Dynamic Maturity Transformation*

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Abstract

We develop an infinite horizon equilibrium model in which banks finance long term assets with non-tradable debt. Banks choose the amount of debt and its maturity taking into account investors' preference for short maturities (which better accommodate their preference shocks) and the risk of systemic liquidity crises (during which refinancing is especially expensive). Unregulated debt maturities are inefficiently short due to pecuniary externalities in the market for funds during crises and their interaction with banks' refinancing constraints. We show the possibility of improving welfare by means of limits to debt maturity, Pigovian taxes, and private and public liquidity insurance schemes.

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1 Introduction

The recent financial crisis has extended the view among regulators that maturity mismatch in the financial system prior to the crisis was excessive and not properly addressed by the existing regulatory framework (see, for example, Tarullo, 2009). When the first losses on the subprime positions arrived in early 2007, investment banks, hedge funds and many commercial banks were heavily exposed to refinancing risk in wholesale debt markets. This risk was a key lever in generating, amplifying, and spreading the consequences of the collapse of money markets during the crisis (Brunnermeier, 2009; Gorton, 2009).

We develop a simple infinite horizon model in which banks finance long-term assets by placing non-tradable debt among unsophisticated savers subject to *preference shocks*.[1] Short maturities are attractive to these savers because they buy bank debt when they are patient but may suffer shocks that turn them impatient, in which case postponing the recovery of the principal until their debt matures is a source of disutility, like in Bryant (1980) and Diamond and Dybvig (1983). With this sole force in action, banks in our model would minimize the cost of their funding by issuing debt of the shortest possible maturity, that would be rolled over by successive generations of initially patient savers.

However, a second force pushes in the opposite direction: the existence of episodes (*systemic crises*) in which all savers turn impatient and banks have to temporarily rely on the more expensive funding provided by some sophisticated investors who have their own outside investment opportunities. The heterogeneity in the outside investment opportunities of these *bridge financiers* produces an upward slopping aggregate supply of funds during crises, so that the excess cost of liquidity in a crisis increases with banks' aggregate refinancing needs.[2]

Banks decide the overall principal, interest rate and maturity of their debt trading off the lower interest cost of short maturity debt with the anticipated excess cost of covering the implied refinancing needs during crises. The resolution of this trade-off leads banks to choose debt maturities associated with lower refinancing needs (i.e. longer maturities) when crisis liquidity is anticipated to be more costly. The intersection between bridge financiers'

[1] By addressing the analysis within an infinite horizon, we state our predictions and normative implications in terms that, to the effects of a possible calibration or quantitative assessment, should be easier to match with real world counterparts than in alternative two- or three-period formulations.

[2] This part of the model plays a role similar to fire-sale pricing in models where levered institutions accommodate their refinancing needs by selling part of their long-term assets (e.g. Allen and Gale, 1998, Acharya and Viswanathan, 2011, and Stein, 2011).

upward sloping supply of funds and banks' downward sloping demand for liquidity in a crisis produces a unique equilibrium excess cost of crisis liquidity.

Importantly, the debt maturity chosen by banks in the unregulated competitive equilibrium is inefficiently short. The reason for this is the combination of pecuniary externalities with the financial constraints faced by banks in their maturity transformation activity.[3] Specifically, banks must have sufficient equity value so as to be able to refinance their maturing debt during crises.[4] This is because, given the non-renegotiable nature of bank debt, the excess cost of crisis liquidity is absorbed by diluting the existing equity. Hence, effectively, guaranteeing the access to bridge financing imposes a limit on bank leverage.

When banks make their uncoordinated, competitive capital structure decisions, they neglect the impact of their refinancing needs on the equilibrium excess cost of crisis liquidity, which tightens the bridge financing constraint of all banks, reduces the total leverage that the banking industry can sustain, and damages the efficiency of the maturity transformation process. A regulator can improve the overall surplus extracted from banks' maturity transformation activities by inducing banks to develop them with a lower use of the intensive margin (i.e. choosing longer maturities) and a larger use of the extensive margin (i.e. issuing more debt). In this sense the unregulated equilibrium is not constrained efficient.[5] We show the possibility of restoring efficiency through the direct regulation of debt maturity or with a Pigovian tax on banks' refinancing needs.

When we extend the analysis to allow for private or public liquidity insurance schemes directed to smoothly spread the excess cost of crisis liquidity across states of the world, the need for maturity regulation does not vanish. Introducing a fairly-priced private liquidity insurance arrangement, if at all feasible, can definitely be welfare-increasing but it is complementary to funding maturity regulation since the basic pecuniary externality that justifies the latter remains present, though in a new form (acting through the competitive cost of

[3]Pecuniary externalities are a common source of inefficiency in models with financial constraints (e.g. Lorenzoni, 2008). The usual emphasis in the existing papers (including the recent contributions of Bianchi and Mendoza, 2011, and Korinek, 2011) is on their potential to cause excessive fluctuations in credit and excessive credit.

[4]We assume that if banks are unable to refinance their debt, they are liquidated. We show that, if the value recovered in case of liquidation is sufficiently low, banks find it optimal to choose capital structures compatible with being refinanced during crises.

[5]We restrict attention to policy interventions involving no subsidization (no net positive use of government funds) and no greater informational requirements than the unregulated competitive equilibrium. Intuitively, lengthening debt maturities implies a transfer of wealth from future bridge financiers to banks' existing owners, but this transfer increases the overall surplus because it relaxes banks' bridge financing constraints.

insuring against crises).

Under a public liquidity insurance arrangement in which the government (e.g. a central bank acting as a lender of last resort) supplies the funds needed by banks during crises, the conclusion is similar. In this case, insurance premia and maturity regulation are complementary tools in attaining the objective of maximizing the surplus generated by banks' maturity transformation activities while covering the costs of the arrangement for the government.

The paper is organized as follows. Section 2 places the contribution of the paper in the context of the existing literature. Section 3 presents the ingredients of the model. Section 4 defines equilibrium and covers the various steps necessary for its characterization. Section 5 examines the social efficiency properties of equilibrium and possible regulatory interventions. Section 6 extends the analysis to the introduction of private or public liquidity insurance schemes. Section 7 discusses robustness and several potential extensions of the analysis. Section 8 concludes. All the proofs are in the appendices.

2 Related literature

Our paper is in the interface of several literature strands. Our work is first related to the contributions in the dynamic capital structure literature that incorporate debt refinancing risk. These include Leland and Toft (1996) who, extending the seminal model of Leland (1994), show that short debt maturities increase the threshold of the firm's fundamental value below which costly bankruptcy occurs and suggest that shorter maturities might have a counterbalancing advantage when shareholders are tempted to undertake risk shifting.[6] He and Xiong (2011a) show the non-trivial connections between liquidity risk and credit risk that arise when shocks to market liquidity increase the cost of debt refinancing. In a related structural model, He and Milbradt (2011) further analyze the feedback between credit risk and the liquidity of the secondary market for corporate debt (modeled as a market with search frictions). He and Xiong (2011b) show that "dynamic debt runs" may occur when lenders stop rolling over maturing debt in fear that future lenders do the same (making off with their repayments and potentially forcing the firm into liquidation) before the debt now offered to them matures. Cheng and Milbradt (2011) consider a setup in which this type of dynamic runs have, up to some point, a beneficial effect on an asset substitution problem, producing an interior value maximizing debt maturity. Finally, in Brunnermeier and Oehmke

[6] See also Leland (1998).

(2011), a conflict of interest between long-term and short-term creditors during debt crises pushes firms to choose debt maturities which are inefficiently short from an individual value maximization perspective.

Of course, many of the underlying themes have also been analyzed in models with a simpler time structures. Flannery (1994) emphasizes the disciplinary role of short-term debt in a corporate finance context, and Calomiris and Kahn (1991), Diamond and Rajan (2001), and Huberman and Repullo (2010) in a banking context. In Flannery (1986) and Diamond (1991), short-term debt allows firms with private information to profit from future rating upgrades, while in Diamond and He (2010) short maturities have a non-trivial impact on a classical debt overhang problem. The emergence of roll-over risk as the result of a coordination problem between short-term creditors is also analyzed by Morris and Shin (2004, 2009) and Rochet and Vives (2004), among other. Various papers, including Acharya and Viswanathan (2011) and Acharya, Gale, and Yorulmazer (2011), study the implications of roll-over risk for risk-shifting incentives, fire sales, and the collateral value of risky securities.

Our work is also connected to recent papers focused on the normative implications of externalities associated with banks' funding decisions. Farhi and Tirole (2011) show that time-consistent, imperfectly targeted liquidity support to distressed institutions during crises (e.g. via central bank lending) makes bank leverage decisions strategic complements, producing excessive short-term borrowing and social gains from the introduction of a cap on such borrowing. Perotti and Suarez (2011) compare the performance of price-based versus quantity-based liquidity regulation alternatives in a reduced-form model where short term funding enables banks to expand their credit activity but generates negative systemic risk externalities. Finally, Stein (2011) develops a three-date model in which banks expand their credit by issuing short-term debt which, if fully safe, offers a money-like convenience yield; short-term debt is kept safe by incurring in asset sales in bad times but these sales cause pecuniary externalities that are detrimental to welfare, providing a rationale for limiting banks' short-term borrowing.[7]

3 The model

We consider an infinite horizon economy in which time is discrete $t = 0, 1, 2, ...$ The economy is populated by two wide classes of long-lived risk-neutral agents: *savers* and *experts*. Agents

[7] And the paper postulates the use of monetary policy as a means to achieve this goal.

of both classes enter the economy in an overlapping generation fashion further described below. Normally, a sufficiently large measure of savers are born *patient*, in which case their per-period discount rate is ρ_P, although they may randomly and irreversibly become *impatient*, in which case their discount rate becomes $\rho_I > \rho_P$. Experts, on the other hand, always discount the future at rate ρ_I, but they are the only agents with the skills needed to extract value from some of the existing investment opportunities and to manage the *banks*.

The banks posses potentially-perpetual illiquid assets, are owned by the experts who manage them (the *bankers*), and obtain external financing by placing non-tradable debt among initially patient savers (thereby profiting from their lower opportunity cost of the funds).[8] Importantly, banks are exposed to *systemic liquidity crises*: random events in which all patient agents become impatient and, as further described below, banks end up covering their refinancing needs by appealing to experts who provide some costly *bridge financing* until the crisis ends.[9]

3.1 Aggregate shocks

We need to differentiate between periods of normality, $s_t = N$, and periods of systemic liquidity crisis, $s_t = C$. For tractability, we assume $\Pr[s_{t+1} = C \mid s_t = N] = \varepsilon$ and $\Pr[s_{t+1} = C \mid s_t = C] = 0$, so that crises have a constant probability of following any normal period but last for just one period (so a period is the standard duration of a crisis).

3.2 Agents

In each period t a continuum of new risk-neutral savers and experts enter the economy, each endowed with a unit of funds. The measure of each class of entrants is large relative to the refinancing and management needs of banks.

[8] In Section 7 we justify and discuss the importance of the assumption that bank debt cannot be traded.

[9] Our results rely on the maintained assumption that, due to unmodeled information and incentive reasons, banks cannot offer to the savers contracts contingent on idiosyncratic and aggregate preference shocks, or that give banks the option to postpone debt repayments at will. These features might help the banks accommodate savers' preferences for liquidity in normal periods while limiting their refinancing problems during systemic crises.

3.2.1 Savers

Except when $s_t = C$, a sufficiently large measure of savers are born patient, with a discount rate ρ_P.[10] In normal periods, patient savers have a purely idiosyncratic (independent) probability $\gamma \in [0,1]$ of turning irreversibly impatient. When $s_t = C$, both entering and existing savers are or become impatient with probability one.

Entering savers decide on whether to invest their endowment in the assets offered by banks (described below) or to consume it. Savers who opt for the first alternative, may face similar (re)investment decisions during their lifetime. Savers who decide to consume their savings become irrelevant from thereon.

We assume that savers learn about their own preferences before learning about the aggregate state of the economy and immediately make consumption plans whose alteration (e.g. in order to postpone consumption) entails a cost $\varkappa$ per unit of planned consumption.[11] The role of this cost will be explained shortly.

3.2.2 Experts

When the impatient experts enter the economy they have the opportunity of undertaking some irreversible private investment project with a cost of one and a net present value (discounted at the rate ρ_I) of z, heterogeneously distributed over the entrants. The distribution of this parameter has support $[0, \overline{\phi}]$. The measure of the population of entering experts with $z \leq \phi$ is described by a differentiable and strictly increasing function $F(\phi)$, with $F(0) = 0$ and $F(\overline{\phi}) = \overline{F}$.

On occasions, especially in crisis periods, *entering experts* have the opportunity of becoming active bankers, in the terms specified below. However, we assume that experts' impatience is always large enough for them not to accumulate any wealth in any form different from their private investments or bank shares, and that each expert can only devote her expertise to a single venture (private project or bank) at a time.[12]

[10] One can interpret ρ_P as the risk-free return of some alternative short-term asset (e.g. government bonds) in which savers can invest and disinvest without the mediation of an expert. In this case, patient savers' "consumption" might correspond to investing such asset until they become impatient, point at which they would definitely consume.

[11] Consumption planning may include the search and ordering of the goods to buy as well as arranging the access to the funds needed to pay for them (e.g. cancelling an automatically renewable term deposit).

[12] This excludes the possibility that experts who undertook private projects in a previous period retain dividends or abandom the projects so as to become bankers during systemic crises.

3.3 The banking sector

The banking sector is initially made up of a measure-one continuum of banks, each with the same fixed amount of assets. Bank assets remain productive if continuously managed by an expert or coalition of experts (bankers), in which case they yield a constant cash flow $\mu > 0$ per period. The best use for unproductive banks assets is liquidation, which yields a residual value of L.

Bankers own 100% of the equity of their banks and decide each bank's initial funding structure at some initial normal period (say, $t = 0$). This initial funding structure is held fixed in between crises and restored immediately after each crisis.[13]

3.3.1 Initial funding and normal times refinancing

Each bank's outside funding consists of a continuum of ex ante equal infinitesimal-size non-tradable debt contracts issued at par which can be collectively described as a triple (D, r, δ), where D is the overall principal, r is the per-period interest rate, and δ is the constant probability with which each infinitesimal contract matures in each period. So debt maturity is random and has the property that the expected time to maturity of any non-matured contract is equal to $1/\delta$. We assume contract maturities to be independent both within banks and across banks.[14] At the level of the bank, this produces essentially the same effect as having the overall debt D made up of uniform perfectly-staggered fixed-maturity contracts which are rolled-over (or replaced by identical contracts) as they mature. Overall this debt will oblige the bank to pay interest equal to rD in each period and to refinance the amount δD resulting from the fraction of contracts that mature.[15]

Due to differences in discount rates all the initial holders of (D, r, δ), if issued in a normal period, will be patient savers. In normal periods, δD will be refinanced by replacing the maturing contracts with identical contracts placed among new or remaining patient savers. The bank will thus have a free cash flow of $\mu - rD$ that can be paid as a dividend to the bankers, who will consume it.[16]

[13] As discussed in Section 7, our setup is such that this assumption entails no loss of generality from a dynamic optimization perspective.

[14] The case of perfectly correlated maturities within a bank (and independent across banks) is as tractable as our benchmark case but implies that banks are more vulnerable to systemic liquidity crises. All results are qualitatively identical to the ones reported below, but banks produce less value to their shareholders.

[15] Debt with random maturity produces trade-offs both for savers and banks very similar to those of (more realistic) fixed-maturity contracts but makes the analytics of the problem much more tractable.

[16] For sufficiently impatient bankers and a sufficiently small likelihood of suffering a systemic crisis, paying

3.3.2 Refinancing during crises

In a systemic crisis, the bank cannot replace its maturing debt with identical debt contracts because there are no old or newly-born patient savers. The following assumptions help define the course of events in crisis periods:

1. *Savers' consumption plans are costly to rectify.* The frequency of systemic crises is low enough for savers to plan to consume their entire savings as soon as they learn to be impatient. Moreover, the rectifying cost $\varkappa$ is larger than the opportunity cost of funds of the relevant marginal entering expert in a crisis period, $z = \phi$, so that banks prefer to directly resort to experts for their crisis funding.[17]

2. *Bankers have consumed their dividends.* Bankers learn about the state of the economy after having received and consumed dividends of $\mu - rD$.[18]

3. *Bankruptcy is worth avoiding.* If the bank were unable to refinance its maturing debt δD, management would be discontinued and the bank's liquidation value L would be divided among creditors. We assume L to be low enough for bankers to choose initial funding structures under which bankruptcy can be avoided.[19]

In these circumstances, banks best alternative in a crisis is to finance the repayment of their maturing debt δD through some of the entering experts. For instance, experts can be offered to *bridge refinance* δD in exchange for an equity stake in the bank.[20] If this arrangement is feasible, the bank operates with lower debt, $(1-\delta)D$, during the crisis period. And once the crisis is over, the bank finds it optimal to restore the original debt structure (D, r, δ) by issuing an additional amount δD of such debt.

out and consuming these dividends is optimal for bankers in normal periods. In other words, it does not pay for bankers to hold precautionary savings (say, remunerated at rate ρ_P) so as to refinance their banks when a crisis comes.

[17] In other words, banks and savers only learn about the occurrence of a systemic crisis when it is too late (too costly) for banks to induce the impatient savers with maturing debt to invest in new (better remunerated) bank debt. This resembles the conditions that justify the well-known sequential service constraint in papers about deposit runs (Wallace, 1988).

[18] This assumption simplifies the algebra and could be removed without material qualitative or quantitative effect on the results. In the numerical examples below, the dividends $\mu - rD$ end up being very small relative to the refinancing needs δD, so their omission would only reduce very marginally the (excess) refinancing costs suffered in a crisis.

[19] In Section 7 we explicitly discuss the exact condition under which avoiding bankruptcy is optimal.

[20] We refer to *equity* here for concreteness. The form of the securities supporting the bridge financing arrangement is not relevant: their overall returns must just be enough to attract the marginal bridge financier.

3.4 The market for bridge financing

When the initial bankers choose debt structures (D, r, δ) compatible with obtaining bridge financing during crises, the bridge financiers receive some fraction α of each bank's equity in each crisis. By virtue of competition, in equilibrium, α will have to be enough to compensate the marginal entering expert for the opportunity cost of her funds, which we denote by ϕ.

The heterogeneity in the value of the private investment opportunities of the entering experts and the size of banks' aggregate refinancing needs, δD, implies that clearing the market for bridge financing requires $F(\phi) = \delta D$. Since $F(\cdot)$ is strictly increasing, we can equivalently write this condition as $\phi = F^{-1}(\delta D) \equiv \Phi(\delta D)$, where $\Phi(\cdot)$ is strictly increasing and differentiable, with $\Phi(0) = 0$ and $\Phi(\overline{F}) = \overline{\phi}$. We will refer to ϕ as the *excess cost of liquidity* during a crisis and to $\Phi(\cdot)$ as the *inverse supply of liquidity* during a crisis.

4 Equilibrium analysis

In this section we stick to the following definition of equilibrium:

Definition 1 *Given the exogenous parameters of the model ε, ρ_P, ρ_I, γ, μ, and the function $\Phi(\cdot)$, an* equilibrium with bridge financing *is a tuple $(\phi^e, (D^e, r^e, \delta^e))$ describing an excess cost of liquidity during a crisis ϕ^e and a debt structure for banks (D^e, r^e, δ^e) such that:*

1. *Patient savers accept the debt contracts involved in (D^e, r^e, δ^e).*
2. *Among the class of debt structures that allow banks to be refinanced during crises, (D^e, r^e, δ^e) maximizes the value of each bank to its initial owners.*
3. *The market for liquidity during crises clears in a way compatible with the refinancing of all banks, i.e. $\phi^e = \Phi(\delta^e D^e)$.*

In the next subsections we undertake the steps necessary to prove the existence and uniqueness of this equilibrium, and establish its properties.

4.1 Savers' required maturity premium

Let us analyze the conditions upon which the debt contracts associated with some debt structure (D, r, δ) are acceptable to savers during normal times. Recall that debt is issued at par and consider a debt contract with a principal of one. Since the bank will fully pay back its maturing debt even in crisis periods, a saver's valuation of such contract does not depend on the aggregate state of the economy *per se* but on whether the saver is patient ($i = P$) or impatient ($i = I$). The value of the contract in each of these individual states, U_P and U_I, must satisfy the following system of equations:

$$\begin{aligned} U_P &= \frac{1}{1+\rho_P}\{r+\delta+(1-\delta)[(1-\varepsilon)(1-\gamma)U_P+((1-\varepsilon)\gamma+\varepsilon)U_I]\}, \\ U_I &= \frac{1}{1+\rho_I}\left[r+\delta+(1-\delta)U_I\right]. \end{aligned} \tag{1}$$

The different discount factors multiply the payoffs and continuation values relevant in each state. The contract pays r with probability one in each next period. Additionally it matures with probability δ, in which case it pays one. With probability $1-\delta$, it does not mature and then its continuation value is U_P and U_I depending on the investor's individual state in the next period. The terms multiplying this variables in the right hand side of the equations reflect the probability of being in each state next period.

When banks issue their debt, patient savers are abundant, so the acceptability of the terms (r, δ) requires

$$U_P(r,\delta) = \frac{r+\delta}{\rho_I+\delta}\frac{\rho_I+\delta+(1-\delta)[(1-\varepsilon)\gamma+\varepsilon]}{\rho_P+\delta+(1-\delta)[(1-\varepsilon)\gamma+\varepsilon]} \geq 1, \tag{2}$$

which uses the solution for U_P arising from (1). Obviously, for any given δ, a bank maximizing its owners' value will offer contracts with the minimal interest rate r that satisfies $U_P(r,\delta) = 1$, i.e.,

$$r(\delta) = \frac{\rho_I\rho_P+\delta\rho_P+(1-\delta)[(1-\varepsilon)\gamma+\varepsilon]\rho_I}{\rho_I+\delta+(1-\delta)[(1-\varepsilon)\gamma+\varepsilon]}. \tag{3}$$

From here, we can state the following result:

Proposition 1 *The minimal interest rates acceptable to patient savers for each maturity parameter δ is given by $r(\delta)$ which is strictly decreasing and convex, with $r(0) = \rho_I\frac{\rho_P+\pi}{\rho_I+\pi} \in (\rho_P, \rho_I)$ and $r(1) = \rho_P$.*

This result evidences the value of offering short debt maturities to the savers in our model. The intuition is quite straightforward. When the expected maturity of the contract, $1/\delta$, gets lengthened, the saver bears the risk of turning impatient and having to postpone his consumption for a longer time (until his contract matures). Compensating the cost of waiting via a larger interest rate generates a maturity premium $r(\delta) - \rho_P > 0$, which is increasing in $1/\delta$. Figure 1 illustrates the behavior of $r(\delta)$ under specific parameter values.[21]

[21] All figures rely on a baseline parameterization in which one period is one month, $\Phi(x) = x^2$, agents'

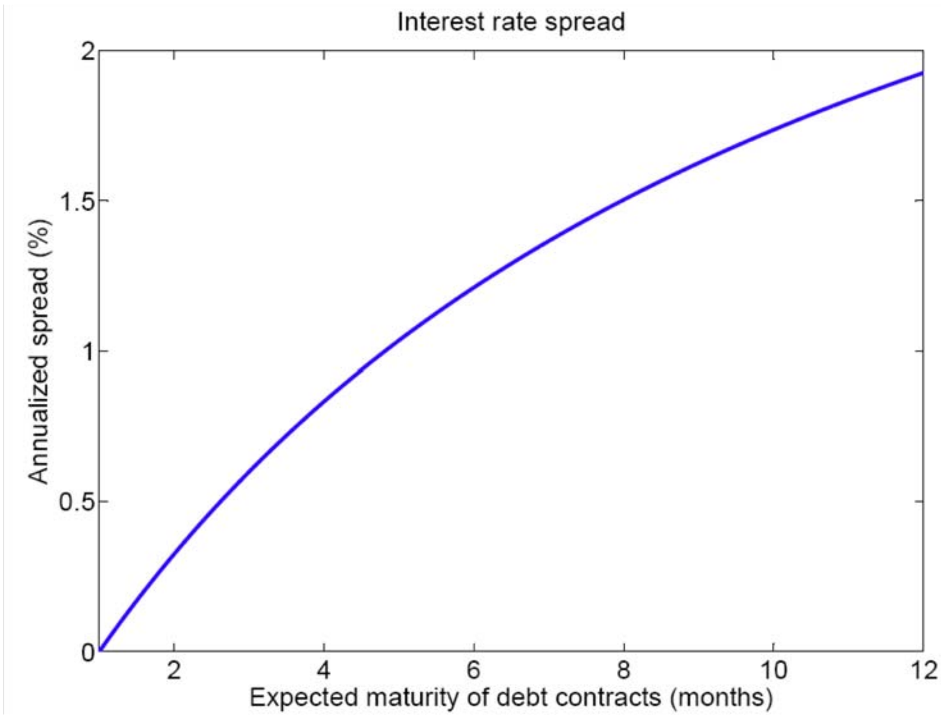

Figure 1: Interest rate spread vs. $1/\delta$

4.2 Banks' optimal debt structures

From now on, we will take savers' participation constraint into account by assuming that the debt structures (D, r, δ) offered by banks always have $r = r(\delta)$. This allows us to refer banks' debt structures as simply (D, δ). In the equations we will keep writing r rather than $r(\delta)$, except when presentationally convenient.

4.2.1 Value of bank equity in normal times

Let $E(D, \delta; \phi)$ be the value of a bank's equity at a normal period immediately after having paid dividends due to cash flows generated in the prior period. This value satisfies the following recursive equation:

$$\begin{aligned} E(D,\delta;\phi) \quad = \quad & \frac{1}{1+\rho_I}\{(\mu - rD) + (1-\varepsilon)E(D,\delta;\phi) + \\ & +\varepsilon(1-\alpha)\frac{1}{1+\rho_I}[\mu - (1-\delta)rD + \delta D + E(D,\delta;\phi)]\}. \end{aligned} \tag{4}$$

annualized discount rates are $\rho_P = 2\%, \rho_I = 6\%$, the annualized yield on bank assets is $\mu = 4\%$, the expected time until the arrival of an idiosyncratic preference shock is 1 year ($\gamma = 1/12$), and the expected time between systemic crises is 10 years ($\varepsilon = 1/120$).

To explain the equation, recall that bankers' discount rate is ρ_I and after all normal periods bankers receive (and immediately consume) the dividend $\mu - Dr$. With probability $1-\varepsilon$, the next period is a normal period and bankers additionally obtain the continuation value $E(D, \delta; \phi)$.[22] With probability ε, a systemic crisis arrives and refinancing the bank involves relinquishing a fraction α of the equity to the bridge financiers.

The factor $\frac{1}{1+\rho_I}[\mu - (1-\delta)rD + \delta D + E(D, \delta; \phi)]$ accounts for the total value of the bank's equity after it gets bridge financed in the crisis period. Such value is expressed in terms of payoffs received one period ahead: $\mu - (1-\delta)rD$ reflects dividends in the period after the crisis (which are inflated by the fact that the bank's debt was temporarily reduced to $(1-\delta)D$), δD reflects the revenue from reissuing the debt that was bridge financed during the crisis period (which is paid to shareholders as a special dividend), and the last term reflects that, one period after the crisis, the bank's original debt structure is fully restored and its equity value is $E(D, \delta; \phi)$ again.

We next discuss how α is determined and its implications for the valuation of equity. Competition between bridge financiers implies that bankers will obtain the funds δD in exchange for the minimal α that satisfies

$$\alpha \frac{1}{1+\rho_I}[\mu - (1-\delta)rD + \delta D + E(D, \delta; \phi)] \geq (1+\phi)\delta D. \tag{5}$$

Since we must have $\alpha \leq 1$, the feasibility of bridge financing eventually requires

$$\mu + E(D, \delta; \phi) \geq [(1+\rho_I)(1+\phi)\delta + (1-\delta)r - \delta]D, \tag{6}$$

which will be referred as the *bridge financing constraint* (BF) in the analysis that follows. Since (5) holds with equality, we can use it to substitute for α in (4). Solving for $E(D, \delta; \phi)$ in the resulting expression yields the following Gordon-type formula for equity value:

$$E(D, \delta; \phi) = \frac{1}{\rho_I}\left[\mu - r(\delta)D - \frac{\varepsilon}{1+\rho_I+\varepsilon}\{[(1+\rho_I)\phi + \rho_I] - r(\delta)\}\delta D\right]. \tag{7}$$

The interpretation is very intuitive. Equity is valued as a perpetuity with payoffs discounted at rate ρ_I:

1. μ is the unlevered cash flow of the bank.

2. $r(\delta)$ is the interest rate paid on debt in normal periods.

[22] Notice that negative cash flows due to maturing debt, δD, are exactly offset by the proceeds from the issuance of an identical amount of replacing debt.

3. $\frac{\varepsilon}{1+\rho_I+\varepsilon}\{[(1+\rho_I)\phi+\rho_I]-r(\delta)\}$ reflects the differential cost of refinancing the amount of maturing debt δD every time a crisis arrives.

4.2.2 Optimal debt structure problem

Clearly, since initial bankers appropriate D out of what savers pay for the bank's debt when issued, optimal debt structures will maximize the total market value of the bank, $V(D,\delta;\phi) = D + E(D,\delta;\phi)$, which using (7) can be expressed as:

$$V(D,\delta;\phi) = D + E(D,\delta;\phi) = \frac{\mu}{\rho_I} + \frac{\rho_I - r(\delta)}{\rho_I}D - \frac{1}{\rho_I}\frac{\varepsilon\{[(1+\rho_I)\phi+\rho_I]-r(\delta)\}}{1+\rho_I+\varepsilon}\delta D. \quad (8)$$

The first term in this expression is the value of the unlevered bank. The second term reflects the value of financing the bank with debt claims held by savers' initially more patient than the bankers (notice that $r(\delta) < \rho_I$, by Proposition 1). The third term reflects the refinancing costs during systemic crises.

The bank's maximization problem is the following:

$$\begin{aligned} \max_{D\geq 0,\ \delta\in[0,1]} \quad & V(D,\delta;\phi) = D + E(D,\delta;\phi) & (9) \\ \text{s.t.} \quad & E(D,\delta;\phi) \geq 0 & \text{(LL)} \\ & \mu + E(D,\delta;\phi) - [(1+\rho_I)(1+\phi)\delta + (1-\delta)r - \delta]D \geq 0 & \text{(BF)} \end{aligned}$$

The first constraint imposes the non-negativity of the bank's equity value in normal periods, and we will refer to it as bankers' limited liability constraint (LL).[23] The second constraint is the bridge financing constraint (6), which can be interpreted as the result of bankers' limited liability in crisis times (since the equity stake of preexisting bankers can, at most, be fully diluted by setting $\alpha = 1$). It can be shown that both constraints impose the same constraint on D for $\delta = 0$, but (BF) is tighter than (LL) for $\delta > 0$.[24] Thus (LL) can be safely ignored.

The following technical assumptions help us prove the existence and uniqueness of the solution to the bank's optimization problem:[25]

Assumption 1 The function Φ is upper bounded by $2\frac{1+\rho_P}{1+\rho_I} - 1$.

Assumption 2 $\pi < \frac{1-\rho_I}{2}$.

Proposition 2 *For any given excess cost of liquidity during a crisis $\phi \leq 2\frac{1+\rho_P}{1+\rho_I} - 1$, the bank's maximization problem has a unique solution (D^*, δ^*). In the solution:*

---

[23] Satisfying (LL) implies in particular the non-negativity of bankers dividends, $\mu - r(\delta)D \geq 0$.

[24] See the proof of Proposition 2 in Appendix A.

[25] We have checked numerically that the results in Proposition 2 below are also true when these assumptions do not hold. In any case, these sufficient conditions do not impose tight restrictions on parameters.

1. *The bridge financing constraint is binding, i.e. in each crisis bridge financiers take 100% of the bank's equity.*

2. *Optimal debt maturity $1/\delta^*$ is increasing in ϕ and the optimal amount of maturing debt per period $\delta^* D^*$ is decreasing in ϕ. In fact, if $\delta^* \in (0,1)$, both δ^* and $\delta^* D^*$ are strictly decreasing in ϕ.*

The intuition for these results is as follows. First, even if the bank does not get involved in maturity transformation ($\delta = 0$), its value is increasing in D, making it interested in choosing the maximum feasible leverage. If maturity transformation generates value, this tendency remains, so (BF) is necessarily binding at the optimum.[26] Second, as the excess cost of liquidity in a crisis ϕ increases, the value of maturity transformation diminishes which implies the choice of a longer expected maturity. The tightening of (BF) forces banks to reduce the amount of funding $\delta^* D^*$ demanded to bridge financiers during crises.[27]

Hence our theory on banks' maturity transformation function has implications in terms of the classical debt-versus-equity capital structure choice. Each bank must keep enough equity value in normal times so as to be able to obtain sufficient bridge financing during a crisis. Figure 2 depicts a bank's optimal equity to total market value ratio in the normal state, E/V, as a function of the excess cost of liquidity in a crisis ϕ. The resulting *capital ratio* is tiny for $\phi = 0$ and strictly increasing in ϕ.[28] Under the illustrated parameterization, the model yields capital ratios in a realistic 4% to 8% range for a wide range of values of ϕ.[29]

4.3 The competitive equilibrium

Banks' optimization problem for any given excess cost of liquidity in a crisis ϕ embeds savers' participation constraint so the only condition for equilibrium that remains to be imposed

[26] The full dilution of the original equity stakes of the bank in each crisis is an implication of the fact that all crises have the same severity. If we introduce heterogeneity in this dimension, for example, by introducing random shifts in the inverse supply of liquidity curve $\Phi(x)$, the bridge financing constraint might only be binding (or even not satisfied, inducing bankruptcy) in the most severe crises.

[27] Although, we have no formal proof regarding total debt D^*, in all our numerical examples D^* is also decreasing in ϕ. Additionally, it is actually possible to prove that δ^* is independent from the asset return μ (which acts very much like a scale parameter), while D^* is increasing in μ.

[28] Even with $\phi = 0$ banks would need to operate with strictly positive equity because bridge financiers would yet demand a return $\rho_I > r$ for the debt financed in a crisis.

[29] Capital ratios in actual banks may be driven by regulatory constraints. In fact the capital ratios depicted in Figure 2 are the minimal ones compatible with banks being able to avoid default during a systemic crises. These might be the relevant regulatory capital ratios imposed on banks in an extended version of the model in which, perhaps without fully internalizing some social costs of bank failures, bankers wanted to expose their banks to default during crises (see Appendix B for a rationalization of when they might wish to do so).

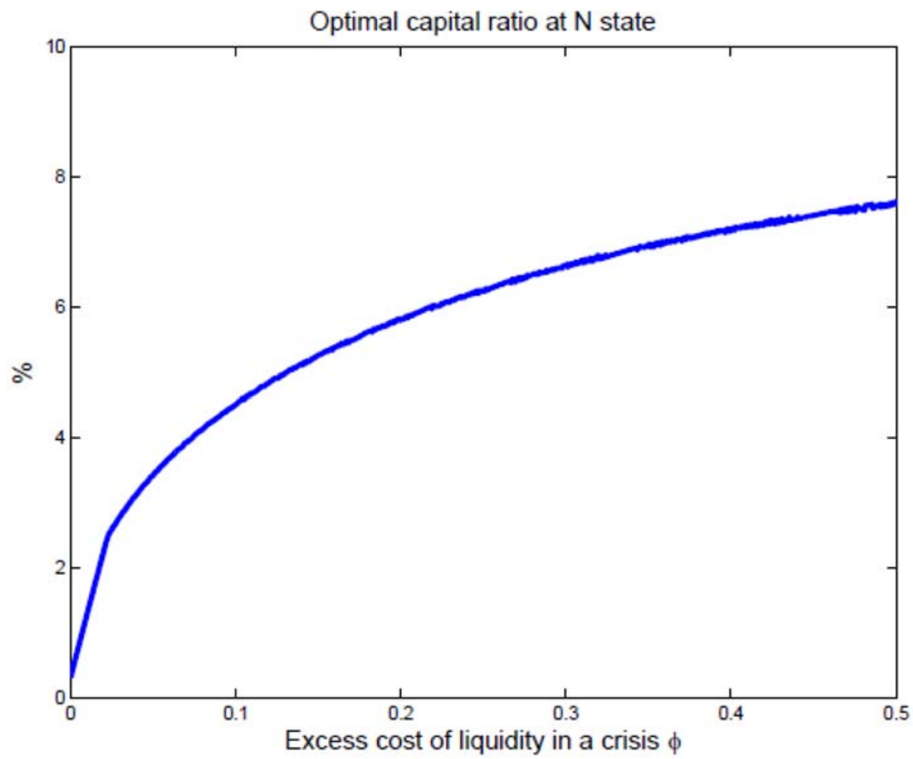

Figure 2: Banks' optimal capital ratio as a function of ϕ

is the clearing of the market for bridge financing in crisis periods. The continuity and monotonicity in ϕ of the function that describes excess demand in such market guarantees that there exists a unique excess cost of crisis liquidity ϕ^e for which the market clears:

Proposition 3 *The equilibrium of the economy* $(\phi^e, (D^e, r^e, \delta^e))$ *exists and is unique.*

The effects on equilibrium outcomes of shifts in the supply of crisis liquidity are summarized in the following proposition:

Proposition 4 *If the inverse supply of liquidity during crises* $\Phi(x)$ *shifts upwards, the equilibrium changes as follows: expected debt maturity* $1/\delta^e$ *increases, total refinancing needs* $\delta^e D^e$ *fall, bank debt yields* r^e *increase, and the cost of liquidity during crises* ϕ^e *increases. If initially* $\delta^e \in (0, 1)$*, all these variations are strict.*

The results in Proposition 4 are illustrated, together with other comparative statics results, in Figure 3, where the inverse supply of crisis liquidity is parameterized as $\Phi_a(x) = ax^2$. Specifically, in the first column of graphs we plot various equilibrium variables (expected

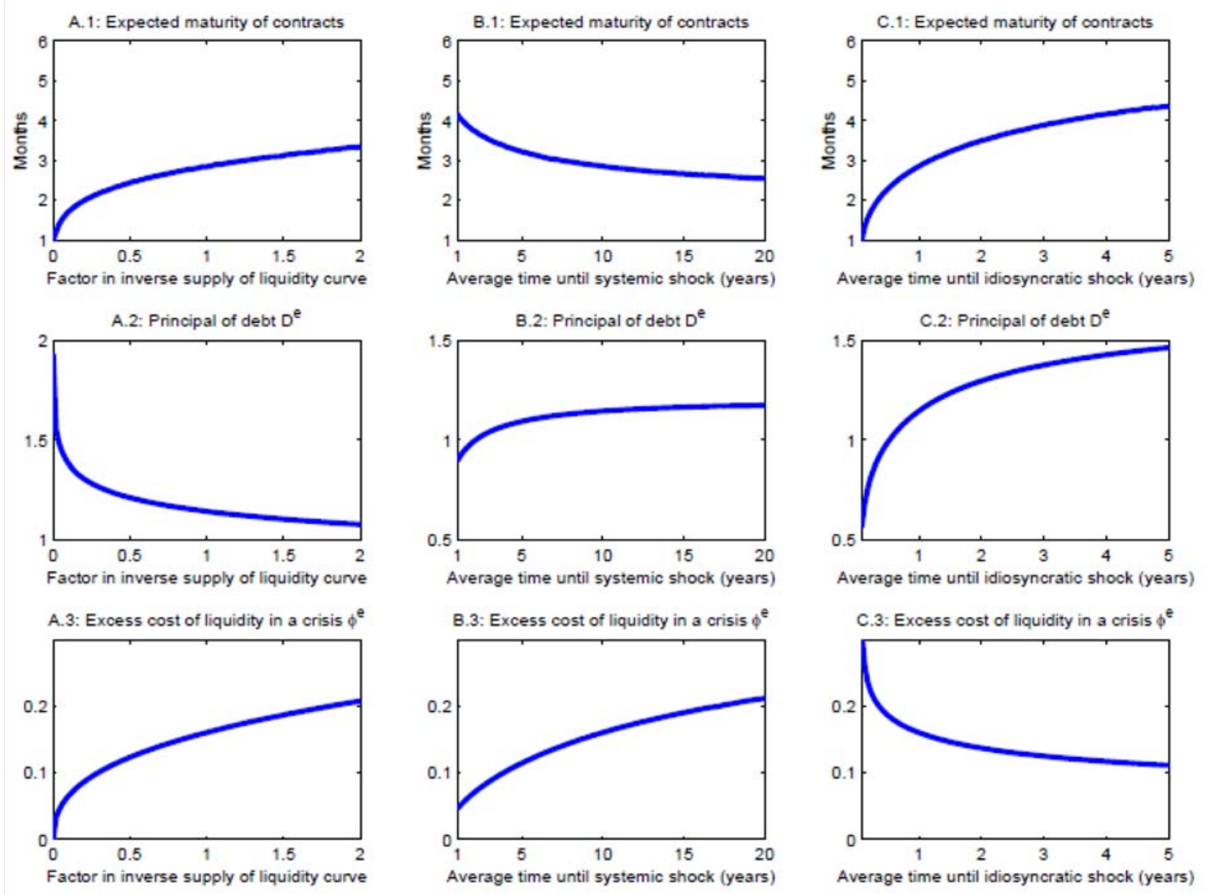

Figure 3: Effect of changes in the parameters on the competitive equilibrium

debt maturity, total debt, and excess cost of crisis liquidity) against the parameter a of such function. The second and third columns of graphs in Figure 3 show the equilibrium effects of increasing the average time to the arrival of a systemic shock ($1/\varepsilon$) and an idiosyncratic shock ($1/\gamma$), respectively. All these effects are intuitive and quite self-explanatory.

5 Efficiency and regulatory implications

In this section we solve the welfare maximization problem of a (constrained) social planner who has the ability to directly control or regulate banks' funding structure decisions subject to the same constraints that banks face when solving their private value maximization problems. We find that the unregulated competitive equilibrium features inefficiently short debt maturities because of the interaction of a pecuniary externality with the constraints faced by banks in their maturity transformation function. We show the possibility of restoring efficiency by directly regulating maturity decisions as well as by means of a Pigovian tax on banks' refinancing needs.

5.1 Inefficiency of the unregulated equilibrium

Let us suppose that a social planner can regulate both the amount D and the maturity parameter δ of banks' debt. In our economy only existing bankers and becoming bankers (bridge financiers) appropriate a surplus. So the natural objective function for the social planner is the present value of such surpluses. Experts who provide bridge financing to banks in crisis periods obtain the difference between equilibrium excess cost of crisis liquidity $\phi = \Phi(\delta D)$ and the net present value of their alternative investment opportunity z. Hence, bridge financiers' surplus in any crisis is:

$$u(D,\delta) = \int_0^{\delta D} \left(\Phi(\delta D) - \Phi(x)\right) dx = \delta D \Phi(\delta D) - \int_0^{\delta D} \Phi(x) dx.$$

Evaluated at a normal period, the present value of the surpluses obtained along all future crises can be written as:

$$U(D,\delta) = \frac{1}{\rho_I}\frac{(1+\rho_I)\varepsilon}{1+\rho_I+\varepsilon} u(D,\delta).$$

Hence, using (8), the objective function of the social planner can be expressed as:

$$\begin{aligned} W(D,\delta) &= V(D,\delta;\Phi(\delta D)) + U(D,\delta) \\ &= \frac{\mu}{\rho_I} + \frac{\rho_I - r(\delta)}{\rho_I}D - \frac{1}{\rho_I}\frac{\varepsilon(\rho_I - r(\delta))}{1+\rho_I+\varepsilon}\delta D - \frac{1}{\rho_I}\frac{(1+\rho_I)\varepsilon}{1+\rho_I+\varepsilon}\int_0^{\delta D}\Phi(x)dx, \quad (10) \end{aligned}$$

which contains four terms: the value of an unlevered bank, the value added by maturity transformation in the absence of systemic crises, the value lost due to financing the bank with impatient agents during liquidity crises, and the value lost due to the fact that these impatient agents are experts that give up the NPV of their own investment projects.

Thus, the social planner's problem can be written as:[30]

$$\begin{aligned} &\max_{D\geq 0,\ \delta\in[0,1]} && W(D,\delta) && (11) \\ &\text{s.t.} && \mu + E(D,\delta;\Phi(\delta D)) - [(1+\rho_I)(1+\Phi(\delta D))\delta + (1-\delta)r - \delta]D \geq 0 && \text{(BF')} \end{aligned}$$

This problem differs from banks' optimization problem (9) in two dimensions. First, the objective function includes bridge financiers' surplus. Second, the social planner internalizes

[30] Recall that the constraint called (LL) in (9) can be ignored because it is implied by the bridge financing constraint.

the effect of banks' funding decisions on the market-clearing excess cost of crisis liquidity, so (BF') contains $\Phi(D\delta)$ in the place occupied by ϕ in individual banks' (BF) constraint (see (6)).

The first result in this section looks at the hypothetical situation in which the social planner were able to regulate δ (or D) without changing some given (perhaps independently regulated) D (or δ):

Proposition 5 *If either the total amount of debt D issued by banks or the expected maturity $1/\delta$ of their debt contracts is exogenously fixed, the competitive equilibrium of the model is socially efficient.*

In other words, moving δ (D) away from the equilibrium value $\overline{\delta}^e$ ($\overline{D}^e$) that would arise in the fixed-D (fixed-δ) situation would not produce any net welfare gain. Changing that sole variable would amount, in the margin, to a pure redistribution of value between bridge financiers and the initial bankers (e.g. a lower δ would reduce ϕ but the induced increase in bankers' surplus V would be exactly offset by the decline in bridge financiers' surplus U).[31]

The result is different if the social planner can influence D and δ simultaneously:

Proposition 6 *If the competitive equilibrium features $\delta^e \in (0,1)$ then a social planner can increase social welfare by choosing a longer expected debt maturity than in the competitive equilibrium, i.e. some $1/\delta^s > 1/\delta^e$.*

The root of the discrepancy between the competitive and the socially optimal allocations is at the way individual banks and the social planner perceive the frontier of the set of maturity transformation possibilities: banks choose their individually optimal (D, δ) along the (BF) constraint (where ϕ^e is taken as given) whereas the social planner does it along (BF') constraint (where $\phi = \Phi(\delta D)$). Each of these constraints and the corresponding decisions are illustrated in Figure 4.

At the equilibrium allocation (D^e, δ^e) both the social planner's and the initial bankers' indifference curves are tangent to (BF). Moreover, (BF) and (BF') intersect at (D^e, δ^e) (since the competitive equilibrium obviously satisfies $\phi^e = \Phi(\delta^e D^e)$). However, the social planner's indifference curve is not tangent to (BF') at (D^e, δ^e), implying that this allocation does

[31] If for whatever reasons the social planner gives more weight in the social welfare function to the initial bankers than to the potential bridge financiers, then, even for fixed D, there might be social gains from imposing some $\overline{\delta} < \overline{\delta}^e$.

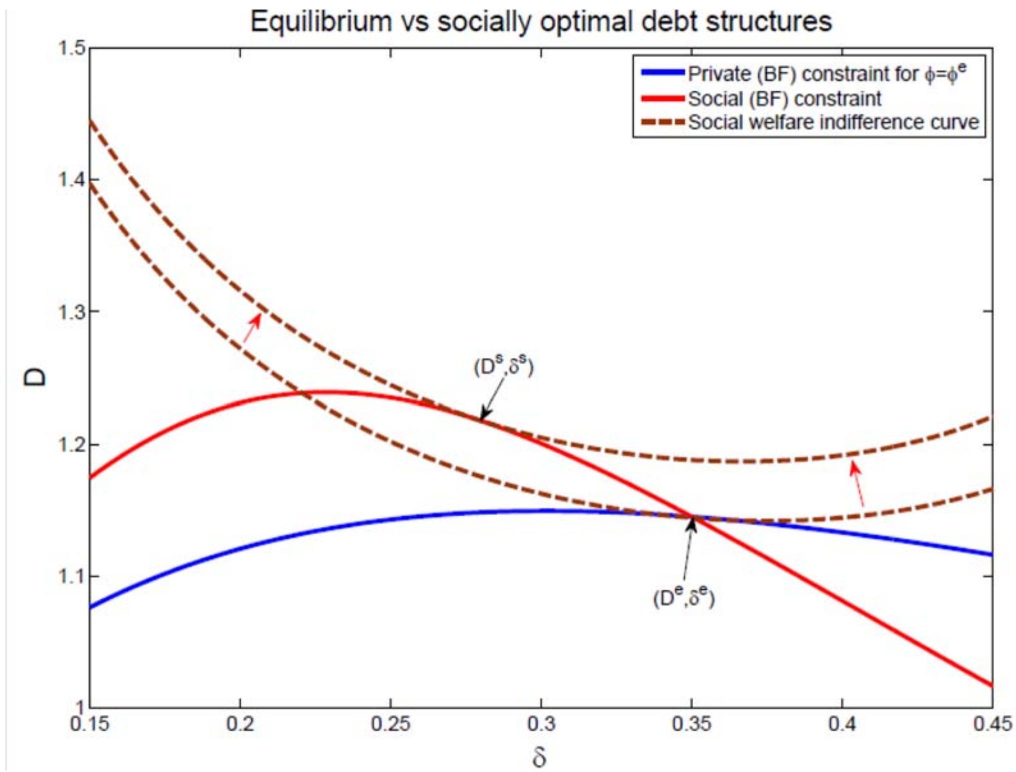

Figure 4: Maturity transformation possibilities from private and social perspectives

not maximize welfare. In the neighborhood of (D^e, δ^e), (BF') allows for a larger increase in D, by reducing δ, than what seems implied by (BF) (where ϕ remains constant). It turns out that maturity transformation can produce a larger surplus with a larger use of its extensive margin (leverage) and a lower use of its intensive margin (short maturities), like at (D^s, δ^s) the figure.[32] Figure 5 illustrates how the differences between the equilibrium and the socially-efficient bank funding structures change with some of the parameters of the model.

5.2 Restoring efficiency with regulation

In order to achieve the socially efficient debt structure (D^s, δ^s) as a regulated competitive equilibrium, the most straightforward intervention in the context of the model would be to impose an upper limit δ^s to banks' maturity decision δ. Given that the inverse of δ is the expected maturity of a bank's debt (and our banks' assets have infinite maturity), such limit

[32] This finding offers a new perspective for the joint assessment of some of the regulatory proposals emerged in the aftermath of the recent crisis, which defend reducing *both* banks' leverage *and* their reliance on short-term funding.

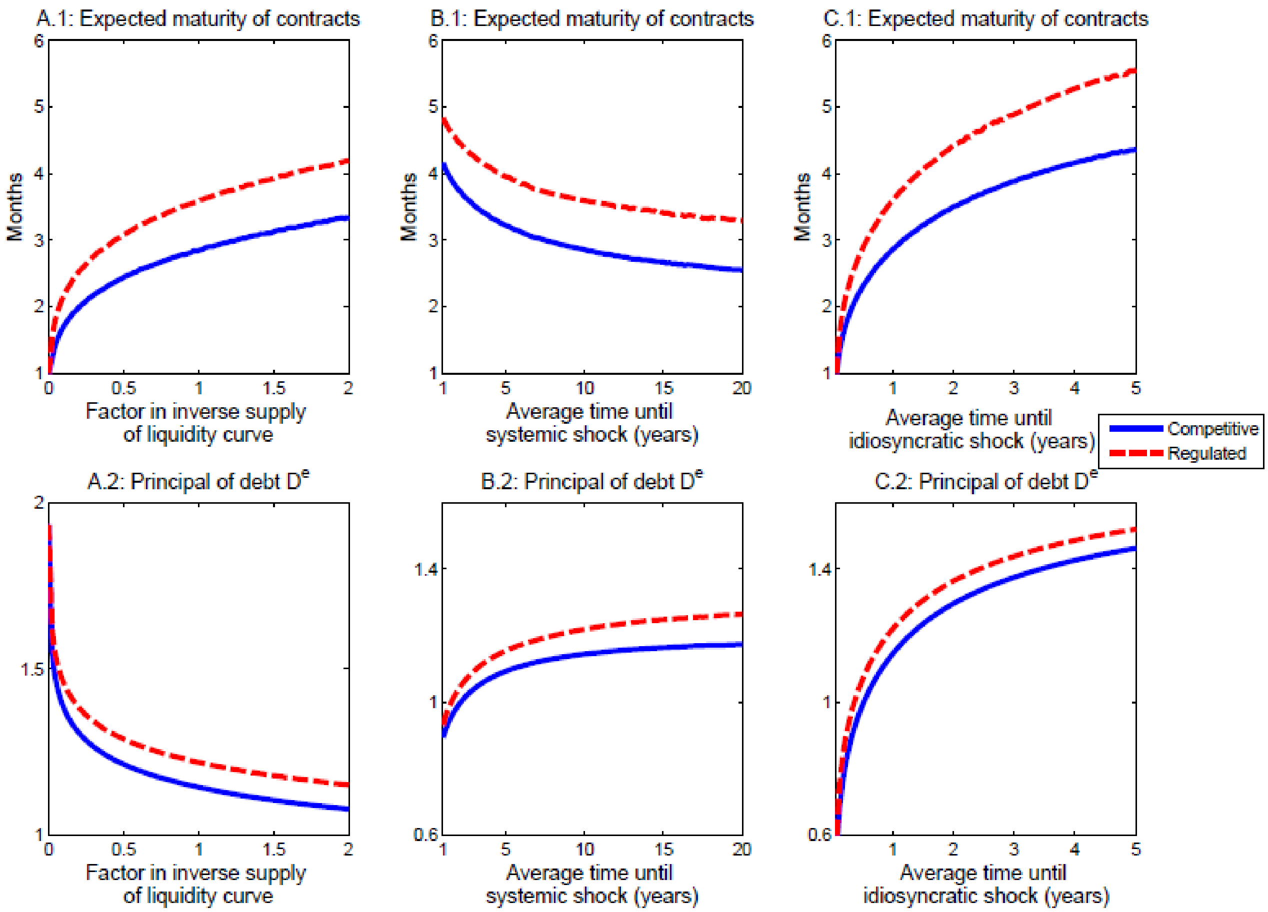

Figure 5: Competitive equilibrium vs. socially-efficient funding structures

could be interpreted as equivalent to introducing a minimum net stable funding ratio like the one postulated by Basel III. Anticipating a cost of liquidity in a crisis ϕ^s, banks in our model would find such a requirement binding and would choose to issue the maximum debt compatible with (BF) given ϕ^s and δ^s, which is D^s.

As shown by Perotti and Suarez (2011), adding unobservable heterogeneity across banks may undermine the efficiency of one-size-fits-all quantity-based liquidity regulation and alternatives such as Pigovian taxes may be superior.[33] With this motivation in perspective (but without explicitly adding heterogeneity), we next check whether a Pigovian tax on banks' refinancing needs might implement (D^s, δ^s) as a regulated competitive equilibrium. We consider the following class of *non-subsidized* Pigovian schemes:

[33]Specifically, these authors show that if banks unobservably differ in their opportunities to extract value from maturity transformation, a flat rate Pigovian tax on refinancing needs can induce the marginal internalization of the relevant externalities while allowing the most efficient banks to operate with larger maturity mismatches than the less efficient ones.

1. Each bank pays a *proportional tax* of rate τ per period on its refinancing needs δD.

2. The social planner pays to each bank a *lump-sum transfer* $M \leq \tau\delta D$ per period.

Notice that $\tau\delta D$ stands for the revenue of the Pigovian tax and $M \leq \tau\delta D$ restricts the possibility of subsidizing banks via the lump-sum transfer M. We can prove analytically the following result:

Proposition 7 *If the unregulated competitive equilibrium features $\delta^e \in (0,1)$, there exists a Pigovian tax scheme (τ^P, M^P) that induces the socially optimal allocation (D^s, δ^s). This scheme satisfies $\tau^P > 0$ and $M^P = \tau^P\delta^s D^s$, and is unique if $\delta^s > 0$.*

The scheme uses some $\tau^P > 0$ to push banks towards funding decisions involving lower refinancing needs than in the unregulated competitive equilibrium. Interestingly, in order to reach the socially efficient allocation, all the revenue raised by the tax τ^P has to be rebated to the banks through M^P. Values of τ and M which induce δ^s but involve $M < \tau\delta^P D^P$ would lower the value of bank equity relative to the situation in which δ^s is directly regulated, which would in turn tighten banks' bridge financing constraint pushing banks towards a leverage D^P strictly lower than D^s.

The need for rebating the revenue from the Pigovian tax is a novel insight relative to the non-pecuniary externality setup of Perotti and Suarez (2011). The general intuition is that, when pecuniary externalities cause inefficiency due to their interaction with financial constraints, regulators must be cautious not to address one of the manifestations of the inefficiency (excessively short maturities) in a way (non-rebated taxes) that, by tightening the relevant constraints (here, reducing banks' equity values) may partly undo the potential gains from the intervention.

6 Liquidity insurance

In the next two subsections we consider the potential welfare contribution of private and government-based liquidity insurance arrangements. In both parts we conclude that liquidity insurance is beneficial but does not eliminate the desirability of debt maturity regulation.

6.1 Private liquidity insurance

The fact that in both the competitive and the regulated allocations banks' bridge financing constraints are binding, while the normal times limited liability constraints are not, suggests that some form of insurance against systemic liquidity crises might increase welfare. In order to introduce insurance, we need to relax the assumption that writing contracts contingent on the realization of systemic crises is unfeasible.[34] To keep things tractable, we focus on simple one-period refinancing insurance arrangements subscribed by individual banks and newly born experts at the beginning of each period, prior to the realization of uncertainty regarding the occurrence of a crisis.

Specifically, the arrangements we consider establish that:

1. Except in the period immediately after each crisis, the bank pays a per-period premium p for each unit of insured refinancing $\lambda\delta D > 0$ to a measure $\lambda\delta D$ of entering experts, where $\lambda \in [0, 1]$ is the insured fraction of refinancing needs.[35]

2. If there is a systemic crisis, the insuring experts supply the bank with funds $\lambda\delta D$ in the period and receive a gross repayment of $[1 + r(\delta) + p]\lambda\delta D$ in the following period.

Under this arrangement the refinancing of $\lambda\delta D$ is just as costly as if no crisis had occurred ($r(\delta)$ is the normal times interest rate). The repayment of $p\lambda\delta D$ to the insuring experts one period after a crisis is included in order to offset the impact on the banks' net income of the fact that insurance is unneeded (and hence not paid for) immediately after a crisis.

For the sale of insurance to be attractive to an entering expert with funds that can earn NPV of z in normal periods and $\max\{z, \phi\}$ in crisis periods, the insurance premium p must satisfy

$$p + (1 - \varepsilon)(1 + z) + \varepsilon \frac{1 + r + p}{1 + \rho_I} \geq (1 - \varepsilon)(1 + z) + \varepsilon \max\{1 + z, 1 + \phi\}. \tag{12}$$

Competition among entering experts will lead to a situation in which (12) is binding for the marginal provider of either insurance or bridge financing, who will have $z = \phi$.[36] Solving for p in such equality yields

$$p = \frac{\varepsilon}{1 + \rho_I + \varepsilon} \{[(1 + \rho_I)\phi + \rho_I] - r(\delta)\}, \tag{13}$$

which is identical to the factor (within the large square brackets) that multiplies δD in (7).

[34] We may interpret the new possibility as associated with the introduction of a macroprudential authority that officially declares the existence of a systemic crisis.

[35] Insurance in the period immediately after a crisis is unneeded because, according to our assumptions, crisis periods are always followed by a normal period.

[36] Clearing the market for liquidity in a crisis requires $\phi = \Phi(\delta D)$ irrespectively of the fraction of δD covered with insurance.

On the other hand, the value of equity at the N state of a bank that decides to insure a fraction λ of its refinancing needs can be written as

$$E(D,\delta,\lambda;\phi)=\frac{1}{\rho_I}\{\mu-r(\delta)D-p\lambda\delta D-\frac{\varepsilon}{1+\rho_I+\varepsilon}\{[(1+\rho_I)\phi+\rho_I]-r(\delta)\}(1-\lambda)\delta D\},$$

which is an extended version of (7). Now, using (13) to substitute for p, it becomes clear that:

$$E(D,\delta,\lambda;\phi)=E(D,\delta,0;\phi)=E(D,\delta;\phi), \tag{14}$$

which can be interpreted as a Modigliani-Miller type result: moving the fraction λ of (fairly priced) insured funding simply redistributes some future cash flows among the insurance takers and the insurers.

Such redistribution is, however, relevant for the bank's overall optimization problem since it alters its bridge financing constraint. The bridge financing constraint in the presence of insurance (BFI) can be written as:

$$\mu+E(D,\delta;\phi)\geq\{(1+\rho_I)(1+\phi)(1-\lambda)\delta+r[1-(1-\lambda)\delta]+\lambda p\delta-(1-\lambda)\delta\}D, \quad \text{(BFI)}$$

which differs from (6) in that now only the uninsured fraction $1-\lambda$ of the bank's refinancing needs have to ex post pay the excess cost ϕ.

It is easy to check that $\lambda=1$ implies the maximal relaxation of this constraint.[37] On the other hand, by (14), the bank's limited liability constraint in a normal period does not depend on λ and thus is identical to (LL) in (6). Therefore, the bank will solve the counterpart of the value maximization problem in (6) by getting fully insured against systemic crises ($\lambda=1$). By doing so, its net cash flow becomes $\mu-rD-p\delta D$ and the constraints (BFI) and (LL) collapse into simply requiring that this cash flow is not negative.

The following proposition describes the positive welfare implications of adding insurance when funding decisions (i.e. δ) are optimally regulated. It also shows that, with liquidity insurance, banks in the unregulated economy would opt for inefficiently short debt maturities.

Proposition 8 *In a* regulated *economy, adding a private liquidity insurance scheme strictly increases welfare. With liquidity insurance, expected debt maturity in the* unregulated *equilibrium is too short.*

[37] It suffices to realize that (13) implies $(1+\rho_I)(1+\phi)-1-r>p$.

Intuitively, when liquidity insurance is introduced, (LL) becomes banks' only relevant constraint, which implies expanding the set of maturity transformation possibilities faced by both them and a possible regulator. Hence, a social planner can definitely produce more social welfare with insurance than without insurance. Instead, in the absence of regulation, the pecuniary externality regarding banks' debt maturity decisions operates qualitatively in the same way as before but through the (LL) constraint: bank decisions affect the excess cost of crisis liquidity ϕ, which in turn affects the cost of insurance p, and ends up tightening the constraint $\mu - rD - p\delta D \geq 0$.[38] So the main policy message from this subsection is that, if arranging for systemic liquidity insurance is at all feasible, it should be promoted but not as a substitute but as a complement to funding maturity regulation.

6.2 Public liquidity provision

We now turn to explore a simple reinterpretation of the model under which the marginal supplier of funds during a crisis is a government-sponsored lender (e.g. a central bank acting as a lender of last resort) which is constrained to offer its funds on a non-subsidized basis. Specifically, suppose that the supply of funds from private agents during crises is small, say zero, but the government is able to obtain alternative funds x with a marginal (excess) opportunity cost that, to save on notation, we describe with the function $\Phi(x)$, which will play a role analogous to that of our previous inverse supply of crisis liquidity.[39]

We are going to compare two public liquidity insurance regimes in which the government commits to cover, on non-subsidized basis, banks' financing needs during liquidity crises:[40]

I. Public liquidity insurance only In each period, the government charges an insurance premium $p\delta D$ to each bank with refinancing needs δD and commits to cover these needs in each crisis period in exchange for a repayment of $[1 + r(\delta)]\delta D$ in the period after

[38] We are not able to prove that the introduction of insurance increases welfare in the unregulated economy, but this is actually the case in all the parameterizations that we have explored. The theoretical ambiguity comes from the fact that, with full insurance, unregulated banks will tend to choose funding structures that put upward pressure on ϕ^e and, in principle, a sufficiently large increase in ϕ^e might fully offset the gains due to the introduction of insurance.

[39] We refer to $\Phi(x)$ as an *excess* cost because it comes on top of the normal opportunity cost of funds implied by assuming that the government has the same discount rate ρ_I as impatient agents. This excess cost may here reflect the NPV of the deadweight losses due to future distortionary taxes.

[40] The government might instead commit to supply liquidity during crises at some fixed excess cost $\widehat{\phi}$. Numerical simulations show that this alternative policy is dominated by the arrangements that we analyze. The reason is that, as in the case of private liquidity insurance analyzed above, spreading the excess cost of crisis liquidity over time expands the set of maturity transformation possibilities.

the crisis.[41]

II. Public liquidity insurance cum maturity regulation In addition to the arrangements of the previous regime, the government regulates banks' maturity decision δ.

As with private insurance, banks' optimal debt structure decisions maximize the total market value of the bank, like in (9), but only subject to the limited liability constraint $\mu - rD - p\delta D \geq 0$. The NPV of the revenues and costs that accrue to the government in its role as an insurer are:

$$G = \frac{1}{\rho_I}\left[p\delta D - \frac{\varepsilon\,(\rho_I - r(\delta))}{1+\rho_I+\varepsilon}\delta D - \frac{(1+\rho_I)\varepsilon}{1+\rho_I+\varepsilon}\int_0^{\delta D}\Phi(x)dx\right], \tag{15}$$

where the last term accounts for the (excess) opportunity cost of the funds lent in a crisis. Aggregate welfare in this setting can be defined as the sum of the total market value of the insured banks, V, and the net present value of the government's stake, G. The resulting expression for welfare is analogous to the expression for W in (10). We assume that the government maximizes W, subject to $G \geq 0$, i.e. we do not allow for positive NPV transfers from the government to the bank owners.[42]

the difference between the two public liquidity insurance regimes described above stems from the tools through which the government may influence banks' decisions. In Regime I, the government can only set p and will do so taking into account the impact of p on $(D, \delta) = (D(p), \delta(p))$. In Regime II, the government can simultaneously set p and δ, taking into account the impact of *both* parameters on $(D, \delta) = (D(p, \delta), \delta)$.

Hence, in Regime I, the premium p has to play the dual role of regulating banks' refinancing needs δD and guaranteeing $G \geq 0$. In the second regime, the direct regulation of δ provides a useful second tool. In fact, when the only tool is p, the welfare maximizing solution involves $G > 0$ in all our simulations while, when the two tools are available, it is possible to prove that the optimum involves $G = 0$.

Based on the same underlying parameterization as in previous figures, Figure 6 illustrates the outcomes under each of the two commented regimes. The horizontal axes represent different excess costs of government funds, measured by the factor a of a cost function specified

[41] Contrarily to the case with (voluntary, one-period) private liquidity insurance, we now assume $p\delta D$ to be paid to the government in *every* period (including periods after a crisis where the probability of suffering another crisis is zero).

[42] Imposing $G \geq 0$ implies that the government must be able to cover the *average* opportunity cost of its funds. This constraint is more flexible than the constraint associated with private liquidity insurance, where the *marginal* competitive provider of insurance must break even.

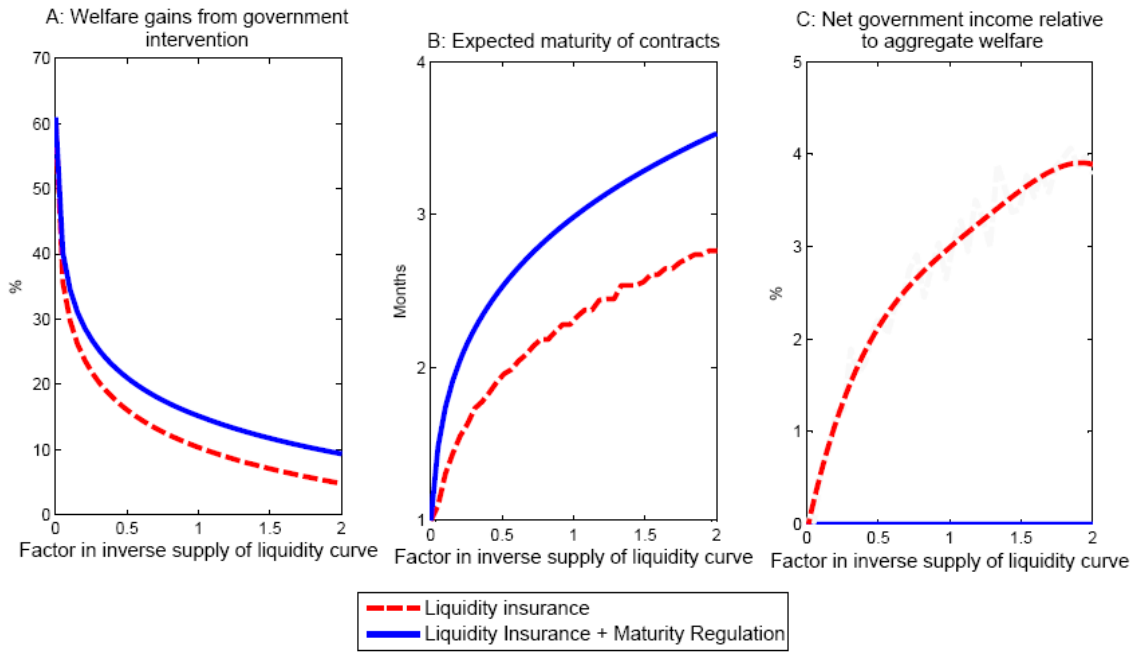

Figure 6: Optimal policy interventions vs. government cost of funds

as $\Phi(x) = ax$. Panel A represents welfare gains relative to a benchmark scenario without liquidity insurance in which banks fail if they have refinancing needs during a crisis.[43] These gains decrease as the excess cost of government funds increases and are significantly greater when maturity regulation is allowed. Quite intuitively, expected debt maturity (depicted in Panel B) is shorter when it can only be regulated using the premium p than when δ is controlled by the regulator. Finally, Panel C shows the positive government surplus $G > 0$ associated with Regime I and the zero surplus $G = 0$ associated with Regime II.[44]

As in the case of private liquidity insurance, the results in this subsection deliver the message that liquidity insurance is not substitute but a complement to maturity regulation. With or without liquidity insurance, unregulated banks tend to choose equilibrium debt maturities that are excessively short, in that they imply refinancing costs during crises that

[43] The liquidation value L in case of default has been calibrated so that it equals 80% of the market value of the bank. This arbitrary choice only affects the scale of the vertical axis in Panel A. Details about the case in which banks fail in crises are provided in subsection 7.1 and Appendix B.

[44] Importantly, the *direct* regulation of δ is not the only way to implement the outcomes associated with Regime II. Consistent with the result in Proposition 7, and by the same reasoning, a solution based on the combination of Pigovian taxes and lump-sum rebates would also work. Intuitively, the Pigovian insurance scheme would charge a higher insurance premium so as to induce the same maturity decision as the one directly regulated in Regime II and it would rebate the government surplus to the banks in a periodic manner so as to compensate the negative effects of the excessive premium on equity values.

tighten banks' financial constraints, impeding them to collectively develop their maturity transformation function in the socially most valuable manner.

7 Discussion and extensions

In this section we comment on the key assumptions and possible extensions of our model.

7.1 Optimality of not defaulting during crises

We have so far assumed that the liquidation value of banks in case of default, L, is small enough for banks to find it optimal to rely on funding structures that satisfy the (BF) constraint. How small L has to be (and what happens if it is not) is discussed next.

If a bank were not able to refinance its maturing debt, it would default, and we assume that this would precipitate its liquidation. For simplicity, we assume that, if the bank defaults, the liquidation value L is orderly distributed among all debtholders, which excludes the possibility of preemptive runs à la He and Xiong (2011a). If a bank were expected to default in a crisis, savers would require r to include a compensation for credit risk.

Based on the derivations provided in Appendix B, Figure 7 depicts for each possible equilibrium excess cost of crisis liquidity, ϕ^e, the maximum liquidation value $L^{\max}(\phi^e)$ for which, when all other banks opt for bridge financing, an individual bank also prefers to rely on bridge financing. The variation of ϕ^e in this figure can be thought of as a general representation of shifts in the inverse supply of liquidity in a crisis $\Phi(\delta D)$ (which affects only the banks opting for bridge financing and only through ϕ^e, which is independent of L). Hence both dimensions of the figure account for shifts in exogenous parameters. For configurations of parameters with $L \leq L^{\max}(\phi^e)$, the candidate equilibrium with bridge financing gets confirmed as an equilibrium.

$L^{\max}(\phi^e)$ is decreasing, so the higher the cost of funds during crises, the stronger the incentives for banks to opt for funding structures that imply defaulting in a crisis. To reinforce intuitions, Figure 7 also shows the total market value in a crisis of a bank that relies on bridge financing, $V^C(\phi^e)$.[45] The fact that $V^C(\phi^e) > L^{\max}(\phi^e)$ reflects that, for the values of L contained between the two curves exposing the bank to liquidation in case of a crisis is ex-ante optimal but ex-post inefficient. Opting for possible liquidation in a crisis,

[45] Since (BF) is binding, the value of a bank's pre-existing equity at a crisis is 0 and thus $V^C(\phi^e) = D^e(\phi^e)$.

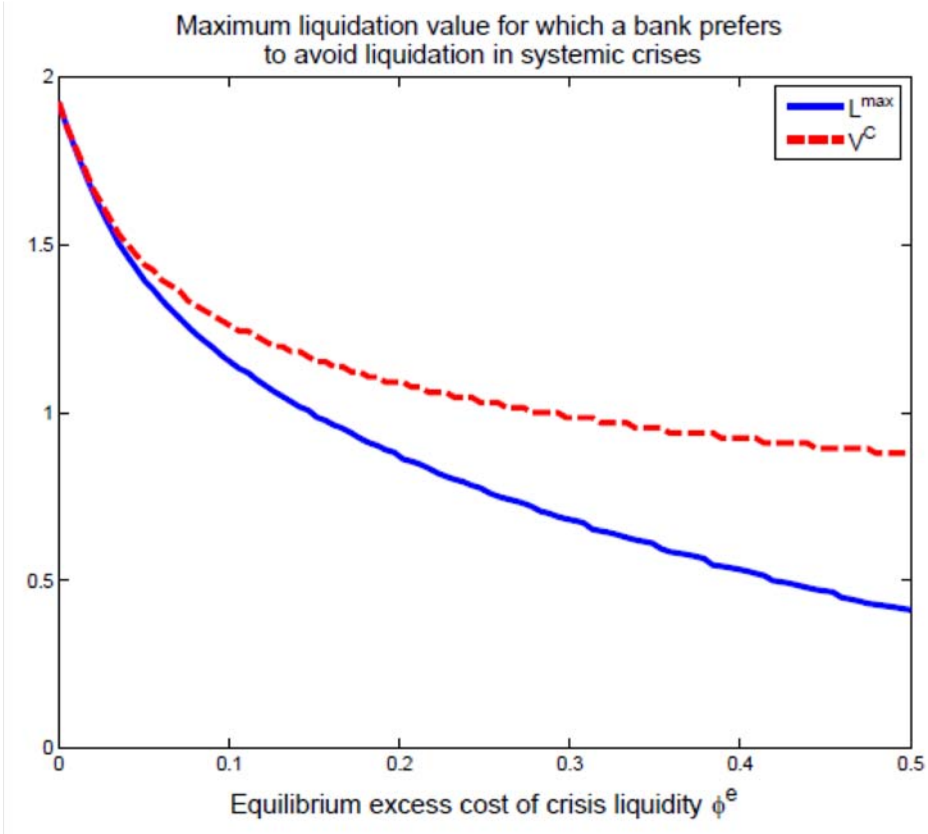

Figure 7: Conditions for the optimality of not defaulting during crises

the bank can get rid of the (BF) constraint in (9) and expand its leverage up to the level allowed by (LL).

In situations with $L > L^{\max}(\phi^e)$ at least some banks will opt for being exposed to liquidation during each systemic crisis. Given the absence of new bank formation in our model, one may wonder whether such a configuration of parameters would lead to the full collapse of the banking sector after sufficiently many crisis. For $L < L^{\max}(0)$, the answer is no, since there is a self-equilibrating mechanism, implied by the adjustment of ϕ throughout the process, that would produce a steady state in which the surviving banks eventually rely on bridge financing.[46]

[46] In such an equilibrium the mass of banks would be $m < 1$ and the excess cost of crisis liquidity would equal the unique ϕ^m that satisfies $L^{\max}(\phi^m) = L$. Now, if (D^m, δ^m) denotes the funding decision under ϕ^m of banks subject to the (BF) constraint, then m can be found as the unique value that solves $\Phi(m\delta^m D^m) = \phi^m$. If the mass of banks were at any point larger than m, then a mass m of banks would use (D^m, δ^m), surviving each crisis, while the remaining ones would be exposed to liquidation in each crisis.

7.2 Deterministic vs random maturity

For tractability we have assumed that debt contracts have random maturity. It would be more realistic to assume that the bank chooses an integer T that describes the deterministic maturity of its debt contracts. In this setting it is possible to determine savers' required maturity premium $r^{\det}(T)$ as we did in Section 4.1. It can also be shown that for $T = 1/\delta$, we have $r^{\det}(T) < r(\delta)$ because discounting is a convex function of time and thus the random variation in maturity realizations produces disutility to impatient savers.

With deterministic maturities, the model would lose some of the Markovian properties that make it tractable. In the period after a crisis the initial funding structure would not be immediately reestablished since, in addition to the debt with principal $\frac{1}{T}D$ that matures and has to be refinanced, the bank would also have to issue the debt with face value $\frac{1}{T}D$ that was bridge financed during the crisis. Thus, in order for the bank to keep a constant fraction $1/T$ of debt maturing in each period, half of the debt issued by the bank in the after-crisis period should have maturity $T-1$, but this would introduce heterogeneity in interest rate payments across the various debts. The description would become further complicated if a new crisis arrives prior to the maturity of the debt with maturity $T-1$.

Therefore, assuming random maturities implies some loss of banks' value but is essential to the simplicity of our recursive valuation formulas. Fortunately, there is no reason to think that deterministic rather than random maturities would qualitatively change any of the trade-offs behind the key results of the paper.

7.3 Resetting debt structures over time

For the sake of clarity, we have assumed that the debt structure (D, δ) decided at $t = 0$ is kept constant along time (except for the fraction δD that is "bridge financed" for just one period during each crisis). What would happen if bankers could reoptimize at some later period?

To narrow down the question, suppose, in particular, that in some given normal period banks had the option to buy all their outstanding debt at market value and then decide on a new debt structure that would be held constant from that moment onwards. It is obvious from the Markovian structure of the model that the bank would not deviate from its initial funding structure.[47] More generally, it is possible to prove that if current bank

[47] The formal argument goes as follows: denote the bank's current debt structure by $(\overline{D}, \overline{\delta})$. In a N state

shareholders are allowed to buy back all outstanding debt and decide a new debt structure at every normal period, their optimal decision (taking all future optimal decisions as given) would also coincide with the one characterized in prior sections.

The even more general case in which at every date the bank could decide to roll-over part of its maturing debt at perhaps some new terms, while keeping constant the structure of its non-maturing debt would not be easy to analyze. We would need a more complicated space of state variables to describe the debt structures that a bank might end up having and, thus, modeling that is out of the scope of this paper. However, there are no reasons to believe that those apparently more general funding structures might be a net source of value to the bank. Intuition from simpler models suggests that altering the terms of new debt as maturing debt is rolled over might only create value to shareholders at the expense of non-maturing debt holders, but this (i) would have a negative repercussion on the value of such a debt when issued (and hence on initial shareholder value) and (ii) could be prevented by including proper covenants in the preexisting debt contracts.

7.4 Tradability of debt

The non-tradability of banks' debt plays a key role in the model. Savers who turn impatient suffer disutility from delaying consumption until their debt matures because there is no secondary market where to sell the debt (or where to sell it at a sufficiently good price). If bank debt could be traded without frictions, impatient savers would try to sell their debts to newly born patient savers, achieving it immediately in normal periods and with one period of delay in crises. Banks could issue perpetual debt ($\delta = 0$) at some initial period and get rid of refinancing concerns. In practice a lot of bank debt, starting with retail deposits, but including also certificates of deposit placed among the public, interbank deposits, debt involved in sales with repurchase agreements (repos), and commercial paper are commonly issued over the counter (OTC) and have no liquid secondary market.

Our model does not contain an explicit justification for the lack of tradability. Arguably, it might stem from administrative, legal compliance, and operational costs associated with

that does not follow a C state, the market value of total outstanding debt is $\overline{D}$. Current shareholders would maximize $V(D, \delta; \phi) - \overline{D}$ subject to the same financing constraints as at $t = 0$ and the optimal solution would be the same as at $t = 0$, since the only difference between the initial optimization problems and the current one is the (constant) $\overline{D}$ now subtracted from the objective function. In a N state that follows a C state, the market value of outstanding debt would be $(1 - \overline{\delta})\overline{D}$ and current shareholders would maximize $V(D, \delta; \phi) - (1 - \overline{\delta})\overline{D}$ but again the solution would not change.

the trading (specially using centralized trade) of heterogenous debt instruments issued in small amounts, with a short life or among a dispersed mass of unsophisticated investors. In fact, if other banks (or some other sophisticated traders) could possess better information about banks than ordinary savers, then costs associated with asymmetric information (e.g. exposure to a winners' curse problem in the acquisition of bank debt) might make the secondary market for bank debt unattractive to ordinary savers (Gorton and Pennacchi, 1990). This view is consistent with the common description of interbank markets as markets where peer monitoring is important (Rochet and Tirole, 1996).

Additionally, the literature in the Diamond and Dybvig (1983) tradition has demonstrated that having markets for the secondary trading of bank claims might damage the insurance role of bank deposits.[48] Yet, Diamond (1997) makes the case for the complementarity between banks and markets when, at least for some agents, the access to markets is not guaranteed.

We believe that our model could be extended to describe situations in which debt is tradable but in a non-centralized secondary market characterized by search frictions (like in the models of OTC markets recently explored by Duffie et al., 2005, Vayanos and Weill, 2008, and Lagos and Rocheteau, 2009). In such setting, shortening the maturity of debt would have the effect of increasing the outside option of an impatient saver who is trying to find a buyer for his non-matured debt.[49] This could allow sellers to obtain better prices in the secondary market, making them willing to pay more for the debt in the first place and encouraging banks to issue short-term debt.[50] In any case developing this extension would constitute another paper.

8 Conclusion

We have developed an infinite horizon equilibrium model in which banks that invest in long-lived assets decide the overall principal, interest rate payments, and maturity of their debt. The model contains a microfoundation for savers' preference for short maturities in line with the traditional Diamond and Dybvig (1983) formulation, which is simplified and adapted to

[48] See von Thadden (1999) for an insightful review of the results obtained in this tradition.

[49] See He and Milbradt (2011), who explicitly model the secondary market for corporate debt as a market with search frictions.

[50] The empirical evidence in Mahanti et al (2008) and Bao, Pan, and Wang (2011), among others, shows that short-term bonds are indeed more "liquid" (as measured by the narrowness of the bid-ask spread) than long-term bonds.

the needs of a recursive dynamic formulation. Banks' incentive not to set debt maturities as short as savers might ceteris paribus prefer, comes from the fact that there are events (called systemic liquidity crises) in which their normal financing channels fail and they have to turn to more expensive sources of funds.

We identify a pecuniary externality that, when combined with the constraints faced by banks for their refinancing, renders the unregulated competitive equilibrium socially inefficient. It turns out that, if a social planner coordinates the banks in the choice of somewhat longer debt maturities, then banks' total leverage and the social value of their overall maturity transformation activity increases.

We have explored alternatives for restoring efficiency, including forcing banks to issue debt of longer maturities or inducing them to do so with a Pigovian tax on their refinancing needs. We have also considered the implications of adding private or public liquidity insurance schemes, finding that the case for regulating maturity decisions does not disappear, so that liquidity insurance and liquidity risk regulation can be considered complements rather than substitutes in dealing with the systemic implications of liquidity crises.

Appendix

A Proofs

This appendix contains the proofs of the propositions included in the body of the paper.

Proof of Proposition 1 Using (3) it is a matter of simple algebra to obtain that:

$$r'(\delta) = \frac{-\pi(1+\rho_I)(\rho_I-\rho_P)}{(\rho_I+\delta+(1-\delta)\pi)^2} < 0,$$

$$r''(\delta) = \frac{2\pi(1-\pi)(1+\rho_I)(\rho_I-\rho_P)}{(\rho_I+\delta+(1-\delta)\pi)^3} > 0.$$

The other properties stated in the proposition are immediate.■

Proof of Proposition 2 The proof is organized in a sequence of steps.

1. If (BF) is satisfied then (LL) is strictly satisfied Using equation (7) we have that (LL) can be written as:

$$0 \le E(D,\delta;\phi) = \frac{1}{\rho_I}(\mu - rD) - \frac{1}{\rho_I}\frac{(1+\rho_I)\varepsilon}{1+\rho_I+\varepsilon}\left(1+\phi-\frac{1+r}{1+\rho_I}\right)\delta D,$$

while (BF) can be written, using (6), as

$$\begin{aligned} 0 &\le \frac{1}{1+\rho_I}(\mu - r(1-\delta)D + \delta D + E(D,\delta;\phi)) - (1+\phi)\delta D = \\ &= \frac{1}{\rho_I}(\mu - rD) - \left(1+\frac{1}{\rho_I}\frac{\varepsilon}{1+\rho_I+\varepsilon}\right)\left(1+\phi-\frac{1+r}{1+\rho_I}\right)\delta D. \end{aligned}$$

Now, since $1+\frac{1}{\rho_I}\frac{\varepsilon}{1+\rho_I+\varepsilon} > \frac{(1+\rho_I)\varepsilon}{\rho_I(1+\rho_I+\varepsilon)}$ we conclude that whenever (BF) is satisfied, (LL) is strictly satisfied.

2. Notation and useful bounds Using equation (7) we can write:

$$V(D,\delta;\phi) = D + E(D,\delta;\phi) = \frac{1}{\rho_I}\mu + D\Pi(\delta;\phi),$$

where

$$\Pi(\delta,\phi) = 1 - \frac{1}{\rho_I}\left[\left(1-\frac{\varepsilon}{1+\rho_I+\varepsilon}\delta\right)r + \frac{(1+\rho_I)\varepsilon}{1+\rho_I+\varepsilon}\delta\left(\phi+\frac{\rho_I}{1+\rho_I}\right)\right]$$

can be interpreted as the value the bank generates to its shareholders per unit of debt. Using Proposition 1 we can see that the function $\Pi(\delta,\phi)$ is concave in δ.

(BF) in equation (6) can be rewritten as:

$$\mu + V(D,\delta;\phi) \ge [(1+\rho_I)(1+\phi)\delta + (1+r)(1-\delta)]D,$$

and if we define $C(\delta,\phi) = (1+\rho_I)(1+\phi)\delta + (1+r)(1-\delta)$, (BF) can be written in the more compact form that will be used from now onwards:

$$\frac{1+\rho_I}{\rho_I}\mu + (\Pi(\delta,\phi) - C(\delta,\phi))D \geq 0. \tag{16}$$

Using Proposition 1 we can see that the function $C(\delta,\phi)$ is convex in δ.

We have the following relationship:

$$\Pi(\delta,\phi) = 1 - \frac{1}{\rho_I}\frac{(1+\rho_I)}{1+\rho_I+\varepsilon}\left[r(\delta) + \frac{\varepsilon}{1+\rho_I}(C(\delta,\phi)-1)\right] \tag{17}$$

The assumption $\phi \leq 2\frac{1+\rho_P}{1+\rho_I} - 1$ implies $(1+\rho_I)(1+\phi) \leq 2(1+\rho_P) \leq 2(1+r(\delta))$ for all δ, and we can check that the following bounds (that are independent from ϕ) hold:

$$\begin{aligned} C(\delta,\phi) &\geq 1 + r(\delta). \\ \frac{\partial C(\delta,\phi)}{\partial\delta} &\leq 2(1+r(\delta)) - (1+r(\delta)) = 1 + r(\delta). \end{aligned} \tag{18}$$

Using the assumption $\pi < \frac{1-\rho_I}{2}$ it is a matter of algebra to check that for all δ:

$$\frac{d^2 r}{d\delta^2} + \frac{dr}{d\delta} \geq 0,$$

and finally from this inequality, $\frac{dr}{d\delta} < 0$ and $r < \rho_I$ we obtain after some algebra:

$$\frac{\partial^2\Pi(\delta,\phi)}{\partial\delta^2} + \frac{\partial\Pi(\delta,\phi)}{\partial\delta} < -\frac{1}{\rho_I}\left(1 - \frac{\varepsilon}{1+\rho_I+\varepsilon}\delta\right)\left(\frac{dr}{d\delta} + \frac{d^2 r}{d\delta^2}\right) \leq 0. \tag{19}$$

To save on notation, we will drop from now on the arguments of these functions when it does not lead to ambiguity.

3. $D^* = 0$ is not optimal It suffices to realize that $\frac{\partial V(D,0;\phi)}{\partial D} = \Pi(0,\phi) = 1 - \frac{r(0)}{\rho_I} > 0$.

4. The solution (D^*,δ^*) of the maximization problem in equation (9) exists, is unique, and satisfies (BF) with equality, i.e. $\frac{1+\rho_I}{\rho_I}\mu + (\Pi(\delta^*,\phi) - C(\delta^*,\phi))D^* = 0$

We are going to prove existence and uniqueness in the particular case that there exist $\delta_\Pi, \delta_C \in [0,1]$ such that $\frac{\partial\Pi(\delta_\Pi,\phi)}{\partial\delta} = \frac{\partial C(\delta_C,\phi)}{\partial\delta} = 0$. This will ensure that the solution of the maximization problem is interior in δ. The other cases are treated in an analogous way but might give rise to corner solutions in δ.[51]

First, since $\Pi(\delta,\phi)$ is concave in δ we have that $\frac{\partial\Pi(\delta,\phi)}{\partial\delta} \geq 0$ iff $\delta \leq \delta_\Pi$. Since $C(\delta,\phi)$ is convex in δ we have that $\frac{\partial C(\delta,\phi)}{\partial\delta} \geq 0$ iff $\delta \geq \delta_C$. It is easy to prove from equation (17) that $\delta_C < \delta_\Pi$.

[51] More precisely, if for all $\delta \in [0,1]$ $\frac{\partial C(\delta,\phi)}{\partial\delta} > 0$ we might have $\delta^* = 0$ and if for all $\delta \in [0,1]$, $\frac{\partial\Pi(\delta,\phi)}{\partial\delta} > 0$ we might have $\delta^* = 1$.

Now, let (D^*, δ^*) be a solution to the maximization problem. The first order conditions (FOC) that characterize an interior solution (D^*, δ^*) are:

$$
\begin{aligned}
(1+\theta)\Pi - \theta C &= 0, \\
(1+\theta)\frac{\partial \Pi}{\partial \delta} - \theta \frac{\partial C}{\partial \delta} &= 0, \\
\theta \left[\frac{1+\rho_I}{\rho_I}\mu + (\Pi - C)D^* \right] &\geq 0, \\
\theta &\geq 0,
\end{aligned}
\tag{20}
$$

where θ is the Lagrange multiplier associated with (BF) and we have used that $D^* > 0$ in order to eliminate it from the second equation.

If $\theta = 0$ then the second equation implies $\delta^* = \delta_\Pi$ and thus $\Pi(\delta^*, \phi) \geq \Pi(0, \phi) > 0$ and the first equation is not satisfied. Therefore we must have $\theta > 0$ so that (BF) is binding at the optimum. Now we can eliminate θ from the previous system of equations, which gets reduced to:

$$
\frac{\partial \Pi(\delta^*, \phi)}{\partial \delta} C(\delta^*, \phi) = \frac{\partial C(\delta^*, \phi)}{\partial \delta} \Pi(\delta^*, \phi), \tag{21}
$$

$$
\frac{1+\rho_I}{\rho_I}\mu = \left[C(\delta^*, \phi) - \Pi(\delta^*, \phi) \right] D^*. \tag{22}
$$

We are going to show that equation (21) has a unique solution in δ. For $\delta \leq \delta_C < \delta_\Pi$, we have $\frac{\partial C}{\partial \delta} \leq 0 < \frac{\partial \Pi}{\partial \delta}$ and thus the left hand side (LHS) of (21) is strictly bigger than the RHS. For $\delta \geq \delta_\Pi > \delta_C$, we have $\frac{\partial \Pi}{\partial \delta} \leq 0 < \frac{\partial C}{\partial \delta}$ and thus RHS of (21) is strictly bigger.

Now, the function $\frac{\partial C(\delta, \phi)}{\partial \delta} \Pi(\delta, \phi)$ is strictly increasing in the interval (δ_C, δ_Π) since both terms are positive and increasing. Thus, it suffices to prove that for $\delta \in (\delta_C, \delta_\Pi)$ the function $\frac{\partial \Pi(\delta, \phi)}{\partial \delta} C(\delta, \phi)$ is decreasing.[52] Using the the bounds in (18), inequality (19) and $\frac{\partial^2 \Pi}{\partial \delta^2} < 0$, $\frac{\partial \Pi}{\partial \delta} > 0$ for $\delta \in (\delta_C, \delta_\Pi)$, we have:

$$
\frac{\partial}{\partial \delta}\left(\frac{\partial \Pi}{\partial \delta} C \right) = \frac{\partial^2 \Pi}{\partial \delta^2} C + \frac{\partial \Pi}{\partial \delta}\frac{\partial C}{\partial \delta} \leq (1+r)\left(\frac{\partial^2 \Pi}{\partial \delta^2} + \frac{\partial \Pi}{\partial \delta} \right) \leq 0.
$$

This concludes the proof on the existence and uniqueness of a δ^* that satisfies the necessary FOC in (21).

Now, for given δ^*, the other necessary FOC (22) determines D^* uniquely.[53]

5. δ^* is independent from μ and D^* is strictly increasing in μ Equation (21) determines δ^* and is independent from μ. Then equation (22) shows that D^* is increasing in μ.

[52] This is not trivial since $C(\delta, \phi)$ is increasing.

[53] Let us observe that for all δ, $C(\delta, \phi) \geq 1 > \Pi(\delta, \phi)$.

6. δ^* is decreasing in ϕ and, if $\delta^* \in (0,1)$, it is strictly decreasing Let $\delta(\phi)$ be the solution of the maximization problem of the bank for given ϕ. Let us assume that $\delta(\phi)$ satisfies the FOC (21). The case of corner solutions is analyzed in an analogous way.

We have proved in Step 3 above that the function $\frac{\partial\Pi}{\partial\delta}C - \frac{\partial C}{\partial\delta}\Pi$ is decreasing in δ around $\delta(\phi)$. In order to show that $\delta(\phi)$ is decreasing, it suffices to show that the derivative of this function w.r.t. ϕ is negative. Using the definitions of $C(\delta,\phi), \Pi(\delta,\phi)$ after some (tedious) algebra we obtain:

$$\frac{\partial}{\partial\phi}\left[\frac{\partial\Pi}{\partial\delta}C - \frac{\partial C}{\partial\delta}\Pi\right] = -(1+\rho_I) - \frac{1}{\rho_I}\frac{1+\rho_I}{1+\rho_I+\varepsilon}\left[(1+\rho_I)\left(\frac{dr}{d\delta}\delta - r\right)+\varepsilon\right].$$

Now we have $\frac{d}{d\delta}\left(\frac{dr}{d\delta}\delta - r\right) = \frac{d^2r}{d\delta^2}\delta \geq 0$ and thus $\frac{dr}{d\delta}\delta - r \geq \frac{dr}{d\delta}\delta - r\big|_{\delta=0} = -r(0)$, and finally:

$$\begin{aligned}\frac{\partial}{\partial\phi}\left[\frac{\partial\Pi}{\partial\delta}C - \frac{\partial C}{\partial\delta}\Pi\right] &\leq -(1+\rho_I) - \frac{1}{\rho_I}\frac{1+\rho_I}{1+\rho_I+\varepsilon}\left[-(1+\rho_I)r(0)+\varepsilon\right] \\ &< -(1+\rho_I) + \frac{1}{\rho_I}(1+\rho_I)r(0) = -(1+\rho_I)\left(1-\frac{r(0)}{\rho_I}\right) < 0.\end{aligned}$$

This conclude the proof that $\frac{d\delta}{d\phi} < 0$.[54]

7. $\delta^* D^*$ is decreasing with ϕ. If $\delta^* > 0$ it is strictly decreasing

Let $\delta(\phi), D(\phi)$ be the solution of the maximization problem of the bank for given ϕ. We have:

$$\frac{1+\rho_I}{\rho_I}\mu = \left[C(\delta(\phi),\phi) - \Pi(\delta(\phi),\phi)\right]D(\phi).$$

Let $\phi_1 < \phi_2$. In Step 5 we showed that $\delta(\phi_1) \geq \delta(\phi_2)$. If $\delta(\phi_2) = 0$ then trivially $\delta(\phi_1)D(\phi_1) \geq \delta(\phi_2)D(\phi_2) = 0$. Let us suppose that $\delta(\phi_2) > 0$. Since trivially $\Pi(\delta(\phi_1),\phi_1)D(\phi_1) \geq \Pi(\delta(\phi_2);\phi_2)D(\phi_2)$, we must have $C(\delta(\phi_1),\phi_1)D(\phi_1) \geq C(\delta(\phi_2),\phi_2)D(\phi_2)$. Now, suppose that $\delta(\phi_1)D(\phi_1) \leq \delta(\phi_2)D(\phi_2)$, then we have the following two inequalities:

$$\begin{aligned}(1+\rho_I)(1+\phi_1)\delta(\phi_1)D(\phi_1) &< (1+\rho_I)(1+\phi_2)\delta(\phi_2)D(\phi_2), \\ (1+r(\delta(\phi_1)))(1-\delta(\phi_1)) &\leq (1+r(\delta(\phi_2)))(1-\delta(\phi_2)),\end{aligned}$$

that imply $C(\delta(\phi_1),\phi_1)D(\phi_1) < C(\delta(\phi_2),\phi_2)D(\phi_2)$, but this contradicts our assumption. Thus, $\delta(\phi_1)D(\phi_1) > \delta(\phi_2)D(\phi_2)$.■

Proof of Proposition 3 Let us denote $(D(\phi),\delta(\phi))$ the solution of the bank's optimization problem for every excess cost of crisis liquidity $\phi \geq 0$. Proposition 2 states that $\delta(\phi)D(\phi)$ is decreasing in ϕ. For $\phi \in [0,\overline{\phi}]$ let us define $\Sigma(\phi) = \Phi(\delta(\phi)D(\phi)) - \phi$. This function represents the difference between the excess cost of liquidity during a crisis by banks' decisions and

[54] In the case of corner solution $\delta^*(\phi) = 1$, we might have $\frac{d\delta^*}{d\phi} = 0$ and obviously for $\delta^*(\phi) = 0$, $\frac{d\delta^*}{d\phi} = 0$.

banks' expectation on such variable. Since Φ is an increasing function on the aggregate demand of funds during a crisis the function $\Sigma(\phi)$ is strictly decreasing. Because of the uniqueness of the solution to the problem that defines $(D(\phi), \delta(\phi))$, the function is also continuous. Moreover, we trivially have $\Sigma(0) \geq 0$ and $\lim_{\phi \to \infty} \Sigma(\phi) = -\infty$. Therefore there exists a unique $\phi^e \in \mathbb{R}^+$ such that $\Sigma(\phi^e) = 0$. By construction $D(\phi^e), \delta(\phi^e), \phi^e$ is the unique equilibrium of the economy.■

Proof of Proposition 4 We are going to follow the notation used in the proof of Proposition 3. Let Φ_1, Φ_2 be two curves describing the inverse supply of liquidity during a crisis and assume they satisfy $\Phi_1(x) > \Phi_2(x)$ for all $x > 0$. Let us denote $\Sigma_i(\phi) = \Phi_i(\delta(\phi)D(\phi)) - \phi$ for $i = 1, 2$. By construction we have $\Sigma_1(\phi_1^e) = 0$. Let us suppose that $\phi_1^e < \phi_2^e$. Then we would have:

$$\Sigma_2(\phi_2^e) = \Phi_2(\delta(\phi_2^e)D(\phi_2^e)) - \phi_2^e \leq \Phi_1(\delta(\phi_2^e)D(\phi_2^e)) - \phi_2^e < \Phi_1(\delta(\phi_1^e)D(\phi_1^e)) - \phi_1^e = \Sigma_1(\phi_1^e) = 0, \tag{23}$$

where in the first inequality we use the assumption $\Phi_2(x) \leq \Phi_1(x)$ for $x \geq 0$, and in the second inequality we use that if $\phi_1^e < \phi_2^e$ then $\delta(\phi_2^e)D(\phi_2^e) \leq \delta(\phi_1^e)D(\phi_1^e)$ (Proposition 2), and that $\Phi_1(\cdot)$ is increasing.

Notice that the sequence of inequalities in (23) implies $\Sigma_2(\phi_2^e) < 0$, which contradicts the definition of ϕ_2^e. We must therefore have $\phi_1^e \geq \phi_2^e$. Now Proposition 2 implies that $\delta_1^e \leq \delta_2^e, \delta_1^e D_1^e \leq \delta_2^e D_2^e, r_1^e \geq r_2^e$. Let us suppose that $\delta_2^e \in (0, 1)$ then the first inequality in (23) is strict, since $\delta_2^e D_2^e > 0$, and we can straightforwardly check that the previous argument implies $\phi_1^e > \phi_2^e$. Now, since $\delta_2^e \in (0, 1)$, Proposition 2 implies that $\delta_1^e < \delta_2^e$, $\delta_1^e D_1^e < \delta_2^e D_2^e$, and $r_1^e > r_2^e$.■

Proof of Proposition 5 We consider two cases.
Case 1: Debt issuance exogenously fixed We are going to follow the notation in the proof of Proposition 2. Using the definition of $W(D, \delta)$ we have

$$\begin{aligned} \frac{\partial W(D, \delta)}{\partial \delta} &= \frac{\partial V(D, \delta; \Phi(\delta D))}{\partial \delta} + \frac{\partial V(D, \delta; \Phi(\delta D))}{\partial \phi} D\Phi'(\delta D) + \frac{\partial U(D, \delta)}{\partial \delta} \\ &= \frac{\partial V(D, \delta; \Phi(\delta D))}{\partial \delta} = D\frac{\partial \Pi(\delta, \Phi(\delta D))}{\partial \delta} \end{aligned} \tag{24}$$

and

$$\begin{aligned} \frac{\partial^2 W(D, \delta)}{\partial \delta^2} &= D\frac{\partial^2 \Pi(D, \delta; \Phi(\delta D))}{\partial \delta^2} + \frac{\partial^2 \Pi(D, \delta; \Phi(\delta D))}{\partial \delta \partial \phi} D^2 \Phi'(\delta D) \\ &= \frac{\partial^2 \Pi(D, \delta; \Phi(\delta D))}{\partial \delta^2} - \frac{1}{\rho_I} \frac{(1 + \rho_I)\varepsilon}{1 + \rho_I + \varepsilon} D^2 \Phi'(\delta D) < 0, \end{aligned}$$

where in the last inequality we have used that $\Pi(D,\delta;\phi)$ is concave in δ and that $\Phi'(\cdot)>0$. Notice that $W(D,\delta)$ is concave in δ.

Denote the exogenous amount of debt referred in the proposition as $\overline{D}>0$. Let (ϕ^e,δ^e) be the equilibrium of the economy in which banks do not decide $\overline{D}$. Let us suppose that $\delta^e\in(0,1)$; the argument if $\delta^e=0,1$ is analogous and will be omitted for brevity. By analogy with the system of equations in (20), the competitive equilibrium is characterized by:

$$\begin{aligned}(1+\theta)\frac{\partial\Pi(\delta^e,\phi^e)}{\partial\delta}-\theta\frac{\partial C(\delta^e,\phi^e)}{\partial\delta} &= 0,\\ \theta\left[\frac{1+\rho_I}{\rho_I}\mu+(\Pi(\delta^e,\phi^e)-C(\delta^e,\phi^e))\overline{D}\right] &\geq 0,\\ \theta &\geq 0,\\ \phi^e &= \Phi(\delta^e\overline{D}).\end{aligned} \tag{25}$$

Now, let δ^s be the solution to the social planner problem. We can distinguish two cases: i) $\theta=0$. In this case the system of equations (25) implies $\frac{\partial\Pi(\delta^e,\phi^e)}{\partial\delta}=0$. Now, if we use equation (24) we have

$$\frac{\partial W(\overline{D},\delta^e)}{\partial\delta}=\overline{D}\frac{\partial\Pi(\delta^e,\Phi(\overline{D}\delta^e))}{\partial\delta}=\overline{D}\frac{\partial\Pi(\delta^e,\phi^e)}{\partial\delta}=0$$

and, therefore, δ^e maximizes the (concave) function $W(\overline{D},\delta)$. Thus, in this case $\delta^s=\delta^e$.
ii) $\theta>0$. In this case, from equation (17) we conclude that $\frac{\partial\Pi(\delta,\phi)}{\partial\delta}<0$ implies $\frac{\partial C(\delta,\phi)}{\partial\delta}>0$. Now, the first equation in the system (25) implies $\frac{\partial\Pi(\delta^e,\phi^e)}{\partial\delta}>0$, $\frac{\partial C(\delta^e,\phi^e)}{\partial\delta}-\frac{\partial\Pi(\delta^e,\phi^e)}{\partial\delta}>0$. So we have

$$\frac{\partial W(\overline{D},\delta^e)}{\partial\delta}=\overline{D}\frac{\partial\Pi(\delta^e,\phi^e)}{\partial\delta}>0,$$

and, since $W(\overline{D},\delta)$ is concave, we have $W(\overline{D},\delta)<W(\overline{D},\delta^e)$ for all $\delta<\delta^e$. Now, given that δ^e satisfies (BF) with equality, in order to prove that $\delta^s=\delta^e$ it suffices to show that for $\delta>\delta^e$ (BF) is not satisfied. This, in turn, is an immediate consequence of the following inequality that is easily checked:

$$\frac{\partial}{\partial\delta}\left[C(\delta,\Phi(\overline{D}\delta))-\Pi(\delta,\Phi(\overline{D}\delta))\right]>0,\ \text{for } \delta\geq\delta^e.$$

Case 2: Average maturity exogenously fixed Once we realize that the function $W(D,\delta)$ is concave in D the proof is completely analogous to the previous one and is omitted here.■

Proof of Proposition 6 We are going to follow the notation used in the proof of Proposition 2. The proof is organized in five steps:

1. Preliminaries We have seen in the proof of Proposition 5 that:

$$\frac{\partial W(D,\delta)}{\partial\delta}=\frac{\partial V(D,\delta;\Phi(\delta D))}{\partial\delta}=D\frac{\partial\Pi(\delta,\Phi(\delta D))}{\partial\delta}. \tag{26}$$

Similarly we have

$$\frac{\partial W(D,\delta)}{\partial D} = \frac{\partial V(D,\delta;\Phi(\delta D))}{\partial D} = \Pi(\delta,\Phi(\delta D)). \tag{27}$$

2. (BF) is binding at the socially optimal debt structure This is a statement that has been done in the main text just before Proposition 6. The proof is analogous to the one for the maximization problem of the bank that we did in Step 4 of the proof of Proposition 2. The only difference is that ϕ is not taken as given but as the function $\Phi(\delta D)$ in D and δ.

3. Definition of function $D^c(\delta)$ and its properties Let $(\phi^e,(D^e,\delta^e))$ be the competitive equilibrium. Let us assume that $\delta^e < 1$. By definition of equilibrium we have $\phi^e = \Phi(\delta^e D^e)$. For every δ let $D^c(\delta)$ be the unique principal of debt such that (BF) is binding, i.e.:

$$\frac{1+\rho_I}{\rho_I}\mu = \left[C(\delta,\phi^e) - \Pi(\delta,\phi^e)\right] D^c(\delta). \tag{28}$$

Differentiating w.r.t. δ:

$$\left[\frac{\partial C(\delta,\phi^e)}{\partial \delta} - \frac{\partial \Pi(\delta,\phi^e)}{\partial \delta}\right] D^c(\delta) + \left[C(\delta,\phi^e) - \Pi(\delta,\phi^e)\right] \frac{dD^c(\delta)}{d\delta} = 0. \tag{29}$$

Using the characterization of δ^e in equation (21), the inequalities $C(\delta,\phi^e) \geq 1 > \Pi(\delta,\phi^e)$ imply $\frac{\partial C(\delta^e,\phi^e)}{\partial \delta} - \frac{\partial \Pi(\delta^e,\phi^e)}{\partial \delta} > 0$ and, then, we can deduce from the equation above that $\frac{dD^c(\delta^e)}{d\delta} < 0$. Since (BF) is binding at the optimal debt structure we can think of the bank problem as maximizing the univariate function $V(D^c(\delta),\delta;\phi^e)$. Hence δ^e must satisfy the necessary FOC for an interior solution to the maximization of $V(D^c(\delta),\delta;\phi^e)$:

$$\frac{dV(D^c(\delta),\delta^e;\phi^e)}{d\delta} = 0 \Leftrightarrow D^c(\delta^e)\frac{\partial \Pi(\delta^e,\phi^e)}{\partial \delta} + \Pi(\delta^e,\phi^e)\frac{dD^c(\delta^e)}{d\delta} = 0, \tag{30}$$

which multiplying by δ^e can be written as

$$D^c(\delta^e)\frac{\partial \Pi(\delta^e,\phi^e)}{\partial \delta^e}\delta^e = \Pi(\delta^e,\phi^e)\left(-\frac{dD^c(\delta^e)}{d\delta}\delta^e\right).$$

Since $\frac{\partial\left(\Pi - \frac{\partial \Pi}{\partial \delta}\delta\right)}{\partial \delta} = -\frac{\partial^2 \Pi}{\partial \delta^2}\delta \geq 0$ and $\Pi(0,\phi) - \frac{\partial \Pi(0,\phi)}{\partial \delta}0 > 0$, we have $\Pi(\delta,\phi) > \frac{\partial \Pi(\delta,\phi)}{\partial \delta}\delta$ for all $\delta \in [0,1]$ and the previous equation implies

$$D^c(\delta^e) > -\frac{dD^c(\delta^e)}{d\delta}\delta^e \Leftrightarrow \left.\frac{d\left(\delta D^c(\delta)\right)}{d\delta}\right|_{\delta=\delta^e} > 0.$$

4. Evaluation of $\left.\frac{d(D^s(\delta))}{d\delta}\right|_{\delta=\delta^e}$ and $\left.\frac{d(\delta D^s(\delta))}{d\delta}\right|_{\delta=\delta^e}$ For every δ, let $D^s(\delta)$ be the unique principal of debt such that (BF) is binding, i.e.

$$\frac{1+\rho_I}{\rho_I}\mu = \left[C(\delta,\Phi(\delta D^s(\delta))) - \Pi(\delta,\Phi(\delta D^s(\delta)))\right] D^s(\delta).$$

Differentiating w.r.t. δ, we obtain

$$\left[\frac{\partial C(\delta,\Phi)}{\partial\delta}-\frac{\partial\Pi(\delta,\Phi)}{\partial\delta}\right]D^s(\delta)+\left[C(\delta,\Phi(\delta D^s(\delta)))-\Pi(\delta,\Phi(\delta D^s(\delta)))\right]\frac{dD^s(\delta)}{d\delta}+$$
$$+\left[\frac{\partial C(\delta,\Phi)}{\partial\phi}-\frac{\partial\Pi(\delta,\Phi)}{\partial\phi}\right]\Phi'(\delta D^s(\delta)))\frac{d\left(\delta D^s(\delta)\right)}{d\delta}=0. \tag{31}$$

By construction, $D^s(\delta^e)=D^c(\delta^e)=D^e$. Now, subtracting equation (29) from equation (31) at the point $\delta=\delta^e$ we obtain

$$[C(\delta^e,\phi^e)-\Pi(\delta^e,\phi^e)]\left(\frac{dD^s(\delta^e)}{d\delta}-\frac{dD^c(\delta^e)}{d\delta}\right)+\left[\frac{\partial C(\delta^e,\phi^e)}{\partial\phi}\right.$$
$$\left.-\frac{\partial\Pi(\delta^e,\phi^e)}{\partial\phi}\right]\Phi'(\delta^e D^e)\left.\frac{d\left(\delta D^s(\delta)\right)}{d\delta}\right|_{\delta=\delta^e}=0. \tag{32}$$

Suppose that $\left.\frac{d(\delta D^s(\delta))}{d\delta}\right|_{\delta=\delta^e}\leq 0$, then we would have $\frac{dD^s(\delta^e)}{d\delta}\geq\frac{dD^c(\delta^e)}{d\delta}$, since trivially $\frac{\partial C(\delta^e,\phi^e)}{\partial\phi}-\frac{\partial\Pi(\delta,\phi^e)}{\partial\phi}>0$. But then

$$\left.\frac{d\left(\delta D^s(\delta)\right)}{d\delta}\right|_{\delta=\delta^e}=D^s(\delta^e)+\frac{dD^s(\delta^e)}{d\delta}\delta^e>D^c(\delta^e)+\frac{dD^c(\delta^e)}{d\delta}\delta^e=\left.\frac{d\left(\delta D^c(\delta)\right)}{d\delta}\right|_{\delta=\delta^e}>0,$$

which contradicts the hypothesis. We must thus have $\left.\frac{d(\delta D^s(\delta))}{d\delta}\right|_{\delta=\delta^e}>0$, in which case equation (32) implies $\frac{dD^s(\delta^e)}{d\delta}<\frac{dD^c(\delta^e)}{d\delta}<0$.

5. Evaluation of $\left.\frac{dW(D^s(\delta),\delta)}{d\delta}\right|_{\delta=\delta^e}$ Using equations (26) and (27), we have:

$$\begin{aligned}\frac{dW(D^s(\delta),\delta)}{d\delta}&=\frac{\partial W(D^s(\delta),\delta)}{\partial\delta}+\frac{\partial W(D^s(\delta),\delta)}{\partial D}\frac{dD^s(\delta)}{d\delta}\\&=D^s(\delta)\frac{\partial\Pi(\delta,\Phi(\delta D^s(\delta)))}{\partial\delta}+\Pi(\delta,\Phi(\delta D^s(\delta)))\frac{dD^s(\delta)}{d\delta}.\end{aligned}$$

And, using $\frac{dD^s(\delta^e)}{d\delta}<\frac{dD^c(\delta^e)}{d\delta}$ and (30), we obtain:

$$\left.\frac{dW(D^s(\delta),\delta)}{d\delta}\right|_{\delta=\delta^e}<D^s(\delta^e)\frac{\partial\Pi(\delta^e,\phi^e)}{\partial\delta}+\Pi(\delta^e,\phi^e)\frac{dD^c(\delta^e)}{d\delta}=0.$$

Summing up, having

$$\left.\frac{dW(D^s(\delta),\delta)}{d\delta}\right|_{\delta=\delta^e}<0,\quad\left.\frac{dD^s(\delta)}{d\delta}\right|_{\delta=\delta^e}<0,\text{ and }\left.\frac{d(\delta D^s(\delta))}{d\delta}\right|_{\delta=\delta^e}>0,$$

implies that a social planner can increase welfare by fixing some $\delta^s<\delta^e$, and suggests that doing so will produce higher leverage and lower refinancing needs than in the unregulated competitive equilibrium.■

Proof of Proposition 7 Let $(\phi^s, (D^s, \delta^s))$ be the socially efficient equilibrium. The sketch of the proof is as follows:

1. For any fixed ϕ banks' optimal choice of δ depends only on τ (and not on the lump-sum tax rebate M) and as τ increases δ decreases. From here we can show that if banks' expectation on the excess cost of liquidity in a crisis is $\phi^s < \phi^e$ there exists a Pigovian tax $\tau^P > 0$ that induces the socially efficient choice of maturity.

2. For $\phi = \phi^s$ and Pigovian tax τ^P defined above, once banks have taken their maturity decision δ^s they issue as much debt D as (BF) allows and at this point the amount of the lump-sum transfer M matters. The effect of the net per period transfer $\tau^P \delta^s D - M \geq 0$ from banks to the SP is to reduce banks' equity value at the N state and thus to strictly tighten (BF) with respect to the situation in which the SP directly regulates maturity to its social optimum $1/\delta^s$ unless there is full rebate of the Pigovian tax, i.e. $M = \tau^P \delta^s D$. More precisely, it can be shown that D is strictly increasing with M and that $D = D^s$ if and only if $M = \tau^P \delta^s D^s$.

3. Our candidate for optimal Pigovian tax scheme is (τ^P, M^P) with $M^P = \tau^P \delta^s D^s$. By construction, under this tax scheme if banks' expectation on the excess cost of liquidity in a crisis is ϕ^s, then banks' optimal funding structure coincides with the socially efficient structure (D^s, δ^s), which in turn satisfies $\phi^s = \Phi(\delta^s D^s)$, confirming ϕ^s as an expectation compatible with the equilibrium.

The most cumbersome details of the proof are analogous to those in the proof of Proposition 2 and are omitted for brevity. They are available from the authors upon demand.■

Proof of Proposition 8 Let us recall that the introduction of (fairly priced) insurance does not change the value of equity at the N state, i.e. $E(D, \delta, \lambda; \phi) = E(D, \delta; \phi)$ for all λ. In addition banks choose full insurance, $\lambda = 1$, and the only financial constraint is (LL) that can be written $E(D, \delta, 1; \phi) = E(D, \delta; \phi) > 0$. For the next steps, we follow the notation introduced in the proof of Proposition 2.

1. Insurance increases social welfare in the regulated economy Let (D^s, δ^s) be the socially optimal debt structure in the absence of insurance. In the proof of Proposition 6 we showed that (BF) is binding at (D^s, δ^s). In fact, we have $\frac{\partial W(D^s,\delta^s)}{\partial D} > 0$. Step 1 in the proof of Proposition 2 states that (LL) is satisfied with slack, i.e.

$$E(D^s, \delta^s; \Phi(\delta^s D^s)) > 0,$$

and thus by continuity there are values $D' > D^s$ such that $E(D', \delta^s; \Phi(\delta^s D')) > 0$ and $W(D', \delta^s) > W(D^s, \delta^s)$. Introducing insurance makes debt structures such as (D', δ^s) feasible and, hence, increases welfare relative to the regulated economy without insurance.

2. Under insurance the competitive expected maturity is shorter than the socially optimal one When insurance is introduced the relevant financial constraint faced both by banks in the unregulated equilibrium (for given ϕ) and by the social planner (for $\phi = \Phi(\delta D)$) is (LL) and is binding. From here, the proof is analogous to that of Proposition 6 and we omit it for brevity.■

B Debt structures that induce default during crises

In this section we examine the possibility that a bank decides to expose itself to the risk of defaulting on its debt obligations and being liquidated during systemic crises. First, we describe the sequence of events following a bank's default. Second, we show how the debt of the bank is valued by savers who correctly anticipate this course of events. Finally, we analyze the bank's decision problem when default during crises is an explicit alternative.

Default and liquidation Liquidation following the bank's inability to satisfy its refinancing needs yields a residual value $L \geq 0$. Suppose that partial liquidation is not allowed and L is distributed equally among all debtholders independently of their contract having just matured or not. This eliminates the type of preemptive runs studied by He and Xiong (2011a). It is easy to realize that if the bank does not want to rely on bridge financing (exposing itself to possible default in a crisis), then it will find it optimal to make its debt mature in a perfectly correlated manner since this minimizes the probability of default during crises. Hence we assume that the debt issued by the bank when getting rid of the (BF) constraint has perfectly correlated maturities.

Savers' required maturity premium when default is anticipated From a saver's perspective, there are three states relevant for the valuation of a given debt contract: personal patience in a normal period ($i = P$), personal impatience in a normal period ($i = IN$), and personal impatience in a crisis period ($i = IC$).

Let $l = L/D < 1$ be the fraction of the principal of debt which is recovered in case of liquidation and let Q_i be the present value of expected losses due to default as evaluated from each of the states i just after the uncertainty regarding the corresponding period has realized and conditional on the debt not having matured in such period. Losses are measured relative to the benchmark case without default in which at maturity savers recover 100% of the principal. These values satisfy the following system of recursive relationships:

$$
\begin{aligned}
Q_P &= \frac{1}{1+\rho_P}\left[\delta\varepsilon(1-l) + (1-\delta)\{(1-\varepsilon)[(1-\gamma)Q_P + \gamma Q_{IN}] + \varepsilon Q_{IC}\}\right], \\
Q_{IN} &= \frac{1}{1+\rho_I}\left[\varepsilon\delta(1-l) + (1-\delta)\left[(1-\varepsilon)Q_{IN} + \varepsilon Q_{IC}\right]\right], \\
Q_{IC} &= \frac{1}{1+\rho_I}(1-\delta)Q_{IN}.
\end{aligned}
$$

These expressions essentially account for the principal $1-l>0$ which is lost whenever the saver's debt contract matures in a state of crisis. First equation reflects that default as well as any of three states i may follow state P. The second equation reflects that impatience is an absorbing state. The last equation reflects that a crisis period can only be followed by a normal period.

The value of a debt contract $(1,r,\delta)$ to a patient saver in a normal period, when default is expected if the bank runs into refinancing needs during a crisis, can then be written as

$$U_P^d(r,\delta)=U_P(r,\delta)-Q_P(\delta),$$

where $U_P(r,\delta)$ is the value of the same contract in the scenario in which the principal is always recovered at maturity, whose expression is given in (2).

Now, let $r^d(\delta)$ be the interest rate yield that the bank offers in the default setting, which satisfies $U_P^d(r^d(\delta),\delta)=1$. Since the non-default yield $r(\delta)$ satisfies $U_P(r(\delta),\delta)=1$, the equation $U_P^d(r^d(\delta),\delta)=U_P(r(\delta),\delta)$ allows us to express $r^d(\delta)$ as the sum of $r(\delta)$ and a default-risk premium:

$$r^d(\delta)=r(\delta)+\frac{1}{D}\frac{(1+\rho_I)\,\delta\varepsilon(D-L)}{1+\rho_I+(1-\delta)\varepsilon}.$$

It is easy to observe that the default-risk premium $r^d(\delta)-r(\delta)$ is increasing and convex in δ, increasing in ε, decreasing in L, and increasing in D. Using Proposition 1 we deduce that $r^d(\delta)$ is convex in δ. However, given that δ increases the probability of default, $r^d(\delta)$ is not necessarily decreasing in δ.

Banks' optimal funding structure inducing default If the bank does not satisfy the bridge financing constraint and thus defaults whenever it faces refinancing needs during a crisis, its equity value in normal times $E^d(D,\delta)$ will satisfy the following recursive equation:

$$E^d(D,\delta)=\frac{1}{1+\rho_I}\left[\mu-r^dD+(1-\varepsilon)E^d(D,\delta)+\{\varepsilon\delta\cdot 0+(1-\delta)\frac{1}{1+\rho_I}[\mu-r^dD+E^d(D,\delta)]\}\right],$$

whose solution yields:

$$E^d(D,\delta)=\frac{1+\rho_I+\varepsilon(1-\delta)}{(1+\rho_I)^2-(1+\rho_I)(1-\varepsilon)-\varepsilon(1-\delta)}(\mu-r^dD).$$

In this context, the problem determining the bank's optimal debt structure decision in the absence of the bridge financing constraint can be written as:

$$\begin{aligned}\max_{D\geq 0,\ \delta\in[0,1]}\quad & V^d(D,\delta)=D+E^d(D,\delta),\\ \text{s.t.}\quad & E^d(D,\delta)\geq 0,\quad \text{(LL)}\end{aligned}\tag{33}$$

where (LL) is trivially equivalent to $\mu-r^dD\geq 0$.

Figure 7 in the main text has been generated by numerically solving this problem for each value of ϕ^e and L, and finding $L^{\max}(\phi^e)$ as the (maximum) value of L for which the total market value of the bank under the best debt structure compatible with (BF) equals the total market value that the bank can attain solving (33).

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