EXPLAINING THE EFFECTS OF GOVERNMENT SPENDING SHOCKS ON CONSUMPTION AND THE REAL EXCHANGE RATE

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ABSTRACT

Using structural VAR analysis, we document that in a panel of industrialized countries, an increase in government purchases leads to an expansion in output and private consumption, a deterioration in the trade balance, and a depreciation of the real exchange rate (i.e., a decrease in the domestic CPI relative to the exchange-rate adjusted foreign CPI). We propose an explanation for these observed effects based on the deep habit mechanism. We estimate the key parameters of the deep-habit model employing a limited information approach. The predictions of the estimated deep-habit model fit remarkably well the observed responses of output, consumption, the trade balance, and the real exchange rate to an unanticipated government spending shock. In addition, the deep-habit model predicts that in response to an anticipated increase in government spending consumption and wages fail to increase on impact, which is consistent with the empirical evidence stemming from the narrative identification approach. In this way, the deep-habit model reconciles the findings of the SVAR and narrative literatures on the effects of government spending shocks.
1 Introduction

In this paper, we present the results of an empirical and theoretical investigation into the effects of government spending shocks on consumption, output, the trade balance, and the real exchange rate. Our empirical analysis uses data from a panel of four industrialized countries, the United States, the United Kingdom, Canada, and Australia, over the post-Bretton Woods period. We employ a structural vector autoregressive representation of the data. Following Fatás and Mihov (2001) and Blanchard and Perotti (2002), we identify government spending shocks by assuming that no variable other than government spending shocks themselves can affect government spending contemporaneously.

We find that a positive innovation in government spending causes an expansion in output, an expansion in consumption, a deterioration of the trade balance, and a depreciation of the real exchange rate (that is, a decline in domestic prices relative to exchange-rate-adjusted foreign prices). The effects of government spending shocks on domestic aggregate activity and private absorption have been extensively studied in the related empirical literature. Our finding that government spending shocks raise output and consumption is consistent with previous studies that have used identification assumptions and estimation techniques similar to those we employ in the present paper (e.g., Rotemberg and Woodford, 1992; Blanchard and Perotti, 2002; Fatás and Mihov, 2001; Perotti, 2004, 2007; and Galí, López-Salido, and Vallés, 2007).

The effects of government spending shocks on the external sector of the economy have received considerably less attention. Notable exceptions are Corsetti and Müller (2006) and Monacelli and Perotti (2006). Our results are most similar to those reported in the second of these two studies. The main difference between our empirical strategy and those adopted in the papers cited above is the pooling of data across countries. We justify a panel analysis by observing that the identified effects of government spending shocks, particularly on consumption and the real exchange rate, are to a large extent qualitatively similar across the individual countries considered. The purpose of our panel approach is to obtain an efficient estimate of a single benchmark against which to evaluate our proposed theoretical explanation of the transmission of government spending shocks.

A central contribution of our investigation is to advance and test a theoretical explanation for the observed effects of government spending shocks based on the deep habit mechanism developed by Ravn, Schmitt-Grohé, and Uribe (2006). To this end, we introduce deep habits into a two-country model. Under deep habits, an increase in aggregate demand provides an incentive for firms to lower markups. Thus, an increase in government spending in the domestic economy leads to a decline in domestic markups relative to foreign markups. In this way, the domestic economy becomes less expensive relative to the foreign economy, or, equivalently, the real exchange rate depreciates. At the same time, a decline in domestic markups shifts the labor demand curve outward, giving rise to an increase in domestic real wages. In turn, the rise in wages induces households to substitute consumption for leisure. This substitution effect may be strong enough to offset the negative wealth effect stemming from the increase in public absorption, resulting in an equilibrium increase in private consumption.
To stress the ability of deep habits to explain the empirical regularities associated with government spending shocks, we deliberately abstract from a number of alternative frictions that have been proposed as potential theoretical explanations in the related literature. Specifically, we abstract from sticky prices, rule-of-thumb consumers, and nonseparabilities between consumption and leisure in preferences.\(^1\)

We estimate the structural parameters defining the deep-habit mechanism using a limited information approach. We find substantial empirical support for the presence of deep habits in private and public consumption. Moreover, the impulse responses of output, consumption, the trade balance, and the real exchange rate predicted by the deep-habit model match remarkably well in size and shape their empirical counterparts.

The empirical finding that consumption and real wages increase in response to a positive government spending shock has been challenged by an empirical literature, pioneered by Ramey and Shapiro (1998), that employs a narrative approach to the identification of government spending shocks. The narrative approach uses news sources to identify dates at which agents learn about increases in government spending. Empirical studies employing the narrative approach find that in response to an increase in government spending consumption and wages fail to increase. These findings therefore stand in sharp contrast to those obtained by the SVAR literature.

In this paper, we argue that the findings of the narrative and SVAR identification schemes do not necessarily contradict each other. The starting point of our argument is the observation that the narrative approach identifies mostly anticipated increases in government spending, whereas the SVAR approach identifies mostly unanticipated innovations in public spending. In effect, empirical studies following the narrative approach find that typically government spending starts rising only two to three quarters after news about the fiscal expansion becomes available. By contrast, under the SVAR approach government spending shocks are by construction orthogonal to past information. It follows that if under the transmission mechanism underlying the actual economy anticipated and unanticipated shocks to public spending have different consequences, one should not expect the estimated effects of fiscal shocks stemming from the narrative and SVAR approaches to coincide. A successful theoretical explanation of the effects of government spending shocks must therefore induce dynamics that are in line with those estimated using the SVAR and the narrative approaches when perturbed by unanticipated and anticipated government spending shocks, respectively.

Indeed, we demonstrate that the predictions of the deep-habit model concerning consumption and wages are consistent not only with the empirical findings of the SVAR approach, but also with those of the narrative method. Specifically, we show that in response to news that government spending will rise two quarters hence consumption and real wages decline on impact. That is, while the deep habit model predicts that consumption and wages rise in response to an unanticipated increase in government spending, it at the same time implies that in response to an anticipated

\(^1\)See, for example, Corsetti and Müller (2006), Linnemann (2006), Monacelli and Perotti (2006), and Galí, López-Salido, and Vallés (2007).
government spending shock, consumption and wages decline.

The remainder of the paper is organized as follows: Section 2 estimates econometrically the effects of government spending shocks on output, consumption, the trade balance, and the real exchange rate. Section 3 presents a two-country, two-good model with deep habits. Section 4 describes the calibration of the nonestimated structural parameters of the model. Section 5 presents the estimation of the structural parameters defining the deep-habit mechanism. Section 6 compares the predicted and estimated impulse response functions. Section 7 reconciles the evidence stemming from the SVAR and narrative approaches through the lens of the deep-habit model. Section 8 explores the robustness of our findings to changes in key structural parameters and detrending technique. Section 9 concludes.

2 The Observed Effects of Government Spending Shocks

In this section, we document the effects of government spending shocks on key macroeconomic variables. The empirical model is a structural vector autoregression of the form

\[
A \begin{bmatrix} \hat{g}_t \\ \hat{y}_t \\ \hat{c}_t \\ \hat{tby}_t \\ \hat{e}_t \end{bmatrix} = B(L) \begin{bmatrix} \hat{g}_{t-1} \\ \hat{y}_{t-1} \\ \hat{c}_{t-1} \\ \hat{tby}_{t-1} \\ \hat{e}_{t-1} \end{bmatrix} + \epsilon_t, \tag{1}
\]

where \( g_t \) denotes real per capita government spending deflated by the GDP deflator, \( y_t \) denotes real per capita GDP, \( c_t \) denotes real per capita private consumption of nondurables and services, \( tby_t \) denotes the net-exports-to-GDP ratio, and \( e_t \) denotes the real exchange rate defined as the ratio of a trade-weighted average of exchange-rate-adjusted foreign CPIs to the domestic CPI. According to our definition, an increase in \( e_t \) means that the real exchange rate of the domestic country depreciates, or that the domestic country becomes cheaper relative to its trading partners. A hat over a variable denotes the log deviation from trend, except for \( tby_t \), for which it indicates the level deviation from trend. All variables are seasonally adjusted, and detrended with a linear and quadratic trend. The variable \( \epsilon_t \) is a mean-zero, serially uncorrelated vector of disturbances with diagonal variance-covariance matrix \( \Sigma_{\epsilon} \). The factor \( B(L) \equiv B_0 + B_1 L + B_2 L^2 + \ldots \) denotes a lag polynomial, with \( L \) denoting the lag operator. The matrices of coefficients \( B_i \) and \( A \) are of size 5 by 5.

Following Blanchard and Perotti (2002), we identify innovations to government spending by assuming that government spending responds with at least one-quarter lag to structural innovations other than innovations to government spending itself. Formally, we impose that the first row of the matrix \( A \) contains unity in its first element and zeros in all other elements.

We estimate the structural VAR pooling quarterly data from Australia, Canada, the United Kingdom, and the United States. Our sample begins in the first quarter of 1975 and ends in
the fourth quarter of 2005. Our choice of countries is guided by our desire to limit attention to industrialized countries, and by the availability of reliable data on aggregate private consumption of nondurable goods and services. The rationale for pooling data is to gain efficiency and to obtain a single benchmark against which to evaluate the performance of our theoretical model to be presented in section 3. We estimate the VAR system by OLS including country dummies.

A potential concern with the panel VAR is the inconsistency of the least squares parameter estimates due to the combination of fixed effects and lagged dependent variables (e.g., Nickell, 1981). However, because the time series dimension of our data is large (124 observations), the inconsistency problem is likely not to be a major concern. A different potential problem is the possibility of correlated residuals across countries. To gauge the importance of these two potential problems (i.e., the Nickell bias and cross-country residual correlations), we also computed impulse response functions by estimating the VAR in differences (to eliminate country fixed effects) and then applied feasible GLS to correct for serial and cross-country correlations in the error terms. The resulting impulse response functions (not shown) are fairly close to their OLS counterparts. Additionally, we confirm that the size of the Nickell bias is small by Monte Carlo analysis. Guided by the likelihood ratio test proposed by Sims (1980), we allow for four lags in the SVAR specification.

Our estimation procedure imposes that the matrices \( A \) and \( B(L) \) are the same across the four countries from which we pool information. This simplifying assumption seems appropriate in light of the fact that estimations using individual country data yield similar results for the dynamic effects of government spending shocks on consumption and the real exchange rate. Our SVAR specification is similar to the one estimated in Monacelli and Perotti (2006). Like these authors, we consider data from the US, the UK, Canada, and Australia, and apply the Blanchard and Perotti (2002) identification strategy. The main differences between our empirical approach and that of Monacelli and Perotti is that we pool data, that we do not include taxes or the nominal interest rate in the SVAR specification, and that our sample is 16 quarters longer per country.

Figure 1 displays with solid lines the impulse response function of government spending, output, consumption, the net-export-to-GDP ratio, and the real exchange rate to a unit innovation in government spending. The figure depicts with broken lines a two-standard error band on each side of the point estimates of the impulse response function computed using the delta method.

The response of government spending is highly persistent, with a half life of about 5 quarters. The one-percent increase in government spending raises output by 0.1 percent. Assuming a government share of 19 percent (the average of government spending over the sample period for the four countries in our sample), the government-spending multiplier, \( \Delta y_t / \Delta g_t \), is 0.52 on impact.

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2 Specifically, we carried out the following experiment. Given the OLS estimates of \( A \), the lag polynomial \( B(L) \), and the country fixed effects, we generated 10,000 artificial data series by bootstrapping the estimated errors. We then estimated by OLS the pooled fixed effects VAR on each of the artificial data series and compared the point estimates of the empirical impulse responses with the median estimates over the 10,000 Monte Carlo experiments. The two estimates are very similar. The results are available from the authors upon request.

3 The test rejects the hypothesis of one or two lags in favor of a longer lag structure. We settle on a lag length of four quarters to maintain comparability with the related literature. The three-lag and four-lag specifications yield virtually identical impulse response functions and error bands.

4 The results are robust to using parametric or nonparametric bootstrap methods for computing error bands.
Figure 1: Estimated Impulse Response To A One-Percent Innovation in Government Spending

Note. All responses are expressed in percent deviations from trend with the exception of the trade-balance-to-GDP ratio, which is expressed in level deviations from trend.
indicating that for each unit increase in public spending output increases by 0.52 units on impact.

Private consumption of nondurables and services experiences a persistent expansion following the increase in public spending. This finding is in line with many other SVAR studies on the effects of government spending inspired by the identification scheme proposed by Fatás and Mihov (2001) and Blanchard and Perotti (2002). The expansion in public spending results in a protracted albeit small deterioration in the trade balance.

The finding that private consumption expands with government purchases is not uncontroversial. A strand of the literature identifies innovations in government spending using the narrative approach pioneered by Romer and Romer (1989). These studies find that in response to news about upcoming military buildups consumption falls (see, Ramey and Shapiro, 1998). In section 7, we argue on theoretical grounds that the effects of government spending shocks estimated using the SVAR and narrative approaches are not necessarily at odds with each other.

The bottom left panel of figure 1 shows that the real exchange rate depreciates by one third of one percent when the economy is hit by a one-percent increase in government spending. That is, an expansion in public consumption causes the domestic country to become cheaper relative to its trading partners. The reaction of the real exchange rate is quite persistent. The peak depreciation occurs only 10 quarters after the innovation to government spending. This result is at odds with the conventional wisdom, according to which an expansion in government consumption is associated with an increase in domestic prices leading to an appreciation of the real exchange rate. The empirical evidence typically drawn upon to support the conventional view is based on raw correlations between government consumption and the real exchange rate. The difficulty with this type of evidence is that, in general, movements in the real exchange rate may be driven by a multitude of shocks. By contrast, in the impulse responses shown in figure 1, movements in all variables are driven by an innovation in government purchases. That is, the figure states that conditional on a positive innovation in government spending the real exchange rate depreciates. It follows that the evidence reported here and that emanating from the analysis of raw correlations are not necessarily contradictory. We note further that other empirical studies have also found that the real exchange rate depreciates in response to a positive government spending shock. For example, Monacelli and Perotti (2006) document this fact for each of the individual countries included in our panel.

Our empirical results deliver four regularities that serve as the basis for evaluating the theory presented in the next section. Namely, in response to an increase in government spending output and consumption increase, the trade balance deteriorates, and the real exchange rate depreciates. These empirical regularities are quite robust. They also emerge in country-by-country estimations, under specifications including additional fiscal variables, such as taxes, and monetary policy variables, such as the nominal interest rate (see Monacelli and Perotti, 2006), and under alternative detrending schemes (see section 8 below).
3 A Two-Country Model of Pricing to Habits

The model economy consists of two countries, the home country and the foreign country. Each country specializes in the production of a set of differentiated goods. We denote by $a$ the set of goods produced by the home country and by $b$ the set of goods produced by the foreign country. All goods are internationally traded. To emphasize the transmission mechanism invoked by deep habits, we abstract from a number of important frictions that are common elements of the related literature, such as sticky prices and wages, distribution costs, nontraded goods, rule-of-thumb consumers, nonseparabilities of preferences across consumption and leisure, and incomplete international asset markets.

3.1 Households

We describe the household’s problem in the domestic economy. The foreign counterpart is a mirror image. The domestic economy is populated by a large number of identical households with preferences described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(x^c_t, h_t).$$

(2)

The variable $x^c_t$ is a composite defined as

$$x^c_t = \chi(x^c_{a,t}, x^c_{b,t}),$$

(3)

where the aggregator function $\chi$ is assumed to be increasing and homogeneous of degree one in both arguments. The variable $x^c_{a,t}$ is a habit-adjusted composite consumption good of varieties of goods of type $a$. Following Ravn, Schmitt-Grohé, and Uribe (2006), we introduce deep habits by assuming that habits form at the level of each individual variety of goods instead of at the level of the aggregate consumption good. We assume that deep habits are external to the individual household (i.e., we model catching up with the Joneses good by good). Formally, $x^c_{a,t}$ is given by

$$x^c_{a,t} = \left[ \int_0^1 \left( c_{i,a,t} - \theta_a s^c_{i,a,t-1} \right)^{1-1/\eta} \frac{1}{1-1/\eta} \right]^{1/(1-1/\eta)}.$$

(4)

Here $c_{i,a,t}$ denotes consumption of variety $i$ of goods belonging to the set $a$ in period $t$. The parameter $\theta_a \in [0,1)$ measures the intensity of deep external habits for consumption goods of type $a$. When $\theta_a$ is equal to zero, preferences for goods of type $a$ display no deep habit formation. The parameter $\eta > 1$ represents the intratemporal elasticity of substitution across varieties. The variable $s^c_{i,a,t}$ denotes the stock of external habit in consumption of variety $i$ of good $a$. This habit stock is assumed to evolve according to the following law of motion:

$$s^c_{i,a,t} = \rho s^c_{i,a,t-1} + (1 - \rho) \tilde{c}_{i,a,t},$$
where $\tilde{c}_{i,a,t}$ denotes the average per capita consumption of variety $i$ of good $a$ in the domestic country; that is, $\tilde{c}_{i,a,t}$ is the integral of $c_{i,a,t}$ over all domestic households. The parameter $1-\rho \in (0,1]$ denotes the rate at which the stock of external habits decays over time.

Similarly, $x^{c}_{b,t}$ is given by

$$x^{c}_{b,t} = \int_{0}^{1} \left( c_{i,b,t} - \theta^{c}_{b} s^{c}_{i,b,t-1} \right)^{1-1/\eta} \frac{1}{d} \left( \int_{0}^{1} (P_{j,b,t})^{1-\eta} \frac{1}{d} \right)^{1/(1-\eta)},$$

with

$$s^{c}_{i,b,t} = \rho s^{c}_{i,b,t-1} + (1-\rho) \tilde{c}_{i,b,t}.$$

To characterize the household’s demands for varieties of type-$a$ and type-$b$ goods, we consider a two-step problem. Suppose the household has determined its desired consumption of the aggregate goods $a$ and $b$, that is, $x^{c}_{a,t}$ and $x^{c}_{b,t}$. Then it is optimal for the household to distribute its purchases of individual varieties to minimize costs, that is,

$$\min_{c_{i,a,t}} \int_{0}^{1} P_{i,a,t} c_{i,a,t} \frac{1}{d}$$

subject to (4). This minimization problem yields the following demand function for variety $i$ of good $a$:

$$c_{i,a,t} = \left( \frac{P_{a,t}}{P_{i,a,t}} \right)^{-\eta} x^{c}_{a,t} + \theta^{c}_{a} s^{c}_{i,a,t-1},$$

where $P_{a,t}$ denotes a price index for goods of type $a$ given by

$$P_{a,t} = \left[ \int_{0}^{1} (P_{j,a,t})^{1-\eta} \frac{1}{d} \right]^{1/(1-\eta)}.$$

Similarly, one can express the demand for variety $i$ of good $b$ as

$$c_{i,b,t} = \left( \frac{P_{b,t}}{P_{i,b,t}} \right)^{-\eta} x^{c}_{b,t} + \theta^{c}_{b} s^{c}_{i,b,t-1},$$

where $P_{b,t}$ is a price index of goods of type $b$ defined as

$$P_{b,t} = \left[ \int_{0}^{1} (P_{j,b,t})^{1-\eta} \frac{1}{d} \right]^{1/(1-\eta)}.$$

Note that the demand for each variety of good $a$, say, is decreasing in its relative price, $P_{i,a,t}/P_{a,t}$, increasing in the level of habit-adjusted consumption of the composite good of type $a$, $x^{c}_{a,t}$, and increasing in the stock of habit of the variety in question $s^{c}_{i,a,t-1}$.

Total expenditure on goods of type $a$ in period $t$ is given by

$$\int_{0}^{1} P_{i,a,t} c_{i,a,t} \frac{1}{d} = P_{a,t} x^{c}_{a,t} + \theta^{c}_{a} \int_{0}^{1} P_{i,a,t} s^{c}_{i,a,t-1} \frac{1}{d}.$$
Let \( \omega_{a,t} \) and \( \omega_{b,t} \) be defined, respectively, as \( \omega_{a,t} \equiv \theta_{a,t} \int_{0}^{1} P_{i,a,t}s_{i,a,t-1}di \) and \( \omega_{b,t} \equiv \theta_{b,t} \int_{0}^{1} P_{i,b,t}s_{i,b,t-1}di \). Note that because habits are assumed to be external, the household takes both \( \omega_{a,t} \) and \( \omega_{b,t} \) as exogenously given. It follows that total expenditure on goods of type \( a \) and \( b \), respectively, can be written as \( \int_{0}^{1} P_{i,a,t}c_{i,a,t}di = P_{a,t}c_{a,t} + \omega_{a,t} \) and \( \int_{0}^{1} P_{i,b,t}c_{i,b,t}di = P_{b,t}c_{b,t} + \omega_{b,t} \).

In each period \( t \geq 0 \), households have access to complete contingent claims markets. Let \( r_{t,t+j} \) denote the stochastic discount factor such that \( E_{t}r_{t,t+j}d_{t+j} \) is the period-\( t \) price of a random payment \( d_{t+j} \) of the (numeraire good) in period \( t+j \). In addition, households are assumed to be entitled to the receipt of pure profits from the ownership of firms, \( \Phi_{t} \). Households pay lump-sum taxes in the amount \( T_{t} \). Then, the domestic representative household’s period-by-period budget constraint can be written as

\[
P_{a,t}x_{a,t} + \omega_{a,t} + P_{b,t}x_{b,t} + \omega_{b,t} + E_{t}r_{t,t+1}d_{t+1} + T_{t} = d_{t} + W_{t}h_{t} + \Phi_{t}. \tag{5}
\]

The variable \( W_{t} \) denotes the wage rate. In addition, households are assumed to be subject to a borrowing constraint of the form \( \lim_{j \to \infty} E_{t}r_{t,t+j}d_{t+j} \geq 0 \), which prevents them from engaging in Ponzi games. The representative household’s optimization problem consists in choosing processes \( x_{a,t}, x_{b,t}, h_{t} \), and \( d_{t+1} \) to maximize the lifetime utility function (2) subject to (3), (5), and the no-Ponzi-game constraint, taking as given the processes for \( \omega_{a,t}, \omega_{b,t}, W_{t}, T_{t} \), and \( \Phi_{t} \) and initial asset holdings \( d_{0} \).

The first-order conditions of the household’s optimization problem are the constraints (3) and (5), the no-Ponzi-game constraint holding with equality, and

\[
\frac{\chi_{a}(x_{a,t}^{c}, x_{b,t}^{c})}{\chi_{b}(x_{a,t}^{c}, x_{b,t}^{c})} = \frac{P_{a,t}}{P_{b,t}},
\]

\[
\frac{U_{b}(x_{a,t}^{c}, h_{t})}{U_{a}(x_{a,t}^{c}, x_{b,t}^{c})} = \frac{W_{t}}{P_{a,t}},
\]

and

\[
\beta U_{x}(x_{t+1}^{c}, h_{t+1}) = \beta U_{x}(x_{t+1}^{c}, h_{t+1}) \frac{U_{a}(x_{a,t+1}^{c}, x_{b,t+1}^{c})}{P_{a,t+1}}. \tag{6}
\]

The first equation states that the marginal rate of substitution between the composite goods \( a \) and \( b \) must equal their relative price. The second equation implicitly defines the supply of labor. It equates the real domestic product wage to the marginal rate of substitution between leisure and consumption of composite good \( a \). The last equation is a standard asset pricing relation equating the price of contingent claims to the intertemporal marginal rate of substitution.

### 3.2 The Government

Like households, the government is assumed to form habits on consumption of individual varieties of goods. This assumption is important for understanding the transmission of government purchases shocks in the context of our model. We motivate the deep-habit formulation in public
spending by assuming that private households value public goods in a way that is separable from private consumption and leisure and that households derive external habits from consumption of government-provided goods. By good-specific external habit formation in the consumption of public goods we mean situations in which the provision of public services in one community—such as street lighting, traffic signals, yard-waste collection—creates the desire in other communities to have access to the same type of service. Alternatively, one can assume that the government forms procurement relationships that create a tendency for it to favor transactions with sellers that supplied public goods in the past.

We treat government habits as external. Conceivably, government habits could be treated as internal to the government even if they are external to their beneficiaries, namely households. This alternative is, however, less tractable, and is therefore not pursued here. In the econometric estimation of the model, presented later in the paper, we let the data tell how much habit formation there is in public spending.

The government is assumed to aggregate individual varieties of domestic and foreign goods to produce two intermediate composite goods denoted \(x^g_{a,t}\) and \(x^g_{b,t}\), using the same aggregator function as the private sector:

\[
x^g_{a,t} = \left[ \int_0^1 (g_{i,a,t} - \theta^g_{a,s} s^g_{i,a,t-1})^{1-1/\eta} di \right]^{1/(1-1/\eta)}
\]

and

\[
x^g_{b,t} = \left[ \int_0^1 (g_{i,b,t} - \theta^g_{b,s} s^g_{i,b,t-1})^{1-1/\eta} di \right]^{1/(1-1/\eta)}.
\]

The parameters \(\theta^g_a, \theta^g_b \in [0, 1]\) measure the degree of habit formation in government consumption of domestic and foreign goods, respectively. The variables \(s^g_{i,a,t}\) and \(s^g_{i,b,t}\) denote the government’s stocks of habit in variety \(i\) of goods \(a\) and \(b\), respectively, and are assumed to evolve over time as

\[
s^g_{i,a,t} = \rho s^g_{i,a,t-1} + (1 - \rho) g_{i,a,t}
\]

and

\[
s^g_{i,b,t} = \rho s^g_{i,b,t-1} + (1 - \rho) g_{i,b,t},
\]

where \(1 - \rho \in (0, 1]\) denotes the rate of depreciation of the stocks of habit. The government combines the intermediate goods \(x^g_{a,t}\) and \(x^g_{b,t}\) to produce a final, public good \(x^g_t\) according to the relationship

\[
x^g_t = \chi(x^g_{a,t}, x^g_{b,t}).
\]

Note that the aggregator function \(\chi\) is the same as the one used by private consumers.

As in the empirical SVAR model of section 2, let \(g_t\) denote total real government spending expressed in units of domestic GDP (i.e., nominal government spending divided by the GDP deflator).
Then, letting $P^y_t$ denote the GDP deflator, to be defined later, we have that

$$g_t \equiv \int_0^1 (P_{i,a,t} g_{i,a,t} + P_{i,b,t} g_{i,b,t}) \, di \cdot (10)$$

To allow for the empirical and the theoretical models to feature the same feedback mechanism and driving process for total government purchases, we assume that fiscal policy takes the form of a feedback rule given by the first equation of the SVAR system given in equation (1). Formally, $g_t$ satisfies

$$\dot{g}_t = B^1(L) \begin{bmatrix} \dot{g}_{t-1} \\ \dot{y}_{t-1} \\ \dot{c}_{t-1} \\ \frac{\hat{g}_t}{\hat{P}_t} \\ \hat{e}_{t-1} \end{bmatrix} + \epsilon^1_t, (11)$$

where $B^1(L)$ and $\epsilon^1_t$ denote, respectively, the first rows of the matrices defining the polynomial $B(L)$ and the first element of the innovation vector $\epsilon_t$. Here, hatted variables denote log-deviations from deterministic steady-state values, except for the variable $\hat{tby}_t$, for which a hat indicates the level deviation of $tby_t$ from its deterministic steady state. Note that the values assigned to $B^1(L)$ are those estimated in section 2. However, the behavior of the endogenous variables appearing in the above law of motion for $g_t$ is dictated by the dynamics of the theoretical model. For this reason, the theoretical and empirical impulse responses of $g_t$ to an innovation in $\epsilon^1_t$ will in general not coincide. Government spending is assumed to be financed by lump-sum taxes.

The government’s problem consists in choosing $g_{i,a,t}$ and $g_{i,b,t}$, $i \in [0,1]$, to maximize $x_i^g$ subject to the budget constraint (10) and the aggregation restrictions (7), (8), and (9), taking as given $g_t$, $P^y_t$, $P_{i,a,t}$, $P_{i,b,t}$, $s_{i,a,t-1}$, and $s_{i,b,t-1}^q$ for all $i \in [0,1]$ and $t \geq 0$.

The government’s problem implies demand functions for individual varieties of goods $a$ and $b$ of the form

$$g_{i,a,t} = \left( \frac{P_{i,a,t}}{P_{a,t}} \right)^{-\eta} x_{a,t}^g + \theta_{i,a,t}^g s_{i,a,t-1}$$

and

$$g_{i,b,t} = \left( \frac{P_{i,b,t}}{P_{b,t}} \right)^{-\eta} x_{b,t}^g + \theta_{i,b,t}^g s_{i,b,t-1}^g.$$ 

### 3.3 Firms

Goods of type $a$ are produced exclusively in the domestic country, and goods of type $b$ are produced exclusively abroad. Each individual variety of good of type $a$ or $b$ is assumed to be produced by a monopolist. Each good $i \in [0,1]$ is manufactured using labor as the sole input with a linear production technology. Specifically domestic output of variety $i$ of type $a$, denoted $y_{i,a,t}$, is produced according to the relationship

$$y_{i,a,t} = h_{i,a,t},$$
where \( h_{i,a,t} \) denotes labor input in producing variety \( i \) of good \( a \).

The producer of variety \( i \) of good \( a \) faces demands from the private and public sectors in the domestic and foreign countries. The private and public domestic demand functions are given by

\[
c_{i,a,t} = \left( \frac{P_{i,a,t}}{P_{a,t}} \right)^{-\eta} x_{a,t}^c + \theta_a^c s_{i,a,t-1},
\]

and

\[
g_{i,a,t} = \left( \frac{P_{i,a,t}}{P_{a,t}} \right)^{-\eta} x_{a,t}^g + \theta_a^g s_{i,a,t-1}.
\]

Letting an asterisk denote a foreign variable or parameter, the foreign private and public components of demand for variety \( i \) of type \( a \) goods are given by

\[
c_{i,a,t}^* = \left( \frac{P_{i,a,t}^*}{P_{a,t}^*} \right)^{-\eta} x_{a,t}^{cs} + \theta_a^{cs} s_{i,a,t-1}^c,
\]

and

\[
g_{i,a,t}^* = \left( \frac{P_{i,a,t}^*}{P_{a,t}^*} \right)^{-\eta} x_{a,t}^{gs} + \theta_a^{gs} s_{i,a,t-1}^g.
\]

Implicit in the above demand functions are the assumptions that firms can price discriminate between the domestic and foreign markets but that they cannot price discriminate between the government and consumers residing in the same country.

A number of important implications for the model’s predictions regarding deviations from the law of one price are evident from inspection of the above demand functions. First, each demand function for an individual variety of goods is of the form

\[
d_t = p_t^{-\eta} x_t + \theta s_{t-1}.
\]

That is, each demand function is the sum of a price-elastic component, \( p_t^{-\eta} x_t \), and a price inelastic component, \( \theta s_{t-1} \). The price elastic component has price elasticity \( \eta \) and is proportional to measures of current aggregate demand, \( x_t \). The price inelastic term is purely habitual in nature. It follows that the price elasticity of each demand function is a weighted average of \( \eta \) and 0, with the weight on \( \eta \) given by the relative importance of the price-elastic, nonhabitual demand component in total demand. An increase in aggregate demand enlarges the importance of the price elastic component of demand increasing the price elasticity. In other words, the price elasticity of each demand function is procyclical. Second, the fact that the price elasticity is procyclical opens the possibility for markups to move countercyclically in equilibrium. Third, because the price elasticity of demand can in principle be different in the domestic and the foreign markets, it follows that firms have an incentive to charge different markups domestically and abroad. We refer to this incentive for price discrimination as ‘pricing to habits’ as it originates from the presence of a habitual demand for individual varieties of goods. More importantly, pricing to habits gives rise to deviations from the law of one price over the business cycle at the level of individual goods traded across borders. Finally, because firms understand that the stock of habit is a weighted average of all past sales, their profit-maximization problem is dynamic in nature. Thus, customer-market and brand-switching cost considerations in
the spirit of Phelps and Winter (1970) and Froot and Klemperer (1989) will endogenously emerge in the pricing behavior of firms, affecting the size and persistence of deviations from the law of one price.

The firm’s problem consists in choosing processes \( \{ P_{i,a,t}, P^*_{i,a,t}, c_{i,a,t}, g_{i,a,t}, c^*_{i,a,t}, g^*_{i,a,t}, s^c_{i,a,t}, g^g_{i,a,t}, s^c*_{i,a,t}, g^g*_{i,a,t}\} \) to minimize

\[
E_0 \sum_{t=0}^{\infty} r_{0,t} \left[ P_{i,a,t}(c_{i,a,t} + g_{i,a,t}) + P^*_{i,a,t}(c^*_{i,a,t} + g^*_{i,a,t}) - W_t h_{i,a,t} \right]
\]

subject to

\[
c_{i,a,t} + g_{i,a,t} + c^*_{i,a,t} + g^*_{i,a,t} = h_{i,a,t}, \\
\]

\[
c_{i,a,t} = \left( \frac{P_{i,a,t}}{P_{a,t}} \right)^{-\eta} x^c_{a,t} + \theta^c_{a} s^c_{i,a,t-1}, \\
g_{i,a,t} = \left( \frac{P_{i,a,t}}{P_{a,t}} \right)^{-\eta} x^g_{a,t} + \theta^g_{a} s^g_{i,a,t-1}, \\
c^*_{i,a,t} = \left( \frac{P^*_{i,a,t}}{P^*_{a,t}} \right)^{-\eta} x^{c*}_{a,t} + \theta^{c*}_a s^c*_{i,a,t-1}, \\
g^*_{i,a,t} = \left( \frac{P^*_{i,a,t}}{P^*_{a,t}} \right)^{-\eta} x^{g*}_{a,t} + \theta^{g*}_a s^g*_{i,a,t-1}, \\
s^c_{i,a,t} = \rho s^c_{i,a,t-1} + (1 - \rho)c_{i,a,t}, \\
s^g_{i,a,t} = \rho s^g_{i,a,t-1} + (1 - \rho)g_{i,a,t}, \\
s^{c*}_{i,a,t} = \rho s^{c*}_{i,a,t-1} + (1 - \rho)c^*_{i,a,t}, \\
s^{g*}_{i,a,t} = \rho s^{g*}_{i,a,t-1} + (1 - \rho)g^*_{i,a,t}, \\
\]

and

\[
s^{g*}_{i,a,t} = \rho s^{g*}_{i,a,t-1} + (1 - \rho)g^*_{i,a,t}.
\]

taking as given the processes \( r_{0,t}, W_t, P_{a,t}, P^*_{a,t}, x^c_{a,t}, x^g_{a,t}, x^{c*}_{a,t}, x^{g*}_{a,t}, \) and the initial conditions \( s^c_{i,a,-1}, s^g_{i,a,-1}, s^{c*}_{i,a,-1}, s^{g*}_{i,a,-1} \). The associated optimality conditions are presented in a separate appendix available on our websites. The foreign firm faces a similar optimization problem.

### 3.4 Symmetric Equilibrium

We assume that given the type of good (a or b), the type of consumer (private or public), and the location of the consumer (domestic market or foreign market), initial habit stocks are identical across different varieties. Then, in a symmetric equilibrium, all firms producing varieties of good a for the domestic market will charge the same price. That is, \( P_{i,a,t} = P_{a,t} \) for all \( i \). Similarly, all firms producing varieties of good a for the foreign market will charge the same price, or \( P^*_{i,a,t} = P^*_{a,t} \) for all \( i \). The same symmetry applies to the foreign produced goods (type b), that is, \( P_{i,b,t} = P_{b,t} \) and \( P^*_{i,b,t} = P^*_{b,t} \) for all \( i \). With prices and initial habit shocks across varieties of a given class of goods
the same, equilibrium consumption will be the same across varieties as well, that is, \( c_{i,a,t} = c_{a,t} \) for all \( i \), and similarly, \( g_{i,a,t} = g_{a,t} \), \( c_{i,b,t} = c_{b,t} \), \( g_{i,b,t} = g_{b,t} \), \( c_{i,a,t}^* = c_{a,t}^* \), \( g_{i,a,t}^* = g_{a,t}^* \), \( c_{i,b,t}^* = c_{b,t}^* \), and \( g_{i,g,t}^* = g_{b,t}^* \).

We define two good-specific real exchange rates. One is the relative price of good \( a \) abroad in terms of units of good \( a \) at home, which we denote by \( e_{a,t} \). The second is the relative price of good \( b \) abroad in terms of units of good \( b \) in the home market, denoted \( e_{b,t} \). Formally, the real exchange rates for goods \( a \) and \( b \), respectively, are given by

\[
e_{a,t} = \frac{P_{a,t}^*}{P_{a,t}}
\]

and

\[
e_{b,t} = \frac{P_{b,t}^*}{P_{b,t}}.
\]

Because firms can price discriminate across domestic and foreign markets, good-specific real exchange rates need not be unity. When the real exchange rate for a particular good is different from one, we say that the law of one price for that good is violated.

At a more aggregate level, the real exchange rate, denoted \( e_t \), is defined as the relative price of foreign consumption in terms of domestic consumption, or

\[
e_t = \frac{P_t^*}{P_t},
\]

where \( P_t \) and \( P_t^* \) denote, respectively, the domestic and foreign consumer price indices. In the model economy under study, however, the presence of habit formation at a good-by-good level implies that there is no natural concept of either an aggregate consumption price index or even aggregate consumption. We therefore define the consumption price index as an expenditure weighted average of the price of final goods:

\[
P_t = \gamma P_{a,t} + (1 - \gamma) P_{b,t},
\]

where \( \gamma \) is a fixed weight defined as

\[
\gamma = \frac{P_a(c_a + g_a)}{P_a(c_a + g_a) + P_b(c_b + g_b)};
\]

where variables without a time subscript represent the deterministic steady state value of their time-subscripted counterparts. We adopt a fixed-weight price index to mimic a common practice in developed countries, whereby consumer price indices take the Laspeyres form. We note that our definition of the consumer price index takes an arithmetic mean of prices in the broad categories \( a \) and \( b \). Within each of these two categories, price indices are constructed as geometric means of individual prices. This convention is in line with the construction of the consumer price index in the United States where, since January 1999, a geometric mean formula has been used to average prices within item categories, while an arithmetic mean formula has been used to average prices
across item categories. The consumer price index in the foreign country is defined in a similar fashion:

\[ P^*_t = \gamma P^*_{a,t} + (1 - \gamma) P^*_{b,t}, \]

with

\[ \gamma^* = \frac{P^*_a(c^*_a + g^*_a)}{P^*_a(c^*_a + g^*_a) + P^*_b(c^*_b + g^*_b)}. \]

Letting

\[ \tau_t \equiv \frac{P_{b,t}}{P_{a,t}} \]

denote the inverse of the domestic terms of trade (or the relative price of imported goods in terms of exportable goods), one can express the real exchange rate in terms of the good-specific real exchange rates and the terms of trade:

\[ e_t = \frac{\gamma e_{a,t} + (1 - \gamma) e_{b,t} \tau_t}{\gamma + (1 - \gamma) \tau_t}. \]

We define aggregate domestic consumption as

\[ c_t = \frac{c_{a,t} + \tau_t c_{b,t}}{\gamma + (1 - \gamma) \tau_t}. \]

Similarly, we define foreign aggregate consumption as

\[ c^*_t = \frac{e_{a,t} c^*_{a,t} + e_{b,t} \tau_t c^*_{b,t}}{\gamma e_{a,t} + (1 - \gamma) e_{b,t} \tau_t}. \]

In equilibrium real GDP, denoted \( y_t \), is given by

\[ y_t = h_t. \]

Market clearing for domestically produced goods requires that

\[ y_t = c_{a,t} + g_{a,t} + c^*_{a,t} + g^*_{a,t}. \]

The GDP deflator \( P^*_t \) is defined as the ratio of nominal GDP to real GDP. Nominal GDP is given by \( P_{a,t}(c_{a,t} + g_{a,t}) + P_{a,t}^*(c^*_{a,t} + g^*_{a,t}) \). Then, the GDP deflator is given by

\[ P^*_t = \frac{P_{a,t}(c_{a,t} + g_{a,t}) + P_{a,t}^*(c^*_{a,t} + g^*_{a,t})}{P_{a,t}(c_{a,t} + g_{a,t}) + P_{a,t}^*(c^*_{a,t} + g^*_{a,t})}. \]

The nominal trade balance is the difference between nominal exports, given by \( P_{a,t}^*(c^*_{a,t} + g^*_{a,t}) \), and nominal imports, given by \( P_{b,t}(c_{b,t} + g_{b,t}) \). The trade balance-to-GDP ratio, \( \frac{tb}{y} \), can then be written as

\[ \frac{tb}{y} = \frac{e_{a,t} (c^*_{a,t} + g^*_{a,t}) - \tau_t (c_{b,t} + g_{b,t})}{(c_{a,t} + g_{a,t}) + e_{a,t} (c^*_{a,t} + g^*_{a,t})}. \]

We close the model by assuming that financial markets are complete and that financial cap-
andal the degrees of habit formation in private and public consumption are the same ($\rho$). Consider the special case in which all stocks of habit depreciate completely after one period. An increase in domestic government spending. To simplify the exposition, in this subsection, we discuss the potential of the pricing-to-habits mechanism to predict a depreciation of the real exchange rate and an expansion in private consumption in response to a change in domestic government spending. We refer to this effect as the intertemporal effect of deep habits. It originates from the fact that when demand increases, the relative importance of the price-elastic (or habitual) component of demand falls. In addition, the markup is decreasing in the present discounted value of a future sale, $\Omega_{a,t}$. We refer to this effect as the price-elasticity effect of deep habits. This effect arises because when the present value of a future sale increases, it pays for the firm to invest in market share today by lowering current markups.

The complete set of equilibrium conditions is given in a separate appendix available on our websites. We note that the variables $g_t, y_t, c_t, tby_t,$ and $e_t$ are conceptually consistent with the homonymous variables used in the empirical analysis of section 2.

3.5 How the Pricing-To-Habits Mechanism Works

We now discuss at an intuitive level the potential of the pricing-to-habits mechanism to predict a depreciation of the real exchange rate and an expansion in private consumption in response to an increase in domestic government spending. To simplify the exposition, in this subsection, we consider the special case in which all stocks of habit depreciate completely after one period ($\rho = 0$) and the degrees of habit formation in private and public consumption are the same ($\theta^c = \theta^g = \theta$). In this case, one can show that the equilibrium markup of price over marginal cost charged on varieties of good $a$ in the domestic market, which we denote by $\mu_{a,t} \equiv P_{a,t}/W_t$, must satisfy

$$
\mu_{a,t} = \left[1 - \frac{1}{\eta (1 - \theta d_{a,t-1}/d_{a,t})} + \theta \Omega_{a,t}\right]^{-1},
$$

where $d_{a,t} \equiv c_{a,t} + g_{a,t}$ denotes aggregate domestic demand for good $a$ and $\Omega_{a,t}$ denotes the present discounted value of a sale in the domestic market in period $t + 1$. Note that in the absence of deep habits ($\theta = 0$), the markup is constant and equal to $1/(1 - 1/\eta)$. The above expression shows that under deep habits, the markup falls in response to expansions in domestic aggregate demand for good $a$, that is, when $d_{a,t}$ increases. We refer to this effect as the price-elasticity effect of deep habits. It originates from the fact that when demand increases, the relative importance of the price-elastic (or habitual) component of demand falls. In addition, the markup is decreasing in the present discounted value of a future sale, $\Omega_{a,t}$. We refer to this effect as the intertemporal effect of deep habits. This effect arises because when the present value of a future sale increases, it pays for the firm to invest in market share today by lowering current markups.
In the foreign market for good \( a \), domestic firms charge a markup \( \mu_{a,t}^{*} \) given by

\[
\mu_{a,t}^{*} = \left[ 1 - \frac{1}{\eta \left( 1 - \theta \frac{d_{a,t-1}}{d_{a,t}^{*}} \right) + \theta \Omega_{a,t}^{*}} \right]^{-1}.
\]

Suppose now that domestic government expenditure increases. This shock increases domestic aggregate demand relative to foreign aggregate demand. By the price elasticity effect of deep habits, firms will lower domestic markups relative to foreign markups. That is, good \( a \) will become relatively cheaper in the domestic country than in the foreign country. Similarly, the increase in government spending leads to an increase in domestic demand for good \( b \), inducing foreign firms to lower domestic markups relative to foreign markups. That is the price of good \( b \) falls domestically relative to the rest of the world. The fact that all goods in the domestic economy (\( a \) and \( b \)) become cheaper relative to the foreign economy implies (in the absence of home bias) that the real exchange rate of the country experiencing the increase in government purchases depreciates.

The decline in markups brought about by the expansion in government spending, is key for the deep-habit model to predict an increase in private consumption. To see this, note first that the increase in government spending produces a negative wealth effect on households, which, all other things equal, induces households to reduce consumption and increase labor effort. In turn, the expansion in the labor supply schedule tends to depress real wages. This is the basic mechanism at work in the standard neoclassical model. Under deep habits, however, the decline in markups that take place following the government spending shock acts as a positive productivity shock that shifts the labor demand upward. This expansion in the demand for labor can be strong enough to cause the real wage to increase. In turn, higher real wages produce a substitution effect whereby households increase consumption and reduce labor supply. This substitution effect may be strong enough to offset the negative wealth effect on consumption. In this case, private consumption increases in response to an expansion in government spending.

4 Calibration and Functional Forms

We adopt the following forms for the period utility function and the aggregator functions:

\[
U(x, h) = \frac{\left[ x_{t}^{\phi}(1 - h_{t})^{1 - \phi} \right]^{1 - \sigma} - 1}{1 - \sigma},
\]

\[
\chi(x_{a}, x_{b}) = \left[ \omega x_{a}^{1 - 1/\xi} + (1 - \omega) x_{b}^{1 - 1/\xi} \right]^{1/(1 - 1/\xi)},
\]

and

\[
\chi^{*}(x_{a}^{*}, x_{b}^{*}) = \left[ (1 - \omega)x_{a}^{*1 - 1/\xi} + \omega x_{b}^{*1 - 1/\xi} \right]^{1/(1 - 1/\xi)}.
\]

Table 1 displays the values we assign to the structural parameters in the baseline calibration of
Table 1: Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Subjective discount factor (quarterly)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1</td>
<td>Intertemporal elasticity of substitution</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.15</td>
<td>Preference parameter</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.5</td>
<td>Preference parameter</td>
</tr>
<tr>
<td>$\xi$</td>
<td>1.5</td>
<td>Elasticity of substitution between home and foreign goods</td>
</tr>
<tr>
<td>$\eta$</td>
<td>5</td>
<td>Elasticity of substitution among varieties of habit-adjusted consumption</td>
</tr>
<tr>
<td>$s_g, s^*_g$</td>
<td>0.2</td>
<td>Steady-state share of government consumption in GDP</td>
</tr>
</tbody>
</table>

The time unit is meant to be one quarter. The discount factor $\beta$ is set at a value consistent with an interest rate of 4 percent per year. The curvature of the period utility function, $\sigma$, is set at 1, which implies that preferences are separable in leisure and consumption. The case of separable preferences in consumption and leisure is of particular interest because it highlights the fact that the pricing-to-habits mechanism does not depend on the assumption of nonseparabilities between leisure and consumption to deliver empirically realistic dynamics for consumption and the real exchange rate in response to public consumption shocks. We pick the parameter $\phi$ of the utility function so that households devote about one fourth of their time to paid work in the deterministic steady state. The parameter $\omega$ of the aggregator function of domestic and foreign goods is set to 0.5. This value allows us to abstract from home-bias effects in the transmission of government spending shocks. It implies a relatively high share of imports in GDP of 50 percent. In our sample, the average share of imports in GDP is 22 percent, which would correspond to a value of $\omega$ of 0.7. We discuss later in section 8 the robustness of our findings to increasing the value of $\omega$. We set the elasticity of substitution between home and foreign goods, $\xi$, equal to 1.5, a value commonly used in business-cycle analysis. We set the elasticity of substitution across habit-adjusted consumption of individual varieties, $\eta$, equal to 5. We assume that in the nonstochastic steady state government consumption represents 20 percent of value added, which is the mean value of the observed government share in our sample. The implied steady-state level of government spending, $g = g^*$, is 0.0487. We calibrate the feedback rule for government spending given in equation (11) using the econometric estimates obtained in section 2. Specifically, we assign the following values:

$$
\begin{bmatrix}
B_0^1 \\
B_1^1 \\
B_2^1 \\
B_3^1
\end{bmatrix} =
\begin{bmatrix}
0.656 & -0.234 & 0.0878 & 0.0198 & 0.0138 \\
0.156 & 0.263 & -0.18 & -0.144 & -0.0632 \\
0.134 & -0.0348 & 0.0671 & 0.189 & 0.0421 \\
-0.0385 & 0.0349 & 0.0494 & -0.0632 & -0.0451
\end{bmatrix}.
$$
5 Estimation of the Deep-Habit Parameters

There exists no readily available evidence on the parameters defining the deep-habit mechanism. For this reason, we proceed to estimate them. We simplify the parameter structure by assuming that the degree of habit formation is common across types of goods and countries. That is, we impose $\theta_a^c = \theta_b^c = \theta_a^c^* = \theta_b^c^* = \theta_a^g = \theta_b^g = \theta_a^g^* = \theta_b^g^* = \theta^c$. We place emphasis on not constraining the parameters $\theta^c$ and $\theta^g$ to be equal to each other. In this way, we allow the data to determine the degrees of private and public deep-habit formation separately. In particular, the estimation procedure does not preclude the possibility that habits are present in private consumption but not in public consumption. In addition, we estimate the parameter $\rho$ measuring the persistence in the stock of habits.

Our estimation procedure consists in assigning values for $\theta^c$, $\theta^g$, and $\rho$ to minimize the distance between the estimated impulse response functions shown in figure 1 and the corresponding theoretical impulse response functions implied by the deep-habit model. We approximate the theoretical impulse response functions up to first order using the log-linearization procedure described in Schmitt-Grohé and Uribe (2004). We consider the first 9 quarters of the impulse response functions of 5 variables (government spending, output, consumption, the trade balance-to-GDP ratio, and the real exchange rate) to a unit innovation in government spending. Specifically, let $\Theta \equiv [\theta^c \theta^g \rho]'$ denote the $3 \times 1$ vector of parameters to be estimated, $IR^e$ the $44 \times 1$ vector of estimated impulse response functions, and $IR^m(\Theta)$ the corresponding vector of impulse responses implied by the theoretical model, which is a function of the three parameters we seek to estimate. Then, the estimate of $\Theta$, denoted $\hat{\Theta}$, is given by

$$\hat{\Theta} = \text{argmin}_{\Theta} (IR^e - IR^m(\Theta))' \Sigma_{IR^e}^{-1} (IR^e - IR^m(\Theta)), \quad (12)$$

where $\Sigma_{IR^e}$ is the $44 \times 44$ variance covariance matrix of $IR^e$ computed using the delta method. This matrix penalizes those elements of the estimated impulse response functions associated with large confidence intervals.

An estimate of the variance-covariance matrix of $\hat{\Theta}$, denoted $\Sigma_{\hat{\Theta}}$, is given by

$$\Sigma_{\hat{\Theta}} = \left[ J_{IR^m}(\hat{\Theta})' \Sigma_{IR^e}^{-1} J_{IR^m}(\hat{\Theta}) \right]^{-1},$$

where $J_{IR^m}(\Theta) \equiv \partial IR^m(\Theta)/\partial \Theta$ denotes the $44 \times 3$ Jacobian matrix of the theoretical impulse response function with respect to the vector $\Theta$.

The estimation results are shown in table 2. The estimated degree of deep habit formation in private consumption is 0.52, which lies well within the range of values estimated on the basis of models featuring superficial habit formation. The estimated degree of deep habit persistence in public consumption is slightly higher than its private counterpart at 0.57. The estimated value of $\rho$

---

5 Specifically, we use the matlab program gx_hx.m available on our websites.

6 The impulse response functions implied by the estimated theoretical model shown below are little changed when we define the weighing matrix as the diagonal of $\Sigma_{IR^e}$ rather than as $\Sigma_{IR^e}$ itself.
Table 2: Estimated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Point Estimate</th>
<th>Standard Deviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta^c$</td>
<td>0.52</td>
<td>0.08</td>
<td>Degree of deep-habit formation in private consumption</td>
</tr>
<tr>
<td>$\theta^g$</td>
<td>0.57</td>
<td>0.15</td>
<td>Degree of deep-habit formation in public consumption</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.9876</td>
<td>0.03</td>
<td>Persistence of deep-habit stock</td>
</tr>
</tbody>
</table>

is 0.9876, which implies that the stock of habits depreciates rather slowly over time. This finding is not uncommon in the related literature. For example, consumption-based models of stock returns typically require a high degree of persistence in the habit stock to fit the data (Campbell and Cochrane, 1999). In section 8 we study the sensitivity of our results to lowering the value of $\rho$. All parameters are estimated to be significantly different from zero. Of particular interest is the fact that the data identifies a nonnegligible amount of deep-habit persistence in public consumption.

6 Comparing Predicted and Observed Impulse Responses

Figure 2 plots with a crossed line the impulse responses to a one-percent increase in government spending predicted by the deep-habit model. In addition, the figure reproduces from figure 1 the estimated impulse responses (solid lines) and their associated two-standard-error bands (broken lines). The deep-habit model predicts an expansion in output and private consumption, a deterioration in the trade balance, and a depreciation of the real exchange rate. The model does a relatively good job at explaining the observed transmission of government spending shocks, as the predicted impulse response functions are remarkably close to their estimated counterparts. All predicted responses fall within the estimated error bands, except for the late transition dynamics of the real exchange rate. As is well known, real exchange rate movements are highly persistent, a fact that in our regressions is reflected in a peak response occurring only 10 quarters after the innovation. Explaining such a high level of persistence in the real exchange rate is a challenge for many macroeconomic models including ours.

An important prediction of the deep habit model is that markups move countercyclically in equilibrium. An increase in domestic government spending induces a decline in markups in all domestically sold goods, regardless of whether they are imported or domestically produced. At the same time, in the foreign economy markups increase as a consequence of a contraction in foreign aggregate demand brought about by the negative wealth effect associated with the increase in domestic government spending (and transmitted via complete international asset markets). The impulse responses of the domestic and foreign markups are shown in figure 3.7 In response to a one-percent increase in domestic government spending, markups in domestic markets fall by 26 basis points on impact and markups in foreign markets rise by 7 basis points.

7Because of our maintained assumption of no home bias ($\omega = 1/2$), the impulse response functions of the domestic markups on imported and domestically produced goods are identical. For the same reason, the impulse response functions of foreign markups on goods produced in the domestic and the foreign countries are also identical.
Figure 2: Predicted and Estimated Impulse Responses To A One-Percent Innovation in Government Spending

Note. All responses are expressed in percent deviations from trend with the exception of the trade-balance-to-GDP ratio, which is expressed in level deviations from trend.
Firms selling in domestic markets find it optimal to reduce markups because the increase in aggregate demand stemming from the local public sector renders the demand for individual goods more price elastic. Recall that in the deep habit model, the price elasticity is an increasing function of the importance of current demand relative to habitual demand. The increase in government spending increases the importance of current demand causing a rise in the price elasticity and a corresponding decline in markups. At the same time, the decline in aggregate demand in the foreign country causes a decline in the price elasticity of demand across all markets inducing sellers to increase their margins.

The implied countercyclicality of markups is crucial in allowing the deep-habit model to capture the observed expansion in private consumption and the observed initial depreciation of the real exchange rate. In effect, the combination of lower domestic markups and higher foreign markups makes the domestic economy cheaper relative to the foreign economy. That is, the domestic real exchange rate depreciates. In fact, the real depreciation of about one third of one percent on impact predicted by the model is equal to the sum of the decline in markups in domestic markets (26 basis points) and the increase in markups in foreign markets (7 basis points). To highlight the role of deep habits in accounting for the observed depreciation of the domestic real exchange rate, figure 4 displays the response of the real exchange rate under deep and superficial habits. In the economy with superficial habits, habits form at the level of each composite good (domestic and imported), as opposed to at the level of each individual variety. The figure shows that the deep habit model
captures well the observed initial real exchange rate depreciation. By contrast, the superficial habits model counterfactually predicts that the real exchange rate is completely unaffected by the government spending shock.

To understand why the real exchange rate is unresponsive under superficial habits, note that in the superficial-habit economy the monopolists producing individual varieties of goods face a static demand function with a constant price elasticity. Therefore, equilibrium markups are constant over time and across countries. Furthermore, because the marginal costs of producing a given variety is independent of destination market, the monopolistic producer will charge the same price in the domestic and the foreign markets. Thus, under superficial habits we have that $P_{i,a,t} = P_{i,a,t}^*$ and $P_{i,b,t} = P_{i,b,t}^*$ for all $i \in [0, 1]$. So that, under the maintained assumption of no home bias ($\omega = 0.5$), the domestic and foreign consumer price indices are identical, or, equivalently, the real exchange rate is constant over time. We note that if in the superficial-habit economy one were to allow for home bias, by setting $\omega > 0.5$, then an increase in government purchases would increase the price of good $a$ relative to good $b$ causing a counterfactual appreciation of the real exchange rate.

A major difficulty of the neoclassical growth model is its inability to explain the observed expansion in private consumption following an increase in public spending. Figure 5 illustrates this problem by depicting the impulse response function of consumption to an innovation in government spending in the economy with superficial habits. The counterfactual predicted decline in consumption is driven by a negative wealth effect brought about by the elevated absorption of resources in the public sector.\(^8\) A central contribution of the deep-habit mechanism is to enable an otherwise standard model to overcome this difficulty. In effect, figure 5 shows that the deep-habit

\(^8\)Government spending shocks also have contractionary effects on consumption in the case of no habits at all.
model predicts not only an expansion in consumption but also one that is similar in magnitude and persistence to the one estimated using actual data. As in the model with superficial habits, in the model with deep habits an increase in government spending creates a negative wealth effect, which tends to depress private consumption spending. However, the deep-habit mechanism generates, at the same time, an increase in wages, driven by a generalized decline in markups, which induces households to substitute away from leisure and into consumption. This substitution effect more than offsets the negative wealth effect, resulting in an equilibrium increase in consumption.

The generalized fall in markups that takes place in the domestic economy following a positive innovation in government spending acts much like a positive technology shock, shifting the demand for labor out and to the right. This increase in the demand for labor tends to push real wages upward. Figure 6 shows that the real wage increases by 0.26 percent in response to a one-percent government spending shock. This prediction of the deep-habit model is qualitatively in line with SVAR evidence based on the Fatas and Mihov (2001) and Blanchard and Perotti (2002) identification assumption. See, for example, Perotti (2007).

7 Anticipated Government Spending Shocks

Thus far, we have focused on understanding the effects of government spending shocks identified using a structural VAR approach. The SVAR identification strategy assumes that government spending is unaffected contemporaneously by innovations to other variables. An important branch of the related empirical literature follows a narrative approach to identify innovations in government spending. The narrative approach consists in finding dates at which agents learn about upcoming

The aforementioned empirical literature has established that when identified using the narrative approach, an innovation in government spending fails to cause increases in private consumption and wages. This finding is at odds with the evidence derived from the SVAR approach. The disparities between the narrative and SVAR approaches to identifying government spending shocks have been interpreted in the related literature as suggesting that the narrative approach gives credence to the neoclassical model, whereas the SVAR approach is consistent with models, like the one developed in this paper, that depart from the neoclassical paradigm, chiefly by featuring a countercyclical markup of prices over marginal costs.

We believe that the empirical evidence stemming from the SVAR and narrative approaches begs a different interpretation than the one summarized in the previous paragraph. To see this, it is important to note that the narrative and the SVAR approaches do not identify the same type of government spending shocks. In particular, the narrative approach identifies anticipated government spending shocks, whereas the SVAR approach identifies unanticipated shocks to government spending. In effect, Ramey and Shapiro (1998), Burnside et al. (2004), and Perotti (2007) show that actual government spending starts rising only two to three quarters after the Ramey-Shapiro
dates and reaches a peak six to eight quarters later, suggesting that these dates indeed identify anticipated increases in government spending. By contrast, the Blanchard-Perotti SVAR approach identifies innovations in government spending that are orthogonal to past information, and thus delivers a measure of unanticipated government spending shocks. It follows that, regardless of what model best captures the underlying transmission mechanism of fiscal shocks, in principle, the SVAR and narrative approaches should not be expected to deliver identical impulse responses to government spending shocks, as long as in the true model agents react differently to anticipated and unanticipated government spending shocks.

We illustrate this point by contrasting the impulse responses implied by the deep-habits model to anticipated and unanticipated government spending shocks. Specifically, we consider a simple process for government spending driven by unanticipated shocks as well as by shocks that are anticipated two quarters in advance. Formally,

\[
\ln \left( \frac{g_t}{\bar{g}} \right) = \rho \ln \left( \frac{g_{t-1}}{g} \right) + \epsilon_0^0 + \epsilon_2^2,
\]

where \( \epsilon_0^0 \) is an unanticipated innovation in \( g_t \) and \( \epsilon_2^2 \) is an anticipated innovation in \( g_{t+2} \). Both innovations, \( \epsilon_0^0 \) and \( \epsilon_2^2 \), are assumed to be i.i.d., mutually uncorrelated and mean zero. This process is deliberately simple and meant only for illustrative purposes. Under this process, the peak response of government spending to an anticipated shock coincides with the first period in which actual government spending increases, namely, the second period after the news. As we will see shortly, this feature induces an abrupt jump in endogenous variables at the time the increase in government spending is realized. A more realistic process would feature a gradual increase in government spending that starts two periods after the news and peaks eight to ten periods after the news.

Figure 7 displays the response of government spending, consumption, the real exchange rate, the product wage rate, and the domestic markup to unit innovations in \( \epsilon_0^0 \) and \( \epsilon_2^2 \). Both innovations occur in period 0. The effects associated with the unanticipated shock are displayed with solid lines. They are similar to those shown in figure 2, albeit not identical because they are generated by a much simpler univariate process for government spending. In particular, the impact effect of the unanticipated government spending shock is an expansion in consumption and a depreciation of the real exchange rate. As explained before, the increase in domestic government spending causes a decline in domestic markups relative to foreign markups. As a result, the domestic economy becomes relatively cheaper than the foreign economy, or, equivalently, the real exchange rate depreciates. At the same time, the decline in domestic markups raises domestic wages inducing a substitution away from leisure and toward current consumption.

The impact response to an anticipated increase in government spending is quite different from the one associated with an unanticipated expansion in public consumption. In figure 7, the impulse response to a two-quarter anticipated government spending shock is depicted with broken lines. On impact, private consumption contracts, product wages fall, and the real exchange rate appreciates. The intuition for this prediction of the deep-habit model is as follows. In period zero, agents
Figure 7: Impulse Responses To a Two-Period Anticipated Innovation in Government Spending

Note. All responses are expressed in percent deviations from trend. The horizontal axis measures quarters after the innovation. The parameter $\rho^g$ takes the value 0.87. All other structural parameters are set as in tables 1 and 2.
learn that government spending will increase by one percent in period two. The arrival of this news causes a negative wealth effect that dampens desired private spending. At the same time, firms selling in the domestic market expect high demand and low per-unit profits (markups) two periods later, when the increase in government spending materializes. Therefore, it is optimal for them to disinvest in domestic market share in the current period by raising domestic markups. In the foreign market, markups are expected to increase in the future, which induces firms to lower current margins as a way to increase market share. The combination of higher domestic markups and lower foreign markups results in an appreciated real exchange rate. Also, the increase in domestic markups means that the real wage falls in the domestic economy.

We interpret the unanticipated innovation \( \epsilon_0^t \) to represent the government spending shock identified using the SVAR approach and the anticipated innovation \( \epsilon_2^t \) to represent the government spending shock identified by the narrative approach. Our findings therefore establish that the deep-habit model can reconcile the observed decline in consumption and wages on impact documented in the empirical literature that follows the narrative approach to identification of government spending shocks with the increase in consumption and wages obtained using the SVAR identification scheme.

8 Sensitivity Analysis

In our baseline model, we assume no home bias in consumption. That is, we assume that the parameter \( \omega \) in the aggregator function of domestic and foreign goods (equation (3)) takes the value 0.5. As we indicated earlier, this value of \( \omega \) implies an import share of 50 percent of GDP, which is large relative to the average import share of 22 percent observed in our panel. When \( \omega \) is exactly 0.5, an increase in domestic aggregate demand does not lead to an increase in the relative price of domestically produced goods. That is, the terms of trade, \( P_{a,t}/P_{b,t} \), are unchanged. It follows that movements in the real exchange rate are entirely due to variations in the deviations from the law of one price, via the deep-habit mechanism, and not due to variations in the terms of trade.

We now set \( \omega \) to 0.7, which implies a steady-state import share that is line with its empirical counterpart in our panel. For this value of \( \omega \), agents in both countries have a bias toward goods produced in their own country. In the presence of home bias, an increase in domestic government spending causes an increase in the price of domestically produced goods relative to foreign-produced goods. That is both \( P_{a,t}/P_{b,t} \) and \( P_{a,t}^*/P_{b,t}^* \) go up, which means that the terms of trade improve for the domestic country and worsen for the foreign country. Because goods of type \( a \) have a larger share in the domestic CPI index than in the foreign CPI index, the improvement of the domestic terms of trade tends, all other things equal, to appreciate the real exchange rate. The response of the real exchange rate to an increase in aggregate demand is then determined by two (opposing) effects, the terms-of-trade effect, which tends to appreciate the real exchange rate and the pricing-to-habits effect, which tends to depreciate it. The upper panel of figure 8 compares the response of the real exchange rate to a positive government spending shock in economies with and without
Figure 8: Sensitivity Analysis: Home Bias and Less Persistent Habit Stock

Note. All responses are expressed in percent deviations from trend.
home bias. In the economy with home bias, all parameters other than $\omega$ take the values shown in tables 1 and 2. Overall, the two theoretical impulse responses for the real exchange rate are fairly similar. In line with the intuition developed above, when home bias is present, the impulse response function of the real exchange rate lies below the one corresponding to the baseline case without home bias.

The lower panel of the figure compares the impulse response of consumption in an economy with and without home bias. The deep-habit model with home bias continues to predict a persistent rise in consumption that tracks the actual response fairly well.

Our estimation of the pricing-to-habits model yields a highly persistent stock of habits with an autoregressive coefficient, given by the parameter $\rho$, of 0.9876. To gauge the sensitivity of our results to a less persistent stock of habits, we now consider the case that $\rho = 0.85$. This value is more than four standard deviations below its point estimate. All other parameters take the values shown in tables 1 and 2. Figure 8 displays with diamonds the impulse responses of the real exchange rate and consumption for this value of $\rho$. As one would expect, the impulse responses of the real exchange rate and consumption are less persistent when the stock of habits itself is less persistent.

Our third sensitivity experiment concerns the detrending method used to compute the empirical impulse responses to a government spending shock. In the baseline case all variables are detrended using a quadratic trend. Here we replace this detrending method with the Hodrick-Prescott filter. Figure 9 shows the empirical impulse response functions obtained after HP filtering the data. Comparing this figure with figure 1, one can see that the empirical impulse responses obtained from HP filtered data are quite similar to those obtained after removing a quadratic trend from the raw data. In particular, a positive innovation in government spending causes an increase in output and consumption, a depreciation of the real exchange rate, and a deterioration of the trade-balance-to-output ratio.

Figure 9 also depicts the impulse responses predicted by the theoretical model, where the structural parameters of the deep-habit mechanism were reestimated to match the impulse responses associated with the HP-filtered data. Inspection of the figure suggests, that the fit of the theoretical model does not appear to be sensitive to whether the empirical impulse responses were estimated from HP filtered or from quadratically detrended data.

9 Conclusion

In this paper, we use data from a panel of four industrialized countries from 1975 to 2005 to identify the effects of government spending shocks on output, consumption, the real exchange rate, and the trade balance. We find that an increase in government spending produces an expansion in output, an expansion in consumption, a deterioration of the trade balance, and a depreciation of the real exchange rate.

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9The resulting point estimates of $\theta^c$, $\theta^p$, and $\rho$ are, respectively, 0.56, 0.48, and 0.99.
Note. All responses are expressed in percent deviations from trend with the exception of the trade-balance-to-GDP ratio, which is expressed in level deviations from trend.
A central contribution of our investigation is to propose and test the hypothesis that deep habits generates a transmission mechanism for government purchases shocks that is consistent with this empirical evidence. The key feature of the transmission channel invoked by deep habits is countercyclical movements in equilibrium markups of prices over marginal costs. In our model, an increase in government spending generates a generalized decline in markups in domestic markets and an increase in markups in foreign markets. Thus, the domestic economy becomes inexpensive relative to the foreign economy, or the real exchange rate depreciates. At the same time, the decline in domestic markups shifts the demand for labor outward pushing real wages up. In turn, the increase in labor remunerations induces households to sacrifice leisure in favor of consumption. In the estimated deep-habit model, this substitution effect dominates the negative wealth effect stemming from the increase in public absorption of resources. As a result private consumption increases in equilibrium.

We estimate the structural parameters defining the deep-habit mechanism and find strong evidence in favor of habit formation at a good-by-good level both in private and public consumption. The predictions of the deep-habit model replicate well the estimated impulse responses of output, consumption, the trade balance, and the real exchange rate. We interpret these results as a step forward in understanding the effect of fiscal policy in the open economy.

Furthermore, we investigate the effect of anticipated increases in government spending from the perspective of the deep habit model. We find that consumption and wages fail to increase upon the release of news about future expansions in public spending. We interpret this result as consistent with the empirical evidence emerging from the narrative approach to identifying government spending shocks. In this way, we establish that the predictions of the deep-habit model can reconcile the findings of the SVAR and narrative literatures on the consequences of government spending shocks on consumption and wages.

We close by noting than in the present study we deliberately abstract from a number of theoretical features that are clearly important for understanding the international transmission of aggregate shocks. In particular, we leave out nontradable goods, capital accumulation, distribution costs, asset-market incompleteness, nominal rigidities, and nonseparabilities between consumption and leisure. It would be of interest to investigate how these features interact with the deep-habit mechanism in shaping the transmission of fiscal shocks.
References


