi$$

so that

$$c = o - \mathfrak{d}k.$$

[20] For each firm j it is the case that $o_j = wl_j + rk_t + \pi_t$. So, the national income identity just sums over all firms.

E Supplemental Material, Adjusting for Period Length

When the model is matched with the US data the 3 phases are of different potential lengths: the newborn (3 years), adolescent (8 years), and adult (∞ years). Some adjustments are required to the labor-market-clearing conditions, discounted profits, and employment.

E.1 Labor-Market-Clearing Condition

From Section 7, the mass of newborn startups is $\mathfrak{m}_n = (1-\mathfrak{s}_n^3)/(1-\mathfrak{s}_n)$, the mass of adolescent startup is $\mathfrak{m}_a = \mathfrak{s}_n^3(1-\mathfrak{s}_a^8)/(1-\mathfrak{s}_a)$, and the mass of adult firms is $\mathfrak{s}_n^3\mathfrak{s}_a^8/(1-\mathfrak{s}_e)$. Given this, the labor-market-clearing condition (23) rewrites as

$$\begin{aligned}
&\mathfrak{m}_n\{\Pr\Big[E[\pi^1|h]-\phi \geq E[\pi^1|n]\Big]E[l_h^1(z_h^1)] + \Pr\Big[E[\pi^1|h]-\phi < E[\pi^1|n]\Big]E[l_n^1(z_n^1)]\}\\
&+\mathfrak{m}_a\Big\{\Pr\Big[E[\pi|h]-\phi \geq E[\pi|n]\Big]\{\Pr[p_h \geq p_h^*]E[l_{hv}^2(z_{\tau v}^2)|p_h \geq p_h^*] + \Pr[p_h < p_h^*]E[l_{hb}^2(z_{hb}^2)|p_h < p_h^*]\}\Big\}\\
&+\mathfrak{m}_a\Big\{\Pr\Big[E[\pi|h]-\phi < E[\pi|n]\Big]\{\Pr[p_n \geq p_n^*]E[l_{nv}^2(z_{nv}^2)|p_n \geq p_n^*] + \Pr[p_n < p_n^*]E[l_{nb}^2(z_{nb}^2)|p_n < p_n^*]\}\Big\}\\
&+\mathfrak{m}_e\Big\{\Pr\Big[E[\pi|h]-\phi \geq E[\pi|n]\Big]\{\Pr[p_h \geq p_h^*]E[l_{hv}^3(z_{hv}^3)|p_h \geq p_h^*] + \Pr[p_h < p_h^*]E[l_{hb}^2(z_{hb}^3)|p_h < p_h^*]\}\Big\}\\
&+\mathfrak{m}_e\Big\{\Pr\Big[E[\pi|h]-\phi < E[\pi|n]\Big]\{\Pr[p_n \geq p_n^*]E[l_{nv}^3(z_{nv}^3)|p_n \geq p_n^*] + \Pr[p_n < p_n^*]E[l_{nb}^3(z_{nb}^3)|p_n < p_n^*]\}\Big\}\\
&= 1
\end{aligned}$$

E.2 Discounted Profits and Employments

The annual survival-adjusted discount factors for each phase are $\delta_n = \mathfrak{s}_n/(1+\iota)$, $\delta_a = \mathfrak{s}_a/(1+\iota)$, and $\delta_e = \mathfrak{s}_e/(1+\iota)$. There are two relevant concepts: discounted profits (left) and employment (right) for each type of newborn startup.

1. Type-τ newborn startups

$$(1+\delta_n+\delta_n^2)\pi_\tau^1 = \frac{1-\delta_n^3}{1-\delta_n}\pi_\tau^1 \text{ and } (1+\mathfrak{s}_n+\mathfrak{s}_n^2)E[l_\tau^1(z_\tau^1)] = \frac{1-\mathfrak{s}_n^3}{1-\mathfrak{s}_n}E[l_\tau^1(z_\tau^1)];$$

2. Type-τf adolescent startups

$$\frac{1-\delta_a^8}{1-\delta_a}\pi_{\tau f}^2 \text{ and } \frac{1-\mathfrak{s}_a^8}{1-\mathfrak{s}_a}E[l(z_{\tau f}^2)];$$

3. Type-τf adult firms

$$\frac{1}{1-\delta_e}\tilde{\pi}_{\tau f}^3 \text{ and } \frac{1}{1-\mathfrak{s}_e}E[l_{\tau f}^3(z_{\tau f}^3)].$$

F Supplemental Material, Firms

Formulas from the firms' problems are derived that are used to compute the model's equilibrium.

F.1 Adult Firms

By combining (2) and (3), the solutions for capital and labor for an adult firm are

$$k_{\tau f}^3(z_{\tau f}^3) = z_{\tau f}^3[(\frac{\kappa}{r})^{1-\lambda}(\frac{\lambda}{w})^{\lambda}]^{\frac{1}{\zeta}}$$

and

$$l^3_{\tau f}(z^3_{\tau f}) = z^3_{\tau f}[(\frac{\kappa}{r})^{\kappa}(\frac{\lambda}{w})^{1-\kappa}]^{\frac{1}{\zeta}}. \tag{29}$$

Accordingly, by substituting these solutions into (1), momentary profits during the adult phase are given by

$$\tilde{\pi}^3_{\tau f}(z^3_{\tau f}) = (1-\kappa-\lambda) z^3_{\tau f}[(\frac{\kappa}{r})^{\kappa}(\frac{\lambda}{w})^{\lambda}]^{1/\zeta}.$$

F.2 Adolescent Startups

From (7), (6), (14), and (15), it can be seen that the generic first-order conditions for an adolescent startup's capital and labor read

$$\lambda(z^2_{\tau f})^{\zeta}(k^2_{\tau f})^{\kappa} l^2_{\tau f}(z^2_{\tau f})^{\lambda-1} = w \tag{30}$$

and

$$\kappa E[(z^2_{\tau f})^{\zeta}(k^2_{\tau f})^{\kappa-1} l^2_{\tau f}(z^2_{\tau f})^{\lambda}|p_\tau] = r.$$

Combining the two first-order conditions yields the following expression for capital investment:

$$k^2_{\tau f} = \left[\left(\frac{\kappa}{r}\right)^{1-\lambda}\left(\frac{\lambda}{w}\right)^{\lambda}\right]^{\frac{1}{\zeta}} E\left[(z^2_{\tau f})^{\frac{\zeta}{1-\lambda}}|p_\tau\right]^{\frac{1-\lambda}{\zeta}}. \tag{31}$$

To use this equation requires knowledge of the conditional expectation $E\left[(z^2_{\tau f})^{\frac{\zeta}{1-\lambda}}|p_\tau\right]$. This can be computed using the properties of the log-normal distribution.

To this end, let $\tilde{z}_{\tau f} \equiv (z^2_{\tau f})^{\frac{\zeta}{1-\lambda}}$. Then,

$$\ln \tilde{z}_{\tau f} = \frac{\zeta}{1-\lambda}\ln z^2_{\tau f} = \frac{\zeta}{1-\lambda}(\ln p_\tau + \ln \varepsilon_{\tau f}).$$

Now, the the sum of two independently distributed normal variables is normal, with mean and variance given by the sum of the means and the variance of the two original distributions. Thus,

$$\ln \tilde{z}_{\tau f}|p_\tau \sim \mathcal{N}(\mu_{\tau f}, \sigma^2_{\tau f}),$$

where $\mu_{\tau f} = \frac{\zeta}{1-\lambda}(\ln p_\tau + \gamma_{\tau f})$ and $\sigma^2_{\tau f} = \frac{\zeta^2}{(1-\lambda)^2}\sigma^2_{\varepsilon_\tau}$ (note at this stage the variance of p_τ is zero). Then, using the formula for the expected value of a variable that distributed according to a log-normal distribution gives

$$E\left[(z^2_{\tau f})^{\frac{\zeta}{1-\lambda}}|p_\tau\right] = E[\tilde{z}_{\tau f}|p_\tau] = \exp\left\{\mu_{\tau f} + \frac{1}{2}\sigma^2_{\tau f}\right\}.$$

Formula (31) for $k^2_{\tau f}$ now appears as

$$k^2_{\tau f}(p_\tau) = p_\tau \exp\left\{\gamma_{\tau f} + \frac{\zeta}{1-\lambda}\frac{\sigma^2_{\varepsilon_\tau}}{2}\right\}\left[\left(\frac{\kappa}{r}\right)^{1-\lambda}\left(\frac{\lambda}{w}\right)^{\lambda}\right]^{\frac{1}{\zeta}}. \tag{32}$$

By substituting (32) into (30), a solution can be derived for labor, $l^2_{\tau f}(z^2_{\tau f})$, conditional both on potential, p_τ, and the productivity shock, $\varepsilon_{\tau f}$. Specifically,

$$l^2_{\tau f}(z^2_{\tau f}) = p_\tau \varepsilon_{\tau f}^{\frac{\zeta}{1-\lambda}} \exp\left\{\frac{\kappa}{1-\lambda}\gamma_{\tau f} + \frac{\kappa\zeta}{(1-\lambda)^2}\frac{\sigma^2_{\varepsilon_\tau}}{2}\right\}\left(\frac{\kappa}{r}\right)^{\frac{\kappa}{\zeta}}\left(\frac{\lambda}{w}\right)^{\frac{\zeta+\lambda\kappa}{(1-\lambda)\zeta}}. \tag{33}$$

Last, the adolescent startup's expected profits, conditional on its potential, read

$$E[\hat{\pi}^2_{\tau f}|p_\tau] \equiv E\left[(z^2_{\tau f})^\zeta \left(k^2_{\tau f}(p_\tau)\right)^\kappa l^2_{\tau f}(z^2_{\tau f})^\lambda - rk^2_{\tau f}(p_\tau) - wl^2_{\tau f}(z^2_{\tau b})|p_\tau\right].$$

These profits may *not* be what either an entrepreneur or a financier earns. By using the policy functions (32) and (33), it follows that

$$E[\hat{\pi}^2_{\tau f}|p_\tau] = p_\tau(1-\lambda-\kappa)\exp\left\{\gamma_{\tau f} + \frac{\zeta}{1-\lambda}\frac{\sigma^2_{\varepsilon_\tau}}{2}\right\}\left[(\frac{\kappa}{r})^\kappa(\frac{\lambda}{w})^\lambda\right]^{\frac{1}{\zeta}}. \tag{34}$$

Expected Selling Value

The selling value of a startup at the end of the adolescent phase, conditional on its potential is $E[\pi^3_{\tau b}|p_\tau]$. Now,

$$\pi^3_{\tau f} = \frac{1}{1-\delta^3}\tilde{\pi}^3_{\tau f} = \frac{1}{1-\delta^3}(1-\lambda-\kappa)z^3_{\tau f}[(\frac{\kappa}{r})^\kappa(\frac{\lambda}{w})^\lambda]^{1/\zeta},$$

where $z^3_{\tau f} = \chi_{\tau f}z^2_{\tau f} = \chi_{\tau f}p_\tau\varepsilon_{\tau f}$. So to compute the expected selling value, $E[\pi^3_{\tau b}|p_\tau]$, requires a solution for $E[z^3_{\tau f}|p_\tau]$. By using the properties of the log-normal, $E[z^3_{\tau f}|p_\tau]$ can be rewritten as

$$E[z^3_{\tau f}|p_\tau] = \chi_{\tau f}p_\tau E[\varepsilon_{\tau f}|p_\tau] = \chi_{\tau f}p_\tau\exp\left\{\gamma_{\tau f} + \frac{1}{2}\sigma^2_{\varepsilon_\tau}\right\}.$$

Thus, the expected selling value of an adolescent startup rewrites as

$$E[\pi^3_{\tau b}|p_\tau] = \frac{1}{1-\delta^3}(1-\lambda-\kappa)\chi_{\tau f}p_\tau\exp\left\{\gamma_{\tau f} + \frac{1}{2}\sigma^2_{\varepsilon_\tau}\right\}[(\frac{\kappa}{r})^\kappa(\frac{\lambda}{w})^\lambda]^{1/\zeta}, \tag{35}$$

where $f = v$, if $p_\tau > p^*_\tau$, and $f = b$, if $p_\tau < p^*_\tau$.

Entrepreneur's Expected Profits, Bank Financing

It is easy to deduce that in the adolescent phase the entrepreneur's expected profits from bank financing are

$$E[\pi^2_{\tau b}|p_\tau] = E[\hat{\pi}^2_{\tau f}|p] + \delta^2 E[\pi^3_{\tau b}|p_\tau],$$

where momentary expected profits are given by $E[\hat{\pi}^2_{\tau f}|p]$ and the discounted expected selling value by $\delta^2 E[\pi^3_{\tau b}|p_\tau]$. Equations (34) and (35) allows this to be modified to

$$E[\pi^2_{\tau b}|p_\tau] = p_\tau(1-\lambda-\kappa)\left[(\frac{\kappa}{r})^\kappa(\frac{\lambda}{w})^\lambda\right]^{\frac{1}{\zeta}}\left[\exp\{\gamma_{\tau b} + \frac{\zeta}{1-\lambda}\frac{\sigma^2_{\varepsilon_\tau}}{2}\} + \frac{\delta^2}{1-\delta^3}\chi_{\tau b}\exp\{\gamma_{\tau b} + \frac{1}{2}\sigma^2_{\varepsilon_\tau}\}\right]. \tag{36}$$

Entrepreneur's Expected Profits, Venture Capital Financing

From the Nash Bargaining problem (5) it is clear that the entrepreneur's expected profits in the adolescent phase from venture capital financing are

$$E[\pi^2_{\tau v}|p_\tau] = E\left[\left(1-s_\tau(p_\tau)\right)[(z^2_{\tau v})^\zeta(k^2_{\tau v})^\kappa l_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}]|p_\tau\right].$$

Equation (8) in turn implies that

$$\begin{aligned} &E\left[\left(1-s_\tau(p_\tau)\right)[(z^2_{\tau v})^\zeta(k^2_{\tau v})^\kappa l_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}]|p_\tau\right] \\ &= (1-\eta)E[\pi^2_{\tau b}|p_\tau] + \eta E\left[(z^2_{\tau v})^\zeta\left(k^2_{\tau v}(p_\tau)\right)^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v}(p_\tau) - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v} - \alpha - \xi p_\tau|p_\tau\right]. \end{aligned} \tag{37}$$

Following steps parallel to the derivation of (36), it can be established that

$$E\left[(z_{\tau v}^2)^\zeta \left(k_{\tau v}^2(p_\tau)\right)^\kappa l_{\tau v}^2(z_{\tau v}^2)^\lambda - rk_{\tau v}^2(p_\tau) - wl_{\tau v}^2(z_{\tau v}^2) + \delta^2\pi_{\tau v}^3 | p_\tau\right] =$$

$$p_\tau(1-\lambda-\kappa)\left[(\frac{\kappa}{r})^\kappa(\frac{\lambda}{w})^\lambda\right]^{\frac{1}{\zeta}}\left[\exp\{\gamma_{\tau v} + \frac{\zeta}{1-\lambda}\frac{\sigma_{\varepsilon_\tau}^2}{2}\} + \frac{\delta^2}{1-\delta^3}\chi_{\tau v}\exp\{\gamma_{\tau v} + \frac{1}{2}\sigma_{\varepsilon_\tau}^2\}\right]. \quad (38)$$

Finally, plugging (38) into (37) and rearranging yields the expected profits from venture capital financing:

$$E[\pi_{\tau v}^2|p_\tau] = p_\tau(1-\lambda-\kappa)\left[(\frac{\kappa}{r})^\kappa(\frac{\lambda}{w})^\lambda\right]^{\frac{1}{\zeta}} \times$$

$$\left\{\exp\{\frac{\zeta}{1-\lambda}\frac{\sigma_{\varepsilon_\tau}^2}{2}\}\left[\eta(e^{\gamma_{\tau v}} - e^{\gamma_{\tau b}}) + e^{\gamma_{\tau b}}\right] + \frac{\delta^2}{1-\delta^3}\exp\{\frac{1}{2}\sigma_{\varepsilon_\tau}^2\}\chi_{\tau b}\left[\eta(\frac{\chi_{\tau v}}{\chi_{\tau b}}e^{\gamma_{\tau v}} - e^{\gamma_{\tau b}}) + e^{\gamma_{\tau b}}\right]\right\} - \eta(\alpha + \xi p_\tau). \quad (39)$$

F.3 Newborn Startups

From (19) the solution for the labor hired by a newborn startup is

$$l_\tau^1(z_\tau^1) = \left[\frac{\lambda}{w}(z_\tau^1)^\zeta m_\tau^\kappa\right]^{\frac{1}{1-\lambda}}. \quad (40)$$

Using this in (18) then gives the momentary profits for a newborn startup:

$$\pi_\tau^1(z_\tau^1) = (1-\lambda)\left[\left(\frac{\lambda}{w}\right)^\lambda (z_\tau^1)^\zeta m_\tau^\kappa\right]^{\frac{1}{1-\lambda}}. \quad (41)$$

Entrepreneur's Expected Profits

For a newborn startup the entrepreneur's unconditional expected profits from using technology τ are given by

$$E[\pi^1|\tau] = E[\pi_\tau^1] + \delta^1 E[\pi_\tau^2],$$

where

$$E[\pi_\tau^2] = \Pr[p_\tau \geq p_\tau^*]E[\pi_{\tau v}^2|p_\tau \geq p_\tau^*] + \left[1 - \Pr[p_\tau \geq p_\tau^*]\right]E[\pi_{\tau b}^2|p_\tau < p_\tau^*].$$

Here $E[\pi_\tau^1]$ is the expected momentary profits from the newborn phase and $E[\pi_\tau^2]$ is the expected profits from the adolescent phase. The terms for $E[\pi_\tau^1]$ and $E[\pi_\tau^2]$ are now computed, starting with the latter.

By using (36) and (39), the expected profits from the adolescent phase are

$$E[\pi_\tau^2] = \left[1 - \Pr[p_\tau \geq p_\tau^*]\right] \times E[p_\tau|p_\tau < p_\tau^*](1-\lambda-\kappa)\left[(\frac{\kappa}{r})^\kappa(\frac{\lambda}{w})^\lambda\right]^{\frac{1}{\zeta}}$$

$$\times\left\{\exp\{\gamma_{\tau b} + \frac{\zeta}{1-\lambda}\frac{\sigma_{\varepsilon_\tau}^2}{2}\} + \frac{\delta^2}{1-\delta^3}\chi_{\tau b}\exp\{\gamma_{\tau b} + \frac{1}{2}\sigma_{\varepsilon_\tau}^2\}\right\}$$

$$+ \Pr[p_\tau \geq p_\tau^*] \times E[p_\tau|p_\tau > p_\tau^*](1-\lambda-\kappa)\left[(\frac{\kappa}{r})^\kappa(\frac{\lambda}{w})^\lambda\right]^{\frac{1}{\zeta}} \times$$

$$\{\left[\exp\{\frac{\zeta}{1-\lambda}\frac{\sigma_{\varepsilon_\tau}^2}{2}\}\left[\eta(e^{\gamma_{\tau v}} - e^{\gamma_{\tau b}}) + e^{\gamma_{\tau b}}\right] + \frac{\delta^2}{1-\delta^3}\exp\{\frac{1}{2}\sigma_{\varepsilon_\tau}^2\}\chi_{\tau b}\left[\eta(\frac{\chi_{\tau v}}{\chi_{\tau b}}e^{\gamma_{\tau v}} - e^{\gamma_{\tau b}}) + e^{\gamma_{\tau b}}\right]\right]$$

$$- \eta\left(\alpha + \xi E[p_\tau|p_\tau > p_\tau^*]\right)\},$$

Next, using the formula for the expected value of variable with a truncated log-normal distribution (see Appendix H) allows the expected values for the potentials, $E[p_\tau|p_\tau < p_\tau^*]$ and $E[p|p > p^*]$, to be put forth as

$$E[p_\tau|p_\tau < p_\tau^*] = E\left[e^{\ln p_\tau}|\ln p_\tau < \ln p_\tau^*\right] = \exp\left\{\mu_{p_\tau} + \frac{\sigma_{p_\tau}^2}{2}\right\} \frac{\Phi_{\mu_{p_\tau}^*,\sigma_{p_\tau}^2}(\ln p_\tau^*)}{\Phi_{\mu_{p_\tau},\sigma_{p_\tau}^2}(\ln p_\tau^*)} \tag{42}$$

and

$$E[p_\tau|p_\tau > p_\tau^*] = E\left[e^{\ln p_\tau}|\ln p_\tau > \ln p_\tau^*\right] = \exp\left\{\mu_p + \frac{\sigma_p^2}{2}\right\} \frac{1-\Phi_{\mu_{p_\tau}^*,\sigma_{p_\tau}^2}(\ln p_\tau^*)}{1-\Phi_{\mu_{p_\tau},\sigma_{p_\tau}^2}(\ln p_\tau^*)}, \tag{43}$$

where $\Phi_{\mu_{p_\tau}^*,\sigma_{p_\tau}^2}$ denotes the cumulative distribution function for a normal distribution with mean $\mu_{p_\tau}^* \equiv \mu_{p_\tau} + \sigma_{p_\tau}^2$ and variance $\sigma_{p_\tau}^2$.

Last, using (41), the unconditional momentary profits from the newborn phase, $E[\pi_\tau^1]$, can be expressed as

$$E[\pi_\tau^1] = (1-\lambda)\left[\left(\frac{\lambda}{w}\right)^\lambda m_\tau^\kappa\right]^{\frac{1}{1-\lambda}} E\left[(z_\tau^1)^{\frac{\zeta}{1-\lambda}}\right].$$

The properties of the bivariate log-normal distribution allow this to read

$$E\left[(z_\tau^1)^{\frac{\zeta}{1-\lambda}}\right] = \exp\left\{\frac{\zeta}{1-\lambda}\mu_{z_\tau^1} + \left(\frac{\zeta}{1-\lambda}\right)^2 \frac{\sigma_{z_\tau^1}^2}{2}\right\}.$$

G Supplemental Material, Data Targets

Formulas for the data targets are presented here. In what follows, let $\Phi_{\mu,\sigma^2}(x)$ represent the cumulative normal distribution for a variable x with mean μ and variance σ^2. The associated density function is denoted by $\phi_{\mu,\sigma^2}(x)$. When subscripts are omitted, $\Phi(x)$ and $\phi(x)$ refer to the cumulative distribution and density functions for the standard normal distribution with mean zero and variance of one. Finally, $\mu_{p_\tau}^* \equiv \mu_{p_\tau} + \sigma_{p_\tau}^2$ where μ_{p_τ} and $\sigma_{p_\tau}^2$ are the mean and variance of the marginal distribution for the log of potential, $\ln p_\tau$.

G.1 Average Employment

Newborn Startups

From equation (40), average employment in a newborn startup can be expressed as

$$E[l_\tau^1] = \left[\frac{\lambda}{w} m_\tau^\kappa\right]^{\frac{1}{1-\lambda}} E\left[(z_\tau^1)^{\frac{\zeta}{1-\lambda}}\right],$$

where by using the properties of a log-normal distribution it can be calculated that

$$E\left[(z_\tau^1)^{\frac{\zeta}{1-\lambda}}\right] = \exp\left\{\frac{\zeta}{1-\lambda}\mu_{z_\tau^1} + \left(\frac{\zeta}{1-\lambda}\right)^2 \frac{\sigma_{z_\tau^1}^2}{2}\right\}.$$

Adolescent Startups, Bank Financing

By using (33), average employment in an adolescent bank-backed startup is

$$E[l_{\tau b}^2] = E[p_\tau|p_\tau < p_\tau^*] E\left[\varepsilon_{\tau b}^{\frac{\zeta}{1-\lambda}}\right] \exp\left\{\frac{\kappa}{1-\lambda}\gamma_{\tau b} + \frac{\kappa\zeta}{(1-\lambda)^2}\frac{\sigma_{\varepsilon_\tau}^2}{2}\right\} \left(\frac{\kappa}{r}\right)^{\frac{\kappa}{\zeta}} \left(\frac{\lambda}{w}\right)^{\frac{\zeta+\lambda\kappa}{(1-\lambda)\zeta}}. \tag{44}$$

Exploiting the properties of the log-normal distribution gives

$$E\left[\varepsilon_{\tau f}^{\frac{\zeta}{1-\lambda}}\right] = \exp\left\{\frac{\zeta}{1-\lambda}\gamma_{\tau f} + \left(\frac{\zeta}{1-\lambda}\right)^2 \frac{\sigma_{\varepsilon_\tau}^2}{2}\right\}. \tag{45}$$

Therefore, substituting in for $E[p_\tau | p_\tau < p_\tau^*]$, by using (42), allows (44) to be rewritten as

$$E[l_{\tau b}^2] = \exp\left\{\gamma_{\tau b} + \mu_{p_\tau} + \frac{\sigma_{p_\tau}^2}{2} + \frac{\zeta}{1-\lambda}\frac{\sigma_{\varepsilon_\tau}^2}{2}\right\}\left(\frac{\kappa}{r}\right)^{\frac{\kappa}{\zeta}}\left(\frac{\lambda}{w}\right)^{\frac{\zeta+\lambda\kappa}{(1-\lambda)\zeta}} \frac{\Phi_{\mu_{p_\tau}^*, \sigma_{p_\tau}^2}(\ln p_\tau^*)}{\Phi_{\mu_{p_\tau}, \sigma_{p_\tau}^2}(\ln p_\tau^*)}. \tag{46}$$

Adolescent Startups, VC Financing

Equation (33) implies that average employment in an adolescent VC-backed startup is

$$E[l_{\tau v}^2] = E[p_\tau | p_\tau > p_\tau^*] E\left[\varepsilon_{\tau v}^{\frac{\zeta}{1-\lambda}}\right] \exp\left\{\frac{\kappa}{1-\lambda}\gamma_{\tau v} + \frac{\kappa\zeta}{(1-\lambda)^2}\frac{\sigma_{\varepsilon_\tau}^2}{2}\right\}\left(\frac{\kappa}{r}\right)^{\frac{\kappa}{\zeta}}\left(\frac{\lambda}{w}\right)^{\frac{\zeta+\lambda\kappa}{(1-\lambda)\zeta}}.$$

Next, using (43) and (45) results in

$$E[l_{\tau v}^2] = \exp\left\{\gamma_{\tau v} + \mu_{p_\tau} + \frac{\sigma_{p_\tau}^2}{2} + \frac{\zeta}{1-\lambda}\frac{\sigma_{\varepsilon_\tau}^2}{2}\right\}\left(\frac{\kappa}{r}\right)^{\frac{\kappa}{\zeta}}\left(\frac{\lambda}{w}\right)^{\frac{\zeta+\lambda\kappa}{(1-\lambda)\zeta}} \frac{1-\Phi_{\mu_{p_\tau}^*, \sigma_{p_\tau}^2}(\ln p_\tau^*)}{1-\Phi_{\mu_{p_\tau}, \sigma_{p_\tau}^2}(\ln p_\tau^*)}.$$

Adult Firms, Bank Financing

Using (29) average employment in an adult bank-backed firm can be expressed as

$$E[l_{\tau b}^3] = \chi_{\tau b} E[p_\tau | p_\tau < p_\tau^*] E[\varepsilon_{\tau b}][(\frac{\kappa}{r})^\kappa (\frac{\lambda}{w})^{1-\kappa}]^{\frac{1}{\zeta}}.$$

The above expression can be rewritten, using (42) and the properties of the log-normal, as

$$E[l_{\tau b}^3] = \chi_{\tau b} \exp\left\{\gamma_{\tau b} + \frac{1}{2}\sigma_{\varepsilon_\tau}^2 + \mu_{p_\tau} + \frac{1}{2}\sigma_{p_\tau}^2\right\}[(\frac{\kappa}{r})^\kappa (\frac{\lambda}{w})^{1-\kappa}]^{\frac{1}{\zeta}} \frac{\Phi_{\mu_{p_\tau}^*, \sigma_{p_\tau}^2}(\ln p_\tau^*)}{\Phi_{\mu_{p_\tau}, \sigma_{p_\tau}^2}(\ln p_\tau^*)}.$$

Adult Firms, VC Financing

Average employment in an adult VC-backed firm can be expressed using (29) as

$$E[l_{\tau v}^3] = \chi_{\tau v} E[p_\tau | p_\tau > p_\tau^*] E[\varepsilon_{\tau v}][(\frac{\kappa}{r})^\kappa (\frac{\lambda}{w})^{1-\kappa}]^{\frac{1}{\zeta}}.$$

Using (43) together with the properties of the log-normal permits the following rewrite:

$$E[l_{\tau b}^3] = \chi_{\tau v} \exp\left\{\gamma_{\tau v} + \frac{1}{2}\sigma_{\varepsilon_\tau}^2 + \mu_{p_\tau} + \frac{1}{2}\sigma_{p_\tau}^2\right\}[(\frac{\kappa}{r})^\kappa (\frac{\lambda}{w})^{1-\kappa}]^{\frac{1}{\zeta}} \frac{1-\Phi_{\mu_{p_\tau}^*, \sigma_{p_\tau}^2}(\ln p_\tau^*)}{1-\Phi_{\mu_{p_\tau}, \sigma_{p_\tau}^2}(\ln p_\tau^*)}.$$

G.2 Variances

Newborn Startups

It is straightforward to see from equation (40) that the variance of log employment for newborn startup is

$$V\left(\ln l_\tau^1\right) = \left(\frac{\zeta}{1-\lambda}\right)^2 \sigma_{z_\tau}^2.$$

Adolescent Startups, Bank Financing

Using (33), the variance of log employment in adolescent bank-backed startup can be written as

$$V(\ln l_{\tau b}^2) = V\left(\ln p_\tau | \ln p_\tau < \ln p_\tau^*\right) + \left(\frac{\zeta}{1-\lambda}\right)^2 \sigma_{\varepsilon_\tau}^2.$$

The first term is

$$V\left(\ln p_\tau | \ln p_\tau < \ln p_\tau^*\right) = E\left[(\ln p_\tau)^2 | \ln p_\tau < \ln p_\tau^*\right] - E[\ln p_\tau | \ln p_\tau < \ln p_\tau^*]^2.$$

Let $\tilde{p_\tau} \equiv (\ln p_\tau - \mu_{p_\tau})/\sigma_{p_\tau}$. Using the formula for the variance of a variable distributed according to a truncated normal distribution (see Appendix H), it transpires that

$$E\left[(\ln p_\tau)^2 | \ln p_\tau < \ln p_\tau^*\right] = \sigma_{p_\tau}^2 + \mu_{p_\tau}^2 - \frac{\sigma_{p_\tau}^2 \tilde{p_\tau}\phi(\tilde{p_\tau})}{\Phi(\tilde{p_\tau})} - 2\mu_{p_\tau}\sigma_{p_\tau}\frac{\phi(\tilde{p_\tau})}{\Phi(\tilde{p_\tau})}.$$

Then, using the formula for the mean of a truncated normal distribution (Appendix H), one can derive that

$$E[\ln p_\tau | \ln p_\tau < \ln p_\tau^*] = \mu_{p_\tau} - \sigma_{p_\tau}\frac{\phi(\tilde{p_\tau})}{\Phi(\tilde{p_\tau})}.$$

Therefore, the variance of log-employment can be written as follows

$$V\left(\ln l_{\tau b}^2\right) = \sigma_{p_\tau}^2 + \mu_{p_\tau}^2 - \frac{\sigma_{p_\tau}^2 \tilde{p_\tau}\phi(\tilde{p_\tau})}{\Phi(\tilde{p_\tau})} - 2\mu_{p_\tau}\sigma_{p_\tau}\frac{\phi(\tilde{p_\tau})}{\Phi(\tilde{p_\tau})} - \left(\mu_{p_\tau} - \sigma_{p_\tau}\frac{\phi(\tilde{p_\tau})}{\Phi(\tilde{p_\tau})}\right)^2 + \left(\frac{\zeta}{1-\lambda}\right)^2 \sigma_{\varepsilon_\tau}^2.$$

Adolescent Startups, VC Financing

This derivation parallels the one above. The variance of log employment in adolescent VC-backed startup can be expressed, using (33), as

$$V(\ln l_{\tau v}^2) = V\left(\ln p_\tau | \ln p_\tau > \ln p_\tau^*\right) + \left(\frac{\zeta}{1-\lambda}\right)^2 \sigma_{\varepsilon_\tau}^2.$$

Rewrite the first term as

$$V\left(\ln p_\tau | \ln p_\tau > \ln p_\tau^*\right) = E\left[(\ln p_\tau)^2 | \ln p_\tau > \ln p_\tau^*\right] - E[\ln p_\tau | \ln p_\tau > \ln p_\tau^*]^2.$$

Again, let $\tilde{p_\tau} \equiv (\ln p_\tau - \mu_{p_\tau})/\sigma_{p_\tau}$. Using the formula for the variance of a truncated normal distribution (Appendix H) delivers

$$E\left[(\ln p_\tau)^2 | \ln p_\tau > \ln p_\tau^*\right] = \sigma_{p_\tau}^2 + \mu_{p_\tau}^2 + \frac{\sigma_{p_\tau}^2 \tilde{p_\tau}\phi(\tilde{p_\tau})}{1-\Phi(\tilde{p_\tau})} + 2\mu_{p_\tau}\sigma_{p_\tau}\frac{\phi(\tilde{p_\tau})}{1-\Phi(\tilde{p_\tau})}.$$

Then, using the formula for the mean of a truncated normal distribution (Appendix H) results in

$$E[\ln p_\tau | \ln p_\tau > \ln p_\tau^*] = \mu_{p_\tau} + \sigma_{p_\tau}\frac{\phi(\tilde{p_\tau})}{1-\Phi(\tilde{p_\tau})}.$$

The variance of log-employment therefore is

$$V\left(\ln l_{\tau v}^2\right) = \sigma_{p_\tau}^2 + \mu_{p_\tau}^2 + \frac{\sigma_{p_\tau}^2 \tilde{p_\tau}\phi(\tilde{p_\tau})}{1-\Phi(\tilde{p_\tau})} + 2\mu_{p_\tau}\sigma_{p_\tau}\frac{\phi(\tilde{p_\tau})}{1-\Phi(\tilde{p_\tau})} - \left(\mu_{p_\tau} + \sigma_{p_\tau}\frac{\phi(\tilde{p_\tau})}{1-\Phi(\tilde{p_\tau})}\right)^2 + \left(\frac{\zeta}{1-\lambda}\right)^2 \sigma_{\varepsilon_\tau}^2.$$

G.3 VC's Profit Share

The formula for a venture capitalist's share of a type-τ adolescent startup's profits is derived now. From (8), it can be seen that a venture capitalist's share of profits is

$$s(p_\tau) = \frac{\eta(\alpha_\tau + \xi p_\tau) + (1-\eta)\left[E\left[(z^2_{\tau v})^\zeta \left(k^2_{\tau v}(p_\tau)\right)^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v}(p_\tau) - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}|p_\tau\right] - E[\pi^2_{\tau b}|p_\tau]\right]}{E\left[(z^2_{\tau v})^\zeta \left(k^2_{\tau v}(p_\tau)\right)^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v}(p_\tau) - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}|p_\tau\right]}. \tag{47}$$

Using (36) and (38), it can be shown that

$$(1-\eta)\frac{\left[E\left[(z^2_{\tau v})^\zeta \left(k^2_{\tau v}(p_\tau)\right)^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v}(p_\tau) - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}|p_\tau\right] - E[\pi^2_{\tau b}|p_\tau]\right]}{E\left[(z^2_{\tau v})^\zeta \left(k^2_{\tau v}(p_\tau)\right)^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v}(p_\tau) - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}|p_\tau\right]} =$$

$$\frac{(1-\eta)}{e^{\gamma_{\tau v}}}\frac{\exp\{\frac{\zeta}{1-\lambda}\frac{\sigma^2_{\varepsilon_\tau}}{2}\}(e^{\gamma_{\tau v}} - e^{\gamma_{\tau b}}) + \frac{\delta^2}{1-\delta^3}\exp\{\frac{1}{2}\sigma^2_{\varepsilon_\tau}\}(\chi_{\tau v}e^{\gamma_{\tau v}} - \chi_{\tau b}e^{\gamma_{\tau b}})}{\exp\left\{\frac{\zeta}{1-\lambda}\frac{\sigma^2_{\varepsilon_\tau}}{2}\right\} + \frac{\delta^2}{1-\delta^3}\chi_{\tau v}\exp\left\{\frac{\sigma^2_{\varepsilon_\tau}}{2}\right\}}. \tag{48}$$

Additionally, one can express

$$\frac{\eta(\alpha_\tau + \xi p_\tau)}{E\left[(z^2_{\tau v})^\zeta \left(k^2_{\tau v}(p_\tau)\right)^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v}(p_\tau) - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}|p_\tau\right]} =$$

$$\frac{\eta(\alpha_\tau p_\tau^{-1} + \xi)}{(1-\lambda-\kappa)e^{\gamma_{\tau v}}\left[(\frac{\kappa}{r})^\kappa(\frac{\lambda}{w})^\lambda\right]^{\frac{1}{\zeta}}\left\{\exp\left\{\frac{\zeta}{1-\lambda}\frac{\sigma^2_{\varepsilon_\tau}}{2}\right\} + \frac{\delta^2}{1-\delta^3}\chi_{\tau v}\exp\left\{\frac{\sigma^2_{\varepsilon_\tau}}{2}\right\}\right\}}. \tag{49}$$

Finally, to compute $E[s(p_\tau)]$ requires solving for $E[1/p_\tau|p_\tau > p^*_\tau]$. Using the formula for the mean of a truncated normal distribution (Appendix H) gives the average share of profits for a type-τ project or

$$E[s(p_\tau)] = \frac{(1-\eta)}{e^{\gamma_{\tau v}}}\frac{\exp\{\frac{\zeta}{1-\lambda}\frac{\sigma^2_{\varepsilon_\tau}}{2}\}(e^{\gamma_{\tau v}} - e^{\gamma_{\tau b}}) + \frac{\delta^2}{1-\delta^3}\exp\{\frac{1}{2}\sigma^2_{\varepsilon_\tau}\}(\chi_{\tau v}e^{\gamma_{\tau v}} - \chi_{\tau b}e^{\gamma_{\tau b}})}{\exp\left\{\frac{\zeta}{1-\lambda}\frac{\sigma^2_{\varepsilon_\tau}}{2}\right\} + \frac{\delta^2}{1-\delta^3}\chi_{\tau v}\exp\left\{\frac{\sigma^2_{\varepsilon_\tau}}{2}\right\}} +$$

$$+ \frac{\eta}{(1-\lambda-\kappa)e^{\gamma_{\tau v}}\left[(\frac{\kappa}{r})^\kappa(\frac{\lambda}{w})^\lambda\right]^{\frac{1}{\zeta}}} \times \frac{\alpha E\left[p_\tau^{-1}|p_\tau > p^*_\tau\right] + \xi}{\left\{\exp\left\{\frac{\zeta}{1-\lambda}\frac{\sigma^2_{\varepsilon_\tau}}{2}\right\} + \frac{\delta^2}{1-\delta^3}\chi_{\tau v}\exp\left\{\frac{\sigma^2_{\varepsilon_\tau}}{2}\right\}\right\}}. \tag{50}$$

H Supplemental Material, Properties of the Normal and Log-Normal Distributions

Three properties of the normal and the log-normal distribution are listed here.[21] In what follows, let $\Phi(x)$ and $\phi(x)$ refer to the standard normal cumulative distribution and density functions with mean zero and variance of one for the random variable x.

1. Expected values for one-sided truncations of a normally distributed variable with mean μ and standard deviation σ:

$$E[x|x > a] = \mu + \sigma\frac{\phi\Big((a-\mu)/\sigma\Big)}{1 - \Phi\Big((a-\mu)/\sigma\Big)}$$

   and

$$E[x|x < a] = \mu - \sigma\frac{\phi\Big((a-\mu)/\sigma\Big)}{\Phi\Big((a-\mu)/\sigma\Big)}.$$

[21] See https://en.wikipedia.org/wiki/Truncated_normal_distribution and https://en.wikipedia.org/wiki/Log-normal_distribution.

2. Variances for one-sided truncations of a normally distributed variable with mean μ and standard deviation σ:

$$V(x|x > a) = \sigma^2\{1 + [(a-\mu)/\sigma]\frac{\phi\Big((a-\mu)/\sigma\Big)}{1-\Phi\Big((a-\mu)/\sigma\Big)} - [\frac{\phi\Big((a-\mu)/\sigma\Big)}{1-\Phi\Big((a-\mu)/\sigma\Big)}]^2\}$$

and

$$V(x|x < a) = \sigma^2\{1 - [(a-\mu)/\sigma]\frac{\phi\Big((a-\mu)/\sigma\Big)}{\Phi\Big((a-\mu)/\sigma\Big)} - [\frac{\phi\Big((a-\mu)/\sigma\Big)}{\Phi\Big((a-\mu)/\sigma\Big)}]^2\}.$$

3. Expected values for one-sided truncations of a log-normally distributed variable x with mean μ and standard deviation σ:

$$E[x|x > a] = e^{\mu+\sigma^2/2}\frac{\Phi\Big((\mu+\sigma^2-\ln a)/\sigma\Big)}{1-\Phi\Big((\ln a-\mu)/\sigma\Big)}$$

and

$$E\,[x|x < a] = e^{\mu+\sigma^2/2}\frac{\Phi\Big((\ln a-\mu-\sigma^2)/\sigma\Big)}{\Phi\Big((\ln a-\mu)/\sigma\Big)}.$$