Macroeconomic Implications of Long-Term Care Policies*

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Abstract

Governments in countries with aging populations consider and implement various policies in response to an increasing number of elderly in need of care. Using data from the Health and Retirement Study we find that in the U.S. caregivers are primarily family members and that economic variables are predictors in determining whether care is provided informally at home or in a nursing facility. We argue that policy analysis needs to take into account the response of families: how will different families react to long-term care policies? Will these policies provide additional insurance or will they merely crowd out informal insurance? Which family members will benefit from different policies? We address these questions in an overlapping-generations economy with heterogeneous, imperfectly-altruistic agents. In the model, a frail elderly person prefers to obtain care from a family member; the provision of care is determined by a bargaining process between generations. Potential caregivers take into account the other's welfare, foregone wages and the cost of nursing homes. We plan to calibrate the model to data from the Health and Retirement Study in order to assess costs and benefits of tax-financed government policies such as subsidizing informal caregivers or formal care.

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1 Introduction

Long-term care is defined as becoming dependent on assistance from another person due to functional limitations such as having difficulties with activities of daily living (e.g. getting in and out of bed, getting dressed, showering, and eating) or with instrumental activities of daily living (e.g. buying groceries, going to the doctor, and going for a walk). Persons with functional limitations typically reside in the community long before moving to a nursing home and it is often family members (spouse or children) who provide long-term care (LTC). Among children, it is primarily daughters in their prime working-age years who take on the role of the main caregiver with possibly adverse consequences on their labor supply, retirement decision, and human capital.[1]

The increase in female labor force participation together with changes in family structure (e.g. increasing divorce rates, fewer children, etc.) puts growing pressure on governments to play a more active role in LTC provision, especially in countries with aging populations.[2] For the U.S. it is projected[3] that the amount of elderly requiring LTC as a fraction of the 25 to 64 year old population will increase from 6.4% in 2010 to 7.4% in 2020 and 9.6% in 2030. Adding urgency to these developments is the fact that LTC is one of the major uninsured financial risks for elderly Americans (see, for example, Brown & Finkelstein, 2011) – means-tested Medicaid and small private LTC insurance markets leave the elderly largely uninsured.

In this paper we want to understand the potential benefits and costs which arise from tax-financed government policies (e.g. subsidizing nursing homes or informal care). In order to do this we argue that it is essential to take into account the response of informal caregivers. Subsidizing formal care may merely crowd out existing informal care and thus provide only limited additional insurance. A positive effect, however, may be that it crowds in informal caregivers into the labor force or retains them in the labor force; after all, taxes are paid on formal and not informal work. Subsidizing informal care may be expensive and ineffective if it goes primarily to infra-marginal families, e.g. retired spouses. Also, potential caregivers with low wages may choose to provide care at the cost of not working. Finally, any kind of subsidy will have to be financed by distortionary taxation, which is costly.

[1] see Johnson & Sasso (2006) and Van Houtven et al. (2013).

[2] In Germany and Japan, for example, the government has already stepped in; Germany has universal LTC insurance and Japan has universal LTC insurance for ages 65 and above; see Gleckman, 2010.

[3] See Johnson et al. (2007).

A further issue we will assess is how policies affect the old and young generations differently. The elderly may prefer subsidies for informal caregiving whereas the young may favor government-financed nursing homes. Subsidizing informal caregivers allows the frail elderly to stay longer in the community which they may deem more desirable (Ameriks et al., 2011) while a subsidy for nursing homes would do the opposite. For the young, however, a nursing-home subsidy could mean that they can stay working and/or do not face the risk of supporting the parent financially.

We first document the importance of family-provided care in the United States using the Health and Retirement Study (HRS). Our key findings are the following. About 82% of respondents with functional limitations reside in the community.[4] The bulk of hours of care these individuals receive stems from informal caregivers, primarily from the (retired) spouse or from a (working-age) daughter; formal home care plays a minor role.[5] The availability of informal caregivers (spouse, child, or sibling) makes it more likely that a frail elderly resides in the community. Furthermore, families with low-earning children are more likely to receive informal care, whereas, families with high-earning children receive care more often from paid sources such as nursing homes.[6] Finally, a majority of children (53%) who provides an *intensive* level of care (at least 19 weekly hours) does not work.

Armed with the key features of the data we build a model in order to study various tax-financed government policies. In order to accommodate the importance of family-provided care we accommodate a life-cycle model populated by overlapping-generations (OLG) with imperfectly-altruistic families. Each family consists of a young and an old household, which interact strategically.[7] In the young household there is a high-productivity and a low-productivity worker. The high productivity worker always provides work in the market place, whereas

[4]Spillman (2004) using the 1999 National Long-Term Care Survey finds that 70% of frail elderly were in the community and 30% in institutions. The discrepancy between these and our numbers is likely due to how stringent disability is defined. Our sample contains individuals that have at least one limitation either with an ADL or an IADL and have a helper due to these. This relatively lax definition of disability helps to account for all the help hours which we observe in the data.

[5]See also Stoller & Martin, 2002; Wolff & Kasper, 2006; etc. Another way of looking at the importance of informal care is by considering imputations of its economic value. P S Arno & Memmott (1999) provide an estimate of the economic value of informal caregiving of $196 billion in 1997. In contrast, national spending for formal home-health care was $32 billion and for nursing-home care it was $83 billion. Thus, the economic value of informal caregiving was equivalent to approximately 18 percent of total national health care spending ($1,092 billion) in 1997; the AARP estimates that in 2009 the economic value of informal care was $450 billion.

[6]See also Johnson (2008).

[7]Here, we build on previous work by Barczyk & Kredler (2012) and Barczyk & Kredler (2013).

the low-productivity worker faces a choice between market work and informal care. There are also two members in the retired household. When one member suffers an LTC shock it is assumed, as observed in the data, that the spouse provides care. This has currently no implication for nursing-home use but will matter when considering subsidizing informal care. Once the LTC shock has been realized the individual faces a mortality hazard. Upon his death, the other member may experience the LTC shock. Now, there is the option that a member of the young household provides care at home. For the young household, there is a trade-off between market work and providing care. We assume that the old can make financial transfers to the young in exchange for care; informal care is provided if it is beneficial for both households, and the size of the transfer is determined through Nash bargaining.

Depending on how altruistic the young and the old households in a family are they internalize the other's situation to some extent. The decision to provide informal care by the member of the young household does not only depend on her own wage but also on the elderly's financial resources, the price of a nursing home, and the elderly's preference for being taken care of at home by a family member. Vice versa, the decision to demand informal care depends in addition to one's own financial resources, the price of the nursing home and one's dislike for nursing homes also on the young's earnings capability and her other financial resources. In the calibration, we plan to identify the preference parameter to be taken care of at home by a family member such that the model replicates the informal-care decision that we find in logistic regressions in the data.

Relative to the existing literature which studies the importance of health expense risk for savings (e.g. Palumbo, 1999; DeNardi et al., 2010), nursing-home expense risk for savings (Kopecky & Koreshkova, 2012), and the implications of financing Medicare (e.g. Borger & Won, 2008; Attanasio et al., 2010), our main contribution is that we study macroeconomic outcomes when informal caregiving arises endogenously within a family. Furthermore, we contribute to this literature by having a family made up of two decision units, namely, a working household and a retired household who are imperfectly-altruistic towards each other. Modelling a family in this way allows us to make statements about how policies affect the old and young generations differently. The interaction between these households is conceptualized using a Markov game; see also Barczyk (2012) and Kaplan (2012).

2 Empirical Facts

Our modelling choice is motivated by empirical findings from the 2002 wave of the Health and Retirement Study (HRS). Here we provide a relatively concise overview of our data findings and refer the reader to the data appendix for a lengthier discussion.

Advantages of using the HRS

A major advantage of using the 2002 wave of the HRS is that it is nationally representative of both the non-institutionalized and the nursing-home populations.[8] Furthermore, the HRS collects information about respondents' informal caregivers. Importantly, it provides data on the frequency and intensity of care provided by various informal caregivers, such as, the spouse, children, relatives, and friends. This allows us to measure the relative importance among the various informal caregivers as well as across informal and formal care.

Measuring care needs

In the context of LTC, care does not refer to *medical services* which require medically-trained workers, e.g., a physical therapist or a nurse. If a nurse, for example, provides *care* it would be counted as formal home-care but if the nurse provides a service requiring medical expertise it is not counted at all. As is common in the medical literature on LTC, we measure the need for care using an index that counts the number of functional limitations a respondent declares (the index ranges from 1-10). It is made up of limitations that pertain to activities of daily living (ADLs), such as having difficulties with dressing, bathing, and going to bed, as well as difficulties that are instrumental in nature, called instrumental activities of daily living (IADLs), which includes having difficulties with grocery shopping, taking medication, and managing money.

Respondents in need of care

If a survey respondent has functional limitations the HRS inquires about helpers available for each limitation. Correspondingly, it establishes a file for each helper (helper-level files)

[8] Initially, a sampled cohort consists of only non-institutionalized individuals. However, respondents who move to nursing homes after the baseline wave are retained in the study and interviewed whenever possible either directly or through a proxy respondent. Currently, the HRS provides sampling weights for the nursing home population for only the 2000 and 2002 waves but plans to add them for other waves in the future. According to the HRS its sample is fairly well representative of the nursing home population. We find that the distribution of weights for those living at home and the nursing-home population are very similar and so over-attrition for nursing-home residents does not seem to be an issue. Kapteyn et al (2006) find that especially the 2002 wave suffers little from attrition.

to document the time each helper provides for each limitation with an (I)ADL. Table 1 in the data appendix documents some basic facts about respondents who have at least one helper or live in a nursing home. About 82% reside in the community (we will refer to community residents as CR) and 18% in a nursing home (we will refer to nursing-home residents as NHR). NHR are older than CR, are more likely to be female, and suffer from more limitations with (I)ADLs.

Table 2 in the data appendix presents estimated odds ratios from a logistic regression on what factors explain why an individual with LTC needs resides in a nursing home. Respondents which have a partner or have children are significantly more likely to reside in the community than in a nursing home. The availability of a sibling also has a statistically significant negative relationship, but less so than having a spouse or children, with nursing-home admission. Unsurprisingly, the number of limitations with (I)ADLs and age are important predictors of nursing home residency. Finally, having more wealth correlates with a higher incidence of community residency.

Who cares?

We now aim to sort informal caregivers into the following three categories: those who are of working age (and thus face an opportunity cost from care) belong to the category "Young" (Y); informal helpers beyond working age belong to the caregiver category "Old" (O) and those for which we don't know whether they are of working age belong to the category "Other" (Ot). The Y-category includes children, step-children, children-in-law, and grandchildren. The spouse/partner, sisters and brothers of the respondent make up the O-category. The Ot-category consists of other relatives and friends. Formal helpers, other than nursing-home helpers, do not play a substantial role in providing care and we therefore combine all formal helpers into one category "Formal" (F).

Table 3 provides an overview of the prevalence of the various caregiver types. It tells us that a large part of the helping population is made up of informal caregivers (Y+O+Ot) and that Y and O are especially common in providing help. It neglects, however, the intensity of care provided.[9] Table 5 summarizes statistics on monthly hours of care provided by informal and formal caregivers, given that they provide a positive number of hours. Informal caregivers provide the lion's share of all hours of care. Among informal caregivers, Y and O

[9]If, for example, a child provides one hour of help per month it would be counted as a helper in the Y category but for our purposes we would deem its importance to be negligible. On the other hand, while formal helpers are relatively infrequent they may be very important in how much time they devote in providing LTC.

are the most significant contributors in terms of care hours. For NHR formal caregivers are most crucial.

We now construct help-intensity categories. A helper who provides 0-7.5 hours per week falls into the help-intensity category "light", one who provides 7.5-19 weekly hours into "medium", and care of more than 19 hours per week qualifies as "heavy". The idea behind the heavy-helper intensity category is that weekly hours of care is equivalent to at least a part-time job. Medium helpers are those that provide at least one hour of help per day and light helpers provide less.

Table 6 shows that the vast majority of all caregiving hours (85.5%) is provided by about one-third of heavy helpers (32.3%). About two-thirds of heavy helpers are informal caregivers and one-third are formal caregivers. Among informal caregivers Y and O are most important and are of roughly the same importance as measured in terms of frequency and hours contributed. Since the fraction of young heavy helpers is larger than that of old heavy helpers, the mean hours provided by a young heavy helper (243.6 hours) is lower than that of an old heavy helper (264.9 hours). The averages are dramatically lower for medium helpers; a young medium helper provides an average of 43.2 hours per month and an old medium helper a monthly average of 43.2 hours.[10] Light helpers are for our purposes negligible.

Caregiving children vs. non-caregiving children

Table 7 shows that almost half (47.9%) of individuals with children have at least one child caregiver and most (79.5%) of these have exactly one. Among the respondents' children 17% are caregivers to at least one parent and almost all of these provide care to only one parent. This suggests that the caregiving task is not shared among children but is primarily the responsibility of one child. What, if anything, distinguishes a caregiving child from those that do not engage in caregiving activities?

Table 8 provides an overview of some pertinent characteristics of helpers and non-helpers. Caregiving children tend to be older, female, without work, slightly more educated, and have a somewhat lower household income than non-caregiving children.

Table 9 shows that among caregiving children roughly half are light helpers ($<$ 7.5 hours per week) and the other half are medium (7.5-19 weekly hours) or heavy helpers ($>$ 19 hours per week). Table 10 is a counterpart to table 8, except that it also breaks down the helper category by the intensity of care. Perhaps unsurprisingly, differences between children who

[10] Part of the reason that this average is low is that there is clustering around 30 hours, presumably, because many respondents simply answer that they obtain help every day for about one hour.

are heavy helpers and non-helpers as well as between heavy helpers and helpers in general (except median age) are even more pronounced than differences between helpers in general and non-helpers as documented in table 8. The most noteworthy differences between heavy helpers and other and non helpers are as follows. Most (79%) are women. They are more likely to co-reside with their parent(s), less likely to work and a substantial fraction has a fairly low household income (< 35k).

A surprising finding of table 6 was that young heavy helpers provide a high monthly average of help hours and their importance in terms of total hours provided is similar to the spouse/partner of the respondent. For our purposes, we would like to know the employment status of young heavy helpers (table 10 has information on this but here we break the heavy-helper category further down).

Table 11 shows that on average, a heavy-helper child which does not work provides 282 monthly hours of care; this average is 226 monthly hours for those who work part-time and 213 for full-time workers. The median monthly hours of care are substantially below the mean values: 180 for those not working, 124 for part-time, and 150 for full-time heavy-helper children. For 50% of out of work heavy-helper children, providing care amounts to a full-time job (i.e. they give at least 45 hours of care per week); 25% of these provide at least 90 hours of weekly care. Even among those who work full-time, there are 50% who provide care which amounts to almost another full-time job; more than 35 hours per week and 25% help more than 60 hours per week.

Nursing home or home care?

The empirical facts presented so far show that informal helpers play a major role in care-giving. It is their availability, as opposed to formal home-care helpers, that seems to be a crucial factor for an elderly in need of care to stay in the community, see table 2. Children (for single respondents) and the spouse (for partnered respondents) are the most important types of informal caregivers. Thus, studying LTC has to take into account the presence of these informal caregivers.

Economic variables, however, also matter in whether home or formal care takes place. Table 12 provides the results of a logistic regression of the binary variable "nursing-home status" on various characteristics of respondents and their children. The explanatory variable "kid income" is the household income of the child with the lowest household income among all children in the family. The intuition is that the child with the lowest household income has the smallest opportunity cost in providing care. The household income of children is a categorical variable which indicates whether the household income is <10k (category=1), 10k-35k (category=2), 35k-70k (category=3), or >70k (category=4). Since children's household income is a categorical variable the estimated odds ratio is relative to a baseline level of household income which is chosen to be the lowest household income (<10k).

The results show that respondents with higher household-income children are more likely to be in a nursing home controlling for other pertinent factors. For example, if a respondent's child has household income of 35k-70k she is three times more likely to be in a nursing home compared to when the respondent's child household income is <10k. In contrast is the respondent's level of wealth: having more wealth makes it less likely that the respondent is a NHR controlling for other pertinent factors. For example, a respondent who is in the third wealth quartile is almost three times less likely to be in a nursing home compared to a respondent in the lowest wealth quartile.

Table 13 provides estimates of odds ratio of a similar logistic regression replacing household income with the child's education. Education is a categorical variable which indicates whether the kid has less than a high school degree (category=1), a high school degree (category=2), more than a high school degree but less than fours years or post-secondary education (category=3), or at least four years of post-secondary education (category=4). If the respondent has a child which has at least four years of post-secondary education the respondent is two and a half times more likely to be in a nursing home compared to a respondent who has a child with less than a high school degree. The estimates in terms of respondent's wealth are similar to those before.

3 The Model

Setting

Time is continuous. We model a life-cycle economy populated by families of child (young) and parent (old) households. A parent household is indexed by j (age or cohort), where $j \in [T_o, T]$; a child household is indexed by i (age or cohort), where $i \in [T_y, T + (T_o - T_y)]$ and $T_o > T_y$. A family is defined as child household i and parent household j, where $j > i$. An old household is retired for $j \geq T_R$ and dies with certainty when $j = T$. A young household is retired for $i \geq T_R$ and dies with certainty when $i = T + (T_o - T_y)$.

The main source of idiosyncratic risk is an LTC-expenditure hazard. Its arrival is governed by a Poisson rate σ_j, which equals zero for $j \leq T_\sigma$, and increases in j. Once the LTC shock has been realized the need for care persists until death.

An old individual faces a j-dependent death hazard $\delta_j^{\text{ltc}} \geq 0$ where ltc $= 0$ means no care is required and ltc $= 1$ means care is required. It is increasing in j, and for a given j, it is such that $\delta_j^1 > \delta_j^0$ so that needing care increases the likelihood of death. The mortality hazard equals zero for $j \leq T_\delta$. We assume that a household's mortality hazard does not become positive before retirement, and that the LTC hazard does not become positive before the mortality hazard has become positive; to summarize, we have that $T_y < T_o < T_R \leq T_\delta \leq T_\sigma < T$. For simplicity, it is assumed that the young household faces neither LTC-expenditure nor mortality risk for $i \leq T$.

Even though the need for care arrives exogenously, one can choose the source of care: formal care, informal care, or Medicaid. Formal care is provided by a representative nursing home. (Since in the data formal home care plays a relatively minor role we neglect it and use nursing-home care and formal care interchangeably.) It has a per unit cost of κ. The price of a unit of nursing-home care, in terms of the consumption good, is denoted by q.

Parent households with $j > T_\delta$ can be single or partnered. When a partnered household experiences the LTC shock it affects only one household member and care is (freely) provided by the partner, consistent with our empirical findings.[11] We do not allow for the possibility that both members in a partnered household are in need of care (as we have seen, this case is quantitatively negligible).

A single parent household can obtain informal care from a member of the child household in exchange for financial transfer Q. This transfer Q is called out by a Walrasian auctioneer, and households decide on demand $h^D \in \{0, 1\}$ and supply $h^S \in \{0, 1\}$ for home care. Home care takes place if and only if both benefit from it, i.e. $h^D = h^S = 1$. In equilibrium Q will be such that it satisfies symmetric Nash bargaining and so any surplus from home care is split equally. Home care can be decided upon in each instant, and there is no cost of switching from home to formal care or vice versa.

The third option to obtain care – besides nursing and informal care– is to enter a Medicaid-financed nursing home. This is an absorbing state, and the elderly has to hand over all assets and social-security benefits to the government. In return she is provided with a consumption floor c_{ma} by the government until her death.

[11] The inclusion of such households is important for two reasons: first, it may be illegal for the government to target informal-care subsidies to families where caregivers are of a certain age. Second, the number of such natural low-opportunity-cost caregivers is likely to decrease in coming decades, most importantly due to high divorce rates. We will later explore how to incorporate such changes into our model.

Young households have two workers, $s \in \{0, 1\}$, for $i < T_R$. Each worker has a one unit time endowment. Young worker 0 faces a discrete-choice problem at each point in time: to work in the goods sector (g), the formal-care sector (f) or to provide home care (h):

$$l_i^0 \in \{g, f, h\}.$$

Young worker 1 can either work in the goods sector or in the formal-care sector:

$$l_i^1 \in \{g, f\}.$$

All workers have the same efficiency at formal and home care: one unit of time equals one unit of care. Workers vary in their productivity in the goods sector, however, and worker 1 is at least as productive as worker 0, $x_i^1 \geq x_i^0$.

Old households with $j < T_R$ are also working. For simplicity, old household members' productivities are not differentiated on an individual level. Old household's j productivity is denoted by x_j. Efficiency units x display a life-cycle profile over i and j. A household obtains social-security benefits for $i, j > T_R$.

There are two goods in the economy, a consumption good and the good nursing-home care. The consumption good is produced using only labor. Total output of the consumption good is given by

$$Y_t = A_t \underbrace{\int (l_i^0 x_i^0 + l_i^1 x_i^1) di}_{\equiv L^Y} + A_t \underbrace{\int x_j dj}_{\equiv L^O}, \quad \text{for} \quad i, j < T_R,$$

and total output of formal care is given by

$$F = \int (f_i^0 + f_i^1) di.$$

We assume that all markets are competitive. The wage rate for worker s is therefore $w_t^s = A_t x_s$ when working in the goods sector and $w^f = q$ in the formal-care sector.

In terms of financial markets, there is a single risk-free asset that pays rate r of interest. A household can hold a non-negative amount of the asset. Markets to insure against idiosyncratic risks are absent. Furthermore, following Barczyk & Kredler (2013), we assume that in each instant households can give voluntary transfers (gifts) $g_t^y \geq 0$ and $g_t^o \geq 0$ to each other. In equilibrium, this will occur only when one household is in a very favorable situation and the poor household is borrowing-constrained. We omit these transfers from the model presentation in order to focus on the elements that are new in our model and will discuss the constrained case later on.[12] Upon the parent household's death any assets left over go to the child household.

The government has two channels of influencing the choice families make regarding formal and informal care. The first one is to subsidize formal care with subsidies s_f. The second is a subsidy on informal care s_h. It finances the subsidies by levying a labor income tax τ. In each period, the government's budget is balanced:

$$\tau_t(A_t L + F) = s_f F + s_h \int h(j)dj + C_{ma} \int I(ma(j))dj + G, \tag{1}$$

where G are other government expenditures.

Preferences

Felicity for young household i is $u_i^y = u(c_i^y)$. For an old household j not requiring care it is $u_j^o = u(c_j^o)$; for an old household j requiring LTC it is $u_j^o + \eta h_j = u(c_j^o) + \eta h_j$, or $u_j^o = u(c_{ma})$ if in a Medicaid-financed nursing home. The parameter η captures the preference for family-provided home care versus residing in a nursing home, and h is an indicator variable equal to one if home care takes place, $h^D = h^S = 1$, and zero otherwise. The consumption equivalent c_{ma} captures not only the consumption value of Medicaid, but also all inconveniences and potential stigma associated with it.

When the parent household does not require care, the child household's flow utility is

$$U_i^{y,h} = u_i^y + \alpha^y u_j^o,$$

and the parent household's flow utility is

$$U_j^{o,h} = u_j^o + \alpha^o u_i^y.$$

When the parent household requires LTC, the child household's flow utility is

$$U_i^{y,h} = u_i^y + \alpha^y [u_j^o + \eta h_j],$$

[12] Following Barczyk & Kredler (2012), we also introduce a noise term into the model that ensures equilibrium existence; we omit its representation in the presentation in order to economize on notation.

and the parent household's flow utility is

$$U_j^{o,h} = u_j^o + \alpha^o u_i^y + \eta h_j.$$

The parameters $\alpha^y, \alpha^o \in [0, 1]$ represent the child and the parent households' respective degrees of altruism for each other. When we speak of imperfect altruism we mean $\alpha^y \in (0, 1)$ or $\alpha^o \in (0, 1)$.

Decision problems

The state for a household is given by the child household's level of wealth (a^y) and income (w^0, w^1), the parent household's level of wealth (a^o) and income (P), as well as the parent household's LTC status (ltc). We denote the state more compactly by (ω_{ij}, ltc_j), where ω_{ij} summarizes economic variables of family ij, that is, $\omega_{ij} = (a_i^y, w_i^0, w_i^1, a_j^o, P_j)$.

The young household chooses consumption (c), gifts (g) and whether to supply home care (h^S). There is no utility from leisure and so members of the young household spend their one unit of time endowment in the sector in which the wage is largest. When the old household is in need of care, worker 0 of the young household chooses whether to spend her one unit of time endowment in the formal or informal sector. The old household announces whether she demands home care, formal care, or the Medicaid option. The timing of events is as follows. First, the frail elderly decides on whether to rely on Medicaid or not. If she does not enter Medicaid the households decide whether home or formal care takes place. If home care takes place the old pays transfer Q to the young. Next, gifts flow and finally consumption choices are made.

For convenience we define after-tax household earnings including transfers and subsidies for home care as:

$$y^*(w^0, w^1; h^*) = (1 - \tau) \max\{w^1, w^f\} + h^*(Q + s_h) + (1 - h^*)(1 - \tau) \max\{w^0, w^f\}. \quad (2)$$

The first term is the after-tax labor income of worker 1; if the wage rate in the goods sector is higher than in the nursing-care sector one unit of time is spend in the goods sector. Worker 0's problem is the same, where we take the decision of home care as given. The choice of whether to provide home care or not is less obvious and its discussion is postponed to section 4.

We will first discuss the characterization of the young and the old households' decision problems when they are unconstrained. Since gifts only flow if one of the households is constrained we set $g^y = 0 = g^o$. We will discuss the constrained case afterwards.

Child and parent households' problems: no LTC required

The following Hamilton-Jacobi-Bellman equation (HJB) characterizes the decision problem of a young household when the parent household does not require care. The young household takes the parent household's policies as given and chooses c^y such that:

$$\begin{aligned} \rho V^y(\omega, 0) = \max_{c^y} \{u(c^y) + \alpha^y u(c^o) + \dot{a}^y(c^y) V^y_{a^y} + \dot{a}^o(c^o) V^y_{a^o}\} + \\ + \delta^0 [W^y(\chi) - V^y(\omega, 0)] + \sigma[Z^y(\omega, 1) - V^y(\omega, 0)], \end{aligned} \tag{3}$$

where

$$\begin{aligned} Z^y(\omega, 1) &= \mathcal{I}_{ma} MA^y(\chi) + (1 - \mathcal{I}_{ma}) V^y(\omega, 1), \\ \mathcal{I}_{ma} &= 1 \quad \text{if} \quad MA^o(\chi) > V^o(\omega, 1). \end{aligned}$$

First, note that the value $V^y(\omega, 0)$ depends on value functions W^y and Z^y. The value W^y is the young household's value conditional on the old household's death. In that case the state is given by family wealth $A = a^y + a^o$, and young's household income $Y = w^0 + w^1$; the variable χ summarizes this state. The value $Z^y(\omega, 1)$ is realized if the old household requires LTC. In this event the old household may or may not choose Medicaid and the young's value is $MA^y(\chi)$ if she does, and $V^y(\omega, 1)$ if not. The old chooses Medicaid only if $MA^o(\chi) > V^o(\omega, 1)$.

Despite the fact that the young household has no LTC and mortality hazards it faces uncertainties due to the link to the parent household. These are captured by the two final terms of the HJB. The first term is the change in value that results from the death of the old household; the second from a switch in the parent household's LTC status.

We will state the laws of motion $\dot{a}$ shortly. It is worthwhile to point out that the young's optimal consumption choice is as in a conventional consumption-savings problem:

$$u_c(c^y) = V^y_{a^y}(\omega, 0).$$

This is significant because it shows us that the decision maker does not have to contemplate all the possible consumption-savings choices the other household makes. In other words, her best response is constant in the contemporaneous actions of the other.

The parent household takes the child household's policies as given and chooses c^o such that:

$$\rho V^o(\omega, 0) = \max_{c^o} \{u(c^o) + \alpha^o u(c^y) + \dot{a}^o(c^o)V^o_{a^o} + \dot{a}^y(c^y)V^o_{a^y}\} + \\ + \delta^0[\alpha^o W^y(\chi) - V^o(\omega, 0)] + \sigma[Z^o(\omega, 1) - V^o(\omega, 0)], \tag{4}$$

where

$$Z^o(\omega, 1) = \max \{MA^o(\chi), V^o(\omega, 1)\}.$$

Compare this equation with the HJB (3) of the young household. The main difference is that the parent household's value conditional on its death is the young household's value in this case multiplied by the strength of the parent's altruism, i.e., $\alpha^o W^y$. If altruism were absent, $\alpha^o = 0$, the parent would not take into consideration anything beyond her own lifetime. The other terms in HJB (4) are analogous to the young's HJB. As mentioned above, the old chooses Medicaid only if $MA^o(\chi) > V^o(\omega, 1)$ which is summarized by value function Z^o.

Equations (3) and (4) share another common feature. Both households take into account the laws of motion for wealth for households in the family. These are given by

$$\dot{a}^y(c^y) = ra^y + y^*(w^0, w^1; 0) - c^y,$$
$$\dot{a}^o(c^o) = ra^o + P - c^o.$$

Over a small increment of time the change in wealth is determined by how much a household consumes out of income from interest payments and labour/retirement income.

Child and parent households' problems: LTC required

We now turn to the case for which a single parent household requires LTC. For a partnered household this case is straightforward since we have assumed that care is provided freely by the partner. The partnered case will only play a role in the calibration of the model and the policy experiments.

Suppose the parent is currently not in a Medicaid-financed nursing home and so the young's value function in the current period is given by $V^y(\omega, 1)$ and the old's by $V^o(\omega, 1)$. Furthermore, recall that according to our timing protocol laid out before the parent in the current period has to first decide whether or not to enter the Medicaid state. Suppose we are past that decision and the parent has decided against relying on Medicaid, i.e. $Z^o(\omega, 1) =$

$V^o(\omega, 1)$ because $V^o(\omega, 1) > MA^o(\chi)$. Now the young and old must decide on home versus formal care.

Taking the parent-household policies as given, the child household chooses consumption c^y and home-care supply h^S such that:

$$\rho V^y(\omega, 1) = \max_{c^y, h^S \in \{0,1\}} \{u(c^y) + \alpha^y[u(c^o) + \eta h^*] + \dot{a}^y(c^y) Z^y_{a^y} - \dot{a}^o(c^o) Z^y_{a^o}\} + \\ + \delta^1[W^y(\chi) - V^y(\omega, 1)], \tag{5}$$

where

$$h^* = h^D h^S, \quad Z^y_{a^y} = V^y_{a^y}(\omega, 1), \quad \text{and} \quad Z^y_{a^o} = V^y_{a^o}(\omega, 1).$$

The marginal values of saving are evaluated using the value function Z. This is because the marginal value of saving depends on whether "next period" the parent household will be in Medicaid. If so $Z^y_{a^o} = MA^y_{a^0} = 0$ because of means testing, and $Z^y_{a^y} = MA^y_{a^y} > 0$. Because we have assumed that she has decided against it we have that $Z^y_{a^y} = V^y_{a^y}(\omega, 1)$ and $Z^y_{a^o} = V^y_{a^o}(\omega, 1)$.

The parent household takes the young-household policies as given and chooses consumption c^o and home-care demand h^D such that:

$$\rho V^o(\omega, 1) = \max_{c^o, h^D \in \{0,1\}} \{u(c^o) + \eta h^* + \alpha^o u(c^y) + \dot{a}^o(c^o) Z^o_{a^o} + \dot{a}^y(c^y) Z^o_{a^y}\} + \\ + \delta^1[\alpha^o W^y(\chi) - V^o(\omega, 1)], \tag{6}$$

where

$$h^* = h^D h^S, \quad Z^o_{a^o} = V^o_{a^o}(\omega, 1), \quad \text{and} \quad Z^o_{a^y} = V^o_{a^y}(\omega, 1).$$

Because she has decided not to enter Medicaid her marginal values are obtained from $V^o(\omega, 1)$. As for the child household, the parent household's consumption and home care choices are independent of each other and so her consumption is the same under home and formal care.

Both households face the laws of motion for wealth given by

$$\dot{a}^y(c^y) = a^y r + y^*(w^0, w^1; h^*) - c^y,$$
$$\dot{a}^o(c^o) = a^o r + P - h^* Q - (1 - h^*)(q - s_f) - c^o.$$

Were home care to take place $h^* = 1$ and the child household's income would be $y^*(w^0, w^1; 1) =$

$(1-\tau)\max\left\{w^1, w^f\right\} + Q + s_h$. We discuss the determination of the home-care transfer Q in section 4.

Finally, we turn our attention to the case for which the old is currently in the Medicaid-financed nursing home. Recall that this is an absorbing state and thus "next period's" value functions are given by MA^y and MA^o just as in the current period.

The young household's HJB is given by

$$\rho MA^y(\chi) = \max_{c^y}\{u(c^y) + \alpha^y u(c_{ma}) + \dot{A}^y(c^y) MA^y_{a^y}\} + \delta^1[W(\chi) - MA^y(\chi)]. \quad (7)$$

The old household's HJB is given by

$$\rho MA^o(\chi) = u(c_{ma}) + \alpha^o u(c^y) + \dot{A}^y(c^y) MA^o_{a^y}\} + \delta^1[\alpha^o W(\chi) - MA^o(\chi)]. \quad (8)$$

In the Medicaid state the old has no choices. Her value is still changing, however, because of her link to the child household who faces a dynamic-decision problem. Both households take the young's law of motion for wealth into account:

$$\dot{A}^y(c^y) = Ar + Y - c^y.$$

When the old is in the Medicaid state she has no wealth so that family wealth is simply given by the young household's wealth, $A = a^y$.

Child's problem: parent household dead

In order to complete the description of the households' problems we still need to consider the decision problem for a child household after the death of the parent household. For our research question these households are irrelevant and so a simpler decision problem will suffice. The aim is to obtain a reasonable continuation value W, in particular, with a reasonable marginal value of saving. The simplification is that we assume that the child household has no link to another household and thus solves a standard consumption-savings problem with LTC and mortality risks. When requiring LTC the household can either choose formal care or Medicaid and the continuation value beyond the household's lifetime is zero. We augment the state χ with a 0 when the young does not require care and with a 1 when LTC is required.

Recall, the young household faces LTC and mortality hazards for $i > T$, that is, beyond the maximum lifetime of the parent. The young household's HJB when no LTC is required is given by

$$\begin{aligned}
\rho W(\chi, 0) &= \max_{c^y} \left\{ u(c^y) + \dot{A}(c^y) W_a \right\} - \\
&\quad - \delta_i^{\text{ltc}} W(\chi, 0) + \sigma_i [Z(\chi, 1) - W(\chi, 0)], \qquad (9) \\
&\text{s.t.} \\
\delta_i^{\text{ltc}} &= 0, \sigma_i = 0, \quad \text{for} \quad i \leq T \\
\dot{A}(c) &= rA + Y - c^y, \\
Z(\chi, 1) &= \max \left\{ W(\chi, 1), MA \right\}.
\end{aligned}$$

We denote the young household's wealth by A instead of a^y to emphasize that wealth includes any assets left over by the parent household. A household $i > T$ can choose to enter Medicaid conditional on suffering an LTC shock; it would do so if $MA > W(\chi, 1)$.

Suppose the young household requires LTC and has already chosen not to make use of Medicaid. The HJB is given by

$$\begin{aligned}
\rho W(\chi, 1) &= \max_{c^y} \left\{ u(c^y) + \dot{A}(c) Z_A \right\} - \delta_i^1 W(\chi, 1), \qquad (10) \\
&\text{s.t.} \\
\dot{A}(c) &= rA + Y - q - c^y, \\
Z_A &= W_A(\chi, 1).
\end{aligned}$$

The household must pay q for nursing-home care. Here, we assume that "next period" the young household does not choose Medicaid which is why $Z_A = W_A$; if the household chooses Medicaid its marginal value of saving is zero because savings will have no use.

A household in Medicaid has the following HJB

$$\rho MA_i = u(c_{ma}) - \delta_i^1 MA_i. \qquad (11)$$

There are no decisions to be made. The value MA_i decreases in i because mortality hazard increases in i.

Equilibrium Definition

An equilibrium is given by value functions $(V_i^y, MA_i^y, W_i, MA_i)$ and (V_j^o, MA_j^o) for all (i, j), policy rules $(c_i^y, h_i^S, l_i^0, l_i^1, \text{MA choice}_i)$ and $(c_j^o, h_j^D, \text{MA choice}_j)$ for all (i, j), densities λ_{ij} of families over the state space for all (i, j), and a home-care pricing function Q_{ij} for all (i, j), such that, given an initial density $\lambda_{T_y T_o}$, prices (q, w^0, w^1, w^f, r), and a government policy (τ, s_h, s_f, G):

1. value functions $(V_i^y, MA_i^y, W_i, MA_i)$ satisfy equations (3), (5), (7), (9), (10), and (11) the maximum being attained by the policies $(c_i^y, h_i^S, l_i^0, l_i^1, \text{MA choice}_i)$, given policy rules $(c_j^o, h_j^D, \text{MA choice}_j)$, and the home-care pricing function Q_{ij}; after-tax household earnings satisfy (2);

2. value functions (V_j^o, MA_j^o) satisfy equations (4), (6), and (8) the maximum being attained by the policies $(c_j^o, h_j^D, \text{MA choice}_j)$, given policy rules $(c_i^y, h_i^S, l_i^0, l_i^1)$, and the home-care pricing function Q_{ij};

3. firms' decisions are optimal taking prices (q, w^0, w^1, w^f, r) as given.

Determination of prices and the tax rate:

1. the home-care pricing function Q_{ij} is the symmetric-Nash-bargaining solution between child household i and parent household j for all ω;

2. the labor markets clear;

3. the tax rate τ is such that the government's budget is balanced, i.e. (1) holds.

Finally:

1. the densities λ_{ij} are obtained from the initial density $\lambda_{T_y T_o}$ and the above given equilibrium objects;

2. the exogenous parameters are $(\lambda_{T_y T_o}, r, s_h, s_f, G)$.

4 Nursing home or home care?

We now study the choice between home and formal care. When both households are unconstrained the characterization is straightforward. When one (or both) household(s) is (are) constrained the analysis is slightly more involved. Thus, we treat the unconstrained and constrained cases separately.

Recall the timing protocol of the game over a short time horizon dt:

1. The parent decides if to enter Medicaid or not.
2. Nature chooses a transfer $Q \geq 0$ (which equals the Nash bargaining solution in equilibrium) and households decide if to supply or demand home care.
3. If home care takes place, Q is transferred from parent to child; whether or not home care takes place, both may give non-negative altruistic gifts g^y and g^o.
4. After all transfers are handed over, players decide on consumption c^y and c^o.

Given knowledge of the value functions, we will now use backward induction to determine the solution of this game. To focus the discussion on home and formal care we assume that the parent does not choose Medicaid in stage 1.

Unconstrained case

We start with the consumption stage 4. From HJBs (5) and (6) we see that the consumption and the home care choice are independent of each other. Thus, no matter whether home or formal care takes place consumption is the same. This simplifies matters significantly. Denote optimal unconstrained consumption by $c^y = u_c^{-1}(V^y_{a_y})$ and $c^o = u_c^{-1}(V^o_{a_o})$.

In stage 3, equilibrium gifts are zero, $g^y = 0 = g^o$, for any Q. In equilibrium, gifts only flow to constrained recipients. We will turn to the constrained case shortly.

In stage 2, nature chooses a non-negative home-care price Q. Suppose that at this price the parent demands home care. The child supplies home care if her surplus from it is non-negative. We can calculate the surplus using HJB (5):

$$\rho V^y = \max \left\{ \underbrace{u(c^y) + \alpha^y[u(c^o) + \eta] + \dot{a}^y(c^y)V^y_{a^y} + \dot{a}^o(c^o)V^y_{a^o}}_{\text{young's flow value home care} \equiv H^{y|h}}, \right.$$
$$\left. \underbrace{u(c^y) + \alpha^y u(c^o) + \dot{a}^y(c^y)V^y_{a^y} + \dot{a}^o(c^o)V^y_{a^o}}_{\text{young's flow value formal care} \equiv H^{y|f}} \right\} + \text{jump terms},$$

where we substitute the optimal consumption and gift choices from stages 3 and 4. Substituting the laws of motion for wealth:

$$\dot{a}^y(c^y) = a^y r + y^*(w^0, w^1; h^*) - c^y,$$
$$\dot{a}^o(c^o) = a^o r + P - h^*Q - (1 - h^*)(q - s_f) - c^o,$$

the young's surplus is non-negative if $H^{y|h} \geq H^{y|f}$:

$$\underbrace{V^y_{a^y}(Q+s_h) + V^y_{a^o}(q-s_f) + \alpha^y\eta}_{\text{Marginal benefit}} \geq \underbrace{V^y_{a^y}(1-\tau)\max\left\{w^0, w^f\right\} + V^y_{a^o}Q}_{\text{Marginal cost}}.$$

The marginal benefit to the young household of providing care is given by the transfer Q and the home-care subsidy s_h; the young values these using her marginal value of saving, $V^y_{a^y}$ (the only factors a selfish agent would consider). Additionally, the old does not have to purchase formal care so that the young benefits from the foregone expenditures, $q - s_f$, valuing them with $V^y_{a^o}$. Finally, it internalizes a fraction of the parent's preference for family-provided care, $\alpha^y\eta$.

The costs the young incurs from providing home care contains the transfer Q because it comes out of the old's wealth which is valued at $V^y_{a^o}$. It also includes worker 0's wage income (the only factor a selfish agent would consider) because she uses her time endowment for home care instead of working in the market sector.

From this expression we can find all Qs for which the child would be willing to provide home care. The child is just indifferent between home and formal care for the Q that leads to $H^{y|h} = H^{y|f}$ given by:

$$\underline{Q}^S = \frac{V^y_{a^y}(w^0 - s_h) - V^y_{a^o}(q - s_f) - \alpha^y\eta}{\underbrace{V^y_{a^y} - V^y_{a^o}}_{\equiv -\mu^y}}. \tag{12}$$

Thus, for any $Q \geq \underline{Q}^S$ nature chooses the child is willing to supply home care. The difference in marginal valuations μ^y is the transfer motive of the young household. In equilibrium it is negative throughout the state space which says that the young has a higher valuation for her own wealth than for the wealth of the parent household, i.e. $V^y_{a^y} > V^y_{a^o}$. Note that our continuous-time framework allows us to obtain this reservation value in a very simple form when we are given the value function.

Analogously, the parent demands home care if at the given price Q her surplus from home care is non-negative. We can use HJB (6) to find this surplus:

$$\rho V^o = \max \left\{ \underbrace{u(c^o) + \eta + \alpha^o u(c^y) + \dot{a}^o(c^o)V^o_{a^o} + \dot{a}^y(c^y)V^o_{a^y}}_{\text{old's flow value home care} \equiv H^{o|h}}, \right.$$
$$\left. \underbrace{u(c^o) + \alpha^o u(c^y) + \dot{a}^o(a^o)V^o_{c^o} + \dot{a}^y(c^y)V^o_{a^y}}_{\text{old's flow value formal care} \equiv H^{o|f}} \right\} + \text{jump terms},$$

where again optimal consumption and gifts from the previous stages are substituted. Inserting the laws of motion for wealth the parent demands home care if $H^{o|h} \geq H^{o|f}$:

$$V^o_{a^y}(Q + s_h) + V^o_{a^o}(q - s_f) + \eta \geq V^o_{a^y}(1 - \tau) \max\left\{w^0, w^f\right\} + V^o_{a^o}Q.$$

The left-hand side of this equation shows the benefits and the right-hand side the costs to the old household of obtaining home care. The old internalizes the fact that the young gets $Q + s^h$ using $V^o_{a^y}$. She values the fact that she does not have to spend $q - s_f$ on nursing-home care and has additional utility, η, from getting family-provided home care. On the cost side, she has to pay for home care and takes into account that the child foregoes her wage.

From this expression we can find all Qs for which the parent demands home care. The parent is just indifferent between home and formal care for the Q that implies $H^{o|h} = H^{o|f}$ given by:

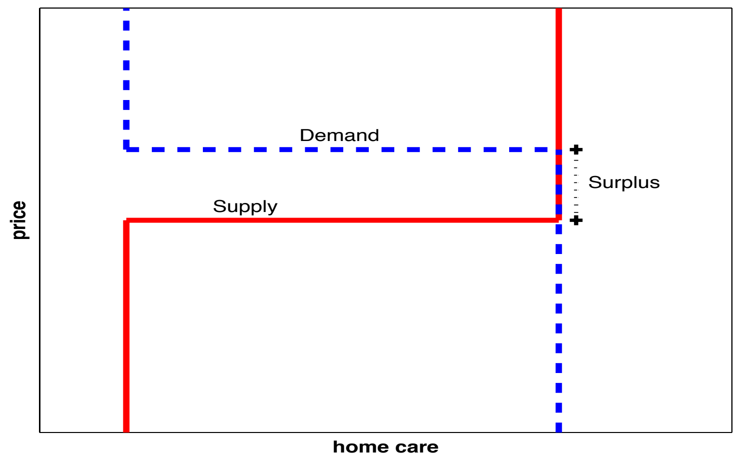
$$\bar{Q}^D = \frac{V^o_{a^y}(w^0 - s_h) - V^o_{a^a}(q - s_f) - \eta}{\underbrace{V^o_{a^y} - V^o_{a^o}}_{\equiv \mu^o}}. \tag{13}$$

For any $Q \leq \bar{Q}^D$ the parent demands home care. The difference in marginal valuations μ^o is the transfer motive of the old household which is negative throughout the state space, i.e. $V^o_{a^o} > V^o_{a^y}$.

Home care will take place if the maximum price the old is willing to pay exceeds the young's reservation price, i.e. $\bar{Q}^D \geq \underline{Q}^S$. If this is the case then any $Q \in [\underline{Q}^S, \bar{Q}^D]$ induces home care. To select one particular price, symmetric Nash-bargaining picks the average of the reservation prices, $Q^* = \frac{1}{2}(\underline{Q}^S + \bar{Q}^D)$. This expression is simple because the payoffs from home care occur over an instant of time are linear.

Figure 1 illustrates a situation in which home care is efficient. The old's reservation value is above the one from the young. The distance between them is the total surplus that arises from home care. With symmetric Nash-bargaining this surplus is shared equally.

Figure 1: Home-care decision



The home-care supply and demand functions. Home-care is a binary choice. It takes place only if the maximum willingness to pay exceeds the reservation price. Because of symmetric Nash-bargaining, the equilibrium price is such that the surplus is equally shared.

Constrained case

We now turn to the determination of the transfer when one (or both) households are at the no-borrowing constraint.

Once again we begin with stage 4. The optimal consumption strategy is either the unconstrained level whenever enough resources are available, or, if not, to consume all available resources (see Barczyk & Kredler, 2012). So players' realized consumption levels c^{o*} and c^{y*} are

$$c^{o*} = \begin{cases} c^o & \text{if } a_o > 0 \\ \min\{c^o, P_j - Q - g^o + g^y\} & \text{if } a_y = 0 \end{cases}, \tag{14}$$

$$c^{y*} = \begin{cases} c^y & \text{if } a_y > 0 \\ \min\{c^y, s_h + P_i^y + Q + g^o - g^y\} & \text{if } a_y = 0. \end{cases} \tag{15}$$

Note that when informal care takes place, the young receives the government subsidy s_h and a pension payment P_i^y if he is retired already ($i > T_R$) as income. The old has the pension P_j as income.

Given these strategies, in the gift-giving stage 3 each agent strives to lift the other agent up to the level of consumption that she desires for the other if the other is below this level. These desired consumption levels are

$$\tilde{c}^o = \begin{cases} \alpha_y^{1/\gamma} c^y & \text{if } a_y > 0, a_o = 0, \\ \min\left\{\alpha_y^{1/\gamma} c^y, \frac{\alpha_y^{1/\gamma}}{1+\alpha_y^{1/\gamma}}(P_j + s_h + P_i^y)\right\} & \text{if } a_y = a_o = 0, \end{cases}$$

$$\tilde{c}^y = \begin{cases} \alpha_o^{1/\gamma} c^o & \text{if } a_y = 0, a_o > 0 \\ \min\left\{\alpha_o^{1/\gamma} c^o, \frac{\alpha_o^{1/\gamma}}{1+\alpha_o^{1/\gamma}}(P_j + s_h + P_i^y)\right\} & \text{if } a_y = a_o = 0. \end{cases}$$

As long as one household in the family has assets available, the desired levels are just a fraction of the household's own unconstrained consumption, their size being governed by the intensity of altruism. When both have run out of assets, the desired levels are given by the agent's desired allocation of the dynasty's flow income $(P_j + s_h + P_i^y)$ under informal care.

Consider first the gift-giving decision of the old. Whenever the transfer Q from the previous stage is not sufficient to lift the young to the desired consumption $\tilde{c}^y$, the old will make up the difference by a subsequent gift g^o. Indeed there is an optimal transfer level Q_o^* for the old: if $Q < Q_o^*$, the old will give a gift $g^o = Q_o^* - Q$ to implement her preferred consumption for the young. If $Q \geq Q_o^*$, the gift is zero. We have

$$g^o = \max\{Q_o^* - Q, 0\},$$

$$\text{where} \quad Q_o^* = \begin{cases} -\infty & \text{if } a_y > 0 \\ \min\{\tilde{c}^y, c^y\} - s_h - P_{y,t} & \text{if } a_y = 0 \end{cases}. \tag{16}$$

We define $Q_o^* = -\infty$ whenever $a_y > 0$ since the old would like to receive an infinitely high flow of transfers since the transfer motive is always negative ($\mu^o < 0$). When $a_y = 0$, we need to make the adjustment $\min\{\tilde{c}^y, c^y\}$ since the donor never wants gifts to go into savings, again since $\mu^o < 0$ (see again Barczyk & Kredler, 2012). We note that Q_o^* may become negative if the young's income $s_h + P_{y,t}$ is high, which would imply that the old would actually like to take away from the young if she could; the optimal gift is thus $g^o = 0$. In the case that the young has no income ($s_h = P_i^y = 0$), Q_o^* is always positive, at least if $\alpha^o > 0$.

Following the same logic for the young, we define the young's desired transfer level as Q_y^* and write

$$g^y = \max\{Q - Q_y^*, 0\},$$
$$\text{where} \quad Q_y^* = \begin{cases} \infty & \text{if } a_o > 0 \\ P_t - \min\{\tilde{c}^o, c^o\} & \text{if } a_o = 0 \end{cases}. \tag{17}$$

The young would like to receive unbounded transfer flows as long as $a_o > 0$, and she would lift the parent up to her desired consumption level (again making sure that transfers do not go into savings since $\mu^y < 0$).

From this it follows that $Q_y^* > Q_o^*$ must hold. Whenever one of the households has positive wealth, this statement is obvious because at least one of the desired transfers is unbounded. When both households are broke, imperfect altruism implies that each player would choose the other to consume less than herself, resulting in the ideal transfer being higher for the young.

We now argue that we only have to consider transfers $Q \in [Q_o^*, Q_y^*]$. To see that we need not consider $Q < Q_o^*$, observe that the old would react to such a low transfer by a gift in the gift-giving stage, lifting up the total amount given to the young to $Q + g^o = Q_o^*$. Thus any transfer $Q < Q_o^*$ will lead to the same consumption-savings allocation and the same surplus as $Q = Q_o^*$, so we may consider these transfers as equivalent and restrict the analysis to $Q \geq Q_o^*$. Similarly, any $Q > Q_y^*$ would be "undone" by a gift from the young, leading to the same allocation and surplus as $Q = Q_y^*$.

A key advantage of restricting the analysis to the interval $Q \in [Q_o^*, Q_y^*]$ is that both agents' surpluses are monotone on this range: the old strictly prefers lower transfers and the young prefers higher transfers, the bounds of the interval being their respective bliss points. Now taking into account the non-negativity constraint on Q, we define the following bounds on the equilibrium transfer:[13]

$$Q_{lb} = \max\{0, Q_o^*\}, \qquad\qquad Q_{ub} = \min\{0, Q_y^*\}. \tag{18}$$

The home-care decision is then characterized as follows:

Proposition (*general characterization of home-care decision*): Let Q_o^* and Q_y^* be defined as in (16) and (17), and let Q_{lb} and Q_{ub} be as defined in (18). Then $Q_o^* < Q_y^*$, and in equilibrium the following hold:

1. If $S_o(Q_{lb}) < 0$ or $S_y(Q_{ub}) < 0$, then $h^* = 0$.

2. If $S_o(Q_{lb}) \geq 0$ and $S_y(Q_{ub}) \geq 0$, then there exist thresholds $\bar{Q}_s \in [Q_{lb}, Q_{ub}]$ and $\bar{Q}_d \in [Q_{lb}, Q_{ub}]$ such that $Q_y(T) \geq 0$ iff $Q \geq \bar{Q}_s$ and $S_o(Q) \geq 0$ iff $Q \leq \bar{Q}_s$.

   (a) If $\bar{Q}_s > \bar{Q}_d$, then $h^* = 0$.

   (b) If $\bar{Q}_s \leq \bar{Q}_d$, then $h^* = 1$ and $Q^* = \max_{T\in[\bar{T}_s,\bar{T}_d]}\{S^y(Q)^\alpha S^o(Q)^{1-\alpha}\}$ and $g^o = 0$. For the young, $g^y = 0$ if $Q_y^* \geq 0$. If $Q_y^* < 0$, then $g^y = -Q_y^* > 0$ and $Q^* = 0$. Consumption is as given in (14) and (15) using $Q = Q^*$.

We now go in detail over the different cases covered by the proposition:

1. If the old is not willing to demand home care even for the lowest-possible transfer (i.e. $S_o(Q_{lb}) < 0$), or the young is not willing to provide care for the highest-possible transfer ($S_y(Q_{ub}) < 0$), then no home care takes place.

2. If both the young and the old are willing to consider home care under some transfer, we can find a reservation transfer $\bar{Q}_s \in [Q_{lb}, Q_{ub}]$ above which the young is willing to set $h_s(Q) = 1$. Note that this reservation transfer may be equal to Q_{lb} and/or to zero if $S_y(Q_{lb}) \geq 0$. Also, there exists a willingness to pay $\bar{Q}_d \in [Q_{lb}, Q_{ub}]$ below which the old sets $h_d = 1$. This willingness to pay may equal Q_{ub} if $S_o(Q_{ub}) \geq 0$. We can distinguish the following two cases according to the ordering of $\bar{Q}_s$ and $\bar{Q}_d$:

[13] In our computations, we also impose an upper bound $Q_{max} < \infty$ on Q_y^* for computational purposes. In regions where the old is wealth-rich but faces only a short time to live, the young can essentially count on possessing all dynasty wealth within little time and thus the timing of transfers becomes inessential for players. In such regions players are essentially pooling their wealth, transfer motives being close to zero. This can lead equilibrium transfers to reach very high levels (see equation (13)), which has no implications on the allocation but slows down our algorithm considerably.

(a) $\bar{Q}_s > \bar{Q}_d$: there is no Q such that both agents have a positive surplus and thus $h^* = 0$.

(b) $\bar{Q}_s \leq \bar{Q}_d$: the surplus is positive for both agents on $Q \in [\bar{Q}_s, \bar{Q}_d]$, thus $h^* = 1$. We can find the Nash-bargaining solution Q^* by evaluating its first-order condition for Q on $Q \in [\bar{Q}_s, \bar{Q}_d]$, which can be shown to be decreasing on $[\bar{Q}_s, \bar{Q}_d]$. The following sub-cases are of interest:

   i. $Q_{lb} = Q_{ub} = 0$: This case arises when the young is not willing to accept a transfer $Q > 0$ from the old and would undo this by an altruistic gift, i.e. $Q_y^* < 0$. In this case we only have to check if both agents prefer home care to formal care for $Q = 0$, in which case home care takes place.

   ii. $Q_{lb} = 0 < Q_{ub}$: The old's bliss point is such that she would prefer not to give any transfer, i.e. $Q_o^* = 0$. In this case a corner solution $Q^* = 0$ may arise, which is characterized by the Nash-bargaining FOC being negative at $Q = 0$.

   iii. $0 < Q_{lb} < Q_{ub}$: In this case we typically find an interior solution, which may be identified by finding the root of the Nash-bargaining FOC on (Q_{lb}, Q_{ub}).[14]

Finally, we note that the case where both players are unconstrained is included as a special case covered in point 2 of the proposition.

5 Calibration

Under construction...

6 Results

To help us understand the key mechanisms of the actual model we begin by studying a simpler version of the model; furthermore, we focus on the stationary equilibrium.

[14] If the donor is able to implement the preferred consumption for the recipient, which amounts $\alpha_y^{1/\gamma} c_o < c_y$ for the old, then a corner solution cannot occur. The reason is that the derivative of the surplus $S^o(Q)$ is zero at $Q^{lb} = Q_o^*$. At this point the surplus of the young could be increased without lowering the surplus of the old, thus ruling it out as the bargaining solution if $\alpha < 1$.

There are two infinitely-lived agents referred to as young and old. Both have deterministic income streams. The only differences between the young and the old agent are that the old may have to acquire a source of LTC and the young may provide home care; the old has a preference for home care; the degrees of altruism may differ.

The young faces a standard consumption-savings problem with the possibility of gifts. When the old requires LTC the young also has the choice between working at the market wage w or provide LTC in exchange for a transfer T from the old. The old also faces a standard consumption-savings problem with the possibility of gifts. When she requires LTC, the old either obtains informal care in exchange for a transfer T, purchases formal care at a per unit price p or gives up assets and income forever and enters an absorbing state in which consumption c_{ma} is provided; this state is interpreted as residing in a Medicaid-financed nursing home (MA).

In this modified setting, the state for a household is given by the young's level of wealth (a^y), the old's level of wealth (a^o) and whether the old requires LTC (1) or not (0).

For ease of presentation we show the HJBs neglecting the choice of gifts. Gifts only flow to constrained individuals and so the HJBs as presented here represent restrictions which need to be satisfied by unconstrained choices. When the old does not require LTC, the old's and young's HJBs are given by:

$$
\begin{aligned}
\rho V^o(\omega,0) &= \max_{c^o}\left\{u(c^o)+\alpha^o u(c^y)+\dot{a}^o V^o_{a^o}(\omega,0)+\dot{a}^y V^o_{a^y}(\omega,0)\right\}+\\
&\qquad +\sigma[\underbrace{\max\left\{V^o(\omega,1),W^o(a^y)\right\}}_{Z^o(\omega)}-V^o(\omega,0)].\\
\rho V^y(\omega,0) &= \max_{c^y}\left\{u(c^y)+\alpha^y u(c^o)+\dot{a}^y V^y_{a^y}(\omega,0)+\dot{a}^o V^y_{a^o}(\omega,0)\right\}+\\
&\qquad +\sigma[\underbrace{\left\{V^y(\omega,1),W^y(a^y)\right\}}_{Z^y(\omega)}-V^y(\omega,0)].\\
&\text{s.t.}\\
\dot{a}^o &= ra^o+P-c^o,\\
\dot{a}^y &= ra^y+w-c^y.
\end{aligned}
$$

The term that is multiplied by σ captures the change in value from the state in which the old does not require LTC and the state in which she does require LTC. For the old this change is given by $[\max\left\{V^o(\omega,1),W^o(a^y)\right\}-V^o(\omega,0)]$. If the old does require LTC she faces the choice between entering the MA state valued at $W^o(a^y)$ or not. Not entering the MA state is valued at $V^o(\omega, 1)$. The young's value changes to $W^y(a^y)$ if the old chooses the MA option and to $V^y(\omega, 1)$ if not.

$V^0(\cdot, 1)$ and $V^y(\cdot, 1)$ satisfy the following HJBs:

$$\begin{aligned}
\rho V^o(\omega,1) &= \max_{c^o, h^D \in \{0,1\}} \{u(c^o) + \eta h^* + \alpha^o u(c^y) + \dot{a}^o(h^*) Z^o_{a^o}(\omega) + \dot{a}^y(h^*) Z^o_{a^y}(\omega)\} \\
\rho V^y(\omega,1) &= \max_{c^y, h^S \in \{0,1\}} \{u(c^y) + \alpha^y[\eta h^* + u(c^o)] + \dot{a}^y(h^*) Z^y_{a^y}(\omega) + \dot{a}^o(h^*) Z^y_{a^o}(\omega)\} \\
&\quad \text{s.t.} \\
&\dot{a}^y(h^*) = r a^y + h^*(T + s_h) + (1 - h^*) w - c^y, \\
&\dot{a}^o(h^*) = r a^o + P - h^* T - (1 - h^*)(p - s_f) - c^o.
\end{aligned}$$

Once the old requires LTC there are no sources of uncertainty. Recall that $h^* = 1$ if home care takes place and 0 otherwise. In addition to the transfer T the young's law of motion now includes a subsidy $s^h \geq 0$ for informal care and the old's law of motion includes a subsidy $s^f \geq 0$ for formal care.

When the old enters the Medicaid-financed nursing home the old and young's value functions satisfy:

$$\begin{aligned}
\rho W^o(a^y) &= u(c_{ma}) + \alpha^o u(c^y) + \dot{a}^y W^o_{a^y}(a^y), \\
\rho W^y(a^y) &= \max_{c^y} \{u(c^y) + \alpha^o u(c^y) + \dot{a}^y W^o_{a^y}(a^y)\}, \\
&\quad \text{s.t.} \\
&\quad \dot{a}^y = r a^y + w - c^y.
\end{aligned}$$

The old faces no more choices but her value still changes due to the presence of the young. The young faces a standard consumption-savings problem.

The young's reservation transfer to supply home care is:

$$\underline{T}^S = \frac{V^y_{a^y}(w - s_h) - V^y_{a^o}(p - s_f) - \alpha^y \eta}{\underbrace{V^y_{a^y} - V^y_{a^o}}_{\equiv -\mu^y}},$$

The old's maximum transfer she is willing to pay for home care is:

$$\bar{T}^D = \frac{\eta + V^o_{a^o}(p - s_f) - V^o_{a^y}(w - s_h)}{\underbrace{V^o_{a^o} - V^o_{a^y}}_{\equiv -\mu^o}}.$$

Table 2 summarizes the baseline parameter values used for the numerical example.

Figure 2: Baseline parameters

Parameter	Value	Parameter	Value
α^o	0.3	w	22
α^y	0.1	P	20
γ	2	p	18
ρ	3%	σ	15%
r	2%	c_{ma}	6
s_h, s_f	0	η	0.007

The wage of the young is w and the pension of the old is P. The price of formal care is p and Medicaid consumption is c_{ma}. The hazard of requiring LTC is σ and home-care preference is η. The home-care subsidy is denoted by s_h and the formal-care subsidy by s_f.

The parameter values are chosen in such a way as to generate an instructive laboratory for a variety of cases which the model can generate. Nonetheless, the chosen values are not unreasonable. In particular, the degrees of altruism are in line with the calibration by Barczyk (2012) who studies an OLG model with young and old imperfectly-altruistic agents. The size of the coefficient of relative risk aversion (γ) is a common choice in macroeconomics. The Medicaid-financed consumption floor has been chosen so that Medicaid is chosen in at least one state (when both are broke) in the benchmark case without subsidies. The hazard of requiring LTC is on the larger side. In this simplified version of the model, however, qualitatively the results would not change when changing this hazard rate. The main leap of faith is taken with respect to the size of the home-care preference parameter. A priory we know very little about what would constitute a reasonable magnitude for this preference parameter. In future versions of the paper we will tackle the calibration of this parameter in a careful manner.

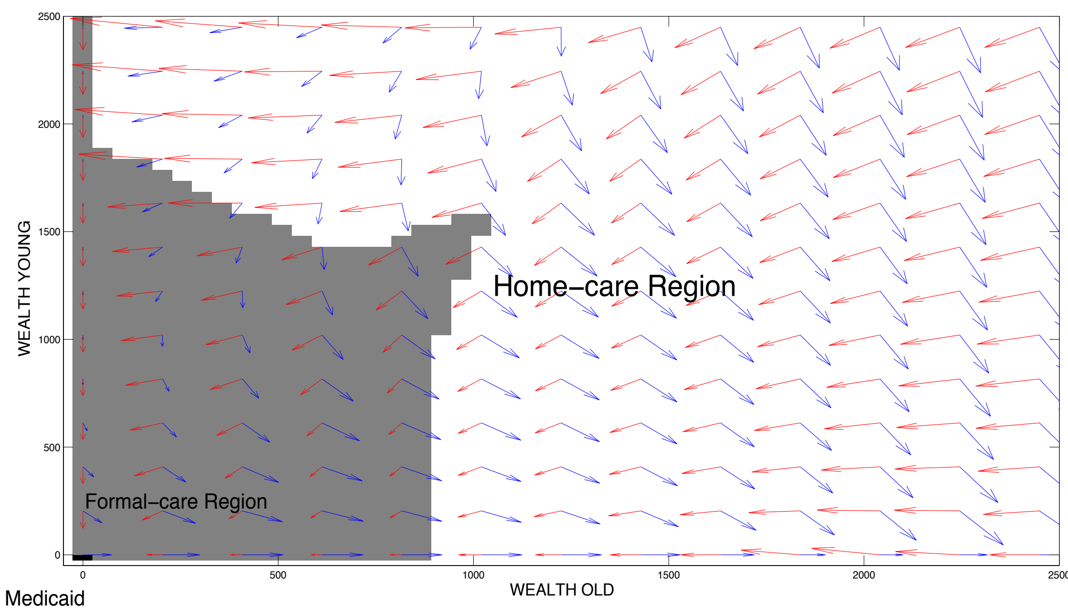
We begin by considering the laws of motion for the young's and the old's wealth in the benchmark economy without subsidies. Figure 3 shows $\dot{a}^y$ and $\dot{a}^o$ when the old does not require LTC (blue) and when she does require LTC (red). The horizontal axis traces out the wealth of the old; along the vertical axis is the young's wealth. An arrow emanates from a particular point in the discretized state space (a^o, a^y). If an arrow points horizontally to the right ($\rightarrow$) it indicates that the old is saving ($\dot{a}^o > 0$) and the young consumes her cash-on-hand ($\dot{a}^y = 0$) for the given combination of wealth levels. If an arrow points vertically up ($\uparrow$) it means that the young is saving ($\dot{a}^y > 0$) and the old consumes her cash-on-hand ($\dot{a}^o = 0$) for the given combination of wealth levels. The length of an arrow signifies the magnitude of the savings behavior; e.g. a longer arrow means that the economy moves more quickly in the direction indicated by the arrow. The area in white corresponds to the region of the state space in which home care takes place; the grey area is the region in which the old makes use of formal care. When both are broke ($a^o = 0 = a^y$) the old enters the Medicaid state. These regions, of course, only exist when the old requires LTC.

First focus on the law of motion in the case when the old does not require LTC (the arrows in blue). We see that along the horizontal axis the old is saving while the young consumes hand-to-mouth. The old displays typical precautionary savings behavior building up a buffer of wealth in anticipation of the LTC shock. After all, the LTC shock is akin to a permanent reduction in the old's income. Furthermore, the old wants to save herself away from the Medicaid state which occurs when both are broke, i.e. when $a^o = 0 = a^y$. The young does not share these concerns. When she has no wealth, she does not, and will not, provide gifts to the old; if she would, she would share the concern for precautionary saving. Also, the young does not expect her income to drop in case the LTC shock hits the old; in contrast, she might expect her income to rise if home care takes place at a transfer which is larger than her wage. Thus, when the young is broke, she simply consumes what she has on hand.

As we move away from the horizontal axis into the state space the same type of consumption-saving behavior persists: the old engages in precautionary savings while the young dis-saves. As a matter of fact, the young dis-saves throughout the entire state space when the old does not require LTC (recall that the rate-of-time preference ρ is larger than the interest rate r). The old's saving behavior is, however, more intricate.

Consider the upper-left corner where the young is rich and the old is getting poorer, say, the area to the left of $a^o = 900$ and above $a^y = 2000$. In this part of the state space the old's saving behavior changes from saving to dis-saving. This is what Barczyk & Kredler (2013) refer to as the *dynamic Samaritan's dilemma*. The old expects that she will eventually obtain gifts from the young, namely, when she is broke and he is very rich. In anticipation of this she consumes down her resources at an inefficiently high rate (figure 9 shows that the young provides gifts to the old when she is broke and he is rich). Furthermore, she knows that if the LTC shock hits she can rely on the young to subsidize her purchase of formal care (again, figure 9 shows that the young provides gifts to the old when she is broke and requires LTC). Thus, her precautionary savings motive is overshadowed in this region of the state space. In essence, the young is doing the precautionary savings on behalf of the old.

Figure 3: Laws of Motion: $w = 22$



The blue arrows represent the law of motion for a^y and a^o when the old does not require LTC and the red arrows represent the law of motion when she requires LTC. The area in white corresponds to the region of the state space in which home care takes place; the grey area is the region in which the old makes use of formal care. When both are broke ($a^o = 0 = a^y$) the old enters the Medicaid state.

As we lower the young's wealth note that the old's savings behavior stirs gradually away from the vertical axis. This happens because for once gifts are decreasing as the young becomes poorer and, second, the time to Medicaid is shortened. Both forces act as an incentive for the old to save.

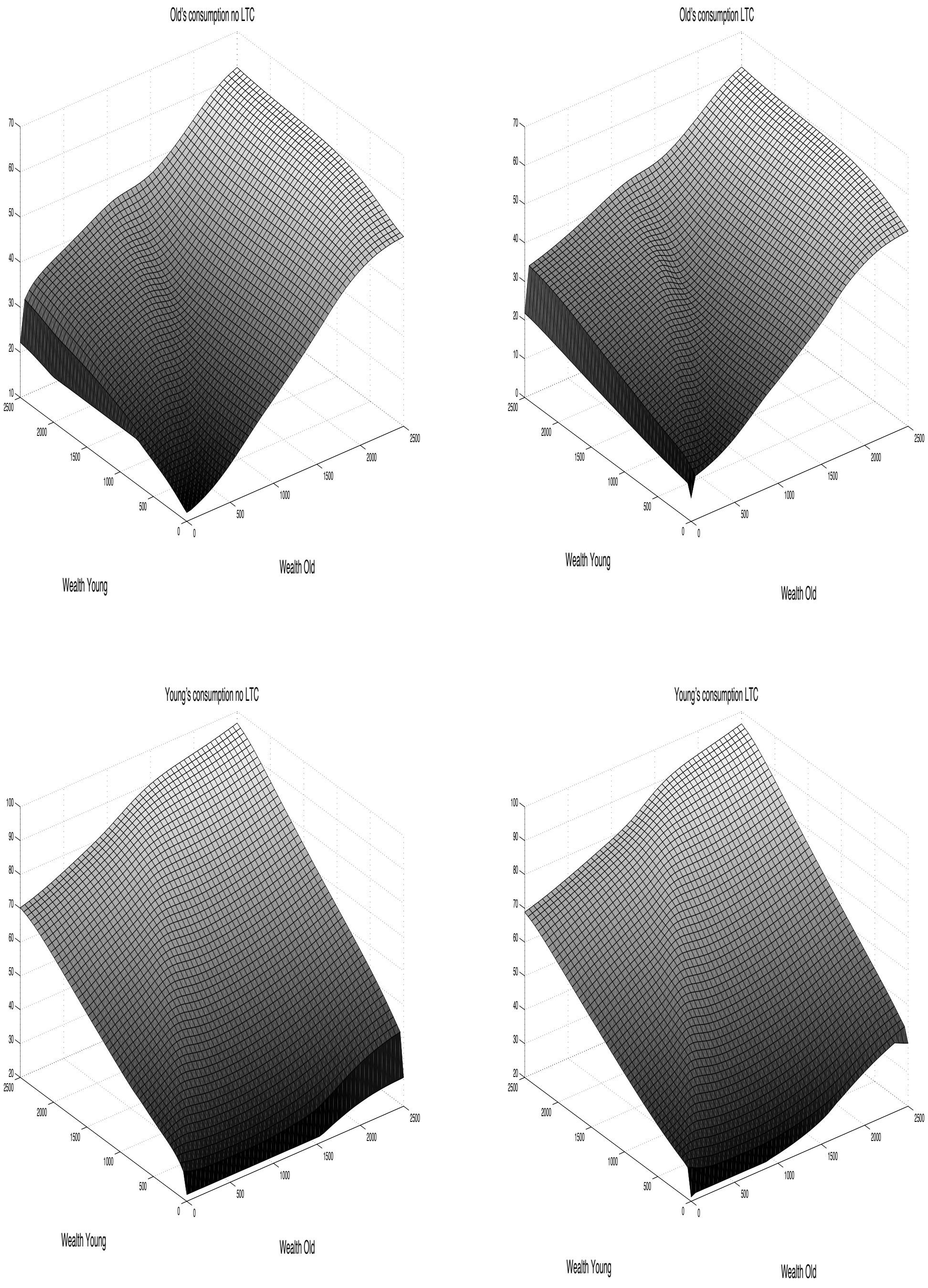
Now focus on the law of motion when the old requires LTC (the arrows in red). In the vast majority of the state space both the young and the old now dis-save. Since there is no "coming back" from the LTC state and the rate-of-time preference exceeds the interest rate this consumption-savings behavior should not come as a big surprise.

It is noteworthy that in regions of the state space in which the intra-family wealth distribution is tilted in favor of the old the young does not stir the economy towards states where she has no wealth. In contrast, when the intra-family distribution is such that the young is relatively rich the economy does head rapidly to states where the old has no wealth. This feature is, once again, the dynamic Samaritan's dilemma. Interestingly, for the old the introduction of exchange-motivated transfers has removed this dilemma; at least in the sense that incentives for the young are removed to stir the economy towards her bankruptcy. The young is still confronted with the dynamic Samaritan's dilemma as can be seen from the arrows pointing towards the old's bankruptcy when the young is rich. The old overconsumes in anticipation of obtaining gifts from the young. This overconsumption is similar to the inefficiency which arises in the tragedy-of-the-commons. However, this overconsumption occurs long before the young and old consume out of common resources, namely, the young's wealth.

The consumption functions shown in figure 4 help to reinforce some of the equilibrium features just discussed. The upper panel shows the old's consumption when she does not require LTC (left) and when she requires LTC (right). The bottom panel is the counterpart for the young's consumption. The x-y plane corresponds to the old's and the young's wealth.

Barczyk & Kredler (2013) thoroughly discuss the qualitative features of the consumption functions in a model with income risk. These are qualitatively the same as those in the upper panel as well as the young's consumption function when the old does not require LTC. The most striking feature of these consumption functions is that in parts of the state space where one of the agents has no wealth and gifts flow there is a discrete drop in consumption. For example, consider the old's consumption function when she requires LTC. When the old has no wealth, the young provides gifts to the old to subsidize the purchase of formal care for all levels of the young's wealth except when she is also broke in which case the old relies only on Medicaid (see figure 9). Upon entering this region, the old's consumption jumps down discretely; this is the counterpart to the well-known Samaritan's dilemma from two-period models. Prior to this happening is what Barczyk & Kredler (2013) refer to as the dynamic Samaritan's dilemma. Distortions from the borders of the state space feed back into the state space causing consumption decisions to be distorted long before actual gifts

Figure 4: Consumption Functions



The top panel displays the old's consumption function when she does not require LTC (left) and when she requires LTC (right). The bottom panel is the counterpart for the young's consumption.

are observed. This also explains the non-monotonicity of the old's consumption as a function of her own wealth (the "bump" in her consumption function).

A novel feature of the current setting can be gleaned from comparing the young's consumption functions when the old does not require and requires LTC. When the old does not require LTC she provides gifts to the young when her wealth level is roughly above 1500 and the young has no wealth. In this case we see that the young's consumption displays the downward jump. In contrast, when the old requires LTC, this downward jump disappears. The reason is that in this part of the state space home care takes place. Consumption for the young does not have to decrease since the young still has some control over the allocation. Without home care the old chooses gifts in such a way which is in her best interest and temporarily becomes the family dictator. With exchange-motivated transfers this is no longer the case.

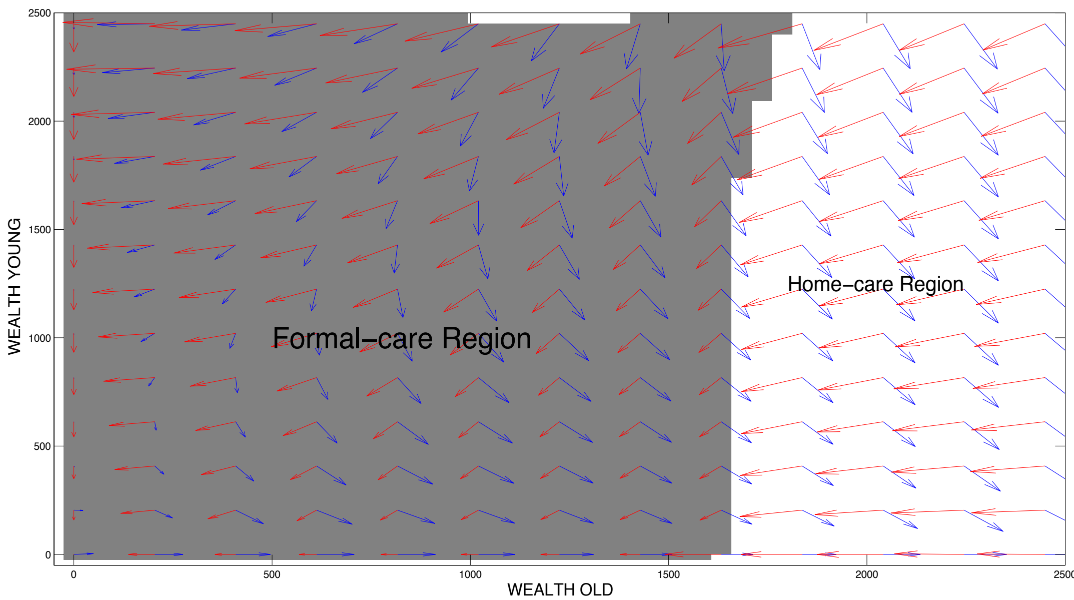
6.1 Comparative Statics

6.1.1 An Increase in the Young's Wage

An important feature of the data is that home care is less likely the more income a potential caregiver has. The model's prediction is consistent with this empirical observation. To demonstrate this we increase the young's wage from $w = 22$ to $w = 30$. Figure 5 shows the laws of motion and the care regions for this case.

Comparing this figure with figure 3 we can quickly see that the formal care region has become substantially larger. Whereas the cutoff value of the old's wealth for home care to take place is roughly $a^o = 900$ when $w = 22$ this cutoff value is now around $a^o = 1700$. The primary reason for the smaller home-care region is that the young's reservation transfer for home care increases when her wage increases; her opportunity cost of providing care has increased. Furthermore, the old's maximum willingness to pay decreases since the old internalizes the young's increased opportunity cost from providing care. The intuition behind the economy's dynamics is the same as discussed for the benchmark economy.

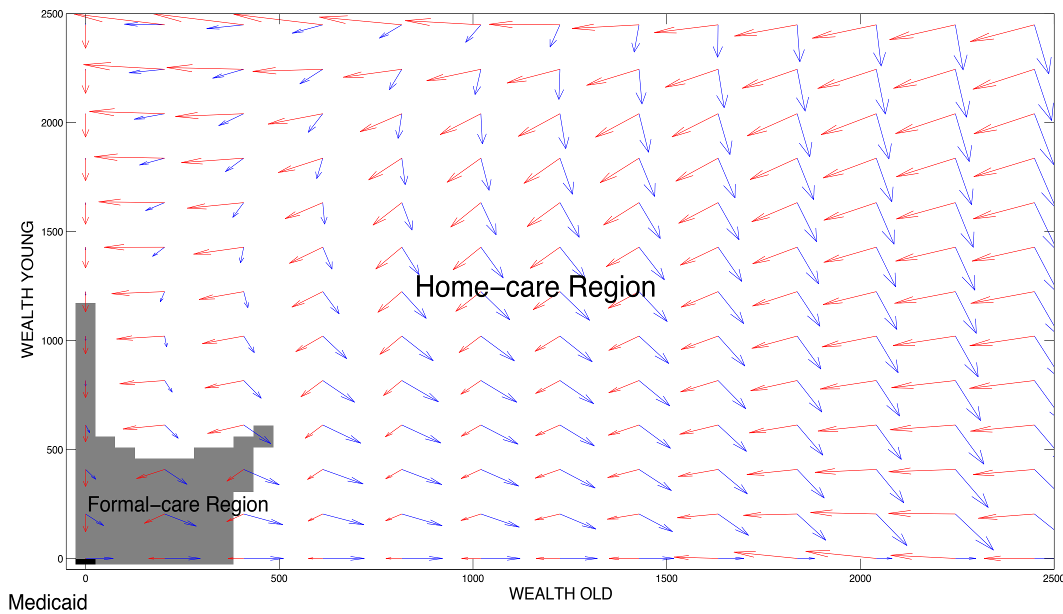
Figure 5: Laws of Motion: $w = 30$



6.1.2 Home-care vs. Formal-care Subsidies

We now return to the baseline parameterization of the economy and study two simple policy experiments. In the first one we introduce a subsidy to only home care and in the second one we introduce a subsidy to only formal care. These experiments are too simple in order to be taken quantitatively seriously (e.g. we neglect the way they are financed for now). Nonetheless they are useful to help us understand the workings of the model. Furthermore, they provide a first approximation as to which policy the young and the old generations would prefer if they were given a choice between the two.

Figure 6: Laws of Motion: Home-care Subsidy



The blue arrows represent the law of motion for a^y and a^o when the old does not require LTC and the red arrows when she requires LTC.

Figure 6 shows the laws of motion and the care regions for the case in which a home-care subsidy of $s_h = 2.5$ is introduced. The home care region increases substantially compared to the one from the benchmark economy. Two main effects account for this dramatic change in the size of the regions. First, the young's reservation transfer for care decreases; providing home care is now associated with a lower opportunity cost. Second, the old's maximum willingness to pay for home care increases since once again she internalizes the reduction in the young's opportunity cost. Both of these changes act in the direction of making home care more a more attractive option. The economy's dynamics are qualitatively unchanged.

Figure 7 presents the laws of motion and the type-of-care regions for the case in which a formal-care subsidy of $s_f = 2.5$ is introduced. The formal care region becomes larger. When formal care is subsidized the old's maximum willingness to pay for home care decreases simply because formal care has become cheaper. The young now knows that when the old purchases formal care less of her resources will be exhausted; the young becomes less willing to provide home care. Both forces act in the direction of making home care less attractive. The economy's dynamics are qualitatively unchanged.

Figure 7: Laws of Motion: Formal-care Subsidy

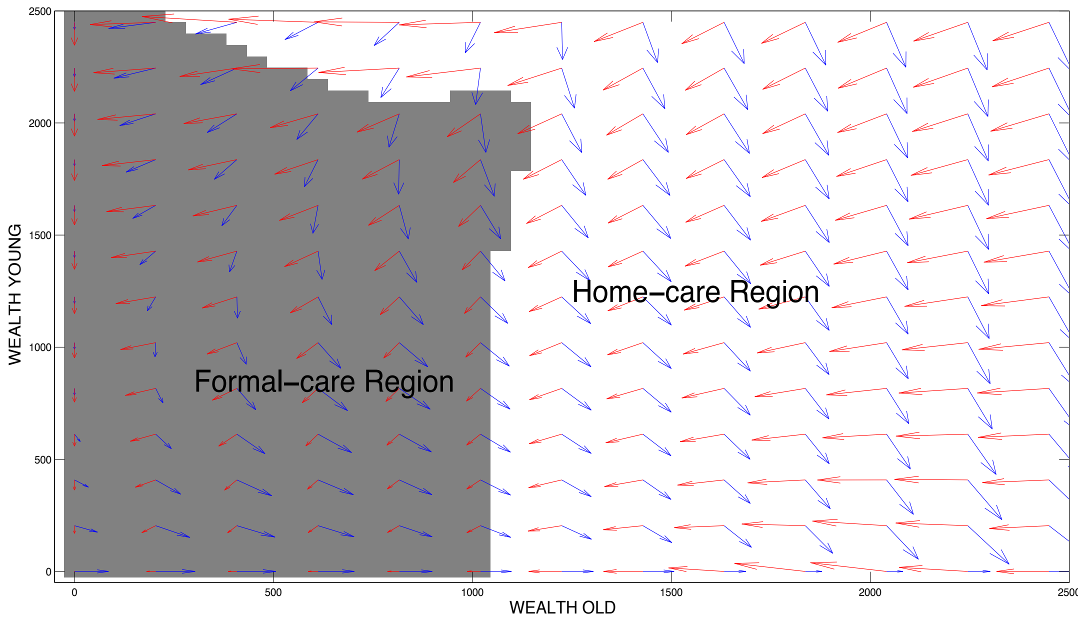
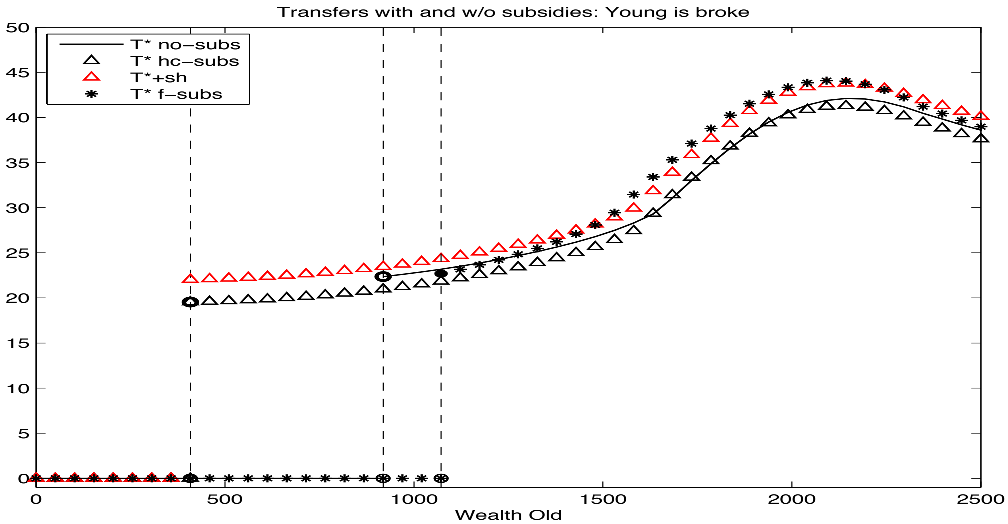


Figure 8 portrays the exchange-motivated transfers along the state space where the young is broke. The vertical-dashed lines demarcate cutoff values of the old's wealth between home and formal care. The solid line traces out transfers without subsidies; the triangles are transfers with a home-care subsidy, the red triangles are what the young receives for providing home care including the subsidy. The stars are transfers under the formal-care subsidy.

The most striking feature of this figure is that the cutoff value of the old's wealth decreases significantly with the introduction of a home-care subsidy. When there are no subsidies the cutoff value for home care to take place is roughly when the old has a level of wealth of 900. This is in contrast to a cutoff value of the old's wealth of about 400 when home care is subsidized and 1100 when formal care is subsidized.

In terms of magnitudes we see that the transfer amount is smallest with a home-care subsidy. In this simple experiment this is unsurprising since the subsidy is financed out of nowhere and therefore has no general equilibrium effects.

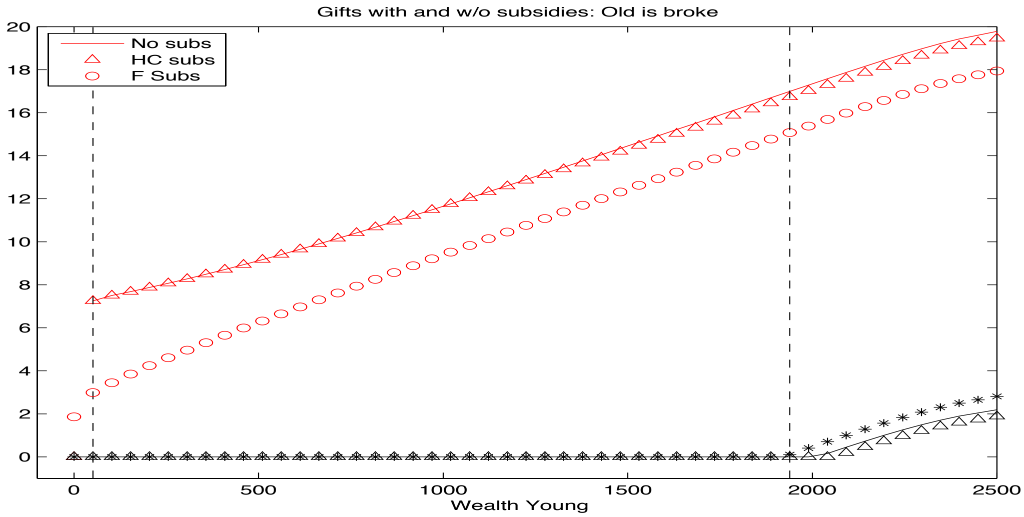
Figure 8: Exchange-Motivated Transfers: Young Broke



The vertical-dashed lines demarcate cutoff values of the old's wealth between home and formal care. The solid line traces out transfers without subsidies; the triangles are transfers with a home-care subsidy, the red triangles are what the young receives for providing home care, which includes the subsidy. The stars are transfers under the formal-care subsidy.

We now consider the part of the state space where the old is broke. Here, a crucial feature is that the young provides gifts to the old. Figure 9 shows the flow of gifts from young to old when the old does not require LTC (black) and when she requires LTC (red). When the old requires LTC the young provides gifts to the old in order to subsidize the purchase of formal care. The circles indicate gifts in the presence of the formal-care subsidy, the triangles when formal care is subsidized and the solid line are gifts without subsidies.

Figure 9: Gifts From Young to Old: Old Broke

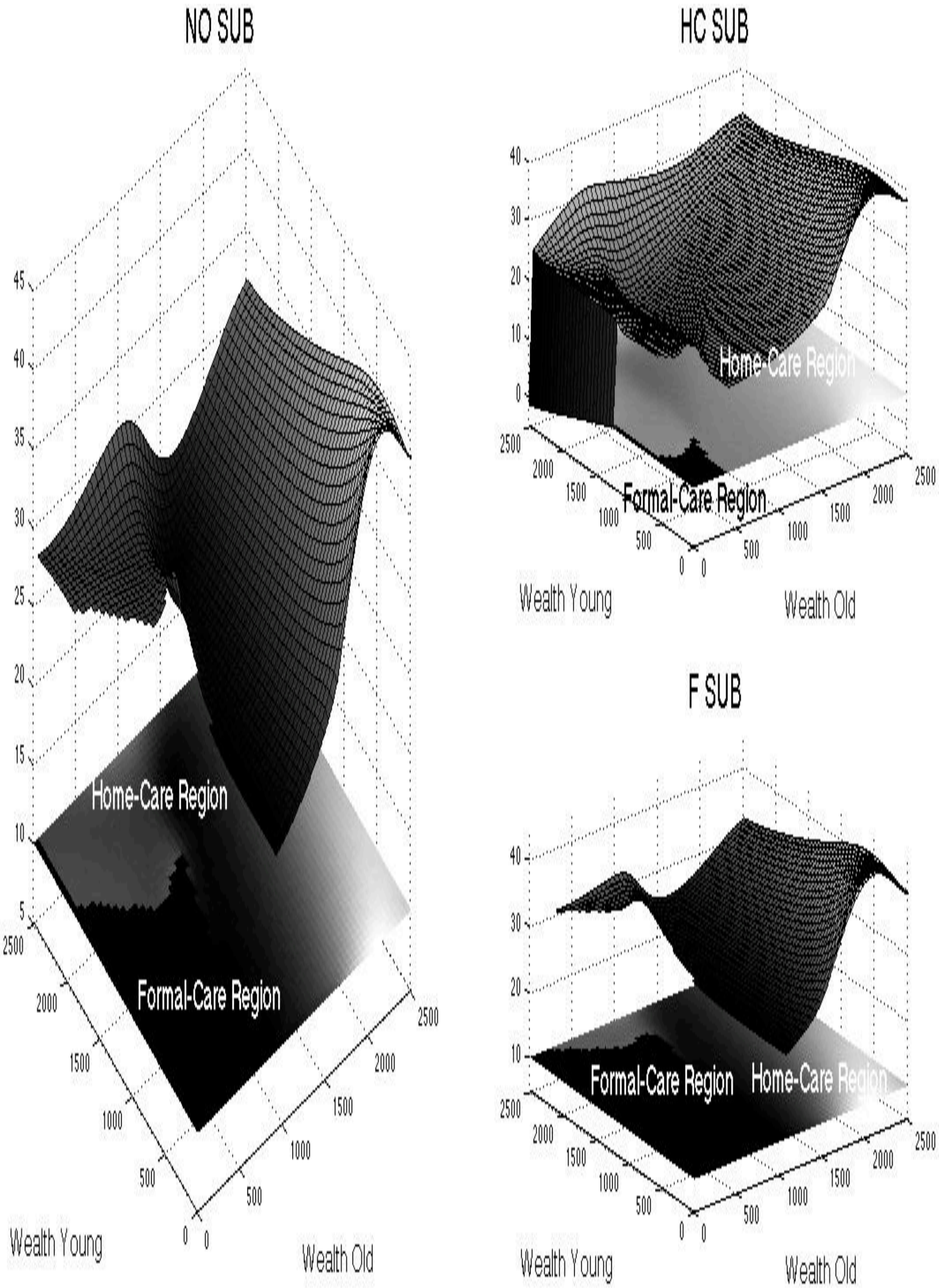


The vertical-dashed lines demarcate the onset of gifts from young to old. Red highlights gifts when the old requires LTC and black when she does not. The circles indicate gifts in the presence of the formal-care subsidy, the triangles when formal care is subsidized and the solid line are gifts without subsidies.

Figure 10 portrays the exchange-motivated transfer functions without subsidies (left), with a home-care subsidy (upper right) and with a formal-care subsidy (lower right). The figure also shows planes to highlight the type of care the old receives. Along the x-y plane are the combinations of the young's and the old's wealth. We know that home care takes place if the transfer policy prescribes a non-zero transfer. The size of the exchange-motivated transfer is given by the policy.

Consider first the transfer policy from the baseline case (the figure on the left). A noticeable feature of the transfer policy is its non-monotonicity, in particular, its shape in the upper-left corner. The dominant effect in this part of the state space is due to the directional derivative $V^y_{a^y} - V^y_{a^o}$. It becomes small relative to its size in the remainder of the state space. The meaning of this directional derivative is the value to the young of taking away one unit of resources from the old ($-V^y_{a^o}$) and adding it to her own resources ($V^y_{a^y}$). The difference between these two values is positive throughout the state space, i.e. $V^y_{a^y} > V^y_{a^o}$. As the young, however, becomes rich relative to the old this difference between marginal valuations of the other's and own wealth decreases.

Figure 10: Exchange-Motivated Transfers



The left-hand graph is the transfer policy without subsidies, $s_h = 0 = s_n$. In the upper-right corner is the transfer policy when home care is subsidized, $s_h = 2.5$, and in the lower-right corner when formal care is subsidized, $s_n = 2.5$. Home care takes place when the transfer policy is non-zero.

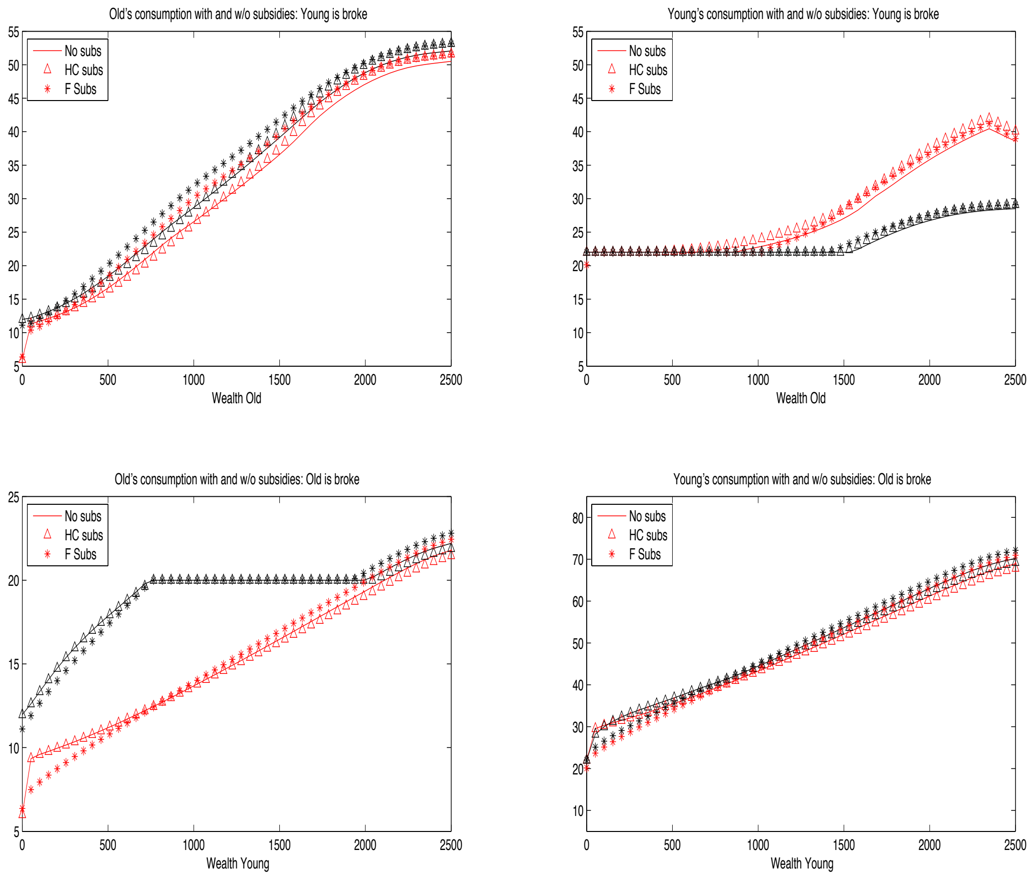
From figure 3 we know that in this part of the state space the economy is heading rapidly to the part of the state space where the old is broke (recall the red horizontal arrows pointing to the left). The reason is that the old has incentives to deplete her wealth since she can count on gifts from the young. Thus, the economy is heading towards a region in which the young and the old consume out of the young's resources. The fact that the young's marginal value for the old's wealth is close to her marginal value for her own wealth is a reflection of this eventual pooling of resources. This pooling will however never take place if the economy is far enough removed from the old's bankruptcy, say, below the 45-degree line (see again figure 3). This mainly explains why the monotonicity of the transfer function does not continue throughout the state space.

Finally, we take a peek at the consumption policies when one (or both) of the generations is (are) broke in the case when the old does not require LTC and when she does. Figure 11 shows these policies. The top panel displays consumption of the old (left) and the young (right) when the young is broke. The bottom panel shows consumption of the old (left) and the young (right) when the old is broke. The solid line traces out consumption without subsidies; the triangles are consumption under a home-care subsidy. The stars are consumption under the formal-care subsidy. Red signifies consumption when the old requires LTC and black when she does not.

The figures suggest that consumption does not change dramatically with the onset of LTC as long as an agent is not broke (see the first and the fourth graph). The differences in consumption levels when the old requires LTC are more stark when an individual is broke (see the second and the third graph).

Consider the young's consumption when she is broke (the second graph). When both are broke the young consumes his income $w = 22$ before and after the LTC shock either when there are no subsidies or with a home-care subsidy. The young consumes less than her income and provides a gift to the old in case of the formal-care subsidy. Only in the case of this subsidy does the old not choose Medicaid when both are broke. She is better off not entering this state since formal care is subsidized and she obtains a gift from the young. In the other two cases she would also obtain a gift from the young if she would not choose Medicaid. But she chooses Medicaid since formal care is too expensive for her and the gift from the young is insufficient to keep her out of Medicaid.

Figure 11: Consumption: One or the Other Broke



The top panel displays consumption of the old (left) and the young (right) when the young is broke. The bottom panel shows consumption of the old (left) and the young (right) when the old is broke. The solid line traces out consumption without subsidies; the triangles are consumption under a home-care subsidy. The stars are consumption under the formal-care subsidy. Red signifies consumption when the old requires LTC and black when she does not.

When the wealth of the old surpasses 400 home care takes place in the case in which it is subsidized. Also, as the old's wealth increases the young's consumption increases because she obtains a larger transfer for providing home care. The young's consumption follows a similar pattern when there are no subsidies and when there is a formal-care subsidy with the difference that home care takes place at higher levels of the old's wealth. When the old does not require LTC (shown in black) the young's consumption eventually exceeds her income since she obtains gifts from the old when the old becomes sufficiently wealthy (approximately above 1500).

Now consider the old's consumption when she is broke (the third graph in figure 11). Let's begin with consumption when she requires LTC and both are broke. When there are no subsidies or when there is a home-care subsidy she enters Medicaid and obtains a consumption level of $c_{ma} = 6$. With the formal-care subsidy she does not enter Medicaid and her consumption is slightly above that provided by Medicaid (6.4). As mentioned above this is because the subsidy and the gift from the young enable her to have a higher level of consumption then she would obtain from Medicaid. In either case the economy is stuck; from then on the old's consumption is given by either $c_{ma} = 6$ or $c = 6.4$ and the young's consumption by either $c = w = 22$ or $c = w - g^y = 20.13$.

While the old's consumption when both are broke is highest in the case of the formal-care subsidy this is not the case as long as the young's level of wealth is positive and less than about 800. In all three scenarios the old obtains formal care in this part of the state space. Additionally, the young provides gifts to the old in order to help purchase this formal care (recall figure 9). The young's gifts are, however, least generous in the case of the formal-care subsidy. The reason is that the young overconsumes in the cases of no subsidies and a home-care subsidy in anticipation of the old entering the Medicaid state; see the fourth figure which shows that the young's consumption is larger in these scenarios. Since the gift amount is directly related to the donor's level of consumption gifts are also relatively large compared to the situation in which the old does not enter Medicaid. Once the young's wealth is large enough (i.e. above 800) the Medicaid state is far enough removed to be relevant.

A related observation is the old's consumption behavior in this part of the state space (i.e. $w^o = 0$ and $0 < w^y < 800$) when she does not require LTC (shown in black). The old consumes less than her income and is therefore saving in anticipation that the LTC shock might hit. In this part of the state space the shock would be particularly unpleasant to her because the low-consumption state would not be too far off. Once the young is wealthy enough (i.e. $w^y > 800$) the low-consumption state is far enough and so she increases her consumption by consuming hand-to-mouth. When the young is very wealthy (i.e. when $w^y > 2000$) she provides gifts to the old which goes into the old's consumption.

6.2 Generations' Policy Preference

In our model generations may differ as to which policy option they prefer if given a choice. The old may prefer a home-care subsidy because it would enable her to remain in her home which yields additional utility. The young may prefer a formal-care subsidy because it enables her to continue working without having to worry about caring for the old.

On the other hand, the old may prefer the formal-care subsidy. She dislikes the fact that the young has to give up her wage and a formal-care subsidy would allow her to obtain care more cheaply and not having the child forego her wage. The young may prefer the home-care subsidy since she does not want the old to spend her resources on formal care but rather on her. Additionally, she also internalizes the old's preference for staying at home.

In what follows we will compare by how much the young and the old would have to be compensated in the baseline parameterization in order to be indifferent to either the formal-care subsidy or the home-care subsidy. The more a generation needs to be compensated in terms of consumption in the baseline scenario the more valuable the policy option is deemed.

We compute the generations' consumption-equivalent variations (CEV) for each point in the state space. A rough sketch on how we compute CEV is as follows. Write the status-quo value function for the old as:

$$V^o(\omega) = E\int_0^\infty (u(c_t^o) + \alpha^o u(c_t^y) + \eta h_t^*)dt = \underbrace{E\int_0^\infty (u(c_t^o) + \alpha^o u(c_t^y))dt}_{\equiv V^{o,c}(\omega)} + \underbrace{E\int_0^\infty \eta h_t^* dt}_{\equiv V^{o,h}}.$$

Now ask by what percentage $1+\beta$ the status-quo consumption would have to change so as to make the old indifferent to a policy. First, multiply the status-quo consumption by $1+\beta$ and substitute the functional form of the utility function:

$$\hat{V}^o(\omega) = (1+\beta)^{1-\gamma}\underbrace{E\int_0^\infty \left[\frac{(c_t^o)^{1-\gamma}}{1-\gamma} + \alpha^o\frac{(c_t^y)^{1-\gamma}}{1-\gamma}\right]dt}_{\equiv V^{o,c}(\omega)} + \underbrace{E\int_0^\infty \eta h_t^* dt}_{\equiv V^{o,h}(\omega)}.$$

Denote the old's value function when home care is subsidized by $V^{o,hc}$. Then $\beta^{hc}(\omega)$ has to satisfy:

$$\begin{aligned}
\hat{V}^o(\omega) &= V^{o,hc}(\omega),\\
(1+\beta^{hc}(\omega))^{1-\gamma}V^{o,c}(\omega) + V^{o,h}(\omega) &= V^{o,hc}(\omega),
\end{aligned}$$

and we can solve for β:

$$\beta^{hc}(\omega) = \left(\frac{V^{o,hc}(\omega) - V^{o,h}(\omega)}{V^{o,c}(\omega)}\right)^{\frac{1}{1-\gamma}} - 1.$$

We can get the CEV when formal care is subsidized β^{f} in the exact same way and can do the analogous calculations for the young. If the β associated with a policy is larger than the one associated with the other policy we will say that a generation prefers the former policy over the latter.

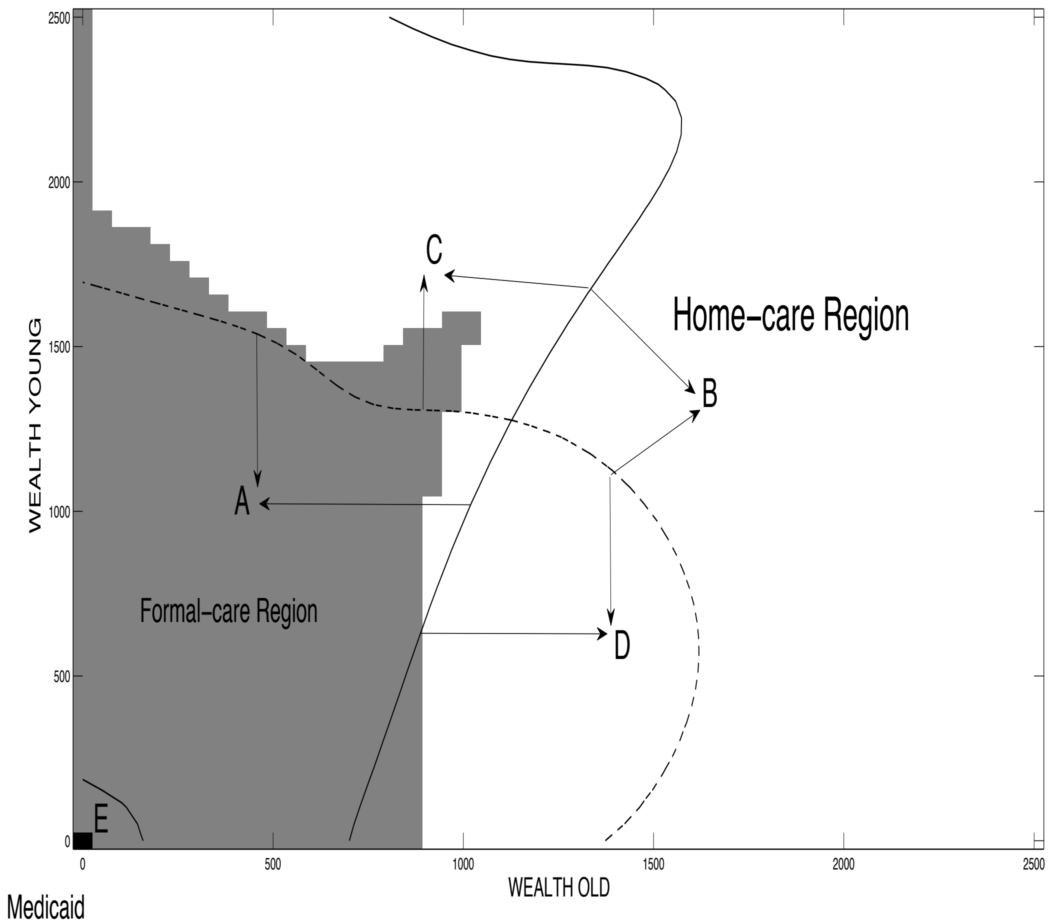
Figure 12 partitions the state space into regions according to which policy the young and the old prefer. Additionally, it portrays the type-of-care regions from the baseline parameterization without subsidies shown in figure 3.

The solid and the dashed curves trace out the points in the state space along which the young and the old are indifferent between the formal- and the home-care subsidies, respectively, e.g. along the solid line the old's β for the home-care subsidy equals the old's β for the formal-care subsidy. In states which are to the right of the two major curves a generation prefers the home-care subsidy e.g. the old's β for the home-care subsidy exceeds the old's β for the formal-care subsidy; in states which are to the left, the formal-care subsidy is preferred. This is not the case for the region *E*. Within this region the young prefers the home-care subsidy over the formal-care subsidy; to the right of it the young prefers the formal-care subsidy.

In region A both prefer the formal-care subsidy over the home-care subsidy and both prefer any subsidy over none. Around the origin formal care takes place even in the presence of the home-care subsidy; refer back to figure 6 which shows the formal-care region in this case. Thus, the home-care subsidy is irrelevant in this part of the state space since home care will never be obtained. A formal-care subsidy is therefore much more valuable. Moving further away from the origin home care does take place eventually. But, the economy is not too far removed from the formal-care region. Since the economy is moving there the formal-care subsidy is preferred by both generations.

The old prefers the formal-care subsidy all the way to the point where both are broke. Only with this policy she never enters Medicaid and thus has a higher consumption level in the long run. For the young the situation around the origin is different. In region E the young would actually prefer no policy or the home-care subsidy (which is essentially no policy since home care does not take place) over the formal-care subsidy. The reason is that her

Figure 12: Generations' Subsidy Preference



The type-of-care regions are those from the baseline parameterization without subsidies shown in figure 3. The solid and the dashed curves trace out the points in the state space along which the young and the old are indifferent between the formal- and the home-care subsidies, respectively. In states which are to the right of the two major curves a generation prefers the home-care subsidy; in states which are to the left, the formal-care subsidy is preferred. This is not the case for the region *E*. Within this region the young prefers the home-care subsidy over the formal-care subsidy; to the right of it the young prefers the formal-care subsidy. A quick summary is the following: (A) Both prefer the formal-care subsidy; (B) both prefer the home-care subsidy; (C) the old prefers the home-care subsidy while the young prefers the formal-care subsidy; (D) the old prefers the formal-care subsidy while the young prefers the home-care subsidy; (E) the old prefers the formal-care subsidy while the young prefers the home-care subsidy.

consumption is somewhat lower when the old does not enter Medicaid; the young actually wants the old to enter Medicaid but cannot force her to do so. For the young it is not credible to threat that she will not provide gifts if the old makes use of formal care instead. Given that the old does not choose Medicaid it is in the young's best interest to provide a gift in order to help the old finance formal care. To summarize, in region E the old prefers the formal-care subsidy while the young prefers the home-care subsidy or simply no subsidy at all.

Region B is characterized by the fact that both prefer the home-care subsidy. In much of this part of the state space there is always home care whether or not it is subsidized. Thus, a formal-care subsidy does not do anything here.

In regions C and D the young and the old have differential preferences with respect to the subsidy. In region C the old prefers the home-care subsidy while the young prefers the formal-care subsidy; in D the old prefers the formal-care subsidy while the young prefers the home-care subsidy.

In region C the old's preference for home care dominates. When home care is subsidized the formal-care region becomes substantially smaller. Considering figure 6 we see that the home-care region even extends along the vertical axis where the old is broke up until the young's wealth is about 1200. The economy is far from the formal-care region and so for the old the benefits of the home-care subsidy outweigh the benefits of the formal-care subsidy in this part of the state space. The young prefers the formal-care subsidy over the home-care subsidy which allows her to obtain the wage rate while knowing that her parent is taken care off.

Finally, in D the old prefers the formal-care subsidy while the young prefers the home-care subsidy. The reason why the old prefers the formal-care subsidy is the same as we discussed for region A above. The young prefers the home-care subsidy in this region since the old becomes relatively wealthier and therefore the old provides higher exchange-motivated transfers.

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