Incomplete Wage Posting*

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Abstract

We consider a directed search model where workers differ in productivity. Productivity becomes observable to firms after assessing their workers on the job, but it is not verifiable. Firms with vacancies choose between posting a non-contingent wage or leaving wages subject to bargaining with the worker. Under wage bargaining, firms cannot optimize the trade-off between paying higher wages and having a larger probability of filling vacancies. But wage bargaining makes wages increasing in worker productivity and so may allow firms to attract better workers into the vacancy. When workers' heterogeneity is large and bargaining powers come close to satisfying Hosios' rule, firms opt for bargaining. Yet, equilibria with bargaining fail to maximize aggregate net income and sometimes are not constrained Pareto optimal.

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1 Introduction

While some companies are committed to rather transparent and well-defined compensation structures, others leave starting salaries and other important components of pay subject to negotiation with each worker. The importance of these negotiations is indirectly evidenced by the abundance of popular literature and web sites that advise workers on how to negotiate better pay for their jobs. However, the use of bargaining seems paradoxical in light of labor models in the *directed search* tradition, including Peters (1991), Montgomery (1991), Moen (1997), Acemoglu and Shimer (1999a, 1999b), Shi (2001, 2002), and Shimer (2005), which predict that firms should post all the terms of the employment contract in advance. By doing so, firms can efficiently optimize the trade-off between paying higher wages to their workers and filling vacancies with higher probability. In contrast, leaving wages subject to ex-post bargaining tends to lead to either too high wages or too few applicants relative to the level that would maximize the value of a vacancy. These *search inefficiencies* related to bargaining always arise, unless the distribution of bargaining power satisfies the well-known condition first derived by Hosios (1990).

In this paper we argue that firms may choose to subject wages to bargaining because, in the jargon of contract theory, posted wage structures are bound to be *incomplete* (see Hart, 1995).[1] The idea is that there are differences in workers' productivity that can be subjectively assessed by managers and supervisors after observing the workers on the job, but these differences cannot be verified by courts, making it impossible to enforce compensation contracts explicitly contingent on them.[2] In this context, an advantage of negotiating wages once the firm assesses the worker on the

[1] The debate on the (lack of) microfoundations of incomplete contracts is still open. See Tirole (1999) and Maskin (2002) for recent assessments of the debate.

[2] To be sure, posted wages can be contingent on verifiable proxies for worker productivity, such as age, formal education, years of service or even some output measure, and yet pay the same to different workers. The human resources (HR) literature shares the view that in most jobs individual performance is hard to measure in an objective, uncontentious manner; see, for instance, Baker et al. (1994) and Baron and Kreps (1999, p. 211).

job is that the resulting wage ends up tailored to the worker's productivity even if courts cannot verify productivity. Thus, bargained wages can be particularly attractive to relatively more productive workers, and firms may offer such wages in order to improve the composition of their pool of job applicants.[3] By the same token, firms that compete by offering a posted wage may be left with relatively less productive workers, thus suffering an *adverse selection* problem.

This paper shows that the adverse selection problem associated with wage posting and the search inefficiencies related to wage bargaining produce a non-trivial trade-off for firms deciding the wage setting mechanism attached to their jobs. We consider a directed search model as in Acemoglu and Shimer (1999a), where firms create vacancies and announce whether they will pay a given wage or set the wage through bargaining after assessing the worker on the job. We find that, in equilibrium, firms opt for wage posting when bargaining powers are far from satisfying Hosios' rule and the heterogeneity in workers' productivity is small. Conversely, wage bargaining emerges when the search inefficiencies induced by bargaining are mild or when workers' productivity is so dispersed that the adverse selection problem of wage posting is severe. Interestingly, when both the search inefficiencies related to bargaining and the adverse selection problem of wage posting are mild, the market gets segmented: some firms set wages through bargaining and attract the most productive workers, while others post a wage and draw the least productive workers.

Equilibria with and without bargaining may coexist, since firms' posting decisions produce externalities on other firms in the market: the composition of the pool of applicants for vacancies with a posted wage worsens if sufficiently many firms offer to bargain their wages. This reduces the profitability of wage posting and reinforces the sustainability of bargaining in equilibrium.

[3] Highhouse et al. (1999) and Lazear (2000) provide direct evidence that linking remuneration to individual productivity helps firms to attract more productive workers. In a study centered on top executives, Michaels et al. (2001) also find that differentiating pay by productivity and offering a fast career progression based on results are effective ways to attract individual high performers.

Relative to wage posting, bargaining redistributes income from low to high productivity workers, but it produces search inefficiencies that generally entail either excessive vacancies or excessive unemployment. Thus, wage bargaining always induces a reduction in aggregate net income.[4] Moreover, when there are multiple equilibria, the equilibrium without bargaining is Pareto dominant.

Our analysis contributes to the directed search literature by noting that, if worker productivity is unverifiable, wage posting suffers from adverse selection and wage bargaining may arise despite its associated search inefficiencies.[5] Ellingsen and Rosen (2003) and Camera and Delacroix (2004) use random search models with unverifiable worker heterogeneity to analyze firms' choice between bargaining and posting. In these models, however, the wage setting mechanism plays no role in attracting workers to vacancies or in tackling search inefficiencies.

Our analysis also relates to the work of McAfee (1993) and Peters (1997), where buyers have heterogeneous private valuations of the exchanged good and sellers can publicly post a pricing mechanism for the good. These papers show that second-price sealed-bid auctions are sellers' preferred pricing mechanism.[6] Taken literally, those auctions imply that each vacancy is bought out by the winning worker, who becomes the residual claimant of the future output. However, when output is not verifiable, as we assume, auctions may not be feasible if workers are wealth constrained or output goes directly to the employer. And auctions may be suboptimal if workers are risk-averse or if having the employer as a residual claimant is convenient for, say, incentive reasons (see Hart and Moore, 1990).

In our model, firms observe their workers' productivity after the matching process is completed.[7] If workers' productivity could be assessed earlier in the recruiting

[4] Aggregate income net of job creation costs is the standard social welfare measure used in the labor search literature (see, for example, Pissarides, 2000, and Shimer and Smith, 2001).

[5] Bester (1993) makes a related point in a product market model where sellers choose the non-contractible quality of their product, which the buyers observe after meeting the seller: he shows that bargaining may better motivate sellers to choose high quality.

[6] For an analysis of the role of auctions in the labor market, see Shimer (1999).

[7] The HR literature acknowledges that the scope for screening job applicants is limited. Arvey

process, firms could rank their applicants, and even vacancies with a posted wage would deliver greater utility to high productivity workers than to low productivity ones. However, the findings in Lang and Dickens (1993) and Lang et al. (2005) suggest that ranking (and the resulting non-degenerate distribution of posted wages) would discriminate across worker types very differently from fully-contingent hiring and compensation policies. We conjecture that such a setup would still leave room for the emergence of wage bargaining (and its coexistence with wage posting) for reasons akin to those explored in this paper.[8]

The rest of the paper is organized as follows. In Section 2 we describe the model. Section 3 defines our notion of equilibrium and provides some important preliminary results. Section 4 characterizes the various possible equilibrium regimes. In Section 5 we compare the various regimes in terms of social welfare. The conclusions appear in Section 6. The Appendix contains all proofs.

2 The model

We consider a static labor market with a unit mass of workers and free entry of firms. Firms and workers are risk neutral and maximize their expected net income. Each firm can create a *vacancy* at a cost $c > 0$. There are two types of workers $i = 0, 1$ in proportions $1 - \mu$ and μ, respectively. The labor services of a low productivity worker ($i = 0$) produce output $y_0 > c$ when delivered to a firm with an open vacancy. In the same circumstances, the labor services of a high productivity worker ($i = 1$) produce $y_1 > y_0$. For simplicity we assume that workers earn no income if unemployed and incur no direct cost in searching for a job.

The process whereby workers and firms match is subject to search frictions. Firms

and Campion (1982) review the vast evidence about the limited reliability of job interviews. Baron and Kreps (1999) question the validity of usual psychometric tests (pp. 352-353) and claim that substantial information about workers' productivity gets disclosed in probation periods (pp. 345).

[8] In a related note (available at http://www.cemfi.es/~michela/note.pdf) we investigate this claim in more detail.

can costlessly announce their vacancies among all workers, but workers have limited capacities to submit job applications and to coordinate their decisions. Specifically, each worker can apply for at most one vacancy. Workers first choose the vacancy announcement they prefer (possibly using a mixed strategy) and then send an application by uniformly randomizing over the firms making the announcement. Thus some firms will receive several applications, while others receive none. Firms with several applicants choose one to whom they make a job offer. For informational problems that we explain below, firms are unable to distinguish applicants' types before the match, so firms with multiple applicants choose one at random. This produces a match.

Both firms and workers face uncertainty about whether they will be successfully matched, and firms also face uncertainty about the productivity of the worker with whom they match. To model the matching uncertainty, we assume that the probability that a firm making a given announcement gets matched is $Q(n)$, which depends on the vacancy's *expected number of applicants*, n. The analogous matching probability for a worker is $P(n)$.[9] We assume that these functions are twice continuously differentiable, with $Q'(n) > 0$ and $P'(n) < 0$, and $P(n) = Q(n)/n$, where the last requirement guarantees constant returns to scale in the matching technology.

Each worker knows his own productivity type from the start, while firms learn that type after matching with the worker. We assume that third parties (say, courts) can verify only labor delivery and wage payments. This guarantees that contracts specifying that "worker A will receive a wage w in exchange for the delivery of his labor to firm B" are enforceable.[10] However, workers' type, their output, the structure

[9] Montgomery (1991) and Peters (1991) provide an explicit probabilistic model of the coordination problem, known as the *urn-ball process*, that leads to $Q(n) = 1 - \exp(-n)$. More general reduced-form functions have been used before by Acemoglu and Shimer (1999a, 1999b). The function $Q(n)$ could also be motivated as the probability implied by a standard *matching function* a la Pissarides (2000). Matching functions appear in the directed search models of Moen (1997), Acemoglu (2001), and Mortensen and Wright (2002).

[10] As in other papers in the literature, we assume that firms have deep pockets and can be forced to pay wages promised upon labor delivery.

of the bargaining between a firm and a worker, the outcome of the matching process (i.e., whether a match has occurred), and workers' application decisions are assumed to be unverifiable. Unverifiability means that courts cannot ascertain those types, outcomes, processes, and decisions at a reasonable cost, and it implies that they cannot enforce rules or provisions concerning them.[11] In this setup, we assume that firms can choose between announcing a wage for "whoever is hired" (wage posting) or announcing that the wage will be set through bargaining between the firm and the worker once they match (wage bargaining).[12]

If a firm posts a wage $x \in \mathbb{R}_+$, the matching between the firm and a worker of type i yields a compensation x to the worker and a profit $y_i - x$ to the firm. If the firm has announced bargaining, which we represent as the posting of $x_\varnothing$, then the worker and the firm bargain on the wage at a point at which the outside options of the worker (becoming unemployed) and the firm (leaving the vacancy empty) are worth zero. The surplus to be split is y_i, which, after the match, is observable to both of them. As in many papers in the random search tradition, we postulate a generalized Nash bargaining solution, implying that the worker and the firm receive βy_i and $(1-\beta)y_i$, respectively, where $\beta \in (0,1)$ represents the worker's bargaining power. Therefore, bargaining implies a higher wage for high productivity workers than for low productivity ones.

We further assume:

A1. $\lim_{n\to\infty} Q(n) = \lim_{n\to 0} P(n) = 1$.

[11] This implies that firms cannot post a wage contingent on the productivity of the worker with whom they match, announce a specific protocol for the bargaining, or promise a payment conditional on the occurrence of the match.

[12] Wage posting can be seen as a firm's announcement that constrains the bargaining. It is the strongest form of commitment and an easily enforceable one. In principle the firm may want to constrain the bargaining set in a less stringent manner, for example, by announcing a minimum or a maximum wage for the job. Under Nash bargaining, one can show that announcing a minimum wage is irrelevant: firms can always mimic its effects by either announcing a posted wage or by offering to bargain. Maximum wage policies are instead useful, but only when workers' bargaining power is high. The possibility of committing to a maximum wage tends to reinforce firms' preference for bargaining and to reduce their incentive to post a wage.

A2. The elasticity of $Q(n)$ with respect to n, $\varepsilon_Q(n) \equiv \frac{Q'(n)n}{Q(n)}$, is weakly decreasing.
A3. $\lim_{n\to\infty} \varepsilon_Q(n) < 1 - c/y_0$.
A1 and A3 help to guarantee the existence of equilibrium.[13] A2 guarantees that the number of posted wages is unique.[14]

To facilitate the diagrammatic representation of equilibria, we describe workers' *demand* for a vacancy whose expected number of applicants is n through the variable $d \equiv 1/P(n) = n/Q(n) \in [1, \infty)$ (the inverse of the workers' employment probability). Because d is a strictly increasing transformation of n, there is a strictly increasing function $n = N(d)$ that allows us to re-express firms' and workers' probabilities of getting matched as $q(d) \equiv Q(N(d))$ and $p(d) \equiv P(N(d))$, respectively. Similarly, the elasticity of $Q(n)$ with respect to n can be written as

$$\eta(d) \equiv \varepsilon_Q(N(d)) = Q'(N(d))\, d, \tag{1}$$

which is decreasing in d, by A2.

3 Equilibrium

We can distinguish two stages in the operation of the labor market. In the first stage, firms simultaneously decide whether to create a vacancy at a cost c and, in that case, which announcement $x \in X \equiv \mathbb{R}_+ \cup \{x_\varnothing\}$ to post. The resulting set of announcements X^* and the measure $v(x)$ of firms posting each $x \in X^*$ are then observed by all workers. In the second stage, workers simultaneously decide (possibly using a mixed strategy) their preferred announcement $x \in X^*$ and submit an application by uniformly randomizing over the firms posting it. For each $x \in X^*$ these actions imply a demand $d(x)$ and an expected proportion of high productivity applicants $\gamma(x)$ for the corresponding vacancies.

[13] In particular A3 ensures that even if all workers had low productivity, creating some vacancies would be socially profitable.

[14] The urn-ball process mentioned in Footnote 9 satisfies A1 and A2.

3.1 Definition of equilibrium

Allocations are described by the set X^* and the functions $v(x)$, $d(x)$, and $\gamma(x)$. Each allocation uniquely determines the expected utility attained by each worker type, U_i, with $i = 0, 1$. Workers' application decisions are based on rational forecasts of $d(x)$ for each announcement $x \in X^*$. Firms' entry decisions are based on rational predictions of $d(x)$ and $\gamma(x)$ for each possible announcement $x \in X$ (including announcements not made by any firm). This requires extending the domain of these functions to the whole set X. Since workers and firms are infinitesimal, they formulate their decisions taking the functions $d(x)$ and $\gamma(x)$ as given.[15]

For simplicity the productivity after a successful match of a worker who fills a vacancy with an announcement $x \in X$ will be denoted by the random variable $\widetilde{y}(x) \in \{y_0, y_1\}$. Analogously his wage is the random variable

$$\widetilde{w}(x) = \begin{cases} x, & \text{if } x \in \mathbb{R}_+, \\ \beta\widetilde{y}(x), & \text{if } x = x_\varnothing. \end{cases} \tag{2}$$

So a firm's net profit from creating a vacancy and announcing x can be expressed as:

$$V(x) = q(d(x))\, E_{\gamma(x)}[\tilde{y}(x) - \tilde{w}(x)] - c, \tag{3}$$

where the subscript in the expectation operator denotes the (possibly degenerate) probability that the hired worker is of the high productivity type.

Definition *An equilibrium is an allocation* $\{X^*, v(x), d(x), \gamma(x), U_0, U_1\}$ *that satisfies:*

C1. *Firms' optimization and free entry:*

$$V(x) = 0 \geq V(x'), \quad \text{for all } x \in X^* \text{ and } x' \in X.$$

[15] This assumption is standard in the directed search tradition. For announcements made by a zero mass of firms, the implicit assumption is that all workers of a given type choose the announcement to which they reply by randomizing over all vacancies that give them the same utility.

C2. *Workers' optimization: For all* $x \in X$ *and* $i = 0, 1$,

$$U_i \geq \frac{E_i[\tilde{w}(x)]}{d(x)} \quad \text{and} \quad [1 - i - \gamma(x)]N(d(x))\left\{U_i - \frac{E_i[\tilde{w}(x)]}{d(x)}\right\} = 0.$$

C3. *Aggregate consistency: For* $i = 0, 1$,

$$\sum_{x \in X^*} [1 - i - \gamma(x)]N(d(x))\,v(x) = 1 - i - \mu.$$

C4. *Balanced expectations: If* $U_0 = U_1$, *then* $\gamma(x) = \mu$ *for all* $x \in \mathbb{R}_+ \backslash X^*$.

Condition C1 requires that posted announcements maximize firms' profits and firms' profits are zero. C2 establishes that workers apply for the vacancies that maximize their utility and imposes the constraint that a vacancy can attract applicants of a given type only if it maximizes their utility. C3 guarantees that the masses of applicants of a given type add up to the exogenous total mass of workers of that type. C4 rules out equilibria based on arbitrary off-the-equilibrium-path beliefs. It requires that, if both worker types are indifferent between an out-of-equilibrium posted wage x and some equilibrium announcement, firms will expect x to attract a proportion μ of high productivity applicants.

3.2 Some intermediate results

The following results greatly simplify the analysis:

Lemma 1

1. *A posted wage* x *attracts a demand* $d(x) = \max\{1, x/U_0\}$.
2. *Wage bargaining attracts a demand* $d(x_\varnothing)$=$\max\{1, \beta y_1/U_1\}$.
3. *Vacancies offering a posted wage* $x > U_0$ *feature* $\gamma(x) = 0$ *if* $U_1 > U_0$ *and* $\gamma(x) = \mu$ *if* $U_1 = U_0$.
4. *The set of equilibrium announcements* X^* *can include at most one posted wage.*
5. *When* $x_\varnothing \in X^*$, *posted wages can attract only low productivity applicants.*

In equilibrium the demand for each vacancy will adjust so as to guarantee that the applying workers obtain their equilibrium utility. Posted wages give the same utility to both types of applicants. However, bargained wages are increasing in the worker's productivity, so they can give higher utility to high productivity workers than to low productivity ones. Thus bargained wages attract high productivity workers first, while low productivity workers are the first to be attracted to vacancies with a posted wage. This explains why demands for vacancies can be related to the equilibrium utilities of low and high productivity workers (Parts 1 and 2 of the lemma). Moreover, vacancies offering a posted wage render a utility of at most U_0; thus, high productivity workers never opt for wage posting when $U_1 > U_0$ (Part 3). The wage-invariant compositions implied by Part 3 together with the properties of the $q(\cdot)$ function under A2 make firms' profits a strictly quasi-concave function of the posted wage; therefore, conditional on posting a wage, the solution to the firms' optimization problem is unique (Part 4).[16] Finally, it turns out that the presence of wage bargaining in equilibrium implies $U_1 > U_0$, which, by Part 3, makes high productivity workers not interested in posted wages (Part 5).

Lemma 1 already shows the disadvantage of wage posting vis-a-vis bargaining in terms of attracting high productivity applicants. This adverse selection problem plays an important role in the analysis below.

4 Characterization of equilibria

Lemma 1 leaves us with three possible equilibrium regimes: (i) *pure posting* (PP), where all firms post a wage, say w_p, (ii) *pure bargaining* (PB), where all firms announce bargaining, and (iii) a *mixed* regime, where some firms post a wage, say w_m, and

[16] This differs from Burdett and Mortensen (1998), where the equilibrium distribution of posted wages is continuous. In those models, firms can hire an unlimited number of workers, and the continuum of equilibrium wages results from the trade-off between raising the number of workers and reducing the wage per worker. In directed search models, posted wages directly affect firms' hiring probability $q(\cdot)$, whose properties, under assumption A2, ensure the strict quasi-concavity of firms' profits, which rules out multiplicity.

others announce bargaining. Actually, depending on the application decisions of low productivity workers, we can distinguish two cases within the mixed regime: (a) *semi-separation* (SS), where low productivity workers are indifferent between the two existing types of vacancies, and (b) *full separation* (FS), where they strictly prefer the vacancies with a posted wage. The emergence of one equilibrium or another is driven by the interaction between the adverse selection problem of wage posting and the standard search inefficiencies related to bargaining.

4.1 When does each equilibrium arise?

The critical value $\beta^* = \eta(d_p)$, where d_p is the demand for a vacancy in the PP equilibrium, plays a central role in the next proposition. This is the value of the workers' bargaining power that satisfies the well-known condition first derived by Hosios (1990), under which the search inefficiencies related to bargaining are nil (at least in a PB equilibrium). The larger the distance between β and β^*, the greater the importance of search inefficiencies. The proposition characterizes when each equilibrium arises:

Proposition 1 *There are some critical values with* $0 < f < p < \beta^* < p' < f' < 1$ *and* $f < s < b < \beta^* < b' < s' < f'$ *such that pure posting is an equilibrium for values of the workers' bargaining power* $\beta \notin (p, p')$*; pure bargaining is an equilibrium for* $\beta \in [b, b']$*; semi-separation is an equilibrium for* $\beta \in [s, b) \cup (b', s']$*; and full separation is an equilibrium for* $\beta \in [f, s) \cup (s', f']$*. Thus an equilibrium always exists and equilibria with and without bargaining coexist for some values of* β*.*

For values of the workers' bargaining power β at or in the proximity of β^*, PB is an equilibrium, while PP is not. When β moves sufficiently farther away from β^*, PB ceases to be an equilibrium, giving way first to SS and then to FS, which are equilibria where vacancies offering bargained wages and posted wages coexist. For sufficiently extreme values of β, equilibria involving bargaining cease to exist.

When search inefficiencies are sufficiently severe, PP is the unique equilibrium. At some intermediate levels of the search inefficiencies, PP and one of the equilibria with bargaining coexist. The possibility of multiple equilibria is due to a negative externality that wage bargaining imposes on the firms that post a wage. By attracting all high productivity workers, the presence of a positive mass of firms that bargain their wages causes a discrete fall in the profitability of posting a wage, relative to the situation where no positive mass of firms opt for bargaining (recall Lemma 1).

We can further illustrate Proposition 1 using some diagrams. We focus on the cases of PP, PB, and FS equilibria. From (3), the net profit from a vacancy with demand d and a posted wage w can be written as $V_\gamma = q(d)[E_\gamma(\tilde{y}) - w] - c$, where γ indicates the probability of hiring a high productivity worker (given by Part 3 of Lemma 1). Part 4 of Lemma 1 implies that iso-profit curves are increasing and concave in the (d, w) plane. Higher levels of profits are reached by moving southeast as well as by increasing γ. Workers' indifference curves have the form $w/d = U$ and so are rays from the origin with slope U. By Parts 1 and 2 of Lemma 1, the demands for vacancies with posted and bargained wages always lie on the indifference curves of levels U_0 and U_1, respectively. An optimal posted wage corresponds to the unique tangency between the relevant iso-profit curve and the ray of slope U_0.

In a PP equilibrium, we have $U_0 = U_1 = U_p$, where U_p can be pinned down by noting that, by free entry, the equilibrium must lie on the iso-profit curve of level zero for $\gamma = \mu$ (point A in Figure 1). For this to be an equilibrium, firms must not profit by offering to bargain. Such a deviation would attract high productivity workers only and would get a demand $d(x_\varnothing) = \beta y_1/U_1$, corresponding to the point on the workers' equilibrium indifference line at which the wage is βy_1. In Figure 1, PP is an equilibrium if βy_1 is either smaller than the wage at point B or larger than the wage at point C. Intuitively, for a very low β the deviation is not profitable because it would attract too few high productivity workers, while for a very high β the deviation is not profitable because hiring a worker would be too expensive.

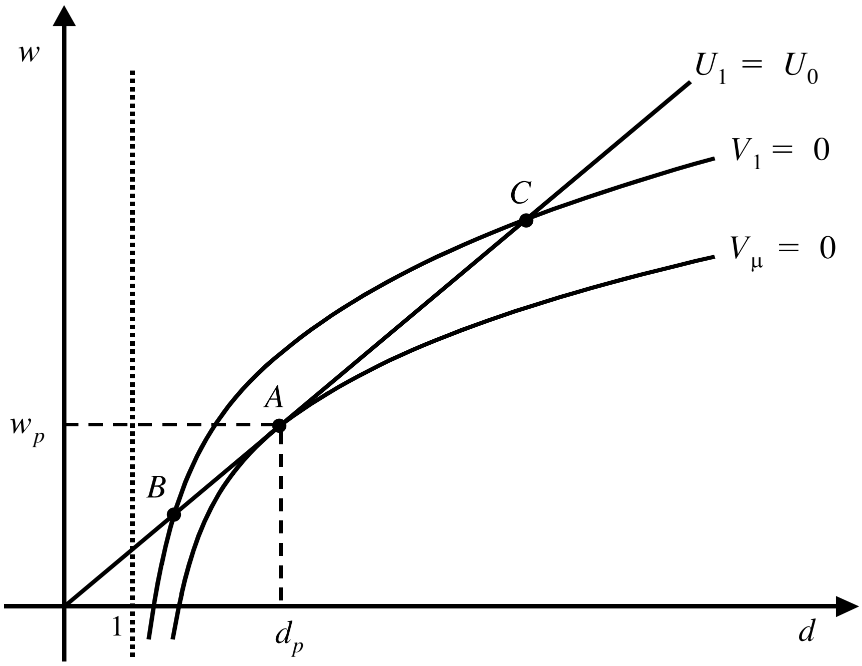

Figure 1: A Pure Posting Equilibrium

The equilibrium demand for vacancies at firms that offer to bargain in a PB equilibrium is d_b, which is the demand on the iso-profit curve of level zero for $\gamma = \mu$ and a wage $\beta E_\mu(\tilde{y})$ (that is, at point A in Figure 2). For this to be an equilibrium, no firm must profit by posting a wage. Since wage posting would attract just low productivity workers, PB is an equilibrium if the iso-profit curve of level zero for $\gamma = 0$ lies entirely below the ray with slope U_0 (like V_0 does in Figure 2). When β becomes close to either zero or one, U_0 falls because either wages are too low (for low β) or demand is too high (for high β). Eventually U_0 becomes low enough to cross the V_0 curve. Intuitively, low productivity workers are so "cheap" to attract that a firm can profit by posting a wage, in which case PB ceases to be an equilibrium.

In an FS equilibrium, the demand and wage of the firms that post a wage correspond to the point where the iso-profit curve of level zero for $\gamma = 0$ is tangent to the ray from the origin that defines U_0 (point A in Figure 3). The demand and the ex-

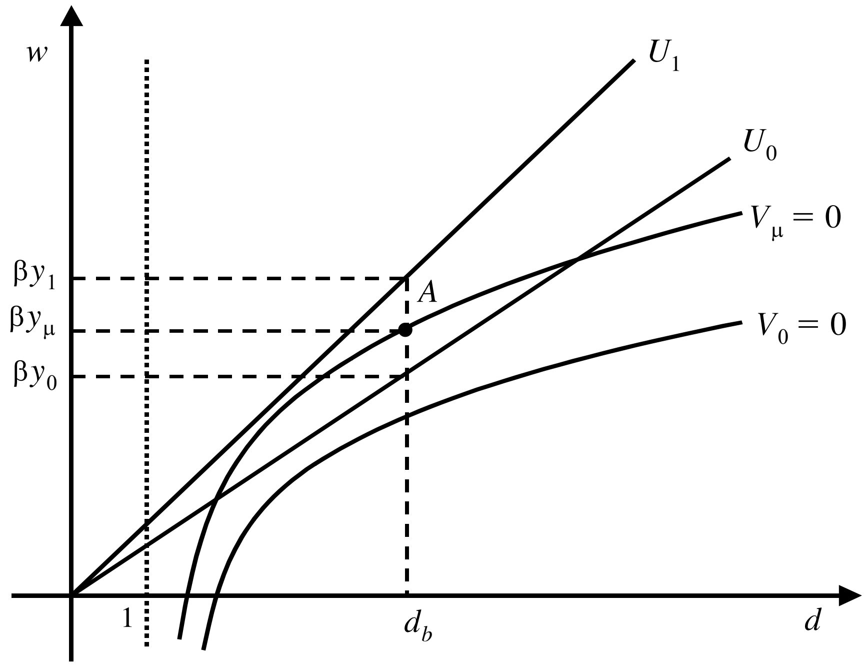

Figure 2: A Pure Bargaining Equilibrium

pected wage corresponding to vacancies with bargained wages are given by the point on the iso-profit curve of level zero for $\gamma = 1$ where the wage is βy_1 (point B in the figure). FS is an equilibrium if (i) low productivity workers do not prefer to negotiate their wage and (ii) high productivity workers do not prefer a posted wage. In Figure 3 both conditions are satisfied, since point C lies below the U_0 ray and point B is such that $U_1 > U_0$. Intuitively, mixed equilibria require that β be neither too close to satisfy the Hosios condition—because otherwise bargaining would be so appealing to the low productivity types that condition (i) would fail—nor too close to zero or one—because otherwise bargaining would no longer appeal to the high productivity types and condition (ii) would fail.

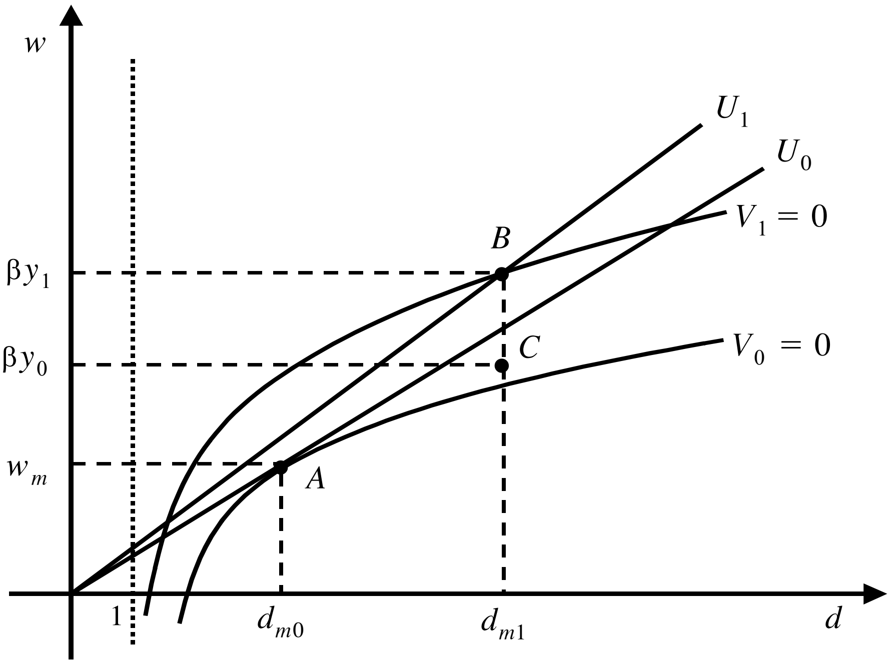

Figure 3: A Fully Separating Equilibrium

4.2 Further comparative statics

In the next proposition we characterize the effects of a mean-preserving increase in the dispersion of workers' productivity (i.e., an increase in y_1 and decrease in y_0 that keeps $E_\mu(\widetilde{y})$ constant):

Proposition 2 *A mean-preserving increase in the dispersion of workers' productivity contracts the region where pure posting is an equilibrium and expands the region where pure bargaining and, more generally, equilibria with bargaining emerge.*

Intuitively, increasing the dispersion in workers' productivity amounts to worsening the adverse selection problem of wage posting. After this change, in a candidate PP equilibrium, a deviation to bargaining becomes more profitable because it involves a greater improvement in the expected productivity of the applicants. Similarly, in equilibria with bargaining, posting a wage now involves a greater fall in the expected productivity of the applicants (relative to that under wage bargaining) and, hence, is less likely to be dominant. So with greater productivity dispersion, the PP equilibrium is more difficult to sustain and bargaining equilibria are more likely to arise.

Next, we consider an increase in the proportion of high productivity workers, μ:

Proposition 3 *Increasing the proportion of high productivity workers expands the pure posting and pure bargaining regions, contracts the semi-separation region, and leaves the full-separation region unaffected.*

As the proportion of high productivity workers in the population increases, sustaining pooling equilibria such as PP and PB becomes easier: the expected productivity of the pool of applicants improves, while the expected productivity of the applicants attracted by deviating from the candidate equilibrium remains unchanged. Thus deviations to bargaining from PP and to posting from PB become less profitable.

In the model, wage inequality is always greater in equilibria with bargaining than under pure posting. Proposition 2 implies that a small increase in the dispersion of workers' productivity can lead to a sharp increase in wage inequality if the equilibrium shifts from PP to one of the equilibria with bargaining. A similar change can occur if high productivity workers become scarcer in the labor force, since Proposition 3 implies that the overall bargaining region (covering PB, SS, and FS equilibria) does not change with μ but the PP region shrinks when μ falls.

5 Efficiency

In this section we compare the social welfare of the various possible equilibria. Following the literature, we first identify social welfare with the sum of all firms' and workers' net income, which, since firms' equilibrium profits are zero, can be expressed as the weighted sum of the equilibrium utilities of each worker type, $\mu U_1 + (1-\mu) U_0$. Bargaining can affect social welfare through two different margins. First, the standard search inefficiencies make aggregate net income fall, because they lead to either excessive vacancies (when $\beta < \beta^*$) or excessive unemployment (when $\beta > \beta^*$). Second, bargaining can promote (full or partial) separation of low and high productivity workers, as in our mixed regimes. This can be a source of efficiency gains since it can help better reproduce the first best allocation: with verifiable worker types, a social planner would set up two submarkets, one for high and one for low productivity workers, with different demand. The next result shows that:

Proposition 4 *The pure posting allocation generates larger aggregate net income than any of the equilibria with bargaining.*

In the PB and PP allocations, the market is not segmented. When search inefficiencies are absent ($\beta = \beta^*$), the PB allocation (which is an equilibrium in such a case) yields the same net income as the PP allocation. But as search inefficiencies increase, the welfare associated with the PP allocation remains constant, while the welfare generated by PB decreases. As search inefficiencies keep increasing, PB ceases to be an equilibrium, and first SS and eventually FS become equilibria (recall Proposition 1), but aggregate welfare continuously declines. This does not mean that mixed regime allocations cannot dominate the PP allocation in terms of social welfare: they can for sufficiently low search inefficiencies.[17] But the SS and FS equilibria can only be sustained for sufficiently large search inefficiencies, because otherwise bargaining is so attractive that low productivity workers no longer accept a posted wage. Thus whenever bargaining emerges in equilibrium, its net welfare contribution is always negative.

Interestingly, the welfare losses induced by search inefficiencies can be so large that not only low but also high productivity workers are better off in a PP equilibrium than in an alternative equilibrium with bargaining. Indeed, we can prove that:

Proposition 5 *Whenever the pure posting equilibrium and an equilibrium with bargaining coexist, the former is Pareto dominant.*

[17] For example, the FS and the first best allocation coincide if the Hosios condition holds in the bargaining segment of the market.

This is because coexistence occurs only when search inefficiencies are sufficiently severe (recall Proposition 1), so severe that either low wages (if $\beta < \beta^*$) or high unemployment probability (if $\beta > \beta^*$) make high productivity workers worse off than in the PP equilibrium.

6 Conclusions

We have presented a tractable directed search model where firms compete for heterogeneous workers by announcing the wage setting mechanism associated with their vacancies. Productivity becomes observable to firms after assessing their workers on the job, but it is not verifiable. So wages bargained after a match can be tailored to a worker's productivity, while posted wages cannot. Firms' decisions are driven by the trade-off between the possibility of using bargaining as a means to attract better applicants to a vacancy and the standard search inefficiencies that bargaining involves. A key prediction of our analysis is that more productive workers bargain their wages, while less productive workers accept a posted wage within the same industry, at least for some parameter values. Other predictions are that the incidence of wage bargaining should be higher in jobs and occupations where the dispersion in workers' productivity is more pronounced and workers' skills are scarcer.

The evidence about these predictions is so far indirect. The challenge is to identify variation in wage setting practices in the data. Unionized firms (as well as firms and agencies that belong to the government sector) are more likely to be constrained to offer wage structures that satisfy the "same job, same pay" principle (see Freeman and Medoff, 1984) and, hence, forced to post wages. If this is the case, we would expect them to attract lower productivity workers than firms in the same industry that can opt for bargaining. Indeed, the evidence in Hirsch (1991) and Booth (1995) suggests that unionized firms have lower productivity. Other studies have analyzed firms' propensity to reward workers through promotions, bonuses, and permanent pay rises based on subjective performance evaluation—practices likely to lead firms and workers to negotiate on pay. MacLeod and Parent (1999) find that these practices are pervasive in jobs that involve a large number of tasks (where the dispersion in workers' productivity is likely to be larger) and more common among professionals, technicians, managers, and administrators (that is, in occupations where workers' skills arguably have larger effects on productivity).

Another interesting implication of the analysis is that wage inequality is always greater in equilibria with bargaining than in a pure posting equilibrium. This suggests that changes in the wage setting regime may explain time and cross-sectional variation in wage inequality. If the dispersion in workers' productivity increases or if high productivity workers become relatively scarcer, the labor market may switch from a pure posting equilibrium to one of the equilibria with bargaining, producing an increase in wage inequality. Processes like these might have contributed to the rise in wage inequality observed in the US over the 1980s and 1990s (see, for instance, Juhn et al. 1993). The IT revolution may have required skills that were (at least temporarily) scarce and dispersed. Some casual evidence suggests that firms' compensation practices have indeed changed in a direction consistent with this interpretation.[18] From a cross-country perspective, the prevalence of predetermined wage structures in Japan (discussed, among others, by Baron and Kreps, 1999 and Klein, 1992) suggests that Japanese firms are closer than US firms to the compensation practices that characterize our pure posting equilibrium. The low level of wage inequality in Japan relative to the US (see Table 3.1 from OECD, 1996) is consistent with this interpretation.

[18] Lemieux et al. (2005) argue that the growing incidence of discretionary bonuses accounts for 30 percent of the growth in male wage inequality experienced in the US between the late 1970s and the early 1990s. The article "Job Candidates Are Promised Quicker Reviews, Likely Raises" (Wall Street Journal, October 6, 1998) reports that the firm Compdata Surveys, after analyzing data covering more than 3.5 million employees in 29 states, concluded that employers show more flexibility in wage determination now than in the past.

Appendix

Proof of Lemma 1 Consider any given X^* and $v(x)$. It is easy to see that workers' optimization (equilibrium condition C2) and (2) necessarily imply that

$$\frac{y_0}{y_1}U_1 \leq U_0 \leq U_1. \tag{4}$$

Recall that $d(x) = 1$ means that workers do not apply for vacancies that announce x, while, by the second part of equilibrium condition C2, $d(x) > 1$ requires $E_i[\tilde{w}\,(x)]/d(x) = U_i$ for at least one of the worker types.

Part 1. Consider first a vacancy with $x \in \mathbb{R}_+$ and $x \leq U_0$. In this case we necessarily have $d(x) = 1$, since the alternative $d(x) > 1$ would imply $x/d(x) < U_0 \leq U_1$, which would contradict equilibrium condition C2. Now, for $x \in \mathbb{R}_+$ and $x > U_0$, we can prove by contradiction that $d(x) = x/U_0$. In fact, if $d(x) < x/U_0$, C2 would be violated for $i = 0$. If $d(x) > x/U_0 > 1$, (4) implies $U_1 \geq U_0 > x/d(x)$, which, given C2, contradicts $d(x) > 1$.

Part 2. If $\beta y_1 \leq U_1$, then (4) and C2 immediately imply that $d(x_\varnothing) = 1$. If $\beta y_1 > U_1$, we can prove by contradiction that $d(x_\varnothing) = \beta y_1/U_1$. In fact, if $d(x_\varnothing) < \beta y_1/U_1$, the first part of C2 would be violated for $i = 1$. If $d(x_\varnothing) > \beta y_1/U_1 > 1$, (4) would also imply $d(x_\varnothing) > \beta y_0/U_0$, which, given C2, would contradict $d(x_\varnothing) > 1$.

Part 3. If $x > U_0$, $E_0[\tilde{w}\,(x)] = E_1[\tilde{w}\,(x)] = x$, by (2), and $d(x) = x/U_0 > 1$, by Part 2 of this lemma. Then if $U_1 > U_0$, C2 for $i = 1$ implies that no high productivity worker will apply for such vacancies, $\gamma(x) = 0$. If $U_0 = U_1$, $d(x) = x/U_0$ implies that both worker types might be willing to apply. If $x \in \mathbb{R}_+ \backslash X^*$, the implication that $\gamma(x) = \mu$ follows directly from the equilibrium condition C4 (balanced expectations). If $x \in \mathbb{R}_+ \cap X^*$, the result that $\gamma(x) = \mu$ can be proved by contradiction. Suppose first that $\gamma(x) < \mu$, then Part 2 of this lemma and C4 would imply that an alternative announcement $x' \in \mathbb{R}_+ \backslash X^*$, arbitrarily close to x, would feature $\gamma(x') = \mu$ and thus yield, thanks to the discrete improvement in the composition of the pool of applicants, $V(x') > V(x)$, which contradicts C1. Suppose alternatively that $\gamma(x) > \mu$, then C3 for $i = 1$ implies that there must be at least one other announcement $x'' \in X^*$ with $\gamma(x'') < \mu$. Moreover, x'' cannot be $x_\varnothing$, since Part 1 of this lemma and the second part of C2 for $i = 0$ imply $\gamma(x_\varnothing) = 1$ when $U_0 = U_1$. But if we have $x'' \in \mathbb{R}_+$ with $\gamma(x'') < \mu$ in the set of equilibrium announcements, then the same contradiction that we showed in the previous subcase arises.

Part 4. Part 2 of this lemma implies $d(x) = x/U_0$ for $x > U_0$, while Part 3 implies $\gamma(x) = \gamma$ for all $x \in \mathbb{R}_+$. From here, if we particularize (3) for $x > U_0$ and derive with respect to $x \in \mathbb{R}_+$, we obtain:

$$V'(x) = q'(d(x))(E_\gamma(\tilde{y}) - x)\frac{1}{U_0} - q(d(x)).$$

Now, if we substitute $x/d(x)$ for U_0 and use (1) to write $q'(d)$ as $\frac{\eta(d)}{1-\eta(d)} \cdot \frac{q(d)}{d}$, we can obtain:

$$V'(x) = \frac{q(d(x))}{1 - \eta(d(x))}\left[\eta(d(x))\frac{E_\gamma(\tilde{y})}{x} - 1\right], \tag{5}$$

where the term out of the brackets is always positive. The sign of the term in brackets is generally ambiguous, but such term is decreasing in x, since $\eta(d)$ is decreasing in d, by A2, and $d(x)$ is increasing in x by Part 1 of this lemma. Thus, as x increases over the range $x > U_0$, the sign of $V'(x)$ can shift from positive to negative at most once, which implies quasi-concavity. Moreover, $\lim_{x\to U_0} V(x) = -c$ and $\lim_{x\to\infty} V(x) = -\infty$. Thus, over the range $x > U_0$, $V(x)$ can be either monotonically decreasing or first increasing and then decreasing. In the first case, no posted wage can yield zero profits so the stated result is true. In the second case, there is a unique critical point satisfying $V'(x) = 0$ that maximizes $V(x)$. This is the candidate to belong to X^*.

Part 5. We prove that $x_\varnothing \in X^*$ implies that $U_0 < U_1$. Then the result emerges as a corollary of Part 3 of this lemma. If X^* includes only wage bargaining, $U_0 < U_1$ follows immediately from the fact that $\beta y_1 > \beta y_0$, see (2). If X^* also includes a posted wage, Part 4 of this lemma implies that this must be unique, say $w \in R_+$. In this case, having $U_0 = U_1$ would lead to a contradiction. If $U_0 = U_1$, Part 3 of this lemma would imply $\gamma(w) = \mu$ and, then, $\gamma(x_\varnothing) = \mu$, by C3. But then C2 would imply that $\beta y_0/d(x_\varnothing) = U_0$ and $\beta y_1/d(x_\varnothing) = U_1$, which contradicts $U_0 = U_1$, since $y_0 < y_1$.$\|$

Proof of Proposition 1 We prove this result by examining the necessary and sufficient conditions for the existence of each of the possible equilibrium regimes:

i. *Pure posting* (PP). In a PP equilibrium, all firms with vacancies post a wage w_p, get a demand d_p, and attract high productivity applicants in the same proportion μ

as they exist in the labor force. Both worker types attain the same utility $U_p = w_p/d_p$. The posted wage must solve

$$w_p = \arg\max_{x\in\mathbb{R}_+} q(\frac{x}{U_p})[E_\mu(\tilde{y}) - x] - c,$$

where we use Part 1 of Lemma 1 and adopt the convention that $q(d) = 0$ when $d < 1$. Moreover, the firms' zero profit-condition implies $w_p > U_p$. Then w_p satisfies a first order condition, which, after using (5), can be written as

$$w_p = \eta(d_p)E_\mu(\tilde{y}), \tag{6}$$

so firms' zero-profit condition simplifies to:

$$q\,(d_p)\,[1 - \eta\,(d_p)]\,E_\mu(\tilde{y}) = c, \tag{7}$$

which uniquely determines d_p and, recursively, w_p and U_p. PP is an equilibrium if $V(x_\varnothing) \leq 0$. Given Part 2 of Lemma 1, $d(x_\varnothing) = \max(1, \beta y_1/U_p)$. Moreover $\gamma(x_\varnothing) = 1$, since with this $d(x_\varnothing)$ low productivity workers would attain an expected income lower than U_p if they applied for this vacancy. Thus, the condition $V(x_\varnothing) \leq 0$ reads as:

$$q(\frac{\beta y_1}{U_p})\,(1-\beta)\,y_1 - c \leq 0.$$

Given that (6) implies $U_p = \eta(d_p)E_\mu(\tilde{y})/d_p$, this condition can be rewritten as

$$P(\beta) \equiv (1-\beta)\,q\left(\frac{\beta y_1 d_p}{\eta\,(d_p)\,E_\mu(\tilde{y})}\right) \leq \frac{c}{y_1}. \tag{8}$$

The function $P(\beta)$ can be shown to be non-negative and quasi-concave and to reach a maximum at $\widehat{\beta} \leq \beta^* \equiv \eta\,(d_p)$. Moreover, $\widehat{\beta} = \beta^*$ and $P(\beta^*) = c/y_1$ in the limit case where $\mu = 1$. Finally $P(\beta)$ is decreasing in μ. Thus, as μ decreases, $P(\beta)$ shifts upward and gives rise to an interval $(p, p') \subset (0,1)$ of values of the bargaining power β where $P(\beta) > c/y_1$ (see Figure 4). Out of that range, (8) holds and, hence, PP is an equilibrium. Since $\widehat{\beta} \leq \beta^*$ and β^* is an increasing function of μ, we also have $\widehat{\beta} \leq \beta^* \leq \beta^*|_{\mu=1}$, which implies that $\beta^* \in (p, p')$.

ii. *Pure bargaining* (PB). In a PB equilibrium, all firms announce bargaining, get a demand d_b, and attract high productivity applicants in the same proportion μ as they exist in the labor force. Given that bargained wages amount to a fraction β

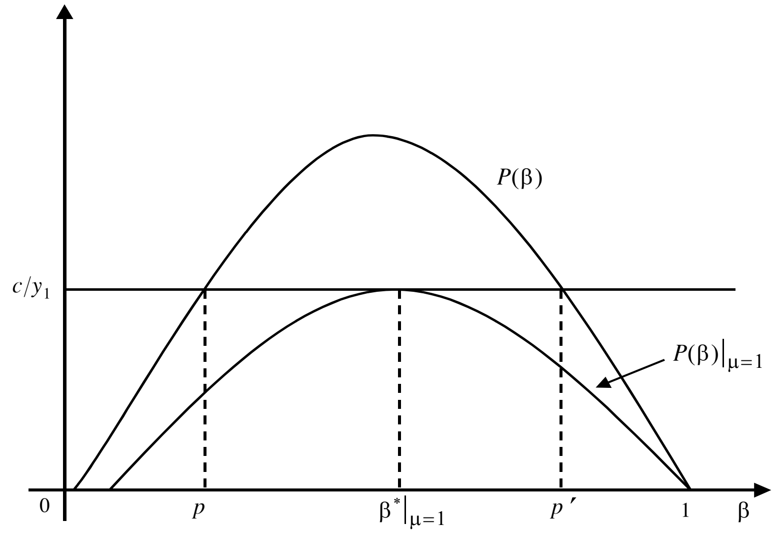

Figure 4: The Prevalence of Pure Posting

of workers' output, the utility of high productivity workers, $U_{b1} = \beta y_1/d_b$, is greater than that of low productivity workers, $U_{b0} = \beta y_0/d_b$. Firms' zero-profit condition is

$$q\,(d_b)\,(1-\beta)\,E_\mu(\tilde{y}) = c, \tag{9}$$

which uniquely determines d_b. Firms' optimization requires $V(x) \leq 0$, $\forall\ x \in \mathbb{R}_+$. The best wage that a firm can post is

$$w' = \arg\max_{x\in\mathbb{R}_+} q(\frac{x}{U_{b0}})\,(y_0 - x) - c,$$

which is always larger than U_{b0} (since $U_{b0} < y_0$). Using (5), the corresponding first order condition becomes:

$$w' = \eta\,(d')\,y_0, \tag{10}$$

where $d' = w'/U_{b0}$ is the demand that w' would generate according to Part 1 of Lemma 1. But since $U_{b0} = \beta y_0/d_b$, we can rewrite (10) as

$$\beta d' = \eta\,(d')\,d_b, \tag{11}$$

which uniquely determines d', since d_b is already determined by (9). With this notation, the condition for the absence of a profitable deviation, $q(\frac{w'}{U_{b0}})\,(y_0 - w') \leq c$, is

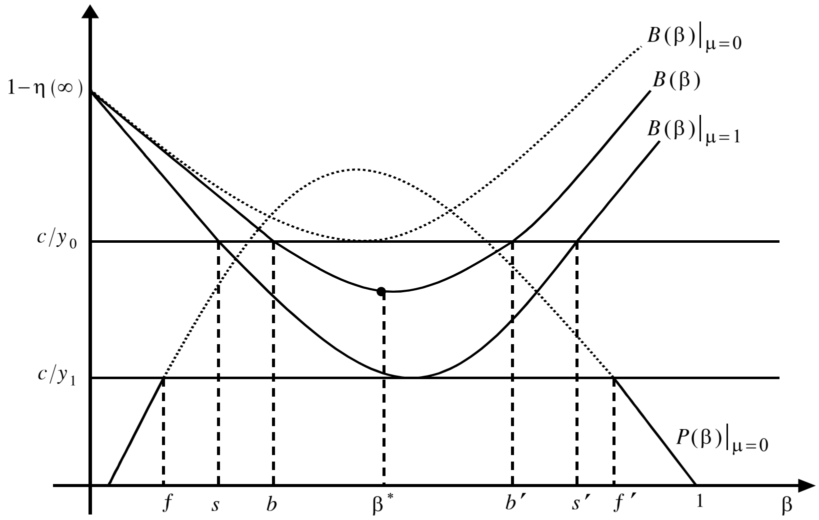

Figure 5: The Prevalence of Bargaining Equilibria

equivalent to

$$B(\beta) \equiv [1 - \eta(d')]\, q(d') \leq \frac{c}{y_0}. \tag{12}$$

The function $B(\beta)$ can be shown to be non-negative and quasi-convex and to reach a minimum at $\beta^* \equiv \eta(d_p)$. Moreover, $B(\beta^*) = c/y_0$ in the limit case where $\mu = 0$, and $B(\beta)$ is decreasing in μ. Thus, as μ increases, $B(\beta)$ shifts downward and gives rise to a range $[b, b'] \subset (0,1)$ of values of β that contains β^* and where (12) holds (see Figure 5). Finally, before proceeding notice that (7) and (9) imply that $d_p = d_b$ when $\beta = \beta^*$, which will be used several times below.

iii. *Mixed regimes.* In a mixed regime, some firms post a wage w_m, receive a demand d_{m0}, and, given Part 5 in Lemma 1, attract just low productivity workers, $\gamma(w_m) = 0$. The remaining firms announce bargaining and receive a demand d_{m1}. Thus low and high productivity workers obtain utility $U_{m0} = w_m/d_{m0}$ and $U_{m1} = \beta y_1/d_{m1}$, respectively. To determine w_m we can use (5) to impose the FOC that yields

$$w_m = \eta(d_{m0}) y_0$$

while the zero-profit condition reads as

$$q\left(d_{m0}\right)\left[1-\eta\left(d_{m0}\right)\right] y_0=c, \tag{13}$$

which uniquely determines d_{m0}. Two equilibrium variables remain to be fixed: the demand for the vacancies offering bargained wages, d_{m1}, and the proportion of high productivity workers among their applicants, say γ, which must lie in the interval $(\mu, 1]$ by C3. The zero-profit condition for the firms announcing bargaining requires:

$$q\left(d_{m1}\right)(1-\beta) E_\gamma(\tilde{y})=c, \tag{14}$$

where $\gamma \in (\mu, 1]$ must be compatible with workers' optimization. Then two mutually exclusive possibilities arise:

a. *Semi-separation* (SS). In this case we must have

$$U_{m0}=\frac{\beta y_0}{d_{m1}}<\frac{\beta y_1}{d_{m1}} \tag{15}$$

and the value γ which solves (14) for $d_{m1}=\beta y_0/U_{m0}$ must lie in the interval $(\mu, 1]$.

b. *Full separation* (FS). In this case we must have

$$\frac{\beta y_0}{d_{m1}}<U_{m0} \leq \frac{\beta y_1}{d_{m1}} \tag{16}$$

where d_{m1} solves (14) for $\gamma=1$.

In an SS equilibrium, firms that announce bargaining behave like firms in the PB equilibrium of an "artificial" economy in which the proportion of high productivity workers is some (endogenously determined) $\gamma \in (\mu, 1]$, rather than μ. Since the firms that post the wage w_m must also break even, condition (12) for the existence of a PB equilibrium must hold with equality in the artificial economy. Then we can identify the values of β for which an SS equilibrium exists by varying the proportion of high productivity in the interval $(\mu, 1]$ and then considering the intersections between the horizontal line c/y_0 and the graph of $B(\beta)$ for the corresponding artificial economy. The values of β that lead to SS equilibria with $\gamma = 1$ correspond to the left and right intersections between the horizontal line c/y_0 and the graph of $B(\beta)\mid_{\mu=1}$ (denoted as s and s' in Figure 5). Conversely, the PB equilibria that emerge with $\beta = b$ and $\beta = b'$ can be seen as degenerated SS equilibria with $\gamma = \mu$. Since increasing μ shifts $B(\beta)$ downward, it is clear that $s < b$ and $b' < s'$, and we can conclude that an SS equilibrium exists for all $\beta \in [s, b) \cup (b', s']$.

To characterize the region where FS is an equilibrium, we need to check condition (16). For $\beta = s, s'$ the SS equilibrium involves $\gamma = 1$ but (16) fails to hold just because $U_{m0} = \beta y_0/d_{m1}$ and so the first inequality holds with equality. However, the value of d_{m1} determined by (14) for $\gamma = 1$ varies with β in such a way that $\beta y_0/d_{m1}$ is a quasi-concave function of β with a maximum at $\beta^*|_{\mu=1} \in [s, s']$. In contrast, U_{m0} is determined in the posting segment of the market and, thus, is independent of β. Therefore, for values of β right below s or right above s', we have $\beta y_0/d_{m1} < U_{m0}$ (while, by continuity, we still have $U_{m0} < \beta y_1/d_{m1}$). Thus an FS equilibrium exists. However, as β moves toward the extremes, $U_{m1} = \beta y_1/d_{m1}$ becomes closer and eventually equal to U_{m0}. In particular, we get to $\beta y_1/d_{m1} = U_{m0}$ when, in the artificial economy with $\mu = 0$, the condition (8) for the existence of PP holds with equality. Graphically, this occurs at the intersections $\beta = f$ and $\beta = f'$ between the horizontal line c/y_1 and the graph of $P(\beta)\mid_{\mu=0}$ (see Figure 5).

Finally notice that, since $P(\beta)$ shifts downward as μ increases, we have that $f < p$ and $p' < f'$, so the full range $[0, 1]$ of possible values of β is covered by one equilibrium or another, and actually there is coexistence between pure posting and one of the equilibria with bargaining for values of β in the intervals $[f, p]$ and $[p', f']$.$\|$

Proof of Proposition 2 Consider the experiment of increasing y_1 and decreasing y_0 without changing workers' average productivity $E_\mu(\widetilde{y})$. It follows from (7) that d_p remains unchanged. Thus, in inequality (8) for the existence of PP, the LHS rises while the RHS falls, so the condition is less likely to hold. In terms of Figure 4, $P(\beta)$ shifts upward while the line c/y_1 shifts downward, so the interval (p, p') expands. Also, it follows from (9) that d_b remains unchanged. Thus, in (12), the LHS remains constant while the RHS increases, so the condition for the existence of PB is more likely to hold. In terms of Figure 5, $B(\beta)$ remains unchanged, while the line c/y_0 moves upward, so the interval (b, b') expands. For similar reasons, the thresholds s and f move toward the left, while s' and f' move toward the right.$\|$

Proof of Proposition 3 In terms of Figures 4 and 5, increasing μ shifts down the graphs of both $P(\beta)$ and $B(\beta)$, so the interval (p, p') contracts, while the interval (b, b') expands, which immediately means that PP and PB are sustainable over larger sets of values of β. On the other hand, the thresholds f, f', s, and s' remain unaffected,

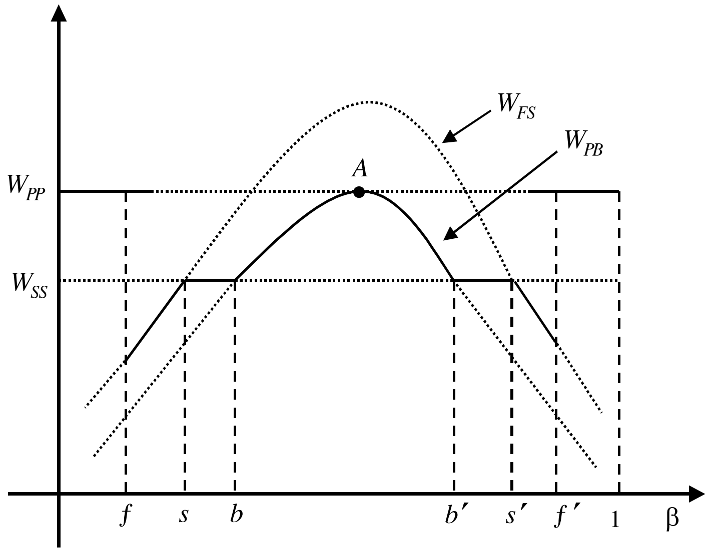

Figure 6: Welfare in the various equilibrium allocations

since the graphs of $P(\beta)$ and $B(\beta)$ in the artificial economies with $\mu = 0$ and $\mu = 1$ do not vary with μ. Thus the ranges of values of β where FS is an equilibrium are unchanged, while the ranges where SS emerges shrink due to the expansion of the interval (b, b').||

Proof of Proposition 4 Let W_j denote the social welfare levels attained at the various possible allocations, $j =$PP, PB, SS, FS. Consider the function $G(\beta, \mu)$, which yields W_{PB} for different values of β and μ:

$$G(\beta, \mu) \equiv \mu U_{b1} + (1 - \mu)U_{b0} = \frac{\beta E_\mu(\tilde{y})}{d_b}, \tag{17}$$

where d_b is implicitly determined by (9). This function can be shown to be quasi-concave in β and to reach a maximum at $\beta = \beta^* \equiv \eta(d_p)$. Such maximum corresponds to point A in Figure 6. When $\beta = \beta^*$, the PP and PB allocations coincide in terms of both the average wage paid and workers' demand (see the proof of Proposition 1). Thus $W_{\text{PP}} = W_{\text{PB}}$ at $\beta = \beta^*$. But the PP allocation and then W_{PP} are independent of β, while W_{PB} falls as β distances from β^*. So we have $W_{\text{PP}} > W_{\text{PB}}$ for all $\beta \neq \beta^*$.

In the SS regime, workers' utilities, and thus social welfare W_{SS}, are independent of β because U_{m0} is determined in the posting segment of the labor market, while

$U_{m1} = \frac{y_1}{y_0} U_{m0}$, by (15). We have already noticed that the SS allocation degenerates into a PB allocation at the critical values $\beta = b, b'$. Hence, $W_{\text{SS}} = G(b, \mu) = G(b', \mu)$ for all β (see Figure 6).

Finally, in the FS regime, the bargaining segment of the labor market attracts all the high productivity workers, and it functions like PB in an economy with $\mu = 1$, so $U_{m1} = G(\beta, 1)$. The posting segment of the market instead attracts all the low productivity workers, and it functions like PP in an economy with $\mu = 0$, so $U_{m0} = \max_\beta G(\beta, 0)$. Thus, $W_{\text{FS}} = \mu G(\beta, 1) + (1 - \mu) \max_\beta G(\beta, 0)$. Importantly, at $\beta = s, s'$ we have $W_{\text{FS}} = W_{\text{SS}}$, since the SS allocation involves $\gamma(x_\varnothing) = 1$. Moreover, since $G(\beta, 1)$ is strictly quasi-concave in β and reaches a maximum at $\beta^*|_{\mu=1} \in (s, s')$, it follows that $W_{\text{FS}} < W_{\text{SS}}$ for all $\beta \in [f, s) \cup (s', f']$.

Given Proposition 1, the solid sections of the curves depicted in Figure 6 identify the values of β for which the corresponding allocation can be sustained as an equilibrium. The result stated in the proposition follows.$\|$

Proof of Proposition 5 Since $U_1 \geq U_0$ it is enough to show that when PP coexists with either FS, SS, or PB, high productivity workers are not better off in the equilibrium with bargaining than in a PP equilibrium. Recall that PP is an equilibrium if and only if

$$q\left(\underline{d}\right)(1 - \beta)\, y_1 \leq c, \tag{18}$$

where $\underline{d} = \max(1, \beta y_1 / U_p)$ from Part 2 in Lemma 1, while U_p is workers' utility in a PP equilibrium. In a bargaining equilibrium high productivity workers earn $U_1 = \beta y_1 / d_\gamma$, where d_γ solves

$$q\left(d_\gamma\right)(1 - \beta)\, E_\gamma(\tilde{y}) = c, \tag{19}$$

and $\gamma = \mu$ in PB, $\gamma \in (\mu, 1]$ in SS, and $\gamma = 1$ in FS. Clearly, (18) and (19) imply that $d_\gamma \geq d_1 \geq \underline{d}$ for all $\gamma \in [\mu, 1]$. But then it follows from the definition of $\underline{d}$ that $d_\gamma \geq \beta y_1 / U_p$ and so $U_p \geq U_1 = \beta y_1 / d_\gamma$ for all $\gamma \in [\mu, 1]$.$\|$

References

Acemoglu, D. (2001), "Good Jobs versus Bad Jobs," *Journal of Labor Economics*, 19, 1-22.

Acemoglu, D. and Shimer, R. (1999a), "Efficient Unemployment Insurance," *Journal of Political Economy*, 107, 893-928.

Acemoglu, D. and Shimer, R. (1999b), "Holdups and Efficiency with Search Frictions," *International Economic Review*, 40, 827-851.

Arvey, R. and Campion, J. (1982), "The Employment Interview: A Summary and Review of Recent Research," *Personnel Psychology,* 25, 281-322.

Baker, G., Gibbons, R. and Murphy, K. (1994), "Subjective Performance Measures in Optimal Incentive Contracts," *Quarterly Journal of Economics*, 109, 1125-56.

Baron, J. and Kreps, D. (1999), *Strategic Human Resources: Frameworks for General Managers,* John Wiley & Sons.

Bester, H., (1993), "Bargaining versus Price Competition in Markets with Quality Uncertainty," *American Economic Review*, 83, 278-88.

Booth, A. (1995), *The Economics of the Trade Union,* Cambridge University Press, Cambridge UK.

Burdett, K. and Mortensen, D. (1998), "Wage Differentials, Employer size, and Unemployment," *International Economic Review*, 39, 257-273.

Camera, G. and Delacroix, A. (2004), "Trade Mechanism Selection in Markets with Frictions," *Review of Economic Dynamics*, 7, 851-868.

Ellingsen, T. and Rosen, A. (2003), "Fixed or Flexible? Wage Setting in Search Equilibrium," *Economica*, 70, 233-250.

Freeman, R. and Medoff, J. (1984), *What Do Unions Do?*, Basic Books, New York.

Hart, O. (1995), *Firms, Contracts and Financial Structure,* Clarendon Press, Oxford.

Hart, O. and Moore, J. (1990), "Property Rights and the Nature of the Firm," *Journal of Political Economy,* 98, 1119-1158.

Highhouse, S., Stierwalt, S., Bachiochi, P. Elder, A. and Fisher, G. (1999), "Effects of Advertised Human Resource Management Practices on Attraction of African American Applicants," *Personnel Psychology* 52, 425-442.

Hirsch, B. (1991), *Labor Unions and the Economic Performance of Firms*, WE Upjohn Institute for Employment Research, Kalamazoo, MI.

Hosios, A. (1990), "On the Efficiency of Matching and Related Models of Search and Unemployment," *Review of Economic Studies*, 57, 279-298.

Juhn, C., Murphy, K. and Pierce, B. (1993), "Wage Inequality and the Rise in Returns to Skill," *Journal of Political Economy,* 101, 410-442.

Klein, E. (1992), "The US Japanese HR Culture Clash," *Personnel Journal,* 71, 30-38.

Lang , K. and Dickens, W. (1993), "Bilateral Search as an Explanation for Labor Market Segmentation and Other Anomalies," NBER Working Paper 4461.

Lang , K., Manove, M. and Dickens, W. (2005), "Racial Discrimination in Labor Markets with Announced Wages," *American Economic Review*, 95, 1327-1340.

Lazear, E. (2000), "Performance Pay and Productivity," *American Economic Review*, 90, 1346-61.

Lemieux, T., MacLeod, B., and Parent, D. (2005), "Bonus Pay and Wage Inequality," mimeo, University of British Columbia.

McAfee, R. P. (1993), "Mechanism Design by Competing Sellers," *Econometrica*, 61, 1281-1312.

MacLeod, B. and Parent, D. (1999), "Job Characteristics and the Form of Compensation," *Research in Labour Economics*, 18, 231-59

Maskin, E. (2002), "On Indescribable Contingencies and Incomplete Contracts," *European Economic Review*, 46, 725-733.

Michaels, E., Handfield-Jones, H. and Axelrod, B. (2001), *The War for Talent*, Harvard Business School Press.

Moen, E. (1997), "Competitive Search Equilibrium," *Journal of Political Economy*, 105, 385-411.

Montgomery, J. (1991), "Equilibrium Wage Dispersion and Interindustry Wage Differentials," *Quarterly Journal of Economics,* 106, 163-79.

Mortensen, D. and Wright, R. (2002), "Competitive Pricing and Efficiency in Search Equilibrium," *International Economic Review*, 43, 1-20.

OECD (1996), *Employment Outlook,* OECD, Paris.

Peters, M. (1991), "Ex-Ante Price Offers in Matching Games Non-Steady States," *Econometrica,* 59, 1425-1454.

Peters, M. (1997), "A Competitive Distribution of Auctions," *Review of Economic Studies,* 64, 97-123.

Pissarides, C. (2000), *Equilibrium Unemployment Theory*, 2nd Edition, MIT Press, Boston.

Shi, S. (2001), "Frictional Assignment, Part I: Efficiency," *Journal of Economic Theory,* 98, 232-260.

Shi, S. (2002), "A Directed Search Model of Inequality with Heterogeneous Skills and Skill-Biased Technology," *Review of Economic Studies, 69,* 467-491.

Shimer, R. (1999), "Job Auctions," mimeo, Princeton University.

Shimer, R. (2005), "The Assignment of Workers to Jobs in an Economy with Coordination Frictions," *Journal of Political Economy*, 113, 996-1025.

Shimer, R. and Smith, L. (2001), "Matching, Search, and Heterogeneity," *Advances in Macroeconomics (BE Journals in Macroeconomics)* 1(1): article 5.

Tirole, J. (1999), "Incomplete Contracts: Where Do We Stand?," *Econometrica*, 67, 741-781.