Defensive Practices and TFP*

Iacopo Varotto

Banco de España

February 23, 2024

Abstract

I study the effect of firms' defensive practices on aggregate total factor productivity (TFP) in a heterogeneous firm model of endogenous technology adoption and oligopolistic competition, in which non-productivity-enhancing intangible investments deter competitors' entry and imitation. I find that defensive practices affect three determinants of TFP: (i) production factor misallocation, (ii) productivity of the intangible investment, and (iii) first-best total production factor productivity (TPFP). I calibrate the model to U.S. economy, and leverage firm size and intangible investment distributions to discipline the quantitative relevance of the channels. The model suggests that preventing defensive practices increases TFP by 1.86 percent. I find that the first-best TPFP channel accounts for the bulk of the rise in the TFP: the increase in the within-firm production efficiency outweighs the fall in product variety. I validate the model prediction in the data.

Key Words: competition, misallocation, firm entry, technology diffusion, TFP.

JEL Classification: E22, D23, D43, L11, L13, L60, O33, O43e.

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*Addresses: iacopo.varotto@bde.es.I am grateful to Tatsuro Senga and Omar Rachedi for their constant and valuable advice, patience and support that allowed me to get through this project. I also thank Giulio Fella, Sang Yoon Lee and Yu Zheng for their suggestions. I also benefited from comments and suggestions from Alessandra Bonfiglioli, Anthony Savagar and Petr Sedláček. All remaining errors are mine. The opinions in this paper are mine and do not necessarily reflect the views of the Banco de España or the Eurosystem.

1 Introduction

A large economic literature has focused on the role of firms' defensive practices, non-productivity-enhancing intangible investments in preemptive patenting or in lobbying on regulation, as an important mechanism that could result in idiosyncratic distortions [1]. Accordingly, these investments, aimed to reduce the technology diffusion and entry, may potentially impact the total factor productivity (TFP).

In this paper I quantify the effect of firms' defensive practices to TFP. I build a quantitative heterogeneous firm model of endogenous technology adoption with variable markups, in which non-productivity-enhancing defensive investments can deter competitors' entry and imitation. In my model, defensive practices affect three determinants of the TFP. First, they affect the first-best total production factor productivity (TPFP) since they alter the average within-firm production efficiency, brand and product variety[2]. Second, they affect the production factor misallocation via cross-sectional markup dispersion, as in Edmond et al. (2015)[3]. Finally, they affect the average productivity of the intangible investment since they influence the total cost of the intangible investment in terms of units of TPFP[4]. The goal of this paper is to quantify the relative contribution of these three forces and to evaluate the overall policy effect of preventing firms' defensive practices on TFP.

To this aim, I develop a general equilibrium model that incorporates the oligopolistic competition framework of Atkeson and Burstein (2008) in a model of endogenous firm dynamics and technology adoption à la Sedláček (2020) and I extend it in several dimensions. Firstly, each industry is populated by a finite number of leaders and followers who strategically compete dynamically to

[1] In the literature, there are various studies on the endogenous distortions aimed to limit competition. See Krusell and Rios-Rull (1996), Parente and Prescott (2002) Bellettini and Ottaviano (2005), Comin and Hobijn (2009), Mukoyama and Popov (2014), Stigler (2021), and Akcigit et al. (2023) for studies on endogenous political barriers to entry and to technology diffusion. Analogously, see Gilbert and Newbery (1982), Farrell and Shapiro (2008), Abrams et al. (2013) and Argente et al. (2020), for studies on preemptive patenting.

[2] I define the total production factor productivity (TPFP) as the value added net of factors directly employed in the production, labor and physical capital.

[3] For other seminal studies on the contribution of factor misallocation to TFP, see Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). For studies on the contribution of within-firm efficiency to TFP, see Howitt (2000), and Klenow and Rodriguez-Clare (2005). For studies on the contribution of product variety to TFP, see Feenstra (1994), Feenstra et al. (1999), and Feenstra and Kee (2008).

[4] I define the average productivity of the intangible investment as the ratio between TPFP and the total intangible investment expressed in units of TPFP.

adopt more productive vintage technologies, playing contests in the form of aggregative games that guarantee the uniqueness of the Nash equilibria[5]. Secondly, leaders' adoption of more productive technologies generates positive spillovers toward followers to catch-up[6]. Nevertheless, leaders can also influence the extent of technology spillovers by conducting two types of defensive practices. First leaders can implement defensive practices that reduce the probability that followers imitate their technologies. Second, leaders can implement defensive practices that reduce the probability of entry of an additional follower[7]. Thirdly, the model features two types of entry. New competitors enter the incumbent industries as additional followers with a new brand or new entrepreneurs enter the economy as leaders of new industries after succeeding in the creation of a new product [8].

What is the effect of preventing defensive investments on TFP[9]? Several forces are at play. First, it has an countervailing effect on first-best TPFP level. On the one hand, imposing restrictions on firms' ability to control their dominant position may improve the average within-firm production efficiency, since it allows more laggard firms to enter the industry with new brands, catch-up, and compete against frontier firms. On the other hand, more competition implies that firms' ability to appropriate the profits generated by their investment is limited, which in turn may discourage firms to adopt newer vintage technologies to expand the technology frontier of the industry or to develop new products. Second, it has an ambiguous effect on the average productivity of the intangible investment. On the one hand, preventing defensive practices may improve the productivity of the intangible investments, because it removes non-productivity-enhancing investments. On the other hand, the lack of defensive practices may encourage intangible investments of imitators and potential entrants, which in turn may reduce the average productivity of the intangible investment. Finally it reduces markup distortions since it creates more industries that are characterized by a head-to-head competition, thereby it diminishes markups and their dispersion, in line with what found Edmond et al. (2015) and Edmond et al. (2023).

[5]In the paper I prove that the Nash equilibria of the different contests are unique with convex cost.

[6]This feature is consistent with the model of industry dynamics of Jovanovic and MacDonald (1994) and the empirical evidence of Bloom et al. (2013). Nevertheless it is crucial for defensive expenditure being increasing in firm size in line with Kerr et al. (2014) and Argente et al. (2020).

[7]Equivalently, The latter form can be seen as the ability of the leaders of preventing potential entrants from imitating the technology followers.

[8]In the spirit of Hoberg and Phillips (2016), I refer to an industry as the set of firms that operate in the same marketplace, whose choices mutually affect each other decisions and pay-offs.

[9]In the paper I use the words defensive investments and defensive practices interchangeably.

I calibrate the model to U.S. economy and I discipline the relative importance of these channels by leveraging the firm size distribution and both the level and the distribution of intangible investment intensity[10]. The model can successfully replicate the elasticity of sales growth rate and product market share to firm size[11].

I find that the hypothetical complete elimination of all types of defensive intangible investments leads to an increase in the TFP by 1.86 percent. More specifically, It turns out that policy implementation either reduces misallocation by 0.36 percent, increases the average productivity of the intangible investments by 0.19 percent, and improves the first-best TPFP by 1.31 percent. I then turn to quantifying the changes in the components that determine the first-best TPFP in my model: (i) average within-firm production efficiency, (ii) product variety, and (iii) brand variety. Overall, the improvements in the average within-firm production efficiency and in brand variety increase the TPF by 6.39 and 0.46 percent, respectively, in contrast to the reduction in the product variety that contributes negatively to the TPF by -5.54 percent. In short I find that, the negative effect of reduction in the product variety is more than compensated by the effects of improvement in the average within-firm production efficiency, and in the expansion in the brand variety.

To disentangle the effect of the entry from the imitation deterrence I conduct two additional experiments. I consider first the hypothetical policy that only eliminates defensive investments that deter the imitation of the leaders' technology, then the policy that only prevents defensive investments that deter entry. I find that the positive effect of the competition policy on the TFP crucially hinges on the elimination of defensive investments that deter the imitation of the leaders' technology. Although the elimination of the imitation deterrence worsens the product variety, it incentivizes the competition in industries of high productive leaders, that in turn, it pushes the adoption of more productive technologies across firms and reduces markup dispersion. Intuitively, defensive investments that deter imitation dampen the leaders' adoption of newer vintage technologies in those industries in which leaders are high productive, since they allow leaders to generate and maintain their large technology gaps with respect to their competitors. In contrast, preventing defensive investments that deter entry reduces

[10] I follow the approach of Edmond et al. (2023), I do not feed into the model separately estimated firm-level defensive expenditures and markups. I prefer to use the defensive expenditures and markups implied by the model due to difficulty to exactly identify them in the data.

[11] I derive the product market share through the product similarity scores of Hoberg and Phillips (2016).

the TFP since it discourages the adoption of newer technologies in industries whose leaders' productivity is low, and it does not enhance the competition on the frontiers of industries with high productive leaders.

I then study how the effect of the competition policy on the TFP varies with different different modeling parameterizations. I find that the positive effect of the competition policy is decreasing in the product and brand substitutability, and in the productivity of the intangible investment technology of the leaders. Conversely, it is increasing in the productivity of the intangible investment technology of the followers, and in the productivity of the new product producers[12].

Finally, I empirically test the prediction of the model by testing whether defensive expenditures may be associated with a higher product differentiation. To this end, I consider lobbying expenditures as proxy of the defensive investment, and the product similarity score of Hoberg and Phillips (2016) as measure of product differentiation. I first find the evidence that lobbying is associated with a future rise in product differentiation at industry level. In the spirit of Gutiérrez and Philippon (2019), I then interact lobbying expenditure and regulation, and I find that the confluence of lobbying and regulation is associated with a reduction in the product similarity in the industry. In other words, the positive correlation between regulation and the change in the future product differentiation of the industry increases in the level of lobbying. Intuitively when lobbying expenditure is high a large part of these expenditures may be used to affect regulations that limit competition. Accordingly, the creation of barriers to competition spurs the product differentiation.

This article is related to several strands of the literature. First, it is closely related to the literature that studies the role of the markup variation as a source of misallocation. For example, Edmond et al. (2015) quantify the gain in TFP from trade or Edmond et al. (2023) gauge the gain in wefare from fiscal policies that lead to the reduction in markup distortions through heterogeneous firm models with oligopolistic competition. My model incorporates endogenous technology adoption and technology spillovers that may qualitatively affect the model prediction of the effect of those policies on the aggregate TFP. More specifically, I show that preventing defensive practices may hinder the product differentiation and the adoption of more productive technologies up to the point of reducing the TFP. In this regard, my paper complements Peters (2020) that shows the effect

[12]Particularly, I find that the competition policy may actually reduce TFP in economies with high brand substitutability and productivity of the intangible investment technology of the leaders.

of the reduction in markup distortions on TFP from a rise in churning is roughly compensated by the reduction in firms' innovation. I expand this previous work in two ways. First, the churning is industry specific since it is determined by the the strategic efforts of leaders and laggards. This feature is critical to understanding the policy effect since it allows to take into how technology diffusion affects the technology adoption and markup dispersion across different kinds of industries. Second, my model includes the product differentiation as an additional channel that affects the TFP. Indeed, I show that the fall in product differentiation substantially dampens the positive effect on TFP of preventing defensive practices.

The paper also contributes to a recent macroeconomics literature on the aggregate implications of the ex-ante start-up heterogeneity. For example, Sterk et al. (2021) investigate how the ex-ante start-up heterogeneity affects the macroeconomic effects of micro-level frictions. De Haas et al. (2022) show how the largest productivity gains can be obtained by reducing taxes for capital-intensive, large, and cash-intensive startup types entry. My model suggests that removing firm defensive practices has important implications for the entry composition. Although preventing defensive practices favours the entry of more productive startups in the incumbent industries, it reduces the entry of startups that are related with the creation of new products.

Finally, my article is related to the literature that studies the effect of political connection on TFP through production factor misallocation. For instance, García-Santana et al. (2020) and Arayavechkit et al. (2018) quantify the TFP loss coming from firms' lobbying activities to obtain subsidies and tax benefits. Differently, in my model defensive practices generate misallocation by increasing the markup dispersion.

The paper is organized as follows. Section 2 presents the model. In section 3 presents the quantitative analysis and the validation of the model prediction. Section 4 concludes.

2 The Economic Environment

I study a stationary heterogeneous firm model of endogenous technology adoption, markups, and entry, in which firms can also implement non-productivity-enhancing intangible investments to deter competitors' entry or imitation.

Household There is a representative household that maximizes her lifetime expected utility $U(C, L)$ where C is consumption and L labor. The household invests in physical capital, K, that depreciates at rate δ, and and she owns all the firms in the economy. The household has CES preferences over the entire set of differentiated products.

Industry timing The economy is composed by a continuum of heterogeneous industries, products, that are populated by a finite number of firms, brands. Specifically, in each industry a finite number leaders, that produce with a superior technology, and finite number of followers, that produce with an inferior technology, strategically compete both in the production, and in the adoption of newer vintage technologies. Accordingly, an industry is defined by its state vector $(\varepsilon_d, \varepsilon_{d-f}, n^L, n^F)$; productivity of leaders $\varepsilon_d \in E^{L,d}$, productivity of followers $\varepsilon_{d-f} \in E^{F,d}$, number of leaders $n^L \in N^L \subset \mathbb{N}^+$, and number of followers $n^F \in N^{F,L} \subset \mathbb{N}^0 \mid n^F \leq N^{\max} - n^L$.

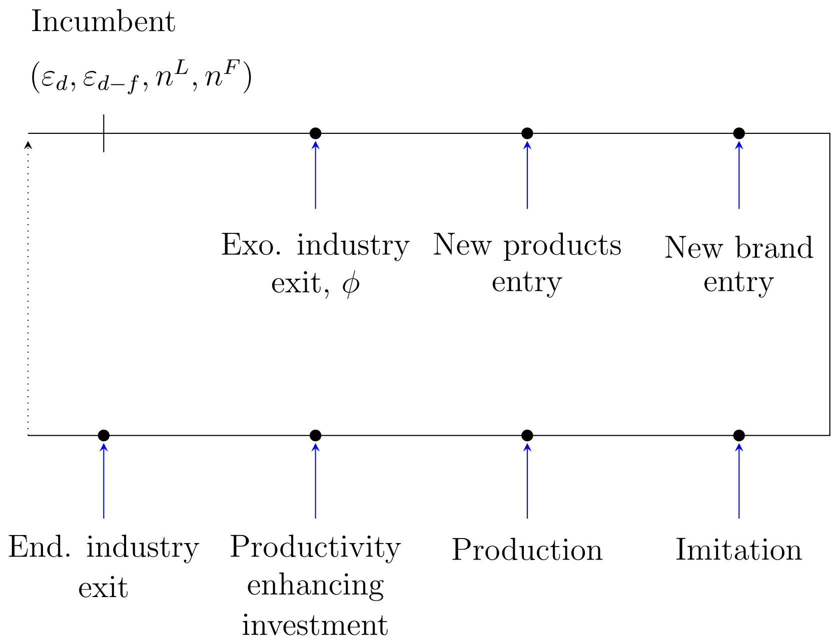
At the beginning of each period, firms enter the economy in two ways. They can invest to create a new product and enter as a leader of a new industry, or enter a market for an existing product as an additional follower. The leaders of each industry can invest in non-productivity-enhancing intangible practices to deter the entry of an additional competitor in their industry.

After entry, followers imitate leaders' technology in order to join the leaders on the industry frontier. In contrast, leaders invest in defensive practices to reduce followers' chances of catch-up.

At the end of the period, production a la Cournot takes place. Only after that leaders also implement productivity-enhancing intangible investments which stochastically advance the industry technological frontier. If any leader succeeds in advancing the frontier, it becomes the new leader next period while all other leaders become followers. The old followers are assumed to be forced out of the market. In case, there are no other leaders, the leader that succeeds it enlarge her technology gap with respect to the followers that continue in the industry. If no leader succeeds in innovation, nothing changes.

Finally, the industries disappear for two reasons. First an industry can be hit by an exogenous destruction shock. Second, the technology of the industry frontier depreciate below the minimum threshold required to continue to produce.

Figure 1: Timing within a period



2.1 Set up

2.1.1 Final good producers

The final good, Y, is produced by a competitive firm using inputs y_j from a continuum of industries

$$Y = \left(\int_0^M y_j^{\frac{(\eta-1)}{\eta}} dj \right)^{\frac{\eta}{\eta-1}}, \tag{1}$$

where $\eta > 1$ is the elasticity of substitution across industries $j \in [0, M]$. Importantly, each industry consists of a finite number of firms. In particular in industry j, output is produced using intermediate inputs of n_j^L leaders, and n_j^F followers

$$y_j = \left(\sum_{i=1}^{N_j} y_{i,j}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \tag{2}$$

where $\rho > \eta$ is the elasticity of substitution across goods i within a particular industry $j \in [0, M]$. In our benchmark model, the number of leaders n_j^L and followers n_j^F in industry j as well as the mass of industries are endogenous and they evolve as I discuss in Section 2.2.

2.1.2 Intermediate Goods Producers

In industry j, both leaders, the most productive firms, and followers rent capital stock, k, and hire labor, l to produce

$$y_{i,j} = \begin{cases} \varepsilon_{d,j} k_{i,j}^{\alpha} l_{i,j}^{1-\alpha}, & \text{if } i \text{ is a leader,} \\ \varepsilon_{d-f,j} k_{i,j}^{\alpha} l_{i,j}^{1-\alpha}, & \text{if } i \text{ is a follower} \end{cases} \tag{3}$$

where within-firm production efficiencies, $\varepsilon_{i,j}, \varepsilon_{i-f,j} \in \{\varepsilon_1, \varepsilon_2, \ldots\ldots, \varepsilon_{N^\varepsilon}\}$ with $f > 1$, evolve as I discuss in detail in Section 2.2 below.

Final good producers buy intermediate goods from leaders at prices $p_{i,j}^L$ and from followers $p_{i,j}^F$. Households buy the final good at price P. Accordingly, a final good producer chooses intermediate inputs $y_{i,j}^L$ and $y_{i,j}^F$ to maximize profits,

$$\max_{y_j} PY - \int_0^M \left(\sum_{i=1}^{n_j^L} p_{i,j}^L y_{i,j}^L + \sum_{i=1}^{n_j^F} p_{i,j}^F y_{i,j}^F \right) dj \tag{4}$$

subject to 1 and 2.

2.2 Entry and Technology Adoption

I now present the evolution in the technology adoption of the firms in the incumbent industries as well as they entry of new products or brands in the economy.

It is assumed that the technology frontier of the economy, $\varepsilon_{N^\varepsilon}$, deterministically grows at the exogenous rate $\bar{\varepsilon}$ that depends on the level of the scientific knowledge in the economy.

Firms invest to adopt newer vintage technologies. The investment into technology adoption is interpreted as the costs of technology adoption related to embodying a newer scientific knowledge in the production technology.

Importantly, leaders can also invest to build barriers to technology adoption of the competitors. In the spirit of Parente and Prescott (2002), these barriers correspond to the various ways by which private firms increase the amount of investment a competitor must make in order to advance its technology level or to enter a specific market[13].

Because the frontier is growing over time, the firms that fail to adopt newer technologies will experience a decline in the relative productivity.

2.2.1 Entry of new products

Potential entrepreneurs of mass Σ choose their effort x^{E^1} to develop a new technology that allows to produce a new product. In case she succeeds, she produces a new good with ε^{E^1} productivity, hence she enters the new industry as a mo-

[13] Most common forms that these barriers take are lobbying on regulation or preemptive patenting.

nopolist.

The probability that the potential entrepreneur i succeeds is given by

$$\pi_i^{E^1}(x_i^{E^1}) = \frac{x_i^{E^1}}{x_i^{E^1} + \bar{Q}}, \tag{5}$$

where the parameter $\bar{Q} > 0$ specifies the likelihood of success.

2.2.2 Entry of new brands

In each industry j, $N^{E^2} = N^{Max} - n_j^L - n_j^F$ potential entrants compete in a contest to adopt the technology that allows to enter the industry as an additional follower. Meanwhile, n_j^L leaders invest in defensive practices to prevent the entry of an additional follower. Let $x_{i,j}^{E^2}$ and $x_{i,j,0}^L$ be the effort of the potential entrants and leaders respectively.

The probability that an additional follower enters industry is given by

$$\pi_j^{E^2}(X_j^{E^2}, X_{j,0}^L) = \frac{\bar{Q}}{X_{j,0}^L + \bar{Q}} \times \frac{X_j^{E^2}}{X_j^{E^2} + \bar{T}} \tag{6}$$

where the parameter $\bar{T} > 0$ specifies the likelihood of winning without defensive barriers, $X_{j,0}^L = \sum_{i=1}^{n_j^L} x_{i,j,0}^L$, and $X_j^{E^2} = \sum_{i=1}^{N^{E^2}} x_{i,j}^{E^2}$ are the total effort of the potential entrants and leaders respectively.

Accordingly, the probability that a potential entrant i enters the industry j is

$$\pi_{i,j}^{E^2}(x_{i,j}^{E^2}, X_{-i,j}^{E^2}, X_{j,0}^L) = \pi_j^{E^2} \times \frac{x_{i,j}^{E^2}}{x_{i,j}^{E^2} + X_{-i,j}^{E^2}} \tag{7}$$

where $X_{-i,j}^{E^2} = \sum_{h \neq i} x_{h,j}^{E^2}$ is the total effort of other potential entrants.

2.2.3 Imitation

In each industry j, n_j^F followers compete in a contest to adopt the technology that allows to become an additional leader of the industry. Meanwhile, n_j^L leaders invest in defensive practices to prevent the catch-up of a follower. Let $x_{i,j}^F$ and $x_{i,j,1}^L$ be the effort of the followers and leaders respectively.

The probability that a follower moves up to the frontier of the industry j is

$$\pi_j^F(X_j^F, X_{j,1}^L) = \frac{\bar{Q}}{X_{j,1}^L + \bar{Q}} \times \frac{X_j^F}{X_j^F + \bar{T}} \tag{8}$$

with $X_{j,1}^L = \sum_{i=1}^{n_j^L} x_{i,j,1}^L$ and $X_j^F = \sum_{i=1}^{n_j^F} x_{i,j}^F$ are the total effort of leaders and followers respectively.

Accordingly, the probability that the follower i becomes an additional leader of the industry j is

$$\pi^F_{i,j}(x^F_{i,j}, X^F_{-i,j}, X^L_{j,1}) = \pi^F_j \times \frac{x^F_{i,j}}{x^F_{i,j} + X^F_{-i,j}} \tag{9}$$

where $X^F_{-i,j} = \sum_{h \neq i} x^F_{h,j}$ is the total effort of other followers.

2.2.4 Technology adoption of leaders

In each industry j, n^L_j leaders compete in a contest to adopt a more productive technology. The winner can enjoy a one-step increase of her productivity. The new technology allows the winner to become the only leader of the industry in next period, to convert the other leaders into followers, and to force the current followers out of the industry. In case there is one leader, the adoption of the new technology increases her productivity gap with respect to the followers that they continue to stay in the industry.

Let $x^L_{i,j,2}$ be the effort of the leaders. The probability that a leader wins the contest in industry j is

$$\pi^L(X^L_{j,2}) = \frac{X^L_{j,2}}{X^L_{j,2} + \bar{Q}}, \tag{10}$$

where $X^L_{j,2} = \sum_{i=1}^{n^L} x^L_{i,j,2}$ is the total effort of the leaders.

Accordingly, the probability of success for the leader i:

$$\pi^L_{i,j}(x^L_{i,j,2}, X^L_{-i,j,2}) = \pi^L(x^L_{i,j,2}, X^L_{-i,j,2}) \frac{x^L_{i,j,2}}{x^L_{i,j,2} + X^L_{-i,j,2}} \tag{11}$$

where $X^L_{-i,j,2} = \sum_{h \neq i} x^L_{h,j,2}$ is the total efforts of other leaders.

2.2.5 Exit

In each period, firms leave the economy for three different reasons. First, each industry faces an exogenous probability of destruction ϕ. Second, firms exit when the technology frontier of the industry depreciates below the productivity level ε_2. Third, in industries with several leaders, followers are forced to leave the industry when a leader adopt a more productive technology.

2.3 Decision Problem

I now present the decision problem for different types of agents in the recursive language. I then decompose the TFP in its determinants. I finally provide a formal definition of a balanced growth path Markov perfect equilibrium.

2.3.1 Household

I assume that the representative household's period utility is the result of indivisible labor (Rogerson (1988)) $U(C, 1-L) = \log C + \kappa(1-L)$ with $\kappa > 0$, as the labour disutility parameter[14]. The household discounts the future utility by a subjective discount factor, β. The optimality conditions are given by

$$\frac{1}{C} = \beta \frac{1}{C'}(R^{'} + 1 - \delta) \tag{12}$$

$$\frac{1}{C} w = \kappa, \tag{13}$$

2.3.2 Firms

For expositional purposes, I collapse the industry characteristics in the firm problems in a single vector of state variable $\Omega \equiv (\varepsilon_d, \varepsilon_{d-f}, n^L, n^F)$. I summarize the distribution of industry in the production by a probability measure, $\mu(\Omega)$, which is defined on a Borel algebra $\mathbb{S} \equiv E^{L,d} \times E^{F,d} \times \mathbb{N}^L \times \mathbb{N}^F$.

I assume that l units of labor are equivalent to an effort level x of $\sqrt{2l}$. Hence, the cost functions $C(x) = \frac{w}{2}x^2$ is increasing and convex in the effort.

Entry of new products. Let V_0^L be the value of being the leader before the contest played by the potential entrants takes place.

The potential entrepreneur problem is

$$V^{E^1} = \max_{x^{E^1}} -C(x^{E^1}) + \mathbb{E}(V_0^L(\Omega')|x^{E^1}) \tag{14}$$

where the expected continuation value is

$$\mathbb{E}(V_0^L(\Omega')|x^{E^1}) = \pi^{E^1}(x^{E^1})V_0^L(\varepsilon^{E^1}, \varepsilon_1, 1, 0)$$

The potential entrepreneur that succeeds enters the economy as a monopolist with $\varepsilon^{E^1} > \varepsilon_1$ productivity.

Entry of new brands. Let V_1^L and V_1^F be the value of being leader and follower respectively, before the followers attempt to adopt leaders' technology.

The problem of the potential entrant i is

$$V_i^{E^2}(\Omega) = \max_{x_i^{E^2}} -C(x_i^{E^2}) + \mathbb{E}_{\Omega'} V_{i,1}^F(\Omega'|x_i^{E^2}, X_{-i}^{E^2}(\Omega), X_0^L(\Omega), \Omega) \tag{15}$$

where the expected continuation value is

$$\mathbb{E}_{\Omega'} V_{i,1}^F(\Omega'|x_i^{E^2}, X_{-i}^{E^2}(\Omega), X_0^L(\Omega), \Omega)$$

$$= \pi_i^{E^2}(x_i^{E^2}, X_{-i}^{E^2}(\Omega), X_0^L(\Omega))V_1^F(\varepsilon_d, \varepsilon_{d-f}, n^L, n^F + 1)$$

The potential entrant that does not succeeds leaves the economy forever.

The problem of the leader i is

$$V_{i,0}^L(\Omega) = \max_{x_{i,0}^L} -C(x_{i,0}^L) + \mathbb{E}_{\Omega'} V_{i,1}^L(\Omega'|X^{E^2}(\Omega), x_{i,0}^L, X_{-i,0}^L(\Omega), \Omega) \tag{16}$$

where the expected continuation value is

[14] The preference specification allows for balanced growth.

$$\mathbb{E}_{\Omega'} V^L_{i,1}(\Omega'|X^{E^2}(\Omega), x^L_{i,0}, X^L_{0,-i}(\Omega), \Omega)$$

$$= \pi^{E^2}(X^{E^2}(\Omega), x^L_{i,0}, X^L_{-i,0}(\Omega), \Omega) V^L_{i,1}(\varepsilon_d, \varepsilon_{d-f}, n^L, n^F + 1) +$$

$$+(1 - \pi^{E^2}(X^{E^2}(\Omega), x^L_{i,0}, X^L_{-i,0}(\Omega), \Omega) V^L_{i,1}(\varepsilon_d, \varepsilon_{d-f}, n^L, n^F)$$

Accordingly, the value of the follower i is

$$V^F_{i,0}(\Omega) = \mathbb{E}_{\Omega'} V^F_{i,1}(\Omega'|X^{E^2}(\Omega), X^L_0(\Omega), \Omega) \tag{17}$$

given the expected continuation value

$$\mathbb{E}_{\Omega'} V^F_{i,1}(\Omega'|X^{E^2}(\Omega), X^L_0(\Omega), \Omega)$$

$$= \pi^{E^2}(X^{E^2}(\Omega), X^L_0(\Omega)) V^F_{i,1}(\varepsilon_d, \varepsilon_{d-f}, n^L, n^F + 1) +$$

$$+(1 - \pi^{E^2}(X^{E^2}(\Omega), X^L_0(\Omega), \Omega) V^F_{i,1}(\varepsilon_d, \varepsilon_{d-f}, n^L, n^F)$$

Imitation. Let V^L_2 and V^F_2 be the value of being leader and follower respectively, before the production takes place .

The problem of the follower i is

$$V^F_{i,1}(\Omega) = \max_{x^F_i} -C(x^F_i) + \mathbb{E}_{\Omega'} V^F_{i,2}(\Omega'|x^F_i, X^F_{-i}(\Omega), X^L_1(\Omega), \Omega) \tag{18}$$

where the expected continuation value is

$$\mathbb{E}_{\Omega'} V^F_{i,2}(\Omega'|x^F_i, X^F_{-i}(\Omega), X^L_1(\Omega), \Omega)$$

$$= \pi^F_i(x^F_i, X^F_{-i}(\Omega), X^L_1(\Omega)) V^L_{i,2}(\varepsilon_d, \varepsilon_{d-f}, n^L + 1, n^F - 1) +$$

$$+(\pi^F(x^F_i, X^F_{-i}(\Omega), X^L_1(\Omega)) - \pi^F_i(x^F_i, X^F_{-i}(\Omega), X^L_1(\Omega))) V^F_{i,2}(\varepsilon_d, \varepsilon_{d-f}, n^L + 1, n^F - 1) +$$

$$+(1 - \pi^F(x^F_i, X^F_{-i}(\Omega), X^L_1(\Omega))) V^F_{i,2}(\varepsilon_d, \varepsilon_{d-f}, n^L, n^F)$$

The follower takes the expectation over three possible events. The first term incorporates the value of becoming an additional leader. The second term incorporates the value of continuing to a be a follower in case another follower becomes an additional leader. The third term incorporates the value of continuing to be a follower in case no follower succeeds in the catch-up.

Proposition 1. *Given x^L_1, let $x^{F\star}$ be a symmetric equilibrium of the game represented by the recursive maximization problem 18, then*

- *$x^{F\star}$ is increasing in the difference*

$$V^L_2(\varepsilon_d, \varepsilon_{d-f}, n^L + 1, n^F - 1) - V^F_2(\varepsilon_d, \varepsilon_{d-f}, n^L, n^F)$$

- *$x^{F\star}$ is increasing in the difference*

$$V_2^L(\varepsilon_d, \varepsilon_{d-f}, n^L+1, n^F-1) - V_2^F(\varepsilon_d, \varepsilon_{d-f}, n^L+1, n^F-1)$$

- *$x^{F\star}$ is decreasing in X_1^L*
- *given X_1^L, $x^{F\star}$ is unique.*

Proof*: see appendix A.2.*

Proposition 1 shows that the effort of the followers depends on the *return of being leader* and the *probability of being blocked*. First, the equilibrium effort, x^F, is increasing in the return of becoming leader: if the productivity at the technology frontier is relatively high, then the followers increase their effort. Second, effort of the followers is decreasing in the total defensive effort of the leaders: if the probability of being blocked is high, the marginal return of the investment is low.

The problem of the leader i is

$$V_{i,1}^L(\Omega) = \max_{x_{i,1}^L} -C(x_1^L) + \mathbb{E}_{\Omega'} V_{i,2}^L(\Omega'|X^F(\Omega), x_{i,1}^L, X_{-i,1}^L(\Omega), \Omega) \tag{19}$$

where the expected continuation value is

$$\mathbb{E}_{\Omega'} V_{i,2}^L(\Omega'|X^F(\Omega), x_{i,1}^L, X_{-i,1}^L(\Omega), \Omega)$$

$$= \pi^F(X^F(\Omega), x_{i,1}^L, X_{-i,1}^L(\Omega))V_{i,2}^L(\varepsilon_d, \varepsilon_{d-f}, n^L+1, n^F-1)+$$

$$+(1-\pi^F(X^F(\Omega), x_{i,1}^L, X_{-i,1}^L(\Omega), \Omega)V_{i,2}^L(\varepsilon_d, \varepsilon_{d-f}, n^L, n^F)$$

Proposition 2. *Given X^F, let $x_1^{L\star}$ be a symmetric equilibrium of the game represented by the recursive maximization problem 19, then*

- *$x_1^{L\star}$ is increasing in the difference*

$$V_2^L(\varepsilon_d, \varepsilon_{d-f}, n^L, n^F) - V_2^L(\varepsilon_d, \varepsilon_{d-f}, n^L+1, n^F-1)$$

- *$x_1^{L\star}$ is increasing in X^F*
- *$x_1^{L\star}$ is unique.*

Proof*: see appendix A.1.*

Proposition 2 shows that the incentive of the leaders in implementing defensive practices has two drivers: *protecting profits* and *credible threat* . First the defensive effort, $x_1^{L\star}$, is increasing in the benefit that leaders have from preventing the catch-up: leaders with large market shares implement a high defensive effort. Second the leader's effort is increasing in the followers' effort.

Proposition 3. *The game, represented by the recursive maximization problems 19 and 18, admits a unique symmetric equilibrium* $(x_l^{L\star\star}, x_l^{F\star\star})$.
Proof: *see appendix A.3.*

The intuition of proposition 3 is straightforward. The leaders' symmetric equilibrium of defensive practices is increasing in the total intangible investment chosen by the followers. Conversely, the symmetric equilibrium of the followers is decreasing in the quantity of defensive practices implemented by leaders.

Finally Proposition 2, 1 and 3 can be directly applied to show that the game between leaders and potential entrants has a unique symmetric equilibrium. Differently from the imitation game, the value of the potential entrant in case it does not succeed is 0.

Production and leaders' productivity-enhancing contest. Leaders and followers compete *à la Cournot* in the industry. Let Y_{-i} be $\sum_{h \neq i} y_h^{\frac{\rho-1}{\rho}}$, the leader (follower) i static maximization problem is

$$\overline{\pi}_i^{L(F)}(\Omega) = \max_{y_i^{L(F)}, k_i, l_i} p_i(y_i^{L(F)}, Y_{-i}(\Omega), \Omega) y_i^{L(F)} - w l_i - R k_i \tag{20}$$

subject to 3.

Given the final producer maximization problem 4 and the cost minimization of firm i, the production problem is

$$\overline{\pi}_i^{L(F)}(\Omega) = \max_{y_i} \Big(\frac{y_j}{Y}\Big)^{\frac{-1}{\eta}} \Big(\frac{y_{i,j}}{y_j}\Big)^{\frac{-1}{\rho}} y_i - \frac{y_i}{\varepsilon_{i,d(d-f)}} \Big(\frac{R}{\alpha}\Big)^{\alpha} \Big(\frac{w}{1-\alpha}\Big)^{1-\alpha} \tag{21}$$

Following leaders play a contest to adopt a more productive technology that advances the technology frontier of the industry.

Accordingly, the problem of the follower i is

$$V_{i,2}^F(\Omega) = \max_{y^{i,F}} \overline{\pi}_i^F(\Omega) + \hat{\beta}(1-\phi)\, \mathbb{E}_{\Omega'}\, V_{i,0}^F(\Omega' | X_{i,2}^L(\Omega), \Omega) \tag{22}$$

where the expected continuation value is

$$\mathbb{E}_{\Omega'}\, V_{i,0}^F(\Omega' | X_2^L(\Omega))$$

$$= \pi^L(X_2^L(\Omega)) \underset{n^L=1}{\mathbb{1}} V_{i,0}^F(\varepsilon_{d+1}, \varepsilon_{\max\{d-f-1,1\}}, 1, n^F) +$$

$$+ (1-\pi^L(X_2^L(\Omega))) \underset{d>2}{\mathbb{1}} V_{i,0}^F(\varepsilon_{d-1}, \varepsilon_{\max\{d-f-1,1\}}, n^L, n^F)$$

The expected continuation value of the follower takes into account two events. The first term of the expected continuation value considers the value in the event that leader wins the contest[15]. The second term considers the value in the event

[15]In the case $n^L > 1$, followers must leave the industry and the value is equal to 0

that no leader succeeds[16]. Importantly, I assume that if $\varepsilon_{d-f} = \varepsilon_1$ and $d > 2$, followers benefit from a positive externality from leaders' technology such that maintain the same relative production efficiency ε_1[17].

Differently, the problem of leader is

$$V_{i,2}^L(\Omega) = \max_{y^{i,L}, x_{i,2}^L} \overline{\pi}_i^L(\Omega) - C(x_{i,2}^L) + \hat{\beta}(1-\phi)\,\mathbb{E}_{\Omega'}\, V_{i,0}^L(\Omega' | x_{i,2}^L, X_{-i,2}^L(\Omega), \Omega) \tag{23}$$

where the expected continuation value is

$$\begin{aligned}
&\mathbb{E}_{\Omega'}\, V_{i,0}^L(\Omega' | x_{i,2}^L, X_{-i,2}^L(\Omega)) \\
&= \pi_i^L(x_{i,2}^L, X_{-i,2}^L(\Omega), \Omega)\Big(\mathbb{1}_{n^L>1} V_{i,0}^L(\varepsilon_{d+1}, \varepsilon_{d-1}, 1, n^L - 1) + (1 - \\
&\mathbb{1}_{n^L>1}) V_{i,0}^L(\varepsilon_{d+1}, \varepsilon_{\max\{d-f-1,1\}}, 1, n^F)\Big) + \\
&+ (\pi^L(x_{i,2}^L, X_{-i,2}^L(\Omega)) - \pi_i^L(x_{i,2}^L, X_{-i,2}^L(\Omega))) V_{i,0}^F(\varepsilon_{d+1}, \varepsilon_{d-1}, 1, n^L - 1) + \\
&+ (1 - \pi^L(x_{i,2}^L, X_{-i,2}^L(\Omega)))\, \mathbb{1}_{d>2} V_{i,0}^L(\varepsilon_{d-1}, \varepsilon_{\max\{d-f-1,1\}}, n^L, n^F)
\end{aligned}$$

The expected continuation value of the leader takes into account three events. The first term of the expected continuation value considers the value of winning the contest[18]. The second term considers the value of loosing the contest. The third term considers the value of being leader when no leader succeeds[19]. Importantly, leaders that do not innovate such as followers will experience a one step decline in their relative productivity.

Proposition 4. *Let $x_2^{L\star}$ be a symmetric equilibrium of the game represented by the recursive maximization problem 23, then*

- $x_2^{L\star}$ *is increasing in*

$$\mathbb{1}_{n^L>1} V_{i,0}^L(\varepsilon_{d+1}, \varepsilon_{d-1}, 1, n^L - 1) + (1 - \mathbb{1}_{n^L>1}) V_{i,0}^L(\varepsilon_{d+1}, \varepsilon_{\max\{d-f-1,1\}}, 1, n^F)) - \mathbb{1}_{d>2} V_{i,0}^L(\varepsilon_{d-1}, \varepsilon_{\max\{d-f-1,1\}}, n^L, n^F)$$

- $x_2^{L\star}$ *is increasing in*

$$\mathbb{1}_{n^L>1} V_{i,0}^L(\varepsilon_{d+1}, \varepsilon_{d-1}, 1, n^L - 1) + (1 - \mathbb{1}_{n^L>1}) V_{i,0}^L(\varepsilon_{d+1}, \varepsilon_{\max\{d-f-1,1\}}, 1, n^F)) - V_{i,0}^F(\varepsilon_{d+1}, \varepsilon_{d-1}, 1, n^L - 1)$$

[16] In the case $d = 2$, the industry disappears hence the value is 0 equal to 0.

[17] This assumption can be easily relaxed. In that case, defensive practices allow leaders to send competitors out of the industry through the technology depreciation.

[18] In the case $n^L = 1$, the leaders only increases the productivity gap and the value of winning the game is $V_{i,0}^L(\varepsilon_{d+1}, \varepsilon_{\max\{d-f-1,0\}}, 1, n^F)$.

[19] In the case $\varepsilon_d = \varepsilon_2$, the industry disappears and value is 0

- $x_2^{L\star}$ *is unique.*

Proof: *see appendix A.4.*

Proposition 4 shows that the incentives of the leaders in adopting a newer vintage technology depends on both benefit of becoming the only new leader and on the loss of becoming a follower.

2.4 Productivity

I define the endogenous aggregate productivity TFP

$$\mathcal{E} = \frac{Y}{K^{\alpha} L^{1-\alpha}} \tag{24}$$

as value added net of aggregate factors employed K and L, capital and labor respectively.

The total factor productivity (TFP) of my economy can be expressed as the the total production factor productivity (TPFP) net of the cost of the intangible investment

$$\mathcal{E} = (1 - \underbrace{\frac{1}{\lambda^L}}_{= \frac{L^{1-\alpha}}{L^{1-\alpha} - L_1^{1-\alpha}} \text{ productivity of intangible investment}}) \times \mathcal{E}^{\star} \tag{25}$$

where the TPFP $\mathcal{E}^{\star}$, the benefit of the intangible investment, is the value added net of factors employed in the production, L_1 and K, and $\frac{1}{\lambda^L}\mathcal{E}^{\star}$ is the total cost of intangible investment in unit of TPFP[20].

As shown in Edmond et al. (2015), markup dispersion generates a wedge between TPTF and its first-best efficiency level

$$\begin{aligned} \mathcal{E}^{\star} &= \Big(\int_0^M \Big(\frac{\mu_j}{\mu}\Big)^{-\eta} \Big(\Big(n_j^L \Big(\frac{\mu_j^L}{\mu_j}\Big)^{-\rho} (\varepsilon_j^L)^{1-\rho} + n_j^F \Big(\frac{\mu_j^F}{\mu_j}\Big)^{-\rho} (\varepsilon_j^F)^{1-\rho} \Big)^{\frac{1}{1-\rho}} \Big)^{1-\eta} dj \Big)^{\frac{1}{1-\eta}} \\ &= (1 - \underbrace{\lambda^{\mu}}_{\frac{\mathcal{E}^{\star}_{efficient} - \mathcal{E}^{\star}}{\mathcal{E}^{\star}_{efficient}} = \% \text{ loss from factor misallocation}}) \times \mathcal{E}^{\star}_{efficient} \end{aligned} \tag{26}$$

where μ, μ_j, μ_j^L, and μ_j^F represent the aggregate, sectoral, and firm level markup respectively. In turn, the efficient total production factor productivity $\mathcal{E}^{\star}_{efficient}$ is given

$$\mathcal{E}^{\star}_{efficient} = \Big(\int_0^M \Big(\Big(\Big(n_j^L (\varepsilon_j^L)^{1-\rho} + n_j^F (\varepsilon_j^F)^{1-\rho} \Big)^{\frac{1}{1-\rho}} \Big)^{1-\eta} dj \Big)^{\frac{1}{1-\eta}} \tag{27}$$

To understand how the competition policy may affect the TFP, It is conve-

[20] In the model the total labor is $L = L_1 + L_2$, where L_2 is the labor employed in the intangible investment.

nient to re-write the total efficient production factor productivity $\mathcal{E}^{\star}_{efficient}$, as the average within-firm production efficiency scaled by the within and across industry variety

$$\mathcal{E}^{\star}_{efficient} = (1+\underbrace{\lambda^{M}}_{\substack{\frac{\mathcal{E}^{\star}_{efficient}-\mathcal{E}^{\star}_{1,efficient}}{\mathcal{E}^{\star}_{1,efficient}} = \\ \text{\% gain from} \\ \text{industry variety}}}) \times (1+\underbrace{\lambda^{n}}_{\substack{\frac{\mathcal{E}^{\star}_{1,efficient}-\mathcal{E}^{\star}_{2,efficient}}{\mathcal{E}^{\star}_{2,efficient}} = \\ \text{\% gain from within} \\ \text{industry variety}}}) \times \underbrace{\bar{\varepsilon}}_{\substack{\mathcal{E}^{\star}_{2,efficient} = \\ \text{firm-level average} \\ \text{productivity}}} \tag{28}$$

where $\mathcal{E}^{\star}_{1,efficient}$, the first-best TPFP without the contribution of the across industry variety, and $\mathcal{E}^{\star}_{2,efficient}$, the first-best TPFP without the contribution of the across and within industry variety, are respectively given

$$\mathcal{E}^{\star}_{1,efficient} = \left(M^{\frac{-1}{1-\eta}} \int_0^M \left(\left(\left(n_j^L(\varepsilon_j^L)^{1-\rho} + n_j^F(\varepsilon_j^F)^{1-\rho}\right)^{\frac{1}{1-\rho}}\right)^{1-\eta} dj\right)^{\frac{1}{1-\eta}} \tag{29}$$

$$\mathcal{E}^{\star}_{2,efficient} = \left(M^{\frac{-1}{1-\eta}} \int_0^M \left(\left((n_j^L+n_j^F)\right)^{\frac{-1}{1-\rho}} \left(n_j^L(\varepsilon_j^L)^{1-\rho} + n_j^F(\varepsilon_j^F)^{1-\rho}\right)^{\frac{1}{1-\rho}}\right)^{1-\eta} dj\right)^{\frac{1}{1-\eta}} \tag{30}$$

Accordingly, combining the equations 24, 26, and 28, the TFP can be written

$$\mathcal{E} = (1-\underbrace{\lambda^{\mu}}_{\substack{\text{production} \\ \text{factor} \\ \text{misallocation}}}) \times (1-\frac{1}{\underbrace{\lambda^{L}}_{\substack{\text{intangible} \\ \text{investment} \\ \text{productivity}}}}) \times \underbrace{(1+\underbrace{\lambda^{M}}_{\substack{\text{product} \\ \text{variety}}}) \times (1+\underbrace{\lambda^{n}}_{\substack{\text{brand} \\ \text{variety}}}) \times \underbrace{\bar{\varepsilon}}_{\substack{\text{average within} \\ \text{firm production} \\ \text{efficiency}}}}_{\text{first-best TPFP}} \tag{31}$$

Hence, preventing firm defensive practices may affect three determinants of TFP: (i) misallocation, (ii) average intangible investment productivity, (iii) first-best TPFP. Importantly, the first-best TPFP depends on three components: (a) average within-firm production efficiency, (b) product variety, and (c) brand variety.

2.5 Balanced Growth and Equilibrium MPE

I focus on the Balanced Growth Path (BGP) Markov perfect equilibrium, where equilibrium strategies depending only on the payoff-relevant state variable Ω and all aggregate variables are growing at the same rate $g^{\bar{\varepsilon}}$[21].

A stationary equilibrium is a set of prices $(R, w, p^L, p^F_{i,j})$, a set of allocations $(Y, C, I, y^L_{i,j}, y^F_{i,j})$, policies $(x^{E^1}_{i,j}, x^L_{i,j,0}, x^{E^2}_{i,j}, x^L_{i,j,1}, x^F_{i,j}, x^L_{i,j,2})$, such that:

1. Given prices, the competitive final good producers maximize their profits.
2. Given Ω_j, $y^L_{i,j}$ and $y^F_{i,j}$ maximize profits of the oligopolistic firms in the

[21] Note that the technology frontier is the only source of growth. Only firm-level and aggregate employment are stationary. All other variables can be stationarized by dividing them with $\varepsilon_{N^{\varepsilon}}$.

industries.

3. x^{E^1} maximizes the value of the potential entrepreneur V^{E^1}.
4. Given Ω_j, $X^L_{j,0}$, and $X^{E^2}_{j,-i}$, $x^{E^2}_{i,j,0}$ maximizes value of the potential entrant $V^{E^2}_i(\Omega_j)$.
5. Given Ω_j, $X^{E^2}_j$ and $X^L_{-i,0}$, $x^L_{i,j,0}$ maximizes the value of the leader i $V^L_{i,0}(\Omega_j)$.
6. Given Ω_j, $X^F_{-i,j}$, and $X^L_{1,j}$, $x^F_{i,j,1}$ maximizes the value of the follower i $V^F_{i,1}(\Omega_j)$.
7. Given Ω_j, X^F_j and $X^L_{-i,j,1}$, $x^L_{i,j,1}$ maximizes the value of the leader i $V^L_{i,1}(\Omega_j)$.
8. Given Ω_j, $X^L_{-i,j,2}$ and, $x^L_{i,j,2}$ maximizes the value of the leader i $V^L_{i,j,2}(\Omega_j)$.
9. Given prices, C satisfies 13.
10. The real interest rate $R = \frac{1+g^{\bar{\varepsilon}}}{\beta} + \delta - 1$.
11. Resource constraint is satisfied:

$$Y = C + I$$

where $I = \delta \int_{\mathbb{S}} (n^L \frac{y^L}{\varepsilon_d} + n^F \frac{y^F}{\varepsilon_{d-f}})[\frac{\alpha}{1-\alpha} \frac{w}{r}]^{1-\alpha} \mu(d[\varepsilon_d \times \varepsilon^{d-f} \times n^L \times n^F])$

12. The industry distribution $\mu(\Omega)$ is is a fixed point where its transition is consistent with the policy functions $(x^{E^1}_{i,j}, x^L_{i,j,0}, x^{E^2}_{i,j}, x^L_{i,j,1}, x^F_{i,j}, x^L_{i,j,2})$[22].

3 Quantitative analysis

In this section, I present the quantitative analysis. I first present my parameterization and calibration strategy. I then quantify the effect of preventing defensive practices on the TFP. Next, I gauge the relative importance of the different channels and examine the mechanisms underlying effects of the competition policy. I finally show empirical evidences that align with my model prediction.

3.1 Calibration

In the model, the size of the gains from the implementation of the competition policy depends on responsiveness and relative strength of three determinats: (i) production factor misallocation, (ii) average productivity of the intangible investment, (iii) within-firm production efficiency, (iv) brand and product variety. I discipline the model by requiring that it reproduces a number of stylized features of the data: firm size distribution and level and distribution of the intangible investment intensity.

[22] I report the time invariant distribution of industry in the appendix B.

3.1.1 Parameters and targets

I divide the parameters of the model economy in two groups. The first group includes parameters that are pinned down targeting the conventional aggregate moments. The second group of parameters are model specific and target moments obtained from the micro data.

Table 1: Calibration common parameters

Parameter	Value	Description	Target	Data	Model
β	0.97	Discount factor	Annual Real interest rate	3%	3%
κ	2.3	Leisure utility	Total hours worked	0.33	0.33
δ	0.1	Depreciation rate	Investment/Capital	10%	10%
α	0.4	Elasticity of output to capital	Capital/Output	2.3	2.3
ϕ	0.04	Exogenous exit probability	Entry Rate	11%	11.3%
$g^{\bar{\varepsilon}}$	0.018	Growth of technology frontier	Average productivity growth	0.018	0.018
Σ	0.09	Mass of Potential Entrepreneurs	Mass of Market Places	1.0	0.996

Common parameters. I calibrate my BE to data for the U.S. at annual frequency. I normalize the units in which output is measured so that P = 1.

The discount factor, β, is set to have an annual interest rate of 3% as in Sedláček (2020). I set the preference parameter of labor disutility, κ, to get the average hours worked of 0.33.

The depreciation rate, δ, is set to 0.1, as in Sedláček (2020). The production parameter, α, is set to be consistent with the average capital to output ratio of 2.3 in the postwar US economy, as in Senga (2015).

The destruction probability, ϕ, is set to match the entry rate as in Sedláček (2020). The growth of the frontier technology $g^{\bar{\varepsilon}}$ targets the average labor productivity as in Sedláček (2020). Finally, I set the mass of potential entrants Σ to have the total mass of products M equals to 1.

Uncommon parameters. To discipline the relative importance of the different channels I set the remaining parameters by leveraging the firm size and intangible investment intensity distribution. To this end, I construct an annual panel of US public firms for the period 1995-2015 and I estimate the firm level intangible investment following Peters and Taylor (2017) [23].

[23] In particular, Using Standard Industry Classification (SIC) codes, I exclude firms in the oil, energy and financial sectors. Specifically, we exclude oil and oil-related firms with SIC codes 2911, 5172, 1311, 4922, 4923, 4924, and 1389; energy firms with SIC code between 4900 and 4940; financial firms with SIC code

Table 2: Benchmark and alternative parametrizations

		Data	BE	$\downarrow \eta$	$\uparrow \rho$	$\uparrow \overline{Q}$	$\uparrow \overline{T}$	$\uparrow \varepsilon^{E^1}$
Features of the economy								
Sales shares top percentiles (pct)	1	35.7	41.9	14.4	45.7	37.1	42.1	36.3
	5	62.1	62.9	31.2	67.3	52.7	63.8	59.4
	10	75.5	70.0	41.5	75.1	60.1	71.7	68.2
	25	90.6	81.4	59.8	85.7	72.3	82.3	80.7
	50	97.8	90.9	79.9	94.9	85.3	91.6	91.6
	75	99.7	97.0	93.8	99.4	94.1	97.3	97.6
Wallmart size (mil)		≈ 1	1.07	0.80	10.0[24]	1.07	1.07	1.07
Start-up relative size		0.29	0.35	0.76	0.29	0.35	0.36	0.54
μ		$1.10 \sim 1.40$	1.17	1.22	1.16	1.16	1.17	1.17
σ_μ(pct)			2.90	5.85	4.71	2.65	2.94	2.83
ξ		$-0.31 \sim -0.17$	-0.24	-0.20	-0.89	-0.12	-0.25	-0.19
$\frac{wL_2}{Y}$ (pct)		≈ 6.50	6.42	8.54	6.53	6.65	6.42	6.56
Parameter values								
	η		5.5	3.8	5.5	5.5	5.5	5.5
	ρ		12.3	59.6	12.3	12.3	12.3	12.3
	$\overline{Q}$		0.03	0.03	0.06	0.03	0.03	0.06
	$\overline{T}$		0.06	0.06	0.06	0.06	0.12	0.06
	ε^{E^1}		ε_2	ε_2	ε_2	ε_2	ε_2	ε_{25}

The elasticity of substitution between products, that governs the relative contribution of the product variety to the first-best TPFP, determines the relationship between firm size and productivity. Accordingly, this elasticity determines the relative importance between defensive and productivity-enhancing invest-

between 6000 and 6999. I eliminate sample firms with missing data items to ensure that the data are valid for all the sample.

ment in the firm strategy: defending the technology advantage is useless when the substitutability is high, since any other firm in other industries can create a similar product using a different technology. I calibrate this elasticity by targeting the output share of the top 1%.

Differently, the elasticity of substitution across brands, determines the relationship between productivity and sale-share within a industry, and, it drives the dispersion of the firm size and markup distributions. I then calibrate this elasticity by targeting the relative employment of the largest firm in the U.S. economy.

In turn, the productivity of the new product producers plays a key role, since it determines the responsiveness of product variety to the competition policy. I pin down this parameter by targeting the relative size of the entrants from Khan et al. (2014).

Regarding the parameters that govern the productivity of the technology of the intangible investment, I target both the level and the distribution of the intangible investment intensity.

In particular, the parameter $\overline{T}$, that governs the productivity of the intangible investment technology of the followers and potential entrants, plays an important role since it determines responsiveness of the competition on the technology frontier of the industry once I implement the policy. Accordingly, I pin down the parameter by requiring that the model reproduces the intangible investment intensity elasticity to firm size.

Finally, the parameter $\overline{Q}$, that governs the productivity of the intangible investment technology of the leaders, is set to match the aggregate intangible intensity of the economy consistently with estimation of David and Gourio (2023). This parameter determines the level of effort required to advance the technology frontier of the industry.

3.1.2 The benchmark economy

The model also performs very well in dimensions not directly targeted in the calibration procedure. In particular, the endogenous dynamic competition can replicate both the relation between firm size and product market share and the variation of sales growth rates across firm size.

Product market share and firm size. I construct a panel data by merging data from Compustat with the product similarity scores of Hoberg and Phillips (2016) for the period 1995-2015. These time-varying firm-by-firm pairwise similarity scores, obtained by parsing the product descriptions from the firm 10Ks,

Table 3: Market share and sale growth rate on firm size

	$\log(\text{Product market share}_{i,j,t})$				Sales growth rate			
	Data			Model	Data			Model
	(1)	(2)	(3)		(1)	(2)	(3)	
$\log(\text{Age}_{i,j,t})$	0.09***	0.15***	0.13***		-0.17***	-0.10***	-0.17***	
$\log(\text{Sales}_{i,j,t})$	0.70***	0.52***	0.74***	0.80				
	(0.01)	(0.00)	(0.01)					
$\log(\text{Sales}_{i,j,t-1})$					-0.26***	-0.02***	-0.27***	-0.33
					(0.00)	(0.00)	(0.00)	
Constant	-5.30***	-4.57***	-5.67***	1.16	1.81***	0.45***	1.84***	0.19
	(0.05)	(0.03)	(0.06)		(0.05)	(0.01)	(0.01)	
Observations	31162	31796	29393		25149	25885	23482	
R2	0.91	0.67	0.92		0.36	0.08	0.44	
Firm f.e.	Y	N	Y		Y	N	Y	
Industry f.e.	N	Y	N		N	Y	N	
Industry × Year f.e.	N	N	Y		N	N	Y	
Year f.e.	Y	Y	N		Y	Y	N	

Standard errors in parentheses
* p<0.10, ** p<0.05, *** p<0.01

represent a continuous measures of product similarity for every pair of firms in Compustat. Particularly, the similarity score between any two firms i and h in the data is a real number in the interval $[0, 1]$ describing how similar the words used by firms i and h are[24].

I then define the firm market share of firm i in the year t as the ratio between firm i sales and the score based product market sales

$$\text{Market share}_{i,t} = \frac{\text{Sales}_{i,t}}{\sum_{h=1}^{n_i} s_{i,h,t}\text{Sales}_{h,t} + \text{Sale}_{i,t}}$$

where $s_{i,h,t}$ is the similarity score between firm i and h in period t and n_i is the total number of competitors with $s_{i,h,j} > 0$.

Table 3 compares the estimations of the elasticity of the product market share to firm size between model and data. I calculate the model implied market share elasticity to firm size with the moments obtained from the stationary distribution of industries of the benchmark economy.

The Table 3 shows that baseline economy performs well in capturing the market share variation across firm size. This dimension is important since it

[24]Importantly, the data provided by the authors only records firms having pairwise similarities with a given firm i that are above a threshold as required based on the coraseness of the three digit SIC classification.

determines the loss in TFP due to the markup dispersion, as shown in equation 26. Intuitively, a higher elasticity of market share to firm size reflects larger TFP loss due to the misallocation of production factors.

Sales growth rate. Using my panel data, I calculate the as

$$\Delta \text{Sales growth rate}_{i,t} = \frac{\text{Sales}_{i,t} - \text{Sales}_{i,t-1}}{0.5(\text{Sales}_{i,t} + \text{Sales}_{i,t-1})}$$

I then simulate the model and I construct the simulated data in exactly the same way as the data are built. Accordingly, I do not consider firms that end up with 0 sales because they leave the economy. Specifically, I simulate 1.7 millions of industries drawn from the stationary distribution of industries for 100 times. In this way, each simulation records 4.5 millions of simulated firms.

Table 3 shows that the model performs well in replicating the variation of sales growth rate across firm size. The baseline economy is quantitatively consistent with the fact a 1 percent increase of firm size is associated with a reduction of 0.003 percent of the firm sales growth rate.

3.2 The competition policy

In this section, I first present the results from the counterfactual analysis in which I prevent leaders from implementing defensive practices. In turn, I study the effect of the different types of defensive practices and I investigate the mechanisms through which the defensive behavior affects the aggregate quantities.

3.2.1 Effect on TFP

How does an implementation of a tougher competition policy affect the total factor productivity in my benchmark economy? In other words, I ask: Which are the most relative important channels through which the policy changes the TFP?

The counterfactual policy exercise consists in comparing the pre-intervention equilibrium where firms are allowed to implement defensive practices, benchmark equilibrium, and the post-intervention equilibrium, which is the new equilibrium reached by the economy after an omnipotent competition authority detects and prevents any implementation of defensive practices. That is, I measure the equilibrium changes in the aggregate economy under each policy relative to the benchmark.

Notice, from equation 32 I can decompose the TFP changes resulting into the changes driven by changes in production factor misallocation, intangible investment productivity as well as due to the change in the first-best TPFP. In addition, the latter can be further decompose into three different component

Table 4: Competition policies and TFP changes, Benchmark model

		Competition policy		
		(1)	(2)	(3)
Features of the counterfactual economy				
Sales shares top percentiles (pct)	1	36.7	35.0	41.8
	5	65.6	65.9	60.9
	10	74.0	74.5	68.4
	25	83.8	85.5	79.3
	50	91.4	91.8	89.2
	75	96.5	96.6	96.0
Start-up relative size		0.84	0.41	0.50
Entry rate (pct)		23.8	13.6	13.6
μ		1.13	1.15	1.15
$\sigma_\mu (pct)$		1.45	2.67	1.90
$\frac{wL_1}{Y}$ (pct)		6.45	5.91	5.53
ΔM (pct)		-22.1	-10.3	-13.3
$\Delta \overline{n}$ (pct)		19.2	-14.3	47.9
$\Delta \overline{\varepsilon}^L$ (pct)		1.5	3.6	-2.0
$\Delta \overline{\varepsilon}^F$ (pct)		5.3	3.9	-0.3
$\Delta \overline{\varepsilon}_j$ (pct)		-0.5	0.6	-1.1
TFP change (pct)				
$\Delta \log(1-\lambda^\mu)$		0.36	0.33	0.09
$\Delta \log(1-\frac{1}{\lambda^L})$		0.19	0.66	1.08
$\Delta \log(1+\lambda^M)$		-5.54	-2.35	-3.16
$\Delta \log(1+\lambda^n)$		0.46	-2.60	3.59
$\Delta \log(\overline{\varepsilon})$		6.39	8.69	-3.44
$\Delta \log(A)$		1.86	4.73	-2.02

Note: The table reports the effects of the competition policies. In particular the competition policy (1) refers to the policy where any types of defensive investment are prevented. The competition policy (2) refers to the policy that precludes only defensive investment that prevent the catch-up. Finally the competition policy (3) reports the effects of precluding defensive investment that prevent entry. $\overline{n}$, $\overline{\varepsilon}^L$, $\overline{\varepsilon}^F$, and $\overline{\varepsilon}_j$ are respectively the average across industries of number of firms, leader and follower's relative productivity, and productivity of the technology frontier. The changes under each policy are relative to the benchmark.

driven by: (i) production variety, (ii) brand variety, and (iii) average within-firm production efficiency.

$$\Delta \log(\mathcal{E}) = \underbrace{\Delta \log(1-\lambda^{\mu})}_{\substack{\Delta\text{production factor}\\ \text{misallocation}}} + \underbrace{\Delta \log(1-\frac{1}{\lambda^{L}})}_{\substack{\Delta\text{intangible investment}\\ \text{productivity}}} + \underbrace{+\underbrace{\Delta \log(1+\lambda^{M})}_{\substack{\Delta\text{product}\\ \text{variety}}} + \underbrace{\Delta \log(1+\lambda^{n})}_{\substack{\Delta\text{brand}\\ \text{variety}}} + \underbrace{\Delta \log(\overline{\varepsilon})}_{\substack{\Delta\text{average within}\\ \text{firm production}\\ \text{efficiency}}}}_{\text{change in first-best TPFP}} \tag{32}$$

As reported in column 1 of Table 4, the model predicts that preventing defensive practices improves the endogenous total factor productivity by 1.86 percent. Specifically, the reduction in the production factor misallocation and in the cost of intangible investment account for 0.36 and 0.19 percent, respectively, while rise in the first-best TPFP accounts for 1.31 percent.

The fall in the production factor misallocation, or equivalently in the markup dispersion, is for two reasons. First, the lack of defensive practices eases the catch-up of the followers that are far from the technology frontier of the industry. Intuitively, in industries of large technology gap between followers and frontier, the leaders have a higher incentive in implementing defensive practices in order to prevent the catch-up. Second, the lack of defensive practices eases the entry of followers close to the technology frontier of the industry. Intuitively, leaders that are threatened by the entry of relatively high productive followers implement aggressive defensive practices in order to prevent their entry.

Focusing on the rise in the first-best TPFP, the negative effect of the reduction in product variety is more than compensate by the positive effects of expansion of the brand varieties and the improvement in the average within-firm production efficiency. More specifically, the rise in the average within-firm production efficiency is driven by the improvement of the production efficiency of both leaders and followers.

Finally, the competition policy improves the average productivity of the intangible investment. Although the first-best TPFP is higher, the share of labor devoted to the intangible investment reduces due to the absence of the non-productivity-enhancing intangible investments.

In short, preventing defensive investment particularly enhances the TFP mainly by improving the within-firm production efficiency. Importantly, the positive effect is dampened by the product variety. Indeed, without considering the effect on product variety the TFP would raise by 7 percent.

3.2.2 Inspection the mechanism

Which mechanism drives the improvement in the TFP and to what extent the two types of defensive investments differ one another? To address this question, I simulate two additional counterfactual competition policies which prevent in turn: (i) defensive investments that prevent the catch-up, column 2, and (ii) defensive investments that prevent the entry, column 3 of Table 4.

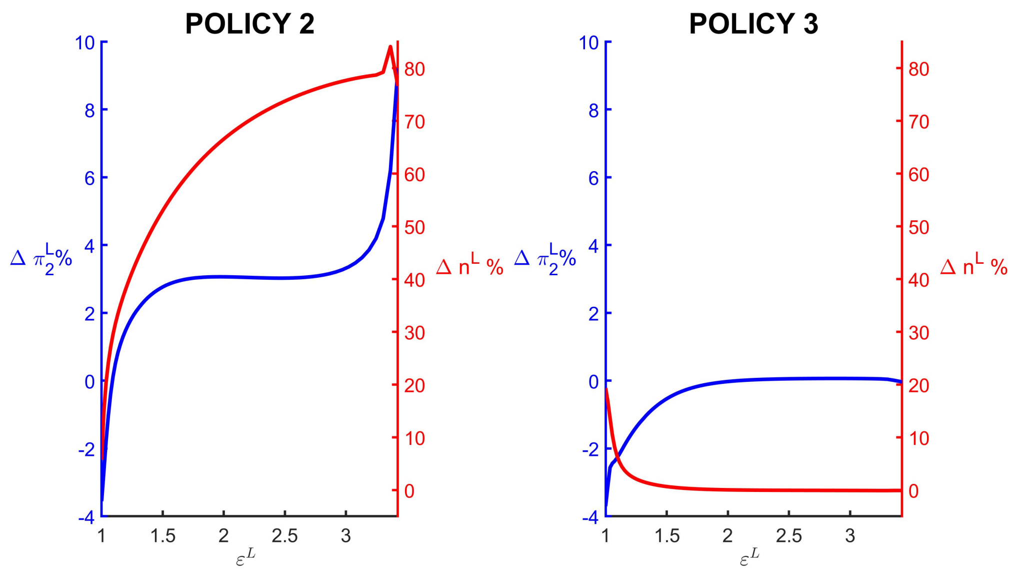
Perhaps surprisingly, I find the rise in TFP total is entirely due to the eradication of the defensive investments that prevent the catch-up. Indeed the policy that only tackles the defensive investment that prevents the catch-up enhances the endogenous total factor productivity by 4.73 percent. In contrast, the policy that tackles defensive investments that limit the entry reduces the TFP by 2.02 percent.

As Table 4 shows, the difference lies on their different effect on the average within-firm production efficiency. While preventing investments that limit the entry reduces the average within-firm production efficiency by 3.44 percent, preventing investments that limit the catch-up enhances it by 8.69 percent. In other words, defensive investments that limit the entry enhance the first-best TPFP and defensive investments that limit the catch-up decrease it.

To understand this result, recall that two types of defensive investments benefit two different types of leaders. Leaders that have a small technology advantage with respect to the potential competitors benefit from defensive investments that limit the entry. By contrast leaders that have a large technology advantage with respect to their competitors benefit from defensive investments that limit the catch-up.

I illustrate the effects of the two policies on the leaders' technology adoption in Figure 2 in which I report how π^L and n^L respond to the implementation of the policy. Notice under policy 2, the leaders' technology adoption particularly improves in industries in which the productivity of the technology frontier is high and in which leaders can potentially enjoy larger technology gap with respect to the current and potential competitors. Intuitively, the positive effect of the huge rise in competition incentivizes the productivity-enhancing investments of the leaders in industries where leaders are high productive. Conversely, under policy 3 the rise in competition, concentrated only in industries of low productive leaders, is moderate and, hence, its effect can not compensate for the effect of the lack of technology appropriability.

Figure 2: Decomposing the effect of the defensive investments.



Note: The blue and red solid lines represent, respectively, the average percentage change of π^L and of n^L — respect to the benchmark economy—as function of the technology frontier of the industry.

3.2.3 Alternative parameterized economies

Which parameter drives the effect of the competition policy? To answer this question, I examine five alternative model parameterizations, shown in Table 2, in which differ in turn for: (i) elasticity of substitution across products η, (ii) elasticity of substitution across brands ρ, (iii) productivity of the technology of the intangible investments of leaders $\overline{Q}$, (iv) productivity of intangible investments technology of followers $\overline{T}$, and (iv) the productivity of the new product producers ε^{E^1}.

Table 5 compares the qualitative effect of a change in each modeling parameter to the impact of the competition policy on the TFP. To isolate the effect of the parameters, each column modifies in isolation a different parameter without recalibrating the model.

I start by comparing the effect of the competition policy in the benchmark economy (Table 4) to an economy where the elasticity of substitution across product is lower, $\downarrow \eta$ (Table 5), and set as in Bilbiie et al. (2012). Although the negative impact of the fall in product variety on the TFP is stronger, the competition policy has a larger positive effect on TFP. The reason is twofold. First, in this economy the defensive intensity of the leaders is higher. The weaker competition across products reduces the relative importance of the productivity-enhancing intangible investment relative to the defensive one. In particular, limiting the entry benefits the leaders more since they can potentially enjoy higher monopolistic markups. Thus, the competition policy enhances the first-best TPFP through a larger expansion of the brand varieties. In addition, the absence of defensive investments improves the TFP through a larger improvement in the intangible investment productivity due to a larger reduction in the share of resources employed in the intangible investment. Second, in line with Edmond et al. (2015), the positive effect of pro-competitive policies on the production factor allocation is stronger when the markup dispersion is higher.

When I focus on an economy in which the elasticity of substitution between brands is larger, $\uparrow \rho$ (Table 5), and set as in Edmond et al. (2023), the effect of the competition policy on TFP is negative. The reasons are two reasons. First the positive effect of the expansion of the brand varieties on the first-best TPFP is absent[25]. Second, the effect of preventing defensive investment has almost no effect of the within-firm production efficiency. The reasons are twofold. Firstly, the responsiveness of the entry in the incumbent industries is weaker since the high competition discourages new firm to enter as laggards. Secondly, the lack of appropriability strongly discourage the productivity-enhancing intangible investments.

When I consider an economy in which the productivity of the intangible investment technology of the leaders is relatively lower $\uparrow \overline{Q}$, the positive effect of the competition policy on the TFP is larger. Not surprisingly, in this environment the competition effect is much stronger than lack of appropriability effect, therefore, the policy has a more powerful effect on the within-firm production efficiency. By contrast, when I consider the opposite case $\uparrow \overline{T}$, the competition policy worsens the TFP.

Finally, when I focus on an economy in which the productivity of the new product producers is higher, $\uparrow \varepsilon^{E^1}$ (Table 5), and set to match the average startup size as in Sedláček (2020), the positive effect of the competition policy on TFP is larger. The reason is due to the lower responsiveness of the product variety. Intuitively, being more productive allows the new product entrants to enjoy higher profits in a highly competitive environment, therefore the absence of defensive investments has a weaker impact on the effort of the potential entrepreneurs to

[25] Although the average number of firms per industry increases the brand variety effect is negative. This result may appear counterintuitive but is, in fact, due to the reduction of the dispersion of distribution of industries over the number of firms. There are less very high populated industries.

create new products.

Table 5: Competition policies and TFP changes, alternative parameterizations

		$\downarrow\eta$	$\uparrow\rho$	$\uparrow\overline{Q}$	$\uparrow\overline{T}$	$\uparrow\varepsilon^{E^1}$
Moments						
Sales shares top percentiles (pct)	1	12.7	37.7	41.2	38.1	36.3
	5	32.3	65.6	61.2	65.0	59.3
	10	43.4	73.5	68.5	73.1	68.2
	25	60.4	83.8	78.5	83.0	80.7
	50	77.8	91.7	88.1	90.9	91.6
	75	90.5	96.8	94.8	96.3	97.6
Start-up relative size		0.98	0.79	0.58	0.97	0.54
μ		1.17	1.08	1.12	1.13	1.17
$\sigma_\mu(pct)$		2.31	2.32	1.43	1.73	2.83
Entry rate (pct)		27.7	18.4	24.1	17.3	14.6
$\frac{wL_2}{Y}$ (pct)		7.80	4.42	6.35	6.53	6.56
ΔM (pct)		−10.5	−26.1	−22.1	−20.1	−12.9
$\Delta\overline{n}$ (pct)		29.2	17.7	17.7	20.3	18.1
$\Delta\overline{\varepsilon}^L$ (pct)		2.6	0.4	2.2	0.5	0.9
$\Delta\overline{\varepsilon}^F$ (pct)		8.1	5.6	2.1	4.7	−6.0
$\Delta\overline{\varepsilon}_j$ (pct)		1.6	−1.5	0.5	−1.8	−1.0
TFP change						
$\Delta\log(1-\lambda^\mu)$		0.88	0.86	0.31	0.35	0.33
$\Delta\log(1-\frac{1}{\lambda^L})$		1.17	2.55	0.44	0.14	0.35
$\Delta\log(1+\lambda^M)$		−6.96	−6.81	−5.56	−5.12	−3.08
$\Delta\log(1+\lambda^n)$		1.90	−0.04	0.76	0.57	0.68
$\Delta\log(\overline{\varepsilon})$		6.64	−0.05	6.88	3.92	5.02
$\Delta\log(A)$		3.63	−3.40	2.83	−0.14	3.30

Note: The table reports the effects of the competition policy that prevents any type of defensive investment in different parameterized economies. $\overline{n}$, $\overline{\varepsilon}^L$, $\overline{\varepsilon}^F$, and $\overline{\varepsilon}_j$ are respectively the average across industries of number of firms, leader and follower's relative productivity, and productivity of the technology frontier. The changes under each policy are relative to the economy pre-policy implementation.

3.3 Validation of the model prediction

My model predicts that defensive investments incentivizes the creation of new products, hence they spur the product differentiation of the economy. I present evidences that aligns with my model prediction.

I first combine my previously build annual panel dataset of accounting information from Compustat, such as sales, and product similarity scores from Hoberg and Phillips (2016), with firm level lobbying expenditure and industry level regulation respectively from LobbyView and RegData database. I first find the evidence that a rise in lobbying expenditure predicts a future rise in the product differentiation of the industry. I then show that the confluence of regulation and lobbying is associated with a reduction in the similarity score of the industry. More specifically, the positive change of the product differentiation associated with the regulation is increasing in lobbying.

3.3.1 Data

Lobbying. The first data source that I merge to my firm level panel dataset is lobbying expenditures at firm level that I extract from LobbyView, a lobbying database that is based on the universe of lobbying reports filed under the Lobbying Disclosure Act of 1995. This dataset allows to investigate firm- and industry-level lobbying activities using the unique identifiers: gvkey and NAICS[26].

Regulation. The second data source that supplement my dataset is RegData, a measure of the total quantity of regulation by industry. RegData U.S. leverage text analysis and machine-learning algorithms to count the individual regulatory restrictions in the Code of Federal Regulations (CFR) and then link them to the industries that are affected. Hence, it also allows me to compare the relative restrictiveness across industries[27].

Product differentiation. I construct five measures of firm-level product differentiation based on the similarity score of Hoberg and Phillips (2016) by considering scores of firms that belong to the same NAICS-4 industry. The first one is the sales-weighted average similarity index of firm i, $\overline{s}_{i,j}$

$$\overline{s}_{i,j} = \frac{\sum_{h=1}^{n_{i,j}} \text{Sales}_{h,j} s_{i,h,j}}{\sum_{h=1}^{n_{i,j}} \text{Sales}_{h,j}}$$

[26] For an introduction to LobbyView see Kim (2017), Kim (2018), and Kim and Kunisky (2021).

[27] For an introduction see Al-Ubaydli and McLaughlin (2017). For a detailed discussion of the database and its limitations see Goldschlag and Tabarrok (2018).

where $n_{i,j}$ is the number of competitors of i, $s_{i,h} > 0$, that belong in the same industry. I then obtain my second similarity index as the highest similarity score of firm i

$$s_{i,j}^{\max} = \max\{s_{i,1,j}, s_{i,2,j}, \ldots, s_{i,h-1,j}, s_{i,h,j}, s_{i,h+1,j}, \ldots, s_{i,n_{i,j}-1,j}, s_{i,n_{i,j},j}\}$$

In addition, I consider other three measures of product differentiation $s_{i,j}^{\text{Top 5}}$, $s_{i,j}^{\text{Top 10}}$, and $s_{i,j}^{\text{Top 20}}$, as the sales-weighted sum of the highest five, ten and twenty, similarity scores of firm i.

I finally build five product similarity indexes at industry level, $\overline{S}_j$, $S_j^{\max}$, $S_j^{\text{Top 5}}$, $S_j^{\text{Top 10}}$, and $S_j^{\text{Top 20}}$, by summing the measure of product differentiation across firms.

3.3.2 Empirical tests of the model prediction

Table 6 presents the results of regressing the change in the logarithm of the similarity index on the past lags of logarithm of lobbying for the period of 2000 through 2015 [28]. Regressions are of the form

$$\Delta \log(S_{j,t}) = \beta_0 + \sum_{k}^{n_k} \beta_k \log(\text{Lobby}_{j,t-k} + 1) + \tau_t + \omega_j + u_{j,t} \tag{33}$$

where $S_{j,t}$ is the product similarity index of four-digit SIC industry j at time t, Lobby is the total expenditure in lobbying, τ_t and ω_j are respectively time and industry fix effect, and $u_{j,t}$ is an error term. The specifications detect a negative relationship between changes in the similarity index of the industry and past lobbying. In turn, the Sum-to-zero test shows that these correlations are statistically significant at the 5% level in four specifications and marginally significant (at the 10% level) in other 3 specifications.

Next I provide some evidence for a plausible channels that may link lobbying to product similarity of the industry. As shown in Gutiérrez and Philippon (2019), lobbying is also used to affect the industry regulation, so that lobbying and regulations are likely to interact with each other. In Table 7, I replicate the regression 33 adding the interaction between regulation and lobbying. Regressions are of the form

$$\begin{aligned} \Delta \log(S_{j,t}) = {} & \beta_0 + \beta_1 \log(\text{Lobby}_{j,t-4} + 1) + \beta_2 \log(\text{Reg}_{j,t-1}) + \\ & + \beta_3 \log(\text{Lobby}_{j,t-4} + 1) \log(\text{Reg}_{j,t-3}) + \\ & + \tau_t + \omega_j + u_{j,t} \end{aligned} \tag{34}$$

where Reg is a measure of regulation.

[28] For presentation purposes I present the regressions with only 3 and 4 lags. The result is similar for the regression with 2 lags.

The estimations detect that the confluence of lobbying and regulation is associated with a rise in the product differentiation. In short, the positive correlation of between regulation and the change in the future product differentiation of the industry increases in the level of lobbying. Intuitively when lobbying is high a large part of these expenditures could then be used to affect regulations to limit competition. Accordingly, the creation of barriers to competition spurs the product differentiation.

Table 6: Lobbying expenditure and product differentiation

	$\Delta\log(\overline{S}_{j,t})$		$\Delta\log(S^{\max}_{j,t})$		$\Delta\log(S^{\text{Top 5}}_{j,t})$		$\Delta\log(S^{\text{Top 10}}_{j,t})$		$\Delta\log(S^{\text{Top 20}}_{j,t})$	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
$\log(\text{Lobby})_{j,t-1}$	-0.080	-0.062	-0.134	-0.102	-0.080	-0.048	-0.066	-0.033	-0.071	-0.037
	(0.08)	(0.08)	(0.11)	(0.11)	(0.11)	(0.10)	(0.11)	(0.10)	(0.11)	(0.10)
$\log(\text{Lobby})_{j,t-2}$	0.041	-0.004	-0.051	-0.092	-0.019	-0.073	-0.049	-0.104	-0.042	-0.098
	(0.10)	(0.10)	(0.13)	(0.13)	(0.13)	(0.13)	(0.13)	(0.13)	(0.13)	(0.13)
$\log(\text{Lobby})_{j,t-3}$	-0.097	-0.158	0.009	-0.013	-0.029	-0.069	-0.025	-0.069	-0.031	-0.081
	(0.08)	(0.10)	(0.11)	(0.13)	(0.10)	(0.13)	(0.10)	(0.13)	(0.10)	(0.13)
$\log(\text{Lobby})_{j,t-4}$		0.096		0.040		0.082		0.087		0.095
		(0.08)		(0.11)		(0.10)		(0.10)		(0.10)
Constant	0.077*	0.092**	0.112**	0.124**	0.084	0.087	0.088*	0.092*	0.091	0.094*
	(0.04)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)
$\sum_k \beta_k = 0$	-0.137**	-0.128**	-0.177**	-0.167 **	-0.129*	-0.107	-0.140*	-0.118	-0.144*	-0.121
	(0.06)	(0.06)	(.08)	(0.08)	(.08)	(0.08)	(0.08)	(0.08)	(0.07)	(0.08)
Observations	590	541	590	541	590	541	590	541	590	541
R2	0.105	0.105	0.163	0.141	0.139	0.137	0.139	0.138	0.139	0.137
Industry f.e.	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Year f.e.	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y

Standard errors in parentheses
* p<0.10, ** p<0.05, *** p<0.01

Table 7: Differentiation on lobbying and regulation

	$\Delta \log(\overline{S}_{j,t})$	$\Delta \log(S_{j,t}^{\max})$	$\Delta \log(S_{j,t}^{\text{Top 5}})$	$\Delta \log(S_{j,t}^{\text{Top 10}})$	$\Delta \log(S_{j,t}^{\text{Top 20}})$
$\log(\text{Lobby}+1)_{j,t-4}$	0.267	0.316	0.477*	0.489**	0.465*
	(0.20)	(0.25)	(0.25)	(0.24)	(0.24)
$\log(\text{Reg})_{j,t-3}$	-0.076	-0.371*	-0.368*	-0.331	-0.332
	(0.17)	(0.22)	(0.21)	(0.21)	(0.21)
$\log(\text{Reg})_{j,t-3} \times \log(\text{Lobby}+1)_{j,t-4}$	-0.032	-0.038	-0.050*	-0.052**	-0.050**
	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)
Constant	0.663	3.159*	3.101*	2.795	2.806
	(1.44)	(1.82)	(1.76)	(1.75)	(1.75)
$\beta_2 + \beta_3 = 0$	-0.108	-0.409*	-0.418**	-0.383 *	-0.382*
	(0.17)	(0.214)	(0.208)	(0.206)	(0.205)
Observations	522	522	522	522	522
Adj. R2	0.116	0.212	0.211	0.211	0.209
Industry f.e.	Y	Y	Y	Y	Y
Year f.e.	Y	Y	Y	Y	Y

Standard errors in parentheses
* p<0.10, ** p<0.05, *** p<0.01

4 Conclusion

The effect of the firms' defensive practices on TFP crucially depends on their effects on markup dispersion, technology adoption, product and brand variety, as well as productivity of the intangible investment. To study the quantitative implication of the defensive practices on endogenous aggregate productivity, I build a heterogeneous firm model of technology adoption in which industry leaders can invest to deter the entry of new competitors or the catch-up of the followers. The model predicts that preventing defensive investments enhances the endogenous total factor productivity mainly by improving the technology adoption across firms and by expanding the brand variety. In addition, the model gives a novel prediction on how sectors react to defensive investments. Higher defensive investments should lead to a larger product differentiation. I find evidence that aligns with this model prediction. I first find the evidence that a rise in lobbying expenditure predicts a future rise in the product differentiation. I then show that the confluence of lobbying and regulation is associated with a reduction in the product similarity.

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A Propositions

A.1 Proposition 1

I can now describe the optimal effort of the defensive practices chosen by the leaders. Let $\Delta\hat{V}^L_{i,2}(\Omega) = V^L_{i,2}(\varepsilon_d, \varepsilon_{d-f}, n^L, n^F) - V^L_{i,2}(\varepsilon_d, \varepsilon_{d-f}, n^L+1, n^F-1)$ be the difference between the value of being leader in a industry with n^L and with n^L+1 leaders before the production takes place.

In the case $\Delta\hat{V}^L_{i,2}(\Omega) \leq 0$, the unique symmetric equilibrium is $x_1^{L*} = 0$. Conversely, if $\Delta\hat{V}^L_{i,2}(\Omega) > 0$, the first and second order conditions of the optimization problem 19 is

$$\frac{\partial\, \mathbb{E}_{\Omega'}\, V^L_{i,2}(\Omega'|X^F(\Omega), x^L_{i,1}, X^L_{-i,1}(\Omega), \Omega)}{\partial x^L_{i,1}} = -wx^L_{i,1} + \frac{X^F}{X^F+\bar{T}} \frac{\bar{Q}\Delta\hat{V}^L_{i,2}(\Omega)}{(x^L_{i,1}+X^L_{-i,1}+\bar{Q})^2} = 0 \tag{A.1}$$

$$\frac{\partial^2\, \mathbb{E}_{\Omega'}\, V^L_{i,2}(\Omega'|X^F(\Omega), x^L_{i,1}, X^L_{-i,1}(\Omega), \Omega)}{\partial x^{L2}_{i,1}} = -w - 2\frac{X^F}{X^F+\bar{T}} \frac{\bar{Q}\Delta\hat{V}^L_{i,2}(\Omega)}{(x^L_{i,1}+X^L_{-i,1}+\bar{Q})^3} < 0 \tag{A.2}$$

Imposing the symmetry to A.1, I derive the equation that describes the symmetric equilibria:

$$x_1^L = f(x_1^L, X^F) = \frac{1}{n^L}\sqrt{\frac{X^F}{X^F+\bar{T}}\frac{\bar{Q}\Delta\hat{V}_2^L(\Omega)}{wx_1^L}} - \frac{\bar{Q}}{n^L} \tag{A.3}$$

Given X^F, A.3 describes a self-map where:

- $\lim\limits_{x\to 0} f(x_1^L, X^F) = +\infty$
- $\lim\limits_{x\to \infty} f(x_1^L, X^F) = -\frac{\bar{Q}}{n^L}$
- $\frac{\partial f(x_1^L, X^F)}{\partial x_1^L} < 0 \quad \forall x_1^L \in R_+$

So there must exist a unique x_1^{L*} such that $x_1^{L*} = f(x_1^{L*}, X^F)$. Moreover

$$\frac{\partial f(x_1^L, X^F)}{\partial X^F} > 0 \quad \forall X^F \in \mathcal{R}_+$$

This implies

$$\frac{\partial x_1^{L*}}{\partial X^F} > 0 \quad \forall X^F \in \mathcal{R}_+ \tag{A.4}$$

A.2 Proposition 2

Let $\Delta\hat{V}^F_{i,2}(\Omega) = [V^L_{i,2}(\varepsilon_d, \varepsilon_{d-f}, n^L+1, n^F-1) - V^F_{i,2}(\varepsilon_d, \varepsilon_{d-f}, n^L, n^F)]$ be the difference between the value of winning and the value when no follower succeeds, and,let $\Delta\tilde{V}_2^F(\Omega) = [V_2^L(\varepsilon_d, \varepsilon_{d-f}, n^L+1, n^F-1) - V_2^F(\varepsilon_d, \varepsilon_{d-f}, n^L+1, n^F-1)]$ be the difference between the value of winning and the value when another follower wins the contest.

In the case $\Delta\hat{V}_{i,2}^F(\Omega) \leq 0$, the unique symmetric equilibrium is $x^{F*} = 0$. Conversely, if $\Delta\hat{V}_{i,2}^F(\Omega) > 0$, the first and second order conditions of the optimization problem 18

$$\begin{aligned} &\frac{\partial\, \mathbb{E}_{\Omega'}\, V_{i,2}^F(\Omega'|x_i^F, X_{-i}^F(\Omega), X_1^L(\Omega), \Omega)}{\partial x_i^F} = \\ &- w x_i^F + \frac{\bar{Q}}{X_1^L + \bar{Q}} \frac{[\bar{T}\Delta\tilde{V}_{i,2}^F(\Omega) + X_{-i}^F \Delta\hat{V}_{i,2}^F(\Omega)]}{(x_i^F + X_{-i}^F + \bar{T})^2} = 0 \end{aligned} \tag{A.5}$$

$$\begin{aligned} &\frac{\partial^2\, \mathbb{E}_{\Omega'}\, V_{i,2}^F(\Omega'|x_i^F, X_{-i}^F(\Omega), X_1^L(\Omega), \Omega)}{\partial x^{F2}} = \\ &- w - 2\frac{\bar{Q}}{x_1^L + \bar{Q}} \frac{[\bar{T}\Delta\hat{V}_{i,2}^F(\Omega) + X_{-i}^F \Delta\tilde{V}_{i,2}^F(\Omega)]}{(x_i^F + X_{-i}^F + \bar{T})^3} < 0 \end{aligned} \tag{A.6}$$

Imposing the symmetry to A.5, I derive the equation that describes the symmetric equilibria:

$$x^F = q(x^F, X_1^L) = -\frac{\bar{T}}{n^F} + \frac{1}{n^F}\sqrt{\frac{\bar{Q}}{X_1^L + \bar{Q}}\left\{\frac{\bar{T}}{w x^F}\Delta\hat{V}_2^F(\Omega) + \frac{(n^F-1)}{w}\Delta\tilde{V}_2^F(\Omega)\right\}} \tag{A.7}$$

Given X_1^L, A.7 describes a self-map where:

- $\lim_{x^F \to 0} q(x^F, x_1^L) = +\infty$
- $\lim_{x^F \to \infty} q(X_1^L, x^F) = -\frac{\bar{T}}{n^F} + \frac{1}{n^F}\sqrt{\frac{\bar{Q}}{X_1^L+\bar{Q}}\frac{(n^F-1)}{w}\Delta\tilde{V}_2^F(\Omega)}$
- $\frac{\partial q(x_1^L, x^F)}{\partial x^F} < 0 \quad \forall x^F \in \mathcal{R}_+$

So there must exist a unique x^{F*} such that $x^{F*} = f(x^{F*}, X_1^L)$. The equilibrium effort chosen by followers, x^{F*}, is increasing $\Delta\hat{V}_2^F(\Omega'|\Omega)$ and $\Delta\tilde{V}_2^F(\Omega'|\Omega)$.

Moreover
$$\frac{\partial q(x^F, X_1^L)}{\partial X_1^L} < 0 \quad \forall X_1^L \in \mathcal{R}_+$$
it implies
$$\frac{\partial x^{F*}}{\partial X_1^L} < 0 \quad \forall X_1^L \in \mathcal{R}_+ \tag{A.8}$$

A.3 Proposition 3

Let $f^* : x^F \to x_1^{L*}$ and $q^* : x_1^L \to x^{F*}$, by A.4 and A.8:

- $\frac{\partial f^*(x^F)}{\partial x^F} > 0 \quad \forall x^F \in \mathcal{R}_+$
- $\frac{\partial q^*(x^F)}{\partial x_1^L} < 0 \quad \forall x^F \in \mathcal{R}_+$

then it implies that there must exist a unique (x_1^{L**}, x^F) such that:

- $x_1^{L**} = f^*(q^*(x_1^{L**}))$
- $x^{F**} = q^*(f^*(x^{F**}))$

A.4 Proposition 4

Define

$\tilde{V}_0^L(\Omega) = \underset{n^L>1}{\mathbb{1}} V_{i,0}^L(\varepsilon_{d+1}, \varepsilon_{d-1}, 1, n^L - 1) + (1 - \underset{n^L>1}{\mathbb{1}})V_{i,0}^L(\varepsilon_{d+1}, \varepsilon_{\max\{d-f-1,1\}}, 1, n^F)$

as the value of the leader of winning the post-production contest. Then define

$$\bar{V}_0^L(\Omega) = V_{i,0}^F(\varepsilon_{d+1}, \varepsilon_{d-1}, 1, n^L - 1)$$

as the value of the leader of not winning the post-production contest when another leader succeeds.

Let $\Delta\hat{V}_0^L(\Omega) = [\tilde{V}_0^L(\Omega'|\Omega) - V_0^L(\Omega'|\Omega)]$ be the difference between the value of winning the post-production contest and the value when no leader succeeds, and let $\Delta\check{V}_0^L(\Omega) = [\tilde{V}_0(\Omega) - \bar{V}_0^F(\Omega)]$ be the difference between the value of winning contest and the value when another leader wins. In case $\Delta\hat{V}_0^L(\Omega) < 0$, the unique symmetric equilibrium is $x_2^{L*} = 0$. Conversely, if $\Delta\hat{V}_0^L(\Omega) > 0$, the first and second order conditions of the recursive optimization problem 23:

$$\frac{\partial \, \mathbb{E}_{\Omega'} \, V_{i,0}^L(\Omega'|x_{i,2}^L, X_{-i,2}^L(\Omega), \Omega)}{\partial x_{i,2}^L} = -wx_{i,2}^L + \frac{[\bar{Q}\Delta\hat{V}_0^L(\Omega) + X_{-i,2}^L\Delta\check{V}_0^L(\Omega)]}{(x_{i,2}^L + X_{-i,2}^L + \bar{Q})^2} = 0 \tag{A.9}$$

$$\frac{\partial^2 \, \mathbb{E}_{\Omega'} \, V_{i,0}^L(\Omega'|x_{i,2}^L, X_{-i,2}^L(\Omega), \Omega)}{\partial x_{i,2}^{L2}} = -w - 2\frac{[\bar{T}\Delta\hat{V}_0^L(\Omega) + X_{-i,2}^L\Delta\check{V}_0^L(\Omega)]}{(x_{i,2}^L + X_{-i,2}^L + \bar{Q})^3} < 0 \tag{A.10}$$

where $\Delta\check{V}_0^L(\Omega) = [\tilde{V}_0(\Omega) - \bar{V}_0^F(\Omega'|\Omega)]$ is the difference between the value of winning contest and the value when another leader wins.

Imposing the symmetry A.9, I derive the equation that describes the symmetric equilibria:

$$x_2^L = \omega(x_2^L) = \frac{1}{n^L}\sqrt{\frac{\bar{Q}}{wx_2^L}\Delta\hat{V}_0^L(\Omega) + \frac{(n^L-1)}{w}\Delta\check{V}_0^L(\Omega)} - \frac{\bar{Q}}{n^L} \tag{A.11}$$

A.3 represents a self-map where:

- $\lim_{x_1^L \to 0} \omega(x_1^L) = +\infty$
- $\lim_{x_1^L \to \infty} \omega(x_1^L) = \sqrt{\frac{(n^L-1)}{w}\Delta\hat{V}_0^L(\Omega)} - \frac{\bar{Q}}{n^L}$
- $\frac{\partial w(x_2^L)}{\partial x_2^L} < 0 \quad \forall x_2^L \in \mathcal{R}_+$

So there must exist a unique x_2^{L*} such that $x_2^{L*} = \omega(x_2^{L*})$.

B Industry distribution

Let $\mu(\varepsilon^L, \varepsilon^F, n^L, n^F)$ be the post-production mass of industries of type Ω of productivity of leaders ε^L, productivity of followers ε^F, number of leaders n^L, and number of followers n^F. Let $\hat{\mu}(\Omega) = (1-\phi)\mu(\Omega)$ be the post-production mass of industries of type Ω that were not hit by the destruction shock. In addition, let there be a fixed mass Σ of potential entrepreneurs attempting to enter the economy in each period. The next-period mass of industries μ' of leader and follower technology $(\varepsilon_d, \varepsilon_{d-f})$ and number of leaders and followers $(n^{L\prime}, n^{F\prime})$ is obtained

For $f = 1$

$$
\begin{gathered}
\mu'(\varepsilon_d, \varepsilon_{d-f}, n^{L\prime}, n^{F\prime}) \\
= \hat{\mu}(\varepsilon_{d+1}, \varepsilon_{d-f+1}, n^{L\prime}, n^{F\prime})(1-\pi^L(\varepsilon_{d+1}, \varepsilon_{d-f+1}, n^{L\prime}, n^{F\prime}))(1-\pi^{E^2}(\varepsilon_d, \varepsilon_{d-f}, n^{L\prime}, n^{F\prime}) \\
(1-\pi^F(\varepsilon_d, \varepsilon_{d-f}, n^{L\prime}, n^{F\prime})) + \mathbb{1}_{n^{L\prime}>1} \hat{\mu}(\varepsilon_{d+1}, \varepsilon_{d-f+1}, n^{L\prime}-1, n^{F\prime}+1) \\
(1-\pi^L(\varepsilon_{d+1}, \varepsilon_{d-f+1}, n^{L\prime}-1, n^{F\prime}+1)) \\
(1-\pi^{E^2}(\varepsilon_d, \varepsilon_{d-f}, n^{L\prime}-1, n^{F\prime}+1)\pi^F(\varepsilon_d, \varepsilon_{d-f}, n^{L\prime}-1, n^{F\prime}+1)+ \\
\mathbb{1}_{n^{L\prime}>1} \hat{\mu}(\varepsilon_{d+1}, \varepsilon_{d-f+1}, n^{L\prime}-1, n^{F\prime}) \\
(1-\pi^L(\varepsilon_{d+1}, \varepsilon_{d-f+1}, n^{L\prime}-1, n^{F\prime}))\pi^{E^2}(\varepsilon_d, \varepsilon_{d-f}, n^{L\prime}-1, n^{F\prime}) \\
\pi^F(\varepsilon_d, \varepsilon_{d-f}, n^{L\prime}-1, n^{F\prime}+1) + \mathbb{1}_{n^{F\prime}>0} \hat{\mu}(\varepsilon_{d+1}, \varepsilon_{d-f+1}, n^{L\prime}, n^{F\prime}-1) \\
(1-\pi^L(\varepsilon_{d+1}, \varepsilon_{d-f+1}, n^{L\prime}-1, n^{F\prime}))\pi^{E^2}(\varepsilon_d, \varepsilon_{d-f}, n^{L\prime}, n^{F\prime}-1) \\
(1-\pi^F(\varepsilon_d, \varepsilon_{d-f}, n^{L\prime}, n^{F\prime}))+ \\
\mathbb{1}_{d-f=1} \Big\{ \hat{\mu}(\varepsilon_{d+1}, \varepsilon_{d-f}, n^{L\prime}, n^{F\prime})(1-\pi^L(\varepsilon_{d+1}, \varepsilon_{d-f}, n^{L\prime}, n^{F\prime}))(1-\pi^{E^2}(\varepsilon_d, \varepsilon_{d-f}, n^{L\prime}, n^{F\prime}) \\
(1-\pi^F(\varepsilon_d, \varepsilon_{d-f}, n^{L\prime}, n^{F\prime})) + \mathbb{1}_{n^{L\prime}>1} \hat{\mu}(\varepsilon_{d+1}, \varepsilon_{d-f}, n^{L\prime}-1, n^{F\prime}+1) \\
(1-\pi^L(\varepsilon_{d+1}, \varepsilon_{d-f}, n^{L\prime}-1, n^{F\prime}+1)) \\
(1-\pi^{E^2}(\varepsilon_d, \varepsilon_{d-f}, n^{L\prime}-1, n^{F\prime}+1)\pi^F(\varepsilon_d, \varepsilon_{d-f}, n^{L\prime}-1, n^{F\prime}+1)+ \\
\mathbb{1}_{n^{L\prime}>1} \hat{\mu}(\varepsilon_{d+1}, \varepsilon_{d-f}, n^{L\prime}-1, n^{F\prime})
\end{gathered}
$$

$$(1-\pi^L(\varepsilon_{d+1},\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime}))\pi^{E^2}(\varepsilon_d,\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime})$$

$$\pi^F(\varepsilon_d,\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime}+1)+\underset{n^{F\prime}>0}{\mathbb{1}}\hat{\mu}(\varepsilon_{d+1},\varepsilon_{d-f},n^{L\prime},n^{F\prime}-1)$$

$$(1-\pi^L(\varepsilon_{d+1},\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime}))\pi^{E^2}(\varepsilon_d,\varepsilon_{d-f},n^{L\prime},n^{F\prime}-1)$$

$$(1-\pi^F(\varepsilon_d,\varepsilon_{d-f},n^{L\prime},n^{F\prime}))\Big\}$$

$$\underset{\varepsilon^{L\prime}=\varepsilon_{N^\varepsilon}}{\mathbb{1}}\Big\{\underset{n^{L\prime}=2}{\mathbb{1}}\sum_{\varepsilon^F}\sum_{n^F}\hat{\mu}(\varepsilon_{N^\varepsilon},\varepsilon^F,n^{F\prime}+2,n^F)$$

$$\pi^L(\varepsilon_{N^\varepsilon},\varepsilon^F,n^{F\prime}+2,n^F)(1-\pi^{E^2}(\varepsilon_{N^\varepsilon},\varepsilon_{N^\varepsilon-1},1,n^{F\prime}+1))\pi^F(\varepsilon_{N^\varepsilon},\varepsilon_{N^\varepsilon-1},1,n^{F\prime}+1)+$$

$$\underset{n^{L\prime}=2}{\mathbb{1}}\sum_{n^F}\sum_{\varepsilon^F}\hat{\mu}(\varepsilon_{N^\varepsilon},\varepsilon^F,n^{F\prime}+1,n^F)$$

$$\pi^L(\varepsilon_{N^\varepsilon},\varepsilon^F,n^{F\prime}+1,n^F)\pi^{E^2}(\varepsilon_{N^\varepsilon},\varepsilon_{N^\varepsilon-1},1,n^{F\prime})\pi^F(\varepsilon_{N^\varepsilon},\varepsilon_{N^\varepsilon-1},1,n^{F\prime}+1)+$$

$$\underset{\substack{n^{L\prime}=1\\n^{F\prime}>0}}{\mathbb{1}}\sum_{n^F}\sum_{\varepsilon^F}\hat{\mu}(\varepsilon_{N^\varepsilon},\varepsilon^F,n^{F\prime},n^F)\pi^L(\varepsilon_{N^\varepsilon},\varepsilon^F,n^{F\prime},n^F)$$

$$\pi^{E^2}(\varepsilon_{N^\varepsilon},\varepsilon_{N^\varepsilon-1},1,n^{F\prime}-1)(1-\pi^F(\varepsilon_{N^\varepsilon},\varepsilon_{N^\varepsilon-1},1,n^{F\prime}))+$$

$$\underset{\substack{n^{L\prime}=1\\n^{F\prime}>0}}{\mathbb{1}}\sum_{n^F}\sum_{\varepsilon^F}\hat{\mu}(\varepsilon_{N^\varepsilon},\varepsilon^F,n^{F\prime}+1,n^F)\pi^L(\varepsilon_{N^\varepsilon},\varepsilon^F,n^{F\prime}+1,n^F)$$

$$(1-\pi^{E^2}(\varepsilon_{N^\varepsilon},\varepsilon_{N^\varepsilon-1},1,n^{F\prime}))(1-\pi^F(\varepsilon_{N^\varepsilon},\varepsilon_{N^\varepsilon-1},1,n^{F\prime}))\Big\}+\underset{\varepsilon^{E^1}=2}{\mathbb{1}}E^1$$

For $f=2$

$$\mu'(\varepsilon_d,\varepsilon_{d-f},n^{L\prime},n^{F\prime})$$

$$=\hat{\mu}(\varepsilon_{d+1},\varepsilon_{d-f+1},n^{L\prime},n^{F\prime})(1-\pi^L(\varepsilon_{d+1},\varepsilon_{d-f+1},n^{L\prime},n^{F\prime}))(1-\pi^{E^2}(\varepsilon_d,\varepsilon_{d-f},n^{L\prime},n^{F\prime})$$

$$(1-\pi^F(\varepsilon_d,\varepsilon_{d-f},n^{L\prime},n^{F\prime}))+\underset{n^{L\prime}>1}{\mathbb{1}}\hat{\mu}(\varepsilon_{d+1},\varepsilon_{d-f+1},n^{L\prime}-1,n^{F\prime}+1)$$

$$(1-\pi^L(\varepsilon_{d+1},\varepsilon_{d-f+1},n^{L\prime}-1,n^{F\prime}+1))$$

$$(1-\pi^{E^2}(\varepsilon_d,\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime}+1)\pi^F(\varepsilon_d,\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime}+1)+$$

$$\underset{n^{L\prime}>1}{\mathbb{1}}\hat{\mu}(\varepsilon_{d+1},\varepsilon_{d-f+1},n^{L\prime}-1,n^{F\prime})$$

$$(1-\pi^L(\varepsilon_{d+1},\varepsilon_{d-f+1},n^{L\prime}-1,n^{F\prime}))\pi^{E^2}(\varepsilon_d,\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime})$$

$$\pi^F(\varepsilon_d,\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime}+1)+\underset{n^{F\prime}>0}{\mathbb{1}}\hat{\mu}(\varepsilon_{d+1},\varepsilon_{d-f+1},n^{L\prime},n^{F\prime}-1)$$

$$(1-\pi^L(\varepsilon_{d+1},\varepsilon_{d-f+1},n^{L\prime}-1,n^{F\prime}))\pi^{E^2}(\varepsilon_d,\varepsilon_{d-f},n^{L\prime},n^{F\prime}-1)$$

$$(1-\pi^F(\varepsilon_d,\varepsilon_{d-f},n^{L\prime},n^{F\prime}))+\mathbb{1}_{n^{L\prime}=2}\sum_{n^F}\sum_{\varepsilon^F}\hat{\mu}(\varepsilon_{d-1},\varepsilon^F,n^{F\prime}+2,n^F)$$

$$\pi^L(\varepsilon_{d-1},\varepsilon^F,n^{F\prime}+1,n^F)(1-\pi^{E^2}(\varepsilon_d,\varepsilon_{d-2},1,n^{F\prime}+1))\pi^F(\varepsilon_d,\varepsilon_{d-2},1,n^{F\prime}+1)+$$

$$\mathbb{1}_{d-f=1}\Big\{\hat{\mu}(\varepsilon_{d+1},\varepsilon_{d-f},n^{L\prime},n^{F\prime})(1-\pi^L(\varepsilon_{d+1},\varepsilon_{d-f},n^{L\prime},n^{F\prime}))(1-\pi^{E^2}(\varepsilon_d,\varepsilon_{d-f},n^{L\prime},n^{F\prime})$$

$$(1-\pi^F(\varepsilon_d,\varepsilon_{d-f},n^{L\prime},n^{F\prime}))+\mathbb{1}_{n^{L\prime}>1}\hat{\mu}(\varepsilon_{d+1},\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime}+1)$$

$$(1-\pi^L(\varepsilon_{d+1},\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime}+1))$$

$$(1-\pi^{E^2}(\varepsilon_d,\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime}+1)\pi^F(\varepsilon_d,\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime}+1)+$$

$$\mathbb{1}_{n^{L\prime}>1}\hat{\mu}(\varepsilon_{d+1},\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime})$$

$$(1-\pi^L(\varepsilon_{d+1},\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime}))\pi^{E^2}(\varepsilon_d,\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime})$$

$$\pi^F(\varepsilon_d,\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime}+1)+\mathbb{1}_{n^{F\prime}>0}\hat{\mu}(\varepsilon_{d+1},\varepsilon_{d-f},n^{L\prime},n^{F\prime}-1)$$

$$(1-\pi^L(\varepsilon_{d+1},\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime}))\pi^{E^2}(\varepsilon_d,\varepsilon_{d-f},n^{L\prime},n^{F\prime}-1)$$

$$(1-\pi^F(\varepsilon_d,\varepsilon_{d-f},n^{L\prime},n^{F\prime}))\Big\}+$$

$$\mathbb{1}_{\substack{n^{L\prime}=2\\(n^{F\prime}>0)}}\sum_{n^F}\sum_{\varepsilon^F}\hat{\mu}(\varepsilon_{d-1},\varepsilon^F,n^{F\prime}+1,n^F)$$

$$\pi^L(\varepsilon_{d-1},\varepsilon^F,n^{F\prime}+1,n^F)\pi^{E^2}(\varepsilon_d,\varepsilon_{d-2},1,n^{F\prime})\pi^F(\varepsilon_d,\varepsilon_{d-2},1,n^{F\prime}+1)+$$

$$\mathbb{1}_{\substack{n^{L\prime}=1\\(n^{F\prime}>0)}}\sum_{n^F}\sum_{\varepsilon^F}\hat{\mu}(\varepsilon_{d-1},\varepsilon^F,n^{F\prime},n^F)\pi^L(\varepsilon_{d-1},\varepsilon^F,n^{F\prime},n^F)$$

$$\pi^{E^2}(\varepsilon_d,\varepsilon_{d-2},1,n^{F\prime}-1)(1-\pi^F(\varepsilon_d,\varepsilon_{d-2},1,n^{F\prime}))+$$

$$\mathbb{1}_{\substack{n^{L\prime}=1\\(n^{F\prime}>0)}}\sum_{n^F>0}\sum_{\varepsilon^F}\hat{\mu}(\varepsilon_{d-1},\varepsilon^F,n^{F\prime}+1,n^F)\pi^L(\varepsilon_{d-1},\varepsilon^F,n^{F\prime}+1,n^F)$$

$$(1-\pi^{E^2}(\varepsilon_d,\varepsilon_{d-2},1,n^{F\prime}))(1-\pi^F(\varepsilon_d,\varepsilon_{d-2},1,n^{F\prime}))+\mathbb{1}_{\varepsilon^{E^1}=3}E^1$$

For $f>2$

$$\mu'(\varepsilon_d,\varepsilon_{d-f},n^{L\prime},n^{F\prime})$$

$$=\hat{\mu}(\varepsilon_{d+1},\varepsilon_{d-f+1},n^{L\prime},n^{F\prime})(1-\pi^L(\varepsilon_{d+1},\varepsilon_{d-f+1},n^{L\prime},n^{F\prime}))(1-\pi^{E^2}(\varepsilon_d,\varepsilon_{d-f},n^{L\prime},n^{F\prime})$$

$$(1-\pi^F(\varepsilon_d,\varepsilon_{d-f},n^{L\prime},n^{F\prime}))+\mathbb{1}_{n^{L\prime}>1}\hat{\mu}(\varepsilon_{d+1},\varepsilon_{d-f+1},n^{L\prime}-1,n^{F\prime}+1)$$

$$(1-\pi^L(\varepsilon_{d+1},\varepsilon_{d-f+1},n^{L\prime}-1,n^{F\prime}+1))$$

$$(1-\pi^{E^2}(\varepsilon_d,\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime}+1)\pi^F(\varepsilon_d,\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime}+1)+$$

$$\mathbb{1}_{n^{L\prime}>1}\hat{\mu}(\varepsilon_{d+1},\varepsilon_{d-f+1},n^{L\prime}-1,n^{F\prime})$$

$$(1-\pi^L(\varepsilon_{d+1},\varepsilon_{d-f+1},n^{L\prime}-1,n^{F\prime}))\pi^{E^2}(\varepsilon_d,\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime})$$

$$\pi^F(\varepsilon_d,\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime}+1)+\mathbb{1}_{n^{F\prime}>0}\hat{\mu}(\varepsilon_{d+1},\varepsilon_{d-f+1},n^{L\prime},n^{F\prime}-1)$$

$$(1-\pi^L(\varepsilon_{d+1},\varepsilon_{d-f+1},n^{L\prime}-1,n^{F\prime}))\pi^{E^2}(\varepsilon_d,\varepsilon_{d-f},n^{L\prime},n^{F\prime}-1)$$

$$(1-\pi^F(\varepsilon_d,\varepsilon_{d-f},n^{L\prime},n^{F\prime}))+$$

$$\mathbb{1}_{d-f=1}\Big\{\hat{\mu}(\varepsilon_{d+1},\varepsilon_{d-f},n^{L\prime},n^{F\prime})(1-\pi^L(\varepsilon_{d+1},\varepsilon_{d-f},n^{L\prime},n^{F\prime}))(1-\pi^{E^2}(\varepsilon_d,\varepsilon_{d-f},n^{L\prime},n^{F\prime})$$

$$(1-\pi^F(\varepsilon_d,\varepsilon_{d-f},n^{L\prime},n^{F\prime}))+\mathbb{1}_{n^{L\prime}>1}\hat{\mu}(\varepsilon_{d+1},\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime}+1)$$

$$(1-\pi^L(\varepsilon_{d+1},\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime}+1))$$

$$(1-\pi^{E^2}(\varepsilon_d,\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime}+1)\pi^F(\varepsilon_d,\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime}+1)+$$

$$\mathbb{1}_{n^{L\prime}>1}\hat{\mu}(\varepsilon_{d+1},\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime})$$

$$(1-\pi^L(\varepsilon_{d+1},\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime}))\pi^{E^2}(\varepsilon_d,\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime})$$

$$\pi^F(\varepsilon_d,\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime}+1)+\mathbb{1}_{n^{F\prime}>0}\hat{\mu}(\varepsilon_{d+1},\varepsilon_{d-f},n^{L\prime},n^{F\prime}-1)$$

$$(1-\pi^L(\varepsilon_{d+1},\varepsilon_{d-f},n^{L\prime}-1,n^{F\prime}))\pi^{E^2}(\varepsilon_d,\varepsilon_{d-f},n^{L\prime},n^{F\prime}-1)$$

$$(1-\pi^F(\varepsilon_d,\varepsilon_{d-f},n^{L\prime},n^{F\prime}))\Big\}+$$

$$\mathbb{1}_{n^{L\prime}=2}\hat{\mu}(\varepsilon_{d-1},\varepsilon_{d-f+1},1,n^{F\prime}+1)$$

$$\pi^L(\varepsilon_{d-1},\varepsilon_{d-f+1},1,n^{F\prime}+1)(1-\pi^{E^2}(\varepsilon_d,\varepsilon_{d-f},1,n^{F\prime}+1))\pi^F(\varepsilon_d,\varepsilon_{d-f},1,n^{F\prime}+1)+$$

$$\mathbb{1}_{n^{L\prime}=2}\hat{\mu}(\varepsilon_{d-1},\varepsilon_{d-f+1},1,n^{F\prime})$$

$$\pi^L(\varepsilon_{d-1},\varepsilon_{d-f+1},1,n^{F\prime})\pi^{E^2}(\varepsilon_d,\varepsilon_{d-f},1,n^{F\prime})\pi^F(\varepsilon_d,\varepsilon_{d-f},1,n^{F\prime}+1)+$$

$$\mathbb{1}_{n^{L\prime}=1}\hat{\mu}(\varepsilon_{d-1},\varepsilon_{d-f+1},1,n^{F\prime}-1)\pi^L(\varepsilon_{d-1},\varepsilon_{d-f+1},1,n^{F\prime}-1)$$

$$\pi^{E^2}(\varepsilon_d,\varepsilon_{d-f},1,n^{F\prime}-1)(1-\pi^F(\varepsilon_d,\varepsilon_{d-f},1,n^{F\prime}))+$$

$$\mathbb{1}_{n^{L\prime}=1}\hat{\mu}(\varepsilon_{d-1},\varepsilon_{d-f+1},1,n^{F\prime})\pi^L(\varepsilon_{d-1},\varepsilon_{d-f+1},1,n^{F\prime})$$

$$(1-\pi^{E^2}(\varepsilon_{d-1},\varepsilon_{d-f},1,n^{F\prime}))(1-\pi^F(\varepsilon_{d-1},\varepsilon_{d-f},1,n^{F\prime}))+$$

$$\mathbb{1}_{d-f=1}\Big\{\mathbb{1}_{n^{L\prime}=2}\,\hat{\mu}(\varepsilon_{d-1},\varepsilon_{d-f},1,n^{F\prime}+1)$$

$$\pi^L(\varepsilon_{d-1},\varepsilon_{d-f},1,n^{F\prime}+1)(1-\pi^{E^2}(\varepsilon_d,\varepsilon_{d-f},1,n^{F\prime}+1))\pi^F(\varepsilon_d,\varepsilon_{d-f},1,n^{F\prime}+1)+$$

$$\mathbb{1}_{n^{L\prime}=2}\,\hat{\mu}(\varepsilon_{d-1},\varepsilon_{d-f},1,n^{F\prime})$$

$$\pi^L(\varepsilon_{d-1},\varepsilon_{d-f},1,n^{F\prime})\pi^{E^2}(\varepsilon_d,\varepsilon_{d-f},1,n^{F\prime})\pi^F(\varepsilon_d,\varepsilon_{d-f},1,n^{F\prime}+1)+$$

$$\mathbb{1}_{n^{L\prime}=1}\,\hat{\mu}(\varepsilon_{d-1},\varepsilon_{d-f},1,n^{F\prime}-1)\pi^L(\varepsilon_{d-1},\varepsilon_{d-f},1,n^{F\prime}-1)$$

$$\pi^{E^2}(\varepsilon_d,\varepsilon_{d-f},1,n^{F\prime}-1)(1-\pi^F(\varepsilon_d,\varepsilon_{d-f},1,n^{F\prime}))+$$

$$\mathbb{1}_{n^{L\prime}=1}\,\hat{\mu}(\varepsilon_{d-1},\varepsilon_{d-f+1},1,n^{F\prime})\pi^L(\varepsilon_{d-1},\varepsilon_{d-f},1,n^{F\prime})$$

$$(1-\pi^{E^2}(\varepsilon_{d-1},\varepsilon_{d-f},1,n^{F\prime}))(1-\pi^F(\varepsilon_{d-1},\varepsilon_{d-f},1,n^{F\prime}))\Big\}+\mathbb{1}_{\varepsilon^{E^1}>3}\,E^1$$

where the mass of new products is

$$E^1=\Sigma\frac{x^{E^1}}{x^{E^1}+\overline{Q}}\Big\{\mathbb{1}_{\substack{n^{L\prime}=2\\ n^{F\prime}=0}}\,\pi^{E^2}(\varepsilon^{E^1},\varepsilon_1,1,0)\pi^F(\varepsilon^{E^1},\varepsilon_1,1,1)+$$

$$\mathbb{1}_{\substack{n^{L\prime}=1\\ n^{F\prime}=1}}\,\pi^{E^2}(\varepsilon^{E^1},\varepsilon_1,1,0)(1-\pi^F(\varepsilon^{E^1},\varepsilon_1,1,1))+$$

$$\mathbb{1}_{\substack{n^{L\prime}=1\\ n^{F\prime}=0}}(1-\pi^{E^2}(\varepsilon^{E^1},\varepsilon_1,1,0))(1-\pi^F(\varepsilon^{E^1},\varepsilon_1,1,1))\Big\}$$

C Calibration

Table C.1: Intangible investment intensity on firm size

	Data				
	(1)	(2)	(3)	(4)	(5)
Firm age	0.02**	-0.01*	0.03**	0.01*	0.09***
	(0.01)	(0.00)	(0.01)	(0.01)	(0.00)
Firm Size	-0.29***	-0.17***	-0.31***	-0.20***	-0.20***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Constant	1.74***	1.23***	1.80***	1.32***	1.13***
	(0.02)	(0.01)	(0.03)	(0.01)	(0.01)
Observations	24826.00	25455.00	24054.00	25467.00	25467.00
Adj. R2	0.88	0.64	0.88	0.53	0.50
Firm f.e.	Y	N	Y	N	N
Industry f.e.	N	Y	N	N	N
Industry × Year f.e.	N	N	Y	N	N
Year f.e.	Y	Y	N	Y	N

Standard errors in parentheses

* p<0.10, ** p<0.05, *** p<0.01