Online Appendix for
"Optimal Dynamic Capital Requirements"

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A Model Details

1.1 Households

Dynasties provide consumption risk sharing to their members and are in charge of taking most household decisions. Each dynasty maximizes

$$E_t \left\{ \sum_{i=0}^{\infty} (\beta_\varkappa)^{t+i} \left[\log (c_{\varkappa,t+i}) + \lambda_{t+i} \upsilon_\varkappa \log (h_{\varkappa,t+i}) - \frac{\varphi_\varkappa}{1+\eta} (l_{\varkappa,t+i})^{1+\eta} \right] \right\} \tag{A.1}$$

with $\varkappa = s, m$, where $c_{\varkappa,t}$ denotes the consumption of non-durable goods and $h_{\varkappa,t}$ denotes the total stock of housing held by the various members of the dynasty (which is assumed to provide a proportional amount of housing services also denoted by $h_{\varkappa,t}$), $l_{\varkappa,t}$ denotes hours worked in the consumption good producing sector, λ_t is a housing preference shock that follows an AR(1) process and is common to both dynasties, $\upsilon_\varkappa$ is a housing preference parameter, $\varphi_\varkappa$ is a leisure preference parameter, and η is the inverse of the Frisch elasticity of labor supply.

1.1.1 Patient Households

The patient households' budget constraint is as follows

$$\begin{aligned} c_{s,t} + q_{h,t} \left[h_{s,t} - (1-\delta_{h,t}) h_{s,t-1} \right] + (q_{k,t} + s_t) k_{s,t} + d_t + B_t \leq \left[r_{k,t} + (1-\delta_{k,t}) q_{k,t} \right] k_{s,t-1} + w_t l_{s,t} \\ + \widetilde{R}_t^d d_{t-1} + R_{t-1}^{rf} B_{t-1} - T_{s,t} + \Pi_{s,t} + \Xi_{s,t} \end{aligned} \tag{A.2}$$

where $q_{h,t}$ is the price of housing, $\delta_{h,t}$ is the rate at which housing units depreciate, and w_t is the wage rate. Savers can hold physical capital $k_{s,t}$ with price $q_{k,t}$, depreciation rate $\delta_{k,t}$, and rental rate $r_{k,t}$, subject to a management cost s_t which is taken as given by households. $T_{s,t}$ is a lump-sum tax used by the DIA to ex-post balance its budget, $\Pi_{s,t}$ are aggregate net transfers of earnings from entrepreneurs and bankers to the household at period t, and $\Xi_{s,t}$ are profits from firms that manage the capital stock held by the patient households.

Each individual saver s can also invest in a risk free asset B_t (in zero net supply) and in a perfectly diversified portfolio of bank debt d_t. The return on such debt has two components. A fraction κ is interpreted as insured deposits that always pay back the promised gross deposit rate R^d_{t-1}. The remaining fraction $1-\kappa$ is interpreted as uninsured debt that pays back the promised rate R^d_{t-1} if the issuing bank is solvent and a proportion $1-\kappa$ of the net recovery value of bank assets in case of default. We assume banks' individual risk profiles to be unobservable to savers, so that they base their valuation of bank debt on the anticipated credit risk of an average unit of bank debt. The return on bank debt for savers can be written as

$$\widetilde{R}^d_t = R^d_{t-1} - (1-\kappa)\Omega_t, \tag{A.3}$$

where Ω_t is the average default loss per unit of bank debt which will be defined in (A.42).

1.1.2 Impatient Households

Impatient households' budget constraint is different from (A.2) in that they borrow, do not invest in capital, and do not receive transfers from entrepreneurs/bankers or capital management firms:

$$c_{m,t} + q_{h,t}h_{m,t} - b_{m,t} \leq w_t l_{m,t} + (1-\Gamma_{m,t}(\overline{\omega}_{m,t}))R^H_t q_{h,t-1}h_{m,t-1} - T_{m,t}, \tag{A.4}$$

where $b_{m,t}$ is the overall amount of mortgage lending granted by banks, $R^H_t = (1-\delta_{h,t})\, q_{h,t}/q_{h,t-1}$ is the gross unlevered return on housing, $(1-\Gamma_{m,t+1}(\overline{\omega}_{m,t}))R^H_t q_{h,t-1}h_{m,t-1}$ is net housing equity after accounting for the fraction of housing repossessed by the bank on the individual housing units that default on their mortgages, and $T_{m,t}$ is the lump-sum tax through which borrowers contribute to the funding of the DIA.

This formulation posits that individual household members default on their mortgages in period t when the value of their housing units, $\omega_{m,t}R^H_t q_{h,t-1}h_{m,t-1}$, is lower than the outstanding mortgage debt, $R^M_{t-1}b_{m,t-1}$, that is when $\omega_{m,t} \leq \bar{\omega}_{m,t} = x_{m,t-1}/R^H_t$, where R^M_t is the gross rate on the corresponding loan and $x_{m,t-1} = R^M_{t-1}b_{m,t-1}/(q_{h,t-1}h_{m,t-1})$ is a measure of household leverage at $t-1$.

The problem of the borrowing households also includes the participation constraint of the bank, which reflects the competitive pricing of the loans that banks are willing to offer for different choices of leverage by the household:

$$E_t\Lambda_{b,t+1}\left[(1-\Gamma_{M,t+1}(\overline{\omega}_{M,t+1}))(\Gamma_{m,t+1}(\overline{\omega}_{m,t+1})-\mu_m G_{m,t+1}(\overline{\omega}_{m,t+1}))R^H_{t+1}\right] q_{h,t}h_{m,t} \geq v_{b,t}\phi_{M,t}b_{m,t}. \tag{A.5}$$

This constraint is further explained below.

1.2 Entrepreneurs and Bankers

In each period some entrepreneurs and bankers become workers and some workers become either entrepreneurs or bankers.[1] Each period can be logically divided in three stages: payment stage, surviving stage, and investment stage. In the payments stage, previously active entrepreneurs ($\varrho = e$) and bankers ($\varrho = b$) get paid on their previous period investments. In the surviving stage, each agent of class ϱ stays active with probability θ_ϱ and retires with probability $1-\theta_\varrho$, becoming a worker again and transferring any accumulated net worth to the patient dynasty. At the same time, a mass $(1-\theta_\varrho)\, x_\varrho$ of workers become new agents of class ϱ, guaranteeing that the size of the population of such agents remains constant at x_ϱ. The cohort of new agents of class ϱ receives total net worth $\iota_{\varrho,t}$, from the patient dynasty. In the investment stage entrepreneurs and bankers provide equity financing to entrepreneurial firms and banks, respectively, and can send their net worth back to the household in the form of dividends.

1.2.1 Individual entrepreneurs

Entrepreneurs are agents that invest their net worth into entrepreneurial firms. The problem of the representative entrepreneur can be written as

$$V_{e,t} = \max_{a_t, \mathrm{div}_{e,t}} \left\{\mathrm{div}_{e,t} + E_t\Lambda_{s,t+1}\left[(1-\theta_e)\, n_{e,t+1} + \theta_e V_{e,t+1}\right]\right\} \tag{A.6}$$

$$a_t + \mathrm{div}_{e,t} = n_{e,t}$$
$$n_{e,t+1} = \int_0^\infty \rho_{f,t+1}(\omega) dF_{f,t+1}(\omega)\, a_t$$
$$\mathrm{div}_{e,t} \geq 0$$

[1] This guarantees that active entrepreneurs and bankers never accummulate enough net worth for them not to be interested in investing all of in equity of firms and banks, respectively (see, e.g. Gertler and Kiyotaki, 2010)

where $\Lambda_{s,t+1} = \beta_s c_{s,t}/c_{s,t+1}$ is the stochastic discount factor of the patient dynasty, $n_{e,t}$ is the entrepreneur's net worth, a_t is the part of the net worth symmetrically invested in the measure-one continuum of entrepreneurial firms further described below, $\mathrm{div}_{e,t} \geq 0$ are dividends that the entrepreneur can pay to the saving dynasty before retirement, and $\rho_{f,t+1}(\omega)$ is the rate of return on the entrepreneurial equity invested in a firm that experiences a return shock ω.

As in Gertler and Kiyotaki (2010), we guess that the value function is linear in net worth

$$V_{e,t} = v_{e,t} n_{e,t}, \tag{A.7}$$

where $v_{e,t}$ is the shadow value of one unit of entrepreneurial equity. Then we can write the Bellman equation in (8) as

$$v_{e,t} n_{e,t} = \max_{a_t, \mathrm{div}_{e,t}} \left\{ \mathrm{div}_{e,t} + E_t \Lambda_{s,t+1} \left[1 - \theta_e + \theta_e v_{e,t+1} \right] n_{e,t+1} \right\}. \tag{A.8}$$

Entrepreneurs find optimal not to pay dividends prior to retirement insofar as $v_{e,t} > 1$, which we verify to hold true under our parameterizations. Finally, (A.8) allows us to define entrepreneurs' stochastic discount factor as

$$\Lambda_{e,t+1} = \Lambda_{s,t+1} \left[1 - \theta_e + \theta_e v_{e,t+1} \right]. \tag{A.9}$$

1.2.2 Entrepreneurial firms

The representative entrepreneurial firm takes a_t equity from entrepreneurs and borrows $b_{f,t}$ from banks at interest rate R_t^F to buy physical capital from capital producers at t. In the next period, the firm rents the available effective units of capital, $\omega_{f,t+1} k_t$, where $\omega_{f,t+1}$ is the firm-idiosyncratic return shock, to capital users and sells back the depreciated capital to capital producers. Firms live for a period and pay out their terminal net worth to entrepreneurs. Hence, assuming symmetry across firms, the problem of the representative entrepreneurial firm can be written as

$$\max_{k_t, R_t^F} E_t \Lambda_{e,t+1} (1 - \Gamma_{f,t+1} \left(\overline{\omega}_{f,t+1} \right)) R_{t+1}^K q_{k,t} k_{f,t} \tag{A.10}$$

subject to the participation constraint of its bank

$$E_t \Lambda_{b,t+1} (1 - \Gamma_{b,t+1} \left(\overline{\omega}_{b,t+1} \right)) \widetilde{R}_{t+1}^F b_{f,t} \geq v_{b,t} \phi_{F,t} b_{f,t} \tag{A.11}$$

where $R_{t+1}^K = \left((1 - \delta_{k,t+1}) q_{k,t+1} + r_{k,t+1} \right) / q_{k,t}$ is the gross return on capital and $b_{f,t} = q_{k,t} k_{f,t} - a_t$ is the loan taken from the bank. As explained when presenting the problem of the borrowing households, the participation constraint of the bank can be interpreted as the equation capturing the competitive pricing of bank loans for different possible decisions on leverage by the firm. Further details on (A.11) appear in subsection 1.2.5.

The payoff that bank F receives from its portfolio of loans to entrepreneurial firms can be expressed as $\widetilde{R}^F_{t+1} b_{f,t} = (\Gamma_{f,t+1}(\overline{\omega}_{f,t+1}) - \mu_f G_{f,t+1}(\overline{\omega}_{f,t+1})) R^K_{t+1} q_{k,t} k_{f,t}$, which takes into account that a firm defaults on its loans when the gross return on its assets, $\omega_{f,t+1} R^K_{t+1} q_{k,t} k_{f,t}$, is insufficient to repay $R^F_t b_{f,t}$, i.e. for $\omega_{f,t+1} < \overline{\omega}_{f,t+1} = x_{f,t}/R^K_{t+1}$,where

$$x_{f,t} = \frac{R^F_t b_{f,t}}{q_{k,t} k_{f,t}} \tag{A.12}$$

is a measure of firms' leverage. Upon default, the bank recovers returns $(1-\mu_f)\omega_{f,t+1} R^K_{t+1} q_{k,t} k_t$, where μ_f is a proportional asset repossession cost.

1.2.3 Law of motion of entrepreneurial net worth

Taking into account effects of retirement and the entry of new entrepreneurs, the evolution of active entrepreneurs' net worth can be described as:[2]

$$n_{e,t+1} = \theta_e \rho_{f,t+1} a_t + \iota_{e,t+1}, \tag{A.13}$$

where $\rho_{f,t+1} = \int_0^\infty \rho_{f,t+1}(\omega) dF_{f,t+1}(\omega)$ is the return on a well-diversified unit portfolio of equity investments in entrepreneurial firms and $\iota_{e,t}$ is new entrepreneurs' net worth endowment, which we assume to be a proportion χ_e of the net worth of the exiting entrepreneurs:

$$\iota_{e,t} = \chi_e (1-\theta_e) \rho_{f,t+1} a_t. \tag{A.14}$$

1.2.4 Individual bankers

Bankers can invest their net worth $n_{b,t}$ into two classes j of competitive banks that extend loans $b_{j,t}$ to either impatient households ($j = M$) or firms ($j = F$). There is a continuum of banks of each class. The problem of the representative banker is

$$V_{b,t} = \max_{e^M_t, e^F_t, \mathrm{div}_{b,t}} \left\{ \mathrm{div}_{b,t} + E_t \Lambda_{s,t+1} \left[(1-\theta_b) n_{b,t+1} + \theta_b V_{b,t+1} \right] \right\} \tag{A.15}$$

[2] To save on notation, we also use $n_{e,t+1}$ to denote the aggregate counterpart of what in (A.6) was an individual entrepreneur's net worth.

$$e_{M,t} + e_{F,t} + \mathrm{div}_{b,t} = n_{b,t}$$
$$n_{b,t+1} = \int_0^\infty \rho_{M,t+1}(\omega)\, dF_{M,t+1}(\omega)\, e_{M,t} + \int_0^\infty \rho_{F,t+1}(\omega)\, dF_{F,t+1}(\omega)\, e_{F,t}$$
$$\mathrm{div}_{b,t} \geq 0$$

where $e_{j,t}$ is the diversified equity investment in the measure-one continuum of banks of class j. $\mathrm{div}_{b,t}$ is a dividend that the banker pays to the saving dynasty at retirement, and $\rho_{j,t+1}(\omega)$ is the rate of return from investing equity in a bank of class j that experiences shock ω.

As in the case of entrepreneurs, we guess that bankers' value function is linear

$$V_{b,t} = v_{b,t} n_{b,t}, \tag{A.16}$$

where $v_{b,t}$ is the shadow value of a unit of banker wealth. The Bellman equation in (A.15) becomes

$$v_{b,t} n_{b,t} = \max_{e_t^M, e_t^F, \mathrm{div}_{b,t}} \left\{ \mathrm{div}_{b,t} + E_t \Lambda_{s,t+1} \left[1 - \theta_b + \theta_b v_{b,t+1}\right] n_{b,t+1} \right\}, \tag{A.17}$$

and bankers will find it optimal not to pay dividends prior to retirement ($\mathrm{div}_{b,t} = 0$) insofar as $v_{b,t} > 1$. From (A.17), bankers' stochastic discount factor can be defined as

$$\Lambda_{b,t+1} = \Lambda_{s,t+1} \left[(1 - \theta_b) + \theta_b v_{b,t+1}\right]. \tag{A.18}$$

From (A.17), interior equilibria in which both classes of banks receive strictly positive equity from bankers ($e_{j,t} > 0$) require the properly discounted gross expected return on equity at each class of bank to be equal to $v_{b,t}$:

$$E_t[\Lambda_{b,t+1} \rho_{M,t+1}] = E_t[\Lambda_{b,t+1} \rho_{F,t+1}] = v_{b,t}, \tag{A.19}$$

where $\rho_{j,t+1} = \int_0^\infty \rho_{j,t+1}(\omega)\, dF_{j,t+1}(\omega)$ is the return of a well diversified unit-size portfolio of equity stakes in banks of class j.

1.2.5 Banks

The representative bank of class j issues equity $e_{j,t}$ among bankers and debt $d_{j,t}$ that promises a gross interest rate R_t^d among patient households, and uses these funds to provide a continuum of identical loans of total size $b_{j,t}$. This loan portfolio has a return $\omega_{j,t+1} \widetilde{R}_{t+1}^j$, where $\omega_{j,t+1}$ is a log-normally distributed bank-idiosyncratic asset return shock and $\widetilde{R}_{t+1}^j$ denotes the realized return on a well diversified portfolio of loans of class j.[3] Banks only live for a period and give back all their terminal net worth, if positive, to the bankers in the payment stage of next period. When a bank's terminal net worth is negative, it defaults. Thus, the DIA takes possession of the returns $(1-\mu_j)\omega_{j,t+1}\widetilde{R}^j_{t+1}b_{j,t}$ where μ_j is a proportional asset repossession cost, pays off the fraction κ of insured deposits in full, and pays a fraction $1-\kappa$ of the reposed returns to the holders of the bank's uninsured debt.

The objective function of the representative bank of class j is to maximize the net present value of their shareholders' stake at the bank

$$NPV_{j,t} = E_t\Lambda_{b,t+1}\max\left[\omega_{j,t+1}\widetilde{R}^j_{t+1}b_{j,t} - R^d_t d_{j,t}, 0\right] - v_{b,t}e_{j,t}, \tag{A.20}$$

where the equity investment $e_{j,t}$ is valued at its equilibrium opportunity cost $v_{b,t}$, and the max operator reflects shareholders' limited liability as explained above. The bank is subject to the balance sheet constraint, $b_{j,t} = e_{j,t} + d_{j,t}$, and the regulatory capital constraint, $e_{j,t} \geq \phi_{j,t}b_{j,t}$,where $\phi_{j,t}$ is the capital requirement on loans of class j.

If the capital requirement is binding (as it turns out to be in equilibrium because partially insured debt financing is always "cheaper" than equity financing), we can write the loans of the bank as $b_{j,t} = e^j_t/\phi_{j,t}$, its deposits as $d_{j,t} = (1-\phi_{j,t})e_{j,t}/\phi_{j,t}$, and the threshold value of $\omega_{j,t+1}$ below which the bank fails as $\overline{\omega}_{j,t+1} = (1-\phi_{j,t})\,R^d_t/\widetilde{R}^j_{t+1}$, since the bank fails when the realized return per unit of loans is lower than the associated debt repayment obligations, $(1-\phi_{j,t})R_{d,t}$. Accordingly, the probability of failure of a bank of class j is $\Psi_{j,t+1} = F_{j,t+1}(\overline{\omega}_{j,t+1})$, which will be driven by fluctuations in the aggregate return on loans of class j, $\widetilde{R}^j_{t+1}$, as well as shocks to the distribution of the bank return shock $\omega_{j,t+1}$.

Using (2) from the body of the paper, the bank's objective function in (A.20) can be written as

$$NPV_{j,t} = \left\{E_t\Lambda_{b,t+1}\left[1-\Gamma_{j,t+1}(\overline{\omega}_{j,t+1})\right]\frac{\widetilde{R}^j_{t+1}}{\phi_{j,t}} - v_{b,t}\right\}e_{j,t}, \tag{A.21}$$

which is linear in the bank's scale as measured by $e_{j,t}$. So, banks' willingness to invest in loans with returns described by $\widetilde{R}^j_{t+1}$ and subject to a capital requirement $\phi_{j,t}$ requires having

$$E_t\Lambda_{b,t+1}\left[1-\Gamma_{j,t+1}(\overline{\omega}_{j,t+1})\right]\widetilde{R}^j_{t+1} \geq \phi_{j,t}v_{b,t}, \tag{A.22}$$

[3] This layer of idiosyncratic uncertainty is an important driver of bank default and is intended to capture the effect of bank-idiosyncratic limits to diversification of borrowers' risk (e.g. regional or sectoral specialization or large exposures) or shocks stemming from (unmodeled) sources of cost (IT, labor, liquidity management) or revenue (advisory fees, investment banking, trading gains).

which explains the expressions for the participation constraints (A.5) and (A.11). These constraints will hold with equality since it is not in borrowers' interest to pay more for their loans than strictly needed.[4] Under the definition $\rho_{j,t+1} = \left[1 - \Gamma_{j,t+1}(\overline{\omega}_{j,t+1})\right] \frac{\widetilde{R}^j_{t+1}}{\phi_{j,t}}$, if (A.22) holds with equality for $j = M, F$, bankers' indifference between investing their wealth in equity of either class of banks, (A.19), is also trivially satisfied.

1.2.6 Law of motion of bankers' net worth

Taking into account effects of retirement and the entry of new bankers, the evolution of active bankers' *aggregate* net worth can be described as:[5]

$$n_{b,t+1} = \theta_b(\rho_{F,t+1}e_{F,t} + \rho_{M,t+1}e_{M,t}) + \iota_{b,t} \tag{A.23}$$

where $\iota_{b,t}$ is new bankers' net worth endowment (received from saving households), which we assume to be a proportion χ_b of the net worth of exiting bankers:

$$\iota_{b,t} = \chi_b(1 - \theta_b)(\rho_{F,t+1}e_{F,t} + \rho_{M,t+1}e_{M,t}). \tag{A.24}$$

1.3 Production Sector

We assume a perfectly competitive production sector made up of firms owned by the patient agents. This sector is not directly affected by financial frictions.

1.3.1 Consumption goods

The representative goods-producing firm produces a single good, y_t, using l_t units of labor and k_t units of capital, according to the following constant-returns-to-scale technology:

$$y_t = z_t \; {l_t}^{1-\alpha} \; {k_{t-1}}^{\alpha}, \tag{A.25}$$

where z_t is an AR(1) productivity shock and α is the share of capital in production.

[4]In fact, any pricing of bank loans leading to $NPV_{j,t} > 0$ would make banks wish to expand $e_{j,t}$ unboundedly, which is incompatible with equilibrium. So, we could have directly written (A.5) and (A.11) with equality, as a sort of zero (rather than non-negative) profit condition.

[5]To save on notation, we also use $n_{b,t+1}$ to denote the aggregate counterpart of what in (A.15) was an individual banker's net worth.

1.3.2 Capital and housing production

Producers of capital (X=k) and housing (X=h) combine investment $I_{X,t}$, with the previous stock of capital and housing, X_{t-1}, in order to produce new capital and housing which can be sold at price $q_{X,t}$.[6] The representative X-producing firm maximizes the expected discounted value to the saving dynasty of its profits:

$$\max_{\{I_{X,t+j}\}} E_t \sum_{j=0}^{\infty} \Lambda_{s,t+j} \left\{ q_{X,t+j} \left[S_X \left(\frac{I_{X,t+j}}{X_{t+j-1}} \right) X_{t+j-1} \right] - I_{X,t+j} \right\} \tag{A.26}$$

where $S_X \left(\frac{I_{X,t+j}}{X_{t+j-1}} \right) X_t + j - 1$ gives the units of new capital produced by investing $I_X, t + j$. The increasing and concave function $S_X(\cdot)$ captures adjustment costs, as in Jermann (1998):

$$S_X \left(\frac{I_{X,t}}{X_{t-1}} \right) = \frac{a_{X,1}}{1 - \frac{1}{\psi_X}} \left(\frac{I_{X,t}}{X_{t-1}} \right)^{1 - \frac{1}{\psi_X}} + a_{X,2}, \tag{A.27}$$

where $a_{X,1}$ and $a_{X,2}$ are chosen to guarantee that, in the steady state, the investment-to-capital ratio is equal to the depreciation rate and $S'_X(I_X, t/X_t - 1)$ equals one (so that the implied adjustment costs are zero).

The law of motion of the corresponding stock is given by

$$X_t = (1 - \delta_{X,t}) X_{t-1} + S_X \left(\frac{I_{X,t}}{X_{t-1}} \right) X_{t-1}, \tag{A.28}$$

where $\delta_{X,t}$ is the time-varying depreciation rate, which follows an AR(1).

1.3.3 Capital management firms

The capital management cost s_t associated with households direct holdings of capital $k_{s,t}$ is a fee levied by a measure-one continuum of firms operating with decreasing returns to scale. These firms have a convex cost function $z(k_{s,t})$ where $z(0) = 0$, $z'(k_{s,t}) > 0$ and $z''(k_{s,t}) > 0$. Under perfect competition, maximizing profits $\Xi_{s,t} = s_t k_{s,t} - z(k_{s,t})$ implies the first order condition

$$s_t = z'(k_{s,t}), \tag{A.29}$$

which determines the equilibrium fees for each $k_{s,t}$. We assume a quadratic cost function, $z(k_{s,t}) = \frac{\xi}{2} k_{s,t}^2$, with $\xi > 0$, so that (A.29) becomes $s_t = \xi k_{s,t}$.

[6] We have examined a variation of the model with a fixed housing stock. The behaviour of the model as well as its policy implications were similar to the ones obtained in the current version.

1.4 Market Clearing Conditions

In equilibrium the following add-up and market clearing conditions must hold. The total mass of households has been normalized to one, so savers and borrowers have measures $x_s = x_w + x_e + x_b = 1 - x_m$ and x_m, respectively, where x_m as well as the composition of x_s are exogenous. The aggregate housing stock equals the house holdings of the two dynasties:

$$h_t = x_s h_{s,t} + x_m h_{m,t}. \tag{A.30}$$

Total demand for households' labor by the consumption good producing firms equals the labor supply of the two dynasties:

$$l_t = x_w l_{s,t} + x_m l_{m,t}. \tag{A.31}$$

total households' consumption equals the consumption of the two dynasties:

$$c_t = x_s c_{s,t} + x_m c_{m,t}. \tag{A.32}$$

Bank debt held by patient households, d_t, must equal the sum of the debt issued by banks making loans to households, $(1 - \phi_{M,t})x_m b_{m,t}$, and to entrepreneurs, $(1 - \phi_{F,t})x_e b_{f,t}$:

$$d_t = (1 - \phi_{M,t})x_m b_{m,t} + (1 - \phi_{F,t})x_e b_{f,t}. \tag{A.33}$$

Equity financing provided by bankers (equal to their entire net worth) must equal the sum of the demand for bank equity from the banks making loans to households, $e_{M,t} = \phi_{M,t} x_m b_{m,t}$, and entrepreneurs, $e_{F,t} = \phi_{F,t} x_e b_{f,t}$:

$$n_{b,t} = \phi_{M,t} x_m b_{m,t} + \phi_{F,t} x_e b_{f,t}, \tag{A.34}$$

where our prior derivations imply

$$b_{f,t} = [q_{k,t} k_{f,t} - a_t] \tag{A.35}$$

and

$$b_{m,t} = \frac{q_{h,t} h_{m,t} x_{m,t}}{R_t^M}, \tag{A.36}$$

so total bank loans are given by

$$b_t = x_m b_{m,t} + x_e b_{f,t}. \tag{A.37}$$

The capital held by patient households and by entrepreneurs must sum up to the total capital stock:

$$x_s k_{s,t} + x_e k_{f,t} = k_t. \tag{A.38}$$

Total output Y_t equals households' consumption c_t, plus the resources absorbed in the production of new housing $I_{h,t}$ and new capital $I_{k,t}$ plus the resources lost in the repossession by banks of the proceeds associated with defaulted bank loans and in the repossession by the DIA and the holders of uninsured bank debt of the proceeds associated with defaulted bank debt:

$$\begin{aligned} Y_t &= c_t + I_{h,t} + I_{k,t} \\ &\quad + x_m \mu_m G_{m,t}\left(\overline{\omega}_{m,t}\right) R_t^H q_{h,t-1} h_{m,t-1} + x_e \mu_f G_{f,t}\left(\overline{\omega}_{f,t}\right) R_t^K q_{k,t-1} k_{f,t-1} \\ &\quad + \mu_b \left[G_{M,t}\left(\overline{\omega}_{M,t}\right) \widetilde{R}_t^M x_m b_{m,t} + G_{F,t}\left(\overline{\omega}_{F,t}\right) \widetilde{R}_t^F x_e b_{f,t} \right]. \end{aligned} \tag{A.39}$$

The risk free asset is assumed to be in zero net supply

$$x_s B_t = 0. \tag{A.40}$$

The total costs to the DIA due to losses caused by M and F banks, and hence the total lump sum tax imposed on agents in order to finance the agency on a balanced-budget basis, are given by

$$T_t = \kappa \Omega_t d_{t-1} \tag{A.41}$$

where Ω_t is average default loss per unit of bank debt, which is the properly weighted average of the losses realized at each class of bank:

$$\Omega_t = \frac{d_{M,t-1}}{d_{t-1}} \Omega_{M,t} + \frac{d_{F,t-1}}{d_{t-1}} \Omega_{F,t} \tag{A.42}$$

with

$$\Omega_{j,t} = \left[\overline{\omega}_{j,t} - \Gamma_{j,t}\left(\overline{\omega}_{j,t}\right) + \mu_j G_{j,t}\left(\overline{\omega}_{j,t}\right)\right] \frac{\widetilde{R}_{j,t}}{1-\phi_{j,t}} \tag{A.43}$$

for $j = M, F$.[7]

Similarly, for reporting purposes, we define banks' average probability of default as

$$\Psi_{b,t} = \frac{d_{M,t-1}}{d_{t-1}} \Psi_{M,t} + \frac{d_{F,t-1}}{d_{t-1}} \Psi_{F,t}, \tag{A.44}$$

[7]Remember that the remaining fraction $1 - \kappa$ of the default losses are directly incurred by the saving households, as reflected in (5) in the body of the paper.

where $\Psi_{j,t} = F_{j,t}(\overline{\omega}_{j,t})$ for $j = M, F$.

The lump-sum tax T_t is paid by households of each class in proportion to their size in the population, implying

$$T_{\varkappa,t} = \frac{x_{\varkappa}}{x_s + x_m} T_t \tag{A.45}$$

for $\varkappa = s, m$.

According to bank accounting conventions, we can find the write-off rate (write-offs/loans) for loans of type j that the model generates, $\Upsilon_{j,t}$, as the product of the fraction of defaulted loans of that type, $F_j\left(\overline{\omega}_{j,t}\right)$, and the average losses per unit of lending experienced in the defaulted loans, which can be found from our prior derivations. For example, in the case of NFC loans, this decomposition produces:

$$\begin{aligned}\Upsilon_{f,t} &= F_{f,t}\left(\overline{\omega}_{f,t}\right)\left[\frac{b_{f,t-1} - \frac{(1-\mu_f)}{F_{f,t}\left(\overline{\omega}_{f,t}\right)}\left(\int_0^{\overline{\omega}_{f,t}} \omega_{f,t} f_{f,t}\left(\omega\right) d\omega\right) R_t^K q_{k,t-1} k_{f,t-1}}{b_{f,t-1}}\right] \\ &= F_{f,t}\left(\overline{\omega}_{f,t}\right) - (1-\mu_f) G_{f,t}(\overline{\omega}_{f,t}) R_t^K \frac{q_{k,t-1} k_{f,t-1}}{b_{f,t-1}}. \end{aligned} \tag{A.46}$$

An expression for the writte-off rate of mortgage loans, $\Upsilon_{m,t}$, can be similarly obtained.

1.5 Sources of Fluctuations

The model economy features eight sources of aggregate uncertainty, namely shocks to productivity, z_t, housing preferences, λ_t, the depreciation of housing, $\delta_{h,t}$, and capital, $\delta_{k,t}$, and the four *risk shocks.* The latter are the shocks to the standard deviation $\sigma_{j,t}$ of the idiosyncratic return shocks experienced by each of the four classes of borrowers $j = m, f, M, F$).[8] All aggregate shocks follow autoregressive processes of order one:

$$\ln \varkappa_t - \ln \bar{\varkappa} = \rho_{\varkappa}\left(\ln \varkappa_{t-1} - \ln \bar{\varkappa}\right) + u_{\varkappa,t}, \tag{A.47}$$

where $\varkappa_t \in \{z_t, v_t, \delta_{h,t}, \delta_{k,t}, \sigma_{m,t}, \sigma_{f,t}, \sigma_{M,t}, \sigma_{F,t}\}$, $\rho_{\varkappa}$ is the corresponding (time invariant) persistence parameter, $\bar{\varkappa}$ is the unconditional mean of ϱ_t, and $u_{\varkappa,t}$ is the innovation to each shock, with mean zero and (time invariant) standard deviation $\sigma_{\varkappa}$.

[8] We refer to the shocks$\{\sigma_{j,t}\}_{j=m,f,M,F}$ as "risk shocks" as in Christiano, Motto and Rostagno (2014).

B Data Used in the Calibration

- Gross Domestic Product: Gross domestic product at market price, Chain linked volumes, reference year 2005, Euro. Source: ESA - ESA95 National Accounts, Macroeconomic Statistics (S/MAC), European Central Bank.

- GDP Deflator: Gross domestic product at market price, Deflator, National currency, Working day and seasonally adjusted, Index. Source: ESA - ESA95 National Accounts, Macroeconomic Statistics (S/MAC), European Central Bank.

- Business Loans: Outstanding amounts at the end of the period (stocks), MFIs excluding ESCB reporting sector - Loans, Total maturity, All currencies combined - Euro area (changing composition) counterpart, Non-Financial corporations (S.11) sector, denominated in Euro. Source: MFI Balance Sheet Items Statistics (BSI Statistics), Monetary and Financial Statistics (S/MFS), European Central Bank.[9]

- Households Loans: Outstanding amounts at the end of the period (stocks), MFIs excluding ESCB reporting sector - Loans, Total maturity, All currencies combined - Euro area (changing composition) counterpart, Households and non-profit institutions serving households (S.14 & S.15) sector, denominated in Euro. Source: MFI Balance Sheet Items Statistics (BSI Statistics), Monetary and Financial Statistics (S/MFS), European Central Bank.

- Write-offs: Other adjustments, MFIs excluding ESCB reporting sector - Loans, Total maturity, All currencies combined - Euro area (changing composition) counterpart, denominated in Euro, as percentage of total outstanding loans for the same sector. Source: MFI Balance Sheet Items Statistics (BSI Statistics), Monetary and Financial Statistics (S/MFS), European Central Bank.

- Housing Investment: Gross fixed capital formation: housing, Current prices - Euro, divided by the Gross domestic product at market price, Deflator. Source: ESA - ESA95 National Accounts, Macroeconomic Statistics (S/MAC), European Central Bank.

[9] All monetary financial institutions in the Euro Area are legally obliged to report data from their business and accounting systems to the National Central Banks of the member states where they reside. These in turn report national aggregates to the ECB. The census of MFIs in the euro area (list of MFIs) is published by the ECB (see http://www.ecb.int/stats/money/mfi/list/html/index.en.html).

- Housing Wealth: Household housing wealth (net) - Reporting institutional sector Households, non-profit institutions serving households - Closing balance sheet - counterpart area World (all entities), counterpart institutional sector Total economy including Rest of the World (all sectors) - Debit (uses/assets) - Unspecified consolidation status, Current prices - Euro. Source: IEAQ - Quarterly Euro Area Accounts, Euro Area Accounts and Economics (S/EAE), ECB and Eurostat.

- Bank Equity Return: Median Return on Average Equity (ROAE), 100 Largest Banks, Euro Area. Source: Bankscope.

- Spreads between the composite interest rate on loans and the composite risk free rate is computed in two steps. Firstly, we compute the composite loan interest rate as the weighted average of interest rates at each maturity range (for housing loans: up to 1 year, 1-5 years, 5-10 years, over 10 years; for commercial loans: up to 1 year, 1-5 years, over 5 years). Secondly, we compute corresponding composite risk free rates that take into account the maturity breakdown of loans. The maturity-adjusted risk-free rate is the weighted average (with the same weights as in case of composite loan interest rate) of the following risk-free rates chosen for maturity ranges:

  - 3 month EURIBOR (up to 1 year)
  - German Bund 3 year yield (1-5 years)
  - German Bund 10 year yield (over 5 years for commercial loans)
  - German Bund 7 year yield (5-10 years for housing loans)
  - German Bund 20 year yield (over 10 years for housing loans).

- Borrowers Fraction: Share of households being indebted, as of total households. Source: Household Finance and Consumption Survey (HFCS), 2010.

- Borrowers Housing Wealth: value household's main residence + other real estate - other real estate used for business activities (da1110 + da1120 - da1121), Share of indebted households, as of total households. Source: HFCS, 2010.

- Fraction of capital held by households: We set our calibration target for this variable by identifying it with the proportion of assets of the NFC sector whose financing is not supported by banks. To compute this proportion we use data from the Euro Area sectoral financial accounts, which include balance sheet information for the NFC sector (Table 3.2) and a breakdown of bank loans by counterparty sector (Tables 4.1.2 and 4.1.3). From the raw NFC balance sheet data, we first produce a "net" balance sheet in which, in order to remove the effects of the cross-holdings of corporate liabilities, different types of corporate liabilities that appear as assets of the NFC sector get subtracted from the corresponding "gross" liabilities of the corporate sector. Next we construct a measure of leverage of the NFC sector

$$LR = \frac{\text{NFC Net Debt Securities} + \text{NFC Net Loans} + \text{NFC Net Insurance Guarantees}}{\text{NFC Net Assets}}$$

and a measure of the bank funding received by the NFC sector

$$BF = \frac{\text{MFI Loans to NFCs}}{\text{NFC Net Assets}}.$$

From these definitions, the fraction of debt funding to the NFC sector not coming from banks can be found as $(LR - BF)/LR$. Finally, to estimate the fraction of NFC assets whose financing is not supported by banks, we simply assume that the financing of NFC assets not supported by banks follows the same split of equity and debt funding as the financing of NFC assets supported by banks, in which case the proportion of physical capital in the model not funded by banks, k_s/k, should just be equal to $(LR - BF)/LR$. This explains the target value of k_s/k in Table 1 of the paper.

C Sensitivity Analysis

In this section we examine how our policy and welfare results depend on the following key model parameters: (i) the rate at which bankers survive and reinvest their net worth as bank capital (θ^b), which determines the scarcity of bank capital in the model; (ii) the insured fraction of bank debt (κ), which measures the importance of the safety net subsidies enjoyed by banks; (iii) the parameter that governs deadweight default losses (μ), which are the key source of first order losses associated with financial fragility. We present our results compactly through the secondary (non solid) lines included in Figures C1 and C2 below (which correspond to Figures 4 and 5 in the paper). Each of those lines show the optimal policy parameters and the associated welfare gains under a ceteris paribus variation in our benchmark calibration. The overall message is that, although the quantitative results get somewhat modified, our main conclusions are remarkably robust.

Cost of equity (dashed-dotted lines). Increasing θ^b so as to reduce the average rate of return on bank equity to 7.2% (from 9.3% in the baseline calibration) leads to higher optimal capital requirements for the two types of loans. Intuitively, with more abundant equity funding, reaching a given level of resilience is less costly in terms of credit supply and the optimal policy reacts to this by demanding banks to operate with more capital. As reflected in Figure C2, in the setup in which equity is more abundant borrowers tend to gain more from optimal capital regulation since the implied credit contraction effects are smaller, while savers' gains are quite similar to those under the baseline calibration.

Safety net guarantees (dashed lines). We look at the interesting polar case where all bank debt is uninsured (κ=0) and hence bank failures produce no tax cost on either savers or borrowers. Importantly, the safety net subsidies disappear but the limited liability distortion associated with the assumption that the cost of bank debt is not explicitly contingent on banks' leverage remains.[10] In these conditions, removing deposit insurance makes banks' debt funding more expensive and more responsive to shocks, reducing banks' resilience and increasing their potential contribution to the propagation of shocks. In response, the optimal capital requirements for both types of loans increase, although the quantitative impact is not dramatic. Without safety net subsidies, savers' gains from optimal capital regulation are smaller (since there are no gains from the reduction in the tax cost of deposit insurance), while, somewhat paradoxically, borrowers' gains from tightening the requirements are larger (because reinforcing banks' resilience when starting from low values reduces bank funding costs and, in general equilibrium, may relax lending standards).

Deadweight default losses (dotted lines). A lower value for the fraction of borrower assets that are lost in the event of bankruptcy (μ) leads to lower capital requirements for both mortgage and corporate loans. Intuitively, capital regulation is the tool used, among other things, to make banks internalize the implications of default for the wider economy. In fact, economizing on these costs is the main source of first order gains from reducing banks' leverage.

[10] Intuitively, we assume that individual banks are too opaque to make their funding costs explicitly contingent on their risk profile. Instead, each individual bank pays an interest rate on its debt which depends on the average risk of the banking system, which is beyond the control of any individual atomistic bank. This provides incentives for individual banks to take on risk in the form of as much leverage as permitted by capital regulation.

Higher capital requirements, through their impact on lending standards and banks' resilience, reduce the default risk of all borrowing sectors. When default implies smaller deadweight losses, the required reinforcement of capital requirements is smaller than under the baseline calibration. As one can see in Figure C2, in this situation, the optimal policies imply lower welfare gains (relative to the regulatory baseline) for both savers and borrowers than under the baseline calibration.

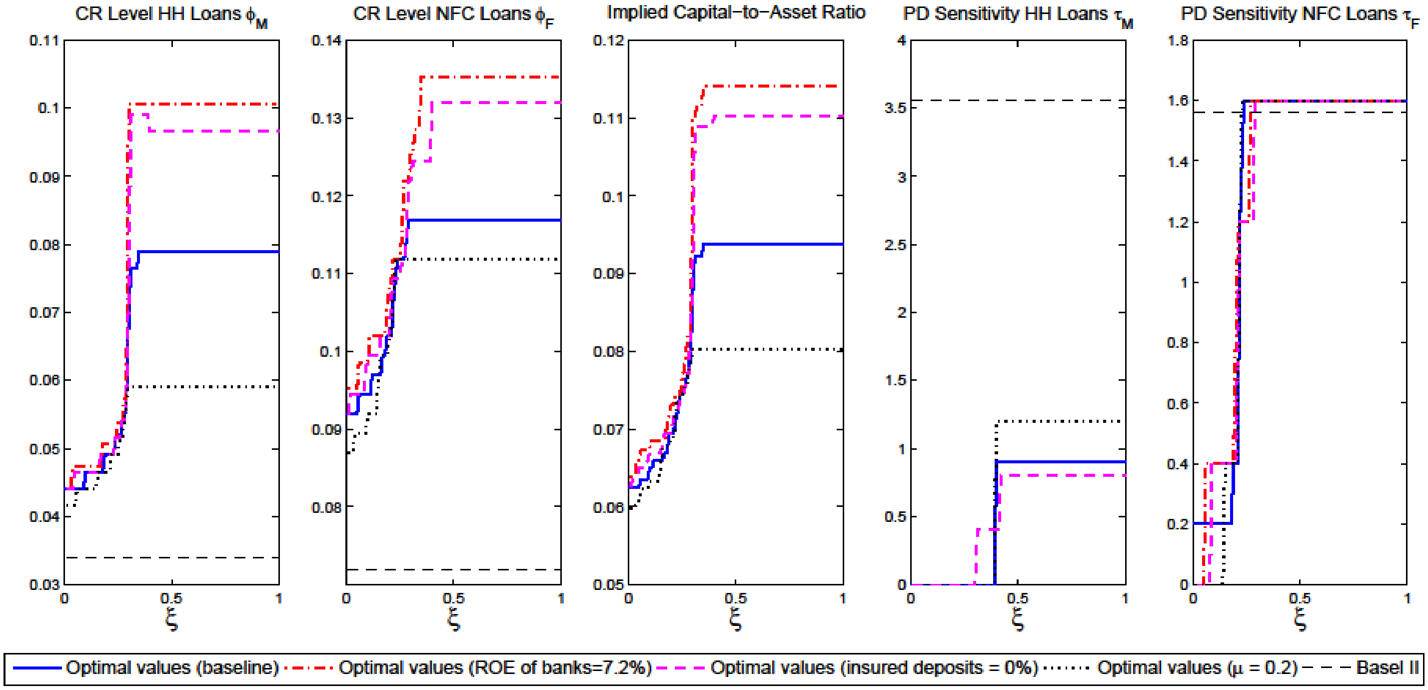

Figure C1. Optimal Dynamic Capital Requirements. Parameters characterizing the welfare maximizing policy rule and the implied average capital-to-asset ratio for each banks are depicted as functions of the weight ξ that the maximized social welfare measure puts on savers' welfare. The two panels on the left describe the optimized values of the parameters that determine the average level of the capital requirements for mortgage (HH) and Corporate (NFC) loans. The two panels on the right describe the optimized PD sensitivities of the requirements to time changes in the PDs of the corresponding loans.

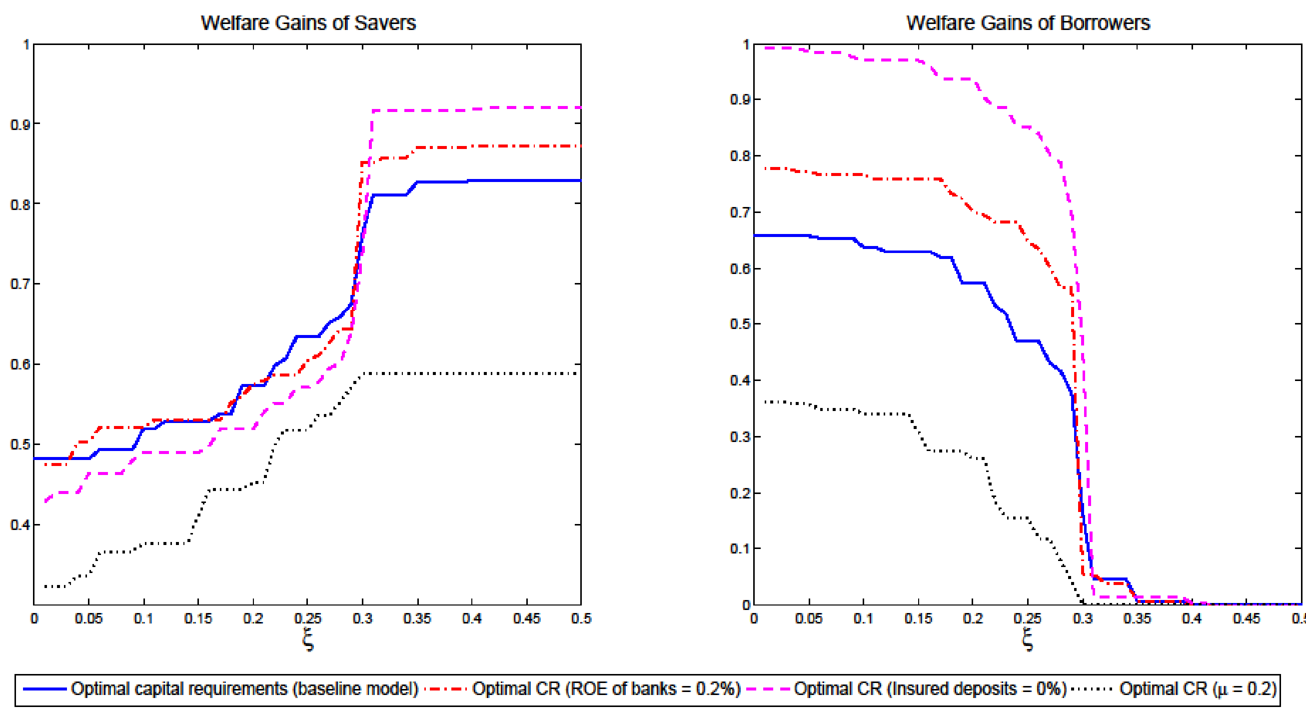

Figure C2. Sensitivity Analysis: Welfare Gains. Individual welfare gains implied by the optimal policy corresponding to each value of the weight ξ that the maximized social welfare measure puts on savers' welfare under the baseline and alternative values of key parameters. The gains are measured in consumption-equivalent terms, as the percentage increase in the consumption of each agent that would make his welfare under the initially calibrated policy rule equal to his welfare under each optimized poliy rule.