# **MATHEMATICS**

September  $1^{st}$ , 2022

### Outline

The goal of this course is to help students understand and use the mathematics required for studying economics at the graduate level. As a way of demonstrating the importance of mathematics in economics, the mathematical concepts studied will be illustrated with applications in economics.

### **Topics**

# Single-variable calculus

- 1 Functions. Continuity, limits, derivatives and differentials. Composite functions and the chain rule.
- 2 Single variable optimization. Integrals.

# Linear algebra

- 3 Vectors and matrices. Matrix operations. Trace, determinant, rank and the inverse of a matrix. Main results on linear systems.
- 4 Eigenvalues, eigenvectors and diagonalization. Quadratic forms. Vector spaces.

#### Analysis in $\mathbb{R}^n$

- 5 Some topologic concepts. Metric spaces. Open and closed sets, interior and closure.
- 6 Sequences. Convergence. Compactness. Cauchy sequences.
- 7 Limits and continuity. Functions. Limit of a function. Continuity. Weierstrass' theorem.

### Multivariable calculus

- 8 Partial and directional derivatives. Differentiability. The gradient.
- 9 Level sets. Chain rule. Higher-order derivatives. Taylor formula. Integration.
- 10 Inverse function theorem. Implicit function theorem. An application: IS-LM model.
- 11 Convex sets. Separating hyperplanes. Convex and concave functions. Quasi-concave and quasi-convex functions.

## Static optimization

- 12 Unconstrained optimization. Some definitions. First-order conditions. Second-order conditions. Global maximum and minimum. An application: OLS estimation.
- 13 Optimization with equality constraints. Lagrange method. Necessary conditions. Sufficient conditions. Interpretation.
- 14 Optimization with inequality constraints. Geometric deduction. Kuhn-Tucker (1951)'s conditions. Formulation in terms of the Lagrangian. The convex case.

### Difference and differential equations

- 15 Introduction to difference equations. Examples. Some useful methods.
- 16 Introduction to differential equations. Examples. Some useful methods.
- 17 Linear difference equations of order n. Difference equations of order n. The linear case. The constant coefficients case. Particular solutions.
- 18 Linear differential equations of order n. Differential equations of order n. The constant coefficients case. Particular solutions.

## Dynamic systems

- 19  $1^{st}$ -order systems of difference equations. The linear case. From a linear difference equation of order n to a  $1^{st}$ -order system. Homogeneous systems. Full systems.
- 20  $1^{st}$ -order systems of differential equations. From a linear differential equation of order n to a  $1^{st}$ -order system. Homogeneous systems. The time-invariant case. Full systems.

#### Grading

The Monday after the end of the course there is a final exam. In addition, there will be a problem set that will be graded. The final grade of the course will be a weighted average of the final exam (80%), homework (15%) and class participation (5%).

#### Basic references

Simon, C.P. and Blume, L. (2010) *Mathematics for Economists*. W.W. Norton, New York.

Sydsaeter, K. and Hammond, P. (2006). Essential Mathematics for Economic Analysis. Second Edition. Prentice Hall, Harlow, England.

Sydsaeter, K. and Hammond, P. (2005). Further Mathematics for Economic Analysis. Prentice Hall, Harlow, England.

#### Additional references

Apostol, T.M. (1974). *Mathematical Analysis*. Second Edition. Addison-Wesley, Reading, Massachusetts.

Binmore, K.G. (1983). Calculus. Cambridge University Press, Cambridge.

Burger, E.B. and Starbird, M. (2005). The Heart of Mathematics. An invitation to effective thinking. Second Edition. Key College Publishing. Emeryville, California.

De la Fuente, A. (2000). *Mathematical Methods and Models for Economists*. Cambridge University Press.

Gilbert, L. and Gilbert, J. (1995). College Algebra with Trigonometry. McGraw-Hill, New York.

Haeussler, E.F. and Paul, R.S. (1996). Introductory Mathematical Analysis for Business, Economics, and the Life and Social Sciences. Eighth Edition. Prentice Hall, Upper Saddle River, New Jersey.

Jacques, I. (1999). *Mathematics for Economics and Business*. Third Edition. Prentice Hall, Harlow, England.

Klein, M.W. (1988). *Mathematical Methods for Economics*. Addison-Wesley, Reading, Massachusetts.

Mas-Colell, A., Whinston, M.D., and Green, J.R. (1995). *Microeconomic Theory*. Oxford University Press.

Novshek, W. (1993). Mathematics for Economists. Academia Press, San Diego.

Woolridge, J. (2006). Introductory Econometrics: A Modern Approach. South-Western College Publishing. Third Edition.