## How Excessive Is Banks' Maturity Transformation?\*

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#### Abstract

We develop an infinite horizon model in which banks finance long term assets with non-tradable debt. Banks choose the amount and maturity of their debt taking into account investors' preference for short maturities and the risk of systemic crises. As in Stein (2012), pecuniary externalities in the market for funds during crises make unregulated debt maturities inefficiently short. We calibrate the model to Eurozone banking data for 2006 and quantify the inefficiencies and the potential welfare gains from optimally regulating bank debt maturities. We also assess the gains from introducing a liquidity insurance scheme and its complementarity with liquidity regulation.

JEL Classification: G01, G21, G28

*Keywords:* liquidity risk, liquidity regulation, liquidity insurance, pecuniary externalities, systemic crises

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## 1 Introduction

The 2007-2009 financial crisis extended the view among regulators that the banking system featured an excessive maturity mismatch in the years leading to the crisis (see, for example, Tarullo, 2009). The heavy reliance on wholesale short-term funding of investment banks, hedge funds, shadow banking vehicles, and many commercial banks was a key lever in amplifying, and spreading the consequences of the subprime crisis and the subsequent collapse of money markets (Brunnermeier, 2009; Gorton, 2009).

Since then, several papers have provided a rationale for liquidity risk regulation based on the idea that banks' refinancing needs during a crisis produce negative pecuniary and non-pecuniary externalities (see, for example, Perotti and Suarez, 2011). This may happen because refinancing needs force banks to undertake fire sales whose impact on asset prices causes a pecuniary externality (Stein, 2012). It can also happen through contagion, because of direct losses coming from interbank positions (Rochet and Tirole, 1996; Allen and Gale, 2000), or through the damage that the disruption of the financial system inflicts on the rest of the economy (Kroszner, Laeven, and Klingebiel, 2007).

In this paper we explore the potential quantitative importance of the pecuniary externalities story in the context of an infinite horizon model that, conceptually, is very similar to the three-period model developed by Stein (2012). The model is calibrated to match aggregate statistics describing the liability structure of the Eurozone banking system in 2006. We perform a model-based assessment of the extent to which pre-crisis debt maturities were excessively short and the size of the welfare gains that would have been associated with regulating liquidity risk in such an environment. The results allow us quantify the effects of introducing regulations such as the Net Stable Funding Ratio (NSFR) of Basel III, which is aimed to limit banks' maturity transformation.

In our recursive infinite horizon model, banks finance long-term assets by placing nontradable debt among unsophisticated savers who are initially patient but may suddenly turn impatient. Short maturities shorten the expected time the savers have to wait up to recovering their funds if they become impatient. In the absence of aggregate uncertainty, banks would satisfy these investors' preferences by issuing debt of the shortest maturity (or, equivalently, demandable debt) that would be repeatedly rolled over among (subsequent cohorts of) investors who are or remain patient. We assume, however, that banks are exposed to systemic liquidity crises: episodes in which they are unable to place debt among the unsophisticated savers and they have to (temporarily) rely on the more expensive funding provided by some crisis financiers. We think of these financiers as sophisticated investors with outside investment opportunities who out of crisis times play no role in funding the banks. The heterogeneity in the value of the investment opportunities of these investors produces an upward sloping aggregate supply of funds during crises.<sup>1</sup>

At the initial (non-crisis) period, banks decide their capital structure by trading off the lower interest cost of shorter debt maturities with their impact on the cost of refinancing during crises. Individual banks choose longer debt maturities (implying smaller refinancing needs) if they anticipate crisis financing to be more costly. The intersection between crisis financiers' supply of funds and banks' aggregate refinancing needs produces a unique equilibrium cost of crisis financing, and some unique bank capital structure decisions associated with it.

Our analysis focuses on parameterizations for which banks want to avoid going bankrupt (which we assume would imply their liquidation) during crises and, thus, choose to keep sufficient equity value in normal times so as to be able to absorb the excess cost of funding in a crisis by diluting their equity. Quite intuitively, guaranteeing their survival in a crisis limits their ex ante leverage and their debt maturity choices, imposing what we call the *crisis financing constraint*.

Importantly, in their uncoordinated, competitive capital structure decisions, banks neglect the impact of their refinancing needs on the equilibrium cost of crisis financing, which tightens the crisis financing constraint of all banks. We show that this pecuniary externality damages the efficiency of the maturity transformation process, making equilibrium maturities excessively short and crisis financing excessively costly, consequently, reducing the aggregate amount of leverage that the banking industry can sustain. By inducing banks to lower the intensive margin of maturity transformation activities (i.e. the choice of *longer* debt maturities), a regulator can increase the use of their extensive margin (i.e. facilitate the issuance of *more* debt) and, as we show, increase the social surplus.

To assess the potential quantitative importance of these externalities and their implication

<sup>&</sup>lt;sup>1</sup>As banks need to tap these financiers to be refinanced during crisis, their upward sloping supply of funds during crises works like a generalized version of a cash-in-the-market constraint  $a \ la$  Allen and Gale (1998), in which the supply of funds is just constant.

for the regulation of banks' maturity transformation function, we calibrate the model to Eurozone data from 2006. We combine information about banks' liability structure and the average maturities of the various debt categories in 2006 to estimate the refinancing needs of a representative Eurozone bank in a crisis. We calibrate the model so as to, among other things, match banks' average debt maturity, which is of 2.4 months.

A key parameter to determine the importance of the underlying externalities is the elasticity of the *inverse* supply of funds during a crisis, for which there are no available estimates.<sup>2</sup> So we report all our normative results as a function of this parameter. We find, for example, that if this elasticity equals one (five), then it would be optimal to regulate banks' debt maturity decisions so as to induce an average debt maturity of 2.9 (3.3) months. Although this increase may look modest, it would allow banks to remain solvent in a crisis with a capital ratio of 3.8% (1.8%) rather than the 5.3% of the unregulated scenario, and would increase the estimated surplus generated by banks about 1.2% (5.1%).<sup>3</sup>

In contrast, a more drastic limitation of banks' maturity transformation such as the one implied by the current formulation of the NSFR of Basel III might be undesirable. Roughly speaking, the NSFR is defined as the ratio between liabilities with maturity longer than one year ("stable funding") to assets with maturity longer than one year ("illiquid assets") and the new regulation will require banks to operate from 1st January 2018 with a NSFR higher than one (see BCBS, 2014). In the context of our model, this amounts to require banks to choose debt maturities above one year, which are far longer than the socially optimal under any value of the elasticity of the inverse supply of funds during crises. In fact, under our calibration introducing this requirement would reduce the social surplus associated with banks' maturity transformation by more than 27% for all elasticities. While there might be reasons for the NSFR and details of its design not captured by our model, this finding provides a call for caution on the unintended consequences of regulations which, on pure qualitative grounds, seem to go in the right direction.<sup>4</sup>

<sup>&</sup>lt;sup>2</sup>Estimating such parameter would require observing banks' excess cost of refinancing during a sufficiently large sample of distinct crises. In models with cash-in-the-market pricing the elasticity of the implied inverse supply of funds is infinity.

 $<sup>^{3}</sup>$ It is reassuring that, without having a parameter with which to specifically match banks' capital ratio, our calibrated model yields a ratio of 5.3% in the unregulated scenario, very close to the average 5.8% observed among Eurozone banks in 2006.

<sup>&</sup>lt;sup>4</sup>For example, some bank liabilities with maturity shorter than one year, such as demand deposits, are assigned a positive weight in the computation of a bank's stable funding, effectively relaxing the restrictions

In a first extension of the model, we analyze the quantitative effects of an unexpected and permanent increase in the severity of a systemic crisis—modeled as the fraction of banks that simultaneously lose access to normal refinancing channels.<sup>5</sup> Assuming that the discovery of the new value of such fraction occurs in a crisis, we compute banks' aggregate capital shortfall (i.e. the amount of equity that a government would have to inject to prevent the affected banks from going bankrupt) and the size of banks' own reaction after learning about the new value (which takes the form of lengthened debt maturity and deleveraging). Beyond its exact details, one purpose of this exercise is to show that the model is suitable to perform simple quantitative analyses of potential policy relevance (e.g. as a complement to less structural stress-testing exercises and quantitative-impact studies).

In a second extension, we study the potential gains from introducing liquidity insurance. Several discussions on the final shape of bank liquidity risk regulation suggest the need to consider it in parallel to explicit liquidity insurance schemes (Stein, 2013). With this motivation, we first extend the theoretical analysis to cover the scenario in which banks have access to a fairly-priced private liquidity insurance arrangement. After noting that the basic pecuniary externality still operates in this context (through the competitive cost of crisis insurance), we evaluate the quantitative effect on debt maturities, capital ratios, and welfare of liquidity insurance and maturity regulation, each in isolation and altogether. We find that, even in the presence of liquidity insurance and unless the cost of crisis financing is very inelastic to changes in aggregate refinancing needs, maturity regulation can significantly increase the social surplus.

The paper is organized as follows. Section 2 places the contribution of the paper in the context of the existing literature. Section 3 presents the ingredients of the model. Section 4 defines equilibrium and covers the various steps necessary for its characterization. Section 5 examines the efficiency properties of the equilibrium. Section 6 describes the calibration of the model and the key quantitative results. Section 7 considers the main extensions. Section 8 discusses robustness and several other extensions. Section 9 concludes. Details on the calibration of the model, proofs of propositions, and other technical derivations are in appendices included at the end.

on maturity imposed by the NSFR (see BCBS, 2014).

<sup>&</sup>lt;sup>5</sup>This fraction is one in the baseline model and in this extension we turn it into a parameter that increases in an unexpected and irreversible manner.

## 2 Related literature

Our paper belongs to the recent literature that emphasizes the normative implications of externalities associated with banks' funding decisions. The closest paper to ours is Stein (2012), in which, like in our paper, the inefficiency in banks' debt maturity choices comes from the combination of pecuniary externalities and financial constraints.<sup>6</sup> The mechanism that in Stein works through banks' need to incur in fire sales during crisis (with an endogenous price that declines with banks' aggregate sales) in our paper works through banks' cost of accessing the funding coming from crisis financiers. The richer time framework of our setup allows us to calibrate the model and to quantify the importance of the pecuniary externalities associated to banks' maturity transformation. The model allows us to evaluate the effect of regulatory proposals aimed to limit banks' liquidity risk, such as the NSFR of Basel III.

In addition to pecuniary externalities, the literature has found other theoretical mechanisms that may justify the introduction of debt maturity regulation. In Perotti and Suarez (2011), banks neglect their contribution to generating systemic risk (which is modeled as a technological externality) when they use short-term funding to expand their credit activity. In Farhi and Tirole (2012), time-consistent liquidity support to distressed institutions during crises (e.g. via central bank lending) makes bank leverage decisions strategic complements, also producing excessive short-term borrowing and potential social gains from limiting such borrowing. Finally, in Brunnermeier and Oehmke (2013), lack of enforceability of debt covenants creates a conflict of interest between the bank's long-term and short-term creditors that pushes the bank to choose debt maturities which are inefficiently short.

Of course, the analysis of the rationale for and the effects of short-term debt financing has a long tradition in the corporate finance and banking literatures, typically using models with very stylized time structures (i.e., three dates). In contributions following Bryant (1980) and Diamond and Dybvig (1983), demand deposits help satisfy investors' idiosyncratic liquidity needs coming from preference shocks but create a maturity mismatch that makes banks

<sup>&</sup>lt;sup>6</sup>Pecuniary externalities are a common source of inefficiency in models with financial constraints (e.g. Lorenzoni, 2008) and more generally in economies with incomplete markets (Geanakoplos and Polemarchakis, 1986, Greenwald and Stiglitz, 1986). Most of the recent papers (including Bianchi and Mendoza, 2011, Korinek, 2011, and Gersbach and Rochet, 2012) emphasize them as a potential cause of excessive fluctuations in credit and/or excessive credit. Bengui (2011) presents a model  $\dot{a}$  la Kiyotaki and Moore (1997) where firms' choice between short-term and long-term debt is inefficient because firms' neglect part of the net worth and asset price stabilization effects of long-term debt.

vulnerable to runs. In papers such as Flannery (1994), in a corporate finance context, and Calomiris and Kahn (1991), Diamond and Rajan (2001), and Huberman and Repullo (2010), in a banking context, short-term debt and the possibility of runs play a disciplinary role. Quite differently, in Flannery (1986) and Diamond (1991), short-term debt allows firms with private information to profit from future rating upgrades, while in Diamond and He (2012) short maturities have a non-trivial impact on a classical debt overhang problem. The rationale for short-term debt in our model is close to the first of these streams of the literature but, rather than assuming bank debt to be demandable, we endogenize its maturity as a decision that banks make by trading off investors' higher valuation of short maturities with banks' concerns about refinancing costs during crises.

Rochet and Vives (2004), Goldstein and Pauzner (2005), and Martin, Skeie, and von Thadden (2013), among others, model the emergence of roll-over risk as the combined result of doubts about the solvency of banks and a coordination problem between short-term creditors. Various papers, including Allen and Gale (1998), Acharya and Viswanathan (2011) and Acharya, Gale, and Yorulmazer (2011), study the implications of roll-over risk and runs for issues such as risk-sharing, risk-shifting, fire sales, and the collateral value of risky securities. In our paper we also study the implications of roll-over risk (on banks' capital structure decisions and overall efficiency) but we abstract from endogenizing the risk of runs or the emergence of liquidity crises. Instead, we model crises as an exogenous "sudden stop" of the type introduced by Calvo (1998) in the emerging markets literature.<sup>7</sup>

From a technical perspective, our work is related to the literature that incorporates debt refinancing risk in infinite-horizon capital structure models. Leland and Toft (1996) study the connection between credit risk and refinancing risk, and show that short debt maturities increase the threshold of a firm's fundamental value below which its costly bankruptcy occurs. He and Xiong (2012a) extend the analysis to a setup in which there are shocks to market liquidity that increase the cost of debt refinancing. He and Milbradt (2014) consider the existence of a secondary market for corporate debt subject to search frictions and explore the interactions between credit risk and the endogenous liquidity of such market. While these papers mainly focus on asset pricing implications and the determinants of credit spreads and market liquidity from a strictly positive standpoint, ours focuses on banks' debt maturity

<sup>&</sup>lt;sup>7</sup>See Bianchi, Hatchondo, and Martinez (2013) for a recent application.

decisions and the extent to which, due to the presence of a pecuniary externality, there is scope to improve the allocation of resources through regulation.<sup>8</sup>

## 3 The model

We consider an infinite horizon economy in which time is discrete t = 0, 1, 2, ... and a special class of *expert agents* own and manage a continuum of measure one of *banks*, which are describable as an exogenous pool of long-term assets. The economy alternates between normal states  $(s_t = N)$  in which banks can roll over their debt among *common savers* and crisis states  $(s_t = C)$  in which they cannot. The crisis states represent systemic liquidity crises in a reduced-form manner. For tractability, we assume  $\Pr[s_{t+1} = C \mid s_t = N] = \varepsilon$  and  $\Pr[s_{t+1} = C \mid s_t = C] = 0$ , so that crises are short-lived episodes with a constant probability of following any normal state. Since crises last for just one period, for calibration purposes one must think of a period as the standard duration of a crisis. Finally, we assume that the economy starts up in a normal state  $(s_0 = N)$ .

#### 3.1 Agents

Both expert agents and common savers are long-lived risk-neutral agents who enter the economy in a steady flow of sufficiently large measure per period and exit it whenever their investment and consumption activities are completed.<sup>9</sup> Each entering agent is endowed with a unit of funds.

#### 3.1.1 Experts

Experts are relatively impatient. They discount future consumption at rate  $\rho_H$ . When entering the economy, each expert has the opportunity to invest his endowment either in bank claims or in an indivisible private investment project with a net present value z which is heterogeneously distributed over the entrants.<sup>10</sup> The distribution of z has support  $[0, \overline{\phi}]$  and

<sup>&</sup>lt;sup>8</sup>Other papers with a positive focus that develop their analyses using infinite horizon models include He and Xiong (2012b), which shows that "dynamic runs" may occur when lenders stop rolling over maturing debt in fear that future lenders would do the same before the currently offered debt matures, and Cheng and Milbradt (2012), which shows that this type of runs may have, up to some point, a beneficial effect on an asset substitution problem.

<sup>&</sup>lt;sup>9</sup>Specifically, the entering agents are assumed to be sufficient to cover banks' refinancing needs, while exit ensures that the measure of active agents remains bounded.

<sup>&</sup>lt;sup>10</sup>The experts who opt for their own projects rather than bank claims exit the economy immediately.

the measure of agents with  $z \leq \phi$  is described by a differentiable and strictly increasing function  $F(\phi)$ , with F(0) = 0 and  $F(\overline{\phi}) = \overline{F}$ .

#### 3.1.2 Savers

Entering savers are initially *patient*. They start discounting next period utility from consumption at rate  $\rho_L < \rho_H$ . However, in every period they face an idiosyncratic probability  $\gamma$ of turning irreversibly *impatient* and starting to discount the utility of any future consumption at rate  $\rho_H$  from that point onwards.

Savers are unsophisticated investors with no other investment opportunity than bank debt. So, in normal states the entering savers decide between buying bank debt or consuming their endowment, while in crisis states they simply consume their endowments. Preexisting savers whose bank debt matures face the same possibilities as the contemporaneous entering savers with respect to the use of their recovered funds.

#### 3.2 Banks

At the initial period (t = 0), each of the banks possesses a pool of long-term assets that, if not liquidated, yields a constant cash flow  $\mu > 0$  per period. If liquidated, bank assets produce a terminal payoff L. For brevity, the experts who own and manage the banks at any given point in time will be called *bankers*.

In order to maximize the value extracted from their existing assets, bankers can profit from the lower discount rates of the patient savers by issuing some non-tradable debt among them.<sup>11</sup> Debt is issued at par in the form of (infinitesimal) contracts with a principal normalized to one. Importantly, bank debt is assumed to be non-tradeable.<sup>12</sup> At the initial period (t = 0), bankers choose a triple ( $r, \delta, D$ ), where r is the per-period interest rate,  $\delta$ is the constant probability with which each contract matures in each period, and D is the overall principal of the debt. So debt maturity is random, which helps for tractability, and

<sup>&</sup>lt;sup>11</sup>To keep the model tractable, we do not consider bank entry, that is, the possibility that existing or new bankers create new bank assets. It should be clear, however, that decisions or policies that in this model allow bankers to extract greater value from their banks would, in a setup with entry, enhance investment and output. And, in an even fuller model in which banks' role as financiers of long-term projects were explicitly formalized, part or all of the gains from their maturity transformation function might be passed through to the owners of those projects in the form of better financing conditions.

<sup>&</sup>lt;sup>12</sup>The lack of tradability might be structurally thought as the result of savers' geographical dispersion and the lack of access to centralized trading. We discuss the importance of this assumption and its connection with the literature in Section 8.5.

has the property that the expected time to maturity of any non-matured contract is equal to  $1/\delta$ .<sup>13</sup> We also assume that contract maturity arrives independently across contracts so that there is constant flow  $\delta D$  of maturing debt in every period. Failure to pay interest or repay the maturing debt in any period leads the bank to be liquidated at value L, which is assumed to be low enough for bankers to choose initial debt structures  $(r, \delta, D)$  under which liquidation is avoided at all times.<sup>14</sup>

In normal periods, the refinancing of maturing debt  $\delta D$  is done by replacing the maturing contracts with identical contracts placed among patient savers. So the bank generates a free cash flow of  $\mu - rD$  that is paid to bankers as a dividend.<sup>15</sup>

In crisis states, refinancing the maturing debt requires bankers to turn to other experts. With the sole purpose of simplifying the algebra, we assume that bankers learn about their banks' refinancing problems after having consumed the normal dividends.<sup>16</sup> Thus, they require  $\delta D$  units of funds. To obtain them, the existing bankers offer a fraction  $\alpha$  of the continuation value of their bank (i.e. of its future free cash flows) to some of the entering experts. After receiving funding from such experts, debt in hands of savers during a crisis reduces to  $(1 - \delta)D$ . Once the crisis is over, the bank restores its original debt structure  $(r, \delta, D)$  by placing an extra amount of debt  $\delta D$  among savers. As later explained in detail, the (paid out) proceeds from such placement are part of the continuation value with which the experts supporting the bank in the crisis are compensated.

<sup>&</sup>lt;sup>13</sup>With  $\delta = 1$ , the debt issued by banks could be interpreted as demand deposits. However, as it will become clear below, if the probability and cost of systemic crises are large enough, this corner solution will not be optimal.

<sup>&</sup>lt;sup>14</sup>In Section 8.2 we explicitly discuss the condition under which bankers find it optimal to avoid liquidation in crises (and then obviously in normal states as well). This condition is easily satisfied under our baseline calibration of the model.

<sup>&</sup>lt;sup>15</sup>We have considered an extension in which banks (or bankers) can invest the free cash flow in a buffer of liquidity with which to partially cover their refinancing needs in a crisis. We have checked that if the probability of suffering a systemic crisis and/or the cost of liquidity in a crisis are not too large, then holding liquidity is strictly suboptimal (this is the case under our calibration of the model). With parameterizations not satisfying this property, analytical tractability is lost.

<sup>&</sup>lt;sup>16</sup>Otherwise, they would find it optimal to cancel the dividends and reduce the bank's funding needs to  $\delta D - (\mu - rD)$ . The algebra in this case would be more tedious but the results would be barely affected because under realistic parameterizations (e.g. our calibration below) the dividends  $\mu - rD$  are very small relative to the refinancing needs  $\delta D$ .

#### 3.3 The cost of crisis financing

By virtue of competition, the fraction  $\alpha$  of the continuation value offered to the funding experts in a crisis will have to be just enough to compensate the marginal entering expert for the opportunity cost of her funds, say  $\phi$ . Given the heterogeneity in experts' private investment opportunities and the size of the aggregate refinancing needs, clearing the refinancing market in a crisis requires  $F(\phi) = \delta D$ . So the market-clearing excess cost of crisis financing can be found as  $\phi = F^{-1}(\delta D) \equiv \Phi(\delta D)$ . The function  $\Phi(x)$  is strictly increasing and differentiable, with  $\Phi(0) = 0$  and  $\Phi(\overline{F}) = \overline{\phi}$ , and we will refer to it as the *inverse sup*ply of crisis financing. In the quantitative part of our analysis, we will adopt the following isoelastic specification:

$$\Phi(x) = ax^{\eta},\tag{1}$$

with  $a \ge 0$  and  $\eta \ge 0$ .

## 4 Equilibrium analysis

We use the following definition of equilibrium:

**Definition 1** An equilibrium with crisis financing is a tuple  $(\phi^e, (r^e, \delta^e, D^e))$  describing an excess cost of crisis financing  $\phi^e$  and a debt structure for banks  $(r^e, \delta^e, D^e)$  such that:

- 1. Patient savers accept the debt contracts involved in  $(r^e, \delta^e, D^e)$ .
- 2. Among the class of debt structures that allow banks to be refinanced during crises,  $(r^e, \delta^e, D^e)$  maximizes the value of each bank to its initial owners.
- 3. The market for crisis financing clears in a way compatible with the refinancing of all banks, i.e.  $\phi^e = \Phi(\delta^e D^e)$ .

In the next subsections we undertake the steps necessary to prove the existence and uniqueness of this equilibrium, and establish its properties.

#### 4.1 Savers' required maturity premia

Let us first analyze the conditions upon which the debt contracts associated with some debt structure  $(r, \delta, D)$  are acceptable to savers in normal states. Since the bank will fully pay back its maturing debt even in crisis periods, a saver's valuation of a contract does not depend on the aggregate state of the economy but only on whether the saver is patient (i = L) or impatient (i = H). The ex-coupon values of the contract in each of these individual states,  $U_L$  and  $U_H$ , must satisfy the following system of equations:

$$U_{L} = \frac{1}{1+\rho_{L}} \{ r+\delta + (1-\delta)[(1-\gamma)U_{L} + \gamma U_{H}] \},$$
  

$$U_{H} = \frac{1}{1+\rho_{H}} [r+\delta + (1-\delta)U_{H}].$$
(2)

These recursive formulas express  $U_L$  and  $U_H$  in terms of the discount factors, payoffs, and continuation values relevant in each state. A non-matured debt contract pays r with probability one in each next period. Additionally it matures with probability  $\delta$ , in which case it pays its face value of one and loses its continuation value. With probability  $1 - \delta$ , it does not mature and then its continuation value is  $U_L$  and  $U_H$  depending on the investor's individual state in the next period. The terms multiplying these continuation values in the right hand side of the equations reflect the probability of each individual state next period.

When banks place debt among savers, patient savers are abundant enough to acquire all the issue, so the acceptability of the terms  $(r, \delta)$  requires

$$U_L(r,\delta) = \frac{r+\delta}{\rho_H+\delta} \frac{\rho_H+\delta+(1-\delta)\gamma}{\rho_L+\delta+(1-\delta)\gamma} \ge 1,$$
(3)

where  $U_L(r, \delta)$  is the solution for  $U_L$  arising from (2). Obviously, for any given  $\delta$ , bankers' value is maximized by issuing contracts with the minimal r that satisfies  $U_L(r, \delta) = 1$ .

**Proposition 1** The minimal interest rate acceptable to patient savers for each maturity  $\delta$  is given by the function

$$r(\delta) = \frac{\rho_H \rho_L + \delta \rho_L + (1 - \delta) \gamma \rho_H}{\rho_H + \delta + (1 - \delta) \gamma},\tag{4}$$

which is strictly decreasing and convex, with  $r(0) = \rho_H \frac{\rho_L + \gamma}{\rho_H + \gamma} \in (\rho_L, \rho_H)$  and  $r(1) = \rho_L$ .

The proofs of all propositions are in Appendix B. Having  $r'(\delta) < 0$  evidences the advantage of offering short debt maturities to a patient saver. For any expected maturity  $1/\delta$  longer than one, the saver bears the risk of turning impatient and having to postpone his consumption until his contract matures. Compensating the cost of waiting generates a ma-

turity premium  $r(\delta) - \rho_L > 0$ , which is increasing in  $1/\delta$ . Figure 1 illustrates the behavior of  $r(\delta)$  under the calibration that we explain in Section 6.<sup>17</sup>



**Figure 1** Annualized interest rate as a function of  $1/\delta$ 

#### 4.2 Banks' optimal debt structures

From now on, we will set  $r = r(\delta)$  and refer to banks' debt structures as  $(\delta, D)$ . And, to further save on notation, we will generally refer to  $r(\delta)$  as simply r.

#### 4.2.1 Value of bank equity in normal times

The continuation equity value of a bank in normal states depends both on its debt structure  $(\delta, D)$  and the fraction  $\alpha$  of the continuation value relinquished to other experts when trying to obtain financing in a crisis. The ex dividend value of *bank equity* in a normal state that follows another normal state,  $E(\delta, D; \alpha)$ , can be found as the solution to the following

<sup>&</sup>lt;sup>17</sup>All our figures correspond to such calibration, where one period is one month, the annualized discount rates of patient and impatient agents is 0.79% ( $\rho_L = 0.000654$ ) and 3.70% ( $\rho_H = 0.003029$ ), respectively, the annualized yield on bank assets is also 3.70% ( $\mu = 0.003029$ ), the expected time until the arrival of an idiosyncratic preference shock is 7.7 months ( $\gamma = 0.13$ ), and the expected time between systemic crises is 10.3 years ( $\varepsilon = 0.0081$ ).

recursive equation:

$$E(\delta, D; \alpha) = \frac{1}{1+\rho_H} \left\{ (\mu - rD) + (1-\varepsilon)E(\delta, D; \alpha) + \varepsilon(1-\alpha)\frac{1}{1+\rho_H} [\mu - (1-\delta)rD + \delta D + E(\delta, D; \alpha)] \right\}.$$
(5)

To explain this formula, recall that bankers' discount rate is  $\rho_H$  and in each normal state they receive a dividend of  $\mu - rD$ . With probability  $1 - \varepsilon$ , the next period is a normal state too and bankers' continuation value is  $E(\delta, D; \alpha)$  once again. With probability  $\varepsilon$ , a systemic crisis arrives and a fraction  $\alpha$  of equity value is relinquished to the crisis financiers.

The factor  $\frac{1}{1+\rho_H}[\mu - (1-\delta)rD + \delta D + E(\delta, D; \alpha)]$  represents the total value of equity after the bank gets refinanced in the crisis. It is expressed in terms of payoffs received one period later, once the crisis is over. So  $\mu - (1-\delta)rD$  is the asset cash flow net of interest payments to common savers (whose debt is reduced to  $(1-\delta)D$  during the crisis),  $\delta D$  is the revenue from reissuing the debt financed by the experts during the crisis (paid as a special dividend), and the last term reflects that, one period after the crisis, the initial debt structure is fully restored and equity value is  $E(\delta, D; \alpha)$  once again.

Competition between entering experts implies that bankers will obtain  $\delta D$  in exchange for the minimal  $\alpha$  that satisfies

$$\alpha \frac{1}{1+\rho_H} [\mu - (1-\delta)rD + \delta D + E(\delta, D; \alpha)] \ge (1+\phi)\delta D, \tag{6}$$

which simply says that the  $\delta D$  refinancing needs in a crisis have to be remunerated so as to compensate each unit of funds at the opportunity cost  $1 + \phi$  of the marginal financier.<sup>18</sup> Under the implied  $\alpha$ , (6) holds with equality and can be used to substitute for  $\alpha$  in (5) and to obtain the following Gordon-type formula for equity value:

$$E(\delta, D; \phi) = \frac{\mu}{\rho_H} - \frac{r(\delta)}{\rho_H} D - \frac{1}{\rho_H} \frac{\varepsilon\{[(1+\rho_H)\phi + \rho_H] - r(\delta)\}}{1+\rho_H + \varepsilon} \delta D.$$
(7)

The interpretation of this expression is very intuitive: Equity resembles a perpetuity in which the relevant payoffs are discounted at the impatient rate  $\rho_H$ ;  $\mu$  is the unlevered cash flow of the bank;  $r(\delta)$  is the interest rate paid on the debt placed among savers; and  $\frac{\varepsilon}{1+\rho_H+\varepsilon}\{[(1 + \epsilon)^2)^2 + \epsilon^2 +$ 

<sup>&</sup>lt;sup>18</sup>Let us highlight that the exact form of the claims that split the continuation value of the bank between the crisis financiers and the bank pre-crisis shareholders in proportions  $\alpha$  and  $1 - \alpha$  is irrelevant due to a Modigliani-Miller type of result.

 $(\rho_H)\phi + \rho_H - r(\delta)$  is the term reflecting the (discounted) differential cost of refinancing each unit of maturing debt each time a crisis arrives.

Finally, taking into account that (6) holds with equality and  $\alpha$  cannot be larger than one, the feasibility of refinancing the bank during crises requires:

$$\mu - (1 - \delta)rD + \delta D + E(\delta, D; \phi) \ge (1 + \rho_H)(1 + \phi)\delta D, \tag{8}$$

which we will call the *crisis financing constraint* (CF). It establishes that the equity value of the bank after the crisis must be no lower than the amount needed to compensate, at the rate  $\rho_H$ , the cost  $1 + \phi$  of each unit of refinancing.

#### 4.2.2 Optimal debt structure problem

Bankers' goal when choosing the bank's initial debt structure is to maximize the total market value of the bank,  $V(\delta, D; \phi) = D + E(\delta, D; \phi)$ , which using (7) can be written as:

$$V(\delta, D; \phi) = \frac{\mu}{\rho_H} + \frac{\rho_H - r(\delta)}{\rho_H} D - \frac{1}{\rho_H} \frac{\varepsilon(\rho_H - r(\delta))}{1 + \rho_H + \varepsilon} \delta D - \frac{1}{\rho_H} \frac{\varepsilon(1 + \rho_H)\phi}{1 + \rho_H + \varepsilon} \delta D.$$
(9)

The first term in this expression is the value of the unlevered bank. The second term is the value obtained by financing the bank with debt claims held by savers' initially more patient than the bankers (notice that  $r(\delta) < \rho_H$ , by Proposition 1). The third term reflects that crisis financing is made by experts (whose discount rate is  $\rho_H$ ) instead of by patient savers (who require a yield  $r(\delta) < \rho_H$ ). The last term accounts for the excess cost coming from having to compensate all crisis financing according to the excess opportunity cost of funds  $\phi$  of the marginal crisis financier.

Bankers solve the following problem:

$$\begin{array}{ll}
\max_{\delta \in [0,1], \ D \ge 0} & V(\delta, D; \phi) = D + E(\delta, D; \phi) \\
\text{s.t.} & E(\delta, D; \phi) \ge 0 & (\text{LL}) \\
& \mu - (1 - \delta)rD + \delta D + E(\delta, D; \phi) \ge (1 + \rho_H)(1 + \phi)\delta D & (\text{CF})
\end{array}$$
(10)

The first constraint imposes the non-negativity of the equity value in normal states and can be thought of as bankers' limited liability constraint (LL) in such states.<sup>19</sup> The second constraint is the crisis financing constraint (8) (or bankers' limited liability in crisis states).

<sup>&</sup>lt;sup>19</sup>Notice that satisfying (LL) requires bankers' dividends,  $\mu - r(\delta)D$ , to be non-negative.

It can be shown that the two constraints boil down to the same constraint on D for  $\delta = 0$ , but (CF) is tighter than (LL) for  $\delta > 0.2^{0}$  Thus (LL) can be ignored.

The following technical assumptions help us prove the existence and uniqueness of the solution to the bank's optimization problem:<sup>21</sup>

A1. 
$$\overline{\phi} < 2\frac{1+\rho_L}{1+\rho_H} - 1$$
.  
A2.  $\gamma < \frac{1-\rho_H}{2}$ .

**Proposition 2** The bank's maximization problem has a unique solution  $(\delta^*, D^*)$ . In the solution: (1) The crisis financing constraint is binding, i.e. crisis financiers take 100% of the bank's equity. (2) Optimal debt maturity  $1/\delta^*$  is increasing in  $\phi$  and the optimal amount of maturing debt per period  $\delta^*D^*$  is decreasing in  $\phi$ . In fact, if  $\delta^* \in (0,1)$ , both  $\delta^*$  and  $\delta^*D^*$  are strictly decreasing in  $\phi$ .

The intuition for these results is that, other things equal, the bank is always interested in maximizing its leverage, so its (CF) constraint is always binding, which in turn means that bankers get fully diluted ( $\alpha = 1$ ) in each crisis.<sup>22</sup> Second, as the excess cost of crisis financing  $\phi$  increases, the value of maturity transformation diminishes and all banks choose a longer expected maturity (a lower  $\delta^*$ ). The tightening of (CF) induces banks to reduce the amount of funding  $\delta^*D^*$  demanded to crisis financiers.<sup>23</sup>

The bank's optimal debt structure decisions  $(\delta^*, D^*)$  determine, as a residual, its *capital* ratio, E/V. As shown in Figure 2, this ratio is strictly increasing in the excess cost of crisis financing  $\phi$ . Intuitively, by (CF), each bank must keep enough equity value in normal states so as to be able to pay its excess cost of funding during a crisis.<sup>24</sup> Notice that under the

 $<sup>^{20}\</sup>mathrm{See}$  the proof of Proposition 2 in Appendix B.

<sup>&</sup>lt;sup>21</sup>A1 and A2 are sufficient conditions that impose rather mild restrictions on the parameters. For instance, for the discount rates  $\rho_L$ ,  $\rho_H$ , used in the calibration of the model (see footnote 17 or Section 6), A1 and A2 impose  $\overline{\phi} < 0.9953$  and  $\gamma < 0.4985$ . Moreover, we have checked numerically that the results in Proposition 2 hold well beyond the region of parameters delimited by these assumptions.

<sup>&</sup>lt;sup>22</sup>This full dilution is an implication of the simplifying assumption that all crises have the same severity. With heterogeneity in this dimension (for example, due to random shifts in  $\Phi(x)$ ), the corresponding crisis financing constraint might only be binding (or even not satisfied, inducing bankruptcy) in the most severe crises, and perhaps leave some equity in hands of the bankers in the mildest crises.

<sup>&</sup>lt;sup>23</sup>In all the numerical examples that we have explored, total debt  $D^*$  is also decreasing in  $\phi$ .

<sup>&</sup>lt;sup>24</sup>Even with  $\phi = 0$  banks need strictly positive equity because crisis financiers demand a return  $\rho_H$  larger than r for their funds.

depicted calibration, capital ratios are in a realistic 3% to 6% range for a wide range of values of  $\phi$ .



Figure 2 Banks' optimal capital ratio E/V in state N

#### 4.3 Equilibrium

Banks' optimization problem for any given excess cost of crisis financing  $\phi$  embeds savers' participation constraint so the only condition for equilibrium that remains to be imposed is the clearing of the market for crisis financing. The continuity and monotonicity in  $\phi$  of the function that describes excess demand in such market guarantees that there exists a unique excess cost of crisis financing  $\phi^e$  for which the market clears:

**Proposition 3** (1) The equilibrium ( $\phi^e$ ,  $(r^e, \delta^e, D^e)$ ) exists and is unique. (2) If the inverse supply of crisis financing  $\Phi(x)$  shifts upwards, (i) expected debt maturity  $1/\delta^e$  increases, (ii) total refinancing needs  $\delta^e D^e$  fall, (iii) bank debt yields  $r^e$  increase, and (iv) the excess cost of crisis financing  $\phi^e$  increases. (3) If initially  $\delta^e \in (0, 1)$ , all these variations are strict.

The proposition also states the pretty intuitive effects associated with a shift in the inverse supply of crisis financing. Other comparative static results are omitted for brevity.<sup>25</sup>

 $<sup>^{25}</sup>$ The interested reader may find them in a previous working paper version of this article (see Segura and Suarez, 2013).

## 5 Efficiency and regulatory implications

In this section we solve the problem of a social planner who has the ability to control banks' funding decisions subject to the same constraints that banks face when solving their private value maximization problems. We show that debt maturity in the unregulated competitive equilibrium is inefficiently short because of the pecuniary externality that operates through the cost of crisis financing and its impact on banks' frontier of maturity transformation possibilities.

Suppose that a social planner can regulate both the amount D and the maturity parameter  $\delta$  of banks' debt. Since in our economy only existing bankers and future crisis financiers appropriate a surplus, a natural objective for the social planner is to maximize the sum of the present value of such surpluses. Crisis financiers appropriate the difference between the equilibrium excess cost of crisis financing,  $\phi = \Phi(\delta D)$ , and the net present value of their alternative investment opportunity,  $z = \Phi(x) < \Phi(\delta D)$  for all  $x < \delta D$ . Hence, their surplus in a crisis is:

$$u(\delta, D) = \int_0^{\delta D} \left(\Phi(\delta D) - \Phi(x)\right) dx = \delta D\Phi(\delta D) - \int_0^{\delta D} \Phi(x) dx.$$
(11)

Evaluated at a normal state, the present value of their expected future surpluses can be written as:<sup>26</sup>

$$U(\delta, D) = \frac{1}{\rho_H} \frac{\varepsilon(1+\rho_H)}{1+\rho_H+\varepsilon} u(\delta, D).$$

From here, using (9), the objective function of the social planner can be expressed as:

$$W(\delta, D) = V(\delta, D; \Phi(\delta D)) + U(\delta, D)$$
  
=  $\frac{\mu}{\rho_H} + \frac{\rho_H - r(\delta)}{\rho_H} D - \frac{1}{\rho_H} \frac{\varepsilon(\rho_H - r(\delta))}{1 + \rho_H + \varepsilon} \delta D - \frac{1}{\rho_H} \frac{\varepsilon(1 + \rho_H)}{1 + \rho_H + \varepsilon} \int_0^{\delta D} \Phi(x) dx, \quad (12)$ 

which contains four terms: the value of an unlevered bank, the value added by maturity transformation in the absence of systemic crises, the value loss due to financing the bank

$$U(\delta, D) = \frac{1}{1 + \rho_H} \left[ (1 - \varepsilon)U(\delta, D) + \varepsilon \left( u(\delta, D) + \frac{1}{1 + \rho_H} U(\delta, D) \right) \right].$$

 $<sup>^{26}</sup>U(\delta, D)$  satisfies the following recursive equation:

The first term in square brackets takes into account that, with probability  $1 - \varepsilon$ , next period is also a normal state and crisis financiers' continuation surplus remains equal to  $U(\delta, D)$ . The second term captures that with probability  $\varepsilon$  there is a crisis, in which case crisis financiers obtain  $u(\delta, D)$  plus the continuation surplus that, one more period ahead, is again  $U(\delta, D)$ .

with impatient experts during liquidity crises, and the value loss due to the sacrifice of the NPV of the investment projects given up by those experts who act as banks' crisis financiers.

With this key ingredient, the social planner's problem can be written as:

$$\max_{\substack{\delta \in [0,1], D \ge 0 \\ \text{s.t.}}} W(\delta, D)$$
(13)  
s.t. 
$$\mu - (1 - \delta)rD + \delta D + E(\delta, D; \Phi(\delta D)) > (1 + \rho_H)(1 + \Phi(\delta D))\delta D$$
(CF')

This problem differs from banks' optimization problem (10) in two dimensions. First, the objective function includes the surplus of the crisis financiers. Second, the social planner internalizes the effect of D and  $\delta$  on the market-clearing excess cost of crisis financing, so (CF') contains  $\Phi(\delta D)$  in the place occupied by  $\phi$  in the (CF) constraint.<sup>27</sup>

Comparing the solution of the planner's problem with the unregulated equilibrium, we obtain the following result:

**Proposition 4** If the competitive equilibrium features  $\delta^e \in (0, 1)$  then a social planner can increase social welfare by choosing a longer expected debt maturity than in the competitive equilibrium, i.e. some  $1/\delta^s > 1/\delta^e$ .

The root of the discrepancy between the competitive and the socially optimal allocations is at the way individual banks and the social planner perceive the frontier of the set of maturity transformation possibilities. As illustrated in Figure 3, banks choose their individually optimal  $(\delta, D)$  along the (CF) constraint (where  $\phi^e$  is taken as given) whereas the social planner does it along the (CF') constraint (where  $\phi = \Phi(\delta D)$ ).

At the equilibrium allocation  $(\delta^e, D^e)$  both the social planner's and the initial bankers' indifference curves are tangent to (CF). Moreover, (CF) and (CF') intersect at  $(\delta^e, D^e)$  since the competitive equilibrium obviously satisfies  $\phi^e = \Phi(\delta^e D^e)$ . However, the social planner's indifference curve is not tangent to (CF') at  $(\delta^e, D^e)$ , implying that this allocation does not maximize welfare. In the neighborhood of  $(\delta^e, D^e)$ , (CF') allows for a larger increase in D, by reducing  $\delta$ , than what seems implied by (CF), where  $\phi$  remains constant. It turns out that maturity transformation can produce a larger surplus with a larger use of its extensive margin (leverage) and a lower use of its intensive margin (short maturities), like at  $(\delta^s, D^s)$ in the figure. Implementing this allocation would simply require imposing  $\delta^s$  as a regulatory upper limit to banks' choice of  $\delta$ .

 $<sup>^{27}</sup>$ The constraint called (LL) in (10) can be ignored in the planner's problem as well because it is implied by the crisis financing constraint.

The results in this section offer a new perspective on regulatory proposals emerged in the aftermath of the recent crisis that defend reducing *both* banks' leverage *and* their reliance on short-term funding. In the context of the current model, once debt maturity ( $\delta$ ) is regulated, limiting banks' leverage (D) would be counterproductive. Our results also indicate that simply limiting banks' leverage (e.g., through higher capital requirements) would not correct the inefficiencies identified above. In fact, as one can see in Figure 3, forcing banks to choose debt lower than  $D^e$  would induce them, in the new regulated equilibrium, to move along (CF') in the direction that implies a shorter expected debt maturity (larger  $\delta$ ), thus lowering welfare even further.



**Figure 3** Equilibrium vs. socially optimal debt structures. The solid curve is the private (CF) constraint in the unregulated equilibrium, the dot-and-dashed curve is the social (CF') constraint. The two dashed curves are indifference curves of the social planner.

## 6 Quantitative results

In this section we calibrate the model to assess the potential quantitative importance of its implications. We use aggregate data from the Eurozone banking sector, for which the ECB regularly publishes data on the structure of its liabilities. This includes information on the maturity profile of bank debt which is essential for our analysis. We choose 2006 as a benchmark year so as to represent a typical Eurozone bank just prior to the first signs of arrival of the 2007-2009 financial crisis. Given the absence of relevant liquidity risk regulations in that year, we will interpret the liability structure observed in 2006 as corresponding to the unregulated equilibrium of previous sections.<sup>28</sup>

#### 6.1 Calibration

Our strategy for the calibration of the model starts with three preliminary observations. First, from the perspective of an individual bank, the asset cash flow  $\mu$  is a pure scale parameter that impacts D and thus the scale of the bank's refinancing needs in a crisis. Since our model is silent about bank entry and exit (or banking sector size) and our analysis will focus on scale-free variables such as r,  $1/\delta$ , E/V, and the percentage welfare gains from liquidity risk regulation, we are going to fix  $\mu$  arbitrarily equal to  $\rho_H$ . This means that our representative bank (out of the assumed measure-one continuum of banks), if unlevered, would feature V = W = 1 (see equations (9) and (12)). This also allows us to interpret the values of the V and W in alternative allocations as measuring proportional gains or losses relative to the unlevered bank benchmark.

Second, we will assume that crises last for one month. Hence, a model period will represent one month. This is consistent with the duration of the "liquidity stress scenarios" that the new liquidity coverage ratio (LCR) of Basel III mandates banks to cover (see BCBS, 2010). Yet, for reporting purposes, some yields and interest rate spreads below will be described in annualized percentage points.

Third, existing data and evidence do not provide an estimate of the inverse supply of crisis financing, which would require sampling such curve at various points, say based on the accumulated experience of several crisis episodes. Still, after having fixed the values of other parameters, we can indirectly infer a value for the equilibrium cost of crisis financing  $\phi^e$  from the observed debt structure decisions of the banks. To reflect our ignorance about the inverse supply function, we will adopt the isoelastic specification given in (1) and report

 $<sup>^{28}</sup>$  Actually, Eurozone banks were subject to capital requirements which, if binding, would interfere with the choice of D in banks' problem. But given that capital requirements are risk-based, fully accounting for the impact of their presence would require a model of simultaneous choice of banks' asset composition and liability structure. For the purposes of our analysis, we will simply assume that capital requirements were not binding in 2006. In the logic of the model, this implies interpreting the voluntary capital buffers that analysts normally attribute to generic precautionary reasons or market pressures as the buffers that banks considered at that time necessary to maintain their refinancing capacity in a liquidity crisis.

all the relevant results as a function of its (unknown) elasticity parameter  $\eta$ .<sup>29</sup>

The parameters that remain to be fixed are then  $\rho_L$ ,  $\rho_H$ ,  $\gamma$ ,  $\varepsilon$ , and  $\phi^e$ . As Appendix A describes in greater detail, we first fix  $\rho_L$ ,  $\rho_H$ , and  $\gamma$  so as to fit the interest rate curve  $r(\delta)$  (which only depends on these parameters, see (4)) to data on the average interest rates paid on term deposits of various maturities by Eurozone banks in 2006. We then fix  $\varepsilon$  and  $\phi^e$  so as to exactly match with their empirical counterparts two model outcomes: the normal-state rate of return on bank equity,  $(\mu - r^e D^e)/E^e$ , and the equilibrium portion of bank debt that matures in each month,  $\delta^e$ . Table 1 shows the resulting values of the parameters.

Table 1							
Parameter values							
(One period is one month)							
Patient agents' discount rate	$ ho_L$	0.000654	(0.8% yearly)				
Impatient agents' discount rate	$ ho_H$	0.003029	(4.0%  yearly)				
Frequency of idiosyncratic preference shocks	$\gamma$	0.13	(every 7.7 months)				
Frequency of crisis	ε	0.0081	(every 10.3 years)				
Equilibrium excess cost of crisis financing	$\phi^e$	0.131	(13.1%  per crisis)				

Note: Expression in parenthesis refer, as indicated, to either equivalent yearly/per-crisis rates or implied expected frequencies.

Based on data on the return on equity observed among Eurozone banks in 2006, we establish a target for the normal-state rate of return on bank equity of 14.2%; this target pins down the value of  $\varepsilon$ .

The most data-intensive part of the calibration exercise is the one referred to estimating the empirical counterpart of  $\delta^e$ . We do it by exploiting the breakdown of aggregate bank liabilities provided by the ECB (and, for specific categories, some complementary survey data). We first compute the average  $\delta^e_i$  of each of the categories i = 1, 2, ...I in the breakdown. Table 2 summarizes our results (see Appendix A for details). Then we calculate our estimate of  $\delta^e$  as a weighted average of those values, with weights that reflect the contribution of each category to total debt. The estimated value of 0.416, which corresponds to an average debt maturity of 2.4 months, is set as our calibration target for  $\delta^e$ .

To cross-check the capacity of our model to reasonably match the data, we can use banks'

<sup>&</sup>lt;sup>29</sup>After calibrating  $\phi^e$  and solving for banks' equilibrium choices ( $\delta^e, D^e$ ), each value of  $\eta$  uniquely identifies the value of the remaining parameter a in (1) under which the whole parameterization is consistent. For brevity, we will not report the corresponding values of a.

capital ratio, E/V, as an overidentifying moment. Under the calibration just described, banks operate in the unregulated equilibrium with a capital ratio of 5.3%, while the average capital ratio of Eurozone banks in 2006 is 5.8%.<sup>30</sup> We interpret the proximity between these two values as reassuring about the capability of the calibrated model to be informative about the potential quantitative importance of its implications.

(One period is one month)							
	Amount	Fraction		$1/\delta_i$			
Debt category	(b€)	(%)	$\delta_i$	(months)			
Deposits from households & NFCs	5,821	27.4	0.563	1.78			
Deposits from other agents	2,906	13.7	0.336	2.98			
Deposits & repos from other banks	7,340	34.6	0.560	1.79			
Other repos	245	1.2	0.693	1.44			
Commercial paper & bonds	4,463	21.1	0.027	37.04			
Eurosystem lending	451	2.1	NA	NA			
Sum / Weighted average	21,225	100.0	0.416	2.40			

Table 2Estimating an average  $\delta$  for Eurozone banks in 2006

Note: NFCs stands for Non-Financial Corporations. The weighted average  $\delta_i$  is computed excluding Eurosystem lending.

#### 6.2 Value of maturity transformation

Table 3 shows the value generated by maturity transformation in the unregulated equilibrium obtained under the parameters described in Table 1. It reports the unlevered value of bank assets (normalized to one) and the additional value gains or losses captured by the other terms of the expression for V in (9).

Maturity transformation allows the representative bank to increase its value by about 96% relative to the unlevered value of its assets (a net 49% of V). Indeed, before subtracting the costs associated with refinancing risk, maturity transformation produces a gross extra value of 124% relative to the unlevered value of assets (63% of V). However, the anticipated discounted costs of crises (almost entirely attributable to having  $\phi^e > 0$ ) represent about 27% of the unlevered value of assets (14% of V). Having enough capacity to pay for those

<sup>&</sup>lt;sup>30</sup>This number is calculated using the aggregate balance sheet of the Eurozone credit institutions as published by the ECB in its Monetary Statistics.

excess costs by diluting the equity every time a crisis occurs explains why E represents 10% of the unlevered value of assets (5% of V).

value of maturity transformation								
(unregulated equilibrium)								
	Value	%						
Value of the unlevered bank (normalized)	1.0000	50.95						
Value gain from maturity transformation if $\varepsilon = 0$	1.2380	63.08						
Value loss from refinancing risk $(\varepsilon > 0)$ if $\phi = 0$	-0.0042	-0.21						
Value loss from excess cost of crisis funding $(\phi > 0)$	-0.2712	-13.82						
Total value of the bank $(V)$	1.9626	100.00						
Debt value of the bank $(D)$	1.8594	94.74						
Equity value of the $bank(E)$	0.1032	5.26						

# Table 3 Value of maturity transformation (unregulated equilibrium)

The total surplus W generated by the bank generally exceeds its total market value V because the former also includes the present expected value of the intramarginal rents appropriated by the expert financiers who finance the bank in each crisis (see (12)). These rents are zero when the inverse supply of crisis financing is inelastic ( $\eta = 0$  or, equivalently, when the supply of crisis financing is perfectly elastic) and increase mechanically as  $\eta$  increases (i.e. as the supply of funds becomes more and more rigid). For instance, under our calibration, they represent 6.9% of bank value (i.e. W/V = 1.069) for  $\eta = 1, 10.4\%$  for  $\eta = 3, and 11.5\%$  for  $\eta = 5$ .

#### 6.3 Quantitative importance of the externality

Next we quantify the effects of the pecuniary externality that renders unregulated banks' debt maturities inefficiently short. Figure 4 compares the equilibria of the unregulated and the optimally regulated economies for different values of  $\eta$ . For  $\eta = 0$ , both equilibria coincide because banks' capital structure decisions have no effect on the cost of crisis financing  $\phi$ .

Panel A shows how as  $\eta$  increases the social planner finds it optimal to push towards longer average debt maturities  $(1/\delta)$ . As it is apparent in the figure, the sensitivity of the optimally regulated  $1/\delta$  to changes in  $\eta$  is higher at lower values of  $\eta$  (and tends to stabilize for  $\eta > 5$ ). It rapidly increases from an implied average debt maturity of 2.4 months to one of above 3 months. Panel B shows the rather sharp response of the excess cost of crisis financing to the lengthening in regulated maturities that occurs as  $\eta$  increases. Panel C describes the discrepancy between banks' refinancing needs in the unregulated and the optimally regulated economies. This discrepancy is non-monotonic and reaches a maximum of 16% for a value of  $\eta$  of about 3.<sup>31</sup>



Figure 4 Comparison between unregulated and regulated equilibria

Panel D shows the importance of the substitutability between the intensive and extensive margins of maturity transformation along (CF') and its dependence on  $\eta$ .<sup>32</sup> It shows that with optimally regulated debt maturities banks would generally need lower equity values to guarantee their financing during crisis; the implied capital ratio falls from 5.3% in the unregulated equilibrium to 3.8% in the regulated equilibrium with  $\eta = 1$ , and to 1.8% in the regulated equilibrium with  $\eta = 5$ . These results suggest that the welfare gains associated with maturity regulation increase sharply with  $\eta$ . Indeed, the optimal regulation of  $\delta$  increases W by 1.2% with  $\eta = 1$  and by 5.1% with  $\eta = 5$ .

<sup>&</sup>lt;sup>31</sup>This non-monotonicity is compatible with the behavior of  $\phi^s$  depicted in Panel B. It occurs because as  $\eta$  increases the inverse supply of crisis financing becomes more and more responsive to reductions in  $\delta D$ .

<sup>&</sup>lt;sup>32</sup>In Figure 3, (CF') is obtained with  $\eta = 3$ .

To a first approximation, these results suggest that regulations such as the NSFR of Basel III, aimed to limit the intensity of banks' maturity transformation, qualitatively point in the right direction. Quantitatively, however, from the perspective of the calibrated version of our model, the NSFR is too drastic. The NSFR is roughly defined as the ratio between liabilities with maturity longer than one year ("stable funding") to assets with maturity longer than one year ("illiquid assets") and the new regulation will require banks to operate with a NSFR higher than one (see BCBS, 2014). In terms of our model, this is equivalent to limiting the choice of the average debt maturity  $1/\delta$  to values higher than 12 months, which is well above the socially optimal value of this variable under any elasticity of the inverse supply of funds during crises. In fact, under  $\delta = 1/12 = 0.083$ , the social surplus generated by banks would decline, for all elasticities, by more than 27% relative to the unregulated equilibrium. Hence, a regulation which, on pure qualitative grounds, seems justified becomes, from the perspective of our analysis, undesirable.

## 7 Extensions

#### 7.1 Impact of an unanticipated rise in systemic risk

As an example of potential quantitative exploitations of the model, we analyze the effects of an unexpected and permanent increase in the severity of the liquidity crises to which banks are exposed.<sup>33</sup> In order to do so, we first extend the model to incorporate a new parameter  $\theta \in [0, 1]$  which determines the fraction of banks simultaneously affected by each crisis event. Importantly, we assume that the risk that an individual bank suffers a liquidity crisis remains equal to  $\varepsilon$  (and invariant to  $\theta$ ) so that  $\theta$  only measures the degree of "systemicity" (or correlation) of the crises suffered by each bank. This "systemicity" may be connected to the degree of (global) integration of the financial system and the global versus local nature of the shocks that cause a crisis. While the baseline model implicitly sets  $\theta = 1$ , we now assume that the economy starts with some  $\theta_0 < 1$  and, when a crisis hits, agents discover that the parameter has unanticipatedly shifted to a value  $\theta_1 \in (\theta_0, 1]$ .<sup>34</sup>

 $<sup>^{33}</sup>$ This exercise is consistent with the view of scholars that have interpreted the 2007-2009 financial crisis as the sudden discovery that the financial system was riskier than initially thought. See Gennaioli, Shleifer, and Vishny (2012) and the references therein.

<sup>&</sup>lt;sup>34</sup>Formally, the extension of the model only requires writing  $\theta \delta D$  in replacement of  $\delta D$  in the expressions (market clearing conditions, (CF'), and W) that reflect that  $\phi$  is a function of banks' aggregate refinancing

If banks make the capital structure decisions  $(\delta_0, D_0)$  that are privately optimal for  $\theta = \theta_0$ but, when a crisis arrives, they realize that the value of  $\theta$  has permanently shifted to  $\theta_1 > \theta_0$ , the first problem they will find is that their aggregate refinancing needs are  $\theta_1 \delta_0 D_0$  rather than  $\theta_0 \delta_0 D_0$ . As long as  $\eta > 0$ , this will also mean that the market clearing cost of crisis financing would be some  $\phi_{01} = \Phi(\theta_1 \delta_0 D_0)$  which is larger than the initially anticipated  $\phi_0 = \Phi(\theta_0 \delta_0 D_0)$ . Since (CF) is binding at  $(\delta_0, D_0)$  under  $\phi_0$ , the banks suffering the run will not have enough equity to warrant their refinancing under  $\phi_{01} > \phi_0$ . Intuitively, they will lack enough equity value to arrange for their refinancing by means of the dilution of their equity value—a bail-in.

The question we address in this section is how large the external injection of equity to banks would have to be to satisfy their (CF) constraint and avoid their liquidation—the size of the required bail-out.<sup>35</sup> We call the required equity injection the *capital shortfall* that emerges upon the discovery of the larger severity of the crisis. To compute it, we take into account the larger cost of the current crisis as well as the fact that, just after the crisis, the banks will reoptimize and choose the new capital structure ( $\delta_1$ ,  $D_1$ ) which is privately optimal given the larger anticipated cost (say  $\phi_1$ ) of subsequent crises.

In Figure 5 we plot the capital shortfall banks experience in the extended economy when  $\theta_0 = 0.5$  and  $\theta_1 = 1$ . We parameterize the economy with  $\theta_0 = 0.5$  in the same way as the baseline economy in Section 6. Clearly, the higher the cost of crisis funds under  $\theta_1 = 1$  relative to what agents expected under  $\theta_0 = 0.5$ , the larger the capital shortfall. Thus the capital shortfall is increasing in the elasticity of the inverse supply of crisis financing  $\eta$ . In fact, it rises very sharply with  $\eta$ . Even for  $\eta = 0.5$  the capital shortfall is already above 5.3% of the total value of the banks, i.e. exceeds 100% of banks' initial equity base. For larger and larger values of  $\eta$ , the shortfall stabilizes at values equivalent to about 350% of banks' initial equity base.

needs.

<sup>&</sup>lt;sup>35</sup>Absent some form of bail-out, some banks would have to default on their maturing debt and be liquidated. Otherwise, the equilibrium cost of crisis financing would not reach a value compatible with the refinancing of the survivors. Liquidating banks would help alleviate the refinancing difficulties of the survivors by reducing the market clearing cost of crisis financing in the current crisis as well as future crises (if there is no entry replacing the failed banks).



Figure 5 Capital shortfall (as a percentage of the initial total value of banks) when systemic risk unexpectedly and irreversibly rises

Figure 6 describes the adjustment in expected debt maturities,  $1/\delta$ , and capital ratios, E/V, that occurs in response to the anticipated greater "systemicity" (severity) of future crises. Banks lengthen their debt maturities and increase their capital ratios (deleverage) quite significantly.



Figure 6 Equilibrium response to a rise in systemic risk

#### 7.2 Private liquidity insurance

Some recent discussions on the final shape of bank liquidity risk regulation suggest the need to consider in parallel the possibility that banks' refinancing needs in a crisis are covered by explicit liquidity insurance schemes (Stein, 2013). In this section we first extend the baseline model to cover the scenario in which banks have access to a fairly priced private liquidity insurance arrangement. Then, relying on our previous calibration of the model, we quantify the social gains from introducing such an arrangement and its complementarity with liquidity regulation.

The fact that banks' crisis financing constraints are binding in equilibrium, while their normal-state limited liability constraints are not, suggests that some form of insurance against systemic liquidity crises be beneficial. To keep things tractable, we focus on simple one-period refinancing insurance arrangements subscribed by individual banks and entering experts at the beginning of each period, prior to the realization of uncertainty regarding the occurrence of a crisis.

Specifically, we consider arrangements where:

- 1. Except in the period immediately after each crisis,<sup>36</sup> the bank pays a per-period premium p for each unit of insured refinancing  $\lambda \delta D > 0$  to a measure  $\lambda \delta D$  of entering experts;  $\lambda \in [0, 1]$  is the insured fraction of refinancing needs.
- 2. If there is a systemic crisis, the insuring experts supply the bank with funds  $\lambda \delta D$  in exchange for receiving a repayment  $[1 + r(\delta) + p]\lambda \delta D$  just after the crisis.

Under this arrangement the refinancing of  $\lambda \delta D$  is just as costly as if no crisis had occurred:  $r(\delta)$  is the normal yield of savers' debt. The part  $p\lambda \delta D$  of the repayment to the insuring experts after the crisis is included to preserve the recursivity of our formulation (specifically, to offset the impact on the banks' net income of the fact that we assume that a crisis never occurs just after another and, thus, paying the insurance premium would in principle not be needed in the period immediately after a crisis).

For the sale of insurance to be attractive to an entering expert, the insurance premium

<sup>&</sup>lt;sup>36</sup>Insurance in the period immediately after a crisis is unneeded because, according to our assumptions, crisis periods are always followed by a normal period.

p must satisfy

$$p + \varepsilon \frac{1+r+p}{1+\rho_H} \ge \varepsilon \max\{1+z, 1+\phi\},\tag{14}$$

where the second term in the LHS is the present value of the post-crisis repayments described above and the RHS controls for the fact that the expert can alternatively either undertake his own investment opportunity (with NPV equal to z) or act as crisis financier (who extracts a NPV of  $\phi$  from his funds). Competition among entering experts will lead to a situation in which (14) is binding for the marginal provider of either insurance or crisis financing, who will have  $z = \phi$ .<sup>37</sup> Solving for p in such equality yields

$$p = \frac{\varepsilon}{1 + \rho_H + \varepsilon} \{ [(1 + \rho_H)\phi + \rho_H] - r(\delta) \},$$
(15)

which is identical to one of the factors that multiply  $\delta D$  in (7).

On the other hand, the value of bank equity in a normal state when a fraction  $\lambda$  of the refinancing needs is insured can be written as

$$E(\delta, D, \lambda; \phi) = \frac{1}{\rho_H} \left\{ \mu - r(\delta)D - p\lambda\delta D - \frac{\varepsilon}{1 + \rho_H + \varepsilon} \{ [(1 + \rho_H)\phi + \rho_H] - r(\delta) \} (1 - \lambda)\delta D \right\},$$

which is an extended version of (7). Now, using (15) to substitute for p, it becomes clear that:

$$E(\delta, D, \lambda; \phi) = E(\delta, D, 0; \phi) = E(\delta, D; \phi), \tag{16}$$

which can be interpreted as a Modigliani-Miller type result: changing the fraction  $\lambda$  of fairly-priced insured funding for given values of D and  $\delta$ , does not per secreate or destroy value.

Insurance is, however, relevant for the bank's overall optimization problem because it helps relax the crisis financing constraint. In the presence of insurance, this constraint can be written as:

$$(\mu - \lambda p \delta D) - [1 - (1 - \lambda)\delta]rD + (1 - \lambda)\delta D + E(\delta, D; \phi) \ge (1 + \rho_H)(1 + \phi)(1 - \lambda)\delta D, \quad (CFI)$$

which differs from (8) in the presence of the insurance premia subtracting from the cash flow  $\mu$  and adaptations that reflect that the bank's effective refinancing needs in the crisis are reduced to the uninsured fraction of its debt,  $(1 - \lambda)\delta D$ .

<sup>&</sup>lt;sup>37</sup>Clearing the market for crisis financing requires  $\phi = \Phi(\delta D)$  irrespectively of the fraction of  $\delta D$  covered with insurance.

It is easy to check that  $\lambda = 1$  implies the maximal relaxation of this constraint.<sup>38</sup> On the other hand, by (16), the bank's limited liability constraint remains the same as (LL) in (10). Therefore, the bank will solve the counterpart of the value maximization problem in (10) by getting fully insured against systemic crises ( $\lambda = 1$ ). By doing so, its net cash flow becomes  $\mu - rD - p\delta D$  and the constraints (CFI) and (LL) collapse into simply requiring that this cash flow is not negative.

The following proposition describes the positive welfare implications of adding insurance when debt maturity ( $\delta$ ) is optimally regulated. It also shows that, with liquidity insurance, banks in the unregulated economy would opt for inefficiently short maturities.

**Proposition 5** In an optimally regulated economy, adding a private liquidity insurance scheme strictly increases welfare. With liquidity insurance, expected debt maturity in the unregulated equilibrium is too short.

Intuitively, when liquidity insurance is introduced,  $\mu - rD - p\delta D \ge 0$  becomes banks' only relevant constraint, which implies expanding the set of maturity transformation possibilities faced by both banks and the social planner. Hence, a social planner can definitely produce more social welfare with insurance than without insurance. However, the pecuniary externality regarding banks' debt maturity decisions remains present, now operating through the  $\mu - rD - p\delta D \ge 0$  constraint. Intuitively, bank decisions affect the excess cost of crisis financing  $\phi$ , which in turn affects the cost of insurance p (see (15)), and ends up tightening this constraint.<sup>39</sup>

Figure 7 shows how the introduction of liquidity insurance affects the unregulated and optimally regulated equilibria under the calibration described in Section 6. Not surprisingly, for very low values of  $\eta$ , the introduction of insurance in the unregulated economy leads to a reduction in debt maturity and an increase in the amount of debt. The reason is that banks are less constrained. As a result, however, their aggregate refinancing needs increase, putting upward pressure on  $\phi$  (whenever  $\eta > 0$ ) and, through it, on the cost of insurance p. As  $\eta$ 

<sup>&</sup>lt;sup>38</sup>It suffices to realize that (15) implies  $(1 + \rho_H)(1 + \phi) - 1 - r > p$ .

<sup>&</sup>lt;sup>39</sup>We are not able to prove that the introduction of insurance increases welfare in the unregulated economy, but this is actually the case in our calibration of the model and all the parameterizations that we have explored. The theoretical ambiguity comes from the fact that, with full insurance, unregulated banks will tend to choose funding structures that put upward pressure on  $\phi^e$  and, in principle, a sufficiently large increase in  $\phi^e$  might fully offset the gains due to the introduction of insurance.

grows, this effect grows, pushing towards the choice of longer maturities. For  $\eta \geq 2.5$  the effect is strong enough for the maturities chosen with insurance to exceed the ones chosen without insurance.



Figure 7 Effect of insurance on regulated and unregulated equilibria

In contrast, in the regulated economy the introduction of insurance always reduces debt maturities (albeit more so for low values of  $\eta$ ). Intuitively, insurance also relaxes the constraints faced by the social planner, allowing the choice of shorter maturities and more debt. And a similar pressure on  $\phi$ , which is growing in  $\eta$ , explains why expected debt maturity grows with  $\eta$ . The difference with respect to the unregulated economy is that the variation of  $1/\delta$  with  $\eta$  also occurs without insurance, keeping a positive difference in favor of the value of  $1/\delta$  chosen without insurance.

It is worth noting that, except for very low values of  $\eta$ , the effect of regulation on  $1/\delta$  is quantitatively much more important than the effect of adding insurance. In welfare terms, however, the comparison requires to also take into account the effect of insurance on the total debt that banks can issue.

Figure 8 shows the welfare gains due to the introduction of insurance (left panel) and regulation (right panel), as well as the additional gains from introducing each of the two when the other is already in place. The left panel shows that the gains from introducing liquidity insurance only are decreasing in the elasticity of the inverse supply of crisis financing. Yet they are important for all values of  $\eta$ , ranging from 6.1% for  $\eta = 0$  down to 2.4% for  $\eta = 5$ . Importantly, except for very low values of  $\eta$ , the welfare gains from the additional introduction of regulation are sizeable, and grow with  $\eta$ . Thus, the conclusion from the left panel of Figure 8 is that, for not too small values of  $\eta$ , regulation is very complementary to liquidity insurance (and increasingly so as  $\eta$  increases).



Figure 8 Marginal welfare gains from insurance and regulation

The right panel in Figure 8 confirms the already commented positive association between  $\eta$  (which measures how important the underlying pecuniary externality is) and the gains from debt maturity regulation. Once debt maturity is optimally regulated, the additional welfare gains from introducing insurance decline with  $\eta$ , although they remain quantitatively significant for all  $\eta$ .

## 8 Discussion and further extensions

In this section we comment on the importance of some of the assumptions of the model and on potential variations of them that could be considered at a cost in terms of tractability.

#### 8.1 Instability of funding during crises

We have assumed that during crises all common savers whose debt matures "run" on the banks. In reality, institutions such as deposit insurance as well as factors such as the existence of long-term relationships between banks and some of their debtholders and the capacity of banks to pledge collateral for their funding may grant stability to (or facilitate the refinancing of) liabilities that are formally demandable during a crisis. This is explicitly recognized in the proposal regarding the liquidity coverage ratio (LCR) in Basel III when establishing the minimum *run-off rates* that should be attributed to each debt category in order to estimate banks' potential refinancing needs during a crisis (BCBS, 2013).<sup>40</sup>

In order to capture these features, the model can be extended by introducing a run-off rate parameter  $\chi \in [0,1]$  that determines the fraction of the total refinancing needs per period  $\delta D$  that actually experience a run in a crisis. This will mean that the financing required to experts in a crisis is  $\chi \delta D$ , while  $(1 - \chi)\delta D$  remains refinanced as in normal times. So the baseline model corresponds to the case  $\chi = 1$  of the extended model. And it is easy to check that to obtain the analytical expressions for the relevant objects in the maximization problem of the bank, namely V and the (CF) constraint, it suffices to replace  $\delta D$  with  $\chi \delta D$  in the corresponding expressions for the baseline model (equations (9) and (8), respectively).<sup>41</sup>

Applying the Basel III run-off rates to the structure of Eurozone bank liabilities in 2006 yields an estimated average value of 0.64 for  $\chi$ .<sup>42</sup> To recalibrate the extended model along the same lines as the baseline model, we only need to adjust  $\phi^e$  to 0.205. Interestingly, the adjustment in  $\phi^e$  is such that it leaves  $\chi \phi^e$  virtually equal to the value of  $\phi^e$  in the baseline model. This reflects that  $\chi \phi^e$  is essentially pinned down (irrespectively of  $\chi$ ) in the attempt to match the target value of  $\delta^e$  because the third term in (9) is quantitatively tiny (see Table 3). Additionally, the isoelastic specification of  $\Phi(x)$  implies that the normative results in the calibrated version of the extended model are also virtually identical to those of the calibrated baseline model.<sup>43</sup> In sum, this is a very tractable extension that does not essentially modify any of our prior results.

 $<sup>^{40}</sup>$ For instance, the LCR assumes a run-off rate for retail demand deposits ranging from 5% to 10%, depending on whether they are covered by an effective deposit insurance scheme and other features that measure their stability. This means that during a one month crisis regulators only expect a fraction from 5% to 10% of these deposits to be withdrawn, even if all of them can be withdrawn. On the other hand, the LCR captures the instability of wholesale funding observed during the past financial crisis by assuming a run-off rate of 100% for interbank deposits or most repos.

<sup>&</sup>lt;sup>41</sup>The same adaptations work for the problem of the social planner in (13).

 $<sup>^{42}</sup>$ This number essentially comes from attributing a 10% run-off rate to the Deposits from households and NFCs in Table 2. Based on the breakdown of the Deposits from other agents and the Basel III rules, this category gets an estimated run-off rate of 94% (coming from attributing a 75% rate to deposits from the general government and a 100% rate to the rest). The remaining categories receive a 100% run-off rate.

<sup>&</sup>lt;sup>43</sup>Except for the results regarding the excess cost of crisis financing, which appear roughly scaled up by the factor  $1/\chi$ , the counterparts of the other panels in Figure 4 are hardly distinguishable from those of the baseline model.

#### 8.2 Optimality of not defaulting during crises

We have so far assumed that the liquidation value of banks in case of default, L, is small enough for banks to find it optimal to rely on funding structures that satisfy the (CF) constraint. How small L has to be (and what happens if it is not) is discussed next.

If a bank were not able to refinance its maturing debt, it would default, and we assume that this would precipitate its liquidation. For simplicity, we assume that, if the bank defaults, the liquidation value L is orderly distributed among all debtholders, which excludes the possibility of preemptive runs à la He and Xiong (2011a). When the bank is expected to default in a crisis, the interest rate paid to savers has to include a compensation for credit risk, complicating the analysis.

Yet, based on the derivations provided in Appendix C, it is possible to compute the maximum liquidation value  $L^{\max}$  for which, when all other banks opt for preventing going bankrupt, an individual bank also prefers to prevent going bankrupt. For configurations of parameters with  $L \leq L^{\max}$ , our candidate equilibrium with crisis financing in which no bank defaults gets confirmed as an equilibrium of the general model that allows banks to potentially default in crisis states.

In the case of our calibrated economy, we have  $L^{\max} = 1.66$ , a value that exceeds the unlevered value of bank assets (normalized to one) by 66% and represents 84% of the total market value of the bank in a normal state (V). Therefore, our focus on situations in which banks do not default in a crisis is consistent with assuming that banks' liquidation value is below 84% of their total market value in normal times. We consider this assumption plausible.<sup>44</sup>

#### 8.3 Deterministic vs. random maturity

For tractability we have assumed that debt contracts have random maturity. It would be more realistic to assume that the bank chooses an integer T that describes the deterministic maturity of its debt contracts. In this setting it is possible to determine savers' required maturity premium  $r^{\text{det}}(T)$  as we did in Section 4.1. It can also be shown that for  $T = 1/\delta$ ,

<sup>&</sup>lt;sup>44</sup>For instance, Bennett and Unal (2014), using data from bank holding companies resolved in the US by the FDIC from 1986 to 2007, estimate an average discounted total resolution cost to asset ratio of 33.18%, a number compatible with our assumption. See Hardy (2013) for related evidence.

we have  $r^{\text{det}}(T) < r(\delta)$  because discounting is a convex function of time and thus the random variation in maturity realizations produces disutility to impatient savers.

With deterministic maturities, the model would lose some of the Markovian properties that make it tractable. In the period after a crisis the initial funding structure would not be immediately reestablished since, in addition to the debt with principal  $\frac{1}{T}D$  that matures and has to be refinanced, the bank would also have to issue the debt with face value  $\frac{1}{T}D$  that was bridge financed during the crisis. Thus, in order for the bank to keep a constant fraction 1/T of debt maturing in each period, half of the debt issued by the bank in the after-crisis period should have maturity T - 1, but this would introduce heterogeneity in interest rate payments across the various debts. The description would become further complicated if a new crisis arrives prior to the maturity of the debt with maturity T - 1.

Therefore, assuming random maturities implies some loss of banks' value but is essential to the simplicity of our recursive valuation formulas. However, we find no obvious reason to think that having deterministic rather than random maturities should significantly alter the trade-offs captured by the current model.

#### 8.4 Resetting debt structures over time

For the sake of clarity, we have assumed that the debt structure  $(\delta, D)$  decided at t = 0 is kept constant over normal periods and restored immediately after each crisis. What would happen if bankers could re-optimize in periods different from t = 0?

To narrow down the question, suppose, in particular, that outstanding debt were exogenously (and unexpectedly) maturing all at a time in a single normal period and bankers were allowed to decide a new debt structure from thereon. It is obvious from the Markovian structure of the model that their optimal decision would coincide with the initial one.<sup>45</sup>

The more general case in which at every date the bank could decide to roll-over part of its maturing debt at perhaps some new terms, while keeping constant the structure of its non-maturing debt, is hard to analyze because describing the debt structures that a bank

<sup>&</sup>lt;sup>45</sup>The formal argument goes as follows: denote the bank's current debt structure by  $(\overline{\delta}, \overline{D})$ . In a N state that does not follow a C state, the market value of total outstanding debt is  $\overline{D}$ . Current shareholders would maximize  $V(\delta, D; \phi) - \overline{D}$  subject to the same financing constraints as at t = 0 and the optimal solution would be the same as at t = 0, since the only difference between the initial optimization problems and the current one is the (constant)  $\overline{D}$  now subtracted from the objective function. In a N state that follows a C state, the market value of outstanding debt would be  $(1 - \overline{\delta})\overline{D}$  and current shareholders would maximize  $V(\delta, D; \phi) - (1 - \overline{\delta})\overline{D}$  but again the solution would not change.

might end up having requires a very complicated space of state variables. However, we find no clear reasons to expect those funding structures to increase the market value of the bank. Intuition from simpler models suggests that altering the terms of new debt as maturing debt is rolled over might only create value to shareholders at the expense of non-maturing debt holders, but this (i) would have a negative repercussion on the value of such a debt when issued (and hence on initial shareholder value) and (ii) could be prevented by including proper covenants in the preexisting debt contracts.<sup>46</sup>

#### 8.5 Tradability of debt

The non-tradability of banks' debt plays a key role in the model. Savers who turn impatient suffer disutility from delaying consumption until their debt matures because there is no secondary market where to sell the debt (or where to sell it at a sufficiently good price). If bank debt could be traded without frictions, impatient savers would sell their debts to patient savers. Banks could issue perpetual debt ( $\delta = 0$ ) at some initial period and get rid of refinancing concerns. In practice a lot of bank debt, starting with retail deposits, but including also certificates of deposit placed among the public, interbank deposits, debt involved in sales with repurchase agreements (repos), and commercial paper are commonly issued over the counter (OTC) and have no liquid secondary market.

Our model does not contain an explicit justification for the lack of tradability. Arguably, it might stem from administrative, legal compliance, and operational costs associated with the trading (specially using centralized trade) of heterogenous debt instruments issued in small amounts, with a short life or among a dispersed mass of unsophisticated investors. In fact, if other banks (or some other sophisticated traders) could possess better information about banks than ordinary savers, then costs associated with asymmetric information (e.g. exposure to a winners' curse problem in the acquisition of bank debt) might make the secondary market for bank debt unattractive to ordinary savers (Gorton and Pennacchi, 1990). This view is consistent with the common description of interbank markets as markets where peer monitoring is important (Rochet and Tirole, 1996).

Additionally, the literature in the Diamond and Dybvig (1983) tradition has demonstrated that having markets for the secondary trading of bank claims might damage the

 $<sup>^{46}</sup>$ See Brunnermeier and Oehmke (2013).

insurance role of bank deposits.<sup>47</sup> Yet, Diamond (1997) makes the case for the complementarity between banks and markets when, at least for some agents, the access to markets is not guaranteed.

Our model could be extended to describe situations in which debt is tradable but in a non-centralized secondary market characterized by search frictions (like in the models of OTC markets recently explored by Duffie, Gârleanu, and Pedersen, 2005, Vayanos and Weill, 2008, and Lagos and Rocheteau, 2009). In such setting, shortening the maturity of debt would have the effect of increasing the outside option of an impatient saver who is trying to find a buyer for his non-matured debt.<sup>48</sup> This could allow sellers to obtain better prices in the secondary market, making them willing to pay more for the debt in the first place and encouraging banks to issue short-term debt.<sup>49</sup>

## 9 Conclusion

We have developed and calibrated an infinite horizon equilibrium model in which banks with long-lived assets decide the overall principal, interest rate payments, and maturity of their debt (and, as residual, their equity financing). Savers' preference for short maturities comes from their exposure to idiosyncratic preference shocks and the lack of tradability of bank debt. Banks' incentive not to set debt maturities as short as savers might, ceteris paribus, prefer comes from the fact that there are episodes (systemic liquidity crises) in which their access to savers' funding fails and their refinancing becomes more expensive. Yet, pecuniary externalities in the market for funds during crises make unregulated debt maturities inefficiently short.

We have calibrated the model to Eurozone banking data for 2006 and assessed the value generated by maturity transformation, the size of the inefficiencies caused by the pecuniary externalities, and the potential welfare gains from optimally regulating bank debt maturities. In one extension we have evaluated the effects of an unanticipated and permanent increase

 $<sup>^{47}</sup>$ See von Thadden (1999) for an insightful review of the results obtained in this tradition.

 $<sup>^{48}</sup>$ He and Milbradt (2014) and Bruche and Segura (2013) explicitly model the secondary market for corporate debt as a market with search frictions.

<sup>&</sup>lt;sup>49</sup>In Bruche and Segura (2013), these trade-offs imply a privately optimal maturity for bank debt. The empirical evidence in Mahanti et al. (2008) and Bao, Pan, and Wang (2011), among others, shows that short-term bonds are indeed more "liquid" (as measured by the narrowness of the bid-ask spread) than long-term bonds.

in the severity of liquidity crises. In another we have analyzed the gains from introducing a liquidity insurance scheme, and documented its complementarity with liquidity regulation.

One surprising result of our analysis on the policy front is the undesirability of the NSFR of Basel III under its current formulation. While qualitatively pointing in the right direction, our quantitative results suggest that the limit it imposes on banks' maturity transformation is excessive. If introduced in the calibrated version of our model, the social surplus associated with the resulting allocation is significantly lower than in the absence of regulation.

## Appendix

## A Data description and calibration details

In this appendix we first describe the data on the structure of Eurozone banks' liabilities in 2006. Then we provide details on the steps followed in the calibration of the model.

#### A.1 Debt categories and outstanding amounts

The data on the outstanding debt liabilities of the aggregate Eurozone banking sector at the end of 2006 comes from the Monthly Bulletin and the Monetary Statistics published by the ECB. The breakdown shown in the first column of Table 2 is chosen to provide a convenient match with the sources of data that allow us to impute an average maturity to each debt category. For the first two categories in our table, we take figures reported in Section 2.5 of the Monthly Bulletin.

We adjust the original figures of the deposits from the various resident and non-resident sectors to exclude, whenever feasible, repurchase agreements, that we group in a separate category. *Deposits from households and non-financial corporations* (NFCs) are deposits (excluding repos) held by the corresponding euro area residents in euro area banks. *Deposits from other agents* includes deposits held by insurance corporations and pension funds (excluding repos), other financial intermediaries (excluding repos), general government, non-bank non-euro area residents, and money market funds (MMFs).<sup>50</sup>

Deposits and repos from other banks is a category created by adding and subtracting several items. It is the result of adding (i) deposits issued by euro area banks with euro area monetary and financial institutions (MFIs) (Monthly Bulletin, Section 2.1, Aggregate balance sheet of euro area MFIs) and (ii) deposits issued by euro area banks with non-euro area banks (Monthly Bulletin, Section 2.5), and subtracting (iii) lending from the Eurosystem to euro area banks (Monthly Bulletin, Section 1.1) and (iv) loans from euro area MMFs to euro area MFIs (Monetary Statistics, Aggregate balance sheet of euro area MMFs).

Other repos includes the repurchase agreements from households, NFCs, insurance corporations and pension funds, and other financial intermediaries. *Commercial paper and bonds* includes the outstanding principal of tradable debt securities issued by euro area banks (Monthly Bulletin, Section 2.7). Finally, *Eurosystem lending* is the lending made by the ECB and the national central banks of the euro area (Monthly Bulletin, Section 1.1).

 $<sup>^{50}</sup>$ For these last three sectors (whose deposits account for 5.9% of total bank debt) it is not possible to distinguish between unsecured deposits and secured deposits (repos).

#### A.2 Average $\delta$ for each debt category

For the first two categories on Table 2 we use the data on the maturity profile of the corresponding deposits (Monthly Bulletin, Section 2.5). The data distinguishes between the following maturities at issuance: (a) overnight, (b) up to two years, (c) more than two years, (d) redeemable at notice of up to three months, and (e) redeemable at notice of more than three months. We assign an average maturity  $\delta_j$  to each of these maturity intervals (j = a,b,c,d,e) and then compute a weighted average to obtain the average  $\delta$  of the corresponding debt category.<sup>51</sup> For the maturity interval (a), the probability that the deposit matures in a one month crisis is  $\delta_a = 1$ . For the maturity interval (b), we assume that the maturity at issuance of the corresponding debt is uniformly distributed in the interval from 0 to 24 months and that the issuance of this debt occurs in a perfectly staggered manner over time, which implies assigning  $\delta_b = 0.18$ .<sup>52</sup> Similar assumptions allow us to impute  $\delta_c = 0.025$ ,  $\delta_d = 0.33$  and  $\delta_e = 0$  to the remaining intervals.

To impute an average  $\delta$  to Deposits and repos from other banks, we use the Euro Money Market Survey of the ECB. This yearly survey reports the average daily volumes of euro denominated interbank borrowing and lending (secured and unsecured) transactions of a sample of European Union (EU) banks.<sup>53</sup> The data is broken down in the following intervals of maturity at issuance: one day, two days to one week, one week to one month, one to three months, three to six months, six to twelve months, more than twelve months. We set  $\delta_j = 1$ for all debt issued with maturity of less than one month and use uniformity assumptions to impute values of  $\delta_j$  to the remaining intervals in the same way we described for the previous

$$\delta_{\rm b} = \frac{1}{24} \left[ 1 + \int_1^{24} \frac{1}{t} dt \right] = 0.18.$$

Similarly, for a maturity interval  $[T_{1j}, T_{2j}]$  with  $T_{1j} > 1$ , we can use the formula

$$\delta_j = \frac{1}{T_{2j} - T_{1j}} \int_{T_{1j}}^{T_{2j}} \frac{1}{t} dt.$$

<sup>&</sup>lt;sup>51</sup>For three of the sectors whose deposits appear in *Deposits from other agents* (general government, non-bank non euro-area residents, and MMFs), there is no data on the maturity profile. To the deposits from the general government (11.3% of this category), we assign an average  $\delta$  equal to that of non-financial corporations. To those from non-bank non euro-area residents (29.9%), which are mostly non-bank financial intermediaries, and MMFs (1.9%) we impute an average  $\delta$  equal to that of the deposits from other financial intermediaries (which are also part of this category).

<sup>&</sup>lt;sup>52</sup>Under the stated assumptions, the probability that an outstanding debt with maturity at issuance of t months matures during a crisis that lasts one month is one if  $t \leq 1$ , and 1/t if t > 1. Integrating these probabilities over  $t \sim U[0, 24]$  we obtain:

 $<sup>^{53}</sup>$ For this and other categories described below, the data source refers to (samples of banks from) the whole EU rather than the Eurozone, and constitutes the best proxy to the reality of Eurozone banks in 2006 available to us.

two categories of deposits. For the more than twelve months interval, we assume a maximum maturity of twenty-four months.

To impute an average  $\delta$  to *Other repos*, we rely on Survey on the European Repo Market conducted by the International Capital Market Association (ICMA). This yearly survey reports the outstanding repo transactions of a sample of European financial groups, mainly banks. The survey distinguishes essentially the same maturity intervals as the Euro Money Market Survey.<sup>54</sup> An important difference is that the reported maturities are times-tomaturity instead of maturities-at-issuance, which implies assigning  $\delta_j = 1$  to all the intervals with time to maturity of less than one month and  $\delta_j = 0$  to residual maturities of more than one month.

To impute an average  $\delta$  to Commercial paper and bonds, we use data from the Risk Dashboard, a report published quarterly by the European Systemic Risk Board. Section 4.6 of the Risk Dashboard provides the outstanding amounts of debt securities issued by EU banks and their breakdown in a number of time-to-maturity intervals: less than one year, one to two years, two to three years, three to four years, four to five years, five to ten years, and more than ten years. With the same reasoning as in previous cases, we assign an average  $\delta_j = 1/12$  to debt in the first interval and  $\delta_j = 0$  to the remaining ones.

Finally, in the absence of precise published data on the maturity profile of *Eurosystem* lending and given that it accounts for only 2.1% of Eurozone bank debt in 2006, we exclude this category from the computation of the overall average  $\delta$  set as a target in our calibration.

#### A.3 Calibration details

In this section we provide details on each of the three steps undertaken for the calibration of the model and which yield the parameter values indicated in Table 1 of the main text.

The interest rate curve According to (4), the interest rate curve  $r(\delta)$  is fully determined by the parameters  $\rho_L, \rho_H$  and  $\gamma$ . Table 4 in Section 4.5 of the ECB's Monthly Bulletin provides the average interest rate  $r_{it}$  on outstanding deposits issued by Eurozone banks and held by domestic households for every month t = 1, 2, ..., 12 in 2006, and three maturity intervals: overnight deposits (j = 1), maturity of up to 2 years (j = 2), and maturity over 2 years (j = 3). To each maturity interval j, we assign a value  $\delta_j$  that corresponds to the probability that deposits in such interval mature within one month. Assuming that the second category is made of deposits with a maturity of one year and the third of deposits with a maturity of three years, we assign  $\delta_1 = 1$ ,  $\delta_2 = 1/12$ , and  $\delta_3 = 1/36$ . Finally set the

<sup>&</sup>lt;sup>54</sup>In fact, it includes an additional category to account for open-ended repos. These contracts can be terminated on demand and thus we assign  $\delta_j = 1$  to them.

calibrated values of  $\rho_L$ ,  $\rho_H$  and  $\gamma$  as those that minimize the sum of the square distances between the observed rates,  $r_{jt}$ , and the interest rates implied by the model,  $r(\delta_j)$ .

The frequency of crises  $\varepsilon$  The model predicts the full dilution of bankers' equity stakes in every crisis ( $\alpha = 1$ ). Setting  $\alpha = 1$  in (5) implies

$$\frac{\mu - rD}{E} = \rho_H + \varepsilon, \tag{17}$$

where  $E = E(D, \delta; 1)$  is the equilibrium value of bank equity in normal states. The LHS of this equation is the ratio of monthly dividends to the market value of equity in normal states. We proxy the value of this ratio in the data with the average return on equity of Eurozone banks in 2006, which reached a yearly value of 14.2% according the FRED database of the Saint Louis Fed. Given the value of  $\rho_H$  already obtained in the previous step, matching 0.142/12 using (17) yields our calibrated value of  $\varepsilon$ .

The equilibrium excess cost of crisis financing  $\phi^e$  As stated in Proposition 2, banks' optimal choice of  $\delta$  is a monotonically decreasing function of  $\phi$ . Given the target value of  $\delta^e$  (which, as previously described, is set so as to match the estimated average maturity of Eurozone banks' debt) and the remaining values of the parameters, we can numerically find the value of  $\phi^e$  under which  $\delta(\phi^e) = \delta^e$ . This is the value reported in Table 1.

## **B** Proofs

This appendix contains the proofs of the propositions included in the body of the paper.

**Proof of Proposition 1** Using (4) it is a matter of algebra to obtain that:

$$r'(\delta) = \frac{-\gamma(1+\rho_H)(\rho_H - \rho_L)}{\left[\rho_H + \delta + (1-\delta)\gamma\right]^2} < 0,$$
  
$$r''(\delta) = \frac{2\gamma(1-\gamma)(1+\rho_H)(\rho_H - \rho_L)}{\left[\rho_H + \delta + (1-\delta)\gamma\right]^3} > 0.$$

The other properties stated in the proposition are immediate.  $\blacksquare$ 

**Proof of Proposition 2** The proof is organized in a sequence of steps.

1. If (CF) is satisfied then (LL) is strictly satisfied Using equation (7), (LL) can be written as:

$$0 \le E(\delta, D; \phi) = \frac{1}{\rho_H} (\mu - rD) - \frac{1}{\rho_H} \frac{(1 + \rho_H)\varepsilon}{1 + \rho_H + \varepsilon} \left( 1 + \phi - \frac{1 + r}{1 + \rho_H} \right) \delta D,$$

while, using (8), (CF) can be written as

$$0 \leq \frac{1}{1+\rho_H} [\mu - r(1-\delta)D + \delta D + E(\delta, D; \phi)] - (1+\phi)\delta D =$$
$$= \frac{1}{\rho_H} (\mu - rD) - \left(1 + \frac{1}{\rho_H} \frac{\varepsilon}{1+\rho_H + \varepsilon}\right) \left(1 + \phi - \frac{1+r}{1+\rho_H}\right) \delta D.$$

Now, since  $1 + \frac{1}{\rho_H} \frac{\varepsilon}{1 + \rho_H + \varepsilon} > \frac{(1 + \rho_H)\varepsilon}{\rho_H(1 + \rho_H + \varepsilon)}$  we conclude that whenever (CF) is satisfied, (LL) is strictly satisfied.

#### 2. Notation and useful bounds Using equation (9) we can write:

$$V(\delta, D; \phi) = \frac{\mu}{\rho_H} + D\Pi(\delta; \phi), \tag{18}$$

where

$$\Pi(\delta,\phi) = 1 - \frac{1}{\rho_H} \left[ \left( 1 - \frac{\varepsilon}{1 + \rho_H + \varepsilon} \delta \right) r + \frac{(1 + \rho_H)\varepsilon}{1 + \rho_H + \varepsilon} \delta \left( \phi + \frac{\rho_H}{1 + \rho_H} \right) \right]$$

can be interpreted as the value the bank generates to its shareholders per unit of debt. Using Proposition 1 we can see that the function  $\Pi(\delta, \phi)$  is concave in  $\delta$ .

(CF) in equation (8) can be rewritten as:

$$\mu + V(\delta, D; \phi) \ge [(1 + \rho_H)(1 + \phi)\delta + (1 + r)(1 - \delta)]D,$$

and if we define  $C(\delta, \phi) = (1 + \rho_H)(1 + \phi)\delta + (1 + r)(1 - \delta)$ , (CF) can be written in a more compact form that will be used from now onwards:

$$\frac{1+\rho_H}{\rho_H}\mu + [\Pi(\delta,\phi) - C(\delta,\phi)]D \ge 0.$$
(19)

Using Proposition 1 we can see that the function  $C(\delta, \phi)$  is convex in  $\delta$ . We have the following relationship:

$$\Pi(\delta,\phi) = 1 - \frac{1}{\rho_H} \frac{1+\rho_H}{1+\rho_H+\varepsilon} \left[ r(\delta) + \frac{\varepsilon}{1+\rho_H} (C(\delta,\phi)-1) \right]$$
(20)

Assumption A1 implies  $(1 + \rho_H)(1 + \phi) \leq 2(1 + \rho_L) \leq 2(1 + r(\delta))$  for all  $\delta$ , and we can check that the following bounds (that are independent from  $\phi$ ) hold:

$$\frac{C(\delta,\phi)}{\partial \delta} \geq 1 + r(\delta).$$

$$\frac{\partial C(\delta,\phi)}{\partial \delta} \leq 2(1+r(\delta)) - (1+r(\delta)) = 1 + r(\delta).$$
(21)

Using assumption A2, it is a matter of algebra to check that, for all  $\delta$ ,

$$\frac{d^2r}{d\delta^2} + \frac{dr}{d\delta} \ge 0$$

And, from this inequality,  $\frac{dr}{d\delta} < 0$ , and  $r < \rho_H$ , it is possible to check that:

$$\frac{\partial^2 \Pi(\delta,\phi)}{\partial \delta^2} + \frac{\partial \Pi(\delta,\phi)}{\partial \delta} < -\frac{1}{\rho_H} \left( 1 - \frac{\varepsilon}{1 + \rho_H + \varepsilon} \delta \right) \left( \frac{dr}{d\delta} + \frac{d^2 r}{d\delta^2} \right) \le 0.$$
(22)

To save on notation, we will drop from now on the arguments of these functions when it does not lead to ambiguity.

**3.**  $D^* = 0$  is not optimal It suffices to realize that  $\frac{\partial V(D,0;\phi)}{\partial D} = \Pi(0,\phi) = 1 - \frac{r(0)}{\rho_H} > 0.$ 

4. The solution  $(D^*, \delta^*)$  of the maximization problem in equation (10) exists, is unique, and satisfies (CF) with equality, i.e.  $\frac{1+\rho_H}{\rho_H}\mu + (\Pi(\delta^*, \phi) - C(\delta^*, \phi))D^* = 0$ 

We are going to prove existence and uniqueness in the particular case that there exist  $\delta_{\Pi}, \delta_C \in [0, 1]$  such that  $\frac{\partial \Pi(\delta_{\Pi}, \phi)}{\partial \delta} = \frac{\partial C(\delta_C, \phi)}{\partial \delta} = 0$ . This will ensure that the solution of the maximization problem is interior in  $\delta$ . The other cases are treated in an analogous way but might give rise to corner solutions in  $\delta$ .<sup>55</sup>

First, since  $\Pi(\delta, \phi)$  is concave in  $\delta$  we have that  $\frac{\partial \Pi(\delta, \phi)}{\partial \delta} \ge 0$  iff  $\delta \le \delta_{\Pi}$ . Since  $C(\delta, \phi)$  is convex in  $\delta$  we have that  $\frac{\partial C(\delta, \phi)}{\partial \delta} \ge 0$  iff  $\delta \ge \delta_C$ . It is easy to prove from equation (20) that  $\delta_C < \delta_{\Pi}$ .

Now, let  $(\delta^*, D^*)$  be a solution to the maximization problem. The first order conditions (FOC) that characterize an interior solution  $(\delta^*, D^*)$  are:

$$(1+\theta)\Pi - \theta C = 0,$$
  

$$(1+\theta)\frac{\partial\Pi}{\partial\delta} - \theta\frac{\partial C}{\partial\delta} = 0,$$
  

$$\theta \left[\frac{1+\rho_H}{\rho_H}\mu + (\Pi - C)D^*\right] \geq 0,$$
  

$$\theta \geq 0,$$
(23)

where  $\theta$  is the Lagrange multiplier associated with (CF) and we have used that  $D^* > 0$  in order to eliminate it from the second equation.

If  $\theta = 0$  then the second equation implies  $\delta^* = \delta_{\Pi}$  and thus  $\Pi(\delta^*, \phi) \ge \Pi(0, \phi) > 0$  and the first equation is not satisfied. Therefore we must have  $\theta > 0$  so that (CF) is binding at the optimum. Now we can eliminate  $\theta$  from the previous system of equations, which gets reduced to:

$$\frac{\partial \Pi(\delta^*, \phi)}{\partial \delta} C(\delta^*, \phi) = \frac{\partial C(\delta^*, \phi)}{\partial \delta} \Pi(\delta^*, \phi), \qquad (24)$$

$$\frac{1+\rho_H}{\rho_H}\mu = [C(\delta^*,\phi) - \Pi(\delta^*,\phi)] D^*.$$
(25)

<sup>&</sup>lt;sup>55</sup>More precisely, if for all  $\delta \in [0, 1]$   $\frac{\partial C(\delta, \phi)}{\partial \delta} > 0$  we might have  $\delta^* = 0$  and if for all  $\delta \in [0, 1]$ ,  $\frac{\partial \Pi(\delta, \phi)}{\partial \delta} > 0$  we might have  $\delta^* = 1$ .

We are going to show that equation (24) has a unique solution in  $\delta$ . For  $\delta \leq \delta_C < \delta_{\Pi}$ , we have  $\frac{\partial C}{\partial \delta} \leq 0 < \frac{\partial \Pi}{\partial \delta}$  and thus the left hand side (LHS) of (24) is strictly bigger than the RHS. For  $\delta \geq \delta_{\Pi} > \delta_C$ , we have  $\frac{\partial \Pi}{\partial \delta} \leq 0 < \frac{\partial C}{\partial \delta}$  and thus RHS of (24) is strictly bigger. Now, the function  $\frac{\partial C(\delta,\phi)}{\partial \delta} \Pi(\delta,\phi)$  is strictly increasing in the interval  $(\delta_C, \delta_{\Pi})$  since both

Now, the function  $\frac{\partial C(\delta,\phi)}{\partial \delta} \Pi(\delta,\phi)$  is strictly increasing in the interval  $(\delta_C, \delta_{\Pi})$  since both terms are positive and increasing. Thus, it suffices to prove that for  $\delta \in (\delta_C, \delta_{\Pi})$  the function  $\frac{\partial \Pi(\delta,\phi)}{\partial \delta} C(\delta,\phi)$  is decreasing.<sup>56</sup> Using the the bounds in (21), inequality (22) and  $\frac{\partial^2 \Pi}{\partial \delta^2} < 0$ ,  $\frac{\partial \Pi}{\partial \delta} > 0$  for  $\delta \in (\delta_C, \delta_{\Pi})$ , we have:

$$\frac{\partial}{\partial \delta} \left( \frac{\partial \Pi}{\partial \delta} C \right) = \frac{\partial^2 \Pi}{\partial \delta^2} C + \frac{\partial \Pi}{\partial \delta} \frac{\partial C}{\partial \delta} \le (1+r) \left( \frac{\partial^2 \Pi}{\partial \delta^2} + \frac{\partial \Pi}{\partial \delta} \right) \le 0.$$

This concludes the proof on the existence and uniqueness of a  $\delta^*$  that satisfies the necessary FOC in (24).

Now, for given  $\delta^*$ , the other necessary FOC (25) determines  $D^*$  uniquely.<sup>57</sup>

5.  $\delta^*$  is independent from  $\mu$  and  $D^*$  is strictly increasing in  $\mu$  Equation (24) determines  $\delta^*$  and is independent from  $\mu$ . Then equation (25) shows that  $D^*$  is increasing in  $\mu$ .

6.  $\delta^*$  is decreasing in  $\phi$  and, if  $\delta^* \in (0, 1)$ , it is strictly decreasing Let  $\delta(\phi)$  be the solution of the maximization problem of the bank for given  $\phi$ . Let us assume that  $\delta(\phi)$  satisfies the FOC (24). The case of corner solutions is analyzed in an analogous way.

We have proved in Step 3 above that the function  $\frac{\partial \Pi}{\partial \delta}C - \frac{\partial C}{\partial \delta}\Pi$  is decreasing in  $\delta$  around  $\delta(\phi)$ . In order to show that  $\delta(\phi)$  is decreasing, it suffices to show that the derivative of this function w.r.t.  $\phi$  is negative. Using the definitions of  $C(\delta, \phi), \Pi(\delta, \phi)$  after some (tedious) algebra we obtain:

$$\frac{\partial}{\partial \phi} \left[ \frac{\partial \Pi}{\partial \delta} C - \frac{\partial C}{\partial \delta} \Pi \right] = -(1 + \rho_H) - \frac{1}{\rho_H} \frac{1 + \rho_H}{1 + \rho_H + \varepsilon} \left[ (1 + \rho_H) \left( \frac{dr}{d\delta} \delta - r \right) + \varepsilon \right]$$

Now we have  $\frac{d}{d\delta} \left( \frac{dr}{d\delta} \delta - r \right) = \frac{d^2r}{d\delta^2} \delta \ge 0$  and thus  $\frac{dr}{d\delta} \delta - r \ge \frac{dr}{d\delta} \delta - r \Big|_{\delta=0} = -r(0)$ , and finally:

$$\frac{\partial}{\partial \phi} \left[ \frac{\partial \Pi}{\partial \delta} C - \frac{\partial C}{\partial \delta} \Pi \right] \leq -(1+\rho_H) - \frac{1}{\rho_H} \frac{1+\rho_H}{1+\rho_H+\varepsilon} \left[ -(1+\rho_H)r(0) + \varepsilon \right]$$
$$< -(1+\rho_H) + \frac{1}{\rho_H} (1+\rho_H)r(0) = -(1+\rho_H) \left( 1 - \frac{r(0)}{\rho_H} \right) < 0.$$

This concludes the proof that  $\frac{d\delta}{d\phi} < 0.^{58}$ 

## 7. $\delta^* D^*$ is decreasing with $\phi$ . If $\delta^* > 0$ it is strictly decreasing

<sup>&</sup>lt;sup>56</sup>This is not trivial since  $C(\delta, \phi)$  is increasing.

<sup>&</sup>lt;sup>57</sup>Let us observe that for all  $\delta$ ,  $C(\delta, \phi) \ge 1 > \Pi(\delta, \phi)$ .

<sup>&</sup>lt;sup>58</sup>In the case of corner solution  $\delta^*(\phi) = 1$ , we might have  $\frac{d\delta^*}{d\phi} = 0$  and obviously for  $\delta^*(\phi) = 0$ ,  $\frac{d\delta^*}{d\phi} = 0$ .

Let  $\delta(\phi)$ ,  $D(\phi)$  be the solution of the maximization problem of the bank for given  $\phi$ . We have:

$$\frac{1+\rho_H}{\rho_H}\mu = \left[C(\delta(\phi),\phi) - \Pi(\delta(\phi),\phi)\right]D(\phi).$$

Let  $\phi_1 < \phi_2$ . In Step 6 we showed that  $\delta(\phi_1) \ge \delta(\phi_2)$ . If  $\delta(\phi_2) = 0$  then trivially  $\delta(\phi_1)D(\phi_1) \ge \delta(\phi_2)D(\phi_2) = 0$ . Let us suppose that  $\delta(\phi_2) > 0$ . Since trivially  $\Pi(\delta(\phi_1), \phi_1)D(\phi_1) \ge \Pi(\delta(\phi_2); \phi_2)D(\phi_2)$ , we must have  $C(\delta(\phi_1), \phi_1)D(\phi_1) \ge C(\delta(\phi_2), \phi_2)D(\phi_2)$ . Now, suppose that  $\delta(\phi_1)D(\phi_1) \le \delta(\phi_2)D(\phi_2)$ , then we have the following two inequalities:

$$(1+\rho_H)(1+\phi_1)\delta(\phi_1)D(\phi_1) < (1+\rho_H)(1+\phi_2)\delta(\phi_2)D(\phi_2), (1+r(\delta(\phi_1)))(1-\delta(\phi_1)) \le (1+r(\delta(\phi_2)))(1-\delta(\phi_2)),$$

that imply  $C(\delta(\phi_1), \phi_1)D(\phi_1) < C(\delta(\phi_2), \phi_2)D(\phi_2)$ , but this contradicts our assumption. Thus,  $\delta(\phi_1)D(\phi_1) > \delta(\phi_2)D(\phi_2)$ .

#### **Proof of Proposition 3** This proof has two parts:

1. Existence and uniqueness of equilibrium. Let us denote  $(\delta(\phi), D(\phi))$  the solution of the bank's optimization problem for every excess cost of crisis financing  $\phi \ge 0$ . Proposition 2 states that  $\delta(\phi)D(\phi)$  is decreasing in  $\phi$ . For  $\phi \in [0, \overline{\phi}]$  let us define  $\Sigma(\phi) = \Phi(\delta(\phi)D(\phi)) - \phi$ . This function represents the difference between the excess cost of financing during a crisis by banks' decisions and banks' expectation on such variable. Since  $\Phi$  is an increasing function on the aggregate demand of funds during a crisis the function  $\Sigma(\phi)$  is strictly decreasing. Because of the uniqueness of the solution to the problem that defines  $(\delta(\phi), D(\phi))$ , the function is also continuous. Moreover, we trivially have  $\Sigma(0) \ge 0$  and  $\Sigma(\overline{\phi}) \le 0$ . Therefore there exists a unique  $\phi^e \in \mathbb{R}^+$  such that  $\Sigma(\phi^e) = 0$ . By construction  $(\phi^e, (\delta(\phi^e), D(\phi^e)))$  is the unique equilibrium of the economy.

2. Comparative statics with respect to a shift in  $\Phi(x)$ . We are going to follow the notation used in the proof of Proposition 3. Let  $\Phi_1$ ,  $\Phi_2$  be two curves describing the inverse supply of financing during a crisis and assume they satisfy  $\Phi_1(x) > \Phi_2(x)$  for all x > 0. Let us denote  $\Sigma_i(\phi) = \Phi_i(\delta(\phi)D(\phi)) - \phi$  for i = 1, 2. By construction we have  $\Sigma_1(\phi_1^e) = 0$ . Let us suppose that  $\phi_1^e < \phi_2^e$ . Then we would have:

$$\Sigma_{2}(\phi_{2}^{e}) = \Phi_{2}(\delta(\phi_{2}^{e})D(\phi_{2}^{e})) - \phi_{2}^{e} \leq \Phi_{1}(\delta(\phi_{2}^{e})D(\phi_{2}^{e})) - \phi_{2}^{e} < \Phi_{1}(\delta(\phi_{1}^{e})D(\phi_{1}^{e})) - \phi_{1}^{e} = \Sigma_{1}(\phi_{1}^{e}) = 0,$$
(26)

where in the first inequality we use the assumption  $\Phi_2(x) \leq \Phi_1(x)$  for  $x \geq 0$ , and in the second inequality we use that if  $\phi_1^e < \phi_2^e$  then  $\delta(\phi_2^e)D(\phi_2^e) \leq \delta(\phi_1^e)D(\phi_1^e)$  (Proposition 2), and that  $\Phi_1(\cdot)$  is increasing. Notice that the sequence of inequalities in (26) implies  $\Sigma_2(\phi_2^e) < 0$ , which contradicts the definition of  $\phi_2^e$ . We must therefore have  $\phi_1^e \geq \phi_2^e$ . And Proposition 2 implies that  $\delta_1^e \leq \delta_2^e$ ,  $\delta_1^e D_1^e \leq \delta_2^e D_2^e$ , and  $r_1^e \geq r_2^e$ , proving all the results in weak terms.

Finally, let us suppose that  $\delta_2^e \in (0, 1)$ . Then the first inequality in (26) is strict, since  $\delta_2^e D_2^e > 0$ , and we can straightforwardly check that the previous argument implies  $\phi_1^e > \phi_2^e$ . In this case, since  $\delta_2^e \in (0, 1)$ , Proposition 2 implies that  $\delta_1^e < \delta_2^e$ ,  $\delta_1^e D_1^e < \delta_2^e D_2^e$ , and  $r_1^e > r_2^e$ .

**Proof of Proposition 4** We are going to follow the notation used in the proof of Proposition 2. The proof is organized in five steps:

1. Preliminaries From first principles, using equations (9) and (12), we can obtain

$$\frac{\partial W(\delta, D)}{\partial \delta} = \frac{\partial V(\delta, D; \Phi(\delta D))}{\partial \delta} = D \frac{\partial \Pi(\delta, \Phi(\delta D))}{\partial \delta}, \tag{27}$$

where the last equality follows from (18). Similarly we can obtain

$$\frac{\partial W(\delta, D)}{\partial D} = \frac{\partial V(\delta, D; \Phi(\delta D))}{\partial D} = \Pi(\delta, \Phi(\delta D)).$$
(28)

2. (CF) is binding at the socially optimal debt structure This is a statement that has been done in the main text just before Proposition 4. The proof is analogous to the one for the maximization problem of the bank that we did in Step 4 of the proof of Proposition 2. The only difference is that  $\phi$  is not taken as given but as the function  $\Phi(\delta D)$  in D and  $\delta$ .

3. Definition of function  $D^{c}(\delta)$  and its properties Let  $(\phi^{e}, (\delta^{e}, D^{e}))$  be the competitive equilibrium. Let us assume that  $\delta^{e} < 1$ . By definition of equilibrium we have  $\phi^{e} = \Phi(\delta^{e}D^{e})$ . For every  $\delta$  let  $D^{c}(\delta)$  be the unique principal of debt such that (CF) is binding, i.e.:

$$\frac{1+\rho_H}{\rho_H}\mu = \left[C(\delta,\phi^e) - \Pi(\delta,\phi^e)\right]D^c(\delta).$$
(29)

Differentiating w.r.t.  $\delta$ :

$$\left[\frac{\partial C(\delta,\phi^e)}{\partial\delta} - \frac{\partial\Pi(\delta,\phi^e)}{\partial\delta}\right] D^c(\delta) + \left[C(\delta,\phi^e) - \Pi(\delta,\phi^e)\right] \frac{dD^c(\delta)}{d\delta} = 0.$$
(30)

Using the characterization of  $\delta^e$  in equation (24), the inequalities  $C(\delta, \phi^e) \geq 1 > \Pi(\delta, \phi^e)$ imply  $\frac{\partial C(\delta^e, \phi^e)}{\partial \delta} - \frac{\partial \Pi(\delta^e, \phi^e)}{\partial \delta} > 0$  and, then, we can deduce from the equation above that  $\frac{dD^c(\delta^e)}{d\delta} < 0$ . Since (CF) is binding at the optimal debt structure we can think of the bank problem as maximizing the univariate function  $V(\delta, D^c(\delta); \phi^e)$ . Hence  $\delta^e$  must satisfy the necessary FOC for an interior solution to the maximization of  $V(\delta, D^c(\delta); \phi^e)$ :

$$\frac{dV(\delta^e, D^c(\delta); \phi^e)}{d\delta} = 0 \Leftrightarrow D^c(\delta^e) \frac{\partial \Pi(\delta^e, \phi^e)}{\partial \delta} + \Pi(\delta^e, \phi^e) \frac{dD^c(\delta^e)}{d\delta} = 0,$$
(31)

which multiplying by  $\delta^e$  can be written as

$$D^{c}(\delta^{e})\frac{\partial\Pi(\delta^{e},\phi^{e})}{\partial\delta^{e}}\delta^{e} = \Pi(\delta^{e},\phi^{e})\left(-\frac{dD^{c}(\delta^{e})}{d\delta}\delta^{e}\right)$$

Since  $\frac{\partial \left(\Pi - \frac{\partial \Pi}{\partial \delta} \delta\right)}{\partial \delta} = -\frac{\partial^2 \Pi}{\partial \delta^2} \delta \ge 0$  and  $\Pi(0, \phi) - \frac{\partial \Pi(0, \phi)}{\partial \delta} 0 > 0$ , we have  $\Pi(\delta, \phi) > \frac{\partial \Pi(\delta, \phi)}{\partial \delta} \delta$  for all  $\delta \in [0, 1]$  and the previous equation implies

$$D^{c}(\delta^{e}) > -\frac{dD^{c}(\delta^{e})}{d\delta}\delta^{e} \Leftrightarrow \left.\frac{d\left(\delta D^{c}(\delta)\right)}{d\delta}\right|_{\delta=\delta^{e}} > 0.$$

4. Evaluation of  $\frac{d(D^s(\delta))}{d\delta}\Big|_{\delta=\delta^e}$  and  $\frac{d(\delta D^s(\delta))}{d\delta}\Big|_{\delta=\delta^e}$  For every  $\delta$ , let  $D^s(\delta)$  be the unique principal of debt such that (CF) is binding, i.e.

$$\frac{1+\rho_H}{\rho_H}\mu = \left[C(\delta, \Phi(\delta D^s(\delta))) - \Pi(\delta, \Phi(\delta D^s(\delta)))\right] D^s(\delta)$$

Differentiating w.r.t.  $\delta$ , we obtain

$$\left[\frac{\partial C(\delta, \Phi)}{\partial \delta} - \frac{\partial \Pi(\delta, \Phi)}{\partial \delta}\right] D^{s}(\delta) + \left[C(\delta, \Phi(\delta D^{s}(\delta))) - \Pi(\delta, \Phi(\delta D^{s}(\delta)))\right] \frac{dD^{s}(\delta)}{d\delta} + \\
+ \left[\frac{\partial C(\delta, \Phi)}{\partial \phi} - \frac{\partial \Pi(\delta, \Phi)}{\partial \phi}\right] \Phi'(\delta D^{s}(\delta))) \frac{d(\delta D^{s}(\delta))}{d\delta} = 0.$$
(32)

By construction,  $D^s(\delta^e) = D^c(\delta^e) = D^e$ . Now, subtracting equation (30) from equation (32) at the point  $\delta = \delta^e$  we obtain

$$\begin{bmatrix} C(\delta^{e}, \phi^{e}) - \Pi(\delta^{e}, \phi^{e}) \end{bmatrix} \left( \frac{dD^{s}(\delta^{e})}{d\delta} - \frac{dD^{c}(\delta^{e})}{d\delta} \right) + \left[ \frac{\partial C(\delta^{e}, \phi^{e})}{\partial\phi} - \frac{\partial \Pi(\delta^{e}, \phi^{e})}{\partial\phi} \right] \Phi'(\delta^{e}D^{e}) \left. \frac{d\left(\delta D^{s}(\delta)\right)}{d\delta} \right|_{\delta = \delta^{e}} = 0.$$
(33)

Suppose that  $\frac{d(\delta D^s(\delta))}{d\delta}\Big|_{\delta=\delta^e} \leq 0$ , then we would have  $\frac{dD^s(\delta^e)}{d\delta} \geq \frac{dD^c(\delta^e)}{d\delta}$ , since trivially  $\frac{\partial C(\delta^e, \phi^e)}{\partial \phi} - \frac{\partial \Pi(\delta, \phi^e)}{\partial \phi} > 0$ . But then

$$\frac{d\left(\delta D^{s}(\delta)\right)}{d\delta}\bigg|_{\delta=\delta^{e}} = D^{s}(\delta^{e}) + \frac{dD^{s}(\delta^{e})}{d\delta}\delta^{e} > D^{c}(\delta^{e}) + \frac{dD^{c}(\delta^{e})}{d\delta}\delta^{e} = \frac{d\left(\delta D^{c}(\delta)\right)}{d\delta}\bigg|_{\delta=\delta^{e}} > 0,$$

which contradicts the hypothesis. We must thus have  $\frac{d(\delta D^s(\delta))}{d\delta}\Big|_{\delta=\delta^e} > 0$ , in which case equation (33) implies  $\frac{dD^s(\delta^e)}{d\delta} < \frac{dD^c(\delta^e)}{d\delta} < 0$ .

5. Evaluation of 
$$\frac{dW(\delta, D^s(\delta))}{d\delta}\Big|_{\delta=\delta^e}$$
 Using equations (27) and (28), we have:  

$$\frac{dW(\delta, D^s(\delta))}{d\delta} = \frac{\partial W(\delta, D^s(\delta))}{\partial\delta} + \frac{\partial W(\delta, D^s(\delta))}{\partial D} \frac{dD^s(\delta)}{d\delta}$$

$$= D^s(\delta) \frac{\partial \Pi(\delta, \Phi(\delta D^s(\delta)))}{\partial\delta} + \Pi(\delta, \Phi(\delta D^s(\delta))) \frac{dD^s(\delta)}{d\delta}.$$

And, using  $\frac{dD^s(\delta^e)}{d\delta} < \frac{dD^c(\delta^e)}{d\delta}$  and (31), we obtain:

$$\frac{dW(\delta, D^s(\delta))}{d\delta}\bigg|_{\delta=\delta^e} < D^s(\delta^e)\frac{\partial\Pi(\delta^e, \phi^e)}{\partial\delta} + \Pi(\delta^e, \phi^e)\frac{dD^c(\delta^e)}{d\delta} = 0.$$

Summing up, having

$$\frac{dW(\delta, D^s(\delta))}{d\delta}\bigg|_{\delta=\delta^e} < 0, \ \left.\frac{dD^s(\delta)}{d\delta}\right|_{\delta=\delta^e} < 0, \text{ and } \left.\frac{d(\delta D^s(\delta))}{d\delta}\right|_{\delta=\delta^e} > 0,$$

implies that a social planner can increase welfare by fixing some  $\delta^s < \delta^e$ , and suggests that doing so will produce higher leverage and lower refinancing needs than in the unregulated competitive equilibrium.

**Proof of Proposition 5** Let us recall that the introduction of (fairly priced) insurance does not change the value of equity at the N state, i.e.  $E(\delta, D, \lambda; \phi) = E(\delta, D; \phi)$  for all  $\lambda$ . In addition banks choose full insurance,  $\lambda = 1$ , and the only financial constraint is (LL) that can be written  $E(\delta, D, 1; \phi) = E(\delta, D; \phi) > 0$ . For the next steps, we follow the notation introduced in the proof of Proposition 2.

1. Insurance increases social welfare in the regulated economy Let  $(\delta^s, D^s)$  be the socially optimal debt structure in the absence of insurance. In the proof of Proposition 4 we showed that (CF) is binding at  $(\delta^s, D^s)$ . In fact, we have  $\frac{\partial W(\delta^s, D^s)}{\partial D} > 0$ . Step 1 in the proof of Proposition 2 states that (LL) is satisfied with slack, i.e.

$$E(\delta^s, D^s; \Phi(\delta^s D^s)) > 0,$$

and thus by continuity there are values  $D' > D^s$  such that  $E(\delta^s, D'; \Phi(\delta^s D')) > 0$  and  $W(\delta^s, D') > W(\delta^s, D^s)$ . Introducing insurance makes debt structures such as  $(\delta^s, D')$  feasible and, hence, increases welfare relative to the regulated economy without insurance.

2. Under insurance the competitive expected maturity is shorter than the socially optimal one When insurance is introduced the relevant financial constraint faced both by banks in the unregulated equilibrium (for given  $\phi$ ) and by the social planner (for  $\phi = \Phi(\delta D)$ ) is (LL) and is binding. From here, the proof is analogous to that of Proposition 4 and we omit it for brevity.

## C Debt structures that induce default during crises

In this appendix we examine the possibility that a bank decides to expose itself to the risk of defaulting on its debt and being liquidated in crisis states. First, we describe the sequence

of events following default. Second, we show how the debt of the bank would be valued by savers who correctly anticipate the possibility of default. Finally, we analyze the bank's capital structure problem when default during crises is an explicit alternative.

**Default and liquidation** Liquidation following the bank's inability to satisfy its refinancing needs yields a residual value  $L \ge 0$ . Suppose that partial liquidation is not allowed and Lis distributed equally among all debtholders independently of their contract having just matured or not. This eliminates the type of preemptive runs studied by He and Xiong (2012b). It is easy to realize that if the bank does not want to rely on crisis financing (exposing itself to possible default in a crisis), then it will find it optimal to make its debt mature in a perfectly correlated manner since this minimizes the probability of default during crises. Hence we assume that the debt issued by the bank when getting rid of the (CF) constraint has perfectly correlated maturities.

Savers' required maturity premium when default is anticipated From a saver's perspective, there are four states relevant for the valuation of a given debt contract: personal patience in a normal period (i = LN), personal patience in a crisis period (i = LC), personal impatience in a normal period (i = HN), and personal impatience in a crisis period (i = HC).

Let l = L/D < 1 be the fraction of the principal of debt which is recovered in case of liquidation and let  $Q_i$  be the present value of expected losses due to default as evaluated from each of the states *i* just after the uncertainty regarding the corresponding period has realized and conditional on the debt not having matured in such period. Losses are measured relative to the benchmark case without default in which at maturity savers recover 100% of the principal. These values satisfy the following system of recursive relationships:

$$\begin{aligned} Q_{LN} &= \frac{1}{1+\rho_L} \left[ \delta \varepsilon (1-l) + (1-\delta) \{ (1-\varepsilon) [(1-\gamma)Q_{LN} + \gamma Q_{HN}] + \varepsilon [(1-\gamma)Q_{LC} + \gamma Q_{HC}] \} \right], \\ Q_{HN} &= \frac{1}{1+\rho_H} \left[ \delta \varepsilon (1-l) + (1-\delta) \left[ (1-\varepsilon)Q_{LN} + \varepsilon Q_{HC} \right] \right], \\ Q_{LC} &= \frac{1}{1+\rho_L} (1-\delta) [(1-\gamma)Q_{LN} + \gamma Q_{HN}], \\ Q_{HC} &= \frac{1}{1+\rho_H} (1-\delta)Q_{HN}. \end{aligned}$$

These expressions essentially account for the principal 1 - l > 0 which is lost whenever the saver's debt contract matures in a state of crisis. First equation reflects that default as well as any of four states *i* may follow state *LN*. The second equation reflects that impatience is

an absorbing state. The third and fourth equations reflect that a crisis period can only be followed by a normal period. We will denote the solution of this linear system of equations  $Q_{LN}(\delta, D; L, \varepsilon)$  in order to highlight its dependence on these variables.

The value of a debt contract  $(r, \delta, 1)$  to a patient saver in a normal period, when default is expected if the bank runs into refinancing needs during a crisis, can then be written as

$$U_L^d(r,\delta) = U_L(r,\delta) - Q_{LN}(\delta,D;L,\varepsilon),$$

where  $U_L(r, \delta)$  is the value of the same contract in the scenario in which the principal is always recovered at maturity, whose expression is given in (3).

Now, let  $r^d(\delta)$  be the interest rate yield that the bank offers in the default setting, which satisfies  $U_L^d(r^d(\delta), \delta) = 1$ . Since the non-default yield  $r(\delta)$  satisfies  $U_L(r(\delta), \delta) = 1$ , the equation  $U_L^d(r^d(\delta), \delta) = U_L(r(\delta), \delta)$  allows us to express  $r^d(\delta)$  as the sum of  $r(\delta)$  and a default-risk premium:

$$r^{d}(\delta) = r(\delta) + \frac{(\rho_{H} + \delta)(\rho_{L} + \delta + (1 - \delta)\gamma)}{\rho_{H} + \delta + (1 - \delta)\gamma} Q_{LN}(\delta, D; L, \varepsilon).$$

One can prove that the default-risk premium  $r^d(\delta) - r(\delta)$  is increasing in  $\delta$ , increasing in  $\varepsilon$ , decreasing in L, and increasing in D. Given that  $\delta$  increases the probability of default,  $r^d(\delta)$  is not necessarily decreasing in  $\delta$ .

**Banks' optimal capital structure inducing default** If the bank does not satisfy the crisis financing constraint and thus defaults whenever it faces refinancing needs during a crisis, its equity value in a normal state  $E^d(\delta, D)$  will satisfy the following recursive equation:

$$E^{d}(\delta, D) = \frac{1}{1+\rho_{H}} \left[ \mu - r^{d}D + (1-\varepsilon)E^{d}(\delta, D) + \varepsilon \{\delta \cdot 0 + (1-\delta)\frac{1}{1+\rho_{H}} [\mu - r^{d}D + E^{d}(\delta, D)] \} \right],$$

whose solution yields:

$$E^{d}(\delta, D) = \frac{1 + \rho_{H} + \varepsilon(1 - \delta)}{(1 + \rho_{H})^{2} - (1 + \rho_{H})(1 - \varepsilon) - \varepsilon(1 - \delta)}(\mu - r^{d}D)$$

In this context, the problem determining the bank's optimal debt structure decision in the absence of the crisis financing constraint can be written as:

$$\max_{\substack{D \ge 0, \ \delta \in [0,1]\\ \text{s.t.}}} V^d(\delta, D) = D + E^d(\delta, D),$$
(34)  
s.t.  $E^d(\delta, D) \ge 0,$ (LL)

where (LL) is trivially equivalent to  $\mu - r^d D \ge 0$ .

To find the maximum value of L consistent with the equilibrium in which banks optimize subject to the (CF) constraint (and, hence, do not default), we proceed in two steps. First, we solve the problem in (34) numerically for an ample grid of values of L. Second, we compare the optimized value of the objective function in (34) with that attained by solving the problem in which (CF) is satisfied under the value of  $\phi^e$  set in our calibration.  $L^{\max}$  is the maximum L for which the solution of the problem with no default dominates.

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