CREDIT AND LIQUIDITY RISK IN SOVEREIGN BONDS

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Abstract

This master thesis, builds on the contributions by Ejsing et al. (2012) and Dubecq et al. (2013) to shed light on the credit and liquidity risk components of sovereign bond yields. Four countries have been studied: France, the Netherlands, Germany and Spain. The task has been faced by considering several state-space models, all of them connected by the idea that the differences in the yields between agency and sovereign bonds mainly correspond to liquidity reasons. The estimated credit and liquidity latent factors capture two distress periods, coinciding with the banking crisis and the sovereign debt crises. At those times we appreciate that the credit factor pushes up the yield to maturity of the bonds, while the liquidity factor exerts downward pressure to it; in general, the credit effect dominates. We also identify a transversal credit effect across countries that allows to distinguish idiosyncratic (credit) patterns.

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1 Introduction

The term structure of sovereign bonds is a key piece of information in economics and finance. It reports how agents are discounting future events and provides a primary signal for the financial shape of a country. Therefore, it is of utmost importance to know about its determinants. A possible approach, quite in vogue in recent years in the aftermath of the crisis, is to decompose sovereign bond yields into credit and liquidity effects. Credit risk accounts for the possible losses that a bond holder would suffer if the issuer defaults. Liquidity risk accounts for the impact of the easiness or difficulty to trade the bond in the price. Following this line, the bond yield of a country will increase with the probability of default and highly traded bonds are expected to offer a lower yield than less traded ones.

This thesis builds on the idea that the yield differences between equally credit risky bonds of similar maturities are due to liquidity reasons. This is a condition met by sovereign and agency bonds if the latter enjoy an explicit guarantee by the government. In other words, they are ex-ante equally credit risky. Four countries will be considered for the analysis: France, Germany, the Netherlands and Spain, whose main agencies are, respectively, CADES (Caisse d’Amortissement de la Dette Sociale; social security debt redemption fund), KfW (Kreditanstalt für Wiederaufbau; Reconstruction Credit Institute), BNG (Bank Nederlandse Gemeenten; Dutch Municipal Bank) and ICO (Instituto de Crédito Oficial; Official Credit Institute).

For this task, we consider several state-space models that explain the spread between the yields and the risk free rate as functions of credit and liquidity latent factors. In all cases these models will be estimated by maximum likelihood using Kalman filtering. The risk free rate will be proxied by the Overnight Indexed Swap rate (OIS). We start initially with a simple benchmark model that only requires as input the agency and sovereign yields plus the OIS rate to identify both factors. We continue by including Credit Default Swap rates (CDS), which basically provide insurance against sovereign default, in this benchmark model to improve the estimation of the credit factors and then looking for a common effect for the Netherlands, Germany and France. In the three cases the spreads are affine functions of the factors. Finally we look at the problem from a more ambitious perspective, where we can provide factor dynamics under the physical and in the risk neutral measures and the spreads are quadratic functions of the factors.

A fundamental feature of all the models considered is that they allow us to quantify the
impact of credit and liquidity risk on the yields. In this sense we claim that the credit factor has in general more influence than the liquidity factor. We capture two distress periods corresponding to the banking crisis and the financial crisis. Interestingly, at those moments the credit factor displayed maximums but its effect on the yield was disguised by the liquidity factor, which accounted for the safe haven flows phenomena typical for highly rated sovereign bonds under tense financial episodes. Furthermore, when we account for a common (credit) factor, we can observe substantial differences in the country specific credit risks. Under this approach, the Netherlands’ idiosyncratic credit factor remains quite stable around zero while France and Germany’s exhibit a mirrored pattern, displaying the widest gap in 2012 and taking positive and negative values respectively.

The rest of the paper continues as follows. Section 2 reviews the previous literature on the topic. Section 3 discusses the data used and the computational methodology for the yield curves. The three affine models comprise section 4 and the fourth model is developed in section 5. Section 6 concludes. The appendix contains all tables, figures and two proofs.

2 Related Literature

This master thesis builds on the contributions of two previous papers: section 4 elaborates on the parametric framework designed by Ejsing et al. (2012) and section 5 consists on an adaptation of a model for the EURIBOR rates by Dubecq et al. (2013) for bond yields. Ejsing et al. (2012)’s work presents a model free and a model based framework to separate the credit and liquidity risk contributions to French and German sovereign yields. In both instances, they exploit the consideration that sovereign and agency bonds are equally credit worthy. In that case the difference between their yields must be due to liquidity reasons, which they use to identify both factors. Dubecq et al. (2013) model the EURIBOR rates as the expected discounted values of the risk free rate plus an intensity parameter. This intensity parameter is a quadratic function of credit and liquidity latent factors and takes different values under the physical and the risk neutral measure.

The finance literature related to the decomposition of term structures into liquidity and credit factors can be classified into two main categories. Firstly, we have the approaches that proxy only one factor and the ones that proxy both of them. Secondly, we can distinguish between models that make interest rates or yields linear approximations of
the factors and models that make them non linear (generally quadratic) functions of the factors.

Within the first category, the paper by Ejsing et al. (2012) is a good example of the papers that only proxy one of the factors (they proxy liquidity and assume credit risk to be the same for the agency and the sovereign). We can also include here Monfort and Renne (2012), who analyze the yields of 8 euro-area countries. They assume perfect co-movement among countries in the liquidity factor and proxy it by the spread between the German Bund and the KfW (German agency) emissions. For the cases it has been possible, we do better in this thesis, as we relax the co-movement assumption in liquidity and proxy this factor by the national sovereign-agency spread. Schwarz (2013) represents an archetype of the papers that proxy both factors. She presents a multi-country European model in which liquidity is proxied by the spread between the sovereign German bond and the KfW bond and proposes a credit measure “defined as the daily spread between actual unsecured interbank borrowing rates paid by banks that are relatively good credit risks versus those that are relatively bad credit risks”.

Another examples are Liu et al. (2006), where they jointly model sovereign, repo and swap term structures as an affine 5 factor model, 2 of them accounting for liquidity and credit risk; Favero et al. (2008), who choose bid-ask spread for liquidity and the spread between the US sovereign yield and the US corporate swap rate for credit; or Dubecq et al. (2013), who proxy liquidity premia by the first principal component of the KfW-Bund spread, Tbill-repo spread and a factor based on a ECB survey, and credit premia by the first principal component of 36 Eurozone banks’ CDS rates.

Into the second category, Ejsing et al. (2012) and Liu et al. (2006) belong to the group that presents yields as linear functions of the factors. Dubecq et al. (2013) make use of the quadratic Kalman filter designed by three of the coauthors (Monfort et al. (2013)) for their work. Other researchers using a quadratic approach are Ahn et al. (2002), who also compare the performance of affine and quadratic term structure models and claim that the second outperform the first and Constantinides (1992), who he presents a model that "allows the term premium to change sign as a function of the state (variables) and the term to maturity and also allows for shapes of the yield curve that are observed (...) but are disallowed in the Cox, Ingersoll and Ross model."\(^1\)

\(^1\)The Cox et al. (1985) model can only deliver strictly increasing, strictly decreasing and hump shaped curves.
3 Data & Yield Curve Computation

We have used daily yield data of fixed coupon bonds of France, Germany, the Netherlands and Spain and of every bond issued by their main agencies (respectively CADES, KfW, BNE and ICO). The data source has been Datastream. These agencies are not fully comparable in terms of their activity, for example, the Spanish agency ICO works mainly providing loans to companies and their German homologue has a much broader scope, including mortgages, environmental and developing projects and also business financing. In any case what matters is that all of them are backed by their respective governments which implies that ex-ante their bonds have the same creditworthiness as the bonds issued by the sovereign.

Figure 1 shows the time to maturity in years of the bonds issued by the four agencies from January 2 2007 to February 27 2014, the time span considered in this thesis. However, due to the scarce number of ICO bonds, we reduced the time frame for Spain to January 2 2010 to February 27 2014. For each day and issuer we have computed a yield curve using the popular Nelson and Siegel (1987) formulae (equation (1)). In their paper, Nelson & Siegel solve a second order differential equation that models the forward rate and make use of the direct relationship between the spot rate and the forward rate to get the first one, which is the yield to maturity $y(\tau)$ in our case. In equation (1), $\beta_0$, $\beta_1$ and $\beta_2$ can be interpreted as three latent factors corresponding to the level, slope and curvature as understood by Litterman and Scheinkman (1991). Another interpretation is that $\beta_0$ is a long term factor (constant loading), $\beta_1$ a short term factor (the loading decreases fast to 0 with time to maturity $\tau$) and $\beta_2$ a medium term factor (hump shaped loading in $\tau$ whose right tail tends to 0). Furthermore, to transform a computationally intensive numerical optimization problem into simple least squares, we set $\lambda = 0.0609$ as suggested by Diebold and Li (2006), which maximizes the loading of $\beta_2$ for $\tau = 29.45$, or roughly speaking, 30 months, in consonance with its medium term interpretation.

$$y(\tau) = \beta_0 + \beta_1 \left( \frac{1 - e^{-\tau\lambda}}{\tau\lambda} \right) + \beta_2 \left[ \frac{(1 - e^{-\tau\lambda})}{\tau\lambda} - e^{-\tau\lambda} \right]$$ (1)

We have disregarded all bonds maturing before one year, to avoid abrupt variations of the yields close to expiration. Besides, yields that are two standard deviations beyond their daily mean are eliminated. For the case of the agencies, we also reject yields that are 200 basis points above or below the yield curve of the corresponding sovereign.
Once all the daily yield curves are computed, we collect the yields to maturity for 2, 5 and 10 years term and keep the weekly median to further eliminate possible outliers. Figures 2, 3 and 4 contain plots of the three of them for all issuers. We can appreciate that both lines move closely and that the yield of the agency lies above the yield of the sovereign in general. As we have already mention, this agency-sovereign spread is explained by their difference in liquidity.

As mentioned in the introduction, we take the OIS rate (at 2, 5 and 10 years) as a proxy for the risk free rate. The OIS is a fixed for floating interest rate swap that places in the floating leg the Euro OverNight Index Average. Before the crisis, it was common to consider interbank offered rates (e.g. LIBOR, EURIBOR...) for this purpose, however, the great recession has heavily questioned the validity of that assertion (Dubecq et al. (2013)). Figure 5 illustrates this fact, displaying the overlapping between the OIS and EURIBOR 3 month rates before 2007 and a gap after. OIS is a better proxy because it does not imply large transactions of capital as no principal is exchanged and also because it enjoys credit and netting enhancement mechanisms, such as margin accounts. The reason as for why CDSs are taken as a proxy for credit risk is straightforward as they basically constitute insurance against sovereign default. The data for the OIS and the CDS rates have also been downloaded from Datastream.

4 Affine Models

In this section we first replicate, with some minor changes, the model by Ejsing et al. (2012) for the four countries considered. We continue with two extensions, the first consists on including into the measurement equation the CDS rates with respect to the sovereign to enhance the estimation of the factors. The second, on using the panel of countries to identify a common European factor.

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2The European Banking Federation defines the EONIA as “... the effective overnight reference rate for the euro. It is computed as a weighted average of all overnight unsecured lending transactions in the interbank market, undertaken in the European Union and European Free Trade Association (EFTA) countries”

3Bomfim (2002) mentions three main credit enhancement mechanisms: “(i) credit triggers clauses, which give the higher-quality counterparty the right to terminate the swap if its counterparty’s credit rating falls below, say, BBB, (ii) the posting of collateral against the market value of the swap, and (iii) requirements to obtain insurance or guarantees from highly-rated third parties”.

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5
4.1 Benchmark Model

We define the spread of bond $i$ at time $t$ as $s_{i,t} = y_{i,t} - r_t$. Where:

- $y_{i,t}$ is the yield to maturity of the bond $i$ at time $t$.
- $r_t$ is the risk free rate at time $t$ (proxied by the OIS).

Ejsing et al. (2012) conceive a model in which the spreads between the yield to maturity of the bonds and the risk free rate are linear functions of the credit risk and the liquidity risk. Instead, we will consider affine functions of the same factors, as presented in the state-space model corresponding to equations (2) and (3). In this way we do not impose the mean of the error term of the measurement equation to be zero, as they do.

\[
\text{Measurement} \quad s_t = \begin{pmatrix} \delta_{sov} \\ \delta_{agn} \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 1 & \theta \end{pmatrix} x_t + \epsilon_t \tag{2}
\]

\[
\text{Transition} \quad x_t = \begin{pmatrix} \alpha_c \\ 0 \\ 0 \end{pmatrix} x_{t-1} + \nu_t \tag{3}
\]

Where:

- $s_t = \begin{pmatrix} s_{sov,t} \\ s_{agn,t} \end{pmatrix}$ is the vector containing the sovereign and agency spreads at time $t$.
- $x_t = \begin{pmatrix} x_{c,t} \\ x_{l,t} \end{pmatrix}$ is the vector containing the credit and liquidity latent factors at time $t$.
- $\epsilon_t \sim iid [0, \sigma^2 I_2]$ and $\nu_t \sim iid \mathcal{N} \left[ 0, \begin{pmatrix} \sigma^2_{\nu_c} & 0 \\ 0 & \sigma^2_{\nu_l} \end{pmatrix} \right]$.
- $\alpha_c$ and $\alpha_l$ are the autoregressive parameters of the latent factors whose modulus we assume to be smaller than one. Notice that we are imposing that the credit risk and liquidity risk factors are conditionally and unconditionally independent. Ejsing et al. (2012) show that this is a necessary condition for identification in this

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\footnote{Ejsing et al. (2012) do allow for different (and constant) variances for $\epsilon_t$. They do also estimate $\sigma^2_{\nu_c}$ and $\sigma^2_{\nu_l}$ and we fix $\mathbb{V} (x_{c,t}) = \mathbb{V} (x_{c,t}) = 1 \forall t$, which implies that $\sigma^2_{\nu_c} = 1 - \alpha^2_c$ and $\sigma^2_{\nu_l} = 1 - \alpha^2_l$. In sum, we propose a more parsimonious model that reduces the number of parameters to be estimated while captures the most relevant aspect of it, meaning the factor dynamics.}
framework. In any case, this orthogonality assumption is empirically sensible, as the correlation and serial correlation between proxies for credit risk and liquidity risk tends to deliver values close to zero.

- $\delta_{sov}$ and $\delta_{agn}$ are time invariant constants and $\theta$ is the liquidity loading for the agency spreads.

Notice that all the loadings but the liquidity one of the agency, on the measurement equation (2) are normalized to 1. This is the key for the identification of both factors. In other words, the contribution of the credit factor to both, agency and sovereign bonds, is the same, while it is different on the liquidity side. For the sovereign it will just be $x_{l,t}$ and for the agency $\theta x_{l,t}$.

To grasp intuition, we can forget about the constant for a moment and represent the spread as in equation (4).

$$\text{spread}_i = \text{credit premium}_i + \text{liquidity premium}_i \text{ for } i = \{sov, agn\}$$

(4)

In general, we should expect less credit-risky bonds to be pricier than more credit-risky bonds; a higher probability of default pushes the yield up and is translated in our model as a positive credit factor, specially under financial distress. On the contrary, more liquid bonds, will experience an appreciation in their price or equivalently a lower yield. This phenomenon is intensified in tense periods, where agents generally want to hold in their portfolio a larger proportion of safe and liquid assets, such as sovereign bonds. This capital flows are known in the finance literature as safe haven flows (Longstaff (2004)). In our model, this effect is captured for sovereign bonds with negative liquidity factors. For the agencies, what matters is the product $\theta x_{l,t}$ and considering that $s_{agn,t} > s_{sov,t}$ in general and that we expect $x_{l,t} < 0$ on average, if we disregard again the constant for intuition purposes, we have:

$$s_{agn,t} - s_{sov,t} = (\theta - 1) x_{l,t} > 0$$

$$\Rightarrow \quad \theta < 1$$

We have estimated the model for maturities 2, 5 and 10 years, however, for conciseness, we only report parameter estimates for the 5 year case in table 1 and the complete
set of estimated $\theta$'s in table 2.\footnote{Notice that this model has both maturity dependent loadings and factors. In section 5 we present a model where only the loadings depend on maturity.} We also present the factors separated by country on figure 6 for 5 year maturities, but all the factors together for the rest of the terms considered in figure 7. For all terms considered we have negative $\theta$'s and statistically different from zero and the liquidity factors tend to be below the zero line most of the time. The constants are not statistically different from zero for all countries but Spain, where they increase with the time to maturity and the relationship $\delta_{sov} < \delta_{agm}$ always holds. The persistence of the autoregressive process is very high, with $\alpha_c$ and $\alpha_l$ taking values very close to 1. The model captures two distress periods in 2009 and 2012, corresponding to the banking crisis and the sovereign debt crisis. A valuable feature of this type of models is that we can quantify the effects of the factors on the observed variable. For example, if we focus on the 5 year time to maturity case (figure 6), we can observe that the credit factor accounts for an increase up to 1.7 percentage points for France sovereign yields or 4.3 for the Spanish case. It is also worth to notice that for all countries, the impact of the credit risk on yields is higher than the impact of liquidity risk.

The nature of the banking crisis differs across countries, due in part to the fact that EU member countries have not fully delegated the regulatory authority to EU institutions and also to the different actions performed by the banks in the preceding years. For example, Dutch banks were the most exposed in Europe to American financial markets (66% of GDP) and after the Lehman bankruptcy in 2008 and the consequent spread of the storm to both sides of the Atlantic, the government had to partially nationalize some institutions, such as Fortis, and took measures to guarantee deposits up to 100,000 € (Masselink and den Noord (2009)). German banks, however, suffered from their originate-to-distribute business model, which basically consists in expanding the lending capacity through collateralized debt obligations and other securities of the sort. By 2009 several German institutions had to be recapitalized by the government and a “bad-bank” was created to transfer all non-performing securitized assets. In all cases, the assistance provided by the national governments contributed to relax the financial tension, though it was translated into increasing amounts of sovereign debt. This, added to the possible understanding of a tacit and implicit guarantee by the government to banks, leads us to the sovereign debt crises three years later. In this case, it were the unconventional monetary policy measures taken by the ECB (e.g. Very Long Term Refinancing Operations, Outright Monetary Transactions...) and a full commitment by
As a validity check of the model, we can compare the CDS rates on the sovereign bonds with the estimated credit factors. Figure 8 does so for the 5 years time to maturity case. CDS rates are a very popular proxy for credit risk of bonds as they essentially constitute insurance against default. As we can see in the picture, the lines move closely, which certifies our findings. We can also observe that the CDS rate is, in general, above the credit factor. This might reflect the illiquidity of these assets, though this assertion must be considered as a tentative explanation; a possible line of research could be started in this direction.

\section*{4.2 Adding CDSs}

Given the implications of figure 8 and the arguments provided in the last paragraph, a natural extension of the previous model would consist in including CDS rates into the measurement equation. In this way, we facilitate the estimation of the factors by considering an additional source of information. The resulting state-space model is characterized by equations (5) and (6). Notice that the credit loading of the CDS rate is the same one as for the bonds (this is 1) and that its liquidity loading is 0. The same considerations about \( \theta \) and the liquidity factor explained in subsection 4.1 apply here too.

\begin{align*}
\text{Measurement} & \quad \begin{pmatrix} s_t \\ \text{cds}_t \end{pmatrix} = \begin{pmatrix} \delta_{sov} \\ \delta_{agn} \\ \delta_{cds} \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 1 & 1 & \theta \\ 1 & 0 \end{pmatrix} x_t + \epsilon_t \\
\text{Transition} & \quad x_t = \begin{pmatrix} \alpha_c & 0 \\ 0 & \alpha_l \end{pmatrix} x_{t-1} + \nu_t
\end{align*}

where:

\begin{itemize}
  \item \( \epsilon_t \sim iid[0, \sigma^2 I_3] \) and \( \nu_t \sim iiN \left[ 0, \begin{pmatrix} \sigma^2_{\nu_c} & 0 \\ 0 & \sigma^2_{\nu_l} \end{pmatrix} \right] \) as before.\(^6\)
  \item \( \text{cds}_t \) denotes the CDS rate against the corresponding sovereign bond at time \( t \) and \( \delta_{cds} \) is the pertinent constant.
\end{itemize}

\(^6\)Again \( \mathbb{V}(x_{c,t}) = \mathbb{V}(x_{t,t}) = 1.\)
Figures 9 and 10 plot the resulting credit and liquidity factors and the CDS rate for maturities 5 and 10 years. Tables 3 and 4 contain the 5 year parameter estimates and all the $\theta$'s respectively. The dynamics of the factors go in line with the findings up to now. We capture the two distress periods corresponding to years 2009 and 2012. The magnitude of the risk premia is comparable to the other model and the credit factor keeps having a higher impact on the yields than the liquidity factor. It is of interest to compare that the difference between Spanish CDS rates and the credit factors is more marked than for the other countries. Furthermore, the 10 year CDS rates/credit factor spreads for all countries are comparatively larger than the 5 year (and 2 year) counterpart. In general, 5 year CDSs are the most liquid ones, which could support the tentative hypothesis stating that this difference is due to liquidity considerations.

Regarding the parameters, we again have intercepts not statistically different from 0 for all countries but Spain, where still the relationship $\delta_{agm} > \delta_{sov}$ holds. In fact we can claim that the order this time is $\delta_{agm} > \delta_{cds} > \delta_{sov}$ for all maturities. Again the factors exhibit a very persistent autoregressive pattern. The main difference here, compared to the pure benchmark model, lies on the sign of the $\theta$ parameters, taking positive values this time. In most occasions they are smaller than one, as it was expected and in all cases statistically different from zero.\(^7\) Including an extra source of information not only helps to improve the identification of the credit factor, it also contributes in the liquidity side of the model, as the smaller standard errors (compared to the benchmark model) of the agency loadings point out.

### 4.3 Joint Model, Looking for a Common Factor

The observed co-movement of the credit and liquidity factors in figures 6 to 10 suggests that a common driver might underlie across countries. Given the panel dimensionality of our data, we can look for a factor that captures this effect, in particular, we will look for a common European credit factor. In this analysis we will exclude Spanish data, because it is more incomplete and it could contaminate the results.

Let us call $x_{eur,t}$ the factor accounting for the common European credit shock at time $t$. Then for country $i$ we will have:

\(^7\)The estimation process did not provide plausible values for the German and Dutch agency liquidity loadings at 2 years time to maturity.
\[ s_{i,sov,t} = x_{eurl,t} + x_{i,c,l,t} + x_{i,l,t} + \epsilon_{i,sov} \]  
\[ s_{i,agn,t} = x_{eurl,t} + x_{i,c,l,t} + \theta_i x_{i,l,t} + \epsilon_{i,agn} \]

Notice that all the loadings are normalized to 1 but the liquidity loading of the agency or to put it differently, the only source of difference between agency and sovereign yields remains to be liquidity. The state-space model representation is customary, to see it please refer to appendix A.

Table 5 contains the parameter estimates for the 5 year case. The main findings provided in the benchmark model still hold here: non statistically significant intercepts for the spreads and negative and statistically significant agency liquidity loadings, endorsed with mostly negative liquidity factors. The autoregressive process remains to be very stable and the spread errors exhibit low volatility. The interest here resides on the factor plots. In figure 13 we can observe that the common factor captures well the distress periods corresponding to the financial crisis and the sovereign debt crisis for all maturities. If we place our attention in figure 11, we can appreciate that the country specific credit effects differ this time. French yields did not experienced a credit premium during the banking crisis, but they did during the sovereign crisis as opposed to Germany, who in those periods, in the light of the results, was considered even safer. The Netherlands’ credit riskiness remains quite stable. The liquidity factors present a similar pattern as in the previous models, which reinforces the assertion that this common factor fundamentally accounts for credit risk considerations. Finally, we want to point out that in general, credit factors have a higher impact than liquidity factors on yields. Notice also that the common factor dominates the idiosyncratic credit factor, as we can see in figures 11 and 12; the latter even takes at some moments negative values, counterbalancing the common effect.

5 Quadratic Model

In this section we adapt the EURIBOR model by Dubecq et al. (2013) for sovereign bond yields. Until now we could interpret that the models seen feature a constant stochastic discount factor (in fact equal to one). This new model provides a more ambitious structure that explicitly provides estimates in the physical \( \mathbb{P} \) and in the risk neutral \( \mathbb{Q} \) measure. Two more important differences with respect to the previous sections are (1)
that we will have non maturity dependent factors (but maturity dependent loadings) and (2) that we will consider quadratic functions of the factors to model yield-OIS spreads.

Let the risk free rate of maturity (the yield to maturity of a risk free bond) at time \( t \) with maturity (in years) \( n \) be:

\[
\hat{r}_{t,n} = \frac{1}{n} \log \left( \mathbb{E}^Q \left( \exp \left\{ \sum_{j=1}^{n} r_{t+j} \right\} \right) \right)
\]

where \( r_t \) is the one year risk free rate.

Let the yield to maturity of bond \( i \) at time \( t \) with time to maturity \( n \) be:

\[
\hat{y}_{i,t,n} = \frac{1}{n} \log \left( \mathbb{E}^Q \left( \exp \left\{ \sum_{j=1}^{n} r_{t+j} + \lambda_{t+j} \right\} \right) \right)
\]

where \( \lambda_t \) is the intensity parameter.

Assuming independence under \( Q \) of \( r_t \) and \( \lambda_t \) the spread of bond \( i \) at time \( t \) with maturity \( n \) is:

\[
\hat{s}_{i,t,n} = y_{i,t,n} - r_{t,n} = \frac{1}{n} \log \left( \mathbb{E}^Q \left( \exp \left\{ \sum_{j=1}^{n} \lambda_{t+j} \right\} \right) \right)
\]

As we did in the last section, we will proxy \( r_{t,n} \) with the corresponding OIS rate.

The intensity parameter is a quadratic function of the factor \( x_t \) which will be the sum of the credit and liquidity latent factors:

\[
\lambda_t = \lambda_0 + \lambda_1 x_t + \lambda_2 x_t^2
\]  

(9)

where \( x_t = x_{c,t} + x_{l,t} \).

The factor \( x_t \) follow a first order autoregressive process:

\[
x_t = \mu^* + \varphi^* x_{t-1} + \epsilon_t^*
\]  

(10)

where \( \epsilon_t^* \sim i.i.d.(0,1) \). For identification reasons, the variance of the factor \( x_t \) is normalized to 1. We also assume that \( |\varphi^*| < 1 \). (The star * indicates a \( Q \) parameter.)

According to financial theory, we will impose that \( \lambda_t \geq 0 \) by setting \( \lambda_2 \geq 0 \) and \( \lambda_0 \geq \frac{\lambda_2^2}{4\lambda_2} \) to make the minimum of the parabola greater or equal to zero. This is one of the reasons for which it is convenient to present the intensity as a quadratic function of the factors, the other is that we make the intensities more flexible as the quadratic function embeds the affine specification as a possibility.

The joint dynamics of \( x_{c,t} \) and \( x_{l,t} \) under \( Q \) follow a VAR(1) process:
\[
\begin{pmatrix}
  x_{c,t} \\
  x_{l,t}
\end{pmatrix} = \begin{pmatrix}
  \mu_c^* \\
  \mu_l^*
\end{pmatrix} + \begin{pmatrix}
  \varphi_c^* & 0 \\
  0 & \varphi_l^*
\end{pmatrix} \begin{pmatrix}
  x_{c,t-1} \\
  x_{l,t-1}
\end{pmatrix} + \epsilon_t^* \tag{11}
\]

where \( \epsilon_t^* \sim i.i.d. \begin{pmatrix} \sigma_c^2 & 0 \\ 0 & \sigma_l^2 \end{pmatrix} \).  

And under \( \mathbb{P} \):

\[
\begin{pmatrix}
  x_{c,t} \\
  x_{l,t}
\end{pmatrix} = \begin{pmatrix}
  \mu_c \\
  \mu_l
\end{pmatrix} + \begin{pmatrix}
  \varphi_c & 0 \\
  0 & \varphi_l
\end{pmatrix} \begin{pmatrix}
  x_{c,t-1} \\
  x_{l,t-1}
\end{pmatrix} + \epsilon_t \tag{12}
\]

where \( \epsilon_t \sim i.i.d. \begin{pmatrix} \sigma_c^2 & 0 \\ 0 & \sigma_l^2 \end{pmatrix} \).

Notice that equations (11) and (9) imply that:

\[
\begin{align*}
\mu^* &= \mu_c^* + \mu_l^* \tag{13} \\
\varphi^* &= \varphi_c^* = \varphi_l^* \tag{14} \\
\sigma_c^2 + \sigma_l^2 &= 1 \tag{15}
\end{align*}
\]

In appendix B.1 we show that there is a one to one mapping from the physical to the risk neutral measure such that

\[
\begin{pmatrix}
  \mu_c^* \\
  \mu_l^*
\end{pmatrix} = \begin{pmatrix}
  \mu_c \\
  \mu_l
\end{pmatrix} + \begin{pmatrix}
  \sigma_c^2 & 0 \\
  0 & \sigma_l^2
\end{pmatrix} \gamma_0 \tag{16}
\]

\[
\begin{pmatrix}
  \varphi_c^* & 0 \\
  0 & \varphi_l^*
\end{pmatrix} = \begin{pmatrix}
  \varphi_c & 0 \\
  0 & \varphi_l
\end{pmatrix} + \begin{pmatrix}
  \sigma_c^2 & 0 \\
  0 & \sigma_l^2
\end{pmatrix} \Gamma_1 \tag{17}
\]

where for computational simplicity we will set \( \gamma_0 = \begin{pmatrix} \gamma_{0,1} \\ \gamma_{0,2} \end{pmatrix} = \begin{pmatrix} \gamma_{0,0} \\ \gamma_{0,0} \end{pmatrix} \) (the upper and lower numbers of the vector coincide) and \( \Gamma_1 = \begin{pmatrix} \gamma_{1,1} & 0 \\ 0 & \gamma_{1,2} \end{pmatrix} \cdot \gamma_{0,0} ; \gamma_{1,1} \) and \( \gamma_{1,2} \)

\( \text{Notice the notational distinction between } \epsilon_t^* \text{ and } \epsilon_t^* \). The first is a vector and the second a scalar.
are parameters to be estimated.\(^9\)

Fortunately, there exist closed form expressions for the spreads. In appendix B.2 we show that

\[
\mathbb{E}^Q \left( \exp \left\{ \sum_{j=1}^{n} \lambda_{t+j} \right\} \right) = \exp \left( a_n + b_n x_t + c_n x_t^2 \right)
\]  

(18)

where

\[
a_n = a_{n-1} - \lambda_0 - \log \sqrt{1 - 2 (c_{n-1} - \lambda_2)} + \frac{1}{2} \frac{(b_{n-1} - \lambda_1)^2}{1 - 2 (c_{n-1} - \lambda_2)} + \mu^* \left[ \frac{2 (b_{n-1} - \lambda_1) (c_{n-1} - \lambda_2)}{1 - 2 (c_{n-1} - \lambda_2)} + b_{n-1} - \lambda_1 \right]
\]

\[
+ \mu^* \left[ \frac{(c_{n-1} - \lambda_2)^2}{1 - 2 (c_{n-1} - \lambda_2)} + c_{n-1} - \lambda_2 \right]
\]

\[
b_n = \varphi^* \left[ \frac{2 (b_{n-1} - \lambda_1) (c_{n-1} - \lambda_2)}{1 - 2 (c_{n-1} - \lambda_2)} + b_{n-1} - \lambda_1 \right]
\]

\[
+ 2 \varphi^* \mu^* \left[ \frac{(c_{n-1} - \lambda_2)^2}{1 - 2 (c_{n-1} - \lambda_2)} + c_{n-1} - \lambda_2 \right]
\]

\[
c_n = \varphi^{*2} \left[ \frac{(c_{n-1} - \lambda_2)^2}{1 - 2 (c_{n-1} - \lambda_2)} + c_{n-1} - \lambda_2 \right]
\]

and \(a_0 = b_0 = c_0 = 0\).

This allows us to rewrite the spread as

\[
\gamma_2 = \frac{\varphi_c + \gamma_{0,1} \sigma_c^2 - \varphi_l}{\sigma_l^2}
\]

which is obtained from equation (17).

The reason to set \(\gamma_0 = \gamma_{0,0} \epsilon_2\) is that

\[
\mu^* = \mu_c^* + \mu_l^*
\]

\[
= \mu_c + \mu_l + \sigma_c^2 \gamma_{0,1} + \sigma_l^2 \gamma_{0,2}
\]

\[
= \mu_c + \mu_l + \gamma_{0,0} (\sigma_c^2 + \sigma_l^2)
\]

safes us from estimating one parameter.
\[ s_{i,t,n} = \theta_{0,n} + \theta_{1,n}x_t + \theta_{2,n}x_t^2 \]  

where \( \theta_{0,n} = a_n/n, \theta_{1,n} = b_n/n \) and \( \theta_{2,n} = c_n/n \). Notice that the \( \theta_{k,n} \)'s, which are functions of \( (\lambda_0, \lambda_1, \lambda_2, \mu^*, \phi^*) \), depend on the time to maturity \( n \).

Instead of using the quadratic Kalman filter formulae (Monfort et al. (2013)) to construct and estimate the state-space model, we first-order Taylor approximate equation (19) around \( x_{t|t-1} \), which denotes the predictions of the factors within the Kalman routine. Therefore we have that:

\[ s_{i,t,n} \approx \theta_{0,n} - \theta_{2,n}x_{t|t-1}^2 + (\theta_{1,n} + 2\theta_{2,n}x_{t|t-1})x_t. \]  

Having this, the measurement and transition equations of the state-space model look as follows:

**Measurement**

\[
\begin{pmatrix}
    s_{sov,t,2} \\
    s_{sov,t,5} \\
    s_{sov,t,10} \\
    cds_{t,5} \\
    pliq
\end{pmatrix}
= 
\begin{pmatrix}
    \theta_{0,2} - \theta_{2,2}x_{t|t-1}^2 \\
    \theta_{0,5} - \theta_{2,5}x_{t|t-1}^2 \\
    \theta_{0,10} - \theta_{2,10}x_{t|t-1}^2 \\
    \pi_{c,0} - \pi_{c,2}x_{c,t|t-1} \\
    \pi_{l,0} - \pi_{l,2}x_{l,t|t-1}
\end{pmatrix}
+ 
\begin{pmatrix}
    \theta_{1,2} - 2\theta_{2,2}x_{t|t-1} \\
    \theta_{1,5} - 2\theta_{2,5}x_{t|t-1} \\
    \theta_{1,10} - 2\theta_{2,10}x_{t|t-1} \\
    \pi_{c,1} - 2\pi_{c,2}x_{c,t|t-1} \\
0
\end{pmatrix}
\begin{pmatrix}
    x_{c,t} \\
    x_{l,t}
\end{pmatrix}
+ \eta_t
\]  

**Transition**

\[
\begin{pmatrix}
    x_{c,t} \\
    x_{l,t}
\end{pmatrix}
= 
\begin{pmatrix}
    \mu_c \\
    \mu_l
\end{pmatrix}
+ 
\begin{pmatrix}
    \phi_c \\
    0
\end{pmatrix}
\begin{pmatrix}
    x_{c,t-1} \\
    x_{l,t-1}
\end{pmatrix}
+ \epsilon_t
\]  

where:

- \( cds_{t,5} = \pi_{c,0} + \pi_{c,1}x_{c,t} + \pi_{c,2}x_{c,t}^2 + \eta_{c,t} \) is the 5 year CDS rate against the sovereign bond, our proxy for credit risk. Notice that it is a quadratic function of the credit factor only. In the measurement equation it appears linearized in the same fashion we did in equation (19).
\[ \text{pliq} \equiv y_{\text{agnt},5} - y_{\text{sov},t,5} = \pi_{t,0} + \pi_{t,1}x_{t,t} + \pi_{t,2}x_{t,t}^2 + \eta_{t,t} \] is the difference between the agency and sovereign 5 year yields, our liquidity proxy. Notice that it is a quadratic function of the credit factor only. In the measurement equation it also appears linearized.

\[ \eta_t \sim iid \begin{bmatrix} 0 \\ \sigma^2_{2,\eta} & 0 \\ 0 & \sigma^2_{5,\eta} \\ 0 & \sigma^2_{10,\eta} \\ \sigma^2_{c} & \sigma^2_{l} \\ \sigma^2_{c} & \sigma^2_{l} \end{bmatrix} \]

Notice that in our measurement equation we include the 2, 5 and 10 year spreads but only 5 year credit and liquidity proxies. There are two main reasons for that. The first, that 5 year CDSs are more liquid compared to the other two. The second is that it reduces significantly the number of parameters to be estimated.\(^{10}\)

To synthesize, we only include the parameter estimates for France as an example in table 7. In any case, some common patterns prevail. In most cases the estimates are statistically different from zero. For all the countries the factors exhibit again very persistent processes with values of \(\varphi_c\) and \(\varphi_l\) very close to 1. Also it is worth to point out that the variances of the spread error terms take quite low values; for France the pricing error for the 2, 5 and 10 year spreads are 34, 5 and 19 basis points respectively.

Contrary to what occurred with the models in section 4, the factors themselves are not very informative about the impact they have on the spreads. For that reason it is convenient to rewrite the spreads in a way that the partial contributions of each factor show off:

\[ s_{i,t,n} = \theta_{0,n} + \theta_{1,n}x_{t,t} + \theta_{2,n}x_{t,t}^2 \]

\[ = \theta_{0,n} + \underbrace{\theta_{1,n}x_{c,t} + \theta_{2,n}x_{c,t}^2}_{\text{credit contribution}} + \underbrace{\theta_{1,n}x_{l,t} + \theta_{2,n}x_{l,t}^2}_{\text{liquidity contribution}} + 2\theta_{2,n}x_{c,t}x_{l,t} \]

The first bracket denotes the credit contribution, the second bracket the liquidity contribution and the third represents the interaction between both factors. Figure 14 plots the 5 year contributions for the four countries considered. The corresponding \(\theta_{k,5}\) loadings can be found in table 8. For the Netherlands, France and Spain we have

\(^{10}\)Also we impose, as in section 4, that \(\mathbb{V}(x_{c,t}) = \mathbb{V}(x_{l,t}) = 1\).
comparable results as with the first two affine models, though the safe-haven liquidity premia is not captured by the liquidity contribution, but by the interaction term. Spain remains without experiencing the downward force on the yields that safer bonds do under stress periods. Interestingly, the factor contributions on German yields are minimal. In my opinion this is due to the fact that this model does not fit well the German case. It happens that the 2 and 5 year German yields to maturity are below the OIS rate through most of the period considered (the respective spread means are -0.1281 and -0.0391), something that does not happen for any other country and term.\textsuperscript{11} This fact is at odds with the assumption of positive intensities $\lambda_t$, therefore we cannot expect plausible results in such a situation.

An interesting feature of this model where the loadings are maturity dependent but not the latent factors is that we can construct a “spread curve”. To do so we just need to compute the loadings for $n = \{1, \ldots, N\}$, recover the latent factor $x_t$ and apply equation (19). Figure 15 presents an example of the Dutch “spread curve” on October 26 2009, where the actual 2, 5 and 10 year yield-OIS spreads are compared with the model output.

6 Conclusions

In this master thesis we have shed light on the impact that credit and liquidity risk have on sovereign bond yields for France, Germany, the Netherlands and Spain. To do so, several state-space models have been estimated by maximum likelihood applying Kalman filtering.

Two distress periods are captured, corresponding to the financial crisis and the sovereign debt crises. The measures taken first by the national governments and later by the ECB contributed to relax the tension in both cases. In general, the credit component dominates, pushing up sovereign yields, however this effect is alleviated by the liquidity premia in some countries, originated by the safe haven flows typical of hectic financial episodes. Both, the affine approach and the quadratic approach, coincide in this results, with the exception of Germany, for which the adaptation of Dubecq et al. (2013) is not suitable. Furthermore, the spread between agency and sovereign yields has proven to be a good liquidity proxy. This, together with the CDS rates comprehend two real time measures of credit and liquidity risks.

\textsuperscript{11}For the 10 year case the mean is positive, taking a value of 0.0532.
The co-movement of the factors across countries lead us to look for a common credit driver. We made use of an extension of the affine model to find that the idiosyncratic credit component significantly differs among sovereigns. A natural step in this direction would be to use a conveniently designed quadratic model for the same purpose. Also recall that for this endeavor we excluded Spain from the analysis, due in part to its different consideration in terms of creditworthiness. Extending our knowledge about this phenomena for peripheral countries (and for all the Eurozone) would also be extremely worthy, though a different model, according to data availability, should be configured.
A Joint Model, state-space representation

To construct the state-space model we define the vectors:

\[
\begin{align*}
s_t^* &= \begin{pmatrix} s_{ger,sov,t} & s_{ger,agn,t} & s_{fra,sov,t} & s_{fra,agn,t} & s_{net,sov,t} & s_{net,agn,t} \end{pmatrix}' \\
x_t^* &= \begin{pmatrix} x_{eur,t} & x_{ger,c,t} & x_{ger,l,t} & x_{fra,c,t} & x_{fra,l,t} & x_{net,c,t} & x_{net,l,t} \end{pmatrix}'
\end{align*}
\]

The resulting state space model is:

**Measurement** \( s_t^* = \begin{pmatrix} \delta_{ger,sov} \\ \delta_{ger,agn} \\ \delta_{fra,sov} \\ \delta_{fra,agn} \\ \delta_{net,sov} \\ \delta_{net,agn} \end{pmatrix} + \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & \theta_{ger} & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & \theta_{fra} \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & \theta_{net} \end{pmatrix} x_t^* + \epsilon_t^* \) (23)

**Transition** \( x_t^* = \begin{pmatrix} \alpha_{eur} \\ \alpha_{ger,c} \\ \alpha_{ger,l} \\ \alpha_{fra,c} \\ 0 \\ \alpha_{net,l} \end{pmatrix} + \begin{pmatrix} \alpha_{eur} \\ \alpha_{ger,c} \\ \alpha_{ger,l} \\ \alpha_{fra,c} \\ 0 \\ \alpha_{net,l} \end{pmatrix} x_{t-1}^* + \nu_t^* \) (24)

Where in this case:

\[
\epsilon_t^* \sim iid \begin{pmatrix} \sigma_{ger,sov}^2 \\ \sigma_{ger,agn}^2 \\ \sigma_{fra,sov}^2 \\ \sigma_{fra,agn}^2 \\ 0 \\ \sigma_{net,sov}^2 \\ \sigma_{net,agn}^2 \end{pmatrix}, \text{ notice that}
\]

this time we relax the assumption of equal variances for the spreads, as we include now several countries.
\[ \nu^*_t \sim \mathcal{N}(0, \Sigma) \]

\[ \Sigma = \begin{pmatrix} \sigma^2_{eur} & \sigma^2_{ger,c} & 0 \\ \sigma^2_{ger,l} & \sigma^2_{fra,c} \\ 0 & \cdots & \sigma^2_{net,l} \end{pmatrix} \]

\section*{B Proofs}

\subsection*{B.1 From Real to Risk Neutral Measure}

Courtesy of Javier Mencía:

Consider the stochastic discount factor

\[ M_{t,t+1} \propto \exp(\gamma_t' \epsilon_{t+1}) \]

where the 2x1 vector \( \gamma_t \) is defined as

\[ \gamma_t = \gamma_0 + \Gamma_1 x_t \]

where \( \gamma_0 \) is a 2x1 vector of parameters, \( \Gamma_1 \) a 2x2 matrix of parameters and \( x_t = (x_{c,t}, x_{l,t})' \). \(^{13}\)

Under the physical measure \( \mathbb{P} \), the density of \( \epsilon_{t+1} \) can be expressed as

\[ f^\mathbb{P}(\epsilon_{t+1}) \propto \exp\left(-\frac{1}{2} \epsilon_{t+1}' \Sigma^{-1} \epsilon_{t+1}\right) \]

where

\[ \Sigma = \begin{pmatrix} \sigma^2_c & 0 \\ 0 & \sigma^2_l \end{pmatrix} \]

The equivalent risk neutral density can be written as

\(^{12}\)We normalize to one the standard deviations of the factors.

\(^{13}\)Notice the notational distinction between \( x_t \) and \( x_t \). The bold symbol denotes the vector comprising the credit and liquidity factors and the other corresponds to the sum of both factors.
\[ f^Q(\epsilon_{t+1}) = M_{t,t+1} f^Q(\epsilon_{t+1}) \]
\[ \propto \exp \left( \gamma'_t \epsilon_{t+1} - \frac{1}{2} \epsilon'_{t+1} \Sigma^{-1} \epsilon_{t+1} \right) . \]

We can exploit the fact that
\[ \gamma'_t \epsilon_{t+1} - \frac{1}{2} \epsilon'_{t+1} \Sigma^{-1} \epsilon_{t+1} = -\frac{1}{2} (\epsilon_{t+1} - \Sigma \gamma_t)' \Sigma^{-1} (\epsilon_{t+1} - \Sigma \gamma_t) + \frac{1}{2} \gamma'_t \Sigma \gamma_t . \]
Considering that given the information at time \( t \) \( I_t \) the last term is a constant we have that
\[ f^Q(\epsilon_{t+1}) \propto -\frac{1}{2} (\epsilon_{t+1} - \Sigma \gamma_t)' \Sigma^{-1} (\epsilon_{t+1} - \Sigma \gamma_t) . \]
Therefore, we obtain that \( \epsilon_{t+1} \sim \mathcal{N}^Q [\Sigma \gamma_t, \Sigma] \) (under the risk neutral measure). In consequence, \( x_{t+1} \) is also multivariate normal under the risk neutral measure given \( I_t \), with parameters
\[
\begin{pmatrix}
\mu_c^* \\
\mu_c^*
\end{pmatrix} = 
\begin{pmatrix}
\mu_c \\
\mu_c
\end{pmatrix} + \Sigma \gamma_0
\]
\[
\begin{pmatrix}
\varphi_c^* & 0 \\
0 & \varphi_t^*
\end{pmatrix} = 
\begin{pmatrix}
\varphi_c & 0 \\
0 & \varphi_t
\end{pmatrix} + \Sigma \Gamma_1
\]
and \( \Gamma_1 \) is diagonal, for the final autoregressive matrix to be diagonal.

### B.2 Recursive Formulae for Spread Loadings

Courtesy of Javier Mencía:

Let \( D_{t,n} = \mathbb{E}^Q \left[ \exp \left\{ \sum_{j=1}^{n} \lambda_{t+j} \right\} \mid I_t \right] \).

By the law of iterated expectations we can rewrite \( D_{t,n} \) as
\[ D_{t,n} = \mathbb{E}^Q \left[ \exp (-\lambda_{t+1}) D_{t+1,n-1} \right] . \quad (25) \]

Now assume that we can parametrize \( D_{t+1,n-1} \) as
\[ D_{t+1,n-1} = \exp \left( a_{n-1} + b_{n-1} x_{t+1} + c_{n-1} x_{t+1}^2 \right). \]  \hspace{1cm} (26)

If we plug equation (25) into (26) and use (9), we obtain

\[ D_{t,n} = \exp (a_{n-1} - \lambda_0) \mathbb{E}^Q \left[ \exp \left( (b_{n-1} - \lambda_1) x_{t+1} + (c_{n-1} - \lambda_2) x_{t+1}^2 \right) \mid I_t \right]. \]  \hspace{1cm} (27)

If we introduce now (10) into (27) we get

\[
D_{t,n} = \exp [(a_{n-1} - \lambda_0) + (\mu^* + \varphi^* x_t) (b_{n-1} - \lambda_1) + (\mu^* + \varphi^* x_t)^2 (c_{n-1} - \lambda_2)] \\
\cdot \mathbb{E}^Q \left[ \exp \left\{ (b_{n-1} - \lambda_1 + 2 (c_{n-1} - \lambda_2) (\mu^* + \varphi^* x_t)) \epsilon_{t+1} + (c_{n-1} - \lambda_2) \epsilon_{t+1}^2 \right\} \mid I_t \right].
\]

Now, we can use the property that

\[
\mathbb{E} \left[ \exp \left( \alpha \epsilon + \beta \epsilon^2 \right) \right] = \frac{1}{\sqrt{1 - 2\beta}} \exp \left( \frac{1}{2} \frac{\alpha^2}{1 - 2\beta} \right)
\]

for \( \epsilon \sim \mathcal{N} \left( 0, 1 \right) \) and \( \beta < 0.5 \). After some algebra we can show that equation (18) holds.
### C Tables

#### C.1 Benchmark Model: Parameters

<table>
<thead>
<tr>
<th></th>
<th>France</th>
<th>Germany</th>
<th>the Netherlands</th>
<th>Spain</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_{sov}$</td>
<td>0.1923 (1.298)</td>
<td>-0.0098 (1.405)</td>
<td>0.1174 (1.344)</td>
<td>2.2810 (0.911)</td>
</tr>
<tr>
<td>$\delta_{agn}$</td>
<td>0.2817 (0.889)</td>
<td>0.2327 (1.942)</td>
<td>0.3583 (1.107)</td>
<td>2.6291 (0.497)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>-0.0215 (0.062)</td>
<td>-1.6388 (0.141)</td>
<td>-0.5990 (0.048)</td>
<td>-0.0633 (0.151)</td>
</tr>
<tr>
<td>$\alpha_c$</td>
<td>0.9986 (0.000)</td>
<td>0.9995 (0.034)</td>
<td>0.9994 (0.034)</td>
<td>0.9720 (0.003)</td>
</tr>
<tr>
<td>$\alpha_l$</td>
<td>0.9993 (0.034)</td>
<td>0.9999 (0.033)</td>
<td>0.9994 (0.034)</td>
<td>0.9935 (0.002)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.0062 (0.003)</td>
<td>0.0150 (0.002)</td>
<td>0.0000 (0.008)</td>
<td>0.0000 (0.021)</td>
</tr>
</tbody>
</table>

Table 1: Benchmark model, 5 year parameter estimates. All countries.

<table>
<thead>
<tr>
<th></th>
<th>2 years</th>
<th>5 years</th>
<th>10 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>-0.2899 (0.075)</td>
<td>-0.0215 (0.062)</td>
<td>-0.1657 (0.039)</td>
</tr>
<tr>
<td>Germany</td>
<td>-3.7972 (0.189)</td>
<td>-1.6388 (0.141)</td>
<td>-0.6477 (0.068)</td>
</tr>
<tr>
<td>the Netherlands</td>
<td>-1.0940 (0.105)</td>
<td>-0.5990 (0.048)</td>
<td>-0.5990 (0.048)</td>
</tr>
<tr>
<td>Spain</td>
<td>0.1497 (0.073)</td>
<td>-0.0633 (0.151)</td>
<td>-1.3664 (0.111)</td>
</tr>
</tbody>
</table>

Table 2: Benchmark model, $\theta$ parameter estimates. All countries and maturities.

#### C.2 Adding CDSs: Parameters

<table>
<thead>
<tr>
<th></th>
<th>France</th>
<th>Germany</th>
<th>the Netherlands</th>
<th>Spain</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_{sov}$</td>
<td>0.1335 (1.284)</td>
<td>-0.1558 (1.402)</td>
<td>0.1098 (1.353)</td>
<td>2.0735 (0.969)</td>
</tr>
<tr>
<td>$\delta_{agn}$</td>
<td>0.3480 (1.155)</td>
<td>0.3502 (1.597)</td>
<td>0.3930 (1.032)</td>
<td>2.4912 (0.875)</td>
</tr>
<tr>
<td>$\delta_{cds}$</td>
<td>0.3437 (0.900)</td>
<td>0.1817 (0.971)</td>
<td>0.1989 (0.957)</td>
<td>2.3381 (0.572)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.7915 (0.016)</td>
<td>1.2533 (0.014)</td>
<td>0.4025 (0.028)</td>
<td>0.8476 (0.032)</td>
</tr>
<tr>
<td>$\alpha_c$</td>
<td>0.9987 (0.000)</td>
<td>0.9997 (0.001)</td>
<td>0.9994 (0.036)</td>
<td>0.9810 (0.002)</td>
</tr>
<tr>
<td>$\alpha_l$</td>
<td>0.9990 (0.000)</td>
<td>0.9998 (0.038)</td>
<td>0.9994 (0.038)</td>
<td>0.9942 (0.001)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.0756 (0.002)</td>
<td>0.1124 (0.002)</td>
<td>0.0760 (0.002)</td>
<td>0.1653 (0.006)</td>
</tr>
</tbody>
</table>

Table 3: Adding CDSs, 5 year parameter estimates. All countries.
Table 4: Adding CDSs, $\theta$ parameter estimates, all countries and maturities.

<table>
<thead>
<tr>
<th></th>
<th>2 years</th>
<th>5 years</th>
<th>10 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>0.7408 (0.026)</td>
<td>0.7915 (0.016)</td>
<td>0.9984 (0.011)</td>
</tr>
<tr>
<td>Germany</td>
<td></td>
<td>1.2533 (0.014)</td>
<td>0.7395 (0.016)</td>
</tr>
<tr>
<td>the Netherlands</td>
<td></td>
<td>0.4025 (0.028)</td>
<td>1.2393 (0.045)</td>
</tr>
<tr>
<td>Spain</td>
<td>0.8010 (0.022)</td>
<td>0.8476 (0.032)</td>
<td>1.4472 (0.071)</td>
</tr>
</tbody>
</table>

C.3 Joint Model: Parameters

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_{ger,sov}$</td>
<td>-0.4787 (1.017)</td>
<td>$\alpha_{ger,l}$</td>
<td>0.9907 (0.006)</td>
</tr>
<tr>
<td>$\delta_{ger,agn}$</td>
<td>0.0057 (1.015)</td>
<td>$\alpha_{fra,c}$</td>
<td>0.9782 (0.011)</td>
</tr>
<tr>
<td>$\delta_{ger,sov}$</td>
<td>-0.0903 (1.149)</td>
<td>$\alpha_{fra,l}$</td>
<td>0.9983 (0.002)</td>
</tr>
<tr>
<td>$\delta_{ger,agn}$</td>
<td>0.1083 (1.045)</td>
<td>$\alpha_{net,c}$</td>
<td>1.0000 (0.066)</td>
</tr>
<tr>
<td>$\delta_{ger,sov}$</td>
<td>-0.2025 (1.416)</td>
<td>$\alpha_{net,l}$</td>
<td>0.9541 (0.015)</td>
</tr>
<tr>
<td>$\delta_{ger,agn}$</td>
<td>0.2005 (1.416)</td>
<td>$\sigma_{ger,sov}$</td>
<td>0.0000 (0.012)</td>
</tr>
<tr>
<td>$\theta_{ger}$</td>
<td>-0.8635 (0.181)</td>
<td>$\sigma_{ger,agn}$</td>
<td>0.0184 (0.003)</td>
</tr>
<tr>
<td>$\theta_{fra}$</td>
<td>-0.4993 (0.230)</td>
<td>$\sigma_{fra,sov}$</td>
<td>0.0167 (0.003)</td>
</tr>
<tr>
<td>$\theta_{net}$</td>
<td>-0.9914 (0.095)</td>
<td>$\sigma_{fra,agn}$</td>
<td>0.0000 (0.005)</td>
</tr>
<tr>
<td>$\alpha_{eur}$</td>
<td>1.0000 (0.037)</td>
<td>$\sigma_{net,sov}$</td>
<td>0.0000 (0.036)</td>
</tr>
<tr>
<td>$\alpha_{ger,c}$</td>
<td>0.9847 (0.008)</td>
<td>$\sigma_{net,agn}$</td>
<td>0.0000 (0.004)</td>
</tr>
</tbody>
</table>

Table 5: Joint model, parameter estimates, 5 years.

<table>
<thead>
<tr>
<th></th>
<th>2 years</th>
<th>5 years</th>
<th>10 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>-2.8308 (0.558)</td>
<td>-0.8635 (0.181)</td>
<td>-1.2078 (0.141)</td>
</tr>
<tr>
<td>France</td>
<td>-2.0547 (0.437)</td>
<td>-0.4993 (0.230)</td>
<td>-0.4904 (0.217)</td>
</tr>
<tr>
<td>the Netherlands</td>
<td>-2.5240 (0.558)</td>
<td>-0.9914 (0.095)</td>
<td>-3.9135 (0.532)</td>
</tr>
</tbody>
</table>

Table 6: Joint model, $\theta$ parameter estimates, all maturities.
C.4 Quadratic Model: Parameters

\[
\begin{align*}
\mu_c &= -0.0943 \pm 0.073, \\
\mu_l &= -0.5270 \pm 0.038, \\
\varphi_c &= -0.9914 \pm 0.002, \\
\varphi_l &= -0.8739 \pm 0.019, \\
\sigma_c &= 0.4347 \pm 0.005, \\
\lambda_0 &= 0.0344 \pm 0.004, \\
\lambda_1 &= 0.0008 \pm 0.001, \\
\lambda_2 &= 0.0015 \pm 0.001, \\
\gamma_{0,0} &= -0.0583 \pm 0.045, \\
\gamma_{1,1} &= -0.3126 \pm 0.009.
\end{align*}
\]

\[
\begin{align*}
\pi_{c,0} &= 0.0064 \pm 0.005, \\
\pi_{c,1} &= 0.0013 \pm 0.001, \\
\pi_{c,2} &= -0.0037 \pm 0.000, \\
\pi_{l,0} &= 0.1480 \pm 0.005, \\
\pi_{l,1} &= 0.0454 \pm 0.006, \\
\pi_{l,2} &= -0.0270 \pm 0.001, \\
\sigma_{\eta_c} &= 0.0561 \pm 0.001, \\
\sigma_{\eta_l} &= 0.0760 \pm 0.002.
\end{align*}
\]

Table 7: Quadratic model, parameter estimates, France.

\[\theta_{k,5}\] loadings

<table>
<thead>
<tr>
<th></th>
<th>France</th>
<th>Germany</th>
<th>the Netherlands</th>
<th>Spain</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\theta_{0,5})</td>
<td>0.0411</td>
<td>0.2661e-3</td>
<td>0.0034</td>
<td>0.6387</td>
</tr>
<tr>
<td>(\theta_{1,5})</td>
<td>0.0021</td>
<td>0.0215e-3</td>
<td>-0.0003</td>
<td>-0.1242</td>
</tr>
<tr>
<td>(\theta_{2,5})</td>
<td>0.0031</td>
<td>0.0058e-3</td>
<td>0.0015</td>
<td>0.0065</td>
</tr>
</tbody>
</table>

Table 8: \(\theta_{k,5}\) loadings.
D Figures

D.1 Maturities, Yields and OIS

Figure 1: Time to maturity of the agency bonds considered.
Figure 2: Sovereign and agency yields, 2 years.
Figure 3: Sovereign and agency yields, 5 years.
(a) France. 
(b) Germany. 
(c) the Netherlands. 
(d) Spain. 

Figure 4: Sovereign and agency yields, 10 years.

Figure 5: 3 month EURIBOR OIS spread.
D.2 Benchmark Model: Factors

Figure 6: Benchmark model, factors by countries, 5 years.
Figure 7: Benchmark model, all factors.
Comparison of the 5 year credit factors with the CDS 5 year rates

(a) France.

(b) Germany.

(c) the Netherlands.

(d) Spain.

Figure 8: Credit factors and CDS rates, 5 years.
D.3 Adding CDSs: Factors

(a) France.

(b) Germany.

(c) the Netherlands.

(d) Spain.

(e) All credit factors.

(f) All liquidity factors.

Figure 9: Adding CDSs, factors, 5 years.
Figure 10: Adding CDSs, factors, 10 years.
D.4 Joint Model: Factors

Figure 11: Joint model, factors by countries, 5 years.
Figure 12: Joint model, all country-specific credit and liquidity factors.
Figure 13: Joint model, common factors, all maturities.
D.5 Quadratic Model

Factors

Figure 14: Quadratic model, 5 year factor contributions.
Figure 15: Example of “spread curve”. the Netherlands, Oct 26 2009.
References

Ahn, D., R. Dittmar, and A. Gallant

Bommim, A.

Constantinides, G.

Cox, J., J. Ingersoll, and S. Ross

Diebold, F. and C. Li

Dubreccq, S., A. Monfort, J. Renne, and G. Rousellet

Dyckmans, A. and V. Stadler

Ejsing, J., M. Grothe, and O. Grothe

Favero, C., M. Pagano, and E. von Thadden

Litterman, R. and J. Sheinkman

Liu, J., F. L. Longstaff, and R. Mandell
Longstaff, F.

Masselink, M. and P. V. den Noord

Monfort, A. and J. Renne

Monfort, A., J. Renne, and G. Roussellet
2013. A Quadratic Kalman Filter.

Nelson, C. and A. Siegel

Noeth, B. and R. Sengupta

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