The Welfare Effects of Complementary Bidding Mechanisms

An Empirical Analysis of the Spanish Wholesale Electricity Market

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Abstract

In many multi-unit auctions, bidders' valuations for a particular good depend on whether or not they also win the auction for another good. In these environments, the auctioneer can allow bidders to reflect these complementarities through additional bidding mechanisms. This paper studies the welfare effects of one such mechanism: "complex bids," a complementary bidding procedure used in wholesale electricity auctions that allows companies to specify a minimum revenue threshold for the day. Complex bids allow generating companies to explicitly link their valuations across different hours of the day, which would not be possible in an otherwise uniform price auction. The welfare implications of introducing such bidding procedures are ambiguous. Allowing for greater flexibility has the potential of improving the efficiency in the market, but also gives bidders an additional dimension through which they may exert market power. I develop a model of complex bidding and estimate its structural parameters in the context of the Spanish electricity market. I then perform a counterfactual analysis in which the original mechanism is compared to one in which complex bids are not allowed. I find that, while firms do exercise market power through complex bids, the positive coordination benefits of complex bidding dominate. The distributional implications of removing complex bids are particularly important. I find that, in the absence of complex bids, increased volatility due to less coordination could increase prices by 7.90%, translating into an increase of over 550M€ of annual payments by consumers.

Keywords: Electricity markets, complex bidding, auction with complementarities.
JEL Classification: L13, L94, D44.

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1 Introduction

Auctions are used to allocate goods in many markets. Among the most commonly studied auction settings, there are first and second price auctions for single goods, and discriminatory and uniform price auctions for multiple goods. However, there are many situations in which the auctioning process departs from these simple rules. In the presence of valuation complementarities, less common rules of bidding are usually introduced. These rules are often tightly related to more common rules: they are an "augmented" auction in which complementary bidding procedures are coupled with more traditional designs.

One of the most studied auctions in which complementarities can be expressed are combinatorial auctions, in which bidders can express their willingness to buy combinations of goods.[1] A well known example of combinatorial auctions is the case of package auctions in Federal Communications Commission (FCC) spectrum auctions (see, for instance, Cramton (2002)). The introduction of combinatorial bidding is motivated by the fact that, in the presence of complementarities across goods, the joint valuation of two or more goods can substantially differ from their individual values.

The welfare implications of augmented bidding procedures are many times theoretically ambiguous in a second best environment.[2] On the one hand, these mechanisms allow bidders to better reflect their valuations, which can increase the efficiency of the auction by improving the allocation of the auctioned goods. On the other hand, it gives bidders another dimension to exert market power, which can lead to a reduction of total welfare. The goal of this paper is to assess this trade-off in the context of a specific form of complementary bidding procedure that is used in wholesale electricity auctions.

Many wholesale electricity markets use complementary bidding procedures in their daily auctions to allow firms to reflect their cost complementarities over time. Some examples are the Pennsylvania-New Jersey-Maryland (PJM) market, the Californian market, the Irish market and the Spanish market. The particularities of the mechanism in each of these markets can vary, but they all have the common denominator of allowing firms to reflect their startup costs in a way that ensures that these costs are potentially recovered in the market. Due to the richness of the bidding data, these markets provide a unique environment in which to analyze the bidding behavior of the firms, measure complementarities and test the effects of these complementary bidding mechanisms on welfare.

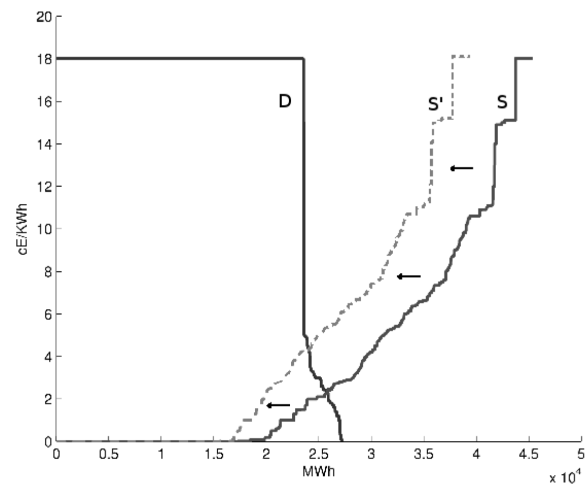
In this paper, I study the benefits and costs of introducing one such mechanism in the Spanish wholesale electricity market. The complementary mechanism used in the Spanish electricity market takes the form of an augmented set of uniform price auctions.[3] For every hour of the day, firms submit offers to produce electricity with step bids as they would do in a uniform price auction, which are called *simple bids*. However, each generator can also express a daily revenue requirement

[1] For a comprehensive treatment, see Cramton et al. (2006).

[2] Cantillon and Pesendorfer (2007) document this ambiguity for the case of the procurement auctions for bus rotes in the United Kingdom, in which bidders are allowed to express a joint preference to serve two or more routes.

[3] In a uniform price auction, the auctioneer crosses demand and supply. The market price is determined by the intersection of the two. All supply units with prices lower or equal to the market price are scheduled to produce.

Figure 1.1: A Uniform Auction in which Offers are Discarded



The augmented bidding procedure used in the Spanish electricity market discards those units for which the minimum revenue requirement is not satisfied, shifting the supply curve to the left (S to S').

on top of these bids, which constitutes its *complex bid*. This minimum revenue requirement makes the simple bids contingent: if the daily gross revenue of a generator is not at least as large as its minimum requirement, its hourly simple bids are taken out from the auction and the generator is not assigned any quantity in the auction. Because the revenue requirement applies to the whole day, this gives firms a mechanism to express their preference regarding joint realizations of demand over the day.

Figure 1.1 gives an intuition for the effects of complex bids on the supply curve at the market for a given hour of the day. The figure plots both demand and supply at the market. The solid supply curve represents the original simple offers made by the firms. However, at the original hourly prices defined by the crossing of the two solid lines, several units do not recover their revenue requirement. These units are taken out iteratively from the aggregate supply curve, shifting the supply curve inwards, until the price is such that the market clears and all minimum revenue requirements are satisfied. Intuitively, this mechanism allows the players to withdraw their capacity from the market if demand realizations are too low, which allows them to bid more flexibly, offering their capacity and letting the mechanism determine whether it is optimal or not to startup a unit. However, if firms have market power, it can allow them to tailor the startup decisions more strategically, taking out their units in situations in which *(i)* it would have been socially optimal to supply them, and *(ii)* they would have not been taken out if complex bids were not allowed.

From a theoretical perspective, the welfare effects of the complex bids are ambiguous (Reguant, 2010). In practice, the potential exercise of market power by means of gaming complex bidding rules has been an important issue of debate. For example, in the initial design of the British electricity market, firms were allowed to submit complex bids that would let them represent their preferences more accurately. However, firms learned to use them to significantly raise prices, which led to the modification of the design towards simpler rules (OFFER, 1999). In New England, during the discussion of the de-regulated electricity market design, scholars were concerned that complex bids would give a wedge to the firms to exercise their market power (Cramton and Wilson, 1998).

In order to empirically quantify the welfare implications of minimum revenue requirements in the Spanish electricity market, I develop a multi-unit auction model in which bidders submit both simple and complex bids. In the model, I take into account the short-run dynamics that are present in the production function as well as the non-convex nature of their costs, which are essential to understanding the role and effects of complex bidding. Then, I estimate the parameters of the model using the first order conditions implied by firm profit-maximizing behavior. By parameterizing the cost function and forward position as well as by exploiting the information contained in complex bids, I am able to estimate both costs and forward contracts jointly. I show that the presence of complex bids helps to identify the startup costs of the firms. Finally, once the fundamentals of the model are obtained, I conduct policy counterfactuals to understand the welfare implications of complex bids in terms of productive efficiency and market power.[4] In particular, I compare the performance of the market with the complex bidding mechanism to the case in which complex bids are not allowed.

The results suggest that the complex bidding mechanism performs better than the simple bidding mechanism, even in the presence of market power. Even though firms could potentially withhold more capacity with complex bids, the mechanism has beneficial coordination effects, effectively increasing the overall elasticity of the supply curve. I find evidence that, in the absence of complex bids, the presence of a more inelastic and uncertain residual demand can increase the incentives of a strategic firm to raise market prices, with an overall negative welfare effect. The distributional implications of removing complex bids are particularly large. I find that, in the absence of complex bids, increased volatility due to less coordination could increase prices by 7.90%, translating into an increase of over 550M€ of annual payments by consumers. Therefore, the results suggest that augmenting the auction with this complementary bidding procedure has a net positive economic effect in this market.

The major contributions of the paper are twofold. First, I extend the estimation of multi-unit auctions by adapting current techniques to a non-standard setting in which players can use augmented forms of bidding. I show that the presence of these complementary bids can help to identify the cost complementarities present in the production function of the firms. Second, with the estimated fundamentals, I construct counterfactuals that contribute to the study of market power in electricity markets in the presence of non-convex costs and non-standard rules of bidding. These mechanisms are common in several electricity markets, but there have not been any attempts to model and estimate their welfare implications.

The rest of the paper is organized as follows. In section 2, I review the literature. In section

[4] The concept of efficiency in an auction setting is usually interpreted as allocative efficiency. In the context of the electricity auctions studied in this paper, this term can be interpreted as productive efficiency. Given the traditionally low elasticity of demand, in the short-run the objective is to minimize the costs of producing a certain demand of electricity.

3, I explain the institutional features of the Spanish electricity market and the data and present a descriptive analysis of the usage of complex bids. Section 4 develops a multi-unit auction model with complex bidding and derives optimality conditions. In section 5 and 6, the estimation strategy and results are presented. In section 7, I detail the counterfactual strategy and perform a set of simulations using the estimated fundamentals. In section 8, I conclude and suggest future lines of research.

2 Related literature

This paper is mostly related to two primary streams of research: the empirical auctions literature, and the work analyzing the exercise of market power in wholesale electricity markets.

Regarding the empirical auctions literature, the paper follows the methods to estimate valuations from the underlying bidding data using the implied optimality conditions, as in Guerre, Perringue and Vuong (2000). It is particularly related to multi-unit auctions studies, such as McAdams (2008), Hortaçsu (2002) and Gans and Wolak (2008), adapting them to the particularities of the complex bidding mechanism.

This paper is also related to the empirical literature on auctions with complementarities. The paper is closely related to the work by Cantillon and Pesendorfer (2007), who also explore the existing tradeoff between efficiency and market power in the context of procurement auctions of bus routes in the United Kingdom. They study an augmented set of first-price auctions, in which players can express joint valuations for the goods in each of the simultaneous auctions. While the auction setting and complementary mechanism that they study is different from the one explored in this paper, they also find that the welfare effects of the mechanism are ambiguous and explore this tradeoff.

A number of previous studies use bidding data to model the strategic behavior of firms in liberalized wholesale electricity market. The model and first-order conditions that I derive are closely related to Hortaçsu and Puller (2008) and Allcott (2009), but adapted to the presence of complex bids. This paper is particularly related to the work by Wolak (2003), who develops an estimator based on best response bidding when bidding data are available, with applications to the study of hedge contracts (Wolak, 2000) and dynamic costs (Wolak, 2007). The estimator and the cost structure that I use are similar to the ones used in Wolak (2007), to which I incorporate startup costs of operation.

Startup costs of operation are an important determinant of electricity costs, but to date they have not been considered in most empirical studies of electricity markets with strategic agents. A few papers have incorporated them in the study of competitive markets. Mansur (2008) studies the effect of dynamic costs of operation. Also in a competitive market, Fowlie (2010) takes into account startup decisions to assess the effects of pollution permits in operational decisions. Cullen (2010b) uses a similar approach to estimate the effects of a CO_2 price in the Texas electricity market.

Cullen (2010a) develops a structural model to investigate the effects of such a policy in a competitive infinite horizon dynamic model. Also in an infinite horizon context, Fershtman and Pakes (2009) develop the concept of Applied Markov Perfect Equilibrium, to account for strategic behavior in the presence of unobserved private information that is correlated over time. They present an application to the electricity market in which firms make strategic maintenance decisions. In contrast to these latter approaches, I use a simplified finite horizon model to capture the dynamic decisions of strategic agents. Even though this is a limitation, it allows me to model very closely the actual auction mechanism in the market, which is the main focus of the paper.[5]

This is the first paper that assesses the effects of complex bids empirically. There are several theoretical papers in the engineering literature that study the optimal design of complex bids in electricity markets. These papers highlight the fact that in the presence of non-convexities, a competitive market equilibrium without complex bids might not exist, as simple bids are not necessarily incentive compatible. These papers address the question of which mechanisms succeed in implementing the first best with competitive players (O'Neill et al., 2005; Gribik et al., 2007). Other papers have suggested the use of a Vickrey-Clarke-Groves (VCG) mechanism when firms have non-convex costs, so that bids are truthful even in the presence of market power (Hobbs et al., 2000). Unlike the previous literature, I take the complex bidding mechanism as given by the auction rules in the electricity market of study, and I evaluate empirically its welfare implications.

3 Institutions and data

The Spanish electricity market is a national market that produces between 15,000 and 45,000 MWh hourly, with around 85,000 MW of installed capacity, serving more than 40 million people.[6] The Spanish territory is interconnected with France, Morocco, Portugal and Andorra. The liberalized electricity market has an annual value of 6 to 8 B€. The Spanish electricity market has been liberalized since 1998 and it shares many features with other liberalized electricity markets.

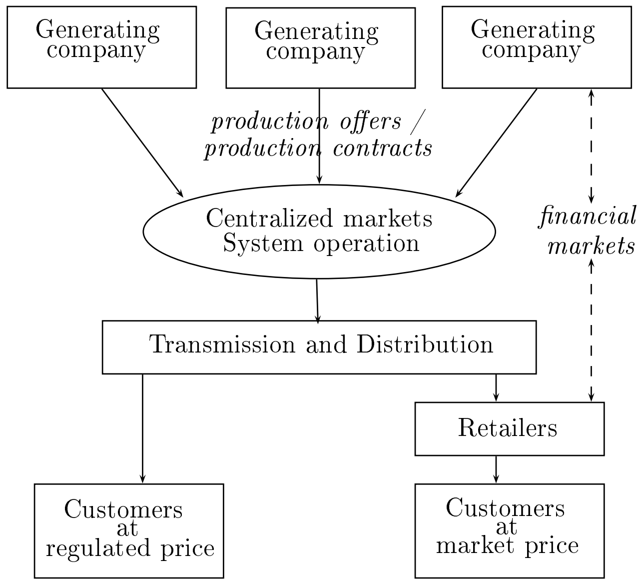
Figure 3.1 presents a schematic representation of the functioning of the electricity market. The electricity market consists of several important segments: generation, transmission, distribution and retailing. Generating firms can sell their electricity either at centralized markets or by means of production contracts. Firms can also establish financial contracts for part of their production. Independently of how the produced quantity is settled financially, all production decisions need to be centralized to ensure the functioning of the overall system. The electricity is then delivered to final consumers by distributors and retailers.

In Spain, most firms are vertically integrated, owning assets in both generation and retailing. Incumbent firms, prior to the liberalization of the market, also provide distribution services, which are fully regulated under incentive regulation. During the period of study, most of the energy is provided to consumers by regulated distributors, and not by retailers. In this paper, I focus on

[5] Modeling the bidding mechanism in a parsimonious way would not be computationally tractable in an infinite horizon game with strategic behavior.

[6] Compared to liberalized electricity markets in the United States, the Spanish electricity market has a size comparable to the Californian electricity market.

Figure 3.1: Market Structure



the generation side, although I account for the retailing share of each firm when performing the analysis.

The technology mix of the generating companies includes several technologies and has changed over the recent years, with the introduction of combined cycle gas plants and renewable energy sources, specially wind resources. During the sample of study, coal was the predominant source of energy (25%), followed by nuclear, natural gas and renewable and special energies, each of them with a share of approximately 20%. Hydraulic energy accounted for approximately 10% of the production. Due to limited cross-border transmission capacity, international imports represent a very small fraction of total production, being only 3%. In the paper, I model the decisions of thermal generators other than nuclear, which account for roughly 45% of the production during the period of study. These are the type of units that use the complementary bidding procedure studied in this paper.[7]

3.1 The decisions of the generating firms

The generating companies in the market make two main decisions. First, they decide their financial position, usually weeks or months in advance. Financial contracts, also known as hedge contracts or contracts for differences, are firm-specific and imply that a certain amount of produced electricity is hedged and, therefore, not subject to the market price. These contracts avoid the risk implied by uncertain prices. This paper does not endogenize the decisions regarding financial

[7]Wind resources do not have startup costs and therefore do not use complex bids. Nuclear plants are usually operating at full capacity whenever not unavailable for maintenance and thus make startup decisions ex-ante. Hydraulic plants do not use complex bids either, as they can operate flexibly.

contracts. However, I take into account their presence, incorporating them in the profits of the firm and estimating them in the empirical section.

Second, firms make decisions on how to operate their plants. The decisions are whether to have a plant running or not, and, conditional on running, how much to produce. In order to take these decisions, firms decide whether to use production offers in the centralized market or production contracts that are arranged ex-ante. Production contracts account for about one third of the electricity produced in Spain. They are linked to a particular production unit and specify that a certain amount is planned to be produced by that given unit. On the contrary, the production offers in the market do not establish a particular amount to be produced. They establish a willingness to produce at different prices, and the final outcomes are resolved in the daily auctions. The focus of this paper is on the use of the auction mechanism as a way to decide production, taking production contracts as given from the data.

3.2 The day-ahead market

I study the most important auction of the centralized markets: the day-ahead market.[8] Firms in the day-ahead market sell electricity to be supplied during the next day. Therefore, the day-ahead market clears essentially homogeneous multi-unit goods for each hour (MWh at hour h), totalling 24 different commodities. Firms submit their bidding strategies for the next day all at once and the 24 commodities are auctioned simultaneously. Roughly 80% of the electricity allocated in centralized markets is sold through this day-ahead market. Financial "hedging" contracts are also often indexed at this price, and therefore, the day-ahead market sets a reference for a large amount of the electricity traded in the electricity market.

Even though I do not endogeneize other contractual forms in the model, I control for the presence of contracts that are not decided in the day-ahead market. Production contracts are observed in the data, given that production contracts need to be communicated to coordinate the operation of the electric system. Regarding financial contracts, they have been found a crucial factor in determining the optimal bids of the agents accurately (Wolak, 2000; Bushnell et al., 2008). Even though they are not observed, I estimate them from the data in the empirical analysis, as in Wolak (2003), Hortaçsu and Puller (2008) and Allcott (2009).

[8] Focusing on the day-ahead market in order to understand firm strategic behavior is common in the literature, see for example Kühn and Machado (2004) for the Spanish electricity market and Borenstein et al. (2002) for the Californian electricity market. There are other centralized markets that are potentially important: the market dealing with congestion in the network (restrictions market) and the congestion market and the sequential intra-day markets. Both the congestion market and the sequential markets open after the day-ahead market has cleared. The effects of these two markets can be substantial, given that firms can potentially adjust their output patterns after making their day-ahead decisions. Whereas adjustments in the intra-day markets tend to be small, more substantial changes arise in congested areas. Generators in congested areas can receive congestion rents if they enter through congestion and therefore do not respond to the same incentives. The study of generators with local market power is a topic that I am currently studying.

3.3 Bidding in the market

Generating firms in the day ahead market bid simultaneously for the 24 hours of the next day to sell electricity.[9] Firms submit bids that are associated to each production unit.[10] Each unit can have both simple and complex bids. Simple bids are step functions that offer a quantity electricity to be produced (MWh) at a certain price for a particular hour of the day. Each hourly step function of each unit in the market can have up to 25 different steps. The price offers need to be positive (or zero) and are capped at 180 €/MWh. Furthermore, the price offers need to be monotonically increasing. Each generating unit has its own bid, which implies that the aggregate supply curve of a given firm can have potentially many steps. For example, large companies such as Iberdrola or Endesa can submit aggregate supply functions that have more than 500 steps. However, in practice, agents do not use all the 25 steps for each unit; generally the bids have no more than 5 to 10 steps.

Only thermal generators can use complex bids. Complex bids complement simple bids and are unique for the whole day. Any unit submitting a complex bid for the whole day still has a simple bid associated to each of the 24 hours. Firms can specify a unit-specific minimum revenue requirement characterized by two bidding parameters: a variable and a fixed component.[11]

The minimum revenue requirement for a given unit j of firm i takes the following form,

$$R_{ij} = A_{ij} + B_{ij}Q_{ij},$$

where R_{ij} represents the implicit revenue requirement, A_{ij} and B_{ij} are the complex bids for unit j, and Q_{ij} represents the total daily quantity associated to that unit in equilibrium, which is endogenously determined. A_{ij} can be interpreted as a bid representing a fixed cost component of the minimum revenue requirement and B_{ij} can be interpreted as a bid representing a marginal cost component.[12]

A given unit is guaranteed not to produce if the revenue obtained by the unit during the whole day is lower than R_{ij}. If the revenue the unit would obtain in the day-ahead market over the whole day with a particular generator is not at least as large as its minimum requirement, the unit is not scheduled to produce for the entire day, even if its simple bids are lower than the market marginal price. Therefore, it is a mechanism by which companies trade off the startup and shut

[9] In the auction, there are also demand bids. Given that the model is focused on the supply side, for clarity purposes I abstract from discussing them in the main text. Note that demand bids do not involve any complementary bidding mechanism, they are only composed by simple bids.

[10] In the case of thermal generation, a production unit is a generator, which in turn might belong to a group of units that constitutes a plant. For renewable sources, units are often an "aggregator," which pools together resources at different locations.

[11] Firms can also submit unit-specific ramping constraints, that respond to the technical constraints of operating a unit, although they do not make use of them very frequently. Only around 6% of the units use ramping constraints. The firm that I study does not use them for any unit during the sample of study. In the empirical analysis, I account for the ramping constraints submitted by the few firms that use them. However, I do not consider the strategic effects of submitting the ramps, given the lack of their usage in revealed strategies.

[12] In fact, in a simplified environment it is ex-post optimal for a competitive firm to set A_{ij} equal to its startup cost and B_{ij} equal to its marginal cost.

down decisions of production units.[13]

When solving for the auction outcome, the market operator uses complex bids as constraints to the simple bids in an iterative fashion. The market clearing outcome is solved as follows. First, optimal quantities and prices are found based on simple bids, crossing demand and supply for each hour of the day independently. Then, the market operator checks that the minimum revenue requirement is satisfied for all units, by comparing their gross revenue with the specified complex bid. If the requirements of some units are not satisfied, they are withdrawn sequentially depending on the magnitude of the violations. The procedure is repeated iteratively until none of the complex bids bind.[14] For a more detailed description of the algorithm, see Appendix A.

3.4 Bidding Data

I construct a new data set from publicly available data from the market and the system operator in Spain (OMEL and REE, respectively).[15] The central piece of the data set are the bidding data from the day-ahead market, which are fully observed and can be mapped to the generating units in the market. I map these generating units to additional data sets that contain characteristics such as type of fuel used, thermal rates, age, location, emissions, etc. These data are coupled with results of the auction outcomes, such as equilibrium prices and assigned quantities. A more detailed explanation of the data sources can be found in Appendix B.

In my empirical analysis, I use data from March 2007 until June 2007. The reason to look at this sample is to ensure that the regulatory benchmark is constant during the period of study. Even though the design of complex bids has not changed since the start of the electricity market, other institutional details that might affect bidding strategies have been changing over time.[16] For this reason, I look at a window of time over which the overall market structure remains constant. Taking into account regulatory changes, it would be possible to extend the analysis to other periods in the data.

In the sample, there are 88 traditional thermal units other than nuclear power plants, which account for most of the thermal units in the Spanish system that are operating during this period.[17] I divide the units in three main categories: coal plants, combined cycle gas plants and traditional gas and oil (peaking) plants.[18]

[13] Firms can also choose their startup decisions by other means, such as with production contracts or extreme simple bids. A more detailed discussion is provided below when discussing the data.

[14] Note that this iterative procedure needs not to be the optimal way to solve the market clearing problem. It also does not guarantee a unique possible solution, which raises a winner's determination problem. It was chosen due to its simplicity and computational tractability when the market was originally conceived. Other liberalized markets use alternative algorithms that compute the market clearing in one step, which have been enabled by advances in the available computational algorithms.

[15] The available data for the Spanish electricity market is more comprehensive than for many other countries, and its transparency and availability has been acknowledged at the European level. According to "DG Competition Report on Energy Sector Inquiry (January 2007)," as reported by www.energiaysociedad.com.

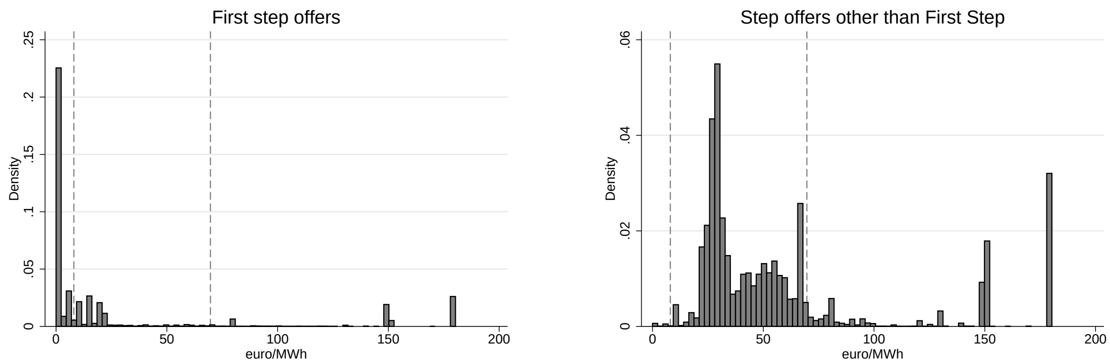
[16] The regulator introduced some sudden changes in the regulatory framework in March 2006, with the approval of the Royal Decree 03/2006, which affected bidding strategies until February 2007. In July 2007, the Spanish electricity market joined the Portuguese market to form the MIBEL market.

[17] Nuclear plants, co-generation plants and new plants that are not online during this period are excluded.

[18] Peaking plants are units that are very expensive to run. They are known as peaking plants because they are used

Figure 3.2: Identification of Operational Parameters

(a) Distribution of Simple Bids for First Step. **(b)** Distribution of Simple Bids for Other Steps.



The distribution of first-step bids appears to be very different from the distribution of "marginal steps." Dashed lines represent minimum and maximum price observed in the whole sample. Firms submit either very low or very high first step bids. Sample from March to June 2007.

3.5 Bidding behavior

Before turning to the model, in which the bidding behavior of firms is more formally discussed, I explore the bidding data and analyze some of the patterns that arise. The main goal of the discussion is to understand how simple and complex bids translate into *(i)* discrete decisions about using a thermal plant or not (startup decisions), and *(ii)* marginal decisions about how much to produce with a given plant.

3.5.1 How do firms use simple bids?

The usual interpretation of simple bids in a multi-unit auction is that they express a marginal willingness to produce. In this sense, they express how much output a firm is willing to produce at different price levels in a given hour. However, if firms have startup costs, simple bids need not to be marginal. In particular, the first step is important to determine whether a unit will run or not at all in a particular hour. Note that conditional on winning the first step, the other steps of simple bids are marginal. Given that the steps need to be monotonically increasing, the unit is already turned on conditional on the first step being accepted.

This fact greatly affects the distribution of bids in the data. Figure 3.2 show the distribution of simple bids for the first step, separated from the rest of the bids. As it can be seen, the distribution of first step bids is polarized in very low and very high bids. On the contrary, steps other than the first one have a more centered distribution around prices that are actually observed in the data.[19]

The figure shows that the first step is a crucial element used to choose running patterns over the day. When using a complex bid, firms submit a zero bid for the first step in most of the

very infrequently and usually for very short intervals, only when demand is at its peak.

[19] Actually, there are some very high prices, that effectively ensure that the unit will not produce at those capacity ranges. One of the reasons for those high bids is the fact that there are ancillary and regulation markets in which those plants can provide other services. I abstract from these other motivations in the model.

hours of the day, which ensures that, conditional on the unit being accepted, it will operate over a continuous period of time. Note that thanks to complex bids they can submit a zero price offer even for very expensive plants, as the complex bid will ensure that those bids are discarded if market prices are too low.[20] This is also true for units with neither complex bids nor production contracts, which have an even more polar distribution of the first step of simple bids (see Figure F.1 in the Appendix). Finally, firms can use production contracts to decide their startup patterns. When firms have production contracts, they cover usually most of the hours of the day, ensuring a smooth schedule. For these units, the first step is not relevant anymore for deciding whether to be on or off and therefore it affects marginal decisions only.

The patterns in the simple bidding data make salient the need to introduce both complex bids and startup costs when analyzing the bidding behavior of the firms in the market. Otherwise, the patterns in the first step might look irrational in a paradigm in which firms only have marginal costs and there are no complex bids.

3.5.2 How do firms use complex bids?

Complex bids are concerned with decisions about using a thermal plant or not, as they only determine whether a unit participates in the market or not. They allow a firm to use its simple bids more flexibly, as they make simple bids contingent to the overall daily revenue being high enough. Complex bids allow the firm to make startup decisions contingent on the daily market prices.

Thermal units make frequent use of complex bids. However, as explained above there are alternatives to complex bids in order to decide whether to run a plant or not. In these data, most firms use either complex bids (66.1%) or production contracts (23.2%), but not both.[21] The exclusivity between the two is intuitive, as firms with production contracts have committed the output of those units by some other contractual arrangement and they do not plan to withdraw them from the market. Units use neither complex bids nor production contracts in 10.7% of the days in the sample. In such case, in order to determine which hours they are running, they use the first step of their bid (see subsection 3.5.1 above).

In all generating subgroups (coal, combined cycle and peaking), units use complex bids more than forty percent of the days on average. However, the frequency is not evenly spread across types. High marginal cost coal plants, natural gas and peaking units use them much more frequently than do lower cost coal plants, which are producing continuously over the year. When units use complex bids, they are discarded in the iterative process used by the system operator over 60% of the days. Remarkably, in these data peaking plants are always rejected due to their high bids.[22] The size of the minimum revenue requirement varies depending on the type of fuel used, as one would expect.

[20] For some non-peak hours of the day, such as the early hours during the night, firms may submit a very high bid, ensuring that they will be turned off.

[21] In the data, only in 1% of the observations a unit has both a production contract and a complex bid.

[22] This is due to the fact that the peaking plants in this market are only economical under extreme market conditions in which demand is very high or there is local congestion. Except for few days during peak season (winter and summer), the units are usually not switched on at the day-ahead market, but in the congestion market.

Table 3.1: Probability of Using Complex Bid by Thermal Units

	By fuel types		Unit Fixed-effects	
	(1)	(2)	(3)	(4)
Constant	0.782	0.352	0.993†	0.5867†
	(0.082)**	(0.102)**	(0.089)	(0.072)
Combined cycle	0.413	0.410		
	(0.082)**	(0.083)**		
Peaking	0.580	0.565		
	(0.083)**	(0.085)		
Unit On Previous Day	-0.058	-0.074	-0.093	-0.115
	(0.065)	(0.066)	(0.021)**	(0.023)**
System hourly price	-0.011	0.001	-0.010	0.001
	(0.003)**	(0.003)	(0.003)**	(0.002)
Weekend Dummy		0.101		0.095
		(0.019)**		(0.018)**
Unit Fixed Effects	No	No	Yes	Yes
Observations	6,041	6,041	6041	6041

Notes: Linear regression. Significance levels at 1% ($**$) and 5% ($*$). †Average value of the unit fixed effect. Dependent variable takes value of one if a given unit submits a positive complex bid. Clustered standard errors in parenthesis, clustering at the plant level. Sample from March to June 2007. Excludes units out for maintenance or outage. Marginal price instrumented with average and maximum hourly demand forecast as given by the System Operator the day before. Input controls include European prices of coal and natural gas.

Conditional on submitting a variable component, peaking plants are the most expensive, with a 96 €/MWh average variable bid. Coal and gas average variable components are more similar to each other, around 30 and 40 €/MWh. The fixed component of peaking plants is also substantially larger than for other plants, expressing a preference for not running on a given day.

In Table 3.1, I present evidence on how firms appear to make their decisions about using complex bids. I present results from a linear probability model to assess which components affect the probability of submitting a complex bid the most, where coal units are the baseline.[23] As explained above, peaking plants and gas plants tend to use complex bids more often than do coal units, the omitted category. Coal plants, instead, use production contracts more frequently. Firms seem to rely less on complex bids when the price is higher. The effect on price seem to be largely explained due to price fluctuations during weekends. Given that complex bids are often substitutes to production contracts, this implies that units tend to have less production contracts in the weekend, when demand is lowest. The last specifications shows that, controlling for unit fixed-effects, firms are less likely to use complex bids if the units are already on. This effect is also

[23]Results using a Probit model are very similar.

Table 3.2: Levels of Complex Bid Components by Thermal Units using Complex Bids

	Fix Component (€, in thousands)		**Var. Component** (in €/MWh)	
	(1)	**(2)**	**(3)**	**(4)**
Constant	-19.502	7.12†	10.866	28.16†
	(16.352)	(3.34)	(4.731)*	(2.45)
Combined cycle	1.436		6.715	
	(0.698)		(0.258)**	
Peaking	33.802		59.085	
	(56.878)		(1.972)**	
Unit On Previous Day	-8.930	-6.400	1.490	-1.226
	(1.966)**	(0.610)**	(0.396)**	(0.245)**
Weekend Dummy	-0.560	0.074	-0.465	0.312
	(0.884)	(0.219)	(0.359)	(0.172)
System Hourly Price	0.207	0.030	0.080	0.138
	(0.186)	(0.049)	(0.061)	(0.032)**
Plate capacity (MW)	0.004			
	(0.002)*			
MW x Combined cycle	-0.005			
	(0.002)*			
MW x Peaking	0.330			
	(0.121)**			
Input controls	Yes	Yes	Yes	Yes
Unit Fixed Effects	No	Yes	No	Yes
Observations	3,348	3,348	3,348	3,348

Notes: Significance levels at 1% (∗∗) and 5% (∗). †Average value of the unit fixed effect. Clustered standard errors in parenthesis, clustering at the unit level. Sample from March to June 2007, only contains units submitting a complex bid. Excludes units out for maintenance or outage. Only includes those units submitting complex bids. Marginal price instrumented with average and maximum hourly demand forecast as given by the System Operator the day before. Input controls include European prices of coal and natural gas.

related to production contracts, which usually affect more than a single day in a row.

Table 3.2 presents correlational evidence about how firms choose the level of their complex bids. The variable and fixed component of the complex bid submitted by the units are regressed on different explanatory variables. Only those units that submit a complex bid are included, and therefore the effects are conditional on using complex bids.[24] The regression confirms the differences in levels across different technologies, with peaking units submitting much larger fixed and variable bids. In specification (1), I include the capacity of a given unit, to show that the fixed component bids are positively correlated with the size of a unit. One can also see that the status of the units the previous day affects their complex bidding behavior. In particular, if the unit is turned on, firms tend to submit a much lower fixed component, many times equal to zero, as the status of the unit the previous day determines whether the unit needs to incur its startup cost or not. The average marginal price at the market, which I instrument with the demand forecast publicly available to firms, affects the variable component of the complex bid, which would suggest firms exercising market power, although this effect is only significant once differences across units are accounted for with unit fixed-effects.

4 The model: a multi-unit auction with complex bids

4.1 The basic game

In order to formalize the decision process of the firms, I represent the electricity market as a multi-unit auction in which bidders submit step functions as well as complex bids. The model considers the bidding decisions of the firm during a given daily auction.[25] The goods auctioned in the market are electricity (MWh) to be produced at each of the H periods of the following day.[26] There are $i = 1, \ldots, N$ firms who maximize their profits. Each firm owns a certain number of units that can produce electricity, indexed by $j = 1, \ldots, J_i$. Units have a limited capacity $\overline{K}_j$ and a minimum level of production $\underline{K}_j$.[27] Producing with each of the units has an associated cost. The overall cost of producing a certain amount of energy is represented by a function $C_i(\mathbf{q}_i)$, where $\mathbf{q}_i$ is an array that represents the quantity assignments for that day.

Firms submit their bidding strategy to the market on a daily basis, represented by σ_i. The strategy is composed by simple and complex bids.

Simple bids are a bidding array of prices and quantities, represented by $\{\mathbf{b}_i, \mathbf{g}_i\}$, which define step functions for each of the units. They contain pairs of price bids ($\mathbf{b}_i$) and generation offers

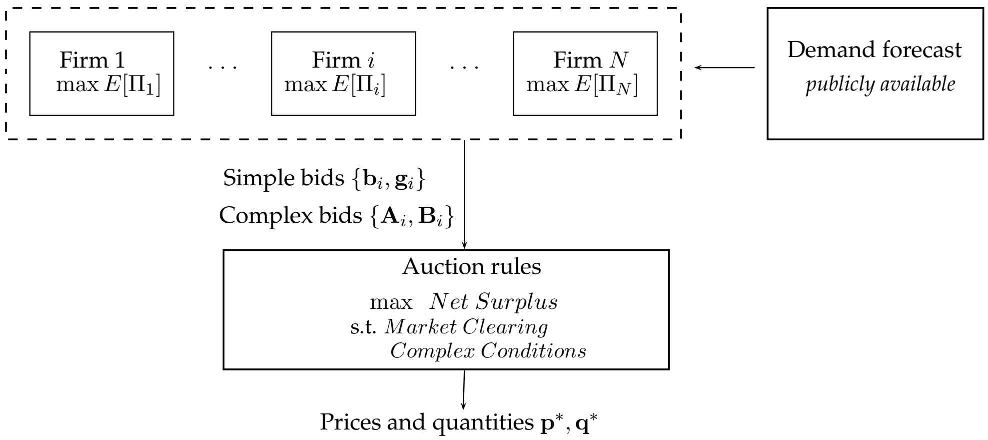
[24] From the previous evidence, this implies that I mainly exclude those units that have a production contract and therefore have decided ex-ante that they want to participate in the market.

[25] In practice, the decision of starting up often involves more than one day. The theoretical model abstracts from this longer horizon, which I discuss later in the empirical and counterfactual sections.

[26] In most electricity markets, H is either 24 or 48. In the Spanish electricity market, quantities are auctioned for hourly intervals, and therefore $H = 24$.

[27] Minimum production levels are a feature of thermal units. in order to operate safely, generators need to produce above a certain level. For this reason, this minimum production level is also referred to as the minimum stable load. This is a source of non-convexities in the production function of the firm.

Figure 4.1: A Game in Two-Stages



The firms in the electricity market maximize their profits taking into account the auction rules and available information.

($\mathbf{g}_i$) for each unit j, each hour $h = 1, \ldots, 24$ and each possible step $k = 1, \ldots, K$, where K is the maximum number of steps and is set by the auctioneer. In practice, firms do not need to use all the allowed steps.

Complex bids are a set of bids, specific to each unit, that implicitly define a minimum revenue requirement for the day. The vector of complex bids is given by $\{\mathbf{A}_i, \mathbf{B}_i\}$, which represent the fixed and the variable component of the minimum revenue requirement, respectively. The revenue requirement is constructed as the sum of the fixed components and the variable component times daily unit output, this is, $R_{ij} = A_{ij} + B_{ij} \sum_{h=1}^{24} q^*_{ijh}$.

Firms maximize their expected profits conditional on their information set and their beliefs about other players' strategies, given by σ_{-i}. The information set contains the common information shared by the firms, such as publicly available demand forecasts, as well as private information, such as private cost shocks. Equilibrium prices and the quantities are determined by the auction rules, taking the bidding strategies of the market participants as given.

In Figure 4.1, I present a diagram with the structure of the game. In a first stage, firms choose their bidding strategies, which include both simple and complex bids. They choose their bidding strategies simultaneously taking into account the demand forecast, made publicly available, as well as potentially other information. Firms anticipate that in a second stage the system operator will determine equilibrium prices and quantities following the auction rules.

The problem of the firm can be represented as follows

$$\begin{aligned} \max_{\sigma_i} \quad & \mathbb{E}_{-i}[\Pi_i(\mathbf{p}^*, \mathbf{q}^*) | \sigma_i, \sigma_{-i}, \mathcal{I}_i] \\ \text{s.t.} \quad & \mathbf{p}^*, \mathbf{q}^* \text{ are determined by the auction rules.} \end{aligned}$$

The equilibrium concept used in this context is a Bayesian Nash equilibrium. Each firm i

chooses a bidding strategy σ_i, composed by a simple bidding array $\{\mathbf{b}_i, \mathbf{g}_i\}$ and a complex bidding array $\{\mathbf{A}_i, \mathbf{B}_i\}$. Firms maximize their expected profits taking the distribution of other firms' bids as given and conditional on a set of information $\mathcal{I}_i$, such as weather forecast, demand predictions or cost shocks. In equilibrium, the beliefs over the distribution of other firms' strategies has to be consistent with the equilibrium play.

4.2 Specification of the main elements

4.2.1 The auction rules

The auction rules in the game follow the ones in the Spanish electricity market, as described in Section 3. Firms submit simple bids to offer their production, $\{\mathbf{b}_i, \mathbf{g}_i\}$; and complex bids $\{\mathbf{A}_i, \mathbf{B}_i\}$, which implicitly define a minimum revenue requirement R_{ij}. A supply curve is constructed using only those units $j \in J$ for which the minimum revenue requirement R_{ij} is satisfied. The rest of offers are not considered. Importantly, this is a daily condition. Therefore, the offers of the unit are taken out from all hours in the market whenever the minimum revenue requirement over the day is not satisfied. The system operator crosses demand and supply once the offers that do not satisfy the complex condition have been discarded, and the price is determined by the last offer accepted.

I define a complex condition $\rho(R_{ij})$ as the function that gives the net difference between the actual revenue obtained by unit j and its minimum revenue requirement R_{ij} implied by the parameters $\{A_{ij}, B_{ij}\}$. A unit is discarded whenever this net revenue is smaller than zero, this is, whenever

$$\rho(R_{ij}) \equiv \sum_{h=1}^{24} p_h^* q_{ijh}^* - R_{ij} < 0.$$

where $R_{ij} = A_{ij} + \sum_{h=1}^{24} B_{ij} q_{ijh}^*$. I assume that otherwise the unit is not discarded and its offers are considered in the auction.[28]

For notational purposes, I define with s each possible combination of complex bids of a given firm being accepted.[29] There is a probability $Pr(s)$ that, once uncertainty is realized, a particular combination s of complex bids is accepted. The probability of a subset of units J_1 being accepted, and another set J_2 being discarded is given by

$$Pr(s|\sigma; \mathcal{I}_i) = Pr\big(\rho(R_{ij}) \geq 0 \ \forall j \in J_1 \cap \rho(R_{ij}) < 0 \ \forall j \in J_2\big),$$

where s defines the event in which only units in the subset J_1 are accepted.

As a result of complex bids, the auction rules imply that the own supply curve of a given firm

[28] Note that in practice, all units are discarded if their minimum revenue requirement is not satisfied, but the converse is not necessarily true. I abstract from this feature of the iterative procedure used in the Spanish electricity, as otherwise it would not possible to embed the mechanism in a stylized theoretical model. This assumption can be interpreted as the firms expecting the unit to be accepted as long as its revenue requirement is satisfied.

[29] An example of a combination of complex bids being accepted is a situation in which firm i with three units A, B and C gets units A and B accepted and the other withdrawn.

is uncertain, as the shape of the supply curve depends on which minimum revenue requirements are satisfied. By choosing each unit's minimum revenue requirement, a firm affects the probability of different combinations of units being accepted in the market. However, conditional on a given set of complex bids being satisfied, the resulting supply curve depends on simple bids only and the game can be treated as a set of simultaneous uniform price auctions.

4.2.2 The profits of the firm

A representation of the expected profits of firm i for a given day is given by

$$\mathbb{E}_{-i}[\Pi_i(\sigma_i, \sigma_{-i})|\mathcal{I}_i] = \sum_{s\in S} Pr(s|\sigma; I_i)\mathbb{E}_{-i}\Big[\ \sum_{h=1}^{24} p_h^* Q_{ih}(q_{ih}(p_h^*), s)\ - \sum_{j=1}^{J} C_j(\mathbf{q}_{ij}^*)\ \Big|\sigma;\ s, I_i\ \Big], \tag{4.1}$$

where $Pr(s)$ defines the probability of a set of complex bids s being accepted, p_h^* represents the equilibrium price and $q_{ih}(p_h^*)$ represents the quantity offered by firm i at that price, $Q_{ih}(q,s)$ is the net quantity position of firm i at hour h when the equilibrium quantity is q, and $C_j(.)$ represents the cost function of a generating unit j, which depends on the allocation of unit quantities over the day, given by $\mathbf{q}_{ij}^*$.

The net quantity $Q_{ih}(q,s)$ reflects the net selling position of the firm, which determines its incentives to drive the price either up or down.[30] In particular, one needs to account for the energy that has been previously contracted, as that quantity is not exposed to the market price and therefore it is not determined in this stage. The net position is defined as

$$Q_{ih}(q,s) = q - q_{ih}^W(q,s) - q_{ih}^D(q) - \tau_{ih} - \nu_{ih},$$

where q is the quantity offer, $q_{ih}^W(q,s)$ represents the infra-marginal quantity withdrawn from the market due to complex bids at state s and quantity offer q, $q_{ih}^D(q)$ represents the retailing position of the firm at q, τ_{ih} represents physical bilateral contracts and ν_{ih} represents financial contracts for that hour. Except for financial contracts ν_{ih}, all other elements are observable. For notational convenience, I represent $Q_{ih}(q,s)$ as the difference between the observed quantities and the unobservable financial contracts, this is $Q_{ih}(q,s) = \tilde{Q}_{ih}(q,s) - \nu_{ih}$.

When there is market clearing, the quantity allocated to the firm to produce in the day-ahead market, given by $QS_{ih}(p_h^*|s) = q_{i,h}^* - q_{i,h}^W(q,s) - q_{ih}^D(q) - \tau_{ih}$, needs to be equal to the residual demand in the market that is not covered by other firms. Therefore, in equilibrium,

$$DR_h(p_h^*|s) = QS_{ih}(p_h^*|s),$$

where $DR(p_h^*|s)$ defines the residual demand and $QS_i(p_h^*|s)$ represents the quantity allocated to the firm after netting out the offers discarded by the complex mechanism, production already

[30] This issue has been explored extensively in the literature. See for example Wolak (2000) and Bushnell et al. (2008).

contracted by means of physical contracts and demand by the firm at that price.

4.2.3 Cost structure

I focus my analysis on the cost structure of thermal plants, which are the ones that primarily submit complex bids. Thermal plants have a dynamic cost structure that makes the valuations of producing in different hours of the day inter-related, making the use of complex bidding specially attractive to them. For example, a firm would like to avoid switching on and off a given plant repeatedly over the day, as this damages equipment and incurs discrete startup costs.

I base my cost structure on engineering models for thermal unit operation (Baíllo et al., 2001). These models take into account short run dynamics involved in electricity production. I assume the following daily cost structure for a thermal plant j,

$$C_j(\mathbf{q}_{ij}) = c_j(\mathbf{q}_{ij}) + \beta_j^{\text{start}} \mathbb{1}_j^{\text{start}},$$

where $c_j(\mathbf{q}_{ij})$ represents the costs of production that depend on the level of output. $\mathbb{1}^{start}$ represents a dummy variable that takes the value of one when a unit gets switched on during that day, which implies incurring a startup cost β^{start}.[31]

The costs of operation are given by

$$c_j(\mathbf{q}_{ij}) = \sum_{h=1}^{24} \Big(\alpha_{j1} q_{ijh} + \frac{\alpha_{j2}}{2} \tilde{q}_{ijh}^2 + \frac{\alpha_{j3}}{4} (q_{ijh} - q_{ij,h-1})^2 + \epsilon_{jh} \Big),$$

where α_{j1}, α_{j2} represent unit-specific marginal costs of production, α_{j3} represents ramping costs, and $\tilde{q}_{ijh}$ represents the quantity over the minimum production level $\underline{K}_j$, i.e. $\tilde{q}_{ijh} = \max\{q_{ijh} - \underline{K}_j, 0\}$. The dynamic structure comes from the ramping costs and the startup costs. Further non-convexities are introduced by the fact that, whenever firms are turned on, they need to produce at least $\underline{K}_j$.

4.3 Optimality conditions

Even though characterizing the full strategy of a multi-unit auction is not possible, I derive optimality conditions that can be used to analyze the optimal behavior of the players.[32] In the context of the auction design considered, one needs to characterize the optimal strategies for both simple and complex bids, which constitute the bidding strategy of the firm.

4.3.1 Optimality conditions for complex bids

In order to derive optimality conditions with respect to complex bids, it is important to note

[31] Note that I have implicitly assumed that a unit switches on or off at most once every day. This is consistent with evidence that units do not switch on or off more than once in the day-ahead market.

[32] The difficulty of solving for the equilibria in a multi-unit auction context is an issue that has not yet been resolved. Therefore, the analysis of multi-unit auction models usually relies on implied optimality conditions.

that they only affect the profits function of the firm through the probability of a given revenue requirement being accepted, given by $Pr(s|\sigma)$ in expression (4.1). A given complex bid for a given unit affects that probability at the point at which the minimum revenue requirement of the unit is just satisfied. The optimality condition with respect to complex bids implies then that the firm chooses parameters A_{ij} and B_{ij} such that the expected profit at the point at which the revenue requirement is just satisfied minus the profit of being out at that point are equalized. This expression is represented by the expected difference in profits from being in and out at the point at which the complex condition equals zero. This is summarized in Proposition 1.

Proposition 1 *Assume $Pr(s)$ is differentiable in A_{ij} and B_{ij}. If $\frac{\partial Pr(s)}{\partial A_{ij}} \neq 0$ or $\frac{\partial Pr(s)}{\partial B_{ij}} \neq 0$, a necessary first-order condition of optimality for R_{ij} is*

$$\mathbb{E}[\Pi_i^{\{j\ in\}} - \Pi_i^{\{j\ out\}} |\rho(R_{ij}) = 0] = 0, \tag{4.2}$$

where $\Pi_i^{\{j\ in\}}$ represents the profit of the firm when unit j's complex bid is accepted and $\Pi_i^{\{j\ out\}}$ represents the profit of the firm when unit j is discarded.

Proof (sketch): Complex bids only affect directly the probability of each complex condition being binding, given by $Pr(s)$. Marginal changes in complex bids affect outcomes along the range at which the complex bid is just binding, given by $\rho(R_{ij}) = 0$. Noting that the complex bid is just binding when a given unit is just accepted or just rejected, the first order condition with respect to A_{ij} is equal to $\frac{\partial Pr(s)}{\partial A_{ij}}\mathbb{E}[\Pi_i^{\{j\text{ in}\}} - \Pi_i^{\{j\text{ out}\}} |\rho(R_{ij}) = 0] = 0$. An analogous argument applies to B_{ij}. □

Proposition 1 states that the firm chooses a complex bid such that the opportunity cost of being accepted versus being rejected are equalized in expectation at the point at which the minimum revenue requirement is just satisfied. This implies that the firm sets the complex bids of a given unit at the point at which the differential value of being accepted, ignoring startup costs, just equals the opportunity cost of incurring such startup.

In a competitive environment in which a firm has only one plant and has no market power, Proposition 1 implies that the optimal complex bid is such that the unit breaks even in expectation when it is just accepted.[33] In fact, equation (4.2) takes the form of

$$\mathbb{E}[\sum_{h=1}^{24} p_h^* q_{jh}^* - C_j(\mathbf{q}_j^*)|\rho(R_{ij}) = 0] = 0,$$

when the minimum revenue requirement is just satisfied. In a simplified environment in which there is only a constant marginal cost and a startup cost, the firm can satisfy this condition in a way that is ex-post optimal by setting A_{ij} equal to its startup cost and B_{ij} equal to its marginal cost, given

[33]In a dynamic context in which different days are interrelated, the complex bid might include an option value of being either turned on or off for the next day. I account for this in the empirical section, but not in this theoretical framework.

that $C_j(\mathbf{q}_j) = \beta_j + \sum_{h=1}^{24} \alpha_{j1} q_{jh}^*$.[34] More generally, the optimal complex bids for a competitive firm are equal to the average expected startup and marginal cost at the point at which the unit is just accepted.

In a strategic environment, Equation (4.2) also captures the fact that the profit of the rest of the units owned by a firm can change depending on whether unit j is accepted or not. If there is a strategic value to withhold capacity, due to the increase in equilibrium prices, then a firm will tend to bid complex bids such that a unit makes positive profits at the point at which it is just accepted. In this case, when the minimum revenue requirement is just satisfied, equation (4.2) takes the form,

$$\mathbb{E}[\sum_{h=1}^{24} p_h^* q_{jh}^* - C_j(\mathbf{q}_j^*) - \Delta\Pi_{i\backslash j}^{out} | \rho(R_{ij}) = 0] = 0,$$

where $\Delta\Pi_{i\backslash j}^{out} = \Pi_{i\backslash j}^{\{j \text{ out}\}} - \Pi_{i\backslash j}^{\{j \text{ in}\}}$. $\Delta\Pi_{i\backslash j}^{out}$ represents the change in profits of firm i from withholding the capacity of unit j, without including the profits of unit j itself. Whenever $\Delta\Pi_{i\backslash j}^{out} > 0$, the firm will tend to overbid when placing its complex bids.

4.3.2 Optimality conditions for simple bids

In order to derive optimality conditions for simple bids, I look at the deviations of changing a bid offer at a given step and quantity. For simple bids to be consistent with the optimality of the strategies, it is a necessary condition that there are no profitable local deviations possible. A firm must be indifferent between raising or lowering the whole bidding step of its supply curve, given by a bid b_{jkh}, for unit j at step k and hour h,

$$\sum_{s \in S} Pr(s) \frac{\partial \mathbb{E}[\Pi_i(\sigma_i, \sigma_{-i})|s]}{\partial b_{jkh}} + \sum_{s \in S} \frac{\partial Pr(s)}{\partial b_{jkh}} \mathbb{E}[\Pi_i(\sigma_i, \sigma_{-i})|s] = 0. \tag{4.3}$$

These optimality conditions are more complicated than the ones found for complex bids, given that simple bids affect, first, market outcomes conditional on a particular set of complex bids being accepted, and second, the probability of a unit being accepted. In order to derive the FOC used in the empirical application, I assume that the FOC with respect to the bid b_{jkh} can be reduced to the effects of simple bids on profits, ignoring its potential effects on the probability of complex bids being binding. The justification for this assumption is both empirical and theoretical, as discussed below.

Assumption 1 *Marginal deviations of simple bids at a single step k and hour h are primarily captured by their marginal effects on conditional profits, this is,*

$$\sum_{s \in S} \frac{\partial Pr(s)}{\partial b_{jkh}} \mathbb{E}[\Pi_i(\sigma_i, \sigma_{-i})|s] \approx 0.$$

[34] Note that when $\rho(R_{ij}) = 0$, $\sum_{h=1}^{24} p_h^* q_{jh}^* = A_{ij} + \sum_{h=1}^{24} B_{ij} q_{jh}^*$.

Discussion: Note that the bid only affects the probability of states s if a given unit's minimum revenue requirement is just satisfied *and* the bid sets the price.Imagine first that there is only one hour in the auction. In that case, these two events will happen simultaneously, if at all. For realizations in which the bid does not set the price, the above term is zero. For realizations in which the bid sets the price, the term will also be zero by the envelope condition implied by the optimal complex bid condition in 4.2. With more hours, this does not necessarily need to be the case, unless the shocks are perfectly correlated across hours. Whenever the shocks are highly correlated, the term tends towards zero. If shocks are not highly correlated, the joint probability of a bid setting the price and the minimum revenue requirement being just satisfied tends to be empirically small, as not only does the bid need to be marginal, but also price realizations in the other twenty-three hours need to be consistent with the minimum revenue requirement being just satisfied. I present evidence in Appendix C.1 that shows that this term is empirically negligible and that the omitted term does not become a source of bias in the estimation.

Assumption 1 is useful to simplify the first order conditions on simple bids, as it allows to treat the decision of the firm over simple bids in a similar manner as a set of simultaneous uniform price auctions. This allows me to derive first order conditions with respect to simple bids that closely resemble the ones usually found in a multi-unit auction with a uniform pricing rule. This result is summarized in Proposition 2.

Proposition 2 *A necessary first order condition for optimality of b_{jkh}, for a given unit j at hour h and bidding step $k > 1$, is given by*

$$b_{jkh} = \overline{\zeta}_{jkh} + \sum_{s \in S|j\ \ in} Pr(s|j\ \ in) \sum_{q \in step\ \ k} \left| \frac{Pr(DR_h^{-1}(q) < b_{jkh} < DR_h^{-1}(q-1)|s)}{Pr'(b_{jkh} < DR_h^{-1}(q-1)|s)} \right| \frac{\tilde{Q}_{ih}(q,s) - \nu_{ih}}{q_{jkh}}, \tag{4.4}$$

where b_{jkh} is the bid offered, $\overline{\zeta}_{jkh}$ represents the average marginal cost at the step, q represents the inframarginal quantity at the step, q_{jkh} represents the quantity at the step and $DR_h^{-1}(q)$ represents the inverse residual demand at level q and period h.

Proof: See Appendix C.2. □

Proposition 2 states that the bid b_{jkh} is equal to the average marginal cost $\overline{\zeta}_{jkh}$ plus a shading factor or markup. The shading factor is composed by a weighted average of the inframarginal quantity produced by the firm at level q and complex state s, given by $Q_{ih}(q, s)$, over the possible states in which the complex bid of unit j is satisfied, represented by $Pr(s|j \text{ in})$. Inframarginal quantities are weighted by the factor $Pr(DR_h^{-1}(q) < b_{jkh} < DR_h^{-1}(q-1)|s)/Pr'(b_{jkh} < DR_h^{-1}(q-1)|s)$, which can be approximated with the inverse of the expected slope of the residual demand. The slope of the residual demand is a summary statistic of the residual demand distribution and indicates the degree of competition faced by firm i.[35]

The first order condition in Proposition 2 can be approximated as,

$$b_{jkh} \approx \overline{\zeta}_{jkh} + \sum_{s \in S|j \text{ in}} Pr(s|j \text{ in}) \frac{\tilde{Q}_{ih}(\overline{q}, s) - \nu_{ih}}{|\mathbb{E}[DR'(b_{jkh})|p = b_{jkh}, s]|},$$

where $\overline{q}$ represents the average inframarginal supply at the step and $\mathbb{E}[DR'(b_{jkh})|p = b_{jkh}, s]$ represents the slope of the residual demand at b_{jkh} when b_{jkh} sets the price. The expression implies that the marginal cost of the unit is equal to its expected marginal revenue at the step.

As in the usual setting, the net quantity supplied by the firm determines the sign of the cost markup. For a positive net quantity $\tilde{Q}_{ih}(\overline{q}, s) - \nu_{ih}$, the firm puts a positive markup to its offer, submitting a bid that is higher than its marginal cost. However, if the firm is a net buyer in the market, either because it has forward contracts or because is also a distributor, the bid stays below the marginal cost of the firm.

This condition is similar to the optimality conditions found in Hortaçsu and Puller (2008) and Allcott (2009). The approximation is exactly satisfied under the assumption that the slope of residual demand at a given bid b_{jkh} is constant, as in Hortaçsu and Puller (2008).[36] One difference in this setup is that the presence of complex bids affects the markup component, which is a weighted average over the possible set of complex bids being binding. Complex bids change the expectations over both inframarginal quantity and residual inverse demand. One other difference is that the first step is not marginal, as it affects startup decisions, and therefore this first order condition is not applicable to it.

In sum, the simple and complex first order conditions derived in this section highlight the major trade-offs faced by the firm when choosing its bidding strategy optimally. They are used in the empirical strategy to infer the fundamentals of the model.

5 Estimation strategy

There are several fundamental parameters that need to be estimated in order to perform the counterfactual simulations. The unknown parameters can be summarized as follows

$$\theta = \{\eta, \alpha, \beta, \gamma\},$$

where η are operational parameters from the production function that can be readily inferred from the data,[37] α and β are marginal and startup costs of operation respectively, and γ represents

[35] Note that this term is not well defined if $Pr'(b_{jkh} < DR_h^{-1}(q-1)|s) = 0$. Therefore, Proposition 2 is only valid if the bid submitted has some positive probability of being marginal.

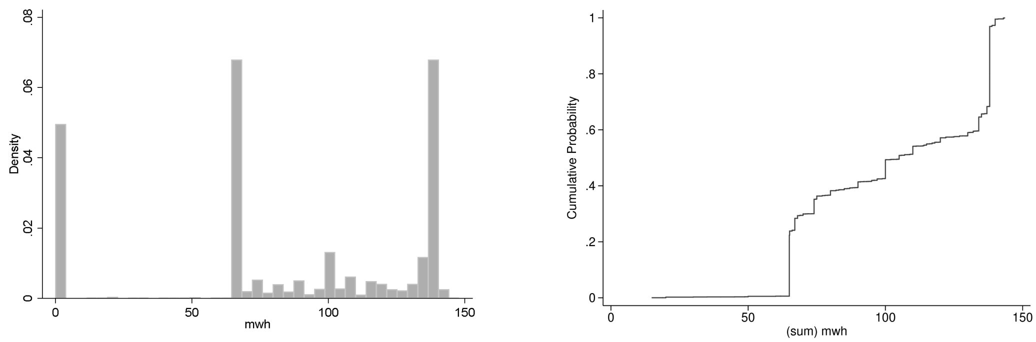
[36] A weaker assumption that also allows the result to hold is that the residual demand has constant slope conditional on the bid setting the price, i.e. $DR'(b_{jkh}|p = b_{jkh}, s) = k$.

[37] These parameters are the minimum stable load of a production unit and its maximum capacity.

Figure 5.1: Identification of Operational Parameters

(a) Production patterns of units over time identify both minimum and maximum production constraints.

(b) Break in CDF Identifies Minimum Stable Load.



the parameters that affect the forward position of the firm. Therefore, $\{\eta, \alpha, \beta\}$ are unit-specific parameters, whereas γ are firm-specific parameters.

The cornerstone of the estimation are the optimality conditions implied by the multi-unit auction bidding game, represented by Proposition 1 and Proposition 2. The estimation proceeds in a two stage fashion. First, marginal costs and forward contracts are inferred from simple bids optimality conditions. With these estimates, startup costs of operation are estimated from complex bids optimality conditions.

I outline here the major techniques used to estimate the unknown parameters in the model and comment on the econometric approach. In the next section, I present the results.

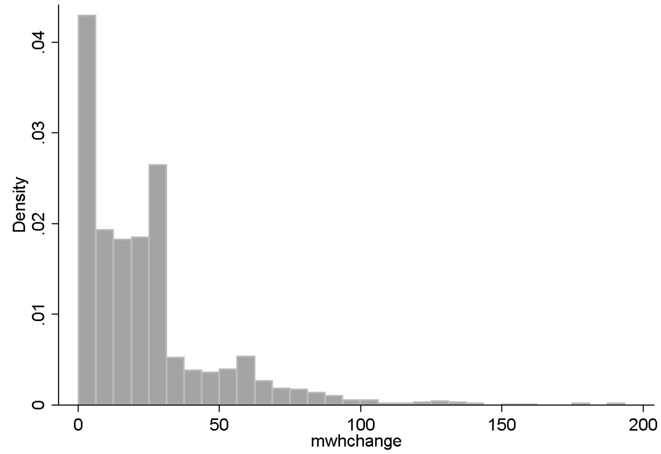
5.1 Operational parameters η

Certain parameters of firm production can be inferred directly from final production data. In particular, the minimum stable load can be observed from actual production patterns over time. Figure 5.1(a) represents the histogram of production of a coal plant. One could infer the minimum production limits of the unit, which in this case are around 65 MWh. In order to estimate it, I search for a break in the cumulative distribution function around the minimum production range that minimizes the sum of squared errors. Figure 5.1(b) shows how the break in the cumulative distribution function identifies the minimum stable load for a particular unit in the data.[38]

There are other operational parameters that could be potentially inferred from the data. For example, by looking at production changes across hours, one can have a sense of the ramping limits of units, both once they are producing and when they are starting up. However, those ramping constraints are not as binding as the minimum stable load: units can choose to ramp up or down at

[38] These minimum stable loads are estimated at the unit-level and are identified as long as the unit produces sometimes at this threshold, which is satisfied in practice. Because minimum stable loads are a fixed parameter of the production function that does not depend on economic variables, longer periods of data can be used whenever this condition is not satisfied.

Figure 5.2: Ramping Up Constraints



Even though firms tend to do relatively small hourly changes of output, ramping constraints are not as clearly identifiable as minimum and maximum production levels.

different speed, although the costs are potentially increasing in the amount of the ramp. Figure 5.2 shows that the limits on ramping do not appear to be binding, compared to minimum production constraints. It is apparent that units make changes that are mostly contained in a particular range, but there is evidence that such limits can be surpassed. For this reason, I estimate these costs from the bidding behavior, following Wolak (2007).

5.2 Marginal cost parameters α and forward contracts γ

I use a generalized method of moments to estimate the unit-specific cost parameters α and the forward parameters γ, using the first order conditions on simple bids implied by Proposition 2, together with an average marginal cost and forwards parametrization.

The simple bidding first order conditions can be expressed as follows from (4.4):

$$\sum_{s \in S | j \text{ in}} Pr(s|j \text{ in})\Big(\overline{\phi}_{jkh}(b_{jkh} - \overline{\zeta}_{jkh}(\alpha)) + \overline{\Phi}_{jkh}(\overline{Q}_{jkh} - \nu_{ih}(\gamma))\Big) = 0,$$

where

$$\begin{aligned}
\overline{\phi}_{jkh} &= \textstyle\sum_{q \in step\ k} Pr'(b_{jkh} < DR_h^{-1}(q-1)|s)/q_{jkh} \\
\overline{\Phi}_{jkh} &= \textstyle\sum_{q \in step\ k} Pr(DR_h^{-1}(q) < b_{jkh} < DR_h^{-1}(q-1)|s)/q_{jkh} \\
\overline{Q}_{jkh} &= \textstyle\sum_{q \in step\ k} q/q_{jkh} - q_{ih}^W(q,s) - q_{ih}^D(q) - \tau_{ih},
\end{aligned}$$

Other than the unobserved parameters α and γ, some of the terms in the above expression are not readily observed in the data, as they involve expressions that depend on the distribution of the residual demands that the firm is facing ($\overline{\phi}_{jkh}, \overline{\Phi}_{jkh}, Pr(s|j \text{ in})$). In order to construct an empirical analog to these terms, I use a simulation procedure to generate expectations over market outcomes, using a bootstrapping procedure similar to Hortaçsu (2002).[39] For a given firm, I randomly draw

[39] An alternative to this bootstrapping technique is to homogenize the bidding data as a function of covariates and

other firms' strategies across similar days. Similar days are pooled depending on whether they are a Monday, another weekday, a Saturday or a Sunday, as well as depending on the maximum hourly levels of predicted demand. By randomly drawing the strategies of other firms under similar conditions, I approximate the uncertainty faced by the firm in the market. This approach is similar to Gans and Wolak (2008), who also pool residual demand across similar days to construct sample analogs of moment conditions implied by profit maximization.

There are three aspects that are different from a standard uniform auction simulation. First, the auction rule is not standard and one needs to create an algorithm to determine which units are accepted and which units are not that closely matches the one used by the ISO. This algorithm is described in Appendix A, in which a Monte Carlo simulation is performed to assess the accuracy of the procedure that mimics the ISO market rules. The Monte Carlo simulations performed show a high level of accuracy. Both the mean and the standard deviation of the simulated prices have on average an error very close to zero.

Second, complex bids change the nature of the uncertainty that firms face when choosing their bidding strategies. In contrast to previous studies in the auction literature, firms face uncertainty over their own supply curve as well, as they do not know ex-ante which complex bids are going to be satisfied. This fact introduces another source of variation when generating random outcomes in the market. Figure 5.3 represents fifteen random draws of market outcomes. As can be observed, firms face uncertainty over both demand and supply.

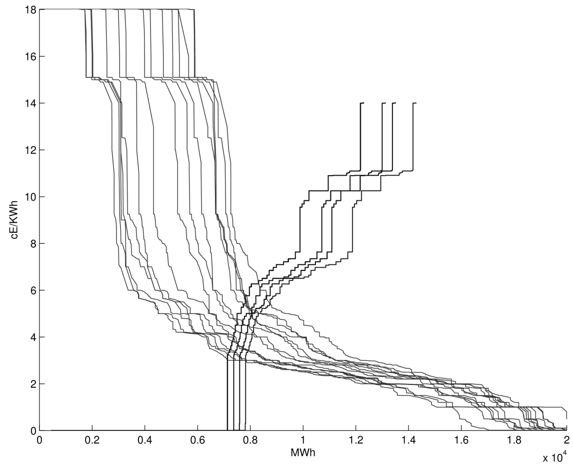
Finally, in contrast to Hortaçsu (2002), in the case of electricity markets it is possible to bootstrap the strategies of other firms maintaining their identities. This is eased even further because the population across different days does not change, as it is mandatory for all generators of a certain size to bid in the market. Therefore, the sampling approach does not require assumptions about the homogeneity of other participants in the market. Because there are several firms in the market as well as many different days observed, I can approximate the distribution as the numbers of similar days increase, without requiring the number of firms to grow.

In Table F.1 and Figure F.2 included in the Appendix, I show that the randomization procedure across similar days together with the ISO algorithm generates a distribution of marginal prices that closely matches the one observed during the period of study. In particular, the distribution captures well the mean prices in the market as well as the different quantiles. It also reflects the fact that the distribution of prices is bounded, as can bee seen in Figure 5.3.

Once market outcomes are simulated, one can construct an empirical analog to the first order condition in (4.4). One of the challenges in doing so is that the probability of a certain bid setting the marginal price might be very small, even if the number of simulated draws is large. Note that for a given realization of residual demand, Φ_{jkh} is either zero or one, taking a value of one only when $p_h = b_{jkh}$. In practice, due to the low probability nature of these events, $\mathbb{1}(p_h = b_{jkh})$ is zero most of the times (if not all), which might make the sample analog very sensitive to the particular

then randomize the remaining error terms, as in Haile et al. (2003) and Allcott (2009). Due the presence of complex bids, in this context such approach would require a probabilistic model about the likelihood of using complex bids and their level conditional on doing so.

Figure 5.3: Generating Random Market Outcomes - April 11th 2007, 5pm



Randomly drawing strategies of other players generates a distribution of expected residual demand. Due to complex bids, the ex-post supply curve of a given firm can depend on the particular realization of other firms' strategies.

randomization draws, even for many bootstrap samples. Similarly, ϕ_{jkh} equals $\mathbb{1}(p_h = b_{jkh})DR'(p_h)$ for a given demand realization, which has the additional problem that $DR'(p_h)$ is not well defined due to the discrete nature of step bids.

In order to address this problem, I follow a smoothing approach as in Wolak (2007) and Gans and Wolak (2008). By means of a smoothing procedure, both the demand and the supply curve are approximated as follows,

$$
\begin{aligned}
DR_h^{bw,bs}(p_h|s^{bs}) &= \sum_{n \in s^{bs}} q_{nh}^{rd} \mathcal{K}\Big(\frac{p_{nh}^{rd} - p_h^{bs}}{bw}\Big), \\
QS_{ih}^{bw,bs}(p_h|s^{bs}) &= \sum_{n \in s^{bs}} q_{nh}^{i} \mathcal{K}\Big(\frac{p_{nh}^{i} - p_h^{bs}}{bw}\Big),
\end{aligned}
$$

where q_{nh}^{rd} and p_{nh}^{rd} are pairs of prices and quantities of the residual demand, q_{nh}^{i} and p_{nh}^{i} are pairs of prices and quantities of the supply of firm i, s^{bs} is the set of offers that are not discarded by complex bids in draw bs, $\mathcal{K}$ is a cumulative weight, such as the standard normal cumulative distribution function, and bw is a bandwidth parameter that determines the degree of smoothing.

Once the residual demand and own supply are approximated, one can compute the residual demand slope by differentiating $DR_h^{bw,bs}$ to get an approximate of $DR'(p_h)$. Similarly, noting that $\Phi_{jkh} = \mathbb{E}[\frac{\partial p_h}{\partial b_{jkh}}]$, one can approximate this term by computing $\partial p_h/\partial b_{jkh}$ using the market clearing condition $DR_h(p_h) = QS_{ih}(p_h)$ and the implicit function theorem on the smoothed versions of residual demand and supply.[40]

[40] The formula is given by $\partial p_h/\partial b_{jkh} = \partial QS_{ih}(p_h)/\partial b_{jkh}/(\partial DR_h(p_h)/\partial p_h - \partial QS_{ih}(p_h)/\partial p_h)$, which can be constructed using the smoothed versions of the firm's supply and residual demand.

The empirical analog to the first order condition of unit j at step k in hour h at a given day t is given by,[41]

$$m_{jkht}(\alpha,\gamma,bw) = \frac{1}{B}\sum_{bs=1}^{B} \mathbb{1}(j \text{ in}) \frac{\partial {p_{ht}}^{bw,bs}}{\partial b_{jkht}} \Big((b_{jkht} - \overline{\zeta}_{jkht}(\alpha)) \frac{\partial DR^{bw,bs}_{ht}}{\partial b_{jkht}} + (\overline{Q}^{bs}_{jkht} - \nu_{ht}(\gamma)) \Big), \quad (5.1)$$

where B represents the number of simulations per day in the bootstrapping algorithm.

The moment conditions are constructed by adding up the empirical analogs to the first order conditions across simulations. Potentially the moments could be constructed at the daily and hourly level, which would provide thousands of moments. To perform the estimation, I pool the moments across subset of hours and time, which provides valid moment conditions as $T \to \infty$ and $B \to \infty$. With these moments, the parameters of the marginal cost function, which appear in the marginal cost $\overline{\zeta}_{jkht}$, and the forward positions of the firm ν_{ht}. Several specifications of $\overline{\zeta}_{jkht}(\alpha)$ and $\nu_{ht}(\gamma)$ as well as ranging parametrizations of the bandwidth parameter bw are considered in the next section, in which results are presented.

Identification As shown in McAdams (2008), the multi-unit auction model is only partially identified. However, by parametrizing the cost function and the forward position, one can point identify the parameters using the optimality conditions at each bidding step.

The parametrization strategy also allows me to identify the marginal costs and the forward position of the firms jointly. Previous work has shown that the forward position cannot be non-parametrically identified in the absence of marginal cost engineering data (Wolak, 2000, 2003; Hortaçsu and Puller, 2008). This has traditionally been a challenge in order to carry out empirical analysis in the context of electricity markets, as engineering cost estimates are needed to perform the analysis, which are often not publicly available. I show that by parametrizing the cost function, one can estimate both costs and forward contracts. The identification makes use of the fact that the forward position is common across units from the same firm, which gives a source of cross-sectional variation. For this reason, forward contracts could be potentially estimated flexibly.

To gain intuition on the identification of the parameters, it is useful to re-write the above GMM estimator as a weighted regression with specification

$$\hat{b}_{jkht} = X'_{jkht}\alpha_j + \frac{\partial \overline{DR}^{bw}_{ht}}{\partial p_{ht}}^{-1} \nu_{ht}(\gamma) + \epsilon_{jkht},$$

where $\hat{b}_{jkht}$ is the offer made by unit j in step k and hour h after substracting the markup component that is observed by the econometrician, given by $\frac{\partial DR^{bw}_{ht}}{\partial b_{jkht}}^{-1} \overline{Q}^{bs}_{iht}(b_{jkht})$; X_{jkht} are covariates of the marginal cost specification, such as a unit fixed effect, offered unit quantity or expected differences of output across hours; and $\frac{\partial DR^{bw}_{ht}}{\partial p_{ht}}^{-1}$ is the expected slope of the inverse residual demand. The

[41] Note that these conditions are analogous to the one found in Wolak (2007), noting that when $b_{jkh} = p_h$, $\partial DR^{bw,bs}_{h} / \partial b_{jkh} = \partial QS_{ih}(p_h)/\partial p_h \cdot \partial p_h / \partial b_{jkh} + \partial QS_{ih}(p_h)/\partial b_{jkh}$.

weights are given by the probability of a given observation being marginal, given by $\frac{\partial p_{ht}^{bw}}{\partial b_{jkht}}$.

The marginal cost parameters α_j are all unit-specific and therefore their identification comes from observing the same unit make different decisions across steps, hours and days for different output levels and market conditions. Because the first step of a given unit is not marginal, one cannot identify the marginal cost of those units that submit a single step. In order to identify both a linear and a quadratic cost component, one requires at least two moments per unit. The identification of forward contracts comes from the cross-section of different units, through variation in the markup components, as well as temporal variation.

The implicit assumption in the identification strategy of both marginal costs and forwards is that its structure is relatively constant over the period of study. Because I look at a reduced period of time with similar weather patterns and forecasted prices, this assumption is not as strong as it could be otherwise. In the results sections, I also consider a specification in which I parameterize marginal costs as a function of input prices to allow for temporal variation. I also consider separating the sample in two, which allows for greater flexibility in both marginal and forward parameters.

The unobserved error might be correlated with the explanatory variables. In particular, when a unit has a large cost shock, it submits higher bids, which implies that it can be possibly located at a higher region of the supply curve. Therefore, $\frac{\partial DR_{ht}^{bw}}{\partial b_{jkht}}^{-1}$ can be potentially correlated with the error term. Similarly, the cost components might include quantity decisions, which might be affected by its cost shocks. In the GMM framework, there are many instruments that can be used. In particular, any variable that belongs to the information set $\mathcal{I}_i$, and is correlated with the aforementioned variables, can be a candidate instrument. I use lags from several periods before the day of the auction, to circumvent possible correlation across cost shocks over time as well as publicly available data on demand forecasts.

Inference In order to construct standard errors for the marginal cost and forward estimates, I use a bootstrapping technique. Due to the temporal nature of the data, I use an empirical block bootstrap estimator at the week level. In order to compute the optimal weighting matrix, I use a Newey-West variance-covariance specification with weekly lags clustered at the unit-step level.

5.3 Startup cost parameters β

The empirical strategy used for simple bids suggests using a similar approach to infer the startup costs of production units. As shown in the previous section, first order conditions with respect to complex bids have a mapping to the startup costs of operation. From the reduced form evidence presented, it was also seen that the fixed component of complex bids is primarily used in those days in which a given unit needs to incur the startup cost. The idea is to exploit the information contained in complex bids to infer this parameter.

Recall that, when firms are strategic, the first order condition with respect to complex bids is given by

$$\mathbb{E}[\sum_{h=1}^{24} p_h^* q_{jh}^* - C_j(\mathbf{q}_j;\alpha,\beta) - \Delta\Pi_{i\backslash j}^{out}(\alpha,\gamma)|\rho(R_{ij})=0]=0.$$

Given an estimate of the marginal costs and forward contracts, $\{\hat{\alpha},\hat{\gamma}\}$, one can construct the sample analog of the above expression. All the elements of the above expression are known except for the startup costs of operation, which enter the cost function $C_j(\mathbf{q}_j;\hat{\alpha},\beta)$. Therefore, one can infer the startup costs of operation as the residual of the above expression when the minimum revenue requirement is just satisfied.

Excluding unit j from the market has the effect of potentially increasing market prices. The unit is trading off starting up and producing versus withholding that capacity. When the minimum revenue requirement is just satisfied, rearranging terms one finds the following expression for the startup cost

$$\beta_j = A_j + B_j \sum_{h=1}^{24} q_{jh}^* - c_j(\mathbf{q}_j;\alpha) - \sum_{h=1}^{24} (p_h^{j\ out} - p_h^{j\ in}) Q_{ih}.$$

All the terms on the right hand side are known at this stage and can be used to calculate the above expression.

Similar to the simple bidding estimation, empirical observations in which the minimum revenue requirement is just satisfied might not be observed, even after augmenting the data with a simulation procedure. One can use a kernel estimator to approximate this term around those demand realizations for which the minimum revenue requirement is close to being just satisfied. The empirical analog to the first order condition for a unit j and day t is given by,

$$\begin{aligned} f_{jt}(\hat{\alpha},\hat{\gamma},\beta,bw) &= \frac{1}{B}\sum_{bs=1}^{B} w_{jt}^{bw,bs}\Big(\sum_{h=1}^{24} p_{ht}^{*,bs} q_{jht}^{*,bs} - C_j(\mathbf{q}_{jt}^{bs};\hat{\alpha},\beta) - \Delta\Pi_{i\backslash j,t}^{off,bs}(\hat{\alpha},\hat{\gamma})\Big) = 0, \quad (5.2) \\ \text{with} \quad w_{jt}^{bw,bs} &= \frac{1}{bs}\kappa\Big(\frac{A_{jt} + \sum_{h=1}^{24}(B_{jt} - p_{ht}^{*,bs}) q_{jht}^{*,bs}}{bw}\Big), \end{aligned}$$

where κ is a density weight such as the standard normal density function. $\Delta\Pi_{i\backslash j}^{off,bs}$ can be calculated by looking at the counterfactual profits difference between excluding unit j and including it. The above equation gives an empirical analog to the first order condition when the minimum revenue requirement is just satisfied. The expression is averaged across different days in the sample to construct the empirical moments.

Identification As shown above, the pointwise estimates identify the startup costs of operation of those units that trade off starting up a unit on a given day. However, there are some units that do not use complex bids at all, and therefore in such case complex bids cannot be used to identify the startup costs. Furthermore, when units are already on, their complex bids might capture the option value of keeping the unit on. Therefore, the pointwise estimates can usually only identify jointly the startup cost and the possible option value associated with being turned on.

In order to separately identify the startup cost, I focus my attention on units that are switched off and decide whether to startup or not a given day.[42] Consider the case in which the unit is turned off and has to decide whether to turn on in a given day. At the point at which the complex bid is just binding, the firm is indifferent between

$$\Pi_i^{j\ in}(\hat{\alpha}, \hat{\gamma}) - \beta_j = \Pi_i^{j\ out}(\hat{\alpha}, \hat{\gamma}),$$

where $\Pi^{j\ in}$ ignores the startup cost, which is separately represented by β. Given first stage estimates, $\Pi^{j\ in}(\alpha, \gamma)$ and $\Pi^{j\ out}(\alpha, \gamma)$ can be computed around those obervations in which the minimum revenue requirement is close to being binding. However, if there is a continuation value due to the dynamic nature of the problem, then the estimate also captures the difference in the continuation value of being on or off. Therefore,

$$\Pi_i^{j\ in}(\hat{\alpha}, \hat{\gamma}) - \Pi_i^{j\ out}(\hat{\alpha}, \hat{\gamma}) = \beta_j - \Delta V_i^{j\ in-out}.$$

Given that $\Delta V_i^{j\ in-out} \geq 0$, this implies that, in the presence of a continuation value, the startup cost estimate will be a lower bound to the actual cost. The intuition behind the above equation is that a unit does not need to recover its startup costs during a single day. Consider for example a competitive unit. The unit might decide to startup if it makes a positive profit during the day, even ignoring the startup cost, if it expects to recover such startup cost in the following days of contiguous operation. A similar argument can be built for the case in which the unit is switched on. Shutting down has the opportunity cost of imposing a startup cost in the future and also has different continuation payoffs.

In order to control for the potential continuation value, I use a reduced form approach using weekday dummies. The idea behind this strategy is to note that the continuation value of starting up a unit depends on the day of the week. For example, a unit starting up on Friday will generally only startup for that given day, as in the weekend there is lower demand for electricity, which implies that the continuation value of that startup can be considered to be approximately zero. Therefore, the Friday fixed effect can be used as a baseline to identify the startup costs of the units. Other specifications are discussed in the results section.

In the context of a more generalized dynamic game, an alternative identifying strategy would be to use moment inequalities implied by Bayesian Nash equilibrium, generating deviations around complex bids. Moment inequalities taken across several days would provide a way to control more explicitly for the continuation value of the startup.[43] For example, observing a unit stay producing at a loss during the weekend to avoid incurring the startup cost again on Monday can provide a lower bound on the magnitude of this startup cost. The reason why I have not followed this route is an empirical one. Even though such bounds can be constructed, I find them to be not very informative in my application. The low power of the bounds is due to the fact that firms do not switch on and off very often, even during a four month sample. Therefore, it is difficult to infer

[42] This approach is similar in spirit to Fowlie (2010).

[43] See Morales et al. (2009).

tight bounds. On the contrary, complex bids are submitted more frequently and can provide direct information on those costs through the pointwise optimality conditions.

Inference In order to create confidence intervals for the pointwise estimates of startup costs, I construct bootstrap standard errors that account for the uncertainty arising from the first step estimates. Due to the selected nature of the sample in the estimation of startup costs, I bootstrap the relevant sample of observations.

6 Estimation results

In this section, I describe the results of the estimation for the marginal and forward parameters as well as for the startup costs. I report the parameter estimates for one of the largest firms in the market, which owns several coal, combined cycle gas and peaking units. This firm has a thermal generating capacity of around 9,000 MW, most of it composed of combined cycle gas plants (∼5,600 MW) followed by peaking units and coal (∼2,050 MW and ∼1,000 MW respectively). Given that peaking plants never produce during the sample of study and are always discarded by their complex bids, I do not include them in the estimation as their costs cannot be properly identified. The firm owns five peaking plants, which submit complex bids that are never accepted during the period of study.[44]

The firm also owns nuclear plants which usually run whenever they are available. The firm has also a large capacity in renewable energy, mainly wind, which usually also runs whenever there is wind available. Finally, the firm has hydro power plants, which can have an important strategic role. In this paper, I do not include these other generating groups in the analysis, holding their decisions as observed in the data. The focus of the paper is on coal and combined cycle gas plants, which are the ones that use complex bids.

6.1 Marginal cost and forward estimates

I estimate the costs of thermal plants as well as the forward position of the firm using of the first order conditions from marginal bids. The cost specification is given by the cost structure presented in Section 4,

$$\overline{\zeta}_{jkht}(\alpha) = \alpha_{j1} + \alpha_{j2}\tilde{q}_{jht} + \alpha_{j3}(2q_{j,h,t} - q_{j,h-1,t} - q_{j,h+1,t}) + \epsilon_{jkht}.$$

The specification for forward contracts assumes that firms hedge in expectation a percentage of their expected output of the next day. I allow forward contracts to differ between peak hour to base hours. This is motivated by the fact that there exist forward contract products that are targeted at peak hours, which range 8am to 8pm. Finally, I also allow weekends to have different

[44] In fact, most of these peaking plants data are unavailable during the period of study or undergoing retirement operations, which gives another reason to exclude them from the sample.

forward positions, given that demand is substantially different in those days. Therefore, the forward position for firm i is specified as

$$\nu_{ht}(\gamma) = (\gamma_1 + \gamma_2 \mathbb{1}_{peak} + \gamma_3 \mathbb{1}_{weekend}) q_{ht} + \varepsilon_{ht},$$

where q_{ht} represents the quantity actually sold at the day-ahead market, which is observed in the data.

The empirical moments are constructed averaging both across simulations and time, as

$$\overline{m}_{jkh}(\alpha, \gamma, bw) = \sum_{t=1}^{T} \sum_{bs=1}^{B} \mathbb{1}(j \text{ in}) \frac{\partial p_{ht}{}^{bw,bs}}{\partial b_{jkht}} \Big((b_{jkht} - \overline{\zeta}_{jkht}(\alpha)) \frac{\partial DR_{ht}^{bw,bs}}{\partial b_{jkht}} + (Q_{jkht}^{bs} - \nu(\gamma)) \Big) = 0$$

I take 100 randomly drawn simulations to simulate market outcomes, i.e. $B = 100$. In the sample of study, $T = 120$. The bandwidth parameter is equal to 3€. Because the number of moments available is very large due to the highly-dimensional strategy space of the firm, which provides over a thousand moment conditions, I experiment with different pooling strategies across first-order conditions. In the baseline specification, the day is divided in six different 4-hour periods and first-order conditions are considered at aggregating at that level, this is, $\overline{m}_{jk\tilde{h}} = \sum_{h \in \tilde{h}} \overline{m}_{jkh}(\alpha, \gamma, bw)$.

The results for each of the units are presented in Table 6.1. I include the estimated linear and quadratic component of the cost function.[45] One can see that there is substantial heterogeneity in marginal cost estimates of combined-cycle gas plants, even though these plants are new and similar. One explanation is that these units sometimes have unit-specific contracts to obtain their inputs. Another plausible explanation is the presence of congestion in some areas. Therefore, the marginal cost estimates do not only capture the cost of the inputs, but also some opportunity cost of giving up congestion rents. In fact, combined cycle units with higher estimated marginal costs in the sample are the ones in systematically congested areas. In this paper, I interpret this opportunity cost as the "marginal cost" of the unit, as I am mainly interested in analyzing the day-ahead market holding other opportunity costs due to other markets fixed, which are not accounted for in the model.[46] Note that, under the assumption of homogeneous marginal costs across similar, newly-built, combined cycle units, the true marginal cost could be easily identified using the estimates of units in uncongested areas.

Table 6.2 provides the estimates summarized at a more aggregate level, by fuel type. The results indicate that average coal costs are around 27.42€/MWh, whereas combined cycle gas costs are around 39.76€/MWh. Combined cycle marginal costs appear to fluctuate more than coal costs over time, which is consistent with the fact that gas prices tend to fluctuate more than coal prices.

[45] In this baseline specification, ramping costs are set to zero in the main specification ($\alpha_{j3} = 0$). Ramping costs are estimated in alternative specifications included in Table F.2 in the Appendix. I find some evidence of ramping costs for coal units, whereas I do not find evidence of ramping costs for combined cycle gas units at the hourly level, specially units with 400MW of capacity. This result is intuitive, as these combined cycle gas plants are new plants that can ramp to full capacity in less than one hour.

[46] Incorporating the actual opportunity cost from the congestion market is a topic that I am currently investigating taking advantage of an institutional change in the congestion market in July 2005.

Table 6.1: Marginal Cost Estimates at the Unit Level

Plant	Type	Size MW	α_{j1} €/MWh	α_{j2} €/MWh^2
ACE3	CCGT	386.0	41.32	2.05E-2
			(2.43)	(6.58E-3)
ARCOS1	CCGT	389.2	44.90	2.52E-2
			(2.82)	(1.41E-2)
ARCOS2	CCGT	373.2	41.07	2.87E-2
			(2.50)	(8.45E-3)
ARCOS3	CCGT	822.8	37.99	7.56E-3
			(2.02)	(2.38E-3)
CTJON2	CCGT	378.9	36.94	2.60E-2
			(3.46)	(6.53E-3)
CTN3	CCGT	782.0	45.19	7.32E-3
			(2.09)	(4.79E-3)
ESC6	CCGT	803.5	33.89	2.20E-2
			(2.49)	(2.54E-3)
STC4	CCGT	396.4	36.78	3.11E-2
			(2.62)	(6.83E-3)
GUA1	CO	148.0	31.37	5.25E-2
			(1.63)	(1.14E-2)
GUA2	CO	350.0	26.60	2.48E-2
			(1.09)	(4.48E-3)
LAD3	CO	155.0	30.62	4.78E-2
			(1.42)	(7.39E-3)
LAD4	CO	350.0	24.54	4.02E-2
			(0.96)	(6.20E-3)
PAS1	CO	214.0	23.95	5.99E-2
			(0.87)	(7.43E-3)

Notes: Sample from March to July 2007. Standard errors computed using block-bootstrap at the week level.

Table 6.2: Marginal Cost and Forward Estimates for Firm 1

	Linear Costs (1)	**Quadratic Costs (2)**	**Ramping Costs (3)**	**Ramping IV (4)**	**Thermal Rates (5)**
Coal (€/MWh) [α_{j1}]	27.23	27.42	27.13	27.50	1.27
	(2.59)	(1.19)	(1.42)	(1.33)	(0.06)
CCGT (€/MWh) [α_{j1}]	36.74	39.76	39.57	40.24	1.75
	(4.23)	(2.55)	(2.74)	(2.65)	(0.14)
Coal X q (€/MWh^2) [α_{j2}]		4.51E-2	4.24E-2	4.34E-2	2.04E-3
		(7.39E-3)	(1.11E-2)	(8.93E-3)	(3.76E-4)
CCGT X q (€/MWh^2) [α_{j2}]		2.11E-2	2.05E-2	2.06E-2	8.40E-4
		(6.53E-3)	(6.63E-3)	(6.60E-3)	(3.14E-4)
Coal ramp (€/MWh^2) [α_{j3}]			4.06E-3	3.82E-3	1.63E-4
			(3.64E-3)	(3.63E-3)	(1.77E-4)
CCGT ramp (€/MWh^2) [α_{j3}]			1.75E-3	7.27E-4	2.57E-5
			(1.15E-2)	(1.71E-3)	(7.30E-5)
Forward Position (%) [γ_1]	82.34	91.41	91.17	92.10	91.19
	(5.56)	(2.54)	(2.71)	(2.88)	(3.00)
Forward Peak (%) [γ_2]	82.51	90.87	90.59	91.36	90.58
	(6.44)	(3.06)	(3.26)	(3.43)	(3.60)
Forward Weekend (%) [γ_3]	84.36	92.21	91.79	92.65	91.81
	(10.70)	(4.30)	(4.47)	(4.63)	(4.82)

Notes: Sample from March to July 2007. Coal and combined cycle (CCGT) estimates present average of unit-specific parameters. Input costs variable in specification (5) constructed with European fuel prices of coal, natural gas and oil. Heat rates as provided in reports by the Spanish Ministry of Industry. Standard errors computed using block-bootstrap at the week level.

The table also includes the forward position at the firm level, which is estimated to be around 91% of the quantity that is sold in the day-ahead market.[47] The forward position appears to be lower during peak hours and slightly higher on weekends, although the estimated differences are very small and not significant.

It is important to note that the specification without quadratic costs differs in the implied forward position of the firms. This is due to the fact that an increasing bid can be rationalized in two ways. First, it can imply that the inframarginal quantity is larger and thus the markup component increases. Second, it can imply an increase in the marginal cost. Because the linear specification does not allow for increasing marginal costs, increasing bids need to be rationalized as higher incentives of firms to markup, given by a lower level of forward contracting. The preferred specification is the one with quadratic costs, given that they are usually considered when modeling the function of these units in engineering terms. The model also gives a better fit from a statistical point of view, according to an F-test of the overall fit of the model.

Specification (4) includes the same specification as in (3), but instrumented with lagged data. One can see that the results do not change significantly. In specifications (5), I use information on thermal rates as well as fuel prices at European markets to control for variation in fuel costs over time. I find that fuel costs computed using thermal rates and European coal prices have an estimated coefficient of 1.27 for coal plants, which is consistent with the fact that coal costs in Spain are higher than in most of European markets.[48] This can be a result of variable operation and maintenance costs as well as transportation costs, not included in fuel prices. One can see that forward contract estimates do not change substantially when variation of fuel prices over time is accounted for in a more parametric fashion. For the rest of the analysis, I use the specification in column (2) as the baseline specification.

Concerns and alternative specifications One of the critiques of the above approach is that the econometrician needs to make a choice on how much to smooth the residual demand in order to construct the moments. The estimates of the inverse residual demand slope could be sensitive to the smoothing parameter. In order to assess this concern, I check several different smoothing parameters, which are relegated to the appendix. A table is included in which the estimates in Table 6.2, column (2), are presented under alternative assumptions of the bandwidth parameter. Results for marginal cost estimates are not too sensitive to the smoothing parameter as long as the smoothing parameter is relatively small.

With a bandwidth parameter between 1 to 5€, the implied forward position ranges between 87 to 97% approximately. Coal estimates range roughly between 26 to 29€, whereas combined cycle gas estimates are more sensitive to the bandwidth parameter, and range between 35 to 43€.In the

[47] Even though 91% might appear to be a large share of forwarded quantity, previous studies in markets in which forward data were available document also large shares of forward contracts. Wolak (2007) documents an average forward position of 88% in the Australian electricity market. It is also consistend with informal discussions with industry participants, who mentioned that they tend to forward a large share of their expected output.

[48] I do not have data of thermal rates for combined cycle gas plants, for which I use an engineering benchmark rate. The results suggest that the thermal rates for gas plants are higher than the ones used.

baseline specification, the bandwidth parameter is set to 3€. In Figure F.3, I show that a bandwidth parameter in that range captures well both the residual demand and its slope. Bandwidth choices that are either too small or too large will tend to exacerbate or attenuate its slope, respectively.

6.2 Startup cost estimates

I perform the estimation of startup costs using the information contained in complex bids. The firm examined makes use of complex bids actively, which allows me to identify the startup cost for all units.

I estimate the startup cost using Equation (5.2). In order to construct a sample of the expected $\Pi^{in}(\alpha,\gamma),\Pi^{out}(\alpha,\gamma)$ at the point at which the minimum revenue requirement is just satisfied, I randomly draw market outcomes and estimate the profits for each unit of either being in or out. I take 100 random samples for every day in order to make sure that observations next to the minimum revenue requirement are sampled. Sampling different random draws provides observations around the point at which the minimum revenue requirement is just satisfied. The simulation procedure results in J moments given by,

$$\overline{f}_j(\hat{\alpha},\hat{\gamma},\beta,bw) = \frac{1}{TB}\sum_{t=1}^{T}\sum_{bs=1}^{B} w_{jt}^{bw,bs}\Big(\sum_{h=1}^{24} p_{ht}^{*,bs} q_{jht}^{*,bs} - C_j(\mathbf{q}_{jt}^{bs}\hat{\alpha},\beta) - \Delta\Pi_{i\setminus j,t}^{off,bs}(\hat{\alpha},\hat{\gamma})\Big) = 0.$$

Figure 6.1 gives a graphical representation of the estimation around the minimum revenue requirement. The difference in profits is represented in the Y-axis against the net revenue of the unit. As one can see, due to uncertainty in the market, different profits can be achieved at the same level of net revenue. The startup cost is estimated as the expected difference in profits around the point at which the net revenue is zero using kernel weights. Given the observed linearity of the difference in profits, I control for the slope induced by the minimum revenue requirement using a locally linear regression.

This sampling process provides an estimate of the difference in profits of having a unit accepted or not. This implies that the startup cost is estimated as,

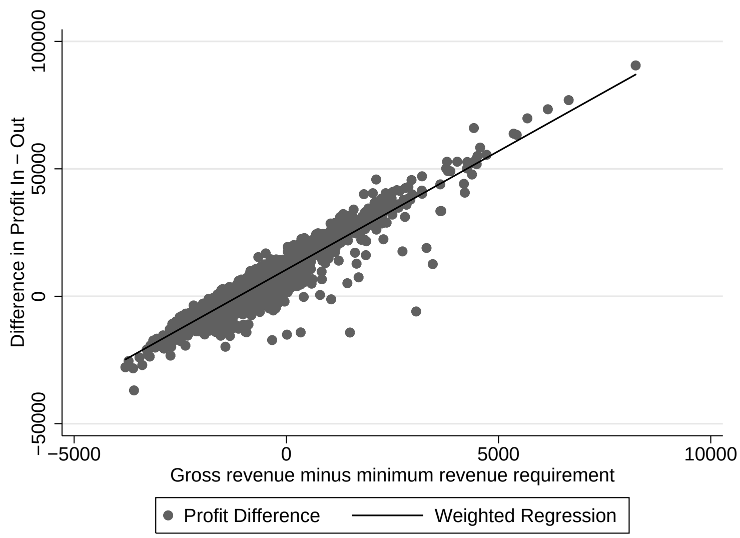
$$\Delta\hat{\Pi}_{jt}^{in-out}(\hat{\alpha},\hat{\gamma}) = \beta_j + \epsilon_{jt}$$

when the minimum revenue requirement is just satisfied. However, as highlighted in the previous section, these estimates will provide in general a lower bound to startup costs due to the presence of the continuation value of the startup.

In order to control for the continuation value, I use a reduced form approach in which I project the estimated profit difference $\Delta\hat{\Pi}^{in-out}$ to a specification with weekday fixed-effects, motivated by the fluctuation of the continuation value due to the weekly cycle. Because there are not enough observations at the unit level to estimate a specification with weekday fixed effects,[49] I pool units

[49]Note that even if I observe complex bids frequently at the unit level, market prices are not always in a range in which the minimum revenue requirement is just satisfied. Furthermore, some units have production contracts and

Figure 6.1: Pointwise Estimation of Fixed Costs



Observations around the point at which the minimum revenue is just satisfied are used to identify fixed startup costs for a given unit (GUA1). The expected intercept at zero provides the estimate of startup costs.

within thermal groups. The coefficients on the covariates are not unit specific, but specific to the type of fuel (combined cycle gas or coal, indexed by f). I include controls for the day of the week, that should capture partially the option value associated to each day. Given that combined cycle units have standard sizes of either 400MW or 800MW approximately, I compute the startup cost estimation separately for the two groups and do not include a size control. For coal plants, I include a coefficient on the size of the plant, which is highly correlated with its startup costs. The specification for coal plants is given by,

$$\Delta\hat{\Pi}_{jt}^{in-out}(\hat{\alpha},\hat{\gamma}) = \beta_{f0} + \beta_{f1} MW_j + \eta_{ft}^{weekday} + \epsilon_{jt}.$$

Results from the startup costs estimation are presented in Table 6.3. In the baseline specification (1), weekday fixed effects are included and Friday is taken as a baseline. Units reportedly unavailable due to maintenance or outages are also not included in the baseline specification, given that their complex bids might reflect this unavailability to produce. Due to the possible bias in the submission of complex bids, units in systematically congested areas during the period of study. Due to the systematic congestion, these units are not included in the specifications in which congested units are excluded.[50]

Specification (1) shows that coal plants have startup costs that are increasing in the plate

do not use complex bids so frequently. Finally, some units are usually turned on and therefore only deciding their startup infrequently.

[50] Congested units are often required to produce after the day-ahead market has been closed, being compensated with congestion rents. Therefore, a given unit that is discarded due to its complex bid, can be actually starting up later on. In such case, the complex bidding decisions do not fully reflect the decision of starting up the unit, but also incorporate incentives from the interaction between the day-ahead and the congestion market.

Table 6.3: Startup Cost Estimates by Type of Fuel

	Friday FE (1)	**No FE (2)**	**Unavailable (3)**	**Congested (4)**	**Non-strategic (5)**
Coal (€)					
cons	-11877.5	-11353.4	-10769.3	-10769.3	-13470.3
	(2424.0)	(2453.6)	(2456.1)	(2456.1)	(1999.7)
mw	125.3	123.7	122.9	122.9	176.8
	(11.1)	(11.2)	(11.0)	(11.0)	(10.4)
- Implied costs:					
150MW	6,915.1	7,199.9	7,662.7	7,662.7	13,056.5
	(1,347.1)	(1,274.6)	(1,340.2)	(1,340.2)	(2,535.6)
350MW	25,707.7	25,753.2	26,094.8	26,094.8	39,583.2
	(1,826.2)	(1,708.2)	(1,724.7)	(1,724.7)	(3,700.3)
CCGT (€)					
400MW	11,999.4	8,923.3	12,667.3	7,459.4	22,397.2
	(11,537.9)	(11,585.0)	(11,282.6)	(8,492.1)	(14,184.6)
800MW	20,116.0	16,163.0	20,523.4	9,895.6	40,924.3
	(14,179.7)	(14,641.5)	(14,346.4)	(10,191.0)	(17,061.7)
Weekday Controls	Y	N	Y	Y	Y
Unavailable Excluded	Y	Y	N	N	Y
Congested Excluded	Y	Y	Y	N	Y
Strategic Firm	Y	Y	Y	Y	N

Notes: Sample from March to July 2007. Dependent variable is the difference in profits of getting one plant in or out from the market. Estimates computed using a locally linear regression around observations for which the minimum revenue requirement is just satisfied. Regression performed by fuel groups controlling different plant sizes. Standard errors include first stage variance. Specification (5) is computed assuming that the firm is not strategic, ignoring the effects of withholding capacity on price.

capacity of the plant. They range approximately between 6,000€ and 26,000€. Coal estimates are not very sensitive to a regression without weekday controls or to the inclusion of congested and unavailable, as can be seen in specifications (2)-(4). The icnlusion of unavailable units has the expected effect of increasing startup cost estimates. On the contrary, if the strategic value of not starting up is ignored, then the implied startup costs are significantly larger, as implied by the confidence intervals in column (5). This highlights the strategic value of capacity withholding through the use of complex bids.

Gas plants have startup costs ranging from 12,000€ (400MW) to 20,000€ (800MW), as shown in column (1). The estimates for gas plants are more sensitive to the removal of weekday fixed effects and to the inclusion of congested units, as can be seen in columns (2) and (4). The effects of congestion on the startup estimates are particularly large. The bias of the estimate is downward, being even negative for the congested units if unit fixed effects are included.[51] Similar to the coal units, ignoring the strategic effects of complex bids lead to larger startup costs, as shown in column (5). In this case, I find that not accounting for strategic behavior doubles the estimated startup costs of combined cycle plants.

The standard errors in the startup costs are tight for coal plants, but are much larger in the estimation of combined cycle plants. The larger standard errors are due to the larger variance in the marginal cost estimates from the first step, which directly affects the startup cost estimates. An estimation ignoring the first step variance produces much tighter standard errors for these plants.

6.3 Replication exercise

In order to assess the validity of the model, I compute the firm's optimal strategy given the forward contract, marginal and startup cost estimates. The model computes the optimal quantity strategy of the firm assuming that there is no uncertainty and firms have perfect foresight.[52] Given the dimensionality of the game and the presence of dynamics, the unit commitment problem of the firm is solved iteratively with a finite horizon model looking at a week ahead.[53] The model should capture, at least in average, the price, quantity and startup distribution that is observed ex-post in the data.

In order to construct confidence intervals around these average simulations, I also simulate optimal outcomes when I take the lower and upper confidence intervals of marginal costs. For startup costs, I take the interquartile range, given the lower power for the combined cycle estimates. I also compute the case in which startup costs are zero. Figure 6.2 shows the daily pattern of four main variables replicated by the model. The first figure shows the evolution of prices over the day. In gray, the original data is plotted. One can see that the model predicts very accurately the evolution of average daily prices. The daily quantity patterns are also captured by the data. In figures 6.2(c) and 6.2(d) I also compute the optimal strategy if I ignore startup costs. The simulations show that it is important to account for startup costs in order to rationalize the startup decisions taken by the firm at the unit level.

Figure 6.3 shows the evolution of the same variables over time. The results show that the model reflects accurately the evolution of prices and quantities over time. In most of the range, average prices and quantities are within the confidence intervals. For certain periods of the sample, particulary the third quarter of the sample, the model does not fit firm decisions as well, understating the number of startups, as can be seen in figure 6.3(c). This might be due to cost shocks at a temporal level that the estimation does not capture. However, the overall fit reflects the evolution

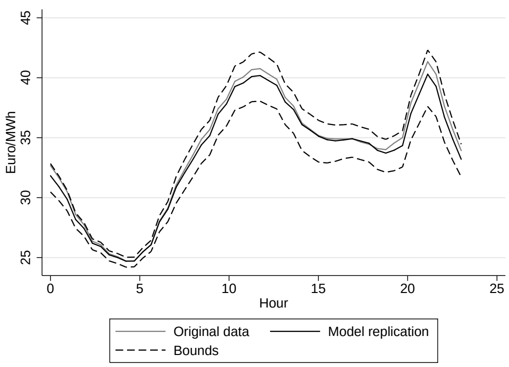
[51] Even though this seems counterintuitive, part of the bias is driven by the fact that marginal costs are overestimated for these units. Using the gas marginal costs of uncongested units in the subset of congested units predicts larger startup costs for congested units, which goes in line with the intuition that these units have some opportunity cost whenever they are accepted, as they give up congestion rents.

[52] In the absence of uncertainty, the bidding problem of the firm can be reduced to the firm choosing quantities for every unit and every hour of the day. In the Section 7, I consider a bidding model in which the firm faces uncertainty.

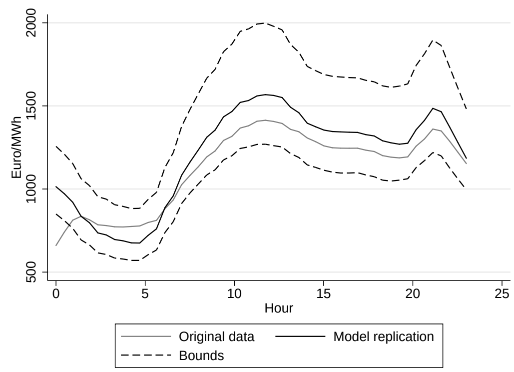
[53] A week ahead horizon is reasonable given that it captures the weekly cycle and is often used in the industry to choose startup decisions.

Figure 6.2: Hourly Patterns from Replication Model

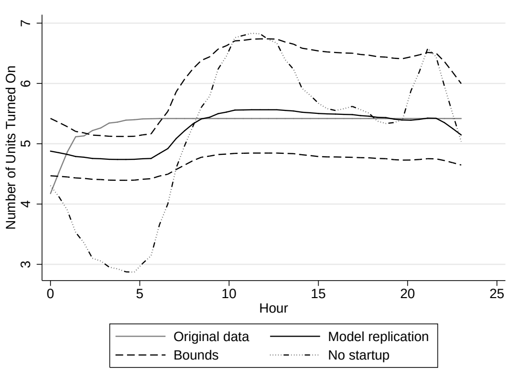
(a) Average Market Price over the Day



(b) Average Thermal Quantity over the Day



(c) Average Number of Started-Up Units over the Day



(d) Average Started-Up Capacity over the Day

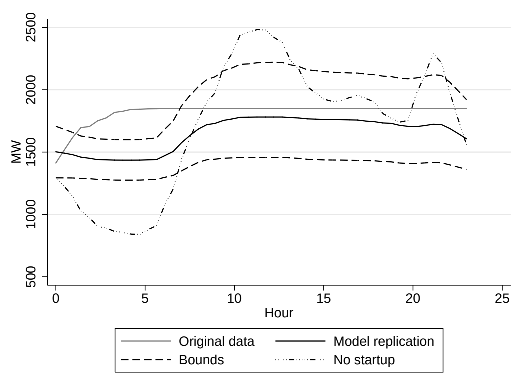
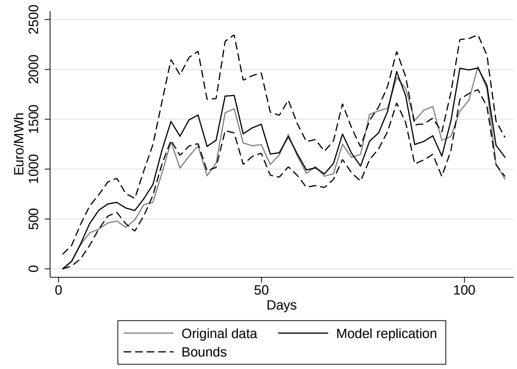


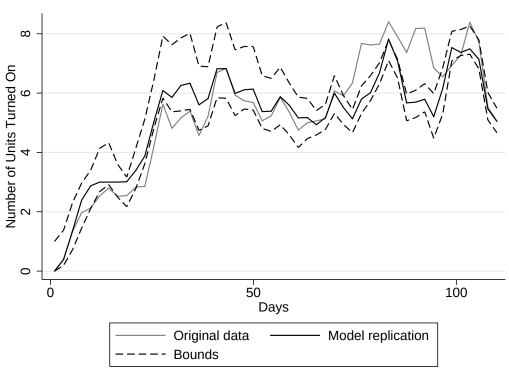
Figure 6.3: Time Patterns from Replication Model

(a) Average Market Price over Time

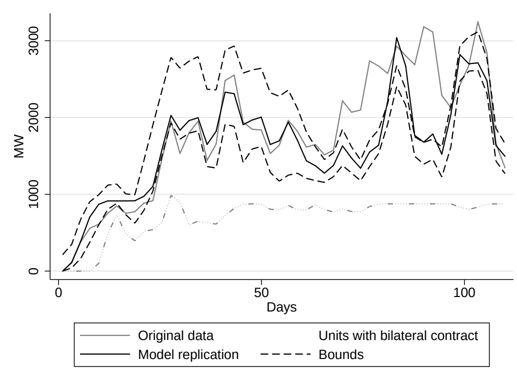
(b) Average Thermal Quantity over Time



(c) Average Number of Started-Up Units over Time



(d) Average Started-Up Capacity over Time



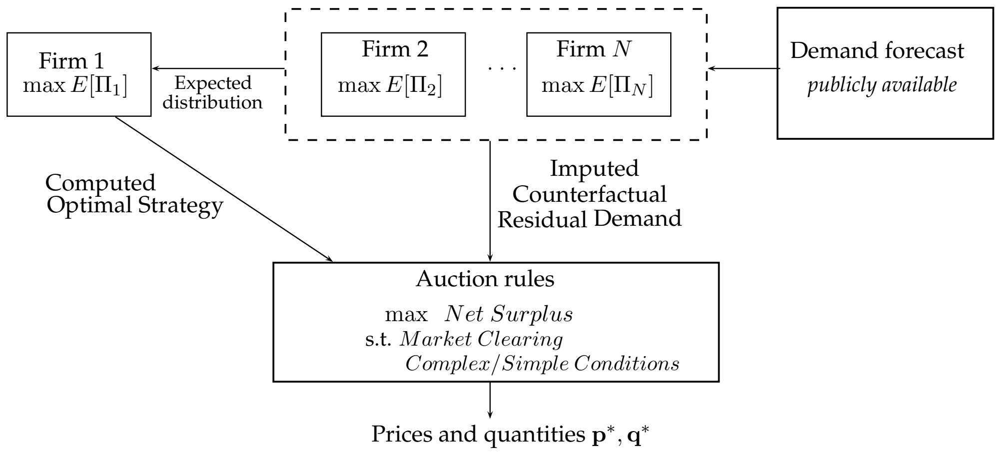
of production patterns over time.

7 Counterfactual simulations

In this section, I present counterfactual simulations to assess the welfare effects of complex bids. The goal of this counterfactual analysis is to separately quantify the efficiency benefits of the complex mechanism versus its market power effects. The exercise consists in comparing market outcomes under the original market design to one in which complex bids are not allowed.

One of the challenges of performing a counterfactual analysis is that, to date, there are no known approaches to characterizing equilibrium strategies in a comprehensive fashion in the context of multi-unit auctions with step bids such as the ones studied here. These difficulties are aggravated in the presence of complex bids and startup costs, which make the cost structure of the firm non-convex. For this reason, the counterfactuals are computed for a strategic firm facing a given residual demand, taking the behavior of the other firms as given. This approach ensures that a global optimum for the firm's strategy is found, as the problem of the firm can be solved exhaustively.

Figure 7.1: Computation of Counterfactuals



The counterfactual model computes the optimal best response of a given firm, given the distribution of other firms' actions. A counterfactual residual demand is computed based on likely changes in rivals' strategies in the absence of complex bids.

To capture more general equilibrium effects of the bidding mechanism change, I adjust the residual demand faced by the strategic firm in a way that seems plausible given the characteristics of the mechanism change, based on the optimal bidder behavior implied by the single firm computational model as well as observed bidding patterns in the market. The main idea behind the endogenization of residual demand changes is based on the fact that, without complex bids, the model predicts that firms will make their on/off decisions ex-ante. This provides me with a simplified way to account for changes in the uncertainty faced by the strategic firm and can be informative about the coordination effects of the mechanism change.

7.1 Counterfactual strategy

Figure 7.1 represents the counterfactual model analyzed. The counterfactual model solves for the optimal strategies of Firm 1, both with and without complex bids. Strategies for other firms are endogenized in a more indirect fashion, based on the actual data from the market as well as a simple prediction of firm behavior under simple bids.

7.1.1 Optimal bids for the strategic firm

Solving the problem for a single firm is in itself a computationally intensive task, due to the dimensionality of the game being played and the presence of non-convexities. In order to implement the counterfactual computation, I use a modified version of the model in Cerisola et al. (2009), which I adapt to introduce complex bidding. The idea underlying the model is to convert the problem of the firm into a linear mixed-integer program (MIP) that can be solved exhaustively in a reasonable amount of time. In order to linearize the problem of the firm, both the residual demand of the firm as well as its gross revenue are approximated with piece-wise linear functions. For a large number of pieces, the approximation gives a modified problem that converges to the original one.

The model solves the day-ahead bidding problem of the firm, in which optimal bidding strategies are computed given the expected residual demand. Expected residual demand is simulated by drawing a set of residual demands from its estimated distribution. The firm solves then for the optimal bidding strategy taking into account the uncertainty in the market. In order to account for the continuation value of the startup, I allow the firm to look five days ahead.

More details regarding the computational implementation can be found in Appendix D.

7.1.2 Counterfactual residual demand

One of the crucial aspects of the simulations is to endogenize the response of residual demand to a change in the bidding mechanism. To adjust the residual demand, I make use of the fact that firms appear to commit their units ex-ante whenever they do not have a complex bid, as explained in section 3.5.

This behavior is consistent with the computational model used for a single firm. Intuitively, if startup costs are large enough and there is enough uncertainty, it can be optimal to decide the status of the unit ex-ante, as it might not be possible to submit bids that are consistent with a smooth pattern of operation across hours. This form of bidding is also consistent in industry short-term scheduling tools for electricity generation, in which firms tend to decide their commitment decisions in the medium run (week) and then take them as given when deciding their daily bids.

Based on these facts, I construct counterfactual strategies for the other firms by bootstrapping strategies across similar days, as originally made in the estimation section.[54] However, instead of randomly sampling all the bids of the firms and solving for the complex bidding auction rule, I only randomly sample the accepted simple bids across days and solve for a uniform auction with no complex bids. This implies that I take the startup decisions that happened ex-post due to the complex bids, as if they had been taken ex-ante.

Because the equilibrium prices will change once I allow the behavior of the strategic firm to change, I iterate the startup decisions of non-strategic firms. Given that I observe their complex bids, which define a contingency plan, I can check which units obtain their minimum revenue requirement at the new prices.[55]

Even though this is a limited experiment, it illuminates the effects of complex bids on coordinating entry, capturing the fact that in the absence of complex bids the distribution of residual demand is more volatile. Further details on the construction of the counterfactual residual demand

[54] Similar days were defined are pooled depending on whether they are a Monday, another weekday, a Saturday or a Sunday, as well as depending on the maximum hourly levels of predicted demand.

[55] Note that this iteration still takes the bidding strategies of other firms as given and, therefore, it has good contraction properties. Even though the iteration needs not to converge to a single point, due to the discrete nature of startup decisions, it has well-defined bounds that in practice are very small. In the results subsection, I report the last iteration obtained.

Table 7.1: Counterfactual Market Results

	Price (€/MWh)	Weighted Price (€/MWh)	Quantity (MWh)
Complex Bids			
with Original Demand	34.24	39.01	1,202
Simple Bids			
with Adjusted Demand	36.95	44.23	1,353
Difference (%)	7.90	13.39	12.60

Notes: Weighted price represents market price weighted by the quantity produced by the firm. Main effect compares the original counterfactual with complex bids to the one in which firms can only use simple bids and the residual demand has been adjusted.

curve as well as a comparison of implied uncertainty of the original and adjusted distributions of residual demand are included in Appendix E.

7.2 Counterfactual results

I present in this section the main results regarding the counterfactual simulations. I compare the market performance with and without complex bids, when a strategic firm faces the original residual demand and when the firm faces an adjusted demand that approximates the responses of other firms to the counterfactual change. I consider four strategic cases: complex bidding with original residual demand, complex bidding with adjusted residual demand, simple bidding with original residual demand, and simple bidding with adjusted residual demand.[56] I also compute the first best equilibrium under both the original and the adjusted residual demand curve.[57] The simulations are computed over the whole sample period, totaling 116 days.[58] I run several simulations with different random draws that correspond to different evolutions of demand realizations over the period. In the discussion, I focus on the comparison between the complex bidding mechanism with the original residual demand and the simple bidding mechanism with the adjusted demand, as this is the comparison that reflects the overall effects of the policy counterfactual. I also comment on the insights coming from the rest of cases.

7.2.1 The effects of complex bids

I examine the equilibrium prices and quantities to assess the performance of the market under complex and simple bids. Table 7.1 presents average measures of prices and quantities. From the

[56] I define the simple bidding mechanism as the auction with no complex bids, which is a set of simultaneous uniform auctions with no complementary bidding.

[57] The first best equilibrium minimizes the costs of producing the residual demand left in the market, assuming that the residual demand represents the marginal costs of producing with units other than the thermal generators of the strategic firm.

[58] Note that the last days cannot be simulated as the simulations look five days ahead to account for the continuation value of the week.

Table 7.2: Counterfactual Aggregate Measures

	Net Revenue	Thermal Profits	Unit Costs	Dispatch Costs	Overall Payments
Complex Bids with Original Demand	-18,640	11,420	35,463	106,791	783,574
Simple Bids with Adjusted Demand	-16,441	19,027	40,832	107,314	847,663
Difference (%)	11.80	66.61	15.14	0.50	8.18

Notes: Average hourly values in €.

table, one can see that the complex mechanism with the original residual demand captures well the average price actually observed in the market. In the simulation with complex bids, the price equals 34.24€/MWh. This is very similar to the average price in the data (34.14 €/MWh) and the average price in the bootstrapped data with complex bids (34.24 €/MWh), as can be seen in Table F.1.

Once complex bids are removed, one can see that the average market price increases, moving from 34.24€/MWh to 36.95€/MWh (7.90%). The increase in prices occurs even though the thermal quantity produced by the firm increases as a response to higher prices due to the more volatile residual demand. In order to assess whether the firm is more or less aggressive in bidding, in Appendix E I perform a naive counterfactual in which I take the decisions of *all* firms as given, and compare market outcomes with the original and the adjusted residual demand. The simulation predicts higher volatility in the market as well, but the increase in market prices is not as large, around 3.79%. This fact highlights the importance of accounting for strategic responses to a more volatile and inelastic residual demand curve.

Table 7.2 presents results regarding overall firm revenues. Note that the average net revenue is negative. This is because the net revenue does not include the revenue coming from forward and production contracts, which are sunk at the time of the day-ahead market. One can see that the firm obtains higher revenues under the simple bidding mechanism, due to the increase in prices.[59]

To get another sense of firm profits, I compute a measure of thermal profits. These profits include the gross revenue obtained with thermal plants minus thermal production costs. One can see that firms make net positive profits with their thermal generators. One can see that the firm experiences an increase in thermal profit in the absence of complex bids, which increase by 66.61%. This is mainly due to the fact that prices are larger in the market, but also due to the increased thermal production.

I also compute aggregate measures that give a sense of the overall efficiency of the allocation and the distributional implications of the implied market prices. Column 4 in Table 7.2 presents the

[59] Note that, if the residual demand were held constant, the firm would be expected to obtain weakly higher net revenues under the complex bidding mechanism, as in this setting with a single firm facing a residual demand complex bids expand the bidding possibilities of the firm.

Table 7.3: Counterfactual Market Results

	Price (€/MWh)	Weighted Price (€/MWh)	Quantity (MWh)
Original Demand			
- Complex Bids	34.24	39.01	1,202
- Simple Bids	34.00	38.40	1,253
Adjusted Demand			
- Complex Bids	37.19	45.19	1,297
- Simple Bids	36.95	44.23	1,353

Notes: Weighted price represents market price weighted by the quantity produced by the firm. Main effect compares the original counterfactual with complex bids to the one in which firms can only use simple bids and the residual demand has been adjusted.

overall cost of serving the residual demand in the market, where the residual demand not covered by the strategic firm is evaluated assuming that the residual demand represents the marginal cost of the alternative generating units. This is a partial measure of cost, as the supply curve of other firms does not necessarily reflect their costs even if they are competitive. For example, as noted above, units might submit zero bids to ensure operating. Taking into account these caveats, the comparison suggests that complex bids induce slightly lower overall costs, although the effects are minor.

Finally, I include in column 5 the total payments made by consumers in the day-ahead market, taking into account the distribution of market prices implied by the two mechanisms. This measure reflects the consequences of higher prices to consumers due to strategic behavior. One can see that differences in payments are of 8.18%, which can imply approximately 550M€ annually.[60] This implies that payments at the electricity markets could increase substantially.

7.2.2 Market power versus startup coordination

In the beginning of the paper, I explained that complex bids could have potentially negative market power effects. Yet, the main result that emerges is that it actually enhances competition by allowing more flexible startup decisions. In this section, I show that, holding the residual demand fixed, a single firm could actually exercise market power and increase prices modestly.

As can be seen in table 7.3, holding the residual demand fixed I find that the simple bidding mechanism induces slightly lower prices on average. With the original residual demand, prices move from 34.24€/MWh to 34.00€/MWh. This reduction in price is due to the fact that the firm tends to withhold less quantity using the simple bidding mechanism. I find similar results when I compare the two mechanisms with the adjusted residual demand. The firm produces 11.29% less when complex bids are allowed, confirming that the complex mechanism allows the firm to withhold its capacity. This translates in a price increase of 0.78%.

[60]Note that the policy counterfactual is evaluated only during four months of the year. Due to seasonality, overall changes might be different if seasonal fluctuations significantly affect price differences across the two counterfactuals.

Table 7.4: First Best Market Results

	Price (€/MWh)	Weighted Price (€/MWh)	Quantity (MWh)
First Best			
- Original demand	32.42	36.18	1,436
Implied markups/changes (%)			
- Complex with original demand vs First Best Original	5.6	7.8	-16.3
- Simple with adjusted demand vs First Best Adjusted	11.6	16.3	-17.1

Notes: Weighted price represents market price weighted by the quantity produced by the firm.

This intermediate effect highlights the tradeoff between market power and efficiency in this market. In the presence of a single firm, holding the uncertainty in the market constant and given the relatively low uncertainty in the market, efficiency savings through complex bids are not sufficient to justify their introduction. However, to the extent that removing complex bids introduces larger volatility in the market, the positive effects of coordination overcome the market power concerns.

7.2.3 Comparison to first best

The methods presented in this paper can be used to compute a measure of market power that accounts for startup costs in both the strategic and the first best scenario. I compare the main results in table 7.1 to the ones in which expected costs are minimized, which are presented in table 7.4. With respect to the first best counterfactual, average prices in the market are 5.6% and 11.6% larger under complex and simple bidding, respectively. Note that these price differences can be interpreted as markups.[61] The increase in these markups when the residual demand becomes more volatile reinforces the thesis of increased incentives to exercise market power in the presence of a more volatile demand.

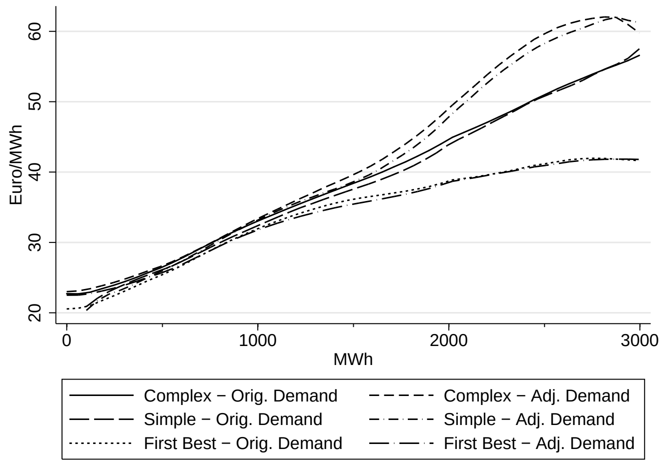
Figure 7.2 represents the equilibrium supply curve of the firm under the six different scenarios considered. One can see that the supply curve in the first best scenario is very similar under both residual demand realizations. Even though removing the complex bidding mechanism increases the volatility of the residual demand, the equilibrium supply curve remains more or less constant. This contrasts with the response of the strategic firm, which increases its markups in the presence of a more volatile and inelastic residual demand.

7.2.4 Discussion of the validity of results

The results of the counterfactuals are subject to different assumptions and modeling decisions that I have taken in the analysis. In this section I discuss some of the main issues and the possible

[61] Markups with respect to marginal costs only can be misleading in this setting in which firms have non-convex costs of operation.

Figure 7.2: Equilibrium Average Supply Curve



Notes: Quadratic fit reflecting equilibrium prices and quantities in the market.

implications with regards to the counterfactual findings.

As discussed above, the analysis does not take into account congestion and sequential markets, which could change the overall production patterns of the units. In the data, I find important adjustment in the congestion markets and relatively minor adjustments in sequential markets (except for those directly related to congestion). However, without complex bids, these patterns could change and sequential markets could gain importance. Such changes would potentially not affect substantially the finding that the day-ahead market prices increase due to volatility, as these adjustments occur once the day-ahead market has cleared. However, changes in production patterns could be important, which could affect the final costs of production.

Another aspect that I do not endogenize is the changes in production and financial contracts, which I hold fix in the simulations. Given that firms seem to use production contracts as a way to schedule their production, one could expect firms increasing this type of arrangements, which could reduce the volatility in the market given that they usually exhibit certain persistence. Regarding financial contracts, an increased volatility could potentially increase the incentives to hedge firms production. Given that the structural model has a reduced form equation for choosing forward contracts, one could potentially endogenize this decision partially and do sensitivity analysis with the estimated forward parameter.[62]

The model does not incorporate the decisions of technologies that do not use complex bids, such as renewable energy, nuclear plants and hydro power plants. Whereas holding production fixed for wind and nuclear resources is a reasonable assumption, the bids of hydro resources could change substantially in the presence of more uncertainty. A treatment of the optimal strategies in

[62]Preliminary analysis in this direction suggests that updating the forward position in an iterative fashion does not have good convergence properties. In future research, it would be interesting to develop a computational method in which production decisions that are consistent with the reduced form equation for forward contracts can be solved simultaneously.

the presence of hydro power plants is beyond the scope of this paper.

Finally, the response of other firms is modeled in a limited fashion, exploiting the fact that complex bids provide a contingency plan even with changing market conditions. While this approach is reasonable for competitive firms, in which complex bids are on average equal to their cost parameters (which should presumably not be affected by the counterfactual change), this is more limited in the presence of other strategic firms. Given the findings for a single strategic firm, in which the firm withholds more capacity when complex bids are not available, one would expect that an iteration of each firm's decisions in this market would actually strengthen the results of increased prices in the absence of complex bids.

8 Conclusions

I study the benefits and costs of introducing augmented forms of bidding in the wholesale electricity auction, often referred as "complex bids." Complex bids allow generating companies to link their valuations across different hours of the day, giving an horizontal dimension to the auction. The introduction of complex bids can potentially improve the efficiency of the market by allowing firms to better express their cost structure. However, in the presence of market power, complex bids also give firms another dimension with which to exert market power. Which of these effects dominates is theoretically ambiguous, so I assess this trade-off empirically in the Spanish electricity market.

Simulated counterfactual analysis of this market suggests that the complex bidding mechanism performs better than the simple bidding mechanism, even in the presence of market power. Holding the residual demand constant, the effect on consumers is an increase in prices and a slight increase in overall dispatch costs. However, once I account for the increased volatility in the market in the absence of complex bids, prices and overall costs increase. I find that, without complex bids, prices increase by 7.90% (2.71€) on average. Overall dispatch costs increase only modestly, suggesting that the distributional concerns of changing the market mechanism are larger than its associated production efficiency losses.

The analysis highlights the importance of complex bids as a mechanism to coordinate entry (startup decisions). If firms commit their units ex-ante, the residual demand becomes more volatile and inelastic, which enhances the incentives to exercise market power. Whereas the results on production costs are more limited, due to the focus of the counterfactuals on a single firm production function, the analysis reveals important distributional implications of removing complex bids, with payment increases of over 550M€. The results suggest that, to the extent that complex bids successfully reduce the volatility in the market, they have positive effects in this market.

There are several extensions that remain open for future work. First, I find that the mechanism studied does not give firms increased incentives to exercise market power and that overall it increases the elasticity of supply in the market, due to its positive coordination benefit. The nature of the problem could change substantially if the complex bidding mechanism were to be used to collude, for example by making it easier to coordinate. It would be interesting to explore whether complex bids could enhance collusion in this setting. Second, there are other complex mechanisms that could be used. In Reguant and Pérez-Arriaga (2010), we explore alternative bidding mechanisms that are used in existing electricity markets, analyzing its efficiency implications in competitive markets. We also consider its long run implications, accounting for fixed investment costs. Understanding the functioning of these alternative complex mechanisms in a strategic setting is a topic of future research.

Finally, I would like to explore how complex bids interact with other markets. In particular, I would like to study how firms use simple and complex bids in the presence of local congestion, given the evidence that I have found of persistent local congestions in this market. I plan to exploit an institutional break in the design of the congestion market. The day-ahead is an augmented uniform price auction, as presented in this paper, whereas the congestion market is a discriminatory auction. Before July 2005, firms had to submit the same bids for both markets. However, after July 2005, firms could separate the decision and bid in the two markets separately. This provides a quasi-natural experiment to understand optimal behavior under these two settings and opens the door to testing the assumption of optimal behavior, which is usually needed for identification. In this context, I would like to understand the antitrust and market monitoring implications of firms in congested areas withholding capacity, paying special attention to the role of complex bids in withholding capacity.

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A The ISO algorithm

I explain here the details of the algorithm that I use to simulate the ISO algorithm, as well as set of simulations to assess the accuracy of this pseudo-algorithm by comparing it to actual prices. First I discuss the actual algorithm used by the ISO and explain how the algorithm is approximated in my application. Finally, I compare predicted and actual market outcomes for a set of 90 days, and assess the magnitude of the bias in the predictions with respect to equilibrium prices as well as number and identity of complex bids that are binding.

A.1 Details of the actual algorithm

The details of the algorithm used by the ISO to compute the market outcome of the day-ahead market are explained in the "Appendix on the Functioning of the Wholesale Electricity Day-Ahead and Intra-day Markets."[63] Here I outline the major steps that are taken to solve for the optimal dispatch.

After receiving and verifying the supply and demand offers made by the market participants, the ISO solves for the optimal dispatch using the following order:

1. Construct aggregate supply and demand curves from simple bids taking into account merit order rules and interconnection constraints.

2. Solve for the optimal dispatch using these aggregate curves (uniform auction rule). Use established rules to deal with indivisible steps and ties.

3. Check ramping constraints at the unit level and change quantities to satisfy them (only check once for ramping up and once for ramping down).

4. Order the units with complex bids whose minimum revenue constraints are not satisfied according to the difference between their average required price and the average price they receive.

5. Discard the unit whose deviation is largest.

6. Repeat 1 to 5 until no complex bid binds. This is the provisional solution.

7. Order units with complex bids that have been discarded according to the difference between their average required price and the average price they would receive at *current* prices. Note that some units that have been discarded might be willing to produce at current prices.

8. Use the resulting merit order from step 7 in step 4 when repeating 1 to 5, again until no complex bid binds, to obtain a new provisional solution.

9. Repeat 7 to 8 until no discarded units would be willing to produce at current prices or stop if time exceeds 30 minutes or number of iteration is larger than 3,000, taking the provisional solution that minimizes the foregone rents of discarded units that would be willing to produce at current prices. This is the final solution.

Mimicking the ISO algorithm poses some challenges. In particular, the ISO algorithm can take up to 30 minutes to complete, which is computationally not feasible in my application, in which I need to simulate hundreds of market outcomes for each firm and each day. Furthermore, the treatment of the interconnections requires some information that I do not have available. For this reasons, it is important to approximate the ISO in an heuristic manner.

[63] Boletín Oficial del Estado num. 128, 05/30/2006, pp. 20157-20192 (in Spanish).

A.2 Details of the pseudo-algorithm

Implementing the ISO algorithm exactly is not possible for two main reasons. On the one hand, I do not have the information necessary to account for congestion at the interconnections. On the other hand, the procedure is computationally very costly. The ISO allows the algorithm to do up to 3,000 iterations during up to 30 minutes. However, for estimation purposes, I need to simulate market outcomes thousands of times. Therefore, there is a need to trade-off the trustworthiness of the pseudo-algorithm with its computational efficiency, but trying to preserve the actual outcomes of the algorithm as much as possible.

I follow García et al. (1999) to implement an heuristic ISO algorithm as a mixed integer linear programming problem. This problems takes into account the ramping constraints submitted by the units, as well as the indivisibility conditions of the steps as a single maximization problem. The minimum revenue requirements are dealt in a similar fashion than the actual ISO algorithm, although I only allow for one iteration. I plan to extend the iteration to better resemble the method used by the ISO.

The pseudo-algorithm is programmed as follows:

1. Solve a mixed-integer linear program that includes indivisibility and ramping constraints.
2. Order the units with complex bids whose minimum revenue requirements are not satisfied according to the difference between their average required price and the average price they receive.
3. Discard the unit whose deviation is largest.
4. Repeat 1 to 3 until no complex bid binds.

The pseudo-algorithm is implemented in Java and the mixed-integer linear program is solved using the commercial solver CPLEX 11.0, which is very efficient for this type of problems.

A.3 Simulations to assess the performance of the pseudo-algorithm

I present a comparison of actual and predicted prices by the pseudo-algorithm in Table A.1. The algorithm predicts the prices accurately and the difference between the two is not significant for any hour of the day. The overall error is small, with a mean close to zero. The predicted prices also present the same standard deviation as actual prices.

I also implement different version of the algorithm, some of which relax the integrality constraints. Some other algorithms allow for a certain degree of iteration, following the minimization criteria used by the ISO. Overall, I find the pseudo-algorithms to replicate very accurately the patterns across the different hours of the day. Results are presented in Table A.1.

Table A.1: Price predicted and simulation error at the hourly level

Hour	**MgPrice**	**Alg 1**	**Alg 2**	**Alg 3**	**Alg 4**	$\Delta 1$	$\Delta 2$	$\Delta 3$	$\Delta 4$
1	33.20	33.11	33.23	33.16	33.21	-0.09	0.03	-0.04	0.01
	(6.98)	(6.91)	(6.96)	(7.04)	(7.06)	(0.83)	(1.19)	(0.95)	(0.97)
2	29.77	29.98	30.07	29.97	29.98	0.20	0.30	0.19	0.21
	(5.40)	(5.39)	(5.36)	(5.46)	(5.45)	(1.03)	(0.96)	(0.88)	(0.86)
3	26.57	26.90	26.90	26.81	26.88	0.33	0.33	0.24	0.30
	(4.24)	(4.39)	(4.33)	(4.44)	(4.33)	(0.75)	(0.64)	(0.71)	(0.70)
4	25.45	25.63	25.67	25.64	25.71	0.18	0.22	0.19	0.26
	(3.86)	(3.99)	(4.02)	(3.97)	(3.81)	(0.70)	(0.76)	(0.68)	(0.67)
5	24.47	24.66	24.66	24.66	24.75	0.19	0.19	0.19	0.28
	(3.98)	(3.98)	(3.99)	(3.98)	(3.91)	(0.59)	(0.61)	(0.59)	(0.52)
6	24.47	24.72	24.71	24.61	24.71	0.25	0.24	0.14	0.24
	(3.53)	(3.49)	(3.49)	(3.50)	(3.35)	(0.56)	(0.58)	(0.47)	(0.50)
7	26.82	27.03	27.00	26.95	27.03	0.21	0.18	0.13	0.21
	(3.76)	(3.57)	(3.56)	(3.62)	(3.41)	(0.89)	(0.88)	(0.72)	(0.70)
8	30.39	30.73	30.70	30.74	30.81	0.33	0.31	0.35	0.42
	(6.28)	(6.25)	(6.29)	(6.28)	(6.23)	(1.09)	(1.34)	(1.07)	(1.12)
9	33.87	34.00	33.83	34.02	34.08	0.13	-0.04	0.15	0.21
	(7.71)	(7.50)	(7.47)	(7.52)	(7.47)	(1.09)	(1.23)	(1.11)	(1.14)
10	36.51	36.69	36.43	36.68	36.73	0.18	-0.07	0.18	0.22
	(7.84)	(7.87)	(7.80)	(7.89)	(7.93)	(1.17)	(1.33)	(1.16)	(1.20)
11	39.93	39.84	39.49	39.86	39.93	-0.10	-0.44	-0.07	-0.00
	(8.08)	(8.23)	(8.17)	(8.23)	(8.25)	(1.26)	(1.27)	(1.25)	(1.24)
12	41.44	41.44	40.99	41.46	41.51	-0.00	-0.44	0.02	0.07
	(8.81)	(8.97)	(8.90)	(8.95)	(8.90)	(1.30)	(1.60)	(1.18)	(1.17)
13	41.97	42.04	41.56	42.05	42.06	0.07	-0.41	0.08	0.09
	(9.40)	(9.50)	(9.42)	(9.49)	(9.47)	(1.28)	(1.64)	(1.20)	(1.19)
14	40.23	40.29	39.91	40.30	40.34	0.06	-0.32	0.07	0.11
	(8.73)	(8.74)	(8.76)	(8.73)	(8.78)	(1.19)	(1.48)	(1.11)	(1.14)
15	37.34	37.41	37.10	37.45	37.47	0.08	-0.23	0.11	0.13
	(8.12)	(8.03)	(8.09)	(8.03)	(8.07)	(1.12)	(1.26)	(1.06)	(1.04)
16	36.12	36.06	35.72	36.07	36.09	-0.06	-0.40	-0.05	-0.03
	(8.59)	(8.60)	(8.57)	(8.61)	(8.62)	(1.14)	(1.22)	(1.11)	(1.07)
17	35.96	35.84	35.44	35.82	35.85	-0.12	-0.52	-0.15	-0.11
	(9.30)	(9.18)	(9.21)	(9.16)	(9.14)	(1.01)	(1.37)	(0.93)	(0.89)
18	36.21	36.02	35.65	36.00	36.01	-0.19	-0.56	-0.21	-0.21
	(9.78)	(9.56)	(9.52)	(9.57)	(9.61)	(1.00)	(1.13)	(0.98)	(0.87)
19	35.00	35.07	34.69	35.08	35.14	0.07	-0.31	0.08	0.14
	(8.87)	(8.63)	(8.65)	(8.63)	(8.61)	(0.96)	(1.21)	(0.93)	(0.92)
20	34.30	34.33	33.97	34.30	34.32	0.03	-0.33	0.00	0.02
	(6.63)	(6.46)	(6.39)	(6.50)	(6.47)	(1.22)	(1.49)	(1.21)	(1.20)
21	35.93	35.33	34.79	35.40	35.51	-0.60	-1.14	-0.53	-0.42
	(6.31)	(6.29)	(6.04)	(6.32)	(6.30)	(1.92)	(2.24)	(1.64)	(1.61)
22	42.35	41.73	41.10	41.78	41.72	-0.62	-1.24	-0.57	-0.63
	(9.12)	(9.40)	(8.91)	(9.31)	(9.19)	(1.90)	(2.25)	(1.70)	(1.90)
23	38.82	38.53	38.11	38.54	38.57	-0.30	-0.72	-0.29	-0.26
	(7.37)	(7.50)	(7.25)	(7.51)	(7.47)	(1.35)	(1.61)	(1.37)	(1.20)
24	32.72	32.82	32.37	32.80	32.81	0.10	-0.34	0.09	0.10
	(5.32)	(5.61)	(5.34)	(5.65)	(5.58)	(1.20)	(1.30)	(1.19)	(1.17)
Total	34.16	34.17	33.92	34.17	34.22	0.01	-0.24	0.01	0.06
	(9.07)	(8.99)	(8.83)	(9.01)	(8.98)	(1.17)	(1.40)	(1.11)	(1.11)

Notes: Monte Carlo simulation covers the period of March-June of 2007. Prices are in €/MWh.

B Data sources

The major part of the data is obtained from the Market Operator website, http://www.omel.es. I obtain the bidding data from bidding files DET and CAB. Physical bilateral contracts are obtained from PDBF files. Congestion restrictions are obtained from PDVD files as well as from the System Operator I90 form, http://www.esios.ree.es. Similarly, unavailability of units as well as the reason of the unavailability are obtained from INDIP files and the I90 form. Outcomes of sequential markets as well as final dispatch data are obtained from PHF files.

Plant characteristics of the plants can be obtained from the annual statistic reports of the System Operator as well as from the structural data in their website. These include maximum capacity, vintage and main type of fuel. I complement the data set with fuel mix obtained from the Ministry of Industry registrar as well as emission rates obtained from the EPER registrar. I also obtain engineering thermal rates for previously regulated plants from several sources from the National Energy Commission and the Ministry of Industry.

I complement the data set with other information available at the System Operator's website. I acquire demand and wind production forecasts, which are made available before the auction is run to reduce balancing needs in real-time. The files are DEMAND_AUX and PREVEOL. I also get commodity price data to include it in the cost estimation. I use NBP day ahead prices for natural gas (UK), API coal indexes, and European ARA prices for low sulfur fuel-oil and gas oil.

C Model first-order conditions

C.1 Assumption regarding effects of simple bidding

This section provides an empirical verification that the term assumed to be negligible in Assumption 1 is indeed small. In other words, I show that the effect of marginal changes of bids on expected profits due to the changes on the probability of a bid being binding is negligible. In addition, I show that ignoring this effect does not seem to be a potential source of bias in the estimation.

First, I calculate the probability of a bid being binding and the revenue requirement being just satisfied. Then, I show that for plausible (estimated) parameter values, the contribution to the total derivative coming from changes in the probability of a bid being binding (holding profits constant) is very small compared to the contribution coming from changes in profits (holding probabilities constant). Finally, I show that the omission of the term coming from changes in probabilities does not correlate with neither simple bids nor with partialled out simple bids that depend on strategic markups.

The probability $Pr(R_{ij} = 0|b_{ijk} = p_h)$. I first compute the empirical probability $Pr(R_{ij} = 0|b_{ijk} = p_h)$. One of the challenges of the simulated outcomes is that usually the above probability will almost always be empirically zero. I use a kernel regression to estimate which bids correspond to revenue requirements just being satisfied. I use as a benchmark bandwidth parameter of 15,000€. Note that this bandwidth parameter is very conservative, as the fixed costs of the units are estimated to have values in similar magnitude. The weights are renormalized so that a probability of one is attributed whenever the revenue requirement is just binding. Using this method, I estimate that at least 30% of the observations have zero probability to both set the price and have minimum revenue requirements just being satisfied.

Assessing relative magnitudes. I now show that the relative contribution of marginal effects due to probability changes are very small when compared to the marginal effects of bids on profits.

$$\frac{\sum_{s\in S} \frac{\partial Pr(s)}{\partial b_{jkh}} \mathbb{E}[\Pi_i(\sigma_i, \sigma_{-i})|s]}{\sum_{s\in S} Pr(s) \frac{\partial \mathbb{E}[\Pi_i(\sigma_i,\sigma_{-i})|s]}{\partial b_{jkh}}},$$

which is the ratio of the contributions to the FOC due to the changes in the probabilities relative to changes on profits. Note that the numerator can be computed using that it can be expressed as

$$Pr(b_{jkh} = p_h) \sum_l Pr(\rho(R_{il}) = 0 | b_j kh = p_h) E[\Pi_i^{lin} - \Pi_i^{lout} | \rho(R_{il}) = 0, b_{jkh} = p_h].$$

This ratio is no greater than 10.55% on average, with an interquartile range with value of zero and median value 0. Trimming the outliers from the simulated outcomes (top and bottom 0.5%), this average ratio is reduced to 1.98%.

Assumption 1 and potential omitted variable bias. Finally, I show that the term

$$\sum_{s\in S} \frac{\partial Pr(s)}{\partial b_{jkh}} \mathbb{E}[\Pi_i(\sigma_i, \sigma_{-i})|s]$$

does not systematically correlate with either simple bids, partialled out simple bids and the main term in the FOC. The raw correlations between these terms and the omitted term is small (0.02-0.05) and not significant, both on average across units and conditional on a given unit.

C.2 First order conditions for simple bids

We need to show that $\frac{\partial \mathbb{E}\Pi_i}{\partial b_{jkh}} = 0$ implies

$$b_{jkh} = \overline{\zeta}_{jkh} + \sum_{s\in S|j \text{ in}} Pr(s|j \text{ in}) \sum_{q\in step\ k} \left| \frac{Pr(DR_h^{-1}(q) < b_{jkh} < DR_h^{-1}(q-1)|s)}{Pr'(b_{jkh} < DR_h^{-1}(q-1)|s)} \right| \frac{Q_{ih}(q,s)}{q_{jkh}}.$$

The expected profit is the sum of revenues minus costs over possible states s and equilibrium quantities q. In order to derive the condition, I first express profit as a sum of discrete incremental profits. This approach follows (McAdams, 2008), but it needs to be adapted to the presence of cost complementarities. Discretizing the quantity increments offered by the firm, I express the profit function as,

$$\mathbb{E}\Pi_i = \sum_{s\in S} Pr(s) \mathbb{E}\Big[\sum_{h=1}^{24} \sum_{q=0}^{Q} Pr(q_{ih}^* = q) \mathbb{E}[p_h^* | q_{ih}^* = q, s] Q_{ih}(q,s) \ - C\Big(q_i^*(s)\Big) \ \Big| \ s, I_i \Big].$$

where s is the state of complex bids accepted and q represents the total quantity of firm i that is offered at a price of p_h^* or less. In a usual setting, q can be interpreted as the total quantity assigned to firm i, but here one needs to take into account that part of it might be discarded by complex bids. $Q_{ih}(q,s)$ represents the actual net quantity that firm i is selling in the market at quantity q.

The revenue expression is separated across hours and quantity outcomes, given the independence of the uniform rule across hours once the complex bids that are accepted have been determined, given by the state s. The cost function cannot be so readily decomposed, due to cost complementarities. However, it can also be expressed as a sum of incremental costs at each period h, conditional on the expectations regarding contiguous periods.

Dropping the i subscript, I define the incremental cost of producing an extra MWh in a particular period h, when the output level is q, as,

$$\zeta_{jkh}(q,s) = \begin{cases} \alpha_{j1} + \alpha_{j2}(q_{jh}(q) - \underline{q}_j) + \alpha_{j3}(2q_{j,h}(q) - \mathbb{E}[q_{j,h-1}|q^*_{ih} = q] - \mathbb{E}[q_{j,h+1}|q^*_{ih} = q]), & \text{if unit } in \text{ at } s \\ 0, & \text{otherwise,} \end{cases}$$

where $q_{j,h}(q)$ represents the quantity produced by the individual output of the marginal unit j and α_j is a vector of unit specific cost parameters. Therefore, the average incremental cost at a given step is given by,

$$\overline{\zeta}_{jkh} = \begin{cases} \frac{1}{q_{jkh}} \sum_{s\in S|j \text{ in}} Pr(s|j \text{ in}) \sum_{q\in step\ k} \zeta_{jkh}(q,s), & \text{if unit } in \text{ at } s \\ 0, & \text{otherwise,} \end{cases}$$

where q_{jkh} is the offered quantity at the step, q represents the total quantity for the firm and, $q_{jh}(q)$ represents the unit output.

I consider the incremental profits for a particular hour h, and drop the firm index i and time index h for ease of notation. Define r_q as the inverse residual demand at q and b_q as the bid offer at q. Then, the expected probability of a given quantity q being accepted and the expected price at that q become,[64]

$$\begin{aligned} Pr(q^* = q|s) &= Pr(b_q < r_{q-1}|s) - Pr(b_{q+1} < r_q|s), \\ \mathbb{E}[p|q^* = q, s] &= \frac{Pr(r_q < b_q < r_{q-1}|s)}{Pr(q^* = q|s)} b_q + \\ &+ \frac{Pr(b_q < r_q < b_{q+1}|s)}{Pr(q^* = q|s)} \mathbb{E}[r_q|b_q < r_q < b_{q+1}, s] \\ &= \frac{Pr(r_q < b_q < r_{q-1}|s)}{Pr(q^* = q|s)} (b_q - \mathbb{E}[r_q|r_q < b_q < r_{q-1}, s]) + \\ &+ \frac{Pr(b_q < r_{q-1}|s)}{Pr(q^* = q|s)} \mathbb{E}[r_q|b_q < r_{q-1}, s] - \frac{Pr(b_{q+1} < r_q|s)}{Pr(q^* = q|s)} \mathbb{E}[r_q|b_{q+1} < r_q, s]. \end{aligned}$$

Substituting these equalities and pooling terms together, the gross profit function can be expressed as,

$$\mathbb{E}[\Pi] = \sum_{s\in S} Pr(s) \sum_{q=0}^{Q} \Pi_{q,s},$$

where

$$\Pi_{q,s} = \begin{cases} Pr(b_q < r_{q-1}|s)\Big(\mathbb{E}[r_q|b_q < r_{q-1}, s] - (q-1)\mathbb{E}[r_{q-1} - r_q|b_q < r_{q-1}, s] - \zeta_q(s)\Big) & \\ \quad -Pr(r_q < b_q < r_{q-1}|s)\Big(\mathbb{E}[r_q|r_q < b_q < r_{q-1}, s] - b_q\Big)Q(q,s)\ , & \text{if unit } in \text{ at } s \\ 0, & \text{otherwise.} \end{cases}$$

Each incremental profit depends only on the bid at the step k, and thus the total effect at the

[64] Note that I make use of the fact that $Pr(b_q < r_q < b_{q+1}|s) = Pr(b_q < r_q) - Pr(b_{q+1} < r_q) = Pr(b_q < r_{q-1}) - Pr(r_q < b_q < r_{q-1}) - Pr(b_{q+1} < r_q))$.

step is given by,

$$\sum_{s\in S} Pr(s) \sum_{q\in step\ k} \frac{\partial \Pi_{q,s}}{\partial b_q}.$$

It is sufficient to show that at each possible q in k, whenever a unit is *in*,

$$\frac{\partial \Pi_{q,s}}{\partial b_q} = \begin{cases} Pr'(b_q < r_{q-1}|s)(\zeta_q(s) - b_q) - Pr(r_q < b_q < r_{q-1}|s)Q(q,s), & \text{if unit } in \text{ at } s \\ 0, & \text{otherwise.} \end{cases}$$

First note that $Pr(r_q < b_q < r_{q-1}|s) = Pr(b_q < r_{q-1}|s) - Pr(b_q < r_q|s)$, given that $r_{q-1} < b_q \Rightarrow r_q < b_q$ and thus $Pr(r_q < b_q \cap b_q < r_{q-1}|s) = Pr(b_q < r_{q-1}) + 1 - Pr(b_q < r_q) - Pr(r_q < b_q \cup b_q < r_{q-1}|s)$. The profit function can be re-written as follows,

$$\begin{aligned} \Pi_{q,s} &= Pr(b_q < r_{q-1}|s)(b_q - \zeta_q(s)) + Pr(b_q < r_q|s)\Big(\mathbb{E}[r_q|b_q < r_q, s] - b_q\Big) \\ &- Pr(b_q < r_{q-1}|s)\mathbb{E}[r_{q-1} - r_q|b_q < r_{q-1}, s](Q(q,s) - 1) \\ &+ Pr(r_q < b_q < r_{q-1}|s)(b_q - \mathbb{E}[r_q|r_q < b_q < r_{q-1}, s])(Q(q,s) - 1). \end{aligned}$$

Note that the derivative of the first line of profits equals

$$Pr'(b_q < r_{q-1}|s)(b_q - \zeta_q(s)) + Pr(b_q < r_{q-1}|s) - Pr(b_q < r_q|s),$$

given that $\frac{\partial Pr(b_q<r_q|s)\mathbb{E}[r_q|b_q<r_q,s]}{\partial b_q} = Pr'(b_q < r_q|s)b_q$.[65] One can show that the partial derivative of the second line and third line of the expression equals to

$$\Big(Pr(b_q < r_{q-1}|s) - Pr(b_q < r_q|s)\Big)(Q(q,s) - 1),$$

which gives the final result given that $Pr(b_q < r_{q-1}|s) - Pr(b_q < r_q|s) = Pr(r_q < b_q < r_{q-1}|s)$. First note that,

$$\begin{aligned} \frac{\partial Pr(r_q < b_q < r_{q-1}|s)b_q(Q(q,s) - 1)}{\partial b_q} &= \Big(Pr(b_q < r_{q-1}|s) - Pr(b_q < r_q|s)\Big)(Q(q,s) - 1) \\ &+ \Big(Pr'(b_q < r_{q-1}|s)b_q - Pr'(b_q < r_q|s)b_q\Big)(Q(q,s) - 1). \end{aligned}$$

Also note that,

$$Pr(b_q < r_{q-1}|s)\mathbb{E}[r_q|b_q < r_{q-1}, s] - Pr(r_q < b_q < r_{q-1}|s)\mathbb{E}[r_q|r_q < b_q < r_{q-1}, s]$$

$$= Pr(b_q < r_q)\mathbb{E}[r_q|b_q < r_q].$$

The derivative of this term together with $-Pr(b_q < r_{q-1})\mathbb{E}[r_{q-1}|b_q < r_{q-1}](Q(q,s) - 1)$, as already seen by Leibniz rule, equals $-\Big(Pr'(b_q < r_{q-1}|s)b_q - Pr'(b_q < r_q|s)b_q\Big)(Q(q,s) - 1)$, which cancels with the above terms. $\square$

D Counterfactual model

In this appendix I describe in detail how the two main pieces of the counterfactual analysis are implemented. First, I discuss how the optimal strategy of a single firm is computed, with

[65] Note that $Pr(b_q < r_q|s)\mathbb{E}[r_q|b_q < r_q, s] = \int_{b_q}^{\infty} r dF(r)$. By Leibniz rule, its derivative is equal to $-b_q f(b_q)$.

and without complex bids. In the next section, I discuss how the residual demand is modified endogenously to capture some of the general equilibrium effects of the mechanism change.

D.1 Baseline problem

The baseline problem of the firm is to maximize profits for each possible realization of residual demand. There are no bidding rules and therefore one can solve the optimal strategy choosing the quantity produced by each unit at each hour of the day for each possible demand realization, taking into account the cost structure of the units.

The goal of the firm is to maximize its expected profit, given by its gross revenue minus the costs of production. The gross revenue depends on the total quantity produced by the firm, which in equilibrium equals the residual demand. The costs depend on the hourly production at the unit level. Units have both a minimum and a maximum capacity. Units incur a startup cost β_j whenever they turn on.

In order to represent the quadratic and ramping costs at the unit level, and in order to preserve the linearity of the problem, the quantity levels at the unit level are discretized into different steps. As the number of steps increases, the solution approximates one in which no linear approximation is being made.

$$\max_{\mathbf{q}_t} \qquad E\Pi_t(\mathbf{q}_t) = \sum_{s=1}^{S_t} Pr(s)\Big(\sum_{h=1}^{24} GR(DR_{tsh}) - \sum_{j=1}^{J} C_j(\mathbf{q}_{tsj})\Big) \qquad s.t.$$

$$\begin{array}{ll}
\text{[Cost function]} & C_j(\mathbf{q}_{tsj}) = \sum_{h=1}^{H}\Big(\alpha_1 q_{jtsh} + \alpha_2(q_{jtsh} - \underline{K}_j) + \beta_j y_{jtsh}\Big),\ \forall j,t,s, \\
\text{[Balance Constraint]} & DR_{tsh} = \sum_{j=1}^{J} q_{jtsh},\ \forall t,s, \\
\text{[Capacity Constraint]} & u_{jtsh}\underline{K}_j \leq q_{jtsh} \leq u_{jtsh}\overline{K}_j,\ \forall j,t,s,h, \\
\text{[Startup Constraint]} & y_{jtsh} = \mathbf{1}(u_{jts,h} > u_{jts,h-1}),\ \forall j,t,s,h, \\
\text{[Integer Constraint]} & u_{jtsh} \in \{0,1\}, y_{jtsh} \in \{0,1\},\ \forall j,t,s,h.
\end{array}$$

where

t	Day index,
s	Demand realization scenarios, $s = 1,...,S_t$,
h	Hours of the day, $h = 1,...,24$,
j	Unit index, $j = 1,...,J$,
$pr(s)$	Probability of demand realization s occurring,
q_{jtsh}	Quantity produced by unit j at day t, scenario s and hour h,
u_{jtsh}	Run indicator, takes value of one if unit is on in day t, scenario s and hour h,
y_{jtsh}	Startup indicator, takes value of one if unit starts up in day t, scenario s and hour h,
DR_{tsh}	Equilibrium residual demand at day t, scenario s and hour h,
$GR(.)$	Gross revenue function (piecewise linear approximation) which depends on residual demand function,
C_{jts}	Daily costs of production.

D.2 Adding simple bids

The above representation does not take into account that firms have constraints when bidding in the market. Firms cannot provide a quantity schedule for every possible demand contingency, but need to represent its preferences by means of bidding functions. In the simple bidding problem, I examine what happens when unit quantity output needs to be increasing with the realized market prices.

One way to introduce bidding in the model is to make the firm choose over bids, instead of quantities directly. A way to introduce a sense of bidding in the above problem is to impose that the quantity produced at the unit level needs to be increasing in the market price.[66] This is,

$$\text{[Simple condition]} \qquad p_{tsh} \leq p_{ts'h} \Rightarrow q_{jtsh} \leq q_{jts'h}, \quad \forall j, t, s, s', h.$$

One of the problems of the first formulation is that it can become computationally very intense, due to the large number of binary variables involved in defining the monotonicity constraints. One solution to this problem, is to reduce the number of scenarios faced by the firm. However, this has important effects in the nature of bidding. If there are not many possible demand realizations, the simple bidding rule will tend to look more flexible than it actually is.

For sufficiently many draws of demand realizations, given the nature of uncertainty in the market and substantial startup costs, choosing the startup decision of the unit ex-ante might can be a profit-maximizing strategy. This is also consistent with observed bidding patterns in the data. Therefore, in order to reduce the dimensionality of the game, I consider the case in which the firm commits its units ex-ante. In such case, the simple bidding restriction is given by

$$\text{[Simple ex-ante commitment]} \qquad u_{jtsh} = u_{jts'h}, \ \forall j, t, s, s', h.$$

The restriction [Simple ex-ante commitment] can be jointly imposed with the [Simple condition] constraints, with the advantage that not as many scenarios are required to capture the actual constraints implied by bidding.

D.3 Adding complex bids

The introduction of complex bids allows the firm to state its preferences more flexibly over the day. In terms of the above problem of the firm, the complex bidding mechanism relaxes the restrictions imposed by simple bidding. For this purpose, the bidding variables A_j, B_j are introduced. The firm can exclude its units by means of these bidding variables. As a simplification, general equilibrium effects to the residual demand function are ignored.[67] Given that the complex bidding mechanism represents the game actually played in the data, the original residual demand should be a good approximation to the residual demand around the equilibrium.

Consider the case in which monotonicity constraints are imposed. The simple bidding rule requires that the quantity produced by a given unit is increasing in the market price. With complex bids, that restriction only applies as long as the unit is accepted in the market. Therefore, the

[66] Note that for a given quantity strategy it is trivial to derive bidding strategies that are consistent with the quantity decisions and satisfy the simple bidding rules. Given the discrete number of demand scenarios, there might exist more than one bidding strategy that is consistent with optimal quantity choices.

[67] A model in which the residual demand is endogenous to the complex bids decisions would require computing a residual demand function for every possible combination of complex bids being binding, which would be prohibitively costly from a computational point of view. The firm own thirteen units, which would imply 2^{13} possible residual demands for every possible demand scenario considered and every day in the sample.

constraint becomes,

$$\text{[Complex condition 1]} \qquad p_{tsh} \leq p_{ts'h} \Rightarrow q_{jtsh} \leq q_{jts'h}, \quad \forall j, t, h, \{s, s'\} \in \tilde{S},$$

where $\tilde{S}$ represents those states of residual demand in which the unit has been accepted. The state $\tilde{S}$ is determined endogenously by means of the complex bids submitted by the firm. Complex bidding conditions are given by,

$$\text{[Complex condition 2]} \qquad \textstyle\sum_{h=1}^{24} p_{tsh} q_{jtsh} < A_{jt} + B_{jt} \sum_{h=1}^{24} q_{jtsh} \Rightarrow \{q_{jtshk} = 0, s \notin \tilde{S}\} \quad \forall j, t, s, h.$$

One of the problems of [Complex condition 2] is that it is a nonlinear function that depends on prices, bids and quantities, which are all endogenous variables. In order to compute the actual equilibrium, I proceed in two-stage fashion. In a first-step I compute the equilibrium of the unrestricted problem. This gives me a baseline hourly production for each unit, $\hat{q}_{jtsh}$. If the unit is not producing in the baseline, I attribute it a total daily quantity equal to the minimum observed daily quantity. Once these quantities are defined, the model can be solved as a linear mixed integer problem, similar to the simple bidding case.

Similar to the simple bidding model, due to the computational limitations of simulating a rich enough spectrum of possible residual demands, the model might overestimate the flexibility introduced by complex bids. As explained in the Section 7, conditional on getting their units accepted, firms appear to make their commitment decisions ex-ante regarding which hours to produce. For this reason, I consider the following constraint, which is analogous to the simple bidding ex-ante commitment constraint bu only applies to states in which the unit has been accepted,

$$\text{[Complex ex-ante commitment]} \qquad u_{jtsh} = u_{jts'h}, \quad \forall j, t, h, \{s, s'\} \in \tilde{S}.$$

D.4 Adding a dynamic dimension to the game

As explained in Section 5, the startup decision of the firm can take a temporal dimension that spans longer than the daily auction. For this reason, it is necessary to account for the continuation value of the startup decisions. In order to account for this intertemporal dimension, I consider a finite horizon game in which the firm is maximizing its profit for the current day, taking into account the expected demand in the future.

The objective function is given by,

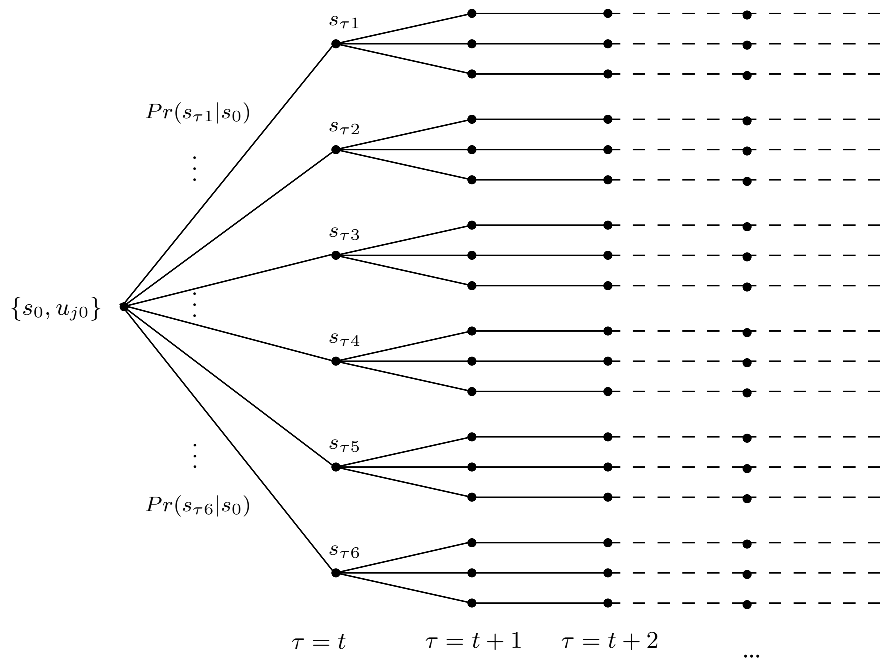
$$\max_{\mathbf{q}} \qquad \sum_{\tau=t}^{t+T} E\Pi_t(\mathbf{q}_t),$$

where T represents the number of periods looking forward. I consider the case in which the firm looks five days ahead, which in principle should be long enough to capture the weekend cycle.

Similar than before, each day there are different possible residual demand scenarios, represented by S_t. A transition matrix is specified in order to account for possible correlation of demand patterns across days. I consider a Markovian structure in which the level of demand today is correlated with the expected demand level tomorrow. In order to reduce the dimensionality of the problem, only uncertainty over today and tomorrow's shocks are considered. This structure is represented in Figure D.1.

In order to reduce the computational dimensionality of the problem, in the computations included in the paper I consider six different possible demand realizations, corresponding to the

Figure D.1: Introduction of Dynamic Interactions



The dynamic game assumes a Markovian process in the evolution of residual demand realizations. Today's demand realizations affect the expectations regarding tomorrow's market outcomes.

intervals ranging the 10, 25, 50, 75 and 90 percentiles.[68] The transitions between the different percentiles across days is estimated based on the actual demand realizations observed in the data. These process should roughly capture the major features of the underlying distribution of market outcomes.

E Counterfactual residual demand

One of the major benefits of complex bids is to coordinate the startup decisions of the firms. In this paper, I capture in a reduced form way these effects on the residual demand faced by a strategic firm. In this section I describe the bootstrapping procedure used.

E.1 Bootstrapping strategy

A distribution of residual demands is constructed for both simple and complex bids. The distribution is computed for 100 different draws, in which bidding strategies are pooled across similar days. The complex and simple bidding bootstrapping strategies differ in that the complex mechanism solves for the actual ISO algorithm in the market to obtain the residual demands, whereas the simple mechanism is based on a simpler uniform auction.

[68]The criterion to classify the residual demand in percentiles is based on the residual demand levels of the different hours of the day in the relevant ranges of production for the firm. The relevant ranges of thermal production are those that range from zero to the total thermal capacity of the firm.

E.1.1 Complex mechanism

To generate residual demand draws for the complex mechanism, I take the original residual demands obtained in the following manner:

1. Randomly select bidding strategies for each firm across similar days.
2. Solve for the market equilibrium under the complex algorithm described in Appendix A.
3. Take only offers that are not discarded by the algorithm.

E.1.2 Simple mechanism

In order to generate the residual demand under simple bidding, I assume that observed accepted bids would have been offered if units were committing their units ex-ante. The process is then:

1. Randomly select bidding strategies that have been accepted for each firm across similar days.
2. Solve for the market equilibrium under a uniform auction.
3. Obtain new prices from strategic firm optimal behavior and repeat 1 and 2 until convergence.

E.2 Comparison of implied volatility levels

Holding the strategies of all firms as given, including the strategic firm, the above procedure generates a first rough assessment of the effects of complex bids in coordinating the startup of units. As it can be seen in Figure E.1, the presence of complex bids helps reduce the volatility in the market. This is due to the fact that firms can contingently decide their startup, whereas with simple bids they take this decision ex-ante. As presented in Table E.1, while the original bootstrapped data reflects accurately the underlying distribution of prices in this market, the bootstrapped data using the simple mechanism only exaggerates the variance in the market.

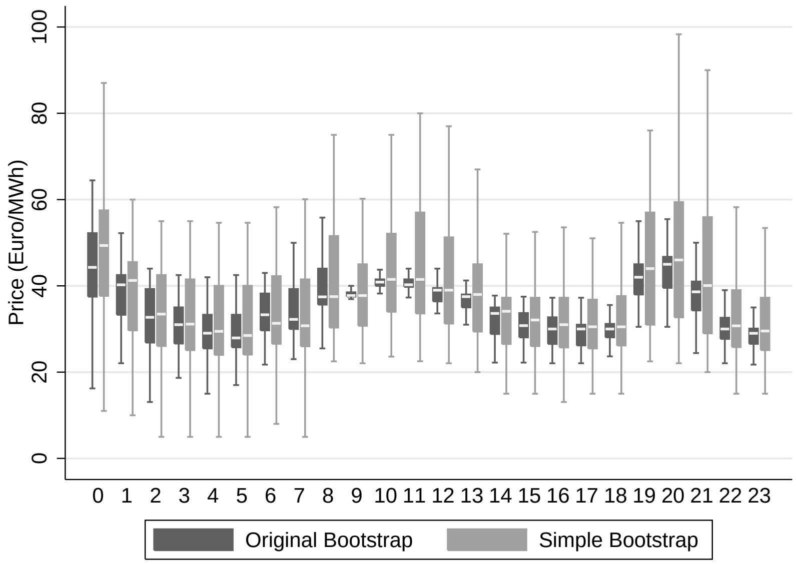
E.3 Estimation of residual demand and gross revenue

Once the distribution of residual demands is obtained, I estimate the 10, 25, 50, 75, and 90 percentile, which are used in the simulations.[69] They are approximated as piece-wise linear functions, so that they can be used in the computational model.

The gross revenue is also estimated as a piece-wise linear function. In some of the simulations, I also impose that the gross revenue is convex in the relevant ranges of production. This allows me to reduce the number of integer variables in the problem, while still capturing the major characteristics of the revenue function. Note that imposing a convex gross revenue function does not impose such restriction to the residual demand shape.

[69] The reason to estimate those percentiles is to ensure that, even if a small number of draws is used, they roughly represent the underlying distribution in the data. Absent computational limitations, more draws could be taken to reflect the underlying distribution more accurately.

Figure E.1: Complex bids reduce the volatility in the market



Distribution of hourly prices in a given day. The price distribution when the mechanism does not use complex bids presents higher variance than the original bootstrapped distribution.

Table E.1: Distribution of Complex and Simple Bootstrapped Prices

Stats	Price (€/MWh)	Price Complex (€/MWh)	Price Simple (€/MWh)
mean	34.14	34.24	35.54
sd	8.99	8.90	14.19
skewness	0.66	0.69	1.53
kurtosis	3.17	3.07	6.82
p5	22.91	23.00	20.07
p25	27.48	27.50	25.83
p50	32.18	32.19	30.98
p75	40.01	40.01	42.00
p95	50.61	50.61	64.10
min	8.03	13.00	5.00
max	69.70	65.00	109.11

Notes: Excludes lower and upper 0.5% of the data. Trimming the data in the extremes avoids including observations in the bootstrapped sample that are not representative from the actual distribution of prices and that could potentially bias the estimates of expected market outcomes. Trimming 1% of the data ensures that no prices at the price cap are observed, which never happens in the data, capturing the actual process very closely.

F Additional tables and figures

Table F.1: Distribution of Bootstrapped Strategies

Stats	Price (€/MWh)	Price Bootstrap (€/MWh)	Non-trimmed Price Bootstrap (€/MWh)
mean	34.14	34.24	34.59
sd	8.99	8.90	10.37
skewness	0.66	0.69	2.16
kurtosis	3.17	3.07	24.86
p5	22.91	23.00	22.22
p25	27.48	27.50	27.50
p50	32.18	32.19	32.83
p75	40.01	40.01	40.06
p95	50.61	50.61	52.02
min	8.03	13.00	0.00
max	69.70	65.00	180.30

Notes: "Price Bootstrap" excludes lower and upper 0.5% of the data. Trimming the data in the extremes avoids including observations in the bootstrapped sample that are not representative from the actual distribution of prices and that could potentially bias the estimates of expected market outcomes. Trimming 1% of the data ensures that no prices at the price cap are observed, which never happens in the data, capturing the actual process very closely.

Table F.2: Marginal Cost Estimates at the Unit Level

Plant	Type	Size MW	α_{j1} €/MWh	α_{j2} €/MWh^2	α_{j3} €/MWh^2
ACE3	CCGT	386.0	41.17	2.00E-2	4.94E-4
			(2.44)	(6.88E-3)	(1.94E-3)
ARCOS1	CCGT	389.2	44.68	2.50E-2	5.68E-4
			(2.91)	(1.43E-2)	(2.01E-3)
ARCOS2	CCGT	373.2	40.90	2.81E-2	8.70E-3
			(3.78)	(8.04E-3)	(7.96E-2)
ARCOS3	CCGT	822.8	37.84	7.16E-3	6.58E-4
			(2.09)	(2.46E-3)	(2.12E-3)
CTJON2	CCGT	378.9	36.72	2.59E-2	2.59E-5
			(3.49)	(6.51E-3)	(1.80E-4)
CTN3	CCGT	782.0	44.92	5.76E-3	2.60E-3
			(2.20)	(4.85E-3)	(4.55E-3)
ESC6	CCGT	803.5	33.75	2.13E-2	9.23E-4
			(2.45)	(2.73E-3)	(1.87E-3)
STC4	CCGT	396.4	36.61	3.10E-2	7.88E-6
			(2.59)	(7.32E-3)	(7.80E-5)
GUA1	CO	148.0	31.27	4.54E-2	8.15E-3
			(1.69)	(1.52E-2)	(6.87E-3)
GUA2	CO	350.0	25.84	2.33E-2	4.73E-3
			(1.95)	(1.70E-2)	(2.81E-3)
LAD3	CO	155.0	30.40	4.60E-2	4.94E-3
			(1.45)	(8.42E-3)	(4.29E-3)
LAD4	CO	350.0	24.37	3.89E-2	9.48E-4
			(1.01)	(6.79E-3)	(2.41E-3)
PAS1	CO	214.0	23.79	5.83E-2	1.54E-3
			(0.99)	(7.81E-3)	(1.83E-3)

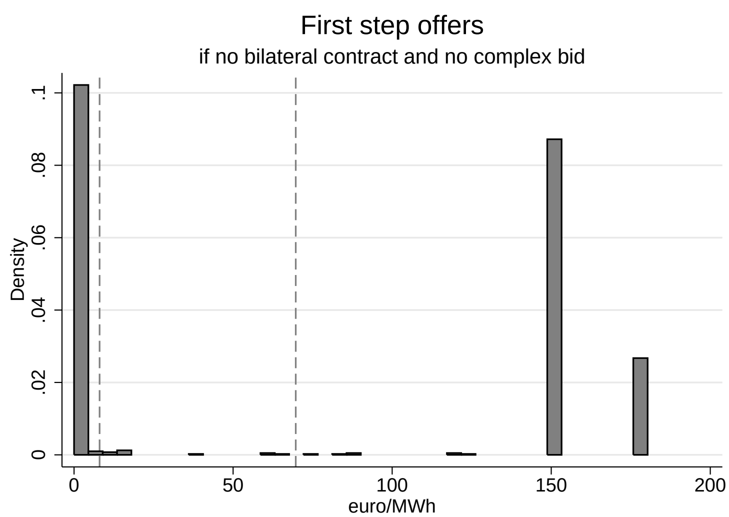
Notes: Sample from March to July 2007. Standard errors computed using block-bootstrap at the week level.

Table F.3: Smoothing Average Marginal Cost and Forward Estimates for Firm 1

	$bw = 1€$	$bw = 2€$	$bw = 3€$	$bw = 4€$	$bw = 5€$
Coal (€/MWh) [α_{j1}]	29.37	28.68	27.42	26.98	26.43
	(0.87)	(0.96)	(1.19)	(1.32)	(1.56)
CCGT (€/MWh) [α_{j1}]	43.57	42.31	39.76	38.74	37.35
	(2.14)	(2.18)	(2.55)	(2.89)	(3.34)
Coal X q (€/MWh^2) [α_{j2}]	5.58E-2	5.16E-2	4.51E-2	4.27E-2	4.08E-2
	(7.45E-3)	(6.15E-3)	(7.39E-3)	(8.09E-3)	(8.92E-3)
CCGT X q (€/MWh^2) [α_{j1}]	3.18E-2	2.73E-2	2.11E-2	1.61E-2	1.32E-2
	(8.11E-3)	(7.07E-3)	(6.53E-3)	(5.51E-3)	(5.44E-3)
Forward Position (%) [γ_1]	97.70	95.79	91.41	89.71	87.55
	(1.57)	(1.55)	(2.54)	(3.33)	(4.45)
Forward Peak (%) [γ_2]	97.64	95.42	90.87	88.80	86.57
	(2.01)	(2.00)	(3.06)	(3.98)	(5.41)
Forward Weekend (%) [γ_3]	99.23	96.80	92.21	89.95	87.97
	(2.70)	(2.86)	(4.30)	(5.59)	(7.11)

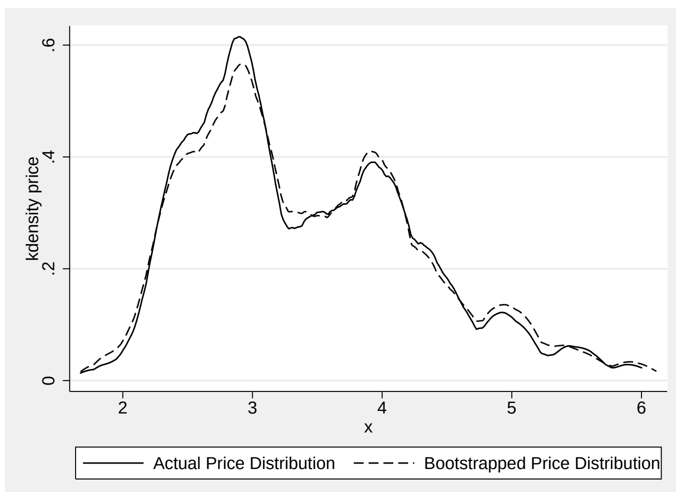
Notes: Sample from March to July 2007. Peak defined according to forward contract products, from 8 a.m. to 8 p.m. Standard errors computed using block-bootstrap at the week level.

Figure F.1: Bidders in the market appear to choose commitment ex-ante



The distribution of first-step bids for units with no bilateral contract and no complex bid shows that firms ensure ex-ante whether a unit will be turned on or not during that day. Dashed lines represent minimum and maximum price observed in the whole sample. Firms submit either very low or very high first step bids. Sample from March to June 2007.

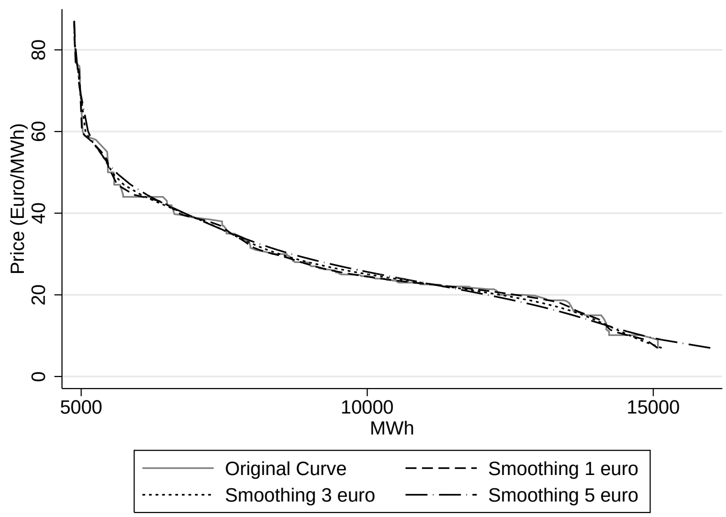
Figure F.2: Distribution of Bootstrapped Strategies



Notes: Excludes 1% lower and upper outliers so that the distribution can be compared at the relevant range. A plot using the non-trimmed price distribution is very similar to the one reported here, except for few observation around the price cap.

Figure F.3: Example of Residual Demand for Different Smoothing Parameters

(a) Residual demand fit for different values in the relevant range of observed bids and prices.



(b) The smoothing technique should also approximate the residual demand slope, which is a key statistic in the construction of the first order conditions. One can see that a low smoothing parameter might produce jagged slope estimates. A large smoothing parameter might flatten out the slope. Note that the original slope is approximated as the slope between the 10 closest bids at each point.