Technological Waves in the Stock Market

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In this paper we analyze a stochastic growth model with lags in the operation of new technologies. The model generates long fluctuations in the aggregate value of firms. Technological innovations arrive exogenously to the economy and impact the value of stocks – even though these innovations cannot be readily implemented. Then local adopters produce new varieties of intermediate goods embodying these new technologies. Episodes of technology adoption may translate into steady increases in the aggregate value of stocks. We fit the model to replicate some basic regularities in economic growth and business fluctuations, and we evaluate its ability to explain long fluctuations in the US stock market.

KEY WORDS: Technological innovation, technology adoption, business fluctuations, stock market.

1 Introduction

In this paper we study a stochastic growth model with lags in the operation of new technologies. Technological innovations arrive exogenously to the economy and impact stocks through their option values. These innovations, however, cannot be readily implemented and undergo a process of adoption that involves the production of new varieties of intermediate goods. Our model is a simplified variant of those in Romer (1990) and Comin and Gertler (2006), but our objectives are quite different. Romer (1990) is concerned with innovations and economic growth and Comin and Gertler (2006) with a quantitative analysis of economic fluctuations. Our main goal is to build a quantitative framework in order to explore the possible channels of influence that technological innovations have on stock prices.

As is commonly realized, financial markets can respond to various types of shocks and new information. Besides the high-frequency volatility that is deeply rooted in stock prices, the US stock market displays protracted episodes of high growth intertwined with shorter spells of price decreases and much lower returns. Jovanovic and Rousseau (2001) associate these long fluctuations in the stock market with three technological revolutions: Electricity, World War II, and IT. These authors document long lags in the operation and diffusion of new technologies. There are, however, plenty of other explanations for the long swings in stock values. Geanakoplos, Magill and Quinzii (2004) contend that these changes are driven by demographic trends, whilst Lustig and Nieuwerburgh (2006) cite credit access from home equity collateral. Our model is also intended to explain the overall evolution of the US stock market.

In contrast, several recent works have focused on the collapse of stocks in the 70s and subsequent periods of recovery. A large body of research [Greenwood and Jovanovic (2001), Hobijn and Jovanovic (2001), Laitner and Stolyarov (2003) and Peralta–Alva (2006)] elaborates on the effects of IT on the values of old and new companies. Mcgrattan and Prescott (2005) attribute fluctuations in stock values to changes in the tax system, whilst Hall (2001) attributes them to intangible investments. We will delve into this literature in Section 2 after discussing some empirical evidence. Of course, it will be instructive to check how these explanations for the behavior of stocks in the 70s may fare in some other time periods.

An important consideration in our model is that asset prices incorporate the option value of technological innovations that have yet to be implemented. Hence, the value of a firm may differ from the book value of its durable factors of production. This disconnection of the stock price from the replacement cost of capital is attained in our model without resorting to commonly used frictions such as borrowing constraints, irreversibility of investment, and adjustment costs. The arrival of new technologies creates new possibilities for existing and potential firms –even though these technologies cannot yet be used. Then, there is a process of wealth creation in which local adopters produce extra varieties of intermediate goods embodying the new technologies. Hence, in our model a long swing in the stock market occurs after a lasting process of technology adoption. This propagation mechanism is likewise present in the partial equilibrium setting of Abel and Eberly (2005), and in the tree economy of Panageas and Yu (2006). None of these papers incorporates explicitly an aggregate production sector.

At a later stage we report some quantitative exercises. The model is solved numerically by a high-order approximation method that picks non-linearities in the evolution of stock values. We assign parameter values which allow the model to fit some basic facts in economic growth and business fluctuations. Most of the discussion will center on those parameters defining the processes of arrival and adoption of new technologies. These parameters are calibrated to replicate the volatility of trademarks registered. Then we perform several numerical exercises to see how technological innovations may affect the dynamics of stock prices and other aggregate variables. The model can account for long-term fluctuations in stock prices although it does not display the short-run volatility firmly established in the data. Hence, technological innovations seem a plausible explanation for long moves in stocks, but other types of factors (i.e., monetary and fiscal policies, international trade, etc.) seem necessary to generate their observed short-term volatility.

The paper will proceed as follows. In Section 2 we gather some facts on the stock market and related macro aggregates. This evidence is the starting point of our analysis, and it becomes handy for a a brief discussion of related contributions. In Section 3 we lay out our model of technology adoption and derive some qualitative properties of the solution with emphasis on a fundamental asset pricing equation that considers the change in the stock price brought about by a technological innovation. Section 4 is devoted to the computation and calibration of the model, and Section 5 reports various numerical experiments. We conclude in Section 6 with a further evaluation of our results.

2 Some Empirical Evidence

For simplicity of the presentation we focus on the Standard and Poors (S&P) composite stock price index although similar remarks may extend to related stock values. Figure 1 plots the evolution of the S&P and the corresponding price-earnings (PE) ratio for the period 1881–2005. Both time series have been detrended by taking out our best fit for a deterministic exponential trend. As we can see both series display similar long-term cyclical behavior.

Indeed, it is well known that stock prices are a main driving force in these trends. These patterns are confirmed in Figure 2 that portrays a centered ten-year moving average of S&P yearly returns. We observe that peak values occur in 1980, 1900, 1925, 1955 and 1995. Therefore, the amplitude of these long-term cycles can be up to 40 years.

To explain these long fluctuations what first comes to mind is the behavior of investment and the stock of capital. But as shown in Figure 3 both investment and the book value of capital remain quite flat over the sample period. Consequently, theories along the lines of the neoclassical growth model are doomed to failure because in this model the stock of capital equals its book value. Extended versions of this model that allow for valuations of capital across vintages seem to deliver marginal improvements. To pick up these vintage effects, most recent literature uses the Gordon index [Greenwood, Hercowitz and Krusell (1997)] but this index does not display enough variability (see Figure 4). Moreover, Figure 3 leaves limited scope for other extensions incorporating market frictions such as borrowing constraints, irreversibility of investment and adjustment costs. These effects would have to be implausibly high. In a stimulating paper, McGrattan and Prescott (2005) attribute the observed discrepancies between market and book values to changes in taxes and regulations. As explained by these authors, taxes directly affect asset pricing equations. The last few decades have witnessed major changes in taxes on corporate distributions and corporate income, and in subsidies to capital. Moreover, McGrattan and Prescott (2005) find no evidence that companies may have changed their distribution policies to hedge against these tax changes. Bian (2006) argues that the aggregate behavior of taxes and regulations can account for the average level of the stock market during some time periods but fails in others. Following Hall (2001), he incorporates intangible investments, but to simulate observed market values in Bian‘s model the stock of intangible capital needs to fluctuate widely and become negative in some periods. Hence, it appears that current models incorporating tax changes and intangible investments fail to account for the existing long-term variability in stock markets.

Another thought–provoking explanation of the behavior of stock markets is encountered in Geanakoplos, Magill and Quinzii (2004). They present a simple model where changes in the valuation of stocks are driven by fluctuations in population trends. In their model the value of the stock market is determined by the ratio of middle aged people to young people

(MY). Because of the different saving habits of these two groups the stock market would be high when the interest rate is low and vice versa. However, this mechanism seems at odds with the data since interest rates comove with stock values (Figure 5). Geanakopplos, Magill and Quinzii (2004) introduce DMY, which is MY in first differences – over nearby time periods. As illustrated in Figure 6 the correlation of the S&P and DMY is striking – especially for the second part of the period. But variable DMY is introduced in their empirical work and is not clearly justified by their basic model. In a more recent study, Lustig and Nieuwerburgh (2006) find that the long term-variability of the stock market can be explained by the evolution of housing values as housing can serve as collateral for some types of borrowing. As with the above papers, this channel is again worth exploring – especially for those countries with wide access to home equity loans. One major objection to this analysis is the direction of causality: Interest rates and stock and housing values should all be endogenously determined. Hence, a more refined theory would integrate all these variables as part of the same explanation.

We close this section with some further evidence in support of our theory regarding technology adoption. First, as mentioned above long lags in the operation of new technologies are documented in Jovanovic and Rousseau (2001). Figure 7 attests that stock prices peaked as electricity use became widespread. Similar patterns emerge for IT when we graph stock prices and computer use intensity (Figure 8). Second, at the micro level stock prices covary with the number of patents granted (Figure 9) and trademarks registered (Figure 10). Even though patents and trademarks may be affected by strategic decisions and many other determinants, these strong correlations lend credibility to the idea that technological innovations are behind stock values. And third, at the macro level as shown in Figure 5 above short-term interest rates comove with stock values. This clearly suggest that investment shocks are positively correlated with stocks. Moreover, Figure 11 illustrates that peaks in the stock market are a leading indicator of increases in labor productivity.

Perhaps, a more informative empirical regularity is the behavior of stocks for incumbents and newcomers upon the arrival of a technological innovation. Figure 12 decomposes the market capitalization relative to GNP into the values of four different groups of companies: (i) the incumbents, (ii) companies originating during 1982-1989, (iii) companies originating during 1990-1994, and (iv) companies originating after 1994. Again, even though we do not deny that technological innovations may impact stocks in some other ways, this illustration makes clear that most value added to stocks is generated by new corporations. These newcomers would correspond in our model to local technology adopters.

There are several recent models of technology adoption [Boldrin and Levine (2001), Jovanovic and Hojbin (2001), Jovanovic and Rousseau (2001), Laitner and Stolyarov (2004) and Peralta-Alva (2006)]. Actually, these studies were conceived to explain the depressed stock values of the 70s, and correspondingly they generate Tobin‘s q below one. Therefore, these models do not seem appropriate to explain long ups and downs in stock values.

3 The Model

As was already pointed out, we consider a simplified analytical framework that borrows several elements from Romer (1990) and Comin and Gertler (2006). There is but one representative household in the economy. At every time $t = 0, 1, \cdots$, this agent holds assets, demands the consumption good, and supplies labor. Asset prices are determined by the flow of dividends as discounted by the marginal rates of substitution of the representative consumer. There are no financial frictions. Hence, the existence of a unique representative agent will yield an equilibrium solution that can be replicated by a complete set of securities. The final consumption good is produced by a single firm with a constant returns to scale technology. Three inputs are involved in the production of this final commodity: Capital accumulated by the firm, labor, and a composite intermediate good. Both the firm and the consumer act competitively in all markets. The sector of intermediate goods, however, is composed of a continuum of monopolistic producers. The production process has been simplified so that each monopolistic producer transforms the final good into a differentiated intermediate product. After the arrival of a new set of technologies, local adopters can expand the available variety of intermediate goods increasing both asset wealth and factor productivity. The remaining source of increasing factor productivity is a shock to the production function of the final good.

3.1 The household

The representative household has preferences over the consumption good and desutility of work. The agent can trade shares a_t of an aggregate stock to transfer income cross states. This aggregate stock yields a dividend d_t at every time period. Preferences are represented by the discounted objective:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\ln(c_t) - \frac{l_t^{1+\chi}}{1+\chi} \right] \right\} \tag{1}$$

with $0 < \beta < 1$ and $\chi > 0$. For given initial asset holdings $\hat{a}$, the optimization problem faced by this agent is to choose a stochastic sequence of consumption, labor, and shares of the aggregate stock $\{c_t, l_t, a_t\}_{t \geq 0}$ that maximizes the objective in (1) subject to the sequence of budget constraints

$$c_t + q_t a_t = \omega_t l_t + (q_t + d_t) a_{t-1} \tag{2}$$

with $a_t \geq 0$. Here, q_t denotes the market price per share and ω_t denotes the unitary wage. Note that for our representative agent economy the assumption of non-negative holdings of the aggregate stock $a_t \geq 0$ entails no restriction of generality.

3.2 The production sector

The firm producing the final good accumulates capital and buys labor and intermediate goods to maximize the discounted sum of net revenues. Total factor productivity of the firm is stochastic, and represented by a random variable θ_t. At every date there is a mass of A_t intermediate goods that enter into the production of the final good. These intermediate goods are bundled together in a composite good M_t defined by a CES technology, $M_t = [\int_0^{A_t} m_t(s)^{\frac{1}{\vartheta}} ds]^{\vartheta}$ where $m(s)$ denotes the amount of intermediate good s bought by the firm and $\vartheta > 1$.

The firm chooses stochastic sequences of investment, labor, and intermediate goods $\{i_t,\ l_t,\ m_t(s)_{s \in (0, A_t)}\}_{t \geq 0}$ so as to solve the optimization program:

$$\max_{\{i_t, l_t, m_t(s)\}_{t\geq 0,\ s\in[0,A_t]}} : E_0 \left\{ \sum_{t=0}^{\infty} \eta_t d_t^f \right\}$$

$$\text{s.t.} \qquad d_t^f \equiv \theta_t \left(k_t^{\alpha} l_t^{1-\alpha}\right)^{1-\gamma} M_t^{\gamma} - i_t - \omega_t l_t - \int_0^{A_t} p(s)m(s)ds \tag{3}$$

$$k_{t+1} = k_t (1-\delta) + i_t \tag{4}$$

$$M_t = [\int_0^{A_t} m_t(s)^{\frac{1}{\vartheta}} ds]^{\vartheta} \tag{5}$$

$$\ln(\theta_t) = \psi_\theta \ln(\theta_{t-1}) + \varepsilon_t^{\theta},\ \varepsilon_t^{\theta} \sim \mathrm{N}(0, \sigma_\theta) \tag{6}$$

$$k_0 > 0 \text{ given},\ (\alpha, \gamma) \in (0,1),\ \vartheta > 1 \tag{7}$$

where η_t is a state price converting income of period t to period 0, and $p(s)$ denotes the price of intermediate good s. Note that the firm producing the final good takes all prices as given. In the intermediate good sector, monopolistic competition prevails. There is a set of monopolistic competitors i indexed over the unit interval $[0,1]$ who have acquired the right to produce the intermediate goods through the adoption process that we describe later. Producer of the intermediate good s, for $s \in [0, A_t]$, selects the optimal quantity $m^i(s)$ considering the inverse demand function of the good – taken as given prices and quantities set up by all other producers of intermediate goods. We denote as $m(s) = \int_0^1 m^i(s)di$ and we assume that a technology s can only be adopted by one agent. The production of the intermediate good is quite simple: One unit of good s requires only one unit of the final good. Thus, the profit obtained at each period by the producer of variety s is given by

$$\pi_t^i(s) \equiv \max_{m_t^i(s)} \left\{ p_t(s) m_t^i(s) - m_t^i(s) \right\} \tag{8}$$

One can easily check that the optimal price is given by $p_t(s) = \left(\frac{m_t^i(s)}{M_t}\right)^{\frac{1-\vartheta}{\vartheta}} p_t$ for $p_t = \left(\int\limits_0^{A_t} p_t(s)^{\frac{1}{1-\vartheta}} ds\right)^{1-\vartheta}$ and $M_t = [\int_0^{A_t} m_t(s)^{\frac{1}{\vartheta}} ds]^{\vartheta}$.

Every intermediate product s may disappear from the market. Let ϕ be the probability of survival of a technology. Then, the value $V_t^i(s)$ of operating a technology from the beginning of time t is the expected present value of profits:

$$V_t^i(s) = E_t \left\{ \sum_{r=t}^{\infty} \frac{\eta_r}{\eta_t} \phi^{r-t} \pi_r^i(s) \right\} \tag{9}$$

3.3 Technology adoption

Technological innovations arrive exogenously to the economy. The total stock of technological innovations Z_t evolves according to the law of motion

$$Z_t = \phi Z_{t-1} + \mu x_t \tag{10}$$

with

$$\ln x_t = \varphi_x \ln x_{t-1} + \varepsilon_t^x,\ \varphi_x \in (0,1),\ \varepsilon_t^x \sim N(0, \sigma_x) \tag{11}$$

where $\mu > 0$.

These technological innovations are put into use by local adopters. We assume that the set of adopters is fixed to capture the idea that adoption skills may be hard to acquire immediately. Hence, adopters may get positive rents. Each agent i can adopt successfully a new technology with probability $\lambda_t(H_t^i)$. This probability is increasing in the amount of resources H_t^i devoted to adoption. The set of technologies adopted by agent i, A_t^i is determined by

$$A_{t+1}^i = \lambda_t(H_t^i)\phi \left[Z_t - A_t\right] + \phi A_t^i \tag{12}$$

Agent i fixes the optimal amount of expenditure H_t^i from the probability of success $\lambda_t(H_t^i)$ and the value of an adopted technology $V_t^i(s)$. Without loss of generality we suppose that $V_t^i(s)$ is the same for all s. Then, optimality requires that the following Bellman equation must be satisfied between the option value of a technology J_t^i and its value once adopted V_t^i,

$$J_t^i = \max_{H_t^i} \left\{ -H_t^i + \phi E_t \left[\frac{\eta_{t+1}}{\eta_t} \left(\lambda_t V_{t+1}^i + (1 - \lambda_t) J_{t+1}^i \right) \right] \right\} \tag{13}$$

Note that J_t^i can be computed recursively by the method of successive approximations.

From these individual values we get the average amount of adopted technologies $A_t = \int_0^1 A_t^i di$, the average value $V_t(s) = \int_0^1 V_t^i(s) di$ of the right to produce good s and the average option value $J_t(s) = \int_0^1 J_t^i(s) di$. Our analysis below will focus on symmetric solutions.

3.4 Equilibrium and Asset Prices

In our model the exogenous state variables are the stock of invented technologies Z_t and the value of total factor productivity θ_t. The remaining variables are determined endogenously as solutions to the model by the above optimization problems and the market clearing conditions. We gather these equations for the computation of an equilibrium solution.

$$Y_t = \theta_t \left(k_t^{\alpha} l_t^{1-\alpha}\right)^{1-\gamma} M_t^{\gamma} \tag{14}$$

$$\ln(\theta_t) = g_\theta + \psi_\theta \ln(\theta_{t-1}) + \varepsilon_t^{\theta} \tag{15}$$

$$k_{t+1} = k_t (1-\delta) + i_t \tag{16}$$

$$Y_t^N = Y_t - A_t^{1-\vartheta} M_t \tag{17}$$

$$Y_t^N = c_t + i_t + H_t (Z_t - A_t) \tag{18}$$

$$(1-\alpha)(1-\gamma)\frac{Y_t}{l_t} = l_t^{\chi} c_t \tag{19}$$

$$1 = E_t \left\{ \frac{\eta_{t+1}}{\eta_t} \left(\frac{d_{t+1} + q_{t+1}}{q_t} \right) \right\} \tag{20}$$

$$1 = E_t \left\{ \frac{\eta_{t+1}}{\eta_t} \left[(1-\gamma)\,\alpha \frac{Y_{t+1}}{k_{t+1}} + 1 - \delta \right] \right\} \tag{21}$$

$$Z_t = \phi Z_{t-1} + \mu x_t \tag{22}$$

$$\ln x_t = \varphi_x \ln x_{t-1} + \varepsilon_t^{x} \tag{23}$$

$$A_{t+1} = \lambda_t \phi \left[Z_t - A_t\right] + \phi A_t \tag{24}$$

$$V_t = \pi_t + E_t \left\{ \frac{\eta_{t+1}}{\eta_t} \phi V_{t+1} \right\} \tag{25}$$

$$J_t = \max_{H_t^i} \left\{ -H_t + \phi E_t \left[\frac{\eta_{t+1}}{\eta_t} \left(\lambda_t V_{t+1} + (1 - \lambda_t) J_{t+1} \right) \right] \right\} \tag{26}$$

$$1 = \rho \frac{\lambda_t}{H_t} \phi E_t \left\{ \frac{\eta_{t+1}}{\eta_t} \left(V_{t+1} - J_{t+1} \right) \right\} \tag{27}$$

Equations (13)-(17) are feasibility and market clearing conditions. Note that Y_t^N in (16) is our measure of output or value added, which is equal to the difference between the final goods firm production and the cost of producing intermediate inputs. Equation (17) reflects that output is devoted to consumption, investment in capital, and expenditures in adopting new technologies. Equations (18)-(20) are first order conditions that must be satisfied by labor and assets for the consumer and by physical capital investment for the firm. Then the last set of equations refer to the exogenous law of motion for technological innovations and the optimal process of technology adoption. Note that the value of a technology is defined from (7) and (8).

In these equilibria we assume that $a_t = 1$ so that q_t corresponds to the value of the stock market. Note that our measure of stock is comprehensive and includes both the value of the firm for producing the final product and the proceeds from technology adoption. The following proposition is central to our study. It decomposes the value of the stock market into the value of capital and existing adopted technologies plus the value of current and future technological innovations.

Proposition 3.1 *In a symmetric equilibrium the market value of stocks is*

$$q_t = k_{t+1} + V_t^+ A_t + J_t^+ (Z_t - A_t) + \xi_t \tag{28}$$

where $V_t^+ \equiv V_t - \pi_t,\ J_t^+ \equiv J_t + H_t,$ *and* $\xi_t \equiv E_t \left\{ \sum_{r=t+1}^{\infty} \frac{\eta_r}{\eta_t} J_r \left(Z_r - \phi Z_{r-1} \right) \right\}.$

Hence, the value of the stock market is given by the sum of four components: the replacement cost of installed capital, the value of adopted technologies, the option value of inventions currently available but not yet implemented, and the present value of future inventions expected to happen. Hence, the value of the stock market is not limited to the book value of capital and adopted technologies, and incorporates the values attached to the the arrival and diffusion of new technologies. This latter components are further sources of volatility in the stock market over the fluctuations in the stock of capital and the value of adopted technologies.

4 Calibration and Computation of the Model

We cannot compute the equilibrium from an optimal planning problem, since producers in the sector of intermediate goods behave non-competitively. Hence, the equilibrium is computed numerically from the system of equations (13)–(26). These equations are solved using a low degree perturbation method.

The parameters of the model are calibrated to match some statistics of medium–term fluctuations observed in the data and the volatility of trademarks registered. The medium–term cycles are defined following Comin and Gertler (2006) as those within a band of frequencies from 2 to 50 years. The filter used to detrend the data is the one recommended by Christiano and Fitzgerald (2003).

Following the RBC literature we choose values for the set of parameters (β, α, δ) such that the deterministic steady state coincides with the average behavior of per–capita U.S. time series[1]. The parameter β is fixed at 0.96, leading to an annual interest rate of 4%. As is usual, we set $\alpha = 0.3$ based on evidence of the average share of labor costs in total costs. Substituting the average investment to capital ratio into (15), implies a depreciation rate δ of 0.098 per year.

The share of materials in gross output γ is assumed to be 0.5 which is consistent with empirical estimates for the manufacturing sector [see Jaimovich (2006)]. We set the inverse of the Frisch elasticity of labor supply χ to 0.5. The intermediate producers' gross markup ϑ, is fixed at 1.4. This is an intermediate value for the range of estimates of this parameter (Rotemberg and Woodford, (1995) provide an overview of microeconomic evidence). We set the value of ϕ to 0.97, ρ to 0.95, and λ to 0.1 following Comin and Gertler (2006). This

[1] We consider output, hours, labor productivity and TFP for the non-farm business sector. The source is the Bureau of Labor Statistics (BLS). Consumption is measured as the sum of non–durables and services and investment is non–residential. Both series are obtained from the Bureau of Economic Analysis (BEA). Each variable is transformed in per–capita terms by dividing it by the population aged 15 to 64. The data are annual and span the period 1948 to 2004.

value for λ implies an average adoption time of ten years. We assume that the probability of adoption obeys

$$\lambda_t(H_t) = \Lambda \left(\frac{A_t}{k_t} H_t \right)^{\rho} \tag{29}$$

with $\Lambda > 0$ and $\rho \in (0, 1)$.

We consider as a proxy for the number of adopted technologies A_t the stock of trademarks registered[2]. The parameter values for the stochastic processes for TFP (6), and the stock of innovations (9)–(10), are selected in order to approximate the variance, correlation, and first–order autocorrelation of TFP and trademarks over medium–term cycles in the data.

5 Quantitative Results

This section contains several numerical experiments. We first compute impulse-response functions for various macro variables after a shock in TFP or the arrival of technology innovation. In a second set of experiments we compare second-order moments generated by the calibrated model with those of the data. The final set of experiments is concerned with the ability of the model to replicate the behavior of the price-dividends ratio (PD) as a regressor for future returns.

Figure 13 exhibits the percentage deviation from the steady state (SS) for the different components of the value of the stock market. The shock considered is one standard deviation in the exogenous process θ_t. This shock generates an initial positive deviation from the SS and a posterior convergence to the initial state for all variables. It is worth noting that an increase in TFP fosters consumption and raises interest rates. But in spite of these increases, investment and all the stock market values go up. In contrast, we can see in Figure 14 that the response to a one standard deviation in the stochastic process lnx_t differs between the variables considered. The stock of capital and the value of installed and not–installed technologies, exhibit an initial negative reaction to the shock, and later on converge to the SS. The arrival of new technologies generates an initial negative effect on the vale of installed

[2]The series considered is the number of registered trademarks obtained from U.S. Patent and Trademark Office for 1970–2004, and from Historical Statistics of the United States series W–107 for 1948 to 1970. We compute the stock of trademarks through equation (23) assuming that the number of new adopted technologies coincides with the number of trademarks registered.

and not–installed technologies because on the one hand it increases expected growth rates of consumption and, as a consequence, implies a reduction in the discount factor. On the other hand, the presence in the future of a larger amount of installed technologies, implies that the profit generated by each one is going to represent a smaller fraction of gross output. But the subsequent increase in productivity generated by new installed technologies offset these negative effects. Given the persistence of the stochastic process lnx_t, the shock increases the expected growth in the arrival of new technologies and accordingly increases the value of ξ_t. As the value of growth options increases strongly, the aggregate value of the stock market presents a small initial negative response to the shock.

Next we describe the results obtained in different simulation experiments. Table 1 exhibits the autocorrelation of the simulated series and the data. (As noted above both series include fluctuations between 2 and 50 years.) As we can see the simulated moments are close to the data. Table 1 also includes standard deviations for various macro variables and the value of the stock market. The major discrepancy is the standard deviation of the stock market where the model yields a standard deviation of 24.25 as opposed to a standard deviation of 31.41 given by the data. There are also noticeable differences in the standard deviations of consumption, investment and hours generated by the model and data, but these differences are probably due to our rather oversimplified modelling of these sectors. Hence, we may conclude that our model generates a sizable volatility in the stock market, and matches rather well the variability of standard macro variables. A further battery of checks is presented in Table 2 that present contemporaneous correlation of output and the stock market with several macro variables. The model reproduces reasonably well the correlation of output, consumption, and investment with the stock market. But it fails in the correlations of hours, TFP and labor productivity. As already has been discussed, there could be important frictions in the labor market which are not integrated in our exercise. Our main interest is to consider a rather simplified model and focus on the behavior of the stock market.

Our final simulation exercise analyzes the behavior of the ratio between the value of the stock market and dividends as a regressor for future returns. Different authors have provided evidence regarding the predictive ability of this ratio [e.g., Campbell and Shiller (1988), Fama and French (1989)]. Given the smoothness of the ratio, there exist no consensus about the statistical significance of this evidence [see Cochrane (2006), for a discussion]. Through simulations we analyze the ability of the model to replicate the empirical regressions. We simulate 1000 series which have the same length as the sample considered (1872–2005), and we run predictive regressions for the stock market return using the price–dividends ratio (PD) as regressor. In Table 3 we present the mean and standard deviations across replications of the regression coefficient and coefficient of determination for different forecasting horizons. For all the horizons considered, the regression coefficient has the right sign. Moreover, the empirical coefficient belongs to the 95% distribution interval generated by the model. The empirical coefficients of determination are lower than the simulated counterparts but belongs to the 95% distribution interval. The PD ratio in the model presents low frequency fluctuations mainly driven by shocks to the arrival of new technologies (lnx_t). The value of the stock market responds positively and dividends respond negatively to shocks in the arrival process. Therefore, during a period of high arrival rate the PD ratio and the value of stocks are above its steady state. As a consequence, the return forecastability results obtained implies that the ratio is capturing the future evolution of the stock price and, as the arrival of new technologies is smooth, the predictability is stronger for longer horizons.

6 Concluding Remarks

In this paper we present a model of technology adoption with a view to analyze the impact of technological innovations on the stock market. The value of the stock market incorporates the option value of the arrival and adoption of future technologies, and hence is it not only determined by the book value of capital. The model can generate long waves in the stock market driven by the processes of technological innovation and adoption.

For the calibration of the model we assigned parameter values to replicate the volatility of the Solow residual; moreover, the process of innovation and adoption of new technologies was required to match the volatility of trademarks. Under these restrictions imposed in our numerical experiments the model was able to generate a sizable volatility in the stock market. Hence, technological innovations seem a major factor to explain the long-term volatility observed in stock markets. Our model failed in some other directions such as in the volatility of hours and investment. To improve along these directions, we need a better modelling of the labor market and investment. Those refinements, however, should not hamper the ability of the model to replicate the volatility of the stock market.

7 Appendix

Proof: The first order condition (19) and the transversality condition $\lim_{T\to\infty} E_t \left\{ \frac{\eta_T}{\eta_t} q_T a_T \right\} = 0$ for the household's problem, imply

$$q_t = E_t \left\{ \sum_{r=t+1}^{\infty} \frac{\eta_r}{\eta_t} d_r \right\} \tag{30}$$

Through the first order condition (20) and the transversality condition $\lim_{T\to\infty} E_t \left\{ \frac{\eta_T}{\eta_t} k_{T+1} \right\} = 0$ for the problem of the final goods firm, and the law of motion for capital (15), we arrive at

$$k_{T+1} = E_t \left\{ \sum_{r=t+1}^{\infty} \frac{\eta_r}{\eta_t} d_r^f \right\} \tag{31}$$

Finally we must prove

$$V_t^+ A_t + J_t^+ \left(Z_t - A_t \right) + \xi_t = E_t \left\{ \sum_{r=t+1}^{\infty} \frac{\eta_r}{\eta_t} d_r^I \right\} \tag{32}$$

to this end, we define the following functions

$$V_{t,T}^+ \equiv E_t \left\{ \sum_{r=t+1}^{T} \frac{\eta_r}{\eta_t} \phi^{r-t} \pi_r \right\} \text{ for } T \geq 1, \text{ and } V_{t,0}^+ = 0 \tag{33}$$

$$\begin{aligned} J_{t,T}^+ \equiv E_t \Bigg\{ \sum_{r=1}^{T} \frac{\eta_{t+r}}{\eta_t} \phi^r \Bigg[\left(\prod_{j=0}^{r-2} \left(1 - \lambda_{t+j}\right) \right)^{I_{(r\geq 2)}} \lambda_{t+r-1} V_{t+r,T-r} \\ - \left(\prod_{j=0}^{r-1} \left(1 - \lambda_{t+j}\right) \right) H_{t+s} \Bigg] \Bigg\} \text{ for } T \geq 1, \text{ and } J_{t,0}^+ = 0 \end{aligned}$$

$$\xi_{t,T} \equiv E_t \left\{ \sum_{r=1}^{T} \frac{\eta_{t+r}}{\eta_t} \left(Z_{t+s} - \phi Z_{t+s-1} \right) J_{t+s,T-s} \right\} \text{ for } T \geq 1 \tag{34}$$

these functions satisfies

$$\mathop{lim}_{T\to\infty} V_{t,T}^+ = V_t^+ \tag{35}$$

$$\underset{T\to\infty}{lim} J_{t,T}^{+} = J_{t}^{+} \tag{36}$$

$$\underset{T\to\infty}{lim} \xi_{t,T}^{+} = \xi_{t}^{+} \tag{37}$$

$$V_{t,T}^{+} A_t + J_{t,T}^{+} \left(Z_t - A_t\right) + \xi_{t,T} = E_t \left\{ \sum_{r=t+1}^{T} \frac{\eta_r}{\eta_t} d_r^I \right\} \tag{38}$$

taking the limit of (37) as T goes to infinity, and using (34)–(36) we obtain (31).

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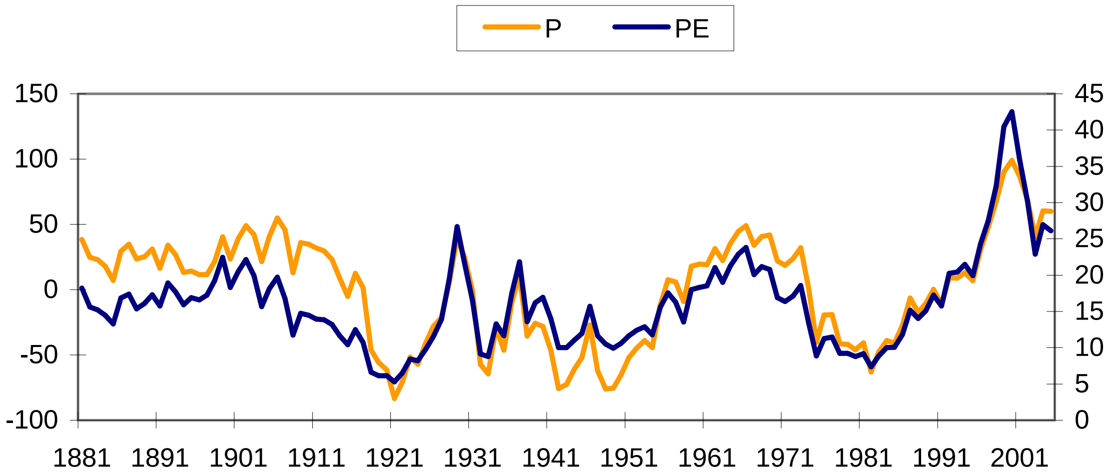
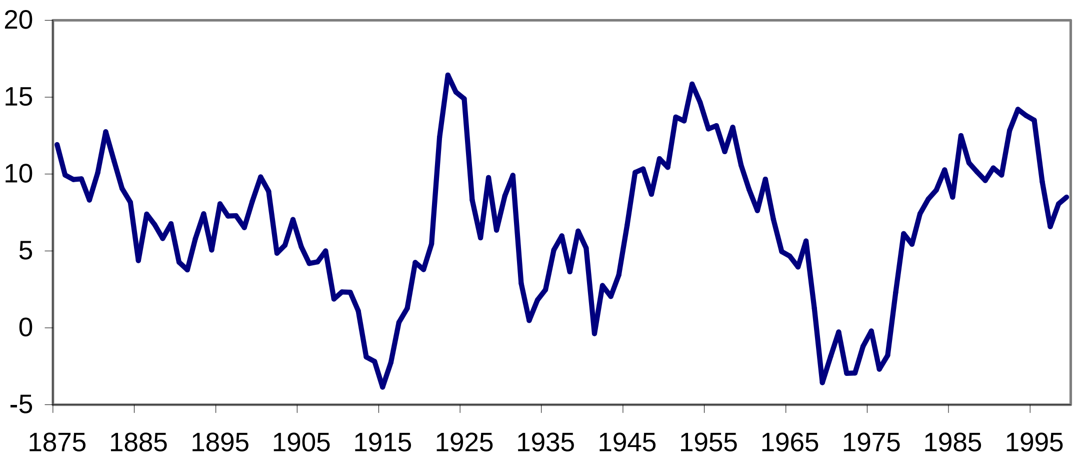
Figure 1: Evolution of S&P and PE



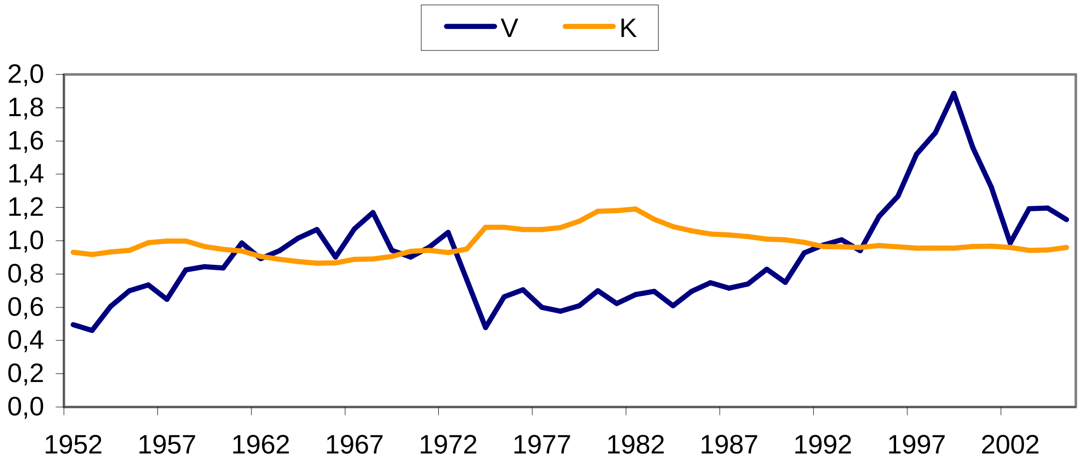
Notes: Both series are percentage deviations from its exponential trend. Annual data from 1881 to 2005.
Source: Robert Shiller's web site: http://www.econ.yale.edu/ shiller/data.htm

Figure 2: Real Rate of Return on S&P



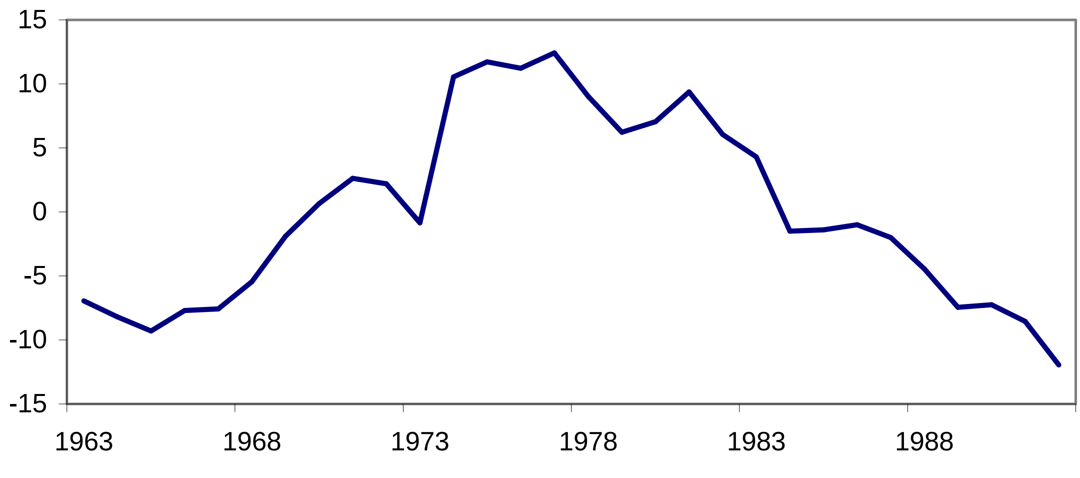
Notes: Centered ten-year moving average of the continuously compounded log return of S&P index. Annual data from 1881 to 2005.
Source: Robert Shiller's web site.

Figure 3: Market Value and Replacement Cost of U.S. Corporate Assets Relative to GDP



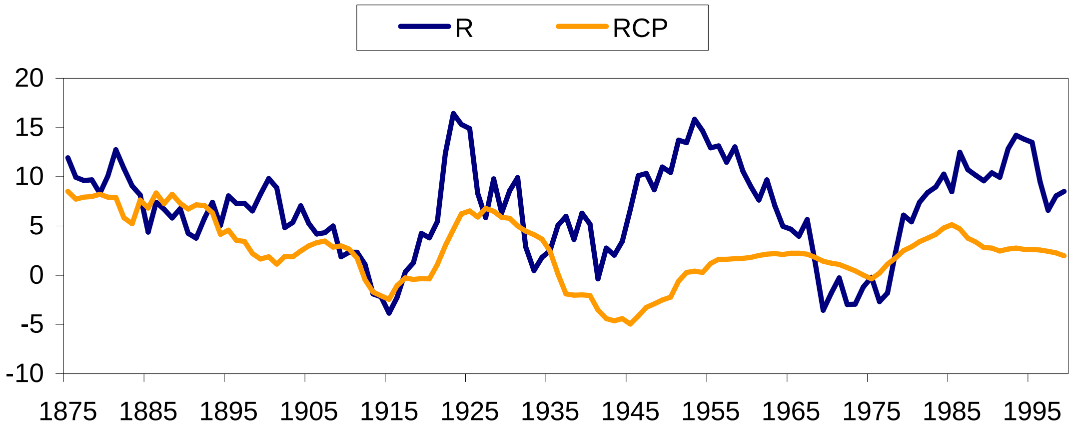
Notes: Both the market value and replacement cost of U.S. corporate tangible assets have been constructed following Peralta–Alva (2006). The measure for the market value considers both the value of equity and debt. Annual data from 1952 to 2005.

Figure 4: Relative Price of Equipment



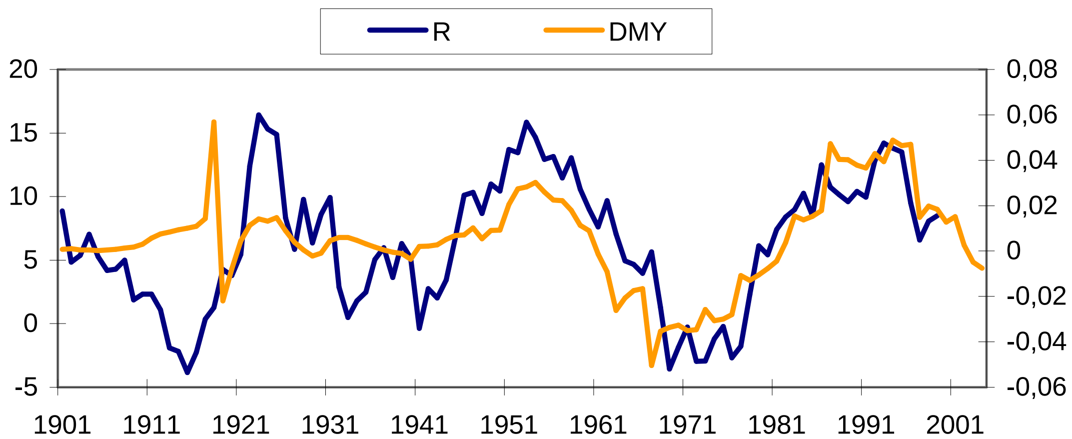
Notes: Percentage deviations from exponential trend. This price index has been adjusted for quality change. Annual data from 1963 to 1992.
Source: Krusel et al. (2000).

Figure 5: Real Rate of Return on S&P and Short–Term Interest Rate



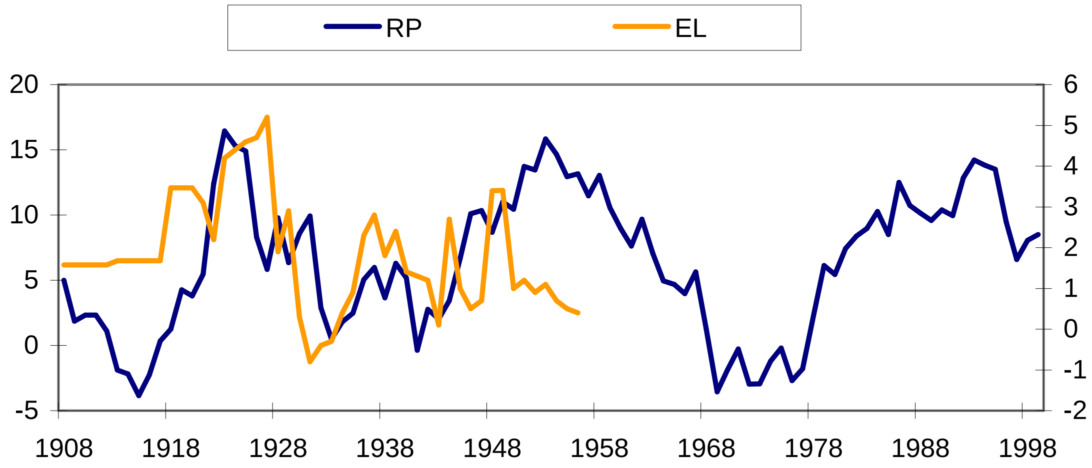
Notes: Centered ten-year moving average of the continuously compounded log annual return of S&P index and the return on 6-month prime commercial paper. Annual data from 1881-2005.
Source: Robert Shiller's web site.

Figure 6: Real Rate of Return of S&P and DMY Variable



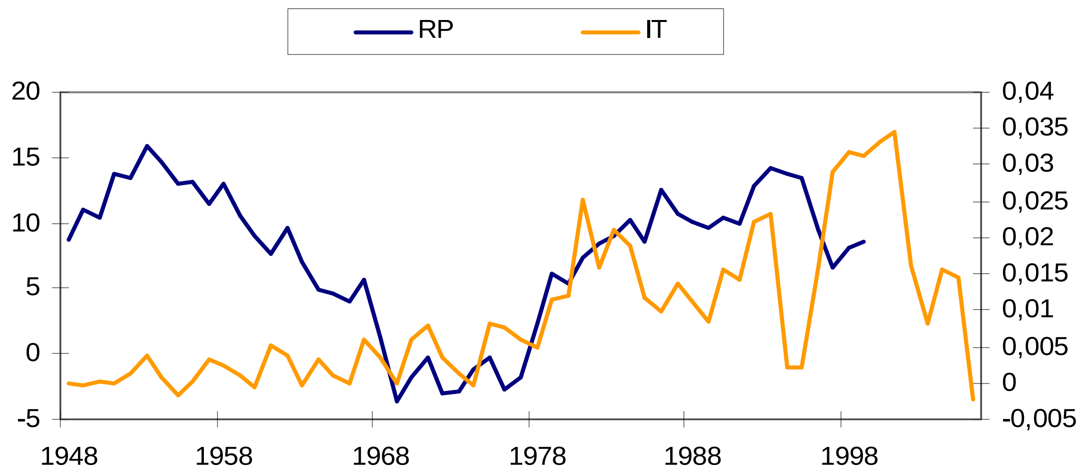
Notes: Centered ten-year moving average of the continuously compounded log annual return of S&P index and DMY variable as defined by Geanakoplos et al. (2004). DMY is the first-order difference of the ratio MY between the cohort aged 40–49 to the cohort aged 20–29. Annual data from 1901 to 2004.
Source: Robert Shiller's web site and U.S. Census Bureau.

Figure 7: Real Rate of Return of S&P Index and Change in the Percent of Households with Electric Service



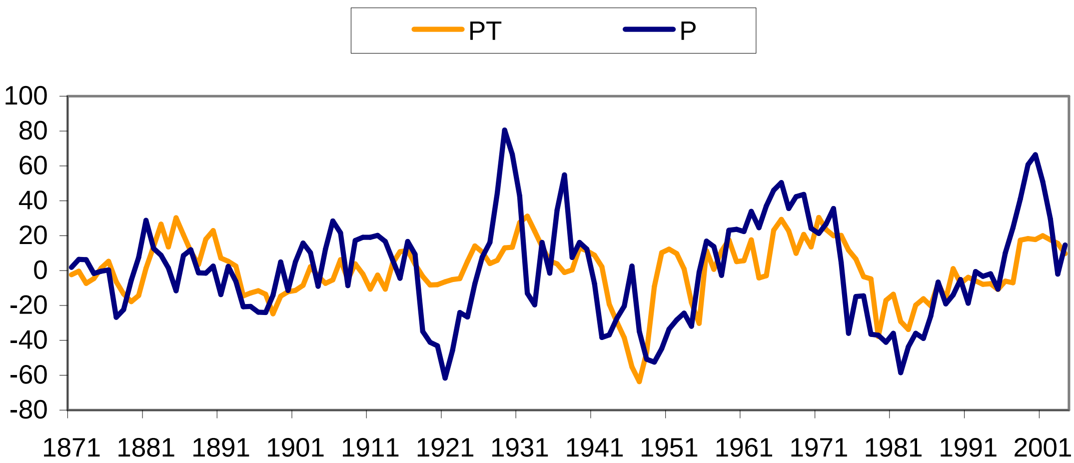
Notes: The Real Rate of Return on S&P is as above. The diffusion of electricity is approximated by the first-order difference of the number of households with electric service. Annual data from 1907 to 2003. Missing data in the first part of the sample are approximated by linear interpolation.
Source: Robert Shiller's web site and Historical Statistics of United States series S–109.

Figure 8: Real Rate of Return of S&P and share of IT Equipment and Software in the Aggregate Capital Stock



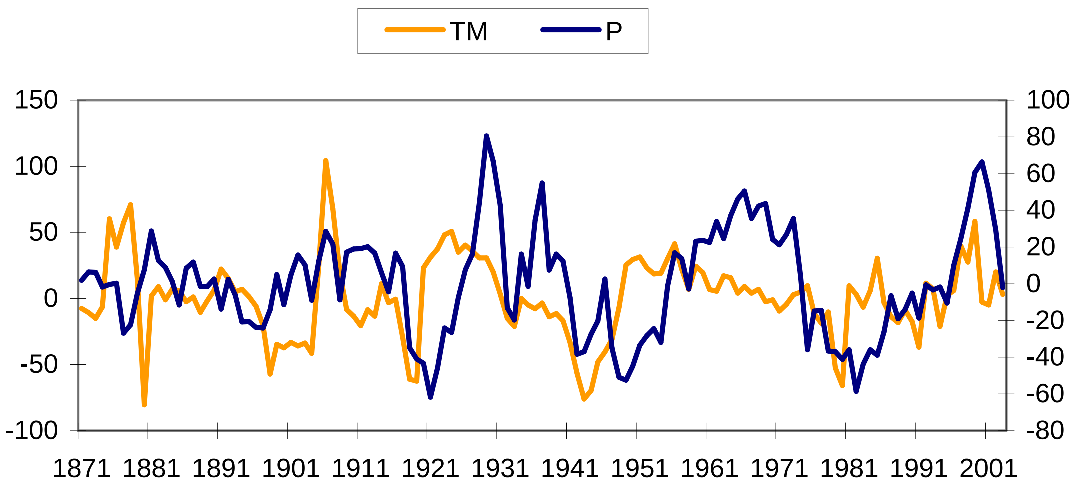
Notes: The Rate of Return of S&P is as above. The measure of the IT equipment and software in the aggregate capital stock is constructed following Jovanovic and Rousseau (2005). Annual data from 1947 to 2005.
Source: Robert Shiller's web site and Bureau of Economic Analysis.

Figure 9: S&P and Patents Issued



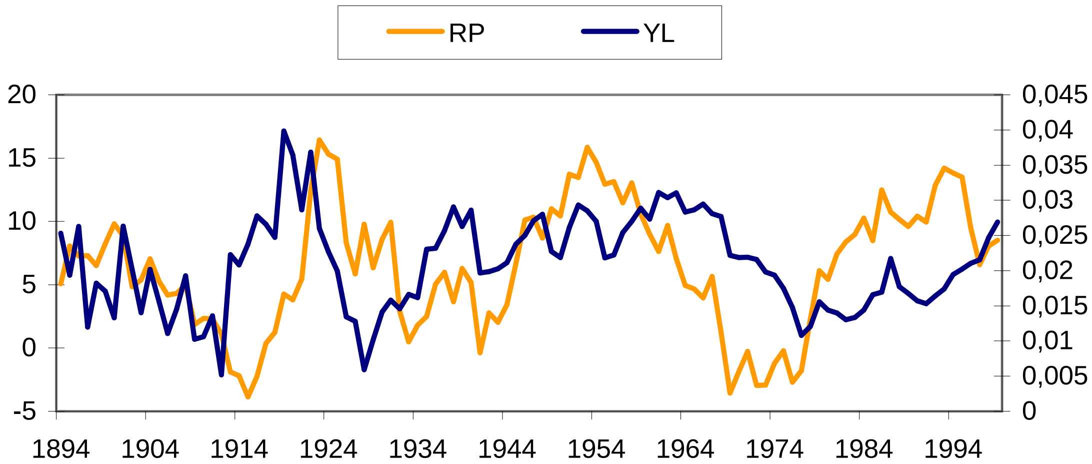
Notes: S&P price index and the number of patents granted. Both series have been filtered with a band pass filter for frequencies between 2 and 50 years. Annual data from 1871 to 2004.
Source: The S&P index is obtained from Robert Shiller's web site. Patents issued are obtained from U.S. Patent and Trademark Office for 1970–2004, and from Historical Statistics of the United States series W–99 for 1871 to 1970.

Figure 10: S&P and Trademarks Registered



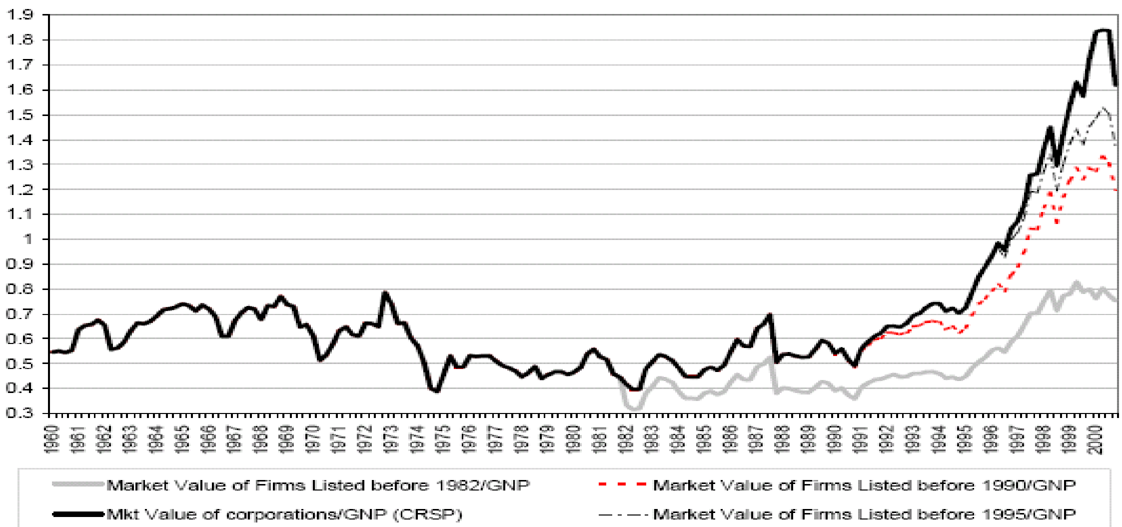
Notes: S&P price index and the number of trademarks Registered. Both series have been filtered with a band pass filter for frequencies between 2 and 50 years. Annual data from 1871 to 2004.
Source: The S&P index is obtained from Robert Shiller's web site. The number of registered trademarks are obtained from Historical Statistics of the United States series W–107 for 1871 to 1970, and from various issues of the Statistical Abstract of the United States for later years.

Figure 11: Real Rate of Return of S&P Index and Growth in Labor Productivity



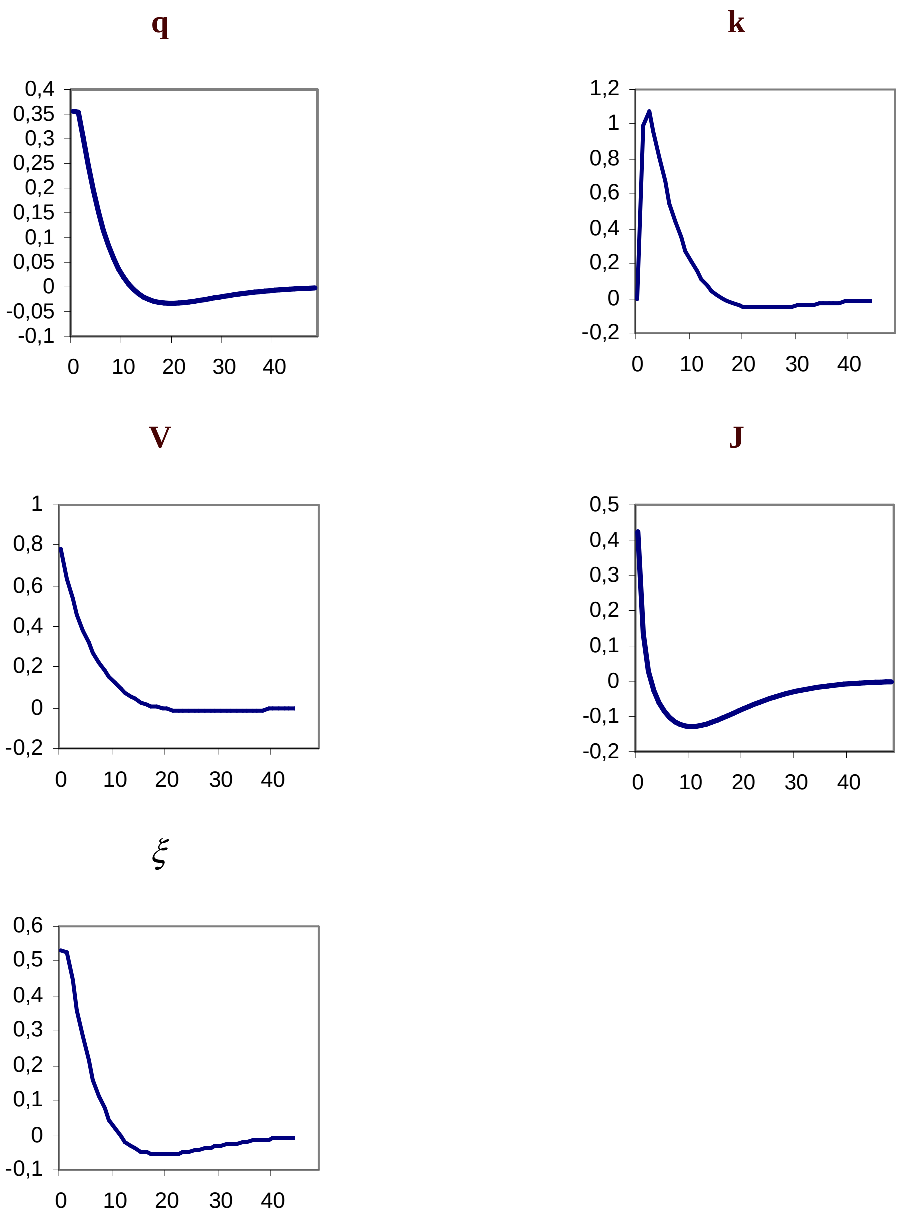
Notes: Both series are centered ten–year moving averages. Labor productivity is output per man-hour in the business, non-farm sector. Annual data from 1889 to 2004.
Source: Robert Shiller's web site, Historical Statistics of the United States and Bureau of Labor Statistics.

Figure 12: Market Value of Different Vintages



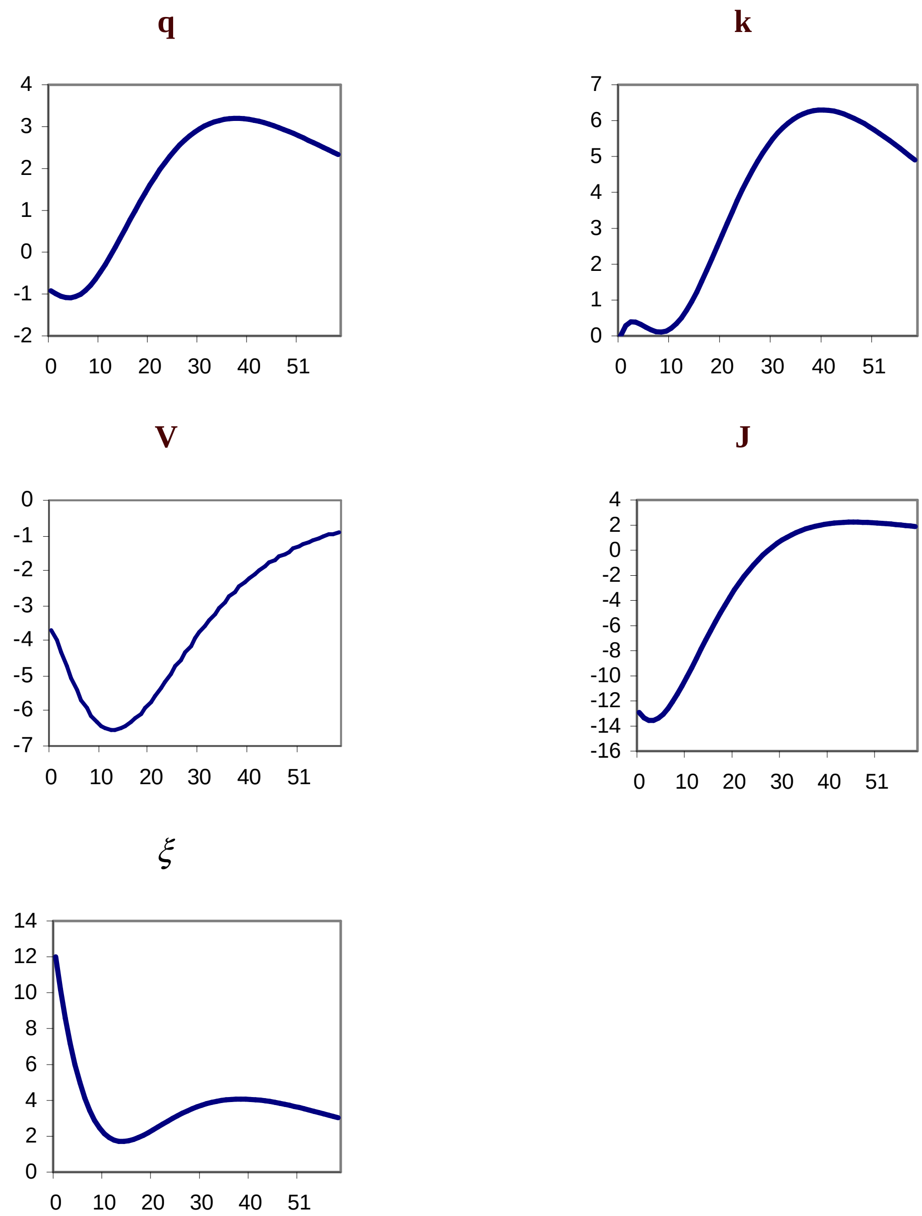
Notes: Market value of securities contained in the CRSP data set.
Source: Kindly provided by Adrian Peralta-Alva.

Figure 13: Impulse–Response Functions to a Shock in θ



Notes: Response to a positive, one–standard–deviation shock to the exogenous variable θ. The variable K denotes capital, V is the value of an installed technology, J is the value of a not–adopted technology, and ξ is the present value of technologies available in the future.

Figure 14: Impulse–Response Functions to a shock in x



Notes: Response of each variable to a positive, one–standard–deviation shock to the exogenous variable lnx_t.

Table 1: Moments of the Model

	Autocorrelation		Standard Deviation	
	Data	Model	Data	Model
Output	0.64	0.61	3.46	3.34
Consumption	0.84	0.96	1.92	2.89
Investment	0.72	0.53	8.86	9.75
Hours	0.71	0.66	3.31	2.06
TFP	0.73	0.73	2.43	2.67
Labor Productivity	0.77	0.86	2.40	2.68
Trademarks	0.94	0.98	5.30	5.30
Stock Market	0.87	0.94	31.41	24.25

Notes: Both series have been filtered with a band pass filter for frequencies between 2 to 50 years. The simulated moments are obtained through a simulation of 3000 years of data, dropping the initial 1000 years.

Table 2: Contemporaneous Correlations

	Correlation with Output		Correlation with Stock Market	
	Data	Model	Data	Model
Output	1	1	0.47	0.23
Consumption	0.74	0.52	0.61	0.70
Investment	0.58	0.69	0.14	0.28
Hours	0.76	0.60	0.52	-0.41
TFP	0.62	0.94	0.09	0.46
Labor Productivity	0.36	0.79	-0.05	0.60
Trademarks	0.04	0.63	-0.02	0.53
Stock Market	0.47	0.23	1	1

Table 3: Predictive Regressions

	Coefficient		R^2	
Horizon (years)	Data	Model	Data	Model
1	-0.05	-0.02	0.01	0.30
		(-0.07, -0.00)		(0.00, 0.44)
2	-0.14	-0.04	0.05	0.39
		(-0.15, -0.01)		(0.01, 0.51)
3	-0.19	-0.07	0.07	0.44
		(-0.23, -0.01)		(0.01, 0.62)
4	-0.28	-0.09	0.10	0.47
		(-0.29, -0.02)		(0.02, 0.75)
5	-0.36	-0.11	0.12	0.49
		(-0.36, -0.02)		(0.01, 0.88)
6	-0.38	-0.13	0.11	0.50
		(-0.44, -0.02)		(0.01, 0.88)
7	-0.42	-0.15	0.11	0.51
		(-0.51, -0.01)		(0.00, 0.88)
8	-0.53	-0.16	0.13	0.51
		(-0.58, -0.01)		(0.00, 0.89)
9	-0.59	-0.18	0.14	0.52
		(-0.65, -0.01)		(0.00, 0.90)
10	-0.63	-0.19	0.15	0.52
		(-0.71, -0.00)		(0.00, 0.90)

Notes: This table contains estimations of the regression of the log real continuously compounded return of the S&P index on the logarithm of S&P over dividends corresponding to the index. The regression is estimated with a constant that is not reported. The series span the period 1872 to 2005. The simulated coefficients are obtained as the average over 1000 simulations of the same length of the data sample. The numbers in parentheses are the 95% confidence interval for the simulated coefficients.
Source: Robert Shiller's web site.