On Anticompetitive Third-Degree Price Discrimination ⋆

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Abstract

Third-degree price discrimination increases output and welfare if certain local demand curvature conditions hold. These curvature conditions, known for nearly a century, have never been evaluated empirically before. To successfully evaluate the output and welfare effects of third-degree price discrimination, demand specification must be sufficiently flexible to allow for curvature heterogeneity across local markets. Otherwise, demand specification bakes-in empirical output and welfare predictions of price discrimination. I show that with the notable exception of logit demand, most other demands families predict output and welfare reductions as their elasticity and curvature are negatively correlated. I use supermarket scanner data to evaluate demand curvature conditions nonparametrically for thousands of chain-store-product combinations and show that, more often than not, third-degree price discrimination (local store pricing) decreases output and welfare relative to uniform pricing (chain-store pricing). Furthermore, I show that using output as a proxy for welfare as Robert Bork suggested overstates potential gains and understates potential damages of price discrimination.

Keywords: Uniform vs. third-degree price discrimination, demand curvature heterogeneity, manifold invariance, retail zone pricing, nonparametric estimation.

JEL Codes: D42, L12, L66

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"Whether the much cruder forms of discrimination that one encounters in the real world lead on average to a greater or smaller output than single-price monopoly is an empirical question."

Antitrust Law, An Economic Perspective
Richard A. Posner, 1976

1 Introduction

A monopolist serving two (or more) separate markets uses *uniform pricing* if he charges a single unit price across locations. Alternatively, he engages in *third-degree price discrimination* (*3DPD*) if he charges different prices in separate locations when preferences are sufficiently heterogeneous and incremental profits are large enough to compensate arbitrage costs. Welfare effects associated to *3DPD* are positive when it helps opening new markets but ambiguous if the number of local markets served remains unchanged. If both local markets are *always* served in equilibrium, the monopolist can increase profits by charging a higher price in the strong market and a lower price in the weak market. The additional profits plus the increase in consumer surplus of low valuation consumers as the weak market expands may or may not compensate the consumer surplus reduction following the exclusion of high valuation customers in the strong market. Figuring out whether charging different prices in separate markets could increase welfare is a long-standing question in economic theory dating back to Robinson (1933, Book V).

The very influential work of Robert Bork (1978) criticized the Robinson-Patman Act for targeting *3DPD* using fairness rather economic efficiency arguments. Bork's work was extremely influential among legal scholars to the point that the Robinson-Patman Act stopped being enforced in the 1980s. What ultimately matters in Bork's view is whether *3DPD* helps expand or restrict the total industry output. He concluded after Joan Robinson's book, that relative demand curvature in the weak and strong markets are likely to facilitate *3DPD* increasing overall sales. Not only legal scholars, but also most economists appear to have a fairly positive view of *3DPD*. However, these well-known demand curvature conditions have yet to be evaluated empirically.

The present paper aims to fill this void. I make two related contributions, one theoretical and one empirical. In order to evaluate the output and welfare effects of *3DPD*, demand specification must be sufficiently flexible to allow for curvature heterogeneity across local markets. Not addressing local demand curvature heterogeneity only produces results that are valid for particular demand systems. Being unaware of this limitation leads empirical economists to bake-in output and welfare results in a similar fashion that demand specification determines the pass-through rate of commodity taxation (Bulow and Pfleiderer, 1983).

Theorists find conditions for *3DPD* to expand overall output and welfare by assuming that infinitesimal price differences across local markets result in local demands with drastically different curvature properties. Absent any "jumps" in local demand curvature, I show that for most common demand families, elasticity and curvature are negatively correlated, so that *3DPD* necessarily reduces output and welfare *regardless* of the nature of the data generating process. The only remarkable exception is the logit demand, which characterized by a positive correlation of elasticity and curvature, always predict an increase in output with *3DPD*. The conclusion of this theoretical analysis of demand systems is that applied economists need to adopt very flexible econometric specifications capable of accommodating demand curvature heterogeneity across local markets, e.g., by allowing for nonlinear price effects interacted with local market indicators.

The empirical goal of this paper is to evaluate whether it is reasonable to assume demand showing drastic curvature variation associated to small variation in prices. To that end, I use the 2008-2011 sample from the IRI Marketing Data Set to test whether the relative curvature demand conditions of theoretical models generally hold in practice. I evaluate nearly 23,000 chain store pricing problems (uniform vs. *3DPD*) using four alternative nonparametric demand specifications for more than 160,000 store-products combinations across ten retail product categories. Results support the view that *3DPD* is generally welfare decreasing across all categories of retail products considered in this study. Thus, theorist's local demand curvature heterogeneity is in practice not large enough for *3DPD* to increase output and welfare. I further show that using output as an empirical proxy for welfare exaggerates the potential gains and underestimates the potential welfare reductions of *3DPD*.

Interestingly, around the time of Bork's writing, Posner (1976, §8) and Schmalensee (1981, footnote 8) speculated about the possibility of *3DPD* being outlawed. Nearly a century after Robinson's seminal work, we still lack any empirical evidence supporting or rejecting the beneficial output and welfare effects of *3DPD*. The robustness of results and the scale of analysis of this paper provides, for the first time, the kind of evidence that legal scholars lacked in the past to decide whether mandating uniform pricing could be socially preferable to *3DPD*.

Theory. I highlight the limitations of parametric models to evaluate the incremental welfare and sales of *3DPD*, as they predetermine demand curvature behavior behind output and welfare effects of *3DPD*. To show these limitations, I make use of the manifold invariance results of Mrázová and Neary (2017, §II.B) in the context of *3DPD*. I adopt the demand manifold framework to illustrate *where* in the space of demand functions the conditions for *3DPD* to increase output and welfare are more likely to hold. The demand manifold framework is very useful to show that without these demand curvature "jumps" in response to infinitesimal price changes, *3DPD* generally leads to reductions in output and welfare relative to uniform pricing as long as demand elasticity and curvature are negatively correlated (downward sloping manifolds).

I also show how particular demand specifications might inadvertently constrain the behavior of demand curvature. This is because demand manifolds might be invariant with respect to some or all parameter estimates. To convey this argument intuitively, suppose that we estimate linear demands to evaluate how markups of a product vary across all stores of a supermarket chain.

The econometrician obtains store-specific intercepts and slopes estimates with this linear demand specification that might even accurately account for differences in price responsiveness and the effect of income differences across locations. But by construction, demand curvature is always zero whenever we use a linear demand specification and the predicted total output is always identical under both, uniform pricing and *3DPD*, if all markets are always served. Furthermore, the predicted welfare is always lower with price discrimination.[1]

Note that this empirical conclusion results *exclusively* from specifying a linear demand and not necessarily because it is a feature of the data generating process. For linear demands, elasticity and curvature are independent of each other. Welfare decreases with *3DPD* for other manifold invariant demands with respect to all parameters, such as semi-logarithmic, linear expenditure system, and translog demands. In all these cases, elasticity and curvature are negatively correlated.

In contrast, logit demand always favors *3DPD* over uniform pricing since, by construction, logit demand ensures that elastic and curvature estimates are positively correlated. This result presents a practical challenge: the widespread use of logistic demand in empirical industrial organization and antitrust might give economists and policymakers the false impression that there is abundant evidence of the beneficial effects of *3DPD*, and thus, support the current lenient treatment of *3DPD* favored by most legal scholars.

I show that many other demand specifications that are not manifold invariant with respect to *all* parameters are also likely to predict output reductions with *3DPD* if their predicted elasticity and curvature are negatively correlated *unless* econometrician allow demand curvature to differ across local markets. Thus, applied economists should use sufficiently flexible demand specifications to obtain robust output and welfare effects associated to *3DPD*.

Empirics. I only focus on the canonical case of a single product monopolist with constant marginal costs always serving all local markets to evaluate the predicted effect of *3DPD* on output and welfare.[2] The case that always raised concerns since the works of Pigou and Robinson is one where the number of local markets is fixed and the monopolist chooses between uniform pricing and *3DPD* because in this environment the misallocation effect might dominate the output effect of price discrimination and reduce welfare.[3]

Economic theory does not provide any guidance on the economic fundamentals driving curvature heterogeneity across markets, but from a practical perspective simply adding nonlinear price effects interacted with local market demographics could, in principle, ensure enough chain demand

[1] This is the misallocation effect first described by Pigou (1932, Part II, Chapter XVII, §13-16) for linear demands.

[2] Despite the evidence against *3DPD* reported in this paper, *3DPD* could still increase overall output and welfare as well as consumer surplus across all local markets if it allows firms to take advantage of economies of scale (Robinson, 1933, §16.2). I do not explore this possibility due to lack of cost information for thousand products.

[3] Welfare increases whenever *3DPD* opens up new markets. This is similar to the increase in pricing options in models of second-degree price discrimination, e.g., Wilson (1993, §8.3). As they increase, options with lower fixed fees help expand the market among low valuation customers while high valuation ones are offered marginal charges closer to the marginal cost, thus promoting efficiency.

curvature flexibility. I instead estimate thousands of store-product demands nonparametrically to avoid any of the specification-induced curvature restrictions discussed above.

I use the 2008–2011 IRI Marketing Data Set where supermarket chains charge nearly uniform prices, as in DellaVigna and Gentzkow (2019). There is however enough price variation across stores and time to allow for separate store-specific estimates. Hence, I evaluate the regularity and curvature conditions of Aguirre, Cowan and Vickers (2010), ACV hereafter, for thousands of chain-store-product combinations. In particular, I directly test ACV's curvature conditions for strong and weak markets after solving for the optimal chain uniform price using the estimated store demands parameters. This approach evaluates curvature conditions locally, in the neighborhood of the optimal uniform price, as ACV's Increasing Ratio Condition for demand curvature conditions to hold globally generally fails.

The resulting evidence shows that output and welfare effects of *3DPD* are far more negative than economists, policy makers, and many legal scholars commonly expected. Output is predicted to increase only for 26% of product-chains (24% of sales). This output proxy overestimates the increase in welfare associated to *3DPD*, which only increases for 19% of product-chains (17% of sales). Output is also predicted to decrease for few cases: 16% of product-chains (15% of sales). These output predictions, however, vastly underestimate missallocation effects, with welfare possibly being lower for 76% of product-chains (78% of sales) with *3DPD*. Similar results hold across all ten product categories studied in this paper.

This empirical evidence challenges the relevance of long-held theoretical views on *3DPD*. Empirical analyses of other datasets using alternative methodologies are needed to confirm the present results. The main contribution of this paper is to show that *3DPD* has the potential to reduce welfare far more often than increases it, which, at the very least, should also question the wisdom of Bork's argument and the current lenient antitrust treatment of *3DPD*.

Related Literature. Robinson (1933) first identified the relative demand curvature conditions driving the overall output and welfare effects of *3DPD* vs. uniform pricing. If they fail, *3DPD* only helps the monopolist increase profits while restricting overall sales or not expanding output enough to compensate for the missallocation effect. While a negative output effect of *3DPD* is sufficient for welfare to decrease, a sales increase is not sufficient to ensure that *3DPD* increases welfare. It needs to be large enough, adding many low-value customers in the weak market to compensate for the exclusion of a few high-value customers in the strong market.

Research on the theory of price discrimination remained mostly dormant until the early 1980s. Schmalensee (1981) first extended Robinson's analysis to the N-market case and proved that *3DPD* cannot enhance welfare unless total output increases in a framework where a constant marginal cost monopolist faces independent local demands across markets. Varian (1985) generalized these results to the case of interdependent demands (imperfect arbitrage) and nondecreasing marginal costs. Schwartz (1990) further generalized the same results for decreasing marginal costs when the cost function depends on total output alone. Mauleg (1983) obtained bounds for welfare under *3DPD* relative to uniform pricing when at least one of the local demands is concave. ACV unified this literature and provided the curvature conditions for *3DPD* to increase output and welfare that I empirically evaluate in this paper. Finally, Bergeman, Brooks and Morris (2015) extended the analysis to calculate output and welfare bounds of all possible market segmentations, including those with non-concave profit functions.

Other important theoretical contributions addressed the case of price discrimination under oligopoly (Borenstein, 1985; Holmes, 1989; Corts, 1998) or in vertical relations where discrimination involves input prices (Katz, 1987; DeGraba, 1990; Miklós-Thal and Shaffer, 2021). Overall, these papers show that some of the results for monopoly markets do not longer hold or are reversed in oligopoly. See Varian (1989) and Stole (2007) for comprehensive review of these extensions.

There are numerous studies that document price discrimination in close geographical areas.[4] However, I am not aware of any empirical study that evaluates the basic tenants of the theory of *3DPD*. As far as I know, applied economists have not yet evaluated whether price discrimination increases output and welfare relative to uniform pricing using an exogenous price regime change. Early empirical work documented the possibility of *3DPD* in oligopolistic markets and the ability of firms to increase prices, either in gasoline retailing (Shepard, 1991) or the airline industry (Borenstein, 1989; Borenstein and Rose, 1994). More recently, empirical studies have evaluated the profitability of *3DPD* but not its potential output and welfare effects. They solidly document, however, that retail chain stores price nearly uniformly (Adams and Williams, 2019; DellaVigna and Gentzkow, 2019; Hitcsh, Hortaçsu and Lin, 2021).

Organization. Section 2 reviews the economic reasoning behind Bork's opinion against regulating the ability of firms to engage in *3DPD*. Section 3 uses the demand manifold framework to (*i*) state output and welfare conditions of *3DPD* in terms of elasticity, curvature, and their derivatives; (*ii*) prove that output and welfare are driven by the chosen demand specification in the absence of local demand curvature heterogeneity; (*iii*) show that common demand specifications that are manifold invariant necessarily predict negative output and welfare effects of *3DPD*; and (*iv*) discuss the curvature restrictions of common demand systems that are not manifold invariant with respect to all parameters. Section 4 presents an econometric model comprising three elements: (*i*) four alternative polynomial specifications for each chain-store-product demand; (*ii*) an equilibrium estimate of a constant chain-product marginal cost; and (*iii*) the numerical computation of the optimal chain-product uniform price used to evaluate the demand curvature conditions. Using the IRI Marketing Data Set, I assess the likelihood that *3DPD* leads to increases of decreases of output and welfare and summarizes results across ten product categories for one specific polynomial demand specification. Section 5 concludes. The Online Appendix reports detailed results for ten product categories and all four polynomial demand specifications.

[4] They include analysis of pricing in movie theaters in New Haven (Davis, 2006); ethnic restaurants in New York City (Davis, Dingel, Monras and Morales, 2019); grocery shops in different neighborhoods of Jerusalem (Eizenberg, Lach and Oren-Yiftach, 2021); gas stations in Quebec (Houde, 2012); and fast food restaurants in Santa Clara County (Thomadsen, 2005).

2 Robert Bork: Output as Proxy for Welfare

The *Robinson-Patman Act* of 1936, RPA hereafter, is the antitrust law dealing with third-degree price discrimination, mostly aimed to intermediate products sold by wholesalers with market power. The RPA aimed at protecting small retailers from large chain stores by limiting wholesale discounts, the so-called *secondary line injury to competition*, ignoring costs and efficiency arguments (Varian, 1989, §3.7). , even if it might end up harming consumers (Breit and Elzinga, 2001, §5). The influential legal scholar Robert Bork, who doubted the RPA as the "Typhoid Mary of Antitrust," led the charge against it:

> "[...] the better guess, it seems to me, is that antitrust policy would do well to ignore price discrimination. That estimate is based upon the judgment that price discrimination is, on balance, probably better for consumers than any rule enforcing nondiscrimination, and upon the belief that law cannot satisfactorily deal with the phenomenon in any event." (Bork, 1978, §20, p.412)

Bork also argued that the law is not very effective at distinguishing true price discrimination from temporary discounts to meet the competition or accounting for cost differences in providing services across local markets. Influenced by Bork's opinion and the work of many economists, the RPA was less frequently enforced beginning in the 1980s.[5]

How did Robert Bork conclude that price discrimination is, on balance, *probably better for consumers*? What was the evidence supporting this opinion? The answer matters because Bork's arguments permeated the opinion of legal scholars for decades. Many legal scholars still believe that *3DPD* generally benefits rather than harms consumers in the aggregate (Hovenkamp, 2017, §1.5b). This may or may not be the case. However, as I document below, this conclusion follows exclusively from Bork's introspective theoretical reasoning rather than from any statistical analysis of actual data.

Bork appealed to efficiency arguments to justify a lenient treatment of *3DPD* and treated the output effect of price discrimination as a *proxy* for consumer welfare:

> "[...] The evil of monopoly is restriction of output and consequent misallocation of resources. The question, therefore, is whether the misallocation will be greater under a rule permitting discrimination or under a rule requiring a single price to all customers. That question, in turn, translates into the question of whether discrimination expands or further restricts the monopolist's output. [...] The impact of discrimination on output, therefore, may be taken as a proxy for its effect on consumer welfare." (Bork, 1978, §20, p.413)

Bork was well aware that *3DPD* excludes some high valuation customers to expand the market among low valuation ones, but acknowledged that antitrust did not have the tools to account for deadweight losses and misallocation across consumers with different valuations (Bork, 1978, footnote, p.413). Using output as a proxy for welfare could sound as a reasonable alternative as,

[5] See O'Brien and Shaffer (1994), Blair and DePasquale (2014), and Schwartz (1986) in addition to Posner (1976) and Bork (1978, §20) himself.

short of opening up new markets, welfare effects of *3DPD* ultimately depend on second derivatives of the direct and inverse demand functions evaluated at the optimal uniform price, i.e., an abstract construct difficult to articulate in legal terms. But the question remains: *Why* did Bork believe that output could increase with *3DPD*?

> "The movement from a single price to a two-price system clearly benefits the seller; the question for antitrust policy is what it does to output. There is no easy answer in this simple two-market case, though Joan Robinson, whose analysis seems as complete as any that has appeared since, *thought it more probable on the whole* that output would be greater under discrimination than non-discrimination." (Bork, 1978, §20, p.415) – cursive added.

Thus, it is not data but Joan Robinson's sole opinion written in 1933, what informs Bork's belief that *3DPD* must be mostly beneficial. His influential legal position thus rests on purely theoretical arguments based on the relationship between demand elasticity and curvature that have so far not been evaluated empirically. Joan Robinson only explored logical conditions for output to increase but did not asses the likelihood of them holding in practice.[6]

An important contribution of the present work is to show empirically that using output as a proxy exaggerates the benefits and underestimates the welfare harm of *3DPD*.

It should be noted that there was a recent push to rehabilitate the enforcement of the RPA claiming that the legislator never cared about economic efficiency but enacted the RPA to pursue fairness by protecting smaller businesses against the "unfair practices" available only to large corporations.[7] This includes two FTC investigations on retail pricing by soda manufacturers and wine and liquor distributors.[8] A recent court decision banned Prestige and Medtech from making promotional payments for advertising and other services of their Clear Eyes drops only to large retailers such as Costco and Sam's Club.[9] The ultimate target appears to be Amazon and its ability to induce wholesalers to price discriminate against smaller retailers.[10]

[6] It could perhaps be argued that Bork was doubly mistaken since Robinson's analysis dealt with discrimination in final product markets while RPA is mainly intended for intermediate goods sold by wholesalers. However, economists had not addressed the welfare effects of input price discrimination by the time Bork wrote his book. Katz (1987) showed that uniform pricing might be more beneficial with intermediate goods nearly a decade after Bork published his influential book.

[7] See Federal Trade Commissioner Alvaro M. Bedoya's (2022) prepared remarks at the Midwest Forum on Fair Markets, `https://www.ftc.gov/system/files/ftc_gov/pdf/returning_to_fairness_prepared_remarks_commissioner_alvaro_bedoya.pdf`.

[8] See the June 12, 2024 WSJ editorial *The FTC Brings Back the 1930s*.

[9] *L.A. International Corp. v. Prestige Brands Holdings Inc. et al.* See "Expect an Increase in Robinson-Patman Act Enforcement" by D. Savrin, N. Kaufman and C. Zeytoonian, on April 29, 2024; and "Eye Drops Must Sell on Even Terms Under Rare Antitrust Win" by B. Koenig, on May 21, 2024, both at `https://www.law360.com/`.

[10] Kim (2021) presents a blueprint of the different strategies that plaintiffs and government agencies could use to bring a secondary-line case against Amazon for using separate vendor programs, Amazon seller Central and Amazon Vendor Central, with different conditions for pricing retailers, listing products, fulfillment and shipping. The article also speculates on effective and admissible evidence to show that these practices harm competition.

3 Theory: Demand Specification and Curvature Conditions

Let's assume that an econometrician has access to detailed price and quantity sales information (p_{sjt}, x_{sjt}) of products j sold in all stores $s \in s(r)$ of chain r across time t. Suppose that the sample contains two pricing regimes. First, chains engage in *3DPD* (store pricing). Then, due to a sudden and unexpected change in regulation, chains are forced to price uniformly across all stores. The econometrician could thus estimate a simple diff-in-diff model to evaluate whether overall output is larger or smaller with *3DPD* or under uniform pricing. A similar approach could be used if regulation moves in the opposite direction, no longer restricting firms to uniform pricing, after accounting for price endogeneity.

In both cases, output effects are evaluated *ex post*, and are not informative for the regulator to decide whether to constrain firms' pricing across locations. My goal is to evaluate the potential effects of *3DPD* vs. uniform pricing *ex ante*, which requires the use of a simple equilibrium model capable of generating thousands of inexpensive robust counterfactuals. The basic elements behind this minimal equilibrium model are the following:

1. I focus on a single-product demand at each store. Despite being a common approach in the literature (DellaVigna and Gentzkow, 2019; Hitcsh et al., 2021), it ignores substitution and strategic pricing decision within and across product categories in order to offer a large number of demand estimates that would be unfeasible otherwise. Consumer identity is not available and I therefore assume they always purchase at the same store location.

2. I specify a constant, product-chain specific, marginal cost. Constant returns ignore the possibility of wholesale quantity discounts, but this is a reasonable assumption as small store sales variations are unlikely to trigger massive discounts. Furthermore, it greatly facilitates the computation of product-store marginal costs as an equilibrium estimate using the chain's profit maximization conditions.

3. Theory of price discrimination evaluates curvature conditions locally at the optimal uniform price. This is the only counterfactual that I need to compute.

This section discusses how parametric specifications might drive output and welfare predictions of *3DPD* within this framework. Section 4 estimates demand nonparametrically to overcome any of the limitations on local curvature heterogeneity highlighted below.

Basic Elements. Let's consider a candidate specification with direct and inverse demand functions that are both positive, continuous, strictly decreasing, and three times differentiable:

$$x = x(p), \qquad \text{s.t.} \quad x' = x_p(p) < 0, \quad \text{and} \quad x(p) \in \mathcal{C}^3\,, \tag{1a}$$

$$p = p(x), \qquad \text{s.t.} \quad p' = p_x(x) < 0, \quad \text{and} \quad p(x) \in \mathcal{C}^3\,. \tag{1b}$$

The elasticity $\varepsilon(x)$ and curvature $\rho(x)$ of the inverse demand are:

$$\varepsilon(x) \equiv -\frac{p(x)}{x \cdot p'(x)} = -\frac{p \cdot x'(p)}{x(p)} = \frac{1}{e(p)} > 0\,, \tag{2a}$$

$$\rho(x) \equiv -\frac{x \cdot p''(x)}{p'(x)} = \frac{x(p) \cdot x''(p)}{[x'(p)]^2} = e(p) \cdot r(p)\,, \tag{2b}$$

where $e(p)$, the reciprocal of the elasticity of the direct demand function is, in equilibrium, the price markup or Lerner index, while $r(p)$ represents the curvature of the direct demand function:

$$r(p) \equiv -\frac{p \cdot x''(p)}{x'(p)} = \frac{p(x) \cdot p''(x)}{[p'(x)]^2} = \varepsilon(x) \cdot \rho(x)\,. \tag{3}$$

Demand Manifolds. A demand manifold is a smooth function relating demand elasticity and curvature. MN[Proposition 1] proves that with the exception of the *CES*, downward sloping direct and inverse demand functions that are three times continuously differentiable lead to a well-defined and smooth equilibrium relationship in the elasticity-curvature space for each demand function:

$$\varepsilon(\rho) = \varepsilon\left(\rho[x(p)]\right), \quad \text{or} \quad \rho(\varepsilon) = \rho\left(\varepsilon[x(p)]\right)\,. \tag{4}$$

Some results proven in the context of demand manifolds, such as their invariance to some or all demand parameters, are relevant for output and welfare predictions of *3DPD*. To illustrate and convey the intuition of the results presented in this paper, Figure 1 depicts the example of demand manifolds associated to the translated *CES* demand system introduced by Pollak (1971):

$$x = \gamma + \delta p^{-\sigma}\,. \tag{5}$$

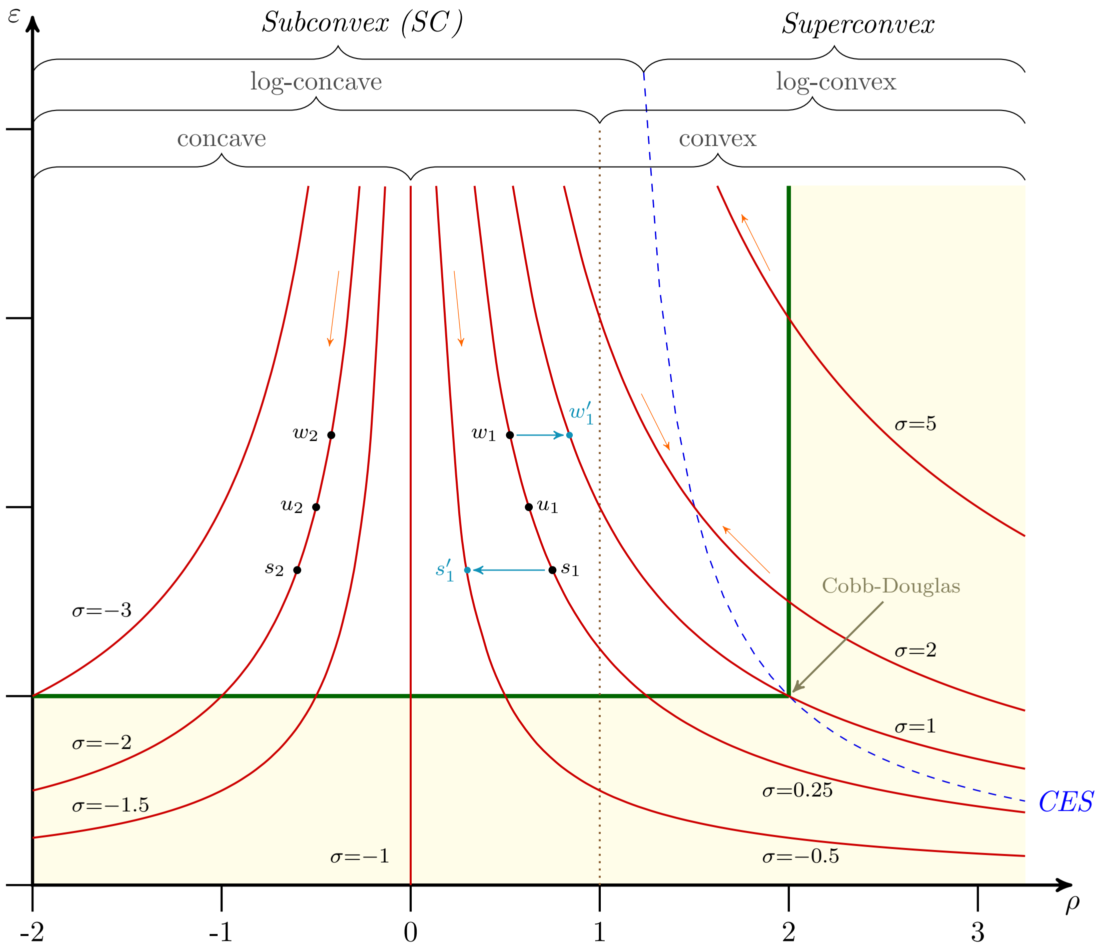
After combining the necessary and sufficient profit maximization conditions using this demand specification, the Pollak demand manifold is:[11]

$$\rho = \frac{\sigma + 1}{\varepsilon}\,. \tag{6}$$

The necessary condition profit maximization requires demand to be elastic in equilibrium for any firm with market power, $\varepsilon > 1$. Similarly, sufficiency requires the profit function to be concave, i.e., that the marginal revenue function is not increasing at the equilibrium prices, or $\rho < 2$. Together, these two conditions restrict the set of admissible combinations of (ε, ρ) for a profit maximizing monopolist to the non-shaded area in Figure 1. Note that in equilibrium, demands can take many different shapes for any elasticity value, ε.

[11] A profit maximizing monopolist with a constant marginal cost c chooses price p so that $p + xp' = p(1 - 1/\varepsilon) = c > 0$ and $2p' + xp'' = p'(2 - \rho) < 0$.

Figure 1: Pollak Demand Manifolds



The Pollak demand system includes both concave and convex demands as well as upward and downward sloping manifolds, an important feature for the evaluation of output and welfare of *3DPD*. When $\rho < 0$ demand is concave; linear for $\rho = 0$; and convex for $\rho > 0$. Among the latter, demand is log-concave when $\rho < 1$ and log-convex if $\rho > 1$, with incomplete or more than complete pass-through rate. When $\rho = 1$, along the dotted vertical line of Figure 1, pass-through rate is exactly 100%.

Figure 1 shows that exponent σ determines the location of the $\{\varepsilon, \rho\}$ manifold for the Pollak family. Parameter σ summarizes in this case all determinants of demand curvature. A value of $\sigma = -1$ identifies the family of linear demands. The manifold is upward sloping for $\sigma < -1$ which, in this particular case, also identifies concave demands functions. Conversely, manifolds are downward sloping for $\sigma > -1$, which includes both log-concave and log-convex demand functions. For any value of σ, a combination of parameters (γ, δ) identifies a single point on each (ε, ρ) manifold.

An important final distinction occurs depending where demand manifold falls relative to the set of blue dashed *CES* elasticity-curvature combinations. For (ε, ρ) combinations to the right (left) of the *CES* demand is superconvex (subconvex). Superconvexity determines how demand elasticity varies with sales (or equivalently, price). A function $f(x)$ is superconvex if $\log[f(x)]$ is convex in $\log(x)$. MN[Online Appendix B] show that:

$$\frac{d\varepsilon(x)}{dx} = \varepsilon_x = -\frac{\varepsilon}{x}\left(1 + \frac{1}{\varepsilon} - \rho\right) > 0\,. \tag{7}$$

Thus, demand is superconvex if for any given price elasticity ε, its associated curvature ρ, exceeds the curvature $\rho_{CES} = 1 + 1/\varepsilon$ of a *CES* demand with the same price elasticity. If demand is superconvex, it becomes more elastic (smaller markups) as sales increase. When $\varepsilon_x > 0$, the small orange arrows to the right of the *CES* curve in Figure 1 point upwards.

Most demand functions are subconvex, with demand becoming less elastic (higher markups) as sales increase. Now $\varepsilon_x < 0$ and the small orange arrows to the left of the *CES* curve in Figure 1 point downwards. Subconvexity corresponds to Marshall's observation that demand should most likely become more elastic at higher prices, i.e., $\varepsilon_p > 0$ (smaller sales). This is the so-called *Marshall's Second Law of Demand* (Marshall, 1920, [Book III, Chapter IV, §2).

3.1 Curvature Conditions and Demand Manifolds

In this section I review how demand curvature conditions across local markets determine output and welfare effects of *3DPD* relative to uniform pricing. I begin by analyzing ACV's increasing ratio condition and then write ACV's output and welfare conditions in terms of demand elasticity and curvature to convey intuitively their meaning with the help of Figure 1. For simplicity, and without loss of generality, assume that there is only one weak and one strong local market so that $p_s > p_w$ when the chain does not engage in uniform pricing, p_u.

The Increasing Ratio Condition. The starting point of ACV's analysis is the *Increasing Ratio Condition* (*IRC*), a property ensuring welfare to vary monotonically with $p_s - p_w$, the price difference between strong and weak markets, or alternatively, showing an interior peak. Let's define $z(p)$ as the ratio of the marginal effect of a price increase on social welfare to the second derivative of the profit function. The ratio $z(p)$ is the product of the markup and pass-through rate of a single-product monopolist (after making use of the Lerner index):

$$z(p) = \frac{(p-c)x'(p)}{2x'(p) + (p-c)x''(p)} = \frac{p-c}{2-\rho[x(p)]}\,. \tag{8}$$

IRC: *The increasing ratio condition holds in every market evaluated at local prices, i.e.,* $z'(p_w) > 0$ *and* $z'(p_s) > 0$.

Differentiating (8), and making use again of the equilibrium Lerner index, $p - c = p/\varepsilon = -x/x'$, as well as the second expression for demand manifold in (4), the *IRC* can be stated as:

$$z'(p) = \frac{(2-\rho) + (p-c)\rho_x \cdot x'}{(2-\rho)^2} = \frac{(2-\rho) - x\cdot\rho_x}{(2-\rho)^2} = \frac{(2-\rho) - x\cdot\varepsilon_x\cdot d\rho/d\varepsilon}{(2-\rho)^2} > 0\,. \tag{9}$$

ACV[Appendix B] claim that many demand functions meet this *IRC* condition. Concavity of a monopolist's profit function requires demand not to be excessively convex, $\rho < 2$. Then, it suffices that $\varepsilon_x \cdot d\rho/d\varepsilon < 0$ to ensure that *IRC* holds. Thus, *IRC* always hold for isocurvature demands (Bulow and Pfleiderer, 1983), , including linear demands with $\rho = 0$, since $\rho_x = d\rho/d\varepsilon = 0$, so that $z'(p) = 1/(2 - \rho) > 0$ for $\rho < 2$.

We can use Figure 1 to assess intuitively for what kind of demand functions the *IRC* holds true. For the Pollak demand system, this sufficient condition holds either for superconvex or concave demands.[12] First, if demand is superconvex, $\varepsilon_x > 0$, manifolds need to be downward sloping, $d\rho/d\varepsilon < 0$. These conditions are fulfilled in the region to the right of the *CES* curve up to the vertical green line of the sufficient profit condition $\rho = 2$ in Figure 1. Alternatively, when demand is subconvex, $\varepsilon_x < 0$, manifolds need to be upward sloping, $d\rho/d\varepsilon > 0$, which is the case for concave Pollak demands.

In practice, most demands fulfill Marshall's Second Law, $\varepsilon_x < 0$, but are also convex, falling between the linear, $\rho = 0$, and *CES* cases. I will show in Section 3.3 that, with the exception of the logit demand, the manifolds of most demand families are downward sloping, $d\rho/d\varepsilon < 0$, and thus the *IRC* condition is not ensured to hold for most subconvex demands. Of course $\varepsilon_x \cdot d\rho/d\varepsilon < 0$ is only a sufficient condition. The *IRC* condition could still hold for subconvex demands with downward sloping manifolds if they are not "too convex." Overall, this becomes an empirical issue that requires to evaluate the joint empirical distribution of (ε, ρ). Result reported in Table 6 of Section 4.4 corroborate that the *IRC* condition fails most often than not.

ACV's Curvature Conditions. I now present ACV's propositions on demand curvature behind output and welfare effects of *3DPD*.

ACV1: *Given the IRC, if the direct demand function in the strong market is at least as convex as that in the weak market at the nondiscriminatory price then discrimination reduces welfare, i.e., welfare decreases if* $r^s(p_u) \geq r^w(p_u)$. The proposition can be rewritten as follows after substituting identity (3): Welfare decreases with *3DPD* when:

$$\underbrace{\varepsilon^s[x(p_u)] \cdot \rho^s[x(p_u)]}_{r^s(p_u)} \geq \underbrace{\varepsilon^w[x(p_u)] \cdot \rho^w[x(p_u)]}_{r^w(p_u)} . \tag{10}$$

If this condition holds, welfare decreases in the neighborhood of p_u. In the unlikely case that *IRC* also holds, welfare reduction will also hold globally, for any price difference across local markets, i.e., for all prices in $[p_w, p_u]$ and $[p_u, p_s]$, respectively. Starting from a situation where the monopolist sets a common price across locations, ACV1 tests whether transitioning from uniform pricing to *3DPD* could reduce welfare.

[12] As I will show in Section 3.3, the shape of demand manifolds for most demand families share many common features with Pollak's demand manifolds. Superconvex demands frequently show unstable behavior with markups increasing monotonically in price. See the discussion on superconvexity in the context of discrete choice demand estimation in Miravete, Seim and Thurk (2023, §4.2).

ACV2: *Given the IRC, if* $z^w(p_w) = (p_w - c)/(2-\rho^w) \geq (p_s - c)/(2-\rho^s) = z^s(p_s)$ *(so inverse demand in the weak market is more convex than that in the strong market at the discriminatory prices, which are close together) then welfare is higher with discrimination.*

ACV suggest using this condition to evaluate a mandatory uniform pricing policy could reduce welfare when the monopolist actually engages in *3DPD*. Again, *IRC* ensures that results are valid globally, for all prices between the local and uniform prices. This proposition could be restated as follows after using Lerner Index equilibrium condition to eliminate the unobservable marginal cost. Given the *IRC*, welfare is higher with discrimination if:

$$\frac{(2-\rho^s[x(p_s)])\cdot\varepsilon^s[x(p_s)]}{p_s} \geq \frac{(2-\rho^w[x(p_w)])\cdot\varepsilon^w[x(p_w)]}{p_w}\,. \tag{11}$$

ACV4(+): *Total output rises if both direct demand and inverse demand are more convex in the weak market than in the strong market.* Evaluated locally at the nondiscriminatory price, output rises with *3DPD* when:

$$\underbrace{\varepsilon^w[x(p_u)]\cdot\rho^w[x(p_u)]}_{r^w(p_u)} > \underbrace{\varepsilon^s[x(p_u)]\cdot\rho^s[x(p_u)]}_{r^s(p_u)}, \quad \text{and} \quad \rho^w[x(p_u)] > \rho^s[x(p_u)]\,, \tag{12}$$

ACV4(–): *Total output does not increase if both direct demand and inverse demand are more (or equally) convex in the strong market than in the weak market.* Evaluated locally at the nondiscriminatory price, output and welfare decreases with *3DPD* when:

$$\underbrace{\varepsilon^s[x(p_u)]\cdot\rho^s[x(p_u)]}_{r^s(p_u)} \geq \underbrace{\varepsilon^w[x(p_u)]\cdot\rho^w[x(p_u)]}_{r^w(p_u)}, \quad \text{and} \quad \rho^s[x(p_u)] \geq \rho^w[x(p_u)]\,. \tag{13}$$

Demand curvatures evaluated at the uniform price are key to determine if *3DPD* could increase sales relative to uniform pricing. In some cases the curvature of the inverse demand function suffices to characterize the output effect. Sales increase with *3DPD* if all demands are convex and $0 < \rho^s[x(p_u)] < \rho^w[x(p_u)]$. Similarly, sales decrease with *3DPD* if all demands are concave and $0 > \rho^s[x(p_u)] \geq \rho^w[x(p_u)]$ (Shih, Mai and Liu, 1988; Cheung and Wang, 1994).

3.2 Demand Curvature Heterogeneity: Output and Welfare Predictions

In this section I use the demand manifold framework to illustrate how these output and welfare conditions may or may not hold across different regions of demand curvature. The slope of demand manifolds plays a key role. The main result of the following analysis is that unless demand curvature heterogeneity across local markets is substantial, *3DPD* reduces output and welfare if demand manifolds are downward sloping.

Homogeneous Curvature Across Markets. I now explore where the curvature conditions behind output and welfare of *3DPD* hold in the (ε, ρ) space. The basic arguments can be conveyed graphically. Dots $\{u, w, s\}$ in Figure 1 represent particular elasticity-curvature combinations (ε, ρ) at strong and weak local markets. They are intended to represent "infinitesimal" deviations from the elasticity and curvature under uniform pricing, (ε_u, ρ_u), along a single manifold. From ACV's conditions we know that for output and welfare to increase with price discrimination, elasticity and curvature need to be positively correlated. This might occur even in the absence of local market demand curvature heterogeneity, i.e., along a single manifold, if manifolds are upward sloping. If demand manifolds are downward sloping, output and welfare decreases with *3DPD* unless the drivers of local demand curvature are exceedingly different across strong and weak markets.

This argument relies on optimal price and elasticity being inversely related for a profit maximizing monopolist. However, demand specification determines if prices increase or decrease with curvature. The monopoly pricing solution is formally identical for the weak and strong market, as well as for the uniform pricing case. The optimal monopoly price is given by:

$$p_j = \frac{\varepsilon_j}{\varepsilon_j - 1}\, c\,, \quad \text{for } j = \{u, w, s\}\,. \tag{14}$$

Local markets are defined as strong or weak if, under *3DPD*, the local price is higher or lower than the optimal uniform pricing solution, $p_s > p_u > p_w$. It follows from the pricing equation (14) that $\varepsilon_s < \varepsilon_u < \varepsilon_w$, i.e., demand is less elastic in the strong market than in the weak one, with demand elasticity of the joint market falling in between (Nahata, Ostaszewski and Sahoo, 1990, Theorem 1).

The strength of the price-curvature connection determines the amount of local market curvature heterogeneity needed for *3DPD* to increase output and welfare. If manifolds are upward sloping, the ordering of elasticities and curvatures in the weak and strong market is the same as points $\{w_2, u_2, s_2\}$ in Figure 1. Demand is always more elastic for the weak than for the strong market, $\varepsilon_{w_2} > \varepsilon_{s_2}$. Because the demand manifold are upward sloping, it is also the case that $\rho_{w_2} > \rho_{s_2}$. Thus, ACV4(+) condition (12), $\varepsilon_{w_2}\rho_{w_2} > \varepsilon_{s_2}\rho_{s_2}$, holds and output increases with price discrimination even in the absence of curvature heterogeneity across local markets, i.e., when local demands have common curvature determinants as in Figure 1 for the $\sigma = -2$ manifold.

A very different outcome occurs when Pollak demands are convex and manifolds downward sloping, e.g., points $\{w_1, u_1, s_1\}$ along the $\sigma = 0.25$ manifold in Figure 1. If local demands still have common curvature determinants $\varepsilon_{s_1} < \varepsilon_{w_1}$, but $\rho_{s_1} > \rho_{w_1}$. Now elasticity and curvature are inversely correlated, which leads to ambiguous rankings of direct demands curvatures.[13]

Consider first the case where manifolds are very steep, $\rho_{s_1} \approx \rho_{w_1}$, corresponding to the case of limited local curvature heterogeneity. Inequality (12), $\varepsilon_{w_1}\rho_{w_1} > \varepsilon_{s_1}\rho_{s_1}$, most likely holds because $\varepsilon_{s_1} < \varepsilon_{w_1}$. Thus, output could still increase with *3DPD* when determinants of local demand

[13] According to Cowan (2016, equation (12)) "captures the intuition that the total output effect is positive if the price elasticity and the curvature measures are positively correlated." Thus if we were to use the *CES* specification of DellaVigna and Gentzkow (2019) for welfare evaluation we will conclude *necessarily* that *3DPD* reduces welfare because for the *CES* demand $\rho = 1 - 1/\varepsilon$, and therefore $\text{corr}(\varepsilon, \rho) < 0$.

curvature are common and the correlation between ε and ρ is negative but sufficiently small. On the other hand, welfare is more likely to decrease with *3DPD* if demand manifolds are relatively flat, i.e., when curvature varies substantially across local markets. Now $\rho_{s_1} >> \rho_{w_1}$ and $\varepsilon^s \rho^s \geq \varepsilon^w \rho^w$. If this is the case, equation (13) implies that output decreases with *3DPD*. Combining (10) and (13) shows that *3DPD* reduces output relative to uniform pricing as well as welfare.

Local Curvature Heterogeneity. For output to increase with *3DPD* it is necessary that $\rho_w > \rho_s$ when $\varepsilon_w > \varepsilon_s$, i.e., that elasticity and curvature are positively correlated. If manifolds are downward sloping this can only happen if an infinitesimal price difference separating strong and weak market results in local demands with drastically different demand curvatures, e.g., for instance, shifting horizontally s_1 to s_1' on the $\sigma = -0.5$ manifold and w_1 to w_1' on the $\sigma = 1$ manifold (gray arrows and nodes on the downward sloping manifolds of Figure 1). This is consistent with the theoretical work on this subject for the past century.[14]

3.3 Implications for Empirical Analysis

To ensure robust output and welfare predictions associated to *3DPD*, applied economists should turn to more flexible demand specifications that do not restrict the behavior of demand curvature and allow for sufficient heterogeneity across local markets, for instance by introducing nonlinear price effects interacted with city or region market demographics.

Parameter σ captures this additional flexibility in the Pollak demand system. A different σ for strong and weak local markets may reverse the *less desirable* effects of a downward sloping manifold with a common σ, i.e., a reduction of output and welfare relative to uniform pricing. For other demand systems, market-specific estimates of demand parameters behind demand curvature might also drive a wedge between the curvature of demand in weak and strong markets. This is not possible at all when demand manifolds are invariant with respect to all parameters of demand. If manifold invariance does not involve *all* parameters, it is possible to accommodate demand curvature heterogeneity across local markets if the econometric specification is flexible enough.

Manifold Invariance and Negative Output and Welfare of 3DPD. There are some important demand specifications where jumps across demand manifolds are not possible at all because the value of the parameter driving curvature properties is a constant common across weak and strong markets. This is perhaps the most interesting result of using the manifold framework in relation with the empirical analysis of *3DPD*: to show that regardless of the data generating process, the choice of some common demand specifications determines necessarily the negative output and welfare prediction associated to *3DPD*. This is a direct consequence of the manifold invariance result of MN[§ II.B].

[14] For instance, Robinson (1933, §15.5) compares linear demands in one market with either a concave or a convex demand in the other. This is also the case in ACV[Example 1], where the authors consider an exponential demand in for the strong market and a linear one over the weak market.

Demand shifts or rotations respond to changes in local demographics or other primitives of each market. In general, we should expect that such changes in demand in the (p, x) space also affect the shape and position of the corresponding demand manifold in the (ε, ρ) space. MN[Proposition 2] present a set of technical conditions ensuring that a change of an arbitrary demand parameter ϕ does not change the shape or position of the associated demand manifold. The demand manifold is then invariant with respect to parameter ϕ.[15]

I illustrate this result with the particular linear case, $x = \gamma + \delta p$. Note that differences in income or price responsiveness across local markets result in different intercept-slope combinations $\{\gamma, \delta\}$, where demand shifts and rotations reflect the particular local market demand conditions. However, all linear demands belong to the same elasticity-curvature manifold $\sigma = -1$ in Figure 1, with a zero-curvature, which in the end lead to the well-known welfare reduction predictions under *3DPD*.

In addition to linear demands, there are other important demand systems with demand manifolds that are invariant to local demand shifts with respect to *all parameters* include the Stone-Geary's linear expenditure system, $x = \gamma + \delta/p$; CARA, $x = \gamma + \delta \log(p)$; and translog demand specifications, $x = (\gamma + \delta \log(p))/p$. For all these cases, even if the econometrician takes great care and estimates separate specifications of these convex demand functions for each local market, the estimated intercepts and slopes across locations correspond to different elasticity-curvature combinations belonging to a *single downward sloping* manifold. Thus, as discussed in the homogeneous curvature case of Section 3.2, regardless of the data generating process, any of these manifold invariant demand specifications will predict that *3DPD* does not increase output and reduces welfare.

Curvature Restrictions of Common Demand Systems. Most demand specifications, even if they are not manifold invariant with respect to all demand parameters, impose important curvature restrictions that determine the sign of correlation between elasticity and curvature estimates that drive the output and welfare effects of *3DPD*. I now documents the manifold curvature properties of nine widely used demand systems. The demand manifolds of the first eight families are downward sloping. Thus, if applied economists fail to model curvature heterogeneity in a sufficiently flexible manner, they may likely conclude that *3DPD* leads to reductions in total sales and welfare. The only exception is the ninth family: logit demand.

Table 1 presents a wide selection of demand families commonly used in empirical research and describes their manifold features, with most of them being almost always downward sloping, i.e., $d\varepsilon/d\rho \leq 0$. The table includes each analytical demand specification, particular cases, important desirable properties, the expression of their demand manifolds and, most importantly, the slope of this manifold for each demand family. See the Appendices of MN for further technical details.

[15] Formally, for demand manifold to be invariant with respect to demand parameter ϕ, elasticity and curvature should depend on (x, ϕ) or (p, ϕ) through a common sub-function of either $F(x, \phi)$ or $G(p, \phi)$.

Table 1: Manifold Slope and Output Effect of Price Discrimination

	Convex Pollak	Inverse *PIGL*	Isoconvex
Specification	$x = \gamma + \delta p^{-\sigma}$, for $\sigma \geq -1$	$p = (\alpha + \beta x^{1-\theta})/x$, where $\theta = (2-\rho)/(\varepsilon - 1) > 0$	$p = \alpha + \beta x^{-\theta}$, where $\theta = \rho - 1$
Cases	linear, $\sigma = -1$; *CES*, $\gamma = 0$; *CARA*, $\sigma \to 0$; *LES* (Stone-Geary), $\sigma = 1$	inverse translog, $\theta \to 1$	linear, $\theta = -1$ log-linear, $\rho \to 0$
Properties	additively separable and quasi-homothetic	constant elasticity of marginal revenue, θ	constant pass-through, $dp/dc = 1/(1-\theta)$
Manifold	$\rho = (\sigma + 1)/\varepsilon$	$\rho = 2 + (1-\varepsilon)\theta$	$\rho = 1 + \theta$
$d\varepsilon/d\rho$	$-(\sigma+1)/\varepsilon^2 < 0$, (if $\sigma > -1$)	$-1/\theta < 0$	0
	CREMR	*CPPT*	*PIGL*
Specification	$p = \beta(x-\gamma)x^{(\sigma-1)/\sigma}/x$, for $1 < \sigma < \infty, x > \gamma\sigma, \beta > 0$	$p = (\beta/x)[x^{(k-1)/k} + \gamma]^{k/(k-1)}$	$x = (\gamma + \delta p^{1-\sigma})p$
Cases	*CES*, $\gamma \to 0$	*LES*, $k = 1/2$	translog, $\sigma \to 1$
Properties	constant revenue elasticity of marginal revenue, $1/(\sigma - 1)$	constant proportional pass-through, $\frac{d \log p}{d \log c} = k > 0$	expenditure function is a translated *CES*
Manifold	$\rho = 2 - (\varepsilon-1)^2/\varepsilon \cdot 1/(\sigma - 1)$	$\rho = 2 - \frac{1}{k} \cdot \frac{\varepsilon - 1}{\varepsilon}$	$\rho = [(\sigma+2)\varepsilon - \sigma]/\varepsilon^2$
$d\varepsilon/d\rho$	$-(\sigma-1)\varepsilon^2/(\varepsilon-1)^2 < 0$	$\frac{-k}{1 + k(2-\rho)} < 0$	$\varepsilon^3/[2\sigma - (\sigma+2)\varepsilon] < 0$, (if $\varepsilon < 2\sigma/(\sigma+2)$)
	QMOR	Inverse Exponential	Logistic
Specification	$x = \gamma p^{-(1-r)} + \delta p^{-(1-r/2)}$	$p = \alpha + \beta \exp(= \gamma x^{\delta})$, where $\gamma > 0, \delta > 0$	$p = a - \log[x/(s-x)]$, for $x \in [0, s]$ and $\rho < 1$
Cases	translog, $r \to 0$; linear, $r = 2$	*LES*, $k = 1/2$	
Properties	demands are homothetic and subconvex	demand slope is proportional to price	manifold independent of market size, s $p = a \ \to \ x = s/2$
Manifold	$\rho = (2-r)(3\varepsilon - 1 + r)/(2\varepsilon^2)$	$\varepsilon = \left[1 + \frac{\alpha}{\beta} \exp\left(\frac{\rho+\delta-1}{\delta}\right)\right]/(\rho + \delta - 1)$	$\varepsilon = [a - \log(1-\rho)]/(2-\rho)$
$d\varepsilon/d\rho$	$\frac{2\varepsilon^3}{4(1-r) - 3\varepsilon} \cdot \frac{1}{2-r} < 0$, (if $r < 2$)	$\left[-\delta + \frac{\alpha}{\beta}(\rho - 1)\exp\left(\frac{\rho+\delta-1}{\delta}\right)\right]/(\rho+\delta-1)^2 < 0$, (if $\alpha > 0, \beta > 0, \rho \leq 1$)	$\left[\frac{2-\rho}{1-\rho} + a - \log(1-\rho)\right]/(2-\rho) > 0$

Notes: Output under *3DPD* also increases in the concave region of the Pollak family ($\sigma < -1$) and for very elastic and concave *PIGL* demands when $\varepsilon > 2\sigma/(\sigma+2)$ as discussed in the Online Appendix. Evidently, the *QMOR* manifolds in decreasing for $r < 2$. The condition $\rho \leq 1$ is sufficient, not necessary, for the manifold of the inverse exponential to be downward sloping.

The first row of Table 1 begins with the subset of convex Pollak demands, with manifolds that are decreasing hyperbolas, e.g., Figure 1. This family includes the linear, *CES*, linear expenditure system (Stone-Geary), and *CARA* demand functions as particular cases. The Inverse *PIGL*, which includes the inverse translog, have manifolds that are all downward sloping straight lines crossing at $(\varepsilon, \rho) = (1, 2)$, the locus of the Cobb-Douglas demand function. Next, the isoconvex demand is the important constant pass-through family of Bulow and Pfleiderer (1983), with vertical manifolds and elasticitcy-invariant curvature across local markets. As in the linear demand case, total output does not change but welfare decreases with *3DPD*.[16]

The second row of Table 1 includes the case of demands with constant revenue elasticity of marginal revenue, *CREMR* of Mrázová, Neary and Parenti (2021), with downward sloping concave manifolds converging at $(\varepsilon, \rho) = (1, 2)$. Demands with constant proportional pass-through, *CPPT*, have downward sloping convex manifolds always crossing at $(\varepsilon, \rho) = (1, 2)$. Last, the price independent generalized linear (*PIGL*) demand system of Muellbauer (1975), which includes the translog demand and the Almost Ideal (*AIDS*) system of Deaton and Muellbauer (1980), have demand manifolds that are downward sloping for nearly all (ε, ρ) combinations.[17]

The first two cases of the last row of Table 1 present demand systems also characterized by downward sloping demand manifolds for all or a wide range of parameters. The manifold of demands with quadratic mean of order r (Diewert, 1976), *QMOR*, is very similar to the manifold of the Pollak family depicted in Figure 1. They are hyperbola-like manifolds that are decreasing for $r < 2$, i.e., for convex demands "to the right" of the linear case, $r = 2$. As for the inverse exponential family, it has downward sloping manifolds if demand is log-concave but not necessarily if they are log-convex. Thus, output might increase with price discrimination for very convex demands.

The ninth family of Table 1, the logistic demand function, is always log-concave and the only case where manifolds are *always* upward sloping and asymptotic to $\rho = 1$. Thus, any estimate of a multinomial logit model not only imposes an incomplete pass-through rate (Miravete et al., 2023, §4), but also, if used to evaluate the effects of price discrimination, predicts, by construction, that *3DPD* leads to increases in output and welfare, a case already noted by Cowan (2016). The analysis of this section indicates, however, that predictions regarding *3DPD* of the logistic demand are not necessarily robust, as they are *implied* by its curvature properties rather than by the behavior of the data. This is an important result to remember given the widespread use of the logit demand in empirical work nowadays because it may lead to an abundance of non-robust evidence showing that *3DPD* increases output and thus wrongly convey the idea that the output and welfare effects of *3DPD* are overwhelmingly positive.

[16] This includes Cowan (2007) with demand $x_j = a_j + b \cdot f(p_j)$, for $j = \{w, s\}$ as curvatures in the weak and strong markets are arbitrarily close when $\delta = p_s - p_w$ is sufficiently small. Thus, total output does not change with *3DPD* and welfare decreases. This welfare reduction should also be predicted by the empirical analysis of Atkin and Donaldson (2015) and Butters, Sacks and Seo (2022), both of whom estimate or assume an isocurvature demand specification.

[17] I show in the Appendix that *PIGL* manifolds become upward sloping only for a very small subset of very elastic and concave demands.

4 Evaluating Output and Welfare Curvature Conditions

Policymakers might be interested in restricting supermarkets' ability to price discriminate across local stores. This might occur after a merger if the new consolidated firm is perceived to have excessive market power. We therefore need to evaluate the optimality of uniform pricing vs. price discrimination *ex ante*, which requires adopting a structural approach for researchers to compute counterfactuals under alternative pricing regimes and predict their output and welfare effects.

The analysis of Section 3 higlights the serious limitations that common parametric demand models impose on output and welfare predictions of *3DPD*. Even if we specify demand in such a way that allows for sufficient demand curvature heterogeneity across local markets solving a full structural model thousands of times might be prohibitive. But this analysis of demand curvature conditions also eases the empirical task: *IRC* and conditions ACV1, ACV2, and ACV4 can be directly evaluated on demand estimates once we determine the optimal uniform pricing, p_u. This is a much simpler problem than solving a full counterfactual evaluation of output and welfare. My approach, detailed below, could thus allow antitrust authorities to evaluate *ex-ante* whether uniform pricing or *3DPD* are socially preferable based on curvature properties of demand obtained using the actual prices charged by chains in each store.

4.1 Estimation: Demand, Costs, and Optimal Uniform Pricing

The estimation focuses on the residual demand of each product in each location, as in DellaVigna and Gentzkow (2019). Consumers are assumed not to change the brand they purchase or the store they purchase them from when evaluated at alternative prices. This restrictive assumption avoids having to estimate a full multiproduct discrete choice demand across stores for all products, but allows me to test local curvature conditions for tens of thousands of chain-store-products.

Data consists of price and quantity observations (p, x) for each product j (UPC) and store s of chain r over t weeks. I adopt flexible demand specifications capable of accommodating curvature heterogeneity across local markets to avoid the limitations discussed in Section 3.3 and estimate the following H-degree Stone-Weierstrass polynomial approximation for each store-product combination:[18]

$$x(p_{sjt}) = \beta_0 + \sum_{h=1}^{H} \beta_h \cdot (p_{sjt})^h + \tau_t + \varepsilon_{sjt}\,, \tag{15}$$

where τ_t denotes the week-of-the-year fixed effects. To address price endogeneity concerns I use Hausman (1996) instruments consisting of the average prices of product j of stores in the own chain $s \in s(r)$ located in other geographic markets. This polynomial regression makes use of the panel data structure to predict store-product estimates for demand and its derivatives, $\hat{x}(p)$, $\hat{x}'(p)$ and

[18] This approach allows for nonlinear price effects that may vary across local markets as demand is estimated for each store-product combination. Data availability conditions this modeling choice. If pooling the data across local markets, another valid approach is to interact local socioeconomic indicators with nonlinear price effects.

$\hat{x}''(p)$, which allows computing estimates of elasticity and curvature, $\hat{\varepsilon}(p)$ and $\hat{\rho}(p)$ characterizing the demand of each product in each store. These are the key ingredients necessary to test for *IRC*, ACV1, ACV2, ACV4 and determine whether *3DPD* increases output and welfare relative to uniform pricing.

The potential for biased predictions of polynomial specifications far from the sample average of the regressors is well-known (Fan and Gijbels, 1996, §1.1). The time span of the IRI data limits the possibility of using other, more flexible nonparametric estimation methods. The number of observations required to nonparametrically estimate derivatives increases exponentially with each additional derivative order (Pagan and Ullah, 1999, §4), something that I cannot credibly achieve with a maximum of 208 weekly observations per store-product combination.

On the positive side, the evaluation of $\hat{x}(p)$, $\hat{x}'(p)$, and $\hat{x}''(p)$ takes place either at the chain uniform price or at the store price, both of which are very close to the price sample mean, as supermarket pricing is very similar across locations. Results are very similar for both, a cubic ($H = 3$) and fourth-degree polynomial ($H = 4$) to evaluate the first two derivatives of demand. Results are very similar when I repeat the analysis using the Müntz-Szász approximation, which Barnett and Yue (1988) favor as it reduces the risk of overfitting the data because it is globally regular (concave) at all orders of approximaiton:[19]

$$x(p_{sjt}) = \beta_0 + \sum_{h=1}^{H} \beta_h \cdot (p_{sjt})^{1/2h} + \tau_t + \varepsilon_{sjt}\,. \tag{16}$$

Marginal Costs. I assume a common constant marginal cost c_{rj} for each product j across all stores $s(r)$ of a chain r. Since prices are not always exactly uniform across stores, I assume that each period t, supermarket chain r sets store prices of product j to maximize total chain profits:

$$\{p^{\star}_{sjt}\} \in \underset{\{p_{sjt}\}}{\operatorname{argmax}}\, \Pi_{rjt}\left(p_{1jt}, p_{2jt}, \ldots p_{s(r)jt}\right) = \sum_{s(r),\, j} \left(p_{sjt} - c_{rj}\right) x(p_{sjt})\,. \tag{17}$$

There are $n_r \times t$ first order profit maximization conditions similar to (14) for each chain, with an identical number of store pricing equations, where n_r is the number of stores in chain r and t the number of weeks when store sales of product j are available for this supermarket chain. After estimating demand for each store-product and computing its predicted store-product-week sales, $\hat{x}(p_{sjt})$, and elasticity estimate $\hat{\varepsilon}_{sj}[\hat{x}(p_{sjt})]$, I use the single-product profit maximization condition

[19] The Stone-Weierstrass approximation theorem ensures that a linear combination of functions $\{1, p, p^2, p^3, \ldots\}$ used in (15) uniformly approximates any continuous demand $x(p)$ on a compact support $[\underline{p}, \overline{p}] \subset \mathbb{R}$. The Müntz-Szász theorem ensures that demand can be uniformly approximated by a linear combination of functions $\{1, p^{\lambda_1}, p^{\lambda_2}, p^{\lambda_3}, \ldots\}$ if $\sum_{h\in\mathbb{N}} \lambda_h^{-1} = \infty$ (Rudin, 1966, §15). I use half the harmonic sequence $(1, 1/2, 1/3, \ldots, 1/h)$ for the power elements of demand specification (16) to approximate demand $x(p)$ with a linear combination of concave functions in $[\underline{p}, \overline{p}] \subset \mathbb{R}$ as each term $\lambda_h = 1/2h \in [0, 1]$ (Barnett and Jonas, 1983).

for each store of a chain and average the $s(r)$ weekly store marginal revenue estimates over all $n_r \times t$ store-weeks to obtain an equilibrium estimate of chain r's marginal cost for each product j:

$$\hat{c}_{rj} = \frac{1}{n_r \times t} \sum_{s(r),\, j} p_{sjt} \left(1 - \frac{1}{\hat{\varepsilon}_{sjt}[\hat{x}(p_{sjt})]}\right). \tag{18}$$

Optimal Uniform Price. Output and welfare conditions ACV1 and ACV4 are evaluated at the uniform price p^u_{rj} while ACV2 is evaluated at the store price p_{sj}. Although supermarket chains price very similarly across stores and time, there are still small differences that need to be taken into account to compare the curvatures of the weak and strong markets. In order to reduce the number of comparisons across weeks, I define p_{sj}, the price of product j in store s as the sales-weighted, weekly average of p_{sjt} in that store. Thus, $\hat{x}(p_{sj})$ and $\hat{\varepsilon}_j(p_{sj})$ denote the within sample prediction of weekly average sales and average elasticity of product j in store $s \in s(r)$, respectively, where both are evaluated at the sales-weighted average store price p_{sj}. Similarly, $\hat{\rho}_j(p_{sj})$ and $\hat{\chi}_j(p_{sj})$ represent the average curvature and average temperance estimates evaluated at the same sales-weighted, weekly average store price.

The estimate $\hat{c}_{rj}$ is necessary to figure out the uniform price that maximizes supermarket chain r's profits of product j. I use the estimated parameters of the chain-store-product demands to predict $\hat{x}(p^u_{rj})$, the weekly average sales of product j in store $s \in s(r)$ evaluated at the optimal uniform chain price p^u_{rj}. I search among the set of prices ensuring non-negative predicted sales for every chain-store to find the optimal uniform price that maximizes chain profits:

$$p^u_{rj} \in \underset{p_j}{\operatorname{argmax}}\, \Pi^u_{rj}(p_j) = \sum_{s(r),j} (p_j - \hat{c}_{rj})\, \hat{x}(p_j) \cdot \mathbf{1}\left[\hat{x}(p_j) \geq 0\right]. \tag{19}$$

Maximizing this objective function mimics the premises of the theoretical model by focusing on cases where all local markets are covered both under uniform pricing and *3DPD*. Since in practice I need to solve this optimal uniform price for nearly twenty-three thousands chain-product combinations, I proceed as follows. I first evaluate the weekly average store sales for each price given by a thousand elements of a uniform sequence between the highest and lowest sales-weighted average price observed in the data, $p_{sj} \in (\underline{p}_{sj}, \overline{p}_{sj})$, for any store in a given chain. I then select p^u_{rj} as the price securing the highest profits on $(\underline{p}_{sj}, p^\circ_{sj}]$, where p°_{sj}, is the highest price in the sequence where sales in all chain stores are positive.

4.2 Testing for Local Demand Curvature

After finding the optimal uniform price p^u_{rj}, I can test for *IRC*, output, and welfare conditions for each store-product within a chain. The *IRC* regularity condition must hold for all local markets evaluated at the local price. As I document below in Table 6, *IRC* fails most of the time. Thus, I only evaluate local versions of the output and welfare conditions. If ACV's conditions fail locally, their global versions also fail.

Generalizing ACV1, ACV2, and ACV4 for more than two markets requires that they hold for all weak and strong markets, i.e., for all pairwise demand curvature comparisons between each store in the weak markets and each store in the strong market. Since $p_s > p_u > p_w$ and $\varepsilon_s < \varepsilon_u < \varepsilon_w$, all pairwise comparisons hold if the infimum of the left hand side of conditions ACV1, ACV2, and ACV4 exceeds the supremum of the right hand side of these conditions. If they hold for the particular pairwise combination where the output and welfare conditions are more similar for weak and strong markets, they will hold for all others. If they do not, there there is at least one pairwise comparison that would violate the curvature condition. I thus evaluate the following hypotheses empirically:

1. **ACV1**. *3DPD reduces welfare* if *IRC* holds for all local markets plus:

$$\min\{r^s[\hat{x}(p_u)] = \varepsilon^s[\hat{x}(p_u)] \cdot \rho^s[\hat{x}(p_u)]\} \geq \max\{\varepsilon^w[\hat{x}(p_u)] \cdot \rho^w[\hat{x}(p_u) = r^w[\hat{x}(p_u)]\}\,. \tag{20}$$

2. **ACV2**. *3DPD increases welfare* if *IRC* holds for all local markets plus:

$$\min\{(2-\rho^s[\hat{x}(p_s)]) \cdot \varepsilon^s[\hat{x}(p_s)]/p_s\} \geq \max\{(2-\rho^w[\hat{x}(p_w)]) \cdot \varepsilon^w[\hat{x}(p_w)]/p_w\}\,. \tag{21}$$

3. **ACV4**(+). *3DPD increases output* if both direct and indirect demands are more convex in the weak than in the strong market:

$$\min\{r^w[\hat{x}(p_u)]\} > \max\{r^s[\hat{x}(p_u)]\} \quad \text{and} \quad \min\{\rho^w[\hat{x}(p_u)]\} > \max\{\rho^s[\hat{x}(p_u)]\}\,. \tag{22}$$

4. **ACV4**(−). *3DPD decreases output and welfare* if both direct and indirect demands are more convex in the strong than in the weak market:

$$\min\{r^s[\hat{x}(p_u)]\} \geq \max\{r^w[\hat{x}(p_u)]\} \quad \text{and} \quad \min\{\rho^s[\hat{x}(p_u)]\} \geq \max\{\rho^w[\hat{x}(p_u)]\}\,. \tag{23}$$

4.3 Supermarket Data

I use weekly sales data from the IRI Marketing Data Set for ten product categories across nearly one thousand stores belonging to seventy-one supermarket chains in fifty medium/large metropolitan areas in the U.S. between 2008 and 2011. Most households purchase one or many of the products included in the IRI Marketing Data Set (Bronnenberg, Kruger and Mela, 2008, Table 2). Products are defined by UPC and they differ by size, flavor, and other attributes. Product categories include beer, breakfast cereal, carbonated beverages, coffee, frozen dinners/entrees, household cleaning products, salty snacks, soup, and yogurt.[20]

[20] Einav, Leibtag and Nevo (2010) document the similarity of the IRI and Nielsen datasets. I restrict the attention to the period 2008-2011 with unique product identifiers across years as in Luco and Marshall (2020).

Table 2: Descriptive Statistics

	Beer	Carb.Bevs.	Coffee	C.Cereal	Fzn.Dinner	Fzn.Pizza	H.Cleaner	S.Snacks	Soup	Yogurt	ALL
Price ($)											
Average Price	11.32	2.22	5.57	3.00	2.36	3.16	2.58	2.56	1.29	1.18	3.52
Std.Dev Price	4.68	1.21	2.26	0.69	1.12	1.68	1.19	0.95	0.52	1.02	1.53
Maximum Price	23.86	6.00	14.58	7.60	12.31	10.07	7.26	7.89	7.34	7.75	23.86
Minimum Price	1.11	0.23	1.10	0.48	0.31	0.70	0.62	0.25	0.14	0.35	0.14
Supermarket Stores											
Avg. Number of UPCs	9.3	38.4	5.6	24.0	22.5	10.8	2.1	14.4	20.0	27.8	17.49
Avg. Weekly Sales (units)	118.5	1,438.2	60.3	381.1	283.5	210.1	16.2	246.5	452.1	860.9	406.74
Avg. Weekly Sales ($)	1,456.86	3,261.20	319.80	1,098.73	587.56	500.53	37.47	589.42	449.31	681.43	898.93
Supermarket Chains											
Avg. Number of Stores	12.5	13.6	13.6	13.6	13.7	13.1	11.2	14.1	13.3	14.0	12.87
Avg. Number of UPCs	18.8	67.9	9.1	45.8	42.8	16.8	4.1	30.2	40.2	59.5	33.52
Avg. Weekly Sales (units)	1,459.3	19,213.2	796.3	5,108.7	3,797.1	2,713.8	171.9	3,382.1	5,952.8	11,784.3	5,437.95
Avg. Weekly Sales ($)	17,944.00	43,568.20	4,225.31	14,730.13	7,868.22	6,464.63	398.02	8,055.83	5,916.16	9327.23	11,849.77
UPC Chain Price Dispersion (%)											
Avg. Coefficient of Variation	1.57	2.51	2.37	2.66	1.98	2.07	1.97	1.61	2.50	2.10	2.13
Max. Coefficient of Variation	16.39	25.18	22.65	18.75	31.03	21.91	25.61	16.96	16.42	30.76	31.03
Min. Coefficient of Variation	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00
Overall Sample											
Total Number of Chains	51	71	65	70	69	71	54	66	71	67	71
Total Number of Stores	636	964	887	954	942	929	607	929	944	937	964
Total Number of UPCs	109	908	129	473	201	192	51	422	249	489	3,223
Total Sales (millions of units)	15.1	273.3	10.5	71.8	52.9	38.6	1.9	45.3	84.6	160.3	754
Total Sales ($ millions)	185.3	619.7	55.8	207.1	109.6	92.0	4.4	108.4	84.1	126.9	1,593
Avg. Number of Weeks	195.5	192.4	175.1	186.4	181.8	185.8	187.1	174.7	190.7	187.8	1,857
Observations (millions)	1.15	7.13	0.88	4.26	3.86	1.86	0.24	2.34	3.60	4.90	30.22
Empirical Analysis per Approximation											
No. Regressions	5,915	37,018	4,967	22,896	21,195	10,033	1,275	13,378	18,880	26,049	161,605
No. Uniform Pricing Problems	959	4,821	592	3,206	2,953	1,193	221	1,993	2,854	3,987	22,779

Notes: Means of the winsorized sample used for the third degree Müntz-Szász series expansion specification of demand of Table 6. The coefficient of variation is the ratio of the within chain standard deviation of prices divided by the mean chain price × 100. The last column presents a simple average/total across product categories. The number of regressions equals the store average number of UPCs times the total number of stores, which is the number of estimated store-product demands. The number of uniform pricing problems solved equals the chain average number of UPCs times the total number of chains, thus accounting for the number of chain-products offered.

Following the sample selection criteria of DellaVigna and Gentzkow (2019), I exclude chains present only in one geographical market as these cases do not allow to exploit within-chain price variation across markets to compute Hausman instruments and properly estimate demand. I also exclude stores switching chains over the sample period and store-product combinations with positive sales for fewer than 104 weeks. To ensure that the estimation uses only products that are widely available, I only include in the sample those items sold at least 80% of store-weeks across all chains. The final sample includes over 3,000 UPCs across ten product categories. Store average weekly price is the result of dividing each store weekly dollar sale by the number of units sold in each store.

Table 2 presents the descriptive statistics of the samples of each product category used in the estimation. It reports descriptive statistics for prices, price variation, number of products, stores, chains, and sales at different levels of aggregation. Overall, magnitudes are similar to those of other retail studies.[21] Prices of sample products range from \$0.14 for a serving of soup to \$23.86 for the most expensive beer. Price dispersion within categories is particularly important for yogurt, with a coefficient of variation of 0.86 across stores (standard deviation of price over mean price), but rather limited for breakfast cereal with 0.23. It is interesting to note that price dispersion is more muted across the stores of a chain, and in many cases nil, with chains frequently charging the same price across all its stores at a moment in time. Sometimes pricing is substantially different in specific locations, where some expensive products are offered. This price heterogeneity across categories is a nice feature of the data. I show that output and welfare conditions are fulfilled in a similar manner across stores, and thus results are robust to price level and price dispersion.

Store sales are closely related to the number of items available They average only \$37.47 per week for the two household cleaning products offered while they amount to an average of \$1,438 per week when selling nearly forty varieties of carbonated beverages. The number of products of each category varies across stores of a chain, e.g., an average of four household cleaning items and nearly seventy carbonated beverages. Weekly chain average sales range from \$398 for household cleaning products to \$43,568 for carbonated beverages. Overall, the data includes information for 3,223 products sold across 964 stores belonging to 71 supermarket chains, with 754 million units sold for all products across these ten categories. Sales amount to \$1.6bn in total.

Table 3 documents how these magnitudes vary across chain size for the case of carbonated beverages, the category with more units sold and with larger sales. Average prices are lower for larger supermarket chains, particularly those with a very large number of stores. They also offer more variety, leading to larger weekly average sales per store. Price dispersion across stores is similar for all chains except for the largest ones. Pricing is very similar if not identical across many of their stores, reducing the overall within chain price dispersion. The most common chain has between eleven and twenty stores. Qualitatively similar, category-specific descriptive statistics are reported in the Online Appendix for all ten product categories used in the empirical analysis.

[21] Following DellaVigna and Gentzkow (2019), I also winsorize the sample by dropping the observations with store-product estimated demand elasticity outside reasonable bounds, $\hat{\varepsilon}_{sjt}[\hat{x}(p_{sjt})] \notin [1.2, 7]$, or with non-concave revenue function, $\hat{\rho}_{sjt}[\hat{x}(p_{sjt})] > 2$. Thus, the final sample size varies slightly across polynomial approximations.

Table 3: Descriptive Statistics: Carbonated Beverages

		Chain Size (number of stores)			
	All	2–5	6–10	11–20	21–88
Price ($)					
Average Price	2.22	2.35	2.38	2.32	2.04
Standard Deviation	1.21	1.25	1.23	1.22	1.15
Maximum Price	6.00	5.21	5.27	6.00	5.23
Minimum Price	0.23	0.23	0.24	0.24	0.29
Supermarket Stores					
Average Number of UPCs	38.4	25.9	31.6	44.3	36.6
Average Weekly Sales (units)	1,438.2	992.8	1,159.1	1,523.8	1,503.1
Average Weekly Sales ($)	3,261.20	2,335.43	2,920.91	3,689.39	3,061.77
Supermarket Chains					
Average Number of Stores	13.6	3.1	8.1	15.7	34.9
Average Number of UPCs	67.9	31.6	59.4	91.0	90.7
Average Weekly Sales (units)	19,213.2	2,840.2	9,123.6	23,172.9	50,229.7
Average Weekly Sales ($)	43,568.20	6,681.11	22,990.59	56,105.18	102,316.33
Within Chain UPC Price Dispersion (%)					
Average Coefficient of Variation	2.51	2.57	2.37	2.35	2.94
Maximum Coefficient of Variation	25.18	17.43	25.18	15.87	13.40
Minimum Coefficient of Variation	0.00	0.02	0.00	0.00	0.08
Overall Data					
Total Number of Chains	71	19	16	25	11
Total Number of Stores	964	58	130	392	384
Total Number of UPCs	908	213	320	604	415
Total Sales (millions of units)	273.3	10.2	29.4	118.8	114.9
Total Sales ($ millions)	619.7	24.0	74.0	287.6	234.1
Average Number of Weeks	192.4	174.5	194.2	192.8	193.3
Observations (millions)	7.13	0.26	0.80	3.35	2.72

Notes: The first block of price information is measured in dollars. The second one reports the average number of products and sales per store. The third block repeats it by chain and includes also the average number of stores. The next one, price dispersion, reports the average coefficient of variation across chains. The last block reports totals to give an idea of the size of the data.

4.4 Results: Anticompetitive Potential of 3DPD

Demand estimation results vary slightly across demand specifications. These estimates also affect the value of the output and welfare conditions, as well as the criteria used to quantify the importance of violating these conditions, e.g., the share of chains or the share of category sales that fulfill them.

Table 4 reports these statistics for the four demand specifications discussed above, i.e., a third and fourth degree Stone-Weierstrass polynomial, and a third and fourth Müntz-Szász series expansion of demand. The third-degree Müntz-Szász series expansion is slightly preferred as fewer observations get dropped after winsorization, both as a share of chain-products included and as a share of chain sales within the carbonated beverages category. This specification also produces the least elastic demand estimates on average, although admittedly, differences are negligible.

I consider both, a local and global version of welfare conditions, ACV1 and ACV2, as equations (20)-(21) are evaluated either by themselves or together with the *IRC* condition. A first important result is that ACV's *IRC* condition does not hold as freuently as previously anticipated

Table 4: Curvature Tests Summary: Carbonated Beverages

	Chain Products (#)				Chain Sales ($)			
	S.W.(3)	S.W.(4)	M.S.(3)	M.S.(4)	S.W.(3)	S.W.(4)	M.S.(3)	M.S.(4)
Average estimated $\hat{\varepsilon}$	3.31	3.38	3.29	3.35	3.31	3.38	3.29	3.35
Std.Dev of estimated $\hat{\varepsilon}$	1.25	1.32	1.27	1.35	1.25	1.32	1.27	1.35
Chain-Products (surviving)	95.15	91.37	96.04	96.15	92.53	86.34	93.76	93.70
IRC holds for all stores in a chain	35.55	49.53	35.89	37.36	38.32	51.90	38.79	40.06
– Welfare decreases globally	8.91	12.52	8.69	9.30	3.91	6.08	3.23	3.48
– Welfare increases globally	9.00	14.36	9.40	9.66	3.13	5.86	4.04	4.20
Potential welfare increase								
– Output increases	23.30	25.72	23.08	23.33	9.43	11.01	9.69	9.76
– Output increases enough	16.77	20.78	16.41	16.17	6.13	7.99	5.26	5.14
Potential welfare decrease								
– Output decreases	15.80	18.91	15.21	15.68	6.09	9.28	5.07	5.55
– Output does not increase enough	79.17	74.55	78.60	78.58	92.32	89.32	91.97	91.59

Notes: Percentage of chain products or chain sales that fulfill each curvature condition for third and fourth degree Stone-Weierstrass polynomials and Müntz-Szász series expansion specifications of demand.

in the economic theory literature. *IRC* fails more often than not, for somewhere between half and two thirds of chain-products or chain sales. I test whether *IRC* plus ACV1 hold simultaneously, i.e., $IRC \cap ACV1$, which occurs only for $8.69\% - 12.52\%$ of chain-products and $3.23\% - 6.08\%$ of chain sales. I therefore conclude that data is consistent with a global reduction of welfare associated to *3DPD*, only for a few cases. Similarly, it is only possible to show that welfare increases globally with *3DPD*, i.e., $IRC \cap ACV2$, for a few cases as well: $9\% - 14.36\%$ of chain-products and $4.04\% - 5.86\%$ of chain sales.

This evidence could be read in different ways. First, it could be thought of being inconclusive: among the subsample where *IRC* holds, welfare increases or decreases globally for 25% of cases, respectively. Results are inconclusive for the remaining 50% of chain-products. Those ambiguous cases represent nearly two thirds of chain sales. The major hurdle to show that welfare may increase or decreases globally with *3DPD* is that *IRC* does not hold most of the time. Thus, data can only prove an unambiguous welfare results for 20% of chain-products and less than 10% of chain sales. For this reason, the rest of the analysis evaluates output and welfare conditions locally, only in the neighborhood of the optimal uniform price. As I have argued above, rejection of ACV's conditions locally also invalidates them globally.

My preferred reading of the evidence is that data do not support Bork's opinion that *3DPD* will *most likely* increase welfare relative to uniform pricing. Remember that Bork's argument actually referred to output rather than welfare as he treated total industry sales as a proxy for welfare. For welfare to increase, it is necessary that output increases, i.e., $ACV4(+)$ should hold. But the increase in output should be large enough to compensate the misallocation effect of excluding some high value consumers in the strong market to increase sales among low valuation consumers in the weak one, i.e., $ACV4(+) \cap ACV2$.

Table 5: Chain Size and Curvature Conditions: Carbonated Beverages

	No. Chain-Products			Chain-Product Sales		
No. Stores:	All	2–10	11+	All	2–10	11+
Chains (%)						
IRC holds for all stores in a chain	37.36	40.69	23.13	40.06	45.51	36.18
– Welfare decreases globally	9.30	11.35	0.56	3.48	7.88	0.34
– Welfare increases globally	9.66	11.61	1.34	4.20	8.08	1.44
Potential welfare increase						
– Output increases	23.33	27.33	6.26	9.76	19.21	3.02
– Output increases enough	16.17	19.30	2.79	5.14	10.94	1.00
Potential welfare decrease:						
– Output decreases:	15.68	18.88	2.01	5.55	11.58	1.24
– Output does not increase enough	78.58	74.79	94.75	91.59	83.49	97.37

Notes: Welfare decreases globally when, *IRC* and ACV1 hold together, i.e., $IRC \cap ACV1$. Similarly welfare increases globally if $IRC \cap ACV2$. For welfare to increase, it is necessary that output increases, i.e., $ACV4(+)$ should hold. The increase in output should be enough to compensate the misallocation effect to ensure that welfare increases, $ACV4(+) \cap ACV2$. Welfare decreases if output decreases, i.e., when $ACV4(-)$ holds. If output decreases, $ACV4(-)$, or does not increase enough, $ACV2$ fails, *3DPD* has the potential to decrease welfare, i.e., when $ACV4(-) \cup \overline{ACV2}$. Results are based on the the third degree Müntz-Szász series expansion specification of demand.

Results show that output increases between 23.08% and 25.72% of chain-products across demand specifications, although the increase in output is large enough to expend welfare only between 16.41% and 20.78% of cases. Similarly, the output proxy criteria predicts an increase of between 9.43% and 11.01% of chain sales, although this increase is only large enough for welfare to expand between 5.26% and 7.99% of sales. Thus, most cases are still ambiguous. However, the evidence is robust across demand specifications which are flexible enough to accommodate curvature heterogeneity across local markets. The important take-away from this analysis is that focusing on output as a proxy for welfare exaggerates the potential benefits of *3DPD*.

What about possible welfare reductions of *3DPD*? Welfare decreases if output does not increase, when $ACV4(-)$ holds. If output decreases or does not increase enough, $ACV2$ fails. *3DPD* has thus the potential to decrease welfare when $ACV4(-) \cup \overline{ACV2}$. Results now indicate that using the output proxy greatly underestimates the potential welfare reduction induced by *3DPD*. While output is predicted to decrease between 15.21% and 18.91% of chain-products and between 5.07% and 9.28% of sales across demand specifications, welfare might get reduced from 74.55% to 79.19% of chain-products and from 89.32% to 92.32% of sales.

Table 5 explores whether these output and welfare conditions are more likely to hold for smaller or larger chains. *IRC* fails frequently, but failure is even more common for products sold by larger chains. This is reasonable as *IRC* is required to hold for every local store. Demands estimated for products sold by larger chains are inconclusive in nearly all cases when welfare is evaluated globally. Output is predicted to increase (decrease) far more often for products sold by smaller chains. However, using output as a proxy for welfare still exaggerates potential gains and underestimate potential losses induced by *3DPD*, a result that holds regardless of the size of supermarket chains.

Table 6: Summary of Output and Welfare Tests

	Beer	Carb.Bevs.	Coffee	C.Cereal	Fzn.Dinner	Fzn.Pizza	H.Cleaner	S.Snacks	Soup	Yogurt
Average estimated $\hat{\varepsilon}$	4.22	3.29	3.70	3.64	3.57	3.92	3.24	3.16	3.26	3.12
Std.Dev of estimated $\hat{\varepsilon}$	1.43	1.27	1.41	1.39	1.36	1.33	1.48	1.32	1.43	1.30
Chains (#)										
Sample after winsorizing	91.17	96.04	95.45	94.61	97.76	96.51	91.04	98.32	90.70	95.96
IRC holds for all stores in a chain	**37.71**	**35.89**	**36.80**	**45.00**	**43.04**	**40.92**	**35.66**	**35.74**	**36.77**	**39.52**
– Welfare decreases globally	10.80	8.69	7.84	9.44	9.84	7.97	10.25	8.04	9.17	8.25
– Welfare increases globally	14.34	9.40	7.84	11.01	11.10	10.35	12.70	11.75	11.43	10.62
Potential welfare increase:										
– Output increases	**29.14**	**23.08**	**21.64**	**24.73**	**23.60**	**26.29**	**25.82**	**29.99**	**27.88**	**25.42**
– Output increases enough	**24.95**	**16.41**	**14.82**	**16.92**	**17.78**	**16.43**	**22.13**	**22.96**	**19.93**	**19.19**
Potential welfare decrease:										
– Output decreases	**20.48**	**15.21**	**14.31**	**15.20**	**15.95**	**13.39**	**19.26**	**14.68**	**17.32**	**15.63**
– Output does not increase enough	**70.86**	**78.60**	**81.09**	**79.10**	**78.27**	**79.54**	**71.72**	**70.75**	**74.97**	**76.65**
Sales ($)										
Sample after winsorizing	89.18	93.76	94.60	92.91	95.75	96.14	90.57	97.93	85.70	89.75
IRC holds for all stores in a chain	**28.02**	**38.79**	**31.83**	**42.34**	**36.16**	**34.79**	**28.67**	**31.56**	**25.95**	**32.77**
– Welfare decreases globally	3.30	3.23	1.29	4.63	3.53	3.66	3.84	4.41	3.35	3.85
– Welfare increases globally	4.90	4.04	1.64	5.14	3.96	4.40	4.50	5.12	5.63	4.84
Potential welfare increase:										
– Output increases	**15.55**	**9.69**	**11.45**	**14.99**	**11.82**	**13.98**	**12.18**	**16.02**	**17.78**	**14.38**
– Output increases enough	**11.66**	**5.26**	**5.69**	**8.11**	**7.22**	**7.29**	**9.54**	**10.76**	**11.03**	**9.96**
Potential welfare decrease:										
– Output decreases	**8.33**	**5.07**	**3.37**	**6.58**	**7.11**	**5.76**	**7.42**	**7.75**	**7.64**	**7.94**
– Output does not increase enough	**86.21**	**91.97**	**92.91**	**89.40**	**90.54**	**89.65**	**85.67**	**84.31**	**85.14**	**86.81**

Notes: Percentage of chain products or chain sales that fulfill each curvature condition. Results are based on the the third degree Müntz-Szász series expansion specification of demand.

Finally, Table 6 summarizes ACV's output and welfare tests for all ten different product categories for the estimates of the third degree Müntz-Szász series expansion of demand. Average estimated price elasticity ranges from 3.12 for yogurt to 4.22 for beer, with all categories showing a very similar empirical distribution of price responsiveness, many of them with a coefficient of variation close to 0.38. Winsorization and the non-negative demand restriction when evaluated at the optimal uniform price eliminate observations comprising up to 9% of chain-products and 15% of sales. Most frequently, however, only 5% of chain-products and 8% of sales are discarded.

IRC fails always for more than half of the chain products and half of the chain sales across all ten product categories. Using output as a proxy always exaggerates welfare increases of *3DPD* across all product categories, both as share of chain-products or share of chain sales. The most extreme cases is frozen pizza: 26.29% (output) vs. 16.43% (welfare) for chain-products and 13.98% vs. 7.29% for chain sales. Similarly to the carbonated beverages case discussed before, output grossly underestimates the potential welfare reductions of *3DPD*, both for products and sales across all categories. Underestimation of potential welfare losses of *3DPD* is most important for coffee: 14.31% vs. 81.09% for chain-products and 3.37% vs. 92.91% for chain sales.

The picture arising from this analysis is very different from the long-held consensus among theoretical economists regarding the potential gains of *3DPD* that informed Robert Bork's position against restricting the practice of price discrimination. I avoid using parametric demand specifications that might constrain the sign of output welfare predictions. The adopted nonparametric approach is capable of handling demand curvature heterogeneity across local markets and evidence hints at *3DPD* reducing welfare more often than expanding it. The empirical evidence also supports in some sense Bork's view that it is more probably on the whole that *3DPD* increases rather than reduces output. However, Table 6 shows that output predictions are ambiguous for most cases and also that the output proxy overestimates the benefits and underestimates the potential damages of *3DPD*, a result that holds across all product categories.

5 Concluding Remarks

Overall, the results reported in the present paper are not very supportive of *3DPD* even though my evaluation relies on a welfare criteria rather than on a loose definition of fairness. My analysis provides evidence against *3DPD* using Bork's preferred consumer welfare standard, which might perhaps be useful to overcome economists' concerns on the potential anticompetitive effects of *3DPD*, if its main effect is to reduce overall sales as a result of market power.

Callaci, Hanley and Vaheesan (2024, § IV.B) dismiss the usefulness of economic models to understanding the consequences of enforcing the RPA robustly with criticisms that squarely apply to the present work. Their view is shared by many legal scholars bent on rehabilitating the RPA.

A first complaint is that I only deal with the simple case of price discrimination vs. no price discrimination that has dominated the economic literature since Robinson (1933) rather than addressing secret discounts. There are compelling reasons to defend the current approach. It is difficult to study discounts that are secret and therefore non-observable to econometricians unless they use a full-fledged structural model, something that would raise a different kind of concerns and limit the analysis to a few products sold in a handful of stores.

Next, it could be argued that I am using a misguided framework for evaluating the performance of *3DPD* by its output and welfare effects rather than adopting the RPA's normative framework "to ensure fair competition and protect suppliers and retailers from unfair exercises of market power." Economic welfare is the natural measure of well-being of the different actors interacting in the market. Any other arbitrary normative criteria could easily be assessed with an exogenous weighting of profits and consumer rents to incorporate whatever is to be considered *fair*.

And lastly, I could be reminded of not focusing on intermediate goods markets. The data of the present study refers to final consumer products rather than intermediate goods transacted between wholesalers and retailers. Although not the common subject of interest for the RPA, focusing on final products has some advantages. Theoretical output and welfare predictions are based on curvature conditions of demands for final products. Showing that output and welfare might not generally increase with *3DPD* has the potential to extend the influence of the RPA to the analysis of final products, if indeed, price discrimination serves as an effective way for firms with market power to reduce sales. Furthermore, Bork appealed to Robinson's analysis of price discrimination in the market for final goods to justify his legal theory against the RPA. The present paper provides robust empirical evidence that should replace the theoretical intuition dating back a hundred years ago used to justify a lenient treatment of *3DPD*. Future empirical analysis using demand for inputs could further test the implications of Katz (1987) and later works on price discrimination on intermediate goods favoring uniform pricing over *3DPD* from a welfare perspective.

Any empirical work is, by definition, limited in one way or another by the availability of data and the computational complexity of the estimation method employed. Thus, for instance, I treat demand for each product in isolation and do not consider consumers' choice of supermarket because of lack of individual consumer purchases data. On the one hand, estimating demand for all retail products sold by a supermarket is not feasible, on the other theoretical models have only studied welfare conditions for single-product demand models. My approach, similar to DellaVigna and Gentzkow (2019), produces abundant evidence based on the estimation of tens of thousands store-product residual demands, which should make results particularly compelling. Of course, additional evidence is always desirable for policymakers to make fully informed decisions. Thus, for instance, in a recent work, Asil (2024) attempts to measure the trade-off between lower consumer prices induced by wholesale discounts to large retailers and the possibility of higher consumer prices if these discounts induce small retailers to leave the market. I hope the present work inspires other researchers to continue evaluating output and welfare effects of *3DPD* in a wide variety of settings.

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Appendix

A PIGL Demand

The Price Independent Generalized Linear demand system (*PIGL*) of Muellbauer (1975) is:

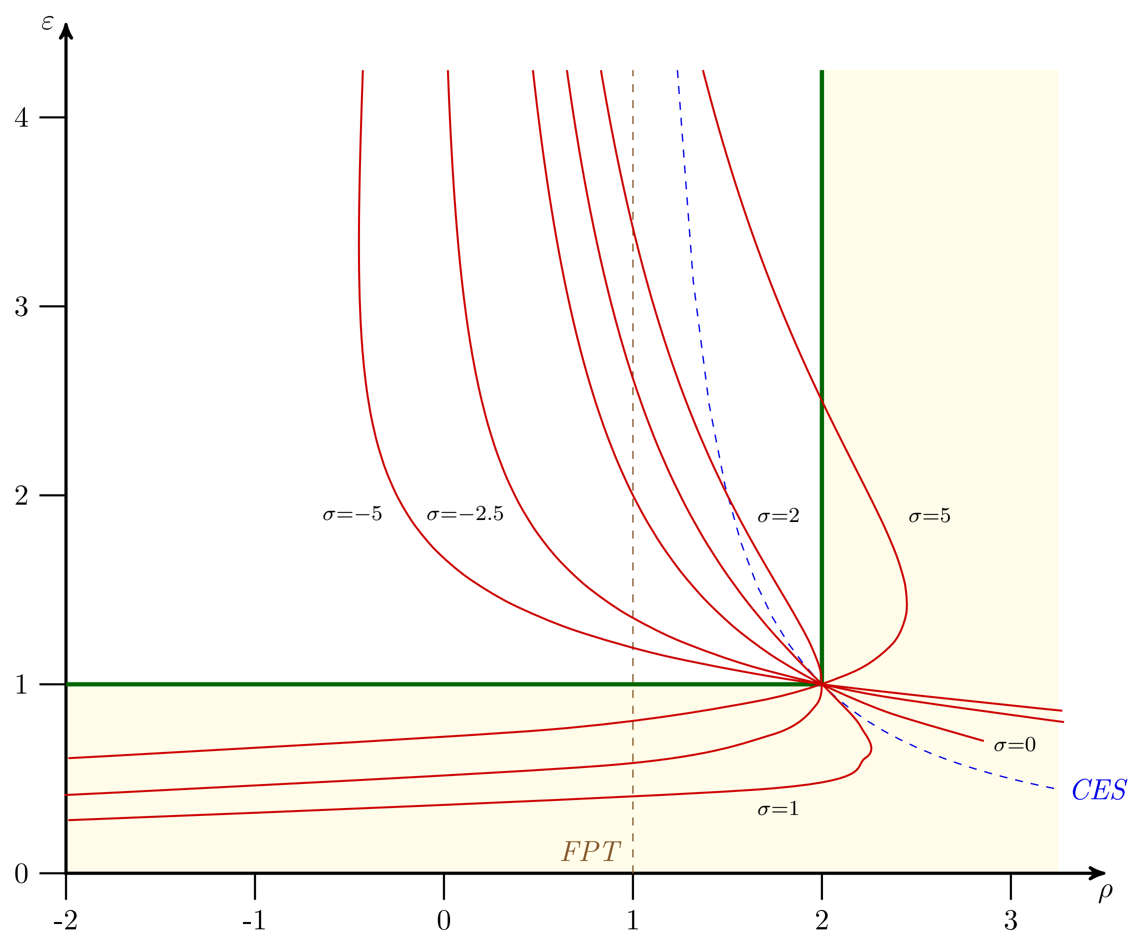
$$x = \frac{\gamma}{p} + \delta p^{-\sigma}\,, \tag{A.1}$$

with demand elasticity and curvature given by:

$$\varepsilon = \frac{\gamma p^{-1} + \sigma\delta p^{-\sigma}}{\gamma p^{-1} + \delta p^{-\sigma}} > 1\,, \qquad \text{and} \qquad \rho = \frac{(\sigma+2)\varepsilon - \sigma}{\varepsilon^2}\,. \tag{A.2}$$

Now the expenditure function is a translated *CES*, $px = \gamma + \delta p^{1-\sigma}$. Among others, *PIGL* demand includes the translog ($\sigma \to 1$) and also the Stone-Geary demand ($\sigma = 0$).

Figure A.1: Demand Manifolds: PIGL



From (A.2) we have:

$$\varepsilon - 1 = \frac{(\sigma-1)\delta p^{-\sigma}}{\gamma p^{-1} + \delta p^{-\sigma}} = (\sigma-1)\frac{\delta p^{1-\sigma}}{\gamma + \delta p^{1-\sigma}} = (\sigma-1)\frac{px-\gamma}{px} > 0\,, \tag{A.3}$$

so that $\text{sign}(\sigma-1) = \text{sign}(\delta) = \text{sign}(px-\gamma)$. In addition, the *PIGL* curvature (A.2) is increasing in σ. For $\sigma > 1$, $\delta > 0$ and demand is more (less) convex than the translog demand for $\sigma < 1$ ($\sigma > 1$). See MN[Appendix B8] for further details on *PIGL* demands.

Differentiating the *PIGL* curvature (A.2) the manifolds bend over when $d\rho/d\varepsilon = 0$, which happens at $\varepsilon = 2\sigma/(\sigma+2)$. After substituting this critical value of ε into (A.2), $\rho = (\sigma+2)^2/4\sigma$. More interestingly for the analysis of *3DPD*, notice that in Figure A.1 manifolds might be upward sloping. This occurs for large values of ε and very negative values of σ in the non-shaded region of interest when $\varepsilon > 2\sigma/(\sigma+2)$ and $\sigma < -2$. For $\sigma = -2$, $\rho = 0$ (linear demand) and the manifold becomes vertical. For smaller values $\sigma < -2$, the manifold becomes upward sloping but demand is always concave, $\rho < 0$. The possibility of upward sloping manifolds thus remains limited to the set of concave and very elastic demands.