

# The Procyclical Effects of Bank Capital Regulation

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## Abstract

We assess the procyclical effects of bank capital regulation in a dynamic equilibrium model of relationship lending in which banks are unable to access the equity markets every period. Banks anticipate that shocks to their earnings as well as the cyclical position of the economy can impair their capacity to lend in the future and, as a precaution, hold capital buffers. We find that under cyclically-varying risk-based capital requirements (e.g. Basel II) banks hold larger buffers in expansions than in recessions. Yet, these buffers are insufficient to prevent a significant contraction in the supply of credit at the arrival of a recession. We show that cyclical adjustments in the confidence level underlying Basel II can reduce its procyclical effects on the supply of credit without compromising banks' long-run solvency targets.

*Keywords:* Banking regulation, Basel II, Business cycles, Capital requirements, Credit crunch, Loan defaults, Relationship banking.

*JEL Classification:* G21, G28, E44

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# 1 Introduction

Discussions on the procyclical effects of bank capital requirements are top in the agenda for reform of financial regulation following the 2007-2009 crisis. The argument whereby procyclical effects may occur is well-known. In recessions, losses erode banks' capital, while risk-based capital requirements (such as those introduced by Basel II) become higher. If banks cannot quickly raise sufficient new capital, their lending capacity can get seriously impaired and a credit crunch may follow. Preventing that possibility would require introducing some cyclical adjustment in the requirements or to arrange for contingent capital injections in bad times.<sup>1</sup>

Skeptics to this view note that banks typically hold capital in excess of the required minima, and argue that what may be binding is not the regulatory requirements but some implicit "economic capital requirement" imposed by the market. Altogether, they seem to suggest that a regulation-induced credit crunch is unlikely and that cyclical adjustments of regulation cannot do very much to reduce the inherent cyclicity of bank credit supply.

This paper presents a tractable dynamic equilibrium model of relationship banking with endogenous capital buffers. The business cycle is modeled as a Markov chain with two states, expansion and recession, that affect expected business failure rates and thus loan default probabilities. Banks decide to raise and keep capital on top of the minimum regulatory requirements precisely because they anticipate that they may need but may not be able to raise capital in the future, and this will entail losing some profitable lending opportunities.

The model is constructed to highlight the direct procyclical effects of capital regulation on the supply of bank credit.<sup>2</sup> We compare the implications of cyclically-varying capital requirements, such as those associated with the internal ratings based (IRB) approach of Basel II (especially under a point-in-time ratings system), with those of essentially flat or cyclically-invariant requirements, such as those of Basel I (or of a perfect through-the-cycle

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<sup>1</sup>See Brunnermeier et al. (2009), Financial Stability Forum (2009), and Kashyap, Rajan, and Stein (2008), among others.

<sup>2</sup>For simplicity, we abstract from demand-side fluctuations and feedback effects, but both could certainly be accommodated in an extended version of the current model.

implementation of Basel II).<sup>3</sup> For comparison, we also report the results in a hypothetical laissez-faire environment without capital regulation.

Our results show that, under cyclically-varying capital requirements, banks indeed choose to hold capital buffers (of up to 5% of assets in our baseline quantitative scenario), but these buffers are not sufficient to fully neutralize the implications of the arrival of a recession, which may cause a very significant reduction in the supply of credit to bank-dependent borrowers (of about 11% on average in the baseline scenario). All in all, our results suggest that the advantages of cyclically-varying capital requirements in terms of preserving banks' solvency over the business cycle are disproportionately small relative to the credit crunch effects that they may produce.

We illustrate the possible use of the model in the assessment of current regulatory reforms by analyzing a way in which Basel II capital requirements can be cyclically adjusted so as to reduce the incidence of a credit crunch, while preserving their notional value-at-risk foundations. Specifically, we show the effectiveness of adjustments that consist in slightly increasing the underlying confidence level during expansions and slightly decreasing it during recessions, holding constant its long-term average at the 99.9% currently targeted by the regulator.

**Further modeling details** Borrowers are overlapping generations of entrepreneurs who demand funds from banks for two consecutive periods. Banks are managed in the interest of their risk-neutral shareholders who are the providers of equity capital. Consistent with the view that relationship banking makes banks privately informed about their borrowers, we assume that (i) borrowers become dependent on the banks with whom they first start a

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<sup>3</sup>In the IRB approach, capital requirements are an increasing function of banks' estimates of the probability of default (PD) and loss given default (LGD) of each loan, and these inputs are likely to rise in downturns. It is unclear whether an implementation of the IRB approach based on using through-the-cycle (rather than point-in-time) estimates of these inputs is at all *feasible* (since it is hard to separate migrations across rating categories due to the business cycle from those due to a change in borrowers' risk profiles) or *desirable* (since through-the-cycle estimates are more manipulable, less informative, and more distant from the estimates that banks use for risk pricing and risk management). However, we do not need to take a position on this, except for the practical interpretation of the results obtained below. For further discussion, see Kashyap and Stein (2004), Catartineu-Rabell, Jackson, and Tsomocos (2005), Gordy and Howells (2006), Saurina and Trucharte (2007), and Repullo, Saurina, and Trucharte (2009).

lending relationship, and (ii) banks with ongoing relationships have no access to the equity market. The first assumption captures the lock-in effects caused by the potential lemons problem faced by banks when a borrower is switching from another bank.<sup>4</sup> The second assumption captures the implications of these informational asymmetries for the market for seasoned equity offerings, which can make the dilution costs of urgent recapitalizations prohibitively costly.<sup>5</sup>

The combination of relationship lending and the inability of banks with ongoing relationships to access the equity market establishes a natural connection between the capital shortages of some banks at a given date and the credit rationing of some borrowers at that date. It also ensures that two necessary conditions for capital requirements to have aggregate procyclical effects on credit supply are satisfied: first, some banks must find it difficult to respond to their capital needs by issuing new equity; second, some borrowers must be unable to avoid credit rationing by switching to other sources of finance.<sup>6</sup>

The market for loans to newly born entrepreneurs is assumed to be perfectly competitive and free from capital constraints. Each cohort of new borrowers is funded by banks that renew their lending relationships at that date, have access to the equity market, and hence face no binding limits to their lending capacity.

**Strategy for the analysis** We characterize equilibrium loan rates and capital holdings in each state of the economy, and derive a number of comparative statics results. However, some important effects cannot be signed. For example, if equity financing is more costly than deposit financing (say, because of the tax advantages of debt financing), capital requirements increase equilibrium loan rates, but have an ambiguous effect on banks' capital holdings. This is because, on the one hand, the higher prospects of ending up with insufficient capital for

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<sup>4</sup>See Boot (2000) for a survey of the relationship banking literature. Several papers explicitly analyze the costs of switching lenders under asymmetric information (e.g., Sharpe, 1990) as well as the trade-offs behind the possible use of multiple lenders as a remedy to the resulting lock-in effects (e.g., Detragiache et al., 2000). We are implicitly assuming that these alternative arrangements are very costly.

<sup>5</sup>This assumption can also be seen as a convenient reduced-form for the observed delays in banks' recapitalizations; see Barakova and Carey (2001).

<sup>6</sup>These conditions have been noted by Blum and Hellwig (1995) and parallel the conditions in Kashyap, Stein, and Wilcox (1993) for the existence of a bank lending channel in the transmission of monetary policy.

the second lending period call for the holding of larger buffers; but, on the other hand, higher capital requirements reduce the profitability of lending, and thus banks' interest in preserving their future lending capacity.

Our analytical expressions suggest that the shape of the distributions of loan losses in different states of the economy matters for determining which effect dominates. This shape is also crucial to the quantitative importance of credit rationing. To clarify these effects, we resort to numerically computing the equilibrium and simulating the supply of second period loans (and the implied credit rationing) under some realistic parameterizations of the model. In particular, we describe the distributions of loan losses according to the single risk factor of Vasicek (2002), which provides the foundation for the risk-based capital requirements of Basel II.

**Related literature** The papers closest to ours are Estrella (2004), Peura and Keppo (2006), and Zhu (2008). Estrella (2004) considers the dynamic optimization problem of a bank whose dividend policy and equity raising processes are subject to quadratic adjustment costs, loan losses follow a second-order autoregressive process, and bank failure is costly. The paper focuses on the comparison between the optimal capital decisions of the bank in the absence of regulation and under a value-at-risk rule, concluding that they are very different. Peura and Keppo (2006) consider a continuous-time model in which raising bank equity takes time. A supervisor checks at random times whether the bank complies with a minimum capital requirement. The paper finds that the bank may hold capital buffers in order to reduce the risk of being closed for holding insufficient capital when audited. Finally, Zhu (2008) adapts the model of Cooley and Quadrini (2001) to the analysis of banks with decreasing returns to scale, minimum capital requirements, and linear equity-issuance costs. Assuming ex-ante heterogeneity in banks' capital positions, the paper finds that for poorly-capitalized banks, risk-based capital requirements increase safety without causing a major increase in procyclicality, whereas for well-capitalized banks, the converse is true. Relative to these papers, we simplify the details of the banks' dynamic optimization problem and embed such problem in the context of an equilibrium model of relationship banking with endogenous

loan rates, focusing on the implications for the dynamics of aggregate bank lending.

**Outline of the paper** The rest of the paper is organized as follows. Section 2 presents the model. In Section 3 we analyze the capital decision of a representative bank. Section 4 defines the equilibrium and provides the comparative statics of equilibrium loan rates and capital holdings. In Section 5 we present the numerical results concerning the size and cyclical behavior of capital holdings, capital buffers, credit rationing, and probabilities of bank failure in a number of parameterizations of the model. In Section 6 we examine adjustments of the Basel II framework that reduce its procyclical effects. Section 7 discusses the robustness of our results to changes in some of the key assumptions of the model. Section 8 concludes. Appendix A contains the proofs of the analytical results, and Appendix B discusses the choice of parameter values for the numerical analysis.

## 2 The Model

Consider a discrete time economy in which time is indexed by  $t = 0, 1, 2, \dots$ . The economy is populated by three classes of risk-neutral agents: entrepreneurs, banks, and investors.

### 2.1 Entrepreneurs

Entrepreneurs belong to overlapping generations formed by a continuum of measure one of ex-ante identical and penniless individuals who remain active for up to two periods (three dates). Each entrepreneur born at date  $t$  has the opportunity to undertake a sequence of two *investment projects* with the following characteristics. The first period project requires a unit investment at date  $t$  and its return at date  $t+1$  is  $1+a$  if it is successful, and  $1-\lambda$  if it fails, where  $a > 0$  and  $0 < \lambda < 1$ . The second period project requires  $\mu$  units of investment at date  $t+1$  and its return at date  $t+2$ , which is independent of the outcome of the first period project, equals  $(1+a)\mu$  if it is successful, and  $(1-\lambda)\mu$  if it fails.

All projects operating from date  $t$  to date  $t+1$  (undertaken by entrepreneurs born at dates  $t-1$  and  $t$ ) have an identical probability of failure  $p_t$ . The outcomes of these projects

exhibit positive but imperfect correlation, so their aggregate *failure rate*  $x_t$  is a continuous random variable with support  $[0, 1]$  and cumulative distribution function (cdf)  $F_t(x_t)$  such that

$$p_t = E_t(x_t) = \int_0^1 x_t dF_t(x_t).$$

For simplicity, we consider the case in which the history of the economy up to date  $t$  only affects  $F_t(x_t)$  (and thus  $p_t$ ) through an observable *state variable*  $s_t$  that can take two values,  $h$  and  $l$ , and follows a Markov chain with

$$q_h = \Pr(s_t = h \mid s_{t-1} = h) \text{ and } q_l = \Pr(s_t = l \mid s_{t-1} = l).$$

Moreover, we assume that the cdfs corresponding to the two states,  $F_h(\cdot)$  and  $F_l(\cdot)$ , are ranked in the sense of first-order stochastic dominance, so that the probabilities of business failure in each state satisfy

$$p_h > p_l.$$

Thus states  $h$  and  $l$  may be interpreted as recession (high business failure) and expansion (low business failure) states, respectively.

## 2.2 Banks

Banks are competitive intermediaries specialized in channeling funds from investors to entrepreneurs. Following the literature on relationship banking, we assume that the financing of an entrepreneur in this economy relies on a sequence of one-period loans granted by the single bank from which the entrepreneur obtains his first loan. We also assume that setting up the relationship with the entrepreneur makes the bank incur some cost  $c$ , to be subtracted from first period revenues.<sup>7</sup> Finally, for simplicity, we abstract from the possibility that part of the required second period investment  $\mu$  is internally financed by the entrepreneur.<sup>8</sup>

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<sup>7</sup>This cost might include personnel, equipment, and other operating costs associated with the screening and monitoring functions emphasized in the literature on relationship banking.

<sup>8</sup>This simplification is standard in relationship-banking models; see, for example, Sharpe (1990) or von Thadden (2004). Moreover, if entrepreneurs' first-period profits are small relative to the required second-period investment, the quantitative effects of relaxing this assumption would be negligible.

Banks are funded with deposits and equity capital, both of which are raised from investors. To simplify the analysis we assume that deposits are fully insured (at a zero premium),<sup>9</sup> and their supply is perfectly elastic at a risk-free rate that we normalize to zero. We also assume that investors require an excess return  $\delta \geq 0$  on each unit of equity capital. The cost of capital  $\delta$  is intended to capture in a reduced-form manner distortions (such as agency costs of equity or debt tax shields) that introduce a comparative disadvantage of equity financing relative to deposit financing—in addition to deposit insurance.<sup>10</sup>

We introduce an important imperfection concerning banks' equity financing: While banks renewing their portfolio of lending relationships can raise new equity in an unrestricted manner, recapitalization is impossible for banks with ongoing lending relationships. This assumption is intended to capture in a simple way the long delays or prohibitive dilution costs that a bank with opaque assets in place might face when arranging an equity injection.<sup>11</sup>

Banks are managed in the interest of their shareholders, who are protected by limited liability. They are subject to a *capital requirement* that obliges them to hold a capital-to-loans ratio of at least  $\gamma_s$  on the loans made when the state of the economy is  $s$ .

This formulation can encompass both flat (or cyclically-invariant) requirements such as those of Basel I (or of a perfect through-the-cycle implementation of Basel II), and cyclically-varying capital requirements, such as those in the IRB approach of Basel II (especially under a point-in-time ratings system). Specifically, for corporate loans Basel I sets  $\gamma_l = \gamma_h = 8\%$ , while the requirement of higher capital for riskier loans in Basel II implies a capital requirement in the high default state,  $\gamma_h$ , higher than the capital requirement in the low

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<sup>9</sup>Under the high target solvency of Basel II, the fair deposit insurance premium would be negligible.

<sup>10</sup>Further to the reasons for the extra cost of equity financing offered by the corporate finance literature, Holmström and Tirole (1997) and Diamond and Rajan (2000) provide agency-based explanations specifically related to banks' monitoring role.

<sup>11</sup>These costs are typically related to asymmetric information. Specifically, in a world in which banks learned about their borrowers after starting a lending relationship (like in Sharpe, 1990) and borrower quality were asymmetrically distributed across banks, the market for seasoned equity offerings might be affected by a lemons problem (like in Myers and Majluf, 1984). Thus, after a negative shock, banks with lending relationships of poorer quality would be more interested in issuing equity at any given price, which explains why the prices at which new equity could be raised may be unattractive to banks with higher-quality relationships and why, in sufficiently adverse circumstances, the market for those SEOs may collapse.



default state,  $\gamma_l$ .<sup>12</sup>

To guarantee that the funding of investment projects is attractive to banks at all dates, we assume that

$$(1 - p_s)(1 + a) + p_s(1 - \lambda) - c > (1 - \gamma_s) + \gamma_s(1 + \delta), \quad (1)$$

for  $s = h, l$ . Thus, in all states of the economy, the expected return per unit of investment, net of the cost  $c$  of setting up the relationship, is greater than the cost of funding it with  $1 - \gamma_s$  deposits and  $\gamma_s$  capital.

### 3 Banks' Capital Decision

We assume that entrepreneurs born at date  $t$  obtain their first period loans from unrestricted banks that can raise capital at this date. This is consistent with the assumption that banks with ongoing lending relationships may face capital constraints, and allows us to analyze the banking industry as if it were made of overlapping generations of banks that operate for two periods, specialize in loans to contemporaneous entrepreneurs, and cannot issue equity at the interim date. Banks are aware of this recapitalization constraint and, in order to accommodate the effect of negative shocks to their first period income or possibly higher second period capital requirements in the case of risk-based capital regulation, they may hold a buffer of equity capital on top of the first period regulatory minimum.

Consider a representative bank that lends to the measure one continuum of entrepreneurs starting up at date  $t$  (it will become obvious that banks that can issue equity face constant returns to scale), possibly refinances them at date  $t + 1$ , and gets liquidated at date  $t + 2$ . Let  $s$  and  $s'$  denote the states of the economy at dates  $t$  and  $t + 1$ , respectively.

At date  $t$  the representative bank raises  $1 - k_s$  deposits and  $k_s \geq \gamma_s$  capital, and invests these funds in a unit portfolio of first period loans. The equilibrium interest rate on these loans, denoted  $r_s$ , will be determined endogenously, but under perfect competition the bank takes it as given. At date  $t + 1$  the bank gets  $1 + r_s$  from the fraction  $1 - x_t$  of performing

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<sup>12</sup>The precise formula that relates the capital requirement  $\gamma_s$  to the loans' probability of default  $p_s$  under Basel II will be described at the beginning of Section 5.

loans (those extended to entrepreneurs with successful projects) and  $1 - \lambda$  from the fraction  $x_t$  of defaulted loans, and incurs the setup cost  $c$ , so its assets are  $1 + r_s - x_t(\lambda + r_s) - c$ , while its deposit liabilities are  $1 - k_s$  (since the deposit rate has been normalized to zero). Thus the capital of the bank at date  $t + 1$  is

$$k'_s(x_t) = k_s + r_s - x_t(\lambda + r_s) - c, \quad (2)$$

where  $x_t$  is a random variable whose conditional cdf is  $F_s(x_t)$ .

The entrepreneurs that start up at date  $t$  demand an amount  $\mu$  of second period loans at date  $t + 1$ .<sup>13</sup> At this stage entrepreneurs are dependent on the bank, so their demand is inelastic as long as the loan interest rate does not exceed the success return of the second period projects. Thus the second period loan rate will be  $a$ . Funding all these projects requires an amount of capital equal to  $\gamma_{s'}\mu$ , where  $\gamma_{s'}$  is the capital requirement at date  $t + 1$  when the state of the economy is  $s'$ .

There are three cases to consider. First, if  $k'_s(x_t) < 0$  the bank fails, the depositors are compensated by the deposit insurer, and the entrepreneurs dependent on the bank cannot undertake their second period projects. Second, if  $0 \leq k'_s(x_t) < \gamma_{s'}\mu$  the bank's available capital cannot support funding all the second period projects, so some entrepreneurs are credit rationed. Third, if  $\gamma_{s'}\mu \leq k'_s(x_t)$  the bank has more capital than the amount required to fund all the second period projects, in which case the bank funds all the second period projects and pays a dividend of  $k'_s(x_t) - \gamma_{s'}\mu$  to its shareholders at date  $t + 1$ .<sup>14</sup>

Which case obtains depends on the realization of the default rate  $x_t$ . Using the definition (2) of  $k'_s(x_t)$ , it is immediate to show that the bank fails when  $x_t > \hat{x}_s$ , where

$$\hat{x}_s = \frac{k_s + r_s - c}{\lambda + r_s}. \quad (3)$$

The bank rations credit to some of the second period projects when  $\hat{x}_{ss'} < x_t \leq \hat{x}_s$ , where

$$\hat{x}_{ss'} = \frac{k_s + r_s - c - \gamma_{s'}\mu}{\lambda + r_s}. \quad (4)$$

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<sup>13</sup>Note that this includes entrepreneurs that defaulted on their initial loans. This is because under our assumptions such default does not reveal any information about the entrepreneurs' second period projects.

<sup>14</sup>Since entrepreneurs born at date  $t + 1$  borrow from banks that can raise equity at that date, the bank may use the excess capital to either pay a dividend to its shareholders or to reduce the deposits to be raised at this date. However, with deposit insurance and  $\delta \geq 0$ , the second alternative is strictly suboptimal.

And the bank funds all the second period projects and pays a dividend to its shareholders when  $x_t \leq \hat{x}_{ss'}$ .

In the next lemma, we provide an expression for the net present value of the bank at date  $t$ . All proofs of formal results appear in Appendix A.

**Lemma 1** *The net present value of the representative bank that in state  $s$  holds capital  $k_s$  and charges an interest rate  $r_s$  on its unit of initial loans is*

$$v_s(k_s, r_s) = \beta E_t[v_{ss'}(x_t)] - k_s, \quad (5)$$

where  $\beta = (1 + \delta)^{-1}$  is the discount factor associated with the cost of capital  $\delta$ ,

$$v_{ss'}(x_t) = \begin{cases} (\beta\pi_{s'} - \gamma_{s'})\mu + k'_s(x_t), & \text{if } x_t \leq \hat{x}_{ss'}, \\ \frac{\beta\pi_{s'}}{\gamma_{s'}} k'_s(x_t), & \text{if } \hat{x}_{ss'} < x_t \leq \hat{x}_s, \\ 0, & \text{if } x_t > \hat{x}_s, \end{cases} \quad (6)$$

is the conditional value of the bank at date  $t + 1$ , inclusive of dividends, and

$$\pi_{s'} = \int_0^1 \max \{ \gamma_{s'} + a - x_{t+1}(\lambda + a), 0 \} dF_{s'}(x_{t+1}) \quad (7)$$

is the expected gross equity return per unit of loans at date  $t + 1$ .

The operator  $E_t(\cdot)$  in (5) takes into account the uncertainty at date  $t$  about both the state of the economy at date  $t + 1$  (which affects the second period capital requirement  $\gamma_{s'}$  and the gross equity return per unit of lending  $\pi_{s'}$ ) and the default rate  $x_t$  of initial loans (which determines the capital  $k'_s(x_t)$  available at that date).

Taking as given the initial loan rate  $r_s$ , the representative bank that first lends to a generation of entrepreneurs in state  $s$  will choose its capital  $k_s$  so as to maximize  $v_s(k_s, r_s)$  subject to the constraint  $\gamma_s \leq k_s \leq 1$ . The following lemma characterizes the solution to this problem.

**Lemma 2** *The bank's capital decision always has a solution, which may be interior or at the corner  $k_s = \gamma_s$ . When the solution is interior, the probability that the bank ends up with excess lending capacity in the low default state  $s' = l$  and rations credit in the high default state  $s' = h$  is strictly positive.*

The existence of a solution follows directly from the fact that  $v_s(k_s, r_s)$  is continuous in  $k_s$ , for any given interest rate  $r_s$ . In the proof the lemma, we show that the function  $v_s(k_s, r_s)$  is neither concave nor convex in  $k_s$ , and its maximization with respect to  $k_s$  may involve interior solutions or corner solutions with  $k_s = \gamma_s$ . The intuition for the positive probability that the bank ends up with excess lending capacity in state  $s' = l$  and rations credit in state  $s' = h$  is the following.<sup>15</sup> If in the two possible states at date  $t + 1$  the bank had probability one of finding itself with excess lending capacity, then it would have an incentive to reduce its capital at date  $t$  in order to lower its funding costs. Conversely, if in the two possible states at date  $t + 1$  the bank had probability one of finding itself with insufficient lending capacity, then it would have an incentive to either increase its capital at date  $t$  (in order to relax the capital constraint at date  $t + 1$ ) or go to the corner  $k_s = \gamma_s$ .<sup>16</sup>

## 4 Equilibrium

In the previous section we have characterized banks' capital and lending decisions at the dates in which they can raise capital, as well as at the dates in which they cannot. This analysis has taken as given the interest rate  $r_s$  at the beginning of a lending relationship in state  $s$ , while the continuation loan rate was set equal to the success return  $a$  of the second period investment projects (because the banks' monopoly position at that stage). In order to define an equilibrium, it only remains to describe how the initial loan rate is determined.

Under perfect competition, the pricing of these loans must be such that the net present value of the bank to its shareholders is zero under its optimal capital decision. Were it negative, no bank would extend loans. Were it positive, banks would have an incentive to expand the scale of their activities. Hence in each state of the economy  $s = h, l$  we must have

$$v_s(k_s^*, r_s^*) = 0, \tag{8}$$

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<sup>15</sup>In fact, excess lending capacity may also occur in state  $s' = h$  and credit rationing may also occur in state  $s' = l$ .

<sup>16</sup>The possible jump to the corner  $k_s = \gamma_s$  is due to the fact that, as shown in the proof of the lemma, in this case the function  $v_s(k_s, r_s)$  is either decreasing or convex in  $k_s$  in the relevant range.

for

$$k_s^* = \arg \max_{\gamma_s \leq k_s \leq 1} v_s(k_s, r_s^*). \quad (9)$$

We define an *equilibrium* as a sequence of pairs  $\{(k_t, r_t)\}$  describing the capital-to-loan ratio  $k_t$  of the banks that can issue equity at date  $t$  and the interest rate  $r_t$  charged on their initial loans, such that each pair  $(k_t, r_t)$  satisfies (8) and (9) for  $s = s_t$ , where  $s_t$  is the state of the economy at date  $t$ . The following result proves the existence of an equilibrium.

**Lemma 3** *The exists a unique  $r_s^*$  that satisfies the equilibrium conditions (8) and (9).*

## 4.1 Comparative statics of the initial loan rate

The structural parameters that describe the economy are the following: The projects' success return  $a$  (which determines the interest rate of second period loans), the scale of the second period projects  $\mu$ , the loss given default  $\lambda$ , the cost of setting up a lending relationship  $c$ , the cost of bank equity capital  $\delta$ , the capital requirements  $\gamma_h$  and  $\gamma_l$ , and the probabilities of transition from each state to the high default state  $q_h$  and  $q_l$ . To complete the description of the economy, one must also specify the state-contingent cdfs of the default rate,  $F_h(x_t)$  and  $F_l(x_t)$ .

Table 1 summarizes the comparative statics of the equilibrium *initial loan rate*  $r_s^*$ , which are derived in Appendix A. The table shows the sign of the derivative  $dr_s^*/dz$  obtained by differentiating (8) with respect to each exogenous parameter  $z$ . The effects of the various parameters on  $r_s^*$  are inversely related to their impact on the profitability of banks' lending activity. Other things equal,  $a$  and  $\mu$  impact positively on the profitability of continuation lending;  $\lambda$  affects negatively the profitability of both initial lending (directly) and continuation lending (directly and by reducing the availability of capital in the second period);  $c$  has a similar negative effect (with no direct effect on the profitability of continuation loans);  $\delta$  increases the cost of equity funding in both periods;  $\gamma_h$  and  $\gamma_l$  increase the burden of capital regulation in the corresponding initial state, as well as in the corresponding continuation state (which will be  $h$  or  $l$  with probabilities  $q_s$  and  $1 - q_s$ , respectively); finally,  $q_s$  decreases the profitability of continuation lending because, in the high default state  $h$ , loan losses are

higher and the corresponding capital requirement  $\gamma_h$  may also be higher.

**Table 1. Comparative statics of the initial loan rate  $r_s^*$**

| $z$                 | $=$ | $a$ | $\mu$ | $\lambda$ | $c$ | $\delta$ | $\gamma_h$ | $\gamma_l$ | $q_s$ |
|---------------------|-----|-----|-------|-----------|-----|----------|------------|------------|-------|
| $\frac{dr_s^*}{dz}$ |     | —   | —     | +         | +   | +        | +          | +          | +     |

## 4.2 Comparative statics of the initial capital

Table 2 summarizes the comparative statics of the equilibrium *initial capital*  $k_s^*$  chosen by the representative bank in an interior solution—obviously, when the solution is at the corner  $k_s^* = \gamma_s$ , marginal changes in parameters other than the capital requirement  $\gamma_s$  do not change  $k_s^*$ . As further explained in Appendix A, the recursive nature of the comparative statics of the system given by (8) and (9) makes it convenient to decompose the effects of the change in any parameter  $z$  into a *direct effect* (for constant  $r_s^*$ ) and a *loan rate effect* (due to the change in  $r_s^*$ ):

$$\frac{dk_s^*}{dz} = \frac{\partial k_s^*}{\partial z} + \frac{\partial k_s^*}{\partial r_s^*} \frac{dr_s^*}{dz}.$$

In Appendix A we show that  $\partial k_s^* / \partial r_s$  is negative, which implies that the signs of loan rate effects are the opposite to those in Table 1. Intuitively, the initial capital  $k_s$  and the initial loan rate  $r_s$  are substitutes in the role of providing the bank with sufficient capital for its continuation lending (see the definition of  $k'_s(x)$  in (2)). In an interior solution, the marginal value of  $k_s$  is decreasing in  $k_s$ , and thus also in  $r_s$ , so a higher  $r_s$  reduces the bank's incentive to hold excess capital.

For the parameters  $a$ ,  $\mu$ , and  $\delta$ , the direct and the loan rate effects point in the same direction, so the total effect can be analytically signed. In essence, higher profitability of continuation lending (captured by  $a$  and  $\mu$ ) and lower costs of capital (captured by  $\delta$ ) encourage banks to hold larger capital buffers in order to better self-insure against the default shocks that threaten its continuation lending. For the setup cost  $c$ , the direct and the loan rate effects have unambiguous but opposite signs, so the total effect is ambiguous. The

positive direct effect comes from the fact that, by the definition of  $k'_s(x)$  in (2),  $c$  subtracts to the bank's continuation lending capacity exactly like  $k_s$  adds to it, without affecting the profitability of such lending and hence the marginal gains from self-insuring against default shocks.

**Table 2. Comparative statics of the initial capital  $k_s^*$**   
(in an interior equilibrium)

|  | $z$ | $=$ | $a$ | $\mu$ | $\lambda$ | $c$ | $\delta$ | $\gamma_h$ | $\gamma_l$ | $q_s$ |
|--|-----|-----|-----|-------|-----------|-----|----------|------------|------------|-------|
| $\frac{\partial k_s^*}{\partial z}$ (direct effect)                      |     |     | +   | +     | ?         | +   | −        | ?          | ?          | ?     |
| $\frac{\partial z}{\partial k_s^*} \frac{dr_s^*}{dz}$ (loan rate effect) |     |     | +   | +     | −         | −   | −        | −          | −          | −     |
| $\frac{dk_s^*}{dz}$ (total effect)                                       |     |     | +   | +     | ?         | ?   | −        | ?          | ?          | ?     |

The direct effects on  $k_s$  of parameters  $\lambda$ ,  $\gamma_h$ ,  $\gamma_l$ , and  $q_s$ , have ambiguous signs. Increasing any of these parameters simultaneously reduces the profitability of continuation lending and impairs the expected capital position of the bank when such lending has to be made. The value of holding excess capital in the initial lending period falls, but the prospects of ending up with insufficient capital increase. So the profitability of continuation lending and the need for self-insurance against default shocks move in opposite directions. The resulting ambiguity of the direct effects extends to the total effects.

The details of the analytical expressions suggest that the shape of the distributions of default rates matter for the determination of the unsigned effects, which eventually becomes a question to be elucidated either empirically or by numerically solving the model under realistic parameterizations. In the rest of the paper, we resort to the second alternative, if only because the experience accumulated with Basel II capital requirements is still very short and the available data is heavily affected by the unprecedented severity of the 2007-2009 financial crisis.

## 5 Numerical Results

To further explore the determinants of banks' initial capital buffers and to compare the performance of flat versus cyclically-varying capital requirements, we numerically solve the model for a range of plausible scenarios. In all scenarios we assume that the state-contingent probability distributions of the default rate, described by the cdfs  $F_h(x_t)$  and  $F_l(x_t)$ , conform to the single risk factor model that underlies the capital requirements associated with the IRB approach of Basel II.<sup>17</sup> This means that we assess the implications of the new capital requirements under the assumption that the regulator's model of reference is correct.<sup>18</sup>

As formally shown in Appendix A, the single risk factor model implies

$$F_s(x) = \Phi \left( \frac{\sqrt{1-\rho_s} \Phi^{-1}(x) - \Phi^{-1}(p_s)}{\sqrt{\rho_s}} \right), \quad (10)$$

for  $s = h, l$ , where  $\Phi(\cdot)$  denotes the cdf of a standard normal random variable, and  $\rho_s \in (0, 1)$  is a parameter that determines the extent of correlation in loan defaults. Following the treatment of corporate loans in Basel II, we further assume that

$$\rho_s = 0.12 \left( 2 - \frac{1 - e^{-50p_s}}{1 - e^{-50}} \right). \quad (11)$$

With this statistical setup in the background, the IRB approach of Basel II requires capital to cover the one-year ahead loan losses with a probability of 99.9%. Hence, it sets the capital requirement in state  $s$  as  $\gamma_s = \lambda F_s^{-1}(0.999)$ , where  $\lambda$  is the loss given default and  $F_s^{-1}(0.999)$  is the 99.9% quantile of the distribution of the default rate. Using (10), the Basel II capital requirement for state  $s$  becomes

$$\gamma_s = \lambda \Phi \left( \frac{\Phi^{-1}(p_s) + \sqrt{\rho_s} \Phi^{-1}(0.999)}{\sqrt{1-\rho_s}} \right), \quad (12)$$

where  $\rho_s$  is given by (11). This is the formula for corporate exposures of a one-year maturity that appears in Basel Committee on Banking Supervision (2004, paragraph 272) and the one

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<sup>17</sup>The single factor model is due to Vasicek (2002) and its use as a foundation for the capital requirements of Basel II is due to Gordy (2003).

<sup>18</sup>See Section 5.6 below for a discussion of what happens when this is not the case.



that we will use below as associated with the Basel II regime.<sup>19</sup> In line with the one-year ahead value-at-risk perspective of Basel II, the parameterization assumes that each model period corresponds to one year.

## 5.1 Benchmark scenarios

Table 3 shows the set of parameter values that define the three benchmark scenarios considered in our numerical analysis. The scenarios differ in the volatility of the state-contingent probabilities of default  $p_l$  and  $p_h$ . We briefly comment on them here, relegating further discussion based on pre-crisis US banking data to Appendix B. Note that because of our normalization of the risk-free rate to zero, all interest rates should be interpreted as spreads over the risk-free rate.

Panel A of Table 3 contains the parameters that are common to the three scenarios. The value of the success return  $a = 0.04$  implies that the interest rate of continuation loans is 4%.<sup>20</sup> The loss given default (LGD) parameter  $\lambda = 0.45$  is taken from the Basel II “foundation IRB” formula for unsecured corporate exposures.<sup>21</sup> The scale of the second period projects  $\mu = 1$  provides just a starting point in the absence of an empirical estimate of the growth rate of loan exposures along a typical corporate lending relationship. The cost of setting up a lending relationship  $c = 0.03$  is chosen so as to yield realistic initial loan rates. The cost of bank capital  $\delta = 0.04$  is intended to capture the tax disadvantages of equity financing relative to deposit financing. The probabilities of transition to the high default state,  $q_l = 0.20$  and  $q_h = 0.64$ , imply expected durations of  $1/q_l = 5$  years for the low default state and  $1/(1 - q_h) = 2.8$  years for the high default state, which we calibrate according to the observed behavior of the charge-off ratio of FDIC-insured commercial banks in the US

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<sup>19</sup>The Basel II formula incorporates an adjustment factor that is increasing in the maturity of the exposure, and equals one for a maturity of one year. Also, Basel II distinguishes between the expected losses,  $\lambda p_s$ , which should be covered with general loan loss provisions, and the remaining charge,  $\lambda(\gamma_s - p_s)$ , which should be covered with capital. However, from the perspective of our analysis, provisions are just another form of equity capital, so the distinction between the expected and unexpected components of loan losses is immaterial.

<sup>20</sup>The success return could be higher, as long as the part that can be pledged to the bank without destroying the entrepreneur’s incentives is set at 4%. See Holmström and Tirole (1997) for a discussion of the concept of pledgeable income.

<sup>21</sup>In the “advanced IRB” approach banks are required to use their own internal models to estimate  $\lambda$ .

during the period 1970-2004.<sup>22</sup> In Section 5.5 we analyze the implications of changing the value of these parameters, one at a time.

**Table 3. Parameter values in the benchmark scenarios**

| A. Common parameters |       |           |      |          |       |       |
|----------------------|-------|-----------|------|----------|-------|-------|
| $a$                  | $\mu$ | $\lambda$ | $c$  | $\delta$ | $q_l$ | $q_h$ |
| 0.04                 | 1.00  | 0.45      | 0.03 | 0.04     | 0.20  | 0.64  |

| B. Probability of default (PD) scenarios |         |               |                       |
|--|---------|---------------|-----------------------|
| Scenarios                                |         | Benchmark PDs | Basel II requirements |
|  |         | $p_s$ (%)     | $\gamma_s$ (%)        |
| Low                                      | $s = l$ | 1.20          | 6.87                  |
| volatility                               | $s = h$ | 2.91          | 10.01                 |
| Medium                                   | $s = l$ | 1.10          | 6.60                  |
| volatility                               | $s = h$ | 3.26          | 10.51                 |
| High                                     | $s = l$ | 1.00          | 6.31                  |
| volatility                               | $s = h$ | 3.62          | 11.00                 |

The three PD scenarios are defined so as to keep the expected capital charge under Basel II equal to 8%, which is the capital requirement under Basel I. Appendix B discusses the choice of parameter values in the light of pre-crisis US banking data.

Panel B of Table 3 shows our choices for the probabilities of default (PDs) in each state,  $p_l$  and  $p_h$ , and the corresponding Basel II capital requirements,  $\gamma_l$  and  $\gamma_h$ , implied by (12). In each scenario, we choose PDs such that the long-run average capital requirement under the cyclically-varying Basel II regime is 8% and, hence, coincides with the flat requirement that corresponds to the Basel I regime.<sup>23</sup> The idea is to allow for a comparison of the cyclical effects of each regime that is not affected by a change in the long-run average level of the

<sup>22</sup>Expected durations are computed as  $q_l + 2(1 - q_l)q_l + 3(1 - q_l)^2q_l + \dots = 1/q_l$  for state  $l$ , and  $(1 - q_h) + 2q_h(1 - q_h) + 3q_h^2(1 - q_h) + \dots = 1/(1 - q_h)$  for state  $h$ . It should be noted that, in line with the empirical findings of Bruche and González-Aguado (2009), in our parameterization credit cycle downturns last longer than typical business cycle downturns.

<sup>23</sup>This long-run average, as well as other unconditional averages reported below, are computed using the unconditional probabilities of visiting the low and the high default state, denoted  $\phi_l$  and  $\phi_h$ . These probabilities are obtained by solving the system of equations  $q_l\phi_l + q_h\phi_h = \phi_h$  and  $\phi_l + \phi_h = 1$ , which gives  $\phi_l = (1 - q_h)/(1 - q_h + q_l) \simeq 0.64$  and  $\phi_h = q_l/(1 - q_h + q_l) \simeq 0.36$ .

capital requirements.<sup>24</sup> The three scenarios only differ in the magnitude of the cross-state variation in the PDs—and all of them are within a range that can be considered empirically plausible.

It is important to note that we are assuming that the LGD parameter  $\lambda$  does not depend on the state  $s$ . Altman et al. (2005), using data on bond defaults, find that LGDs are positively correlated with default rates. In the context of our model, cyclical variation in LGDs has similar effects as the cyclical variation in PDs. Hence our conservative assumption on LGDs could be offset by simply placing more weight on the results corresponding to the scenarios with more volatile PDs.

## 5.2 Loan rates and capital buffers

Table 4 shows initial loan rates  $r_s^*$ , initial capital  $k_s^*$ , and the implied capital buffers  $\Delta_s^* = k_s^* - \gamma_s$ , for  $s = h, l$ , in each of the three scenarios described in Table 3 and under the two regulatory regimes that we want to compare: Basel I, with a flat capital requirement of 8%, and Basel II, with the requirements given by (12). As a reference, we also include the results in a laissez-faire environment without capital requirements ( $\gamma_h = \gamma_l = 0$ ).

**Table 4. Initial loan rates, capital, and capital buffers**  
(all variables in %)

| Scenarios  |         | Basel I |         |              | Basel II |         |              | Laissez-faire |         |              |
|------------|---------|---------|---------|--------------|----------|---------|--------------|---------------|---------|--------------|
|            |         | $r_s^*$ | $k_s^*$ | $\Delta_s^*$ | $r_s^*$  | $k_s^*$ | $\Delta_s^*$ | $r_s^*$       | $k_s^*$ | $\Delta_s^*$ |
| Low        | $s = l$ | 1.2     | 11.0    | 3.0          | 1.2      | 11.9    | 5.0          | 0.7           | 5.2     | 5.2          |
| volatility | $s = h$ | 2.4     | 11.2    | 3.2          | 2.5      | 12.2    | 2.2          | 1.8           | 5.4     | 5.4          |
| Medium     | $s = l$ | 1.2     | 11.0    | 3.0          | 1.2      | 11.7    | 5.1          | 0.6           | 5.1     | 5.1          |
| volatility | $s = h$ | 2.7     | 11.2    | 3.2          | 2.8      | 12.5    | 1.9          | 2.1           | 5.3     | 5.3          |
| High       | $s = l$ | 1.1     | 10.9    | 2.9          | 1.1      | 10.7    | 4.3          | 0.5           | 5.0     | 5.0          |
| volatility | $s = h$ | 3.0     | 11.1    | 3.1          | 3.1      | 12.6    | 1.6          | 2.3           | 5.2     | 5.2          |

The parameters that define the three scenarios and the associated Basel II capital requirements are described in Table 3. The capital requirement in Basel I is 8% and in laissez-faire is 0%.

<sup>24</sup>With this parameterization strategy, the results for the Basel I regime can also be interpreted as (roughly) those that would emerge under a perfect through-the-cycle implementation of Basel II (if it were at all feasible), and one can compare them with those of the point-in-time implementation assumed in our Basel II regime.

The results show that initial loan rates are always higher in the high default state, reflecting the need to compensate banks for both a higher PD and a lower prospective profitability of continuation lending (since the high default state  $h$  is more likely to occur after state  $h$  than after state  $l$ ). In the high default state, the loan rates obtained under Basel II are slightly higher than those obtained under Basel I; in the low default state, loan rates are roughly the same under both regimes.<sup>25</sup> These effects are explained by the fact that Basel II significantly increases banks' capital in the high default state, but has a smaller impact on capital in the low default state.

The results also show that, in order to preserve their future lending capacity, banks hold sizeable capital buffers. Under Basel I, the cyclical variation in PDs has a rather small impact on capital decisions, although excess capital tends to be larger in the high default state (where loan losses can be expected to cause a larger reduction in future lending capacity) than in the low default state.<sup>26</sup> Under Basel II the cross-state variability in PDs visibly translates into greater variability of both total capital and capital buffers. Interestingly, the cyclical pattern of the buffers gets reversed, from very slightly countercyclical in the Basel I regime to strongly procyclical in the Basel II regime. The main reason for this reversal is that under Basel II banks in state  $l$  anticipate that if the economy switches to state  $h$  the capital requirement will increase from  $\gamma_l$  to  $\gamma_h$ . This jump in capital requirements implies a reduction in their lending capacity, so to preserve continuation lending they have an incentive to hold larger precautionary capital buffers than under Basel I. Symmetrically, under Basel II banks in state  $h$  anticipate that if the economy switches to state  $l$  the capital requirement will decrease from  $\gamma_h$  to  $\gamma_l$ , so they have an incentive to hold smaller capital buffers than under Basel I. The numerical results for the three scenarios show that the first effect (higher buffers in state  $l$ ) turns out to be more important than the second effect (lower buffers in state  $h$ ), which implies that the long-run average level of the capital buffers (computed with

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<sup>25</sup>The small loan pricing effects of Basel II are in line with previous results obtained in a static framework; see Repullo and Suarez (2004).

<sup>26</sup>This is consistent with the existing evidence about the behavior of capital buffers under Basel I—including Ayuso et al. (2004) with Spanish data, Lindquist (2004) with Norwegian data, and Bikker and Metzmakers (2004) with data from 29 OECD countries—and raises doubts about the interpretation that such evidence reflects banks' myopia.

the unconditional probabilities of visiting each state) is higher under Basel II than under Basel I.<sup>27</sup>

As for the laissez-faire environment, the results in Table 4 confirm that, under our parameterization, the “economic capital” chosen by banks starting lending relationships is well below the regulatory capital of any of the two Basel frameworks, but significantly different from zero (and very similar across states), reflecting banks’ interest in preserving their valuable continuation lending which would be lost if they fail.<sup>28</sup> The lower initial loan rates in both states (relative to the Basel frameworks) reflect the savings on the costs of equity financing due to the use of less capital in the two lending periods.

### 5.3 Credit rationing

Table 5 compares the cyclical behavior of credit rationing under Basel I and Basel II, as well as in the laissez-faire environment. Lending in any given period is made up of initial loans, whose quantity is always one, and continuation loans, whose quantity varies with the lending capacity of the banks that are unable to issue equity in that period. We define *credit rationing* as the expected percentage of continuation projects that cannot be undertaken because of banks’ insufficient lending capacity. Table 5 shows credit rationing in state  $s' = l, h$  when it is reached from state  $s = l, h$  according to any of the four possible sequences  $(s, s')$ .

In the Basel I regime, as well as in the laissez-faire environment, credit rationing does not depend on whether the arrival state  $s'$  is a high or a low default state, since the capital requirement is constant (at 8% and 0%, respectively). Rationing only depends on the profits realized during the previous period, which determine the capital available for continuation lending. The distribution of this random variable depends on the state  $s$  of the economy in the previous period. This explains why the figures for Basel I (and the laissez-faire environment) in Table 5 only vary with  $s$  in each scenario, and are smaller for  $s = l$  than for  $s = h$ .

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<sup>27</sup>The difference in the medium volatility scenario is of 0.88 percentage points.

<sup>28</sup>Elizalde and Repullo (2007) discuss the concept of economic capital and its relationship with the regulatory capital in a model where banks are concerned about the loss of charter values upon failure.

**Table 5. Credit rationing**  
(all variables in %)

| Scenarios          | Credit rationing in state $s'$ |          |               |
|--------------------|--------------------------------|----------|---------------|
|                    | Basel I                        | Basel II | Laissez-faire |
| Low volatility     |                                |          |               |
| $(s, s') = (l, l)$ | 1.4                            | 0.3      | 2.2           |
| $(s, s') = (l, h)$ | 1.4                            | 4.9      | 2.2           |
| $(s, s') = (h, h)$ | 2.5                            | 3.8      | 4.5           |
| $(s, s') = (h, l)$ | 2.5                            | 0.7      | 4.5           |
| Unconditional      | 1.8                            | 1.7      | 3.0           |
| Medium volatility  |                                |          |               |
| $(s, s') = (l, l)$ | 1.4                            | 0.3      | 2.1           |
| $(s, s') = (l, h)$ | 1.4                            | 10.7     | 2.1           |
| $(s, s') = (h, h)$ | 2.7                            | 4.5      | 5.2           |
| $(s, s') = (h, l)$ | 2.7                            | 0.6      | 5.2           |
| Unconditional      | 1.9                            | 2.6      | 3.2           |
| High volatility    |                                |          |               |
| $(s, s') = (l, l)$ | 1.3                            | 0.4      | 2.0           |
| $(s, s') = (l, h)$ | 1.3                            | 24.4     | 2.0           |
| $(s, s') = (h, h)$ | 3.0                            | 5.3      | 6.1           |
| $(s, s') = (h, l)$ | 3.0                            | 0.5      | 6.1           |
| Unconditional      | 1.9                            | 4.6      | 3.5           |

The parameters that define the three scenarios and the associated Basel II capital requirements are described in Table 3. The capital requirement in Basel I is 8% and in laissez-faire is 0%. Credit rationing is the expected percentage of continuation projects that cannot be undertaken because of banks' insufficient lending capacity. The rows show credit rationing in state  $s'$  when it is reached from state  $s$  according to the sequence  $(s, s')$  in the first column. Rows labeled 'unconditional' show weighted averages based on the unconditional probabilities of each state.

In the Basel II regime, the impact of bank profits is also present, but the overall effects on credit rationing are dominated by the cross-state variation of capital requirements, and its endogenous effects on capital buffers. Thus the sequences with  $(s, s') = (l, h)$ , and next those with  $(s, s') = (h, h)$ , systematically exhibit the largest credit rationing. Intuitively, the high default state  $h$  is more likely to occur after state  $h$  than after state  $l$ , so banks hold less capital in state  $s = l$  than in state  $s = h$  (see Table 4). But then if the economy ends up in state  $s' = h$ , the combination of a lower capitalization in the previous period and a

higher current requirement explains the very sizable contractions in lending capacity shown in Table 5.<sup>29</sup> In particular, for the medium volatility scenario, when the economy goes from the low to the high default state (and despite of the fact that banks hold a capital buffer of 5.1%) an average of 10.7% of the continuation projects are rationed, a figure that goes up to 24.4% in the high volatility scenario.

Thus the Basel II regime implies significantly larger cyclical variation in credit rationing (and consequently in investment and output) than the Basel I regime. Its incidence on the average level of credit rationing, shown in the rows labeled ‘unconditional’ in Table 5, depends on the volatility of PDs along the cycle. For the medium volatility scenario, the extra cost of Basel II in terms of long-run average credit rationing amounts to 0.7% of the potential continuation investment, going up to 2.7% in the high volatility scenario.

The behavior of credit rationing in the laissez-faire environment is very much an amplified version of what obtains under Basel I, with levels of rationing that are between 50% and 100% larger. In relation with Basel II, the comparison depends on the volatility of PDs along the cycle: except in the low volatility scenario, the laissez-faire exhibits lower cyclical variation and, in the high volatility scenario, it even exhibits lower unconditional credit rationing.

## 5.4 Banks’ solvency

We next compare the various regulatory regimes in terms of banks’ solvency. Table 6 reports the probability of failure of the representative bank for each of the three scenarios described in Table 3. These probabilities are different for (first period) banks making initial loans and (second period) banks making continuation loans. Unlike in the results on credit rationing, these probabilities do not depend on the state of the economy in the previous period, and hence we only report their conditional-on- $s$  and unconditional values.

Table 6 shows that the probabilities of bank failure are much more uniform across states under the risk-based capital requirements of Basel II than under the constant 8% capital requirement of Basel I. Conditional on the state of the economy, the link between the level

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<sup>29</sup>Interestingly, for the sequences with  $s' = l$  (which entail the lowest credit rationing under Basel II), the effect of profits becomes visible again, producing lower rationing in the  $(l, l)$  sequence than in the  $(h, l)$  sequence.

of the requirements and the level of solvency of second period banks is direct (since these banks hold no capital buffers and the net interest income earned on performing loans is the same in both states), so not surprisingly the Basel II regime implies a significantly higher solvency in the high default state  $h$  and a slightly lower solvency in the low default state  $l$ , with the unconditional effect being clearly positive.

**Table 6. Banks' solvency**  
(all variables in %)

| Scenarios                    | Probability of bank failure |          |               |
|------------------------------|-----------------------------|----------|---------------|
|                              | Basel I                     | Basel II | Laissez-faire |
| Low volatility               |                             |          |               |
| First period banks: $s = l$  | 0.025                       | 0.016    | 2.185         |
| $s = h$                      | 0.094                       | 0.051    | 4.492         |
| Unconditional                | 0.050                       | 0.028    | 3.013         |
| Second period banks: $s = l$ | 0.008                       | 0.014    | 1.023         |
| $s = h$                      | 0.054                       | 0.018    | 5.721         |
| Unconditional                | 0.024                       | 0.015    | 2.710         |
| Medium volatility            |                             |          |               |
| First period banks: $s = l$  | 0.022                       | 0.014    | 2.080         |
| $s = h$                      | 0.115                       | 0.054    | 5.210         |
| Unconditional                | 0.056                       | 0.029    | 3.203         |
| Second period banks: $s = l$ | 0.006                       | 0.014    | 0.867         |
| $s = h$                      | 0.074                       | 0.019    | 7.195         |
| Unconditional                | 0.030                       | 0.015    | 3.139         |
| High volatility              |                             |          |               |
| First period banks: $s = l$  | 0.019                       | 0.023    | 1.968         |
| $s = h$                      | 0.140                       | 0.059    | 6.126         |
| Unconditional                | 0.063                       | 0.036    | 3.461         |
| Second period banks: $s = l$ | 0.005                       | 0.013    | 0.723         |
| $s = h$                      | 0.099                       | 0.019    | 8.895         |
| Unconditional                | 0.039                       | 0.015    | 3.657         |

The parameters that define the three scenarios and the associated Basel II capital requirements are described in Table 3. The capital requirement in Basel I is 8% and in laissez-faire is 0%. Rows labeled 'unconditional' show weighted averages based on the unconditional probabilities of each state.

For first period banks there are additional effects coming from the endogenous capital buffers and loan interest rates. Table 4 shows that interest rates vary very little across



regulatory regimes, so the comparison between them is dominated by the behavior of the total holdings of capital. Basel II entails higher solvency of first period banks in state  $h$ , and (in the low and medium volatility scenario) also higher solvency in state  $l$ , despite imposing lower requirements than the 8% of Basel I. All in all, Basel II roughly halves the probabilities of bank failure associated with Basel I, and makes the risk of failure more evenly distributed over time. This suggests that the risk-based capital requirements of Basel II have a payoff in terms of the long-term solvency of the banking system.

It is worth noting that the probabilities of bank failure are very small in the two Basel regimes—unconditionally, they range between 0.024% and 0.063% under Basel I, and between 0.015% and 0.036% under Basel II. Interestingly, the combination of capital buffers and net interest income earned on performing loans makes the latter much lower than the 0.1% implied by the 99.9% confidence level of Basel II. This combination also explains why the much higher probabilities of bank failure in the laissez-faire environment—which unconditionally range between 2.710% and 3.657%—are lower than one might have initially expected.

## 5.5 Effect of parameter changes

We now consider the effects on initial loan rates, capital buffers, and credit rationing of changes in the structural parameters that describe the economy. We focus on the effects for the medium volatility scenario under the Basel II regime. The first row of Table 7 reproduces the results for the benchmark parameter values, while the remaining rows presents the effect of changing each of the indicated parameters at a time.

The qualitative effects on initial loan rates  $r_s^*$  correspond to those in Table 1. Increases in the success return  $a$  and the scale of the second period projects  $\mu$  increase the profitability of banks' lending and, consequently, reduce  $r_s^*$ . The quantitative effect of  $a$  is particularly strong, reflecting the impact of second period loan rates on ex ante competition. Increases in the loss given default  $\lambda$ , the cost of setting up a lending relationship  $c$ , and the cost of equity capital  $\delta$  reduce the profitability of banks' lending and, consequently, increase  $r_s^*$ . The quantitative results show that changes in  $c$  are fully passed through to initial loan rates.

Finally, changes in the transition probabilities,  $q_l$  and  $q_h$ , imply only modest changes in the associated initial loan rates.

**Table 7. Effect of parameter changes in the medium volatility scenario under Basel II**  
(all variables in %)

|                           | Credit rationing in state $s'$ |         |              |         |           |          |          |          |
|---------------------------|--------------------------------|---------|--------------|---------|-----------|----------|----------|----------|
|                           | $r_s^*$                        |         | $\Delta_s^*$ |         | $(s, s')$ |          |          |          |
|                           | $s = l$                        | $s = h$ | $s = l$      | $s = h$ | $(l, l)$  | $(l, h)$ | $(h, h)$ | $(h, l)$ |
| Benchmark                 | 1.2                            | 2.8     | 5.1          | 1.9     | 0.3       | 10.7     | 4.5      | 0.6      |
| $\Delta a = 0.01$         | 0.3                            | 1.9     | 6.8          | 3.4     | 0.1       | 3.8      | 3.0      | 0.4      |
| $\Delta \mu = 0.10$       | 0.9                            | 2.6     | 6.3          | 3.2     | 0.2       | 10.7     | 4.1      | 0.4      |
| $\Delta \lambda = 0.05$   | 1.4                            | 3.2     | 4.1          | 1.4     | 0.6       | 20.7     | 5.4      | 0.7      |
| $\Delta c = 0.01$         | 2.2                            | 3.8     | 5.2          | 2.0     | 0.3       | 10.1     | 4.5      | 0.7      |
| $\Delta \delta = 0.01$    | 1.4                            | 3.0     | 3.6          | 1.3     | 0.8       | 23.4     | 5.9      | 0.8      |
| $\Delta(1/q_l) = 1$       | 1.1                            | 2.8     | 4.4          | 1.9     | 0.5       | 18.0     | 4.5      | 0.6      |
| $\Delta(1/(1 - q_h)) = 1$ | 1.2                            | 2.9     | 5.1          | 2.0     | 0.3       | 10.7     | 4.1      | 0.6      |

The parameters that define the medium volatility scenario are described in Table 3.  $1/q_l$  and  $1/(1 - q_h)$  are the expected durations of the low and the high default states, respectively.

The qualitative effects on initial capital  $k_s^*$  and capital buffers  $\Delta_s^* = k_s^* - \gamma_s$  of parameters  $a$ ,  $\mu$ , and  $\delta$  correspond, obviously, to the effects already signed in Table 2. For the other parameters, Table 7 reveals signs that were analytically ambiguous. For instance, increasing  $\lambda$  turns out to decrease the capital buffers, which means that the negative profitability effect dominates the positive self-insurance effect. Consequently, the problem of credit rationing worsens in all sequences of states, an especially in  $(s, s') = (l, h)$  and, to a lesser extent, in  $(s, s') = (h, h)$ . The capital buffers increase with  $c$ , barely vary with the expected duration of the high default state, and significantly decrease with the expected duration of the low default state (which illustrates the dominance in this case of the negative self-insurance effect over the positive profitability effect). The impact on credit rationing is particularly strong for the last parameter: an increase in the expected duration of the low default state from 5 to 6 years leads to a jump in credit rationing from 10.7% to 18.0%.

Although the sign of the effects of increasing the cost of bank capital are analytically predictable, the size of them is worth commenting. An increase in  $\delta$  from 4% to 5% reduces the capital buffer maintained in both states (and especially, in the low default state where it declines from 5.1% to 3.6%) and has a strong impact on credit rationing, especially in the high default state—where it jumps from 10.7% to 23.4% in the  $(l, h)$  sequence, and from 4.5% to 5.9% in the  $(h, h)$  sequence.

## 5.6 Effect of higher default correlation

The results obtained so far are based on the assumption that the regulators’ model of reference, which is used to compute the Basel II capital requirements, is correct. We now examine what happens if the regulator’s assumption (11) on the correlation parameter  $\rho_s$  that appears in the Basel II formula (12) does not coincide with the *true* correlation parameter, denoted  $\rho_s^*$ . Specifically, we assume that  $\rho_s^*$  is state-independent and equal to 0.25 (i.e., 0.01 higher than the highest value yielded by (11)). Given that lower correlation implies lower Basel II capital requirements, this formulation means that the regulator underestimates the capital required to cover the losses associated with the 99.9% quantile of the true distribution of the default rate. Additionally, given that  $\rho_s$  is decreasing in  $p_s$ , it also means that the downward bias is more significant in the high default state.

Table 8 shows the effects of this change in the correlation assumption for the medium volatility scenario. The columns labeled “Benchmark economy” and “Higher correlation economy” show the results for the economies with  $\rho_s^* = \rho_s$  and  $\rho_s^* = 0.25 > \rho_s$ , respectively. The differences in the initial loan rates  $r_s^*$  across these economies are negligible. In contrast, the differences in terms of initial capital  $k_s^*$  and capital buffers  $\Delta_s^* = k_s^* - \gamma_s$  are quite significant. Specifically, in the higher correlation economy the capital buffers are more procyclical than in the benchmark economy, reflecting banks’ additional effort to protect their second period lending when the economy switches from the low to the high default state.

The effects on credit rationing are driven by the interaction between the more dispersed distribution of the default rate implied by the higher correlation parameter and the (offsetting) endogenous responses of the capital buffers. Indeed, the higher  $\Delta_l^*$  and slightly

lower  $\Delta_h^*$  explain why credit rationing in the higher correlation economy is, relative to the benchmark economy, more severe in the  $(h, h)$  sequence than in the  $(l, h)$  sequence.

**Table 8. Effect of higher default correlation in the medium volatility scenario under Basel II**  
(all variables in %)

|                                |                    | Benchmark<br>economy | Higher correlation<br>economy |
|--------------------------------|--------------------|----------------------|-------------------------------|
| Initial loan rates             | $s = l$            | 1.2                  | 1.2                           |
|                                | $s = h$            | 2.8                  | 2.8                           |
| Capital buffers                | $s = l$            | 5.1                  | 5.7                           |
|                                | $s = h$            | 1.9                  | 1.7                           |
| Credit rationing in state $s'$ |                    |                      |                               |
|                                | $(s, s') = (l, l)$ | 0.3                  | 0.4                           |
|                                | $(s, s') = (l, h)$ | 10.7                 | 4.8                           |
|                                | $(s, s') = (h, h)$ | 4.5                  | 6.7                           |
|                                | $(s, s') = (h, l)$ | 0.6                  | 2.2                           |
|                                | Unconditional      | 2.6                  | 2.6                           |
| Probability of bank failure    |                    |                      |                               |
| First period banks:            | $s = l$            | 0.014                | 0.051                         |
|                                | $s = h$            | 0.054                | 0.581                         |
|                                | Unconditional      | 0.029                | 0.240                         |
| Second period banks:           | $s = l$            | 0.014                | 0.064                         |
|                                | $s = h$            | 0.019                | 0.289                         |
|                                | Unconditional      | 0.015                | 0.145                         |

The parameters that define the medium volatility scenario are described in Table 3. In the higher correlation economy  $\rho_s^*$  is state-independent and equal to 0.25. Credit rationing is the expected percentage of continuation projects that cannot be undertaken because of banks' insufficient lending capacity. The rows show credit rationing in state  $s'$  when it is reached from state  $s$  according to the sequence  $(s, s')$  in the first column. Rows labeled 'unconditional' show weighted averages based on the unconditional probabilities of each state.

The last rows in Table 8 show the effects of the correlation assumption on banks' solvency. The probabilities of bank failure in the benchmark economy were, not surprisingly, very low, since the Basel II capital requirements were constructed to guarantee probabilities of bank failure below 0.1% (actually strictly lower than that because of the capital buffers held in

the first period and the interest rate margin earned in both periods). With a higher true value of the correlation parameter the probabilities of bank failure are much higher: up to 0.58% in the high default state.

These results show how failures in the calibration of the probability distributions of the default rate (and specifically in the correlation parameter  $\rho_s$ ) may produce significant deviations from the solvency levels targeted by Basel II. Additionally, they show that if the true correlation is higher than what is implicitly assumed in the Basel II formulas, the relative magnitude of the key elements in the trade-off implied by the procyclical effects of risk-based capital requirements (the incidence of credit rationing in the high default states vis-a-vis the probability of bank failure in those states) becomes more balanced than in the benchmark economy. This will be relevant to put the results obtained in the next section into perspective.

## 6 Policy analysis

Our previous results show that the supply of bank credit is much more cyclical under risk-based capital requirements than under flat capital requirements. Specifically, in the Basel II regime there is a strong reduction in banks' lending capacity (and a rise in credit rationing) when the economy goes into a recession. They also show that bank solvency is higher in the Basel II than in the Basel I regime.

These results suggest that, even in our stylized setup with a representative class of loans per period, assessing the net advantages of risk-based regulation in welfare terms is not straightforward and will crucially depend on the (structural or reduced-form) imputation of a social cost to bank failures.<sup>30</sup> Although in principle the model could be extended to perform such a welfare analysis, the discussion in this section will focus on a less normative point: we will show that it is possible to reduce the procyclical effects of Basel II without renouncing to risk-sensitiveness or to the targeted long-term confidence. The adjustments

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<sup>30</sup>A complete welfare assessment should also take into account the advantages of risk-based capital requirements in the cross-sectional treatment of exposures with heterogenous risk profiles. Repullo and Suarez (2004) perform this type of welfare analysis in a static setup where procyclicality is not a concern, but capital requirements imply a deadweight loss due to the extra cost of equity financing.

that we consider consist of modifying the cyclical profile of the confidence levels set by the regulator in such a way that their long-term average equals 99.9%—the target level in the IRB approach of Basel II—but lessens the confidence level in those states (or sequences of states) where credit rationing turns out to be the highest under the Basel II regime.

Table 9 shows the results of two specific policy experiments of this kind. Both are performed under the parameterization of the medium volatility scenario described in Table 3. To facilitate the comparison, we reproduce the results for the Basel II regime already shown in previous tables. Policy 1 reduces the confidence level in the high default state  $h$  to 99.8% and increases the confidence level in the low default state  $l$  to  $\eta > 99.9\%$  so as to maintain the long-run average at 99.9%.<sup>31</sup> Such a small adjustment causes a relevant change in capital requirements (from 6.6% to 7.9% for  $\gamma_l$  and from 10.5% to 9.3% for  $\gamma_h$ ). As show in Panel A, this policy reduces the need for self-insurance and consequently modifies banks' optimal buffers in a direction that partly offsets its intended effects. Yet, as shown in Panel B, the policy is overall effective in smoothing the cyclicity of credit rationing. In the sequences  $(l, h)$  and  $(h, h)$  credit rationing falls from 10.7% and 4.5%, respectively, to less than 4%. Unconditionally, it falls from 2.6% to 1.9% (which coincides with its unconditional value under Basel I in Table 5). Interestingly, although the probabilities of bank failure in the high default state  $h$  obviously increase, they remain lower than 0.08% in all cells, and their unconditional average only increases from 0.029% to 0.040% for first period banks, and from 0.015% to 0.017% for second period banks.

In Policy 2 we confine the reduced 99.8% confidence level to periods where state  $h$  occurs after state  $l$ . The objective is to reduce the credit rationing detected in the second period of sequences with  $(s, s') = (l, h)$ . The capital requirement when  $h$  occurs after  $h$  is left unchanged, while the confidence level applied to the second period of the sequences  $(l, l)$  and  $(h, l)$  is increased so as to to keep the long-run average confidence level at 99.9%. By construction, Policy 2 makes smaller adjustments in the Basel II capital requirements, so we can expect it to be less effective than Policy 1 in terms of smoothing credit rationing, but it

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<sup>31</sup>Thus  $\eta$  solves  $\eta \times \phi_l + 0.998 \times \phi_h = 0.999$ , where  $\phi_l$  and  $\phi_h$  are, respectively, the unconditional probabilities of the low and the high default state derived in footnote 23.

will also have a smaller impact on banks' solvency. The results in Table 9 show that, although Policy 2 also modifies optimal capital buffers in a direction that partly offsets its intended effect on credit rationing, its impact in the  $(l, h)$  sequence is still substantial. Meanwhile the unconditional probabilities of bank failure are almost unchanged relative to those of Basel II.

**Table 9. Procyclicality correction**  
(all variables in %)

| A. Capital buffers in state $s'$ |          |          |          |          |  |
|----------------------------------|----------|----------|----------|----------|--|
| $(s, s') =$                      | $(l, l)$ | $(l, h)$ | $(h, h)$ | $(h, l)$ |  |
| Basel II                         | 5.1      | 1.9      | 1.9      | 5.1      |  |
| Policy 1                         | 3.5      | 2.5      | 2.5      | 3.5      |  |
| Policy 2                         | 4.3      | 3.1      | 1.9      | 4.7      |  |

| B. Credit rationing in state $s'$ |          |          |          |          |               |
|-----------------------------------|----------|----------|----------|----------|---------------|
| $(s, s') =$                       | $(l, l)$ | $(l, h)$ | $(h, h)$ | $(h, l)$ | Unconditional |
| Basel II                          | 0.3      | 10.7     | 4.5      | 0.6      | 2.6           |
| Policy 1                          | 0.8      | 3.7      | 3.6      | 1.6      | 1.9           |
| Policy 2                          | 0.5      | 4.4      | 4.4      | 0.6      | 1.9           |

| C. Probability of failure of first period banks in state $s'$ |          |          |          |          |               |
|---|----------|----------|----------|----------|---------------|
| $(s, s') =$   | $(l, l)$ | $(l, h)$ | $(h, h)$ | $(h, l)$ | Unconditional |
| Basel II  | 0.014    | 0.054    | 0.054    | 0.014    | 0.029         |
| Policy 1  | 0.017    | 0.079    | 0.079    | 0.017    | 0.040         |
| Policy 2  | 0.019    | 0.054    | 0.054    | 0.019    | 0.031         |

| D. Probability of failure of second period banks in state $s'$ |          |          |          |          |               |
|--|----------|----------|----------|----------|---------------|
| $(s, s') =$  | $(l, l)$ | $(l, h)$ | $(h, h)$ | $(h, l)$ | Unconditional |
| Basel II   | 0.014    | 0.019    | 0.019    | 0.014    | 0.015         |
| Policy 1   | 0.007    | 0.035    | 0.035    | 0.007    | 0.017         |
| Policy 2   | 0.011    | 0.035    | 0.019    | 0.014    | 0.016         |

The parameters and associated Basel II capital requirements are those of the medium volatility scenario described in Table 3. Policy 1 reduces the confidence level in the Basel II formula to 99.8% in state  $h$  and increases it in state  $l$  so as to keep the unconditional average at 99.9%. Policy 2 sets the confidence level at 99.8% only when state  $h$  occurs after state  $l$ , compensating it when state  $l$  occurs so as to keep the unconditional average at 99.9%. Columns labeled 'unconditional' show weighted averages based on the unconditional probabilities of each state.

All in all, our policy experiments show the feasibility of achieving significant gains in terms of lower credit rationing without major costs in terms of banks' long-term solvency. This can be achieved with cyclical adjustments that preserve the value-at-risk foundation of the IRB approach of Basel II. The choice between Policy 1 and Policy 2 (or the fine tuning of their details) should eventually depend on the trade-off between the gains in terms of a smoother and lower credit rationing, and the losses in terms of less smooth and slightly higher probabilities of bank failure. The experiments also show the importance of treating capital buffers as endogenous variables in this type of policy analyses.

## 7 Discussion

In this section we discuss some simplifying features of our model, including the distribution of banks' market power in the first and second period of the lending relationships, the use of short-term loan contracts, and the assumption that banks with ongoing relationships have no access to the equity market. We consider the possible effects of relaxing these assumptions, noting that in some cases our conclusions about the procyclicality of Basel II would be strengthened.

### 7.1 Competition and market power

In our model banks are perfectly competitive in the market for first period loans and act as monopolists in the market for second period loans. These assumptions are made to guarantee tractability and internal consistency. They are consistent with our justification of lock-in effects as a result of some (unmodeled) asymmetric information problem (e.g., one that makes borrowers searching for a new bank after one period look like lemons in the refinancing market). Perfect competition in the market for initial loans avoids the complications due to having market power on potential borrowers even before starting a relationship with them. In such a scenario, loans from different banks would not be perfect substitutes for the initial borrowers, and banks could attach positive expected value to even non-started relationships with prospective borrowers. So, on the one hand, some initial borrowers would suffer the



capital constraints experienced by their most preferred bank and, on the other, banks would have to take the value of these prospective relationships into account when deciding their capital buffers. Ignoring the effects of these complications do not seem to obviously bias our results in one direction or another, while it is clear that the complications would break the OLG structure of the current model.

As for second period loans, we have opted for an all-or-nothing modeling of the lock-in effects for several reasons. The most obvious is again tractability: if the market for continuation loans were more competitive, the degree of effective competition and the resulting loan rates would vary with the degree of credit rationing. Even in the polar case of perfect competition, banks would be able to appropriate scarcity rents from their (non-rationed) borrowers over the range of poor realizations of the risk factor that lead to rationing.<sup>32</sup> Relative to the current modeling, a more competitive market for second period loans would entail lower continuation rents for banks, and hence lower incentives for them to keep capital buffers. This provides a second justification for our assumption: it yields a conservative estimate of the procyclicality induced by capital regulation.

## 7.2 Short-term loan contracts

We have described the relationship between entrepreneurs and banks as instrumented by a sequence of one-period loans. One might wonder whether focusing on short-term contracts builds in the “imperfection” that drives our main results. The answer is yes and no. With long-term contracts, or more generally contracts that differ from the sequence of one-period loans on which we focus, there might be room for improving over the credit allocation outcomes obtained in our analysis. For example, for given capital buffers, setting higher loan rates in the first period and lower loan rates in the second would reduce the incidence of credit rationing, since banks would have more capital to support their second period lending. But in the context of our model, long-term contracts pose important commitment and enforcement problems. In particular, they would have to specify the loan rates in the

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<sup>32</sup>In the context of the single risk factor model of Basel II, either all banks or no bank are capital constrained in the market for second period loans, so the second period loan rate would be either the monopoly rate  $a$  or some break-even rate that makes continuation lending a zero net present value investment for the banks.

first and the second period, as well as the rationing scheme to be used in those cases where the bank ends up with insufficient lending capacity (since otherwise the bank might try to renegotiate the terms of the second period loans by threatening entrepreneurs to put them in the pool of rationed borrowers). This means that default rates in the first period would have to be verifiable, and banks would have to be restricted in their ability to pay dividends. They might also have to commit to maintain a capital buffer in the first period. The reason for this is that when competing for borrowers in the first period, banks internalize the whole surplus that the underlying investment projects generate over the two periods, but once the relationship starts, banks only internalize the return of their current and future lending. If the second period loan rate is lower than  $a$ , then banks do not take into account part of the continuation surplus when deciding how much capital to hold—recall that first period capital buffers are held for the purpose of reducing credit rationing in the second period. All in all, the previous considerations suggest that there are serious commitment and enforcement problems that limit the possible improvements in the allocation of credit that long-term contracts might bring about.

### 7.3 Imperfect access to the equity market

The assumption that banks with ongoing relationships have no access to the equity market is obviously crucial for our results. With perfect, frictionless access to the equity market in the interim period, there would be no credit rationing among second period borrowers, except in the rare event that their bank fails (and it is not recapitalized by its shareholders). Banks in such a context would most likely hold no buffers, at least under the high capital requirements of Basel I or Basel II. Given the ample evidence on capital market imperfections, the key question is whether the specificities of our approach—that ties these imperfections to the informational asymmetries associated with relationship lending—drive the results.

A more general way of capturing capital market imperfections would be to assume that access can occur with some (exogenous) probability  $v < 1$ . Changes in parameter  $v$  could then be used to evaluate the marginal effects of the friction on capital buffers and credit rationing. One could also explore situations in which  $v$  is contingent on the state  $s'$  of the

economy at the interim date. This extension would probably reinforce our conclusions about the procyclicality of risk-based capital requirements. If with some probability banks can access the equity market in the interim period, they would have lower incentives to keep capital buffers, so depending on parameter values the incidence of credit rationing could be higher.

It should also be noted that the really strong assumption that we are making is that banks can frictionless access the equity market when they renew their stock of lending relationships. This assumption is instrumental to achieving a tractable OLG structure in the context of an infinite horizon problem that otherwise would be characterized by longer memory and more complex dynamics (namely, credit rationing might also affect first period loans and the pricing of these loans might depend on the full history of the economy). A modeling alternative would be to assume a structure similar to the one in the popular Calvo (1983) model of staggered price setting, i.e., that in each period a fraction of the banks can issue new equity. In this context one would have to discuss the allocation of the newly born entrepreneurs to the existing banks. Would they demand loans to the recapitalizing banks only? If not, how would the pricing of the new loans be determined and what would be the effect on banks' incentives to hold excess capital?

## 7.4 Other extensions

The framework used in this paper could also be extended in a number of other directions. First, we could consider lending relationships that extend over more than two periods. If relationships last for  $T$  periods and banks cannot raise equity for the whole length of the relationship, the qualitative results should be very similar to ours. Such a model would, of course, yield richer dynamics, as the effect of a shock would propagate over several periods. Second, we could incorporate cyclically-varying demand. One easy way of doing this would be to let the scale  $\mu$  of second period projects vary with the state  $s$  of the economy. A bigger challenge would be to introduce some downward sloping aggregate demand for loans, which could be done by assuming that entrepreneurs are heterogeneous in their opportunity cost of becoming active in the first period. The already existing variability in projects' success

probabilities would tend to produce (procyclical) variability in the demand for loans. Further cyclical variability might be introduced by replacing the current success return  $a$  for some  $a_s$ . Finally, one could allow for feedback effects from constrained to unconstrained entrepreneurs by letting  $a_t = a(I_t)$  instead of  $a$ , where  $a(I_t)$  is an increasing (and possibly concave) function of  $I_t$ , the aggregate investment at date  $t$ . This would capture demand externalities or technological complementarities similar to those studied in endogenous growth theory. Analyzing these alternative models is beyond the scope of this paper.

## 8 Concluding Remarks

In many early reports on the implications of Basel II, it was standard to first recognize the potential cyclical effects of the risk-based capital requirements and then qualify that, given that most banks held capital in excess of the regulatory minima, the practical incidence of the procyclicality problem was likely to be small if not negligible. In light of the experience accumulated during the current crisis, the temptation now is to presume that the buffers voluntarily chosen by the banks will never be sufficient to prevent the procyclicality of bank lending, and to conclude that a radical adjustment of the regulation is needed.

While many of the early writings did not have the extension or the technical nature required to elaborate on the foundations of their claims, others unveiled two related misconceptions. The first misconception is that the holding of capital buffers means that capital requirements are “not binding.” Under a purely static perspective this would be tautologically true. In a convex optimization problem, it would also be true that small changes in the level of the requirements would not alter the optimal capital holdings. In a dynamic problem, however, this need not be the case: banks may hold capital buffers because they wish to reduce the risk of facing a statically “binding” requirement in the future. Perhaps these precautions make future requirements “not binding” when the time comes, but clearly their presence alters banks’ capital decisions and the whole development of future events. So observing that banks hold capital buffers does not mean that capital requirements do not matter.

A second related misconception is to accept that the cyclical behavior of capital buffers observed under some particular regulatory regime can be somehow extrapolated, without much qualification, to other regulatory regime. The results in this paper suggest that the behavior of buffers can radically differ across regimes. In other words, if buffers are endogenously affected by the prevailing regulation (even if they appear not to “bind”), reduced-form predictions of what might occur under different regulatory regimes do not resist the Lucas’ critique.

Our model provides a tractable framework in which it is possible to evaluate the cyclical effects of bank capital regulation without incurring in these misconceptions. To keep the analysis as transparent as possible, we simplify on a number of dimensions. For example, we abstract from demand side fluctuations and aggregate feedback effects that might mitigate and exacerbate, respectively, the supply-side effects that we identify. One could take our model as a building block for a fuller dynamic general equilibrium model with a production sector partly composed of entrepreneurial firms that rely on relationship bank lending. One could also think about extensions that generalize our modeling of the frictions related to banks’ access to equity financing. Our contribution is to show that the interaction of relationship lending (which makes some borrowers dependent on the lending capacity of the specific bank with which they establish a relationship) with frictions in banks’ access to equity markets (which makes some banks’ lending capacity a function of their historically determined capital positions and the capital requirements imposed by regulation) has the potential to cause significant cyclical swings in the supply of credit.

Under realistic parameterizations, the model produces capital buffers and equilibrium loan rates whose levels and cyclicity under flat capital requirements such as those of Basel I are in line with those observed in the data. The same parameterizations when applied to the cyclically-varying risk-based capital requirements of Basel II yield a substantial increase in the procyclicality of bank lending. Specifically, despite banks taking precautions and holding larger buffers during expansions in order to have a reserve of capital for the next recession (when capital requirements rise), the arrival of recessions is normally associated with a sizeable credit crunch, as capital-constrained banks are forced to ration credit to some

of their dependent borrowers.

Having a model that explicitly accounts for the endogenous determination of capital buffers and equilibrium loan rates is important for a rigorous analysis of the type of countercyclical adjustments that recent proposals on regulatory reform advocate. As an illustration, we have shown that small cyclical adjustments in the confidence level of Basel II would substantially reduce the incidence of credit rationing over the business cycle without compromising the long-run solvency targets implied in the original regulation.

# Appendix

## A Proofs of analytical results

**Proof of Lemma 1** To derive the expressions provided in the statement of the lemma, we first compute the expected discounted value of shareholders' continuation payoffs at date  $t+1$ , after the default rate on the loans originated at date  $t$  realizes. If the bank fails ( $x_t > \hat{x}_s$ ), this value is obviously zero. When there is insufficient lending capacity ( $\hat{x}_{ss'} < x_t \leq \hat{x}_s$ ), the bank finances  $k'_s(x_t)/\gamma_{s'}$  loans with  $[k'_s(x_t)/\gamma_{s'}] - k'_s(x_t)$  deposits and  $k'_s(x_t)$  capital. At date  $t+2$  the bank gets  $1+a$  from the fraction  $1-x_{t+1}$  of performing loans and  $1-\lambda$  from the fraction  $x_{t+1}$  of defaulted loans, so its assets are  $[1+a-x_{t+1}(\lambda+a)]k'_s(x_t)/\gamma_{s'}$ , while its deposit liabilities are  $(1-\gamma_{s'})k'_s(x_t)/\gamma_{s'}$ . Thus shareholders' expected payoff, conditional on the state of the economy at date  $t+1$ , can be expressed as  $\pi_{s'}k'_s(x_t)/\gamma_{s'}$ , where  $\pi_{s'}$  is the expected gross equity return per unit of loans given by (7). The value of shareholders' stake in the bank at date  $t+1$  is

$$v_{ss'}(x_t) = \frac{\beta\pi_{s'}}{\gamma_{s'}} k'_s(x_t). \quad (13)$$

Assumption (1) guarantees that  $\beta\pi_{s'} > \gamma_{s'}$ , so shareholders strictly prefer to keep  $k'_s(x_t)$  invested in the bank rather than pay a dividend.<sup>33</sup>

When there is excess lending capacity ( $x_t \leq \hat{x}_{ss'}$ ), the bank finances  $\mu$  loans using  $(1-\gamma_{s'})\mu$  deposits and  $\gamma_{s'}\mu$  capital, and pays a dividend of  $k'_s(x_t) - \gamma_{s'}\mu$  to its shareholders. At date  $t+2$  the bank gets  $1+a$  from the fraction  $1-x_{t+1}$  of performing loans and  $1-\lambda$  from the fraction  $x_{t+1}$  of defaulted loans, so its assets are  $[1+a-x_{t+1}(\lambda+a)]\mu$ , while its deposit liabilities are  $(1-\gamma_{s'})\mu$ . Thus shareholders' expected payoff, conditional on the state of the economy at date  $t+1$ , is  $\mu\pi_{s'}$ , where  $\pi_{s'}$  is given by (7). The value of shareholders' stake in the bank at date  $t+1$ , inclusive of the dividend  $k'_s(x_t) - \gamma_{s'}\mu$ , can be written as

$$v_{ss'}(x_t) = (\beta\pi_{s'} - \gamma_{s'})\mu + k'_s(x_t), \quad (14)$$

The first term in (14) measures the net present value contribution of the capital that remains invested in the bank up to date  $t+2$  (which is positive, since  $\beta\pi_{s'} > \gamma_{s'}$ ).

Putting together the three possible cases, we can express the value of the bank at date  $t+1$ , inclusive of dividends, as in (6), where  $v_{ss'}(x_t)$  is a continuous and piecewise linear

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<sup>33</sup>To see this, notice that  $\pi_{s'} > \int_0^1 [\gamma_{s'} + a - x_{t+1}(\lambda+a)] dF_{s'}(x_{t+1}) = \gamma_{s'} + a - p_{t+1}(\lambda+a)$ , but assumption (1) implies  $a - p_{t+1}(\lambda+a) > \gamma_{s'}\delta$  and hence  $\pi_{s'} > \gamma_{s'}(1+\delta) = \gamma_{s'}/\beta$ .

function of  $x_t$ . After taking expectations, discounting, and subtracting the initial capital contribution, the net present value of the bank at date  $t$  is given by (5). ■

**Proof of Lemma 2** The existence of a solution to the bank's capital decision problem comes directly from the fact that the net present value  $v_s(k_s, r_s)$  is continuous in  $k_s$ , for any given interest rate  $r_s$ . This value can be written as

$$v_s(k_s, r_s) = q_s v_{sh}(k_s, r_s) + (1 - q_s) v_{sl}(k_s, r_s), \quad (15)$$

where

$$v_{ss'}(k_s, r_s) = \beta \left[ \int_0^{\hat{x}_{ss'}} [(\beta \pi_{s'} - \gamma_{s'}) \mu + k'_s(x)] dF_s(x) + \frac{\beta \pi_{s'}}{\gamma_{s'}} \int_{\hat{x}_{ss'}}^{\hat{x}_s} k'_s(x) dF_s(x) \right] - k_s. \quad (16)$$

By the definitions (3) and (4) of  $\hat{x}_s$  and  $\hat{x}_{ss'}$ , the function  $v_{ss'}(k_s, r_s)$  has the following properties:

1. For  $k_s \leq c - r_s$  we have  $\hat{x}_{ss'} < \hat{x}_s \leq 0$ , so

$$\frac{\partial v_{ss'}}{\partial k_s} = -1 < 0.$$

2. For  $c - r_s < k_s \leq c - r_s + \gamma_{s'} \mu$  we have  $\hat{x}_{ss'} \leq 0 < \hat{x}_s$ , so

$$\frac{\partial v_{ss'}}{\partial k_s} = \frac{\beta^2 \pi_{s'}}{\gamma_{s'}} F_s(\hat{x}_s) - 1 \leq 0, \text{ and } \frac{\partial^2 v_{ss'}}{\partial k_s^2} = \frac{\beta^2 \pi_{s'} F'_s(\hat{x}_s)}{\gamma_{s'}(\lambda + r_s)} > 0.$$

3. For  $c - r_s + \gamma_{s'} \mu < k_s < c + \lambda + \gamma_{s'} \mu$  we have  $0 < \hat{x}_{ss'} < 1$ , so

$$\frac{\partial v_{ss'}}{\partial k_s} = \frac{\beta}{\gamma_{s'}} [\beta \pi_{s'} F_s(\hat{x}_s) - (\beta \pi_{s'} - \gamma_{s'}) F_s(\hat{x}_{ss'})] - 1 \leq 0,$$

and

$$\frac{\partial^2 v_{ss'}}{\partial k_s^2} = \frac{\beta}{\gamma_{s'}(\lambda + r_s)} [\beta \pi_{s'} F'_s(\hat{x}_s) - (\beta \pi_{s'} - \gamma_{s'}) F'_s(\hat{x}_{ss'})] \leq 0.$$

4. For  $c + \lambda + \gamma_{s'} \mu \leq k_s$  we have  $1 \leq \hat{x}_{ss'} < \hat{x}_s$ , so

$$\frac{\partial v_{ss'}}{\partial k_s} = \beta - 1 < 0.$$

Hence the function  $v_{ss'}(k_s, r_s)$  is linearly decreasing or strictly convex for  $k_s \leq c - r_s + \gamma_{s'} \mu$ , linearly decreasing for  $k_s \geq c + \lambda + \gamma_{s'} \mu$ , and may be increasing or decreasing, and concave or convex for  $c - r_s + \gamma_{s'} \mu < k_s < c + \lambda + \gamma_{s'} \mu$ . Introducing the constraint  $\gamma_s \leq k_s \leq 1$  (and assuming that parameter values are such that  $c + \lambda + \gamma_{s'} \mu < 1$ ) it follows that the problem



$\max_{\gamma_s \leq k_s \leq 1} v_{ss'}(k_s, r_s)$  has either a corner solution with  $k_s = \gamma_s$ , or an interior solution with  $c - r_s + \gamma_{s'}\mu < k_s < c + \lambda + \gamma_{s'}\mu$ . In the latter case we have  $0 < \hat{x}_{ss'} < 1$ , so there is a positive probability  $F_s(\hat{x}_{ss'})$  that the bank has excess lending capacity in state  $s'$ , and a positive probability  $1 - F_s(\hat{x}_{ss'})$  that the bank has insufficient lending capacity in state  $s'$ . Since  $\gamma_l \leq \gamma_h$  implies  $c - r_s + \gamma_l\mu \leq c - r_s + \gamma_h\mu$  and  $c + \lambda + \gamma_l\mu \leq c + \lambda + \gamma_h\mu$ , we conclude that the problem  $\max_{\gamma_s \leq k_s \leq 1} [q_s v_{sh}(k_s, r_s) + (1 - q_s) v_{sl}(k_s, r_s)]$  has either a corner solution with  $k_s = \gamma_s$ , or an interior solution with  $c - r_s + \gamma_l\mu < k_s < c + \lambda + \gamma_h\mu$ . In the latter case, there must be a positive probability that the bank has insufficient lending capacity in state  $s' = h$  (and possibly also in state  $s' = l$ ), and a positive probability that the bank has excess lending capacity in state  $s' = l$  (and possibly also in state  $s' = h$ ). ■

**Proof of Lemma 3** Differentiating (8) we have

$$\frac{dv_s}{dr_s} = \frac{\partial v_s}{\partial k_s^*} \frac{dk_s^*}{\partial r_s} + \frac{\partial v_s}{\partial r_s},$$

where the first term is zero, by the envelope theorem, and the second is positive, because  $r_s$  has a positive impact on  $k'_s(x_t)$  and hence on  $v_{ss'}(x_t)$ . Moreover, for sufficiently low interest rates we have  $v_s(k_s^*, r_s) < 0$ , while for  $r_s = a$  assumption (1) implies  $v_s(k_s^*, r_s) > 0$ . Hence we conclude that there is a unique  $r_s^*$  that satisfies  $v_s(k_s^*, r_s^*) = 0$ .<sup>34</sup> ■

**Comparative statics of the initial loan rate** The sign of  $dr_s^*/dz$  for  $z = a, \mu, \lambda, c, \delta, q_s, \gamma_h, \gamma_l$  is obtained by total differentiation of (8):

$$\frac{\partial v_s}{\partial k_s^*} \frac{dk_s^*}{dz} + \frac{\partial v_s}{\partial r_s} \frac{dr_s^*}{dz} + \frac{\partial v_s}{\partial z} = 0. \quad (17)$$

When  $k_s^*$  is interior, the first-order condition for a maximum that follows from (9) gives  $\partial v_s / \partial k_s = 0$ , so the first term in (17) vanishes. Hence we have

$$\frac{dr_s^*}{dz} = - \left( \frac{\partial v_s}{\partial r_s} \right)^{-1} \frac{\partial v_s}{\partial z}, \quad (18)$$

In the proof of Lemma 3 we have noted that  $\partial v_s / \partial r_s > 0$ . so the sign of  $dr_s^*/dz$  is just opposite to the sign of  $\partial v_s / \partial z$ .

Similarly, in a corner solution with  $k_s^* = \gamma_s$  we have  $dk_s^*/dz = 0$  for all  $z \neq \gamma_s$ , in which case the first term in (17) also vanishes and (18) obtains again. Finally, for  $z = \gamma_s$ , we have  $dk_s^*/d\gamma_s = 1$ , which implies

$$\frac{dr_s^*}{d\gamma_s} = - \left( \frac{\partial v_s}{\partial r_s} \right)^{-1} \left( \frac{\partial v_s}{\partial \gamma_s} + \frac{\partial v_s}{\partial k_s} \right),$$

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<sup>34</sup>However, since the function  $v_s(k_s, r_s)$  is not concave in  $k_s$ , there may be multiple optimal values of  $k_s$  corresponding to  $r_s^*$ .

where  $\partial v_s / \partial k_s \leq 0$ , since otherwise fixing  $k_s^* = \gamma_s$  would not be optimal. With these expressions in mind, the results in Table 1 can be immediately related to the (self-explanatory) signs of the partial derivatives of  $v_s(k_s^*, r_s^*)$  that we summarize in Table A1 (and whose detailed expressions we omit, for brevity).

**Table A1. Effects on the net present value of the bank**

|                                   | $z$ | $=$ | $r_s$ | $a$ | $\mu$ | $\lambda$ | $c$ | $\delta$ | $\gamma_h$ | $\gamma_l$ | $q_s$ |
|-----------------------------------|-----|-----|-------|-----|-------|-----------|-----|----------|------------|------------|-------|
| $\frac{\partial v_s}{\partial z}$ |     |     | +     | +   | +     | -         | -   | -        | -          | -          | -     |

**Comparative statics of the initial capital** When the optimal initial capital in state  $s$  is at the corner  $k_s^* = \gamma_s$ , with  $\partial v_s / \partial k_s < 0$ , marginal changes in any parameter other than  $\gamma_s$  have no impact on  $k_s^*$ , while obviously  $dk_s^* / d\gamma_s = 1$ . Thus, in what follows we focus on the more interesting interior solution case.<sup>35</sup>

The sign of  $dk_s^* / dz$  for  $z = a, \mu, \lambda, c, \delta, q_s, \gamma_h, \gamma_l$  is obtained by total differentiation of the first-order condition  $\partial v_s / \partial k_s = 0$  that characterizes an interior equilibrium:

$$\frac{\partial^2 v_s}{\partial k_s^2} \frac{dk_s^*}{dz} + \frac{\partial^2 v_s}{\partial k_s \partial r_s} \frac{dr_s^*}{dz} + \frac{\partial^2 v_s}{\partial k_s \partial z} = 0. \quad (19)$$

By the second-order condition we have  $\partial^2 v_s / \partial k_s^2 < 0$ , which gives

$$\frac{dk_s^*}{dz} = - \left( \frac{\partial^2 v_s}{\partial k_s^2} \right)^{-1} \left( \frac{\partial^2 v_s}{\partial k_s \partial z} + \frac{\partial^2 v_s}{\partial k_s \partial r_s} \frac{dr_s^*}{dz} \right).$$

Hence the sign of  $dk_s^* / dz$  coincides with the sign of the second term in brackets, which has two components: the direct effect of  $z$  on  $k_s^*$  (for constant  $r_s^*$ ) and the loan rate effect (due to the effect of  $z$  on  $r_s^*$ ). The signs of the direct effects shown in the first row of Table 2 coincide with the signs of the cross derivatives  $\partial^2 v_s / \partial k_s \partial z$  summarized in Table A2 (whose detailed expressions we omit, for brevity).

<sup>35</sup>The case with  $k_s^* = \gamma_s$  and  $\partial v_s / \partial k_s = 0$  is a mixture of both cases since, depending on the sign of the effect of the marginal variation in a parameter, the optimal decision might shift from being at the corner to being interior. A similar complexity may occur if the change in a parameter breaks some underlying indifference between an interior and a corner solution (or between two interior solutions). We will omit the discussion of these cases, for simplicity.

**Table A2. Effects on the marginal value of capital**

|  | $z$ | $=$ | $r_s$ | $a$ | $\mu$ | $\lambda$ | $c$ | $\delta$ | $\gamma_h$ | $\gamma_l$ | $q_s$ |
|--|-----|-----|-------|-----|-------|-----------|-----|----------|------------|------------|-------|
| $\frac{\partial^2 v_s}{\partial k_s \partial z}$ |     |     | —     | +   | +     | ?         | —   | —        | ?          | ?          | ?     |

The loan rate effects shown in the second row of Table 2 can be easily determined from the results summarized on Table 1 and the fact that  $\partial k_s^* / \partial r_s$  is negative. To see this, differentiate the first-order condition that characterizes  $k_s^*$  in an interior solution:

$$\frac{\partial^2 v_s}{\partial k_s^2} \frac{\partial k_s^*}{\partial r_s} + \frac{\partial^2 v_s}{\partial k_s \partial r_s} = 0.$$

The term that multiplies  $\partial k_s^* / \partial r_s$  is negative, by the second-order condition that characterizes  $k_s^*$  in an interior solution. As for the second term, differentiating (15) and (16) one can show that

$$\frac{\partial^2 v_s}{\partial k_s \partial r_s} = q_s \frac{\partial^2 v_{sh}}{\partial k_s \partial r_s} + (1 - q_s) \frac{\partial^2 v_{sl}}{\partial k_s \partial r_s} < 0, \quad (20)$$

where

$$\frac{\partial^2 v_{ss'}}{\partial k_s \partial r_s} = \frac{\beta}{\gamma_{s'}(\lambda + r_s)} [\beta \pi_{s'}(1 - \hat{x}_s) F'_s(\hat{x}_s) - (\beta \pi_{s'} - \gamma_{s'})(1 - \hat{x}_{ss'}) F'_s(\hat{x}_{ss'})].$$

To prove this, notice that the second-order condition  $\partial^2 v_s / \partial k_s^2 < 0$  implies

$$\beta f_s(\hat{x}_s) \left[ q_s \frac{\pi_h}{\gamma_h} + (1 - q_s) \frac{\pi_l}{\gamma_l} \right] < q_s \frac{\beta \pi_h - \gamma_h}{\gamma_h} F'_s(\hat{x}_{sh}) + (1 - q_s) \frac{\beta \pi_l - \gamma_l}{\gamma_l} F'_s(\hat{x}_{sl}).$$

Hence using the definitions (3) and (4) of  $\hat{x}_s$  and  $\hat{x}_{ss'}$ , together with the fact that  $\gamma_l \leq \gamma_h$ , we have  $1 - \hat{x}_s < 1 - \hat{x}_{sl} \leq 1 - \hat{x}_{sh}$ , so we conclude that

$$\beta(1 - \hat{x}_s) f_s(\hat{x}_s) \left[ q_s \frac{\pi_h}{\gamma_h} + (1 - q_s) \frac{\pi_l}{\gamma_l} \right] < q_s \frac{\beta \pi_h - \gamma_h}{\gamma_h} (1 - \hat{x}_{sh}) F'_s(\hat{x}_{sh}) + (1 - q_s) \frac{\beta \pi_l - \gamma_l}{\gamma_l} (1 - \hat{x}_{sl}) F'_s(\hat{x}_{sl}),$$

which after some reordering proves that (20) holds.

**The single risk factor model** The derivation of the cdf (10) of the default rate  $x_t$  can be summarized as follows. Suppose that the project undertaken by entrepreneur  $i$  at date  $t$  fails if  $y_{it} < 0$ , where  $y_{it}$  is a latent random variable defined by

$$y_{it} = \alpha_t + \sqrt{\rho_t} u_t + \sqrt{1 - \rho_t} \varepsilon_{it},$$

where  $\alpha_t$  is a parameter determined by the state of the economy at date  $t$ ,  $u_t$  is a single factor of systematic risk,  $\varepsilon_{it}$  is an idiosyncratic risk factor, and  $\rho_t \in (0, 1)$  is a state-contingent parameter that determines the extent of correlation in project failures. It is assumed that  $u_t$  and  $\varepsilon_{it}$  are  $N(0, 1)$  random variables, independently distributed from each other and over time, as well as, in the case of  $\varepsilon_{it}$ , across projects. Conditional on the information available at date  $t$ , the probability of failure of project  $i$  is  $p_t = \Pr(y_{it} < 0) = \Phi(-\alpha_t)$ , since  $y_{it} \sim N(\alpha_t, 1)$ , which implies  $\alpha_t = -\Phi^{-1}(p_t)$ .

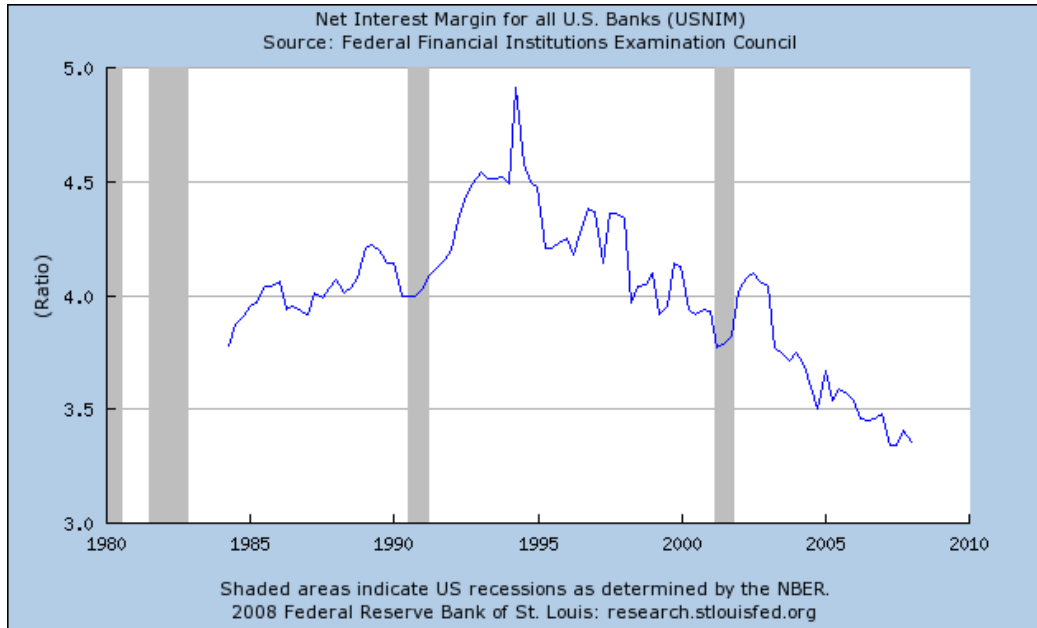
With a continuum of projects, the failure rate  $x_t$  is only a function of the realization of the systematic risk factor  $u_t$ . Specifically, by the law of large numbers,  $x_t$  coincides with the probability of failure of a (representative) project  $i$  conditional on the information available at  $t$  and the realization of  $u_t$ :

$$x_t = g_t(u_t) = \Pr\left(-\Phi^{-1}(p_t) + \sqrt{\rho_t} u_t + \sqrt{1 - \rho_t} \varepsilon_{it} < 0 \mid u_t\right) = \Phi\left(\frac{\Phi^{-1}(p_t) - \sqrt{\rho_t} u_t}{\sqrt{1 - \rho_t}}\right).$$

The cdf of the failure rate is  $F_t(x_t) = \Pr(g_t(u_t) \leq x_t) = \Pr(u_t \geq g_t^{-1}(x_t))$  (since  $g'_t(u_t) < 0$ ), so using the definition of  $g_t(u_t)$  and the fact that  $u_t \sim N(0, 1)$  we get (10).

## B Discussion of parameter values

**Interest rate on continuation loans:**  $a = 0.04$ . The interest rates on banks' marginal lending and borrowing activities are not available in standard statistical sources. A common approach is to proxy them with implicit average rates computed from accounting figures. According to the FDIC Statistics on Banking for the years 2004 to 2007 (available at <http://www2.fdic.gov/SDI/SOB/>), *Total interest income* of all US commercial banks represents, on average, 5.74% of *Earning assets*, while *Total interest expense* represents 2.32% of *Total liabilities*. This yields an average net interest margin of 3.42%. Yet *Service charges on deposit accounts* are 0.55% of *Total deposits*, which implies that deposit-funded activities yield an average intermediation margin of 3.97%. This number is very close to our assumed 4%. (See Figure 1 for quarterly data on the net interest margin of US banks over a longer period.)



**Cost of setting up a lending relationship:**  $c = 0.03$ . This is a rather conservative estimate of the importance of intermediation costs. According to the FDIC Statistics on Banking for the years 2004 to 2007, *Total non-interest expense* of all US commercial banks represented an average of 3.97% of *Total assets*.

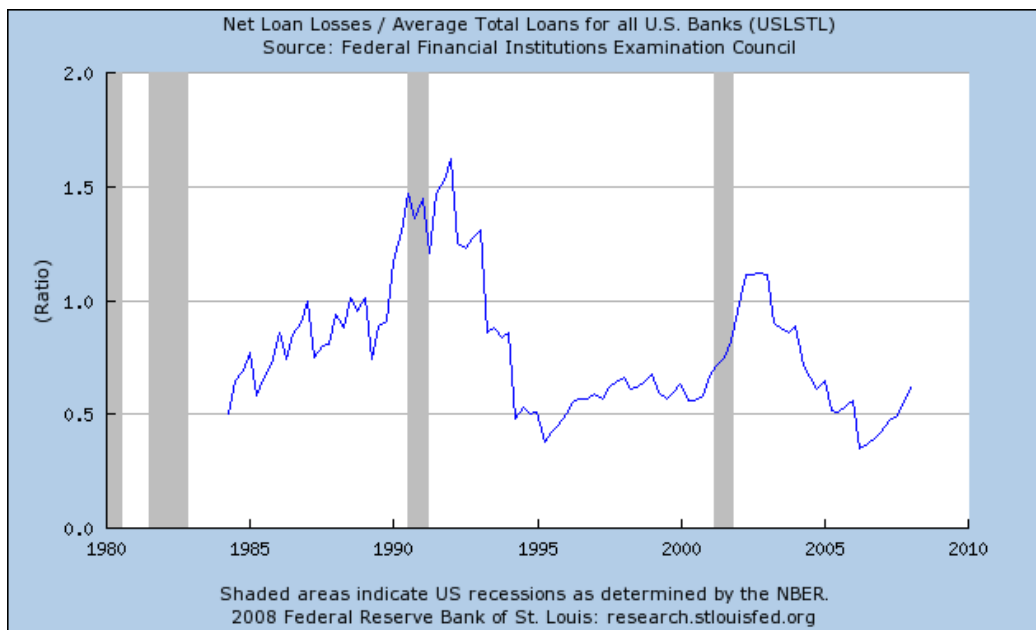
**Cost of bank capital:**  $\delta = 0.04$ . Based on the estimates of Graham (2000) for non-financial corporations, an annual discount rate of 4% is a rather conservative estimate for the tax

disadvantage of equity financing. To see this, consider the standard measure of the marginal tax shield of debt financing, net of personal taxes:  $MTS = [(1 - \tau_i) - (1 - \tau_c)(1 - \tau_e)] / (1 - \tau_i)$ , where  $\tau_i$ ,  $\tau_c$ , and  $\tau_e$  are the marginal tax rates on personal interest income, corporate income, and personal equity income, respectively. As in Hennessy and Whited (2007), set  $\tau_i = 0.29$  and consider  $\bar{\tau}_c = 0.40$  as an upper bound to  $\tau_c$  (based on the combination of the top statutory federal rate and the average state rate as reported by Graham, 2000). Suppose, conservatively, that  $\tau_e = 0$ , so that the marginal investor manages to make its equity income fully exempt from personal taxation. Then we get  $MTS \simeq 0.04$  for  $\tau_c = 0.32$ , where this last choice is consistent with US data. In particular, for US commercial banks over the period 2004-2007, *Applicable income taxes* represented, on average, 31.7% of *Pre-tax net operating income*. As in Hennessy and Whited (2007), this number can be seen as the expected tax rate in a situation in which the representative bank earns positive taxable income and hence faces an effective corporate tax rate of  $\bar{\tau}_c$  with a probability of 80%, while it faces an effective zero tax rate with a probability of 20%.

**Probabilities of transition to the high default state:**  $q_l = 0.20$  and  $q_h = 0.64$ . In our Markov switching setup, the expected durations of states  $l$  and  $h$  are  $1/q_l$  and  $1/(1 - q_h)$ , respectively. We calibrate these durations using data from the FDIC Historical Statistics on Banking (available at <http://www2.fdic.gov/hsob/index.asp>). Specifically, we compute the annual ratio of *Net loan and lease charge-offs* to *Gross loans and leases* for FDIC-insured commercial banks over the period 1969-2004, and we detrend the series using the standard HP-filter for annual data. The resulting series includes 20 below-average observations in 4 complete low default phases (implying an average duration of  $20/4 = 5$  years) and 14 above-average observations in 5 complete high default phases (implying an average duration of  $14/5 \simeq 2.8$  years). The observations corresponding to 1969 and 2004 belong to censored below-average phases that are not taken into account. The imputed expected durations are in line with Koopman et al. (2005), that identify a stochastic cycle in US business failure rates with a period of between 8 and 11 years.

**PD scenarios:**  $p_l \in [1.00, 1.20]$  and  $p_h \in [2.91, 3.62]$ . In the Special Report “Commercial Banks in 1999” (available at <http://www.philadelphiafed.org/files/bb/bbspecial.pdf>), the Federal Reserve Bank of Philadelphia offers data on the experience of US commercial banks during the full cycle of the 1990s. Following the 1990-1991 recession, the aggregate ratio of *Non-performing loans* to *Total loans* was slightly above 3% in years 1990-1993, declined to slightly above 2% in 1993, and remained below 1.5% (with a downward trend) for the rest

of the decade. It is also possible to check the realism of our PD scenarios by looking at the ratio of *Loan losses* to *Total loans*, whose quarterly evolution over recent years appears in Figure 2. Notice that under our assumption about the value of the LGD parameter,  $\lambda = 0.45$  (which we take from the “foundation IRB” approach of Basel II), the average default rate behind the series in Figure 2 should be  $1/0.45 \simeq 2.22$  times the ratio depicted there, which again suggests the realism of our choice of PDs slightly above 1% in low default states and around 3% in high default states.



## References

- Ayuso, J., D. Pérez, and J. Saurina (2004), “Are Capital Buffers Procyclical? Evidence from Spanish Panel Data,” *Journal of Financial Intermediation*, 13, 249-264.
- Altman, E., B. Brady, A. Resti, and A. Sironi (2005), “The Link between Default and Recovery Rates: Theory, Empirical Evidence, and Implications,” *Journal of Business*, 78, 2203-2227.
- Barakova, I., and M. Carey (2001), “How Quickly Do Troubled U.S. Banks Recapitalize? With Implications for Portfolio VaR Credit Loss Horizons,” Federal Reserve Board.
- Basel Committee on Banking Supervision (1988), *International Convergence of Capital Measurement and Capital Standards*, Basel: Bank for International Settlements.
- Basel Committee on Banking Supervision (2004), *International Convergence of Capital Measurement and Capital Standards. A Revised Framework*, Basel: Bank for International Settlements.
- Bernanke, B., and C. Lown (1991), “The Credit Crunch,” *Brookings Papers on Economic Activity*, 2, 205-239.
- Bikker, J., and P. Metzmakers (2004), “Is Bank Capital Procyclical? A Cross-Country Analysis,” De Nederlandsche Bank WP No. 009/2004.
- Blum J., and M. Hellwig (1995), “The Macroeconomic Implications of Capital Adequacy Requirements for Banks,” *European Economic Review*, 39, 739-749.
- Boot, A. (2000), “Relationship Banking: What Do We Know?,” *Journal of Financial Intermediation*, 9, 7-25.
- Bruche, M., and C. González-Aguado (2009), “Recovery Rates, Default Probabilities and the Credit Cycle,” *Journal of Banking and Finance*, forthcoming.
- Brunnermeier, Markus, Andrew Crockett, Charles Goodhart, Avi Persaud, and Hyun Shin (2009), “The Fundamental Principles of Financial Regulation”, *Geneva Reports on the World Economy*, 11, preliminary draft.
- Calvo, G. (1983), “Staggered Prices in a Utility-Maximizing Framework,” *Journal of Monetary Economics*, 12, 383-398.



- Catarineu-Rabell, E., P. Jackson, and D. Tsomocos (2005), "Procyclicality and the New Basel Accord – Banks Choice of Loan Rating System," *Economic Theory*, 26, 537-557.
- Cooley, T., and V. Quadrini (2001), "Financial Markets and Firm Dynamics," *American Economic Review*, 91, 1286-1310.
- Detragiache, E., P. Garella, and L. Guiso (2000) "Multiple versus Single Banking Relationships: Theory and Evidence," *Journal of Finance*, 55, 1133-1161.
- Diamond, D. W., and R. G. Rajan (2000), "A Theory of Bank Capital," *Journal of Finance*, 55, 2431-2465.
- Elizalde, A., and R. Repullo (2007), "Economic and Regulatory Capital in Banking: What is the Difference?," *International Journal of Central Banking*, 3, 87-117.
- Estrella, A. (2004), "The Cyclical Behavior of Optimal Bank Capital," *Journal of Banking and Finance*, 28, 1469-1498.
- Financial Stability Forum (2009), *Addressing Procyclicality in the Financial System*. Available at [http://www.financialstabilityboard.org/publications/r\\_0904a.pdf](http://www.financialstabilityboard.org/publications/r_0904a.pdf).
- Gordy, M. (2003), "A Risk-Factor Model Foundation for Ratings-Based Bank Capital Rules," *Journal of Financial Intermediation*, 12, 199-232.
- Gordy, M., and B. Howells (2006), "Procyclicality in Basel II: Can We Treat the Disease Without Killing the Patient?," *Journal of Financial Intermediation*, 15, 395-417.
- Graham, J. (2000), "How Big Are the Tax Benefits of Debt?," *Journal of Finance*, 55, 1901-1941.
- Hennessy, C., and T. Whited (2007), "How Costly Is External Financing? Evidence from a Structural Estimation," *Journal of Finance*, 62, 1705-1745.
- Holmström, B., and J. Tirole (1997), "Financial Intermediation, Loanable Funds, and the Real Sector," *Quarterly Journal of Economics*, 112, 663-691.
- Kashyap, A., R. Rajan, and J. Stein (2008), "Rethinking Capital Regulation," *Federal Reserve Bank of Kansas City Economic Policy Symposium*, 431-471.

- Kashyap, A., and J. Stein (2004), "Cyclical Implications of the Basel II Capital Standards," *Federal Reserve Bank of Chicago Economic Perspectives*, 1st Quarter, 18-31.
- Kashyap, A., J. Stein, and D. Wilcox (1993), "Monetary Policy and Credit Conditions: Evidence from the Composition of External Finance," *American Economic Review*, 83, 151-195.
- Koopman, S. J., A. Lucas, and P. Klaassen (2005), "Empirical Credit Cycles and Capital Buffer Formation," *Journal of Banking and Finance*, 29, 3159-3179.
- Lindquist, K.-G. (2004), "Bank's Buffer Capital: How Important Is Risk?," *Journal of International Money and Finance*, 23, 493-513.
- Myers, S., and N. Majluf (1984), "Corporate Financing and Investment Decisions when Firms Have Information that Investors Do Not Have," *Journal of Financial Economics*, 13, 187-221.
- Peura, S., and J. Keppo (2006), "Optimal Bank Capital with Costly Recapitalization," *Journal of Business*, 79, 2163-2201.
- Repullo, R., J. Saurina, and C. Trucharte (2009), "Mitigating the Procyclicality of Basel II," CEPR Discussion Paper No. 7382.
- Repullo, R., and J. Suarez (2004), "Loan Pricing under Basel Capital Requirements," *Journal of Financial Intermediation*, 13, 496-521.
- Saurina, J., and C. Trucharte (2007), "An Assessment of Basel II Procyclicality in Mortgage Portfolios," *Journal of Financial Services Research*, 32, 81-101.
- Sharpe, S. (1990), "Asymmetric Information, Bank Lending, and Implicit Contracts: A Stylized Model of Customer Relationships," *Journal of Finance*, 45, 1069-1087.
- Vasicek, O. (2002), "Loan Portfolio Value," *Risk*, 15, December, 160-162.
- Von Thadden, E.-L. (2004), "Asymmetric Information, Bank Lending and Implicit Contracts: The Winner's Curse," *Finance Research Letters*, 1, 11-23.
- Zhu, H. (2008), "Capital Regulation and Banks' Financial Decisions," *International Journal of Central Banking*, 4, 165-211.