Twin Defaults and Bank Capital Requirements*

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Abstract

We examine optimal capital requirements in a quantitative general equilibrium model with banks exposed to non-diversifiable borrower default risk. Contrary to standard models of bank default risk, our framework captures the limited upside, but significant downside risk of loan portfolio returns (Nagel and Purnanandam, 2020). This helps to reproduce the frequency and severity of *twin defaults*: simultaneously high firm and bank failures. Hence, the optimal bank capital requirement, which trades off a lower frequency of twin defaults against restricting credit provision, is higher than under default risk models, which underestimate the impact of borrower default on bank solvency.

Keywords: Financial Intermediation, Macroprudential Policy, Default Risk, Bank Assets Returns.

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More than a decade after the 2008-2009 financial crisis, the optimal level of bank capital requirements still remains an open question. Bank capital is considered the best way to protect individual banks and the aggregate economy against the risk of bank insolvencies. When bank capital ratios are low, abnormally high default rates among bank borrowers lead to sharp declines in bank net worth and increases in bank failures. The resulting fall in bank lending further amplifies the real and financial implications of credit losses. Thus, many academics and policy-makers have made the case for significantly higher capital requirements (see e.g. Admati and Hellwig, 2013; The Federal Reserve Bank of Minneapolis, 2017). However, when banks' capacity to raise equity is limited, lowering the frequency of severe bank insolvencies may come at the cost of restricting bank credit provision in normal times (see, e.g. Calomiris, 2013). Quantifying this trade-off is crucial for the assessment of optimal capital requirements and requires a framework that captures well the behavior of the economy in normal times– including normal expansions and recessions – as well as the frequency and severity of financial recessions – i.e., episodes of simultaneously high levels of borrower and bank defaults (twin defaults).

This paper studies this important trade-off in a quantitative macro-banking model in which financial recessions are particularly severe contractions that endogenously arise in response to shocks to bank borrowers that are not fully diversifiable at the bank level. The main distinguishing feature of the model is to account for the special structure of bank asset risk (see Nagel and Purnanandam, 2020). Specifically, in our model banks hold portfolios of risky loans whose risk of default is not fully diversifiable at the bank level. As a result, bank solvency problems arise endogenously from high default rates among bank borrowers. Such defaults cause losses for banks, depleting their net worth and ultimately leading to bank undercapitalization and insolvency. Hence, credit supply contracts, amplifying the drop in firm investment and production associated with the initial impact of the shock. In addition, default entails bankruptcy costs, and twin default episodes impose huge deadweight economic losses.[1] Hence, for the same level of bank insolvencies, our model implies optimal capital requirements that are six percentage points higher than under specifications of bank asset returns, which overlook the impact of borrowers' default on banks' default.

As noted by Gornall and Strebulaev (2018) and Nagel and Purnanandam (2020) in a partial equilibrium setup, capturing bank default risk dynamics requires a structural model of bank asset returns. In our framework, bank assets are portfolios of loans subject to non-diversifiable default risk. As a result, bank asset returns exhibit limited upside potential but significant downside risk. Importantly, the asymmetry in these returns arises endogenously, and loan performance is the main driver of bank insolvencies. This aspect of the model is essential to reproduce the high and positive correlation between borrower and bank defaults and generate the frequency and severity of twin default crises observed in the data.

Existing macro-banking papers on the optimal level of capital requirements instead typically represent bank asset returns like in the standard default model of Merton (1974), which implies that banks earn equity-like returns with an unbounded upper tail. Some of the models fully abstract from the default of bank borrowers and assume that banks invest directly in productive capital (e.g. Van Den Heuvel, 2008; Begenau and Landvoigt, 2017; Begenau, 2020).[2] Others adopt a "double-decker" framework where banks explicitly provide defaultable loans to firms, but, for tractability, model bank default as the result of shocks that affect bank asset returns independently of the performance of individual loans (e.g., Clerc et al., 2015; Mendicino et al., 2018, 2020; Elenev, Landvoigt and Nieuwerburgh, 2020).[3] We show that, for this reason, standard Merton-type models of bank default underestimate the

[1] As in the financial accelerator literature (e.g. Bernanke and Gertler, 1989), the reliance on debt contracts and the deadweight bankruptcy costs could be justified as in the costly state verification model of Gale and Hellwig (1985).

[2] This approach is similar to the one adopted in seminal macro-banking models (e.g. Gertler and Kiyotaki, 2010; He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014) whose focus is neither on bank default risk nor the optimal level of capital requirements.

[3] Existing papers generally assume idiosyncratic shocks to bank revenues. A single risk factor specification (Vasicek, 2002) or aggregate shocks could also generate bank default risk. In the absence of idiosyncratic shocks to bank revenues, these approaches, however, would not produce heterogeneity in default outcomes across banks.

correlation between firm and bank defaults and the frequency of twin defaults observed in the data. Hence, in these frameworks, bank insolvencies are associated with remarkably lower deadweight losses and contractions in economic activity than in our model. This biases downward the net benefits of higher capital requirements and, thus, underestimates their optimal level.[4]

In our quantitative framework, banks extend loans to firms using insured deposits and equity and are subject to regulatory capital requirements. Firms produce the final good using capital and labor and pay for their production inputs partly using external financing in the form of bank loans. Both firms and banks operate under limited liability and can default on their debt obligations. As in Baron, Verner and Xiong (2021), bank equity declines are the key driver of bank solvency crises in our model.[5] Banks are exposed to default risk because firms' performance is affected by shocks that are not fully diversifiable at the bank level. Specifically, we assume that credit markets are segmented into islands: a bank can only grant loans to a continuum of firms on a given island.[6] Each firm on the island is exposed to both firm- and island-idiosyncratic productivity shocks. Banks can diversify away firm-idiosyncratic shocks by lending to all firms on the island. But island-idiosyncratic shocks affect all firms operating on the island in the same way and, hence, are not diversifiable at the bank level.[7] Thus, island risk generates heterogeneity in banks' asset returns and default outcomes.

[4]The comparison is based on a calibrated Merton-type variant of our model. Specifically, we assume that the default risk of banks comes from exogenous disturbances that directly hit banks' loan returns. Importantly, the average probability of bank default and its standard deviation are the same in both models.

[5]Using historical data, Baron, Verner and Xiong (2021) find bank equity losses to predict subsequent contractions in bank credit and aggregate economic activity. Their evidence clearly shows that while panics are an amplification mechanism, they are unnecessary for a banking crisis to have severe economic consequences. Our analysis, therefore, abstracts from the complications associated with the modeling of panics.

[6]The segmentation of the loan market into islands is a shortcut to specialization and, hence, ex-post heterogeneity in bank asset returns. Crucially, the segmentation does not apply to any other market, including the funding of banks.

[7]Our assumption on the exposure of banks to non-diversifiable risk is consistent with the evidence in Galaasen et al. (2020), which using matched bank-firm data for Norway shows that idiosyncratic borrower risk is an economically significant source of non-diversifiable risk affecting banks' loan portfolio returns.

The asset returns of individual banks depend on the island-idiosyncratic productivity shock in a highly non-linear manner. In islands with high realizations of this shock, a large fraction of borrowers repay the contractual amount. In islands with low realizations, more borrowers default, and banks make significant losses. Thus, asset returns of individual banks are characterized by limited upside risk but significant downside risk. While the firm-specific and island-specific productivity shocks affecting borrowers are assumed to be log-normally distributed, individual bank asset returns endogenously feature highly left-skewed and asymmetric returns.

In the quantitative part of the paper, we show that our macro-banking model of default risk can reproduce relevant features of the data, including the positive correlation between bank and firm defaults and the frequency and severity of twin default episodes. To generate aggregate fluctuations in macroeconomic and financial variables, the model includes aggregate shocks: total factor productivity (TFP) shocks, as well as firm- and island-risk shocks. Firm- and island-risk shocks affect the variance of the idiosyncratic productivity shocks to firms and islands, respectively, and resemble the risk and uncertainty shocks commonly used in the literature (see Bloom, 2009; Christiano, Motto and Rostagno, 2014). In our model, they are crucial to generate fluctuations in firm and bank defaults.[8]

We estimate the model parameters using the generalized method of moments, targeting a large set of unconditional moments in macro, banking, and financial euro area (EA) data over the period 1992-2016. To capture the non-linearity intrinsic in the returns on bank loans in a tractable way, we use a higher-order perturbation solution method. Our model matches well the targeted mean and standard deviation of firm and bank defaults, as well as the positive correlation that these rates exhibit in the data. In contrast, as mentioned above, the standard Merton-type model of bank default risk commonly used in the literature underestimates the correlation between the default rates of banks and their borrowers.

[8]Our results are also consistent with Alfaro, Bloom and Lin (2023), who find that financial frictions amplify the effects of uncertainty shocks.

We also validate the performance of the model in terms of empirical moments describing the relationship between firm and bank defaults and GDP growth not targeted in the estimation. In the data, the overall positive correlation between the two default rates hides substantial non-linearity in their co-movement. Quantile regressions clearly show that the sensitivity of bank default to firm default is higher in the upper quantiles of bank default. Once bank default risk is very high, its sensitivity to an increase in borrowers' default is higher than in good times. In addition, there is a strong negative link between GDP growth and bank default at lower quantiles of GDP growth, consistent with the importance of financing conditions as a determinant of the economy's downside risk (Adrian, Boyarchenko and Giannone, 2019). Contrary to the Merton-type approach, our model can mimic these non-linearities well thanks to the non-linear structure of bank asset returns, which enables it to reproduce the frequency and severity of the twin default episodes and the associated macroeconomic outcomes.

In addition to helping match the data, the structural link between the solvency of firms and banks constitutes a powerful amplification mechanism that allows the model to generate twin default episodes without the need for large exogenous aggregate shocks to banks. In fact, these episodes are the result of sequences of small negative island-risk shocks that become increasingly amplified as the probability of bank failure grows. Intuitively, the non-linearity in bank asset returns implies that once banks have a high risk of failure, the marginal impact of additional credit losses on banks' solvency and the macroeconomy is much larger than in normal times.

After validating the quantitative implications of the model, we turn to the assessment of the optimal level of capital requirements as well as their optimal degree of dynamic adjustment in response to credit growth. The rationale for bank capital requirements in our setup stems from the presence of safety net guarantees for banks and aggregate externalities derived from the deadweight losses caused by defaults.[9] Banks' outside funding comes from

[9]See Kareken and Wallace (1978) for an early reference.

insured deposits which pay an interest rate that is independent of banks' leverage choices. This gives banks with limited liability an incentive to under-price borrower risk, as they do not internalize the effects of their individual choices on the social costs of their failures. In addition, they also neglect their impact on the aggregate dynamics of bank equity, which is key to determining the lending capacity of the whole banking sector and, hence, the dynamics of the real economy. Thus, the model combines conventional micro- and macro-prudential rationales for regulatory capital requirements.

Higher bank capital requirements limit bank risk-taking incentives and make the banking sector more resilient to credit losses. This reduces the probability of twin defaults and, hence, the negative impact of high firm and bank defaults on welfare. However, higher capital requirements also imply a higher average cost of funding for banks, which translates into higher average borrowing costs for firms and lower average equilibrium credit levels. Assessing the optimal level of the capital requirements that maximizes social welfare requires quantifying this trade-off.

In our estimated model, a sixteen percent bank capital requirement brings the probability of twin defaults close to zero and maximizes social welfare. This is about six percentage points higher than the optimal level of capital requirements implied by the Merton-type model of bank default risk, which underestimates the probability of twin defaults. While in the Merton-type model, firm default is not the primary driver of bank default, in our framework, bank insolvencies are endogenously driven by high levels of defaults among banks' borrowers. Hence, bank default events are significantly more severe in our model compared to the Merton-type framework. For the same level of bank insolvencies, our model predicts higher costs for society as the economy experiences deadweight default losses and equity declines not only for banks but also for firms. This result underscores the importance of modeling bank default risk in a structural way. Failing to generate the right frequency and severity of twin defaults understates the costs associated with bank default and, hence, biases downwards the net benefits of higher capital requirements.

Finally, we analyze the role of dynamic capital adjustments. We show that allowing capital requirements to be higher in good times than in bad times stabilizes loan provision in bad times at the expense of reducing banks' resilience. Therefore, a strong degree of adjustment to credit growth is only optimal for a sufficiently high level of capital requirements so that banks remain highly solvent even when their borrowers' default risk is high and the requirements are lowered. [10]

Related literature This paper contributes to several strands of the macro-finance literature. First, from a modeling perspective, we contribute to the macro-banking literature by showing how shocks to bank borrowers can endogenously trigger financial recessions. In an important departure from the standard financial accelerator literature (e.g., Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997; Jermann and Quadrini, 2012), we assume that banks are not a veil and are subject to borrower default risk. We share with earlier papers the assumption that the returns on the firms' productive projects are log-normally distributed, as in the classical Merton model of corporate default (Merton, 1974). But, in line with Gornall and Strebulaev (2018) and Nagel and Purnanandam (2020), the returns on the portfolio of defaultable loans feature limited upside but unlimited downside risk. This appears endogenously in our model due to the incidence of non-diversifable borrower default risk on bank asset returns. Sufficiently high borrower defaults in our model can trigger high bank default rates and lead to financial recessions. This structural modeling of bank default risk is crucial for replicating the correlation between firm and bank default rates and the large economic contraction associated with financial recessions.[11]. This natural but non-trivial extension of the standard framework distinguishes our model from those in which banks directly hold productive assets (e.g. Gertler and Kiyotaki, 2010; He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014; Piazzesi, Rogers and Schneider, 2019) as well as

[10]We thank one referee for suggesting this interesting addition to the paper.

[11]Our findings are consistent with the emphasis on banks as amplifiers of crises in Giesecke et al. (2014). We also find that financial recessions have large macroeconomic effects, as in Krishnamurthy and Miur (2017)

from standard double-decker models of bank default risk.[12]

The tractability of the Merton-type approach to bank default risk is useful when solving large models which include, for instance, different types of intermediaries (e.g., Begenau and Landvoigt, 2017) or loans (e.g., Mendicino et al., 2018), long-term debt (e.g., Jermann, 2019; Elenev, Landvoigt and Nieuwerburgh, 2020), liquidity interventions (e.g., Gete and Melkadze, 2020) and monetary policy (e.g., Mendicino et al., 2020).[13] However, our structural approach is better suited to understanding the importance of credit losses for bank insolvencies and, hence, for financial recessions, which, differently from previous work, arise without the need to introduce exogenous shocks to bank asset returns that are independent of the performance of bank borrowers.[14]

Second, from a normative perspective, our findings challenge the common result in the existing macro-finance literature (e.g. Van Den Heuvel, 2008; Clerc et al., 2015; Begenau, 2020; Corbae and D'Erasmo, 2019; Davydiuk, 2019; Mendicino et al., 2018, 2020; Elenev, Landvoigt and Nieuwerburgh, 2020) that the optimal level of capital requirements is only few percentage points different from the baseline levels in place before the 2008-2009 financial crisis.[15] By properly capturing the simultaneously high levels of bank and borrower defaults in financial recessions and properly accounting for the large deadweight losses associated with bank insolvencies, our framework prescribes substantially higher capital requirements. In addition, for sufficiently high levels of capital requirements, it also rationalizes the optimality

[12] Gete (2018), Rampini and Viswanathan (2019), Ferrante (2019) and Villacorta (2020), among others, develop double-decker models of banks' and borrowers' net-worth which, however, abstract from bank default.

[13] In these models, banks are only exposed to aggregate risk and the ex-post heterogeneity in bank asset returns arises from shocks that affect directly the aggregate returns on the loan portfolio of the bank and not the performance of the individual loan/borrower. We share with this earlier literature the focus on banking crises without panics (Baron, Verner and Xiong (2021)).

[14] Another important contribution relative to our previous work (Mendicino et al., 2018, 2020) concerns the modeling of endogenous voluntary buffers and dividend payouts, which, in the current framework, play a role in the transmission of shocks.

[15] Regardless of the differences in the underlying frictions, the majority of papers in this strand of the literature suggests gains from higher capital requirements. An exception is Elenev, Landvoigt and Nieuwerburgh (2020), whose results point to an optimal level of capital requirements one percentage point below the baseline level.

of dynamic capital requirements.

Third, our structural approach to bank asset risk also contributes to the understanding of how financial vulnerabilities lead to downside risks to GDP. Consistent with recent evidence on the link between financial vulnerabilities and downside risks to GDP (e.g. Adrian, Boyarchenko and Giannone, 2019), we show that bank default risk is a strong determinant of the economy's downside risk. In our model, when the risk of bank insolvencies is high, small shocks to banks' non-diversifiable risk have a magnified negative impact on aggregate macroeconomic outcomes.

Finally, our focus on the non-linearities due to the special structure of bank asset risk and its impact on bank default risk adds a complementary perspective to the literature that emphasizes other non-linear aspects of financial crises. Aspects analyzed by prior work include asset price feedback loops (He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014), occasionally binding constraints (Mendoza, 2010; Benigno et al., 2013; Bianchi, 2016), bank panics (Gertler, Kiyotaki and Prestipino, 2019), liquidity problems (Bigio, 2015; De Fiore, Hoerova and Uhlig, 2018), systemic risk (Martinez-Miera and Suarez, 2014), time varying risk-premia (Coimbra and Rey, 2019) and sovereign defaults (Arellano, 2008; Bocola, 2016).

1. The Model

We consider a discrete-time, infinite horizon economy in which dates are indexed by t.

Household. The model economy is populated by a representative household that works, consumes, invests savings in bank deposits, and owns the representative corporate holding company (CHC) and the representative bank holding company (BHC). The CHC and BHC operate intertemporally, managing the equity investments (ownership stakes) of the household in firms and banks, respectively. Households also directly own the capital-producing firms that operate at each date.

Firms and banks. Final good-producing firms (denoted just firms, for brevity) and banks operate between two consecutive dates. Firms produce the final good and pay for the inputs of production in advance. Both firms and banks obtain external financing by issuing non-contingent debt in the form of bank loans and (fully insured) deposits, respectively.[16] They operate under limited liability and default when, after a period of operation, the owning CHC or BHC optimally decides not to cover the gap between their debt obligations and their terminal asset value if strictly positive. In case of default of a firm or bank, creditors take possession of their assets at a cost. After a period of operation, non-defaulted firms and banks pay their terminal net worth to the CHC and the BHC, respectively. Finally, at each date, the BHC pays lump-sum taxes to the deposit guarantee scheme (DGS) so as to cover the losses on the insured deposits of the banks that defaulted as a result of their previous period of operation.

Island setup. There exists a continuum of measure one of the islands that we index by $j \in (0, 1)$. In each island, there is a continuum of measure one of ex-ante identical firms that we index by $i \in (0, 1)$ and a representative bank. Firms are subject to both firm- and island-idiosyncratic shocks, whose realizations affect their terminal asset value. Banks cannot lend across islands, so they diversify their lending across firms in their island but not across islands. The island structure is a metaphor for bank specialization, which may be geographical (e.g. in a country or in a region of a country), sectoral (e.g. real estate) or even in lending to individual large firms. We do not model the reasons for specialization but merely capture the fact that it leads to imperfect diversification, exposing banks to risks

[16] The focus of our paper on bank lending to firms is consistent with the important role of EA banks in lending to non-financial corporations (NFCs) and the importance of NFC defaults as drivers of credit losses in Europe (EBA, 2018). Our model could be adapted to consider the case in which bank borrowers are households to finance house purchases with mortgages. However, such a setup would be less relevant in the EA since the recourse nature of most European mortgages makes the default rates of these loans very low, even in bad times.

that are idiosyncratic to the region and industry they operate in.[17] In our model, this is captured by the way bank asset returns depend on the realization of the island-idiosyncratic shock. Island-based market segmentation only applies to the bank loan market. All factors of production, the final output, and the deposit and equity funding of banks are freely mobile across islands.

Aggregate Shocks. In addition to the firm- and island-idiosyncratic shocks, the economy faces an aggregate TFP shock and aggregate firm- and island-risk shocks. These last two shocks are modeled as shocks to the standard deviation of each class of idiosyncratic shocks. Aggregate shocks are described in Internet Appendix A together with aggregation and market clearing conditions as well as the full set of equilibrium conditions.

1.1 The Household

The household chooses consumption, C_t, hours worked, H_t, and insured bank deposits, D_t, to maximize the present discounted value of utility

$$\mathbb{E}_t \sum_{s=t}^{\infty} \left(\beta^s \log\left(C_s\right) - \frac{\varphi}{1+\eta} H_s^{1+\eta} \right)$$

subject to the budget constraint

$$C_t + D_t = w_t H_t + R_{d,t-1} D_{t-1} + \Xi_{f,t} + \Xi_{b,t} + \Xi_{k,t} \tag{1}$$

where η is the inverse of the Frisch elasticity of labor supply, φ is the weight of labor supply in the utility of households, w_t is the real hourly wage and $R_{d,t-1}$ is the gross rate

[17] In Europe, banks operate largely within national borders, and many specialize in lending to particular industries and sectors (Guiso, Sapienza and Zingales, 2004; De Bonis, Pozzolo and Stacchini, 2011; Behr and Schmidt, 2016; De Jonghe et al., 2020).Geographic and sectoral specialization is also a feature of US small and medium-sized banks (Deyoung et al., 2015; Regehr and Sengupta, 2016) and banks in Peru (Paravisini, Rappoport and Schnabl, 2023).

of deposits. The last three terms in Equation (1) represent the net dividends paid by the CHC, $\Xi_{f,t}$, and the BCH, $\Xi_{b,t}$, and the profits of the capital producing firms, $\Xi_{k,t}$. We are interested in a symmetric equilibrium, hence, we assume that the household invests its deposits symmetrically in all the (ex-ante identical) banks in the economy. All the variables in the the problem of the household represents aggregate variables. First order conditions (FOCs) of this problem are in Internet Appendix A.

1.2 Firms

The representative CHC manages the equity investments in all individual firms. Individual firms are ex-ante identical, although ex-post they are hit by the different firm- and island-idiosyncratic shocks denoted ω_i and ω_j. Across any two consecutive dates $t-1$ and t, the CHC solves a two-stage problem. In the first stage (in date $t-1$), it chooses how to invest its equity $N_{f,t-1}$ across its subsidiary firms and how much these firms should lever up using a bank loan. At this stage, the CHC also chooses the combination of labor h_{t-1} and physical capital k_{t-1} to be used as inputs in the production of output at t.[18] Thus, each firm finances its input cost, $w_{t-1}h_{t-1} + q_{t-1}k_{t-1}$, with the equity from CHC $N_{f,t-1}$ and a bank loan with principal $B_{f,t-1}$ and a promised gross interest rate $R_{f,t-1}$ agreed between the firm and its bank. We define firms' leverage as the ratio

$$\theta_{t-1} = \frac{N_{f,t-1}}{w_{t-1}h_{t-1} + q_{t-1}k_{t-1}}.$$

At date t shocks realize and the second stage of the CHC's decision problem begins: each portfolio firm obtains revenue $\omega_i\omega_j\left[y_t + (1-\delta)\, q_t k_{t-1}\right]$ from net production $y_t = A_t h_{t-1}^{\alpha} k_{t-1}^{1-\alpha}$, with $\alpha \in (0,1)$ and $A_t > 0$, and the sale of depreciated physical capital $(1-\delta)\, q_t k_{t-1}$. In the absence of financial assistance from the CHC, the individual firm is insolvent if its revenues

[18] Our formulation is equivalent to allowing firms to choose their leverage ratio and inputs of production without any agency friction between them and the CHC.

are lower than its loan repayment obligations $R_{f,t-1}B_{f,t-1}$. So firms in island j are solvent without assistance if

$$\omega_i \geq \hat{\omega}^j_{f,t} = \frac{R_{f,t-1}(1-\theta_{t-1})}{\omega_j R^k_t},$$

where

$$R^k_t = \frac{y_t + (1-\delta)\, q_t k_{t-1}}{w_{t-1}h_{t-1} + q_{t-1}k_{t-1}}.$$

Firms with $\omega_i < \hat{\omega}^j_{f,t}$ are in financial distress and can only avoid default if CHC arranges an emergency equity injection. The CHC can raise the extra equity from the household at a cost $1+\gamma$ per unit of net funds. On the contrary, letting the firm default implies a net worth loss of $\mu_F B_{f,t-1}$ to the CHC. This loss captures reputational costs, losses of goodwill, or any other organizational losses suffered by the CHC. In addition, default produces a repossession cost μ_f per unit of assets to the lending bank.[19]

Figure 1: Timeline of CHC decisions

Stage 1 (date $t-1$)	**Stage 2** (date t)
CHC starts with net worth $N_{f,t-1}$	Aggregate shocks and idiosyncratic shocks ω_i, ω_j realize
Firm-level input and financing decisions dated $t-1$ made $(h_{t-1}, k_{t-1}, B_{f,t-1}, R_{f,t-1}, \theta_{t-1})$	For distressed firms, CHC decides on letting them default ($d^{ij}_{f,t}=1$) or not ($d^{ij}_{f,t}=0$)
	Non-defaulting firms pay net worth back to CHC
	CHC decides gross dividend yield $x_{f,t}$

[19] The CHC fails to internalize these repossession costs when deciding on firms' default. These costs, however, affect the terms at which the bank grants the loans in the first place and, hence, firms' decisions in the first stage. Interpreting asset repossession costs as state-verification or enforcement costs, the contractual frictions in this formulation are consistent with those that rationalize the optimality of debt contracts in Gale and Hellwig (1985) and Krasa and Villamil (1992).

We assume that the cost γ of emergency equity is high enough for the CHC to only consider the minimum recapitalization necessary to avoid the default of the distressed firm, and we denote by $d^{ij}_{f,t}$ the CHC's binary decision, for firm i in island j, on whether to provide an equity injection to avoid default ($d^{ij}_{f,t} = 0$) or to let the firm go under ($d^{ij}_{f,t} = 1$).

In the second stage, the CHC also decides how much dividend to pay to the household, out of the gross equity returns received from all its non-defaulting portfolio firms. We assume that the CHC has an exogenous target $\overline{\delta}_f$ for its gross dividend yield, $x_{f,t}$, defined as a proportion of the equity invested in firms in the previous period $N_{f,t-1}$. Deviating from the target implies a penalty $\frac{\psi_f}{2}(\overline{\delta}_f - x_{f,t})^2 N_{f,t-1}$ that directly reduces the net dividends received by the household at date t but allows the CHC to optimally choose the net worth retained to invest in its portfolio firms for one more period.[20] Figure 1 represents the timeline for the decisions of the CHC regarding firms.

1.2.1 Firm second-stage decisions

We start from the second stage before moving on to the first stage of the CHC problem. The second stage problem (firm default and CHC dividend decisions) at time t can be represented by the following value function:

$$\begin{aligned} V_{f,t}(N_{f,t-1}) &= \max_{d^{ij}_{f,t}\in\{0,1\}, x_{f,t}} \Bigg(\Bigg(x_{f,t} - \frac{\psi_f}{2}(\overline{\delta}_f - x_{f,t})^2 \\ &- \frac{(1+\gamma)\int_0^\infty \int_0^{\hat{\omega}^j_{f,t}} (1-d^{i,j}_{f,t}) \left[R_{f,t-1}(1-\theta_{t-1}) - \omega_i \omega_j R^k_t \right] dF_{i,t}(\omega_i) dF_{j,t}(\omega_j)}{\theta_{t-1}} \Bigg) N_{f,t-1} \\ &+ \mathbb{E}_t(\Lambda_{t+1} V_{f,t+1}(N_{e,t})) \Bigg), \end{aligned} \tag{2}$$

[20] The cost of missing the target dividend can be interpreted as capturing in reduced form the informational and agency frictions that motivate the prevalence of target payout ratios and dividend smoothing among real-world firms.

where $F_{i,t}$ and $F_{i,t}$ are the cumulative distribution functions of the idiosyncratic firm and island shocks, respectively. Equation (2) states that the value of the CHC which had invested net worth of $N_{f,t-1}$ in the previous period is given by dividends net of the costs of missing the payout target and of the costs of recapitalizing any firms with negative residual net worth plus the continuation value of the CHC. CHC net worth invested in portfolio firms $N_{f,t}$ follows the law of motion:

$$\begin{aligned} N_{f,t} &= \left(\int_0^\infty \left\{ \int_{\hat{\omega}_{f,t}^j}^\infty \left[\omega_i \omega_j R_t^k - R_{f,t-1}(1-\theta_{t-1}) \right] dF_{i,t}(\omega_i) \right.\right. \\ & \left.\left. - \int_0^{\hat{\omega}_{f,t}^j} d_{f,t}^{i,j} \mu_F dF_{i,t}(\omega_i) \right\} dF_{j,t}(\omega_j) - x_{f,t} \right) \frac{N_{f,t-1}}{\theta_{t-1}} \end{aligned} \tag{3}$$

and Λ_t is the household's stochastic discount factor. $N_{f,t}$ is equal to the gross residual value of its portfolio firms net of the losses to the CHC from portfolio firm defaults and net of any dividend payments.

We guess and verify that the value function of the CHC is linear in equity invested in its portfolio firms at $t-1$, that is, $V_{f,t}\left(N_{f,t-1}\right) = v_{f,t} N_{f,t-1}$. Substituting this in Equation (2) produces the following functional equation for the value of a unit of equity $v_{f,t}$:

$$\begin{aligned} v_{f,t} &= \max_{d_{f,t}^{i,j} \in \{0,1\}, x_{f,t}} \left\{ x_{f,t} - \frac{\psi_f}{2} \left(\overline{\delta}_f - x_{f,t} \right)^2 \right. \\ &- \frac{1+\gamma}{\theta_{t-1}} \int_0^\infty \int_0^{\hat{\omega}_{f,t}^j} (1 - d_{f,t}^{i,j}) \left[R_{f,t-1}(1-\theta_{t-1}) - \omega_i \omega_j R_t^k \right] dF_{i,t}(\omega_i) dF_{j,t}(\omega_j) \\ &+ \mathbb{E}_t \left[\Lambda_{t+1} v_{f,t+1} \left(\frac{1}{\theta_{t-1}} \int_0^\infty \left\{ \int_{\hat{\omega}_{f,t}^j}^\infty \left[\omega_i \omega_j R_t^k - R_{f,t-1}(1-\theta_{t-1}) \right] dF_{i,t}(\omega_i) \right.\right.\right. \\ & \left.\left.\left.\left. - \int_0^{\hat{\omega}_{f,t}^j} d_{f,t}^{i,j} \mu_F dF_{i,t}(\omega_i) \right\} dF_{j,t}(\omega_j) - x_{f,t} \right) \right] \right\}. \end{aligned} \tag{4}$$

Throughout the analysis, we will assume that the cost of raising emergency equity exceeds the CHC's marginal cost of cutting down the dividend yield ratio, that is, $\gamma > \psi\left(\overline{\delta}_f - x_{f,t}\right)$, for relevant values of $x_{f,t}$.[21] This means that emergency equity is only optimally used at the minimal scale needed to avoid the default of some of the firms in distress but not to increase $N_{f,t}$.

Finally it is helpful to define $p_{f,t} \equiv \mathbb{E}_t(\Lambda_{t+1} v_{f,t+1})$ as the (expected discounted) continuation value of one unit of equity invested by the CHC in any of its portfolio firms at t. Together with μ_F, this shadow value is a key determinant of the franchise value that the CHC may preserve by recapitalizing firms in distress.

Firm default decisions. Avoiding the default of a distressed firm is optimal if and only if

$$p_{f,t}\mu_F \geq (1+\gamma)\left[R_{f,t-1}(1-\theta_{t-1}) - \omega_i \omega_j R_t^k\right],$$

that is when the shadow value of the losses that default would inflict on the CHC exceeds the cost of the equity injection required to restore solvency. This condition determines the threshold value of the idiosyncratic firms shock below (above), which a firm in island j defaults (is recapitalized):

$$\bar{\omega}_{f,t}^{j} = \frac{(1+\gamma)\, R_{f,t-1}(1-\theta_{t-1}) - p_{f,t}\mu_F}{(1+\gamma)\omega_j R_t^k}.$$

Thus, as long as $p_{f,t}\mu_F > 0$, firms have charter value and $\bar{\omega}_{f,t}^{j} < \hat{\omega}_{f,t}^{j}$. It is, therefore, optimal to prevent the default of some distressed firms using an emergency equity injection. The shadow value of equity is an important determinant of the recapitalization decision: when the equity of the CHC is more scarce ($p_{f,t}$ is high), more firms avoid default using these injections.[22]

[21] We numerically check that this is the case.

[22] In this formulation, the net worth of the CHC works like inside equity, while the emergency equity injection that avoids the default of distressed firms work like outside equity. Incentives to raise outside equity increase when inside equity is more scarce.

Firm dividend decision. The FOC with respect to the dividend yield $x_{f,t}$ implies the following dividend policy function $x_{f,t} = \overline{\delta}_f - \frac{p_{f,t}-1}{\psi_f}$, which implies $x_{f,t} < \overline{\delta}_f$ for $p_{f,t} > 1$.[23] In this formulation, deviating from the dividend yield target $\overline{\delta}_f$ has a cost that reduces the "net dividend" effectively received by the household. Taking additionally into account the costs to the household of the optimal firm recapitalizations, we can express the net payout from the CHC to the household as:

$$\Xi_{f,t} = \left\{ \overline{\delta}_f - \frac{1}{2\psi_f}(p_{f,t}^2 - 1) - \frac{1+\gamma}{\theta_{t-1}} \int_0^{\infty} \int_{\bar{\omega}_{f,t}^j}^{\hat{\omega}_{f,t}^j} \left[R_{f,t-1}(1-\theta_{t-1}) - \omega_i \omega_j R_t^k \right] dF_{i,t}(\omega_i) dF_{j,t}(\omega_j) \right\} N_{f,t-1}.$$

Intuitively, the negative term in $p_{f,t}^2$ accounts for the (quadratic) cost of optimally reducing the dividend yield below its target $\overline{\delta}_f$ when the shadow value of bank equity is higher than one. In fact, $\overline{\delta}_f - \frac{1}{2\psi_f}(p_{f,t}^2 - 1)$ can turn negative (implying an equity injection from the household to the CHC) if the the internally accumulated net worth of the CHC is sufficiently scarce (which, in equilibrium, can happen after sufficiently large corporate net worth losses at the aggregate level).

1.2.2 Firm first-stage decisions

In the first stage problem (at date $t-1$), the CHC decides how to allocate equity across firms. The linearity of Equation (4) implies indifference with respect to the allocation of equity across firms. Since we are interested in a symmetric equilibrium, we assume that the allocation is symmetric. In this stage, the CHC also decides on firms' leverage and on their capital and labor inputs. The optimal leverage and input decisions of the portfolio firms

[23] This will be the case at and around the steady state in our calibration. The restriction on the cost of emergency equity and the FOC for the optimality of $x_{f,t}$ requires having $p_{f,t} < 1 + \gamma$ a condition that holds under our calibration.

emerge from the solution to the following Bellman equation:

$$\begin{aligned} p_{f,t-1} &= \max_{\theta_{t-1},h_{t-1},k_{t-1},R_{f,t-1}} \mathbb{E}_{t-1}\{\Lambda_t[(\overline{\delta}_f + \frac{1}{2\psi_f}) - (\overline{\delta}_f + \frac{1}{\psi_f})p_{f,t} + \frac{1}{2\psi}p_{f,t}^2 \\ &- \frac{1+\gamma}{\theta_{t-1}} \int_0^\infty \int_{\bar{\omega}_{f,t}^j}^{\hat{\omega}_{f,t}^j} \left[R_{f,t-1}(1-\theta_{t-1}) - \omega_i\omega_j R_t^k\right] dF_{i,t}(\omega_i)dF_{j,t}(\omega_j) \\ &+ \frac{p_{f,t}}{\theta_{t-1}} \int_0^\infty \{\int_{\hat{\omega}_{f,t}^j}^\infty \left[\omega_i\omega_j R_t^k - R_{f,t-1}\left(1-\theta_{t-1}\right)\right] dF_{i,t}(\omega_i) - \mu_F F_{i,t}(\bar{\omega}_{f,t}^j)\} dF_{j,t}\left(\omega_j\right)]\}, \end{aligned} \tag{5}$$

subject to a constraint (called the bank's participation constraint) that describes how the representative bank extends credit to the firm modifies the loan rate $R_{f,t-1}$ depending on the leverage and input decisions made by the firm:

$$\mathbb{E}_{t-1}\left(\Lambda_t \Pi_{b,t}\right) = p_{b,t-1}\phi_{t-1}. \tag{6}$$

As specified in detail in Internet Appendix A, in the participation constraint of the bank, $\Pi_{b,t}$ is the value at t of the net equity payoffs that the household and BHC receive at date t for each unit of lending to firms at date $t-1$, $p_{b,t-1}$ is the shadow value of the BHC's net worth at date $t-1$, and ϕ_{t-1} is the capital ratio of the banks (that is, the fraction of each unit of bank lending financed with equity provided by the BHC). This equation describes the loan supply schedule of perfectly competitive banks in our model. It describes combinations of loan amounts, loan rates and other relevant decisions by firms (e.g. labor and capital inputs) under which banks break even. In other words, it describes loan contracts under which the expected discounted value of the net equity returns received by the bank owners at t per unit of lending at t-1 is just enough to compensate them for the opportunity cost $p_{b,t-1}$ of the fraction ϕ_{t-1} of equity funding devoted to each unit of lending.The FOCs of the CHC's first-stage decision problem are also in Internet Appendix A.[24]

[24] These FOCs define the optimal contracting terms between the firm and its bank as in the costly state verification formulation of Bernanke, Gertler and Gilchrist (1999).

The right-hand side of the Bellman equation follows from two steps. First, we use Equation (4) and the expression of the optimal dividend yield of the CHC to write a recursive expression for $v_{f,t}$, that is, the value at t of net worth resulting from the investment of one unit of corporate equity at $t-1$. Second, we plug the resulting expression in the definition of $p_{f,t-1} \equiv \mathbb{E}_{t-1}(\Lambda_t v_{f,t})$.

1.3 Banks

The representative BHC manages the equity investments of the households in individual banks located on all the islands. All individual banks in island j are ex-ante identical, although ex-post island-idiosyncratic shocks produce heterogeneity in the performance of their loan portfolios. Because we are interested in a symmetric equilibrium, we assume a representative bank per island. Similarly to the CHC, the BHC solves a two-stage problem. In the first stage (date $t-1$), the BHC chooses how to invest its equity $N_{b,t-1}$ across its portfolio banks and how much these banks should lever up using insured deposits d_{t-1} to make loans $b_{f,t-1} = N_{b,t-1} + d_{t-1}$ to the firms in their island.[25] This results in a "capital ratio" (the regulatory name for the equity to asset ratio) defined as

$$\phi_{t-1} = \frac{N_{b,t-1}}{N_{b,t-1} + d_{t-1}}. \tag{7}$$

Prudential capital regulation constrains banks to operate with a capital ratio no lower than a minimum capital requirement $\underline{\phi}$.

At date t shocks realize and the second stage of the BHC decision problem begins. As derived in the description of the firms' problem above, a firm in island j pays in full the loan received at $t-1$ if it experiences a firm-idiosyncratic shock no lower than $\overline{\omega}_t^j$. If the firm-idiosyncratic shock is lower, the firm defaults, and the bank recovers a fraction $1-\mu_f$

[25]This is equivalent to banks directly making these choices without any agency frictions between banks and the BHC.

Figure 2: Timeline of BHC decisions

Stage 1 (date $t-1$)	**Stage 2** (date t)
BHC starts with net worth $N_{b,t-1}$	Aggregate shocks and idiosyncratic shocks ω_i, ω_j realize
Bank-level funding and lending decisions dated $t-1$ made $(d_{t-1}, b_{f,t-1}, \phi_{t-1})$	For undercapitalized banks, BHC decides on letting them fail ($d^j_{b,t}=1$) or not ($d^j_{b,t}=0$)
	Non-failed banks pay net worth back to BHC
	BHC decides gross dividend yield $x_{b,t}$

of the firm's asset value, $y_t + (1-\delta)q_t k_{t-1}$, due to asset repossession costs. Hence, the gross return on assets of the bank in island j is:

$$\widetilde{R}_{f,t}(\omega_j) = \frac{(1-\mu_f)\omega_j[y_t + (1-\delta)q_t k_{t-1}]}{b_{f,t-1}} \int_0^{\overline{\omega}^j_t} \omega_i dF_{i,t}(\omega_i) + R_{f,t-1} \int_{\overline{\omega}^j_t}^{\infty} dF_{i,t}(\omega_i). \tag{8}$$

Thus the bank in island j obtains gross loan repayments $\widetilde{R}_{f,t}(\omega_j)b_{f,t-1}$ which are increasing in the realization of the island idiosyncratic shock ω_j at date t.

We assume that a bank supervisor regards each bank as ex-post well-capitalized if it complies with the minimum capital requirement on an ex post basis, that is, $\widetilde{R}_{f,t}(\omega_j)b_{f,t-1} - R_{d,t-1}d_{t-1} \geq \underline{\phi} b_{f,t-1}$,where $R_{d,t-1}d_{t-1}$ are the bank's deposit repayment obligations and, hence, the left-hand side represents the net worth with which the bank arrives at t. This condition is equivalent to having $\omega_j \geq \hat{\omega}_{b,t}$, where $\hat{\omega}_{b,t}$ solves:

$$\widetilde{R}_{f,t}(\hat{\omega}_{b,t}) - R_{d,t-1}(1-\phi_{t-1}) = \underline{\phi}. \tag{9}$$

A bank that is not well-capitalized ($\omega_j < \hat{\omega}_{b,t}$) can only avoid its failure by covering its capital shortfall with an emergency equity injection. As in the case of the CHC, the BHC can arrange the required injection from the household at a cost $1+\gamma$ per unit of raised net funds. If, alternatively, the bank is allowed to fail, the BHC suffers (i) the loss of any residual net worth of the failed bank (which is appropriated by the DGS in the process of liquidation of the bank) and (ii) an additional loss equal to $\mu_B b_{f,t-1}$ which captures all failure-related costs incurred by the BHC. These may include reputation-restoration costs, litigation losses, the re-acquisition of informational capital lost with the failure of the portfolio bank, or the cost of replacing the failed bank. These losses effectively work as the charter (or continuation) value of the potentially failing bank that the BCH can be preserved with the emergency recapitalization of the bank.

Thus, in the second stage of its problem, the BHC must decide which realizations of $\omega_j < \hat{\omega}_{b,t}$ its undercapitalized banks should avoid failure using an emergency equity injection, $d^j_{b,t} = 0$, and for which one's banks should be allowed to fail, $d^j_{b,t} = 1$. In this stage, after receiving all the gross equity returns from its non-failed banks, the BHC also decides how much dividend to pay to the household. As in the case of the CHC, we assume that the BHC has an exogenous target $\overline{\delta}_b$ for its gross dividend yield $x_{b,t}$ defined as proportion of the equity invested in banks in the previous period $N_{b,t-1}$. Deviating from the target implies a penalty $\frac{\psi}{2}(\overline{\delta} - x_{b,t})^2 N_{b,t-1}$ that directly reduces the net dividends paid to the household at date t but allows the BHC to optimally choose the net worth retained to invest in its portfolio banks for one more period. Figure 2 represents the timeline for the decisions regarding banks.

1.3.1 Bank second-stage decisions

We start from the second stage of the BHC problem before moving to the first stage. The second stage problem (bak failure and BCH dividend choice) decisions at time t can be represented by the following value function:

$$V_{b,t}(N_{b,t-1}) = \max_{d^j_{b,t}\in\{0,1\},x_{b,t}} \{[x_{b,t} - \frac{\psi_b}{2}(\overline{\delta}_b - x_{b,t})^2]N_{b,t-1} \tag{10}$$
$$- \frac{1+\gamma}{\phi_{t-1}}\left\{\int_0^{\hat{\omega}_{b,t}}(1-d^j_{b,t})[\underline{\phi} + R_{d,t-1}(1-\phi_{t-1}) - \widetilde{R}_{f,t}(\omega_j)]dF_{j,t+1}(\omega_j)\right\}N_{b,t-1}$$
$$+ \mathbb{E}_t(\Lambda_{t+1}V_{b,t+1}(N_{b,t}))\},$$

where $N_{b,t}$ follows the law of motion

$$N_{b,t} = \left\{\frac{\int_{\hat{\omega}_{b,t}}^{\infty}[\widetilde{R}_{f,t}(\omega_j) - R_{d,t-1}(1-\phi_{t-1})]dF_{j,t}(\omega_j) + \int_0^{\hat{\omega}_{b,t}}[(1-d^j_{b,t})\underline{\phi} - d^j_{b,t}\mu_B]dF_{j,t}(\omega_j)}{\phi_{t-1}} - x_{b,t}\right\}N_{b,t-1}, \tag{11}$$

Equation (10) states that the value of the BHC which had invested net worth of $N_{b,t-1}$ in the previous period is given by dividends net of the costs of missing the payout target and of the costs of recapitalizing any undercapitalized banks plus the continuation value of the BHC with invested net worth $N_{b,t}$. Net worth invested in portfolio banks $N_{b,t}$ is equal to the gross profits of its portfolio banks plus the net worth of recapitalized banks net of bank default costs and dividend payouts, see Equation (11).

We guess and verify that the BHC's value function is linear in $N_{b,t-1}$, that is, $V_{b,t}(N_{b,t-1}) = v_{b,t}N_{b,t-1}$. Substituting this in Equation (10) produces the following functional equation for the per-unit value of equity:

$$v_{b,t} = \max_{d^j_{b,t}\in\{0,1\},x_{b,t}} \{x_{b,t} - \frac{\psi_b}{2}(\overline{\delta}_b - x_{b,t})^2 - \frac{1+\gamma}{\phi_{t-1}}\int_0^{\hat{\omega}_{b,t}}(1-d^j_{b,t})[\underline{\phi} + R_{d,t-1}(1-\phi_{t-1})$$
$$-\widetilde{R}_{f,t}(\omega_j)]dF_{j,t+1}(\omega_j) + \mathbb{E}_t[\Lambda_{t+1}v_{b,t+1}(\frac{1}{\phi_{t-1}}\{\int_0^{\hat{\omega}_{b,t}}[(1-d^j_{b,t})\underline{\phi} - d^j_{b,t}\mu_B]dF_{j,t}(\omega_j)$$
$$+\int_{\hat{\omega}_{b,t}}^{\infty}[\widetilde{R}_{f,t}(\omega_j) - R_{d,t-1}(1-\phi_{t-1})]dF_{j,t}(\omega_j)\} - x_{b,t})]\}. \tag{12}$$

Throughout the analysis, we will assume that the cost of raising emergency equity exceeds the BHC's marginal cost of cutting down the dividend yield ratio, that is, $\gamma > \psi_b\left(\overline{\delta}_b - x_{b,t}\right)$, for relevant values of $x_{b,t}$.[26] This means that emergency equity is only optimally used at the minimal scale needed to avoid the failure of some portfolio banks but not to increase $N_{b,t}$.

Finally it is helpful to define $p_{b,t} \equiv \mathbb{E}_t(\Lambda_{t+1} v_{b,t+1})$ as the (expected discounted) continuation value of one unit of the equity invested by the BHC in its portfolio banks at t. Together with μ_B, this shadow value is a key determinant of the charter value that the BHC aims to protect when recapitalizing its undercapitalized portfolio banks. $p_{b,t}$ also appears as the shadow value of bank equity in the participation constraint of the bank that determines the pricing of bank loans in firms' problem; see Equation (6).

Bank recapitalization decision. Avoiding the failure of an under-capitalized bank ($d^j_{b,t} = 0$) is optimal if and only if $p_{b,t}(\mu_B + \underline{\phi}) \geq (1+\gamma)\{\underline{\phi} - [\widetilde{R}_{f,t}(\omega_j) - (1-\phi_{t-1})R_{d,t-1}]\}$ where the left-hand side is the value of the BHC's net worth that is preserved by avoiding the failure of the undercapitalized bank and the right hand side is the cost of the emergency equity injection needed to avoid such a failure.

The above condition determines a threshold $\bar{\omega}_{b,t}$ for the island idiosyncratic shock below (above), which a bank is allowed to fail (recapitalized) by the BHC. The threshold solves

$$\widetilde{R}_{f,t}(\bar{\omega}_{b,t}) - (1-\phi_{t-1})R_{d,t-1} = \underline{\phi} - \frac{p_{b,t}(\underline{\phi} + \mu_B)}{1+\gamma}. \tag{13}$$

and depends negatively on $p_{b,t}(\underline{\phi} + \mu_B)$, which works as the charter value (per unit of loans) of the undercapitalized bank. This charter value is increasing in the shadow value of bank capital $p_{b,t}$, meaning that when a bank's net worth is scarcer and, hence, more valuable, it is profitable to save even more deeply undercapitalized banks.

[26] We numerically check that this is the case.

Bank dividend decision. The FOC with respect to the dividend yield ratio $x_{b,t}$ leads to the dividend policy function $x_{b,t} = \overline{\delta}_b - \frac{p_{b,t}-1}{\psi_b}$, which implies $x_{b,t} < \overline{\delta}_b$ for $p_{b,t} > 1$.[27] In this formulation, deviating from the dividend yield ratio target $\overline{\delta}_b$ has a cost that reduces the "net dividend" effectively received by the household. Taking additionally into account the costs of the optimal emergency equity injections, we can express the optimal net payout from banks to the household as

$$\Xi_{b,t} = \left\{ \overline{\delta}_b - \frac{1}{2\psi_b}(p_{b,t}^2 - 1) - \frac{1+\gamma}{\phi_{t-1}} \int_{\bar{\omega}_{b,t}}^{\hat{\omega}_{b,t}} [\underline{\phi} + R_{d,t-1}(1-\phi_{t-1}) - \widetilde{R}_{f,t}(\omega_j)] dF_{j,t}(\omega_j) \right\} N_{b,t-1}.$$

Intuitively, the negative term in $p_{b,t}^2$ accounts for the (quadratic) cost of optimally reducing the dividend yield below its target $\overline{\delta}_b$ when the shadow value of bank equity is higher than one. In fact, $\overline{\delta}_b - \frac{1}{2\psi_b}(p_{b,t}^2 - 1)$ can turn negative (implying an equity injection from the household to the BHC) if bank equity is sufficiently scarce (which can occur after sufficiently large bank net worth losses at the aggregate level).

1.3.2 Bank first-stage decisions

In the first stage problem (at date $t-1$), the BHC decides how to allocate equity across banks and also on how leveraged the banks should be that is, their capital ratio ϕ_{t-1}. The linearity of Equation (10) implies indifference with respect to the allocation of equity across banks. Since we are interested in a symmetric equilibrium, we assume that the allocation is symmetric. The optimal leverage of the portfolio banks emerges from the solution to the

[27] This will be the case at and around the steady state in our calibration. The restriction on the cost of emergency equity, and the FOC requires having $p_{e,t} < 1+\gamma$ a condition that holds our calibration.

following Bellman equation:

$$
\begin{aligned}
p_{b,t-1} \quad = \quad & \max_{\phi_{t-1}\geq\underline{\phi}} \mathbb{E}_{t-1}[(\Lambda_t(\overline{\delta}_b + \frac{1}{2\psi_b}) - (\overline{\delta}_b + \frac{1}{\psi_b})p_{b,t} + \frac{1}{2\psi_b}p_{b,t}^2 \\
- \quad & \frac{1+\gamma}{\phi_{t-1}} \int_{\bar{\omega}_{b,t}}^{\hat{\omega}_{b,t}} \{\underline{\phi} - [\widetilde{R}_{f,t}(\omega_j) - R_{d,t-1}(1-\phi_{t-1})]\} dF_{j,t}(\omega_j) \\
+ \quad & \frac{p_{b,t}}{\phi_{t-1}} \{ \int_{\hat{\omega}_{b,t}}^{\infty} [\widetilde{R}_{f,t}(\omega_j) - R_{d,t-1}(1-\phi_{t-1})] dF_{j,t}(\omega_j) + \underline{\phi}\, [F_{j,t}(\hat{\omega}_{b,t}) - F_{j,t}(\bar{\omega}_{b,t})] \\
& -\mu_B F_{j,t}(\bar{\omega}_{b,t})\})].
\end{aligned} \tag{14}
$$

The right-hand side of the Bellman equation follows from two steps. First, we use Equation (12) and the expression of the optimal dividend to write a recursive expression for $v_{b,t}$. Second, we plug the resulting expression in the definition of $p_{b,t-1} \equiv \mathbb{E}_{t-1}(\Lambda_t v_{b,t})$. The FOCs of this problem are in Internet Appendix A.

1.4 Capital Production

At each date t, capital producers combine the final good, I_t, with the last period capital goods, K_{t-1}, in order to produce new capital goods that competitively sell to firms at price q_t. Capital producers face adjustment costs, $S\left(\frac{I_{k,t}}{K_{t-1}}\right)$, as in Jermann (1998).[28] The law of motion of the capital stock can be written as

$$K_t = (1-\delta)\,K_{t-1} + S\left(\frac{I_t}{K_{t-1}}\right)K_{t-1}. \tag{15}$$

where δ is the capital depreciation rate. The FOC for the maximization of the profits from capital production implies:

$$q_t = \left(S'\left(\frac{I_t}{K_{t-1}}\right)\right)^{-1}. \tag{16}$$

[28] We adopt the form $S\left(\frac{I_{k,t}}{K_{t-1}}\right) = \frac{a_{k,1}}{1-\frac{1}{\psi_k}}\left(\frac{I_t}{K_{t-1}}\right)^{1-\frac{1}{\psi_k}} + a_{k,2}$, where $a_{k,1}$ and $a_{k,2}$ are chosen to guarantee that in the steady state the investment-to-capital equals the depreciation rate and $S'\left(I_t/K_{t-1}\right)$ equals one.

1.5 Deposit Guarantee Scheme

The deposit guarantee scheme (DGS) guarantees bank deposits in full and ex-post balances its budget in each period by charging a lump-sum tax to the BHC. The lump sum tax T_t covers the difference between the gross deposit repayments that the DGS has to honor in failing banks and the repossession value of the loan portfolio of those banks:

$$T_t = \left(F_{j,t}\left(\bar{\omega}_{j,t}\right) R_{d,t-1} - \frac{1-\mu_b}{1-\phi_{t-1}} \int_0^{\bar{\omega}_{b,t}} \tilde{R}_{f,t}\left(\omega_j\right) dF_{j,t}\left(\omega_j\right) \right) d_{t-1}, \tag{17}$$

where μ_b is the cost of repossessing bank assets.This expression uses the fact that equilibrium bank lending at $t-1$ can be written as $b_{f,t-1} = d_{t-1}/\left(1-\phi_{t-1}\right)$.

1.6 Aggregate Shocks

We assume the following AR(1) law of motion for the TFP shock

$$log(A_{t+1}) = \rho_A log(A_t) + \sigma_A \epsilon_{A,t+1}, \tag{18}$$

where $\epsilon_{A,t+1}$ is normally distributed with mean zero and variance one.

The standard deviation of the distribution of each idiosyncratic shock is time-varying and evolves as an AR(1) process

$$\log\left(\frac{\sigma_{\omega_\vartheta,t+1}}{\bar{\sigma}_{\omega_\vartheta}}\right) = \rho_{\sigma_i} \log\left(\frac{\sigma_{\omega_\vartheta,t}}{\bar{\sigma}_{\omega_\vartheta}}\right) + \sigma_\vartheta \epsilon_{\omega_\vartheta,t+1} \tag{19}$$

for $\vartheta = i, j$, where $\epsilon_{\omega_\vartheta,t+1}$ is normally distributed with mean zero and variance one.[29] Shocks to the variance of Firm- and Island-priductivity shocks are common across firms and islands. These shocks resemble the risk and uncertainty shocks commonly used in the literature (Bloom, 2009; Christiano, Motto and Rostagno, 2014). We will refer to them as firm- and

[29] This specification is similar to the one adopted in Christiano, Motto and Rostagno (2014).

island-risk shocks. In the next sections we will show that these shocks are important source of aggregate risk in the model and will be vital to generate fluctuations in firm and bank defaults.

1.7 Aggregation, Market Clearing, and Equilibrium

Aggregate Shocks, model aggregation, and market clearing conditions, as well as the exhaustive list of equilibrium conditions of the model, are reported in Internet Appendix A.

2. Solution, Estimation and Model Validation

We now present the solution of the model, the estimation, and the validation results.

2.1 Solving the Model

We solve the system of stochastic difference equations implied by the equilibrium conditions using a pruned state-space system for the third-order approximation around the steady state as defined in Andreasen, Fernandez-Villaverde and Rubio-Ramirez (2017). This approach eliminates explosive sample paths and greatly facilitates inference. In particular, it ensures the existence of unconditional moments. This enables us to estimate the parameters of the model by applying the generalized method of moments (GMM).

In order to use perturbation methods to approximate the solution to the model, we need to compute the expected aggregate loan returns that banks generate conditional on not defaulting, defined here as $R_{p,t+1}$. Namely:

$$R_{p,t+1} \equiv \int_{\bar{\omega}_{j,t+1}}^{\infty} \tilde{R}_{f,t+1}\left(\omega_j\right) dF_{j,t+1}\left(\omega_j\right). \tag{20}$$

As already mentioned in the previous section, the bank's loan return $\tilde{R}_{f,t+1}\left(\omega_j\right)$ is not log-normally distributed because ω_j enters non-linearly in its definition. This introduces a complication: the integral in Equation (20), as well as its derivatives, cannot be written as an explicit function of the state variables. We overcome this challenge by (i) splitting this integral into the sum of integrals taken over smaller intervals and (ii) computing a series of quadratic Taylor approximations of $\tilde{R}_{f,t+1}(\omega_j)$ around a mid-point of each interval. Because the powers of log-normally distributed variables are themselves log-normally distributed, the quadratic approximation to the bank profit function is itself approximately log-normally distributed and the expected profits as well as its derivatives can be computed as explicit functions of the state variables.[30] This approach is tractable and highly accurate. More details are provided in Internet Appendix B.

2.2 Model Estimation

The estimation of the model follows a two-step procedure. First, prior to the estimation procedure, some parameters are set to commonly used values in the literature. Second, we estimate the rest of the parameters using quarterly euro area (EA) data between 1992:Q1 and 2016:Q4. See Internet Appendix C for further details on data details.

First Step. Since we use quarterly data, the discount factor of the households, β, is set to 0.995, the Frisch elasticity of labor supply, η, to one, the value of capital depreciation, δ, to 0.025, and the capital-share parameter of the production function, α, to 0.30.[31] The bankruptcy parameters μ_f and μ_b are all set equal to 0.30, in line with empirical studies (e.g. Alderson and Betker, 1995; Djankov et al., 2008; Granja, Matvos and Seru, 2017).[32]

[30]The state variables of the model are $\mathbf{w_t} = \left(D_t, K_t, H_t, N_{e,t}, N_{b,t}, q_t, w_t, R_{f,t}, R_{d,t}, A_{t-1}, \sigma_{\omega_j,t-1}, \sigma_{\omega_i,t-1}\right)$.

[31]The discount factor β of 0.995 implies the steady-state level of the deposit rate of 2%. This value was chosen to match the average level of the 3-month Euribor rate in the Euro Area.

[32]Similar values of firm default costs are used, among others, in Carlstrom and Fuerst (1997), who refer to the evidence in Alderson and Betker (1995), where estimated liquidation costs are as high as 36 percent of asset value. Among non-listed bank-dependent firms, these costs can be expected to be larger than those among the highly levered, publicly traded US corporations studied in Andrade and Kaplan (1998), where estimated financial distress costs fall in the range of 10 percent to 23 percent. Our choice of 30 percent is consistent with the large foreclosure, reorganization, and liquidation costs found in some of the countries

Table 1: Estimated Parameters

Par.	Description	Value	Par.	Description	Value
$\bar{\sigma}_{\omega_i}$	Mean firm-risk shock	0.36	$\bar{\sigma}_{\omega_j}$	Mean island-risk shock	0.37
$\bar{\delta}_f$	Firm dividend target	0.39	$\bar{\delta}_b$	Bank dividend target	0.025
ψ_f	Firm equity adj. cost	0.15	ψ_b	Bank equity adj. cost	7.50
μ_F	Cost of firm distress	0.01	μ_B	Cost of bank undercapitalization	0.04
ψ_k	Capital adjustment cost	2.00	γ	Cost of emergency eq. issuance	0.60
σ_A	Std TFP shock	0.004	ρ_A	Persistence TFP shock	0.98
σ_i	Std firm-risk shock	0.07	ρ_{σ_i}	Persistence firm-risk shock	0.60
σ_j	Std island-risk shock	0.08	ρ_{σ_j}	Persistence island-risk shock	0.41

Notes: The reader should note that σ_i is not the standard deviation of firm-risk shock, which is $\frac{\sigma_i}{\sqrt{1-\rho^2_{\sigma_i}}}$ The same applies for the standard deviation of the island-risk shock.

The capital requirement level, $\underline{\phi}$, is set to be 0.08, which was the regulatory minimum in the Basel II regime. Finally, the labor utility parameter, φ, which only affects the scale of the economy, is normalized to one.

Second Step. We estimate the parameters summarized in Table 1 by targeting a number of macroeconomic, financial and banking moments. We target the standard deviations of GDP, investment and consumption growth, the mean ratio of corporate loans to GDP (B_t/GDP_t in the model) along with the standard deviation of loan growth, the mean and standard deviation of the loan spread ($R_{f,t} - R_t$ in the model).[33] Additionally, we also target the mean and standard deviation of net payout ratio for both banks and firms, the mean total capital ratio of banks, and the share of undercapitalized banks and distressed firms.[34] Finally, we match the mean and standard deviation of the conditional expectation of firm and bank default rates and the unconditional correlation between

analyzed by Djankov et al. (2008). Our choice is also in line with Granja, Matvos and Seru (2017), who find that the average FDIC loss from selling a failed bank is 28% of assets.

[33]Bankruptcy costs in the model are reflected in output. Hence, we define GDP_t as $GDP_t = C_t + I_t$.

[34]Net payout ratio is defined as (stock repurchases + dividends - stock issuance + real asset growth) as a share of previous quarter book equity from the Euro Area Flow of Funds data. We consider a bank to be undercapitalized if its total capital ratio falls below the regulatory requirement of 8%. We compute the average total Firms in distress as the proportion of firms that were downgraded to a rating CCC or below.

the two default probabilities. The conditional expectation of firm defaults is defined as $DF_t = \mathbb{E}_t \left(\int_0^\infty \int_0^{\bar{\omega}_{t+1}(\omega_j)} dF_{i,t+1}(\omega_i)\, dF_{j,t+1}(\omega_j) \right)$, while the conditional expectation of bank defaults is $DB_t = \mathbb{E}_t \left(\int_0^{\bar{\omega}_{j,t+1}} dF_{j,t+1}(\omega_j) \right)$.

Table 2 shows that our model matches the data targets reasonably well, including the bank- and firm-level moments, along a number of important dimensions. First, the net payout ratio and its volatility for banks and firms implied by the model are close to their empirical counterparts. Second, the model can match jointly the banks' average expected default frequency (EDF) of about 0.69%, the endogenous bank capital buffer of around 4% above the regulatory capital minimum of 8%, and the average share of undercapitalized banks. The model also does a good job of matching the distribution of firm riskiness, including both the firm default and financial distress frequency. Finally, the model is able to reproduce the positive unconditional correlation between firm and bank default (0.64 in the data versus 0.62 in the model).[35] Matching this correlation turns out to be of first-order importance when drawing conclusions about optimal bank capital requirements.

2.3 Model Validation

As shown in Table 2, the model is able to match the unconditional moments related to defaults and macroeconomic variables targeted in the calibration. In this section, we perform model validation by comparing the model's implications for important untargeted conditional moments of firm and bank defaults and GDP growth. This is a relevant step since the assessment of the benefits and costs of higher capital requirements hinges upon the ability of the model to match key features of the data, including the frequency and severity of bank insolvency crises.

[35] The expected firm and bank default variables in the data are measured using the asset-weighted average of EDFs within one year provided by Moody's KMV for individual EA non-financial corporations and banks. The expected firm default variable captures EA banks' exposure to small and medium-sized enterprises (SMEs) and large firms. A similar correlation can be observed in US data.

Table 2: Targeted Moments: Baseline Model

Variable	Data	Model	Variable	Data	Model
STD GDP growth	0.69	0.78	STD Cons. growth	0.56	0.67
MEAN Loans/GDP	2.44	2.29	STD Loan growth	1.20	1.97
MEAN Loan spread	1.24	1.07	STD Loan spread	0.68	0.66
MEAN Firm default	2.64	2.44	STD Firm default	1.10	1.34
MEAN Bank default	0.66	0.69	STD Bank default	0.84	0.68
CORR (DF & DB)	0.64	0.62	STD Inv. growth	1.39	1.24
MEAN Share undercap banks	1.18	0.89	MEAN Share distress firms	0.25	0.09
MEAN Dividend Ratio Banks	0.91	0.71	STD Dividend Ratio Banks	0.78	0.53
MEAN Dividend Ratio Firms	1.16	1.58	STD Dividend Ratio Firms	0.69	0.51
MEAN Capital Ratio	12.01	12.47			

Notes: Interest rates, equity returns, default rates, and spreads are reported in annualized percentage points. The standard deviations (Std) of GDP growth, Investment (Inv), and Loan growth are reported in quarterly percentage points.

2.3.1 Defaults and economic performance in the data

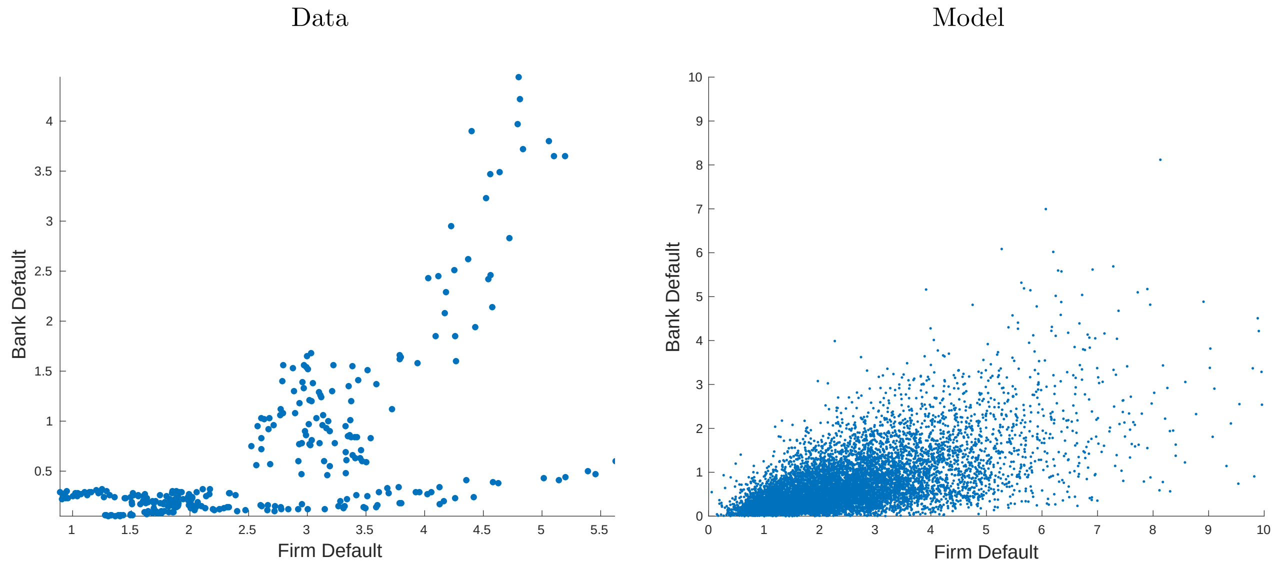
Firm and bank defaults are positively correlated, as successfully matched in the estimation. However, as Figure 3 reveals, the overall positive correlation between the two default rates hides substantial non-linearity in their co-movement. The figure displays a scatter plot of the average EDFs of firms and banks in the euro area (EA) over the period 1992-2016.[36]

Broadly speaking, one can identify three main regimes in the relationship between firm and bank default. In the most frequent regime, the default rates of both firms and banks are low. In another regime, the firm default rate is high, but the bank default rate is modest. The last regime is one in which the default rates of both firms and banks are elevated.[37] We deem the EDFs of firms and banks to be "high" when they are above their respective 90th percentile in the data.

[36]Each dot represents a monthly average of the corresponding probabilities of default over one year. The underlying EDFs are estimates provided by Moody's.

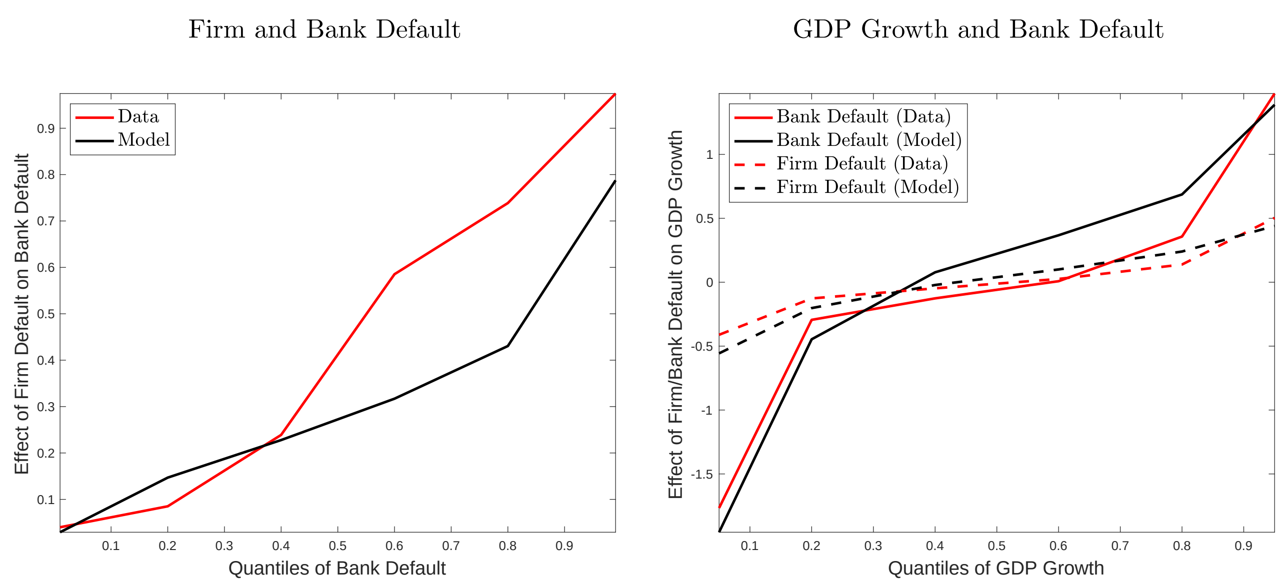
[37]The same pattern can be observed in other countries, including the US.

Figure 3: Firm and Bank Default



Left panel: Scatter plot of Moody's cross-sectional average of the EDFs within one year for the 1992:M1 to 2016:M12 (monthly frequency) sample of firms (non-financial corporations) and banks in the EA; series in percent. Right panel: scatter plot of firm and bank default produced with the baseline model.

Figure 4: Quantile Regression: Data vs Model



Notes: The left panel of this figure presents coefficients ζ_τ from the quantile regression in Equation (21). The right panel of this figure presents coefficients β_τ from the quantile regression in Equation (22). Both equations are estimated on EA data (1992-2016) and on simulated data from the baseline model.

Another way of representing the state-contingent relationship between firm and bank default risk is through quantile regressions of the following form:

$$\text{BankDef}_t(\tau) = \zeta_\tau \text{FirmDef}_t, \tag{21}$$

where FirmDef_t is firm EDF and BankDef_t is bank EDF.

The left panel of Figure 4 (red line) plots the quantile regression coefficients ζ_τ in Equation (21). The non-linearity in the relationship between the two defaults is clearly visible and highly statistically significant. At higher levels of bank default risk, the coefficient obtained by regressing bank on firm defaults is higher. The quantile regression coefficients indicate that the correlation between firm and bank default is state-dependent and increases with the bank default rate.[38]

Table 3: Average Quarterly GDP Growth

	High Firm Def.	Twin Defaults
Euro Area	-0.0466	-0.5842
Germany	-0.2550	-0.6690
France	-0.0718	-0.6605
Italy	-0.0242	-0.5471
Netherlands	-0.5043	-2.1904
Belgium	-0.3645	-0.4051
US	-0.0781	-0.9790

Notes: First column refers to periods of high firm defaults and low bank defaults, whereas the second column uses periods of twin defaults. GDP growth rates (demeaned) are reported in quarterly rates. Sample: EA 1992Q1-2016Q4, US: 1940:Q1-2016:Q4.

Next, we explore the relationship between aggregate economic activity and firm and bank defaults, respectively. A simple way to analyze this relationship is to look at GDP growth during the different firm and bank default regimes discussed above. As documented in Table 3, the growth rates of GDP in the EA, the US, and a number of European countries are below normal when firm default is high but much lower when firm and bank defaults are

[38] The variance of firm and bank defaults is roughly constant across bank default quantiles.

both high. This is consistent with standard definitions of a systemic financial crisis and the large bank default rates and output losses associated with them (see, e.g., Laeven and Valencia, 2013).[39]

We investigate the relationship between firm and bank defaults and GDP growth using quantile regressions of the following form:

$$\Delta y_t(\tau) = \beta_\tau \text{Def}_{t-1} + \gamma_\tau \Delta y_{t-1}, \tag{22}$$

where Def_{t-1} can either be FirmDef_{t-1} or BankDef_{t-1} and Δy_t represents GDP growth. This exercise is similar in spirit to the one performed in Adrian, Boyarchenko and Giannone (2019), which runs a quantile regression of GDP growth on lagged GDP growth and an index of financial conditions using US data. Firm and bank defaults are the main proxies for financial conditions in our framework. Hence, we regress GDP growth on the lagged GDP growth and the lagged level of default (Def_{t-1}) of either firms or banks. The right panel of Figure 4 plots the coefficients for either firm (the dashed red lines) or bank (the solid red lines) default in the corresponding quantile regressions estimated on EA data.

The results highlight three key features of the non-linear relationship between defaults and real activity. First, the link between both defaults and economic growth is weak for GDP growth quantiles close to the median. This suggests that defaults (whether bank or firm) have only a weak correlation with GDP growth in normal times. Second, the negative relationship between bank default and GDP growth becomes quantitatively more negative for the bottom quantiles. Increases in bank defaults have a larger (negative) impact on GDP growth when the economy is already in a recession (i.e. at the bottom quantile for GDP growth). Third, the above relationship does not hold for firm default. In sharp contrast to the non-linear pattern between bank default and economic activity, the impact of corporate defaults on GDP growth is small and flat across all GDP growth quantiles. Thus, Figure 4

[39] Average growth rates have been demeaned using the unconditional mean of GDP growth for each country.

(right panel) clearly shows that it is the risk of bank failures that are driving the deterioration in macroeconomic performance during periods of twin defaults identified in Table 3. This link between bank default and economic performance during the twin default crises will explain the importance of capital regulation in mitigating the downside risk to the real economy.

2.3.2 Defaults and economic performance in the model

Section 2.3.1 established a number of important data facts regarding the state-dependent co-movement between default rates and GDP growth. We learned that the marginal impact of corporate failures on bank solvency is stronger when banks are weaker. We saw that twin defaults are associated with deeper recessions. Finally, our results established that the correlation of bank (but not firm) defaults with real activity is higher in recessions. We now test the model's performance in reproducing these important empirical regularities not targeted in the estimation.

In the previous section, we used a 90th percentile-based criterion to identify the low default, high firm default, and twin default regimes in the EA data. Here, we use DF_t and DB_t as the model counterparts for firm and bank EDFs, respectively, and we employ the same criterion to split the model-simulated time series into the three regimes.

Table 4 compares the model-simulated (Baseline Model) and EA data (Data) averages for firm default, bank default, and GDP growth within the different regimes. The baseline model does a good job in reproducing these untargeted conditional moments thanks to its capacity to generate an empirically realistic positive relationship between firm and bank default rates, as shown in the right panel of Figure 3. First, the model reproduces the frequency of the default regimes remarkably well. Second, it reproduces the same ranking observed in the data in terms of the drop in GDP growth across regimes. The twin default regimes feature by far the worst GDP growth realizations, whereas the high firm default regime features a relatively mild recession despite the fact that firms' default rates are very similar across

Table 4: The Default Regimes in the Data and theModel

	Frequency	GDP growth	Bank default	Firm default
Low Default Regime				
Data	86.0%	0.0923	0.4346	2.3480
Baseline Model	84.7%	0.10111	0.5042	2.0568
Merton-type Model	81.0%	0.076144	0.56438	2.3997
High Firm Default Regime				
Data	4.00%	-0.0466	0.4033	4.8500
Baseline Model	5.35%	-0.38327	0.87401	5.0714
Merton-type Model	9.00%	-0.32938	0.52211	4.9994
High Bank Default Regime				
Data	3.00%	-0.6744	2.1056	3.7604
Baseline Model	5.35%	-0.65094	2.2649	4.2974
Merton-type Model	9.00%	-0.29141	2.7337	2.4591
Twin Defaults Regime				
Data	7.00%	-0.8189	3.0224	4.6076
Baseline Model	4.65%	-0.84251	2.5536	5.6439
Merton-type Model	1.00%	-0.58292	2.7013	5.127

Notes: This table compares the model and data averages for firm default, bank default and GDP growth within default regimes for the EA data and the simulated data from different models. Merton-type Model corresponds to the model in which the Merton-type specification of bank asset returns is adopted. Twin Defaults episodes are defined as the simultaneous occurrence of firm and bank default above their respective 90th percentiles. High Firm (Bank) Default are episodes with firm (bank) default above the 90th percentile and bank (firm) default below the 90th percentile. In Low Default episodes, both bank and firm default are below the 90th percentile. The default thresholds used to define the three regimes in the Merton-type model and the 1st Order App. model are the ones determined by the baseline model. Model results are based on 1,000,000 simulations. GDP growth is demeaned.

these two regimes. The table also reports a fourth regime where the bank default rate is above the 90th percentile, but firm default is below the 90th percentile.[40]

In the previous section, we also used quantile regressions to characterize the non-linear relationships between the two default series and GDP growth. The black lines in Figure 4 show that our model can replicate both quantile regressions well.[41] The model is qualitatively and quantitatively consistent with the key facts identified in our description of the quantile regressions on EA data. The correlation between firm and bank default is higher when banks are more fragile, and their probability of default is high. During times of average GDP growth, neither firm nor bank defaults affect economic performance in a significant manner. Bank (but not firm) defaults have a large and negative impact on GDP growth when the economy is already in recession.[42]

Both the island-idiosyncratic productivity shocks and the island-risk shocks are vital in generating realistic conditional and unconditional correlation patterns between firm and bank defaults and economic activity. When the non-diversifiable risk is constant (no island-risk shocks), the relationship between the two defaults and between bank default and GDP growth is significantly weakened. Hence, non-diversifiable risk shocks are essential to reproduce the non-linearities observed in the data well. When the non-diversifiable risk is absent (no island-idiosyncratic productivity shocks), banks do not default in our calibrated model.[43] In the absence of island-idiosyncratic shocks, banks are only exposed to aggregate shocks, and their net worth evolves ex-post in a fully symmetric manner. Bank default could only occur as a result of implausibly large aggregate shocks that would, thus, happen with a very low

[40]Even though the average firm default in this regime is below the 90th percentile, it remains at elevated levels (on average at about the 85th percentile in the model).

[41]Regression coefficients for the model are obtained using simulations of the model for 100,000 periods. When reporting moments generated by the model, we use realized firm and bank default rates, DF_t and DB_t, as the model counterparts for firm and bank EFDs, respectively.

[42]The model also produces a non-linear relationship between bank equity returns and future GDP as in Baron, Verner and Xiong (2021). Internet Appendix D shows the model-implied relationship between GDP at time t and bank return on equity (RoE) at time $t-1$. Very low bank RoE is associated with a much larger decline in future GDP compared to the increase associated with a strong RoE.

[43]Internet Appendix E explores the importance of each of these shocks in detail.

probability. Additionally, this would imply that either all banks default at the same time or none does, which would be counterfactual.

Given the non-linearity in bank asset returns with respect to non-diversifiable borrower risk, a crucial element for the ability of our model to reproduce the non-linearities observed in the data is the use of a higher-order solution method. Internet Appendix E shows that the model solved with linear approximation methods underestimates (overestimates) the severity of the twin defaults (high firm default) regime in terms of GDP growth.[44]

Overall, our model reproduces well the importance of financial vulnerabilities as determinants of the economy's downside risk (see Adrian, Boyarchenko and Giannone, 2019). In particular, it reproduces the fact that a deterioration in bank default risk corresponds to an increase in the downside risk to GDP growth, consistent with what is observed in EA data. In the next section, we will show that the reason why our model can replicate these non-linearities observed in the data is because it features a non-linear structure of bank asset returns. This is essential for the model to generate the right frequency and severity of the twin default episodes and the associated macroeconomic outcomes.

3. Understanding the Model: Bank Asset Returns

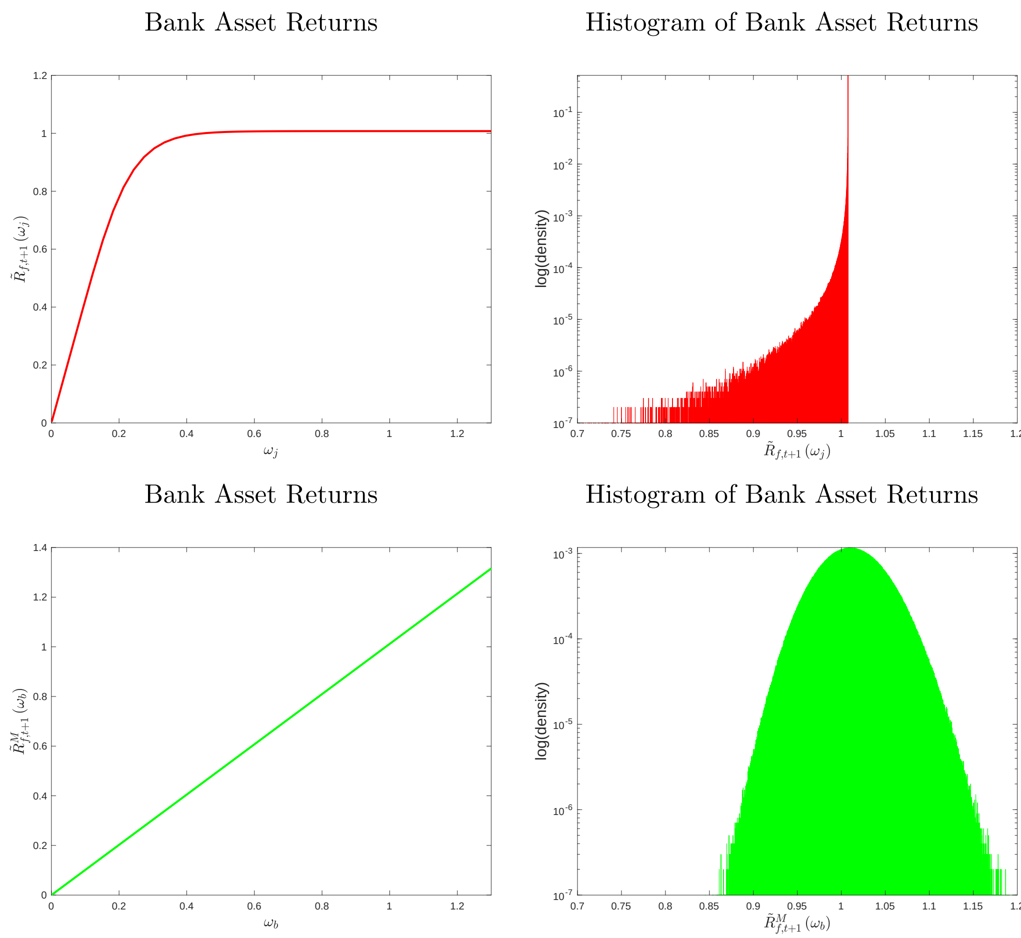
Results above show that our structural model of bank default risk is able to replicate well the frequency and severity of the twin default episodes and the associated macroeconomic outcomes. Next, we confirm the result of Nagel and Purnanandam (2020) and Gornall and Strebulaev (2018), who show that a reduced-form approach to bank default risk that uses a Merton-type formulation – with bank asset returns following a log-normal distribution – cannot capture the downward skewness in loan portfolio returns and, hence, the frequency and severity of twin defaults.[45]

[44] Internet Appendix E shows that the second-order model fails to match the non-linearities in the data.

[45] Baron, Verner and Xiong (2021) document that at the start of banking crises, the distribution of bank equity returns is considerably more left-skewed than that of non-financial equity returns.

Bank asset returns in our model. A distinguishing feature of our model is the structural approach to loan default risk whereby banks fail only when a significant fraction of the borrowers in their imperfectly diversified loan portfolios default. Hence, even if the banks' finance underlying projects with log-normal returns, the distribution of bank asset returns is downwardly skewed. If borrowers repay, they repay a fixed contractual amount. If they default, the loan recovery value is a fraction of the firms' asset values.

Figure 5: Bank Asset Returns: Baseline vs Merton-type Model



Notes: The top panels of this figure present bank asset gross returns as a function of the non-diversifiable island shock ω_j (left plot) and the histogram of bank asset returns (right plot) in the baseline model. The bottom panels of this figure present bank asset returns as a function of the bank-idiosyncratic shock ω_b (left plot) and the histogram of bank asset returns (right plot) in the Merton-type version of our model.

The top left panel of Figure 5 depicts the gross loan returns of the representative bank of island j, $\tilde{R}_{f,t+1}(\omega_j)$ as a function of the island-idiosyncratic shock, ω_j. This clearly shows that bank asset returns are highly non-linear in the island-idiosyncratic productivity shock (ω_j). When ω_j is very high, all borrowers repay, and the bank receives the promised repayment, including interest, from all its borrowers. But the upside is limited for the lender, as is naturally the case under a standard debt contract. However, the presence of default creates downside risk for the bank. As the island's idiosyncratic shock takes lower and lower values, the fraction of defaulting firms on the island increases, and bank asset returns decline in a highly non-linear fashion.

The top right panel Figure 5 depicts the distribution of $\tilde{R}_{f,t+1}(\omega_j)$.[46] Importantly, this distribution is not log-normal, even though the underlying idiosyncratic shocks are assumed to be log-normally distributed, as standard in the literature. The density of the returns spikes at the level at which all borrowers repay. Bank asset returns are left-skewed with a long left tail of low-return realizations caused by high firm defaults. Considering the asymmetric distribution of loan returns in general equilibrium is a distinctive feature of our macro-banking framework.[47]

Comparison to the Merton-type model.

A common approach in the macro-banking literature is to consider banks with (ex-ante) perfectly diversified loan portfolios and to capture the heterogeneity in bank asset returns by introducing bank-specific shocks that affect ex-post the aggregate performance of their loan portfolio returns without directly affecting the performance of the underlying borrowers. This approach makes loan returns and their implications for bank equity returns and bank failure similar to the classical Merton (1974) approach to corporate default.

[46] For these figures, we have fixed $q_{t+1}, k_t, y_{t+1}, b_{f,t}, R_{d,t}, d_t$ to their steady-state values obtained with the parameter values described in Section 2.2. We set $\sigma_{\omega_\vartheta,t+1}$ such that the expected bank default rate equals its targeted value from Table 2. We use 10,000,000 draws of ω_j to plot the histograms.

[47] In Internet Appendix Section D, we show that the distribution of asset returns for major European banks is asymmetric and left skewed as in our model.

To create a Merton-type version of our model, we modify Equation (8) in two ways. First, we remove the impact of the island-idiosyncratic shocks by setting them to unity at all times $\omega_j = 1$. This is equivalent to assuming that banks are perfectly diversified across islands. Second, to introduce ex-post heterogeneity in bank default outcomes, we include a log-normally distributed bank-idiosyncratic shock to bank revenues ω_b. The loan portfolio returns under this specification are determined by

$$\widetilde{R}_{f,t}(\omega_b) = \omega_b \left(\frac{(1-\mu_f)[y_t + (1-\delta)q_t k_{t-1}]}{b_{f,t-1}} \int_0^{\overline{\omega}_t^i} \omega_i dF_{i,t}(\omega_i) + R_{f,t-1} \int_{\overline{\omega}_t^i}^{\infty} dF_{i,t}(\omega_i) \right). \quad (23)$$

Identically to the island-idiosyncratic shock, the standard deviation of the distribution of the bank-idiosyncratic shock, ω_b, is also time-varying and evolves as described in Internet Appendix A. Importantly, we parametrize the Merton-type model to match the set of moments in Table 2. We keep the rest of the model identical. Hence, both in the Merton-type model and in our model, firms issue non-recourse, non-contingent debt in the form of bank loans. The main difference concerns the way heterogeneity in bank asset returns is introduced via firm- and island-productivity shocks, which directly hit bank borrowers in our structural model, versus via bank-idiosyncratic shocks that do not directly affect borrowers as in the Merton-type version of the model.

The bottom panels of Figure 5 depict the gross loan portfolio returns, $\tilde{R}_{f,t+1}(\omega_b)$, as a function of the idiosyncratic shock to bank loan revenues (left panel) and its distribution (right panel). The Merton-type model produces bank asset returns that are linear in the bank-idiosyncratic shock that produces heterogeneity in bank performance. [48] Thus, banks symmetrically experience upside and downside shocks. Since ω_b is log-normal, $\tilde{R}_{f,t+1}(\omega_b)$ is log-normal too. Thus, bank returns feature a much smaller left tail.

[48]This linearity is not the result of the type of claims held by banks (e.g., because banks hold equity claims instead of debt claims) but of the way heterogeneity in bank asset returns is introduced (e.g., via bank idiosyncratic shocks to aggregate loan returns).

Characterizing bank asset returns in an accurate manner is essential when studying the relationship between firm and bank defaults. In particular, Internet Appendix F shows that the Merton-type model fails to reproduce the non-linearity in the relationship between firm and bank defaults along several dimensions. Table 4 shows that the frequency of the twin default regime implied by the Merton-type model is much lower than the one observed in the data and in our baseline model (while the frequency of both the high firm default regime and high bank default regime are overestimated). The Merton-type approach also underestimates (overestimates) the severity of the twin defaults (high firm default) regime in terms of GDP growth.[49] In Section 5., we will show that, as a result of this, the Merton-type variant of our model also implies a lower optimal level of capital requirements than our baseline model.

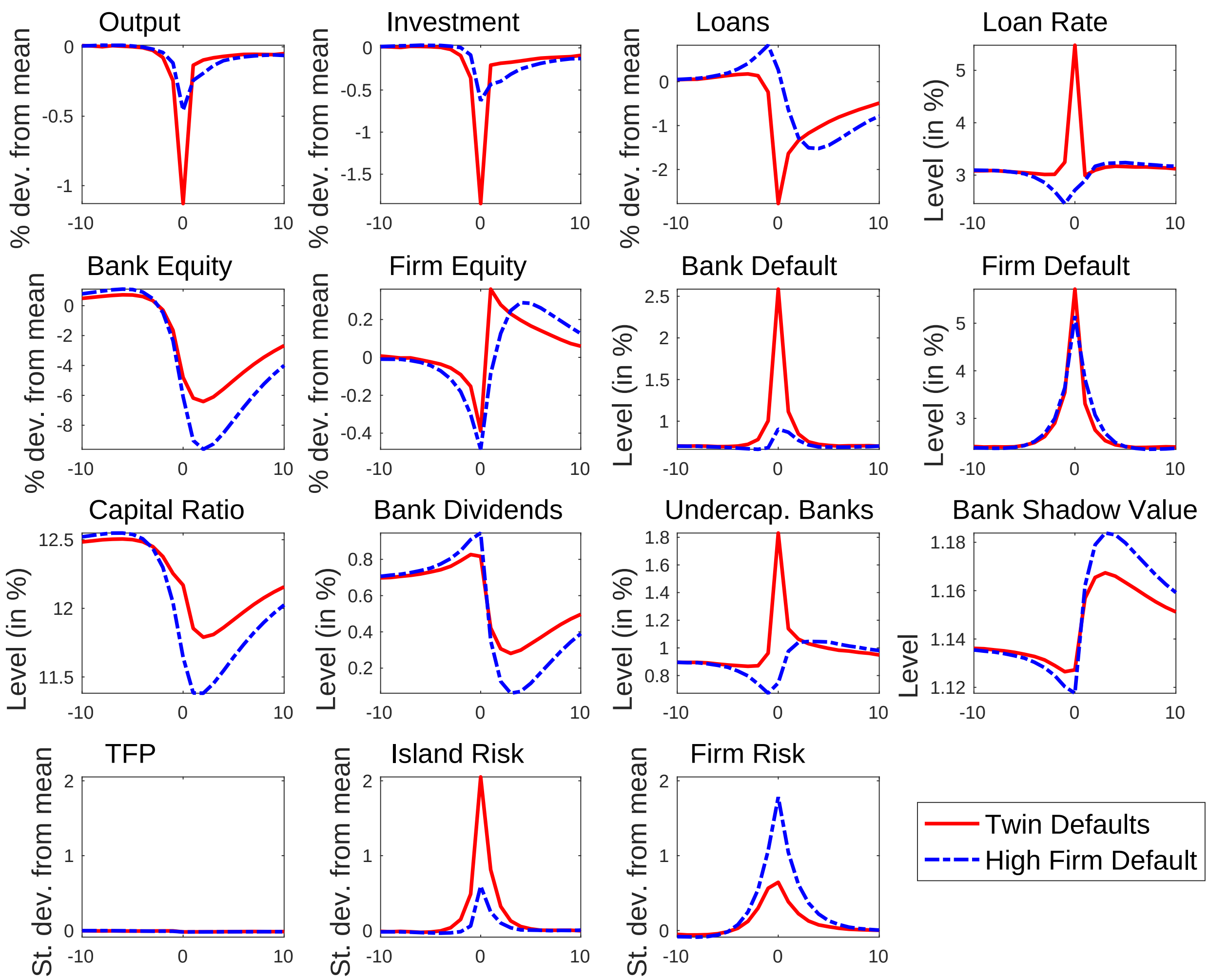
4. The Anatomy of Twin Default Crises

After validating the quantitative implications of our framework, we are well-equipped to understand the factors that lead to financial recessions in our model. An appealing feature of our setup is that episodes of simultaneously high firm and bank defaults appear due to sequences of small negative island-risk shocks that become increasingly amplified as the probability of bank failure increases. Intuitively, the non-linearity in bank asset returns implies that once banks have a high risk of failure, the marginal impact of additional credit losses on banks' solvency is much larger than in normal times. When the probability of twin defaults is high, even small shocks to bank borrowers can have a severe contraction in credit and economic activity.

Figure 6 shows the average path leading to high firm default (blue line) and twin de-

[49] Bank asset returns would also be linear if, in our island context, banks were holding equity claims instead of debt claims on the firms that they finance. Models in which banks directly hold productive assets (e.g. Gertler and Kiyotaki (2015)) and suffer bank-idiosyncratic shocks to the performance of those assets will feature returns very much like in the Merton-type formulation described above. With aggregate shocks being the only source of risk for banks, the model would instead deliver a degenerate distribution of bank asset returns.

Figure 6: Paths to Crises



Notes: This figure shows the average path leading to high firm default (blue dashed line) and twin defaults (red solid line) regimes. The figure is generated by simulating the model for 1,000,000 periods, identifying periods in which defaults are above the 90th percentile, and then computing the average realizations of shocks and endogenous variables for twenty periods before and after the crisis periods. We define twin defaults as the simultaneous occurrence of firm and bank default above their respective 90th percentiles. High firm default periods are those where firm default is above the 90th percentile and bank default is below the 90th percentile. TFP, Island Risk and Firm Risk represent the level of A_t, $\frac{\sigma_{\omega_{j,t+1}}}{\bar{\sigma}_{\omega_j}}$ and $\frac{\sigma_{\omega_{i,t+1}}}{\bar{\sigma}_{\omega_i}}$ in their respective standard deviation units.

faults (red line) regimes.[50] Two facts are noteworthy. First, the model implies that twin default episodes generate output falls that are more than two times larger than high firm default events. Second, the model captures the evolution of bank defaults for both regimes remarkably well. Bank defaults rise to around 2.5 percent during twin defaults, which is close to what we observe in the EA data during the recent financial crisis. In contrast, bank failures barely increase in the episodes of high firm defaults. Both cases are very close to the evidence reported in Table 4. The declines in output, investment, and lending are more pronounced in the case of twin defaults than in cases of high firm defaults.

The island-risk shocks are crucial to generate twin defaults. The increase in the volatility of the island-idiosyncratic productivity shocks (island risk) leads to high rates of firm default, which affects banks in several important ways. First, as the evolution of bank equity and banks' capital ratios show, the shock causes bank losses, depleting their net worth. Second, the riskier environment characterized by the jump in bank defaults and the share of under-capitalized banks encourages banks to contract credit provision in a precautionary manner. Hence, lending rates increase sharply, and loan volumes fall. This amplifies the drop in firm investment and production associated with the initial impact of the shock. In addition, the bankruptcy costs associated with the joint realization of high firm and bank default rates cause very large deadweight economic losses, which also weigh on the economy's performance. Once the economy starts to recover from the crisis, firm defaults fall, banks contract dividends, and their equity starts to recover gradually together with loan supply.

In contrast, firm-risk shocks alone can only give rise to high firm default events. The increase in the volatility of the firm-idiosyncratic productivity shocks (firm risk) also leads to high firm default, causing bank losses but in a more evenly distributed manner. Consequently, the share of under-capitalized banks does not increase, and banks actually relax

[50] The figure is generated by simulating the model for 1,000,000 periods, identifying periods in which defaults are above the 90th percentile, and then computing the average realizations of shocks and endogenous variables for twenty periods before and after the crisis periods.

lending standards by cutting lending rates and allowing their capital ratios to fall further instead of cutting lending massively. As a result, bank default only increases marginally, and the impact on the real economy is considerably reduced compared to twin default episodes.

The figure also shows that, on average, TFP remains broadly unchanged in both episodes. The two risk shocks instead play an important role. The rise in the firm-risk shocks plays a role in both high firm defaults and twin defaults, while the rise in the island-risk shocks plays a key role in generating twin default crises.

Finally, the model does not need very large risk shocks to generate a financial recession. These episodes occur following a sequence of small and positive risk shocks that accumulate into a two-standard deviation increase. Thus, in addition to matching the data, the microfounded link between the solvency of banks and firms also introduces an amplification mechanism that allows the model to generate crisis episodes without the need for large exogenous aggregate shocks.

Internet Appendix E shows the importance of these non-linearities, which is corroborated by the fact that if solved to a first-order approximation, the model only generates twin default crises if hit by implausibly large realizations of the island-risk shock. The strong non-linear effects of island-risk shocks can also be demonstrated using generalized impulse response (GIRFs) functions as in Andreasen, Fernandez-Villaverde and Rubio-Ramirez (2017). The GIRFs show that island-risk shocks have a much larger impact when conditioning on either twin defaults or a high firm default episode. In contrast, the GIRFs conditional only on high firm default show much less amplification than when we condition on a twin default episode.

5. Implications for Capital Requirements

After documenting the quantitative performance of our model and analyzing how twin defaults arise, this section provides implications for the optimal level and dynamic adjustment of capital requirements. The rationale for capital requirements in this model is related to the presence of safety net guarantees and externalities associated with the cost of bank failures and the disruption of bank lending during twin default crises. The presence of safety net guarantees modeled in the form of insured deposits makes the interest rate on deposit funding independent of banks' leverage choices. Further, banks operate under limited liability and do not internalize the social cost of their failures and the effects of their choices on the bank equity returns and, hence, on the next period aggregate lending capacity of the banking sector.[51] Feedback effects operating at the general equilibrium level provide a clear rationale for the macroprudential calibration of bank capital requirements.

5.1 Optimal Capital Requirement Level

We first assess the implications of different capital requirement levels, $\underline{\phi}$, on the mean of the ergodic distribution of selected variables for our baseline model.[52] Figure 7 shows that the imposition of higher capital requirements implies a trade-off between reducing the probability of twin default crises and maintaining the supply of bank credit. Higher capital requirements reduce bank leverage and make banks better protected against non-diversifiable risk, contributing to preserving aggregate bank net worth and, hence, reducing bank default.[53] Banks hedge against this risk by charging higher interest rates to leveraged borrowers and endogenously increasing their voluntary capital buffers. This endogenous reaction to higher capital requirements further protects banks' solvency. As a result, twin default crises become less frequent, and deadweight losses associated with bankruptcy costs decline.

Higher capital requirements are, however, also costly for the economy. They increase

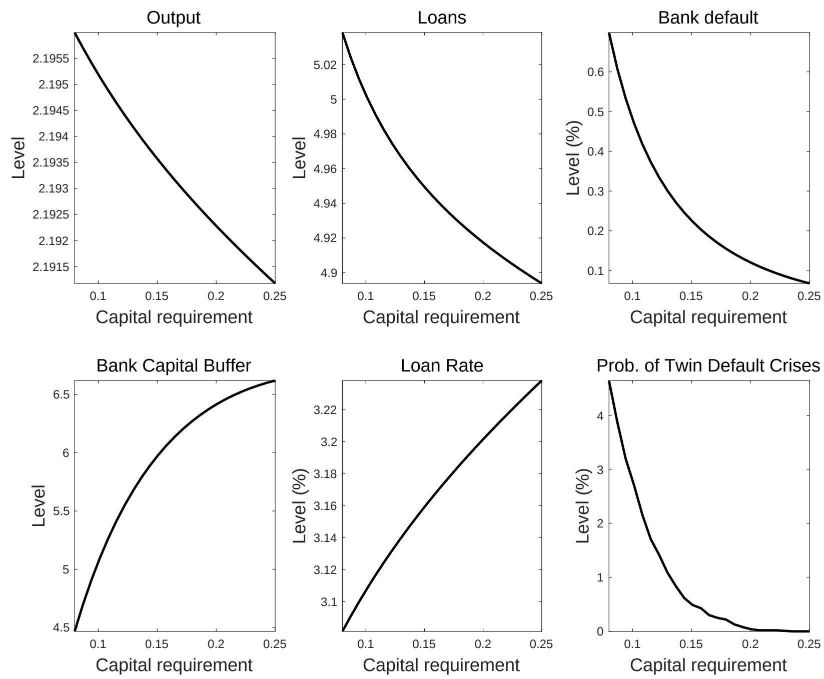
[51] In our model, the market segmentation does apply to the funding of banks. In every period, the aggregate net worth of the whole banking sector is invested in a portfolio of equity of the continuum of banks.

[52] We change $\underline{\phi}$ while keeping all other parameters unchanged and equal to the baseline calibration.

[53] As shown in the Internet Appendix F (see Figure E.4 and related discussion), leveraged banks with a positive risk of default engage in risk shifting by underpricing risky loans and by choosing lower voluntary buffers over the minimum capital requirements. Higher capital requirements reduce banks' probability of failing and discourage them from risk shifting. Once banks are subject to more stringent minimum capital requirements, the probability of failing becomes very small, but the risk of failing to satisfy the new higher capital requirements on an ex-post basis increases.

the relative scarcity of bank net worth and, hence, the average cost of bank funding. This implies, on average, a reduction in the provision of bank credit, higher borrowing costs, and lower firm investment.

Figure 7: Comparative Statics with Respect to Capital Requirement Level



Notes: This figure shows the implications of different values of the capital requirement $\underline{\phi}$ on the mean of the ergodic distribution of selected variables for our baseline model.

This trade-off is reflected in the overall effects of higher capital requirements on social welfare. The solid black line in Figure 8 reports the ergodic mean of household welfare as a function of the level of bank capital requirements. A 16 percent minimum capital requirement is optimal, implying a total capital ratio (including the voluntary capital buffers) of around 22 percent. Moving the minimum capital ratio from the baseline to the optimum would bring welfare gains of approximately 0.1 percent in certainty equivalent consumption terms relative to the baseline model, which features a minimum bank capital requirement of 8 percent and a total capital ratio of 12.5 per cent.[54]

Starting from the 8 percent minimum capital requirement, welfare first increases because the gains from the reduction in the probability of bank default outweigh the losses from imposing higher funding costs on banks. At the optimum, the probability of bank default is around 0.2 percent, and further reductions in bank failures have a limited impact on welfare. For a capital requirement above 16 percent, the negative effect of elevated borrowing costs for firms dominates, and welfare declines.

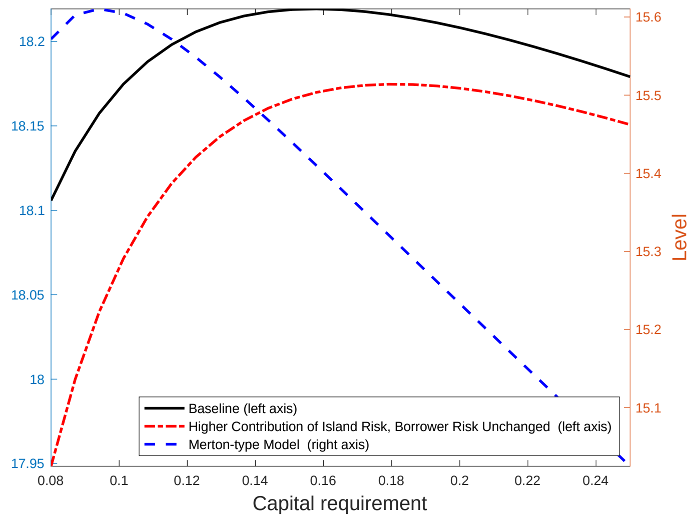
In order to understand the implications of higher capital requirements on the emergence of twin default crisis, Internet Appendix Figure E.2 compares the baseline path to a crisis with the minimum bank capital requirements of 8 percent (Baseline) with the path implied by a capital requirement level closer to the optimum (Higher Cap. Req).[55] We find that under higher bank capital requirements (dashed line) the model needs a much larger (3 standard deviation) increase in the island-risk shock to generate a twin default episode than under the minimum capital requirement of 8 percent. This is because, with higher capital requirements, the economy experiences a much lower frequency of the twin default regime. These results are corroborated by the implications of a higher level of capital requirements for the performance of the model in terms of untargeted conditional moments also reported in Internet Appendix Table E.1.[56]

[54]Due to the assumption of log utility, our results provide a lower bound of the welfare gains delivered by higher capital requirements. To the best of our knowledge, model-based estimates of the welfare gains of optimal macroprudential policy range from a modest 0.1% to a substantial 1.4% in terms of the annual consumption equivalent (e.g. Bianchi (2011); Van der Ghote (2021)). Importantly, our results refer to gains relative to the baseline capital requirement level, while the larger numbers in the literature refer to gains over the laissez-faire economy.

[55]As in Table 4, the thresholds used to define the high firm and bank regimes are the same as in the baseline model.

[56]Internet Appendix F also reports robustness of the results to a lower calibration of bank default costs, μ_j. The bank capital requirement that brings the probability of twin defaults close to zero and maximizes social welfare is always substantially higher than the baseline level and higher than in the standard Merton-type formulation, regardless of the calibration of bankruptcy costs.

Figure 8: Welfare Effects of the Capital Requirement in Different Scenarios



Notes: This figure reports the ergodic mean of household welfare as a function of the level of bank capital requirements in different scenarios. The baseline (black solid line) corresponds to our baseline model. Merton-type model (blue dashed line) corresponds to the model in which the Merton-type specification of bank asset returns is adopted. Higher Contribution of Island Risk, Borrower Risk Unchanged (red dashed-dotted line) corresponds to the model in which we increase the average standard deviation of the island-idiosyncratic shock and reduce the average standard deviation of the firm-idiosyncratic shock while keeping the probability of firm default unchanged.

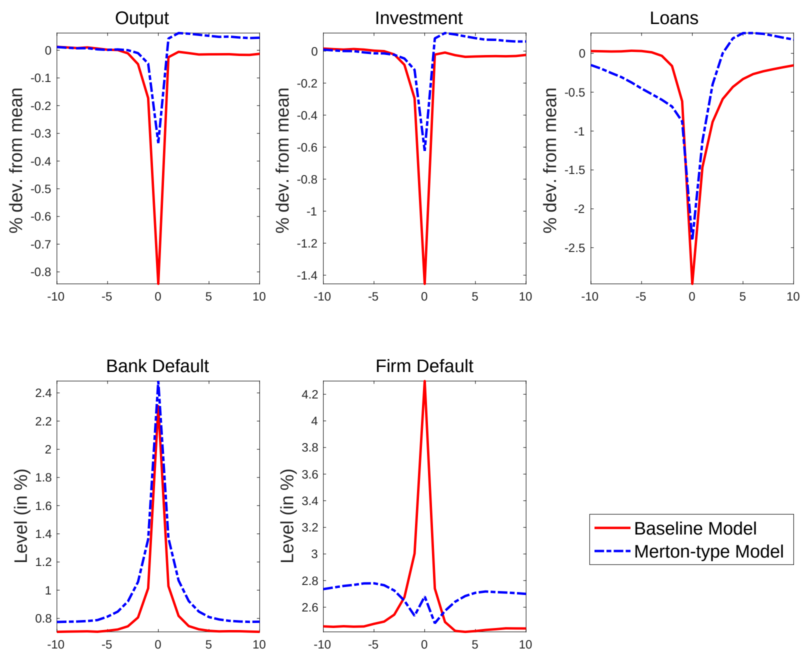
5.2 The Role of Non-diversifiable Bank Risk

To gain some insights regarding the importance of properly quantifying the impact of borrower default risk on bank insolvencies, we consider two counterfactual experiments. First, firm default risk is assumed to be less diversifiable at the bank level than in the calibrated model. Hence, the link between the default of firms and banks is much stronger, implying a much higher probability of twin defaults. This is obtained by increasing the average standard deviation of the island-idiosyncratic shock and reducing the average standard deviation of the firm-idiosyncratic shock while keeping the probability of firm default unchanged.[57]

[57]The firm default rate is the same as in the calibrated model. The only difference between the two versions of the model is the composition of diversifiable and non-diversifiable (firm- vs island-idiosyncratic) firm default risk for banks. The average standard deviation of the island-idiosyncratic shock is increased by

This reduces the extent to which banks can diversify away firm default risk. The red dashed line in Figure 8 shows social welfare as a function of the capital requirement level in this counterfactual scenario. As expected, when firm default risk is less diversifiable at the bank level, the optimal capital requirement needs to be higher, i.e., close to 19 percent.

Figure 9: Paths to high bank default episodes



Notes: This figure shows the average path leading to a high bank default episode under the baseline model (red solid line) and under a Merton-type model (blue dashed line). The figure is generated by simulating the model for 1,000,000 periods, identifying periods of high bank defaults, and then computing the average realizations of shocks and endogenous variables for ten periods before and after the crisis periods. We define a high bank default episode as a bank default above the 90th percentile.

In the second experiment, we assume that all firm default risk is ex-ante diversifiable at the bank level (no island-idiosyncratic risk), and the default risk of banks comes from

10 percent, whereas the average standard deviation of the firm-idiosyncratic shock is reduced by 6.3 percent. While the average probability of firm default remains equal to 2.25 percent, the probability of bank default increases from 0.59 percent to 1.03 percent. The probability of twin default crises increases from 5.9 percent to 8.8 percent.

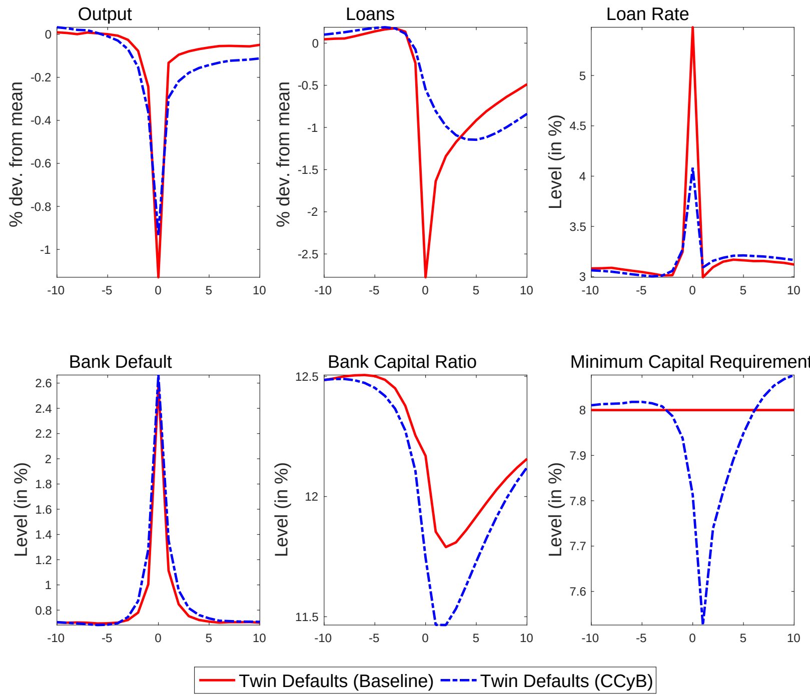
an exogenous disturbance that directly hits the banks' loan returns. This aligns with the standard Merton-type model of bank default risk used in the previous literature. Figure 8 reports welfare as a function of the capital requirement also under this alternative specification (blue dashed line). Even though the probability of bank default is the same in both models, under the Merton-type formulation, the optimal capital requirement is more than six percentage points lower, i.e. just under 10 percent.[58] In addition, the welfare gain from imposing higher minimum capital requirements is also much smaller at 0.01 percent.

Figure 9 reports the difference in key variables in the high bank default episodes in each model version. It clearly shows that, in the Merton-type model, high bank defaults are not accompanied by an increase in firm defaults and the deadweight losses associated with them. Therefore, the overall losses associated with bank default are lower, translating into a less sizable drop in economic activity. This explains why the standard model of bank default risk underestimates the welfare gains from increasing capital requirements compared to our baseline model.

Overall, our results show that capturing the special nature of bank asset returns and their implications for bank default risk is essential to provide accurate prescriptions on the optimal level of capital requirements. Indeed, microfounding the relationship between firm and bank defaults is crucial to reproduce the frequency and severity of twin defaults observed in the data and, thus, properly account for the costs associated with bank insolvencies and the net benefits of higher capital requirements.

[58] We parametrize the Merton-type version of the model so as to ensure that the mean and standard deviation of bank default are the same as in our baseline model. However, since firm default is not the main driver of bank default in such a model version, the probability of twin defaults drops to 1 percent, which is considerably lower than what is observed in the data and produced by our baseline model. In addition, the symmetric normal distribution of bank asset returns in the Merton-type model means that higher bank capital requirements are unrealistically effective at making banks safe compared to our downward-skewed fat-tailed bank asset return distribution. See Figure F.2 in Internet Appendix F.

Figure 10: Paths to Twin Defaults with and without CCyB

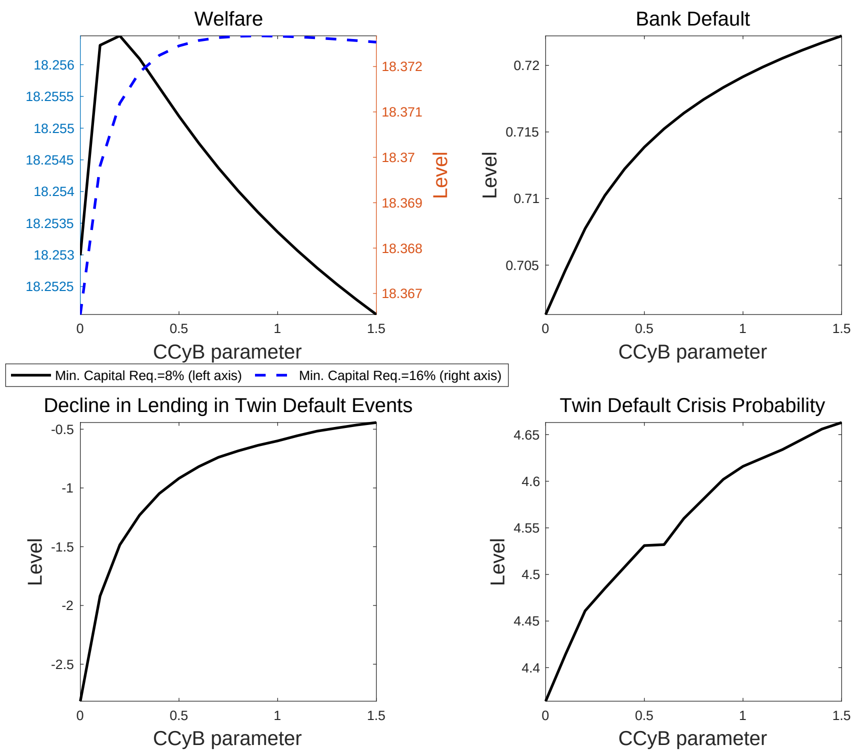


Notes: This figure shows the average path leading to a twin default episode under different policy assumptions. The baseline (red solid line) corresponds to our baseline model with a constant capital requirement set to 8 percent ($\underline{\phi} = 0.08$). 'CCyB' (blue dashed line) corresponds to the model with a capital requirement policy rule that adjusts the capital requirement in response to credit growth. Dynamic adjustment parameter κ is set to 1. The figure is generated by simulating the model for 1,000,000 periods, identifying periods of twin defaults, and then computing the average realizations of shocks and endogenous variables for twenty periods before and after the crisis periods. We define a twin default episode as the simultaneous occurrence of firm and bank default above their respective 90th percentiles. The 90th percentile default thresholds used to define the three regimes in the three models are always the ones determined by the baseline model.

5.3 Dynamic Adjustment of Bank Capital Requirements

In what follows, we evaluate dynamic adjustment of bank capital requirements, i.e. requirements that are higher in good times than in bad. In line with the prescription of the Basel III framework, we focus on the role of a Counter-cyclical capital buffer (CCyB). We assume a rule that adjusts the minimum capital requirement in response to the credit growth as follows: $\underline{\phi}_t = \phi_{SS} + \kappa(log(B_{f,t}) - log((B_{f,t-1})))$, where ϕ_{SS} is the steady-state level of the requirement and κ controls its dynamic adjustment in response to the credit growth.

Figure 11: Comparative Statics with respect to the degree of countercyclical adjustment



Notes:This figure reports the ergodic mean of household welfare, the probability of bank default, the probability of a twin default crisis, and the average decline of lending in a crisis as a function of the degree of adjustment of bank capital requirements in response to credit growth (CCyB). 'Min. Capital Req. = 8 %' (black solid line) corresponds to our baseline model with an 8 percent minimum capital requirement. 'Min. Capital Req. = 16 %' (blue dashed line) corresponds to the 16 percent capital requirement, which maximizes expected social welfare.

Figure 10 compares the path to crisis simulation under the baseline model with a constant minimum capital ratio of 8 % and under a dynamic capital ratio, which is adjusted counter-cyclically. The figure reveals the trade-offs introduced by using the CCyB. It is beneficial because it allows bank capital ratios to fall by more during financial recessions, thus dampening the increase in lending rates and mitigating the fall in credit and GDP. The cost of this measure is that it leads to a slightly larger increase in bank failures.

Figure 11 examines the welfare impact of the strength of the countercyclical response both in the baseline with an 8% minimum capital ratio (solid line) and when the capital ratio is at its optimal steady-state value of 16% (dashed line). We see that the dashed line peaks at a much higher responsiveness of the CCyB to the credit cycle than the solid line. In other words, strong counter-cyclical adjustment of minimum capital ratios is more beneficial only when the level of bank capital ratios is already very high. Indeed, adjusting bank capital ratios helps to dampen the fall in bank lending during crises at the cost of slightly increasing the likelihood of bank failure and the probability of a twin default crisis. Once banks are very well capitalized, bank failures and twin defaults become very low probability events, which are less sensitive to prevailing bank capital ratios. It, therefore, becomes optimal to use the CCyB more aggressively.

6. Conclusions

The assessment of the benefits and costs of higher capital requirements requires a framework that adequately quantifies the trade-off between a lower frequency of bank insolvency crises and a more limited provision of credit to the wider economy. Thus, it crucially hinges upon the ability of the model to match key features of the data, including the frequency and severity of twin defaults, i.e. episodes characterized by deep recessions and abnormally high default rates among both banks and their borrowers.

With this purpose in mind, we build a quantitative structural general equilibrium model of bank default risk in which bank solvency problems arise endogenously from high default rates among bank borrowers. Our paper represents the first quantitative exploration of the way bank borrowers' default translates into rare but severe episodes of bank insolvencies and the large output losses associated with them.

Microfounding the link between bank and firm solvency allows our framework to capture a very important aspect of bank loan portfolios: they deliver asymmetrically distributed payoffs that feature limited upside potential but significant downside risk due to borrowers defaults. This feature allows our model to reproduce the non-linearities associated with firm and bank defaults and macroeconomic outcomes observed in the data. Thus, our model captures the behavior of the economy well, not only in normal times but also in twin defaults.

We show that our model implies higher optimal capital requirements than standard Merton-type models of bank default risk, which neglect or underestimate the impact of borrower default on bank solvency. Thus, our results suggest that a structural approach to bank default risk is crucial for the assessment of the net benefits of higher capital requirements.

References

Admati, A. R. and M. Hellwig (2013). *The Bankers New Clothes: What's Wrong With Banking and What To Do About It.* Princeton University Press.

Adrian, T., N. Boyarchenko, and D. Giannone (2019). Vulnerable growth. *American Economic Review 109*, 1263–89.

Alderson, M. J. and B. L. Betker (1995). Liquidation costs and capital structure. *Journal of Financial Economics 39*, 45–69.

Alfaro, I., N. Bloom, and X. Lin (2023). The finance uncertainty multiplier. *Journal of Political Economy*.

Andrade, G. and S. N. Kaplan (1998). How costly is financial (not economic) distress: evidence from highly leveraged transactions that became distressed. *Journal of Finance 53*, 1443–1493.

Andreasen, M. M., J. Fernandez-Villaverde, and J. F. Rubio-Ramirez (2017). The pruned state-space system for non-linear DSGE models: theory and empirical applications. *Review of Economic Studies 85*, 1–49.

Arellano, C. (2008). Default risk and income fluctuations in emerging economies. *American Economic Review 58*, 273–289.

Baron, M., E. Verner, and W. Xiong (2021). Banking Crises Without Panics. *Quarterly Journal of Economics 136*, 51–113.

Begenau, J. (2020). Capital requirements, risk choice, and liquidity provision in a business-cycle model. *Journal of Financial Economics 136*(2), 355–378.

Begenau, J. and T. Landvoigt (2017). Financial regulation in a quantitative model of the modern banking system. Working paper.

Behr, P. and R. H. Schmidt (2016). The German banking system. In *The Palgrave Handbook of European Banking*, pp. 541–566. Springer.

Benigno, G., H. Chen, C. Otrok, A. Rebucci, and E. Young (2013). Financial crises and macro-prudential policies. *Journal of Interntional Economics 89*, 453470.

Bernanke, B. and M. Gertler (1989). Agency costs, net worth, and business fluctuations. *American Economic Review 79*, 14–31.

Bernanke, B. S., M. Gertler, and S. Gilchrist (1999). The financial accelerator in a quantitative business cycle framework. *Handbook of macroeconomics 1*, 1341–1393.

Bianchi, J. (2011, December). Overborrowing and systemic externalities in the business cycle. *American Economic Review 101*(7), 3400–3426.

Bianchi, J. (2016). Efficient bailouts. *American Economic Review 106*, 36073659.

Bigio, S. (2015). Endogenous liquidity and the business cycle. *American Economic Review 105*(6), 1883–1927.

Bloom, N. (2009). The impact of uncertainty shocks. *Econometrica 77*, 623–685.

Bocola, L. (2016). The pass-through of sovereign risk. *Journal of Political Economy 124*, 879–926.

Brunnermeier, M. K. and Y. Sannikov (2014). A macroeconomic model with a financial sector. *American Economic Review 104*, 379–421.

Calomiris, C. W. (2013). Reforming banks without destroying their productivity and value. *Journal of Applied Corporate Finance*.

Carlstrom, C. T. and T. S. Fuerst (1997). Agency costs, net worth, and business fluctuations: A computable general equilibrium analysis. *American Economic Review 87*, 893–910.

Christiano, L. J., R. Motto, and M. Rostagno (2014). Risk shocks. *American Economic Review 104*, 27–65.

Clerc, L., A. Derviz, C. Mendicino, S. Moyen, K. Nikolov, L. Stracca, J. Suarez, and A. Vardoulakis (2015). Capital regulation in a macroeconomic model with three layers of default. *International Journal of Central Banking 11*, 9–63.

Coimbra, N. and H. Rey (2019). Financial cycles with heterogeneous intermediaries. Working paper.

Corbae, D. and P. D'Erasmo (2019). Capital requirements in a quantitative model of banking industry dynamics. Technical report, National Bureau of Economic Research.

Davydiuk, T. (2019). Dynamic bank capital requirements. Working paper.

De Bonis, R., A. F. Pozzolo, and M. Stacchini (2011). The Italian banking system: Facts and interpretations. Working paper.

De Fiore, F., M. Hoerova, and H. Uhlig (2018). Money markets, collateral and monetary policy. NBER Working Paper 79.

De Jonghe, O., H. Dewachter, K. Mulier, S. Ongena, and G. Schepens (2020). Some borrowers are more equal than others: Bank funding shocks and credit reallocation. *Review of Finance 24*, 1–43.

Deyoung, R., A. Gron, G. Torna, and A. Winton (2015). Risk overhang and loan portfolio decisions: small business loan supply before and during the financial crisis. *Journal of Finance 70*, 2451–2488.

Djankov, S., O. Hart, C. McLiesh, and A. Shleifer (2008). Debt enforcement around the world. *Journal of Political Economy 116*, 1105–1149.

EBA (2018). EU-wide stress test results. Technical report, European Banking Authority.

Elenev, V., T. Landvoigt, and S. V. Nieuwerburgh (2020). A macroeconomic model with financially constrained producers and intermediaries. *Econometrica, forthcoming*.

Ferrante, F. (2019). Risky lending, bank leverage and unconventional monetary policy. *Journal of Monetary Economics 101*, 100–127.

Galaasen, S., R. Jamilov, R. Juelsrud, and H. Rey (2020). Granular credit risk. Nber working paper 27994.

Gale, D. and M. Hellwig (1985). Incentive-compatible debt contracts: The one-period problem. *Review of Economic Studies 52*, 647–663.

Gertler, M. and N. Kiyotaki (2010). Financial intermediation and credit policy in business cycle analysis. *Handbook of Monetary Economics 3*, 547–599.

Gertler, M. and N. Kiyotaki (2015, July). Banking, liquidity, and bank runs in an infinite horizon economy. *American Economic Review 105*, 2011–43.

Gertler, M., N. Kiyotaki, and A. Prestipino (2019). A Macroeconomic Model with Financial Panics. *Review of Economic Studies*.

Gete, P. (2018). Lending standards and macroeconomic dynamics.

Gete, P. and G. Melkadze (2020). A quantitative model of international lending of last resort. *Journal of International Economics 123*.

Giesecke, K., F. A. Longstaff, S. Schaefer, and I. A. Strebulaev (2014). Macroeconomic effects of corporate default crisis: A long-term perspective. *Journal of Financial Economics 111*, 297–310.

Gornall, W. and I. A. Strebulaev (2018). Financing as a supply chain: The capital structure of banks and borrowers. *Journal of Financial Economics 129*, 510–530.

Granja, J., G. Matvos, and A. Seru (2017). Selling failed banks. *The Journal of Finance 72*(4), 1723–1784.

Guiso, L., P. Sapienza, and L. Zingales (2004). Does local financial development matter? *Quarterly Journal of Economics 119*, 929–969.

He, Z. and A. Krishnamurthy (2013). Intermediary asset pricing. *American Economic Review 103*, 732–70.

Jermann, U. (2019). Is SOFR better than LIBOR? Working paper.

Jermann, U. and V. Quadrini (2012). Macroeconomic effects of financial shocks. *American Economic Review 102*, 238–271.

Jermann, U. J. (1998). Asset pricing in production economies. *Journal of Monetary Economics 41*, 257–275.

Kareken, J. H. and N. Wallace (1978). Deposit insurance and bank regulation: a partial-equilibrium exposition. *Journal of Business 51*, 413–438.

Kiyotaki, N. and J. Moore (1997). Credit cycles. *Journal of Political Economy 105*, 211–248.

Koop, G., M. H. Pesaran, and S. M. Potter (1996). Impulse response analysis in nonlinear multivariate models. *Journal of Econometrics 74*, 119–147.

Krasa, S. and A. P. Villamil (1992). Monitoring the monitor: an incentive structure for a financial intermediary. *Journal of Economic Theory 57*, 197–221.

Krishnamurthy, A. and T. Miur (2017). How credit cycles across a financial crisis. NBER Working Paper 23850.

Laeven, L. and F. Valencia (2013). Resolution of banking crises. In *Handbook of Safeguarding Global Financial Stability*, pp. 231–258. Elsevier.

Martinez-Miera, D. and J. Suarez (2014). Banks' endogenous systemic risk taking. Working paper.

Mendicino, C., K. Nikolov, J. Suarez, and D. Supera (2018). Optimal dynamic capital requirement. *Journal of Money Credit and Banking 50*, 1271–1297.

Mendicino, C., K. Nikolov, J. Suarez, and D. Supera (2020). Bank capital in the short and in the long run. *Journal of Monetary Economics 115*, 64–79.

Mendoza, E. G. (2010). Sudden stops, financial crises, and leverage. *American Economic Review 100*, 1941–66.

Merton, R. C. (1974). On the pricing of corporate debt: The risk structure of interest rates. *Journal of Finance 29*, 449–470.

Nagel, S. and A. Purnanandam (2020). Banks risk dynamics and distance to default. *The Review of Financial Studies 33*, 2421–2467.

Paravisini, D., V. Rappoport, and P. Schnabl (2023). Specialization in bank lending: Evidence from exporting firms. *The Journal of Finance 78*(4), 2049–2085.

Piazzesi, M., C. Rogers, and M. Schneider (2019). Money and banking in a new keynesian model. Working paper.

Rampini, A. A. and S. Viswanathan (2019). Financial intermediary capital. *The Review of Economic Studies 86*, 413–455.

Regehr, K. and R. Sengupta (2016). Sectoral loan concentration and bank performance (2001-2014). *Federal Reserve Bank of Kansas City Working Paper* (16-13).

The Federal Reserve Bank of Minneapolis (2017). The Minneapolis plan to end too big to fail. Technical report.

Van Den Heuvel, S. (2008). The welfare cost of bank capital requirements. *Journal of Monetary Economics 55*, 298–320.

Van der Ghote, A. (2021, January). Interactions and coordination between monetary and macroprudential policies. *American Economic Journal: Macroeconomics 13*(1), 1–34.

Vasicek, O. (2002). The distribution of loan portfolio value. *Risk 15*, 160–162.

Villacorta, A. (2020). Business cycles and the balance sheets of the financial and non-financial sectors.

Internet Appendix

Twin Defaults and Bank Capital Requirements

A Model Details

A.1: First Order Conditions

Household. The household's problem yields the following FOCs with respect to consumption,

$$U_{C_t} = \lambda_t, \tag{24}$$

labor supply,

$$-U_{H_t} = w_t \lambda_t, \tag{25}$$

and demand for the portfolio of insured deposits,

$$1 = \mathbb{E}_t \left(\Lambda_{t+1} \right) R_{d,t}, \tag{26}$$

where λ_t is the Lagrange multiplier of the budget constraint and $\Lambda_{t+1} \equiv \beta \frac{\lambda_{t+1}}{\lambda_t}$ is the household's stochastic discount factor.

Firm first-stage decisions. The model section states the CHC's first-stage problem in term of decisions made at date $t-1$ by firms that produce at t. For greater coherence with the coding of the model, here the write the FOCs for the decisions made at t by the firms that produce at $t+1$. To make expressions more compact, let

$$\begin{aligned}
\Pi_{f,t+1} \quad = \quad & -(1+\gamma) \int_0^\infty \int_{\bar{\omega}^j_{f,t+1}}^{\hat{\omega}^j_{f,t+1}} \left\{ R_{f,t} B_{f,t} - \omega_i \omega_j \left[y_t + (1-\delta)\, q_t k_{t-1} \right] \right\} dF_{i,t+1}(\omega_i) dF_{j,t+1}(\omega_j) \\
& + p_{f,t+1} \int_0^\infty \int_{\hat{\omega}^j_{f,t+1}}^\infty \left\{ \omega_i \omega_j [y_{t+1} + (1-\delta)\, q_{t+1} k_t] - R_{f,t} B_{f,t} \right\} dF_{i,t+1}\left(\omega_i\right) dF_{j,t+1}\left(\omega_j\right) \\
& - p_{f,t+1} \int_0^\infty \mu_F F(\bar{\omega}^j_{f,t+1}) \left(w_t h_t + q_t k_t \right) dF_{j,t+1}\left(\omega_j\right),
\end{aligned} \tag{27}$$

and

$$\begin{aligned}\Pi_{b,t+1} \quad &= \quad -\frac{1+\gamma}{\phi_t}\int_{\bar{\omega}_{b,t+1}}^{\hat{\omega}_{b,t+1}}[\underline{\phi}-\widetilde{R}_{f,t+1}(\omega_j)+R_{d,t}(1-\phi_t)]dF_{j,t+1}(\omega_j) \\ &+ \quad \frac{p_{b,t+1}}{\phi_t}\{\int_{\hat{\omega}_{b,t+1}}^{\infty}[\widetilde{R}_{f,t+1}(\omega_j)-R_{d,t}(1-\phi_t)]dF_{j,t+1}(\omega_j)+\underline{\phi}[(F_{j,t+1}(\hat{\omega}_{b,t+1}) \\ &\quad -F_{j,t+1}(\bar{\omega}_{b,t+1})]-\mu_B F_{j,t+1}(\bar{\omega}_{b,t+1})\},\end{aligned} \tag{28}$$

which collect the terms affected by first-stage decisions in the per-period payoffs of the CHC and the BHC in date t versions of Equations (5) and (14), respectively.

The FOCs of the CHC with respect to capital, labor, bank borrowing, and interest rates on loans can be expressed as

$$\mathbb{E}_t\left(\Lambda_{t+1}\frac{\partial\Pi_{f,t+1}}{\partial k_t}\right)+\zeta_{f,t}q_t-\xi_{f,t}\mathbb{E}_t\left(\Lambda_{t+1}\frac{\partial\Pi_{b,t+1}}{\partial k_t}\right)=0, \tag{29}$$

$$\mathbb{E}_t\left(\Lambda_{t+1}\frac{\partial\Pi_{f,t+1}}{\partial h_t}\right)+\zeta_{f,t}w_t-\xi_{f,t}\mathbb{E}_t\left(\Lambda_{t+1}\frac{\partial\Pi_{b,t+1}}{\partial h_t}\right)=0, \tag{30}$$

$$\mathbb{E}_t\left(\Lambda_{t+1}\frac{\partial\Pi_{f,t+1}}{\partial B_{f,t}}\right)-\zeta_{f,t}-\xi_{f,t}\mathbb{E}_t\left(\Lambda_{t+1}\frac{\partial\Pi_{b,t+1}}{\partial B_{f,t}}\right)+\xi_{f,t}p_{b,t}\phi_t=0, \tag{31}$$

and

$$\mathbb{E}_t\left(\Lambda_{t+1}\frac{\partial\Pi_{f,t+1}}{\partial R_{f,t}}\right)-\xi_{f,t}\mathbb{E}_t\left(\Lambda_{t+1}\frac{\partial\Pi_{b,t+1}}{\partial R_{f,t}}\right)=0, \tag{32}$$

where $\zeta_{f,t}$ is the Lagrange multiplier associated to the balance sheet constraint (imposing $w_t h_t + q_t k_t = b_{f,t} + N_{f,t}$) and $\xi_{f,t}$ is the Lagrange multiplier of the bank's participation constraint; the date t version of Equation (6). This second constraint links the leverage and input decisions of each firm to the loan rate $R_{f,t}$ agreed with its bank. Since banks in the model are perfectly competitive, this loan rate is the minimum one that allows the bank to obtain a RoE that compensates for the opportunity cost of the bank equity required to finance the loan.

Bank first-stage decisions. The model section states the BHC's first-stage problem in terms of decisions made at date $t-1$ by banks that receive their returns from lending at t. For greater coherence with the coding of the model, here we write the optimality conditions for the decisions made at t by the banks that receive lending returns at $t+1$. If banks' optimal leverage at date t is interior ($\phi_t > \underline{\phi}$), it must satisfy the following FOC:

$$\frac{\partial\mathbb{E}_t(\Lambda_{t+1}\Pi_{b,t+1})}{\partial\phi_t}=0. \tag{33}$$

But if the solution to (33) is below $\underline{\phi}$, assuming the relevant second-order condition holds, the solution is at the corner $\phi_t = \underline{\phi}$.

A.2: Firm equity return.

Prior equations imply that the gross book return on corporate equity, $\rho_{f,t} \equiv \frac{\Xi_{f,t}+N_{f,t}}{N_{f,t-1}}$, which is used in the calibration of the model can be written as follows:

$$\begin{aligned}\rho_{f,t} &= x_{f,t} - \frac{\psi}{2}\left(\bar{\delta} - x_{f,t}\right)^2 - \frac{1+\gamma}{\theta_{t-1}}\int_0^\infty \int_{\bar{\omega}^j_{f,t}}^{\hat{\omega}^j_{f,t}} \left[R_{f,t-1}\left(1-\theta_{t-1}\right) - \omega_i\omega_j R^k_t\right] dF_{i,t}(\omega_i) dF_{j,t}(\omega_j) \\ &+ \frac{1}{\theta_{t-1}}\int_0^\infty \{\int_{\hat{\omega}^j_{f,t}}^\infty \left[\omega_i\omega_j R^k_t - R_{f,t-1}(1-\theta_{t-1})\right] dF_{i,t}\left(\omega_i\right) - \mu_F F_{i,t}(\bar{\omega}^j_{f,t})\} dF_{j,t}\left(\omega_j\right) - x_{f,t}\end{aligned} \tag{34}$$

where the first three terms contain the net dividend yield components in Equation (4) and the last two contain the gross rate of growth of book equity $N_{f,t}/N_{f,t-1}$, taken also from the continuation value part of Equation (4).

A.3: Bank equity return.

Prior equations imply that the gross book return on bank equity, $\rho_{b,t} \equiv \frac{\Xi_{b,t}+N_{b,t}}{N_{b,t-1}}$, which is used in the calibration of the model can be written as follows

$$\begin{aligned}\rho_{b,t} &= x_{b,t} - \frac{\psi}{2}\left(\bar{\delta} - x_{b,t}\right)^2 - \frac{1+\gamma}{\phi_{t-1}}\int_{\bar{\omega}_{b,t}}^{\hat{\omega}_{b,t}} [\underline{\phi} - \widetilde{R}_{f,t}(\omega_j) + R_{d,t-1}(1-\phi_{t-1})] dF_{j,t}(\omega_j) \\ &+ \frac{1}{\phi_{t-1}}\{\int_{\hat{\omega}_{b,t}}^\infty [\widetilde{R}_{f,t}(\omega_j) - R_{d,t-1}(1-\phi_{t-1})] dF_{j,t}(\omega_j) + \underline{\phi}[F_{j,t}(\hat{\omega}_{b,t}) - F_{j,t}(\bar{\omega}_{b,t})] \\ &-\mu_B F_{j,t}(\bar{\omega}_{b,t})\} - x_{b,t},\end{aligned} \tag{35}$$

where the first three terms contain the net dividend yield components in Equation (12) and the last two terms contain the gross rate of growth of equity, $\frac{N_{b,t}}{N_{b,t-1}}$, taken also from the continuation value part of Equation (12).

A.4: Aggregate Shocks

We assume the following AR(1) law of motion for the TFP shock $log(A_{t+1}) = \rho_A log(A_t) + \sigma_A \epsilon_{A,t+1}$, where $\epsilon_{A,t+1}$ is normally distributed with mean zero and variance one. The standard deviation of the distribution of each idiosyncratic shock is time-varying and evolves as an

AR(1) process

$$\log\left(\frac{\sigma_{\omega_\vartheta,t+1}}{\bar{\sigma}_{\omega_\vartheta}}\right) = \rho_{\sigma_i}\log\left(\frac{\sigma_{\omega_\vartheta,t}}{\bar{\sigma}_{\omega_\vartheta}}\right) + \sigma_\vartheta\epsilon_{\omega_\vartheta,t+1}$$

for $\vartheta = i, j$, where $\epsilon_{\omega_\vartheta,t+1}$ is normally distributed with mean zero and variance one.[59] These shocks resemble the risk and uncertainty shocks commonly used in the literature (Bloom, 2009; Christiano, Motto and Rostagno, 2014). We will refer to them as firm- and island-risk shocks. Our results show that these shocks are important sources of aggregate risk in the model and vital to generate fluctuations in firm and bank defaults.

A.5: Model Aggregation and Market Clearing

In this subsection, we describe model aggregation and market clearing conditions. To help explain the statement of these conditions, notice that in writing up the model and the equations that follow, we have differentiated quantities on each side of the market by using capital letters for one side and small letters for the other. Typically we have used small letters for decisions made by either firms or banks. In the case of the loan market, where firms and banks interact, $B_{f,t}$ denotes loan demand by firms and $b_{f,t}$ loans supply by banks.

Final good The clearing of the market for final goods requires

$$Y_t = y_t, \tag{36}$$

where aggregate output Y_t equals household consumption, C_t, plus the investment in the production of new capital, I_t, plus the resources absorbed by the deadweight losses associated with firm and bank default, the emergency recapitalization of distressed firms and undercapitalized banks to prevent default, and deviations from target dividend ratios:

$$Y_t = C_t + I_t + \Sigma_{f,t} + \Sigma_{b,t}, \tag{37}$$

[59]This specification is similar to the one adopted in Christiano, Motto and Rostagno (2014).

where

$$
\begin{aligned}
\Sigma_{f,t} &= \mu_F \int_0^\infty F_{i,t}(\bar{\omega}_{f,t}(\omega_j)) dF_{j,t}(\omega_j)(w_t H_{t-1} + q_{t-1}K_{t-1}) \\
&+ \mu_f \int_0^\infty \int_0^{\bar{\omega}^j_{f,t}} \omega_i \omega_j \left[Y_t + (1-\delta) q_t K_{t-1}\right] dF_{i,t}(\omega_i) dF_{j,t}(\omega_j) \\
&+ \gamma \int_0^\infty \int_{\bar{\omega}^j_{f,t}}^{\hat{\omega}^j_{f,t}} \left\{R_{f,t-1}B_{f,t-1} - \omega_i \omega_j \left[Y_t + (1-\delta) q_t K_{t-1}\right]\right\} dF_{i,t}(\omega_i) dF_{j,t}(\omega_j) \\
&+ \frac{\psi_f}{2}(\bar{\delta}_f - x_{f,t})^2 N_{f,t-1}
\end{aligned}
\tag{38}
$$

and

$$
\begin{aligned}
\Sigma_{b,t} &= \mu_B F_{j,t}(\bar{\omega}_{b,t}) + \mu_b \int_0^{\bar{\omega}_{b,t}} \tilde{R}_{f,t}(\omega_j) B_{f,t-1} dF_{j,t}(\omega_j) \\
&+ \gamma \int_{\bar{\omega}_{b,t}}^{\hat{\omega}_{b,t}} [\underline{\phi} - \tilde{R}_{f,t}(\omega_j) + R_{d,t-1}(1-\phi_{t-1})] B_{f,t-1} dF_{j,t}(\omega_j) \\
&+ \frac{\psi_b}{2}(\bar{\delta}_b - x_{b,t})^2 N_{b,t-1}.
\end{aligned}
\tag{39}
$$

Each of these expression comprise, for firms and banks, respectively, losses suffered by the corresponding holding company due to the default of their subsidiaries, asset repossession costs suffered by banks or the DGS when firms or banks default, emergency equity injection costs, and costs from deviating from target dividend ratios.

Labor The clearing of the labor market requires

$$H_t = h_t. \tag{40}$$

Physical capital The clearing of the market for physical capital requires

$$K_t = k_t. \tag{41}$$

Loans The clearing of the market for loans, requires

$$B_{f,t} = b_{f,t}. \tag{42}$$

Bank deposits The clearing of the market for bank deposits, requires

$$D_t = d_t. \tag{43}$$

Profits from capital production Profits received by households from capital-producing firms are

$$\Xi_{k,t} = q_t S\left(\frac{I_t}{K_{t-1}}\right) K_{t-1} - I_t. \tag{44}$$

A.6: Model Equilibrium Conditions

In this section we provide the exhaustive list of equilibrium conditions for our model, which altogether define an Equilibrium. We begin with the equilibrium conditions related to the household, then the CHC and firms, then the BHC and banks, then the capital production sector, and finally the market clearing conditions.

Household Using equations (24) and (25) we obtain

$$-\frac{U_{H_t}}{U_{C_t}} = w_t, \tag{45}$$

Equation (26) is part of the Equilibrium conditions. Hence, we have

$$1 = \mathbb{E}_t(\Lambda_{t+1}) R_{d,t}. \tag{46}$$

CHC Equations (5) is part of the Equilibrium conditions

$$\begin{aligned} p_{f,t} &= \mathbb{E}_t\{\Lambda_{t+1}[(\bar{\delta}_f + \frac{1}{2\psi_f}) - (\bar{\delta}_f + \frac{1}{\psi_f})p_{f,t+1} + \frac{1}{2\psi}p_{f,t+1}^2 \\ &- \frac{1+\gamma}{\theta_t}\int_0^\infty \int_{\bar{\omega}_{f,t+1}^j}^{\hat{\omega}_{f,t+1}^j} \left[R_{f,t}(1-\theta_t) - \omega_i\omega_j R_{t+1}^k\right] dF_{i,t+1}(\omega_i) dF_{j,t+1}(\omega_j) \\ &+ \frac{p_{f,t+1}}{\theta_t}\int_0^\infty \{\int_{\hat{\omega}_{f,t+1}^j}^\infty \left[\omega_i\omega_j R_{t+1}^k - R_{f,t}\left(1-\theta_t\right)\right] dF_{i,t+1}(\omega_i) - \mu_F F_{i,t+1}(\bar{\omega}_{f,t+1}^j)\} dF_{j,t+1}\left(\omega_j\right)]\}, \end{aligned} \tag{47}$$

where the next two definitions are also part of the Equilibrium conditions

$$\theta_t = \frac{N_{f,t}}{\omega_t H_t + q_t K_t} \tag{48}$$

and

$$R_t^k = \frac{Y_t + (1-\delta)\, q_t K_{t-1}}{w_{t-1}H_{t-1} + q_{t-1}K_{t-1}}. \tag{49}$$

The law of motion of the net worth of the CHC in Equation (3) is also part of the Equilibrium:

$$\begin{aligned} N_{f,t} &= \left(\int_0^\infty \left\{ \int_{\hat{\omega}^j_{f,t}}^\infty \left[\omega_i \omega_j R^k_t - R_{f,t-1}(1-\theta_{t-1}) \right] dF_{i,t}(\omega_i) \right.\right. \\ &\quad \left.\left. - \int_0^{\hat{\omega}^j_{f,t}} d^{i,j}_{f,t} \mu_F dF_{i,t}(\omega_i) \right\} dF_{j,t}(\omega_j) - x_{f,t} \right) \frac{N_{f,t-1}}{\theta_{t-1}} \end{aligned} \tag{50}$$

The production functions, and the definitions of $\hat{\omega}^j_{f,t}$, and $\bar{\omega}^j_{f,t}$ are also part of the Equilibrium conditions:

$$Y_{t+1} = A_{t+1} K^\alpha_t H^{1-\alpha}_t, \tag{51}$$

$$\hat{\omega}^j_{f,t} = \frac{R_{f,t-1}(1-\theta_{t-1})}{\omega_j R^k_t}. \tag{52}$$

$$\bar{\omega}^j_{f,t} = \frac{(1+\gamma)\, R_{f,t-1}(1-\theta_{t-1}) - p_{f,t}\mu_F}{(1+\gamma)\omega_j R^k_t}. \tag{53}$$

The optimal condition for dividends is also part of the Equilibrium conditions:

$$x_{f,t} = \overline{\delta}_f - \frac{p_{f,t} - 1}{\psi_f}. \tag{54}$$

The FOCs for capital, labor, bank borrowing, and loan interest rates are also part of the Equilibrium conditions. They require consideration of the definition

$$\begin{aligned} \Pi_{f,t+1} &= p_{f,t+1} \int_0^\infty \int_{\hat{\omega}^j_{f,t+1}}^\infty \left(\omega_i \omega_j \left(Y_{t+1} + (1-\delta)\, q_{t+1} K_t \right) - R_{f,t} B_{f,t} \right) dF_{i,t+1}(\omega_i)\, dF_{j,t+1}(\omega_j) \\ &- p_{f,t+1} \int_0^\infty \mu_F F(\bar{\omega}^j_{f,t+1}) \left(w_t H_t + q_t K_t \right) dF_{j,t+1}(\omega_j) \\ &- (1+\gamma) \int_0^\infty \int_{\bar{\omega}^j_{f,t+1}}^{\hat{\omega}^j_{f,t+1}} \left[R_{f,t} B_{f,t} - \omega_i \omega_j \left(Y_t + (1-\delta)\, q_t K_{t-1} \right) \right] dF_{i,t+1}(\omega_i) dF_{j,t+1}(\omega_j), \end{aligned} \tag{55}$$

the balance sheet constraint

$$B_{f,t} + N_{f,t} = w_t H_t + q_t K_t, \tag{56}$$

and the FOCs with respect to the four decision variables:

$$\mathbb{E}_t \left(\Lambda_{t+1} \frac{\partial \Pi_{f,t+1}}{\partial K_t} \right) + \zeta_{f,t} q_t - \xi_{f,t} \mathbb{E}_t \left(\Lambda_{t+1} \frac{\partial \Pi_{b,t+1}}{\partial K_t} \right) = 0, \tag{57}$$

$$\mathbb{E}_t\left(\Lambda_{t+1}\frac{\partial \Pi_{f,t+1}}{\partial H_t}\right)+\zeta_{f,t}w_t-\xi_{f,t}\mathbb{E}_t\left(\Lambda_{t+1}\frac{\partial \Pi_{b,t+1}}{\partial H_t}\right)=0, \tag{58}$$

$$\mathbb{E}_t\left(\Lambda_{t+1}\frac{\partial \Pi_{f,t+1}}{\partial B_{f,t}}\right)-\zeta_{f,t}-\xi_{f,t}\mathbb{E}_t\left(\Lambda_{t+1}\frac{\partial \Pi_{b,t+1}}{\partial B_{f,t}}\right)+\xi_{f,t}p_{b,t}\phi_t=0, \tag{59}$$

and

$$\mathbb{E}_t\left(\Lambda_{t+1}\frac{\partial \Pi_{f,t+1}}{\partial R_{f,t}}\right)-\xi_{f,t}\mathbb{E}_t\left(\Lambda_{t+1}\frac{\partial \Pi_{b,t+1}}{\partial R_{f,t}}\right)=0. \tag{60}$$

BHC The equations of the BHC problem that enter the Equilibrium conditions are described next. Equation (14) gives us:

$$\begin{aligned}
p_{b,t} \;&=\; \mathbb{E}_t[\Lambda_{t+1}((\overline{\delta}_b+\frac{1}{2\psi_b})-(\overline{\delta}_b+\frac{1}{\psi_b})p_{b,t+1}+\frac{1}{2\psi_b}p_{b,t+1}^2\\
&-\; \frac{1+\gamma}{\phi_t}\int_{\bar{\omega}_{b,t+1}}^{\hat{\omega}_{b,t+1}}\{\underline{\phi}-[\widetilde{R}_{f,t+1}(\omega_j)-R_{d,t}(1-\phi_t)]\}dF_{j,t+1}(\omega_j)\\
&+\; \frac{p_{b,t+1}}{\phi_t}\{\int_{\hat{\omega}_{b,t+1}}^{\infty}[\widetilde{R}_{f,t+1}(\omega_j)-R_{d,t}(1-\phi_t)]dF_{j,t+1}(\omega_j)+\underline{\phi}\,[F_{j,t+1}(\hat{\omega}_{b,t+1})-F_{j,t+1}(\bar{\omega}_{b,t+1})]\\
&-\mu_B F_{j,t+1}(\bar{\omega}_{b,t+1})\})].
\end{aligned} \tag{61}$$

The law of motion of $N_{b,t}$ in Equation (11) is part of the Equilibrium:

$$N_{b,t}=\left\{\frac{\int_0^{\hat{\omega}_{b,t}}[(1-d_{b,t}^j)\underline{\phi}-d_{b,t}^j\mu_B]dF_{j,t}(\omega_j)+\int_{\hat{\omega}_{b,t}}^{\infty}[\widetilde{R}_{f,t}(\omega_j)-R_{d,t-1}(1-\phi_{t-1})]dF_{j,t}(\omega_j)}{\phi_{t-1}}-x_{b,t}\right\}N_{b,t-1}, \tag{62}$$

The definitions of $\widetilde{R}_{f,t}$, $\hat{\omega}_{b,t}$, and $\bar{\omega}_{b,t}$ in Equations (8), (9), and (13) are part of Equilibrium:

$$\widetilde{R}_{f,t}(\omega_j)=\frac{(1-\mu_f)\omega_j[y_t+(1-\delta)q_tk_{t-1}]}{b_{f,t-1}}\int_0^{\overline{\varpi}_t^j}\omega_i dF_{i,t}(\omega_i)+R_{f,t-1}\int_{\overline{\varpi}_t^j}^{\infty}dF_{i,t}(\omega_i). \tag{63}$$

$$\widetilde{R}_{f,t}(\hat{\omega}_{b,t})-R_{d,t-1}(1-\phi_{t-1})=\underline{\phi}, \tag{64}$$

and

$$\widetilde{R}_{f,t}(\bar{\omega}_{b,t})-(1-\phi_{t-1})R_{d,t-1}=\underline{\phi}-\frac{p_{b,t}(\underline{\phi}+\mu_B)}{1+\gamma}. \tag{65}$$

The optimal condition for dividends is also part of the Equilibrium conditions:

$$x_{b,t}=\overline{\delta}_b-\frac{p_{b,t}-1}{\psi_b}. \tag{66}$$

The conditions for the optimality of bank leverage implied by equations (28) and (33) are also part of the Equilibrium:

$$\begin{aligned}\Pi_{b,t+1} &= -\frac{1+\gamma}{\phi_t}\int_{\bar{\omega}_{b,t+1}}^{\hat{\omega}_{b,t+1}}[\underline{\phi} - \widetilde{R}_{f,t+1}(\omega_j) + R_{d,t}(1-\phi_t)]dF_{j,t+1}(\omega_j) \\ &+ \frac{p_{b,t+1}}{\phi_t}\{\int_{\hat{\omega}_{b,t+1}}^{\infty}[\widetilde{R}_{f,t+1}(\omega_j) - R_{d,t}(1-\phi_t)]dF_{j,t+1}(\omega_j) + \underline{\phi}[(F_{j,t+1}(\hat{\omega}_{b,t+1}) \\ &-F_{j,t+1}(\bar{\omega}_{b,t+1})] - \mu_B F_{j,t+1}(\bar{\omega}_{b,t+1})\},\end{aligned} \tag{67}$$

and

$$\frac{\partial \mathbb{E}_t(\Lambda_{t+1}\Pi_{b,t+1})}{\partial \phi_t} = 0. \tag{68}$$

Finally, the balance sheet of the banks,

$$b_{f,t} = N_{b,t} + d_t, \tag{69}$$

and the definition of capital ratio ϕ_t in Equation (7),

$$N_{b,t} = \phi_t b_{f,t}, \tag{70}$$

are also part of the Equilibrium conditions.

Capital production The evolution of capital is controlled by the FOC of the capital producer and the law of motion of capital, i.e. Equations (15) and (16)

$$q_t = \left[S'\left(\frac{I_t}{K_{t-1}}\right)\right]^{-1} \text{ and} \tag{71}$$

$$K_t = (1-\delta)\,K_{t-1} + S\left(\frac{I_t}{K_{t-1}}\right)K_{t-1}. \tag{72}$$

Deposit insurance costs By using $D_t = d_t$ we can write the deposit insurance costs in Equation (17) as:

$$T_t = \Omega_t D_{t-1}, \tag{73}$$

where

$$\Omega_t = F_{j,t}\left(\bar{\omega}_{j,t}\right)R_{d,t-1} - \frac{1-\mu_b}{1-\phi_t}\int_0^{\bar{\omega}_{b,t}} \tilde{R}_{f,t}\left(\omega_j\right)dF_{j,t}\left(\omega_j\right). \tag{74}$$

Market clearing The aggregate resource constraint equation (37) can be written as

$$Y_t = C_t + I_t + \Sigma_{b,t} + \Sigma_{f,t}. \tag{75}$$

B Approximating Banks' Expected Profits

In order to use perturbation methods to approximate the solution to the model, we need to compute the bank's expected return on the loan portfolio (conditional on not defaulting), defined here as $R_{p,t+1}$, which is part of Equation (14) and is given by the integral defined in Equation (20).

We take $q_{t+1}, k_t, y_{t+1}, b_{f,t}, R_{d,t}, d_t$ as given and use the notation of $\tilde{R}_{f,t+1}$ to be the function of island shock, ω_j, only. From the analysis in Section 3., it should be clear that the bank's loan return $\tilde{R}_{f,t+1}(\omega_j)$ is not log-normally distributed. Mathematically, this is due to the fact that $\Gamma_{i,t+1}(\bar{\omega}_{t+1}(\omega_j))$ and $G_{i,t+1}(\bar{\omega}_{t+1}(\omega_j))$ which enter into $\tilde{R}_{f,t+1}(\omega_j)$ are both non-linear functions of ω_j. As a result of the highly non-linear shape of $\tilde{R}_{f,t+1}(\omega_j)$, the integral in Equation (20) cannot be computed as an explicit function of the state variables and perturbation methods cannot be applied. We overcome this challenge by (i) splitting this integral into the sum of integrals taken over smaller intervals, (ii) computing a series of quadratic Taylor approximations of $\tilde{R}_{f,t+1}(\omega_j)$ around a mid-point of each interval.

Formally, we split the domain of ω_j into N intervals of equal length defined on $N+1$ points x_k ranging from $x_1 = \bar{\omega}_{j,t+1}$ to $x_{N+1} = \omega_j^{\max}$ where the highest point $\omega_j^{\max}$ is chosen such that $\tilde{R}_{f,t+1}\left(\omega_j^{\max}\right) = R_{f,t}$ almost surely. Given those assumptions, $R_{p,t+1}$ is approximately given by:

$$R_{p,t+1} \approx \sum_{k=1}^{N}\left(\int_{x_k}^{x_{k+1}} \Theta^k(\omega_j)\, dF_{j,t+1}(\omega_j)\right) + \left[1 - F_{j,t+1}(x_{N+1})\right] R_{f,t} \tag{76}$$

where $\Theta^k(\omega_j)$ is a Taylor approximation of $\tilde{R}_{f,t+1}(\omega_j)$ around a point $\omega_j = \bar{x}_k \equiv \frac{x_{k+1}+x_k}{2}$ and is given by

$$\Theta^k(\omega_j) = \tilde{R}_{f,t+1}(\bar{x}_k) + \tilde{R}'_{f,t+1}(\bar{x}_k)(\omega_j - \bar{x}_k) + \frac{1}{2}\tilde{R}''_{f,t+1}(\bar{x}_k)(\omega_j - \bar{x}_k)^2 \tag{77}$$

All the derivatives of $\tilde{R}_{f,t+1}$ are with respect to ω_j and can be computed as an explicit function of the state variables. Using the simplified expression for $\Theta^k(\omega_j)$ we can rewrite $\int_{x_k}^{x_{k+1}} \Theta^k(\omega_j)\, dF_{j,t+1}$ as follows:

$$\int_{x_k}^{x_{k+1}} \Theta^k(\omega_j)\, dF_{j,t+1} = Q_0(\bar{x}_k) + Q_1(\bar{x}_k)\int_{x_k}^{x_{k+1}} \omega_j dF_{j,t+1} + Q_2(\bar{x}_k)\int_{x_k}^{x_{k+1}} \omega_j^2 dF_{j,t+1} \tag{78}$$

where: $Q_i\left(\bar{x}_k\right)$ are just constants given by:

$$Q_0\left(\bar{x}_k\right) = \left[F_{j,t+1}\left(x_{k+1}\right) - F_{j,t+1}\left(x_k\right)\right]\left[\tilde{R}_{f,t+1}\left(\bar{x}_k\right) - \bar{x}_k\tilde{R}'_{f,t+1}\left(\bar{x}_k\right) + \frac{1}{2}\bar{x}_k^2\tilde{R}''_{f,t+1}\left(\bar{x}_k\right)\right],$$

$$Q_1\left(\bar{x}_k\right) = \left[F_{j,t+1}\left(x_{k+1}\right) - F_{j,t+1}\left(x_k\right)\right]\left[\tilde{R}'_{f,t+1}\left(\bar{x}_k\right) - \frac{1}{2}\bar{x}_k\tilde{R}''_{f,t+1}\left(\bar{x}_k\right)\right],$$

$$Q_0\left(\bar{x}_k\right) = \left[F_{j,t+1}\left(x_{k+1}\right) - F_{j,t+1}\left(x_k\right)\right]\left[\frac{1}{2}\tilde{R}''_{f,t+1}\left(\bar{x}_k\right)\right],$$

Given our assumption of log-normally distributed island shock, ω_j, we have expressions for $\int_{x_k}^{x_{k+1}}\omega_j dF_{j,t+1}$ and $\int_{x_k}^{x_{k+1}}\omega_j^2 dF_{j,t+1}$ as explicit functions of the state variables. Consequently, we can easily derive, very accurately, the approximation of $R_{p,t+1}$ in Equation (76) as an explicit function of the state variables.

C Data

- Investment: Gross Fixed Capital Formation, Millions of euros, Chain linked volume, Calendar, and seasonally adjusted data, Reference year 1995, Source: the Area Wide Model (AWM) dataset.

- Gross Domestic Product (GDP): we define the GDP as the sum of Consumption and Investment.

- Loans: Outstanding amounts of loans at the end of quarter (stock) extended to non-financial corporations by Monetary and Financial Institution (MFIs) in EA, Source: MFI Balance Sheet Items Statistics (BSI Statistics), Monetary and Financial Statistics (S/MFS), European Central Bank.

- Loan Spread: Spread between the composite interest rate on loans and the composite risk-free rate. We compute this spread in two steps.

  1. Firstly, we compute the composite loan interest rate as the weighted average of interest rates at each maturity range (up to 1 year, 1-5 years, over 5 years).
  2. Secondly, we compute corresponding composite risk-free rates that take into account the maturity breakdown of loans. The maturity-adjusted risk-free rate is the weighted average (with the same weights as in the case of composite loan interest rate) of the following risk-free rates chosen for maturity ranges:
     - 3 month EURIBOR (up to 1 year).
     - German Bund 3 year yield (1-5 years).

  - German Bund 10-year yield (over 5 years for commercial loans).
  - German Bund 7-year yield (5-10 years for housing loans).
  - German Bund 20-year yield (over 10 years for housing loans).

  Source: MFI Interest Rate Statistics of the European Central Bank, Bloomberg.

- Return on equity of banks: Bank Equity Return (after tax), EA. Source: Global Financial Development, World Bank.

- Expected default of Banks: Asset weighted average of expected default frequency (EDF) within one year for the sample of banks in EA. The series is available on a monthly basis and aggregated at quarterly frequency by averaging the monthly series within a quarter.[60] Source: Moody's KMV.

- Expected default of non-financial firms: we compute it using the individual expected default frequency (EDF) series by Moody's KMV for non-financial corporations in the EA. To mimic the exposure to small and medium-sized enterprises (SMEs) and large firms in the loan portfolio of banks in the EA, we proceed in two steps. First, use the individual non-financial corporations' EDFs provided by Moody's KMV to construct two separate EDF indices: i) for SMEs ii) for large firms.[61] Second, we build an aggregate default series for non-financial firms as a weighted average of EDF indices for SMEs and large firms. As weights we use the share of loans extended by banks in EA to SMEs and large firms.[62] The EDF data are available on a monthly basis. We aggregate it to quarterly series by averaging the monthly series within a quarter.

- Net payout ratio is computed as the sum of stock repurchases and dividends, net of equity issuance, as a share of the previous quarter's book equity. Because our model is stationary, we calculate issuance as net of real asset growth of the respective sector. This yields a net payout ratio = repurchases + dividends - issuance + real asset growth. The time-series data on the aggregate ingredients needed to compute the payout ratio (dividends, stock repurchases, equity issuance, book equity, total assets, deflator) comes from the Euro Area Flow of Funds. Banks are defined as monetary financial institutions, and firms are defined as non-financial corporations. Source: Eurostat Flow of Funds

- Bank total capital ratio and share of undercapitalized banks: we collect the data on the total capital ratio (total capital/risk-weighted assets) of individual Euro Area banks

[60] See detailed EDF description on the Moody's webpage.

[61] EDF indices are constructed as asset weighted average of EDF within one year for non-financial firms within each size category. We define SMEs as firms with average total assets below €43 m (following the definition of the European Commission).

[62] The data on the share of SMEs loans in total loans is from the Financing SMEs and Entrepreneurs database of OECD.

from BankScope and BankFocus. We compute the aggregate banking sector capital ratio as the average capital ratio of individual banks weighted by total assets. The share of undercapitalized banks is computed as the proportion of banks whose total capital ratio fell below the regulatory requirement of 8%. Source: BankScope and BankFocus

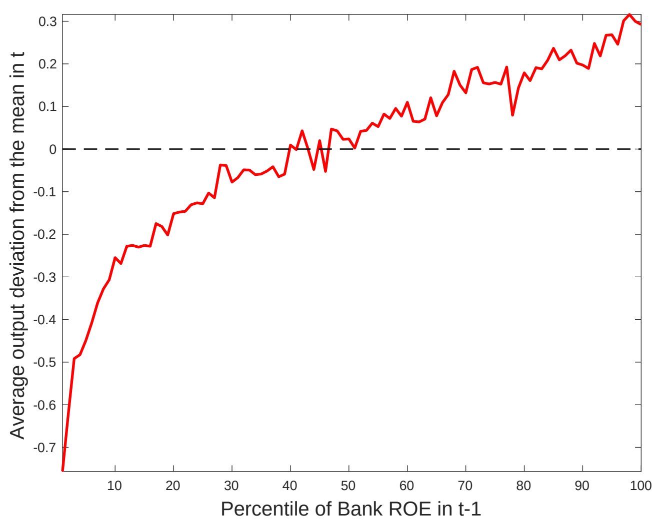
- Share of distressed firms: We compute the average share of firms in distress as the proportion of firms that were downgraded to a rating CCC or below in a given quarter. Source: Moody's KMV

D Additional Empirical Results

D.1: Relationship between GDP and banks' Rate of Return on Equity

This appendix shows the model-implied relationship between GDP at time t and the bank's rate of return on equity (RoE) at time $t-1$. Very low bank RoE is associated with a much larger decline in future GDP compared to the increase associated with a strong RoE out-turn.

Figure D.1: GDP at different percentiles of banks' Rate of Return on Equity

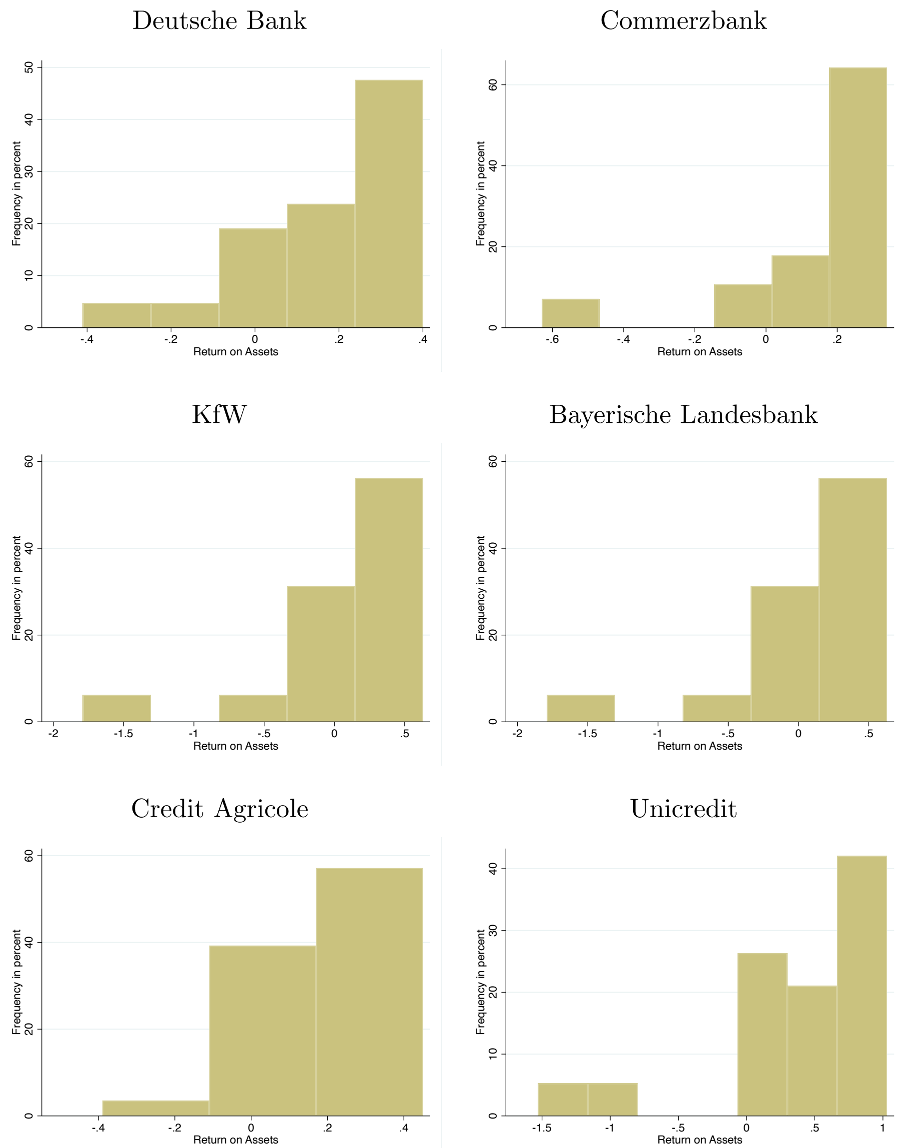


Notes: This figure shows the model-implied relationship between GDP at time t at different percentiles of the bank's rate of RoE at time $t-1$. GDP is defined as the output deviation from the mean. The figure is generated by simulating the model for 1,000,000 periods.

D.2: Empirical distribution of bank asset returns

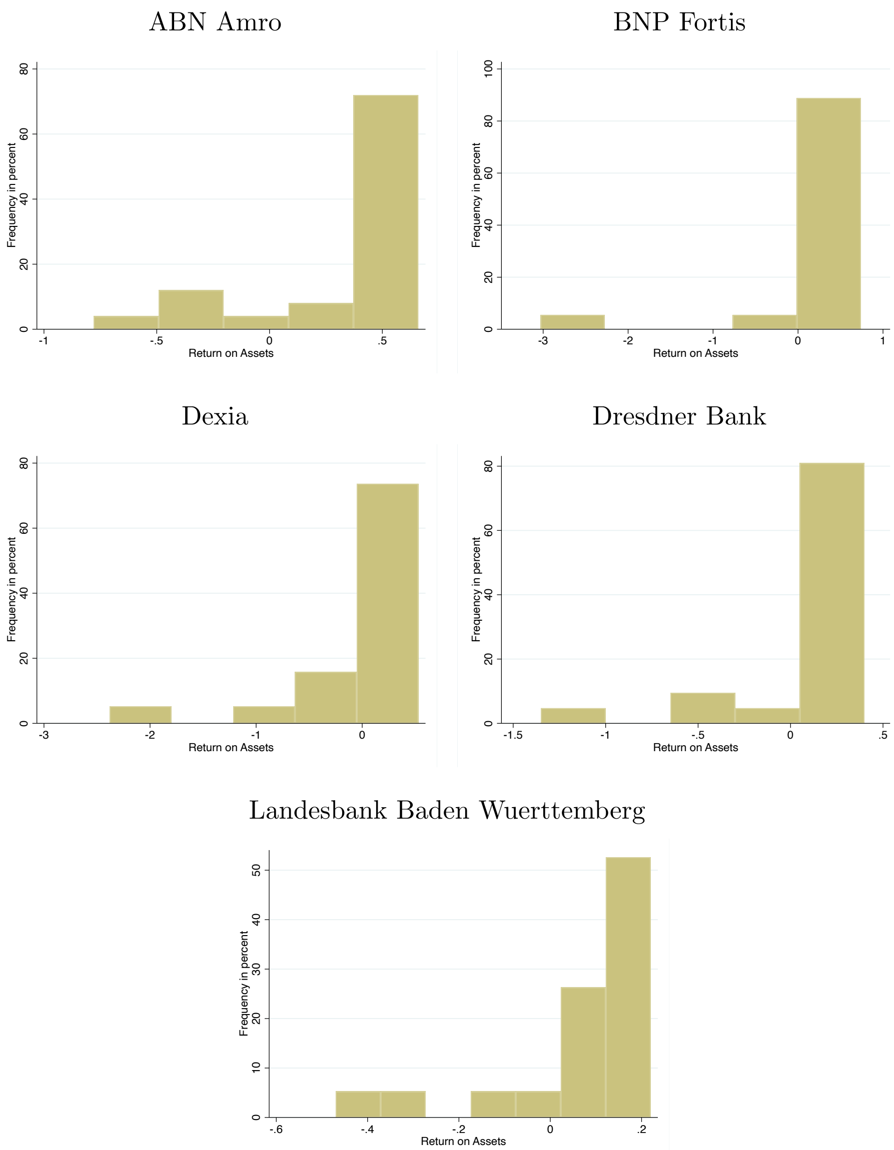
Here, we show that the distribution of asset returns for major European banks is asymmetric and left skewed as in our model

Figure D.2: Histograms of Return on Assets of individual European banks: Part 1



Notes: Return on Assets of major European banks.

Figure D.3: Histograms of Return on Assets of individual European banks: Part 2



Notes: Return on Assets of major European banks.

E Additional Model Results

Our model, with its endogenous connection between firm and bank solvency, features a number of idiosyncratic and aggregate risk shocks that are important for the transmission of firm defaults to bank defaults and to the macroeconomy at large. In this section, we investigate the importance of each of these shocks. We do this by removing them on an individual basis and then examining the extent to which this deteriorates the model's performance in replicating the quantile regressions in Section 2.1. We also show the importance of solving the model non-linearly by reporting the results that one would obtain by solving it using a first-order (instead of a third-order) approximation. Further, we also document non-linearities in the transmission of shocks in the model using generalized impulse response functions (GIRFs). We examine the importance of modeling bank asset returns in a structural way compared to the Merton-type model of banks. We also perform the sensitivity of our results on the optimal level of capital requirement to changes in the bank default costs and report the empirical distribution of bank asset returns.

E.1: Importance of the higher approximation order

Table E.1 reports the performance of the model solved with linear approximation methods (1st Order App.) in terms of untargeted conditional moments in the three default regimes. The frequency of the twin default regime reproduced by the linearized model is lower than the one observed in the data and in our baseline model.

The importance of the non-linearities introduced by the structural modeling of bank asset returns is underlined by the fact that, if solved to a first-order approximation, the model only generates twin default crises if hit by larger realizations of the island-risk shock. The dotted-dashed line in Figure E.2 shows that, in the first-order approximation, island-risk shocks need to increase by more than 2.5 standard deviations in the baseline. The thresholds used to define the three regimes are always the ones determined by the baseline model. Nevertheless, despite the large shocks, the first-order approximation cannot generate a realistic increase in the probability of bank default. This is consistent with the fact that, as shown in Table E.1, the linear model can only produce twin defaults with a 0.6% probability. In contrast, the frequency implied by the baseline model is 4.7%, which is much closer to the 7% observed in the data.

Finally, we investigate the role of our solution method by comparing the quantile regressions implied by our baseline model (which is solved using third-order approximation) with the quantile regressions implied by first-order (green lines) or second-order (blue lines) approximate solutions. We estimate the parameters of the first- and second-order approximation versions of our model to match the set of moments presented in Table 2. The bottom panels of Figure E.1 shows the results. Both the linear and the second-order models clearly fail to match the non-linearities found in the data. They generate flat quantile regression coefficients in both panels. Intuitively, a model solved to first or second order works well in normal times but fails to generate the sharp and non-linear deterioration of economic and financial conditions in crises or recessions. In contrast, a third-order approximation captures the non-linearity in the co-movements of firm and bank defaults and economic activity.

We have already discussed in Section 3. modeling bank portfolios as consisting of defaultable loans introduces an important non-linearity into bank asset returns and hence into bank default realizations. It is, therefore, natural that a non-linear solution method is needed to capture such non-linearities in an accurate manner. Our results show that a third-order solution is sufficient for this purpose.

E.2: Importance of the island-idiosyncratic and island-risk shocks

We start with the island-idiosyncratic and island-risk shocks. In our framework banks default when they experience abnormally low realizations of the island-idiosyncratic shock. Our model also allows aggregate fluctuations in the non-diversifiable (island) risk by means of island-risk shocks, i.e., shocks to the dispersion of the island-idiosyncratic risk. These shocks increase the probability of very low realizations of the island-idiosyncratic shocks, making banks more vulnerable.

The results of eliminating island-idiosyncratic and island-risk shocks are shown in the top panels of Figure E.1. The figure presents the quantile regression coefficients for Equations (21) and (22) for the model without island-idiosyncratic shocks (green line), i.e., when the island-idiosyncratic shock is set to one, and without the island-risk shocks (blue line). The red and black lines correspond to our baseline model and the data, respectively.[63]

The figure shows that both island-idiosyncratic and island-risk shocks are vital in generating a realistic sensitivity of bank default to firm default and of GDP growth to bank defaults. In the model without island-idiosyncratic shocks, the quantile regression coefficients go to zero because banks are perfectly diversified, and their loan portfolio returns are very stable. Firms continue to default because of the firm-idiosyncratic shocks, but banks are diversified against these shocks. And while aggregate shocks induce some fluctuations in firm default, these are too small to make banks fail since our banks' solvency is protected by their equity buffers. Thus, if the bank is fully diversified, bank defaults do not happen and cannot possibly affect GDP growth. The model without island-risk shocks shows that, although eliminating this shock does not lead to fully diversified banks, keeping the non-diversifiable risk (and hence the probability of bank default) low and relatively constant reduces the model's capability to match the sensitivity of bank default to firm default and of GDP growth to bank default. Clearly, the model without island-risk shocks, although it does a better job than the model without island-idiosyncratic shocks, fails to generate the state-dependent relationship between firm and bank defaults and economic activity that we

[63]Note that when we eliminate the island-idiosyncratic shocks, the island-risk shocks become irrelevant.

see in the data.

This experiment clearly indicates the importance of both island-idiosyncratic and island-risk shocks in generating realistic conditional and unconditional correlation patterns between firm and bank defaults and economic activity. When the non-diversifiable risk is constant (no island-risk shocks), bank defaults are rare, they are mostly unaffected by firm defaults, and they do not affect real economic activity. When the non-diversifiable risk is absent (no island-idiosyncratic shocks), banks do not default.

E.3: Importance of the firm-idiosyncratic and firm-risk shocks

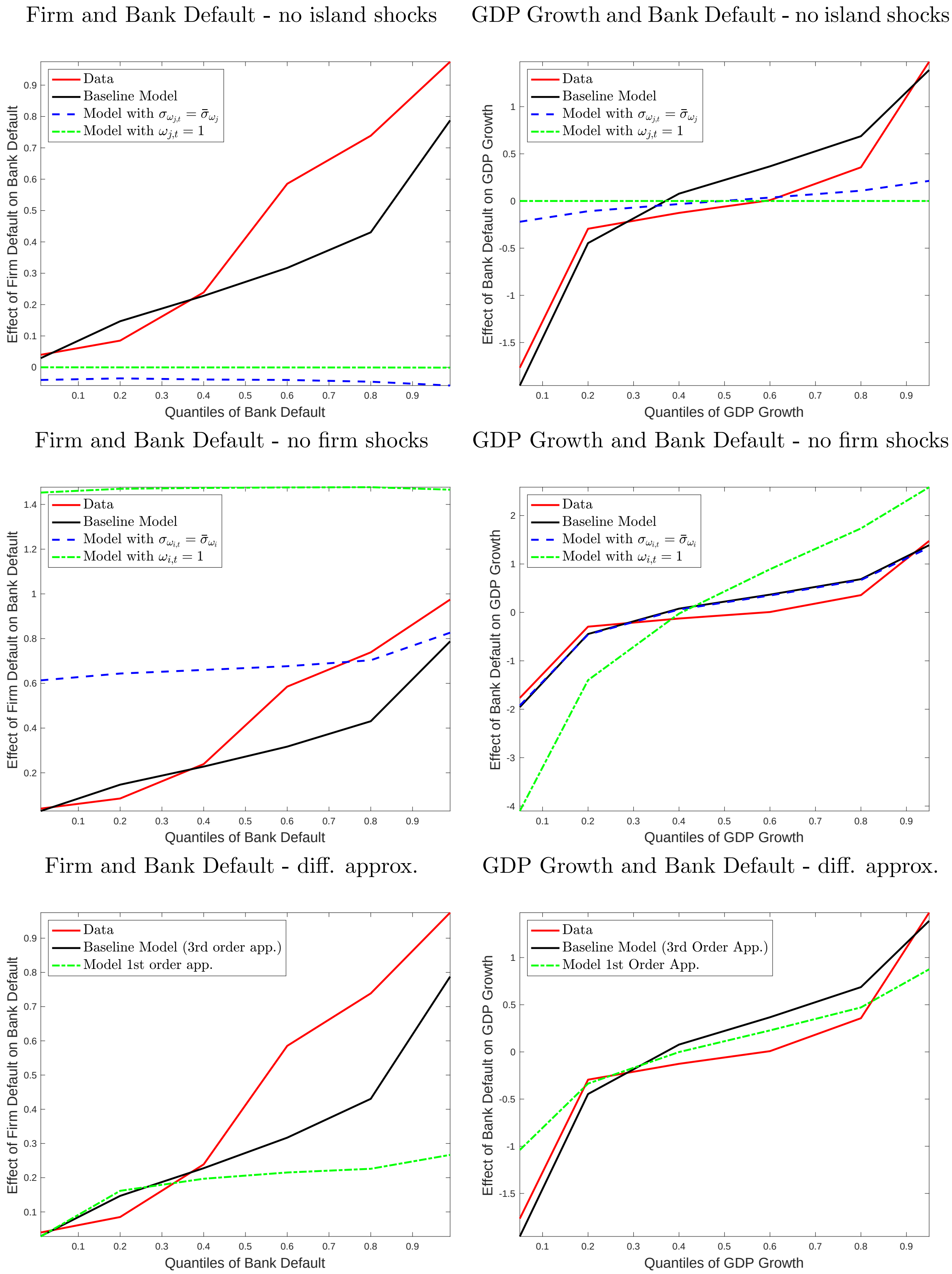
The other source of risk to firms in our model comes from firm-idiosyncratic and firm-risk shocks, i.e. shocks to the dispersion of the firm-idiosyncratic risk. These shocks capture risks to individual firms that are diversifiable at the individual bank level. Firm-risk shocks increase firm defaults, but they affect different banks evenly rather than concentrating the bulk of losses on a few unlucky banks, as is the case for island-risk shocks.

In this section, we investigate how the model's ability to replicate the quantile regression coefficients for Equations (21) and (22) changes when we eliminate the firm-idiosyncratic and -risk shocks. The middle panels of Figure E.1 show the results. This time, the green line displays the quantile regression coefficients in the model where we set the firm-idiosyncratic shock equal to unity for all firms, while the blue line presents the results from the model where firm-risk shocks are shut down.

Both the green and blue lines display a relationship between firm and bank defaults. Intuitively, the green lines in the middle panels of Figure E.1 correspond to an economy with fully non-diversified banks in which the defaults of banks and firms are almost perfectly correlated. This makes the sensitivity of bank default to firm default very large and rather constant over states. The impact of shutting down the firm-risk shocks is qualitatively similar to the elimination of the firm-idiosyncratic shocks but not as quantitatively large with respect to the quantile regression coefficients for Equation (21). The right middle panel shows that eliminating firm-idiosyncratic generates a state-dependent relationship between bank defaults and economic activity that is too strong compared both with the data and the implications of our baseline model.

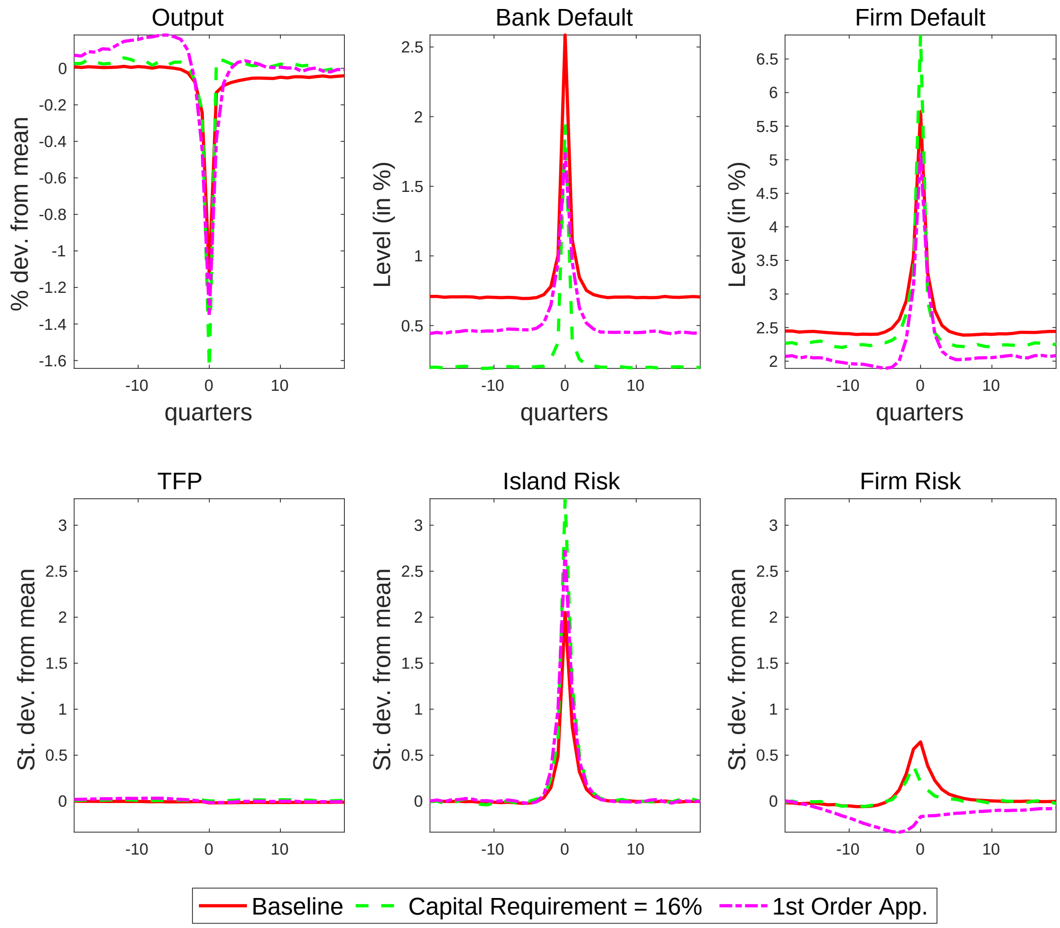
This experiment clearly indicates the importance of both firm and island shocks in generating realistic conditional and unconditional correlation patterns between firm and bank defaults and economic activity. When we eliminate non-diversifiable risk (no island shocks), the conditional and unconditional correlation between firm and bank default is too small. Instead, when we eliminate diversifiable risk (no firm shocks), the conditional and unconditional correlation between firm and bank default is too large. In both instances, the conditional and unconditional correlation between bank default and economic activity is too low.

Figure E.1: Quantile Regressions: Key Model Features



Notes: The figure explores the importance of non-diversifiable risk (top panels), diversifiable risk (middle panels) and approximation order (bottom panels). The left column presents coefficients ζ_τ from the quantile regression in Equation (21), while the right column presents coefficients β_τ from the quantile regression in Equation (22).

Figure E.2: Path to Twin Defaults in Different Scenarios



Notes: This figure shows the average path leading to a twin default episode under different model assumptions. Baseline (red solid line) corresponds to our baseline model with the capital requirement set to 8 percent ($\underline{\phi} = 0.08$) and solved with third-order perturbation methods. Capital Requirement = 16% (green dashed line) corresponds to the model with the capital requirement set to 16 percent ($\underline{\phi} = 0.16$). 1st Order App. (pink dashed-dotted line) corresponds to the model solved with a first-order perturbation method. The figure is generated by simulating the model for 1,000,000 periods, identifying periods of twin defaults, and then computing the average realizations of shocks and endogenous variables for twenty periods before and after the crisis periods. We define a twin default episode as the simultaneous occurrence of firm and bank default above their respective 90th percentiles. The 90th percentile default thresholds used to define the three regimes in the three models are always the ones determined by the baseline model. TFP, Island Risk and Firm Risk represent the level of A_t, $\frac{\sigma_{\omega_{j,t+1}}}{\bar{\sigma}_{\omega_j}}$ and $\frac{\sigma_{\omega_{i,t+1}}}{\bar{\sigma}_{\omega_i}}$ in their respective standard deviation units.

Table E.1: The Default Regimes in the Model

	Frequency	GDP growth	Bank default	Firm default
Low Default Regime				
Data	86.0%	0.0923	0.4346	2.3480
Baseline Model	84.7%	0.10111	0.5042	2.0568
Linear Model	95.9%	0.029577	0.40248	1.9874
Higher Cap. Req. Model	92.4%	0.051184	0.16423	2.0235
High Firm Default Regime				
Data	4.0%	-0.0466	0.4033	4.8500
Baseline Model	5.35%	-0.38327	0.87401	5.0714
Linear Model	3.26%	-0.64025	1.1559	4.6815
Higher Cap. Req. Model	7.31%	-0.58188	0.58711	5.2541
High Bank Default Regime				
Data	3.0%	-0.6744	2.1056	3.7604
Baseline Model	5.35%	-0.65094	2.2649	4.2974
Linear Model	0.239%	-0.69736	1.6654	3.7619
Higher Cap. Req. Model	0.0115%	-0.90339	1.7183	3.8298
Twin Defaults Regime				
Data	7.0%	-0.8189	3.0224	4.6076
Baseline Model	4.65%	-0.84251	2.5536	5.6439
Linear Model	0.643%	-0.90121	1.7428	5.0967
Higher Cap. Req. Model	0.327%	-1.4192	1.9443	6.8585

Notes: This table compares the model and data averages for firm default, bank default and GDP growth within three default regimes for the EA data and the simulated data from different models. The baseline model corresponds to a capital requirement set to 8 percent ($\phi = 0.08$) and solved with third-order perturbation. Merton-type Model corresponds to the model in which the Merton-type specification of bank asset returns is adopted. 1st Order App. corresponds to the model solved with first-order perturbation methods. Higher Cap. Req. corresponds to the model with a capital requirement set to 16 percent ($\underline{\phi} = 0.16$). Twin default episodes are defined as the simultaneous occurrence of firm and bank default above their respective 90th percentiles. High Firm Defaults are episodes with firm defaults above the 90th percentile and bank defaults below the 90th percentile. In Low Default episodes, both bank and firm default are below the 90th percentile. The default thresholds are used to define the three regimes in the Merton-type model and the 1st Order App. model are the ones determined by the baseline model. Model results are based on 1,000,000 simulations. GDP growth is demeaned.

E.4: Generalized Impulse Response Functions to an Island-risk Shock

We now use the Generalized Impulse Response Functions (GIRFs) to show that the economy "accelerates" into a twin default event as the impact of additional island-risk shocks grows. Following Koop, Pesaran, and Potter (1996), the GIRF for any variable in the model var in period $t + l$ following a disturbance to the n^{th} shock of size ν_n in period $t + 1$ is defined as

$$GIRF_{var}(l, \epsilon_{n,t+1} = \nu, \mathbf{w_t}) = \mathbb{E}\left[var_{t+l}|\mathbf{w_t}, \epsilon_{n,t+1} = \nu\right] - \mathbb{E}\left[var_{t+l}|\mathbf{w_t}\right], \tag{79}$$

where $\precapprox_t$ are the value of the state variables of the model at time t (The state variables of the model are $\mathbf{w_t} = \big(D_t, K_t, H_t, N_{e,t}, N_{b,t}, q_t, w_t, R_{f,t}, R_{d,t}, A_{t-1}, \sigma_{\omega_j,t-1}, \sigma_{\omega_i,t-1}\big)$) and $n \in \{A, \delta, i, j\}$. Hence, the GIRF depends on the value of the state variables when the shocks hit. For example,

$$GIRF_{\Delta \log Y_t}(4, \epsilon_{A,t+1} = -3, (1.1D_{ss}, 0.9K_{ss}, \dots, 1.01\bar{\sigma}_{\omega_i}))$$

is the GIRF of GDP growth, $\Delta \log Y_t$, at period $t + 4$, after a TFP shock of value -3 in period $t + 1$, when D_t was 10 percent above the steady state, K_t was 10 percent below the steady state, ... , and $\sigma_{\omega_i,t}$ was one percent above steady state.

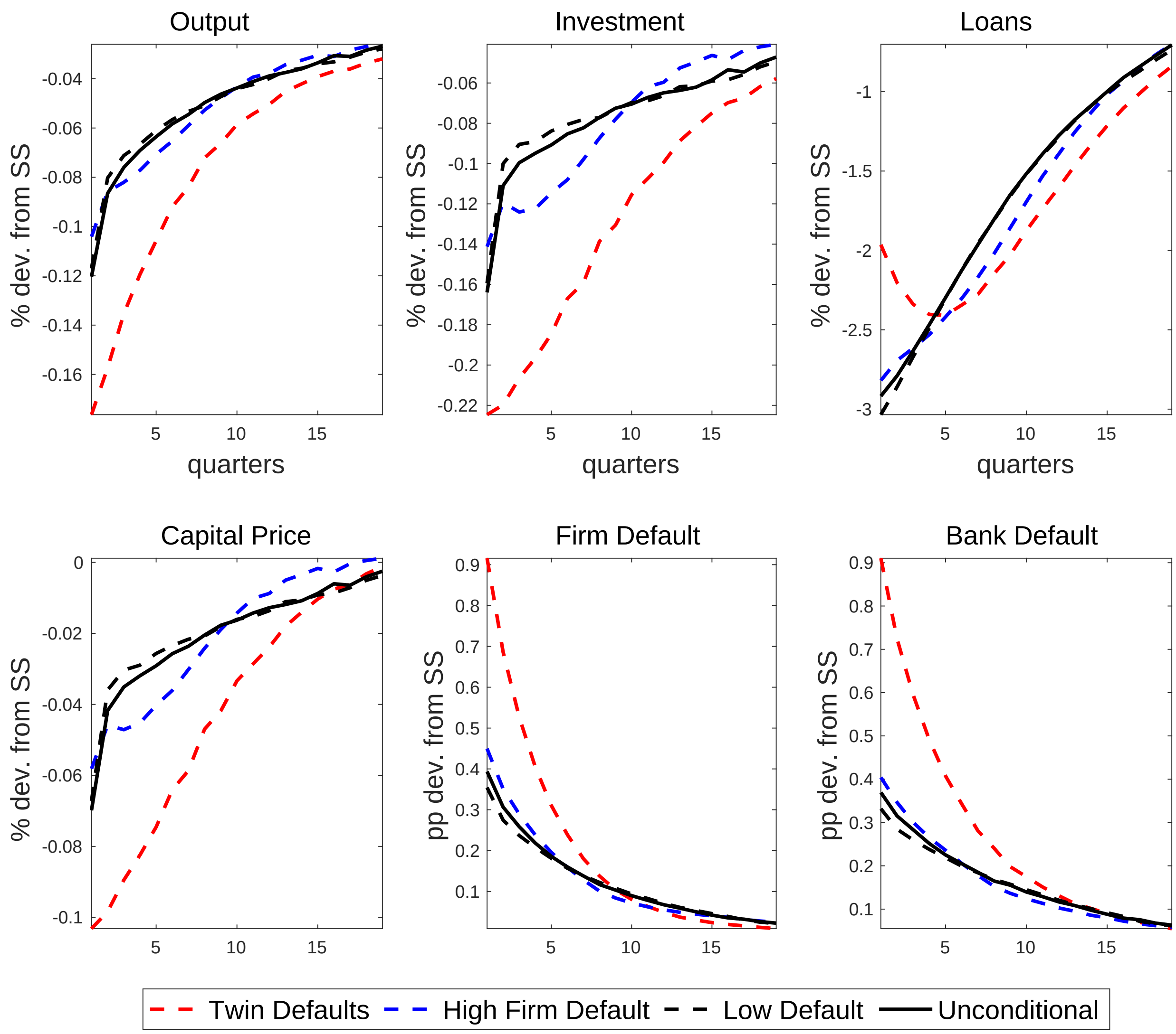
But GIRF defined in Equation (79) is conditioned on the value of the state variables when the shocks hit. In what follows, instead, we want to compute GIRFs that are conditioned on the values of observables when the shocks hit. For example, we would condition on the expected default rate of firms, $ED_{f,t}$, to be above one percent at the time of the shock. In this case, we want to compute the following GIRF

$$GIRF_{var}(l, \epsilon_{n,t+1} = \nu, ED_{f,t} > 0.01) = \int \mathbf{1}_{\{ED_{f,t}>0.01\}}\left(\mathbf{w_t}\right) GIRF_{var}(l, \epsilon_{n,t+1} = \nu, \mathbf{w_t}) f\left(\mathbf{w_t}\right) d\mathbf{w_t}, \tag{80}$$

where $\mathbf{1}_{\{ED_{f,t}>0.01\}}\left(\mathbf{w_t}\right)$ takes a value equal to one if the state variables at time t are such that he expected default rate of firms is above one percent at time t and zero otherwise and where $f\left(\mathbf{w_t}\right)$ is the unconditional density of the state variables. Of course, Equation (80) needs to be computed by simulation.

Figure E.3 reports three sets of GIRFs to a one standard deviation island-risk shock. The solid line shows the unconditional GIRF, the blue dashed line shows the GIRF conditional on the economy being at a high firm default episode, and the red dashed line shows the GIRF conditional on the economy being in a twin default episode. The internal propagation shown in the figure helps us understand why the model generates twin default crises without the need for huge shocks.

Figure E.3: Conditional Impulse Response Functions: Island Risk Shock



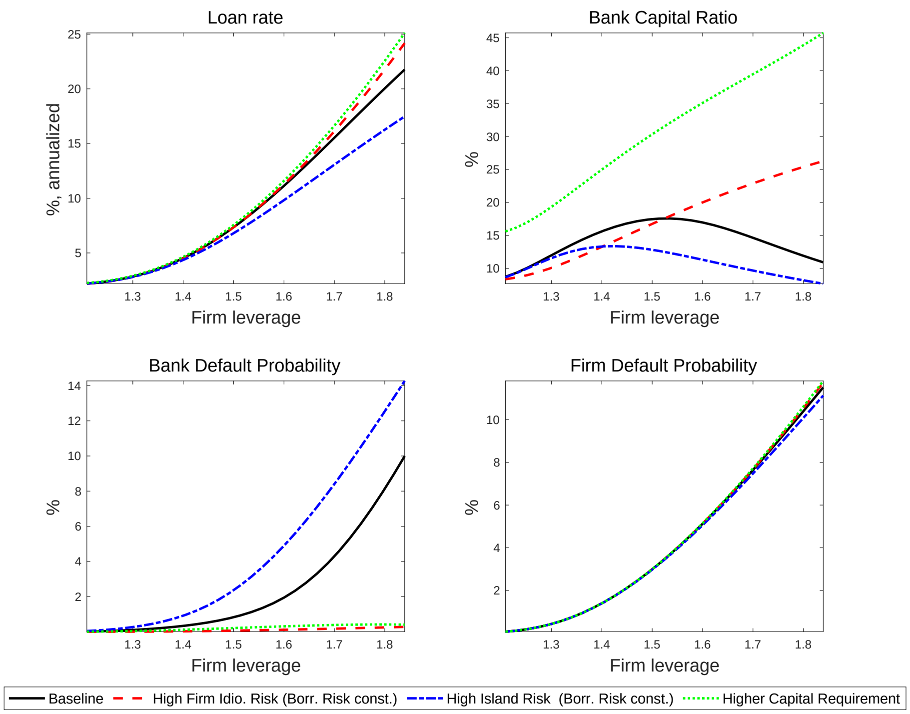
Notes: This figure reports three sets of generalized impulse response functions (GIRFs) to a one standard deviation island-risk shock. For comparability with other shocks, we set the persistence of each shock to 0.9. The solid black line shows the unconditional GIRF, the black dashed line shows the GIRF conditional on the economy being in a low default episode, the blue dashed line shows the GIRF conditional on the economy being at a high firm default episode, and the red dashed line shows the GIRF conditional on the economy being in a twin default episode. We define a twin default episode as the simultaneous occurrence of firm and bank default above their respective 90th percentiles. High firm default episodes are those where firm default is above the 90th percentile and bank default is below the 90th percentile. In the low default regime, both bank and firm default are below the 90th percentile.

The shock has a much larger impact when conditioning on a twin defaults.[64] The GDP drop is much larger than the effect in the unconditional GIRF. The same is true for the drop in investment and the price of capital and for the impact on firm and bank defaults. This shows how the model solved with a third-order approximation is able to amplify island-risk shocks during crisis times differently than during normal times. In our model, once the economy finds itself in a situation of twin defaults, it becomes very vulnerable to island-risk shocks.

[64] It is important to note that the traditional linear IRFs are independent of the state of the economy.

E.5: Determinants of Bank Risk Taking

Figure E.4: Determinants of Bank Risk Taking



Notes: This figure shows a representative bank's loan pricing schedule, i.e., the relationship that determines the interest rate that borrowing firms must pay for each given leverage choice. We also show the associated leverage choice by the bank on a partial equilibrium basis – i.e. assuming that all prices and funding costs, with the exception of the loan rate, are fixed to their steady-state values. The figure considers four scenarios: i) baseline case in which firm idiosyncratic and island risk are set to calibrated values and minimum capital requirement is 8% (black solid line), ii) higher firm idiosyncratic risk and lower island specific risk while keeping total borrower risk unchanged (red dashed line), iii) higher island risk and lower firm idiosyncratic risk while keeping total borrower risk unchanged (blue dash-dotted line), iv) higher minimum capital requirements of 15% (green dotted line).

In our model, bank failures are costly both because they cause deadweight costs ex post and because they create a limited liability/deposit insurance subsidy to bank risk-taking. To delve deeper into the mechanism that makes capital requirements a potentially welfare-improving policy tool in this set-up, we compare banks' loan pricing schedules (and other associated decisions) under a number of alternative settings in which the bank's decision is examined in partial equilibrium. In the top left panel of Figure E.4, we consider the loan rates that a perfectly competitive bank would apply (on a partial equilibrium basis) for different leverage choices of its borrowing firms. In other words, we consider the combination of loan rates and firm leverage that lie on the bank's participation constraint (equation (16) in the main text). The top right panel shows the bank's own capital ratio choice that would accompany each possible leverage (and loan rate) choice by the borrowing firms. The bottom row shows the resulting default probabilities for the bank (bottom left panel) and its borrowing firms (bottom right panel).

The black solid line is produced by varying firm leverage under the baseline calibration of the model. The upward-sloping loan rate schedule is typical of papers in the tradition of (see e.g. Bernanke, Gertler and Gilchrist, 1999), but a crucial difference is that the lending in our model is done by with limited liability and deposit insurance which distort their pricing of the underlying default risk. Starting from low levels of leverage, loan rates are first insensitive to changes in leverage as borrowers' probability of default is close to zero. Under our baseline calibration, further increases in firm leverage are associated with higher firm and bank defaults. The top right panel also shows that the bank's own capital ratio first increases as firms become riskier and banks deleverage in order to protect their charter value. However, this prudent behaviour by banks is reversed (i.e. the voluntary buffers on top of the regulatory minimum fall) once risk increases sufficiently.

To shed more light on banks' loan pricing behavior and leverage choice, it is useful to draw a distinction between the two sources of higher borrowers' riskiness in our model - firm idiosyncratic shocks and island shocks. Figure E.4 compares the baseline parameterization of the model with two other parameterizations that reduce the importance of the two sources of borrowers' riskiness, one at a time. The red dashed line depicts a version of the model where the variance of the firm idiosyncratic shocks is higher while the variance of island shocks is lower in a way that leaves overall firm riskiness unchanged. The blue dashed line corresponds to a version of the model in which the opposite is true - the island shocks have a higher variance at the expense of firm idiosyncratic shocks. The figure clearly indicates that higher firm leverage is associated with higher bank default only in the presence of significant island risk. Indeed, island-specific shocks make the returns of all firms in a given island volatile and, thus, *non-diversifiable* at the island/bank level. Higher firm leverage increases the probability that a large fraction of borrowers of an island-specific bank becomes ex-post insolvent. This increases the probability of banks becoming under-capitalized or failing.

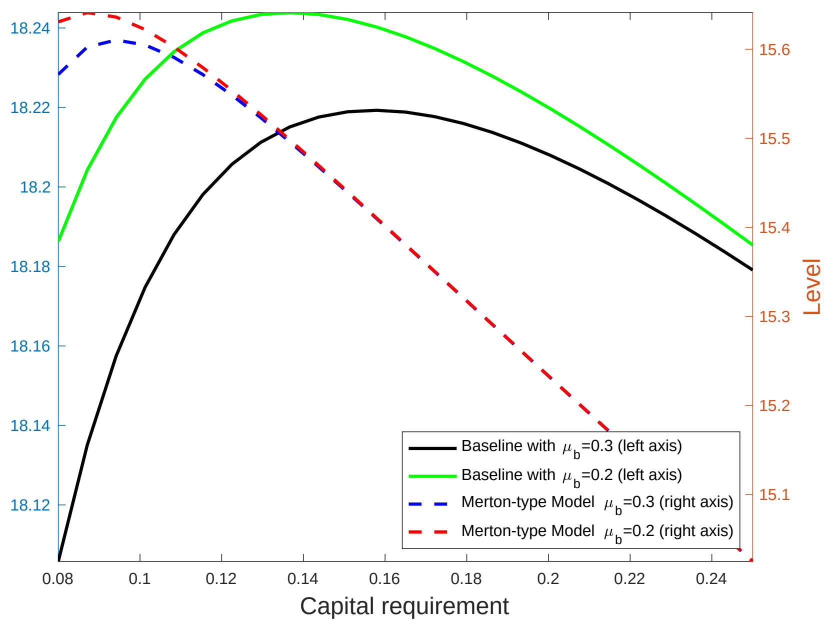
The top left panel of the figure shows that banks that are more likely to fail (the blue dashed line) display a greater tendency to under-price borrowers' risk. Banks set the interest rate on loans by maximizing the expected discounted value of returns on the portfolio of loans under limited liability – i.e. disregarding the losses (due to low loan returns) incurred when they default. As a result of this mispricing of risk, the unconditional expectation of the loan return declines with firm leverage (not shown in the graphs). When the loan pricing curve shifts downwards, it indicates that banks are happy to take more risk for a given expected return. The same message is corroborated by the behavior of the bank's capital ratio in the blue dashed line (high volatility of island risk). Despite facing a greater risk than in the baseline, banks choose a lower capital ratio which is another indication of risk shifting.

The situation is very different when the increase in leverage happens when island risk is lower, as shown in the red dashed line. Higher firm leverage increases the probability of individual borrowers' default and hence reduces the returns obtained by the bank on the corresponding loans. The bank's reaction indicates no risk shifting. The loan rate is increased sharply to protect banks from the higher risk of losses on individual loans. The bank's capital ratio now actually increases with higher firm riskiness to make sure they can absorb the potentially larger losses. Because island risk is low, banks have an almost zero probability of failing, Instead, they worry about becoming under-capitalized and having to raise costly equity to bring the bank back to full compliance with the minimum capital ratio. The bank wants to avoid this, which is why it increases the ex-ante capital ratio. With a zero probability of failure but a positive probability of becoming under-capitalized, small increases in risk actually make banks more risk-averse, leading them to tighten lending standards and deleverage. This is in contrast with its risk-shifting behavior under high levels of island risk.

Finally, Figure E.4 also shows the behavior of the model under the baseline level of island and idiosyncratic risk but under a higher minimum capital requirement (15 percent). The tighter regulation forces the bank to increase its capital ratio, thus bringing its failure probability close to zero. Risk shifting disappears, and high-leverage/high-risk loans are no longer under-priced. Moreover, the bank now responds to an increase in firm leverage by increasing its capital ratio to protect itself from the risk of becoming under-capitalized due to bad loan return out-turns. This demonstrates a clear benefit of higher capital requirements: it makes banks safer, thus helping to avoid deadweight default losses, but it also improves their incentives to price risky loans appropriately.

E.6: Sensitivity to the size of deadweight costs of bank failure: μ_j

Figure E.5: Welfare Effects of the Capital Requirement: sensitivity to μ_j

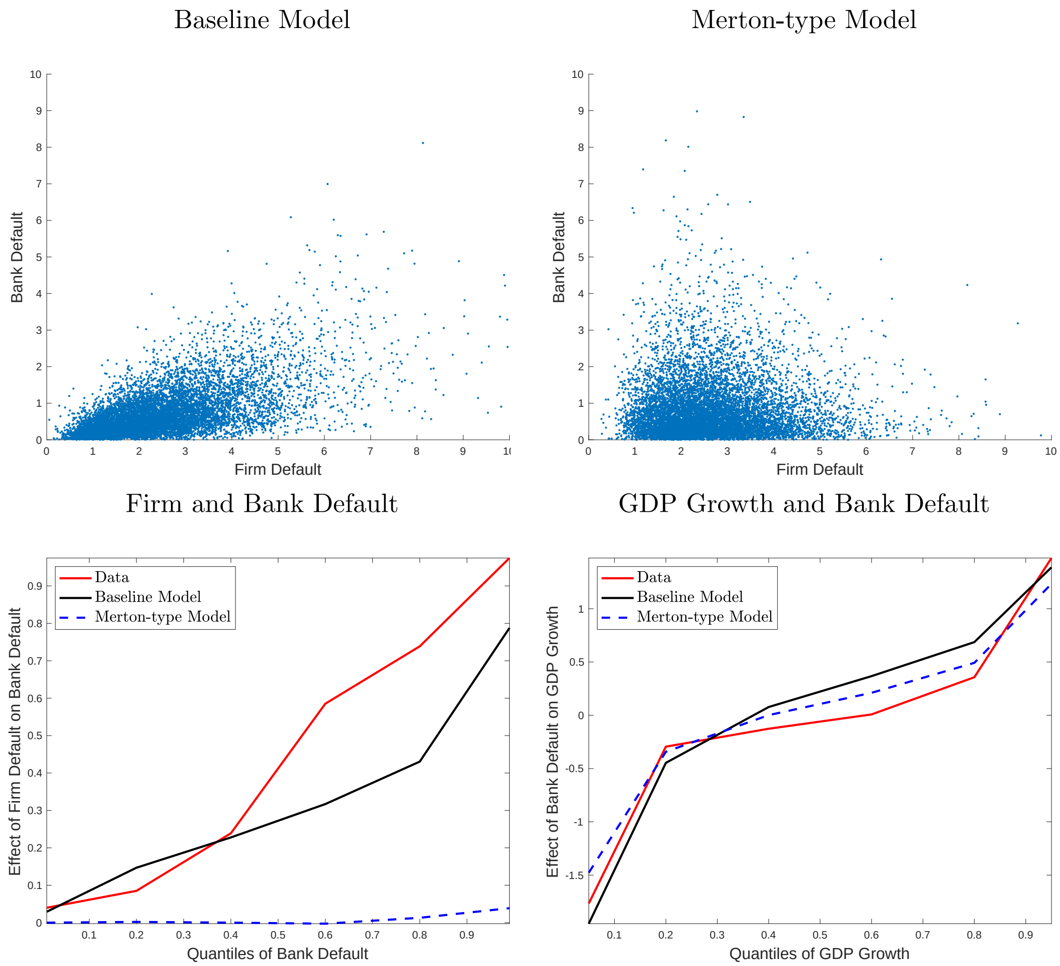


Notes: Ergodic mean of household welfare under different values of μ_j.

In the main body of the paper, we showed that the model implies an optimal capital requirement of 16 percent under the baseline calibration. In this section, we examine the sensitivity of the optimal capital requirements to the size of deadweight costs of bank failure (μ_j). Figure E.5 compares expected household welfare under the baseline value of $\mu_j = 0.3$ (black solid line) and under a lower value of $\mu_j = 0.2$ (green solid line). We see that when deadweight losses from bank default are lower, the optimal bank capital requirement declines to 14 percent. The optimal capital requirement in the Merton model (dashed lines in the Figure) is also affected by μ_j falling to 9 percent when $\mu_j = 0.2$.

F Additional Results Merton-type Model

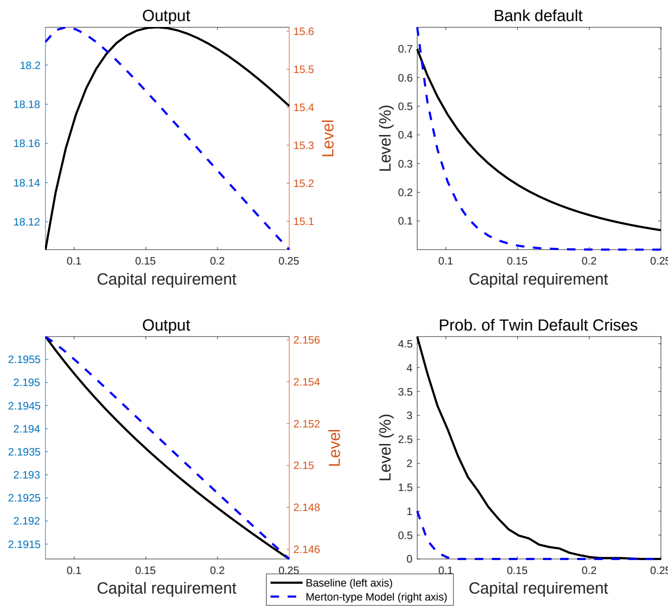
Figure F.1: Scatter Plots and Quantile Regressions: Baseline vs Merton-type Model



Notes: The top panels display the scatter plot of firm and bank default produced with the baseline model (top left plot) and the Merton-type version of our model (top right). The bottom left panel presents coefficients ζ_τ from the quantile regression in Equation (21), whereas the bottom right panel presents coefficients β_τ from the quantile regression in Equation (22) for the Merton-type model (blue line), the data (red line) and the baseline model (black line).

Characterizing bank asset returns in an accurate manner is essential when studying the relationship between firm and bank defaults. The Merton-type model fails to reproduce the non-linearity in the relationship between firm and bank defaults along several dimensions. The top right panel of Figure F.1 presents a scatter plot of firm and bank defaults implied by the Merton-type model, which uses $\tilde{R}^M_{f,t+1}(\omega_b)$ instead of $\tilde{R}_{f,t+1}(\omega_j)$. Contrary to what is implied by our model (top left panel), the standard Merton-type representation of bank asset returns implies a very low correlation between firm and bank failures.[65] Our mechanism treats instead the two defaults as intimately linked, endogenously generating an empirically realistic positive relationship between them.

Figure F.2: Comparative Statics with Respect to Capital Requirement Level for baseline model and for Merton-type model



Notes: This figure shows the implications of different values of the capital requirement $\underline{\phi}$ on the mean of the ergodic distribution of selected variables for our baseline and Merton-type model.

Another way to examine the relationship between firm and bank defaults is through quantile regressions. The bottom panels of Figure F.1 compare the quantile regression coefficients for Equations (21) and (22) in our model (black line) to those obtained from its Merton-type variant (blue line). For completeness, we also include the estimated coefficient using EA data (red line). Yet again, in the Merton-type model, the relationship between firm and bank defaults is very weak at all quantiles of bank default. The standard approach to bank default risk also fails to match the relationship between GDP growth and bank default at both the top and the bottom quantiles of GDP growth.

[65] Following the existing literature, heterogeneity in bank default outcomes is generated by adding exogenous idiosyncratic disturbances to banks' asset values directly rather than to the performance of banks' borrowers. If such shocks follow log-normal distributions, bank asset returns are distributed exactly as in Merton (1974). The standard reduced-form approach implies that banks can default when borrowers repay in full. This generates a close to zero correlation between the default rates of banks and their borrowers.