The Role of Trade in Structural Transformation*

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Abstract

High agricultural productivity is considered a necessary condition for the start of industrialization and modern economic growth. This paper argues that international trade in agricultural goods can accelerate the structural transformation of countries with low agricultural productivity, and shows that trade played an important role in the structural transformations of the United Kingdom and South Korea. To do the analysis, I introduce international trade into a neoclassical growth model with two sectors, agriculture and nonagriculture. A key feature of the model is the low-income elasticity of the agricultural good. Consequently, in the closed economy model, as countries get richer labor moves out of agriculture and into the other sector. International trade accelerates this transition for countries with positive net agricultural imports. I calibrate and simulate the model to show it can match three different structural transformations: the United States in the 20th century, the United Kingdom in the 19th century, and South Korea for the last 50 years. The results show that in the United Kingdom international trade played a very important role in its industrialization and that, without trade, the agricultural employment at the beginning of the 19th century would have been almost as large as the one observed in the United States at that time. In South Korea, the results show that international trade also had a positive impact on its structural transformation but it could have played a much larger role if the country had not introduced agricultural protection policies, without which the agricultural employment share would have dropped below 10% twenty years earlier than it did.

Keywords: Structural Transformation, International Trade, Agricultural Productivity.

JEL classification: O41, F11, Q18.

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1 Introduction

Low agricultural productivity is considered responsible for part of the huge cross-country differences in the size of the agricultural sector and the level of per capita income, and high agricultural productivity is regarded as a necessary condition for the start of industrialization and modern economic growth.[1] The reason is that, in the absence of food trade, countries have to produce the entirety of the food they consume and, as a result, when agricultural productivity is low they have to allocate their productive resources in agriculture. This paper shows that international trade in agricultural goods can accelerate the structural transformation of countries with low agricultural productivity because it gives them the possibility of importing part of their food consumption and reduce the size of their agricultural sector.[2]

This paper studies the structural transformations of the United States, the United Kingdom and South Korea, and shows that trade played an important role in the structural transformations of the United Kingdom and South Korea. Comparing these three cases is an interesting exercise because the time periods are very different, and the countries also have important differences in terms of their comparative advantage and attitude towards agricultural trade. As we can see in figure 1, at the beginning of the 19th century, the share of workers employed in agriculture in the United Kingdom was less than 40%, while in the United States it was around 80% and it did not drop below 40% until the end of the 19th century. It has been continuously decreasing over the last 200 years, and it is now less than 2% in both countries. In South Korea, the agricultural employment share was still above 80% around 1950 but it was already below 10% at the end of the 20th century, a fall that took the United States more than 150 years. When agricultural employment is plotted against

[1] See Caselli (2005), Gollin, Parente, and Rogerson (2007) and Restuccia, Yang, and Zhu (2008) for some arguments in this direction. See also Sachs (2001) , Córdoba and Ripoll (2006) , Vollrath (2009), and Lagakos and Waugh (2012) for some evidence and some possible explanations on the fact that poor countries productivity is particularly low in agriculture.

[2] This point was first formalized by Matsuyama (1992), which argues that the early industrialization of Belgium and New England was possible because of their agricultural imports from Holland and the South of the United States respectively. Similarly, Matsuyama (2009) argues that if a country is open to international trade, high productivity growth in a sector does not have to affect negatively employment in that sector.

income per capita, however, we can see that in South Korea the size of the agricultural sector at each income level is similar to the one in the United States, so the two transitions are not that different when the evolution of income is taken into account. In the United Kingdom, on the other hand, the agricultural employment share is significantly lower at all the income levels studied, which indicates that the transition was started much earlier and had some important differences.

There is a large body of literature that, like my paper, analyzes the structural transformation of countries. Until recently, however, most of the articles studied it in a closed economy context, while my paper does it in an open economy context.[3] Some authors like Mundlak (2000) have long recognized the fact that agricultural products are tradeable, and there are is now an emerging literature studying the structural transformation of countries in an open economy context. Most of these articles are mainly theoretical, like Matsuyama (1992), Echevarria (2008), Galor and Mountford (2008), Matsuyama (2009) or Yi and Zhang (2010), while the main contributions of my article are about applying the theory to the data and getting some quantitative predictions. On the quantitative side, some articles analyzing the structural transformation of open economies have been written in the recent years, all of which are obviously related my analysis at some level. The most important one for my paper is clearly Stokey (2001), which uses a three-sector growth model to quantify the effects of international trade and technological change on the industrial revolution of Great Britain over the period 1780-1850. Another important article for my analysis is Shin (1990), which studies the relationship between structural change and economic development in South Korea and the United States, and concludes that including trade in the model proposed might be very useful to explain the South Korean experience. Hayashi and Prescott (2008) considers the possibility of food being a tradeable good in its quantitative study of the Japanese structural transformation, but the main focus of the paper, as well as the main

[3] Some important examples are Echevarria (1997), Kongsamut, Rebelo, and Xie (2001), Caselli and Coleman II (2001), Hansen and Prescott (2002), Lucas (2004), Gollin, Parente, and Rogerson (2007), Restuccia, Yang, and Zhu (2008). Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008), or Herrendorf, Rogerson, and Valentinyi (2009).

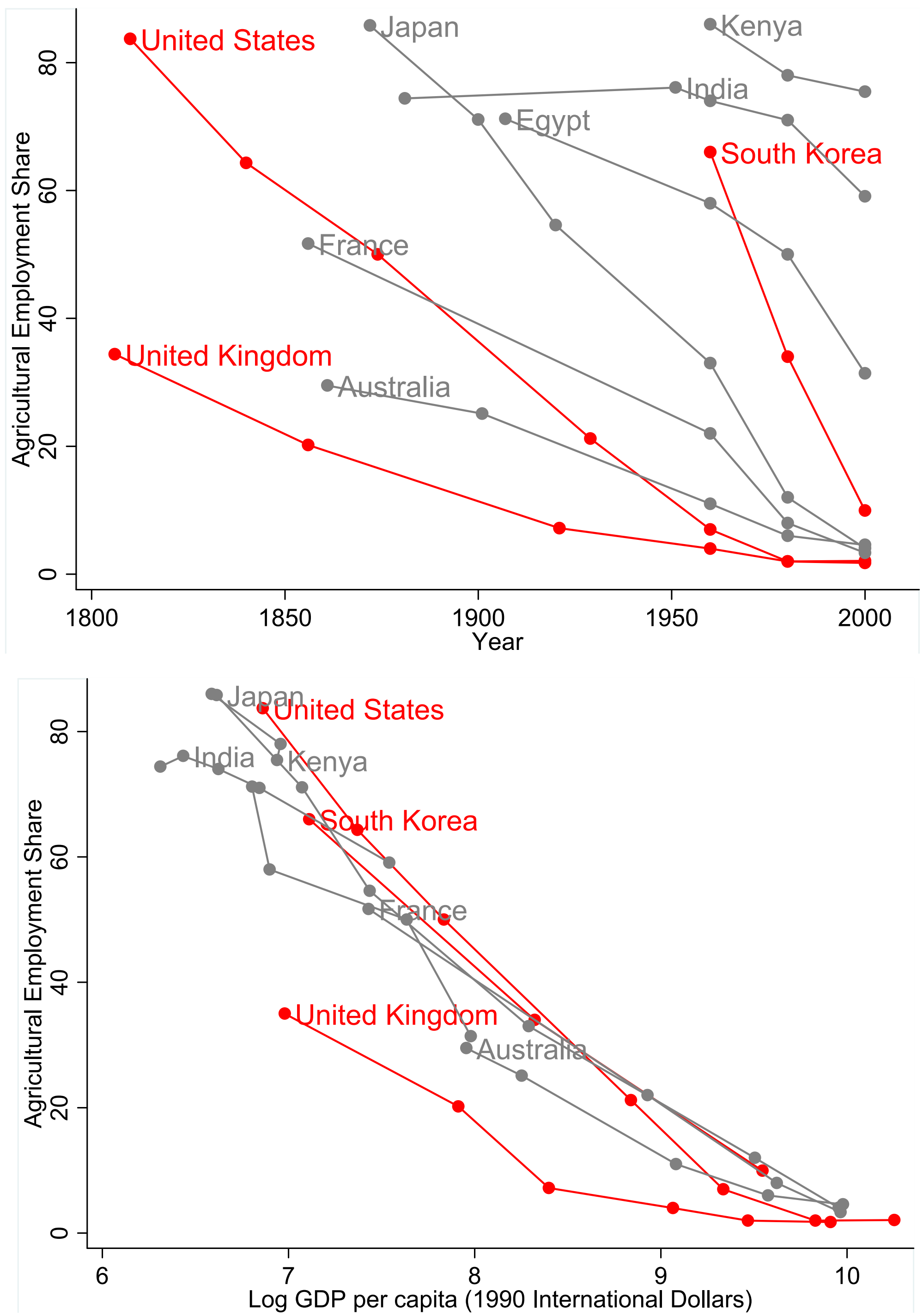

Figure 1: **United States, United Kingdom, South Korea comparison**
Data sources: Agricultural employment: Kuznets (1966), World Bank WDR1984, Food and Agriculture Organization; GDP per capita: Maddison (2006).

conclusions, assume that food was nontraded. More recently, and in parallel to my work, Mo Choi and Ma (2011) and Betts and Verma (2011) present analysis of the South Korean transition using quantitative multi-sector growth models.Mo Choi and Ma (2011) presents a two-sector growth model with learning-by-doing externalities at the country level in the non-agricultural sector, a knowledge spillover from the non-agricultural to the agricultural sector, and labor frictions to that trade can accelerate urbanization and is able to generate "growth miracles" like the ones observed in some East Asian economies in the last 50 years. And Betts and Verma (2011) shows that the quantitative predictions of the open economy version of their two-country, three-sector growth model is able to match the South Korean data better than the closed economy version.[4]

This paper has three main contributions with respect to the existing literature. First, it develops a model that makes possible to determine the importance of international agricultural trade in the structural transformation of a country. Second, it shows that the closed economy version of the model is able to replicate the structural transformation data of the United States during the period 1890-2007, and that the open economy version of the model is able to match the structural transformation of the United Kingdom during the period 1800-1900 and of South Korea during the period 1963-2007. And third, it quantifies the role played by international trade in the transformations of the United Kingdom South Korea in these time periods.

To do the analysis, I introduce international trade into a general equilibrium, neoclassical growth model with two sectors, agriculture and the rest of the economy. In the model, preferences are such that consumers spend a large fraction of their income in the agricultural good when they are poor. Under autarky, then, a low income country employs most of its productive resources in agriculture. As technological change occurs and capital accumulates, consumers get richer and productive resources are reallocated from the agricultural sector to

[4]Stefanski (2011) and Ungor (2009) are other quantitative articles studying the sectoral reallocations of countries, although they do it in different environments. Other articles like Young (1995), Hsieh (2002), Connolly and Yi (2009) and Deardorff and Park (2010) are also related to this paper because they study the growth experience of South Korea too, although using significantly different methodologies.

the nonagricultural one. Under international trade, if the relative price of the agricultural good is higher domestically than in the international markets, the country imports the agricultural good and, as a results, its agricultural sector shrinks and its transformation gets accelerated.

The model is then calibrated and used to study the three structural transformations. The results show that in the case of the United States, its high agricultural productivity growth was the key factor for its transformation. In the case of the United Kingdom, trade was extremely important for its early structural transformation and it had huge effects on the intertemporal welfare of households. If the country had been in autarky, the agricultural employment share in 1800 would have been around 80% instead of 35%. and the intertemporal welfare gains from international trade were equivalent to a 5.5% increase in the yearly consumption expenditures. In South Korea, trade played a positive role in its structural transformation but its effects were smaller than in the United Kingdom; the initial employment share in the agricultural sector was 62% and would have been 72% in autarky, and the gains in intertemporal welfare are equivalent to a 0.4% increase in the autarky consumption expenditures. Moreover, if South Korea had not adopted policies to protect its agricultural sector from foreign competition, the volume of agricultural trade would have been much larger and the country would have experienced an even faster transformation. The agricultural employment share would have dropped below 10% by 1979 instead of 2002 and the intertemporal gain in intertemporal welfare from being open to international trade in this case would have been almost as large as in the United Kingdom.

The rest of the paper is organized as follows. Section 2 describes the two-sector growth model, for both closed and open economies. Section 3 simulates the model and compares its performance with the three different structural transformation episodes. Section 4 quantifies the role played by international trade in the cases of South Korea and the United Kingdom, as well as the role trade would have played in South Korea under different agricultural polices. Section 5 concludes.

2 Structural Transformation Model

2.1 Model Setup

In this section, I present and analyze the two-sector growth model used in this paper, first its closed economy version and then its small open economy version.[5] One of the sectors in the economy is agriculture, which produces a good that is only used for consumption; the other is the nonagricultural sector, which produces a good that is used for consumption as well as investment.

Households' description. In the model, there is a representative household with $N(t)$ infinitely-lived members, who derive utility from consuming the agricultural and the nonagricultural good, as described in the following utility function:

$$U(0) = \int_0^\infty e^{-(\rho-\nu)t}\left[u\left(c_a(t)\right) + \log(c_n(t))\right] dt \tag{1}$$

where

$$u(c_a) = \begin{cases} \mu_0 \log\left(c_a(s) - \underline{c_a}\right) & \text{if } c_a(s) \leq c_a^* \\ B + \mu_1 \log\left(c_a - c_a^* + A\right) & \text{if } c_a(s) > c_a^* \end{cases} \tag{2}$$

and the variables $c_a(t)$ and $c_n(t)$ denote the amount of agricultural and nonagricultural good consumed by each member, while the parameter ρ is the intertemporal discount rate and ν the population growth rate.[6] The instantaneous utility function is a variation of the Stone-Geary utility function, with $\underline{c_a}$ denoting the agricultural good minimum consumption, which can be interpreted as the subsistence level. When agricultural consumption is lower than the threshold c_a^*, its relative weight in the preferences is μ_0, and it becomes μ_1, when agricultural consumption is higher than c_a^*, where $\mu_1 < \mu_0$.[7] The constants A and B are

[5] My model is similar to one used by Hayashi and Prescott (2008) and it is also related to other two-sector growth models in the literature like Echevarria (1997), Echevarria (2008), Gollin, Parente, and Rogerson (2007), or Matsuyama (1992).

[6] $N(0)$ is set equal to 1 without loss of generality.

[7] My analysis is for economies where agricultural consumption is higher than the subsistence level $\underline{c_a}$, but one may think of $\underline{c_a}$ as the minimum consumption people need to survive. Thus, if population is such that

defined as $A \equiv \frac{\mu_1}{\mu_0}\left(c_a^* - \underline{c_a}\right)$ and $B \equiv \mu_0 \log\left(c_a^* - \underline{c_a}\right) - \mu_1 \log\left(\frac{\mu_1}{\mu_0}\left(c_a^* - \underline{c_a}\right)\right)$, and they are introduced to guarantee that the function $u(.)$ is continuous and differentiable at c_a^*.

Figure 2 illustrates graphically the main properties of the preferences, with the parameters μ_0 and μ_1 taking the same values as in simulations of sections 3 and 4. The change in the relative weight increases the concavity of the function $u(.)$ at c_a^*, and decreases the marginal rate of substitution at c_a^*, as the first the first plot shows. The second plot shows that the agricultural consumption expenditure is continuous and decreasing both below and above c_a^*.[8]

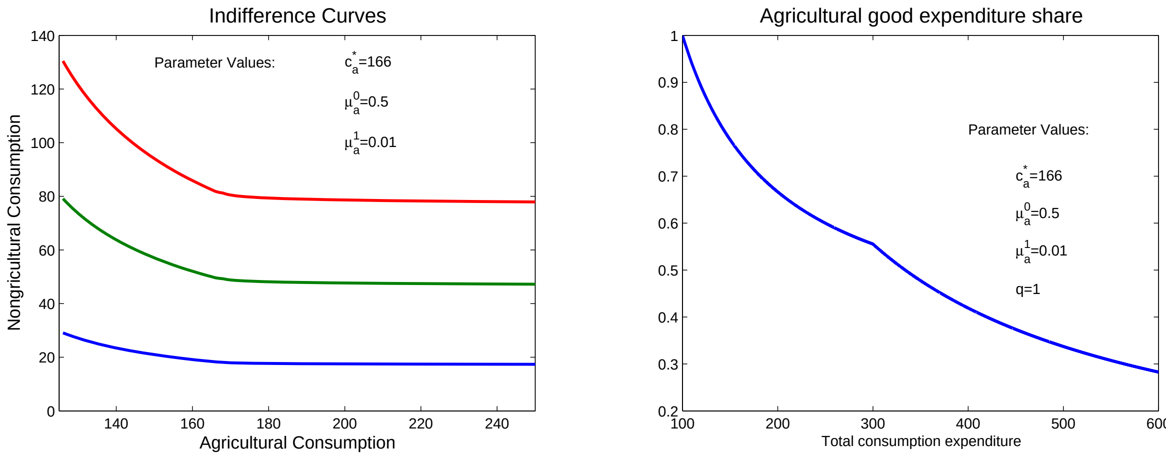

Figure 2: **Graphical Description of Preferences**

Households own capital, land and labor, which are supplied inelastically to firms together with their labor endowment.[9] At each period, households choose the amount to save, the amount of both goods to consume, and the sector to rent the inputs they own. Since both labor and capital are perfectly mobile across sectors, there is a unique wage rate $w(t)$ and a unique capital rental rate $r(t)$ in the economy. Households' income also includes the land

the amount of agricultural good available is not less than $\underline{c_a}$, population would adjust. However, this is beyond the analysis of this paper because in this paper population evolves exogenously.

[8]These preferences are able to fit the data in section 3 better than similar preferences used in the literature like the ones with a constant weight on agricultural consumption, like Hayashi and Prescott (2008), or the ones that assume a satiation level in the agricultural good consumption, as in Gollin, Parente, and Rogerson (2007), Laitner (2000), or Stokey (2001).

[9]In the model, all the members of the household work so that total employment is equal to total population. In the simulations, the variables employment and population are taken from the data, so they are not necessarily the same.

rents, which are denoted by $p_l(t)L$, with $p_l(t)$ denoting the price per unit of land and L denoting the total land available. Thus, the per-capita budget constraint is

$$\frac{dk}{dt} = w + (r - \delta - \nu)k + p_l L e^{-\nu t} - qc_a - c_n, \tag{3}$$

with the price of the nonagricultural good normalized to 1, k denoting the per-capita capital stock, δ the capital depreciation rate, ν the population growth rate, and q the relative price of the agricultural good at time t. Note that all variables depend on time, except for the parameters δ, ν, and the land stock L. The optimization problem of the representative household consists of choosing $[c_a(t), c_n(t), k(t)]_{t>0}$ to maximize equation (1) subject to the budget constraint in (3) and k_0 given.[10]

Firms' description. In the model, there are also many identical firms in each sector, which hire labor and capital to maximize profits taking the relative price as given q. The production function for the agricultural good in per capita terms is

$$y_a = A_a k_a^{\eta} n_a^{\beta} \left(L_a e^{-\nu t}\right)^{1-\eta-\beta} \tag{4}$$

where A_a denotes the total factor productivity, k_a the capital stock per capita, n_a the labor input per capita, and L_a the total land employed in that sector.

The production for the nonagricultural sector is

$$y_n = A_n k_n^{\alpha} n_n^{1-\alpha}, \tag{5}$$

with A_n denoting total factor productivity in the nonagricultural sector, k_n the per capita stock of capital, and n_n the labor input per capita used in that sector.[11] Productivity grows

[10]The optimization conditions of households and firms, as well as a detailed description of the dynamic equilibrium, both for the closed and open economies, is available in appendix A of this paper.

[11]Since firms exhibit constant returns to scale and there is perfect competition, the total number of firms is irrelevant for the equilibrium. Therefore, we can solve the equilibrium as if there was a representative firm in each sector.

exogenously in both sectors, with the rates γ_a and γ_n being potentially different.

2.2 Closed Economy Competitive Equilibrium

A competitive equilibrium for the closed economy, given k_0 and the exogenous variables (A_a, A_n, N), are prices $[q, w, r, p_l]_{t>0}$ and quantities $[y_a, y_n, c_a, c_n, n_a, n_n, k_a, k_n, k]_{t>0}$, satisfying the consumers' optimization conditions, the firms' optimization conditions, and the market clearing conditions which guarantee that the amount of goods and inputs supplied domestically is equal to the amount demanded domestically.

As technology grows in both sectors, capital accumulates and production of both goods increases. Moreover, as in Hayashi and Prescott (2008), if

$$(1-\eta-\beta)\,\nu < \frac{\eta}{1-\alpha}\gamma_n + \gamma_a, \tag{6}$$

per capita consumption increases over time, not only for the nonagricultural good but also for the agricultural good.[12] As a result, the nonhomothetic component of the preferences becomes less and less important over time, and in the limit consumers behave as if their preferences were homothetic and equal to

$$u\left(c_a, c_n\right) = \mu_1 \log\left(c_a\right) + \log\left(c_n\right). \tag{7}$$

For the preferences in (7), the equilibrium system is consistent with a Balanced Growth Path where all the variables grow at constant but not necessarily common rates, and the solution to the equilibrium system is the unique path that converges asymptotically to this BGP. If equation (6) is satisfied, the solution to the equilibrium system of the model with the preferences in (2) is the unique path that converges to the Balanced Growth Path of the model with the preferences in equation (7).

[12] This condition states that population growth cannot be very high relative to productivity growth because there are decreasing returns to scale in agricultural good production.

Structural Transformation in a Closed Economy. As consumers get richer, their agricultural consumption expenditures share decreases. As a result, in a closed economy, the fraction of labor and capital employed in the agricultural sector tends to decrease over time.[13] In other words, structural transformation takes place. With respect to the relative price of the agricultural good, in a closed economy, it tends to move in the same direction as the agricultural employment share, although other variables also affect it, like TFP, population or capital.[14]

2.3 Small Open Economy Competitive Equilibrium

A competitive equilibrium for the small open economy, given k_0 and the exogenous variables (A_a, A_n, N, q), are prices $[w, r, p_l]_{t>0}$ and quantities $[y_a, y_n, c_a, c_n, x_a, x_n, n_a, n_n, k_a, k_n, k]_{t>0}$, satisfying the consumers' optimization conditions, the firms' optimization conditions, together with the inputs' market equilibrium conditions and the open-economy goods' market equilibrium conditions. In the open economy scenario considered here, there is balanced trade in final goods, so there is no international borrowing or lending of capital. Moreover, since the economy is small relative to the world, it does not affect the world prices so both the relative price q and its evolution $\gamma_q \equiv \dot{q}/q$ are exogenous to the country. The model endogenously determines the value of exports and imports at each period.

The optimization conditions of consumers and firms, as well as the market clearing conditions for inputs, are the same as in the closed economy, but the market clearing conditions of the final goods are obviously different.[15] They state that total production of agricultural good is equal to consumption plus net agricultural exports x_a, and total production of

[13]With positive economic growth, we would only observe an increase in the agricultural employment share n_a if c_n/y_n increased enough to offset the other effects.

[14]An increase in the capital stock -provided that the nonagricultural sector is more capital intensive-, or an increase in total population, or an increase in the ratio A_n/A_a affect the relative price growth rate positively.

[15]The firms' optimization conditions are different than in the closed economy if the international price is such that the country specializes in the agricultural good production. Note that specialization in the nonagricultural good is not possible in the short run because the production factor land can only be used in the agricultural production.

nonagricultural good is equal to consumption plus net exports x_n plus total investment.

As it was the case for the closed economy model, if population growth is not too high compared to the productivity growth rates, consumers behave asymptotically as if their preferences were the ones presented in equation (7). As a result, the solution to the competitive equilibrium of the small open economy studied here is the unique path that converges to the Balanced Growth Path of preferences in equation (7), where all the variables grow at constant rates.

During the transition path, both goods are produced unless the agricultural price is high enough and the economy specializes in the agricultural good production. In the Balanced Growth Path, the specialization pattern of the economy depends on the relationship between the relative price growth rate, γ_q, and $\widetilde{\gamma} \equiv (1-\eta-\beta)\,\nu + \frac{1-\eta}{1-\alpha}\gamma_n - \gamma_a$. If $\gamma_q = \widetilde{\gamma}$, the economy produces both goods in the BGP, if $\gamma_q > \widetilde{\gamma}$ the economy specializes in the agricultural good, while if $\gamma_q < \widetilde{\gamma}$ the economy specializes in the nonagricultural good.

Structural Transformation in an Open Economy. When countries open up to international trade, the fraction of productive inputs allocated to the agricultural sector increases or decreases depending on the relation between domestic and international prices. Countries with low land endowment or low agricultural productivity are likely to become agricultural importers, and, as a result, their structural transformation is likely to get accelerated because of trade. Since q is not constant over time, however, the comparative advantage of a country may change over time.

3 Analysis of Three Structural Transformations

In this section, I simulate the model and compare the results to data of the three structural transformation episodes. The closed economy version is used to analyze the structural transformation of the United States during the period 1890-2007, and the small open economy version is used to analyze the transformations of South Korea during the period 1963-2007 and the United Kingdom during the period 1800-1900.[16] To simulate the model, one must first specify the parameter values, which are the same in the three simulations[17]. Then, it is necessary to specify the path of the exogenous variables, both for the sample period and for future periods, and give the initial condition for the capital stock.[18]

3.1 Model Parametrization

The parameters of the model are the production functions exponents (η, β, α), the depreciation rate δ, and the preferences parameters (ρ, μ_0, μ_1, $\underline{c_a}$, c_a^*).

The value used for the labor intensity in the nonagricultural good production function, $1-\alpha$, is 2/3. This is the customary value used in the literature, and it is exactly equal to the average labor income share on total nonagricultural income for South Korea during the period 1963-1995.[19] The value used for the labor exponent in the agricultural good production function, β, is 0.5. This is approximately the average labor income share in total agricultural income for South Korea in the period 1963-1995, and it is within the range of the values used in the literature for other countries.[20] The agricultural capital exponent in the agricultural production function, η is 0.1, which is the average income share in the agricultural sector for South Korea the period 1963-1995. This implies an exponent for land in the agricultural production function equal to 0.4, which is also in the range of values used in the literature.[21]

[16] Although in the real world the United States is not a closed economy, I use the closed economy version of the model to study the United States transformation because its net agricultural exports are low in relation to its production. During the period 1910 - 2007, the ratio of net agricultural exports over agricultural output is between -5% and 5% for most years, and only during the period 1975-1995 the ratio gets over 10%.

[17] As explained below, the parameter $\underline{c_a}$ takes different values for each country, since the units of the three exercises are not directly comparable.

[18] It is necessary to specify the exogenous variables' values for periods beyond the sample because the model is simulated for a longer horizon than the sample.

[19] The income shares series are taken from Kim and Hong (1997). See page 79 for the nonagricultural income shares, and page 67 for the agricultural income shares.

[20] Hayashi and Prescott (2008), for instance, use 0.545 for Japan, Caselli and Coleman II (2001) use 0.6 for the United States, and Stokey (2001) uses 0.387 for the United Kingdom.

[21] Hayashi and Prescott (2008), for instance, use 0.1932 for land in Japan, Caselli and Coleman II (2001) use 0.19 for the United States, and Stokey (2001) uses 0.45 for the United Kingdom.

The value used for the depreciation rate parameter δ is 0.1, which corresponds to the average replacement rate of nonresidential structures and producer durables, as discussed in Jorgenson (1995). The value used for the intertemporal discount factor ρ is 0.06, which makes the long run capital output ratio in the model match that of the United States data. This value is slightly larger than the one used in Cooley (1995), which is 0.054, the reason being that the capital data I will use does not include consumer durables or residential buildings.

The value of the preference parameter μ_1 is 0.01, which is chosen to match the long run agricultural employment share as observed in the United States. The preference parameter μ_0 is set equal to 0.5 to fit the agricultural consumption data for South Korea, given that this parameter only plays a role at low levels of income and does not affect the simulations for the other countries considered. The consumption threshold c_a^* is set equal to 1.5 times $\underline{c_a}$, which is approximately the relation between the two parameters in the South Korean data. Since the units of the data are not comparable across countries, the consumption subsistence level parameter $\underline{c_a}$ is independently chosen for each country from their measured consumption data. For the case of the United States, the value used for $\underline{c_a}$ is about 60% of the agricultural consumption in 1890, for South Korea it is 85% of its agricultural good consumption in 1963, and for the United Kingdom it is 91% of its agricultural good consumption in 1800.

3.2 The United States' Structural Transformation

There are two main reasons to start the analysis with the United States. First, it illustrates the elements behind industrialization of an economy under autarky, such as sectoral productivity growth in the two sectors and capital accumulation. Second, its equilibrium system is very close to its Steady State values by the end of the period, which makes possible to get the value of the parameters that determine the long run equilibrium.

Exogenous variables specification. For the United States simulations, the exogenous variables are total population, total employment, agricultural TFP, and nonagricultural TFP. The agricultural sector equivalent in the data is the *Farm sector*, and the nonagricultural sector is the rest of the economy.[22]

The initial value for capital is the actual value of nonresidential capital stock from the data for the year 1890, which is approximately equal to two times the nonagricultural output. The time series used in the simulation for total population N is obtained directly from the data but eliminating the high-frequency fluctuations. Its growth rate is almost 2% at the beginning of the period and decreases to around 1% by the end of thw period, which is also the value assumed for future periods.

The employment-to-population ratio is approximated using a linear trend. It takes an initial value of 35% and increases during the sample period until 45%, which is assumed to be the value for future periods.

Since there is no direct data on total factor productivity, I infer it from the available data using the Cobb-Douglas production functions from equations (4) and (5), as well as data on value added and employment by sector. These series obtained are then smoothed to eliminate the high frquency fluctuations due to measurement error. Agricultural and nonagricultural capital stocks are inferred from data on aggregate capital assuming that both labor and capital are efficiently allocated across sectors.[23] The growth rates are increasing during the sample period, with the agricultural TFP growth rate starting at 0.5% and increasing to 6%, and the nonagricultural growth rate starting at 0.2% and increasing to 1.5%. The growth rates used for future periods are the average of the last 10 years, which is 5.7% for the agricultural TFP and 1.44% for the nonagricultural TFP.

[22] A more detailed description of the variables used in the simulations of the three countries, as well as their data sources, is available in appendix B of the paper.

[23] It is possible to obtain data on sectoral capital stocks for some periods, but it is not complete and the different data sources do not seem to be compatible.

3.2.1 United States Simulation Results

In Figure 3 we can see how the results of the United States simulations compare to the actual data on agricultural employment, agricultural production, nonagricultural production, aggregate capital stock, and agricultural relative price.

The model is able to successfully replicate the agricultural employment share data, although it seems to slightly underpredict it in the initial years. The model also matches closely the agricultural production data, although it overpredicts the data during the period 1950 - 1980, and the nonagricultural production data is also well matched, with the exception of the period 1930 -1950 which corresponds to the Great Depression and the second world war. The fit of the model is also good in terms of the aggregate capital stock and the agricultural price data of the last 60 years. The simulated model, however, overpredicts the price data during the Great Depression and underpredicts it during the second world war, since the nonagricultural sector TFP is below its trend during the Great Depression years and above its trend during the second world war years.

3.3 South Korea Structural Transformation

Exogenous variables specification. The agricultural sector is defined in the South Korean data as *Agriculture, Forestry and Fishing*, and the nonagricultural sector is composed by all other parts of the economy. The initial capital stock of the simulations is taken directly from the data, and it is equal to 2.87 times the nonagricultural output level. The exogenous variables that need to be specified for the South Korea simulations are total population, total employment, agricultural TFP, and nonagricultural TFP, as well as the agricultural relative price. Note that the initial value of the agricultural production per capita, the nonagricultural production per capita and the capital stock are normalized to 100 to facilitate the reading of the figures.

Total population, N, is taken directly from the data, although the high frequency fluc-

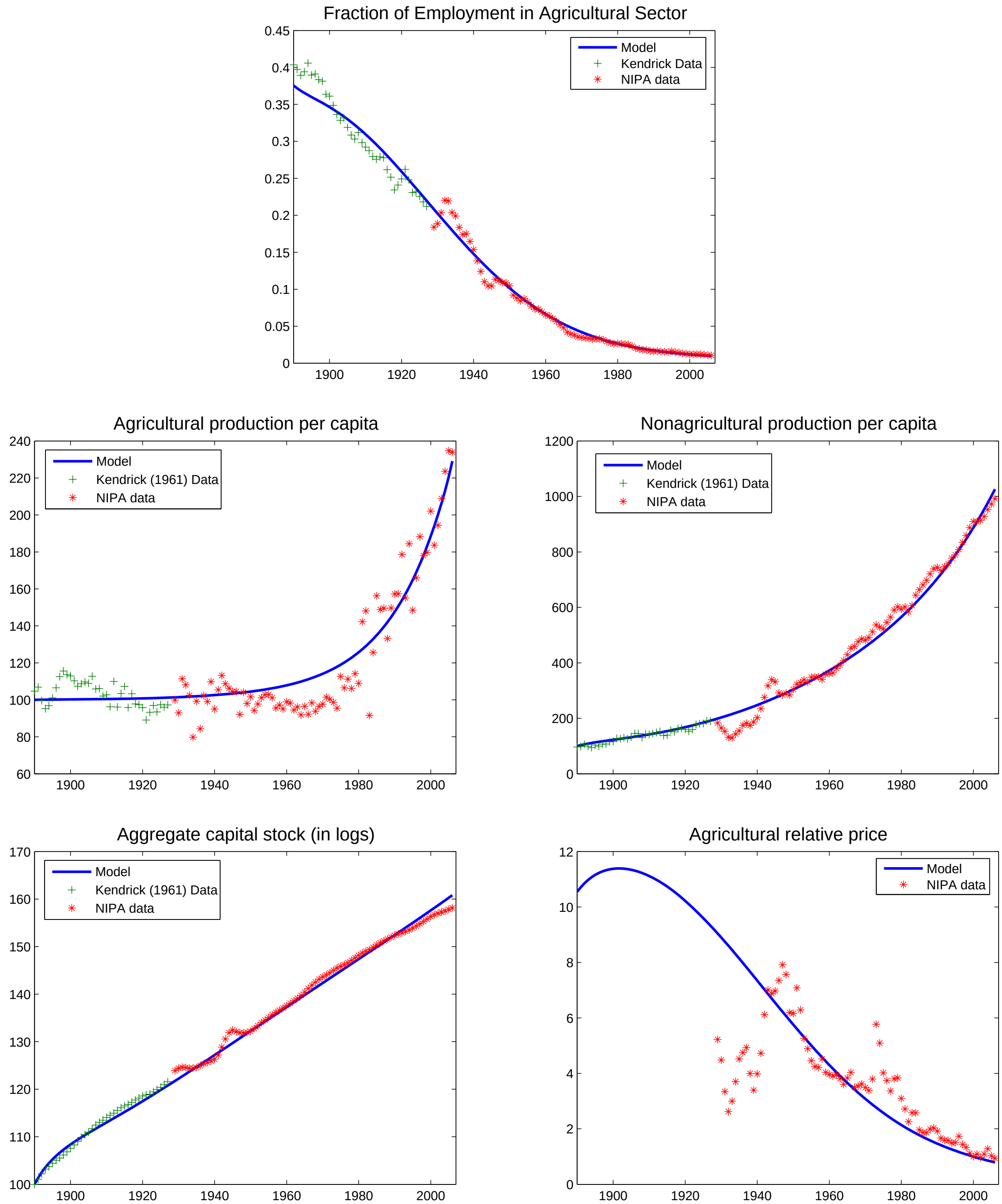

Figure 3: **Model Simulation vs United States Data**
Datasources: Kendrick (1961), Bureau of Economic Analysis - National Income and Product Accounts.

tuations are eliminated using the Hodrick-Prescott filter. Its growth rate is around 2.5% initially and 0.5% by the end of the period, which is also the value assumed for future periods. The employment-to-population ratio is approximated with a linear trend, and its initial and final values are 27% and 50% respectively.

Agricultural and nonagricultural TFPs are assumed to have constant growth, since the data does not show any significant trend. The growth rates chosen for both are the ones that make the simulated production variables fit the actual production variables, but they are very close to the growth rates measured in the data.[24] Finally, the relative agricultural price is approximated by dividing the agricultural GDP deflator by the nonagricultural GDP deflator, eliminating the high frequency fluctuations using the Hodrick-Prescott filter. Its growth rate during the sample period ranges form -0.023 to -0.012, and its future growth is assumed to be -0.0216, which is equal to the average growth of the last 10 years.

Government policy variables. There are many studies documenting the efforts of the Korean government to protect the agricultural sector and increase the income of agricultural producers. The two main policy tools have been agricultural production subsidies and the agricultural import tariffs.[25] To improve the fit of the model, a production agricultural subsidy σ_a is introduced, together with a lump sum tax to households τ to ensure that the government budget is balanced every period, so that $\sigma_a(t)\, q(t)\, Y_a(t) = N(t)\, \tau(t)$.

In the simulations, I use $\sigma_a = 0$ until 1972 because no subsidies seem to be used prior to that date, and $\sigma_a = 0.10$ from 1973 onwards because higher values make the model clearly overpredict agricultural production and agricultural employment. With respect the agricultural import tariff, denoted t_a, I use 104% for the year 1990, as estimated by Diao,

[24]For agricultural TFP, the average growth rate of the measured time series is 0.033 and the value used in the simulations is 0.032. For nonagricultural TFP, the average growth rate of the measured time series is 0.02 and the value used in the simulations is 0.021.

[25]According to the OECD report "Agricultural Policies at a Glance (2008)", agricultural production subsidies were equal to 56% in 1980 and 65% in 2000, but other studies estimate lower values. And, according to the United States Department of Agriculture report Diao, Dyck, Skully, Somwaru, and Lee (2002), the agricultural sector tariff equivalent rate, which is computed taking into account not only explicit import tariff rates but also quantitative restrictions such as direct government bans and quotas, is equal to 104% in 1990.

Dyck, Skully, Somwaru, and Lee (2002).[26] I assume that the import tariffs do not affect the households budget constraint, which implies that either the tariffs do not generate revenues or that the revenues are not transferred to households. In fact, according to the USDA article mentioned above, "support to Korean agriculture is principally manifested by border measures" and, for example "the tariff rate on rice in 1975 and 1990 (5 percent) does not reflect the extremely high barrier posed by complete government control over rice imports or direct subsidies to production".[27]

3.3.1 South Korea Simulation Results

Figure 4 compares the outcome of the simulated model with the actual South Korean data for six different variables: fraction of employment in the agricultural sector, agricultural production per capita, agricultural consumption per capita, agricultural net imports, nonagricultural production per capita, and aggregate capital stock.[28]

In the first two subplots, one can easily see the effects of the agricultural production subsidy in 1972; both the agricultural employment share and the agricultural production per capita have a sudden increase. Obviously, the subsidy also affects the net agricultural imports, since the sudden increase in agricultural production also leads to a sudden decrease in net agricultural imports. The introduction of the subsidy helps the model to the data, especially in the case of the agricultural production, but the main conclusions are not affected by it. In the agricultural consumption subplot, one can see the consequences of the preferences used; around 1980 agricultural consumption per capita reaches the level c_a^* and it starts growing more slowly after that date.

[26]This value does not affect the results of the baseline simulations because I am already using data on the domestic relative price. It matters, however, for the counterfactual with no agricultural policies, as explained in next section.

[27]See Diao, Dyck, Skully, Somwaru, and Lee (2002), page 3.

[28]Note that the variables agricultural production per capita and nonagricultural production per capita are plotted starting at 100 to facilitate the reading of the figures.

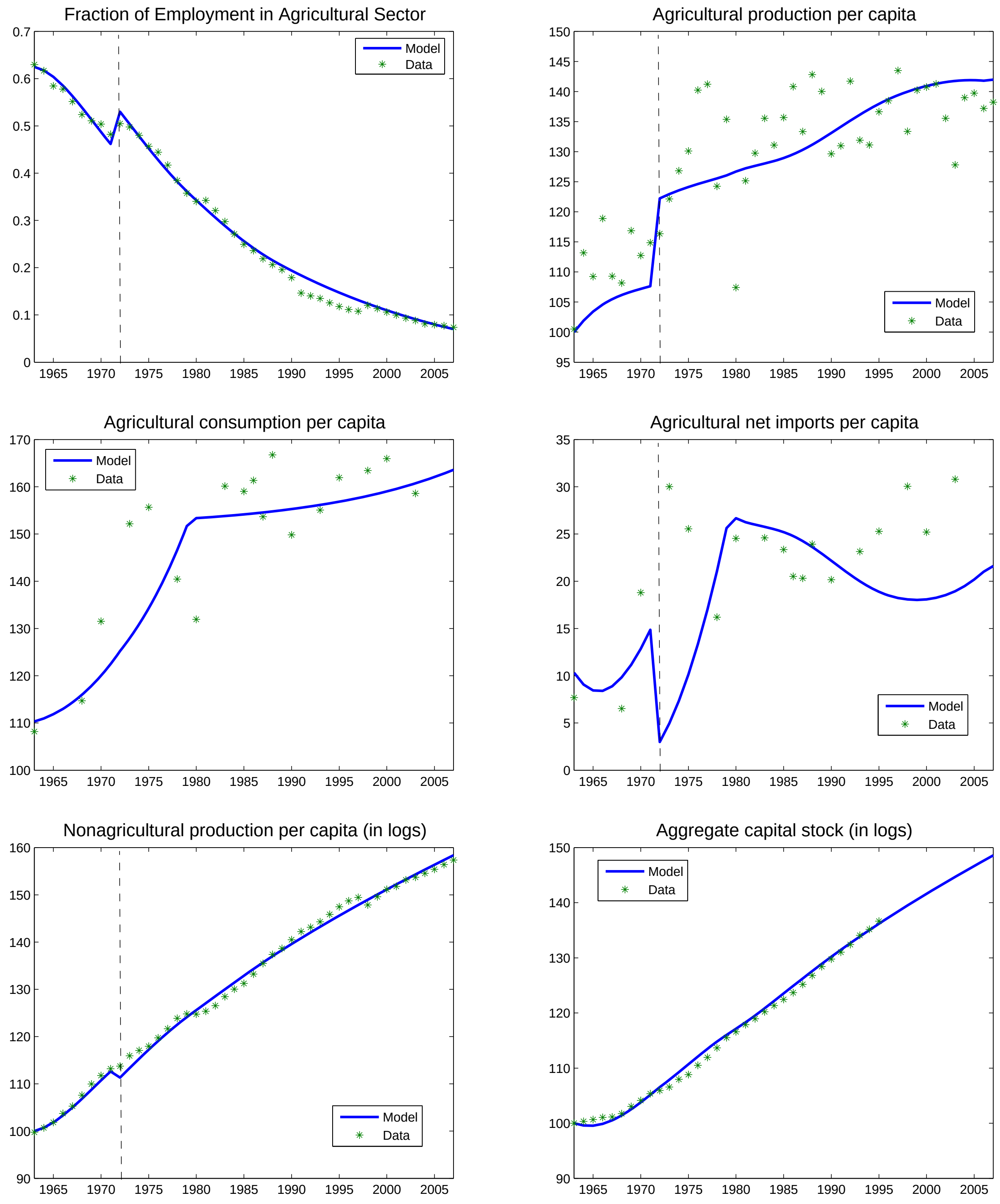

Figure 4: **Model Simulation vs South Korea Data**
Datasources: Statistical Yearbooks of the Republic of Korea, Bank of Korea - Economic Statistics System, Korea Statistical Information Service, Kim and Hong (1997).

3.4 United Kingdom Structural Transformation

Exogenous variables specification. The procedure followed to obtain the exogenous variables for the United Kingdom simulations is exactly the same as the for South Korea. Thus, the agricultural sector is defined as *Agriculture, Forestry and Fishing* and the nonagricultural sector is all the other sectors of the economy. The initial capital stock is taken from the data, and it is about 2.5 times the nonagricultural output level. Total population, N, is the HP-filtered data from different sources, the employment-to-population ratio is approximated with a linear trend, and the sectoral TFPs are assumed to have a constant growth rate.

The population growth rate starts at around 1.6% at the beginning of the sample period, and it is equal to 0.8% by the end of the period. After 1900, it is assumed to be constant and equal to 1%, which is the average population growth of the last 10 years of the sample period. The initial employment participation is 28% while the final one is close to 45%. The TFP growth rates used in the simulations are 1.25% for the agricultural sector and 0.65% for the nonagricultural one. They are chosen to get a good fit of the model, but they are almost identical to the ones measured using the Cobb-Douglas production functions and data on value added and employment by sector. Finally, the relative agricultural price is obtained by fitting a constant-growth curve to the ratio of the agricultural GDP deflator over the nonagricultural GDP deflator. Its growth rate both during the sample period and in future periods is equal to -0.43%.

3.4.1 United Kingdom Simulation Results

The outcome of the simulated model is compared with the actual data in figure 5 for six different variables: fraction of employment in the agricultural sector, agricultural production per capita, agricultural consumption per capita, agricultural net imports, nonagricultural production per capita, and aggregate capital stock. Note that the initial value of the agricultural production per capita, the nonagricultural production per capita and the capital stock are normalized to 100 to facilitate the reading of the figures.

The first plot shows the agricultural employment share. We can see that the model is able to capture the main features of the data, although it overpredicts the data initially and then it underpredicts it around 1850. The variable in the second plot is agricultural production per capita, which shows that the model is also able to get close to the data for that variable, although it is very volatile in this case. The third plot compares the net agricultural imports series from the model simulation and from the data, and it shows a good fit except for the last observation available. The data availability, however, is shorter for the case of this variable than for the others. The fourth and fifth plot show the model is also able to match the data for the nonagricultural production variable and the aggregate capital stock.

4 Analysis of the Role Played by International Trade

4.1 The Role of Trade in South Korea

To evaluate the importance of international trade in the structural transformation of South Korea, I perform two counterfactual exercises. The first consists of comparing the baseline simulation of South Korea presented in the previous section with a situation where South Korea is not open to international trade and has to produce all the agricultural good consumed by itself. The second consists of comparing the baseline simulation of South Korea with a *free trade* situation where no agricultural policies are implemented to protect the agricultural sector.

The autarky counterfactual exercise is performed by simulating the closed economy model keeping all parameters and exogneous variables identical to the baseline simulation. Figure 6 shows that if South Korea had been under autarky, the agricultural relative price would

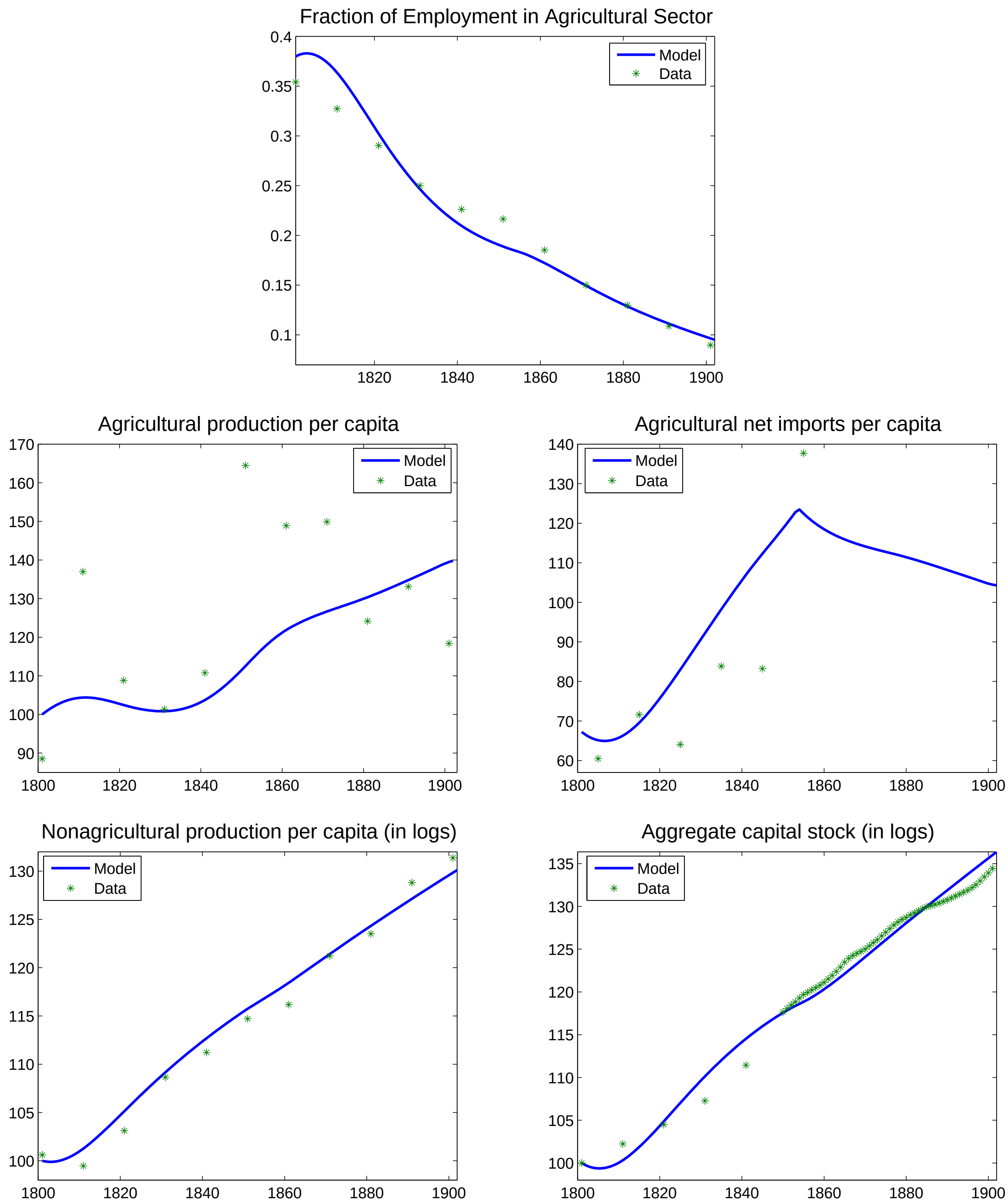

Figure 5: **Model Simulation vs United Kingdom Data**
Datasources: Mitchell (1962), Deane and Cole (1969), Davis (1979), Feinstein (1988).

have been significantly larger, the agricultural employment share would have been somewhat larger, the agricultural consumption would have been slightly lower, the nonagricultural consumption would have been approximately the same, and the capital stock would have been lower.

The second counterfactual exercise, labeled as "No Agricultural Policy", predicts what would have been the South Korea development process if the government had not introduced policies attempting to protect the agricultural sector. To do so, I remove the agricultural production subsidy used from the baseline simulation of South Korea and infer the free trade relative price by removing the agricultural import tariff. To generate this relative price under free trade, I use the average growth rate of the relative agricultural price in the United States data for the period 1963–2007 -which is equal to -3.7%- with the underlying assumption that the prices in the US are the same as in the world markets. Then, I choose the level of the relative price to match the implicit tariff rate estimated by the USDA for the year 1990, which is equal to 104%.

Figure 7 shows that if South Korea had no production subsidies and no import tariffs it would have had a much higher level of net agricultural imports, a much lower agricultural employment share, a significantly higher agricultural consumption per capita, a somewhat higher nonagricultural consumption per capita, as well as a significantly higher capital stock per capita.

4.2 The Role of Trade in the United Kingdom

To quantify the importance of international trade for the structural transformation of the United Kingdom, the baseline simulation presented above is compared to the closed economy simulation, which are done using the same exogenous variables and initial conditions.

Figure 8 shows that both the agricultural relative price and the agricultural employment share would have been significantly larger, while the agricultural consumption level, the

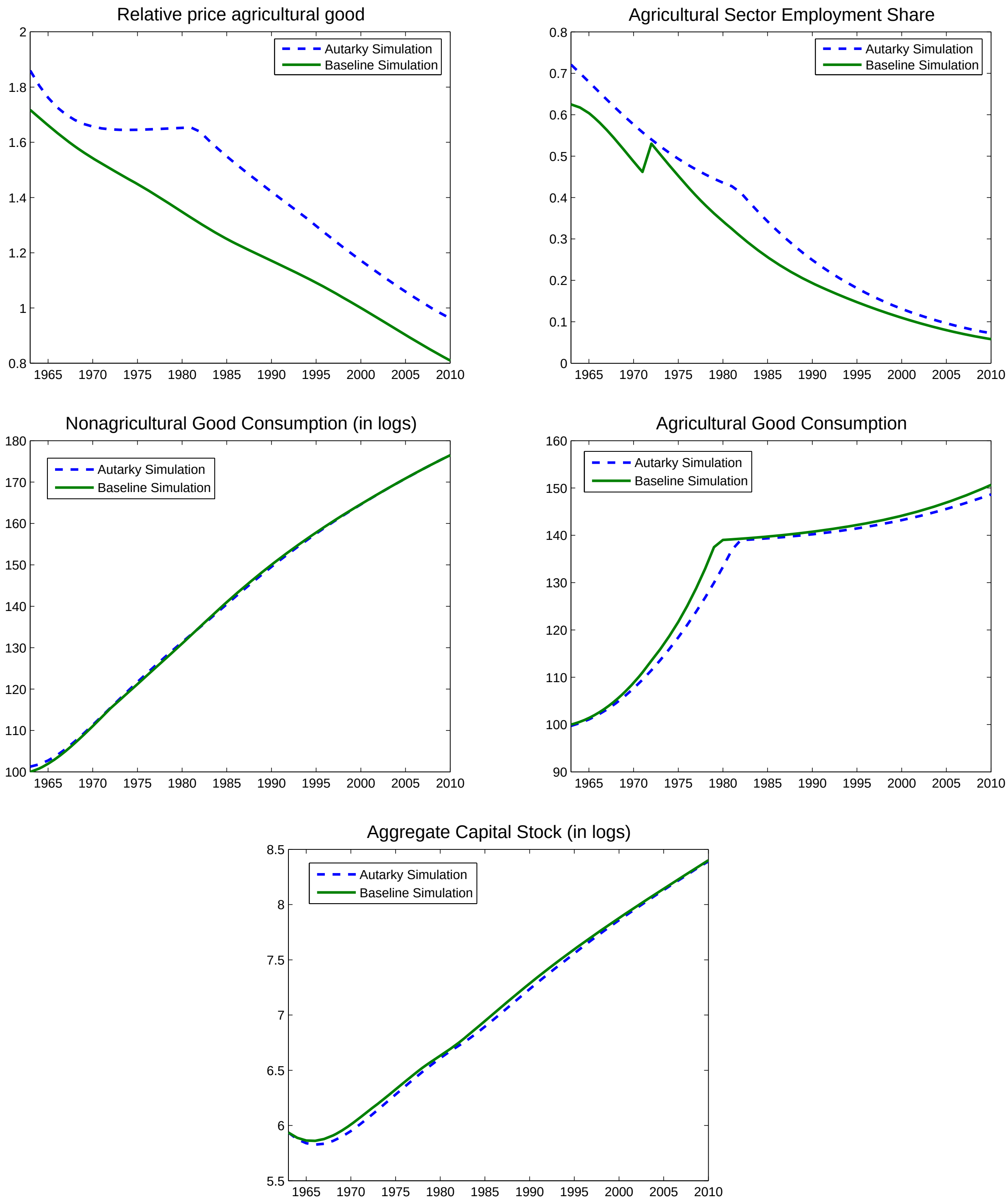

Figure 6: **South Korea: Baseline Simulation vs Autarky Simulation**

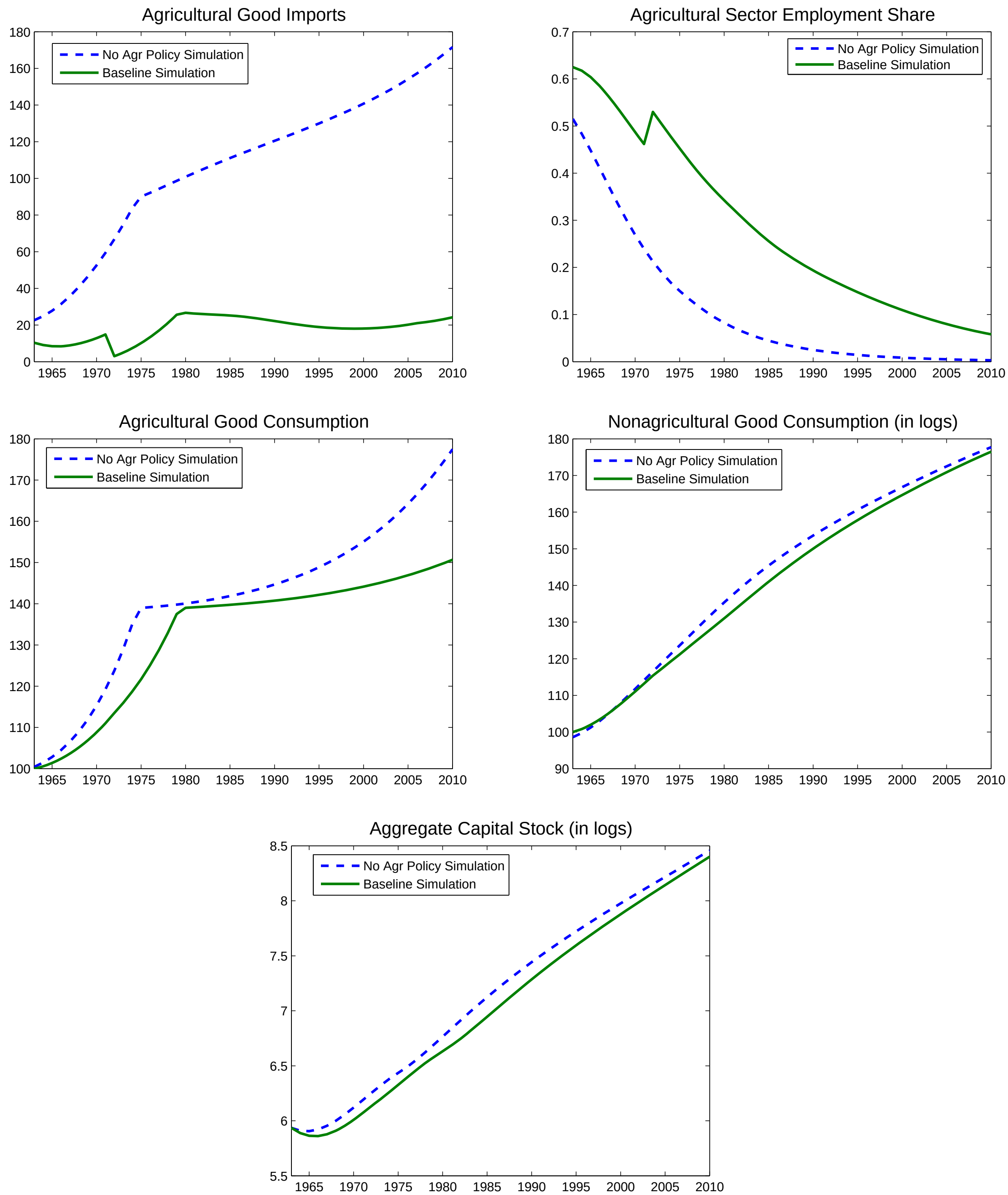

Figure 7: **South Korea: Baseline Simulation vs No Agricultural Policy Simulation**

nonagricultural consumption level and the capital stock would have been significantly lower. The model also predicts that the differences in capital stock, nonagricultural consumption, and agricultural employment share would vanish over time, but not the differences in agricultural consumption level and the agricultural relative price.

4.3 Results summary and effects on income growth and welfare

Figure 9 plots the agricultural employment share at each income level in the data and in the counterfactual simulations. In the first plot we can see that if the United Kingdom had been under autarky, its agricultural employment share at each income level would have been very similar to the one observed in the United States. In the second plot we can see that if South Korea had been under autarky, its agricultural employment share at each income level would have been even higher than the one observed in the United States. If it had been completely open to international trade, on the other hand, its agricultural employment share at each income level would have been much lower and very similar to the one observed in the United Kingdom.

Economic Growth Analysis. In South Korea, the average income growth rate during the sample period, computed as the nominal income growth minus the change in the consumption price index, is 4.71% per year in the baseline simulation. If the country had remained under autarky, the real income growth rate would have been slightly lower, 4.5%. On the other hand, it would have been significantly higher, 5.5% per year, if the country had not introduced either agricultural production subsidies or agricultural import tariffs, but it would not have changed at all by removing only the production subsidies. In the United Kingdom, the average real income growth rate during the 19th century was 1.44% per year in the baseline simulation, which is 1.13 times higher the one under autarky, 1.28% per year.

Welfare Analysis. To measure the intertemporal welfare gain of the representative household with respect to autarky, I compute the increase in the autarky yearly consump-

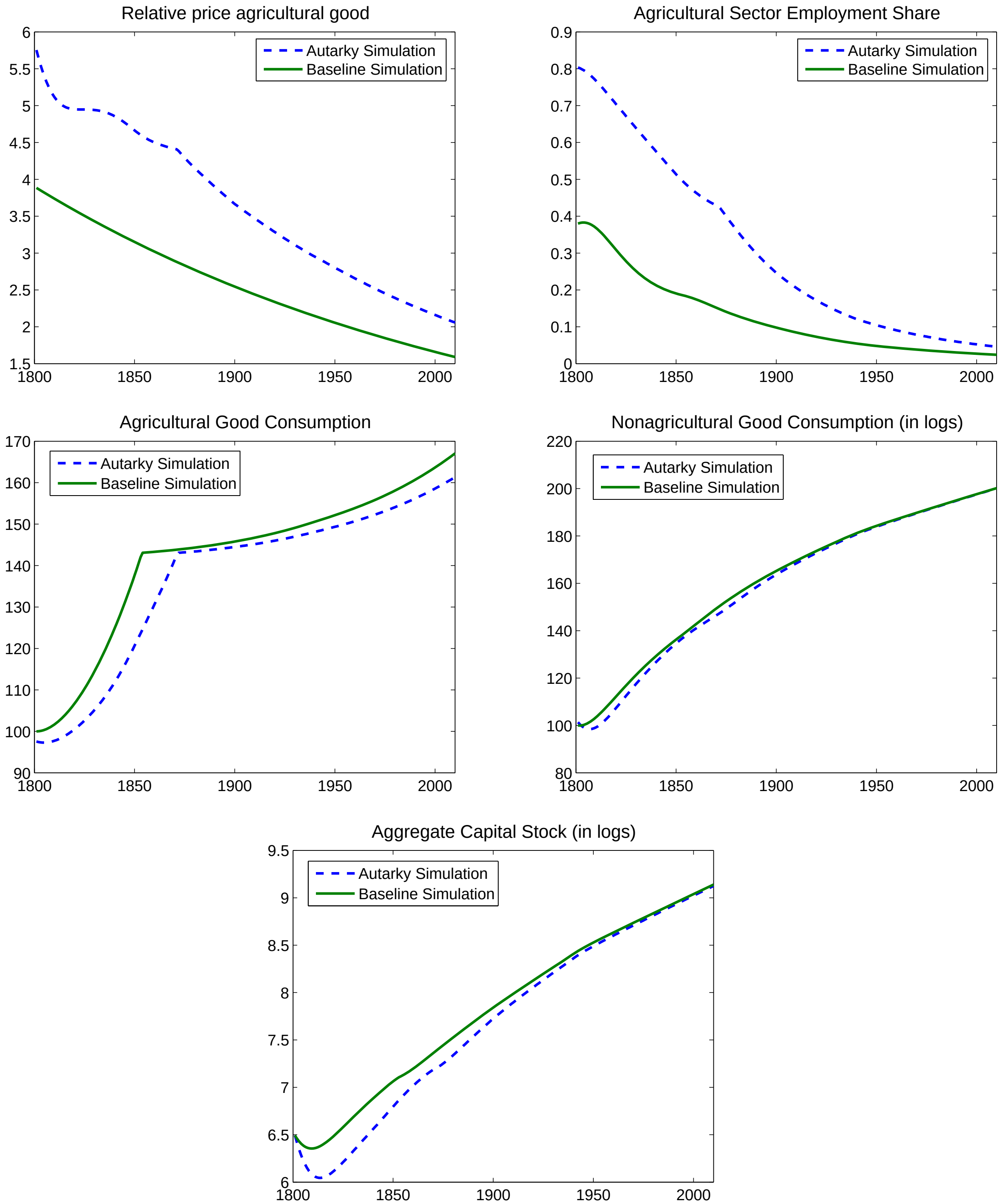

Figure 8: **United Kingdom: Baseline Simulation vs Autarky Simulation**

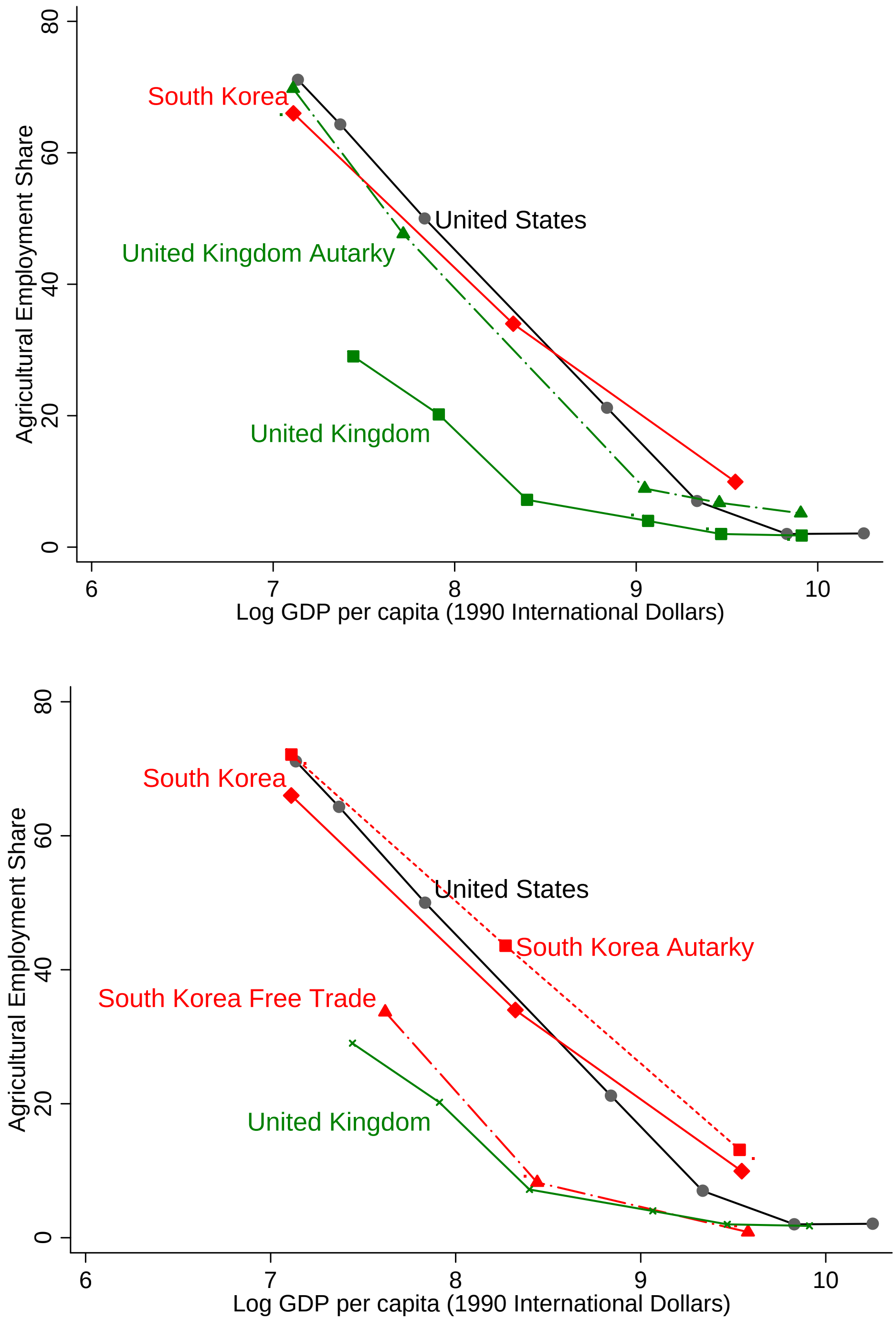

Figure 9: **Main Results - United States, United Kingdom and South Korea**
Data sources: Kuznets (1966), World Bank WDR1984, Food and Agriculture Organization, Maddison (2006), and own calculations.

tion expenditures that delivers the same increase in intertemporal welfare as opening up to international trade. In South Korea, the gain in intertemporal welfare experienced by the representative household with respect to autarky is equivalent to a 0.4% increase in the autarky yearly consumption expenditures. In other words, the representative consumer of South Korea in 1963 is indifferent between getting open to international trade or remaining under autarky but increasing the consumption expenditure by 0.4% every year. If there had not been agricultural production subsidies, the welfare gain in South Korea would have been almost the same, equivalent to a 0.5% annual increase in the consumption expenditures, and if there had been neither agricultural production subsidies nor agricultural import tariffs, the gain in intertemporal welfare with respect to autarky would have been equivalent to a 5.4% increase in the annual consumption expenditures. In the United Kingdom, on the other hand, the extra intertemporal welfare enjoyed by the representative household because the country was open to international trade is equivalent to a 5.5% annual increase in the autarky consumption expenditures.

5 Conclusion

To study the importance of international trade in the structural transformation process of countries, this paper analyzes the closed and open economy versions of a neoclassical, two-sector growth model. The model is calibrated and simulated using data from the United States, South Korea and the United Kingdom, and it is shown to be able to match the structural transformation data of the three countries. The model is then used to quantify the importance of agricultural imports in the transformations of South Korea and the United Kingdom. In the case of the United Kingdom during the 19th century, the results show that international trade played a very significant role in its industrialization, confirming the results in Stokey (2001) in a somewhat different model. In the case of South Korea during the last 50 years, the results show that international trade also had a positive impact on its structural transformation but it would have played a much larger role if the country had not introduced agricultural production subsidies in the early 1970s and had not applied tariffs to agricultural imports during the entire sample period.

In terms of policy implications, this paper suggests that policies aimed at increasing international trade in agricultural goods can be very beneficial, and that protectionism towards the agricultural sector is likely to reduce welfare and slow down income growth. Through the examples of the United Kingdom and South Korea we can see that there are large potential gains from agricultural trade. This is specially true for poor countries, since they have much larger agricultural sectors even if they do not seem to have comparative advantage in that sector.[29] In 1985, for instance, the average share of employment in agriculture was over 85% in the lowest income decile countries, while it was only 5% in the highest decile countries. At the same time, the lowest decile of countries were on average fifty times less productive than the highest decile of countries in agriculture, while the ratio in the aggregate was *only* thirty. Similarly, the relative price of the agricultural sector good was, on average, two times larger in the lowest decile of countries than in the highest decile.[30]

[29] Agricultural trade flows are usually small, but the data show that poor countries tend to be food importers. According to Aksoy and Ng (2008), in 2004, 72% of poor countries were net food importers and 28% were net food exporters, while 61% of industrial economies were net food importers and 39% net food exporters.

[30] See, for example, Caselli (2005), Gollin, Parente, and Rogerson (2004), and Restuccia, Yang, and Zhu (2008). Data sources: Heston, Summers, and Aten (2006) for the aggregate prices and income per capita, Rao et al. (1993) for the agricultural sector prices.

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Appendices

A Equilibrium Analysis

A.1 Agents' Optimization

Households choose the amount to consume of each good, the amount to save and invest, as well as the sector to rent the inputs they own for all time periods. From the consumers' optimization problem, we can derive the following two conditions:

$$\begin{aligned} \frac{1}{\mu_0}\frac{\left(c_a-\underline{c_a}\right)}{c_n} &= q \text{ if } c_a \leq c_a^* \\ \frac{1}{\mu_1}\frac{\left(c_a-c_a^*+A\right)}{c_n} &= q \text{ if } c_a > c_a^* \end{aligned} \tag{8}$$

$$\frac{dc_n}{dt}=c_n\left(r-\delta-\rho\right) \tag{9}$$

Equation (8) gives the optimal consumption mix, while equation (9) gives the optimal growth rate of nonagricultural consumption. These, together with equation (3), which gives the optimal evolution of the capital stock, and the boundary conditions $k\left(0\right)=k_0$ and the following transversality condition:

$$\lim_{t\to\infty}\left\{e^{-\int_0^t\left(r(s)-\nu-\delta\right)ds}\frac{k\left(t\right)}{c_n\left(t\right)}\right\}=0. \tag{10}$$

With respect to firms, in order to maximize their profits, they choose the amount of inputs to hire at each period so that the value of their marginal productivity is equal to its price in both sectors

$$r=q_j\frac{\partial y_j}{\partial k_j},\ j=a,n \tag{11}$$

$$w=q_j\frac{\partial y_j}{\partial n_j},\ j=a,n \tag{12}$$

$$p_l=q\frac{\partial y_a}{\partial\left(L_a e^{-\nu t}\right)} \tag{13}$$

where q_n is normalized to 1, and q_a is denoted by q.

A.2 Closed Economy Equilibrium Analysis

For both the closed and open economy, the production factor markets clear at every point in time:

$$n_a+n_n=1 \tag{14}$$

$$k_a + k_n = k \tag{15}$$

$$L_a = L. \tag{16}$$

If we define $\kappa_a \equiv \frac{k_a}{k}$, equation (15) implies that $\frac{k_n}{k} = 1 - \kappa$.

When the economy is closed to international markets, the endogenous relative price adjusts so that the amount of both goods supplied domestically is equal to the amount demanded domestically. The two output market clearing conditions for a closed economy are:

$$A_a f^a \left(k_a, n_a, L_a e^{-\nu t}\right) = c_a \tag{17}$$

$$A_n f^n \left(k_n, k_n\right) = c_n + \frac{dk}{dt} + \nu k + \delta k. \tag{18}$$

Definition 1. *A competitive equilibrium for the closed economy, given k_0 and the exogenous variables (A_a, A_n, N), are prices and quantities $[q,\ w,\ r,\ p_l,\ y_a,\ y_n,\ c_a,\ c_n,\ n_a,\ n_n,\ k_a,\ k_n,\ k]_{t>0}$, satisfying the consumers' optimization conditions (8) - (10), the firms' optimization conditions (11) -(13), and the market equilibrium conditions (14) -(18).*

The equilibrium conditions of the closed economy can be simplified to the two dynamic equations plus the three static equations below. Nonagricultural good consumption c_n is the control variable and per capita capital stock k the state variable, both of which depend on the other endogenous variables n_a and $\kappa_a \equiv \frac{k_a}{k}$.

$$\frac{\dot{c_n}}{c_n} = \alpha A_n \left(\left(1-\kappa_a\right) k\right)^{\alpha-1} \left(1-n_a\right)^{1-\alpha} - \delta - \rho \tag{19}$$

$$\dot{k} = A_n^{\alpha} \left(\left(1-\kappa_a\right) k\right) \left(1-n_a\right)^{1-\alpha} - \left(\delta+\nu\right) k - c_n \tag{20}$$

$$q\eta \frac{A_a n_a^{\beta} \left(Le^{-\nu t}\right)^{1-\eta-\beta}}{\left(\kappa_a k\right)^{1-\eta}} = \alpha \frac{A_n \left(1-n_a\right)^{1-\alpha}}{\left(\left(1-\kappa_a\right) k\right)^{1-\alpha}} \tag{21}$$

$$q\beta \frac{A_a \left(\kappa_a k\right)^{\eta} \left(Le^{-nt}\right)^{1-\eta-\beta}}{n_a^{1-\beta}} = \left(1-\alpha\right) \frac{A_n^{\alpha} \left(\left(1-\kappa_a\right) k\right)}{\left(1-n_a\right)^{\alpha}} \tag{22}$$

$$A_a \left(\kappa_a k\right)^{\eta} n_a^{\beta} \left(Le^{-\nu t}\right)^{1-\eta-\beta} = \begin{cases} c_a + \mu_0 \frac{c_n}{q} & \text{if } c_a \leq c_a^* \\ c_a^* + \mu_1 \frac{c_n}{q} & \text{if } c_a > c_a^* \end{cases} \tag{23}$$

As explained in the lemmas below, the equilibrium solution is the path that solves this dynamic system and converges to the Balanced Growth Path of preferences in equation (7). Hence, if we detrend the dynamic system variables by their long-run growth rates, the solution is the path that converges to the Steady State of the detrended system.

Lemma 1. *If the preferences are the ones in equation (7), then there exists a Balanced Growth Path where all the variables grow at constant rates, shown in equations (24) - (27):*

$$\frac{\dot{n_a}}{n_a} = \frac{\dot{\kappa_a}}{\kappa_a} = 0 \tag{24}$$

$$\frac{\dot{k}}{k} = \frac{\dot{c_n}}{c_n} = \frac{1}{1-\alpha}\gamma_n \tag{25}$$

$$\frac{\dot{c_a}}{c_a} = \gamma_a + \frac{\eta}{1-\alpha}\gamma_n - (1-\eta-\beta)\,\nu \tag{26}$$

$$\frac{\dot{q}}{q} = (1-\eta-\beta)\,\nu + \frac{1-\eta}{1-\alpha}\gamma_n - \gamma_a \tag{27}$$

Lemma 2. *Let* $\left\{\widehat{k}, \widehat{c_n}, \widehat{q}\right\}$ *be defined as follows:*

$$\begin{aligned}
\widehat{k} &\equiv \frac{k}{A_n^{\frac{1}{1-\alpha}}} \\
\widehat{c_n} &\equiv \frac{c_n}{A_n^{\frac{1}{1-\alpha}}} \\
\widehat{q} &\equiv \frac{q}{A_n^{\frac{1-\eta}{1-\alpha}}\left(Le^{-\nu t}\right)^{1-\eta-\beta}}
\end{aligned}$$

The equilibrium system defined above in equations (19) -(22), together with the initial condition for k_0 *and the transversality condition in equation (10), can be rewritten in terms of this new set of normalized or detrended variables:*

$$\dot{\widehat{c_n}} = \widehat{c_n}\left[\alpha\left((1-\kappa_a)\,\widehat{k}\right)^{\alpha-1}(1-n_a)^{1-\alpha} - \delta - \rho - \frac{1}{1-\alpha}\gamma_n\right] \tag{28}$$

$$\dot{\widehat{k}} = \left((1-\kappa_a)\,\widehat{k}\right)^{\alpha}\left((1-n_a)\right)^{1-\alpha} - \left(\delta + \nu + \frac{1}{1-\alpha}\gamma_n\right)\widehat{k} - \widehat{c_n} \tag{29}$$

$$\widehat{q}\eta\frac{n_a^{\beta}}{\left(\kappa_a\widehat{k}\right)^{1-\eta}} = \alpha\frac{(1-n_a)^{1-\alpha}}{\left((1-\kappa_a)\,\widehat{k}\right)^{1-\alpha}} \tag{30}$$

$$\widehat{q}\beta\frac{\left(\kappa_a\widehat{k}\right)^{\eta}}{n_a^{1-\beta}} = (1-\alpha)\,\frac{\left((1-\kappa_a)\,\widehat{k}\right)^{\alpha}}{(1-n_a)^{\alpha}} \tag{31}$$

$$\begin{aligned}
\left(\kappa_a\widehat{k}\right)^{\eta}(n_a)^{\beta} &= \left\{\begin{array}{c} z(t)\,\underline{c_a} + \mu_0\frac{\widehat{c_n}}{\widehat{q}} \text{ if } c_a \leq c_a^* \\ z(t)\,c_a^* + \mu_1\frac{\widehat{c_n}}{\widehat{q}}\ \ c_a > c_a^* \end{array}\right. \\
\text{where } z(t) &\equiv \frac{\left(Le^{-\nu t}\right)^{\eta+\beta-1}}{A_a A_n^{\frac{\eta}{1-\alpha}}}
\end{aligned} \tag{32}$$

$$\lim_{t\to\infty}\left\{e^{-\int_0^t (r(s)-\delta-\nu)ds}\frac{\widehat{k}}{\widehat{c_n}}\right\} = 0 \tag{33}$$

Lemma 3. *If the preferences are the ones in equation (7), then the detrended equilibrium system defined in equations (28) - (32) has a Steady State, where all the variables are constant. The solution to the detrended equilibrium system consists of the path that converges to this Steady State.*

$$\alpha\left(\left(1-\kappa_a^{ss}\right)\widehat{k}^{ss}\right)^{\alpha-1}\left(\left(1-n_a^{ss}\right)\right)^{1-\alpha}=\delta+\rho+\frac{1}{1-\alpha}\gamma_n \tag{34}$$

$$\widehat{c_n}^{ss}=\left(\left(1-\kappa_a^{ss}\right)\widehat{k}^{ss}\right)^{\alpha}\left(\left(1-n_a^{ss}\right)\right)^{1-\alpha}-\left(\delta+n+\frac{1}{1-\alpha}\gamma_n\right)\widehat{k}^{ss} \tag{35}$$

$$\widehat{q}^{ss}\eta\frac{\left(n_a^{ss}\right)^{\beta}}{\left(\kappa_a^{ss}\widehat{k}^{ss}\right)^{1-\eta}}=\alpha\frac{\left(1-n_a^{ss}\right)^{1-\alpha}}{\left(\left(1-\kappa_a^{ss}\right)\widehat{k}^{ss}\right)^{1-\alpha}} \tag{36}$$

$$\widehat{q}^{ss}\beta\frac{\left(\kappa_a^{ss}\widehat{k}^{ss}\right)^{\eta}}{\left(n_a^{ss}\right)^{1-\beta}}=\left(1-\alpha\right)\frac{\left(\left(1-\kappa_a^{ss}\right)\widehat{k}^{ss}\right)^{\alpha}}{\left(1-n_a^{ss}\right)^{\alpha}} \tag{37}$$

$$\left(\kappa_a^{ss}\widehat{k}^{ss}\right)^{\eta}\left(n_a^{ss}\right)^{\beta}=\mu_1\frac{\widehat{c_n}^{ss}}{\widehat{q}^{ss}} \tag{38}$$

Lemma 4. *For the equilibrium system with the preferences defined in equation (2), if* $\left(1-\eta-\beta\right)\nu<\gamma_a+\frac{\eta}{1-\alpha}\gamma_n$, *then the detrended equilibrium system defined in (28) - (32) converges to same equilibrium as the equation (7) preferences case. Therefore, the Competitive Equilibrium defined in equations (8) - (10), (11) -(13) and (14) -(18) has a unique Balanced Growth Path, where the fraction of labor and capital allocated to both sectors are positive and constant, and the growth rates of capital, consumption and the relative price are*

$$\frac{\dot{k}}{k}=\frac{\dot{c}_n}{c_n}=\frac{1}{1-\alpha}\gamma_n$$

$$\frac{\dot{c}_a}{c_a}=\gamma_a+\frac{\eta}{1-\alpha}\gamma_n-\left(1-\eta-\beta\right)\nu$$

$$\frac{\dot{q}}{q}=\left(1-\eta-\beta\right)\nu+\frac{1-\eta}{1-\alpha}\gamma_n-\gamma_a$$

Moreover, for any initial condition, the economy converges asymptotically to this Balanced Growth Path.

Lemma 5. *In a closed economy, the agricultural employment share of a country,* n_a, *can be expressed as*

$$\begin{aligned}\frac{n_a}{1-n_a}\left(1-\frac{\underline{c_a}}{c_a}\right) &= \mu_0\frac{\beta}{1-\alpha}\frac{c_n}{y_n} \quad \textit{if } c_a\le c_a^* \\ \frac{n_a}{1-n_a}\left(1-\frac{c_a^*-A}{c_a}\right) &= \mu_1\frac{\beta}{1-\alpha}\frac{c_n}{y_n} \quad \textit{if } c_a> c_a^*\end{aligned}$$

Hence, n_a shrinks as households raise their consumption of agricultural good, unless $\frac{c_n}{y_n}$ increases enough to make n_a decrease.

Lemma 6. *The relative price of the agricultural good can be expressed*

$$q = \frac{A_n}{A_a}\left(Le^{-\nu t}\right)^{\eta+\beta-1} k^{\alpha-\eta}\Psi\left(n_a\right) \tag{39}$$

where

$$\Psi\left(n_a\right) = \frac{n_a^{1-\eta-\beta}}{\left(\frac{\beta}{\eta} - n_a\left(\frac{\beta}{\eta} - \frac{1-\alpha}{\alpha}\right)\right)^{\alpha-\eta}},\ \Psi'\left(n_a\right) > 0.$$

Hence, if the TFP ratio, the capital stock, and the population are constant, the relative price of the agricultural good moves in the same direction as the agricultural employment share. Also, an increase in total population has a positive effect on the agricultural price q; at the same time, if the nonagricultural sector is more capital intensive than the agricultural one, then capital accumulation has a positive effect on the agricultural price q; finally, an increase in the nonagricultural productivity relative to the agricultural productivity, has a positive effect on the relative price q.

A.3 Open Economy Equilibrium Analysis

In the open economy, the two market clearing conditions for the final goods are

$$A_a f^a\left(k_a, n_a, L_a e^{-nt}\right) = c_a + x_a \tag{40}$$

$$A_n f^n\left(k_n, n_n\right) = c_n + x_n + \frac{dk}{dt} + \nu k + \delta k. \tag{41}$$

where x_a denotes the value of net agricultural exports and x_n denotes the value of net nonagricultural exports. Because of the balanced trade assumption, the value of net agricultural exports plus the value of net nonagricultural exports is zero at every date:

$$qx_a + x_n = 0 \tag{42}$$

Definition 2. *A competitive equilibrium for the small open economy, given k_0 and the exogenous variables $(A_a,\ A_n,\ N,\ q)$, are prices and quantities $[w,\ r,\ p_l,\ y_a,\ y_n,\ c_a,\ c_n,\ x_a,\ x_n,\ n_a,\ n_n,\ k_a,\ k_n,\ k]_{t>0}$, satisfying the consumers' optimization conditions (8) - (10), the firms' optimization conditions (11) -(13), and the inputs' market equilibrium conditions (14) - (16) and the goods' market equilibrium conditions (40) - (42).*

The open economy equilibrium can be simplified to two dynamic equations - (43) and (44) below - and four static equations, which vary depending on the country's specialization pattern, as shown below.

$$\frac{\dot{c_n}}{c_n} = r - \delta - \rho \tag{43}$$

$$\dot{k} = A_n^{\alpha} (k - k_a)^{\alpha} (1 - n_a)^{1-\alpha} - (\delta + \nu) k - c_n - x_n \tag{44}$$

If the international relative price $q(t)$ is such that the country optimally produces both goods, then the static equations describing the equilibrium are equation (45) below - which defines the interest rate r -, together with equations (21) and (22) above - which state that the value of the marginal product of both factors has to be equal across sectors-, and equation (40) below - which is the agricultural good market clearing condition written in terms of nonagricultural consumption and nonagricultural exports.

$$r = \alpha \frac{A_n (1 - n_a)^{1-\alpha}}{((1 - \kappa_a) k)^{1-\alpha}} \tag{45}$$

$$A_a (\kappa_a k)^{\eta} n_a^{\beta} \left(Le^{-\nu t}\right)^{1-\eta-\beta} = \begin{cases} \mu_0 \frac{c_n}{q} + \underline{c_a} - \frac{x_n}{q} \text{ if } c_a \leq c_a^* \\ \mu_1 \frac{c_n}{q} + c_a^* - A - \frac{x_n}{q} \text{ else} \end{cases} \tag{46}$$

If the international relative price $q(t)$ is such that only the agricultural good is produced, then the interest rate is defined by equation (47), the agricultural good market clearing condition in equation (46) does not change, and the two other equations defining the equilibrium are (48) and (49) below.

$$r = q\eta \frac{A_a (n_a)^{\beta} (Le^{-\nu t})^{1-\eta-\beta}}{(\kappa_a k)^{1-\eta}} \tag{47}$$

$$n_a = 1 \tag{48}$$

$$\kappa_a = 1 \tag{49}$$

Note that during the transition to the Balanced Growth Path, specialization in the nonagricultural good is not possible because land is only useful as a production input in the agricultural sector.

These equations, together with the boundary conditions $k(0) = k_0$ and the transversality condition in equation (10), are the ones used to find the solution of the model. As in the closed economy case, the equilibrium solution is the path that solves this dynamic system and converges to the Balanced Growth Path of preferences in equation (7). Hence, if we detrend the dynamic system variables by their long-run growth rates, the solution is the path that converges to the Steady State of the detrended system.

Lemma 7. *If the preferences are the ones in equation (7), then the open economy equilibrium is consistent with a Balanced Growth Path, in which all the variables grow at a constant rate. The growth rates of the endogenous variables in the BGP are the ones showed in equations (50) -(52):*

$$\frac{\dot{n_a}}{n_a} = \frac{\dot{\kappa_a}}{\kappa_a} = 0 \tag{50}$$

$$\frac{\dot{k}}{k} = \frac{\dot{c_n}}{c_n} = \frac{\dot{x_n}}{x_n} = \begin{cases} \frac{1}{1-\alpha}\gamma_n \ \textit{if}\ \gamma_q \leq \widetilde{\gamma} \\ \frac{1}{1-\eta}\gamma_a + \frac{1}{1-\eta}\gamma_q - \frac{1-\eta-\beta}{1-\eta}\nu \ \textit{if}\ \gamma_q > \widetilde{\gamma} \end{cases} \tag{51}$$

$$\frac{\dot{c_a}}{c_a}=\begin{cases}\frac{1}{1-\alpha}\gamma_n-\gamma_q \text{ if } \gamma_q\leq\widetilde{\gamma}\\ \frac{1}{1-\eta}\gamma_a+\frac{\eta}{1-\eta}\gamma_q-\frac{1-\eta-\beta}{1-\eta}\nu \text{ if } \gamma_q>\widetilde{\gamma}\end{cases}\tag{52}$$

where $\widetilde{\gamma}\equiv(1-\eta-\beta)\nu+\frac{1-\eta}{1-\alpha}\gamma_n-\gamma_a$. *If the growth rate of the exogenous relative price* q*, denoted by* γ_q*, is equal to* $\widetilde{\gamma}$*, then the Balanced Growth Path has positive production of both goods, i.e.* $0<n_a^{ss}<1$ *and* $0<\kappa_a^{ss}<1$*). If* $\gamma_q>\widetilde{\gamma}$*, then only the agricultural good is produced in the Balanced Growth Path, i.e.* $n_a^{ss}=1$ *and* $\kappa_a^{ss}=1$*. Finally, if* $\gamma_q<\widetilde{\gamma}$*, then only the nonagricultural good is produced in the Balanced Growth Path, i.e.* $n_a^{ss}=0$*,* $\kappa_a^{ss}=0$*).*

Lemma 8. *Let* $\widehat{k}$*,* $\widehat{c_n}$*, and* $\widehat{x_n}$ *be defined as in equations (53) - (55). In the Balanced Growth Path,* $\widehat{k}$*,* $\widehat{c_n}$*, and* $\widehat{x_n}$*, as well as* n_a *and* κ_a *are constant.*

$$\widehat{k}\equiv\begin{cases}\frac{k}{A_n^{1/(1-\alpha)}} \text{ if } \gamma_q\leq\widetilde{\gamma}\\ \frac{k}{\left(A_a q(Le^{-\nu t})^{1-\eta-\beta}\right)^{1/(1-\eta)}} \text{ if } \gamma_q>\widetilde{\gamma}\end{cases}\tag{53}$$

$$\widehat{c_n}\equiv\begin{cases}\frac{c_n}{A_n^{1/(1-\alpha)}} \text{ if } \gamma_q\leq\widetilde{\gamma}\\ \frac{c_n}{\left(A_a q(Le^{-\nu t})^{1-\eta-\beta}\right)^{1/(1-\eta)}} \text{ if } \gamma_q>\widetilde{\gamma}\end{cases}\tag{54}$$

$$\widehat{x_n}\equiv\begin{cases}\frac{x_n}{A_n^{1/(1-\alpha)}} \text{ if } \gamma_q\leq\widetilde{\gamma}\\ \frac{x_n}{\left(A_a q(Le^{-\nu t})^{1-\eta-\beta}\right)^{1/(1-\eta)}} \text{ if } \gamma_q>\widetilde{\gamma}\end{cases}\tag{55}$$

Lemma 9. *The equilibrium system defined in equations (43) -(46) can be rewritten in terms of the detrended variables* $\widehat{k}$*,* $\widehat{c_n}$*, and* $\widehat{x_n}$*, as shown in the equations below. If the country is not specialized and both goods are produced, then the detrended equilibrium system consists of equations (56) - (60). If only the agricultural good is produced, the detrended equilibrium system consists of equations (56), (57), and (61) together with equations* $n_a=\kappa_a=1$*.*

$$\dot{\widehat{c_n}}=\begin{cases}\widehat{c_n}\left[\alpha\left((1-\kappa_a)\widehat{k}\right)^{\alpha-1}(1-n_a)^{1-\alpha}-\delta-\rho-\frac{1}{1-\alpha}\gamma_n\right] \text{ if } \gamma_q\leq\widetilde{\gamma}\\ \widehat{c_n}\left[\begin{array}{c}\eta\left(\kappa_a\widehat{k}\right)^{\eta-1}n_a^{\beta}\left(Le^{-\nu t}\right)^{1-\eta-\beta}\\ -\delta-\rho-\frac{1}{1-\eta}\gamma_a-\frac{1}{1-\eta}\gamma_q-\frac{1-\eta-\beta}{1-\eta}\nu\end{array}\right] \text{ if } \gamma_q>\widetilde{\gamma}\end{cases}\tag{56}$$

$$\dot{\widehat{k}}=\begin{cases}\left[\begin{array}{c}\left((1-\kappa_a)\widehat{k}\right)^{\alpha}(1-n_a)^{1-\alpha}-\widehat{c_n}\\ -\widehat{x_n}-\left(\delta+n+\frac{1}{1-\alpha}\gamma_n\right)\widehat{k}\end{array}\right] \text{ if } \gamma_q\leq\widetilde{\gamma}\\ \left[\begin{array}{c}\left((1-\kappa_a)\widehat{k}\right)^{\alpha}(1-n_a)^{1-\alpha}-\widehat{c_n}-\widehat{x_n}\\ -\left(\delta+n+\frac{1}{1-\eta}\gamma_a+\frac{1}{1-\eta}\gamma_q+\frac{1-\eta-\beta}{1-\eta}\nu\right)\widehat{k}-\widehat{c_n}-\widehat{x_n}\end{array}\right] \text{ if } \gamma_q>\widetilde{\gamma}\end{cases}\tag{57}$$

$$\left(\kappa_a\widehat{k}\right)^{\eta}(n_a)^{\beta}=\begin{cases}\left[\begin{array}{c}\left(\mu_0\widehat{c_n}-\widehat{x_n}\right)\left(\frac{q}{\frac{A_n^{(1-\eta)/(1-\alpha)}}{A_a\left(Le^{-\nu t}\right)^{1-\eta-\beta}}}\right)^{-1}\\+\underbrace{\left(\frac{\left(Le^{-\nu t}\right)^{1-\eta-\beta}}{A_aA_n^{\frac{\eta}{1-\alpha}}}\right)}_{\equiv z^1(t)}\underline{c_a}\end{array}\right] & if\ c_a\le c_a^*\\ \left(\mu_1\widehat{c_n}-\widehat{x_n}\right)\left(\frac{q}{\frac{A_n^{(1-\eta)/(1-\alpha)}}{A_a\left(Le^{-\nu t}\right)^{1-\eta-\beta}}}\right)^{-1}+\underbrace{\left(\frac{\left(Le^{-\nu t}\right)^{\eta+\beta-1}}{A_aA_n^{\frac{\eta}{1-\alpha}}}\right)}_{\equiv z^1(t)}c_a^* & else\end{cases}\tag{58}$$

$$\frac{q}{\frac{A_n^{(1-\eta)/(1-\alpha)}}{A_a(Le^{-\nu t})^{1-\eta-\beta}}}\eta\frac{(n_a)^{\beta}L^{1-\eta-\beta}}{\left(\kappa_a\widehat{k}\right)^{1-\eta}}=\alpha\frac{(n_a)^{1-\alpha}}{\left((1-\kappa_a)\widehat{k}\right)^{1-\alpha}}\tag{59}$$

$$\frac{q}{\frac{A_n{}^{(1-\eta)/(1-\alpha)}}{A_a(Le^{-\nu t})^{1-\eta-\beta}}}\beta\frac{\left(\kappa_a\widehat{k}\right)^{\eta}}{(n_a)^{1-\beta}}=(1-\alpha)\frac{\left((1-\kappa_a)\widehat{k}\right)^{\alpha}}{(1-n_a)^{\alpha}}\tag{60}$$

$$\left(\kappa_a\widehat{k}\right)^{\eta}(n_a)^{\beta}=\begin{cases}\left(\mu_0\widehat{c_n}-\widehat{x_n}\right)+\underbrace{\left(\frac{\left(Le^{-\nu t}\right)^{\eta+\beta-1}}{q^{\eta}A_a}\right)^{\frac{1}{1-\eta}}}_{\equiv z^2(t)}\underline{c_a} & if\ c_a\le c_a^*\\ \left(\mu_1\widehat{c_n}-\widehat{x_n}\right)+\left(\frac{\left(Le^{-\nu t}\right)^{\eta+\beta-1}}{q^{\eta}A_a}\right)^{\frac{1}{1-\eta}}c_a^* & if\ c_a>c_a^*\end{cases}\tag{61}$$

Lemma 10. *The detrended equilibrium system defined in equations (56) - (61) has a Steady State, where all the variables are constant. The solution to this detrended equilibrium system consists of the path that leads to this Steady State.*

Lemma 11. *If $(1-\eta-\beta)\nu<\gamma_a+\frac{\eta}{1-\alpha}\gamma_n$, then the detrended equilibrium system defined in (56) - (61) converges to same equilibrium as the equation (7) case, so the equilibrium defined in equations (8) - (10), (11) -(13), (14) - (16), and (40) - (42) has a unique Balanced Growth. For any initial condition, the economy converges asymptotically to this Balanced Growth Path, which is described next:*

- *If $\gamma_q\le\widetilde{\gamma}\equiv(1-\eta-\beta)\nu+\frac{1-\eta}{1-\alpha}\gamma_n-\gamma_a$*

$$\frac{\dot{k}}{k}=\frac{\dot{c}_n}{c_n}=\frac{\dot{x}_n}{x_n}=\frac{1}{1-\alpha}\gamma_n$$

$$\frac{\dot{c}_a}{c_a}=\frac{1}{1-\alpha}\gamma_n-\gamma_q$$

Moreover, if the relation holds with equality, the fraction of inputs allocated to both sectors is positive and constant, and if the relation holds with inequality the economy specializes in the production of the nonagricultural good.

- *If* $\gamma_q > \widetilde{\gamma} \equiv (1-\eta-\beta)\, n + \frac{1-\eta}{1-\alpha}\gamma_n - \gamma_a$*, the economy specializes in the production of the agricultural good, and*

$$\frac{\dot{k}}{k} = \frac{\dot{c}_n}{c_n} = \frac{\dot{x}_n}{x_n} = \frac{1}{1-\eta}\gamma_a + \frac{1}{1-\eta}\gamma_q - \frac{1-\eta-\beta}{1-\eta}\nu$$

$$\frac{\dot{c}_a}{c_a} = \frac{1}{1-\eta}\gamma_a + \frac{\eta}{1-\eta}\gamma_q - \frac{1-\eta-\beta}{1-\eta}\nu.$$

B Exogenous Variables and Data Sources

B.1 Unites States exogenous variables

This section of the appendix describes the construction and data sources of the exogenous variables used in the United States simulations. The information about the data sources is summarized in table 1, and the main exogenous variables are plotted in figure 10.

Variable	Description	Period	Source
N	Total Population	1890-2007	Maddison (2006)
E	Total Employment	1890-1928 1929 - 2007	J.W. Kendrick (1961) National Income and Product Accounts
Y, Y_a	Real GDP by Sector	1890-1928 1929 - 2007	J.W. Kendrick (1961) National Income and Product Accounts
PY	Nominal GDP	1890-1928 1929 - 2007	J.W. Kendrick (1961) National Income and Product Accounts
P_aY_a	Agriculture Nominal GDP	1890:10:1900 1929 - 2007	Historical Statistics of the United States National Income and Product Accounts
N_a, N_n	Employment by Sector	1890 - 1928 1929 - 2007	J.W. Kendrick (1961) National Income and Product Accounts
K	Real Net Capital Stock by Sector	1890 - 1928 1929 - 2007	J.W. Kendrick (1961) National Income and Product Accounts
x_a	Net agricultural exports	1910 - 1949 1950 - 2007	Historical Statistics of the United States US Department of Agriculture, ERS

Table 1: **Sources United States Data**

Data on total population for the entire sample period 1890-2007 is available in Maddison (2006). Data on total employment is available in Kendrick (1961) for the subperiod 1890-1928, and in the National Income and Product Accounts for the subperiod 1929-2007.[31]

[31] http://www.bea.gov.

Measures for agricultural and nonagricultural Total Factor Productivity are obtained using the production functions defined in equations (4) and (5),together with data on sectoral real GDP, sectoral employment and sectoral real capital.

Data on real GDP by sector also comes from these two different sources: for the period 1890-1928 data on constant dollars gross value added is available in Kendrick (1961) for both the farm sector and the aggregate economy, and for the period 1929-2007 data on chained dollars gross value added is also available for both the farm sector and the aggregate economy in the National Income and Product Accounts.

Data on gross value added in current prices is available in Kendrick (1961) for the subperiod 1890-1928, and in the National Income and Product Accounts for the subperiod 1929-2007. Data on farm sector gross value added in current prices is available in the Historical Statistics of the United States for the years 1890 and 1900, and in the National Income and Product Accounts for the subperiod 1929-2007.[32] Data on total employment and farm sector employment is available in Kendrick (1961) for the subperiod 1890-1928, and in the National Income and Product Accounts for the subperiod 1929-2007.

Finally, the data used for the aggregate capital stock for the period 1890-1928 is the Real Capital Stock Domestic Economy series minus Farm, Forest and Park Land series, minus Monetary Gold and Silver, and minus Residential Capital Stock in Kendrick (1961). This corresponds to the sum of Structures, Equipment, and Inventories. The data used for aggregate capital stock for the period 1929-2007 is the Chain-Type Quantity Indexes for Net Stock of total fixed series assets minus the Chain-Type Quantity Indexes for Net Stock of private residential assets series, minus the Chain-Type Quantity Indexes for Net Stock of government residential assets in the National Income and Product Accounts. The data used for the agricultural sector capital stock for the period 1890-1928 is the Real Capital Stock Farm Economy series minus Farm Land from Kendrick (1961), and for the period 1929-2007 is the series Chain-Type Quantity Indexes for Net Stock of Private Farms Fixed Assets from the National Income and Product Accounts.[33]

B.2 South Korea exogenous variables

This section of the appendix describes the construction and data sources of the exogenous variables used in the simulations of South Korea -which are total population, total employment, agricultural relative price, agricultural TFP and nonagricultural TFP- as well as the data sources of other time series used to compare the fit of the model with the actual data are also explained. The information is summarized in table 2, and the plots of the main variables are shown in figure 11.[34]

[32] Historical Statistics of the United States, Millennial Edition Online.

[33] Farm land data is not available for all the years of the 1890-1928 period in Kendrick (1961), but it is estimated to be 92% of Farm, Forest and Park land data series. The data capital stock data from Kendrick (1961) includes inventories, while the data from the National Income and Product Accounts only includes fixed assets. This is probably not a big problem for the aggregate capital stock, but it may make a significant differnce in the agricultural capital stock. As a result, the capital stock data by sector may not be completely compatible, which is one of the reasons why it is not used to measure sectoral TFP.

[34] The first plot in the last row shows the relative price data and the variable used in the baseline simulation, and the second plot shows the relative price variable of the baseline simulation with the one used in the "No

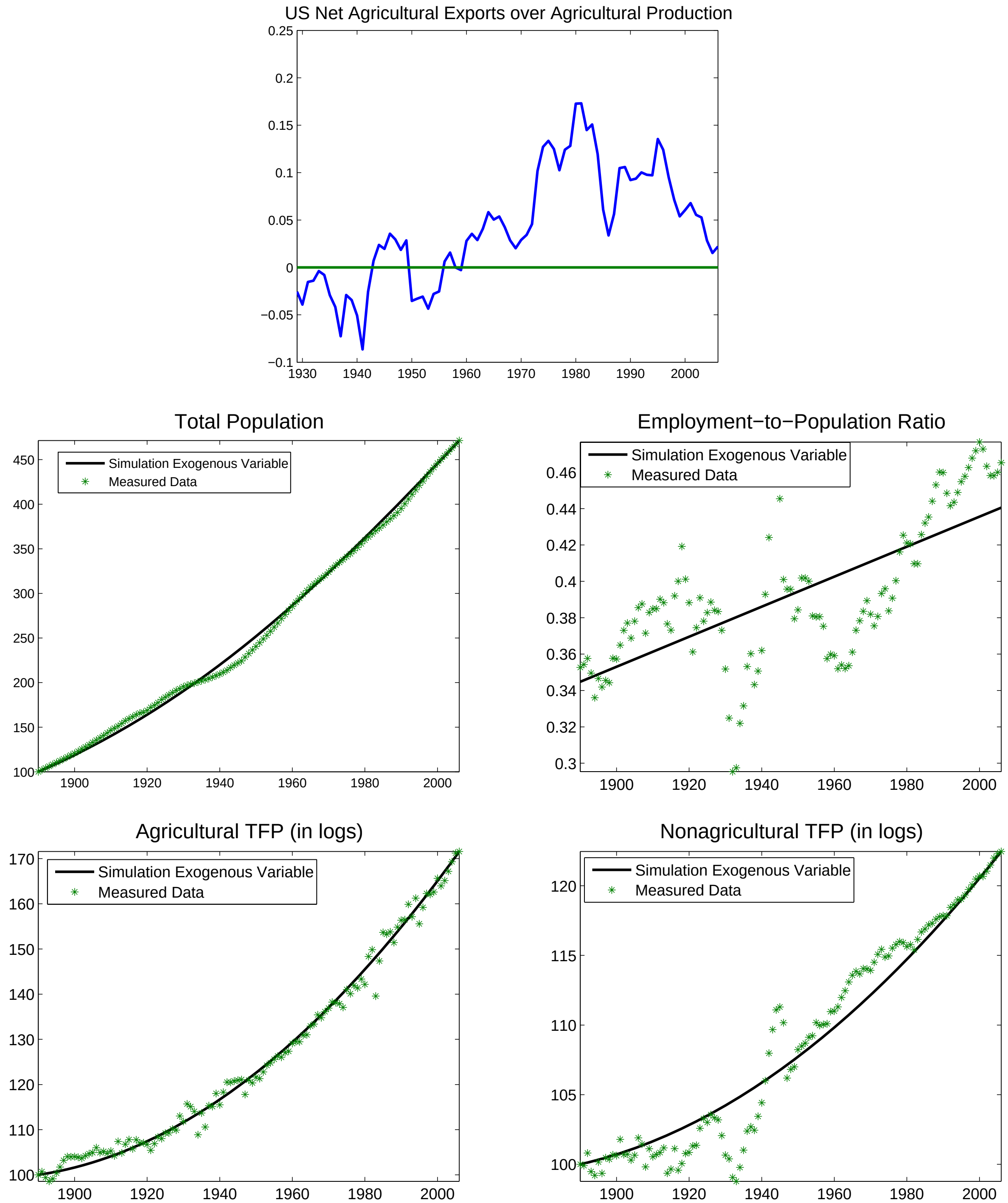

Figure 10: **Exogenous Variables - United Sates Data**
Data sources: Maddison (2006), Kendrick (1961), BEA - NIPA, Historical Statistics of the United States, US Department of Agriculture - ERS.

Variable	Variable description	Period	Source
N	Total Population	1950 - 1970 1960 - 2007	Statistical Yearbooks Bank of Korea
E	Total Employment	1957 - 1970 1963 - 2007	Statistical Yearbooks Korea Statistical Information
P_aY_a, P_nY_n	Nominal GDP by industry	1953 - 2007	Bank of Korea
Y_a, Y_n	Real GDP by industry	1953 - 2007	Bank of Korea
N_a, N_n	Employment by industry	1963 - 2007	Korea Statistical Information Service
P_ax_a	Agricultural net exports	1960 - 2003 (various years)	Statistical Yearbooks Bank of Korea
K	Real net capital stock	1963 - 1995	Korea Development Institute

Table 2: **Sources South Korea Data**

Data on total population is available from 1944 to 1966 in the Economic Statistics Yearbook 1970 (Bank of Korea), and from 1960 onwards from the Korean Statistical Information Service.[35] Data on total employment is available from 1963 onwards from the Korean Statistical Information Service, and it is available from 1957 in the Statistical Yearbook of the Republic of Korea, year 1960.

Data on the relative price of the agricultural good is not directly available. The way I construct it is by dividing the agricultural sector GDP deflator by the GDP deflator of the rest of the economy, where the sectoral GDP deflator is obtained by dividing nominal GDP data by real GDP data for each sector. Data on current and constant prices GDP by industry is available from the Economic Statistics System of the Bank of Korea starting at 1953.[36] Agricultural sector production is Agriculture, Forestry and Fishing GDP, and nonagricultural sector production is total GDP minus the agricultural sector GDP.

Real GDP data for each sector is available from the Economic Statistics System of the Bank of Korea from 1953 onwards, as just explained. Data on total employment and employment in Agriculture, Forestry and Fishing is available from the Korean Statistical Information Service from 1963 onwards. For the period 1957-1960 employment data is available in the Statistical Yearbook of the Republic of Korea, year 1961. Data for aggregate physical capital is obtained from Kim and Hong (1997, Korea Development Institute) for the period 1962-1995. The capital time series used is the sum of the Net Fixed Capital Stock of Nonresidential Business at 1990 constant prices plus Total Inventories for Nonresidential Business at 1990 Constant Prices. Capital stocks for the agricultural and the nonagricultural sector are also available from the same publication, but instead of using them I created alternative series assuming that both employment and capital are efficiently allocated across sectors.

The Sectoral TFPs variables used in the simulations have constant growth rates and they are equal to 0.0315 in the agricultural sector and 0.0215 in the nonagricultural one. Data on agricultural and nonagricultural Total Factor Productivity is obviously not directly available, but one could infer them using equations (4) and (5) together with data on sectoral

agricultural policy" counterfactual simulation.

[35]http://www.kosis.kr/eng/index.htm

[36]http://ecos.bok.or.kr/EIndex_en.jsp

real GDP, sectoral employment and sectoral real capital.[37] As argued in the main text and shown in figure 11, the measured TFPs growth rates are quite similar to the ones used in the simulations.

Finally, data on net agricultural exports is needed to compute agricultural consumption (which is defined as the sum of the domestic production plus the net agricultural exports). Data on net agricultural exports is obtained from the Input-Output tables published in the Economic Statistics System of the Bank of Korea for many years between 1970 and 2003. Data for the years 1960, 1963 and 1968 is form the Input-Output tables published in the Economic Statistics Yearbook of the Bank of Korea (years 1965, 1966, 1970). Agricultural net exports are defined here as the net exports of crops, livestock breeding, forestry products, and fishery products.

B.3 United Kingdom Exogenous Variables and Data Sources

This section of the appendix describes the construction of the exogenous variables used in the simulations of the United Kingdom, as well as their data sources. Table 3 summarizes the data sources, and figure 12 plots the exogenous variables used in the simulations together with the measured data.

Variable	Variable description	Period	Source
$\boldsymbol{N}$	Total Population	1800 1820-1900	Mitchell (1962) Maddison (2003)
$\boldsymbol{E}$	Total Employment	1800(10)1900	Deane, Cole (1969)
$\boldsymbol{N}_a$	Agriculture Employment	1800(10)1900	Deane, Cole (1969)
$\boldsymbol{PY}$	Nominal GDP	1800(10)1850 1855-1900	Deane, Cole (1969) Mitchell (1962)
$\boldsymbol{P}_a\boldsymbol{Y}_a$	Nominal GDP Agriculture	1800(10)1900	Deane, Cole (1969)
$\boldsymbol{Y}$	Real GDP	1800(10)1850 1855-1900	Deane, Cole (1969) Mitchell (1962)
$\boldsymbol{P}_a$	Agriculture Price Level	1800-1900	Mitchell (1962)
$\boldsymbol{P}_a\boldsymbol{x}_a$	Nominal Agr Net Exports	1805(10)1855	Davis (1979)
$\boldsymbol{K}$	Real Net Capital Stock	1800(10)1850-1900	Feinstein (1988)

Table 3: **Sources United Kingdom Data**

To construct the exogenous variables total population and total employment, only raw data is used. Data on total population is available for the year 1800 in Mitchell (1962), and for the years 1820-1900 in Maddison (2003), and data on total employment is available in Deane and Cole (1969) for the period 1800 - 1900 at a frequency of ten years.

The data used for the aggregate capital stock, which is available in Feinstein (1988) corresponds to total net stock of domestic reproducible fixed assets minus the category

[37]Note that the the measured agricultural TFP in this case corresponds the agricultural TFP defined in equation (4) times total land to the power of $(1-\eta-\beta)$, but this is not a problem because total land is assumed to be constant.

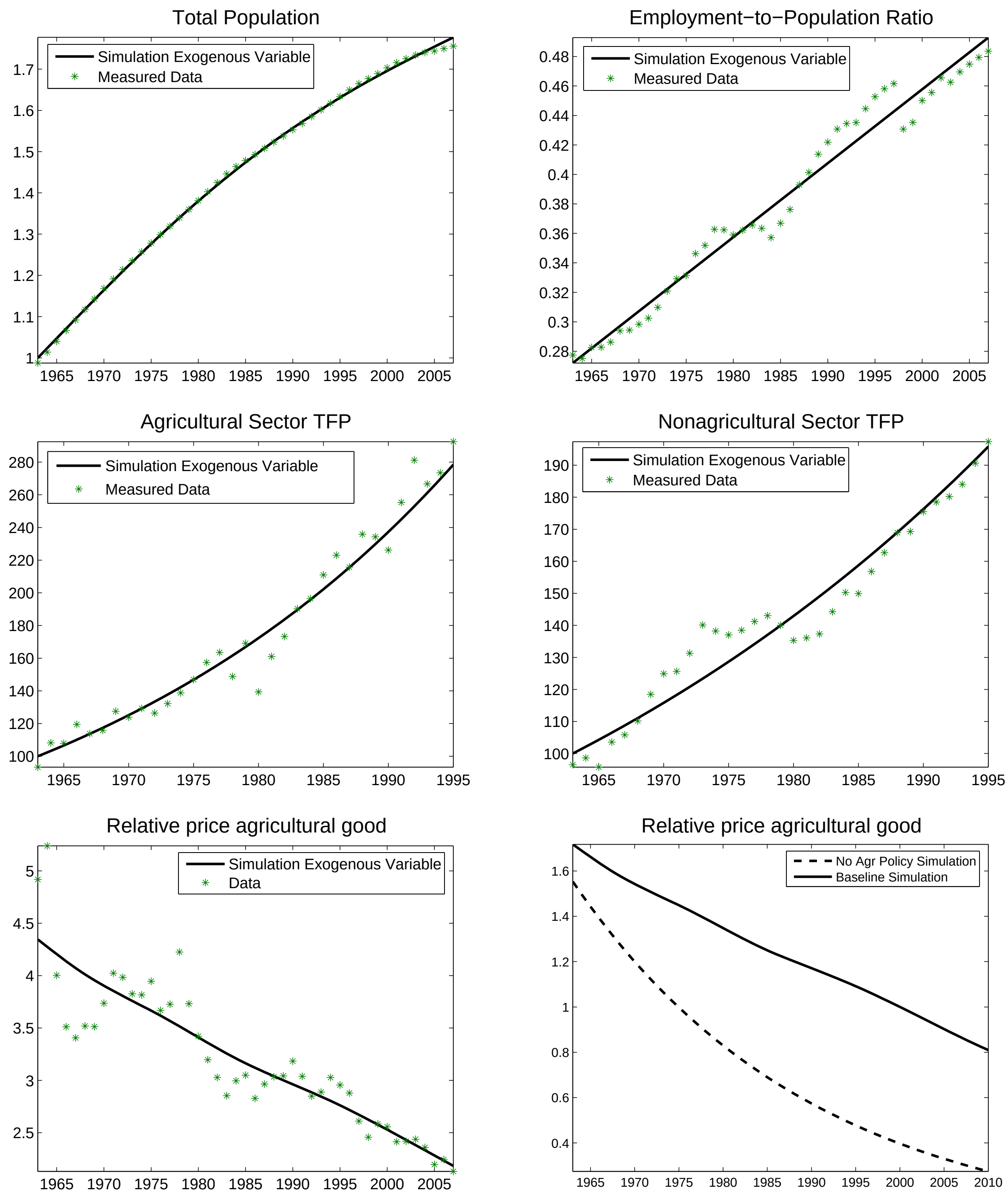

Figure 11: **Exogenous Variables - South Korea Data**
Data sources: Statistical Yearbooks of the Republic of Korea, Korea Statistical Information Service, Kim and Hong (1997); own calculations (counterfactual relative price).

dwellings. This is the sum of industrial and commercial buildings, other nonresidential buildings and works, plant machinery and equipment, rolling stock and vehicles, and ships.

Real GDP in agriculture (which corresponds to Agriculture, Forestry and Fishing) can be obtained by dividing nominal GDP in agriculture, which is available from Mitchell (1962), by the price level in agricultural, which is also available from Mitchell (1962) for the sample period 1800 - 1900. Real GDP in the nonagricultural sector can be obtained by subtracting the real GDP in agriculture from aggregate real GDP, which is available in Deane and Cole for the period 1800 - 1850 and in Mitchell (1962) for the period 1855 - 1900. Data on employment by sector is available in Deane and Cole (1969) for the entire sample period, and data on aggregate capital stock is available in Feinstein (1988) for the entire sample period. Using data on capital and employment by sector it is possible to infer the level of capital by sector, by assuming that capital is efficiently allocated. It is possible to construct the exogenous variables agricultural and nonagricultural TFP using the production functions defined equations (4) and (5), together with data on real GDP by sector and employment by sector. The TFP growth rates used in the simulations are 1.25% for the agricultural sector and 0.65% for the nonagricultural one, which are almost identical to the measured ones.

To compute the relative price of the agricultural good I divide the agricultural price level, which is available in Mitchell (1962) for the period 1800 - 1900 by the GDP deflator of the nonagricultural sector. The latter is constructed by dividing nominal GDP outside agriculture by the real GDP outside agriculture, both of which are obtained by subtracting the agricultural GDP to the aggregate GDP. Finally, data on net agricultural imports is also necessary to get agricultural consumption series. Davis (1979) provides data on net agricultural imports, defined as foodstuffs plus raw materials, for the years 1805 - 1855.

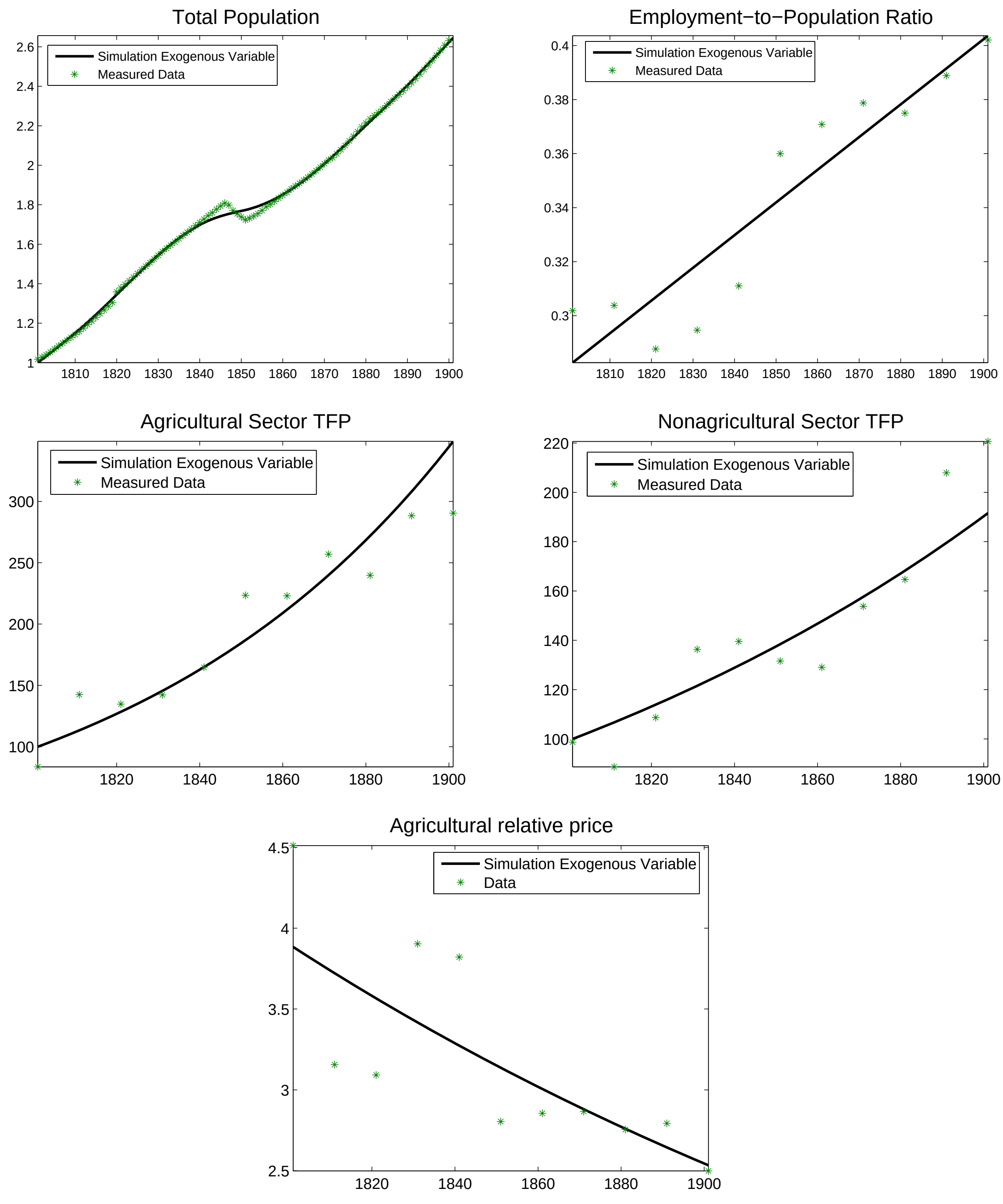

Figure 12: **Exogenous Variables - United Kingdom Data**
Data sources: Mitchell (1962), Deane and Cole (1969), Davis (1979), Feinstein (1988).