Online Appendix for:
How Excessive Is Banks' Maturity Transformation?

This online appendix contains the proofs of the formal propositions of the paper and provides details on the analysis and extensions described in its subsections 7.1-7.4. Since for all of them the analysis follows closely that in the baseline model, we will heavily rely on notation and arguments already made in the paper and focus attention here on the new elements of each extension. In particular, when referring to banks' (CF) constraint we mean equation (7) in the paper. For convenience, equations and formal results in this appendix are numbered sequentially to the equations and formal results in the paper.

A Proofs

This appendix contains the proofs of the propositions included in the body of the paper.

Proof of Proposition 1 Using (3) it is a matter of algebra to obtain that:

$$r'(\delta) = \frac{-\gamma(1+\rho_H)(\rho_H-\rho_L)}{[\rho_H+\delta+(1-\delta)\gamma]^2} < 0,$$

$$r''(\delta) = \frac{2\gamma(1-\gamma)(1+\rho_H)(\rho_H-\rho_L)}{[\rho_H+\delta+(1-\delta)\gamma]^3} > 0.$$

The other properties stated in the proposition are immediate.■

Proof of Proposition 2 The proof is organized in a sequence of steps.

1. If (CF) is satisfied then (LL) is strictly satisfied. Using equation (6), (LL) can be written as:

$$0 \leq E(\delta, D; \phi) = \frac{1}{\rho_H}(\mu - rD) - \frac{1}{\rho_H}\frac{(1+\rho_H)\varepsilon}{1+\rho_H+\varepsilon}\left(1+\phi-\frac{1+r}{1+\rho_H}\right)\delta D,$$

while, using (7), (CF) can be written as

$$0 \leq \frac{1}{1+\rho_H}[\mu - r(1-\delta)D + \delta D + E(\delta, D; \phi)] - (1+\phi)\delta D =$$
$$= \frac{1}{\rho_H}(\mu - rD) - \left(1+\frac{1}{\rho_H}\frac{\varepsilon}{1+\rho_H+\varepsilon}\right)\left(1+\phi-\frac{1+r}{1+\rho_H}\right)\delta D.$$

Now, since $1+\frac{1}{\rho_H}\frac{\varepsilon}{1+\rho_H+\varepsilon} > \frac{(1+\rho_H)\varepsilon}{\rho_H(1+\rho_H+\varepsilon)}$ we conclude that whenever (CF) is satisfied, (LL) is strictly satisfied.

2. Notation and useful bounds. Using equation (8) we can write:

$$V(\delta, D; \phi) = \frac{\mu}{\rho_H} + D\Pi(\delta; \phi), \tag{16}$$

where

$$\Pi(\delta,\phi)=1-\frac{1}{\rho_H}\left[\left(1-\frac{\varepsilon}{1+\rho_H+\varepsilon}\delta\right)r+\frac{(1+\rho_H)\varepsilon}{1+\rho_H+\varepsilon}\delta\left(\phi+\frac{\rho_H}{1+\rho_H}\right)\right]$$

can be interpreted as the value the bank generates to its shareholders per unit of debt. Using Proposition 1 we can see that the function $\Pi(\delta,\phi)$ is concave in δ.

(CF) in equation (7) can be rewritten as:

$$\mu+V(\delta,D;\phi)\geq[(1+\rho_H)(1+\phi)\delta+(1+r)(1-\delta)]D,$$

and if we define $C(\delta,\phi)=(1+\rho_H)(1+\phi)\delta+(1+r)(1-\delta)$, (CF) can be written in a more compact form that will be used from now onwards:

$$\frac{1+\rho_H}{\rho_H}\mu+[\Pi(\delta,\phi)-C(\delta,\phi)]D\geq 0. \tag{17}$$

Using Proposition 1 we can see that the function $C(\delta,\phi)$ is convex in δ.

We have the following relationship:

$$\Pi(\delta,\phi)=1-\frac{1}{\rho_H}\frac{1+\rho_H}{1+\rho_H+\varepsilon}\left[r(\delta)+\frac{\varepsilon}{1+\rho_H}(C(\delta,\phi)-1)\right]. \tag{18}$$

Assumption A1 implies $(1+\rho_H)(1+\phi)\leq 2(1+\rho_L)\leq 2(1+r(\delta))$ for all δ, and we can check that the following bounds (that are independent from ϕ) hold:

$$\begin{aligned} C(\delta,\phi) &\geq 1+r(\delta),\\ \frac{\partial C(\delta,\phi)}{\partial\delta} &\leq 2(1+r(\delta))-(1+r(\delta))=1+r(\delta). \end{aligned} \tag{19}$$

Using assumption A2, it is a matter of algebra to check that, for all δ,

$$\frac{d^2r}{d\delta^2}+\frac{dr}{d\delta}\geq 0.$$

And, from this inequality, $\frac{dr}{d\delta}<0$, and $r<\rho_H$, it is possible to check that:

$$\frac{\partial^2\Pi(\delta,\phi)}{\partial\delta^2}+\frac{\partial\Pi(\delta,\phi)}{\partial\delta}<-\frac{1}{\rho_H}\left(1-\frac{\varepsilon}{1+\rho_H+\varepsilon}\delta\right)\left(\frac{dr}{d\delta}+\frac{d^2r}{d\delta^2}\right)\leq 0. \tag{20}$$

To save on notation, we will drop from now on the arguments of these functions when it does not lead to ambiguity.

3. $D^*=0$ is not optimal It suffices to realize that $\frac{\partial V(D,0;\phi)}{\partial D}=\Pi(0,\phi)=1-\frac{r(0)}{\rho_H}>0$.

4. The solution (D^*,δ^*) of the maximization problem in equation (9) exists, is unique, and satisfies (CF) with equality, i.e. $\frac{1+\rho_H}{\rho_H}\mu+(\Pi(\delta^*,\phi)-C(\delta^*,\phi))D^*=0$. We are going to prove existence and uniqueness in the particular case that there exist $\delta_\Pi,\delta_C\in[0,1]$ such that $\frac{\partial\Pi(\delta_\Pi,\phi)}{\partial\delta}=\frac{\partial C(\delta_C,\phi)}{\partial\delta}=0$. This will ensure that the solution of the maximization problem is interior in δ. The other cases are treated in an analogous way but might give rise to corner solutions in δ.[62]

First, since $\Pi(\delta,\phi)$ is concave in δ we have that $\frac{\partial \Pi(\delta,\phi)}{\partial \delta} \geq 0$ iff $\delta \leq \delta_\Pi$. Since $C(\delta,\phi)$ is convex in δ we have that $\frac{\partial C(\delta,\phi)}{\partial \delta} \geq 0$ iff $\delta \geq \delta_C$. It is easy to prove from equation (18) that $\delta_C < \delta_\Pi$.

Now, let (δ^*, D^*) be a solution to the maximization problem. The first order conditions (FOC) that characterize an interior solution (δ^*, D^*) are:

$$\begin{aligned} (1+\theta)\Pi - \theta C &= 0, \\ (1+\theta)\frac{\partial \Pi}{\partial \delta} - \theta \frac{\partial C}{\partial \delta} &= 0, \\ \theta \left[\frac{1+\rho_H}{\rho_H}\mu + (\Pi - C)D^* \right] &\geq 0, \\ \theta &\geq 0, \end{aligned} \tag{21}$$

where θ is the Lagrange multiplier associated with (CF) and we have used that $D^* > 0$ in order to eliminate it from the second equation.

If $\theta = 0$ then the second equation implies $\delta^* = \delta_\Pi$ and thus $\Pi(\delta^*, \phi) \geq \Pi(0,\phi) > 0$ and the first equation is not satisfied. Therefore we must have $\theta > 0$ so that (CF) is binding at the optimum. Now we can eliminate θ from the previous system of equations, which gets reduced to:

$$\frac{\partial \Pi(\delta^*,\phi)}{\partial \delta} C(\delta^*,\phi) = \frac{\partial C(\delta^*,\phi)}{\partial \delta} \Pi(\delta^*,\phi), \tag{22}$$

$$\frac{1+\rho_H}{\rho_H}\mu = \left[C(\delta^*,\phi) - \Pi(\delta^*,\phi)\right] D^*. \tag{23}$$

We are going to show that equation (22) has a unique solution in δ. For $\delta \leq \delta_C < \delta_\Pi$, we have $\frac{\partial C}{\partial \delta} \leq 0 < \frac{\partial \Pi}{\partial \delta}$ and thus the left hand side (LHS) of (22) is strictly bigger than the RHS. For $\delta \geq \delta_\Pi > \delta_C$, we have $\frac{\partial \Pi}{\partial \delta} \leq 0 < \frac{\partial C}{\partial \delta}$ and thus RHS of (22) is strictly bigger.

Now, the function $\frac{\partial C(\delta,\phi)}{\partial \delta}\Pi(\delta,\phi)$ is strictly increasing in the interval (δ_C, δ_Π) since both terms are positive and increasing. Thus, it suffices to prove that for $\delta \in (\delta_C, \delta_\Pi)$ the function $\frac{\partial \Pi(\delta,\phi)}{\partial \delta} C(\delta,\phi)$ is decreasing.[63] Using the the bounds in (19), inequality (20) and $\frac{\partial^2 \Pi}{\partial \delta^2} < 0$, $\frac{\partial \Pi}{\partial \delta} > 0$ for $\delta \in (\delta_C, \delta_\Pi)$, we have:

$$\frac{\partial}{\partial \delta}\left(\frac{\partial \Pi}{\partial \delta} C\right) = \frac{\partial^2 \Pi}{\partial \delta^2} C + \frac{\partial \Pi}{\partial \delta}\frac{\partial C}{\partial \delta} \leq (1+r)\left(\frac{\partial^2 \Pi}{\partial \delta^2} + \frac{\partial \Pi}{\partial \delta}\right) \leq 0.$$

This concludes the proof on the existence and uniqueness of a δ^* that satisfies the necessary FOC in (22).

Now, for given δ^*, the other necessary FOC (23) determines D^* uniquely.[64]

[62] More precisely, if for all $\delta \in [0,1]$ $\frac{\partial C(\delta,\phi)}{\partial \delta} > 0$ we might have $\delta^* = 0$ and if for all $\delta \in [0,1]$, $\frac{\partial \Pi(\delta,\phi)}{\partial \delta} > 0$ we might have $\delta^* = 1$.

[63] This is not trivial since $C(\delta,\phi)$ is increasing.

[64] Let us observe that for all δ, $C(\delta,\phi) \geq 1 > \Pi(\delta,\phi)$.

5. δ^* **is independent from** μ **and** D^* **is strictly increasing in** μ. Equation (22) determines δ^* and is independent from μ. Then (23) shows that D^* is increasing in μ.

6. δ^* **is decreasing in** ϕ **and, if** $\delta^* \in (0,1)$**, it is strictly decreasing.** Let $\delta(\phi)$ be the solution of the maximization problem of the bank for given ϕ. Let us assume that $\delta(\phi)$ satisfies the FOC (22). The case of corner solutions is analyzed in an analogous way.

We have proved in Step 3 above that the function $\frac{\partial \Pi}{\partial \delta}C - \frac{\partial C}{\partial \delta}\Pi$ is decreasing in δ around $\delta(\phi)$. In order to show that $\delta(\phi)$ is decreasing, it suffices to show that the derivative of this function w.r.t. ϕ is negative. Using the definitions of $C(\delta,\phi), \Pi(\delta,\phi)$ after some (tedious) algebra we obtain:

$$\frac{\partial}{\partial \phi}\left[\frac{\partial \Pi}{\partial \delta}C - \frac{\partial C}{\partial \delta}\Pi\right] = -(1+\rho_H) - \frac{1}{\rho_H}\frac{1+\rho_H}{1+\rho_H+\varepsilon}\left[(1+\rho_H)\left(\frac{dr}{d\delta}\delta - r\right) + \varepsilon\right].$$

Now we have $\frac{d}{d\delta}\left(\frac{dr}{d\delta}\delta - r\right) = \frac{d^2 r}{d\delta^2}\delta \geq 0$ and thus $\frac{dr}{d\delta}\delta - r \geq \frac{dr}{d\delta}\delta - r\big|_{\delta=0} = -r(0)$, and finally:

$$\begin{aligned}\frac{\partial}{\partial \phi}\left[\frac{\partial \Pi}{\partial \delta}C - \frac{\partial C}{\partial \delta}\Pi\right] &\leq -(1+\rho_H) - \frac{1}{\rho_H}\frac{1+\rho_H}{1+\rho_H+\varepsilon}\left[-(1+\rho_H)r(0) + \varepsilon\right] \\ &< -(1+\rho_H) + \frac{1}{\rho_H}(1+\rho_H)r(0) = -(1+\rho_H)\left(1 - \frac{r(0)}{\rho_H}\right) < 0.\end{aligned}$$

This conclude the proof that $\frac{d\delta}{d\phi} < 0$.[65]

7. $\delta^* D^*$ **is decreasing with** ϕ. **If** $\delta^* > 0$ **it is strictly decreasing.** Let $\delta(\phi), D(\phi)$ be the solution of the maximization problem of the bank for given ϕ. We have:

$$\frac{1+\rho_H}{\rho_H}\mu = \left[C(\delta(\phi),\phi) - \Pi(\delta(\phi),\phi)\right]D(\phi).$$

Let $\phi_1 < \phi_2$. In Step 6 we showed that $\delta(\phi_1) \geq \delta(\phi_2)$. If $\delta(\phi_2) = 0$ then trivially $\delta(\phi_1)D(\phi_1) \geq \delta(\phi_2)D(\phi_2) = 0$. Let us suppose that $\delta(\phi_2) > 0$. Since trivially $\Pi(\delta(\phi_1),\phi_1)D(\phi_1) \geq \Pi(\delta(\phi_2);\phi_2)D(\phi_2)$, we must have $C(\delta(\phi_1),\phi_1)D(\phi_1) \geq C(\delta(\phi_2),\phi_2)D(\phi_2)$. Now, suppose that $\delta(\phi_1)D(\phi_1) \leq \delta(\phi_2)D(\phi_2)$, then we have the following two inequalities:

$$\begin{aligned}(1+\rho_H)(1+\phi_1)\delta(\phi_1)D(\phi_1) &< (1+\rho_H)(1+\phi_2)\delta(\phi_2)D(\phi_2), \\ (1+r(\delta(\phi_1)))(1-\delta(\phi_1)) &\leq (1+r(\delta(\phi_2)))(1-\delta(\phi_2)),\end{aligned}$$

that imply $C(\delta(\phi_1),\phi_1)D(\phi_1) < C(\delta(\phi_2),\phi_2)D(\phi_2)$, but this contradicts our assumption. Thus, $\delta(\phi_1)D(\phi_1) > \delta(\phi_2)D(\phi_2)$.■

Proof of Proposition 3 This proof has two parts:

1. Existence and uniqueness of equilibrium. Let us denote $(\delta(\phi), D(\phi))$ the solution of the bank's optimization problem for every excess cost of crisis financing $\phi \geq 0$. Proposition 2 states that $\delta(\phi)D(\phi)$ is decreasing in ϕ. For $\phi \in [0,\overline{\phi}]$ let us define $\Sigma(\phi) = \Phi(\delta(\phi)D(\phi)) - \phi$. This function represents the difference between the excess cost of financing during a crisis by

[65] In the case of corner solution $\delta^*(\phi) = 1$, we might have $\frac{d\delta^*}{d\phi} = 0$ and obviously for $\delta^*(\phi) = 0$, $\frac{d\delta^*}{d\phi} = 0$.

banks' decisions and banks' expectation on such variable. Since Φ is an increasing function on the aggregate demand of funds during a crisis the function $\Sigma(\phi)$ is strictly decreasing. Because of the uniqueness of the solution to the problem that defines $(\delta(\phi), D(\phi))$, the function is also continuous. Moreover, we trivially have $\Sigma(0) \geq 0$ and $\Sigma(\overline{\phi}) \leq 0$. Therefore there exists a unique $\phi^e \in \mathbb{R}^+$ such that $\Sigma(\phi^e) = 0$. By construction $(\phi^e, (\delta(\phi^e), D(\phi^e)))$ is the unique equilibrium of the economy.

2. Comparative statics with respect to a shift in $\Phi(x)$. We are going to follow the notation used in the proof of Proposition 3. Let Φ_1, Φ_2 be two curves describing the inverse supply of financing during a crisis and assume they satisfy $\Phi_1(x) > \Phi_2(x)$ for all $x > 0$. Let us denote $\Sigma_i(\phi) = \Phi_i(\delta(\phi)D(\phi)) - \phi$ for $i = 1, 2$. By construction we have $\Sigma_1(\phi_1^e) = 0$. Let us suppose that $\phi_1^e < \phi_2^e$. Then we would have:

$$\Sigma_2(\phi_2^e) = \Phi_2(\delta(\phi_2^e)D(\phi_2^e)) - \phi_2^e \leq \Phi_1(\delta(\phi_2^e)D(\phi_2^e)) - \phi_2^e < \Phi_1(\delta(\phi_1^e)D(\phi_1^e)) - \phi_1^e = \Sigma_1(\phi_1^e) = 0, \tag{24}$$

where in the first inequality we use the assumption $\Phi_2(x) \leq \Phi_1(x)$ for $x \geq 0$, and in the second inequality we use that if $\phi_1^e < \phi_2^e$ then $\delta(\phi_2^e)D(\phi_2^e) \leq \delta(\phi_1^e)D(\phi_1^e)$ (Proposition 2), and that $\Phi_1(\cdot)$ is increasing. Notice that the sequence of inequalities in (24) implies $\Sigma_2(\phi_2^e) < 0$, which contradicts the definition of ϕ_2^e. We must therefore have $\phi_1^e \geq \phi_2^e$. And Proposition 2 implies that $\delta_1^e \leq \delta_2^e$, $\delta_1^e D_1^e \leq \delta_2^e D_2^e$, and $r_1^e \geq r_2^e$, proving all the results in weak terms.

Finally, let us suppose that $\delta_2^e \in (0, 1)$. Then the first inequality in (24) is strict, since $\delta_2^e D_2^e > 0$, and we can straightforwardly check that the previous argument implies $\phi_1^e > \phi_2^e$. In this case, since $\delta_2^e \in (0, 1)$, Proposition 2 implies that $\delta_1^e < \delta_2^e$, $\delta_1^e D_1^e < \delta_2^e D_2^e$, and $r_1^e > r_2^e$.$\blacksquare$

Proof of Proposition 4 We are going to follow the notation used in the proof of Proposition 2. The proof is organized in five steps:

1. Preliminaries From first principles, using equations (8) and (12), we can obtain

$$\frac{\partial W(\delta, D)}{\partial \delta} = \frac{\partial V(\delta, D; \Phi(\delta D))}{\partial \delta} = D\frac{\partial \Pi(\delta, \Phi(\delta D))}{\partial \delta}, \tag{25}$$

where the last equality follows from (16). Similarly we can obtain

$$\frac{\partial W(\delta, D)}{\partial D} = \frac{\partial V(\delta, D; \Phi(\delta D))}{\partial D} = \Pi(\delta, \Phi(\delta D)). \tag{26}$$

2. (CF) is binding at the socially optimal debt structure This is a statement that has been done in the main text just before Proposition 4. The proof is analogous to the one for the maximization problem of the bank that we did in Step 4 of the proof of Proposition 2. The only difference is that ϕ is not taken as given but as the function $\Phi(\delta D)$ in D and δ.

3. Definition of function $D^c(\delta)$ and its properties Let $(\phi^e, (\delta^e, D^e))$ be the competitive equilibrium. Let us assume that $\delta^e < 1$. By definition of equilibrium we have $\phi^e = \Phi(\delta^e D^e)$. For every δ let $D^c(\delta)$ be the unique principal of debt such that (CF) is binding, i.e.:

$$\frac{1 + \rho_H}{\rho_H}\mu = [C(\delta, \phi^e) - \Pi(\delta, \phi^e)]\, D^c(\delta). \tag{27}$$

Differentiating w.r.t. δ:

$$\left[\frac{\partial C(\delta,\phi^e)}{\partial\delta}-\frac{\partial\Pi(\delta,\phi^e)}{\partial\delta}\right]D^c(\delta)+\left[C(\delta,\phi^e)-\Pi(\delta,\phi^e)\right]\frac{dD^c(\delta)}{d\delta}=0. \tag{28}$$

Using the characterization of δ^e in equation (22), the inequalities $C(\delta,\phi^e)\geq 1>\Pi(\delta,\phi^e)$ imply $\frac{\partial C(\delta^e,\phi^e)}{\partial\delta}-\frac{\partial\Pi(\delta^e,\phi^e)}{\partial\delta}>0$ and, then, we can deduce from the equation above that $\frac{dD^c(\delta^e)}{d\delta}<0$. Since (CF) is binding at the optimal debt structure we can think of the bank problem as maximizing the univariate function $V(\delta,D^c(\delta);\phi^e)$. Hence δ^e must satisfy the necessary FOC for an interior solution to the maximization of $V(\delta,D^c(\delta);\phi^e)$:

$$\frac{dV(\delta^e,D^c(\delta);\phi^e)}{d\delta}=0\Leftrightarrow D^c(\delta^e)\frac{\partial\Pi(\delta^e,\phi^e)}{\partial\delta}+\Pi(\delta^e,\phi^e)\frac{dD^c(\delta^e)}{d\delta}=0, \tag{29}$$

which multiplying by δ^e can be written as

$$D^c(\delta^e)\frac{\partial\Pi(\delta^e,\phi^e)}{\partial\delta^e}\delta^e=\Pi(\delta^e,\phi^e)\left(-\frac{dD^c(\delta^e)}{d\delta}\delta^e\right).$$

Since $\frac{\partial\left(\Pi-\frac{\partial\Pi}{\partial\delta}\delta\right)}{\partial\delta}=-\frac{\partial^2\Pi}{\partial\delta^2}\delta\geq 0$ and $\Pi(0,\phi)-\frac{\partial\Pi(0,\phi)}{\partial\delta}0>0$, we have $\Pi(\delta,\phi)>\frac{\partial\Pi(\delta,\phi)}{\partial\delta}\delta$ for all $\delta\in[0,1]$ and the previous equation implies

$$D^c(\delta^e)>-\frac{dD^c(\delta^e)}{d\delta}\delta^e\Leftrightarrow\left.\frac{d\left(\delta D^c(\delta)\right)}{d\delta}\right|_{\delta=\delta^e}>0.$$

4. Evaluation of $\left.\frac{d(D^s(\delta))}{d\delta}\right|_{\delta=\delta^e}$ and $\left.\frac{d(\delta D^s(\delta))}{d\delta}\right|_{\delta=\delta^e}$ For every δ, let $D^s(\delta)$ be the unique principal of debt such that (CF) is binding, i.e.

$$\frac{1+\rho_H}{\rho_H}\mu=\left[C(\delta,\Phi(\delta D^s(\delta)))-\Pi(\delta,\Phi(\delta D^s(\delta)))\right]D^s(\delta).$$

Differentiating w.r.t. δ, we obtain

$$\begin{aligned}&\left[\frac{\partial C(\delta,\Phi)}{\partial\delta}-\frac{\partial\Pi(\delta,\Phi)}{\partial\delta}\right]D^s(\delta)+\left[C(\delta,\Phi(\delta D^s(\delta)))-\Pi(\delta,\Phi(\delta D^s(\delta)))\right]\frac{dD^s(\delta)}{d\delta}+\\&\qquad+\left[\frac{\partial C(\delta,\Phi)}{\partial\phi}-\frac{\partial\Pi(\delta,\Phi)}{\partial\phi}\right]\Phi'(\delta D^s(\delta)))\frac{d\left(\delta D^s(\delta)\right)}{d\delta}=0.\end{aligned} \tag{30}$$

By construction, $D^s(\delta^e)=D^c(\delta^e)=D^e$. Now, subtracting equation (28) from equation (30) at the point $\delta=\delta^e$ we obtain

$$\begin{aligned}&\left[C(\delta^e,\phi^e)-\Pi(\delta^e,\phi^e)\right]\left(\frac{dD^s(\delta^e)}{d\delta}-\frac{dD^c(\delta^e)}{d\delta}\right)+\left[\frac{\partial C(\delta^e,\phi^e)}{\partial\phi}\right.\\&\qquad\left.-\frac{\partial\Pi(\delta^e,\phi^e)}{\partial\phi}\right]\Phi'(\delta^e D^e)\left.\frac{d\left(\delta D^s(\delta)\right)}{d\delta}\right|_{\delta=\delta^e}=0.\end{aligned} \tag{31}$$

Suppose that $\left.\frac{d(\delta D^s(\delta))}{d\delta}\right|_{\delta=\delta^e} \leq 0$, then we would have $\frac{dD^s(\delta^e)}{d\delta} \geq \frac{dD^c(\delta^e)}{d\delta}$, since trivially $\frac{\partial C(\delta^e,\phi^e)}{\partial\phi} - \frac{\partial\Pi(\delta,\phi^e)}{\partial\phi} > 0$. But then

$$\left.\frac{d\left(\delta D^s(\delta)\right)}{d\delta}\right|_{\delta=\delta^e} = D^s(\delta^e) + \frac{dD^s(\delta^e)}{d\delta}\delta^e > D^c(\delta^e) + \frac{dD^c(\delta^e)}{d\delta}\delta^e = \left.\frac{d\left(\delta D^c(\delta)\right)}{d\delta}\right|_{\delta=\delta^e} > 0,$$

which contradicts the hypothesis. We must thus have $\left.\frac{d(\delta D^s(\delta))}{d\delta}\right|_{\delta=\delta^e} > 0$, in which case equation (31) implies $\frac{dD^s(\delta^e)}{d\delta} < \frac{dD^c(\delta^e)}{d\delta} < 0$.

5. Evaluation of $\left.\frac{dW(\delta,D^s(\delta))}{d\delta}\right|_{\delta=\delta^e}$ Using equations (25) and (26), we have:

$$\begin{aligned}\frac{dW(\delta, D^s(\delta))}{d\delta} &= \frac{\partial W(\delta, D^s(\delta))}{\partial\delta} + \frac{\partial W(\delta, D^s(\delta))}{\partial D}\frac{dD^s(\delta)}{d\delta} \\ &= D^s(\delta)\frac{\partial\Pi(\delta, \Phi(\delta D^s(\delta)))}{\partial\delta} + \Pi(\delta, \Phi(\delta D^s(\delta)))\frac{dD^s(\delta)}{d\delta}.\end{aligned}$$

And, using $\frac{dD^s(\delta^e)}{d\delta} < \frac{dD^c(\delta^e)}{d\delta}$ and (29), we obtain:

$$\left.\frac{dW(\delta, D^s(\delta))}{d\delta}\right|_{\delta=\delta^e} < D^s(\delta^e)\frac{\partial\Pi(\delta^e, \phi^e)}{\partial\delta} + \Pi(\delta^e, \phi^e)\frac{dD^c(\delta^e)}{d\delta} = 0.$$

Summing up, having

$$\left.\frac{dW(\delta, D^s(\delta))}{d\delta}\right|_{\delta=\delta^e} < 0, \quad \left.\frac{dD^s(\delta)}{d\delta}\right|_{\delta=\delta^e} < 0, \text{ and } \left.\frac{d(\delta D^s(\delta))}{d\delta}\right|_{\delta=\delta^e} > 0,$$

implies that a social planner can increase welfare by fixing some $\delta^s < \delta^e$, that is associated with debt $D^s > D^e$ and with refinancing needs $\delta^s D^s < \delta^e D^e$.

6. Implementation of $\delta^s D^s$ We have that $\frac{\partial V(D,\delta^s;\phi)}{\partial D} = \Pi(\delta^s, \phi) = 1 - \frac{r(\delta^s)}{\rho_H} > 0$. As a result, once the social planner fixes banks' choice of δ at δ^s banks will find optimal to issue as much debt D^* as is compatible with their (CF). It is straightforward to prove that in equilibrium this implies $D^* = D^s$.■

B Debt structures that induce default during crises

In this section we examine the possibility that a bank decides to expose itself to the risk of defaulting on its debt and being liquidated in the crisis state. First, we describe the sequence of events following default. Second, we show how the debt of the bank would be valued by savers who correctly anticipate the possibility of default. Finally, we analyze the bank's capital structure problem when default during crises is an explicit alternative.

Default and liquidation Liquidation following the bank's inability to satisfy its refinancing needs yields a residual value $L \geq 0$. Suppose that partial liquidation is not feasible and in case of liquidation L is first used to repay $\overline{D}_R$ to retail depositors and only if $L > \overline{D}_R$ the residual value is equally distributed among the wholesale debtholders. Assume further, for simplicity, that $L > \overline{D}_R$ so that retail deposits are riskless.[66] It is easy to realize that if the bank exposes itself to default in a crisis (rather than relying on crisis financing), then it will find it optimal to opt for debt contracts that mature in a perfectly correlated manner since this minimizes the probability of defaulting during crises. Hence we assume that the debt issued by the bank when getting rid of the (CF) constraint has perfectly correlated maturities.

Pricing of wholesale debt in the presence of default risk From the perspective of the investor in wholesale debt, there are four states relevant for the valuation of a non-matured debt contract: patience in the normal state ($i = LN$), patience in the crisis state ($i = LC$), impatience in the normal state ($i = HN$), and impatience in the crisis state ($i = HC$).

Let $l = (L - \overline{D}_R)/D < 1$ be the fraction of the principal of wholesale debt which is recovered in case of liquidation and let Q_i be the present value of expected losses due to default as evaluated from each of the states i just after the uncertainty regarding the corresponding period has realized and conditional on the debt not having matured in such period. Losses are measured relative to the benchmark case without default in which debtholdes recover the full principal (one unit) at maturity. These values satisfy the following system of recursive relationships:

$$Q_{LN} = \frac{1}{1+\rho_L}\left[\delta\varepsilon(1-l) + (1-\delta)\{(1-\varepsilon)[(1-\gamma)Q_{LN} + \gamma Q_{HN}] + \varepsilon[(1-\gamma)Q_{LC} + \gamma Q_{HC}]\}\right],$$

$$Q_{HN} = \frac{1}{1+\rho_H}\left[\delta\varepsilon(1-l) + (1-\delta)\left[(1-\varepsilon)Q_{HN} + \varepsilon Q_{HC}\right]\right],$$

$$Q_{LC} = \frac{1}{1+\rho_L}(1-\delta)[(1-\gamma)Q_{LN} + \gamma Q_{HN}],$$

$$Q_{HC} = \frac{1}{1+\rho_H}(1-\delta)Q_{HN}.$$

These expressions essentially account for the principal $1 - l > 0$ which is lost if the bank's wholesale debt matures in the crisis state (pushing the bank into default). The first equation reflects that default as well as any of the four non-default states i may follow state LN. The second and fourth equations reflect that impatience is an absorbing state. The third and fourth equations reflect that after the crisis state the economy reverts deterministically to the normal state. We will denote the solution for Q_{LN} associated with this linear system of equations by $Q_{LN}(\delta, D; L)$ in order to highlight its dependence on δ, D and L.

The value of a debt contract $(r, \delta, 1)$ to a patient investor in the normal state, when default is expected if the bank's debt matures during a crisis, can then be written as

$$U_L^d(r,\delta) = U_L(r,\delta) - Q_{LN}(\delta, D; L),$$

[66] Under our calibration, this inequality is satisfied by the maximum liquidation value $L^{\max}$ for which banks prefer to avoid defaulting during crises.

where $U_L(r,\delta)$ is the value of the same contract in the benchmark senario in which banks do not default in crises, given by (2).

The interest rate yield that the bank offers when default may occur, $r^d(\delta)$, satisfies $U_L^d(r^d(\delta),\delta) = 1$, while the non-default yield $r(\delta)$ satisfies $U_L(r(\delta),\delta) = 1$. Thus, we have $U_L^d(r^d(\delta),\delta) = U_L(r(\delta),\delta)$, which allows us to express $r^d(\delta)$ as the sum of $r(\delta)$ and a default-risk premium:

$$r^d(\delta) = r(\delta) + \frac{(\rho_H+\delta)\,(\rho_L+\delta+(1-\delta)\gamma)}{\rho_H+\delta+(1-\delta)\gamma} Q_{LN}(\delta,D;L).$$

The above equations imply that the default-risk premium $r^d(\delta) - r(\delta)$ is increasing in δ and D, and decreasing in L. Given that δ increases the probability of default, $r^d(\delta)$ is not necessarily decreasing in δ.

Optimal capital structure when debt maturity in crises leads to default If the bank does not satisfy the crisis financing constraint and thus defaults whenever it faces refinancing needs during a crisis, its equity value in the normal state $E^d(\delta,D)$ will satisfy

$$E^d(\delta,D) = \frac{1}{1+\rho_H}\left[\mu\text{-}\rho_L\overline{D}_R\text{-}r^dD+(1\text{-}\varepsilon)E^d(\delta,D)+\varepsilon\{\delta\cdot 0+(1\text{-}\delta)\frac{1}{1+\rho_H}[\mu\text{-}r^dD+E^d(\delta,D)]\}\right],$$

whose solution yields:

$$E^d(\delta,D) = \frac{1+\rho_H+\varepsilon(1-\delta)}{(1+\rho_H)^2-(1+\rho_H)(1-\varepsilon)-\varepsilon(1-\delta)}(\mu-\rho_L\overline{D}_R-r^dD).$$

And the bank's optimal capital structure decision can be described as

$$\begin{aligned} \max_{D\geq 0,\ \delta\in[0,1]} \quad & V^d(\delta,D) = \overline{D}_R + D + E^d(\delta,D), \\ \text{s.t.} \quad & E^d(\delta,D) \geq 0, \quad \text{(LL)} \end{aligned} \tag{32}$$

where (LL) is trivially equivalent to $\mu - \rho_L\overline{D}_R - r^dD \geq 0$.

To find the maximum value of L which, under our calibration of the model, is consistent with banks' optimizing subject to the (CF) constraint (and, hence, with not getting exposed to default), we proceed in two steps. First, we solve the problem in (32) numerically for an ample grid of values of L. Second, we compare the optimized value of the objective function in (32) with the equilibrium market value of the bank in the scenario with no default. $L^{\max}$ is the maximum L for which the solution of the problem with no default dominates.

C Crises that lead to default

In this section we cover the extension described in subsection 7.2 of the paper.

C.1 Set up

Let us suppose that the liquidation value L of the bank under default satisfies $L > L^{\max}$, where $L^{\max}$ is defined in subsection 7.1. Then, according to the results obtained there, the equilibrium in which all banks obtain crisis financing is not sustainable.

To allow for default in equilibrium, we extend the definition equilibrium to allow for a mixed-strategies regime in which at the initial date some banks choose debt structures that satisfy (CF) while other choose debt structures that do not satisfy it. We will refer to the former as safe banks (since they never default), and to the latter as risky banks (since may default when experiencing a crisis). We denote by $x \in [0, 1]$ the measure of safe banks. And we assume that banks defaulting in crises (and therefore liquidated) are replaced in the following period by identical new banks (created at a cost c), so that the total measure of banks is always equal to one.

The optimal debt structure of risky banks is described in section B of this appendix and is independent of x. Let V^d denote the market value of a bank that chooses such structure, while we keep using $V(\delta, D; \phi)$ to denote the market value during normal times of a safe bank under a given debt structure (δ, D) and crisis cost of funds ϕ (equation (8) in the paper). Finally, to make the entry of new banks profitable, we assume $c \leq V^d$.

We use the following notion of equilibrium:

Definition 2 *An* equilibrium *is a tuple $(x^e, \phi^e, (\delta^e, D^e))$ describing the measure x^e of safe banks, the excess cost of crisis funds ϕ^e and the debt structure of safe banks (δ^e, D^e) such that*

1. *Among the class of debt structures that satisfy (CF) given ϕ^e, (δ^e, D^e) maximizes the safe bank's value to its initial owners.*
2. *Banks' decisions to become safe or risky are optimal, i.e.*

$$x^e = 1 \text{ if } V(\delta^e, D^e; \phi^e) > V^d \text{ and } x^e = 0 \text{ if } V(\delta^e, D^e; \phi^e) < V^d.$$

3. *The market for crisis financing clears in a way compatible with the refinancing of safe banks, i.e. $\phi^e = \Phi(x^e \delta^e D^e)$.*

Note that if $x^e = 1$, the tuple $(\phi^e, (\delta^e, D^e))$ is an equilibrium with crisis financing (and no bank default) as defined for the baseline model.

C.2 Equilibrium analysis

The equilibrium analysis in the paper easily extends to this more general setup. Let us denote by $(\phi^e(x), (\delta^e(x), D^e(x)))$ the equilibrium with crisis financing that emerges in our economy if the measure of safe banks is $x \in [0, 1]$; Proposition 3 in the paper ensures this equilibrium exists and is unique. Let $V^e(x)$ be the equilibrium market value of a bank in such economy. And, to guarantee that the mixed-strategy equilibrium is non-trivial (and eventually involves $x > 0$), assume $V^e(0) > V^d$, which, taking into account that $\phi^e(0) = 0$, means that if the excess cost of crisis funds were zero then all banks would prefer to be safe.

We have that:

Lemma 1 *If $\delta^e(x) > 0$, then both $\delta^e(x)$ and $V^e(x)$ are strictly decreasing in x. If $\delta^e(x) = 0$, then they are constant in x.*

Proof The proof is analogous to that of statement (2) of Proposition 3 in the paper.■

Using this result we can prove the analogous to Proposition 3 in the paper:

Proposition 5 *The equilibrium of the economy exists and is unique.*

Proof From the assumption that $V^e(0) > V^d$, it suffices to prove that either there exists a unique $x \in [0, 1]$ such that $V^e(x) = V^d$ (in which case such x is the equilibrium measure x^e of safe banks) or $V^e(x) > V^d$ for all $x \in [0, 1]$ (in which case $x^e = 1$).

Let us distinguish two cases:

1. $\phi^e(1) = 0$. Then $V^e(1) = V^e(0) > V^d$ and using the previous lemma we have $V^e(x) > V^d$ for all x.

2. $\phi^e(1) > 0$. Then a fortiori $\delta^e(1) > 0$ and using the lemma $\delta^e(x) > 0$ for all x. Using again the lemma, $V^e(x)$ is strictly decreasing in the interval $[0, 1]$. As a result either $V^e(1) \leq V^d$ and there exists a unique solution to $V^e(x) = V^d$ with $x \in [0, 1]$, or $V^e(1) > V^d$ and $V^e(x) > V^d$ for all $x \in [0, 1]$.■

C.3 Efficiency and regulatory results

Suppose that the social planner (SP) can choose x and the debt structures of safe and risky banks. And assume, for simplicity, that the cost of entry of replacing banks satisfies $c = V^d$ so that we can abstract from the welfare implications associated with rents obtained from entering. Under this assumption the privately optimal debt structure choices of risky banks involve no externalities and hence coincide with the socially optimal ones. The social value generated by a measure $1 - x$ of risky banks is simply $(1 - x)V^d$.

Extending the notation in Section 5 of the paper, let $W(x, \delta, D)$ denote the social surplus generated by the economy when the SP chooses x banks to be safe with debt structure (δ, D). We have:

$$W(x, \delta, D) = xV(\delta, D; \Phi(x\delta D)) + U(\Phi(x\delta D)) + (1 - x)V^d, \tag{33}$$

where $U(\Phi(x\delta D))$ is the present value of experts' surplus from refinancing safe banks during crises (a function of the excess cost of crisis funds as given by (11) in the paper).

The determination of the optimal SP decision could be conceptually split into two steps. First, for every possible x, the SP chooses the debt structure (δ, D) that maximizes the social surplus generated by safe banks. The SP internalizes the effect of this decision on the excess cost of crisis funds in the way described in the paper. Second, and taking the first step into account, the SP decides x so as to maximize aggregate surplus. The novel externality the SP internalizes in the second step is that each safe bank relies on experts' funds during crises, marginally increasing their cost and tightening the (CF) constraint of other safe banks. The next proposition (analogous to Proposition 4 in the paper) states that in this context the SP will regulate x in addition to regulating δ:

Proposition 6 *Suppose that in the unregulated equilibrium $x^e \in (0,1)$ or $\delta^e \in (0,1)$. Then the equilibrium is inefficient and at the socially efficient tuple $(x^s, (\delta^s, D^s))$ we have*

$$V(\delta^s, D^s; \Phi(x^s\delta^s D^s)) > V^d.$$

To induce efficiency, the SP can charge an entry fee $\tau = V(\delta^s, D^s; \Phi(x^s\delta^s D^s)) - V^d$ to banks allowed to access experts' funding during crises and limit their expected debt maturity to $1/\delta^s$.

Proof The proof relies on notation introduced and results derived in the proof of Proposition 4 in the paper. We first prove the inequality. For every x and δ we can define the function $D^S(x,\delta)$ as the maximum debt a bank can choose so that its (CF) constraint is satisfied with $\phi = \Phi(x\delta D^S(x,\delta))$. For a given x, δ, $D^S(x,\delta)$ is the debt choice of safe banks that maximizes social surplus. Taking this into account (and with some notational abuse), for each x, δ, we write $W(x,\delta) = W(x,\delta,D^S(x,\delta))$.

For simplicity, we make our arguments implicitly assuming that the optimal solution of the maximization problem of the SP problem is interior, i.e. $x^s, \delta^s \in (0,1)$. The arguments can be easily adapted to the case of corner solutions.

For a given x the value of δ that maximizes $W(x,\delta)$ satisfies the following FOC:

$$x\left[\frac{\partial V}{\partial \delta} + \frac{\partial V}{\partial D}\frac{\partial D^S}{\partial \delta} + \frac{\partial V}{\partial \phi}\frac{\partial \Phi}{\partial y}\left(xD^S + x\delta\frac{\partial D^S}{\partial \delta}\right)\right] + \frac{\partial U}{\partial \phi}\frac{\partial \Phi}{\partial y}\left(xD^S + x\delta\frac{\partial D^S}{\partial \delta}\right) = 0,$$

where $\frac{\partial \Phi}{\partial y}$ is the derivative of the inverse supply of funds function $\Phi(y)$ and is evaluated at the point $x\delta D^S(x,\delta)$. Taking into account that $x\frac{\partial V}{\partial \phi} = -\frac{\partial U}{\partial \phi}$ the FOC simplifies to:

$$\frac{\partial V}{\partial \delta} + \frac{\partial V}{\partial D}\frac{\partial D^S}{\partial \delta} = 0, \tag{34}$$

where the partial derivatives of V wrt δ and D are computed at $(\delta, D^S(x,\delta), \Phi(x\delta D^S(x,\delta)))$.

Now, the FOC for the optimal measure x of safe banks can be obtained by fully differentiating the expression in (33) with respect to x. We have that:

$$\begin{aligned} V - V^d + +x\Bigg[&\frac{\partial V}{\partial \delta}\frac{d\delta^S}{dx} + \frac{\partial V}{\partial D}\left(\frac{\partial D^S}{\partial \delta}\frac{d\delta^S}{dx} + \frac{\partial D^S}{\partial x}\right) + \\ &+ \frac{\partial V}{\partial \phi}\frac{\partial \Phi}{\partial y}\left(\delta^S D^S + x\frac{d\delta^S}{dx}D^S + x\delta^S\frac{\partial D^S}{\partial \delta}\frac{d\delta^S}{dx} + x\delta^S\frac{\partial D^S}{\partial x}\right)\Bigg] + \\ &+ \frac{\partial U}{\partial \phi}\frac{\partial \Phi}{\partial y}\left(\delta^S D^S + x\frac{d\delta^S}{dx}D^S + x\delta^S\frac{\partial D^S}{\partial \delta}\frac{d\delta^S}{dx} + x\delta^S\frac{\partial D^S}{\partial x}\right) = 0. \end{aligned}$$

Using $x\frac{\partial V}{\partial \phi} = -\frac{\partial U}{\partial \phi}$ and (34) (which amounts to applying the Envelope Theorem) we can simplify this FOC to:

$$V - V^d + x\frac{\partial V}{\partial D}\frac{\partial D^S}{\partial x} = 0. \tag{35}$$

Finally, taking into account that $\frac{\partial V}{\partial D} > 0$ and that $\frac{\partial D^S}{\partial x} < 0$ (which can be easily proven from the (CF) constraint) we deduce that at an interior optimum $(x^s, (\delta^s, D^s))$ of the SP problem:

$$V(\delta^s, D^s; \Phi(x^s\delta^s D^s)) > V^d.$$

The wedge between the profits of safe and risky banks arises due to the negative externality a new safe bank creates on the measure x of safe banks: it forces them to marginally reduce their debt by $\frac{\partial D^S}{\partial x}$ and each unit of debt reduction has a profit impact $\frac{\partial V}{\partial D}$.

From here the proof of the remaining results is almost straightforward. If $x^e \in (0,1)$ then $V(\delta^e, D^e; \Phi(x^e \delta^e D^e)) = V^d$ and a fortiori $(x^e, (\delta^e, D^e))$ is not efficient. If $x^e = 1$ and $\delta^e \in (0,1)$ then Proposition 4 in the paper shows that $(x^e, (\delta^e, D^e))$ is not efficient. Finally, the proof of the statement regarding the implementation of the socially efficient tuple is straightforward and omitted here for brevity.■

When $x^e \in (0,1)$ the proposition states that, as opposed to the unregulated economy in which banks are indifferent between being safe and risky, in the optimally regulated economy safe banks' expected profits are strictly higher than those of risky banks. This is because internalizing the negative externalities that safe banks cause to other safe banks via the cost of crisis financing requires restricting entry. So beyond limiting the expected debt maturity of banks that wish to opt for funding during crises, the SP must ensure that not too many banks go for that option. Charging an entry fee to safe banks is a way to achieve this purpose.

Finally, notice that the proposition leaves it open the comparison between the socially efficient and laissez-faire measures of safe banks, x^s and x^e, since on the one hand controlling the externality pushes for $x^s < x^e$ but the fact that the externality can also be controlled by fixing $\delta^s < \delta^e$ points in the opposite direction.

D Asset risk

In this section we cover the asset risk extension described in subsection 7.3 of the paper.

D.1 Set up

In addition to the set up of the baseline model, we assume that each crisis destroys a fraction $\chi \in (0,1)$ of banks' assets (and the corresponding share of future cash flows). We also assume that following a crisis banks can replenish their asset base at a cost c. Banks can pay such cost with the proceeds from reestablishing their pre-crisis debt structure, δD, and/or experts' direct contribution. For simplicity, we assume that during normal times experts' funds have a constant opportunity cost ρ_H. Moreover, we assume $\chi\mu/(\rho_H + \varepsilon\chi) \geq c$ so that replenishing the asset base is a positive NPV investment even on an unlevered basis.

To eliminate a potential debt overhang problem, we assume that each bank operates under a covenant that forces it to replenish its asset base immediately after each crisis. As in the baseline model, each bank decides its debt structure (δ, D) at an initial date and we focus on equilibria with crisis financing (no default), whose definition is analogous to that in the baseline model.[67]

[67] As discussed in subsection 7.1 of the paper, when the liquidation value of assets is sufficiently small, banks optimally choose debt structures (δ, D) that protect them from defaulting.

D.2 Equilibrium analysis and regulatory results

The analysis is very similar to that of the baseline model. Here we focus on key expressions that experience some changes.

Equity value of the bank during normal times Conditional on the bank obtaining crisis financing and replenishing its asset base after each crisis, we can find the following recursive expression for the continuation value of the bank to its shareholders:

$$\begin{aligned} E^{AR}(\delta, D; \phi) &= \frac{1}{1+\rho_H}\Big\{(\mu - rD) + (1-\varepsilon)E^{AR}(\delta, D; \phi) + \\ &\quad \varepsilon\frac{1}{1+\rho_H}[(1-\chi)\mu - (1-\delta)rD - c + \delta D + E^{AR}(\delta, D; \phi)] - \varepsilon(1+\phi)\delta D\Big\}. \end{aligned} \tag{36}$$

This equation is similar to equation (4) in the baseline model. The differences reflect that in the period just after each crisis the still unrepaired assets yield only $(1-\chi)\mu$ and the bank incurs the replenishment cost c.

From the equation above we can obtain the following closed form expression

$$\begin{aligned} E^{AR}(\delta, D; \phi) &= \frac{\mu}{\rho_H} - \frac{r(\delta)}{\rho_H}D - \frac{1}{\rho_H}\frac{\varepsilon\{[(1+\rho_H)\phi + \rho_H] - r(\delta)\}}{1+\rho_H+\varepsilon}\delta D \\ &\quad - \frac{\varepsilon\chi}{\rho_H(1+\rho_H+\varepsilon)}\mu - \frac{1}{\rho_H}\frac{\varepsilon}{1+\rho_H+\varepsilon}c, \end{aligned} \tag{37}$$

which is analogous to (6) in the paper. The additional last two terms capture, respectively, the present value of the cash flow losses and replenishing costs incurred just after each crisis.

Crisis financing constraint From previous expressions, the crisis financing constraint becomes

$$(1-\chi)\mu - (1-\delta)rD + \delta D - c + E^{AR}(\delta, D; \phi) \geq (1+\rho_H)(1+\phi)\delta D, \tag{38}$$

which is similar to (CF) in the baseline model, differing from it in the adjustments again due to the reduced cash flow $(1-\chi)\mu$ and replenishment cost c that accrue in the period after the crisis.

Financing of asset replenishment If the bank's debt structure satisfies (38), the bank is able to obtain experts' funds during crises. In the period after each crisis, bank assets generate a cash-flow $(1-\chi)\mu$, interest payments involve an outflow $-(1-\delta)rD$, net debt issuance yields δD, and the asset replenishment cost is c. When $(1-\chi)\mu-(1-\delta)rD+\delta D - c \geq 0$, the net positive cash flow from the after crisis period contributes (together with the continuation value of equity $E^{AR}(\delta, D; \phi)$) to cover the excess refinancing costs $(1+\rho_H)(1+\phi)\delta D$ implied by the crisis. Otherwise, the negative cash flows just after the crisis add to the costs of the crisis that need to be "paid for" by leaving the residual equity value $E^{AR}(\delta, D; \phi)$ in hand of the crisis financiers.

Using previous expressions, the results in Propositions 2, 3 and 4 of the paper can be replicated for this extension with very minor adaptations in the proofs. We omit the details for brevity.

E Bailout expectations and the regulation of leverage

In this section we develop the formal details of the extension on bailout expectations and regulation of leverage described in subsection 7.4 of the paper.

E.1 Set up

We add to the baseline set up a government that can subsidize the refinancing of banks by experts during crises. In our baseline model, if for a given excess cost of crisis funds ϕ some bank did not satisfy its (CF) constraint, then experts would not refinance its maturing debt. This would trigger default and the liquidation of the bank (which yields L). Assume now that the failure of the bank also causes some external social cost $C \geq 0$ and that, to avoid bank failure, the government can promise to the financing experts a subsidy τ paid just after the crisis with funds obtained by taxing the savers. If this subsidy τ is sufficiently large, experts will be willing to refinance the bank and its liquidation will be avoided. Assume further that this bailout intervention has an extra cost $\lambda > 0$ on top of τ (e.g. a reputational or political cost) and that due to unmodeled political constraints the government cannot give a subsidy larger than the minimum which prevents the bank from defaulting in the crisis.

We assume that the government's bailout decisions are time consistent, which means that in each crisis the government decides on bailing out banks unable to obtain crisis financing so as to maximize the continuation social surplus generated by the banking system, taking future bailout decisions as given. We will write $\sigma = 1$ if the government bails out a bank in distress, and $\sigma = 0$ if it does not. In addition, to keep the aggregate size of the banking sector constant, we assume that each liquidated bank is replaced by an identical new bank that incurs an entry cost c (small enough for the optimality of replacing failed banks with new ones).[68]

In what follows, we denote the interest rate function for riskless debt given by (3) in the paper as $\widehat{r}(\delta)$ and we describe a debt structure (δ, D) as *feasible* if $\mu - \widehat{r}(\delta)D \geq 0$, i.e. when, if priced as riskless debt, it satisfies the (LL) constraint. More generally, we assume that banks choosing initial debt structures that violate (LL) are not allowed to operate. The set of feasible debt structures is a compact subset of $[0,1] \times [0, \frac{\mu}{\rho_L}]$.

A symmetric stationary equilibrium is now defined as follows:

Definition 3 *An* equilibrium *of the economy with possible bailouts is a tuple $(\phi^e, (\delta^e, D^e), (r^e(\delta, D)), (\sigma^e(\delta, D)))$ describing an excess cost of crisis financing ϕ^e, a debt structure for banks (δ^e, D^e), an interest rate $r^e(\delta, D)$ required by savers for each bank debt structure (δ, D), and a government bailout decision $\sigma^e(\delta, D)$ for each bank debt structure (δ, D) not satisfying (CF), such that*

1. *For every (δ, D) the interest rate on banks debt $r^e(\delta, D)$ is such that savers break even taking into account that the debt is safe unless (δ, D) does not satisfy (CF) given ϕ^e and $\sigma^e(\delta, D) = 0$.*

2. *Among the class of debt structures satisfying banks' limited liability (LL) constraint during normal times, i.e. with $\mu - r^e(\delta, D)D \geq 0$, (δ^e, D^e) maximizes the bank's value to its initial owners given ϕ^e and $(\sigma^e(\delta, D))$.*

[68] A straightforward sufficient condition is $c < \frac{\mu}{r(0)}$.

3. *If (δ^e, D^e) does not satisfy (CF) given ϕ^e, in each crisis the government on-the-equilibrium-path bailout decision $\sigma^e(\delta^e, D^e)$ maximizes social welfare taking as given future bailout decisions.*

4. *For each bank debt structure $(\delta, D) \neq (\delta^e, D^e)$ not satisfying (CF) given ϕ^e, the government bailout decision $\sigma^e(\delta^e, D^e)$ maximizes social welfare given other banks' debt structure decisions (δ^e, D^e), the government's bailout decisions regarding other banks $\sigma^e(\delta^e, D^e)$, if relevant, and future bailout decisions.*

5. *The cost of crisis financing is determined as $\phi^e = \Phi(\delta^e D^e)$, that is, the cost that would clear the market if all banks were asking for crisis financing.*

Most of these conditions are self-explanatory so we will only comment on a few features. First, the interest rate function $r^e(\delta, D)$ may include a default premium; this is the case if the debt structure does not satisfy (CF) given ϕ^e and no government bailout is expected, $\sigma^e(\delta, D) = 0$.[69] Second, government bailout decisions on and off the equilibrium path are assumed to be time consistent, that is, to maximize continuation social surplus, taking the pattern of future bailout decisions as given. Third, we simplify the analysis by imposing that the cost of crisis financing in the equilibrium above, ϕ^e, is determined as the one that would clear the market if all banks were to satisfy their refinancing needs in the crisis. Strictly speaking, this implies that ϕ^e is not a market clearing price if banks choose debt structures that violate (CF) and the government subsequently decides not to bail them out. However, given our focus below on parameter values leading to equilibria with bailouts (see Lemmas 2 and 3, and Propositions 7 and 8 below), this limitation seems admissible.

E.2 Equilibrium analysis

To characterize the equilibrium of the model we first analyze the government decisions' regarding the bailout during a crisis of banks that do not satisfy their (CF) constraint. We will focus on situations in which the social cost of bank default C is sufficiently large with respect to the government intervention cost λ. When this is the case, the following lemmas show that in equilibrium the government will bail out any bank that is unable to obtain experts' funds without public support.

The first formal result characterizes the optimal government bail out decisions for a large C in an equilibrium in which banks do not satisfy their (CF) constraint (in which case the government decides whether or not to bailout all the banks) while the second formal result characterizes the government bailout decision to a bank that would deviate from equilibrium and choose a debt structure not satisfying the (CF) constraint (in which case the government decides whether or not to bailout the atomistic deviating bank).

We have formally that:

Lemma 2 *(Optimal bailouts on the equilibrium path) For every government's intervention cost λ, if the social cost of bank default C is sufficiently large then if an equilibrium debt structure (δ^e, D^e) does not satisfy (CF) given ϕ^e, the government finds optimal in equilibrium to bail banks out during crises, i.e. $\sigma^e(\delta^e, D^e) = 1$.*

[69] In these cases savers' required interest rate $r^e(\delta, D)$ would be determined as in section B of this appendix.

Proof Let $\phi^e, (\delta^e, D^e), r^e := r^e(\delta^e, D^e), \sigma^e := \sigma^e(\delta^e, D^e)$ be elements of an equilibrium tuple that does not satisfy (CF) given ϕ^e. We want to prove that for C sufficiently large relative to λ we must have $\sigma^e = 1$. To prove this we can follow a two step approach. First we show that $\sigma^e = 1$ is an optimal time-consistent decision. Second, we show that $\sigma^e = 0$ is not an optimal time-consistent decision. We use a one-shot deviation principle in both cases.

1. $\sigma^e = 1$ is an optimal decision. A government deciding in a crisis whether to bail banks out takes as given that in the future governments will support banks during crises.

If the government bails out the banks ($\sigma = 1$), in the normal period following the crisis the banks reestablish their original debt structures and the government makes them a transfer τ out of savers' funds satisfying:

$$\frac{1}{1+\rho_H}[\mu - (1-\delta^e)r^e D^e + \tau + \delta^e D^e + E^e] = (1 + \Phi(\delta^e D^e))\delta^e D^e, \tag{39}$$

where E^e is the continuation equity value of the bank in the normal state just after the crisis which, taking into account that bank shareholders are fully diluted in every crisis, satisfies $E^e = \frac{\mu - r^e D}{\rho_H + \varepsilon}$ with $r^e = \widehat{r}(\delta^e)$. The expression above is analogous to (CF) in the paper with an additional term capturing the transfer τ necessary to make experts willing to finance the bank in the crisis.

If banks are bailed out the debtholders whose debt matures in the crisis are entirely repaid, obtaining $\delta^e D^e$. Taking into account that in future crisis banks will be bailed out the remaining $(1-\delta^e)D^e$ non-matured banks' debt is safe and will be repaid entirely at maturity. Still, the current debtholders valuation of this non-matured debt will be below its face value since some of the current bank debtholders will be impatient. Let A denote banks' debtholders valuation of their (safe) non-matured debt.[70]

Taking the previous reasoning into account, the social surplus when $\sigma = 1$ is:

$$\begin{aligned} S_{\sigma=1} &= \delta^e D^e + A - \lambda - \frac{1}{1+\rho_H}\tau + \delta^e D^e \Phi(\delta^e D^e) - \int_0^{\delta^e D^e} \Phi(z)dz + \\ &+ \frac{1}{\rho_H}\frac{\varepsilon}{1+\rho_H+\varepsilon}\left(-\lambda - \frac{1}{1+\rho_H}\tau + \delta^e D^e \Phi(\delta^e D^e) - \int_0^{\delta^e D^e} \Phi(z)dz\right). \end{aligned} \tag{40}$$

The terms in the first line capture the social benefits and costs of current bailouts: holders of banks' maturing debt are repaid $\delta^e D^e$, holders of non-matured bank debt value it in A, government intervention has a cost λ, in the normal period just after the crisis savers pay taxes τ (with τ given by (39)), and finally experts obtain some rents from financing all the banks at an excess cost $\Phi(\delta^e D^e)$. The term in the second line captures the social benefits and costs that these bailed-out banks will generate in the future taking into account that they will also be bailed-out in future crises. Since savers buying banks' debt do not obtain rents, the social benefits amount to the rents obtained by the financing experts in the future crises, while the social costs comprise the intervention costs and transfers associated to bank bail outs in future crises.

[70] The expression for A as a function of δ^e, D^e could be easily derived from the ergodic proportions of patient and impatient savers in a given debt structure. Since its analytical expression is not necessary for our purposes we ommit this derivation.

Using (39) we can isolate τ and replace it in the previous expression to obtain

$$\begin{aligned} S_{\sigma=1} &= A - \lambda + \frac{1}{1+\rho_H}[\mu - (1-\delta^e)r^e D^e + \delta^e D^e + E^e] - \int_0^{\delta^e D^e} \Phi(z)dz + \\ &+ \frac{1}{\rho_H}\frac{\varepsilon}{1+\rho_H+\varepsilon}\left(-\lambda - \frac{\rho_H}{1+\rho_H}\delta^e D^e + \frac{1}{1+\rho_H}[\mu - (1-\delta^e)r^e D^e + E^e] - \int_0^{\delta^e D^e} \Phi(z)dz\right). \end{aligned} \tag{41}$$

For the sake of compactness, let $\widehat{S}$ denote the term in the second line. If the government were one-shot deviating to not bailing out the banks ($\sigma = 0$), banks would default and in the normal period following the crisis new banks would enter (and be bailed out in future crises). The social surplus from the deviation would then be:

$$S_{\sigma=0} = L - C + \frac{1}{1+\rho_H}(D^e + E^e - c) + \widehat{S}. \tag{42}$$

The two first terms capture the asset liquidation proceeds and the social costs of bank default. The third term in turn captures that in the normal period following the crisis new banks will be created at a cost c and generating a value for their owners $D^e + E^e$, and the last term captures the net surplus that the newly created banks would generate in the future (which is $\widehat{S}$ since future bailout decisions are assumed to involve $\sigma = 1$).

Subtracting the expressions for the social surplus for $\sigma = 1$ and $\sigma = 0$ we obtain:

$$S_{\sigma=1} - S_{\sigma=0} = A + \frac{\mu - (1-\delta^e)r^e D^e}{1+\rho_H} + C + \frac{1}{1+\rho_H}c - \lambda - \int_0^{\delta^e D^e} \Phi(z)dz - L - \frac{1}{1+\rho_H}(1-\delta^e)D^e \tag{43}$$

We have the following immediate bounds on the terms in the expression above involving endogenous variables: $A + \frac{\mu-(1-\delta^e)r^e D^e}{1+\rho_H} \geq 0$, $\delta^e D^e \leq \frac{\mu}{\rho_L}$, $\frac{1}{1+\rho_H}(1-\delta^e)D^e \leq \frac{1}{1+\rho_H}\frac{\mu}{\rho_L}$. Using them we deduce that for any λ, if C is sufficiently large we have $S_{\sigma=1} - S_{\sigma=0} > 0$ and hence the government finds optimal to choose $\sigma = 1$. Thus $\sigma^e = 1$ is an optimal time-consistent bailout policy.

2. We cannot have $\sigma^e = 0$. The analysis is very similar to that conducted in the previous case so we will skip some steps. A government deciding in a crisis whether to bail banks out takes as given the bailout decisions of future governments, which in this case involve $\sigma = 0$.

If the government bails out the banks ($\sigma = 1$), in the normal period following the crisis the banks reestablish their original debt structures and the government makes them a transfer τ out of savers' funds satisfying (39) with the only difference that in this case $r^e = r^e(\delta^e, D^e)$ will not coincide with the riskless interest rate $\widehat{r}(\delta^e)$ because it will include a default risk premium.

If banks are bailed out the debtholders whose debt matures in the crisis are entirely repaid. In contrast to the previous case, the claims of the savers whose banks' debt has not yet matured are exposed to the possibility of default losses in future crises. Let B denote banks' debtholders valuation of their (risky) non-matured debt.[71]

[71] As in the previous case an analytical expression for B as a function of δ^e, D^e could be easily derived.

Analogously to (40), we can derive the expression for social surplus if the government chooses $\sigma = 1$ for the current crisis:

$$\begin{aligned} S_{\sigma=1} &= B - \lambda + \frac{1}{1+\rho_H}[\mu - (1-\delta^e)r^e D^e + \delta^e D^e + E^e] - \int_0^{\delta^e D^e} \Phi(z)dz + \qquad (44) \\ &+ \frac{1}{\rho_H}\frac{\varepsilon}{1+\rho_H+\varepsilon}\left(-C + \frac{1}{1+\rho_H}\left(D^e + E^e - c\right)\right), \end{aligned}$$

where the term in the second line measures the social benefits and costs of future crises taking into account that banks will not be bailed out. Let $\widehat{S}'$ denote such term.

Analogously to (42), we obtain an expression for social surplus if the government chooses $\sigma = 0$ for the current crisis:

$$S_{\sigma=0} = L - C + \frac{1}{1+\rho_H}(D^e + E^e - c) + \widehat{S}'. \qquad (45)$$

And from here we can prove as before that for any given λ, if C is sufficiently large we have $S_{\sigma=1} - S_{\sigma=0} > 0$ and hence the government finds optimal to choose $\sigma = 1$. Thus $\sigma^e = 0$ is not an optimal bailout policy.■

Lemma 3 *(Optimal bailouts off the equilibrium path) For every government's intervention cost λ, if the social cost of bank default C is sufficiently large and a deviating bank chooses a debt structure (δ, D) that does not satisfy (CF) given the equilibrium ϕ^e, then the government finds it optimal to bail out the deviating bank during crises, i.e. $\sigma^e(\delta, D) = 1$.*

Proof The proof is similar although slightly more involved than that of Lemma 2. We will follow some of the notation introduced in the proof of that lemma and will skip the interpretation of expressions similar to those derived there. Let $\phi^e, (\delta^e, D^e)$ be elements of an equilibrium tuple and let $(\delta, D) \neq (\delta^e, D^e)$ be a feasible debt structure not satisfying (CF) given ϕ^e chosen by a bank deviating from equilibrium. Suppose that C is large enough so that the previous Lemma is satisfied.

We use a one-shot deviation principle to prove that, when C is large enough, the government would find optimal to bail out the deviating bank, $\sigma^e(\delta, D) = 1$, and then prove that not bailing it out is not optimal.

1. $\sigma^e(\delta, D) = 1$ is an optimal decision. In this case condition 1 of the definition of equilibrium implies $r^e(\delta, D) = \widehat{r}(\delta)$. When a crisis arrives the deviating bank does not satisfy (CF) and the government has to decide whether to bail it out or not. The government takes as given that if it chooses $\sigma = 1$ then the bank will be always bailed out in the future, while if it chooses $\sigma = 0$ the bank will be replaced by a new entering bank choosing debt structure (δ^e, D^e). Importantly, since the bank is atomistic the current bailout decision does not affect the equilibrium excess cost of funds during the current nor future crises and hence this decision does not affect the social surplus generated by the non deviating banks (and the banks that replace them if they default during crises). The social surplus that the government wants to maximize is hence that generated by the deviating bank and, should it default, also by the bank or banks that will replace it in the future.

If the government chooses $\sigma = 1$, the normal period following the crisis the bank reestablishes its original debt structure and the government makes a transfer τ' to crisis financiers in the period after the crisis that satisfies:

$$\frac{1}{1+\rho_H}[\mu - (1-\delta)\widehat{r}(\delta)D + \tau' + \delta D + E] = (1+\phi^e)\delta D. \tag{46}$$

This expression is analogous to (39).

The social surplus the deviating bank generates is:

$$S'_{\sigma=1} = \delta D + A' - \lambda - \frac{1}{1+\rho_H}\tau + \frac{1}{\rho_H}\frac{\varepsilon}{1+\rho_H+\varepsilon}\left(-\lambda - \frac{1}{1+\rho_H}\tau\right),$$

where A' is the analogous for debt structure (δ, D) to the term A defined in the proof of Lemma 2 (with the differences reflecting that the bailout decision of an atomistic bank does not affect the rents obtained by experts during crises). Isolating τ' in (46) and replacing in the expression above we obtain:

$$\begin{aligned} S'_{\sigma=1} &= A' - \lambda + \frac{1}{1+\rho_H}[\mu - (1-\delta)\widehat{r}(\delta)D + \delta D + E] - \phi^e\delta D + \\ &+ \frac{1}{\rho_H}\frac{\varepsilon}{1+\rho_H+\varepsilon}\left(-\lambda - \frac{\rho_H}{1+\rho_H}\delta D + \frac{1}{1+\rho_H}[\mu - (1-\delta)\widehat{r}(\delta)D + E] - \phi^e\delta D\right), \end{aligned} \tag{47}$$

which is similar to (41). The terms involving $\phi^e\delta D$ account for the value of the positive NPV investment opportunities forgone by experts that finance banks during crises. Using $E = \frac{\mu - \widehat{r}(\delta)D}{\rho_H+\varepsilon}$ we obtain:

$$\begin{aligned} S'_{\sigma=1} &= A' - \lambda - \phi^e\delta D + \frac{\mu - (1-\delta)\widehat{r}(\delta)D + \delta D}{1+\rho_H} + \\ &\frac{1}{1+\rho_H}\left[\frac{\mu - \widehat{r}(\delta)D}{\rho_H} - \frac{1}{\rho_H}\frac{(1+\rho_H)\varepsilon}{1+\rho_H+\varepsilon}\frac{\rho_H - \widehat{r}(\delta)}{1+\rho_H}\delta D - \frac{1}{\rho_H}\frac{(1+\rho_H)\varepsilon}{1+\rho_H+\varepsilon}\phi^e\delta D\right] \\ &- \frac{1}{\rho_H}\frac{\varepsilon}{1+\rho_H+\varepsilon}\lambda. \end{aligned} \tag{48}$$

This expression is very intuitive. The terms in the first line have already been explained. The terms grouped in the second line coincide, after discounting by $\frac{1}{1+\rho_H}$, with the continuation equity value of a bank in normal times (shown in (6) in the paper). In the baseline model that expression is only valid when the debt structure satisfies the (CF) constraint. Not surprisingly, when banks are bailed out that expression reemerges in the social surplus evaluation of the government. However, the expression is not necessarily positive. An immediate lower bound for it is $-\frac{1}{\rho_H}\frac{\varepsilon}{1+\rho_H+\varepsilon}\Phi\left(\frac{\mu}{\rho_L}\right)\frac{\mu}{\rho_L}$. The last term in the expression above is the present value of the future intervention costs.

If the government deviated to not bailing out the bank ($\sigma = 0$), the bank would default and be replaced, after the crisis, by a new bank with debt structure (δ^e, D^e).

Let us distinguish two cases:

(i) (δ^e, D^e) does not satisfy (CF). Then we know from the previous Lemma that $\sigma^e(\delta^e, D^e) = 1$ and, similarly to (48), social surplus can be written as:

$$\begin{aligned} S'_{\sigma=0} &= L - C - \frac{1}{1+\rho_H}c + \frac{D^e}{1+\rho_H} \qquad (49) \\ &+ \frac{1}{1+\rho_H}\left[\frac{\mu - \widehat{r}(\delta^e)D^e}{\rho_H} - \frac{1}{\rho_H}\frac{(1+\rho_H)\varepsilon}{1+\rho_H+\varepsilon}\frac{\rho_H - \widehat{r}(\delta^e)}{1+\rho_H}\delta^e D^e - \frac{1}{\rho_H}\frac{(1+\rho_H)\varepsilon}{1+\rho_H+\varepsilon}\phi^e\delta^e D^e\right] \\ &- \frac{1}{\rho_H}\frac{\varepsilon}{1+\rho_H+\varepsilon}\lambda. \end{aligned}$$

The terms in the first line account for the liquidation of bank assets, the social costs of default, the cost of bank creation in the next period and the debt issued by newly created banks. The remaining terms are analogous to those in (48). An immediate upper bound to the term in the second line is $\frac{1}{1+\rho_H}\frac{\mu}{\rho_H}$.

Subtracting (49) from (48) we have:

$$S'_{\sigma=1} - S'_{\sigma=0} = A' + \frac{\mu - (1-\delta)\widehat{r}(\delta)D + \delta D}{1+\rho_H} + C + \frac{1}{1+\rho_H}c - \lambda - \phi^e\delta D - L - \frac{D^e}{1+\rho_H} + \Delta,$$

where Δ denotes the difference between the second lines in (48) and (49). We have the following immediate bounds to the terms that depend on endogenous variables: $A' + \frac{\mu-(1-\delta)\widehat{r}(\delta)\delta D+\delta D}{1+\rho_H} \geq 0$, $\phi^e\delta D \leq \Phi(\frac{\mu}{\rho_L})\frac{\mu}{\rho_L}$, $\frac{1}{1+\rho_H}D^e \leq \frac{1}{1+\rho_H}\frac{\mu}{\rho_L}$, $\Delta \geq -\frac{1}{\rho_H}\frac{\varepsilon}{1+\rho_H+\varepsilon}\Phi(\frac{\mu}{\rho_L})\frac{\mu}{\rho_L} - \frac{1}{1+\rho_H}\frac{\mu}{\rho_H}$. Using them we deduce that for any λ, if C is sufficiently large we have $S'_{\sigma=1} - S'_{\sigma=0} > 0$ and hence the government finds optimal to choose $\sigma = 1$. We conclude that bailing out the deviating bank is optimal, $\sigma^e(\delta, D) = 1$.

(ii) (δ^e, D^e) satisfies (CF). The expression for social surplus in this case coincides with that in (49) except for the absence of the last term, which reflects that the replacing bank will not need government bailouts in future crises. Using the bounds above, we can replicate the result that for large C bailing out the deviating bank is optimal, $\sigma^e(\delta, D) = 1$.

2. We cannot have $\sigma^e(\delta, D) = 0$. Assume, on the contrary, $\sigma^e(\delta, D) = 0$. In this case the interest rate $r^e(\delta, D)$ would include a default risk premium. If the government bails out the bank ($\sigma = 1$), in the normal period following the crisis the bank reestablishes its original debt structure and the government makes a transfer τ' to crisis financiers such that

$$\frac{1}{1+\rho_H}[\mu - (1-\delta)r^e(\delta, D)D + \tau' + \delta D + E] = (1+\phi^e)\delta D. \qquad (50)$$

An additional subtle issue may emerge in this case: since $r^e(\delta, D)$ includes a default risk premium it could be the case that $\mu - r^e(\delta, D)D < 0$, i.e. that (δ, D) does not satisfy the (LL) constraint, generating a negative net cash flow per period. When this is the case, in order to make the bank able to pay all the interest payments associated with its refinancing the government will have to contribute an additional per period transfer $\tau'' = r^e(\delta, D)D - \mu$. We assume that this additional transfer is part of the bailout package to which the government commits in the crisis and it is paid until the arrival of the next crisis, when additional support would be needed and, since $\sigma^e(\delta, D) = 0$, it is anticipated that the future government would not provide it.

As in the previous analysis, the social surplus will depend on whether the equilibrium debt structure (δ^e, D^e) of the eventual replacing bank involves government bailouts.

(i) (δ^e, D^e) does not satisfy (CF). Then we know that $\sigma^e(\delta^e, D^e) = 1$ and the social surplus generated by the bank if bailed out is:

$$\begin{aligned} S'_{\sigma=1} &= \delta D + B' - \lambda - \frac{1}{1+\rho_H}\tau - \frac{1}{1+\rho_H}\frac{\max\{r^e(\delta,D)D-\mu,0\}}{\rho_H+\varepsilon} \\ &\quad + \frac{1}{1+\rho_H}\frac{\varepsilon}{\rho_H+\varepsilon}\left(-C+\frac{1}{1+\rho_H}\widetilde{S}\right), \end{aligned} \tag{51}$$

where B' is analogous to B as used in (44) and the last term captures that in the following crisis the bank will not be bailed out and will hence default, generating a social cost C and being replaced by another bank. $\widetilde{S}$ is the social surplus generated by the new bank:

$$\begin{aligned} \widetilde{S} &= -c + D^e + \frac{\mu-\widehat{r}(\delta^e)D^e}{\rho_H} - \frac{1}{\rho_H}\frac{(1+\rho_H)\varepsilon}{1+\rho_H+\varepsilon}\frac{\rho_H-\widehat{r}(\delta^e)}{1+\rho_H}\delta^e D^e - \frac{1}{\rho_H}\frac{(1+\rho_H)\varepsilon}{1+\rho_H+\varepsilon}\phi^e\delta^e D^e \\ &\quad -\frac{1}{\rho_H}\frac{(1+\rho_H)\varepsilon}{1+\rho_H+\varepsilon}\lambda. \end{aligned} \tag{52}$$

This expression is similar to that in (49). Isolating τ' in (50) and taking into account that the continuation equity value of the bank in that equation is $E = \frac{\max\{\mu-r^e(\delta,D)D,0\}}{\rho_H+\varepsilon}$ we can rewrite (51) as:

$$\begin{aligned} S'_{\sigma=1} &= B' - \lambda + \frac{\mu-(1-\delta)r^e(\delta,D)D+\delta D}{1+\rho_H} + \frac{\mu-r^e(\delta,D)D}{\rho_H+\varepsilon} - \phi^e\delta D \\ &\quad + \frac{1}{1+\rho_H}\frac{\varepsilon}{\rho_H+\varepsilon}\left(-C+\frac{1}{1+\rho_H}\widetilde{S}\right). \end{aligned} \tag{53}$$

On the other hand, the social surplus generated by the bank if it were not bailed out is:

$$S'_{\sigma=0} = L - C + \frac{1}{1+\rho_H}\widetilde{S}. \tag{54}$$

Subtracting (54) from (53) we have:

$$\begin{aligned} S'_{\sigma=1}-S'_{\sigma=0} &= B' + \frac{\mu-(1-\delta)r^e(\delta,D)D+\delta D}{1+\rho_H} + \frac{\mu-r^e(\delta,D)D}{\rho_H+\varepsilon} - \lambda - \phi^e\delta D - L \\ &\quad + \left(1-\frac{1}{1+\rho_H}\frac{\varepsilon}{\rho_H+\varepsilon}\right)\left(C-\frac{1}{1+\rho_H}\widetilde{S}\right), \end{aligned} \tag{55}$$

where the key new term is the last one. It captures the gains from postponing the default of the bank until the next crisis (note that $1-\frac{1}{1+\rho_H}\frac{\varepsilon}{\rho_H+\varepsilon} > 0$). Using as in the other cases immediate bounds on the terms that involve endogenous variables, we conclude that $S'_{\sigma=1} - S'_{\sigma=0} > 0$ for sufficiently large C.[72] This contradicts having $\sigma^e(\delta, D) = 0$. Notice that

[72] Note that under the assumption $\sigma^e(\delta, D) = 0$ the continuous function $\frac{\mu-r^e(\delta,D)D}{\rho_H+\varepsilon}$ has a lower bound independent from C in the compact space of feasible debt structures.

the reason why the government finds optimal to bail out a bank even if it expects future governments not to do so is that by doing so the social cost of bank default gets postponed.

(ii) (δ^e, D^e) satisfies (CF). The analysis is very similar to the previous one. The only change is that in the analogous expression for $\widetilde{S}$ the last term capturing expected government intervention costs disappears. We omit the rest of the details for the sake of brevity.■

The previous lemmas show that for large C the government will always bail-out banks that are unable to obtain experts' funds during crises. Since this is anticipated by all agents the banks can issue debt at a riskless rate regardless of whether their debt structure satisfies or not the (CF) constraint. As a result, in order to minimize the interest rate paid on debt banks will find optimal to choose debt of the shortest maturity ($\delta^e = 1$), which implies an interest rate $r^e = \rho_L$. And banks will issue as much debt as compatible with their (LL), i.e. $D^e = \frac{\mu}{\rho_L}$. This claim is stated and proved formally in the next proposition:

Proposition 7 *For every government's intervention cost λ there exists $\overline{C}$ such that if the social cost of bank default C satisfies $C > \overline{C}$ then the equilibrium is unique and satisfies $\delta^e = 1, D^e = \frac{\mu}{\rho_L}$ and $\sigma^e(\delta^e, D^e) = 1$.*

Proof Suppose C is sufficiently large relative to λ. Let us consider an equilibrium with excess cost of crisis funds ϕ^e. Let us define Σ_{ϕ^e} to be the set of debt debt structures (δ, D) satisfying (CF) given ϕ^e, and $\Sigma^c_{\phi^e}$ its complementary in the set of feasible debt structures. The two lemmas on optimal bailouts state that in equilibrium for $(\delta, D) \in \Sigma^c_{\phi^e}$ we have $\sigma^e(\delta, D) = 1$. As a result for all (δ, D) the interest rate paid on debt is $\widehat{r}(\delta)$ and all feasible debt structures satisfy the (LL) constraint.

The market value $V(\delta, D; \phi^e)$ of a bank with debt structure $(\delta, D) \in \Sigma_{\phi^e}$ is given in (8) in the paper. It is straightforward to realize that $\max_{\Sigma_{\phi^e}} V(\delta, D; \phi^e) < \frac{\mu}{\rho_L}$. Moreover, the market value $V(\delta, D)$ of a bank with debt structure $(\delta, D) \in \Sigma^c_{\phi^e}$ is

$$V(\delta, D) = D + \frac{\mu - \widehat{r}(\delta)D}{\rho_H}.$$

The expression is trivially increasing in D and δ and as a result it is maximized at the upper right corner of the set of feasible debt structures, i.e. for $(\delta, D) = (1, \frac{\mu}{\rho_L})$, for which we have $V(\delta, D) = \frac{\mu}{\rho_L}$. We deduce that in equilibrium we must have $(\delta^e, D^e) = (1, \frac{\mu}{\rho_L})$.■

E.3 Efficiency and regulatory results

Let us suppose that the SP can choose banks' debt structure at the initial date. Let us define Σ_s to be the set of debt debt structures (δ, D) satisfying (CF') in (13) in the paper, and Σ^c_s its complementary in the set of feasible debt structures. By construction, if the SP fixes a debt structure $(\delta, D) \in \Sigma_s$ for all banks then the excess cost of crisis funds is $\phi = \Phi(\delta D)$ and the banks are able to obtain experts' funds during crisis. In the set Σ_s the debt structure maximizing social surplus coincides with that in the baseline model, that we denoted (δ^s, D^s). Let us denote W^s the social surplus associated to this debt structure. Its expression can be found in (12) in the paper.

Also by construction, if the SP fixes a debt structure $(\delta, D) \in \Sigma_s$ for all banks then in the absence of government support banks will not be able to obtain experts' funds during crises. Suppose that C is sufficiently large relative to λ. Then Lemma 2 shows that the government will find optimal to bail-out the banks during crises, $\sigma(\delta, D) = 1$. Taking this into account and using expressions derived in the previous section we have that social surplus is given by $\widetilde{W}(\delta, D|\lambda) = \widetilde{V}(\delta, D) + \widetilde{U}(\delta, D|\lambda)$ with

$$\widetilde{V}(\delta, D) = D + \frac{\mu - \widehat{r}(\delta)D}{\rho_H + \varepsilon},$$

and

$$\begin{aligned}\widetilde{U}(\delta, D|\lambda) &= \frac{1}{\rho_H}\frac{(1+\rho_H)\varepsilon}{1+\rho_H+\varepsilon}\Bigg[-\lambda - \frac{\rho_H}{1+\rho_H}\delta D \\ &\quad + \frac{1}{1+\rho_H}\left(\mu - (1-\delta)\widehat{r}(\delta)D + \frac{\mu - \widehat{r}(\delta)D}{\rho_H+\varepsilon}\right) - \int_0^{\delta D}\Phi(z)dz\Bigg].\end{aligned}$$

The first component of $\widetilde{W}$ captures the surplus obtained by initial bankers and takes into account that banks will be bailed out in crises and hence their debt is riskless and pays the interest $\widehat{r}(\delta)$. Also, the expression for the equity value of the bank takes into account that initial bank owners will be fully diluted in the first crisis. The second component of $\widetilde{W}$ captures the present value of the social costs and benefits of the financing of banks during crises with government support. The expression for $\widetilde{W}$ can be simplified to obtain:

$$\begin{aligned}\widetilde{W}(\delta, D|\lambda) &= \frac{\mu}{\rho_H} + \frac{\rho_H - \widehat{r}(\delta)}{\rho_H}D - \frac{1}{\rho_H}\frac{\varepsilon(\rho_H - \widehat{r}(\delta))}{1+\rho_H+\varepsilon}\delta D - \frac{1}{\rho_H}\frac{\varepsilon(1+\rho_H)}{1+\rho_H+\varepsilon}\int_0^{\delta D}\Phi(x)dx \quad (56) \\ &\quad - \frac{1}{\rho_H}\frac{(1+\rho_H)\varepsilon}{1+\rho_H+\varepsilon}\lambda,\end{aligned}$$

which is easy to compare with the expression for social surplus $W(\delta, D)$ for $(\delta, D) \in \Sigma_s$ in (12) in the paper. This allows us to rewrite $\widetilde{W}$ as:

$$\widetilde{W}(\delta, D|\lambda) = W(\delta, D) - \frac{1}{\rho_H}\frac{(1+\rho_H)\varepsilon}{1+\rho_H+\varepsilon}\lambda,$$

where the last (negative) term captures the present value of the government bail-out costs along all future crises. Not surprisingly, the redistributional transfers associated with the financing of bank bailouts per se do not affect our additive measure of social surplus.

From here it is immediate to prove the following result:

Proposition 8 *Suppose that both the government intervention cost λ and the social cost of bank default C are sufficiently large. Then the equilibrium of the unregulated economy exhibits $\delta^e = 1, D^e = \frac{\mu}{\rho_L}$ and banks are bailed out during crises, while the SP would choose the socially efficient debt structure (δ^s, D^s) of the baseline model and banks would not need government support during crises. Moreover, $D^s < D^e$ and, as long as $\delta^s < 1$, the SP needs to regulate both δ and D to achieve (δ^s, D^s) because if it only regulated δ (D) then banks would end up choosing an excessively large D (δ) that would lead the government to bail them out during crises.*

Proof Let $\overline{\Sigma}_s^c$ be the closure of the set Σ_s^c (i.e. the minimum closed set containing Σ_s^c). Since $\overline{\Sigma}_s^c$ is a compact set and $\widetilde{W}(\delta, D|\lambda)$ is a continuous function in that set its maximum exists. Indeed, due to the additive decomposition found in (56) we have that there exists a constant K such that for all λ

$$\max_{\overline{\Sigma}_s^c} \widetilde{W}(\delta, D|\lambda) = K - \frac{1}{\rho_H} \frac{(1+\rho_H)\varepsilon}{1+\rho_H+\varepsilon} \lambda.$$

From here we have that for λ sufficiently large

$$W^s > \max_{\overline{\Sigma}_s^c} \widetilde{W}(\delta, D|\lambda) \geq \sup_{\Sigma_s^c} \widetilde{W}(\delta, D|\lambda).$$

Moreover, for such large λ, then from Lemma 2 we have that if C is also sufficiently large then for $(\delta, D) \in \Sigma_s^c$ the government will bail out banks during crises and hence $\widetilde{W}(\delta, D|\lambda)$ corresponds to the social surplus on the set Σ_s^c. As a result, the SP finds optimal to choose the (δ, D) that maximizes social surplus on the set Σ_s, as in the baseline model.

Suppose that the SP only fixes one of the debt structure variables. To fix our ideas, let us suppose that it fixes $\delta = \delta^s$. Then a similar game to the one described in the previous section would be played with the only difference that banks are only allowed to choose at the initial date their debt D, since the per period probability of debt maturity has been fixed to δ^s by the SP. Since C is large compared to λ the same straightforward arguments in the proof of Proposition 7 could be reproduced to show that in the unregulated equilibrium banks would choose $D^e_{\delta^s} = \frac{\mu}{r(\delta^s)}$, they would be bailed out during crises, and their (LL) constraint would be binding. Since $(\delta^s, D^s) \in \Sigma_s$, this debt structure strictly satisfies (LL), which implies that $D^s < D^e_{\delta^s}$ and the SP would need to regulate also D to achieve (δ^s, D^s).

Similarly, if the SP fixes $D = D^s$ then banks will end up choosing $\delta^e_{D^s} = 1$ and as long as $\delta^s < 1$ the socially efficient debt structure is not achieved. Finally, since $(\delta^s, D^s) \in \Sigma_s$ we trivially have that $D^s < \frac{\mu}{\rho_L} = D^e$.$\blacksquare$

The proposition shows that when the government cannot commit not to bail out banks the regulation of both banks' debt maturity and their leverage is social welfare maximizing if bank default costs and government intervention costs are large. If the government could commit not to bail out banks then, as in the baseline model, only the regulation of $1/\delta$ would suffice to achieve the socially optimal debt structure. It is the interaction of pecuniary externalities and lack of commitment not to bail out banks which leads to the need for regulating both $1/\delta$ and D.