

# Entry Deterrence and Learning Prevention on eBay \*

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### Abstract

Internet auctions (such as eBay) differ from the traditional auction format in that participants 1) typically face a choice over several simultaneous auctions and 2) often have limited information about rival bidders. Since existing economic models do not account for these features of the bidding environment, it should not be surprising that even casual empiricism reveals a sharp discrepancy between the predictions of existing theory and the actual behavior of bidders. In particular, despite the second-price structure of eBay auctions (which, along with private values, implies that bidders submit a single bid), eBay bidders frequently bid multiple times over the course of a single auction and cluster their bids at the very end. In this paper, I show that the presence of multiple, contemporaneous auctions for similar items coupled with uncertainty regarding rival entry can explain both features. The multi-auction structure of eBay induces two types of aggressive bidding behavior: entry deterrence and learning prevention. While the incentive to deter entry leads to aggressive early bidding, the incentive to prevent rivals from learning the unobserved features of the auction by observing the price process can lead to late bidding. I analyze these features in a continuous-time stochastic auction model with endogenous entry, in which bidder types are differentiated by their initial information regarding the entry process. I establish non-parametric identification of the structural model and develop a computationally attractive method for estimating its parameters. Empirical estimates using eBay auctions of pop-music CDs confirm my theoretical prediction that the rate of entry depends on price. I then test my model against alternative explanations of observed bidding behavior using a detailed field experiment. The method that I develop for identification and estimation of continuous-time interaction models is applicable to many settings in which agents compete in large dynamic markets.

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# 1 Introduction

Since it went online in 1995, eBay has both provided consumers with an efficient market for the exchange of goods and researchers with fertile ground for testing academic theories about auctions. Although empirical evidence from this rich environment has validated many aspects of standard auction theory, in general, auction theory provides little information about optimal strategic behavior in a dynamic multiple-auction model. Many observed features of bidding behavior contradict predictions from the standard auction models, suggesting that these models provide an incomplete picture of online auctions. In particular, it has been widely observed that bidders on eBay tend to bid multiple times for the same object and cluster their bids in the final seconds of a given auction.<sup>1</sup> Such behavior is inconsistent with a standard second-price auction, whereby bidders simply submit their valuations once and for all. This paper instead proposes a dynamic model of competing auctions that can both rationalize the observed behavior and provide additional testable restrictions. These restrictions are then evaluated using data from a novel field experiment.

The existence of many simultaneous auctions is a defining feature of eBay. It is also the primary reason why static auction theory cannot explain observed online bidding patterns. Because similar items are frequently auctioned in close temporal proximity, the number of entrants into a particular auction will depend on how the price in that auction relates to prices in other auctions. The endogenous nature of entry, coupled with bidders' uncertainty regarding the number of rivals they expect to face in a given auction, creates two opposing strategic incentives that are absent from the simple static framework: entry deterrence and learning prevention. On the one hand, should a bidder find an object for which there are no current bids (given a reasonably attractive starting price), she has a strong incentive to bid immediately, thereby deterring the entry of potential rivals. On the other hand, once an auction has been entered, more informed bidders will have an incentive to delay their bids until the final seconds of the auction, in order to prevent their private information from being incorporated into the price. I will show that the tension between these two opposing forces can explain the coexistence of both early and late bidding. The structure of the model also highlights the importance of learning, even in settings where bidder valuations are uncorrelated.

To formalize this intuition, I propose and empirically estimate a continuous-time structural auction model with endogenous entry. I first estimate the structural parameters of the model using data from a single auction category (pop music CDs). I then conduct a controlled experiment aimed at identifying the impact of entry deterrence on optimal bidding behavior. The rationale behind this experiment turns on the fact that the incentive to deter entry depends on the thickness of the market. In thicker markets (which have more auctions per bidder), entry into individual auctions should be significantly more sensitive to price than in markets that are thinner. Therefore, an exogenous increase in the number of objects being auctioned should be associated with a significant increase in early bidding.

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<sup>1</sup>See, for example, Bajari and Hortacsu (2003).

Consistent with the predictions of the model, the experiment reveals that early bidding is indeed significantly more common when the number of contemporaneous auctions increases.

This paper contributes to both the auction literature and a growing field of research aimed at estimating models of dynamic strategic interaction. The theoretical model illustrates the importance of considering auctions within the context of market interactions with endogenous entry. I show that even if bidder's valuations are uncorrelated, market thickness and the composition of bidder types in the market each have a significant effect on aggressive bidding. In particular, the strategic behavior of individual bidders depends on the characteristics of every available auction, as well as all available information regarding potential rivals. This creates a complex learning process in which bidders must infer the number of potential entrants and their valuations from observations on price. Although solving the full strategic game in a high dimensional space is clearly intractable, by reformulating the problem with a competitive-type mechanism and working in continuous time, I am able to reduce the burden significantly. In particular, I assume that instead of reacting to particular rivals, bidders formulate a best-response to the price movement. This structure allows me to reduce the individual bidding problem to one of single-agent optimal control and analyze an equilibrium similar in flavor to the competitive equilibria studied in the stochastic finance literature.<sup>2</sup> The techniques developed here can easily be extended to other dynamic settings. In Nekipelov (2007b), I use a similar approach to tackle the dynamic principal-agent model of Holmstrom and Milgrom (1987).

By modeling entry as a Poisson process with a price-dependent frequency, I am able to capture the key features of endogenous entry without solving the full strategic game. The model employs a competitive-type mechanism in which bidders directly influence only the price, which then affects the payoffs of rival bidders through their market interaction. Uncertainty is captured as unobserved heterogeneity across auctions using a one-dimensional "visibility" parameter. While bidders have different a priori information regarding the visibility of the auction, they can only learn its value during the auction by observing the price path. Although I assume that the auction visibility is a fixed parameter in a single auction, it is used as a reduced-form characteristic of an equilibrium in a multiple auction environment. Therefore, visibility is endogenous in a general equilibrium context.

Although the bid function in my model does not yield a closed-form solution, I am able to reduce the computational burden significantly by formulating the model in continuous time and exploiting a convenient linear algorithm to solve the model numerically. My estimation method employs a form of indirect inference and contains a method of solving for individual bidding strategies that only requires a fixed number of iterative steps, yielding a much faster procedure than traditional value function iteration. The approach is based on matching the empirically-observed distribution of equilibrium responses to its theoretical counterpart. In an auction, if bidders respond optimally to the movement of price, their bids should replicate the price process over time. This idea is implemented

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<sup>2</sup>See, for instance Cochrane (2001), and Shiryaev (1999).

by minimizing the distance between these two distributions. I establish consistency and asymptotic normality of the structural parameter estimates, as well as non-parametric identification of the auction model emphasized here.

To estimate the model, I use a unique dataset of auctions for pop-music CDs on eBay - a collection of auctions over uniform items for which bidders arguably have uncorrelated valuations. The estimation results validate the predictions of my theoretical model. Specifically, I observe that the entry rate is a decreasing function of price, while the average entry rate is closely related to the average number of bidders per auction. In addition, the model predicts that prices should adjust more frequently towards the end of the auction, which also coincides with the empirical observations. The structural parameters also allow me to perform a counterfactual analysis to examine how equilibrium bidding responds to the elasticity of entry with respect to price and bidders' beliefs. In particular, I quantify the effect of strategic early bidding in the auctions and find that it contributes more to the observed bidding patterns than non-strategic uninformed bidding.

There have, of course, been other attempts at explaining the observed features of eBay bidding behavior.<sup>3</sup> To distinguish my model from competing theories, I conduct a field experiment that highlights a unique feature of my framework: the relationship between entry deterrence and market thickness. Specifically, according to my model, any event that shifts the rate of entry into a group of auctions should also change optimal bidding strategies. In particular, the incentive to deter entry should depend on the thickness of the market (other things being equal). In thicker markets (holding the number of potential bidders constant), entry should be significantly more sensitive to price, making entry deterrence (and the early bidding associated with it) more common in larger markets. This prediction is not shared by the alternative models listed above. Therefore, I can directly test the validity of my hypothesis against these alternative models via a controlled experiment that artificially increases the size of the market for a specific CD. My empirical results reveal that an expansion in the size of the market indeed increases the frequency of early bidding, confirming my predictions.<sup>4</sup>

The structure of the paper is as follows. Section 2 describes the data used in the structural estimation, and Section 3 describes the model and derives the optimal bidding strategy. Section 4 outlines the identification strategy and explains the estimation procedure. Section 5 contains the main

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<sup>3</sup>Existing attempts to explain the eBay data include the tacit collusion proposed in Ockenfels and Roth (2002), technical features of eBay (lack of information about the proxy bidding mechanism) offered in Ockenfels and Roth (2002), multiplicity of listings in Jank and Shmueli (2002) and Peters and Severinov (2004)) and behavioral explanations, such as bidder's irrationality or uncertainty about own private value as in Rasmusen (2003) and Hossain (2004), risk-aversion of bidders in Akerberg, Hirano, and Shahriar (2006) and the possibility of common values in Bajari and Hortacsu (2003).

<sup>4</sup>A unique feature of my field experiment is that it focuses on both homogeneous and inexpensive items. Therefore, issues specific to common-value items, such as the winner's curse, are unlikely to arise. Most previous field experiments on eBay have focused on common-value items. For instance, Yin (2003) studies bidding for computers on eBay, other studies such as Reiley (2006), Bajari and Hortacsu (2003) and Ockenfels and Roth (2002) of eBay discuss bidding for collectible items which also possess common value components.

structural estimates of the model obtained using the data from eBay and the results of counterfactual experiments. Section 6 presents the results of the field experiment and a broader discussion. Section 7 concludes.

## 2 Bidding for pop-music CDs on eBay

My theoretical framework focuses on private value auctions. While there is no common-value component, individual bidders have to learn about the attractiveness of competing auctions by observing the price process. To carry out the analysis I will need to identify a class of eBay auctions for which this assumption is appropriate.<sup>5</sup> I have chosen to study music CDs from the recording artist Madonna for which, I will argue below, the private value assumption is appropriate. First, a CD is a relatively inexpensive item, so its purchase should have minor wealth effects. Second, variation in the market price of CDs sold outside of eBay is likely to be small, implying that most bidders are consuming CDs for their intrinsic value, rather than buying them for resale purposes. Finally, focusing on a single recording artist should further limit unobserved heterogeneity in the bidder population and the uncertainty in the quality of individual items.

Data from eBay have been used by economists in several empirical studies, mainly in the context of common values.<sup>6</sup> My dataset has a unique feature that it contains information about both the auctioned objects (including information about the seller) as well as the bidding process itself. The data was collected using a *Perl* "spider" program, which gathered information about both the items and the bidding histories.<sup>7</sup> The information on bidding behavior characterizes the bid profile for each item, along with the bid history, includes an item identifier, bid amounts, and a bidder identifier. The sample statistics (timing of bids and their quantities) for this dataset are contained in Table 1.<sup>8</sup> In Table 1, the duration of all auctions is normalized to 1 the reason for which will be explained in a subsequent section). Table 1 reveals that the average bid is close to \$14 and the average bidding time is very close to the end of the auction. The abundance of information in my dataset facilitates the

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<sup>5</sup>A private value structure requires that valuations of different bidders be uncorrelated. In practice, I will assume that a private value setting is appropriate so long as these correlations are sufficiently small compared to the variance of valuations.

<sup>6</sup>The eBay computer category studied in Yin (2003) has an issue with product authenticity. Common value component is essential for bidding behavior in many other empirical studies such as Bajari and Hortacsu (2004) and Ockenfels and Roth (2002).

<sup>7</sup>The program was designed to work as follows. First it submitted a search query for music CDs sold through auctions (no eBay stores), and sorted the items according to the auction ending time. The program selects up to 20 pages of items so that the auctions for those items end in at most 4 hours. The program browses individual pages for those items and saves the exact time when the auction ends, the rating and the nickname of the seller, the characteristics of the auction for a CD. Then the program sleeps for 4 hours and goes to the page where eBay saves the bidding history. The process repeats the necessary number of times.

<sup>8</sup>The tables described in this and further sections can be found in Appendix

estimation of a structural continuous-time model and a related field experiment.

Along with the bidding profile, I collected additional information about each auction in the dataset. The variables, summarized in Table 1, include the duration of the auction, the buy-it-now price (if offered), the shipping cost, the condition of the CD (used or new), the seller's feedback score, the proportion of the seller's positive feedback, and a dummy variable indicating that the item's page includes a photo of the CD cover. Most sellers in the dataset have very high scores, and positive feedback levels. The average duration of the auction is 6.5 days, reflecting the fact that sellers can avoid additional listing fees by choosing an auction lasting less than 7 days. The buy-it-now price is very close to the average bid and the buy-it-now option is offered approximately half of the time. The condition variable reveals that half of the CDs on the market are used, while the picture dummy indicates that most auctions include a photo of the CD. Approximately 83% of the sellers in my dataset were located in the United States. Other sellers were primarily located in Argentina, Australia, Europe and Japan. I should also note that the four best-selling Madonna albums (according to Amazon.com) accounted for 16% of the eBay market. A "top 4" indicator variable will therefore be used to control for additional heterogeneity across auctions.

In Table 2, I examined some features of the data that are related to the bidding behavior describing my model. To describe these features, I first constructed two additional variables: a variable corresponding to the arrival rate of bids (i.e. the number of bids in the auction per unit of time) and another variable equal to the size of the price jump (in the beginning of the auction, it is simply the first bid). I analyzed how these variables are affected by the auction duration, the current prices and other characteristics of auctions. The first column contains the output from a fixed effect regression that explains the observed size of bid increments. The estimates reveal that bids become more incremental toward the end of the auction. Moreover, the price tends to jump higher when the price level is high.

In column 2 the dependent variable is the number of bidders who are active over the course of the auction. The estimates from this regression reveal that price is a significant determinant of entry: the rate of entry of bidders is lower in auctions with higher starting prices. These findings imply that ignoring the dependence of bidding behavior on price in eBay auctions, as in a static private value auction model, will lead to incorrect conclusions.

The regression analysis reported in Table 2 not only confirms that the basic features of the auction data are consistent with my theoretical model but also suggests relevant auction observables that can be used to control for cross-auction heterogeneity in my dataset. These characteristics will be used in the structural econometric model which I develop later in this paper. Column 3 examines the intensity of bidding in the auctions. Here, we can see that the number of bids is lower if starting price is high and higher for the most recent albums, while a higher feedback score (of the seller) increases the number of bids. Similarly, Column 4 shows that the results for the number of bidders are close

to the results for the number of bids in Column 3. Therefore, we can conclude that the observed characteristics of cross-auction variation are significant and have sensible coefficients in these simple regressions. For this reason, I will use them as shifters in my structural model.

## 3 Theoretical model

### 3.1 The Setup

eBay is a large Internet auction site where similar items are often available in multiple, simultaneous auctions. eBay bidders don't just choose how much to bid in a given auction, but also which auction to participate in. The choice of auction clearly depends on both the current posted price as well as the relative chance of winning, requiring bidders to form expectations regarding the likely outcome of the bidding process and then update these beliefs as the auction unfolds. In my model I consider independent private value auctions. Learning behavior in my model comes from the learning about the evolution of the future price process rather than the object itself.

In my model I analyze a single auction within a context of a large auction market. It is a continuous time auction model with both continuous actions (bids) and states (prices). Entry into a particular auction is characterized by a stochastic time-heterogeneous Poisson process that determines the number of bidders that enter the auction per unit of time. The arrival rate or frequency of this Poisson process can be thought of as the "instantaneous demand" for the object on auction. Submitted bids cannot be withdrawn; non-participants simply submit bids of zero. Withdrawal of bids is almost never observed in the dataset, the only two incidences of bid withdrawal was due to submission of very large bids by mistake<sup>9</sup>. The price in this auction is described as a continuous-time heterogeneous Poisson process (which is a Markov process). A bidder who arrives in the auction best responds to the entire path of price process rather than to actions of particular rival bidders.

In my model the behavior of sellers is taken as exogenously given. In general, it is also possible that sellers strategically choose the time to list their item(s), as well as the relevant auction parameters, including a reserve price (which can be kept secret) and the "buy-it-now" option<sup>10</sup>. I will consider relaxing this assumption in my future work. However, tackling the full two-sided market with endogenous buyers and sellers is substantially more complex to analyze<sup>11</sup>.

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<sup>9</sup>Entry and exit from in auctions corresponding to the withdrawal of bids is discussed for the case of a static English auction in Izmalkov (2003). The setting with arrival of new buyers in the bargaining context has been considered in Fuchs and Skrzypacz (2007).

<sup>10</sup>Many eBay sellers choose to list their items as "buy-it-now" only, meaning that we observe both items sold in the auction format and items sold at fixed prices. The formation of such an equilibrium is consistent with the theory of search and matching as examined in Pissarides (1990), Baye, Morgan, and Scholten (Forthcoming) and Lagos (2000), which study search frictions in different markets. In particular, it has been shown that in a costly search environment, a distribution of different prices for the same item can be supported in equilibrium.

<sup>11</sup>In the dynamic game describing the auction market, the first stage of the game characterizes optimal reserve price



The role of bidders' searching behavior is captured in my paper by modeling the entry process directly. The instantaneous demand for the object is clearly a function of both price and time, but it also depends on bidders' beliefs regarding the ultimate outcome of the auction process (i.e. whether they will win and how much they will have to pay). This in turn depends on the overall "visibility" of the auction: how attractive it is to enter. This notion of visibility is reflected in an idiosyncratic characteristic that is specific to each individual auction. Note that this structure captures the two salient features of the eBay environment noted above: multiple listings of identical items and heterogeneity in bidder information. The dependence of instantaneous demand on the price of the object is motivated by the role of search. Specifically, if the entry decision is based on the reservation value from a search process, a bidder will enter into an auction where the price of the object is lower (*ceteris paribus*) with higher probability. This means that the instantaneous demand will be a decreasing function of price (holding all other characteristics of the item constant). This structure suggests that the price process in general depends to a family of Poisson processes parametrized by the visibility. To best-respond to the price movement bidders will need to identify the visibility and, thus, the price process *per se*.

The visibility parameter is intended to capture idiosyncratic differences in entry rates for different auctions of similar items. It is fixed for a specific auction and does not change over time. The visibility is closely related to the elasticity of entry into auctions with respect to price. By introducing this parameter, I am able to capture a form of unobserved heterogeneity that is fixed across auctions. Intuitively, the visibility of the auction should reflect heterogeneous effects coming from the supply-side, in particular, the extent of a seller's participation on eBay. For instance, if a seller is selling a variety of items (or perhaps owns an eBay store), some of her items might be cross-listed. As a result, bidders who enter into her auction might follow a link from one of her other items, rather than via a purposeful search, giving her greater visibility. All else equal, cross-listed auctions should have more bidders than auctions with no cross-listings. Another source of heterogeneity of entry coming from the supply-side effects are listing errors by the seller. For example, a seller might misspell the name of the item or make an error in the item's profile. As a result, the item might not appear in the search results by keyword or appear lower in the queue than it would have had it not been misspelled. Finally, some bidders could simply be loyal; if some sellers have a long history of producing satisfied customers, it's likely that they will have repeat customers. These supply-side forces will all influence the visibility of the auction.

The optimal strategy of the bidders depends on the visibility parameter. Bidders do not observe the exact value of this parameter <sup>12</sup>, they have prior beliefs in the form of probability distributions setting by sellers and search for an auction by buyers. In the second stage the buyers compete with each other in particular auctions. A similar setup where the second stage is described by a static second-price auction is considered in Peters and Severinov (1997).

<sup>12</sup>The bidders can distinguish between more visible and less visible auctions, but they cannot predict the exact quan-



over its value. They can then update these beliefs (i.e., learn) over the course of the auction. Initial heterogeneity in beliefs is captured by assuming that bidders possess heterogeneous priors, possibly due to differences in their experience or sophistication. It is important to emphasize that an auction's visibility is not related to the properties of the object itself: uncertainty regarding visibility is present even if every bidder is completely certain about the properties of the object. In this way, the learning concept considered in this paper is different from the typical common value interpretation, in which bidders are learning about the quality of the object on auction. This assumption emphasizes a specific feature of online auctions: even if the bidder is completely certain about the quality of the object itself, she can be uncertain about the group of potential rivals. The bidder will then need to adjust her strategic behavior in response to the distribution of rival bidding behavior.

### 3.2 The Components of the Model

The continuous-time auction model considered here describes a price process that starts at time 0 and ends at a time  $T$ . Consider the strategic behavior of a single representative bidder who competes against her rivals by submitting multiple bids  $b \in \mathbb{R}_+$  at any moment of time  $t \in [0, T]$ .

The auction has a second-price structure, so that the price of the object -  $p \in \mathbb{R}_+$  - at a given time  $t$  is equal to the second highest bid among the bids submitted before time  $t$ . A bidder observes neither the identities nor the number of her rivals. She knows only that entry into the auction is characterized by a Poisson process with frequency  $\lambda(t, p, \theta_0)$ , a function of time, price and the visibility parameter  $\theta_0$ <sup>13</sup>, while its functional form is a common knowledge. This function is the instantaneous demand described above, whose dependence on price is motivated by equilibrium search considerations.

The visibility parameter  $\theta_0$  is exogenous and fixed for a specific auction. The set of possible visibility values  $\Theta$  is convex, compact, and known by the bidders. Although  $\theta_0$  is not observed, bidders have initial beliefs regarding its value. These beliefs take the form of normal priors, truncated to the set  $\Theta$ , with means  $\mu_\theta$  and variances  $\sigma_\theta^2$  drawn from the distributions  $G_\mu(\cdot)$  and  $G_\sigma(\cdot)$  respectively. Bidders with small mean-squared errors of the initial belief  $(\mu_\theta - \theta_0)^2 + \sigma_\theta^2$  are considered "more informed", while bidders with large mean-squared errors are "less informed".

In addition to being uncertain about the visibility of the auction the bidders also have only imperfect observations of the price process. Due to the inability to continuously monitor the price in the auction (late at night, for example), a price change occurring at time  $t$  will be observed by the bidder at time  $t + \epsilon$  where  $\epsilon$  is independent and drawn from the same (non-negative) distribution across the bidders. Bidders are assumed to observe their own  $\epsilon$  and it is constant throughout the auction for the given bidder. The instantaneous demand for an individual bidder with an observation delay  $\epsilon$  will

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titative effect for a specific auction.

<sup>13</sup>Baldi, Frascini, and Smyth (2003) observed that the number of hits at popular Internet sites can be described by a Poisson process

be denoted  $\lambda_\epsilon(t, p, \theta)$ . For example, one of possible forms for the transformation of  $\lambda(\cdot)$  into  $\lambda_\epsilon(\cdot)$  is  $\lambda_\epsilon(t, p, \theta) = \alpha e^{a\epsilon} \lambda(t, p, \theta)$  for fixed  $a$  and  $\alpha$ .

The bidder's valuation for the object is  $v$  and she is risk-neutral: hence if she obtains the object for price  $p$ , then her utility is  $v - p$ . The valuations of the bidders are independently drawn from the distribution  $F(v)$ . The bidder maximizes the expected utility<sup>14</sup> from winning the auction  $E_0 \{(v - p_T) \mathbf{1}[b_T > p_T]\}$ . This expected utility is positive only if the bid of the bidder under consideration at the end of the auction is the highest (and, thus, is higher than the price of the object equal to the second highest bid). The strategy of the representative bidder is characterized by a bidding function  $b_{v\epsilon}(t, p, \mu, \sigma)$ , which gives the optimal bid value at time  $t$  for a bidder with valuation  $v$  and observation delay  $\epsilon$  if the price of the object is  $p$  and the beliefs of the bidder about the visibility have mean  $\mu$  and variance  $\sigma$ . To facilitate derivation, I suggest the following decomposition of the bidding function:

$$b_{v\epsilon}(t, p, \mu, \sigma) = p + \eta(t, p, \mu, \sigma). \quad (1)$$

The function  $\eta(\cdot)$  will be referred to as the bid increment. In general  $\eta(\cdot)$  depends on  $\epsilon$  and  $v$ , but to facilitate the notation we will dropping these indices wherever it clear which bidder is described. As noted above, the bidder wins the auction if her bid increment is positive at the end of the auction.

The price is always equal to the second highest bid. From the perspective of a particular bidder who submits at least a minimum required bid higher than the current price, the price is equal to the minimum of her own bid and the highest bid of her rivals. In this way, if she submits a bid and is the current highest bidder, then the current price is determined by the highest bid of the remaining bidders. A new bidder determines the price if her bid is between the current highest bid and the second highest bid. Any particular bid of a bidder can determine the price only once per auction. The correspondence between the bid increment and the price as observed by an individual bidder can be characterized by the following stochastic differential equation describing the price process:

$$dp_{t,\epsilon} = h(t, p_t, \eta) dJ_\epsilon(t, p_t, \theta_0), \quad (2)$$

where  $dp_t$  are changes in the price over infinitesimal time intervals  $dt$ , and  $dJ_\epsilon(t, p_t, \theta_0)$  are increments of the Poisson process with frequency  $\lambda_\epsilon(t, p_t, \theta_0)$ .<sup>15</sup> This equation reveals that the price evolves in jumps. The size of the jump at time  $t$  is equal to  $h(t, p_t, \eta)$  where the bid increment  $\eta$  is defined at (1)

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<sup>14</sup>It can be argued that in some cases individuals might bid on eBay for reasons other than maximizing the expected surplus from winning when, for instance, they like gambling. A variety of possible other targets are discussed in Kagel (1995). In this paper, I focus only on maximization of expected surplus.

<sup>15</sup>A more formal treatment of the conditions providing the existence of the function in the described model is given in Nekipelov (2007a). I will be using footnotes in this section to give short comments regarding the formal properties of the functions in the model. To describe the model formally I consider a process defined on a complete probability space  $(\Omega, \mathbb{F}, \mathbf{Q})$  and  $\{\mathfrak{F}_t, t \in [0, T]\}$ , a filter such that  $\mathfrak{F}_t \subset \mathbb{F}$  and  $J(t, x)$ , and a Poisson measure for  $t \in [0, T]$  adapted to filter  $\{\mathfrak{F}_t\}$ .

while the timing of the jumps is governed by the Poisson process  $J_\epsilon(\cdot)$ , so that  $n_t^J = \int_0^t dJ_\epsilon(\tau, p_\tau, \theta_0)$  is the number of price jumps from the beginning of the auction up to time  $t$ <sup>16</sup>. Thus if  $n_t^J$  is the number of price jumps up to  $t$  and  $t_i$  are the times when jumps occur for  $i = 1, \dots, n_t^J$ , then the price can equivalently be written as:

$$p_t = \sum_{i=1}^{n_t^J} h(t_i, p_{t_i}, \eta). \quad (3)$$

This expression shows that the price of the object at time  $t$  is equal to the total sum of the price jumps up to time  $t$ .

Second, the timing of the price jumps is determined by the rest of the bidders, while the observed timing is contaminated by observation delay error. In the absence of observation delays, bidders would coordinate their bidding such that all participants would bid at the same time when new information arrived.

Equation (2) contains the visibility parameter  $\theta_0$ , which is unobserved by the bidders. Because a bidder forms her bidding strategy to optimally influence the auction price, it is important that she predict the visibility parameter precisely. This prediction incorporates prior information, which the bidders can have from previous bidding experience and from observing the behavior of the price in the auction. In this way, as the price evolves during the auction, the precision of the bidder's estimate of the visibility of the auction increases. This process can be the result of strategic learning. One way to describe the estimation of the visibility parameter by the bidder is to consider Bayesian updating of the prior beliefs given the stochastic movement of the price.<sup>17</sup>

To compute the dynamics of these beliefs I use the linear filtration method developed in Van Schuppen (1977). While an optimal linear filter (as shown in Lipster and Shiryaev (2001)) is infinite-dimensional, for convenience of computing the bidder's estimate of the visibility of the auction, I

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<sup>16</sup>Formally we need to make sure that the solution to this stochastic differential equation exists for given  $h(\cdot)$  in terms of a stochastic Itô integral. For that purpose we impose two requirements on  $h(\cdot)$ : (i)  $\int_0^T |h(t, x_t, \eta)|^2 \lambda_\epsilon(t, x_t) dt < +\infty$  with probability 1; (ii) The compensated process  $\int_0^t h(t, x_t, \eta) \overline{J_\epsilon(dt, dx)}$  is a local square integrable martingale adapted to the filter  $\{\mathfrak{F}_t\}$  with piecewise - continuous sample functions.

<sup>17</sup>The specific problems of construction of Bayesian estimates for Poisson - type processes are described, for example in Kutoyants (1998) and Karr (1986). For the Gaussian processes the solution to nonlinear mean-square inference problem may be found, for example in Lipster and Shiryaev (2001) in the form of the system of stochastic differential equations. In a more general case of point-stochastic processes, a solution becomes complicated, relies heavily on martingale properties of the underlying stochastic processes and, more importantly, becomes a computationally intensive problem. A constructive way of building linear Bayesian forecasts for the parameters of a Poisson process with a variable frequency is shown in Grandell (1972). The generalization of Grandell's method follows from the integral representation of martingales and is discussed in detail, for example, in Lipster and Shiryaev (2001). Here I will discuss an easier and more intuitive approach which, despite being less rigorous, provides a straightforward way to construct linear Bayesian estimates for the unknown parameters.

restrict the analysis to the first two moments<sup>18</sup>. Nekipelov (2007a) shows that the posterior distribution for  $\theta_0$  can be described by a normal posterior distribution with mean  $\mu_t$ , and variance parameter  $\sigma_t$ . The mean and variance parameter of the bidder's belief regarding the auction's visibility change over time according to the following system of stochastic differential equations:

$$\begin{aligned} d\mu_t &= \frac{\partial \lambda(t, p_t, \mu_t)}{\partial \theta} \frac{\sigma_t}{\psi(t, p_t, \mu_t)} \{dp_t - \psi(t, p_t, \mu_t) dt\}, \\ d\sigma_t &= -\frac{1}{\lambda(t, p_t, \mu_t)} \left( \frac{\partial \lambda(t, p_t, \mu_t)}{\partial \theta} \right)^2 \sigma_t^2 dt. \end{aligned} \quad (4)$$

In this expression  $\psi(t, p_t, \mu_t) = h(t, p_t, \eta_t(t, p_t, \mu_t, \sigma_t)) \lambda_\epsilon(t, p_t, \mu_t)$  characterizes the expected growth rate of price per unit of time.

### 3.3 The Optimal Bidding Problem

In the previous subsection, I described the components of the model of behavior of individual bidders on eBay. I assumed that bidders maximize their expected utilities from winning the auction by submitting optimal bids. Their bidding strategies are described by the bid increment functions  $\eta(t, p, \mu, \sigma) \in \mathbb{R}_+$  defined as the difference between the submitted bid of the bidder and the current price. Expected utility maximization is constrained by the dynamics of the price, described by the stochastic differential equation (2). The visibility of the auction  $\theta_0$  is unobserved by the bidders, but they try to infer its value from observing the actual movement of price in the auction. I assume that the bidders infer the visibility of the auction by forming priors and update their beliefs as the price in the auction changes. This allows me to write the problem of the bidder as:

$$\begin{aligned} &\max_{\eta(\cdot)} E_0\{(v - p_T) \mathbf{1}[\eta_T > 0]\} \\ &dp_t = h(t, p_t, \eta) dJ_\epsilon(t, p_t, \mu_t), \\ &d\mu_t = \frac{\partial \lambda(t, p_t, \mu_t)}{\partial \theta} \frac{\sigma_t}{\psi(t, p_t, \mu_t)} \{dp_t - \psi(t, p_t, \mu_t) dt\}, \\ &d\sigma_t = -\frac{1}{\lambda(t, p_t, \mu_t)} \left( \frac{\partial \lambda(t, p_t, \mu_t)}{\partial \theta} \right)^2 \sigma_t^2 dt. \\ &\theta_t|_{t=0} \sim N(\mu_0, \sigma_0), \quad p_t|_{t=0} = p_0. \end{aligned} \quad (5)$$

The first equation of this dynamic optimization problem represents the objective of the bidder, which is the expected utility of winning the auction. This objective reflects the fact that the bidder obtains a positive utility from the auction only if she wins it (so that her bid is the highest, implying that the bid increment at the end of the auction is strictly positive  $\eta_T > 0$ ). The expectation  $E_t[\cdot]$  is taken over the information set up to time  $t$  which includes the paths  $\{p_t, \mu_t, \sigma_t\}_{t=0}^T$ .

The second equation describes the dynamics of the price as a jump process with Poisson-driven jumps. The frequency of price jumps is  $\lambda_\epsilon(t, p_t, \mu_t)$ . The bidders observe the price jumps with a random delay  $\epsilon$ . The mean of the belief of the bidder about the visibility of the auction is  $\mu_t$ .

<sup>18</sup>In other words if  $\mathfrak{F}_{xt}$  are the  $\sigma$ -algebras generated by the sample trajectories of the price process, then my assumption is that  $E\{(\theta_t - \theta^*)^3 | \mathfrak{F}_{xt}\} = 0$  for any  $t \in [0, T]$ .

The third and the fourth equations represent the evolution of the mean and variance of the bidder's beliefs regarding visibility. These beliefs are driven by the price changes so that the mean of the distribution shifts when the auction price jumps. Moreover, if the expected price growth rate is elastic with respect to visibility, then the variance of the bidder's beliefs will decrease over time.

The last line in the dynamic optimization problem (5) sets the initial conditions. Bidders' initial beliefs about the visibility parameter,  $\theta_0$ , take the form of a normal distribution with mean  $\mu_0$  and variance  $\sigma_0$ . The price at the beginning of the auction is the reserve price for the object set by the seller. It is assumed to be observable by the bidders and equal to  $p_0$ .

### 3.4 The Solution to the Optimal Bidding Problem

My approach to solving the problem (5) uses the Bellman equation formulation, which has been studied extensively in the literature on stochastic dynamic optimal control. Here, we can define the value function of the bidder as:

$$V(t, p, \mu, \sigma) = E_t \{ (v - p_T) \mathbf{1} [\eta_T > 0] \}.$$

The function  $V(\cdot)$  specifies the expected surplus of the bidder from winning the auction given that, at time  $t \in [0, T]$ , the price in the auction is  $p$  and the belief about the visibility of the auction has mean  $\mu$  and variance  $\sigma$ . The value function at the end of the auction is the utility from winning the auction (to win the auction the bidder must have the highest bid, but the price is determined by the second highest bid, so  $\eta_T = b - p_T > 0$ ). The vector of state variables  $(p_t, \mu_t, \sigma_t)'$  forms a sufficient statistic for the dynamics information embedded in the entire price path up to time  $t$ . The dynamic evolution for the state variables  $(p_t, \mu_t, \sigma_t)'$  over time is a Markov process.

The optimal behavior of the representative bidder is derived by applying the Itô calculus to the value function. Intuitively, the derivation uses the fact (due to the Bellman principle) that the expected surplus of the bidder at time  $t$  is equal to the maximum over all possible bid increments of the expected surplus at time  $t + dt$ . The expectation is taken over the distribution of all possible price changes in the interval of time  $dt$ . The bidders optimally influence the price process to maximize their expected payoff. As a result, we can represent the law of motion of the value function of the bidder as:

$$\begin{aligned} \frac{\partial V(t, p, \mu, \sigma)}{\partial t} + \sup_{\eta \in \Xi} \left[ -\frac{\sigma}{h} \frac{\partial \lambda}{\partial \theta} \frac{\partial V}{\partial \mu} - \frac{\sigma^2}{\lambda} \left( \frac{\partial \lambda}{\partial \theta} \right)^2 \frac{\partial V}{\partial \sigma} + \right. \\ \left. + V(t, p + h(t, p, \eta), \mu + \frac{\partial \lambda}{\partial \theta} \frac{\sigma}{\lambda}, \sigma) \lambda_\epsilon(p + h(t, p, \eta), \mu + \frac{\partial \lambda}{\partial \theta} \frac{\sigma}{\lambda}, t) - \right. \\ \left. - V(t, p, \mu, \sigma) \lambda_\epsilon(p, \mu, t) \right] = 0, \end{aligned} \quad (6)$$

$$V(T, p, \mu, \sigma) = \sup_{\eta \in \Xi} \{ (v - p) \mathbf{1} [\eta > 0] \}. \quad (7)$$

In this equation, the space of the control functions  $\Xi$  limits my analysis to the bounded and piecewise-continuous bid increment functions, simplifying further derivations<sup>19</sup>. Equation (6) is a partial differential equation for the value function of the bidder  $V(\cdot)$  with boundary condition (7) which implies that the value function of the bidder at the end of the auction has to be equal to her utility from winning the auctioned object<sup>20</sup>. Note that the boundary condition in (6) implies that the optimal bid increment at the end of the auction can take a range of values  $\eta_{v\epsilon}(T, p, \mu, \sigma) \in (0, +\infty)$ . If we analyze the optimal strategy in the interval  $[T - \tau, T]$  then the first equation in the boundary problem (6) suggests that as  $\tau \rightarrow 0$  then  $v - p$  is optimal for  $\eta_{v\epsilon}(T, p, \mu, \sigma)$  if  $v > p$ . Therefore, it is optimal for the bidder to submit a bid equal to her true valuation at the last moment of the auction, analogous to the behavior in a static second-price auction.

### 3.5 Equilibrium

In reality, auction markets are complex and involve repeated strategic interactions between buyers, sellers and the auction company. The equilibrium that I examine in this paper considers the behavior of a buyer in a single auction and takes as given the behavior of the sellers and the auction engine itself.

In my model, each bidder competes against the market – as embodied in the price process – instead of the other individual bidders. In this way the equilibrium constructed in this paper is a sequential Bayesian equilibrium where beliefs are updating according to (4). The main components of the equilibrium are the price in the auction, the latent number of participating bidders, and the beliefs of the bidders at each point in time. In equilibrium these three components should be self-consistent so that movements in the price determine the number of participants and their beliefs. Conversely, on the basis of their beliefs the participating bidders submit bids, which determine the observed price movements. This concept is formally presented in the definition below.

**Definition 1** *A sequential Bayesian equilibrium with Markov strategies is a collection of price trajectories  $p_t$ , a number of bidders who entered into the auction  $N_t$  by the time  $t$ , and means and variances of the beliefs of the bidders  $\{\mu_t^i, \sigma_t^i\}_{i=1}^{N_t}$ . Bidders are indexed  $i = 1, \dots, N_t$ . Then for each moment of time  $t \in [0, T]$  :*

- *The entry of the bidders is a Poisson process with the frequency  $\lambda(t, p, \theta_0)$  for a given  $\theta_0 \in \Theta$  so that the number of bidders who entered up to time  $t$  is  $N_t$  and is unobserved by the participating*

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<sup>19</sup>In fact I require that  $\eta \in \Xi$  are continuous and differentiable almost everywhere functions defined on  $[0, T] \times [0, \bar{X}] \times \Theta \times \Sigma$ . I assume that these functions are bounded by some  $\bar{X} < \infty$  and that  $h(\cdot, \eta(\cdot))$  are measurable functions of finite variance with respect to the Poisson measure.

<sup>20</sup>Technically speaking, I have obtained a boundary problem for the expected surplus only. This determines the behavior of the function on one of the boundaries but does not provide the information about the derivatives. Such a boundary problem is often classified as a Dirichlet problem.

*bidders*

- The valuations of the bidders are drawn from the distribution  $v^i \sim F(\cdot)$ , the initial parameters of the beliefs are  $\mu_0^i \sim G_\mu(\cdot)$  and  $\sigma_0^i \sim G_\sigma(\cdot)$ , and the observation delay errors are uniformly distributed  $\epsilon^i \sim U[0, 1]$  for all bidders  $i = 1, \dots, N_t$  and for each  $t \in [0, T]$ <sup>21</sup>
- The bidders solve the dynamic optimization problem (5), which generates the optimal bidding functions  $\eta_{v^i \epsilon^i}(t, p, \mu, \sigma)$  and the paths of beliefs  $\{\mu_t^i, \sigma_t^i\}_{t \in [0, T]}$  with  $\mu_t^i|_{t=0} = \mu_0^i$  and  $\sigma_t^i|_{t=0} = \sigma_0^i$  for the bidders  $i = 1, \dots, N_t$  for each  $t \in [0, T]$
- For each  $t$  the change in the price of the auctioned object is equal to the change in the second highest bid:

$$dp_t = \max_{i=1, \dots, N_t, i \neq J} \eta_{v^i \epsilon^i}(t, p_t, \mu_t^i, \sigma_t^i), \quad (8)$$

for  $J = \operatorname{argmax}_j \eta_{v^j \epsilon^j}(t, p_t, \mu_t^j, \sigma_t^j)$ .

It is important to note that in my model bidders can submit multiple bids, and equation (8) refers to the last bid of every bidder who entered into the auction up to time  $t$ . This equilibrium concept suggests that the price increase at time  $t$  is driven by the second order statistic of the bid increment function computed at price  $p_t$ . In this way, a competitive dynamic equilibrium is a solution of the stochastic differential equation (8).

Let me first consider the steps that establish the existence of equilibrium in the continuous-time auction model. It is straightforward to establish existence and uniqueness of the fixed point by considering the individual optimization problem of the bidder. An equilibrium in this setting corresponds to the solution of a collection of individual bidding problems of the participating bidders. This collection is a system of partial differential equations describing the law of motion of the value functions of the bidders in the auction. Nekipelov (2007a) shows that, by certain transformations, this system of differential equations can be represented as a contraction mapping. As a result, existence of the equilibrium is straightforward.

To prove uniqueness of the equilibrium, we must account for entry of bidders into the auction. By assumption, this entry is driven by a Poisson process with variable frequency. If this frequency is finite for any value of the price, time, and visibility parameter, then the number of bidders entering into the auction per unit of time is finite with probability one. This means that the equilibrium in the continuous-time auction can be represented as a finite collection of optimal bidding problems of individual bidders with probability one. As a result, this collection has a unique fixed point that is the equilibrium in the continuous-time auction. A more detailed treatment of this result is presented in

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<sup>21</sup>As I will show in the next section, the distribution of the observation error and the frequency of entry are not separately identified. For this reason, I assume here that the distribution of observation errors is uniform to estimate the frequency of entry.



Nekipelov (2007a). I will use this uniqueness result to describe the likelihood of the continuous-time auction model.

### 3.6 Discussion of the Model

A collection of individual bidding problems generates a dynamic equilibrium with Markov strategies. This individual bidding problem is represented by the boundary problem (6) and (7). I will first describe the general properties of this problem and then discuss the interpretation of the individual components of the decision problem.

The problem (6), (7) has a two-component structure. The component in the *supremum* describes optimal bidding at time  $t$ . This component allows me to compute the bidding strategy for a fixed instant as it produces the optimal response of the bidder at time  $t$  to the current price, and the mean and variance of the bidder's beliefs regarding auction visibility. After the optimal bidding strategy is computed at time  $t$ , I can compute the time derivative of the value function of the bidder and, as a result, obtain the value function for the previous instant of time<sup>22</sup>. Having discussed the general structure of the bidder's problem, let us now proceed with the interpretation of the individual components describing bidding behavior at a specific point of time in the auction.

The last two terms in equation (6) describe the main strategic tradeoffs faced by each bidder. These strategic tradeoffs highlight the role of visibility in the bidder's behavior: higher visibility will in general imply that the bidders will bid more aggressively. The information about the visibility influences the bidding frequency and allows me to distinguish the more informed bidders from the less informed ones. Let me consider these two terms by components. The component

$$V(t, p + h(t, p, \eta), \mu, \sigma) \lambda_\epsilon(t, p + h(t, p, \eta), \mu) - V(t, p, \mu, \sigma) \lambda_\epsilon(t, p, \mu)$$

describes the expected change in the value function due to the price jump. The tradeoff of the bidder is between bidding early and bidding late. Bidding early deters entry, decreasing the probability of price jumps and resulting in a higher value function at the end of the auction. On the other hand,

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<sup>22</sup>This two component structure of the problem naturally enables me to use the standard Euler algorithm to compute the optimal behavior of the bidder. The algorithm would begin from the boundary condition which states that at the end of the auction ( $T$ ) the value function of the bidder is equal to her utility from winning the auction. Then I make a step backwards in time  $\tau$  and recompute the value function at time  $T - \tau$ . The step size  $\tau$  determines the precision of the computed value function. For this value function I compute the optimal bid increment of the bidder and evaluate the time derivative of her value function. This process repeats until it reaches the beginning time 0. This approach has a significant computational advantage over the value function iteration approach which is used to compute the optimal behavior in an infinite horizon discrete time optimization problem. As compared to the value function iterations, there is no need to prove the convergence of the iteration method and wait until the convergence is reached. In my case the number of steps is fixed and is equal to the number of grid points of time while the convergence of Euler's steps for the differential equations is well known. Moreover, such an approach allows me to use a continuous state space which is not always feasible in the discrete time case.

the direct effect of bidding early is to raise the price throughout the auction, which decreases the value function of the bidder at the end of the auction. The optimal time to bid occurs when these two effects offset each other. The movement of the value function is determined by the product of the current value function and the rate of entry into the auction  $\lambda_\epsilon(\cdot)$  (instantaneous demand). As a result, the bidder maximizes the total value of the entry rate (value times demand) and extracts a surplus from bidding by choosing the optimal point on the demand schedule by means of bidding early. This behavior constitutes entry deterrence.

The component

$$V\left(t, p, \mu + \frac{\partial \lambda}{\partial \theta} \frac{\sigma}{\psi}, \sigma\right) \lambda_\epsilon\left(t, p, \mu + \frac{\partial \lambda}{\partial \theta} \frac{\sigma}{\psi}\right) - V(t, p, \mu, \sigma) \lambda_\epsilon(t, p, \mu)$$

reveals the role of information in the strategic decision of the bidders. Note that the size of this component depends on the variance of a bidder's belief regarding the visibility of the auction, so that the effect will be different for bidders with different variances of prior beliefs. If the variance of the prior is large, the bidder does not have much information about the visibility of a specific auction. As a result, the value function of the bidder will significantly depend on the behavior of price. As the variance of the bidder's belief diminishes (towards the end of the auction), the value function will become less sensitive to the new information and the strategy of this bidder will converge to the strategy of the fully informed bidder. Therefore bidders with large prior variances will prefer rapid changes in price because this allows them to reduce the variance of their beliefs faster and extract a higher surplus from bidding. The other type of bidders includes those who initially have low variances of beliefs. In the limit, if the variance of the prior is zero, these bidders will not change their strategies as price changes. Specifically, if this type of bidder deters the entry of the other potential bidders at the beginning of the auction (or the instantaneous demand is initially low), she will wait until the end of the auction to submit her final bid since there are no incentives for her to bid more frequently. In this way, she will not allow her information about the visibility of the auction to be absorbed by the price. This type of behavior constitutes learning prevention.

The component  $-\left\{\frac{\sigma}{\lambda} \frac{\partial \lambda}{\partial \theta}\right\} \frac{\partial V(t, p, \mu, \sigma)}{\partial \mu}$  can be interpreted as the effect of the mean of the bidder's beliefs on the value function. Specifically, this effect depends on the variance of beliefs and the sensitivity of the entry rate  $\lambda(\cdot)$  with respect to the changes in the visibility. If the variance of the belief distribution  $\sigma$  is small, then the effect of a change in the mean of bidder's beliefs is small. Similarly, if the instantaneous demand  $\lambda$  is very sensitive to changes in visibility, then the value function of the bidder will be more sensitive to changes in the mean of bidder's beliefs.

The component  $-\frac{\sigma^2}{\lambda} \left(\frac{\partial \lambda}{\partial \theta}\right)^2 \frac{\partial V(t, p, \mu, \sigma)}{\partial \sigma}$  can be interpreted the effect of the variance of a bidder's beliefs on her value function. This indicates that as bidders obtain more information about entry into the auction over time, their beliefs regarding visibility become closer to the truth. As a result, an increase in the variance of the bidder's belief will be followed by a decrease in the bidder's value function. Intuitively, this implies that the bidder values more precise information from the auction.

It is important to mention that my model is designed to analyze a partial equilibrium in a single auction within a context of a large auction market. In general equilibrium in the market visibility is endogenous characteristic: entry rate into particular auctions will depend on the total number of bidders in the market, the number of sellers, and characteristics of an auction (such as a position in the search contents, spelling errors in the name, number of cross-listed items).

### 3.7 Identification

The continuous-time auction model considered in this paper has a complex structure. In the theoretical section, I represented this structure in an algorithmic form which allowed me to describe the behavior of bidders as a reaction to a stochastically changing price. The reaction of the bidders is summarized by the second highest bid which is equal to the equilibrium price. In order to fit the model to the data, one makes a "guess" about the structural functions of the model (the instantaneous demand, the size of the price jumps, the distribution of valuations, and the distribution of bidders' beliefs), simulates the optimal behavior of the bidders, computes the second highest bid at each instant, and matches the path of the simulated second highest bid to the price path that is actually observed. In this section I verify that the model is identified and the result of the specified matching procedure establishes a one-to-one correspondence between the simulated distribution and structural parameters.

**Definition 2** *The continuous-time auction model of the continuous-time auction is identified if there is a one-to-one correspondence between the joint probability distribution of price jump sizes and the jump times*

$\mathbf{P} \left\{ \int h(t, p_t, \eta) dJ(t, p_t, \theta) < \pi, \int dJ(t, p, \theta) < \tau \right\}$  and a specific collection of characteristics of the theoretical model: instantaneous demand function  $\lambda(t, p_t, \theta)$ , price jump size function  $h(t, p_t, \eta)$ , the distribution of valuations  $F(v)$ , and the distributions of initial beliefs  $G_\mu(\mu_0)$  and  $G_\sigma(\sigma_0)$

This definition implies that if one simulates the equilibrium behavior of the bidders, this will also determine the characteristics of the theoretical model that produce the same simulated distribution of prices. According to this definition, identification is achieved if no other set of characteristics of the model produces the same distribution of simulated prices.

My strategy now will be to provide a set of conditions that allow me to demonstrate that the model is non-parametrically identified using Definition 2. As a result, the potential outcome of the estimation procedure will be the complete functional form of the instantaneous demand function, the price jump function, and the distributions of bidders' valuations and beliefs. From the point of view of Definition 2, the model will be identified if two conditions hold. First, the observable distributions should contain sufficient information about the structural functions of the model. The known observational outcomes are the joint distribution of timing and sizes of the price jumps. The structural functions of the model can be recovered by using some method of inversion of the joint distribution of timing and sizes of

the price jumps. As a result, identification of the model is partially assured by the structure of the data and partially assured by the functional form assumptions that allow me to invert the estimated distribution.

The data collected from eBay contain the complete price paths for multiple auctions. My random data collection methodology assures that there is no selection bias in the distribution of the visibility in the sample of auctions that I am using for estimation. Namely, the collected sample of auctions will be considered free of selection bias (i.e. I did not pre-select the auctions with certain visibility). Moreover, I will assume that the data are collected for very uniform and similar items, guaranteeing that the functions driving the continuous-time auction (the instantaneous demand, the size of the price jumps, and the distribution functions) are the same across auctions, conditional on individual-specific covariates. These data collection assumptions assure that the dataset possesses a certain degree of independence and homogeneity, so that observing repetitions of the auction with certain parameters is equivalent to observing a cross-section of simultaneous similar auctions. As a result, my data have two dimensions: on the one hand, I have a complete record of individual bids corresponding to the continuous-time observations of the price process over time. On the other hand, for each instant I can see the distribution of prices across the auctions. In this way, under certain assumptions I will be able to disentangle the characteristics which are constant over time by looking at the cross sectional dimension, and then recover the remaining characteristics from the data over time.

I will now formulate a set of assumptions which will allow me to establish identification of the theoretical model.

- *Assumption 1.*

The support of the joint distribution of prices and the price jumps

$$\mathbf{P} \left\{ \int_0^t h(t, p_t, \eta) dJ(t, p_t, \theta) < \pi, \int_0^t dJ(t, p, \theta) < \tau \right\}$$

is a convex compact set for any  $t \in [0, T]$ .

- *Assumption 2.*

The instantaneous demand function observed by the bidders  $\lambda_\epsilon(t, p, \theta)$  is decreasing with respect to the observation delay  $\epsilon$ .

- *Assumption 3.*

The instantaneous demand function observed by the bidder  $\lambda_\epsilon(t, p, \theta)$  is strictly increasing in the visibility of the auction.

- *Assumption 4.*

The bid increment function  $\eta(t, p_t, v, \mu, \sigma)$  is non-decreasing in the bidder's valuation  $v$  and the mean of the prior belief of the bidder  $\mu$  about the visibility of the auction, while bidder beliefs are independent from their valuations. Moreover, for each  $v$  there is a price  $\bar{p}$  such that for all prices  $p$  higher than  $\bar{p}$  we have  $\eta(t, p, v, \mu, \sigma) = 0$ .

- *Assumption 5.*

The bid increment function  $\eta(t, p, v, \mu, \sigma)$  is less sensitive to changes in the price for bidders with smaller initial variance of beliefs about the visibility of the auction. That is for  $\sigma' < \sigma$  and a sufficiently small  $\Delta p > 0$ :

$$|\eta(t, p + \Delta p, v, \mu, \sigma') - \eta(t, p, v, \mu, \sigma')| \leq |\eta(t, p + \Delta p, v, \mu, \sigma) - \eta(t, p, v, \mu, \sigma)|.$$

Before beginning the proof of identification, I will discuss the role of each assumption. The first assumption suggests that there are no "holes" in the joint distribution of the number of price jumps and the price. In this way, if this distribution is observed at any moment of time, one can invert it to get the expected number of price jumps and the expected price.

The second assumption allows me to specify the direction in which the observation delay influences the instantaneous demand perceived by the individual bidders. This assumption suggests that the bidders with higher observation delays should be observing the price jumps less frequently.

The third assumption formalizes the notion of the visibility of the auction as a measure of "attractiveness" of a specific auction to the bidders. This enables me to estimate the visibility parameter from multiple auctions based on ranking them by the number of active bidders.

The fourth assumption allows me to distinguish aggressive bidding by bidders with high valuations from aggressive bidding by bidders with high beliefs regarding the visibility. This separation allows me to use information from different parts of the support of the bid increment distribution to estimate the distributions of valuations and the initial beliefs.

The fifth assumption allows me to identify bidder types based on their different reactions to the price jumps. Type indicates the precision of the initial information that the bidder has regarding the visibility of the auction.

The assumptions allow me to uniquely recover from the data the set of structural functions of my model. I will now outline the identification strategy and then proceed to a formal proof of identification.

For any given instantaneous demand function and the price jump size function, we can compute the optimal bidding function for individual bidders. The bidding function reflects the optimal bid value for the bidder given her valuation, beliefs, and the current price of the object for each moment of the auction. Existing papers such as Guerre, Perrigne, and Vuong (2000), Campo, Guerre, Perrigne, and Vuong (2003), and Athey and Haile (2002) establish non-parametric identification of static structural auction models. My strategy in this paper extends the identification results to the continuous-time auction model. If one assumes that the bidding function is monotone with respect to the valuation at each moment of time and that the beliefs are independent from valuations, then the distribution of the number of active bidders across auctions given the price will reflect the distribution of valuations. As a result, a set of cross-sectional observations will identify the distribution of valuations.

Once the distribution of valuations is available, it becomes possible to identify the distribution of beliefs. Two facts are important for the identification of this distribution. First, according to

Assumption 5 strategic learning occurs faster for bidders whose beliefs are close to the true visibility. We will then be able to identify the mean beliefs of the bidders by the distance between their observed bidding patterns given their information set and the optimal bidding pattern (computed for an auction with given visibility and given structural functions). Second, as the model predicts that bidders with more diffuse priors bid more frequently, we will be able to sort the bidders according to the relative number of their bids and, in this way, separate bidders by size of variance of initial beliefs. This allows me to identify the distribution of the bidders' beliefs from the observations across time by measuring the relative frequencies of bidding of different bidders and the distance of their bidding patterns from the optimal one.

Finally, given the distributions of valuations and beliefs, for a given instantaneous demand and price jump size function, I will be able to simulate the path of the second highest bid. If we match the distribution of the simulated second highest bid and the distribution of actually observed prices over time, we reduce the estimation of the model to the estimation of parameters of the compound Poisson process. Such estimation is described in detail in the literature (for instance, in Møller and Waagepetersen (2004), Lipster and Shiryaev (2001), and Karr (1986)) and provides a unique outcome under conditions which are assumed to be satisfied here.

In addition to assumptions 1-5, I will assume that the distribution of the observation delay  $\epsilon$  is known (and assume that it is uniform on  $[0, 1]$ ). I need to impose this assumption because this distribution is not separately identified from the instantaneous demand. Consider the individually perceived instantaneous demand  $\lambda_\epsilon(t, p_t, \theta)$ . Cumulative bidding behavior is associated with the "effective" frequency of price jumps  $\int \lambda_\epsilon(t, p_t, \theta) dF(\epsilon)$  because  $\epsilon$  was assumed to be independent across the auctions. For any differentiable monotone transformation  $\varphi(\cdot)$  we can find a constant  $C$  and a function  $\phi(\cdot)$  such that:

$$\int \lambda_\epsilon(t, p_t, \theta) dF(\epsilon) = \int \varphi(\lambda_\epsilon(t, p_t, \theta)) \frac{1}{C} \phi(\epsilon) dF(\epsilon),$$

and  $\frac{1}{C} \int \phi(\epsilon) dF(\epsilon) = 1$ . Thus, the distribution of the observation delay errors cannot be identified separately from the instantaneous demand function. For this reason I set this distribution to be uniform and impose the normalizations  $\lambda_\epsilon(t, p, \theta)|_{\epsilon=0} = \lambda(t, p, \theta)$  and  $\lambda_{\epsilon'}(t, p, \theta) < \lambda_\epsilon(t, p, \theta)$  if  $\epsilon' > \epsilon$ . This normalization implies that the instantaneous demand observed without any delay is equal to the actual instantaneous demand.

The formal identification argument is provided in the following theorem.

**Theorem 1** *Suppose that we observe distributions of the form*

$$\mathbf{P} \{p_t, J_t\}, \quad \text{for each } t \in [0, T],$$

*where  $p_t$  is the price in the auction at time  $t$  and  $J_t$  is the number of price jumps from the beginning to time  $t$ . Then under assumptions 1-5 and given that  $\epsilon \sim U[0, 1]$  the following results are valid for identification from this collection of distributions:*

- The price jump size and the instantaneous demand function are identified;
- The distribution of valuations of the bidders is identified;
- The distributions of the beliefs of the bidders are identified up to the scale provided by the visibility of the auction:  $G_\mu\left(\frac{\mu}{\theta_0}\right)$  and  $G_\sigma\left(\frac{\sigma}{\theta_0}\right)$ .

**Proof:** The argument for the first statement follows from the generic identifiability of time-heterogeneous Poisson processes.

Now consider the last two statements. By my assumption, the distribution of the observation delay errors is uniform on  $[0, 1]$ . The instantaneous demand is assumed to be monotone with respect to the observation delay errors. In this case the conditional probability of the second highest bid given valuations and beliefs of bidders will be a monotone transformation of the bid increment function. For example, if the instantaneous demand is a multiplicative function of the observation delay:

$$\mathbf{P}_\epsilon\left(\max_{(2)} \eta_{v\epsilon}(t, p, \mu_0, \sigma_0) < \pi\right) = \sum_{k=2}^{\bar{N}} \binom{\bar{N}}{k} \left[ \frac{\pi}{\eta_{v\epsilon=1}(\cdot) - \eta_{v\epsilon=0}(\cdot)} \right]^k \left[ 1 - \frac{\pi}{\eta_{v\epsilon=1}(\cdot) - \eta_{v\epsilon=0}(\cdot)} \right]^{\bar{N}-k}$$

Here  $\max_{(2)}$  denotes the second highest bid increment. The bid increment function is non-decreasing in the valuation of the bidder. This formula shows that the probability  $\mathbf{P}_\epsilon(\max_{(2)} \eta_{v\epsilon}(t, p, \mu_0, \sigma_0) < \pi)$  is a monotone one-to one map from the bid increment function to the conditional distribution of the second highest bid. As a result, I can conclude that the conditional probability of the bid increment function is increasing in the valuations of the bidders. Then for two subsets  $\mathbb{V}_1$  and  $\mathbb{V}_2$  of  $\mathbb{R}_+$  such that  $\mathbb{V}_1 \leq \mathbb{V}_2$ , the probability distributions of the second highest bid given the initial belief are also ordered:

$$\mathbf{P}_\epsilon\left(\sum_{\tau < t} \max_{(2)} \eta_{v_1\epsilon}(t, p, \mu_0, \sigma_0) < \pi\right) \leq \mathbf{P}_\epsilon\left(\sum_{\tau < t} \max_{(2)} \eta_{v_2\epsilon}(t, p, \mu_0, \sigma_0) < \pi\right),$$

for all  $v_1 \in \mathbb{V}_1$  and  $v_2 \in \mathbb{V}_2$ , and the summation  $\sum_{\tau < t}$  is over all discrete moments of price jumps up to time  $t$ . This means that we can partition the distribution of valuations  $F(v)$  into very small regions. If the unconditional distributions (with respect to  $\epsilon$ ) of bid increments in two distinct small regions are the same, the density of valuations in the region with higher valuations is lower. In this way we will be able to uncover the distribution of valuations over the entire state space using such indifference conditions.

Now let me consider whether I can provide a similar ordering for the conditional distribution of the second highest bid with respect to the parameters of the initial bidder's beliefs. By assumption 4, a higher mean of the belief distribution implies that the bidder expects a higher demand for the object. As it follows from the model, the bid increment function is non-decreasing with the instantaneous demand, implying that the bid increment function is non-decreasing in the mean of the bidder's belief.



In this case, for two subsets of  $\Theta$  such that  $\mathbb{T}_1 \leq \mathbb{T}_2$  we should have that:

$$\mathbf{P}_\epsilon \left( \sum_{\tau < t} \max_{(2)} \eta_{v\epsilon}(\tau, p_\tau, \mu_1, \sigma_0) < \pi \right) \leq \mathbf{P}_\epsilon \left( \sum_{\tau < t} \max_{(2)} \eta_{v\epsilon}(\tau, p_\tau, \mu_2, \sigma_0) < \pi \right),$$

such that  $\mu_1 \in \mathbb{T}_1$  and  $\mu_2 \in \mathbb{T}_2$ .

The last parameter of interest is the variance of the bidders' beliefs at the beginning of the auction. By Assumption 5, the bid increment of bidders with more diffuse priors is more sensitive to the price than that of bidders with less diffuse priors. By monotonicity, the distribution of the second highest bid is more concentrated if the variance of the initial belief is smaller. In this way, the more informed bidders (those with lower variance of beliefs regarding visibility) will be using bidding strategies that are close to the optimal one. As a result, we can write that for two sets  $\Sigma_1 \leq \Sigma_2$ :

$$\begin{aligned} & \int \mathbf{P}_\epsilon \left( \sum_{\tau < t} \max_{(2)} \eta_{v\epsilon}(\tau, p, \mu_0, \sigma_1) < \pi \right) \mathbb{J}(d\tau, dp) \\ & \leq \int \mathbf{P}_\epsilon \left( \sum_{\tau < t} \max_{(2)} \eta_{v\epsilon}(\tau, p, \mu_0, \sigma_2) < \pi \right) \mathbb{J}(d\tau, dp), \end{aligned}$$

if  $\sigma_1 \in \Sigma_1$  and  $\sigma_2 \in \Sigma_2$ . The integral is taken over a Poisson measure  $\mathbb{J}(dt, dp)$  which means that we integrate the distribution of the second highest bid over the entire price path. If we defined the kernel function  $K_\pi(\cdot)$  by the integral

$$\int \mathbf{P}_\epsilon \left( \sum_{\tau < t} \max_{(2)} \eta_{v\epsilon}(\tau, p, \mu, \sigma) < \pi \right) \mathbb{J}(d\tau, dp) = K_\pi(v, \mu, \sigma),$$

I have provided a set of assumptions assuring that it will be monotone with respect to its arguments for each  $\pi$ . Moreover, given the specific Poisson measure  $\mathbb{J}(t, p)$  we can compute this kernel function. As a result, we reduce the problem of finding the unknown distribution to the problem of solving the integral equation:

$$\mathbf{P}\{p_t < \pi\} = \int \int \int K_\pi(v, \mu, \sigma) dF(v) dG_\mu(\mu) dG_\sigma(\sigma)$$

with a monotone kernel. This is a Volterra-type integral equation for the unknown distribution functions, which is known to have a unique solution. This suggests that given the price process and the visibility parameter we can reproduce the distribution of the valuations and the beliefs of the bidders. To find the unknown distributions, I find the subsets of valuations and the parameters of the bidders corresponding to the same cumulative probability of the auction price. To equate the resulting probability distributions of the auction prices, the probability density of beliefs and valuations in the region with a larger kernel should then be smaller than that in the region with a smaller kernel. This allows me to associate ranks with the probability masses in all regions for the distributions under consideration. As the probability masses must sum to one across all the regions, they are automatically determined by the rank of each region.

Finally, consider a shift in the visibility of all auctions by the same constant. Given the monotonicity of the instantaneous demand with respect to the visibility of the auction, this will result in an equivalent shift of the bidder's beliefs. In this case, if the visibility parameter is not fixed, we cannot separately identify the distribution of beliefs and the visibility parameter. Thus the visibility is identified only up to a location shift.

The instantaneous demand function and the size of price jumps which can be estimated from the collection of price trajectories represent the characteristics of the price process averaged over unobserved characteristics of the bidders: valuations, initial beliefs and observation errors. Therefore, they are informative of the conditional moments of the price process - average frequency and average jump magnitude given time and price. Nonparametric identification of such a system of moments can be formally justified using arguments from the identification theory for nonseparable models with an exogenous regressor or an independent instrument (such as in Chesher (2003), Matzkin (2003), and Imbens and Newey (2002)). I have shown above that the structure of the simulations from the model is quite similar to nonseparable triangular systems of moments considered in Chesher (2003).

*Q.E.D.*

## 4 Estimation

To estimate my model I need to solve the optimal dynamic control problem faced by a bidder. This problem can be represented by the differential equation (6), which rarely has a closed form solution and it needs to be solved numerically. To simulate the complete equilibrium model, it is necessary to solve a large set of individual bidding problems.

Simulation-based estimation methods used for estimation of structural methods cannot be applied to my continuous-auction models. Examples of these methods are simulated method of moments (including indirect inference as a special case<sup>23</sup>) and two-stage inference (including the hedonic approach<sup>24</sup>). They all require simulations from the structural model given the parameters. Such simulations, however, are not feasible in case of my continuous-time model<sup>25</sup>. Therefore, to estimate my

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<sup>23</sup>This method usually referred to as indirect inference (II) was analyzed in Gourieroux, Monfort, and Renault (1993) and elaborated later to its efficient version adapted to the score estimation method with the auxiliary model generated by the semiparametric estimate of the likelihood of the data, described for example in Gallant and Tauchen (2002) and Gallant and Tauchen (1996).

<sup>24</sup>For instance the two-stage method proved to be useful for estimation of models of imperfect competition as in Bajari, Benkard, and Levin (Forthcoming) and Pesendorfer and Schmidt-Dengler (2003) of dynamic games as in Hotz and Miller (1991) and, auctions as in Guerre, Perrigne, and Vuong (2000), differentiated product markets as in Berry, Levinsohn, and Pakes (1995) and Bajari and Benkard (2002).

<sup>25</sup>The simulated methods of moments relies on the specific choice moment equations, which is unclear in the model under consideration. The two-stage methods have not been developed for continuous-time models and they can have extremely large standard errors.

structural model I develop a new estimation method that is based on the simulation of the response of the bidders to the observed price process.

The idea of the estimation method is the following. First of all, I can reconstruct the entire continuous price path from the observed actual bid retrospectively. Then for each point of the price path I can simulate the number of entrants into the auction. For each entrant and the incumbent bidders who have entered in the previous time periods, I can compute the optimal response to the price movement. The components of the structural model (entry rate, distributions of valuations and the initial beliefs) can be parametrized or represented non-parametrically. Structural parameters affect the simulated optimal behavior: parametrization of the entry rate affects the simulated number of bidders and parametrizations of distributions of valuations and beliefs affect the individual bids. From simulated entry of bidders and the optimal response for each bidder, we can compute the second highest bid for the specified group of bidders given the structural parameters, which I will call the response of the structural model to the price. In equilibrium, this response should coincide with the price. As the process of entry is stochastic, the distribution of this response should coincide with the distribution of the price. The "true" parameter of the structural model is estimated by minimizing the distance between the distribution of the price and the distribution of the response of the structural model<sup>26</sup>.

Let me now describe the estimation algorithm in more detail. While most existing empirical research on auctions uses individual bids as observations, a unique feature of this paper is that I consider the entire price path in the auction as the unit of observation. This feature captures the continuous-time structure of the model, where the equilibrium is described by the stochastic behavior of price.

The estimation procedure has three components. In the first component I estimate the joint distribution of the data. The auction proceeds from time 0 to time  $T$ , and at each  $t$  let  $p_t$  be the observed price and  $N_t$  be the number of bidders. Let  $f(p_t, N_t, t, \gamma_0)$  denote the joint distribution of the object price and the time when the price jumps<sup>27</sup>. This distribution is characterized by a structural parameter  $\gamma_0 \in \Gamma$ . Since it is possible to observe prices of multiple auctions, it is possible to record several price trajectories and estimate the marginal distribution of price  $p_t$  and jump time  $t$  using a kernel estimator:

$$\hat{f}(p, t) = \frac{1}{n} \sum_{k=1}^n \frac{1}{h_p h_t} \sum_{i=1}^{I_k} \kappa \left( \frac{p_i^{(k)} - p}{h_p} \right) \kappa \left( \frac{t_i^{(k)} - t}{h_t} \right), \quad (9)$$

where  $n$  is the number of observed auctions,  $k$  is the index of an auction,  $I_k$  is the number of price jumps in the auction  $k$ ,  $\kappa(\cdot)$  is a kernel function, and  $h_p$  and  $h_t$  are bandwidth parameters. Conditions

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<sup>26</sup>The idea of matching distributions for parameter estimation by indirect inference is discussed in Gallant (2003), where the author suggests using  $\mathbf{L}^2$  distance between distributions.

<sup>27</sup>I assume that the joint distribution of realizations of stochastic processes  $t$ ,  $p_t$  and  $N_t$  exists. Formal conditions can be found in Gihman and Skorohod (1979).

to ensure consistency and asymptotic normality of the estimates are provided in Nekipelov (2007a) and impose several restrictions on the kernel function and bandwidth parameters<sup>28</sup>.

A problem for estimation in a continuous-time auction is that the movement of the price depends on the entry of bidders, which is considered latent. If there is a large number of simultaneous auctions (so that entry of bidders to the auction can be considered independent across auctions), the kernel estimator will still yield the correct estimate of the marginal distribution of prices and the moments of price jumps. Formally this implies that, under the aforementioned restrictions:

$$\widehat{f}(p, t) \xrightarrow{n \rightarrow \infty} \int f(p, N, t, \gamma_0) dN,$$

where  $N$  denotes the total latent number of bidders who have entered into the auction up to time  $t$ . In the technical companion Nekipelov (2007a), I gave conditions under which the obtained density estimates are pointwise asymptotically normal:

$$\sqrt{nh_t h_p} \left( \widehat{f}(p, t) - f(p, t) \right) \xrightarrow{d} N \left( 0, f(p, t) \left\{ \int_0^\infty \kappa^2(\psi) d\psi \right\}^2 \right). \quad (10)$$

This is an important result establishing that even in the continuous-time case, the density estimate possesses the property of asymptotic normality.

The second component of the estimation procedure is concerned with estimation of the distribution of the response of the model given structural parameters and the data. First, simulate the entry of bidders  $N_t$  given parameter vector  $\gamma$  for every instant  $t$ . Given the structural parameter  $\gamma$  and the optimal bidding problem for each price, I can then calculate the second highest bid in the auction at any given instant. This determines the "response" of the structural model to the data:

$$\widehat{p}_i^{(k)}(\gamma) = \varphi_{\widehat{N}, \gamma} \left( p_i^{(k)} \right).$$

Given the price process and the simulated optimal response  $\widehat{p}(\gamma)$  for every  $t$  given parameter vector  $\gamma$  we can estimate the density of the model response as:

$$\widehat{f}_\gamma(p, t) = \frac{1}{n} \sum_{k=1}^n \frac{1}{h_t h_p} \sum_{i=1}^{I_k} \kappa \left( \frac{\widehat{p}_i^{(k)}(\gamma) - p}{h_p} \right) \kappa \left( \frac{t_i^{(k)} - t}{h_t} \right).$$

The structure of this estimator is the same as the structure of the estimator for the density of the data but the observed price is substituted by the simulated price given the parameter vector  $\gamma$ . By construction, the total number of bidders who have entered into the auction  $\widehat{N}$  is independent across auctions. This suggests that:

$$\widehat{f}_\gamma(p, t) \xrightarrow{p} \int f(p_t, N, t, \gamma) dN,$$

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<sup>28</sup>These restrictions specify that the density that I am trying to estimate actually exists, the kernel function is very smooth and decreases at a fast rate to suppress the influence of "outliers", and bandwidth parameters go to zero as the sample size increases so that there is no asymptotic bias in the estimates.

where convergence is justified by the same arguments as in (10).

The measure distance between the observed joint distribution of the observed and the simulated price and time of the price jumps is chosen to be a Kullback-Leibler information criterion (KLIC), which can be used both for estimation and model selection<sup>29</sup>. The idea now will be to compare the empirical model with the structural model (based on the simulated response of bidders). At the point when the parameter of the structural model is  $\gamma = \gamma_0$ , both models should work equally well.

Let  $I_k$  be the number of price jumps in the auction  $k$  and  $n$  be the total number of auctions. The Kullback - Leibler Information Criterion takes the form:

$$\widehat{\text{KLIC}} = \frac{1}{n} \sum_{k=1}^n \sum_{i=1}^{I_k} \log \left[ \frac{\widehat{f}(p_i^{(k)}, t_i^{(k)})}{\widehat{f}_\gamma(p_i^{(k)}, t_i^{(k)})} \right] \quad (11)$$

Minimizing the KLIC is equivalent to maximizing

$$L_n(\gamma) = \frac{1}{n} \sum_{k=1}^n \sum_{i=1}^{I_k} \log \left[ \widehat{f}_\gamma(p_i^{(k)}, t_i^{(k)}) \right],$$

which represents the pseudo-likelihood of the model. To compute this pseudo-likelihood we don't need to estimate the density of the observed data. The latter is needed to make specification testing of the model. In Nekipelov (2007a), it is shown that the estimate for the structural parameter  $\gamma$  obtained from minimization of the KLIC function will be asymptotically normal, so that:

$$\sqrt{nh_t h_p} (\widehat{\gamma} - \gamma_0) \xrightarrow{d} N(0, Q^{-1} \Omega Q^{-1}), \quad (12)$$

where

$$Q = E \left\{ \int_0^T \frac{\partial f(p_\tau, \tau, \gamma_0)/\partial \gamma}{f(p_\tau, \tau, \gamma_0)} dJ(\tau, p_\tau) \right\} \quad \text{and} \quad \Omega = 2 \left( \int_0^\infty \kappa^2(\psi) d\psi \right)^2 E \left\{ \int_0^T \frac{dJ(\tau, p_\tau)}{f(p_\tau, \tau, \gamma_0)} \right\}.$$

A significant problem that might arise in this context is that optimization of (11) requires simulating the model response  $\widehat{p}$  for each parameter value  $\gamma$ . For especially large models, this simulation can be extremely time consuming. However, the computational burden can be reduced substantially by using Bayesian methods for simulation. In fact, unlike deterministic optimization procedures, a Bayesian estimation procedure utilizes all of the intermediate output (such as function and parameter values) to form the posterior distribution<sup>30</sup>.

Let me first describe the procedure for estimation through Markov Chain Monte Carlo (MCMC) and then characterize the asymptotic properties of the obtained estimates. First, we obtain a non-parametric estimate of the density of the data  $\left\{ \left( p_i^{(k)}, t_i^{(k)} \right)_{i=1}^{I_k} \right\}_{k=1}^n$ . Then we construct a random

<sup>29</sup>Vuong (1989) and Chen, Hong, and Shum (2005) show that the KLIC is a powerful tool for model selection.

<sup>30</sup>Chernozhukov and Hong (2004) have shown that for specific classes of loss functions in minimum distance - type procedures, the first and second moments of posterior distribution formed from the minimum distance criterion have the same asymptotic properties as the maximizer to the minimum distance criterion and its variance.

walk sampler from the density  $\varphi(\gamma) = \zeta \exp \left\{ -\widehat{KLIC}(\gamma) \right\}$  for some normalizing constant  $\zeta$ , where  $\widehat{KLIC}$  refers to the data-driven estimate of the KLIC. The random walk sampler is paired with the Metropolis-Hastings method for sampling from an arbitrary density. The method is performed as follows.

The algorithm is initialized at some parameter value  $\gamma^{(0)}$ . One then proposes a parameter vector  $\gamma$  drawn from the auxiliary distribution. The constraints on the parameter support are easily implementable by a particular choice of a prior distribution. If the KLIC function decreases at the new parameter value, the draw is "accepted" (i.e. we add it to the database of the draws from  $\varphi(\gamma)$ ) with probability  $p = \exp \left\{ \widehat{KLIC}(\gamma^{(0)}) - \widehat{KLIC}(\gamma) \right\}$  and the new draw  $\gamma^{(1)} = \gamma$ . Otherwise, with probability  $1 - p$  the draw will be rejected and  $\gamma^{(1)} = \gamma^{(0)}$ . This process is repeated sufficiently many times to achieve convergence (meaning that the Markov chain of  $\gamma$  becomes stable)<sup>31</sup>. Markov chains generated by this algorithm will be stationary under conditions described in Robert and Casella (2006). These conditions are satisfied for posteriors generated by distance functions with a single minimum and a distance measure increasing at a high enough rate.

Chernozhukov and Hong (2004) show that the asymptotic behavior of the posterior mean and variance is the same as the asymptotic behavior of the estimates implied by the minimum distance function. In Nekipelov (2007a) it is shown that the asymptotic behavior of the MCMC estimate of  $\gamma$  under a fixed, non-parametric estimation of the data density is:

$$\sqrt{nh_t h_p} (\hat{\gamma}_{MCMC} - \gamma_0) \xrightarrow{d} N \left( 0, \frac{1}{2} Q^{-1} \Omega Q^{-1} \right) \quad (13)$$

As compared to the asymptotic variance, the variance of the MCMC estimate has an additional factor of  $\frac{1}{2}$ . This occurs because the variance of the non-parametric estimate of the data density is not taken into account in the MCMC procedure. To obtain the correct variance estimate, we need to double the MCMC variance estimate.

Although the form of the variance matrix for the sampled parameters is non-standard, it does not imply that the "mean-squared Hessian" is not equal to the variance of the "mean-squared score" of the objective function. This result is presented formally in Nekipelov (2007a). Here I will give a simple argument providing intuition for such result. The pseudo-likelihood for the model can be represented

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<sup>31</sup>The fact that the resulting values of  $\gamma$  will represent the distribution with the density  $\varphi(\cdot)$  is possible to establish in the following way. Let  $\gamma_0 \sim \varphi(\gamma)$ . Then acceptance of  $\gamma$  is justified by the fact that  $\exp \left\{ \widehat{KLIC}(\gamma_0) - \widehat{KLIC}(\gamma) \right\} < z$ . The probability of this event:

$$\mathbf{P} \{ \gamma \text{ is accepted} | \gamma_0 \} = \int_0^1 \mathbf{1} \left\{ e^{\left\{ \widehat{KLIC}(\gamma_0) - \widehat{KLIC}(\gamma) \right\}} < z \right\} dz = \exp \left\{ \widehat{KLIC}(\gamma_0) - \widehat{KLIC}(\gamma) \right\}$$

The unconditional density is thus expressed as:  $f(\gamma) = \mathbf{P} \{ \gamma \text{ is accepted} | \gamma_0 \} \varphi(\gamma_0) = \zeta e^{\left\{ -\widehat{KLIC}(\gamma) \right\}}$ , which is exactly the density  $\varphi(\gamma)$ . This implies that  $\gamma$  is a draw from this density.

as:

$$L_n(\gamma) = \frac{1}{n} \sum_{k=1}^n \int_0^T \log \left[ \hat{f}_\gamma(p^{(k)}, t^{(k)}) \right] dJ(p^{(k)}, t^{(k)}).$$

To derive the asymptotic behavior of the estimate  $\hat{\gamma}$  we need to consider the difference between  $L_n(\gamma)$  and its population value for the true structural parameter:

$$E \{L(\gamma_0)\} = E \left\{ \int_0^T \log [f_{\gamma_0}(p, t)] dJ(p, t) \right\}.$$

The difference between these values can be represented as a sum of three error components: the component due to the deviation of the estimated parameter from the true one, the component due to the deviation of the estimated density from the true one and the sampling error in the stochastic integral. The last component has a smaller order than the first two ones<sup>32</sup>. The second order components will have similar structure as the first order components. In fact, if  $\gamma$  changes then the estimated density  $\hat{f}_\gamma(\cdot)$  changes as well. Thus, in the second order term the error associated with the density estimation will dominate. Therefore, the expected second-order component will have the same structure as the variance of the first-order component.

## 5 Results of structural estimation

In the previous sections I constructed a continuous-time auction model and proved that the model is non-parametrically identified. I also developed a method for flexible estimation of the model using a Markov Chain Monte Carlo approach. In this section I describe a particular model specification I adopt for my dataset and report the empirical results.

To perform inference, I need to estimate the posterior distribution of the structural parameters using a sequence of simulations. However, the computational burden of the simulation increases with the dimension of the parameter vector, requiring more evaluations of the distance function. For this reason, I need to specify the structural model parsimoniously enough to allow the chain to explore the posterior parameter distribution in an efficient manner. These parametric assumptions are made for computational convenience and are not required for model identification.

The baseline structural functions describing the dynamics of the price are the instantaneous demand and the size of price jumps. The specification of the instantaneous demand as observed by the individual bidder (given the observation error  $\epsilon$ ) is assumed to have a logistic form:

$$\lambda_\epsilon(t, p, \theta) = \alpha_\lambda \epsilon \exp(\theta) \frac{\exp(\mathcal{P}(t, p))}{1 + \exp(\mathcal{P}(t, p))} = \alpha_\lambda \epsilon e^\theta \mathcal{L}(\mathcal{P}(t, p)), \quad (14)$$

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<sup>32</sup>This result is valid because the counting integral in the pseudo-likelihood function belongs to a Donsker class as shown in Nekipelov (2007a).



where  $\mathcal{P}(t, p)$  is a polynomial in time and price,  $\mathcal{L}(\cdot)$  is the logistic function, and  $\epsilon$  is uniformly distributed on  $[0, 1]$ . The logistic structure of the frequency is not only convenient but also ensures that the frequency stays reasonably bounded<sup>33</sup>. Flexibility in the structure is captured by a polynomial form in the argument of the logistic function.

The magnitude of the price jumps is generated by the logistic form, similar to that for the instantaneous demand function

$$h(t, p, \eta) = \alpha_h \mathcal{L}(\mathcal{Q}(t, p, \eta)), \quad (15)$$

where  $\mathcal{Q}(\cdot)$  is a polynomial function of time, price and the individual bid increment. This polynomial structure allows a semi-parametric representation of both the instantaneous demand and the size of the price jumps.

The non-parametric flexibility of the structural model increases with the power of the polynomials  $\mathcal{P}$  and  $\mathcal{Q}$ . However, this also increases the number of unknown coefficients in the polynomial representations, leading to a decrease in computation speed. For this reason the degrees of polynomials  $\mathcal{P}$  and  $\mathcal{Q}$  were restricted to 2. The polynomial  $\mathcal{P}(\cdot)$  in the instantaneous demand function is thus quadratic in time  $t$  and current price  $p$ , taking the form:

$$\mathcal{P}(t, p) = a_0 + a_1 p + a_2 p t + a_3 p^2. \quad (16)$$

The price jump size function  $h$  is a logistic function of the quadratic function of time  $t$ , price  $p$ , and the bid increment  $\eta$ :

$$\mathcal{Q}(t, p, \eta) = b_0 + b_1 p + b_2 t + b_3 \eta p + b_4 \eta + b_5 \eta^2. \quad (17)$$

The parameters characterizing individual bidders in the continuous-time auction model are their valuations and the parameters of their prior beliefs regarding the visibility of the auction. The valuation of each player was assumed to take the form  $v = \underline{v} + \sigma_v \xi^2$ , where  $\xi$  is a standard normal random variable. The parameters  $\underline{v}$  and  $\sigma_v$  are estimated. Since this implies that the normal variables  $\xi$  are independent across bidders, the bidders have private valuations as in the theoretical model.

To simplify the computations of the individual bidding problem, I assume that there are only three types of bidders (the intermediate type includes all bidders except for the bidders with degenerate beliefs), and each entering bidder is exogenously assigned a type with certain probability. The first type of bidders - the non-strategic bidders - bid their valuations immediately after entry. The probability of the first type is denoted  $\Delta_u$ . The second type of bidders - the learning bidders - have imprecise information about the visibility of the auction. I estimate the parameters of the distribution of the initial beliefs of these bidders - the mean  $\mu_\theta$  and variance  $\sigma_\theta$ . The distributions of initial beliefs across bidders were taken to be normal, truncated to be positive. The third type of bidders - the experienced

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<sup>33</sup>For large frequencies the solution to the individual bidding problem becomes unstable.

bidders - are assumed to know the visibility of the auction exactly and behave according to the optimal strategy. The probability of the third type is the complement  $\Delta_i$ .

I then applied my MCMC estimation procedure to data for 1281 eBay auctions of pop-music CDs. The estimation procedure was organized in the following way. For each auction I simulate up to 50 potential bidders and assign to each of them the observation errors drawn from a uniform distribution on  $[0, 1]$  and their private values drawn from the distribution of valuations. Then for the proposed values of the structural parameters  $\alpha = (\alpha_\lambda, \alpha_h)$ ,  $a_i$   $i = 1, \dots, 3$ ,  $b_i$   $i = 1, \dots, 5$ ,  $\underline{v}$ ,  $\sigma_v$ ,  $\Delta_u$  and  $\Delta_i$ , I compute the optimal bid for each bidder and identify the second highest bid. For every auction this simulation process is repeated for the entire price path. On the basis of this bid distribution, I compute the value of the KLIC. The KLIC, non-parametrically computed from the simulated path, is then parsed through the Metropolis-Hasting procedure which generates 50,000 quasi-posterior draws after an initial stabilization period. The estimated parameters which are computed from the mean of the MCMC chain together with the standard deviations are reported below.

$$\begin{aligned}
\mathcal{P}(t, p) &= 7.3145 - 1.7595p - .7334t + 5.3492tp + 2.1066p^2 \\
&\quad (1.9049) \quad (1.1613) \quad (2.1024) \quad (2.1919) \quad (2.4269) \\
\lambda(t, p, \theta) &= 2.7475 \quad \epsilon e^\theta \mathcal{L}(\mathcal{P}) \\
&\quad (1.0376) \\
\mathcal{Q}(t, p, \eta) &= 2.5286 + 4.2515p + 5.1823t - .9114\eta + .3195\eta^2 \\
&\quad (3.7750) \quad (1.8670) \quad (2.2704) \quad (1.2372) \quad (.0701) \\
h(t, p, \eta) &= 1.0776 \quad \mathcal{L}(\mathcal{Q}) \\
&\quad (.4990)
\end{aligned}$$

$$\begin{aligned}
E[\mu_\theta] &= 0.4733 & \text{var}[\mu_\theta] &= 0.7846 \\
&\quad (0.3598) & &\quad (0.1868)
\end{aligned}$$

$$\begin{aligned}
E[\sigma_\theta] &= 5.8977 & \text{var}[\sigma_\theta] &= 1.6799 \\
&\quad (2.8188) & &\quad (2.9275)
\end{aligned}$$

$$\begin{aligned}
\Delta_i &= .2981 & \Delta_u &= .1116 \\
&\quad (.0420) & &\quad (.0900)
\end{aligned}$$

$$\begin{aligned}
\underline{v} &= .5385 & \sigma_v &= 4.5286 \\
&\quad (1.7838) & &\quad (1.6418)
\end{aligned}$$

Consider several features of these structural estimates. The first equation, which shows the estimated parameters for the polynomial arguments of the instantaneous demand, suggests that the instantaneous demand is price-dependent. The coefficient for interaction between price and time is positive and

significant, implying that elasticity of instantaneous demand is increasing over time. This suggests that the incentive to bid to deter entry changes along the price path.

The second equation describes the instantaneous demand function in terms of the "scale" parameter  $\alpha_\lambda$ . This parameter can be interpreted as the average expected number of bidders who enter the auction. The estimated parameter is equal to 2.74, consistent with the observed bidding patterns since the average number of bidders in an auction is 2.02 in the data.

The third equation represents the polynomial in the price jump size function. The coefficients for time and price are positive, implying as the price increases and time approaches the end of the auction, price jumps approach to their "cap" values. By logistic construction of price jump function, the "cap" value is determined by the factor in the fourth equation. Most of the influence of individual bid increments on price jumps is captured by the last quadratic term. If price and time are fixed, then the influence of the individual bids increases with the size of the individual bid increments until it approaches its "cap". This is reflected in the negative coefficient on the quadratic term of the individual bid increment.

The last four values are the parameters of the distribution of valuations and the shares of the first two types of bidders. The parameter for the distribution of valuations suggests that the lower bound of the support of valuations is statistically indistinguishable from zero. This also implies that the expected valuation of the bidders is equal to \$9.04 (from the mean of the  $\chi^2$  distribution). The average price in the auctions is \$6.08. This implies that the bidders are left with a surplus of approximately 30% of the final price. Another key parameter is the proportion of non-strategic bidders  $\Delta_u$ . The value of the estimated proportion and the corresponding standard error suggest that I cannot reject the hypothesis that the actual number of non-strategic bidders is zero. From parameter  $\Delta_i$  the proportion of "informed" bidders is 29.81% and is statistically significant. Given these results, I can claim that the estimates of the structural model suggest that the majority of bidders are in fact behaving strategically. The first type bidders are basically naive and they are not behaving strategically. The estimates show that there are very very few of such bidders. The second type bidders are strategic but not experienced enough to have precise information about the visibility of the auction. The estimates show that roughly 2/3 of all bidders are that type. The bidders of the third type are both experienced and strategic and the estimates show that the bidders of this type constitute a large proportion of the bidders population.

## 5.1 Counterfactual simulations of optimal behavior

I now use the parameter estimates obtained in the previous section to demonstrate the properties of the model. I also illustrate how bidding behavior changes in response to exogenous changes in the determinants of the rate entry into the auction and bidder beliefs (i.e., comparative statics). The results of the analysis in this section quantify the effect of the parameters of the model (specifically,

the parameters of the entry rate and the bidder beliefs) on the timing and sizes of bids.

Two kinds of aggressive bidding behavior are present in my model. The first is entry deterrence, when bidders raise the price in the auction early to prevent the entry of other bidders. The second is learning prevention, when bidders who are more informed about the unobserved visibility of the auction tend to bid late to hide their information. These two types of aggressive behavior are reflected by the timing of bids. I will analyze how changes in the structure of the model affect the distribution of timing and sizes of bids (relative to the final price).

For the purpose of my counterfactual simulations it is sufficient to look at the behavior of a single bidder because the behavior of other bidders in the auction is fully described by the Poisson price jumps. In my simulation procedure I first produce a "representative" sample of bidders (in terms of their valuations and beliefs) and record the fraction of early bids submitted by a bidder as well as the size of her bids (as proportion of the final price)<sup>34</sup>. This procedure consistently describes the characteristics of the model by looking at the features of their bidding<sup>35</sup>. I simulate the model as described above at the estimated and the alternative parameter values which allows me to find how the bidding behavior is affected by the structure of the model. In the subsequent discussion, I analyze the model by changing the appropriate coefficients proportional to their values at the benchmark (estimated parameter values).

The first feature of interest is the motive for learning prevention. In my model the variance of the bidder beliefs plays an important role in the incentive to prevent learning of other bidders. On the one hand, a bidder who has a large variance of beliefs engages in the process of updating her beliefs from the price observations referred to as strategic learning. On the other hand, the behavior of bidders with small variance of beliefs is associated with learning prevention. In the first experiment, I simulate the model using different values for the variance of the initial beliefs of the bidders. Table 3 shows changes in early bidding behavior and sizes of bids if the average variance of bidders' beliefs is set to 0.5, 1.5 and 2.0 times the benchmark value. If the average variance of the bidder beliefs increases, the fraction of early bids tends to increase and bid sizes tend to diminish. This implies that bids

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<sup>34</sup>For convenience of exposition I define an early bid as a bid submitted during the first 60% of the auction duration. Additional simulations, however, show that the results are not qualitatively different if the timing of early bidding moves up to 95% of the auction duration.

<sup>35</sup>A concrete implementation of the counterfactual simulations will be organized in the following way. (i) choose specific structural parameters which identify the model. The benchmark model is the model with the parameters obtained from the structural estimation. (ii) given the structural parameters draw the characteristics of the bidder: valuation, mean and variance of the initial beliefs about the visibility of the auction, and observation delay error. The distributions of these characteristics are defined by the structural parameters. (iii) for fixed characteristics of the bidder compute the optimal bidding function. The optimal bidding function reflects the value of the optimal bid given time, price and current beliefs about the visibility of the auction. (iv) simulate the price behavior taking into account the optimal bidding function. This allows me to record the timing and the values of bids of the bidder of interest. I repeat steps one to four sufficiently many times (in this analysis - 1000 times) to build a representative sample of the bidders.

become more frequent and more incremental in the presence of strategic learning if the variance of initial beliefs is high. On the other hand, early and frequent bidding becomes rare due to learning prevention if the beliefs of a representative bidder are precise. Table 3 shows that strategic learning and learning prevention are important even if less than 20% of the bidders belong to the learning type.

Entry deterrence behavior in my model is determined by the price sensitivity of the instantaneous demand, which is characterized by the frequency of the Poisson entry process  $\lambda(\cdot)$ . The scale factor  $\alpha_\lambda$  and the coefficient  $a_1$  for the linear price term in the quadratic polynomial are used to control for the price sensitivity of the entry process in my model (see equations (14) and (16)). For both coefficients an increase in either of the coefficients leads to an increase in the marginal entry rate. Table 3 shows the simulation results, when  $\alpha$  and  $a_1$  are set to 0.5, 1.5 and 2.0 times the benchmark values respectively. In general, increases in both coefficients lead to increases in early bidding and the average sizes of bids. The numbers, however, show that the response to changes in the scale factor  $\alpha$  is larger. The results of simulations confirm the theoretical prediction that an increase in the sensitivity of entry with respect to price increases leads to more frequent early bidding to prevent entry of other bidders.

Simulation analysis of entry deterrence behavior reported in Table 3 produces an important testable implication of the model. Consider observations on two markets with different price sensitivities but otherwise similar. In this case we should expect more early bidding in this market. In general, the entry rate reflects the search equilibrium in the auction market, partly determined by the set of auctions that are available to bidders. Taking the number of bidders as given, an increase in the number of choices translates into an increase in the probability of entry into a specific auction. In addition, a high price in the auction negatively affects the surplus of the entering bidders. Therefore the probability of entry into the auction with a high price decreases if the number of choices grows. As a result, the sensitivity of entry with respect to price decreases if the the number of available auctions increases and the average number of bidders in the market remains the same. One way of empirically testing this pattern is to analyze the variation in the early bidding behavior across different categories of items on eBay with different thickness of markets, controlling for the number of bidders. *Ceteris paribus* the frequency of early bidding should be positively correlated with the number of listed auctions. Another way of testing this implication for early bidding is to run a field experiment where the number of auctions in the market is increased for a period of time. The simulation results imply that we should observe an increase in early bidding behavior in these circumstances. The results of such experiment are discussed in the next section.

## 6 Bidding for musical CDs on eBay: A Field Experiment

### 6.1 Methodology

According to my model, the level of entry deterrence should depend on the sensitivity of the entry rate to price. In fact, the structural estimates presented above indicate that the entry rate is a decreasing function of price. The field experiment described in this section will allow me to exogenously shift the entry rate and change the entry deterrence behavior, providing a strong test of the underlying theoretical model.

Using structural estimation, I found that the rate of entry into the auction decreases as the price grows. I also found that a significant portion (up to 70%) of bidders are the "learning" type - they update their beliefs by observing the evolution of the auction price. These findings suggest that if the bidders are behaving optimally (from the point of view of the model), then we should be able to observe the predicted features of bidding.

Because entry into the auction depends on price, bidders can deter entry by bidding early. Therefore, if it is possible to change the structure of this dependence, we should be able to observe a change in this early bidding behavior. In particular, the rate of entry should depend on the size of the market. If the market is increased (i.e. the number of listed items increases), then the probability of entry into auctions with high prices should decrease, since the chance of winning the item in an auction with no other bidders is higher. As a result, we should expect that early bidding to become more attractive.

The incentive for entry deterrence may also vary across bidders due to differences in their experience. Past bidding experience allows the bidders to predict the entry rate into the auction by forecasting the visibility of the auction. This has different implications for bidders with different prior experience. On the one hand, less experienced bidders will want to "experiment" with the auction to increase their information regarding visibility. On the other hand, more experienced bidders will have an incentive to submit bids only at the end of the auction to conceal their information. For this reason, bidders with different prior experience should respond differently to the exogenous changes in the market, including as an increase in market size. As I argued before, an increase in market size increases the incentive to bid early. If the bidder is experienced, but does not bid early in the market of original size, she may start bidding early if the market expands. As a result, an increase in market size should increase the probability of early bidding by more experienced bidders. Since most of the bidders appear to be experienced, the change in bidding behavior should be substantial.

Field experiments on eBay have been used by several applied researchers to study bidding in Internet auctions. Examples include numerous studies analyzing static models of eBay bidding behavior, such as the effects of seller's reputation (Melnik and Alm (2002))), revenue equivalence (Lucking-Reiley (1999)), and reserve prices (Reiley (2006)). This is the first paper to study the implications of entry and experience on dynamic bidding behavior. The experiment presented in this section has a unique

feature in that it studies the implications of dynamic bidding behavior on early bidding. I perform the experiment by exogenously changing the number of items listed in a particular category on eBay, and observing the change in early bidding behavior.

For my experiment, I chose a specific small item category on eBay and increased the total number of items in this category by listing additional auctions. I formed a control sample of auctions by observing the market prior to treatment. To form a treatment sample of auctions, I listed enough additional auctions to double the size of the market relative to the control group. According to my model, if the treatment increases the sensitivity of entry to price (the elasticity of instantaneous demand increases) then we should observe the entry deterrence feature of bidding behavior discussed above. Specifically, the probability of early bidding across all types of bidders should increase, but it should grow more for more experienced bidders.

In my experiment, I look at bidding on eBay for the Robbie Williams' CD "The Greatest Hits", which was released in 2004. The supply for this CD is quite stable, with an average of 24 items listed each day. This number of listed items allows me to shift the supply significantly by listing additional items. On the other hand, the number of items is sufficiently large to yield a dataset of about two hundred items over a period of two months. I form the dataset by first observing the unperturbed market. Then I double the size of the market and look at the change in bidding behavior.

I next analyze the frequencies of early jump bidding in the treatment (i.e., post-experiment) and control (pre-experiment) groups. According to the theoretical model, the frequency of early jump bidding should increase with the size of the market. I construct a dummy variable for each auction equal to 1 if there is an early jump bid and zero otherwise. Conditional on the auction characteristics (such as seller's feedback score and the location of the seller), we should expect a positive relationship between this dummy variable and the treatment group dummy.

The second relationship studies the dependence between multiple bidding and the bidder's experience. The model predicts that less experienced bidders should bid more frequently than more experienced bidders. Moreover, if the instantaneous demand is low, then experienced bidders will bid only at the last moment of the auction. If the instantaneous demand grows, then the experienced bidders should start submitting early jump bids to prevent entry by potential rivals. The incentive to prevent entry of other bidders offsets the incentive to prevent learning by other bidders. As a result, in the regression across bidders of the number of early bids submitted by a specific bidder on the treatment dummy, experience of the bidder and the interaction of the treatment dummy with experience indicator (conditional on the auction characteristics), there should be a positive coefficient for the interaction term, and a negative coefficient on experience.



## 6.2 Control dataset

The control dataset in my study is a set of auctions for the same CDs before the market was inflated. In total, I collected data for 136 auctions, the earliest auction starting on August 23, 2006 and the latest ending on October 4, 2006. 15 auctions had items located in North America, 69 in Europe (predominantly the UK), 43 in Asia (mainly China and Taiwan), and 9 in Australia. The shipping cost can be substantial if the item is located very far from the winning bidder. In these circumstances, the shipping costs might play an important role in bidding behavior and I will use regional indicators to capture this effect in the data.

I collected the following auction-specific characteristics: buy-it-now price, starting bid, picture dummy, duration (in days), a dummy equal to 1 if the item is available only in the country of origin (regional auction dummy), shipping cost, the percentage of positive feedback for the seller, seller's feedback score, and a dummy equal to 1 if the seller has a store. One minor issue with the prices and costs is that the auctions in the UK and Australia use, British pounds and Australian dollars respectively. All prices and shipping costs were converted to US dollars on the basis of daily average FOREX exchange rate on the day of transaction. Since there were no significant fluctuations of the US dollar exchange rate to these currencies, this conversion should not significantly impact the results. Table 4 presents summary statistics for the variables in the control sample. Note that the highest buy-it-now prices and the starting bids correspond to rare autographed CDs which might be considered collectibles. These were included in the dataset because they still possess the basic properties of the standard CDs. Most of the items in the sample have a picture. It is worth noting that, in most cases, this is a picture that is offered by eBay automatically and corresponds to the cover of the album <sup>36</sup>. The average duration of the auction is 7 days, with a range of 3 to 10 days<sup>37</sup>. While 70% of CDs in the sample are new, most of the used CDs were auctioned from the UK (40 in total) and none of them were auctioned from Asia. The average shipping cost (consisting of the cost of postage and a handling fee) is close to the average sale price of the CD. This makes the total highest prices of the item on eBay close to the lowest prices available at online retail stores (such as Amazon.com). The largest shipping costs are for the farthest distance between seller and buyer. The shipping cost shown in the table corresponds to the shipping cost to the United States, while the empirical analysis uses the bidder-specific shipping cost. The feedback variable does not seem to be very informative as a majority of sellers in the sample have a 100% positive feedback, probably due to the existing eBay culture of leaving negative feedback only in the case of a very bad experience. A more informative measure is

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<sup>36</sup>This picture and the list of songs can be generated automatically on eBay website if the seller uses the search option allowing to track the album information from the bar code on the cover of the disc. For this reason, it is possibly an unimportant factor for the bidders.

<sup>37</sup>This can be explained by the fact that there is no additional listing fee on eBay for a 7-day listing (and if the item was not sold in can also be relisted without extra charge), while a 10 day listing costs an extra 40 cents, and a 3 day listing requires a certain level of feedback of the seller.

the feedback *score*, which indicates the experience of the eBay participant and is proportional to the number of transactions made by the user on eBay. Most of the sellers in the sample have very high feedback scores, and, as the mean of the store dummy indicates, almost 60% of the sellers have stores. This suggests that most sellers in the sample are quite experienced.

Turning to the characteristics of the bidders in the control sample, Table 5 summarizes the main parameters of the bidders. First of all, we can see that the feedback scores of the bidders are significantly smaller than the feedback score of the sellers. This implies that the sellers in my sample have more experience on average. The percentage of positive feedback, similar to that of the sellers, is equal to 100% for most of the bidders. The last two variables are the dummy "experience" indicators constructed from the observations of past purchase histories of the bidders. The first is equal to one if a specific bidder has won an auction for any music CD on eBay during the last three months. The second variable is equal to 1 if a bidder has won any auction on eBay within the last three months. The CD "experience" variable shows that, in my sample, 45% of the bidders have won a CD on eBay, while 94% of bidders have won any auction on eBay during the last three months. These variables are informative from the point of view of my theoretical model, since they indicate that the bidders are familiar with the price behavior on eBay during the auction and, thus, can use this information to improve their bidding. The last variable reported in the table is a dummy equal to 1 if the auctioned item and the bidder are located in the same region. As one can see, 62% of bidders preferred to bid on items on the same continent.

### 6.3 Treatment dataset

To construct the treatment dataset, I expanded the market by listing additional items. I used 5 different sellers' accounts and listed from 5 to 10 CDs on each one. This allowed me to double the average number of listed CDs per day in the auction. During the experiment, I sold 60 music CDs. Some of these CDs had to be re-listed because there were no bids in the auctions. The actual dates of the experiment ran from October 4, 2006 to November 10, 2006. To avoid "transition effects" in the beginning, and to take into account that I did not have enough CDs at the end to double the market, I only used the data from 20 days of the experiment. I used the data for 156 auctions with the first auction starting on October 8, 2006 and the last auction ending on October 28, 2006. In total, 70 items were located in North America (these were predominantly the items listed from my accounts), 56 in Europe, 25 in Asia, and 5 in Australia.

I collected the same variables for the auctions in the treatment dataset as in the control dataset. Table 4 presents the statistics for the auction characteristics in the treatment sample. The characteristics of the items in the treatment sample are quite similar to those in the control sample. The duration of the auctions in the treatment sample is close to 7 days. Because all the CDs which I listed were new, the percentage of new CDs is higher in the treatment dataset. There are also fewer auctions

which restrict the shipping to the country of item location, because my CDs were available to overseas bidders.

Table 7 reports the characteristics of the bidders in the treatment sample. The average experience characteristics, the dummies for CD purchases and all purchases of bidders on eBay as well as their feedback scores, are very similar in the treatment and control samples. The number of bidders in the treatment sample is smaller because the treatment sample covers a shorter period of time (20 days as compared to 31 days in the control sample). The only visible difference between the treatment and the control sample is that the fraction of bidders who are located in the same region as the item is larger in the treatment sample. This can be explained by the fact that, before my experiment, many bidders from the U.S. were bidding for items in Europe, predominantly in the U.K.

## 6.4 Analysis of early bidding

Using the data from my experiment, I analyze the correspondence between the predictions of my theoretical model and the results of the experiment. The model predicts that bidding early becomes more attractive to bidders due to the increased size of the market. To capture this effect, I constructed an early jump bid dummy variable. This dummy variable is equal to one if, during the first 80% of the duration of the auction<sup>38</sup>, there was a bid which increased the price above a certain threshold. To verify the robustness of my analysis, I set the thresholds of price increases determining the jump bid at 10%, 15%, and 20% of the final price in the auction respectively. For this dummy, I then estimated probit models across auctions using the combined dataset of treatment and control groups. While the dependent variables in these probit models are the jump bid dummies, the independent variables include the treatment effect dummy and a set of auction-specific regressors including availability of picture, duration, availability outside the country of item location, seller's feedback and feedback score, and an indicator that the seller has a store. The results of the probit regressions are presented in Table 8.

The estimates in Table 8 suggest that the treatment dummy is significant for every jump bid threshold. Therefore, the results of the experiment support the model's prediction that an increase in supply leads to increased early bidding. This effect is quite significant in absolute terms: the probability of early bidding in auctions with doubled supply is 11% higher than in auctions without this treatment.

According to Table 8, early bidding is affected by the characteristics of the auctions. Specifically, one can see that including a picture of the CD increases early bidding. On the other hand, longer auctions tend to have early jump bids less frequently than short auctions. The increase in the shipping cost decreases overall bidding and, in this way, decreases early bidding as well. Finally, one can see that early bidding is more likely in auctions where the seller has a store. This last result coincides

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<sup>38</sup>I also used other thresholds such as 30% and 50% of the duration but the results were quite similar

with my interpretation of the visibility of the auction. Specifically, if eBay stores have many cross-listed items, then auctions via eBay stores are more "visible". As a result, we should expect that the instantaneous demand in these auctions to be higher and more price-elastic, meaning that there is a greater incentive to bid early. This is confirmed by the data from my field experiment.

The information about individual bidders, specifically the experience dummies and the feedback scores, allows us to examine the effect of bidding experience on early bidding. My model predicts that bidders have an incentive to deter entry and prevent learning. These incentives have the opposite effect on experienced bidders. The entry deterrence incentive leads bidders to bid early to discourage other bidders from entering the auction. The learning prevention incentive leads bidders to bid late to prevent learning. The relative effect of these two incentives should change during my experiment. We know that the increase in supply makes the instantaneous demand more price - elastic. In this case, for more experienced bidders, the incentive to deter entry should dominate the learning prevention incentive. As a result, we should expect an increase in early bidding by experienced bidders. To analyze the effect of increased supply on bidding, I constructed a variable equal to the number of bids that a single bidder submitted in the first 80% of the auction duration. To measure experience, I used the experience dummy variables described in Section 6.2. In addition, I also used the feedback score of the bidder, which is approximately equal to the total number of transactions the bidder has had on eBay. I ran OLS regressions of the number of early bids on the experience indicators and interactions between the treatment dummy and the experience indicator (controlling for the bidder location). For a given market size, more experienced bidders bid less frequently than less experienced bidders. My model predicts that while all bidders tend to bid more frequently when the market size increases, the largest increase should occur for more experienced bidders.

Table 9<sup>39</sup> contains the coefficient estimates for the models with different bidding experience indicators. In all three cases, I find a positive and significant coefficient on the interaction term, confirming the predictions of the theoretical model. Moreover, we can see that for the bidding experience indicators, the coefficient is negative, confirming that more experienced bidders indeed submit early bids less frequently due to the dominance of the learning prevention incentive. Analysis of these models with additional explanatory variables confirms that the results are robust with respect to changes in the model specification.

## 7 Conclusion

While it is not uncommon to use standard second-price auction models to study Internet auctions, the co-existence of multiple simultaneous auctions for similar items together with the information heterogeneity inherent in Internet auctions significantly affects bidding behavior. In this paper, I show

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<sup>39</sup>Feedback score variable was normalized by 10,000 to obtain reasonable coefficient values

that in such an environment two types of aggressive behavior by bidders can occur: entry deterrence and learning prevention. To deter rival entry, bidders submit early aggressive bids to raise the price in the auction, thereby diverting the entry of rivals to alternative similar items. To prevent their rivals from learning the true "visibility" of the auction, informed bidders will bid late. The observed bidding behavior arises from the interplay between these two forces.

In this paper, I develop a methodology for modeling auctions in continuous-time, allowing for both endogeneity and uncertainty in the entry process, two central characteristics of on-line auctions. I estimate the model using data from auctions for pop-music CDs, validating my theoretical model with empirical data. As an independent test of the model, I conduct a field experiment on eBay. The experiment tests the prediction of my model that, in thicker markets (with more items per bidder), entry deterrence should be more frequent. I increased the number of listed items in the market for a particular music CD and found a significant increase in early bidding. This result both validates my dynamic model of endogenous entry and uncertainty, and also distinguishes it from alternative models that lack similar predictions.

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## Appendix

Table 1: Summary statistics for auctions for Madonna's CDs

<i>Variable</i>	<i>N. obs</i>	<i>Mean</i>	<i>Std. deviation</i>	<i>Min</i>	<i>Max</i>
<i>Auctions</i>					
Buy-it-now price	249	14.7676	46.7879	0.01	577
Starting bid	1132	5.0984	9.2041	0.01	175
Condition	1281	0.5097	0.5001	0	1
Picture	1281	0.9633	0.1880	0	1
Duration	1281	6.5011	1.3270	1	10
Store	1281	0.3575	0.4794	0	1
% seller's positive feedback	1281	98.6860	8.6711	10	100
Seller's feedback score	1281	9131.411	34726.98	0	325614
<i>Bidding</i>					
time	1591	0.7775	0.3010	0.0015	1
bid	1591	13.8287	28.3903	0.01	699

Table 2: Results of regression analysis.

<i>Dependent variable</i>	<i>price jump</i>	<i>bids per time</i>	<i>number of bids</i>	<i>number of bidders</i>
<i>Regressors</i>	(1)	(2)	(3)	(4)
Time	-16.141 <b>(-6.43)**</b>	-	-	-
Bid	1.152 <b>(38.36)**</b>	-	-	-
Starting price	-	-0.091 <b>(-2.17)**</b>	-0.004 <b>(-2.11)*</b>	-0.005 <b>-1.86</b>
Confessions CD	-	10.27 <b>(3.10)**</b>	0.513 <b>(2.80)**</b>	1.075 <b>(2.45)*</b>
Confessions Tour CD+DVD	-	32.263 <b>(3.22)**</b>	1.197 <b>(3.37)**</b>	1.642 <b>(3.07)**</b>
Duration	-	-6.224 <b>(-4.15)**</b>	-0.003 <b>-0.09</b>	-0.033 <b>-0.54</b>
Picture	-	2.644 <b>0.92</b>	0.154 <b>0.87</b>	0.158 <b>0.41</b>
Shipping cost	-	0.625 <b>-1.1</b>	0.008 <b>-0.31</b>	0.096 <b>-1.6</b>
% seller's positive feedback	-	-0.035 <b>-0.34</b>	-0.003 <b>-0.54</b>	-0.002 <b>-0.19</b>
Seller's feedback score	-	1.232 <b>(5.17)**</b>	0.072 <b>(6.10)**</b>	0.118 <b>(4.27)**</b>
Constant	-0.752 <b>-0.37</b>	50.528 <b>(3.52)**</b>	0.866 <b>-1.35</b>	0.878 <b>-0.81</b>
Observations	1591	1280	1280	1280
F-statistic	737.6	28.78	7.64	4.75

t-statistics are given in the parenthesis. Coefficients for which standard errors are marked by \*\* are significant at 1%

confidence level marked by \* are significant at 5% confidence level.

Table 3: Response to changes in beliefs and the instantaneous demand

Changes in average variance of beliefs $E[\sigma_\theta]$				
<i>Factor</i>	0.5	1.0	1.5	2.0
<i>Fraction of early bids</i>	2.1% (0.8)	3.4% (1.1)	7.5% (3.2)	9.4% (7.0)
<i>Average ratio of bid and the final price</i>	12.0% (6.0)	12.2% (5.2)	7.9% (3.3)	10.0% (5.5)
Changes in the coefficient $\alpha_\lambda$				
<i>Factor</i>	0.5	1.0	1.5	2.0
<i>Fraction of early bids</i>	2.2% (1.1)	3.4% (1.5)	6.0% (2.3)	10.2% (1.1)
<i>Average ratio of bid and the final price</i>	10.0% (3.9)	12.2% (5.2)	13.8% (7.4)	16.7% (9.0)
Changes in the coefficient $a_1$				
<i>Factor</i>	0.5	1.0	1.5	2.0
<i>Fraction of early bids</i>	4.3% (1.8)	3.4% (1.1)	7.0% (5.1)	10.0% (7.2)
<i>Average ratio of bid and the final price</i>	8.8% (5.0)	12.2% (5.2)	16.0% (6.5)	15.7% (8.0)

Standard errors are in the parentheses

Table 4: Characteristics of the auctions for "Greatest hits" CDs in the control group

variable	N obs.	mean	stdev	min	max
Buy it now price (US \$)	80	14.97	17.60	1.89	63.72
Picture dummy	136	0.8823529	0.3233808	0	1
Duration (days)	136	7.632353	1.919911	3	10
Shipping cost (\$)	136	7.269701	6.490984	0	37.2
Seller's feedback score	136	13414.46	22421.4	0	164546
Store dummy	136	0.6029412	0.4910972	0	1
Condition (new=1)	136	0.6911765	0.4637162	0	1
Number of bids	136	1.875	3.009707	0	10
Number of bidders	136	1.279412	1.95388	0	8
Average bid	53	3.984911	3.505953	0.01	25.04667

Table 5: Characteristics of the bidders for "Greatest hits" CDs in the control group

variable	N obs.	mean	stdev	min	max
Feedback score	230	209.7217	557.9766	0	4611
% positive feedback	230	99.63739	1.183508	92.6	100
Bought CD last 3 months	230	0.4565217	0.4991924	0	1
Bought from eBay	230	0.9130435	0.2823859	0	1
Europe dummy	230	0.8521739	0.3557016	0	1
Australia dummy	230	0.0652174	0.2474476	0	1

Table 6: Characteristics of the auctions for "Greatest hits" CDs in the treatment group

variable	N obs.	mean	stdev	min	max
Buy it now price (US \$)	45	7.98	7.33	2.98	34.37
Picture dummy	156	0.8205128	0.3849957	0	1
Duration (days)	156	7.685897	2.23406	3	10
Shipping cost (\$)	156	4.924754	2.894402	1.59	18.1
Seller's feedback score	156	34448.39	65604.64	0	176262
Store dummy	156	0.4615385	0.5001241	0	1
Condition (new=1)	156	0.9102564	0.2867346	0	1
Number of bids	156	0.6858974	1.751698	0	12
Number of bidders	156	0.5192308	1.127207	0	7
Average bid	46	3.393384	3.278704	0.98	14.99

Table 7: Characteristics of the bidders for "The Greatest hits" CDs in the treatment group

variable	N obs.	mean	stdev	min	max
Feedback score	102	276.4412	716.481	0	4795
% positive feedback	102	99.64902	1.370751	87	100
Bought CD last 3 months	102	0.4313725	0.4977137	0	1
Bought from eBay	102	0.9117647	0.2850375	0	1
Europe dummy	102	0.6568627	0.4771014	0	1
Australia dummy	102	0.0980392	0.2988362	0	1

Table 8: Effect of market expansion on early bidding: dependent variable - early bidding dummy

<i>Variable</i>	Threshold		
	10%	15%	20%
Treatment dummy	0.6379 ( <b>0.2776</b> )**	0.8134 ( <b>0.2980</b> )***	0.7091 ( <b>0.2835</b> )**
Condition	-0.9041 ( <b>0.2960</b> )***	-0.9609 ( <b>0.3053</b> )***	-0.7330 ( <b>0.3040</b> )**
Duration	-0.0914 ( <b>0.0545</b> )	-0.0895 ( <b>0.0541</b> )*	-0.0840 ( <b>0.0556</b> )
Shipping cost	0.0024 ( <b>0.0221</b> )	0.0141 ( <b>0.0195</b> )	-0.0015 ( <b>0.0247</b> )
Seller's feedback score	-0.0053 ( <b>0.0260</b> )	-0.0071 ( <b>0.0257</b> )	-0.0098 ( <b>0.0259</b> )***
Store dummy	-0.5876 ( <b>0.2526</b> )**	-0.5088 ( <b>0.2533</b> )**	-0.3891 ( <b>0.2486</b> )
Constant	-0.0011 ( <b>0.4715</b> )	-0.2262 ( <b>0.4588</b> )	-0.3783 ( <b>0.4677</b> )***
N. obs.	292	292	292
pseudo $-R^2$	0.1530	0.1637	0.119

Table 9: Multiple bidding as a function of experience

<i>Variable</i>	Model		
	(1)	(2)	(3)
Treatment	.0968 <b>(.0704)</b>	.0306 <b>(.0980)</b>	.0298 <b>(.0528)</b>
Bought CD last 3 months	-.0421 <b>(.0218)*</b>	-	-
Bought CD $\times$ Treatment	.1218 <b>(.0430)***</b>	-	-
Bought from eBay	-	-.0870 <b>(.0411)**</b>	-
Bought from eBay $\times$ Treatment	-	0.1351 <b>(.0642)*</b>	-
Feedback score	-	-	-.4969 <b>(.1577)**</b>
Feedback score $\times$ Treatment	-	-	.4702 <b>(.2556)*</b>
Europe dummy	-.2937 <b>(.0533)***</b>	-.3042 <b>(.0521)***</b>	-.3013 <b>(.0528)***</b>
Australia dummy	-.2336 <b>(.0691)***</b>	-.2497 <b>(.0670)</b>	-.2578 <b>(.0662)</b>
Constant	.6143 <b>(.0551)***</b>	.6845 <b>(.0640)***</b>	.6114 <b>(.0537)***</b>
N. obs.	332	332	332
<b>R</b> <sup>2</sup>	.2361	.2359	.2580