# Monetary Policy with Heterogenous Agents and Borrowing Constraints*

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**Abstract**

This paper assesses the effect of monetary policy in economies where heterogeneous households face borrowing constraints, and can partially self-insure against individual income shocks by using capital holdings and real balances.

First, we show theoretically that inflation has a long-run real effect as long as borrowing constraints are binding. A rise in inflation triggers endogenous heterogeneity in money demand across constrained and unconstrained households, increasing capital accumulation via precautionary saving motives. Second, we quantify the importance of this new channel in incomplete market economies which closely match the wealth distribution and the share of borrowing-constrained households in the United States. Inflation turns out to have a sizeable positive impact on output which is three times higher in the benchmark incomplete market set-up compared to the traditional complete market economy. Third, the average welfare costs of inflation are similar across incomplete and complete market economies, but inflation does not have the same effect on wealth-rich and wealth-poor agents.

*JEL*: E2, E5

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# 1 Introduction

One of the best-known propositions in textbook monetary economics is that concerning the long-run neutrality of money, first shown by Sidrauski (1967). Yet there is growing empirical evidence that long-run changes in the level of inflation do in fact have real effects. A small increase in the rate of money growth in economies with initially low inflation rates is found to increase the long-run levels of capital stock (Kahn et al., 2006, Loayza, et al., 2000) and output (Bullard and Keating, 1995)[1]. In order to reconcile this apparent gap between traditional monetary theory and empirical evidence, a number of papers have re-evaluated the hypotheses under which long-run inflation neutrality holds. Potential long-run real effects of monetary policy have been considered via inflation's redistribution of seigniorage rents across households (Grandmont and Younès, 1973; and Kehoe et al., 1992) or across generations (Weiss, 1980; Weil, 1991), and inflation's distortionary effect on capital taxation (Phelps, 1973 and Chari et al., 1996 among others) or labor supply (Den Haan, 1990).

This paper proposes a new channel for the non-neutrality of money transiting via borrowing constraints. If households can use both fiat money and capital to partially self-insure against individual income shocks, they may substitute away from real balances towards financial assets when inflation rises and the return to money falls. However, if there are asset market imperfections, borrowing-constrained households will not be able to undertake such portfolio adjustment and will adjust their money holdings differently compared to unconstrained households. Inflation thus triggers endogenous intra-period heterogeneity in money holdings when borrowing constraints are binding, providing incentives for unconstrained households with positive income shocks to increase their savings in order to smooth consumption between periods[2]. Hence inflation may affect aggregate capital and output in the long run. Since the tightness of borrowing constraints is a well-established empirical fact (Jappelli 1990, Budria Rodriguez et *al.*, 2002), this new channel may well account for a sizeable quantitative impact of inflation on the real economy and household welfare.

To investigate this effect, we model capital market imperfections in a production economy in which ex-ante identical infinitely-lived agents face idiosyncratic income shocks. They can accumulate interest-bearing financial assets in the form of capital to partially insure against income risks, but they face borrowing constraints. In this framework we embed money in the utility

[1] This relationship is generally found to be non-linear. A permanent increase in the money growth rate in economies with initially high rates of inflation (above 6-8 percent according to Khan et al., 2006) has detrimental consequences for long-run real activity (see also Barro, 1995).

[2] The effect of inflation on portfolio allocation between money and capital is close in spirit to that in Tobin (1965), although the borrowing constraints channel through which portfolio composition affects the real economy is different from Tobin's.

function. Money is valued both for its liquidity and as a store of value which provides additional insurance against labor-market risks. In this set-up, individuals are completely identical ex-ante. Heterogeneity in money demand emerges endogenously due to borrowing constraints[3].

The first contribution of this paper is theoretical. In an economy with deterministic income shocks à la Woodford (1990), we show that inflation has a long-run effect as long as borrowing constraints are binding. This occurs even in the absence of the other potential real effect channels which have been proposed in the existing literature, such as capital tax distortions, labor supply distortions, or distortionary redistribution of the seigniorage rent.

Second, we quantitatively evaluate the impact of borrowing constraints in explaining the potential long-run real effect of inflation. We do this by embedding Sidrauski's money in the utility model into a fully-fledged incomplete market set-up à la Aiyagari (1994), in which heterogeneous agents face idiosyncratic income risks and borrowing constraints. One key element of this quantitative analysis is that we consider a wealth distribution, and in particular a fraction of borrowing-constrained households, which closely resembles that in the United States. We first gauge the specific quantitative role played by borrowing constraints and incomplete markets by eliminating all of the other potential frictions. Next, we quantify the potential interactions between incomplete markets and borrowing constraints, on the one hand, and the distortions put forward in the existing literature on the other. Specifically, we disentangle the quantitative real effects of inflation transiting through i) the non-neutral redistribution of the seigniorage rent across households, ii) the distorting effect on the capital tax, and eventually iii) the distorting effect on labor supply. We evaluate the contribution of incomplete markets with borrowing constraints to these real effects by comparing the outcomes to those from corresponding complete market economies.

One of our main results is that incomplete markets with borrowing constraints have a quantitatively sizeable effect. Following a permanent one point rise in inflation from 2 to 3 percent[4], borrowing constraints *per se* yield a 0.08 percent increase in output. This result pertains when we eliminate all other channels through which inflation might be expected to have a long-run real effect. Namely, this framework abstracts from potential redistributive effects of the seigniorage rent, distorting tax on capital, and adjustment of labor supply (by assuming exogenous hours).

[3] A non-neutrality of money result is trivial if we assume exogenous heterogenity in money demand, due to differences in preferences across housholds for instance.

[4] This paper focuses on low-inflation policy experiments, such as those currently found in OECD countries. It reproduces the positive effect of inflation on capital and output, consistent with the first part of the non-linear relation between inflation and economic activity estimated by Bullard and Keating (1995) and Khan et al. (2006). The second part of the relation, with episodes of high inflation detrimental to economic activity, is beyond the scope of this article and might require different features than those found in the simple growth monetary model.

Note that in this set-up, inflation has no long-run real effect on output without financial market imperfections. We then show that borrowing constraints amplify the real effect of inflation previously estimated. First, with respect to distortionary taxes on capital, it has long been recognized that the seigniorage rent could alleviate capital taxes and induce greater capital accumulation. Yet, this so-called Phelps effect (Phelps, 1973; Chari *et al.* 1996) is quantitatively much larger in an incomplete market set-up, since the presence of borrowing constraints gives rise to precautionary savings motives. A one point rise in the inflation rate triggers an increase in aggregate output by 0.213 percent in the incomplete market set-up, as against 0.104 in the representative agent economy. Regarding the distorting effect of inflation on labor supply, we show that this matters quantitatively much more with incomplete markets: inflation triggers greater precautionary saving in the presence of borrowing constraints, which leads in turn to a rise in labor productivity and the remuneration from working. A one point increase in the inflation rate leads to a 0.44 percent rise in output via labor supply, which is three times higher than that in the corresponding economy with perfect financial markets. Consequently, borrowing constraints can account for the apparent significant positive effect of inflation on output found in low-inflation countries (Bullard and Keating, 1995).

Third, we reassess the welfare effect of inflation with borrowing constraints. It first turns out that the average welfare cost of inflation is around 0.18 percent of permanent consumption in the benchmark economy. This seems lower than Lucas estimates (2000), but consistent with recent micro estimates which explicitly take into account differences in interest-bearing assets and heterogeneity in wealth (see Attanasio et *al.*, 2002). The average welfare cost of inflation is in general of the same order under incomplete and complete markets for similar steady-state comparisons. The explanation is that the sharper reduction in money holdings in incomplete market economies is offset by a steady increase in output and consumption. This finding challenges Imrohoroglu's (1992) result of much higher welfare costs of inflation under incomplete market economies compared to the representative agent framework. This paper suggests that Imrohoroglu's measure was overestimated by considering incomplete market economies in which money is the only store of value and by excluding potential positive real effects of money on steady-state capital.

Last, we show that inflation has strong welfare redistributive effects, which have been ignored so far in the representative agent literature. It turns out that the wealth-poor gain from inflation while the wealth-rich lose. This result is mainly due to a general equilibrium price effect. Inflation leads to higher labor productivity and wages in general equilibrium, due to a higher steady-state capital stock. This effect benefits the wealth-poor, whose income mainly comes from labor. In contrast, the return on financial assets decreases in general equilibrium, which affects the wealth-rich negatively.

*Related literature*

To the best of our knowledge, this paper is the first to provide theoretical and quantitative evidence on the real effect of inflation stemming from borrowing constraints and incomplete markets *per se*.

Various papers (following the seminal articles of Bewley, 1980 and 1983) have studied monetary policy in endowment economies with borrowing constraints. But as Bewley's goal was mainly to provide foundations for the theory of money, this asset was considered as the only store of value. As a consequence, heterogeneity in money demand and the resulting real effects are not found in this type of model, as households do not substitute money for other assets. In the same way, Kehoe, Levine and Woodford (1992) and Imrohoroglu (1992) analyzed the welfare effect of inflation in endowment economies, but only measured the redistributive effect of inflation and not its real effect on production. Akyol (2004) analyzed the welfare effect of inflation in an incomplete market set-up where borrowing constraints are binding in equilibrium, but in an endowment economy. He actually assumes a specific type of money demand, so that only high-income agents hold money in equilibrium. Further, the analysis is carried out in an endowment rather than a production economy, excluding any analysis of the long-run real effect of inflation on capital accumulation and output.

A paper closer in spirit to ours is Erosa and Ventura (2002), who analyzed the distributional impact of inflation in a production economy with incomplete markets and two potential assets. The authors find that inflation may have a real effect on savings, but their result relies on the assumption of a specific transaction technology rather than the presence of incomplete markets and borrowing-constrained households. As such, they do not disentangle the specific role of borrowing constraints and incomplete markets in assessing the real effects of monetary policy. Similarly, Heer and Sussmuth (2006) recently quantified the interaction between inflation and the tax system in a OLG economy with heterogeneous agents and borrowing constraints. The non-neutrality of inflation in their context does not stem from incomplete markets and borrowing constraints on their own, as the authors introduce distortionary taxes and exogenous heterogeneity through the OLG structure.

Our paper is organized as follows. Section 2 first provides a simple model with deterministic individual shocks to show analytically the non-neutrality of money transiting only through borrowing constraints. Section 3 lays out the full model with stochastic individual shocks. Section 4 quantifies the real effect of inflation and its implied welfare costs.

# 2 A Simple Model

## 2.1 The model

In this section, we provide a theoretical model to show that inflation is no longer long-run neutral in a production economy with binding borrowing constraints. To obtain closed-form solutions, we set out a simplified version of the fully-fledged model used in the following quantitative section. The model draws upon a standard heterogeneous agent production economy à la Aiyagari in which agents face individual income fluctuations and borrowing constraints. But we make the key assumption that households alternate *deterministically* between the different labor market states. This liquidity-constrained model has been used, for instance, by Woodford (1990) to study the effect of public debt and by Kehoe and Levine (2001) to characterize the equilibrium interest rate. We extend this framework to monetary policy issues by taking account of the value of money in the utility function. We show analytically that Sidrauski's neutrality result no longer holds when borrowing constraints are binding in this framework. Inflation affects the long-run interest rate, even when seigniorage revenue is redistributed in the most neutral way, and regardless of any other potential frictions.

*Preferences and technology*

Households are infinitely-lived and have identical preferences[5]. Each household is in one of two states, $H$ or $L$. In state $H$ (resp. $L$), households have a high labor endowment $e^H$ (resp. $e^L$). For the sake of simplicity we assume that $e^H = 1$ and $e^L = 0$. Households alternate deterministically between state $H$ and $L$ at each period. At the initial date, there is a unit mass of the two household types. Type 1 households are in state $H$ at date 1, type 2 households are in state $L$ at date 1. Consequently, type 1 (resp. 2) households are in state $H$ (resp. $L$) every odd period and in state $L$ (resp. $H$) every even period. Type $i$ ($i = 1, 2$) households seek to maximize an infinite-horizon utility function over consumption $c^i$ and real money balances $m^i$ which provide liquidity services. The period utility function $u$ of these households is assumed to have the simple form

$$u\left(c_t^i, m_t^i\right) = \phi \ln c_t^i + (1-\phi) \ln m_t^i$$

where $1 > \phi > 0$ weights the marginal utility of consumption and money. For the sake of simplicity we use a log-linear utility function in this section, but the results hold for very general

[5]These assumptions are key in cancelling out the potential real effects of inflation stemming from the OLG structure or from exogenous heterogeneity in preferences. Significantly, we do not use Kiotaky and Moore's (1997) assumption of different discount factors which ensure that credit constraints are binding in equilibrium in this kind of model. However, we will establish sufficient conditions under which credit constraints are binding in our simple framework with identical preferences.

utility functions, as shown in Ragot (2005).

At each period $t \geq 1$, a type $i$ household can use her revenue for three different purposes. She can first buy an amount $c_t^i$ of final goods. We denote by $P_t$ the price of the final good in period $t$, and $\Pi_{t+1}$ is the gross inflation rate between period $t$ and period $t+1$, that is $\Pi_{t+1} = P_{t+1}/P_t$. She also saves an amount $a_{t+1}^i$ of financial assets yielding a return of $(1 + r_{t+1})\, a_{t+1}^i$ in period $t+1$, where $1+r_{t+1}$ is the gross real interest rate between period $t$ and period $t+1$. A borrowing constraint is introduced in its simplest form, in that we assume that no household is able to borrow: $a_{t+1}^i \geq 0$. Finally, type $i$ households buy a nominal quantity of money $M_t^i$, which corresponds to a level of real balances $m_t^i = M_t^i/P_t$. This yields revenue $m_t^i/\Pi_{t+1}$ in period $t+1$. In addition to labor income and to the return on her assets, each household receives by helicopter drop a monetary transfer from the State, denoted $\mu_t^i$ in nominal terms.

The problem of the type $i$ household, $i = 1, 2$, is given by

$$\max_{\{c_t^i, m_t^i, a_{t+1}^i\}_{t=1..\infty}} \sum_{t=1}^{\infty} \beta^t u\left(c_t^i, m_t^i\right) \text{ with } 0 < \beta < 1 \tag{1}$$

$$\text{s.t.} \quad c_t^i + m_t^i + a_{t+1}^i = (1 + r_t)\, a_t^i + \frac{m_{t-1}^i}{\Pi_t} + w_t e_t^i + \frac{\mu_t^i}{P_t} \qquad \text{with} \qquad a_t^i, c_t^i, m_t^i \geq 0 \tag{2}$$

where $\beta$ stands for the discount factor, $a_1^i$ and $M_0^i = P_0 m_0^i$ are given, and $a_t^i$ and $m_t^i$ are subject to the standard transversality conditions.

The production function of the representative firm has a simple Cobb-Douglas form $K^\alpha L^{1-\alpha}$ where $L$ stands for total labor supply and $K$ is the amount of total capital which fully depreciates in production. Profit maximization is given by $\max_{K_t, L_t} F(K_t, L_t) - (1 + r_t) K_t - w_t L_t$, and yields the standard first-order conditions

$$1 + r_t = \alpha K_t^{\alpha-1} L_t^{1-\alpha}, \;\; w_t = (1 - \alpha) K_t^\alpha L_t^{-\alpha} \tag{3}$$

In period $t \geq 1$, financial market equilibrium is given by $K_{t+1} = a_{t+1}^1 + a_{t+1}^2$. Labor market equilibrium is $L_t = e_t^1 + e_t^2 = 1$. Goods market equilibrium implies $F(K_t, L_t) = K_{t+1} + c_t^1 + c_t^2$.

*Monetary policy with neutral redistribution*

Let $\bar{M}_t$ denote the nominal quantity of money in circulation and $\Sigma_t = \bar{M}_t/P_t$ the real quantity of money in circulation at the end of period $t$. Money market equilibrium implies $m_t^1 + m_t^2 = \Sigma_t$ in real terms and $M_t^1 + M_t^2 = \bar{M}_t$ in nominal terms.

Monetary authorities provide a new nominal quantity of money in period $t$, which is proportional to the nominal quantity of money in circulation at the end of period $t - 1$. As a result, $\mu_t^1 + \mu_t^2 = \pi \bar{M}_{t-1}$, where the initial nominal quantity of money, $\bar{M}_0 = M_0^1 + M_0^2$, is given. The law of motion of the nominal quantity of money is thus

$$\bar{M}_t = (1 + \pi)\, \bar{M}_{t-1} \tag{4}$$

In order to focus on the specific role of borrowing constraints in the non-neutrality of inflation, it is assumed that monetary authorities follow the "most" neutral rule, which is to distribute by lump-sum transfer the exact amount of resources paid by private agents due to the inflation tax. Obviously this assumption is unrealistic and its only aim is to stress the specific role of borrowing constraints independent of any redistributive effects. As a consequence, new money is distributed proportionally to the level of beginning-of-period money balances. In period $t$, type $i$ agents hold a beginning-of-period quantity of money $M_{t-1}^i$. Hence, we assume that $\mu_t^i = \pi M_{t-1}^i$, and the real transfer is

$$\frac{\mu_t^i}{P_t} = \frac{\pi}{\Pi_t} m_{t-1}^i \tag{5}$$

## 2.2 Stationary Equilibrium

Given the initial conditions $a_1^1$, $a_1^2$, $M_0^1$, and $M_1^2$, and given $\pi$, an equilibrium in this economy is a sequence $\{c_t^1, c_t^2, m_t^1, m_t^2, a_{t+1}^1, a_{t+1}^2, P_t, r_t, w_t\}_{t=1...\infty}$ which satisfies the households' problem (1), the first-order condition of the firms' problem (3), and the different market equilibria. More precisely, we focus on symmetric stationary equilibria[6], where all real variables are constant, and where all agents in each state $H$ and $L$, denoted $H$ and $L$ households, have the same consumption and savings levels. The variables describing agents in state $H$ will be denoted $m^H, c^H, a^H$, and those in state $L$ will be described by $m^L, c^L, a^L$. As a consequence, since the real quantity of money in circulation $\Sigma = \bar{M}_t/P_t$ is constant in a stationary equilibrium, equation (4) implies that the price of the final good grows at rate $\pi$, and hence $\Pi = 1 + \pi$.

Note that under our assumption of a neutral redistributive monetary policy, we can use the budget constraint (2), and the amount $\mu_t^i/P_t$ given by (5), to obtain the budget constraints of $H$ and $L$ households at the stationary equilibrium

$$\text{H households} \quad : \quad c^H + m^H + a^H = (1+r)\, a^L + m^L + w \tag{6}$$

$$\text{L households} \quad : \quad c^L + m^L + a^L = (1+r)\, a^H + m^H \tag{7}$$

The inflation rate does not appear in these equations since the creation of new money does not imply any transfer between the two types of households. The redistributive effects of the seigniorage rent analyzed for instance by Kehoe et al. (1992) and Imrohoglu (1992) are cancelled out.

Using standard dynamic programming arguments, the households' problem can be solved easily. This is carried out in Appendix A.

[6] In liquidity-constraint models, the path of the economy converges toward a steady state, or even begins at a steady state if a period 1 transfer is made to households consistent with steady state values (Kehoe and Levine, 2001)

For $H$ households, we have the following optimality conditions

$$u_c'\left(c^H, m^H\right) = \beta\left(1+r\right)u_c'\left(c^L, m^L\right) \tag{8}$$

$$u_c'\left(c^H, m^H\right) - u_m'\left(c^H, m^H\right) = \frac{\beta}{\Pi}u_c'\left(c^L, m^L\right) \tag{9}$$

Equation (8) is the Euler equation for $H$ households, who can smooth their utility thanks to positive savings. $H$ households are high-income and are never borrowing constrained. The second equation is the arbitrage equation, which determines the demand for real money balances. $H$ households set the marginal cost of holding money in the current period (i.e. the left-hand side of equation 9) equal to the marginal gain of transferring one unit of money to the following period when they are in state $L$ (i.e. the right-hand side of equation 9). The marginal utility of money shows up here as a decrease in the opportunity cost of holding money. And the gain from money holdings takes into account the real return $1/\Pi$ of cash.

The solution of the program of $L$ households depends on whether borrowing constraints are binding or not. If borrowing constraints are binding, the solution is $a^L = 0$ and

$$u_c'\left(c^L, m^L\right) > \beta\left(1+r\right)u_c'\left(c^H, m^H\right) \tag{10}$$

$$u_c'\left(c^L, m^L\right) - u_m'\left(c^L, m^L\right) = \frac{\beta}{\Pi}u_c'\left(c^H, m^H\right) \tag{11}$$

The first inequality shows that $L$ households would be better off if they could transfer some income from the next period to the current period. The second equation involves the same trade-off as that for $H$ households discussed above. Finally, if borrowing constraints are not binding for $L$ households, inequality (10) becomes an equality and $a^L > 0$.

Using expression (8) together with condition (10), we find that borrowing constraints are binding if and only if $1+r < 1/\beta$. If borrowing constraints are not binding, equation (8) and the relationship (10) taken with equality imply $1+r = 1/\beta$. The following proposition[7] summarizes this standard result.

**Proposition 1** *Borrowing constraints are binding for L households if and only if* $1+r < 1/\beta$. *If borrowing constraints are not binding then* $1+r = 1/\beta$.

When borrowing constraints are binding, the gross real interest rate $1+r$ is lower than the inverse of the discount factor. As a result, there is always capital over-accumulation due to the precautionary saving motive, which is a standard result in this type of liquidity-constrained model (see Woodford, 1990; Kehoe and Levine, 2001, amongst others). The next section establishes sufficient conditions for borrowing constraints to be binding in this simple framework.

[7] Note that $1+r$ cannot be lower than $1/\Pi$, otherwise financial markets cannot clear. As such, an equilibrium with binding credit constraints can exist only if $1/\Pi < 1/\beta$. Moreover, we assume that the surplus left for consumption is positive at the Friedman rule, which implies $\alpha < 1/\Pi$.

### 2.3 Monetary Policy with binding borrowing constraints

*Perfect financial markets*

As a starting point, we present the conditions required to produce Sidrauski's neutrality result in this simple framework. If markets were complete and borrowing constraints were not binding, the Euler equation would hold with equality whatever the state of the labor market. In this case, money demand would be identical across households of types H and L. Using a log utility specification and taking the Euler equation with equality, we can rewrite money demand as follows

$$\frac{m^H}{c^H} = \frac{m^L}{c^L} = \frac{1-\phi}{\phi} \frac{1}{1 - \frac{1}{\Pi} \frac{1}{1+r}} \tag{12}$$

In this case, whatever the current state and the history of the labor market, the ratio of money over consumption is determined only by the preference parameters and the opportunity cost of holding money. To see this, assume that $r$ and $\pi$ are small, so that $1 - 1/(1+\pi)(1+r) \simeq r - (-\pi)$, which is the difference between the real net return on financial titles and the real net return on money or, in other words, the nominal interest rate.

In this case, inflation has no real effect on savings since households adjust their money demand in exactly the same proportion following a rise in inflation. Inflation does not then bring about any intra-period heterogeneity between household H and L; it therefore has no effect on saving patterns for inter-period smoothing motives, or on the equilibrium interest rate. This is the traditional Sidrauski result regarding the long-run neutrality of money.

*Binding borrowing constraints*

This long-run neutrality result no longer holds in this simple framework when borrowing constraints are binding.

Since $H$ households are never borrowing-constrained and profit from their good employment state to accumulate a buffer financial stock, their Euler equation always holds with equality. The money demand of $H$ households is therefore still only determined by the opportunity cost of holding money

$$\frac{m^H}{c^H} = \frac{1-\phi}{\phi} \frac{1}{1 - \frac{1}{\Pi} \frac{1}{1+r}} \tag{13}$$

By contrast, the money demand of $L$ households might be affected, depending on whether borrowing constraints are binding since the Euler equation no longer holds with equality. When borrowing constraints are binding, that is when $1+r < \frac{1}{\beta}$, we have the following money demand equation from (8) and (11):

$$\frac{m^L}{c^L} = \frac{1-\phi}{\phi} \frac{1}{1 - \frac{\beta^2}{\Pi}(1+r)} \tag{14}$$

The equilibrium ratio for $L$ households is not simply determined by the opportunity cost of holding money, but by the difference between consumption in the current period and the return on money holdings two periods hence. The ratio $\beta^2(1+r)/\Pi$ is the discounted value of one unit of money held in state $L$, transferred to state $H$, and then saved via financial market on to the next period, where the household is in state $L$ again. As this ratio rises, $L$ households increase the ratio of their money holdings over their consumption. $L$ households then increase the relative demand for money as the real interest increases, contrary to $H$ households. The real interest rate appears here as the remuneration of future savings and not as the opportunity cost of holding money. The following proposition summarizes this key property of the model. The proof can be found in the Appendix.

**Proposition 2** *If $\alpha < 1/(2+\beta)$, there exists an unique equilibrium with binding borrowing constraints. In such an equilibrium, the real interest rate falls as inflation rises.*[8]

When borrowing constraints are binding, a rise in inflation triggers a heterogeneous response in money demand across households. $L$ households decrease their money holdings $m^L$ proportionately less than do $H$ households, because money is their only available store of value. As a result, $H$ households have more resources since their budget constraint is $c^H+a^H = w+m^L-m^H$. An increase in inflation thus provides an incentive for $H$ households to save more in order to smooth consumption between periods. Thus in this simple framework with binding borrowing constraints, inflation unambiguously favors capital accumulation and output, in line with the traditional result of Tobin (1965).

This simple model has shown that imperfections on financial markets give rise to heterogeneity in money demand, which is at the core of the non-neutrality of inflation. The next section provides a quantitative evaluation of this new channel.

# 3 The General Model

We now describe a fully-fledged model including more general assumptions about idiosyncratic risks, endogenous labor supply and distorting taxes in order to investigate quantitatively the role of inflation. The economy considered here is based on the traditional heterogeneous agent framework *à la* Aiyagari (1994). However, we embed money in the utility function in this framework. This section presents the most general model. Different specifications of this model will be used in the simulation exercise to disentangle the various channels through which inflation affects the real economy.

[8] Under this condition, we have 1+r<$1/\beta$. Note that this condition holds for fairly standard parameter values such as $\alpha = 1/3$, $\beta < 1$ and $\Pi > 1$.

## 3.1 Agents

### 3.1.1 Households

The economy consists of a unit mass of *ex ante* identical and infinitely-lived households. They maximize expected discounted utility from consumption $c$, from leisure and real balances $m = \frac{M}{P}$. Labor endowment per period is normalized to 1, working time is $l$ and thus leisure is $1 - l$. For the sake of generality, we follow the literature which directly introduces money $m$ in the utility function of private agents to capture its liquidity services. For the benchmark version of the model, we assume that the utility function has a general CES specification, following Chari *et al.* (2000). The utility of agent $i$ is given by:

$$u(c_i, m_i, l_i) = \frac{1}{1-\sigma}\left[\left(\omega c_i^{\frac{\eta-1}{\eta}} + (1-\omega)\, m_i^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} (1-l_i)^{\psi}\right]^{1-\sigma} \tag{15}$$

where $\omega$ is the share parameter, $\eta$ is the interest elasticity of the demand for real balances, $\psi$ is the weight of leisure and $\sigma$ is risk aversion.

Individuals are subject to idiosyncratic shocks to their labor productivity $e_t$. We assume that $e_t$ follows a three-state Markov process over time with $e_t \in E = \left\{e^h, e^m, e^l\right\}$, where $e^h$ stands for high productivity, $e^m$ for medium productivity, and $e^l$ for low productivity. The productivity process follows a $3 \times 3$ transition matrix[9] $Q$. The probability distribution across productivity is represented by a vector $n_t = \{n_t^h, n_t^m, n_t^l\}$: $n_t \geq 0$ and $n_t^h + n_t^m + n_t^l = 1$. Under technical conditions, that we assume to be fulfilled, the transition matrix has a unique vector $n^* = \{n^h, n^m, n^l\}$ such that $n^* = n^* Q$. Hence, $n_t$ converges toward $n^*$ in the long run. $n^*$ is distribution of the population in each state. For instance, $n^h$ is the proportion of the population with high labor productivity. In the general model, there is endogenous labor supply for each productivity level.

Markets are incomplete and no borrowing is allowed. In line with Aiyagari (1994), households can self-insure against employment risks by accumulating a riskless asset $a$ which yields a return $r$. But they can also accumulate real money assets $m = M/P$, which introduces a new channel compared to the previous heterogeneous agent literature. With the price level of the final good at period $t$ being denoted $P_t$, the gross inflation rate between period $t-1$ and period $t$ is $\Pi_t = \frac{P_t}{P_{t-1}}$. If a household holds a real amount $m_{t-1}$ of money at the end of period $t-1$, the real value of her money balances at period $t$ is $\frac{m_{t-1}}{\Pi_t}$. As long as $\Pi_t > \frac{1}{1+r_t}$, money is a strictly-dominated asset, but which will nonetheless be in demand for its liquidity services. Households are not

[9] This assumption is important to match empirical features of the employment process and wealth distribution. See the section devoted to the specification of the model parameters.

allowed to borrow and cannot issue any money.

The budget constraint of household $i$ at period $t$ is given by:

$$c_t^i + m_t^i + a_{t+1}^i = (1 + r_t)\, a_t^i + \frac{m_{t-1}^i}{\Pi_t} + w_t e_t^i l_t^i \qquad t = 0, 1, .. \tag{16}$$

where $(1 + r_0)\, a_0^i$ and $m_{-1}^i$ are given The sequence of constraints on the choice variables is

$$a_{t+1}^i \geq 0,\ 1 \geq l_t^i \geq 0, c_t^i \geq 0,\ m_t^i \geq 0 \quad t = 0, 1... \tag{17}$$

The value $r_t$ is the after-tax return on financial assets, $e_t^i$ is the productivity level of the worker in period $t$, and $w_t$ is after-tax labor income per efficient unit.

For the sake of realism, we assume that there is a linear tax on private income. The tax rate on capital at period $t$ is denoted $\chi_t^a$ and the tax rate on labor is denoted $\chi_t^w$. Letting $\tilde{r}_t$ and $\tilde{w}_t$ denote capital cost and labor cost per efficient unit, the returns for households then satisfy the following relationships

$$\begin{aligned} r_t &= \tilde{r}_t (1 - \chi_t^a) \\ w_t &= \tilde{w}_t (1 - \chi_t^w) \end{aligned}$$

Let $q_t^i$ denote total wealth in period $t$

$$q_t^i = (1 + r_t)\, a_t^i + \frac{m_{t-1}^i}{\Pi_t}$$

With this definition, the program of agent $i$ can be written in the following recursive form

$$v\left(q_t^i, e_t^i\right) = \max_{\{c_t^i, m_t^i, l_t, a_{t+1}^i,\}} u\left(c_t^i, m_t^i, l_t^i\right) + \beta E\left[v\left(q_{t+1}^i, e_{t+1}^i\right)\right]$$

$$\text{s.t} \quad c_t^i + m_t^i + a_{t+1}^i = q_t^i + w_t e_t l_t^i$$

with the sequence of constraints on the choice variables in (17) and the transition probabilities for labor productivity given by the matrix $Q$.

Since the effect of inflation on individual behavior depends heavily on whether borrowing constraints are binding, we distinguish two cases.

- *Non-binding borrowing constraints*

In this case, the first-order conditions of agent $i$ are as follows

$$u_c'\left(c_t^i, m_t^i, l_t^i\right) = \beta\,(1 + r_{t+1})\, E\left[v_1'\left(q_{t+1}^i, e_{t+1}^i\right)\right] \tag{18}$$

$$u_c'\left(c_t^i, m_t^i, l_t^i\right) - u_m'\left(c_t^i, m_t^i, l_t^i\right) = \frac{\beta}{\Pi_{t+1}} E\left[v'\left(q_{t+1}^i, e_{t+1}^i\right)\right] \tag{19}$$

$$u_l'\left(c_t^i, m_t^i, l_t^i\right) = -w_t e_t u_c'\left(c_t^i, m_t^i, l_t^i\right) \tag{20}$$

Equation (20) only holds if the solution satisfies $l_t^i \in [0;1]$. Otherwise, $l_t^i$ takes on a corner value, and the solution is given by (18) and (19).

Let $\gamma_{t+1}$ denote the real cost of money holdings

$$\gamma_{t+1} \equiv 1 - \frac{1}{\Pi_{t+1}} \frac{1}{(1 + r_{t+1})}$$

This indicator measures the opportunity cost of holding money. When the after-tax nominal interest rate $r_{t+1}^n$, defined by $1 + r_{t+1}^n = \Pi_{t+1}(1 + r_{t+1})$, is small enough, then $\gamma_{t+1} \simeq r_{t+1}^n$. With this notation and the expression of the utility function given in (15) above, the first-order conditions (18) and (19) yield

$$m_t^i = \left( \frac{1-\omega}{\omega} \frac{1}{\gamma_{t+1}} \right)^\eta c_t^i$$

This equation shows that the money demand of unconstrained households is only affected by the substitution effect, which depends on the opportunity cost of holding money.

- *Binding borrowing constraints*

When the household problem yields a negative value for financial savings, borrowing constraints are binding, $a_{t+1} = 0$, and the first-order condition yields the inequality

$$u_c' \left( c_t^i, m_t^i, l_t^i \right) > \beta (1 + r_{t+1}) E \left[ v_1' \left( q_{t+1}^i, e_{t+1}^i \right) \right]$$

The first-order conditions of the constrained problem are given by

$$u_c' \left( c_t^i, m_t^i \right) - u_m' \left( c_t^i, m_t^i \right) = \frac{1}{\Pi_{t+1}} \beta E \left[ v' \left( \frac{m_t^i}{\Pi_{t+1}}, e_{t+1}^i \right) \right] \tag{21}$$

$$u_l' \left( c_t^i, m_t^i, l_t^i \right) = -w_t e_t u_c' \left( c_t^i, m_t^i, l_t^i \right) \tag{22}$$

There is no simple expression for money demand in the case of binding constraints. The static trade-off between money demand and consumption demand appears on the left-hand side of (21). Were money not to be a store of value, this expression would be equal to 0. However, as money allows individuals to transfer income to the next period, this introduces an additional motive for holding money.

The right-hand side of equation (21) makes clear that inflation has two opposing effects on the demand for money by borrowing-constrained households. On the one hand, inflation induces a substitution effect which serves to decrease money demand as inflation rises (represented by the term $1/\Pi_{t+1}$); on the other hand, as inflation enters the value function via a revenue effect, there might be an increase in money demand as inflation increases.

The core reason for this result is that money is the only store of value which can be adjusted if households are borrowing-constrained. If the function $v$ is very concave, and for realistic parameter values, this second effect may dominate, and the demand for money can increase with inflation. We will show in the quantitative analysis that this result holds for the poorest agents. As a consequence, this case proves that the change in money demand resulting from inflation, the so-called *Tobin effect*, can be decomposed into a revenue effect and a substitution effect for borrowing-constrained households.

Finally, working hours are determined by equation (22). If the value of $l_t$ from (22) is negative, then $l_t = 0$ and the first-order condition (22) holds with inequality.

The solution of the households' program provides a sequence of functions which yield at each date the policy rules for consumption, financial savings, money balances and leisure as a function of the level of labor productivity and wealth:

$$\left.\begin{array}{r} c_t(.,.) : E \times \mathbb{R}^+ \longrightarrow \mathbb{R}^+ \\ a_{t+1}(.,.) : E \times \mathbb{R}^+ \longrightarrow \mathbb{R}^+ \\ m_t(.,.) : E \times \mathbb{R}^+ \longrightarrow \mathbb{R}^+ \\ l_t(.,.) : E \times \mathbb{R}^+ \longrightarrow \mathbb{R}^+ \end{array}\right\} t = 0, 1, ...$$

### 3.1.2 Firms

We assume that all markets are competitive and that the only good consumed is produced by a representative firm with aggregate Cobb-Douglas technology. Let $K_t$ and $L_t$ stand for aggregate capital and aggregate effective labor used in production respectively. It is assumed that capital depreciates at a constant rate $\delta$ and is installed one period ahead of production. Since there is no aggregate uncertainty, aggregate employment and, more generally, aggregate variables are constant at the stationary equilibrium

Output is given by

$$Y_t = F\left(K_t, L_t\right) = K_t^{\alpha} L_t^{1-\alpha} \quad 0 < \alpha < 1$$

Effective labor supply is equal to $L_t = L_t^h e^h + L_t^m e^m + L_t^l e^l$, where $L_t^h, L_t^m$ and $L_t^l$ are the aggregate demands for each type of labor. Prices are set competitively:

$$\begin{aligned} \tilde{w}_t &= (1-\alpha)\left(K_t/L_t\right)^{\alpha} && (23) \\ \tilde{r}_t + \delta &= \alpha(K_t/L_t)^{\alpha-1} && (24) \end{aligned}$$

As high, medium and low productivity workers are perfect substitutes with different productivities, we necessarily have

$$\tilde{w}_t^h = e^h \tilde{w}_t, \qquad \tilde{w}_t^m = e^m \tilde{w}_t, \qquad \tilde{w}_t^l = e^l \tilde{w}_t \tag{25}$$

The aggregate demand for capital is given by

$$K_t = L_t(\alpha/(\tilde{r}_t + \delta))^{\frac{1}{1-\alpha}}$$

### 3.1.3 Government

The government levies taxes to finance a public good, which costs $G$ units of final goods in each period. Taxes are proportional to the revenue of capital and labor, with coefficients $\chi_t^a$ and $\chi_t^w$ in period $t$. In addition, the government receives the revenue of the new money created at period $t$, which is denoted $\tau_t^{tot}$ in real terms. It is assumed that the government does not issue any debt. The government budget constraint is given by

$$G = \chi_t^a \tilde{r}_t K_t + \chi_t^w \left( L_t^h e_t^h + L_t^l e_t^l + L_t^m e_t^m \right) \tilde{w}_t + \tau_t^{tot} \tag{26}$$

### 3.1.4 Monetary Policy

Monetary policy is assumed to follow a simple rule. In each period, the monetary authorities create an amount of new money which is proportional with factor $\pi$ to the nominal quantity of money in circulation, $P_t\Omega_t = P_{t-1}\Omega_{t-1} + \pi P_{t-1}\Omega_{t-1}$. As is standard in the monetary literature, we assume that the State receives all the revenue from the inflation tax[10], which is a more realistic assumption than the helicopter drops of money. As a result the real quantity of money in circulation at period $t$ is

$$\Omega_t = \frac{\Omega_{t-1}}{\Pi_t} + \pi \frac{\Omega_{t-1}}{\Pi_t} \tag{27}$$

The real value of the inflation tax in period $t$ is

$$\tau_t^{tot} = \pi \frac{\Omega_{t-1}}{\Pi_t} \tag{28}$$

Note that if the real quantity of money in circulation is constant (which is the case in equilibrium), equation (27) implies that $\Pi = 1 + \pi$, and hence $\tau^{tot} = \frac{\pi}{1+\pi}\Omega$, which is the standard expression for the inflation tax.

## 3.2 Equilibrium

*Market Equilibria*

Let $\lambda_t : E \times \mathbb{R}^+ \longrightarrow [0,1]$ denote the joint distribution of agents over productivity and wealth. Aggregate consumption $C_t$, aggregate real money holdings $M_t$, aggregate effective labor $L_t^s$ and aggregate financial savings $A_{t+1}$ are respectively given by

$$\begin{aligned}
C_t &= \int\int c_t \left(e^k, q\right) d\lambda_t \left(e^k, q\right) \\
m_t^{tot} &= \int\int m_t \left(e^k, q\right) d\lambda_t \left(e^k, q\right) \\
L_t^s &= e^h \int l_t \left(e^h, q\right) \lambda_t \left(e^h, q\right) dq + e^l \int l_t \left(e^l, q\right) \lambda_t \left(e^l, q\right) dq + e^m \int l_t \left(e^m, q\right) \lambda_t \left(e^m, q\right) dq \\
A_{t+1} &= \int\int a_{t+1} \left(e^k, q\right) d\lambda_t \left(e^k, q\right)
\end{aligned}$$

[10] In practice, the profits of Central Banks are redistributed to the State and are not used for specific purposes.

Equilibrium in the final good market implies

$$C_t + K_{t+1} + G_t = Y_t + (1 - \delta) K_t \tag{29}$$

Equilibrium in the labor market is

$$L_t = L_t^s$$

Equilibrium in the financial market implies

$$K_{t+1} = A_{t+1} \tag{30}$$

Last, money-market equilibrium is defined by

$$m_{tot} = \Omega_t \tag{31}$$

where $\Omega_t$ is the real quantity of money in circulation at period $t$.

*Competitive equilibrium*

A stationary competitive equilibrium for this economy consists of constant decision rules $c(e, q)$, $m(e, q), l(e, q)$ and $a(e, q)$ for consumption, real balances, leisure and capital holdings respectively, the steady-state joint distribution over wealth and productivity $\lambda(e, q)$, a constant real return on financial assets $r$, a constant real wage $w$, the real return on real balances $1/\Pi$, and tax transfers $\chi^a$ , $\chi^w$ , consistent with the exogenous supply of money $\pi$ and government public spending $G$ such that

1. The long-run distribution of productivity is given by a constant vector $n^*$.
2. The functions $a(.,.), c(.,.), m(.,.), l(.,.)$ solve the households' problem
3. The joint distribution $\lambda$ over productivity and wealth is time invariant.
4. Factor prices are competitively determined by equations (23)-(25).
5. Markets clear: equations (29)-(31).
6. The quantity of money in circulation follows the law of motion (27).
7. The tax rates $\chi^a$ and $\chi^w$ are constant and are defined to balance the budget of the State (26), where the seigniorage rent from the inflation tax $\tau^{tot}$ is given by (28).

Note that equilibrium on the money market and stationarity of the joint distribution imply that the real quantity of money in circulation is constant.

### 3.3 Calibration

We have parameterized the model using data from the US economy. The model period is one year, the most appropriate horizon for considering monetary policy changes[11]. The key targets

[11] This horizon allows us to take into account the fact that liquid and illiquid assets are not perfect substitutes and that there are some adjustment costs in the portfolio following a permanent monetary policy change. Note

of this parametrization are the wealth distribution, including the share of borrowing-constrained individuals, income fluctuations, the interest-elasticity of money demand and the key capital-output and money-consumption ratios in the American economy. In what follows we focus on the benchmark incomplete market economy with endogenous prices, proportional taxes, and endogenous labor supply at an initial inflation rate $\pi$ of 2 percent.

*Technology and Preferences*

Table 2 shows the parameters we have used for preferences and technology. The parameters relating to the production technology and the discount factor are standard: capital's share $\alpha$ is set equal to 0.36, the capital depreciation rate is 0.1 and the discount factor is set to 0.96.

We choose parameter values for the utility function (15) as follows. For $\omega$ and $\eta$ we draw on the money-demand literature. Interest elasticity $\eta$ is set to $\eta = 0.5$, which is close to traditional estimates (e.g. Chari *et al.* 2000, Holman 1998, Hoffman, Rasche and Tieslau 1995), and which can be microfounded in a Baumol-Tobin type model of money demand. The share parameter $\omega$ of consumption relative to money is then set to $\omega = 0.98$. This figure yields a ratio of M1 over consumption of 0.78, which is close to that found in the data over the period 1960-2000 which is traditionally used for the estimation of interest elasticity.

The weight on leisure $\psi$ is set to reproduce a steady state fraction of labor of 33 percent of total time endowment. Last, we set the risk-aversion parameter to a value different to that from the log utility function but close enough to one in order to match the observed capital-ouput ratio.

Table 1: Benchmark calibration

| Parameters | $\beta$ | $\alpha$ | $\delta$ | $\omega$ | $\eta$ | $\psi$ | $\sigma$ |
|---|---|---|---|---|---|---|---|
| Values | 0.96 | 0.36 | 0.1 | .98 | 0.5 | 2 | 1.1 |

*Employment Process*

With respect to employment, the main goal of calibration is to find a stylized process for wages which is both empirically relevant and able to replicate the US wealth distribution, in particular the share of people who are borrowing-constrained.

We follow Domeij and Heathcote (2004) who estimated a stylized process to match some of these criteria (see Castaneda *et al.*, 2003, for a relevant alternative strategy). Domeij and Heathcote found that at least three employment states are needed to obtain realistic earnings

however that this time period might lead to an overestimation of the effect of the inflation tax on welfare (see Erosa and Ventura, 2002).

fluctuations and to match the skewness of the wealth distribution. For instance Budria Rodriguez *et al.* (2002) estimated that the two poorest quintiles of the distribution hold no more than 1.35 percent of total wealth and that the Gini coefficient is close to 0.78. Thus the set of employment states is represented by $E = \{e^h, e^m, e^l\}$ where $e^h$ stands for high productivity, $e^m$ for medium productivity, and $e^l$ for low productivity. The ratio between the different productivity levels and the transition probabilities are set to match an autocorrelation of 0.9 and a standard deviation of 0.224 of individual earnings, as estimated on PSID data by Domeij and Heathcote (2004). The implied ratio of productivity values are $e_1/e_2 = 6.09$ and $e_2/e_3 = 5.04$. The Markov chain consistent with the observed earning process is $p_{1,1} = p_{3,3} = 0.9$ and $p_{2,2} = 0.98$.

$$Q = \begin{bmatrix} p_{1,1} & 1 - p_{1,1} & 0 \\ \frac{1-p_{2,2}}{2} & p_{2,2} & \frac{1+p_{2,2}}{2} \\ 0 & 1 - p_{1,1} & p_{1,1} \end{bmatrix}$$

*Key Features of the Equilibrium Distribution*

This specification yields a Gini coefficient in wealth of 0.76, which is fairly close to the recent findings of Budria Rodriguez *et al.* (2002); the Gini coefficient in consumption is 0.30, consistent with Krueger and Perri (2005).

A key point is that our specification yields a substantive fraction of households, around 7.5 percent, who are borrowing constrained. Naturally the empirical measure of this fraction heavily depends on the choice of measure. By using information on the number of borrowing requests which were rejected in the Survey of Consumer Finance (SCF), Jappelli showed that up to 19 percent of families are liquidity constrained. But by using updated SCF data, Budria Rodriguez et al. (2002) reported that only 2.5 percent of households have zero wealth, which might correspond to our theoretical borrowing limit in the model. Obviously this figure does mean that these households are liquidity-constrained. In particular, Budria Rodriguez et al. (2002) also report that 6 percent of households have delayed their debt repayments for two months or more, which could be used as another proxy for liquidity constraints. To this extent, our measure of 7.5 percent of liquidity-constrained individuals in our model can be considered as an intermediate value, which prevents us from over-estimating the effect of borrowing constraints on the non-neutrality of inflation. [12]

[12] It is worth noting at this point that our model with capital and real balances yields quite naturally a positive number of people who are liquidity-constrained with the employment process at stake. This result is due to the introduction of real balances in the traditional Aiyagari model. For instance Domeij and Heathcote (2004) and Heathcote (2005) found that no-one is borrowing-constrained for the same kind of employment process. By

Table 2 reports the main statistics reproduced by our model under the benchmark specification with endogenous prices, distorting inflation taxes and endogenous labour supply. The benchmark specification closely matches the key observed ratios of capital $K/Y = 3$ and public spending $G/Y = 0.24$. Moreover, we reproduce the average value of the consumption velocity of money, $(M1/P)/C$, which is 0.79 over the period 1960-2000. In the benchmark calibration we assume that the tax rates on capital and labor are identical: $\chi^a = \chi^w \equiv \chi$. The calibration yields an average tax rate on labor and capital $\chi = 0.34$ which is close to that observed (Domeij and Heathcote, 2004). Significantly, the benchmark set-up matches the Gini coefficients of both wealth and consumption and replicates both the upper- and lower tails of the wealth distribution.

Table 2: Benchmark calibration

| Values | Data | Benchmark economy |
|---|---|---|
| K/Y (%) | 3 | 3 |
| (M/P)/C (%) | 0.79 | 0.78 |
| G/Y (%) | 0.20 | 0.24 |
| $\chi$ (%) | 0.33 | 0.34 |
| Gini (Wealth) | 0.78 | 0.76 |
| Gini (Consumption) | 0.30 | 0.30 |
| Borrowing constraints (%) | [2-19] | 7.5 |

contrast, introducing money in the utility function naturally entails that wealth-poor people need to carry over real balances into the next period in order to be able to consume. They thus draw down their financial assets to zero to be able to keep a positive amount of real balances when they are affected by negative labor productivity shocks. Note that the previous literature generally uses stochastic discounting factors to fit this dimension (Krusell and Smith, 1998, Carroll, 2000). We do not follow this strategy since the goal of this paper is to look at the specific role of credit constraints and incomplete markets in the non-neutrality of money regardless of any additional heterogeneity, in particular with respect to preferences.

# 4 Results

## 4.1 Individual policy rules

We start by discussing the impact of inflation on individual policy rules in the benchmark economy with endogenous hours and taxes.

Figure 1 illustrates the main policy rules in the benchmark economy with an inflation rate of $\pi = 2$ percent. Consumption, real balances and financial assets are an increasing function of labor productivity and current total wealth $q$, made up of financial assets and cash. But due to the presence of borrowing constraints, the value functions and the implied policy rules for consumption and money demand are concave at the low values of wealth and productivity. Moreover the policy rule for financial assets held by medium- and low-productivity workers displays kinks at low levels of wealth, indicating that these two types of workers are net-dissavers. By contrast high productivity workers are net-savers in order to smooth consumption across less favorable productivity states.

Figure 2 shows the impact of a one-point permanent increase in inflation, from $\pi = 2\%$ to $\pi = 3\%$, on next-period asset holdings and money balances as a function of total beginning of period wealth. The focus is on policy rules around the kink where the main non-linearity lies. We focus on the high- and low-productivity states, as households in the medium state have similar policy rules to low-productivity households. For the high value of productivity, an increase in inflation provides more incentives to save via financial assets at the expense of real money balances whose value has been slashed by inflation. This behavior stands in sharp contrast with that of households in lower productivity states. These households are borrowing-constrained on asset holdings at the low level of total wealth. In this case they have no alternative but to carry over higher level of money balances following a rise in inflation in order to sustain their level of consumption. Money is used as a store of value, and the revenue effect dominates the substitution effect when wealth is low, as explained in the discussion of equation (21). Their level of real-money balances decreases only at the higher level of total wealth for which borrowing constraints on financial assets are no longer binding and households can thus use their capital as a buffer stock. This contrasting effect suggests that the impact of inflation on the real economy and welfare crucially depends on borrowing constraints. Moreover, these policy rules show that wealth-poor households hold a higher fraction of their wealth in real balances compared to wealth-rich households. This endogenous outcome is consistent with the data (see Erosa and Ventura, 2002).

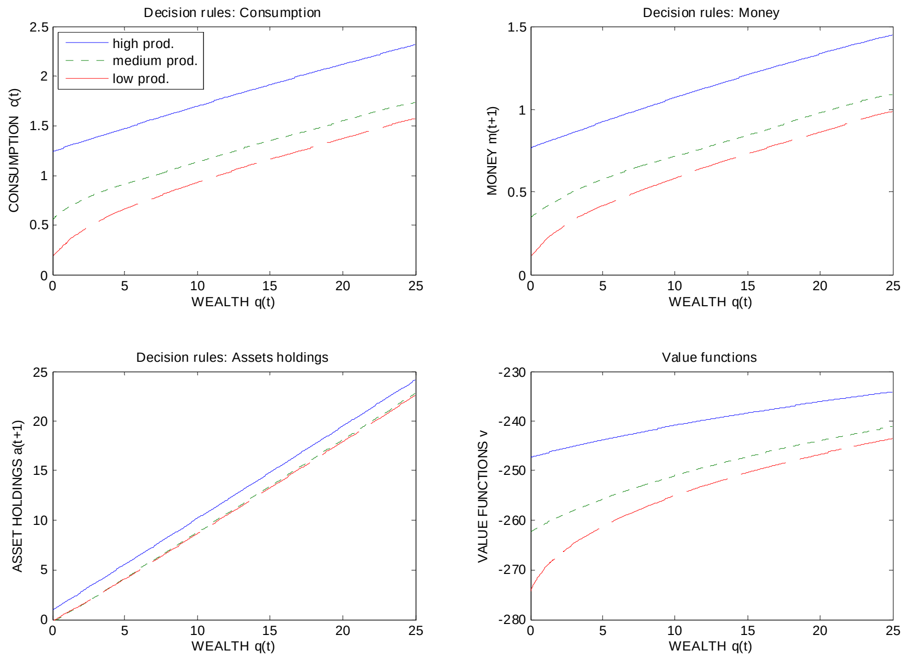


Figure 1: Individual policy rules

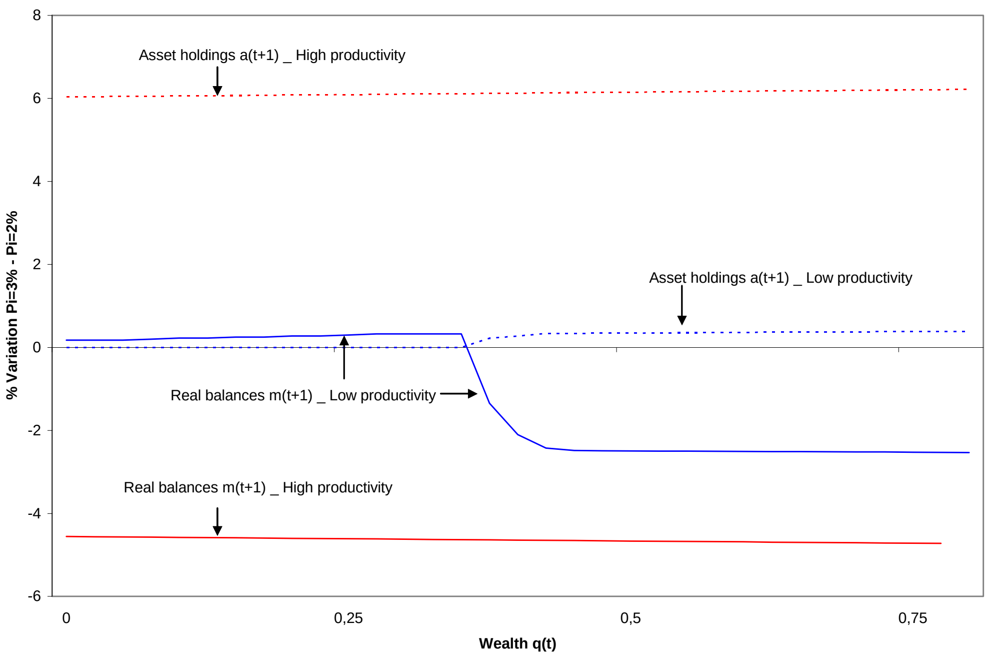


Figure 2: Effect of inflation on individual policy rules

## 4.2 Aggregate results

This section quantifies the impact of monetary policy on the real economy and welfare. We look at a policy experiment in which the inflation rate rises by one point from $\pi = 2$ percent to $\pi = 3$ percent. The quantitative theoretical analysis proceeds as follows: we quantify the aggregate impact of inflation depending on different assumptions made regarding the redistribution of the seigniorage rent, the tax structure and the adjustment of labor supply, to be able to disentangle the various effects of inflation in this economy.

First we consider a version of the model in which hours are exogenous and money creation is made by helicopter drops. The new money is redistributed proportionally to the begining-of-period real balances of households, who consider these transfers as lump-sum. We thus abstract from any redistributive and distortionary issues discussed in the previous literature. Consistent with our theoretical results in section 2, this set-up allows us to quantify the non-neutrality of monetary policy which *only* transits through borrowing constraints. This framework is thus mainly illustrative since the neutrality of money would apply under these assumptions were markets to be complete.

Second, we take into account the traditional redistributive and distortionary effects of inflation which will interact with borrowing constraints. Labor supply is still assumed to be exogenous but there are now proportional taxes on labor and capital income. In this case, borrowing constraints give rise to two new inflation effects. The redistributive effect is due to the seigniorage rent being redistributed unevenly across wealth-poor and wealth-rich agents. The distortionary tax effect is due to the the seigniorage rent allowing a reduction in capital taxes and thus increasing incentives to save. This traditional Phelps effect is amplified by the presence of borrowing constraints via precautionary savings motives. We assess the contribution of borrowing constraints to the magnitude of these effects by comparing complete markets and incomplete markets with borrowing constraints.

Third, we extend the model by introducing endogenous labor supply. Due to borrowing constraints, inflation gives rise to heterogeneous labor supply responses depending on the endogenous heterogeneity in wealth. We measure this new effect by comparing the incomplete and complete market set-ups with endogenous hours of work and distortionary taxation.

For each economy, we change the parameters relating to the household productivity process so that each economy matches the same targeted feature of the US wealth distribution[13]. The

[13] For each environment with exogenous hours, we keep exactly the same parameter values for the incomplete and complete market models. Actually, these two set-ups provide very similar initial steady-states. By contrast (and as in Heathcote, 2005), we use a smaller labor supply elasticity $\psi = 0.75$ in the representative economy with endogenous hours in order to start from the same steady state values for $L$, K/Y, (M/C) and taxes $\chi$ as those found in the incomplete market frameworks.

calibration described below refers to the benchmark model with endogenous labor supply.

### 4.2.1 Lump-Sum Transfers

*Environment*

In the first stage of the analysis, we define a special case of our model in which the real effect of monetary policy only transits through borrowing constraints, regardless of other potential distortions. To do so we consider the following environment. First, we assume that each household supplies inelastically $l = \bar{l}$ hours of labor. We set $\bar{l} = 0.33$, which corresponds to the steady state value of labor with endogenous labor supply at $\pi = 2$ percent. Second, we assume that there are no taxes on labor and capital, and all (net) transfers are lump-sum. Third, government spending is equal to 0, and the government distributes new money proportionally to the beginning of period level of real balances held by each household. This environment corresponds to the simple model presented in Section 2 but with a more general labor income process.

The budget constraint (16) of household $i$ can be written in this case as

$$c_t^i + m_t^i + a_{t+1}^i = (1 + \tilde{r}_t)\, a_t^i + \tilde{w}_t e_t^i \bar{l} + \frac{m_{t-1}^i}{\Pi_t} + \tau_t^i$$

where $\tau_t^i$ stands for the lump-sum transfer of the seigniorage tax, and $\tilde{r}_t$ and $\tilde{w}_t$ are the levels of the interest rate and wages paid by the firm (without tax), defined by equations (23) and (24) respectively.

The seigniorage tax is redistributed *ex-post* to each agent as a lump-sum transfer proportional to beginning of period money holdings

$$\tau_t^i = \frac{\pi}{\Pi_t} m_{t-1}^i$$

As a consequence, the *ex-post* individual budget constraint is

$$c_t^i + a_{t+1}^i + m_t^i = (1 + \tilde{r}_t)\, a_t^i + \tilde{w}_t e_t^i \bar{l} + m_{t-1}^i$$

Here inflation no longer appears in the individual budget constraint. But since the seigniorage tax is redistributed *ex-post*, the inflation rate is still taken into account by households as the anticipated inflation rate affects the arbitrage conditions to hold money.

*Aggregate results*

We now consider the aggregate impact of a one-point rise in the inflation rate from $\pi = 2$ percent to 3 percent in this environment. The aggregate results for the economy with neutral lump-sum taxes and exogenous hours are reported in Line 1 of Table 3. We focus on the main aggregate variables: output $Y$, capital $K$, real balances $M/P$ , aggregate consumption $C$ and prices $w$ and $r$. In this table, we show the percentage change in each variable compared to its value with inflation of 2%.

With complete markets and non-distortionary taxes, inflation has no real effect on the stationary values of the real aggregate variables. Each household adjusts its demand for real money and financial asset holdings in the same proportions, leading to a neutral effect of inflation on aggregate consumption, capital and output. The effect of inflation only transits through nominal variables, with the aggregate stock of money decreasing by 6.97 percent

By contrast, when markets are incomplete and households face borrowing constraints, those who are borrowing constrained cannot adjust their money and capital holdings in the same way as unconstrained agents, as illustrated in figure 2. Constrained agents have no choice other than to increase their demand for money to restore their level of real balances and to be able to consume tomorrow. At the other extreme, unconstrained agents increase their level of financial assets, whose returns relative to cash increase with inflation. Aggregate capital rises by 0.23 percent, leading to an increase in output and consumption of 0.08 percent and 0.02 percent respectively. As households have a greater incentive to save in financial assets in the incomplete market set-up, the reduction in real balances is much sharper compared to that with complete markets.

#### 4.2.2 Redistributive effects of seigniorage rent

*Environment*

We now consider the sensitivity of the role played by borrowing constraints in the non-neutrality of money when we take into account additional distortions in the inflation tax. We thus introduce proportional taxes in line with the benchmark incomplete market model described in section 3, and compare the results to those from a complete market economy. We still avoid the labor supply channel by assuming that the number of hours is fixed at its stationary level $\bar{l} = 0.33$. In this case, the individual and government budget constraints are respectively

$$c_t^i + m_t^i + a_{t+1}^i = (1 + \tilde{r}_t(1 - \chi_t^a))a_t^i + (1 - \chi_t^w)\tilde{w}_t e_t^i \bar{l} + \frac{m_{t-1}^i}{\Pi_t}$$

and

$$G = \chi_t^a \tilde{r}_t K_t + \chi_t^w \left( n^h e_t^h + n^m e_t^l + n^m e_t^m \right) \bar{l} \tilde{w}_t + \tau_t^{tot}$$

with $\tau_t^{tot} = \pi \frac{\Omega_t}{\Pi_t}$.

*Redistributive distortion of the inflation tax*

We first focus on the redistributive effect of the seigniorage rent. In particular we assume that the seigniorage rent is redistributed proportionally to labor income. To isolate this redistributive effect, we need to cancel out the Phelps effect which works via a reduction in capital tax. We then assume that the distorting proportional tax on capital $\chi^a$ is not affected by inflation. This tax is held constant at its stationary value, corresponding to an inflation rate of 2 percent. Yet the rise in the seigniorage tax $\tau_t^{tot}$ allows a reduction in the proportional tax on labor $\chi^w$. Thus everything works as if the government was engineering a transfer of the seigniorage rent proportionally to labor income. As we assume in this section that labor supply is exogenous, these transfers are not distortionary.

Line 2 of Table 3 shows that the tax on labor sharply decreases by 1.04 due to higher seigniorage rents. However, since the redistribution of the seigniorage rent is proportionally more favorable to high-productivity workers, these latter have a greater incentive to save in order to smooth their consumption. The increase in aggregate capital and output (by 0.34 and 0.12 percent respectively) is greater compared to the previous environment with neutral redistribution of the seigniorage rent. Monetary policy remains neutral in the complete markets economy since labor supply is still assumed to be exogenous and taxes on labor are thus non-distortionary.

### 4.2.3 Capital taxation distortion

We now discuss the interplay between borrowing constraints and distortionary taxes on capital. We retain the same environment as above with exogenous hours. However, we do take into account the adjustment of capital tax following a rise in inflation. Due to the seigniorage rent, inflation allows a reduction in the capital tax rate required to balance the government budget constraint. This phenomenon, traditionally known as the Phelps effect, interacts in our framework with the borrowing constraints which amplify incentives to save via the precautionary savings motive. We quantify the contribution of borrowing constraints to this traditional Phelps effect by comparing the incomplete to the complete market set-up.

Line 3 of Table 3 first indicates that the tax on capital decreases by 0.94 percent with incomplete markets. This provides a greater incentive to save. Significantly, the precautionary saving motive due to the existence of borrowing constraints amplifies the rise in aggregate capital. The Phelps effect turns out to be twice as high in incomplete markets compared to complete markets, with aggregate capital rising by 0.58 percent and 0.29 percent respectively. This produces a proportional increase in output and consumption of 0.15 percent and 0.12 percent in incomplete markets. Conversely, the greater incentive to save with borrowing constraints leads to a sharper reduction in real money balances with incomplete markets (8.78 percent) compared to 6.8 percent in the representative agent economy.

Table 3: Aggregate impact of inflation: economies with proportional taxes and exogenous hours

| Economies | Percentage change following a rise in inflation $\pi = 2\% \rightarrow 3\%$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $Y$ | $K$ | $M/P$ | $C$ | $L$ | $\chi^w$ | $\chi^a$ | $\tilde{r}$ | $\tilde{w}$ |
| $Neutral\ \ lump-sum\ tax$ | | | | | | | | | |
| $Exogeneous\ hours$ (1) | | | | | | | | | |
| Incomplete markets | 0.08 | 0.23 | -9.69 | 0.02 | 0 | 0 | 0 | -0.67 | 0.06 |
| Complete markets | 0 | 0 | -6.97 | 0 | 0 | 0 | 0 | 0 | 0 |
| $Redistributive\ distortion$ | | | | | | | | | |
| $Exogeneous\ hours$ (2) | | | | | | | | | |
| Incomplete markets | 0.12 | 0.34 | -8.85 | 0.06 | 0 | -1.04 | 0 | -0.80 | 0.12 |
| Complete markets | 0 | 0 | -6.97 | 0 | 0 | -0.91 | 0 | 0 | 0 |
| $Capital\ tax\ distortion$ | | | | | | | | | |
| $Exogeneous\ hours$ (3) | | | | | | | | | |
| Incomplete markets | 0.21 | 0.58 | -8.78 | 0.11 | 0 | -0.94 | -0.94 | -1.26 | 0.19 |
| Complete markets | 0.10 | 0.29 | -6.88 | 0.08 | 0 | -0.80 | -0.80 | -0.46 | 0.10 |
| $Benchmark\ economy$ (4) | | | | | | | | | |
| $Distort.tax - Endog.hours$ | | | | | | | | | |
| Incomplete markets | 0.44 | 0.94 | -11.15 | 0.35 | 0.16 | -1.16 | -1.16 | -2.94 | 0.28 |
| Complete markets | 0.15 | 0.32 | -6.86 | 0.10 | 0.05 | -1.36 | -1.36 | -0.47 | 0.09 |

### 4.2.4 Endogenous labor supply

We finish this analysis by taking into account the interplay between borrowing constraints and labor supply. Line 4 of Table 3 compares the benchmark incomplete market economy described in section 3 with a complete market set-up. Note that taxes on labor and capital income are now both distortionary.

The primary channel through which inflation affects labor is by altering the productivity of labor (measured by wages) and thus the marginal rate of substitution between leisure and consumption. As aggregate capital increases, the productivity of labor rises and wages increase by 0.28 percent. This entails a substitution effect in labor supply, which rises by 0.16 percent. Conversely, the rise in labor supply increases capital productivity and provides greater incentives to save. This effect leads to a sizeable increase in aggregate capital and output of 0.94 percent and 0.44 percent respectively. This effect is three times higher than in complete market economies, allowing much greater steady-state consumption. Yet this sharp substitution between capital and real balances with incomplete markets triggers a reduction in real balances which is almost twice as high under borrowing constraints as in an economy with complete markets.

### 4.3 Welfare

We conclude by assessing the welfare costs of inflation in incomplete markets compared to that in the traditional representative agent literature. This analysis is carried out in the benchmark model with endogenous hours and proportional taxes. We use the standard Aiyagari-McGrattan average welfare criterion, defined as the expected discounted sum of utilities under the equilibrium stochastic stream of consumption and real balances of infinitely-lived agents. Henceforth we focus on steady state welfare analysis in order to compare our results with those from previous studies carried out in the representative agent framework (see Lucas, 2000) or in incomplete market economies but where the potential positive real effect of inflation on capital was not taken into account (see Imrohoroglu, 1992).[14]

Following Lucas' tradition, we measure the welfare gain of inflation as the percentage of consumption we need to give to households living in an environment with low inflation to make them indifferent to living in another economy with higher inflation.[15] The monetary policy experiment is the same as above and consists in an increase of one point in the inflation rate from $\pi = 2\%$ to $\pi = 3\%$. Let $c(\epsilon, q)$, $m(\epsilon, q)$ and $l(\epsilon, q)$ be the level of consumption, real balances and labor supply of households with labor productivity $\epsilon$ and wealth $q$. These quantities are defined at the stationary equilibrium under the benchmark level of inflation $\pi = 2\%$ used in the calibration. Let $c^{\Delta\pi}(\epsilon, q)$, $m^{\Delta\pi}(\epsilon, q)$ and $l^{\Delta\pi}(\epsilon, q)$ be the level of these quantities after a change in the inflation rate, and let $\lambda^{\Delta\pi}$ be the new stationary joint distribution after a change in inflation.

The average welfare loss $\Delta^{av}$ is thus defined as

$$\int\int u\left((1+\Delta^{av})c(e,q), m(e,q), l(e,q)\right) d\lambda(e,q) = \int\int u\left(c^{\Delta\pi}(e,q), m^{\Delta\pi}(e,q), l^{\Delta\pi}(e,q)\right) d\lambda^{\Delta\pi}(e,q)$$

We also look at the redistributive impact of inflation, depending on the level of wealth. More precisely, let denote $\Lambda^{\epsilon}_{X\%}$ the range of financial wealth of the poorest $X\%$ of type $\epsilon$ workers in

[14] Naturally, taking into account transitions would increase the welfare costs of inflation. However, this analysis requires us to take into account changes in prices and the wealth distribution along the transition path, which is beyond the scope of this article.

[15] The welfare criterion is based on steady-state comparisons à la Lucas in order to obtain a benchmark comparison with the representative agent literature.

the benchmark economy and define by $\Lambda'^{\epsilon}_{X\%}$ the range of financial wealth of the poorest $X\%$ of type $\epsilon$ workers in the modified economy. Thus the welfare cost of inflation for the poorest $X\%$ of type $\epsilon$ workers is given by

$$\int_{\Lambda^{\epsilon}_{X\%}} u\left((1+\Delta_{X,}\epsilon)c(\epsilon,q), m(\epsilon,q), l(e,q)\right) dq = \int_{\Lambda'^{\epsilon}_{X\%}} u\left(c^{\Delta\pi}(\epsilon,q), m^{\Delta\pi}(\epsilon,q), l^{\Delta\pi}(e,q)\right) dq$$

Here $\Delta_X$ is the additional consumption required for the poorest $X\%$ of type $\epsilon$ workers in order to make them indifferent between living in the benchmark economy and becoming the poorest $X\%$ type $\epsilon$ workers in the modified economy. The welfare cost of inflation for the richest $Y\%$ is defined analogously.

Table 4 shows the welfare costs generated by a one point increase in the inflation rate from $\pi = 2$ percent to $\pi = 3$ percent. We look both at the average cost and at the cost for the poorest 5 percent and wealthiest 5 percent of households. We also decompose the welfare costs by labor productivity, distinguishing the two polar cases of high- and low-productivity workers.

Column 1 of Table 4 compares the average welfare cost of inflation under incomplete and complete markets. The welfare cost in consumption equivalent due to the change in inflation is 0.185% percent in the incomplete market economy. This negative impact is mainly due to the sharp reduction in real balances and the decline in leisure induced by inflation, as shown in Table 3. This negative effect more than offsets the positive impact of inflation on aggregate capital, which allows higher stationary values of consumption. Remarkably, this number is fairly close to that estimated by Attonasio et al. (2002) on individual data which explicitly took into account wealth heterogeneity and differences between real balances and interest-bearing assets.

Moreover, we find that this welfare cost is slightly smaller than that which pertains in a complete market economy. In the latter, inflation triggers a less-pronounced decrease in real balances compared to that under incomplete markets. However, the potential positive effect of inflation on welfare, transiting through the rise in output and consumption, is also lower in complete market economies. It is worth underlining that our findings are at odds with the traditional result of Imrohoroglu (1992), where the welfare cost of inflation increases by a factor of four in an incomplete market economy in which money is the only store of value. Our results suggest that this previous finding was overestimated by ignoring the real positive effect of inflation on capital and output due to incomplete markets and borrowing constraints.

Last, we show that inflation does not affect all households in the same way. Columns 2 and 3 of Table 4 show that the wealth-poor gain from inflation while the wealth-rich are hurt by inflation. This mainly results from the price composition effect. As suggested by Table 3, inflation has a significant positive impact on labor productivity and wages by triggering greater capital accumulation. In the benchmark economy, a one-point rise in the inflation rate was found to increase wages by 0.28 percent. Conversely, the induced higher accumulation of capital entailed a sharp reduction in the interest rate by 2.94 percent. Inflation then increases the welfare of the wealth-poor whose income is mainly made up of labor income. This effect is all the more important for high-productivity workers whose welfare increases by 0.7569 percent, as against 0.296 percent for the low-skilled. By contrast, inflation lowers the welfare of the wealth-rich since their total income is mainly made up of financial assets, whose return decreases.

Table 4: Welfare costs of inflation: benchmark economy with proportional taxes and endogeneous hours

| Economies | Average (1) | High skill (2) | | Low skill (3) | |
|---|---|---|---|---|---|
| | | Poorest 5% | Richest 5% | Poorest 5% | Richest 5% |
| Incomplete Markets | -0.185 | 0.756 | - 0.090 | 0.296 | - 0.189 |
| Complete Markets | -0.192 | | | | |

## 5 Conclusion

This paper has proposed a new theoretical and quantitatively significant channel for the non-neutrality of money which directly transits through borrowing constraints. Here, higher inflation induces heterogeneity in money demand to the extent that borrowing-constrained households cannot substitute away their real balances for financial assets in the same way as unconstrained households do. This endogenous heterogeneity across households due to borrowing constraints gives rise to a real effect of inflation on aggregate capital for precautionary savings motives. We have shown that this specific channel has a sizeable quantitative impact in incomplete market economies with an empirically-relevant wealth distribution. Not only do incomplete markets and borrowing constraints have a real quantitative impact on their own, they also amplify significantly the other potential distorting channels of inflation compared to the case in complete market economies. Finally, we suggest that the previous measures of the welfare costs of inflation have been overestimated by excluding the potential positive impact of inflation on steady-state capital and consumption due to binding borrowing constraints and incomplete markets.

This paper has focussed on the long-run properties of money with borrowing constraints. A promising route for future research would be to analyze the short-run effects of monetary shocks in this type of incomplete market economy with borrowing constraints. This framework could provide a new relevant channel to account for the persistence and non-neutrality of monetary shocks, presenting an alternative to traditional sticky-price models. Borrowing constraints and household heterogeneity would likely offer a useful framework for the analysis of the short-run redistributive effects and the transition costs of monetary policies.

# A Solution to the Households' Problem

Using the Bellman equations, the households' problem can be written in recursive form. Stationary solutions satisfy, of course, the usual transversality conditions. As a consequence, we can focus on the first-order condition of the households' problem. This is given by the program

$$
\begin{aligned}
V\left(q_t^i, e_t^i\right) &= \max_{\{c_t^i, m_t^i, a_{t+1}^i\}} u\left(c_t^i, m_t^i\right) + \beta V\left(q_{t+1}^i, e_{t+1}^i\right) \\
c_t^i + m_t^i + a_{t+1}^i &= q_t^i + w_t e_t^i + \frac{\mu_t^i}{P_t} \qquad (32) \\
q_{t+1}^i &= R_{t+1} a_{t+1}^i + \frac{m_t^i}{\Pi_{t+1}} \qquad (33) \\
c_t^i, m_t^i, a_{t+1}^i &\geq 0 \qquad (34)
\end{aligned}
$$

where $q_1^1, q_1^2$ are given, $R_{t+1} = 1 + r_{t+1}$, and income shocks are deterministic: $e_{t+1}^i = 0$ if $e_t^i = 1$, and $e_{t+1}^i = 1$ if $e_t^i = 0$. Using (32) and (33) to substitute for $c_t^i$ and $q_{t+1}^i$, we can maximize only over $a_{t+1}^i$ and $m_t^i$. Using the first-order conditions, together with the envelope theorem (which yields in all cases $V'\left(q_t^i, e_{t+1}^i\right) = u_c'\left(c_t^i, m_t^i\right)$), we have

$$
\begin{aligned}
u_c'\left(c_t^i, m_t^i\right) &= \beta R_{t+1} u_c'\left(c_{t+1}^i, m_{t+1}^i\right) \qquad (35) \\
u_c'\left(c_t^i, m_t^i\right) - u_m'\left(c_t^i, m_t^i\right) &= \frac{\beta}{\Pi_{t+1}} u_c'\left(c_{t+1}^i, m_{t+1}^i\right) \qquad (36)
\end{aligned}
$$

If the equations above yield a quantity $a_{t+1}^i < 0$, then the borrowing constraint is binding and the solution is given by $a_{t+1}^i = 0$ and $u_c'\left(c_t^i, m_t^i\right) > \beta R_{t+1} u_c'\left(c_{t+1}^i, m_{t+1}^i\right)$, together with (36). In a stationary equilibrium, all $H$ agents become $L$ agents the next period and *vice versa.* Since $H$ agents are in the good state, they always take the opportunity to save for precautionary motives and their borrowing constraints are never binding (see next section). We can rewrite the previous equations using the state of the households instead of their type. With the logarithmic utility function, this yields the expressions given in section 2.

# B Proof of Proposition 2 on binding borrowing constraints and the non-neutrality of money

In this proof, we assume as a first step that borrowing constraints are binding for $L$ households to derive the equilibrium interest rate. In a second step, we check that borrowing constraints are actually binding for $L$ agents but not for $H$ agents. By using proposition 1, it will suffice to check that the equilibrium interest rate satisfies $1 + r < \frac{1}{\beta}$

First, by using the first-order condition (8), we obtain $\frac{c^L}{c^H} = \beta(1+r)$. Equilibrium on the goods market implies that $c^H + c^L = K^\alpha - K$, and the first-order conditions of the firm imply that $1+r=\alpha K^{\alpha-1}$ and $w=(1-\alpha)K^{\alpha}$. Substituting for $c^H, w$ and $K$ we obtain

$$c^L=\beta\frac{1+r-\alpha}{\beta(1+r)+1}\left(\frac{\alpha}{1+r}\right)^{\frac{\alpha}{1-\alpha}}$$

The budget constraint of $L$ agents, given by (7), yields

$$\frac{m^L}{c^L}-\frac{m^H}{c^H}\frac{c^H}{c^L}=\frac{a^H(1+r)-c^L}{c^L}$$

Using the value of the ratio $\frac{c^L}{c^H}=\beta(1+r)$ and the expressions (13) and (14), one finds

$$f(r)=g(r,\Pi) \tag{37}$$

with

$$f(r)\equiv\frac{\phi}{1-\phi}\left(\alpha\frac{\beta(1+r)+1}{1+r-\alpha}-\beta\right) \text{ and } g(r,\Pi)\equiv\frac{\beta}{1-\frac{\beta^2}{\Pi}(1+r)}-\frac{1}{1+r-\frac{1}{\Pi}}$$

Equation (37) determines the equilibrium interest rate as a function of the parameters of the model and $\Pi$. We now have to prove the existence and uniqueness of the equilibrium.

*Existence of a solution with binding borrowing constraints*

Recall that we assume that $\alpha<1/\Pi<1/\beta$. We then look for the existence of a solution $r^*$ such that $1+r^*\in(1/\Pi;1/\beta)$. If such a solution exists, borrowing constraints are binding and both money and financial titles are held in equilibrium.

Note that $f(r)$ is continuous in $r$, for $1+r\in(\frac{1}{\Pi};\frac{1}{\beta})$ and $f$ takes finite values at the boundaries $\frac{1}{\Pi}$ and $\frac{1}{\beta}$. For a given value of $\Pi$, $g(r,\Pi)$ is continuous in $r$ for $1+r\in(\frac{1}{\Pi};\frac{1}{\beta})$. However, $g(r,\Pi)\longrightarrow-\infty$ when $1+r\in(\frac{1}{\Pi};\frac{1}{\beta})$ and $1+r\longrightarrow 1/\Pi$. And $g\left(\frac{1}{\beta}-1,\Pi\right)=0$. As a result, a sufficient condition for an equilibrium to exist is $f\left(\frac{1}{\beta}-1\right)<0$. This condition is equivalent to $\alpha<1/(2+\beta)$. Hence, if $\alpha<1/(2+\beta)$, there exists an equilibrium interest rate $r^*$ such that $1+r^*\in(1/\Pi;1/\beta)$. From proposition 1, borrowing constraints are binding in such an equilibrium. QED

*Uniqueness and variations*

Note that $f(r)$ is decreasing in $r$ when $1+r\in(\frac{1}{\Pi};\frac{1}{\beta})$ as $\alpha<1/\Pi$ (a simple derivative of $f$). We can show that $g(r,\Pi)$ is increasing in $r$. As a result, the solution is unique, for continuity reasons. Finally, we can show that $g(r,\Pi)$ is increasing in $\Pi$. Define a function $h$ such that

$$h(y)=\frac{y^3(1+r)^3}{\left(1+r-\frac{y^2}{\Pi}(1+r)^2\right)^2} \tag{38}$$

The function $h$ is positive and increasing in $y$. Now, the derivative $g'_{\Pi}(r,\Pi)$ can be written as $g'_{\Pi}(r,\Pi)=\frac{1}{\Pi^2}\left(h\left(\frac{1}{1+r}\right)-h(\beta)\right)$. At the equilibrium $1/(1+r^*)>\beta$, and hence we have $g'_{\Pi}(r^*,\Pi)>0$.

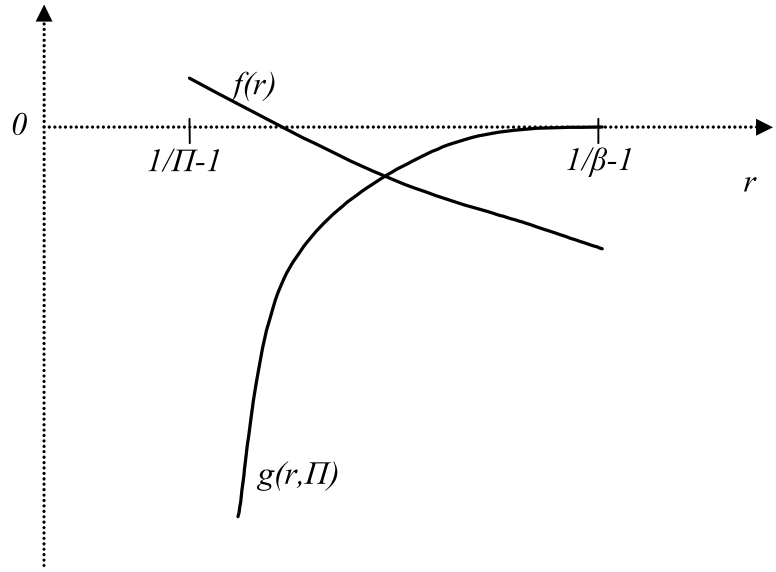


Figure 3: Existence of an Equilibrium with $1{+}r{<}1/\beta$

Consequently, by the implicit function theorem $f(r^*) = g(r^*, \Pi)$ defines implicitly $r^*$ as a decreasing function of $\Pi$. Figure 3 illustrates the existence and the uniqueness of the equilibrium with binding borrowing constraints. QED

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