The Role of Liquidity Standards in Optimal Lending of Last Resort Policies[*]

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Abstract

We consider a model in which banks vulnerable to liquidity crises may receive support from the lender of last resort (LLR). Higher liquidity standards, though costly to banks, give the LLR more time to find out the systemic implications of denying support to the banks in trouble. By modifying banks' prospects of being supported in a crisis, liquidity standards affect banks' adoption of precautions against a crisis. We show that this effect is positive (negative) when banks are ex ante perceived to be systemically important (unimportant). We analyze the implications of these results for the design of liquidity standards and LLR policies.

Keywords: Liquidity standards, lender of last resort, constructive ambiguity

JEL Classification: G01, G21, G28

1 Introduction

Prior to the Great Recession, the focus of bank regulation was on bank capital. However, the liquidity problems that banks experienced since the onset of the financial crisis in 2007 brought to the forefront a debate about the potential value of regulating banks' liquidity.[1] In this paper, we contribute to this debate by presenting a novel theory of banks' liquidity standards.

Our theory builds on what we believe is a distinctive feature of an instrument such as the liquidity coverage ratio of Basel III: Once a liquidity crisis starts, the presence of liquidity standards gives time to the lender of last resort (LLR) to ascertain the potential implications of an early, disordered liquidation of the bank in trouble. Key to our theory is the assumption that banks can tap on the liquidity buffers associated with the standards in crises periods. We identify circumstances under which liquidity standards can improve the ex ante efficiency of the LLR policy by adding credibility to the strategy of denying support to banks.

We consider a model in which liquidity crises may lead banks into failure, unless they are able to borrow from the LLR. In making this lending decision, the LLR faces the classical problem that some of the banks seeking liquidity support may be fundamentally insolvent.[2] While it is optimal to grant liquidity to solvent banks, in the case of an insolvent bank, LLR support may still make sense if the negative externalities that will arise with the bank's early liquidation are sufficiently large. But assessing these externalities in real time is quite difficult. Following this view, we assume that the LLR is generally uncertain about the extent of the external costs of an early liquidation but, and this is where liquidity standards come into play, the presence of liquidity standards increases the likelihood of the LLR knowing whether its decision to deny liquidity support will lead to a systemic crisis. The idea is that liquidity standards, by lengthening the time a bank can sustain a liquidity shock, give the LLR more time to find out the systemic implications of a bank's default.

The buying-time role of liquidity standards is also relevant for another reason: moral

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[1]See Gorton (2009) and Shin (2010) for a discussion on the role of banks' liquidity problems during the most recent financial crisis.

[2]In fact, the model simplifies along this dimension and assumes all the banks in seek of support are insolvent.

hazard. If banks anticipate generous support from the LLR during a crisis, they are likely to undertake lower precautions against a crisis. When LLR's support policy is set in a time-consistent manner, the extent to which the LLR knows the systemic implications of a bank's failure when deciding on it will have an impact on the credibility of a threat not to support certain types of banks.[3]

In our setting, if the LLR knows the external costs of a bank failure, ex post it is optimal for the LLR to refuse liquidity support when these costs are low (the bank is not systemically important) and to provide liquidity support when they are high (the bank is systemically important). When the LLR is uncertain about the external costs of a bank failure, it will have to base its decision on the expected costs of refusing liquidity support, that is, on the ex ante likelihood of the bank being systemically important. For non-systemic banks, the LLR will not provide support, which will favor the adoption of larger ex ante precautions by bankers when compared to systemic banks that the LLR is expected to unconditionally support.

When the LLR knows the external costs of refusing liquidity support, for example, by virtue of the information acquired during the time bought by the presence of liquidity standards, his support decision will depend on the size of those costs and can be ex post more efficient. However, the implications for bankers' adoption of precautions against a crisis (and hence ex ante efficiency) are not necessarily positive. In the case of non-systemic banks, the informed LLR increases the prospect of support relative to the uniformed LLR, inducing the choice of lower precautions by bankers and producing a trade-off between ex ante and ex post efficiency. In contrast, in the case of systemic banks, the incentive effects of having an informed LLR are positive and such trade-off does not exist.

Our model shows that the case for liquidity standards (that buy time for the LLR during a crisis) is more clear when we are in the presence of systemic banks, which makes unconditional support the preferred alternative for an uninformed LLR. In light of this insight, the lessons of the Lehman Brothers debacle (i.e. learning about the large size of the external costs

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[3]In an extension, we analyze whether the outcomes obtained with a time-consistent LLR can be strictly improved if the LLR can commit ex ante to different ex post policies (e.g. through the exercise of some "constructive ambiguity").

resulting from its bankruptcy, the proximate cause of which was a rapid exit by its repo and other short-term creditors) and the concerns about the moral hazard implications of implementing too generous support policies in future crises can rationalize the importance of liquidity standards. Buying the time needed to make better informed support decisions when confronting a crisis will not only improve the efficiency of the ex post decisions, but it will also reduce banks' prospects of unconditional support in a future crisis and the moral hazard problem associated with them.

Until recently, there was no consensus among policy makers about the need for liquidity regulation. This was in contrast with an existing body of academic research that pointed to the existence of inefficiencies in worlds with a strictly private provision of liquidity, via either interbank markets (Bhattacharya and Gale 1987) or credit line agreements (Holmström and Tirole 1998). A common view was that liquidity regulation was costly for banks in spite of results pointing to its welfare enhancing effects, e.g. by reducing fire-sale effects in crises (Allen and Gale 2004) or the risk of panics due to coordination failure (Rochet and Vives 2004). Another view was that the effective action by the LLR rendered liquidity standards unnecessary.[4] Another view yet was that although the financial system was vulnerable to panics (Allen and Gale 2000), there were positive incentive effects of the implied liquidation threat (Calomiris and Kahn 1991, Chen and Hasan 2006, Diamond and Rajan 2005).

The severity of banks' liquidity problems during the recent crisis led to a consensus among policy makers about the need to introduce some form of liquidity regulation for banks.[5] Those problems also motivated new academic papers analyzing bank liquidity standards. Perotti and Suarez (2011), for example, rationalize liquidity regulation as a response to the existence of systemic externalities and analyze the relative advantages of price-based vs. quantity-based instruments. Calomiris, Heider, and Hoerova (2012), in turn, show that liquidity requirements may substitute for capital requirements in a moral hazard setup.

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[4]See Flannery (1996), Freixas, Parigi and Rochet (2000), and Rochet and Vives (2004).

[5]Banks' liquidity problems appear to have started in the summer of 2007 following the collapse of the asset-backed commercial paper (ABCP) market. These problems grew larger with the collapse or near collapse of several other markets, including the repo and the financial commercial paper markets, and even several segments of the interbank market, and with banks' shortages of collateral in part due to downward spirals in market and funding liquidity (Brunnermeier 2009, Brunnermeier and Pedersen 2009).

These studies, however, are unable to explain the potential dominance of liquidity standards *a la* Basel III over relevant alternatives such as capital or Pigovian charges for liquidity risk or the effective provision of emergency liquidity by the LLR.[6]

We contribute to close this gap in the literature with a theory that relies on a novel way of thinking about liquidity requirements – an instrument that, by making banks better able to withstand the initial phases of a crisis, allows the LLR to be better informed when making his decision.[7] Our paper is also related to papers about moral hazard and the potential value of commitment to be tough in the context of lending of last resort or bank rescue policies, including Mailath and Mester (1994), Perotti and Suarez (2002), Acharya and Yorulmazer (2007, 2008), Ratnovski (2009), and Farhi and Tirole (2012). We add to this literature by showing the implications for these issues of having a LLR who is more or less informed about troubled banks at the time of deciding whether to support them or not. Finally, our paper is also novel at providing a setup in which one can analyze the implications of identifying systemically important financial institutions ahead of time.

The rest of the paper is organized as follows. Section 2 introduces our model. Sections 3 and 4 describe the optimal policies according to our model when banks are unlikely and likely to be systemically important, respectively. Section 5 discusses the value of adding constructive ambiguity to the policy followed by the lender of last resort. Section 6 discusses further issues, including the implications of identifying systemically important financial institutions ahead of time. Section 7 contains some concluding remarks. The Appendix contains the proofs of lemmas and propositions.

2   The model

Consider an economy with three dates $t = 0, 1, 2$ and two risk neutral agents with a discount rate normalized to zero: a representative bank managed in the interest of its owners (the

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[6]See Basel Committee on Banking Supervision (2010) for a description of Basel's proposed liquidity standards.

[7]Nosal and Ordoñez (2013) describe a setup in which a government delays intervention in order to learn more about the systemic dimension of a crisis. Their analysis focuses on the strategic interaction between banks, which can restrain from risk taking in order to avoid getting into trouble earlier than their peers, i.e. at a time in which the government is still not supporting the banks in trouble.

bankers) and a benevolent lender of last resort (LLR) who maximizes the overall social net present value associated with banking activities.

2.1 Model ingredients

The bank has some illiquid assets in place at $t = 0$ and outstanding debt obligations associated with a promised repayment of D at $t = 2$. A significant fraction of the bank's debt is demandable at $t = 1$ (e.g. consists of uninsured demand deposits or short-term debt that needs to be rolled over), making the bank vulnerable to a liquidity crisis at $t = 1$. Specifically, there is a probability $1 - p$ that a crisis occurs at $t = 1$ and the bank gets in trouble because it suffers a run on its debt. A run will force the bank into default and the early liquidation of its assets unless it receives support from the LLR.

The assets of an untroubled bank are worth $a_H > D$ at $t = 2$, so an untroubled bank is fundamentally solvent. In contrast and to streamline the presentation, we assume that the assets of a troubled bank are worth $a_L < D$ if the bank continues in operation up to $t = 2$, and $L \in (a_L, D)$ if the bank is early liquidated at $t = 1$. So a troubled bank is fundamentally insolvent and its intrinsic value is larger if early liquidated. The latter could be because its managers are tempted to gamble, destroying value in the process, or because in the absence of a drastic restructuring the bank will start losing business opportunities and clients, or be vulnerable to predatory actions by its competitors.

Early liquidation of the bank, however, has two additional effects. First, it causes negative externalities of a random size $x \geq 0$ to the rest of the economy, where $x = X$ with probability ε and $x = 0$ with probability $1 - \varepsilon$. These externalities capture the differential external costs of the bank's default at $t = 1$ relative to its more orderly termination at $t = 2$ (e.g. when part of the bank's repayment obligations are held by the LLR and a recapitalization of the bank may have taken place). We will interpret ε as the unconditional probability with which not supporting a troubled bank results in a *systemic* crisis.

A second implication of liquidation is the loss of some private control benefits B to bankers. This loss is important for bankers incentives to adopt precautions against a crisis and for the interaction of those precautions with liquidity standards. The loss B also

guarantees that bankers never want to voluntarily liquidate a troubled bank at $t = 1$ and always request support from the LLR, if available. To avoid making B interfere with welfare calculations, we consider this loss as either purely redistributional (say, because B can also be appropriated by those who take control of the bank assets in case of liquidation) or negligible at the aggregate level.

To model the uncertainty regarding the external costs of bank default at $t = 1$, we assume that with probability q the LLR knows whether its decision to deny liquidity support will lead to a systemic crisis or not, and with probability $1 - q$ the LLR only learns the systemic implications of such a decision after making it. Importantly, we assume that q is an increasing function of the liquidity standards imposed on the bank at $t = 0$, because they increase the expected time span during which a troubled bank can withstand a liquidity crisis without support.[8] Implicitly, this assumes the bank is allowed to use some of the implied liquidity buffer to meet its liquidity needs once the crisis starts. We also assume that satisfying the standards has a cost $c(q)$, with $c' > 0$, $c'' > 0$, $c'(0) = 0$ and $\lim_{q \to 1} c'(q) = +\infty$, to the bank. These costs might be the result of, for instance, the need to forego some valuable investment opportunities to meet the liquidity standards. So liquidity standards are a costly way to buy time for the LLR to figure out the systemic implications of refusing to lend to the bank in trouble.

Finally, to capture moral hazard regarding the adoption of funding and investment strategies that protect the bank against a liquidity crisis in a more fundamental way, we assume that the probability of not getting into trouble, $p \in [0, 1]$, is affected by some unobservable choice made by the bank at $t = 0$ at a cost given by a function $\psi(p)$, with $\psi' > 0$, $\psi'' > 0$, $\psi'(0) = 0$ and $\lim_{p \to 1} \psi'(p) = +\infty$. The bank's decisions that determine p are made after the LLR has set the liquidity standards q and with expectations about how the LLR will decide on its support to troubled banks. Our baseline analysis assumes the LLR's support policy to be set in a time-consistent manner, i.e. maximizing the expected continuation social net

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[8]Under the current formulation, avoiding default requires the eventual support of the LLR. Liquidity standards simply postpone the intervention. In a world with some transitory liquidity crises (e.g. crises triggered by false alarms about bank solvency), liquidity standards might also increase the proportion of crises that self-resolve.

present value as evaluated at the time of making the decision. In Section 5 we investigate how our key findings would change if the LLR could commit ex ante to different ex post policies.

Figure 1 summarizes the sequence of events in the model. The order of the rows in each column reflects the chronology, if relevant:

$t = 0$	$t = 1$	$t = 2$
Assets in place	Bank suffers a run w. pr. $1–p$	Realization & distribution of:
Debt obligations D	LLR knows $x \in \{0,X\}$ w. pr. q	– Unliquidated asset values
LLR decides q	LLR decides on bank support	– Relevant external costs
Bank decides $p \in [0,1]$	Unsupported bank liquidated for L	

Figure 1 The time line

2.2 Solving the model

Our model can be solved by backward induction. To set the basis for the organization of the rest of the analysis, we will close this section on the model by characterizing the support decisions made by the (time consistent) LLR at $t = 1$. In later sections we will move backwards, first, to bankers adoption of precautions p at $t = 0$ and, then, to the LLR decision on the liquidity standards q also at $t = 0$.

To analyze the time-consistent decision of the LLR at $t = 1$, suppose the bank suffers a run, and consider first the case in which the LLR knows x. If the bank is not supported, it gets liquidated and generates a social net present value of $L - x$. If the bank gets supported, it remains in operation until $t = 2$ and generates a social net present value of a_L. Hence, supporting the bank is ex post optimal for $x = X > L - a_L$, and allowing it to default is ex post optimal for $x = 0 < L - a_L$. When the LLR does not know x, the trade-offs are exactly the same, but the relevant calculations are based on the expected size of the externalities εX rather than the realized size x. The following result is self-explanatory and requires no formal proof:

Proposition 1 *If the LLR knows x prior to his decision, he will just follow the ex post optimal course of action, supporting the bank if and only if $x = X$. If the LLR does not*

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know x, he supports the bank if and only if $E(x) = \varepsilon X > L - a_L$.[9]

In light of the result regarding how the LLR decides when he does not know x, we split our analysis in the next two sections between the case in which the bank is *likely non-systemic*, in the sense that the expected value of the externalities caused by its disordered failure are low ($\varepsilon X \leq L - a_L$), and the case in which the bank is *likely systemic*, in the sense that the expected externalities caused by its disordered failure are high ($\varepsilon X > L - a_L$). To make our narrative more compact, we will identify these two types of banks as *ex ante non-systemic* and *ex ante systemic*, respectively. This is in contrast to the case in which x is known by the LLR at $t = 1$. In this case, we will call the bank to be *non-systemic* if $x = 0$ and to be *systemic* if $x = X$.

3   Liquidity standards for ex ante non-systemic banks

Consider the case of a bank with $\varepsilon X < L - a_L$. In this case, bankers will expect the troubled bank to be liquidated when the LLR does not know x (which occurs with probability $1 - q$) and to obtain support if and only if the bank is systemic ($x = X$) when the LLR knows x. So the value of the bank at $t = 0$ can be written as

$$V_I = -c(q) - \psi(p) + p[(a_H - D) + B] + (1 - p)q\varepsilon B, \tag{1}$$

where the first term reflects the value implications of complying with the liquidity standards and the second term is the cost of adopting more fundamental precautions against a liquidity crisis. The third term is the probability of not suffering a crisis times the expected value of an untroubled bank to the bankers, which includes the residual equity payoffs $a_H - D$ and the control benefits B. The fourth term accounts for the control benefits retained by the bankers when the LLR knows that the bank is systemic and, hence, supports it. Notice that bankers obtain a zero residual equity payoff both from the unsupported troubled bank (because $L < D$) and the supported troubled bank (because $a_L < D$).

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[9]We adopt the innocuous tie-breaking rule that, when indifferent, the LLR does not support the bank.

3.1 Banks' precautions against a liquidity crisis

Under the Inada conditions satisfied by $\psi(p)$, the banker's decision, p_I, will be determined by the following first order condition (FOC):

$$(a_H - D) + (1 - q\varepsilon)B = \psi'(p_I), \tag{2}$$

which equals the banker's marginal benefits from increasing p to the marginal cost $\psi'(p)$. The marginal benefits include receiving with a larger probability both the residual equity payoff of an untroubled bank $a_H - D$ and the control benefits B.

Using (2), together with the assumed properties of $\psi'(\cdot)$, it is possible to show that among ex ante non-systemic banks, the precautions against a liquidity crisis adopted by bankers, p_I, are increasing in the untroubled asset value a_H and the control benefits B, and decreasing in the leverage D, the liquidity standards q, and the probability ε that the liquidation of the bank causes large systemic externalities.

The effects of a_H, B and D on p_I reflect the conventional effects of profitability, continuation rents, and leverage on a costly decision that enhances performance and the probability of continuation. The somewhat more intriguing effects of ε and q on p_I are connected to the impact on these parameters on the prospect of receiving LLR support if in trouble, which is detrimental to the incentives to take precautions against a liquidity crisis. Other things equal, a bank that expects the LLR to consider it systemic with larger probability will take lower precautions against a crisis. As for the liquidity standards represented by q, we have

$$\frac{dp_I}{dq} = -\frac{\varepsilon B}{\psi''(p_I)} < 0, \tag{3}$$

because, in the case of an ex ante non-systemic bank, the uninformed LLR does not support the troubled bank, while the informed LLR supports it with probability $\varepsilon > 0$. So liquidity standards in this first case are detrimental to bankers' adoption of precautions against a crisis.[10]

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[10]It is worth noting that the impact of the liquidity standards q on bankers' precautions p depends on bankers experiencing a strictly positive loss of private benefits $B > 0$ when the bank is liquidated. In a formulation similar to ours but with $B = 0$, Repullo (2005) claims that the prospects of LLR support do not modify bankers' precautions.

3.2 Optimal liquidity standards

Let us now turn to the stage at $t = 0$ in which the LLR establishes the liquidity standards that determine $q \in [0, 1]$. In the case of an ex ante non-systemic bank ($\varepsilon X \leq L - a_L$), the overall social net present value associated with banking activities can be expressed as

$$W_I = -c(q) - \psi(p_I) + p_I a_H + (1 - p_I)\{(1 - q)(L - \varepsilon X) + q[(1 - \varepsilon)L + \varepsilon a_L]\} \qquad (4)$$

where the first two terms reflect the value losses due to the cost of satisfying the liquidity standards and the cost of adopting precautions against liquidity crises. The third term reflects the value generated by bank assets when the bank remains untroubled. The fourth term is the value (net of external costs) generated by the bank when in trouble. The first term within braces is the net value generated when the LLR is forced to make his decision without observing x. The second term within braces is the net value coming from deciding to liquidate the bank when the LLR knows that it is non-systemic ($x = 0$) and to support it when he knows that it is systemic ($x = X$).

Notice that the LLR is assumed to care about the overall value of bank activities. So W_I contains no direct reference to the bank's debt obligations D or the bankers' control benefits B since these variables only affect the *distribution* of the total social value generated by the bank. We write p_I (rather than p) in W_I to indicate the fact that, when the LLR decides on q, he takes into account the impact of q on the precautions p_I subsequently adopted by the bankers according to (2).

Lemma 1 *For ex ante non-systemic banks ($\varepsilon X < L - a_L$), the optimal liquidity standards q_I, if interior, will satisfy the FOC*

$$\frac{\partial W_I}{\partial q} + \frac{\partial W_I}{\partial p_I}\frac{dp_I}{dq} = 0, \qquad (5)$$

which includes the (direct) net informational gain from increasing q,

$$\frac{\partial W_I}{\partial q} = -c'(q) + (1 - p_I)\varepsilon[X - (L - a_L)], \qquad (6)$$

and an (indirect) net incentive gain from increasing q which is the product of

$$\frac{\partial W_I}{\partial p_I} = -\psi'(p_I) + a_H - \{(1 - q)(L - \varepsilon X) + q[(1 - \varepsilon)L + \varepsilon a_L]\} \qquad (7)$$

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and $dp_I/dq < 0$ as given by (3).

The following lemma identifies (plausible) circumstances in which $\partial W_I/\partial p_I$ is positive and, hence, increasing q has a welfare-reducing effect on bankers' incentives. The proof is in the Appendix.

Lemma 2 *Among ex ante non-systemic banks ($\varepsilon X < L - a_L$), if the control benefits B are not too large, the incentive effect associated with rising the liquidity standards q is negative, $\frac{\partial W_I}{\partial p_I}\frac{dp_I}{dq} < 0$.*

Intuitively, for not too large values of the control benefits B, the presence of debt and a positive probability of default, as well as the lack of internalization of the externalities caused by liquidation if the bank is systemic, will push bankers to adopt lower precautions against a liquidity crisis than it would be socially optimal. In our model, the prospect of receiving support by the LLR in a crisis (and preserving B in spite of all) worsens this moral hazard problem. Higher liquidity standards make the LLR more likely to know that a bank is systemic at the time of making his decision and, hence, more likely to provide his support, making the ex ante moral hazard problem more severe.

The negative incentive effects of increasing the liquidity standards produces a trade-off with the (positive, except for the cost $c'(q)$) informational effects, explaining the following result (proven in the Appendix):

Proposition 2 *Among ex ante non-systemic banks ($\varepsilon X < L - a_L$), if bankers' control benefits B are not too large, the overall socially optimal value of the liquidity standards, q_I, is strictly* smaller *than if liquidity standards were set with the sole objective of maximizing the net value of the information relevant to the decision of the LLR, $\hat{q}_I$. In fact, with a sufficiently strong incentive effect, a corner solution with $q_I = 0$ may emerge.*

Thus, in the case of ex ante non-systemic banks, the presence of moral hazard regarding bankers' adoption of precautions against a liquidity crisis implies lower liquidity standards than in a world in which p were exogenously fixed at its equilibrium level. Adopting those

lower liquidity standards allows the LLR to credibly commit to a tougher-on-average support policy, which is good for incentives. Credibility comes from the fact that with a lower q the LLR will be more frequently uninformed about x at $t = 1$, in which case he will opt for not supporting the troubled bank.

The above equations allow us to analyze the comparative statics of q_I. We can explore how q_I responds to changes in parameters such as a_H, a_L, L, D, B, ε, and X, as well as to shifts in the functions $c(q)$ and $\psi(p)$. The presence of direct and indirect effects with potentially opposite signs complicates the analysis, implying that, for some of the shifts, signing the response would require the full numerical parameterization of the model. The following proposition (proven in the Appendix) summarizes the analytically unambiguous results.

Proposition 3 *Among ex ante non-systemic banks ($\varepsilon X < L - a_L$), if bankers' control benefits B are not too large and $\psi'''(p_I)$ is negative or not too large, the optimal liquidity standards q_I, if strictly positive, are increasing in the continuation value of a troubled bank a_L and the bank's leverage D, and decreasing in the value of the untroubled bank a_H, the bankers' control benefits B, and the marginal cost of the standards $c'(q)$.*

The effects of a rise in the bank's leverage D are specially relevant to discuss because such a rise may be interpreted as the result of a relaxation in some binding capital standards and be used to evaluate the possible substitutability (or complementarity) between capital standards and liquidity standards from the perspective of the LLR. The case of D is, a priori, among the simplest to analyze in this model because D does not directly enter into (4) and thus only affects (5) through the incentive effect (i.e. the effect on the precautions p_I adopted by the bankers). Moreover, it is clear from (2) that D has a negative impact on p_I. In spite of this, the effect of a decline in p_I on the marginal net social value of liquidity standards, as captured in (5) is hard to sign. On the one hand, a lower p_I increases the probability that banks get in trouble and, hence, the importance of the ex post efficiency of LLR decisions, which would improve by rising q. On the other hand, increasing q makes the moral hazard problem more severe. The detailed derivations behind Proposition 3, however, allow us to

state the following corollary:

Proposition 4 *Among ex ante non-systemic banks ($\varepsilon X < L - a_L$), if bankers' control benefits B are not too large and $\psi'''(p_I)$ is negative or not too large, liquidity standards, if strictly positive, should behave as a* substitute *for binding capital standards, i.e. should rise (decline) in response to a relaxation (tightening) of the capital standards.*

Notice that this substitutability between capital standards and liquidity standards from the perspective of the setter of the latter holds subject to important qualifications. It holds when, in response to the decline in p_I brought by the lower capital standards, the "improving ex post efficiency" effect of moving q dominates the "worsening incentives" effect (since a rise in q_I will make p_I decline even further). Yet, for a large positive $\psi'''(p_I)$ and a large value of B, the possibility of boosting p_I by reducing q_I might change the sign of the response of the optimal liquidity standards to the change in capital standards, making the former behave as a *complement* of the latter.

4 Liquidity standards for ex ante systemic banks

We now turn to the case of banks that are ex ante systemically important in the sense that failing to support them, if in trouble, would involve large expected externalities ($\varepsilon X > L - a_L$). In this case, according to Proposition 1, bankers will expect troubled banks to be supported by the LLR if he does not know the size of the externalities x when making his decision at $t = 1$. As we will see, the switch in expectations regarding what the uninformed LLR will decide when he does not know x has dramatic implications for the incentive effects of tightening the liquidity standards and, hence, for the resolution of the trade-offs driving the socially optimal choice of the standards.

When bankers expect the troubled bank to be supported whenever the LLR remains uninformed about x, which occurs with probability $1 - q$, and to otherwise support the troubled bank if and only if it is known to be systemic ($x = X$), the value of the bank to bankers at $t = 0$ can be written as

$$V_{II} = -c(q) - \psi(p) + p[(a_H{-}D) + B] + (1-p)[1 - q(1{-}\varepsilon)]B, \tag{8}$$

where the only difference with respect to the expression in (1) is the fourth term, which reflects that, in case of trouble, the bank will be supported in all circumstances except when the LLR knows that the bank is non-systemic ($x = 0$).

4.1 Banks' precautions against a liquidity crisis

Under the Inada conditions satisfied by $\psi(p)$, bankers' decision, p_{II}, will now be determined by the following FOC:

$$(a_H - D) + q(1 - \varepsilon)B = \psi'(p_{II}), \tag{9}$$

which is similar to (2) but contains important differences in the term that captures the marginal impact of p on the preservation of the control benefits B. Precautions help avoid liquidation but only in proportion to the probability $q(1 - \varepsilon)$ with which the LLR knows that the bank is non-systemic and, thus, declines to support it.

Using (9), together with the assumed properties of $\psi'(\cdot)$, it is possible to show that among ex ante systemic banks, the precautions against a liquidity crisis adopted by bankers, p_{II}, are increasing in the untroubled asset value a_H, the control benefits B, and the liquidity standards q, and decreasing in the leverage D and the probability ε that the liquidation of the bank causes large systemic externalities.

Compared to the case of ex ante non-systemic banks, the key qualitative difference is the sign of the effect of the liquidity standards. Specifically, we now have

$$\frac{dp_{II}}{dq} = \frac{(1 - \varepsilon)B}{\psi''(p_{II})} > 0, \tag{10}$$

because, in this case, the uninformed LLR always supports the troubled bank, while the informed LLR only supports it with probability $\varepsilon < 1$. So increasing the probability q with which the LLR knows x at the time of his decision reduces the prospect of support and, thus, encourages bankers to adopt greater precautions.

Comparing the FOCs (2) and (9) has direct implications for the ranking between p_I and p_{II} near the boundary of the parameter space that divides the cases of low and high average externalities (the proof is in the Appendix):

Proposition 5 *In the neighborhood of $\varepsilon X = L - a_L$, for a given value of the liquidity standards q, the precautions adopted by bankers in an ex ante systemic bank are strictly lower than those adopted in an ex ante non-systemic bank.*

What this reflects is that, other things equal, an uninformed LLR who systematically denies support to troubled banks generates less moral hazard than one who is systematically supportive. For here, one can derive some empirical implications about the severity of the moral hazard problem in a setup where liquidity standards remain constant while a reduction in L, an increase in a_L or an increase in ε imply a transition from the ex ante non-systemic scenario to the ex ante systemic scenario. Any of those evolutions lead unambiguously to a reduction in both p_I and p_{II} (i.e. within each scenario) as well as a downward jump in p with the shift from p_I to p_{II}.

4.2 Optimal liquidity standards

In the case of ex ante systemic banks, the overall social net present value associated with banking activities can be expressed as

$$W_{II} = -c(q) - \psi(p_{II}) + p_{II}a_H + (1 - p_{II})\{(1 - q)a_L + q[(1 - \varepsilon)L + \varepsilon a_L]\} \qquad (11)$$

where there are two differences with respect to the expression for W_I in (4). First, bankers' precautions against a liquidity crisis are now p_{II} as determined by (9). Second, the first term within braces reflects that the decision of the uninformed LLR is now to support the troubled bank. The following lemmas are parallel to those enunciated for the case of ex ante non-systemic banks. The first is an immediate application of the Chain Rule and requires no proof; the second is proven in the Appendix.

Lemma 3 *Among ex ante systemic banks ($\varepsilon X > L - a_L$), the optimal liquidity standards q_{II}, if interior, will satisfy the FOC*

$$\frac{\partial W_{II}}{\partial q} + \frac{\partial W_{II}}{\partial p_{II}}\frac{dp_{II}}{dq} = 0, \qquad (12)$$

which includes the (direct) net informational gain from increasing q,

$$\frac{\partial W_{II}}{\partial q} = -c'(q) + (1 - p_{II})(1 - \varepsilon)(L - a_L), \qquad (13)$$

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and an (indirect) net incentive gain from increasing q which is the product of

$$\frac{\partial W_{II}}{\partial p_{II}} = -\psi'(p_{II}) + a_H - \{(1-q)a_L + q[(1-\varepsilon)L + \varepsilon a_L]\} \tag{14}$$

and $dp_{II}/dq > 0$ as given by (10).

Lemma 4 *Among ex ante systemic banks ($\varepsilon X > L - a_L$), if the control benefits B are not too large, the incentive effect associated with rising the liquidity standards q is positive, $\frac{\partial W_{II}}{\partial p_{II}} \frac{dp_{II}}{dq} > 0$.*

Thus, like in the case of non-systemic banks, rising q has informational and incentive effects. However, in the current case, when bankers' control benefits B are not too large, the sign of the incentive effect is positive rather than negative, modifying the sign of the distortion previously emphasized in Proposition 2 (the proof is in the Appendix):

Proposition 6 *Among ex ante systemic banks ($\varepsilon X > L - a_L$), if bankers' control benefits B are not too large, the overall socially optimal value of the liquidity standards, q_{II}, is interior and strictly* larger *than if liquidity standards were set with the sole objective of maximizing the net value of the information relevant to the ex post decision of the LLR, $\widehat{q}_{II}$.*

Thus, in the case of ex ante systemic banks, the presence of moral hazard regarding bankers' adoption of precautions against a liquidity crisis calls for higher liquidity standards than in a world in which p were exogenously fixed at its equilibrium level. These standards are also higher than if the LLR could commit not to support a troubled bank when uninformed about the size of the externalities x that its failure may provoke. Adopting higher liquidity standards allows the LLR to credibly commit to a tougher-on-average support policy, which is good for bankers' incentives to protect the bank against a liquidity crisis. The credibility gained by increasing q comes from the fact that ex ante systemic banks are always supported in case of trouble by the uninformed LLR, whereas they are supported by the informed LLR only if the relevant externalities x are learned to be large ($x = X$).

The following proposition (proven in the Appendix) refers to how the optimal liquidity requirements behave when crossing the boundary between the cases of non-systemic and systemic banks:

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Proposition 7 *In the neighborhood of $\varepsilon X = L - a_L$, when bankers' control benefits B are not too large, the liquidity standards that maximize social welfare among ex ante systemic banks are strictly* higher *than those that maximize social welfare among ex ante non-systemic banks.*

This result is a by-product of the fact that moral hazard problems call for relaxing liquidity standards when the average externalities associated with the default of a troubled bank are low and reinforcing the standards when the externalities are high. But the intuition here is not that liquidity standards per se reduce the size of the potential externalities (not in this model). What they do is to modify (in different directions in each case) the prospects that the LLR will end up supporting the troubled bank, changing bankers' incentives to adopt greater precautions against a liquidity crisis.

The analysis of the effects on the optimal standards q_{II} of changes in the various parameters, as well as shifts in the functions $c(q)$ and $\psi(p)$, is subject to signing complications similar to those discussed for the low externalities case. The following proposition (proven in the Appendix) summarizes the analytically unambiguous results:

Proposition 8 *Among ex ante systemic banks ($\varepsilon X > L - a_L$), if bankers' control benefits B are not too large and $\psi'''(p_{II})$ is positive or not too small, the optimal liquidity standards q_{II} are increasing in the bank's leverage D, and decreasing in the continuation value of a troubled bank a_L and the marginal cost of the standards $c'(q)$.*

Hence, relative to ex ante non-systemic banks, the effect on the optimal liquidity requirements of increases in the value of the untroubled bank a_H and the bankers' control benefits B are now ambiguous (rather than negative), the marginal effect of the size of the systemic externalities X becomes zero (rather than ambiguous), and the effect of increasing the continuation value of a troubled bank a_L is negative (rather than positive). Only the positive effect of the bank's leverage D, the negative effect of the marginal cost of the liquidity standards $c'(q)$, and the ambiguity of the effects of moving the liquidation value L and the ex ante probability that the troubled bank is systemic ε remain the same as in the case of ex ante non-systemic banks.

The main differences with respect to such case can be explained as a result of one or several of the following reasons:

1. The "improving ex post efficiency effect" that calls for higher liquidity standards becomes differently important due to the different support policy followed by the uninformed LLR. This is what happens when rising a_L: It improves the value of the troubled bank under the (unconditional support) decision of the uninformed LLR, reducing the value of allowing him to make a more informed decision.

2. The "incentive effect" of increasing the liquidity standards is now positive rather than negative, producing a conflict between the signs of the two relevant effects in cases where there was no conflict before. This is what happens when rising a_H or B. On the one hand, as before, these shifts increase the precautions taken by the bankers, make the bank less likely to get in trouble and, hence, reduce the need to care about ex post efficiency. On the other hand, any of those shifts now also increase the effectiveness with which the moral hazard problem could be ameliorated by rising q. So it is analytically ambiguous in which direction q should eventually move.

For the relevant case of a shift in D, we have that, as in the case of ex ante non-systemic banks, D does not directly enter into (11) and, hence, only affects the choice of q_{II} through the indirect effects identified in (12). Moreover, it is clear from (9) that D has a negative impact on bankers' precautions p also in this case. Intuition suggests that the effect of shifts in p on the marginal net social value of liquidity standards, as captured in (12), should now be less ambiguous than in the case with low average externalities. This is because in this case strengthening the liquidity standards has the virtue of simultaneously improving the ex post efficiency of LLR decisions (more relevant when p is lower) and the moral hazard problem (bankers' choice of p). The following conclusion is a corollary of Proposition 8:

Proposition 9 *Among ex ante systemic banks ($\varepsilon X > L - a_L$), if bankers' control benefits B are not too large and $\psi'''(p_{II})$ is positive or not too small, liquidity standards should behave as a* substitute *for binding capital standards, i.e. should rise (decline) in response to a relaxation (tightening) of the capital standards.*

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5 The value of constructive ambiguity

The analysis in prior sections is based on a fully time-consistent specification of the conduct of the LLR. We now explore the possibility of achieving gains in the overall social net present value associated with bank activities by making the LLR to ex ante commit to a different conduct. Specifically, we consider the room for improvement around the situation without commitment in which liquidity standards are optimally set as characterized in prior sections.

In principle, commitment might allow changing the time-consistent policy of the LLR along any of its dimensions: (i) the support provided to troubled banks when the LLR does not know the size of the systemic externalities x, and (ii) the support provided under each realization of x when this is learned in advance. However, the presence of moral hazard regarding the precautions p that bankers can adopt against a liquidity crisis makes it evident that the only deviations relative to the ex post optimal LLR policies that might increase welfare are those that, by lowering the prospects of support to a bank in trouble, ameliorate the moral hazard problem. Hence, we formally assess the value of denying support with some positive probability in each of the instances in which the time-consistent LLR policy implies supporting the troubled bank. In other words, we check the welfare effects of adding some "constructive ambiguity" to the support policies characterized in prior sections.

5.1 The case of ex ante non-systemic banks

In this case, the time-consistent LLR only supports a troubled bank when it knows it to be systemic (i.e. when the LLR learns that $x = X$). Thus, constructive ambiguity might only make sense in the form of reducing by some amount $\alpha > 0$ the probability of granting support to the bank in such an instance. The following equations extend the expressions of a bank's ex ante value to the bankers, V_I, and to the society as a whole, W_I, to show their dependence on α:

$$V_I = -c(q) - \psi(p) + p[(a_H - D) + B] + (1 - p)q(1 - \alpha)\varepsilon B, \tag{15}$$

$$\begin{aligned} W_I ={}& -c(q) - \psi(p_I) + p_I a_H + (1 - p_I)\{(1 - q)(L - \varepsilon X) \\ & + q[(1 - \varepsilon)L + \varepsilon(1 - \alpha)a_L + \varepsilon\alpha(L - X)]\}, \end{aligned} \tag{16}$$

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where p_I denotes, as before, the value of p that maximizes (15) given the value of all other parameters and variables, including the liquidity standards q. The two expressions above are straightforwardly related to (1) and (4), respectively. The presence of $\alpha > 0$ in (15) reduces the probability that bankers appropriate the control rents B in a troubled bank which is known to be systemic. It also modifies the FOC for the precautions adopted by bankers in a direction that increases p_I and thus ameliorate the moral hazard problem. This suggests that constructive ambiguity, $\alpha > 0$, might improve the resulting social welfare W_I. The following proposition shows, however, that such room for improvement does not exist if the liquidity standards q are set optimally (i.e. to maximize W_I).

Proposition 10 *In the case of ex ante non-systemic banks, if liquidity standards are set optimally, introducing constructive ambiguity in LLR policies is* detrimental *to social welfare.*

The proof of Proposition 10 (in the Appendix) is based on showing that having $dW_I/dq = 0$ (which is the FOC for the optimal liquidity standards q_I, if interior) implies having $dW_I/d\alpha < 0$, so that constructive ambiguity is detrimental to social welfare and one should set $\alpha = 0$.[11] Intuitively, in the case of ex ante non-systemic banks, the sole rationale for imposing the cost of strictly positive liquidity standards is to give the chance to the LLR to support those troubled banks identified as systemic. However, if for incentive reasons it were convenient to reduce troubled banks' prospects of support, the social planner could just reduce the standards q and save on its costs $c(q)$ rather than playing around with constructive ambiguity after having found out $x = X$. This means that, irrespectively of whether the LLR can commit to time inconsistent policies or not, the socially optimal liquidity standards are those characterized in Section 3, q_I, and there is no value associated with constructive ambiguity.[12]

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[11] The only other possibility is having $q_I = 0$, but in this case the value of α is completely irrelevant to the resulting allocation.

[12] Under the current formulation of the model the liquidity standards q_I can be reduced, if needed, all the way down to zero (in which case the LLR never knows x at the time of making its decision). In a formulation in which zero standards still imply a positive probability that the LLR learns x, there might be circumstances (e.g. if the moral hazard problem is very severe) in which setting $\alpha > 0$ is socially valuable.

5.2 The case of ex ante systemic banks

In this case, the time-consistent LLR supports a troubled bank both if it does not know its type and if it knows the bank to be systemic ($x = X$). Thus, we must discuss the effects of constructive ambiguity in these two different instances. Denoting by β and γ the probabilities with which support is denied in each of them, the expressions in (8) and (11) for a bank's ex ante value to the bankers, V_{II}, and to the society as a whole, W_{II}, can be extended as follows:

$$V_{II} = -c(q) - \psi(p) + p[(a_H - D) + B] + (1 - p)[(1 - q)(1 - \beta) + q\varepsilon(1 - \gamma)]B, \qquad (17)$$

$$
\begin{aligned}
W_{II} = \ & -c(q) - \psi(p_{II}) + p_{II}a_H + (1 - p_{II})\{(1 - q)[(1 - \beta)a_L + \beta(L - \varepsilon X)] \\
& + q[(1 - \varepsilon)L + \varepsilon(1 - \gamma)a_L + \varepsilon\gamma(L - X)]\},
\end{aligned}
\qquad (18)
$$

where p_{II} denotes, as before, the value of p that maximizes (17) given the value of all other parameters and variables, including q. The presence of $\beta > 0$ or $\gamma > 0$ in (17) reduces the probability that bankers appropriate the control rents B if their bank gets in trouble. So increasing any of these parameters affects the FOC for the precautions adopted by bankers in a direction that increases p_{II} and thus ameliorates the moral hazard problem. This is a necessary (but not sufficient) condition for constructive ambiguity in any of these two forms to possibly improve the resulting social welfare W_{II}.

Opposite to the case of ex ante non-systemic banks, we do not have a general proof against the potential optimality of constructive ambiguity under optimal liquidity standards, but a result showing that the necessary and sufficient condition for $\beta > 0$ to decrease welfare (relative to the case without constructive ambiguity) is tighter than the necessary and sufficient condition for $\gamma > 0$ to decrease welfare.

Proposition 11 *In the case of ex ante systemic banks, if liquidity standards are set at their optimal level q_{II}, the condition*

$$-c'(q_{II}) + (1 - p_{II})(1 - \varepsilon)\varepsilon X > 0 \qquad (19)$$

is necessary and sufficient for the introduction of constructive ambiguity in the state in which the LLR does not know the type of the troubled bank ($\beta > 0$) to decrease *welfare. Likewise, the less stringent condition*

$$-c'(q_{II}) + (1 - p_{II})(1 - \varepsilon)X > 0 \tag{20}$$

is necessary and sufficient for the introduction of constructive ambiguity in the state in which the LLR knows that the bank is systemic ($\gamma > 0$) to decrease *welfare.*

Clearly, condition (19) is tighter than (20) for all $\varepsilon < 1$. And, it can be checked through specific numerical examples that there are parameterizations of the model under which both conditions, only the second or none of the two are satisfied, implying that no ambiguity, some ambiguity when the LLR does not know x, or some ambiguity in the two relevant cases may be optimal. An intuitive driver through the various possibilities is the value of the systemic externalities X, which in this scenario must satisfy $\varepsilon X > L - a_L$. Interestingly, when the moral hazard problem bites in equilibrium (see Lemma 4), the FOC for an interior q_{II} requires, using (12) and (13),

$$-c'(q_{II}) + (1 - p_{II})(1 - \varepsilon)(L - a_L) < 0. \tag{21}$$

Thus, for values of εX sufficiently close to $L - a_L$, condition (19) will not hold. Similarly, for even lower values of X, condition (20) will not hold either.

Figure 1 illustrates these possibilities by comparing the welfare obtained under alternative values of the liquidity standards q with and without constructive ambiguity. To make the effects visible, constructive ambiguity is represented by setting $\beta = 0.2$ (and $\gamma = 0$) instead of $\beta = 0$ (and $\gamma = 0$). Cases 1 and 2 only differ in the value of X.[13]

Figure 1 shows that when X is high enough (Case 1), constructive ambiguity adds no value, not only under q_{II} (which is the value of q at which the depicted curve reaches its

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[13]The parameterization behind Figure 1 is as follows. In common across the two panels, we set $a_H = 100$, $a_L = 50$, $L = 80$, $D = 90$, $\varepsilon = 0.2$, $B = 10$, and the cost functions $c(q) = 0.25q/(1 - q)$ and $\psi(p) = 15p^2$. The externalities caused by the systemic bank X equal 300 in Case 1 and 170 in Case 2. In both cases we have $\varepsilon X > L - a_L$ so as to remain in a scenario of banks likely to be systemically important. The functional form chosen for $\psi(p)$ does not satisfy $\lim_{p \to 1} \psi'(p) = +\infty$ but we take care numerically of possible corner solutions with $p = 1$ (although they are not relevant in equilibrium).

maximum) but under any possible q. Instead, for a lower value of X (Case 2), constructive ambiguity is welfare improving and, again, not only under q_{II} but (especially) under suboptimal values of q.

As usual, exercising constructive ambiguity requires some degree of commitment capacity on the side of the LLR, and achieving this commitment capacity is quite tricky when the optimal degree of ambiguity is "interior" (i.e. involves $\beta \in (0,1)$ or $\gamma \in (0,1)$) since the obvious alternative of establishing some short of statutory (or constitutional) limit to what the LLR can or cannot do is not the solution. If in the limit one must choose between unambiguous support or unambiguous lack of support to troubled (unknown-type or systemic) banks, then the region in which the latter option dominates will be smaller than the region in which conditions (19) or (20) fail to hold.

6 Other discussions

6.1 Identifying SIFIs

One distinctive feature of some recent regulatory reforms is the identification of *systemically important financial institutions* (SIFIs): institutions "whose distress or disorderly failure, because of their size, complexity and systemic interconnectedness, would cause significant disruption to the wider financial system and economic activity."[14] Supervisors across the globe are working on developing a framework according to which financial institutions ex ante identified as SIFIs can be subject to closer supervisory scrutiny, more stringent requirements, enhanced coordination between the relevant supervisors, and, if needed, especial resolution regimes. While a comprehensive assessment of these developments exceeds the aims of this paper, it is still interesting to discuss them through the lens provided by our model.

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[14]See Point 3 in the document "Policy Measures to Address Systemically Important Financial Institutions," issued by the Financial Stability Board on 4 November 2011.
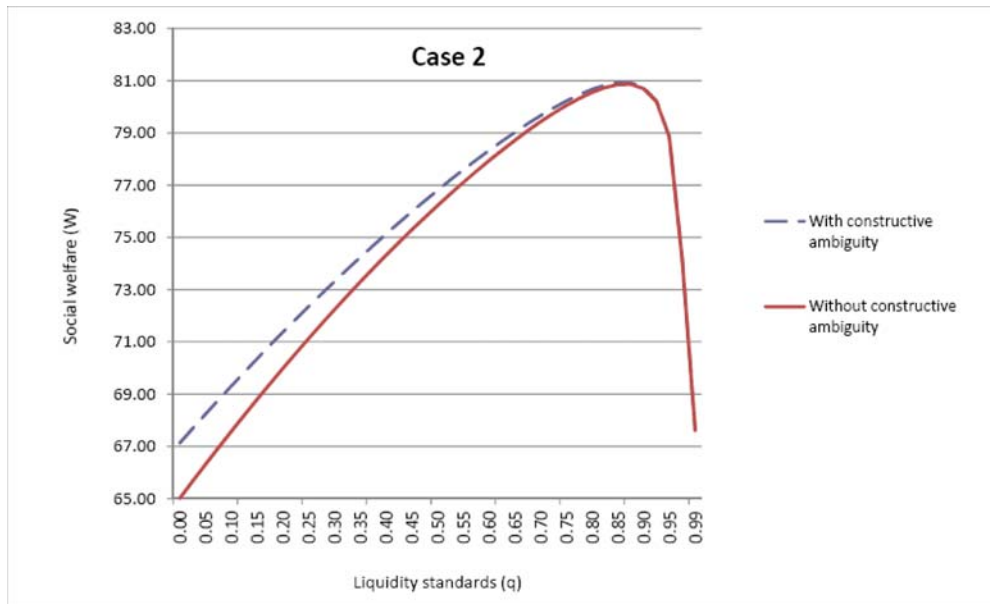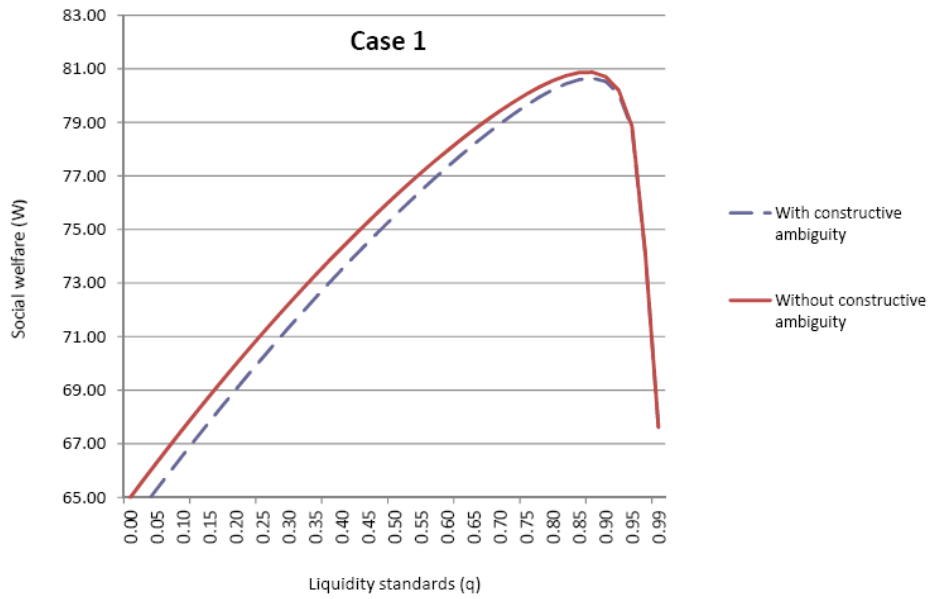
Figure 2 Welfare implications of constructive ambiguity

The two panels represent social welfare W_{II} as a function of the liquidity standards q in the case of ex ante systemic banks. The only parameter that differs across panels is X, which is higher in Case 1 than in Case 2. The solid lines represent W_{II} without constructive ambiguity ($\beta = \gamma = 0$). The dashed lines represent W_{II} under $\beta = 0.2$ and $\gamma = 0$.

Our model contains ingredients consistent with the standard narrative about why SIFIs constitute a problem: "(A)uthorities have all too frequently had no choice but to forestall the failure of such institutions through public solvency support. As underscored by this crisis, this has deleterious consequences for private incentives and for public finances."[15] In terms of our model, the support to banks ex ante identified as SIFIs is explained by the likelihood ε or the size X of the potential externalities and the negative implications for "private incentives" are reflected in the impact of support prospects on bankers' precautions p. Under our interpretation that liquidity standards q "buy time" for the LLR to discover a troubled bank's true systemic importance, we have shown that the trade-offs involved in setting the optimal q crucially depends on the expected size of the systemic externalities of a bank whose type remains unknown.

Consistent with this approach, we can think of identifying SIFIs as producing a partition in a set of banks initially treated in a homogenous manner, and having the possibility to set different liquidity standards (and effectively different LLR policies) on each of the resulting subsets. Specifically, in what follows we consider the partition of a set of banks with some initial probability ε of being systemic (like in our baseline model) into two subsets with probabilities ε_L and ε_H, respectively, of being systemic. We do not attribute any other merit to the identification of SIFIs but just the fact that banks listed as SIFIs are systemic with a probability $\varepsilon_H > \varepsilon$, while those not identified as SIFIs feature $\varepsilon_L < \varepsilon$.

It turns out that the comparative statics of our model with respect to changes in ε within each of the two cases in which we structured the discussion in the preceding sections is not clear cut. The optimal level of the liquidity standards in our model depends on the value of the information achieved by buying time and on the incentive effects coming from altering bankers' prospects of obtaining support if they get in trouble. A partition like the one described above will make the banks within each group more homogenous, which leads to a reduction in the value of acquiring information ex post about their exact type. So there is a value-of-information effect which points to a reduction in the optimal liquidity standards

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[15] See Point 4 in the document "Policy Measures to Address Systemically Important Financial Institutions," issued by the Financial Stability Board on 4 November 2011.

for either of the two groups.

As for the incentive effects, the answer needs further considerations. If banks in the group of non-SIFIs are as our "ex ante non-systemic" banks (in the sense of having $\varepsilon_L X < L - a_L$), while banks in the initial set were "ex ante systemic" ($\varepsilon X > L - a_L$), then there is an incentive effect which also points to the reduction of the liquidity standards to which the non-SIFIs should be subject to (recall Proposition 2). In contrast, if banks in the group of SIFIs are "ex ante systemic" ($\varepsilon_H X > L - a_L$), while banks in the initial set were "ex ante non-systemic" ($\varepsilon X < L - a_L$), then the incentive effect would point in the direction of increasing the liquidity requirements for the group of SIFIs (recall Proposition 6). Putting the ex post efficiency and incentive effects together, this discussion means that for banks classified as SIFIs, the optimal liquidity standards might (but do not necessarily have to) be higher that in the situation prior to the partition.

To see the complexity of the discussion, consider the polar case in which the group of banks identified as SIFIs features $x = X$ with probability one. Then there is no informational value associated with liquidity requirements, and there is no incentive effect either, since banks in this group will be sure about receiving support from the LLR irrespective of their choice of q. Similarly, in the (somewhat less plausible) scenario in which all the banks not identified as SIFIs feature $x = 0$ with probability one, liquidity standards will again have no information and no incentive effects attached. It is worth noting, however, that in this polar case there is a clear welfare gain coming from the fact that the banks in the non-SIFI group of the partition will take higher precautions than in a world in which they may end up being supported with positive probability.

With the caveats due to the difficulty to state a result for the general case, the discussion of the previous polar case suggests a potential lack of consistency between the efforts to identify SIFIs and the reinforcement of liquidity standards across the whole spectrum of financial institutions, at least in a world in which liquidity standards play the roles emphasized in this paper.

6.2 Solvent vs. insolvent banks

Most of the analysis and its conclusions could be re-stated in terms closer to Bagehot's classical view, where what is crucial to determine whether a troubled bank should be supported or not is whether it is solvent or insolvent.[16] The main casuistics for the analysis would then be whether banks are ex ante more or less likely to be solvent. When banks are unlikely to be solvent (or "ex ante insolvent"), the default decision by the LLR (if he does not know whether a specific bank is solvent or insolvent) would be not to support the bank, which is good for bankers' incentives. The opposite happens when banks are likely to be solvent (or "ex ante solvent"). In the first (second) scenario, liquidity standards have a negative (positive) effect on the moral hazard problem. Mutatis mutandi the analytical results and policy insights would be very similar to the ones obtained under the baseline model.

6.3 Asymmetric information

What if banks know their own type at the time of deciding on p? As in the variation mentioned in the previous subsection, several details in the analytics of the model would change but the main qualitative and policy insights would prevail.

Specifically, when banks know their type, they can better predict what the LLR will do with them. In the absence of specific knowledge about the systemic or non-systemic status of a bank, the LLR will follow its default policy, irrespective of a bank's true type. However, the systemic (non-systemic) bank can now be sure that the LLR will support if the LLR ends up discovering its type. This means that liquidity standards will (i) erode systemic banks' incentives when banks are ex ante considered unlikely to be systemic, and (ii) reinforce non-systemic banks' incentives when banks are ex ante considered likely to be systemic. So the effects on banks' *average* incentives are essentially the same that we identify in the baseline model.

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[16]Formally, the presence of solvent or insolvent banks could be introduced by making $x = 0$ (no externalities) and by replacing the current assumption that a troubled bank that continues is worth $a_L < L$ with probability one by a new assumption that such bank may be worth-continuing ("solvent") and yield $a_M \in (D, L)$ with probability ε or not-worth-continuing ("insolvent") and yield $a_L < L$ with probability $1 - \varepsilon$.

7   Final remarks

We have presented a model in which banks are vulnerable to liquidity crises that may force them into failure, unless they receive support from the LLR. We postulate that liquidity standards, though costly to banks, lengthen the time a bank can withstand a liquidity crisis and, thus, give the LLR more time to find out the systemic implications of its failure. From this perspective, we have analyzed the implications of liquidity standards for the ex post efficiency of LLR policies and for bankers' adoption of precautions against a crisis.

Our model shows that liquidity standards are beneficial from both perspectives when banks are ex ante considered likely to be systemic, but only from the first one (and counter-productive from the second one) when banks are ex ante considered unlikely to be systemic. This means that, other things equal, the optimal liquidity standards for banks which are likely to be systemically important should be higher. Our analysis helps rationalize the proposals for the reinforcement of liquidity standards emerged after the Lehman debacle (interpreted as a dramatic way to learn about the size of the potential externalities associated with the disordered liquidation of some banks). It also helps structure the discussions about the logical interconnections between liquidity regulation and other important issues in the regulatory debate: lending of last resort policies, capital requirements, and the identification of SIFIs

Appendix

Proof of Lemma 2 Using (2) to substitute for $-\psi'(p_I)$ in (7), we can write

$$\frac{\partial W_I}{\partial p_I} = D - (1 - q\varepsilon)B - \{(1 - q)(L - \varepsilon X) + q[(1 - \varepsilon)L + \varepsilon a_L]\}, \tag{22}$$

whose sign seems, in principle, ambiguous under the assumptions adopted so far. However, having $D > a_L$ and $D > L$, and negative externalities X generated whenever a liquidated bank happens to be systemic (but neglected by the bankers when deciding on p_I) implies $\partial W_I / \partial p_I > 0$ for $B \to 0$. To guarantee that $\partial W_I / \partial p_I$ remains positive for $B > 0$ the necessary and sufficient condition is having

$$B < \frac{D - \{(1 - q)(L - \varepsilon X) + q[(1 - \varepsilon)L + \varepsilon a_L]\}}{1 - q\varepsilon}, \tag{23}$$

which imposes a positive upper bound to B. If this condition holds we have $\frac{\partial W_I}{\partial p_I}\frac{dp_I}{dq} < 0$, i.e. the incentive effect of rising the liquidity standards is negative.∎

Proof of Proposition 2 The Inada conditions satisfied by the cost function $c(q)$ guarantee that there is a unique interior value of q, say $\widehat{q}_I$, for which $\partial W_I / \partial q = 0$, i.e. that would be optimal if the incentive effect were zero or if liquidity standards were set to maximize the net value of the information relevant to the ex post decision of the LLR. However, by Lemma 2, the incentive effect of rising q contributes an additional negative term in the left hand side of (5), implying $q_I < \widehat{q}_I$. In fact, its is possible to produce examples in which the incentive effect is so strong that leads to a corner solution with $q_I = 0$.∎

Proof of Proposition 3 Consider the case in which the socially optimal liquidity standards q_I are strictly positive and denote any of the exogenous parameters by z. Let the system of equations given by (5) and (2), which characterizes q_I when it is interior, be abstractly described as

$$G(q_I, p_I, z) = 0 \tag{24}$$

and

$$p_I = H(q_I, z), \tag{25}$$

respectively. By fully differentiating this system and substituting into the differentiated version of (24) the expression for dp obtained from the differentiated version of (25), we can

express

$$(G_q + G_p H_q)dq + (G_z + G_p H_z)dz = 0, \tag{26}$$

which implies

$$\frac{dq_I}{dz} = -\frac{G_z + G_p H_z}{G_q + G_p H_q}, \tag{27}$$

where the sign of the denominator is negative by the second order condition (SOC) for the maximization of W_I at q_I. So the sign of dq_I/dz will coincide with the sign of $G_z + G_p H_z$.

Now, by deriving in the left hand side of (5) with respect to p_I, we obtain

$$G_p = -\varepsilon[X - (L - a_L) - B] - \frac{\partial W_I}{\partial p_I} \frac{dp_I}{dq} \frac{\psi'''(p_I)}{\psi''(p_I)}, \tag{28}$$

where we know that $\partial W_I / \partial p_I > 0$ under (23), by Lemma 2, and $dp_I/dq < 0$, by (3). Hence, if B is small enough relative to $X - (L - a_L) > 0$ and $\psi'''(p_I) \leq 0$, we must necessarily have $G_p < 0$, (whereas $G_p > 0$ may arise if B is large enough and/or $\psi'''(p_I) > 0$). If $G_p < 0$, the sign of dq_I/dz will then be necessarily positive if $G_z \geq 0$ and $H_z \leq 0$, necessarily negative if $G_z \leq 0$ and $H_z \geq 0$, and potentially ambiguous whenever G_z and H_z share the same strict sign.

Inspecting the expressions for G_z and H_z for each of the relevant parameters and for upward shifts in the marginal cost function $c'(q)$ (that we omit for brevity), one can obtain the results contained in the following table, where the signs $+$ and $-$ stand for unambiguously positive and negative effects, respectively, and ? stands for effects with a potentially ambiguous sign:

Effects of an upward shift in z (if $G_p < 0$)

z	a_H	a_L	L	D	B	ε	X	$c'(q)$
G_z	$-$	$+$	?	0	$-$	?	?	$-$
H_z	$+$	0	0	$-$	$+$	$-$	0	0
dq_I/dz	$-$	$+$	?	$+$	$-$	?	?	$-$

Table A1 Comparative statics of the optimal liquidity
standards for ex ante non-systemic banks

The non-ambiguous results in Table A1 prove the proposition.∎

Proof of Proposition 5   It is immediate to check that the second term in the left hand side of (2) is strictly larger than the corresponding term in (9), which implies $p_I > p_{II}$ in the boundary case $\varepsilon X = L - a_L$. By the continuity of p_I and p_{II} in the relevant parameters, the strict inequality remains true for the values p_I and p_{II} that prevail, near the boundary, in the regions with $\varepsilon X < L - a_L$ and $\varepsilon X > L - a_L$, respectively.∎

Proof of Lemma 4   Using (9) to substitute for $-\psi'(p_{II})$ in (14), we can write

$$\frac{\partial W_{II}}{\partial p_{II}} = D - q(1 - \varepsilon)B - \{(1 - q)a_L + q[(1 - \varepsilon)L + \varepsilon a_L]\}, \tag{29}$$

which is positive for $B \to 0$ because $D > a_L$ and $D > L$. To guarantee that $\partial W_{II}/\partial p_{II}$ remains positive for $B > 0$ the necessary and sufficient condition is having

$$B < \frac{D - \{(1 - q)a_L + q[(1 - \varepsilon)L + \varepsilon a_L]\}}{q(1 - \varepsilon)}, \tag{30}$$

which, like (23) for the case with low average externalities, imposes a positive upper bound to B.∎

Proof of Proposition 6   The Inada conditions satisfied by the cost function $c(q)$ guarantee that there is a unique interior value of q, say $\widehat{q}_{II}$, for which $\partial W_{II}/\partial q = 0$, i.e. that would be optimal if the incentive effects were zero or if liquidity standards were set to maximize the net value of the information relevant to the ex post decision of the LLR. However, by Lemma 4, the incentive effect of rising q contributes an additional positive term in the left hand side of (5). So the overall socially optimal value of q in this scenario, q_{II}, is still interior (because $\lim_{q \to 1} c'(q) = +\infty$) but strictly larger than $\widehat{q}_{II}$.∎

Proof of Proposition 7   Consider the boundary case with $\varepsilon X = L - a_L$ and suppose that B is low enough to satisfy (23) for $q = q_I$ and (30) for $q = q_{II}$. Notice first that if $q_I = 0$ the result is clearly true since $q_{II} > 0$ by Proposition 6. So let us consider the situation in which the optimal liquidity standards for the case of ex ante non-systemic banks are $q_I > 0$ and let us analyze the implications of setting some $q^* \leq q_I$ as the liquidity standards for ex ante systemic banks. Let p_I^* and p_{II}^* denote the precautions adopted by bankers in each of

the cases, i.e. under $q = q_I$ and $q = q^*$, respectively. Then, it follows from Proposition 5 that $p_I^* > p_{II}^*$ and, from (6) and (13), that we would subsequently have:

$$\left.\frac{\partial W_{II}}{\partial q}\right|_{q^*} = -c'(q^*) + (1 - p_{II}^*)(1 - \varepsilon)(L - a_L) > \left.\frac{\partial W_I}{\partial q}\right|_{q_I} = -c'(q_I) + (1 - p_I^*)\varepsilon[X - (L - a_L)] \quad (31)$$

since $\varepsilon[X - (L - a_L)] = (1 - \varepsilon)(L - a_L)$ when $\varepsilon X = L - a_L$. Now, (5) together with Lemma 2 implies:

$$\left.\frac{\partial W_I}{\partial q}\right|_{q_I} = -\left.\frac{\partial W_I}{\partial p_I}\frac{dp_I}{dq}\right|_{q_I} > 0, \quad (32)$$

and, hence, by (31),

$$\left.\frac{\partial W_{II}}{\partial q}\right|_{q^*} > 0, \quad (33)$$

for any $q^* \leq q_I$. However, (12) together with Lemma 4 implies that the optimality of q_{II} requires

$$\left.\frac{\partial W_{II}}{\partial q}\right|_{q_{II}} = -\left.\frac{\partial W_{II}}{\partial p_{II}}\frac{dp_{II}}{dq}\right|_{q_{II}} < 0, \quad (34)$$

so q_{II} cannot be equal to any $q^* \leq q_I$ and, hence, must be strictly larger than q_I. By the continuity of p_I^* and p_{II}^* (and the expressions involved above) in the relevant parameters, the strict inequalities established in (31) and (33)—and hence the result that $q_I < q_{II}$—remain true near the boundary between the regions with $\varepsilon X < L - a_L$ and $\varepsilon X > L - a_L$. $\blacksquare$

Proof of Proposition 8   Denote any of the exogenous parameters by z and let the system of equations given by (12) and (9), which characterizes q_{II}, be abstractly described as

$$J(q_{II}, p_{II}, z) = 0 \quad (35)$$

and

$$p_{II} = K(q_{II}, z), \quad (36)$$

respectively. Following the same steps as in the proof of Proposition 3, we can express

$$\frac{dq_{II}}{dz} = -\frac{J_z + J_p K_z}{J_q + J_p K_q}, \quad (37)$$

where $J_q + J_p K_q < 0$ by the SOC for the maximization of W_{II} at q_{II}. Hence dq_{II}/dz will have the same sign as $J_z + J_p K_z$. Moreover, deriving in the left hand side of (12) with respect to p_{II}, we obtain

$$J_p = -(1 - \varepsilon)[(L - a_L) + B] - \frac{\partial W_{II}}{\partial p_{II}}\frac{dp_{II}}{dq}\frac{\psi'''(p_{II})}{\psi''(p_{II})}, \quad (38)$$

where we know that $\partial W_{II}/\partial p_{II} > 0$ under (30) and $dp_{II}/dq > 0$ by (10). Hence, for $\psi'''(p_{II}) \geq 0$, we must necessarily have $J_p < 0$. The same will be true, by continuity, if $\psi'''(p_{II}) < 0$ but its absolute value is not too large. With $J_p < 0$, the sign of dq_{II}/dz is then positive if $J_z \geq 0$ and $K_z \leq 0$, negative if $J_z \leq 0$ and $K_z \geq 0$, and potentially ambiguous if J_z and K_z share the same strict sign.

Examining the expressions for J_z and K_z for each of the relevant parameters (and for upward shifts in the marginal cost function $c'(q)$), one can obtain the results contained in the following table, where the signs $+$, $-$, and ? have the same meaning as in Table A1, and the superscript * is used to indicate differences with respect to the results obtained for ex ante non-systemic banks (see Table A1).

Effects of an upward shift in z (if $J_p < 0$)

z	a_H	a_L	L	D	B	ε	X	$c'(q)$
J_z	$+^*$	$-^*$	?	0	$+^*$	?	0^*	$-$
K_z	$+$	0	0	$-$	$+$	$-$	0	0
dq_{II}/dz	$?^*$	$-^*$	?	$+$	$?^*$	?	0^*	$-$

Table A2 Comparative statics of the optimal liquidity
standards for ex ante systemic banks

The non-ambiguous results in Table A2 prove the proposition.∎

Proof of Proposition 10 Consider first the simpler case in which the optimal liquidity standards q_I are interior for all the relevant values of α. Using the same notation as in Lemma 1, an interior q_I will satisfy (5), which implies

$$\frac{\partial W_I}{\partial p_I} = \frac{-\frac{\partial W_I}{\partial q}}{\frac{dp_I}{dq}}, \tag{39}$$

while the marginal welfare effect of changing α can be evaluated through the total derivative

$$\frac{dW_I}{d\alpha} = \frac{\partial W_I}{\partial \alpha} + \frac{\partial W_I}{\partial p_I}\frac{dp_I}{d\alpha}, \tag{40}$$

which, since $dp_I/d\alpha > 0$, will be negative if and only if

$$\frac{\partial W_I}{\partial p_I} < \frac{-\frac{\partial W_I}{\partial \alpha}}{\frac{dp_I}{d\alpha}}. \tag{41}$$

Now, (39) and (41) together imply that constructive ambiguity is detrimental to social welfare if and only if

$$\frac{\frac{\partial W_I}{\partial q}}{\left(-\frac{dp_I}{dq}\right)} < \frac{-\frac{\partial W_I}{\partial \alpha}}{\frac{dp_I}{d\alpha}}, \tag{42}$$

where we have shifted the signs of the numerator and the denominator of the fraction in the left hand side to make the denominator positive. After evaluating each term using (15) and (16), (42) becomes

$$\frac{-c'(q_I) + (1 - p_I)\varepsilon(1 - \alpha)[X - (L - a_L)]}{\frac{(1 - \alpha)\varepsilon B}{\psi''(p_I)}} < \frac{(1 - p_I)q\varepsilon[X - (L - a_L)]}{\frac{q\varepsilon B}{\psi''(p_I)}}. \tag{43}$$

And, it is a matter of algebra to reduce this inequality to $-c'(q_I)q_I < 0$, which is indeed true for any $q_I > 0$.

As for possible non-interior values of q_I, our assumption that $\lim_{q \to 1} c'(q) = +\infty$ rules out the possibility of having $q_I = 1$, and leaves $q_I = 0$ as the only possibly optimal corner solution (which might indeed arise if the moral hazard problem affecting bankers' decision on p were extremely severe).[17] However, with $q_I = 0$, the choice of α is irrelevant, so we can claim that setting $\alpha = 0$ (i.e. exercising no constructive ambiguity) is socially optimal without loss of generality.■

Proof of Proposition 11 When banks are ex ante systemic, our assumptions that $c'(0) = 0$ and $\lim_{q \to 1} c'(q) = +\infty$ allow us to rule out the possibility of corner solutions for q_{II}.[18] Using the same notation as in Lemma 3, an interior q_{II} will satisfy (12), which implies

$$\frac{\partial W_{II}}{\partial p_{II}} = \frac{-\frac{\partial W_{II}}{\partial q}}{\frac{dp_{II}}{dq}}, \tag{44}$$

while the marginal welfare effect of changing $z = \beta, \gamma$ can be evaluated through the total derivative

$$\frac{dW_{II}}{dz} = \frac{\partial W_{II}}{\partial z} + \frac{\partial W_{II}}{\partial p_{II}}\frac{dp_{II}}{dz}. \tag{45}$$

which, since $dp_{II}/dz > 0$ for $z = \beta, \gamma$ will be negative if and only if

$$\frac{\partial W_{II}}{\partial p_{II}} < \frac{-\frac{\partial W_{II}}{\partial z}}{\frac{dp_{II}}{dz}}. \tag{46}$$

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[17]In the case of ex ante non-systemic banks, the assumption that $c'(0) = 0$ is not enough to rule out a corner solution with $q_I = 0$ because increasing q has the indirect negative welfare effect of worsening the moral hazard problem, and such an effect does not go to zero for $q \to 0$.

[18]Notice that in the case of ex ante systemic banks, q_{II} is never lower than $\widehat{q}_{II}$, and $c'(0) = 0$ guarantees that $\widehat{q}_{II} > 0$.

Now, (44) and (41) together imply that marginally increasing the corresponding z (from a benchmark value of zero) will be detrimental to social welfare if and only if

$$\frac{-\frac{\partial W_{II}}{\partial q}}{\frac{dp_{II}}{dq}} < \frac{-\frac{\partial W_{II}}{\partial z}}{\frac{dp_{II}}{dz}}. \tag{47}$$

After evaluating each term using (17) and (18), (47) becomes

$$\frac{c'(q) + (1 - p_{II})\{(1 - \beta)a_L + \beta(L - \varepsilon X) - (1 - \varepsilon)L - \varepsilon(1 - \gamma)a_L - \varepsilon\gamma(L - X)\}}{\frac{[(1-\beta)-\varepsilon(1-\gamma)]B}{\psi''(p_{II})}}$$

$$< \frac{(1 - p_{II})(1 - q)[\varepsilon X - (L - a_L)]}{\frac{(1-q)B}{\psi''(p_{II})}}$$

for $z = \beta$, and

$$\frac{c'(q) + (1 - p_{II})\{(1 - \beta)a_L + \beta(L - \varepsilon X) - (1 - \varepsilon)L - \varepsilon(1 - \gamma)a_L - \varepsilon\gamma(L - X)\}}{\frac{[(1-\beta)-\varepsilon(1-\gamma)]B}{\psi''(p_{II})}}$$

$$< \frac{(1 - p_{II})q\varepsilon[X - (L - a_L)]}{\frac{q\varepsilon B}{\psi''(p_{II})}}$$

for $z = \gamma$. And, it is a matter of algebra (and evaluating the expressions at $\beta = \gamma = 0$) to reduce these inequalities to conditions (19) and (20), respectively.∎

References

Acharya, Viral, and Tanju Yorulmazer (2007), "Too many to fail—An analysis of time-inconsistency in bank closure policies," *Journal of Financial Intermediation* 16, 1–31.

Acharya, Viral, and Tanju Yorulmazer (2008), "Cash-in-the-Market Pricing and Optimal Resolution of Bank Failures," *Review of Financial Studies* 21, 2705–2742.

Allen, Franklin, and Douglas Gale (2000), "Financial Contagion," *Journal of Political Economy* 108, 1–33.

Allen, Franklin, and Douglas Gale (2004), "Financial Intermediaries and Markets", *Econometrica* 72, 1023–1061.

Basel Committee on Banking Supervision (2010), Basel III: International framework for liquidity risk measurement, standards and monitoring, Bank for International Settlements.

Bhattacharya Sudipto, and Douglas Gale (1987), "Preference Shocks, Liquidity and Central Bank Policy," in W. Barnett and K. Singleton (eds.), *New Approaches to Monetary Economics*, Cambridge: Cambridge University Press.

Brunnermeier, Markus, and Lasse Pedersen (2009), "Market Liquidity and Funding Liquidity," *Review of Financial Studies* 22, 2201–2238.

Calomiris, Charles, Florian Heider, and Marie Hoerova (2012), "A Theory of Bank Liquidity Requirements," European Central Bank, mimeo.

Calomiris, Charles, and Charles Kahn (1991), "The Role of Demandable Debt in Structuring Optimal Banking Arrangements," *American Economic Review* 81, 497–513.

Chen, Yehning, and Iftekhar Hasan (2006), "The Transparency of the Banking System and the Efficiency of Information-Based Bank Runs," *Journal of Financial Intermediation* 15, 307–331.

Diamond, Douglas, and Raghuram Rajan (2005), "Liquidity Shortages and Banking Crises," *Journal of Finance* 60, 615–647.

Farhi, Emmanuel, and Jean Tirole (2012), "Collective Moral Hazard, Maturity Mismatch and Systemic Bailouts," *American Economic Review* 102, 60–93.

Flannery, Mark (1996), "Financial Crises, Payment System Problems and Discount Window Lending," *Journal of Money Credit and Banking* 28, 805–824.

Freixas, Xavier, Bruno Parigi, and Jean-Charles Rochet (2000), "Systemic Risk, Interbank Relations, and Liquidity Provision by the Central Bank," *Journal of Money, Credit and Banking* 32, 611–638.

Gorton, Gary (2009), "Slapped in the Face by the Invisible Hand: Banking and the Panic of 2007," prepared for the Federal Reserve Bank of Atlanta's 2009 Financial Markets Conference: Financial Innovation and Crisis, Jekyll Island, Georgia, May 11–13.

Holmström, Bengt, and Jean Tirole (1998), "Private and Public Supply of Liquidity," *Journal of Political Economy* 106, 1–40.

Mailath, George, and Loretta Mester (1994), "A Positive Analysis of Bank Closure," *Journal of Financial Intermediation* 3, 272–299.

Nosal, Jaromir, and Guillermo Ordoñez (2013), "Uncertainty as Commitment," University of Pennsylvania, mimeo.

Perotti, Enrico, and Javier Suarez (2002), "Last Bank Standing: What Do I Gain if You Fail?," *European Economic Review* 46, 1599–1622.

Perotti, Enrico, and Javier Suarez (2011), "A Pigovian Approach to Liquidity Regulation," International Journal of Central Banking, 7, 3–41.

Ratnovski, Lev (2009), "Bank Liquidity Regulation and the Lender of Last Resort," *Journal of Financial Intermediation*, 18, 541–558.

Repullo, Rafael (2005), "Liquidity, Risk-Taking, and the Lender of Last Resort," *International Journal of Central Banking* 1, 47–80.

Rochet, Jean-Charles, and Xavier Vives (2004) "Coordination Failures and the Lender of Last Resort: Was Bagehot Right after All?," *Journal of the European Economic Association* 6, 1116–1147.

Shin, Hyun (2010), *Liquidity and Risk*, Oxford: Oxford University Press.