Liquidity standards and the value of an informed lender of last resort*

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Abstract

We consider a dynamic model in which receiving support from the lender of last resort (LLR) may help banks to weather investor runs. We show the need for regulatory liquidity standards when the underlying social trade-offs make the uninformed LLR inclined to support troubled banks during a run. Liquidity standards increase the time available before the LLR must decide on supporting the banks. This facilitates the arrival of information on banks' financial condition and improves the efficiency of the decision taken by the LLR, a role that can be reinforced but not replaced with the use of capital regulation.

Keywords: Liquidity standards, lender of last resort, bank runs

JEL Classification: G01, G21, G28

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1 Introduction

Prior to the Great Recession, the focus of bank regulation was on bank capital. However, the liquidity problems that banks experienced since the onset of the financial crisis in 2007 brought to the forefront a debate about the potential value of regulating banks' liquidity.[1] Those problems also reignited the debate on the challenges that uncertainty about the financial condition of banks pose to the lender of last resort (LLR).[2] In this paper, we contribute to these debates by presenting a novel theory of banks' liquidity standards.

Our theory builds on what we believe is a distinctive feature of an instrument such as the liquidity coverage ratio (LCR) of Basel III:[3] Once a crisis starts, liquidity buffers provide banks the capability to autonomously accommodate potential debt withdrawals for some time. Having time to resist without LLR support is valuable; it allows for the discovery of information on the bank's financial condition that is useful for the LLR's decision on whether to grant support or not. This generally improves the efficiency of the decisions regarding the continuation of the bank as a going concern or its liquidation and, on occasions, allows for a resolution of the crisis without explicit intervention of the LLR.

We consider a model in which a bank ex ante decides how to allocate its funds across liquid and illiquid assets. Illiquid assets are ex ante more profitable than liquid assets but their quality is vulnerable to the realization of an interim shock to the bank's financial condition. If assets get damaged by the shock, the bank turns fundamentally insolvent and its early liquidation is efficient.[4] In contrast, if assets do not get damaged, the bank remains fundamentally solvent and its early liquidation is inefficient. A crucial problem is that discerning whether the assets are damaged or not may take time.

The bank is funded with equity and short-term debt, and faces rollover risk because each

[1] See Gorton (2009) and Shin (2010) for a discussion on the role of banks' liquidity problems during the most recent financial crisis.

[2] Bagehot (1873) advocates that central banks should extend liquidity support to banks experiencing liquidity problems provided they are solvent. However Goodhart (1999) argues that the feasibility of establishing a clear-cut distinction between illiquidity and insolvency on the spot is a myth.

[3] For a description of the LCR, see Basel Committee on Banking Supervision (2010).

[4] Early liquidation may help reverse unprofitable investment strategies, stop "evergreening" strategies with respect to a portfolio of bad loans or any other form of gambling for resurrection.

period a portion of investors are entitled to decide whether to rollover their short-term debt.[5] Under these conditions, the shock to the bank's financial condition can trigger a run among these investors, which if sustained for long enough, may lead the bank into failure, unless it can borrow from the LLR. In making its lending decision, the LLR faces the classical problem that the bank seeking liquidity support might be fundamentally insolvent. While it is optimal to grant liquidity to solvent banks, early liquidation would be preferable in the case of an insolvent bank.

In general, assessing the financial condition of the bank in real time is quite difficult. Following this view, we assume that the LLR is initially uncertain about the financial condition of the bank (the quality of its illiquid assets) but may obtain the relevant information over time. Thus, liquidity standards, which lengthen the time a bank can sustain a liquidity shock without outside support, allow for more information on the bank's financial condition to come out prior to the LLR decision on whether to extend its emergency lending to the bank. Such information is valuable because it improves the efficiency of the implied continuation versus liquidation decision regarding the bank's illiquid assets.[6] Our model, therefore, shows that postponing the time at which the bank needs liquidity support from the LLR may be conducive to a more efficient resolution of the crisis.

Our model also shows that, when the potential support received from the LLR involves positive implicit subsidies, the liquidity standards voluntarily adopted by bank owners may be lower than those that a regulator might like to set.[7] Specifically, if bank owners expect

[5] We focus on short-term debt different from retail demand deposits that are typically protected with deposit insurance and, hence, more stable than certain categories of wholesale debt financing such as interbank loans, commercial paper, repos, etc.

[6] For some parameter values, liquidity standards may help sustain what we define below as a *late run equilibrium*, in which investors do not start running right after the shock to the bank's financial condition, but only when further news confirm its illiquid assets to be bad. Intuitively, liquidity standars reduce investors' incentives to run by increasing their prospect of recovering value out of their debt claims when the bank's assets turn out to be damaged. Under these circumstances, it is more likely that the crisis self-resolves without the intervention of the LLR and in the most efficient terms regarding the continuation versus liquidation of the bank's illiquid assets.

[7] In Section 7.1 we show that, for certain classes of (modestly strong) banks, supporting the bank is overall efficient but cannot be made on an ex post break-even basis, so it must necessarily involve some degree of subsidization. In this sense, it might be argued that in the baseline case, the LLR consolidates two forms of government support: (unsubsidized) emergency lending and (subsidized) capital support. This does not undermine the interest of the analysis since most bank crises involve both types of support.

support to be granted if the LLR remains uninformed about the quality of the assets once the bank exhausts its cash, they may prefer to opportunistically hold less liquidity than it would be socially optimal. By doing this, they shorten the spell over which the bank can resist the run without support and, thus, the chances of being supported by the LLR. In this case, introducing a minimal regulatory liquidity standard can increase overall efficiency relative to the laissez faire benchmark.

Although some of the key trade-offs of our analysis could be illustrated in reduced form using a standard banking model with three dates, our model is formulated in continuous time. This allows us to study explicitly the implications of liquidity buffers during the course of a run. During a run, the gradual withdrawal of funds by the holders of maturing debt reduces the bank's residual capacity to resist the run, while simultaneously the discovery of relevant information about the quality of its illiquid assets (modeled as a Poisson process) may arrive and either lead the run to self-resolve or render the LLR better informed when called to act. This modeling also allows us to formally examine the impact of liquidity on debtholders' incentive to rollover their debts or not taking into account the subsequent unfolding of the run, as in the dynamic runs studied by He and Xiong (2012).

In the main extension of the baseline model, we endogenize the bank's capital structure and the impact on its fundamental solvency, which allows us to uncover some interesting interactions between liquidity standards and capital requirements. We show that capital requirements reinforce solvency but are not a perfect substitute for liquidity standards in our setting. In another extension, we show that requiring the LLR to lend on an expected break-even basis has important implications, but it does not eliminate the rationale for liquidity standards that we put forth. Yet in another extension, we show that allowing the LLR to extend temporary liquidity assistance to allow for information on the bank's financial condition to be revealed does not necessarily improve upon the outcomes obtained when LLR support is treated as irreversible.

Until recently, there was no consensus among policy makers about the need for liquidity regulation. This was in contrast with an existing body of academic research that pointed to the existence of inefficiencies in worlds with a strictly private provision of liquidity, via either interbank markets (Bhattacharya and Gale 1987) or credit line agreements (Holmström and Tirole 1998). A common view was that liquidity regulation was costly for banks in spite of results pointing to its welfare enhancing effects, e.g. by reducing fire-sale effects in crises (Allen and Gale 2004) or the risk of panics due to coordination failure (Rochet and Vives 2004). Another view was that the effective action by the LLR rendered liquidity standards unnecessary.[8] There was also the view that, although the financial system was vulnerable to panics (Allen and Gale 2000), there were positive incentive effects of the implied liquidation threat (Calomiris and Kahn 1991, Chen and Hasan 2006, Diamond and Rajan 2005).

The severity of banks' liquidity problems during the recent crisis led to a consensus among policy makers about the need to introduce some form of liquidity regulation for banks.[9] Those problems also motivated new academic papers analyzing bank liquidity standards. Perotti and Suarez (2011), for example, rationalize liquidity regulation as a response to the existence of systemic externalities and analyze the relative advantages of price-based vs. quantity-based instruments. Calomiris, Heider, and Hoerova (2014) show that liquidity requirements may substitute for capital requirements in a moral hazard setup. Diamond and Kashyap (2016), in turn, show that liquidity holdings help deter runs that might otherwise occur as a probabilistic sunspot equilibrium and defend the need for a regulator that continuously monitors banks' liquidity positions. These studies, however, do not rationalize the specific time-dimension of requirements such as the LCR of Basel III and do not discuss the interaction between liquidity regulation and the provision of emergency liquidity by the LLR.

We contribute to close this gap in the literature with a theory that relies on a novel way of thinking about liquidity requirements – an instrument that, by making banks better able to withstand the initial phases of a crisis, allows the LLR to be better informed when it gets called into action.[10] Our paper is also related to the literature on the value of commitment

[8] See Flannery (1996), Freixas, Parigi and Rochet (2000), and Rochet and Vives (2004).

[9] Banks' liquidity problems appear to have started in the summer of 2007 following the collapse of the asset-backed commercial paper (ABCP) market. These problems grew larger with the collapse or near collapse of several other markets, including the repo and the financial commercial paper markets, and even several segments of the interbank market, and with banks' shortages of collateral in part due to downward spirals in market and funding liquidity (Brunnermeier and Pedersen 2009).

[10] Nosal and Ordoñez (2013) describe a setup in which a government delays intervention in order to learn

to be tough in the context of lending of last resort or bank rescue policies (Mailath and Mester 1994, Perotti and Suarez 2002, Repullo 2005, Acharya and Yorulmazer 2007, 2008, Ratnovski 2009, Farhi and Tirole 2012, and Chari and Kehoe 2013), and to Acharya and Thakor (2016) who argue that the prospects of (unconditional) LLR support undermine investors' incentives to generate information about banks. We add to this literature by analyzing how liquidity holdings and liquidity requirements allow LLR support to be based on better information and, thus, less frequently unconditional, hence leaving a larger residual role for market discipline.

Finally, our paper is related to the literature that studies investors' incentives to run on banks, including Diamond and Dybvig (1983), Jacklin and Bhattacharya (1988), Goldstein and Pauzner (2005), He and Xiong (2012), and He and Manela (2014). We share with He and Manela (2014) the modeling of a slow moving run that may be reversed by the arrival of good news, but instead of focusing on investors' incentives to acquire information along the run, we introduce a LLR and examine the impact of banks' cash holdings on the timing and efficiency of its support decisions.

The rest of the paper is organized as follows. Section 2 introduces our dynamic model of runs. Section 3 analyzes several issues relevant for solving the model: the effects of information on crisis resolution, the time a bank can resist a run, and debtholders' expected payoffs in case of early liquidation. Section 4 characterizes the *early run equilibrium*: the situation in which investors start canceling their debt immediately after the shock to the bank's financial condition. Section 5 considers social welfare and the rationale for liquidity standards when such an equilibrium is anticipated. Section 6 presents the case in which the bank's capital structure is endogenous and liquidity standards interact with capital standards. Section 7 discusses two other extensions. Section 8 concludes the paper. All proofs are in the Appendix. The Online Appendix analyzes the possibility of sustaining other types of equilibria in greater detail, expands the analysis on the determinants of optimal liquidity standards, and discusses some additional extensions of the baseline model.

more about the systemic dimension of a crisis. Their analysis focuses on the strategic interaction between banks, which can restrain from risk taking in order to avoid getting into trouble earlier than their peers, i.e. at a time in which the government is still not supporting the banks in trouble.

2 The model

Consider a continuous time model of an individual bank in which time is indexed by $t \in \mathbb{R}$. There are three classes of agents: bank owners, investors, and a lender of last resort (LLR). All agents are risk neutral and discount future payoffs at a zero rate. Bank owners and investors care about the expected net present value of their own payoffs. The LLR is a benevolent maximizer of total expected net present value, with proper consideration of the cost of the subsidies embedded in its lending activity. The model puts particular focus on what happens to the bank after some shock arriving at $t = 0$ weakens its perceived solvency.

The bank exists from a foundation date, say $t = -1$. At that date, the bank's initial owners invest in assets of total size one, issue debt and equity among competitive investors, and, hence, appropriate as a surplus the difference between the value of the securities sold to the investors and the unit of funds needed to start up the bank.

2.1 Assets and liabilities of the bank

The assets of the bank consist of an amount C of a liquid asset (*cash*) and an amount $1 - C$ of *illiquid assets*. The illiquid assets pay some potentially risky per-unit final return equal to $\tilde{a}$ at termination and to $\tilde{q}$ in case of early liquidation. Early liquidation is feasible at any date prior to termination but cannot be partial.

The debt issued by the bank at t=–1 is uniformly distributed among a measure-one continuum of debtholders. Each debtholder is given the option to "put" her debt back to the bank in exchange for an early repayment of D at some exercise dates over the life of her contract. Alternatively, investors can obtain a late repayment $B = (1 + b)D$, with $b > 0$, at termination. Debt puttability is a convenient way to make investors face rollover decisions and banks face rollover risk similar to those that would emerge in a more complex environment with overlapping issues of short-term debt with fixed maturity.[11]

To facilitate tractability, we assume that both the illiquid assets and the uncanceled debt

[11] This debt is not intended to represent demand deposits, whose stability is in practice guaranteed by the existence of deposit insurance, but the short-term wholesale debt at the root of liquidity problems during the 2007-2008 Global Financial Crisis.

of the bank mature at $T \to \infty$, which is a practical way to capture "the long run" in this model.[12] We also assume that debtholders' chances to put their debt arrive according to independent Poisson processes with intensity δ, so that $1/\delta$ can be interpreted as the average maturity of bank debt.[13] This parameter will determine the speed of the run and, therefore, the length of time the bank can survive the run with its available cash.

2.2 Sequence of events after $t = 0$

To focus the analysis of the model on the possibility of bank runs, and the way the bank and the LLR cope with them, we assume that the bank has a quiet life between dates t=-1 and t=0. So the bank keeps the same debt and liquidity as initially chosen. At $t = 0$, there is a probability ε that the bank suffers a shock that impairs the quality of its illiquid assets, otherwise its life continues quiet forever.

The illiquid assets of the bank can be good (g) or bad (b). The final per-unit returns of good and bad assets are a_g and a_b, respectively, and their per-unit liquidation returns are $q_g < 1$ and $q_b < 1$, respectively. In the absence of the shock, assets are good with probability one. But when the shock hits, assets are good with probability μ and bad with probability $1 - \mu$. We assume:

$$a_b < q_b \leq q_g < D < B < a_g, \tag{1}$$

so that the efficient continuation decision depends on the quality of the assets and gets compromised by the possibility of runs. Specifically, a good bank that invests only in risky assets ($C = 0$) is *fundamentally solvent* at termination ($a_g > B$), and its assets are worth more if continued than if early liquidated ($a_g > q_g$). In contrast, a bad bank that invests only in risky assets is *fundamentally insolvent* at termination (since $B > a_b$), and its assets are worth more if early liquidated ($q_b > a_b$). By continuity, these properties remain true

[12] Equivalently, we can think of both assets and debt maturing randomly according to Poisson arrival processes with intensities going to zero (so that their expected life-spans go to infinity), which means that any other arrival process with positive intensity will arrive earlier on with probability one.

[13] One can think of the puttability of bank debt as a feature that under "normal circumstances" allows investors to cease their investment in the bank for idiosyncratic reasons (that the model abstracts from). In those circumstances, the bank would have no problem in simply replacing the exiting debtholders with new debtholders who would buy debt identical to the one canceled. See Segura and Suarez (2017) for a model with this type of recursive refinancing structure.

when the bank holds liquidity ($C > 0$) insofar as C is not very large.

To formally guarantee this, we assume that C is always such that

$$a_g(1 - C) + C \geq B, \tag{2}$$

so that the good bank is fundamentally solvent, but

$$q_g(1 - C) + C < D, \tag{3}$$

so that it is vulnerable to runs. For further use, let $\bar{C}$ be the highest value in the interval $[0, 1]$ compatible with (2).

If the shock hits the bank at $t = 0$, debtholders' decisions regarding the exercise of their put options become non-trivial. If they start exercising their put options, the bank will begin consuming its cash holdings. Once the bank runs out of cash, the LLR decides whether to support it ($\xi = 1$) or not ($\xi = 0$). If the bank gets supported, all the residual debtholders are paid D.[14] Otherwise, the bank is forced into liquidation and its liquidation value gets proportionally divided between the residual debtholders.

When the bank is hit by the shock at $t = 0$, a process of potential revelation of the true quality of its illiquid assets starts. We assume that the arrival of news publicly revealing such quality follows a Poisson process with intensity λ. The speed of arrival of information implied by λ and the speed of the run implied by δ determine the buying time effectiveness of cash holdings. As shown below, under our assumptions, learning that the illiquid assets are good at any time before the cash gets exhausted leads the crisis to self-resolve because the LLR is willing to support the bank if it runs out of cash (since $a_g > q_g$) and, in anticipation of this, debtholders no longer find optimal to redeem their debt. So the bank (efficiently) continues with its illiquid assets until termination. In contrast, when the news is bad, exercising their puts is a dominant strategy for debtholders. The bank eventually runs out of cash, the LLR does not support it (since $a_b < q_b$), and the illiquid assets end up (efficiently) liquidated.

[14] In equilibrium, the LLR will only assist the bank when the quality of its assets remains unknown at the time of the intervention. To justify why debtholders recover just the early repayment D (rather than the termination payoff B), we can assume that market signals about the quality of the illiquid assets become uninformative after the LLR intervenes and that debtholders, afraid of ending up getting less than D at maturity, keep exercising their put options until getting rid of all their debt.

The decision of the LLR is less trivial when the quality of the illiquid assets remains uncertain after the bank runs out of cash. In this scenario, the LLR has to decide by comparing the *expected* continuation value of the illiquid assets, $\bar{a} = \mu a_g + (1 - \mu)a_b$, and their *expected* liquidation value, $\bar{q} = \mu q_g + (1 - \mu)q_b$. Thus, the bank is supported if and only if

$$\mu > \bar{\mu} = \frac{q_b - a_b}{(a_g - q_g) + (q_b - a_b)}. \tag{4}$$

We will refer to the *strong bank* case and *weak bank* case depending on whether (4) holds or not. In both cases, the continuation vs. liquidation decision made by the LLR in the absence of news about asset quality is, with some probability, less efficient than the one attainable if the news had arrived on time.

In the baseline case analyzed below, emergency lending is assumed to be made at the zero risk-free rate. In Section 7.1 we extend the analysis to the case in which the LLR is only allowed to lend on an expected break-even basis and show that in that case having $\mu > \bar{\mu}$ is not a sufficient condition to guarantee support, implying that some *modestly strong* banks (with $\mu \in (\bar{\mu}, \hat{\mu})$ for some $\hat{\mu} \in (\bar{\mu}, 1)$) would be inefficiently liquidated.

2.3 Strategy for the analysis

To simplify the exposition, the core of our analysis focuses on the case in which the realization of the shock at $t = 0$ gives rise to an *early run (ER) equilibrium*: the situation in which debtholders start exercising their puts from $t = 0$. After establishing conditions that guarantee the existence of this equilibrium, we will discuss the impact of the bank's liquidity C and the expected intervention of the LLR on equilibrium outcomes.[15]

We will then move backwards, to discuss the trade-offs regarding the choice of the liquidity holdings C at $t = -1$ from the perspective of both the LLR (ex ante social welfare) and the initial owners (ex ante total market value of the bank). The choice of C is supposed to be observable at $t = -1$ prior to the issuance of the bank's debt, meaning that bank owners when deciding on C take into account the impact of this variable on the issuance value of

[15] An alternative equilibrium configuration in which debtholders only start exercising their put options when further news confirm that the illiquid assets of the bank are bad is discussed in Appendix A.

debt and the residual value of equity.

To keep the analysis simple, we first treat the capital structure of the bank, as defined by D, b, and δ, as exogenously given. In Section 6 we assume that the initial owners have some initial wealth $w < 1$ with which they can provide (inside) equity funding $k \leq w$ to their bank, financing the rest of the initial assets, $1 - k > 0$, with the puttable debt described above. This allows us to endogenize D as the result of initial owners choice of k and to discuss the role of capital requirements.[16]

3 Solving the model

As the model will be solved by backward induction, it is convenient to start analyzing what happens when news reveal the type of the bank's illiquid assets during a run (i.e. prior to the exhaustion of the bank's cash); in the remaining terminal nodes, the situation is more trivial and will be described in due course. It is also convenient to get familiar with the role of Poisson processes in helping us obtain expressions for the time span during which the bank can resist a run that starts at $t = 0$, and for the probability that news arrive prior to the point in which its cash gets exhausted.

3.1 News and ex post efficiency

We want to show that the arrival of good news during a run stops the run, whereas the arrival of bad news implies that the bank gets liquidated once it fully consumes its cash. This implies that the arrival of news induces ex post efficient outcomes regarding the continuation vs. liquidation of the bank's illiquid assets.

Let the good news arrive at some date $t > 0$ in which the residual fraction of bank debtholders is n_t and the available cash is $C_t = C - (1 - n_t)D \geq 0$ (which reflects that a fraction $1 - n_t$ of the initial debt has been canceled using cash). Then, if the run stops at t, the terminal value of assets is $a_g(1 - C) + [C - (1 - n_t)D]$, while the residual debt promises

[16] Endogenizing b and δ would require attributing some value to the puttability of bank debt. The literature offers abundant rationales for each of these features, but capturing them here in a fully structural way would blur the essence of our contribution.

to pay $n_t B$ at termination. Now, we can establish the following chain of inequalities:

$$a_g(1-C) + [C - (1-n_t)D] \geq B - (1-n_t)D = n_t B + (1-n_t)bD > n_t B, \tag{5}$$

where the first inequality follows from (2) (we are just subtracting the consumed cash from both sides of it) and the second inequality follows from having $b > 0$.

Equation (5) means that, insofar as the bank can accommodate the run using its cash, the bank with good assets remains fundamentally solvent and a Nash equilibrium in which residual debtholders do not exercise their put options is sustainable after the good news. Specifically, waiting to be paid $B = (1+b)D$ at termination rather than recovering D prior to termination is a best response for any individual debtholder who expects no other debtholder to exercise her put.[17]

Upon the arrival of bad news, the situation is more straightforward. The inequalities contained in (1) imply that the bank with bad assets is insolvent both if early liquidated and if continued, and irrespectively of the available cash or the fraction of residual debtholders. Moreover, all agents anticipate that the LLR will not support the bank. Debtholders with the opportunity to put their debt before the bank exhausts its cash find it optimal to recover D because, as shown in detail in subsection 3.3, the payoff to residual debtholders at liquidation is lower than D. These results are summarized in the following proposition:

Proposition 1 *The arrival of good news during the early run stops the run, allowing the illiquid assets of the bank to continue up to termination. In contrast, the arrival of bad news does not stop the run and leads to the full liquidation of the bank once its cash gets exhausted.*

3.2 How long will the bank resist a run?

Suppose debtholders start exercising their puts immediately after the shock realizes at date 0 and assume that no good news arrive that interrupt the run. Let n_t denote as before

[17] In the absence of a LLR, a second subgame perfect Nash equilibrium might also exist, based on the self-fulfilling prophecy that debtholders' run continues and the good bank is forced to liquidate its assets. This is because, as in e.g. Diamond and Dybvig (1983), liquidating the illiquid assets produces insolvency. However, in our setup the possibility of such an equilibrium is removed by the expectation that, if the occasion arrived, the LLR would support the bank whose assets are known to be good. Eventually, then, the run stops as soon as the good news arrive, the LLR intervention is unneeded on the equilibrium path, and the bank can preserve any cash available when the news arrive.

the fraction of debtholders who have not exercised their put options by an arbitrary date $t \geq 0$. Since the opportunities to exercise the puts arrive among debtholders as independent Poisson processes with intensity δ, the dynamics of n_t is driven by

$$\dot{n}_t = -\delta n_t, \tag{6}$$

with the initial condition $n_0 = 1$. Integrating in (6) implies $n_t = \exp(-\delta t)$. So the bank will exhaust its cash at the date τ such that $(1 - n_\tau)D = C$, that is, when

$$[1 - \exp(-\delta\tau)]D = C. \tag{7}$$

Solving for τ yields the following result:

Proposition 2 *Once a run starts, the bank can resist it without assistance for a maximum time span of length*

$$\tau \equiv -\frac{1}{\delta}\ln(\frac{D-C}{D}), \tag{8}$$

which is greater than zero for $C > 0$, increasing in C, and decreasing in δ and D.

3.3 How much is recovered when the bank gets liquidated?

The bank is liquidated when its cash gets exhausted and the LLR does not support it ($\xi = 0$). At liquidation, the value of bank assets is $q_i(1-C)$, where $i = g, b$ denotes their quality, and the fraction of residual debtholders is $n_\tau = \exp(-\delta\tau) = (D-C)/D$ as already explained above. So the amount recovered by each residual debtholder, conditional on asset quality i, can be written as

$$Q_i = \frac{q_i\,(1-C)}{n_\tau} = \frac{q_i\,(1-C)}{D-C}D < D, \tag{9}$$

where the last inequality follows from (1) and (3).

Thus the payoff received by the fraction $1 - n_\tau = 1 - \exp(-\delta\tau) = C/D$ of debtholders who manage to recover D prior to liquidation is strictly larger than the payoffs of those trapped at the bank when liquidated. This explains why the former will prefer to exercise their put options whenever the probability that the bank ends up liquidated is sufficiently high.

In the context of a run, whether debtholders manage to get paid D or Q_i is just a matter of luck. Then, from the perspective of the date at which the run starts, the expected payoffs accruing to debtholders, conditional on the quality of the illiquid assets being i and the bank ending up liquidated, can be computed as a weighted average of each of the outcomes:

$$[1-\exp(-\delta\tau)]D+\exp(-\delta\tau)Q_i = C + q_i\,(1-C)\,, \tag{10}$$

which, quite intuitively, equals the total value of bank assets conditional on liquidation.[18]

4 The early run equilibrium

We define the *early run equilibrium* as the subgame perfect Nash equilibrium of the game that starts after the bank gets hit by a shock at $t=0$ in which, unless and until good news stop the run, all debtholders exercise their put options as soon as they have the opportunity to do so. In this equilibrium, the logic pushing debtholders to take D whenever possible is that D is higher than the expected value of waiting for the next occasion, if any, to get back D, for the end of the run or for the liquidation of the bank, whichever comes first.

Let $V_t^{ER}(C)$ denote a residual debtholder's value of not exercising the put option at date $t\in[0,\tau]$ when the bank's *initial* cash holding is C, when no news have yet revealed the quality of the illiquid assets and when, in all subsequent opportunities, residual debtholders are assumed to exercise their puts unless good news stop the run. Having $V_t^{ER}(C)\leq D$ for all $t\in[0,\tau]$ means that recovering D, if having the occasion to do so, is a debtholder's best response to the strategies followed by the subsequent players in the game (debtholders who have not yet canceled their debt and the LLR if called upon to act). Thus,

Proposition 3 *An early run equilibrium is sustainable if and only if a residual debtholder's value of not putting her debt at some date t during an early run satisfies $V_t^{ER}(C)\leq D$ for all $t\in[0,\tau]$.*

As shown in detail in the proof of the following proposition, in order to find out the expression for $V_t^{ER}(C)$, it is convenient to think of it as the weighted average, using weights

[18] (10) obtains directly from (7) and (9).

μ and $1-\mu$, of the expected payoffs that a debtholder not exercising her put option at date t would obtain conditional on the illiquid assets of the bank being good and bad, respectively. The result is the following:

Proposition 4 *A residual debtholder's value of not putting her debt at some date $t \in [0, \tau]$ during an early run can be written as follows*

$$\begin{aligned} V_t^{ER}(C) &= D + \mu[1 - \exp(-(\delta+\lambda)(\tau - t))]\frac{\lambda}{\delta+\lambda}(B - D) \\ &\quad - \exp(\delta t)\{\mu \exp(-\lambda(\tau - t))[D - C - q_g(1-C)] + (1-\mu)[D - C - q_b(1-C)]\} \\ &\quad + \xi \exp(\delta t)\exp(-\lambda(\tau - t))[D - C - \bar{q}(1-C)]. \end{aligned} \tag{11}$$

Equation (11) reflects that the holder of one unit of debt during an early run does not always recover D. Specifically, its second term says that if the assets are good and the news come on time, the debtholder recovers B instead of D. The third term says that, if the debtholder gets trapped at the bank and the illiquid assets end up liquidated, her payment is lower than D. The sub-term multiplied by μ reflects that, if the illiquid assets are good, liquidation only happens if no news arrive prior to date τ (and no LLR support is received at τ). The sub-term multiplied by $1-\mu$ reflects that, in contrast, a bad bank not supported by the LLR will get liquidated irrespectively of the possible arrival of news prior to date τ. Finally, the last term in (11) captures the gains, relative to the liquidation payoffs that we have just described, associated with receiving LLR support ($\xi = 1$) at date τ.

The reasoning that may lead to having $V_t^{ER}(C) \leq D$ is a combination of what explains why a debtholder might find it profitable to recover D even if no other debtholders were trying to subsequently recover D (a *fundamental run*), the logic of a *dynamic run* a la He and Xiong (2012) (where each debtholder's incentive to run is reinforced by the fear that, if subsequent debtholders are also early runners, the bank will be consuming its cash and the chances to recover D at a later date will be declining), and distortions to that logic that come from the potential support received from the LLR.

In the weak bank case ($\mu \leq \bar{\mu}$), debtholders anticipate that the uninformed LLR will not support the bank ($\xi = 0$) and the He and Xiong (2012) effect unambiguously reinforces debtholders' incentives to run. It is easy to check that, in this case, $V_t^{ER}(C)$ is decreasing in t. So having $V_t^{ER}(C) \leq D$ for all $t \in [0, \tau]$ only requires having $V_0^{ER}(C) \leq D$.

However, in the strong bank case ($\mu > \bar{\mu}$), the expectation of support from the uninformed LLR ($\xi = 1$) creates a countervailing effect: the bank is more likely to be supported the closer the bank is to exhaust its cash (since this makes less likely the potential revelation that its assets are bad). Due to the third term in (11), the expectation of being supported increases as time passes, making $V_t^{ER}(C)$ increasing when t approaches τ. For simplicity we will focus the core of our analysis on parameter configurations for which having $\mu > \bar{\mu}$ is compatible with having $V_t^{ER}(C) \leq D$ for all $t \in [0, \tau]$, so that the ER equilibrium exists. Alternative configurations of equilibrium are discussed in Section A of the Online Appendix.

5 Welfare and optimal liquidity holdings

Assessing ex ante welfare in the early run equilibrium, $W_{-1}^{ER}(C)$, is equivalent to properly accounting for the returns that the bank's initial assets end up producing over the various future paths that the bank can follow. Building on the analysis that led us to obtain an expression for $V_t^{ER}(C)$ in Proposition 4, we obtain the following result:

Proposition 5 *The ex ante welfare associated with the early run equilibrium is*

$$\begin{aligned} W_{-1}^{ER}(C) \;&=\; C + \{[1 - \varepsilon(1-\mu)]a_g + \varepsilon(1-\mu)q_b\}(1-C) \\ &\quad -\varepsilon \exp(-\lambda\tau)[\mu(1-\xi)(a_g - q_g) + (1-\mu)\xi(q_b - a_b)](1-C), \qquad (12) \end{aligned}$$

where $\exp(-\lambda\tau) = [(D-C)/D]^{\lambda/\delta}$ *by (8).*

The first two terms in (12) represent the returns that the bank would generate in a full information scenario in which its illiquid assets were continued or liquidated according to the ex post most efficient rule (that is, depending on whether they are good or bad, respectively). The third term represents the deadweight losses due to the uninformed nature of the decision made by the LLR when the bank exhausts its cash at date τ and no news on the quality of the illiquid assets has been received. In our model, consistent with Bagehot's doctrine, LLR support ($\xi = 1$) is welfare enhancing if the illiquid assets are good and welfare reducing if they are bad. But, in the absence of news about asset quality by date τ, the LLR decision involves either type I error (good assets are liquidated) or type II error (bad assets are not liquidated). As reflected in (12), type I error occurs, with a cost proportional to $a_g - q_g > 0$, in the weak bank case ($\xi = 0$), while type II error occurs, with cost proportional to $q_b - a_b > 0$, in the strong bank case ($\xi = 1$).

Is there a social value to postponing the LLR support decision? The quick answer is yes. To see this, consider a notional ceteris paribus increase in τ. Such change would reduce the absolute size of the third term of $W_{-1}^{ER}(C)$ (which is negative) and, thus, be good for welfare. Intuitively, it would increase the probability that news arrive prior to date τ and reduce the type I or II errors potentially associated with the otherwise uninformed decision of the LLR. The right answer, however, requires an important qualification. In our setup, τ can only be increased by increasing C, which implies forgoing part of the bank's investment in illiquid assets, which is its only potential source of strictly positive net present value.[19]

To formally analyze the dependence of $W_{-1}^{ER}(C)$ with respect to C, it is convenient to rewrite it as

$$W_{-1}^{ER}(C) = C + A_H(1-C) - A_L\left(\frac{D-C}{D}\right)^{\lambda/\delta}(1-C) \tag{13}$$

where $A_H = [1 - \varepsilon(1-\mu)]a_g + \varepsilon(1-\mu)q_b$, $A_L = \varepsilon[\mu(1-\xi)(a_g - q_g) + (1-\mu)\xi(q_b - a_b)]$, and $((D-C)/D)^{\lambda/\delta}$ replaces $\exp(-\lambda\tau)$. Intuitively, A_H can be interpreted as the *fundamental per-unit value* of illiquid assets at t =–1 (the gross expected return that they would generate under efficient full-information decisions on continuation vs. liquidation), which must be greater than one for the investment in the bank to be a source of social surplus.[20] A_L are the per-unit differential losses on illiquid assets incurred due to the uninformed decision of the LLR, if it happens.

We can prove the following result:

[19] Mathematically, τ could also be reduced, without affecting other terms in (12), by reducing D or δ, which we are treating as exogenously fixed parameters for the time being. Setting $C = D = 0$ or $C = \delta = 0$ in the current model would trivially maximize $W_{-1}^{ER}(C)$ but this is because, to keep things simple, we are not explicitly modeling the gains from financing the bank with debt or from making bank debt redeemable.

[20] Otherwise, $W_{-1}^{ER}(C)$ would be trivially maximized at $C = 1$, where $W_{-1}^{ER}(1) = 1$.

Proposition 6 *The ex ante welfare associated with the early run equilibrium,* $W_{-1}^{ER}(C)$*, is a strictly concave function of* C*, which, depending on parameters, may be increasing or decreasing at* $C = 0$*. If it is decreasing at* $C = 0$*,* $W_{-1}^{ER}(C)$ *is maximized at* $C^* = 0$*. If it is strictly increasing at* $C = 0$*,* $W_{-1}^{ER}(C)$ *reaches a maximum over the interval* $[0, \bar{C}]$ *at some unique* $C^* > 0$*.*

As shown in the proof of the proposition, having strictly positive optimal cash holdings, $C^* > 0$, requires the net present value of the assets of the bank under the liquidation policy induced with $C = 0$, which is $A_H - A_L - 1$, to be small relative to the losses, A_L, that can be avoided by having enough time to obtain the relevant information during a run. It also requires that the effectiveness of cash holdings as a means for gaining the relevant information (which at $C = 0$ is directly proportional to $\lambda/(\delta D)$) is large enough. Quite intuitively this says that, ceteris paribus, liquidity holdings make more sense in situations in which the rate of arrival of information during a run is high relative to the rate at which debt gets canceled.

5.1 Total market value and the need for liquidity standards

Before turning to providing numerical examples of the possibility that the welfare-maximizing liquidity holdings C^* are strictly positive, it is worth clarifying the relationship between ex ante welfare $W_{-1}^{ER}(C)$ and the ex ante total market value of the bank, $TMV_{-1}^{ER}(C)$, which would be the driver of the decision on C of the bank's initial owners in the absence of regulation.

At t=–1 the initial owners place the bank's debt and equity among investors and start up the bank, thus appropriating the difference between the market value of the securities sold to investors or retained as their own investment, $TMV_{-1}^{ER}(C)$, and the required unit of investment as a profit. Both debt and equity are assumed to be competitively priced by the risk neutral investors under each choice of C by the bank, the (given) values of the parameters D, b, and δ that describe the puttable debt contract, and the anticipated course of events in subsequent stages of the game. Equityholders anticipate that they will receive the part of the total expected cash flows generated by bank assets which is not owed to the debtholders or, if applicable, to the LLR.

In the weak bank case ($\xi = 0$), the LLR never intervenes on the equilibrium path and, thus, $TMV_{-1}^{ER}(C)$ is made of the expected value of exactly the same cash flows taken into account when computing $W_{-1}^{ER}(C)$; the capital structure simply divides such value among security holders. Therefore, we have $TMV_{-1}^{ER}(C) = W_{-1}^{ER}(C)$, which means that the initial owners fully internalize the net social gains associated with their choice of C. So in the weak bank case there is no obvious rationale for imposing C^* by means of regulation.[21]

In the strong bank case (where $\xi = 1$), things are different because, when the early run takes place and no news arrive prior to date τ, the LLR intervenes, providing a net value transfer of $(D - C) - a_b(1 - C) > 0$ to investors if the illiquid assets of the bank are bad.[22] This value transfer is appropriated by debtholders, who incorporate it in the valuation of the debt at t=–1 and, hence, in $TMV_{-1}^{ER}(C)$.

Encompassing the two cases, the total market value of the bank can be written as

$$TMV_{-1}^{ER}(C) = W_{-1}^{ER}(C) + \xi\varepsilon \exp(-\lambda\tau)(1 - \mu)[(D - C) - a_b(1 - C)], \qquad (14)$$

where the term multiplied by ξ contains the expected net subsidy received by a strong bank. This term is decreasing in C, both because cash prolongs the time τ over which the strong bank can resist a run (increasing the likelihood that LLR does not have to intervene) and because the value transfer received in case of intervention is decreasing in C (since $a_b < q_b < 1$). Hence, the marginal value of liquidity holdings is lower for the owners of the strong bank than for an ex ante social welfare maximizer.

In fact, as shown in the proof of the following proposition, when $\xi = 1$, the decline with C of the value transfer term related to LLR support exceeds the increase with C of the information gains included in $W_{-1}^{ER}(C)$ (the last term in (13)). This implies that, in the

[21] Of course we could always make the argument, as in Diamond and Kashyap (2016), that debtholders might have difficulties in directly assessing the bank's liquidity position, justifying a monitoring role for the supervisor. This is because non-invigilated shareholders might be tempted to opportunistically distribute C as a dividend at some point after t=–1.

[22] Specifically, the LLR advances $D - C$ at the zero risk-free rate at $t = \tau$ and only recovers $a_b(1 - C)$ at termination. In contrast, if the assets are good, having $a_g(1 - C) > D - C$, by (2) and $D < B$, guarantees the full repayment of the emergency lending.

strong bank case, $TMV_{-1}^{ER}(C)$ is strictly decreasing in C.

Proposition 7 *The ex ante total market value of the bank associated with the early run equilibrium, $TMV_{-1}^{ER}(C)$, coincides with $W_{-1}^{ER}(C)$ in the weak bank case ($\xi = 0$), while it is strictly larger than $W_{-1}^{ER}(C)$ and strictly decreasing in C in the strong bank case ($\xi = 1$).*

The fact that $TMV_{-1}^{ER}(C)$ is strictly decreasing in C when $\xi = 1$ has the important implication that if $C^* > 0$, it will be socially optimal to impose a regulatory liquidity requirement of the form $C \geq C^*$, which will be binding in equilibrium. Intuitively, the initial owners of a strong bank anticipate that LLR support will be granted if the bank exhausts its cash prior to the revelation of the quality of its illiquid assets. And they foresee that the payoffs to security holders in such a situation are better than in the alternative situation in which the quality of the illiquid assets is discovered on time, so they choose the lowest possible liquidity.

The total market value of the bank at t=–1 can be easily broken down into the issuance value of debt and the issuance value of equity. In particular, by first principles, the value of debt at t=–1 can be written as

$$V_{-1}^{ER}(C) = (1-\varepsilon)(1+b)D + \varepsilon V_0^{ER}(C), \tag{15}$$

which uses the fact that debtholders get the full repayment $B = (1 + b)D$ at termination if the bank is not hit by a shock at $t = 0$, while they obtain expected payments equal to $V_0^{ER}(C)$ otherwise. From here, the value of equity at t=–1, $E_{-1}^{ER}(C)$, can be simply found as a residual:

$$E_{-1}^{ER}(C) = TMV_{-1}^{ER}(C) - V_{-1}^{ER}(C). \tag{16}$$

5.2 Numerical examples

In this subsection we introduce some parameterizations under which the socially optimal liquidity holdings C^* are interior. Such parameterizations appear in Table 1 together with the implied C^*. We will use variations of these examples when we endogenize the bank's capital structure below and also in the Online Appendix (when discussing the possible existence of equilibria other than the early run equilibrium and when analyzing numerically the determinants of C^*). For concreteness, we refer to one month as the relevant unit of time but the values of the parameters are purely illustrative, so the results only tell about qualitative properties of the model.

Table 1
Numerical examples
(one unit of time = one month)

	ε	μ	a_g	a_b	q_g	q_b	λ	δ	b	D	C^*
Strong bank	0.2	0.50	1.2	0.5	0.85	0.75	2.5	0.167	0.01	1	0.0613
Weak bank	0.2	0.25	1.2	0.5	0.85	0.75	2.5	0.167	0.01	1	0.0486

The above parameters imply $\bar{\mu} = 0.416$ and, hence, allow to obtain a strong bank example or a weak bank example by merely fixing the value of μ above or below such threshold. For instance, with $\mu = 0.5$, the bank is strong and its socially optimal liquidity holdings C^* are roughly 6% of total assets, although its owners would choose $C = 0$ unless liquidity standards force them to do otherwise. Instead, with $\mu = 0.25$ the bank is weak and it is in the best interest of its owners to choose the socially optimal liquidity holdings C^*, which in this case are about 5% of total assets, without the need for regulation. In Section B of the Online Appendix we build on these examples to provide the numerical comparative statics of C^*, that is, to discuss the dependence of the optimal liquidity holdings on the main parameters of the model.[23]

6 Adding capital

Up to now we assumed the bank's capital structure was exogenously fixed at $t = -1$. This simplified the analysis, but it precluded us from considering the role of capital regulation. In this section, we modify our model to investigate the interplay between the capital and liquidity regulations. To that end, we assume that the owners of the bank have a fixed endowment of w at $t = -1$, out of which they decide to contribute $k \leq w$ to finance the

[23] As also discussed in the Online Appendix, for some of the parameters, the sign of such dependence can be established analytically, but for other there are interesting non-monotonicities (for instance, C^* is first increasing and then decreasing in μ, δ, and λ), which are documented and explained there.

bank, so that $1 - k$ needs to be funded externally. External funding is in the form of debt as the one captured in the baseline model. Regarding such debt, we endogenize D but keep treating b and δ as exogenous parameters.

As in, e.g., Holmström and Tirole (1997), we link the bank's solvency to its capital structure through a moral hazard problem affecting some costly managerial actions that determine the probability μ with which the illiquid assets remain good when the bank is hit by the solvency shock at $t = 0$. Specifically, we assume that bank owners decide the unobservable value of μ at t=–1, right after the bank has decided on how much to invest in cash C and on how much of their own wealth to contribute as equity funding k. Thus, when the bank owners decide on μ, the debt D needed to raise $1 - k$ externally has already been issued.

The choice of μ implies a private non-pecuniary cost to bank owners $\Psi(\mu)$, with $\Psi' > 0$ and $\Psi'' > 0$. For concreteness, the following functional form is explored in the numerical examples:

$$\Psi(\mu) = \psi \left(\frac{\mu}{1 - \mu} \right)^2, \tag{17}$$

with $\psi > 0$, which guarantees solutions for μ in the interior of the $[0, 1]$ interval. Taking this cost into account requires modifying the expression for the social and private value of the bank. This involves subtracting $\Psi(\mu)$ from W_{-1}^{ER} in (12) and (13), so that

$$W_{-1}^{ER} \mid_{new} = W_{-1}^{ER} \mid_{old} - \Psi(\mu); \tag{18}$$

with this modification, the expressions for TMV_{-1}^{ER} in (14) and E_{-1}^{ER} in (16), already based on W_{-1}^{ER}, would still be valid.[24]

In this extended version of the model, any choice (C, k) of liquidity and capital at t=–1 (either by the bank owners or by the regulator) induces a sequential game with three subsequent moves (i) bank owners set optimally the value of D (and thus $B = (1+b)D$) required to attract funds $1 - k$ from debtholders, (ii) bank owners unobservably choose μ, and (iii) if relevant, the LLR decides whether to support or not the bank. Let $(D(C, k), \mu^*(C, k), \xi(C, k))$

[24] As in the analysis of the baseline model, we focus the discussion on the situation in which, when the bank suffers a solvency shock, an early run starts. So in all the examples used in the discussion below we will assume and verify that the condition established in Proposition 3 ($V_t^{ER} \leq D$ for all $t \in [0, \tau]$) holds for the whole range of values of C and k explored in the analysis.

denote the subgame perfect Nash equilibrium (SPNE) of this game. Such equilibrium can be found as the tuple compatible with the conditions reflecting the relevant players' best response in each stage. In stage (i) bank owners will choose the minimal D that satisfies

$$V_{-1}^{ER} = 1 - k, \tag{19}$$

under the anticipated subsequent choices of μ^* and ξ. In stage (ii) bank owners will set

$$\mu^* = \arg\max_{\mu} E_{-1}^{ER}, \tag{20}$$

where D and the LLR's subsequent choice of ξ are taken as given.[25] In stage (iii), the LLR will choose $\xi = 1$ if and only if it believes the bank to be strong, that is, if $\mu^* > \bar{\mu}$.

Our main interest is to analyze situations where such SPNE involves strong banks ($\xi = 1$). In practical terms, we can fix $\xi = 1$, solve for the candidate equilibrium in (D, μ^*), and check that $\mu^* > \bar{\mu}$. Likewise, to explore a SPNE with weak banks, we can fix $\xi = 0$, solve for the candidate equilibrium in (D, μ^*), and check that $\mu^* \leq \bar{\mu}$.

The final objective of the discussion is to clarify which value of (C, k) would maximize the social ex ante value of the bank, W_{-1}^{ER}, or the private ex ante value of the bank to its owners, TMV_{-1}^{ER}. Comparing the socially and privately optimal choices of (C, k) will clarify the need or not for liquidity and capital regulation.

6.1 Strong bank results

Figure 1 has been produced with the parameters in Table 1, excluding μ and D which are now endogenous, and setting the parameter of the private cost of improving asset quality as ψ=0.005.[26] Under this parameterization, the bank is strong over the whole depicted range of values of (C, k). As reflected in the μ panel of the figure, bank owners' choose values of μ above $\bar{\mu}$, which is the level indicated in the 3D graph by the red rectangle at the bottom. Quite intuitively, μ is increasing in k as a larger share of owners' financing reduces the bank's

[25] Notice that the unobservability of μ to the LLR effectively makes (ii) and (iii) part of a simultaneous-move subgame betwen the bank and the LLR.

[26] In fact, this choice of ψ makes the endogenous values of μ and D under $(C, k) = (0, 0)$ approximately equal to those used in Table 1 for the strong bank baseline example.

leverage and increases insiders' incentives to guarantee that bank assets are resilient to the solvency shock—a standard skin-in-the-game effect.

In contrast, μ is slightly decreasing in C due to the combined impact of two forces that, in this case, push in the same direction. The first and more straightforward is that C reduces the share of illiquid assets held by the bank, which other things equal diminishes owners' incentives to invest in increasing μ. The second force operates through the cost of debt financing. As shown in the $D/(1-k)$ panel, increasing the liquidity holdings C increases quite significantly the promised repayments D needed to raise any given value of funding $1-k$ at t=–1. Debtholders anticipate that a strong bank endowed with more liquidity gives more time to the LLR to discover, in the event of a run, the quality of its illiquid assets and, hence, increases the chances that the bank ends up resolved (rather than blindly supported), in which case they will experience losses. The higher cost of debt acts as a reverse skin-in-the-game effect on bank owners' incentives regarding μ.

Since μ is below its first best level in this extension, the overall negative effect of C on μ identifies a novel second best cost of liquidity standards, one that further deteriorates the bank's fundamental solvency and reinforces the costs behind the existence of a limited socially optimal level of C in the baseline model. Indeed, as reflected in the W_{-1}^{ER} panel, the social surplus generated by the bank is unambiguously increasing in k—the maximum inside-equity-participation principle common to moral hazard models with risk neutrality applies here—but its relationship with C has an inverted U-shape.

By arguments already exposed when bankers' only choice was C, the strong bank needs both liquidity and solvency regulation. As reflected in the TMV_{-1}^{ER} panel, the owners of a strong bank would maximize their wealth at t=–1 by choosing $(C,k)=(0,0)$ even if their wealth $w>0$ would allow them to provide some equity financing. Intuitively, TMV_{-1}^{ER} is decreasing in both C and k because of the distortions associated with the prospect of benefiting from blind LLR support in a run.[27]

[27] In the $D-V_0^{ER}$ panel, we verify that for the whole range of depicted values of (C,k) the necessary condition for an early run, $D>V_0^{ER}$, holds. We have also numerically checked the corresponding condition for all $t\in(0,\tau)$.

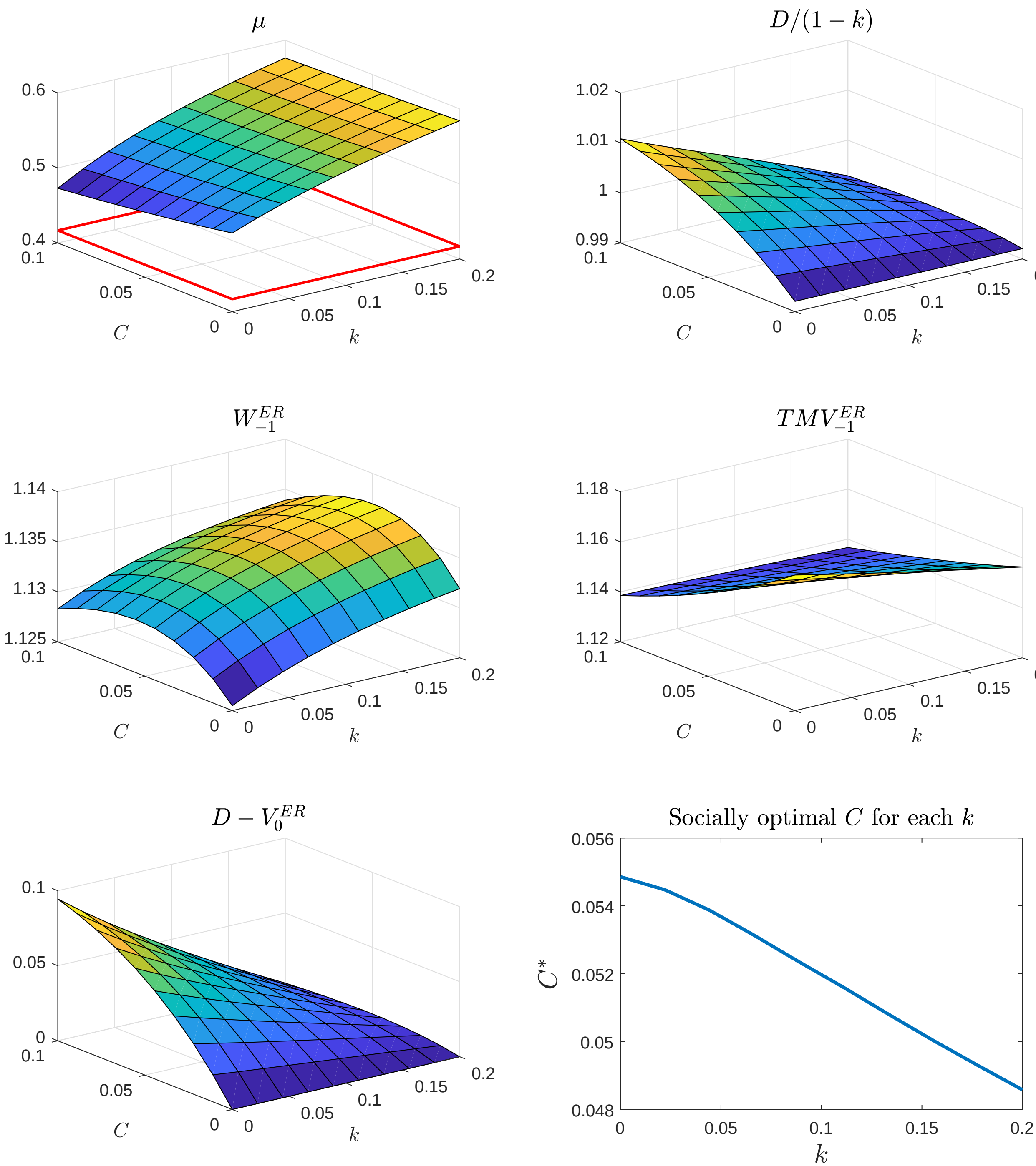

Figure 1. Strong bank example of the extended model

As in the baseline model, lowering C increases the chances of such support. Since LLR support makes debt financing effectively subsidized, bank owners in this example prefer debt financing to equity financing in spite of the advantages of the latter in regards to the moral hazard problem that affects their choice of μ. Regulation in this framework should therefore force bank owners to contribute their wealth to the financing of the bank by setting a minimum capital standard of $k = w$ and accompany it with a liquidity standard that fixes the liquidity holdings which are optimal for such level of k.

Those liquidity standards are shown in the $C(k)$ panel of Figure 1, which is an immediate by-product of the W_{-1}^{ER} panel. In this example, the optimal liquidity standard is decreasing in k, so capital standards work as a substitute for liquidity standards at the margin. Intuitively, as k increases, the quality of the bank's illiquid assets increases and the likelihood of type II errors associated with blind LLR support (i.e. the support of an insolvent bank) diminishes and, with it, the informational value of liquidity holdings. Nevertheless capital standards are a rather imperfect substitute for liquidity standards: in this example, increasing k from 0% to 20% only reduces the optimal liquidity holdings $C(k)$ by about 70 basis points. This is because the impact of k on μ is far from bringing it to a level in which the informational value of liquidity holdings goes to zero.[28]

6.2 Weak bank results

Figure 2 has also been produced with the parameters in Table 1, again excluding the now endogenous μ and D, and setting the parameter of the private cost of improving asset quality as $\psi = 0.02$ (four times bigger than in the strong bank example above). Under this parameterization, the bank is weak over the whole depicted range of values of (C, k). As shown in the μ panel, bank owners' choose values of μ below $\bar{\mu}$, which is the level indicated by the red rectangle that appears at the top of the 3D graph.

As in the strong bank case, μ is increasing in k due to the standard skin-in-the-game effect. Opposite to the strong bank case, however, μ is increasing in C over the depicted range. While it is still the case that a larger C reduces the share of assets affected by the choice of μ, there are now two other effects operating in the direction of increasing μ. First, since the uninformed LLR does not support a weak bank, increasing C increases the chances

[28]Having said that, there are discontinuities that would become relevant over a wider range of values of k and confirm observers intuition that by-and-large sufficiently capitalized banks might not need liquidity standards at all. As the $D - V_0^{ER}$ panel suggests, if k could be set higher than 20%, the early run equilibrium might no longer exist and, in the absence of an early run, the optimal C in our setup would be zero.

that the illiquid assets get discovered to be good during the run (in which case debtholders end up repaid in full and equityholders obtain a strictly positive terminal payoffs) and, with it, bank owners' incentives to increase the likelihood that assets remain good in a crisis.[29] Second, the burden of debt repayments, as reflected in the $D/(1-k)$ panel is now slightly decreasing in C, since debtholders discount the larger probability of being fully repaid and, even if not, recovering a higher liquidation value on the bank's illiquid assets.

Explaining the remaining panels of Figure 2 is easier than for Figure 1, since in the case of the weak, as in our baseline model, the absence of distortions associated with the prospect of subsidized LLR support makes bank owners' objective function, TMV_{-1}^{ER}, equivalent to the full social surplus generated by the bank, W_{-1}^{ER}, rendering liquidity and capital regulation unnecessary.[30] As reflected in the corresponding panels, both measures are increasing in k—the maximum inside-equity-participation principle applies— and are related to C in an inverted U-shape manner.

What bank owners (or likewise a social planner) would do in this setup is to set $k = w$ and accompany this capital ratio with the associated socially and privately optimal liquidity holdings $C(k)$ which appear depicted in the corresponding panel of Figure 2. Interestingly, in this weak bank example, $C(k)$ is first increasing and then decreasing in k. So, opposite to the strong bank example, there is a low range of values of k over which the capital ratio is complementary at the margin to the optimal liquidity holdings. This is because increasing k increases μ and, opposite to the strong bank case, increasing μ increases, other things equal, the information value of liquidity holdings because the LLR default decision is to resolve the weak bank and raising C increases the chances of discovering that the illiquid assets are good and the bank should not be liquidated. Beyond some point, however, the informational gains decline and the force leading $C(k)$ to become locally decreasing is the rising opportunity cost of investing in liquidity rather than in the increasingly valuable illiquid assets.

[29] In fact, with C=0 bank owners choose μ=0, because, without time for asset quality to be revealed, the weak bank gets immediately resolved and bank owners obtain zero payoffs irrespectively of their prior choice of μ.

[30] The same caveat as in Footnote 21 applies here and could be extended to the need to verify that the bank's leverage remains at its initially set level.

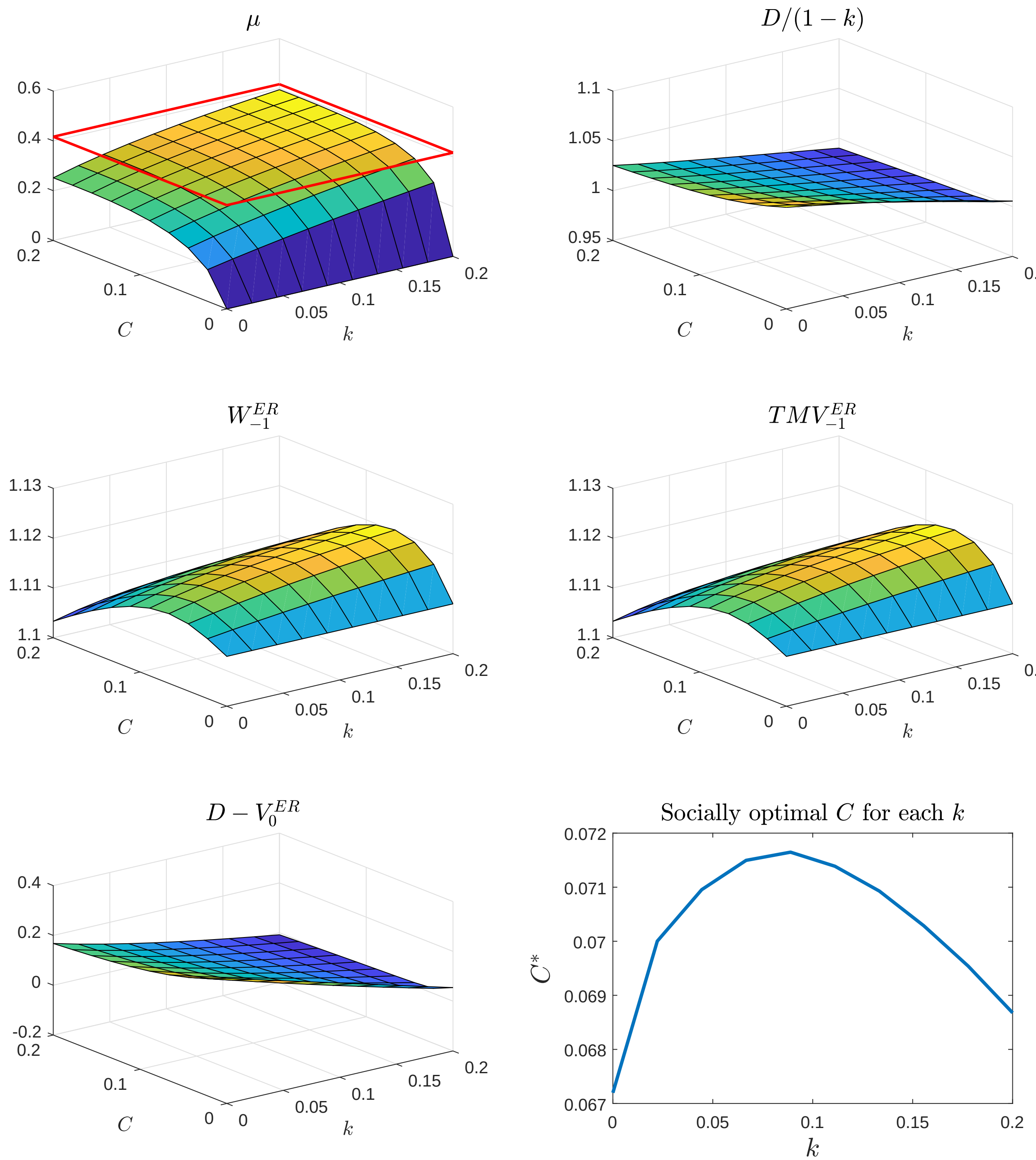

Figure 2. Weak bank example of the extended model

7 Further extensions

In this section, we present the results of two additional extensions to our baseline model (with exogenous D and μ). Subsection 7.1 discusses the implications of forcing the LLR to lend on an expected break-even basis. Subsection 7.2, in turn, considers the possibility of using temporary LLR support instead of liquidity holdings for the purpose of buying time for the arrival of information.[31]

7.1 Fair pricing of LLR support

What are the implications of forcing the LLR to lend on an expected break-even basis, i.e. at terms that imply no subsidization of the supported banks? In the baseline model, banks that exhaust their cash prior to the discovery of the quality of their illiquid assets get supported ($\xi = 1$) if and only if, conditional on the probability μ that the assets are good, their continuation yields higher overall expected value than liquidation ($\bar{a} > \bar{q}$). This means that banks get only supported in the strong bank case, $\mu > \bar{\mu}$. Importantly, the LLR lends at the (zero) risk-free rate, implying that its support involves a positive expected subsidy, as reflected in the second term at the RHS of equation (14). Such expected subsidy is the source of the discrepancy between the social and the private value of the bank, $W_{-1}^{ER}(C)$ and $TMV_{-1}^{ER}(C)$, respectively, and the reason for imposing liquidity requirements on a strong bank.

Suppose alternatively that the LLR is obliged to lend at terms that imply no expected subsidy or tax. LLR support implies advancing $D - C$ to a bank that, if its assets are good, will yield $a_g(1 - C) > D - C$ at termination, while, if its assets are bad, will yield $a_b(1 - C) < D - C$. Thus, the feasibility of break-even LLR support requires the existence of $F \leq a_g(1 - C)$ such that

$$\mu F + (1 - \mu)a_b(1 - C) = D - C, \tag{21}$$

where F is the repayment due to the LLR. It is immediate to see that the existence of such F requires

$$\mu \geq \hat{\mu} \equiv \frac{(D - C) - a_b(1 - C)}{(a_g - a_b)(1 - C)}, \tag{22}$$

[31] In Section C of the Online Appendix, we provide a discussion of two additional issues: (i) bankers' incentives to produce information on their condition, and (ii) a variation of our model in which what needs to be discovered about the bank is not the quality of its assets but the potential systemic importance of its failure.

where, using assumption (1), one can easily prove that $\hat{\mu} \in (\bar{\mu}, 1)$. Therefore, getting support on a break-even basis is unfeasible for *modestly strong* banks with $\mu \in (\bar{\mu}, \hat{\mu})$ and feasible for *sufficiently strong* banks with $\mu \in [\hat{\mu}, 1]$.

Thus, if the LLR lends on an expected break-even basis, $\xi = 1$ can only occur for $\mu \in [\hat{\mu}, 1]$. Formally, the (conditional on ξ) expressions for $V_t^{ER}(C)$ and $W_{-1}^{ER}(C)$ remain valid, but the absence of subsidies now implies $TMV_{-1}^{ER}(C) = W_{-1}^{ER}(C)$ for all values of the parameters. These modifications of the baseline analysis have several important implications:

1. The incentives of bank owners and the social planner with respect to C are fully aligned for all values of μ. Thus, regulatory liquidity requirements would no longer be needed.

2. Differently from the case in which LLR lending occurs at the risk-free rate, there is an additional range $(\bar{\mu}, \hat{\mu})$ of values of the probability that assets are good for which the early run will lead to liquidation if the quality of bank assets remains unknown at $t = \tau$. However, in this range liquidating banks involves a net efficiency loss (the cost of higher type I error exceeds the gain from lower type II error). Thus, the baseline arrangement that combines subsidized LLR support and liquidity requirements is superior in ex ante welfare terms.

3. For $\mu \in (\bar{\mu}, \hat{\mu})$, the arrangement based on break-even LLR support will lead banks to voluntary hold liquidity higher than C^*.[32] This is because the above-mentioned efficiency losses can be reduced, on expectation, by "buying additional time" for the information on asset quality to possibly arrive during the run.

If the concern leading to opt for a break-even LLR arrangement rather than the baseline arrangement is about the financing of the subsidy involved in the latter, the society could

[32] To see this, assume a situation in which the baseline arrangement implies an interior socially optimal amount of liquidity $C^* > 0$. Then, C^* will satisfy the first order condition $\partial W_{-1}^{ER}/\partial C = 0$ and the second order condition $\partial^2 W_{-1}^{ER}/\partial C^2 < 0$, where for $\mu \in (\bar{\mu}, \mu^*)$ the baseline arrangement implies $\xi = 1$. Now, for the sake of the argument, consider the effect of a marginal decrease in the prospect of receiving support, $d\xi < 0$. We can assess the impact on the optimal liquidity choice by differentiating the first order condition:

$$\underbrace{\frac{\partial^2 W_{-1}^{ER}}{\partial C^2}}_{-} dC^* + \frac{\partial^2 W_{-1}^{ER}}{\partial C \partial \xi} \underbrace{d\xi}_{-} = 0,$$

where one can check that $\partial^2 W_{-1}^{ER}(C^*)/\partial C \partial \xi < 0$ for $\mu > \bar{\mu}$. Thus dC^* must be positive.

still opt for some form of ex ante liquidity insurance arrangement in which banks pay ex ante for the expected cost of the support that they may receive if an early run happens. Liquidity insurance cum liquidity requirements would then be an ex ante break-even arrangement superior to the one based on forbidding the LLR to lend on an ex post subsidized basis.

7.2 Buying time through temporary LLR support

In our baseline model, LLR support becomes irreversible once granted. This irreversibility might have various causes. For example, unmodeled political or reputational considerations might make it too costly to acknowledge that a previously supported bank is no longer considered solvent. The political cost might also come from the implied unequal treatment of debtholders who manage to exercise their puts before support is canceled and those who do not. Another reason could be that, under LLR support, the information about the quality of bank assets ceases to arrive (or becomes too noisy), e.g. because market participants no longer have incentives to discover it or because the relevant market prices get distorted by the presence of LLR support.[33]

In this section we examine the implications of relaxing the assumption of irreversibility. For tractability, we focus on the case in which the "buying time" role played by liquidity holdings gets fully replaced by some intendedly temporary support from the LLR. Specifically, we consider the case in which banks carry no liquid assets ($C = 0$) and the LLR supports them as soon as an early run starts, but with the intention to just wait until the quality of the illiquid assets gets discovered. The plan is that, like in Proposition 1, if the assets are good, the run self-resolves and LLR support does not need to continue, while if the assets are bad, support is withdrawn so as to (efficiently) force the bank into resolution. However, to make the comparison with the baseline liquidity-based arrangement non-trivial, we assume that there is a probability $\pi > 0$ that the temporary support becomes permanent, implying that the bank with bad assets is inefficiently continued up to termination.[34]

[33] See Acharya and Thakor (2016) for an explicit model of this channel.

[34] Having $\pi > 0$ might also reflect that the information arriving about the quality of the assets of a supported bank is more noisy than that received about the quality of the assets of a non-supported bank—our insights would be very similar if the noise could also lead to the inefficient liquidation of good assets. Having $\pi > 0$ can also capture the possibility that information arrives so late that it is no longer possible to

In this new setup, we could derive an expression for debtholders' value of not putting their debt at $t \geq 0$ (conditional on the quality of assets not having yet been revealed) similar to $V_t^{ER}(C)$ but adapted to the case in which the bank carries no cash ($C = 0$) and gets supposedly temporary LLR support from $t = 0$. Call it $\hat{V}_t^{ER}$. Assuming, realistically, that the LLR is senior to other debtholders, for values of π sufficiently lower than one, we would get $\hat{V}_t^{ER} < D$ for all $t \geq 0$ confirming the sustainability of an early run that starts at $t = 0$. Intuitively, the early run would occur because debtholders would fear that by the time the quality of the illiquid assets gets revealed, there is a high enough chance that the bank is bad and gets resolved, and they receive much less than D.[35]

The ex ante welfare associated with the operation of this arrangement can be written as the expected value of the payoffs extracted from bank assets in each of the possible final states:

$$\hat{W}_{-1}^{ER} = [1 - \varepsilon(1-\mu)]a_g + \varepsilon(1-\mu)\left[q_b - \pi(q_b - a_b)\right], \tag{23}$$

where the first term contains the payoffs extracted when there is no run or the run ends with the good bank unresolved, while the second term shows the value extracted in the run when the bank is bad (which is affected by the probability $\pi > 0$ of mistakenly leaving the bad bank unresolved).

We can establish whether the arrangement based on temporary LLR support dominates or not the baseline arrangement examined before by comparing the expressions for $\hat{W}_{-1}^{ER}$ in (23) and $W_{-1}^{ER}(C)$ in (12). In fact, we can extract some general lessons by looking at a few polar scenarios:

- Consider a strong bank which is not subject to liquidity requirements. In such case, under the baseline arrangement, the bank chooses $C = 0$ (implying $\tau = 0$) and, in an early run, gets supported for sure, $\xi = 1$. So the implied ex ante welfare is

$$W_{-1}^{ER}(0)_{|\xi=1} = [1 - \varepsilon(1-\mu)]a_g + \varepsilon(1-\mu)a_b < \hat{W}_{-1}^{ER}, \tag{24}$$

extract the value from an "early" liquidation of bad assets.

[35] They may receive much less than D not just because $q_b < D$, by assumption (1), but also because, in the event of liquidation, the debt with the LLR is senior to that with the residual debtholders.

for all $\pi < 1$, since $a_b < q_b$, by (1). Thus, for a strong bank not subject to liquidity requirements, the arrangement based on temporary LLR support would be strictly superior for all $\pi < 1$.

- Consider now an either weak or strong bank that holds the socially optimal amount of liquidity C^* under the baseline arrangement. In the polar case with $\lambda \to \infty$ (i.e. when information arrives arbitrarily close to $t = 0$) we have $C^* \to 0$ and

$$\lim_{\lambda\to\infty} W_{-1}^{ER}(C^*) = [1-\varepsilon(1-\mu)]a_g + \varepsilon(1-\mu)q_b > \hat{W}_{-1}^{ER}, \tag{25}$$

for all $\pi > 0$. Thus, by continuity, provided that information arrives at a sufficiently high rate and banks hold the socially optimal liquidity buffers, the baseline arrangement is strictly superior.

- Finally, consider again an either weak or strong bank that holds the socially optimal amount of liquidity C^* under the baseline arrangement. In the alternative polar case with $\lambda \to 0$ (i.e. when information arrives arbitrarily slowly) we also have $C^* \to 0$, but in this case

$$\lim_{\lambda\to 0} W_{-1}^{ER}(C^*) = [1-\varepsilon(1-\mu)]a_g + \varepsilon(1-\mu)q_b - \varepsilon[\mu(1-\xi)(a_g-q_g) + (1-\mu)\xi(q_b-a_b)].$$

Hence, we can distinguish two subcases. If the bank is strong ($\xi = 1$), we have

$$\lim_{\lambda\to 0} W_{-1}^{ER}(C^*) = [1-\varepsilon(1-\mu)]a_g + \varepsilon(1-\mu)a_b < \hat{W}_{-1}^{ER}, \tag{26}$$

as in (24). If the bank is weak ($\xi = 0$), we have

$$\lim_{\lambda\to 0} W_{-1}^{ER}(C^*) = (1-\varepsilon)\, a_g + \varepsilon\mu q_g + \varepsilon(1-\mu)q_b,$$

which is strictly lower than $\hat{W}_{-1}^{ER}$ if and only if

$$\pi < \frac{\mu}{1-\mu}\frac{a_g-q_b}{q_b-a_b}. \tag{27}$$

In fact, $\frac{\mu}{1-\mu}\frac{a_g-q_b}{q_b-a_b} \leq 1$ if and only if the bank is weak. Thus, by continuity, we can generally establish that, if information arrives at sufficiently low rates (and, in the weak bank case, (27) holds), the temporary support arrangement is strictly superior to the baseline arrangement.

All in all, the message is that the dominance of one arrangement over the other crucially depends on the comparison between the risk of irreversibility of (or poorer quality of information under) temporary LLR support (as measured by π) and the "buying time" effectiveness of liquidity holdings (as measured by, e.g., λ).[36]

8 Conclusions

We provided in this paper a novel rationale for banks' liquidity standards, one which builds on the idea that liquidity buffers make banks capable to deal with debt withdrawals for some time before they have to seek support from the LLR. This ability to wait before seeking LLR support is valuable because it allows for the release of information on the bank's financial condition that is useful for the LLR's decision on whether to grant support. Specifically, it generally improves the efficiency of the decisions regarding the continuation of the bank as a going concern or its liquidation. Importantly, as shown in the main extension of the baseline model, capital regulation can contribute to reinforce the bank's fundamental solvency but is unable to fully mimic the informational role of liquidity standards during a run.

In addition to the extensions of the baseline model that we consider, our analysis points to some avenues that would be interesting to explore in future research. For example, we have assumed in our model that the arrival of information on the bank's financial condition following a shock is exogenous. However, in general the nature and the speed at which information on the bank's financial condition is produced and disclosed is endogenous and depends on the entity responsible for this activity. Further, the bank may not have the proper incentives to disclose that information in a timely manner. This provides a rationale for entrusting an agency with the authority to produce information about the bank's financial condition. Importantly, this information would have to be made available not only to the LLR but also to the bank's investors, as it is key for their decision to rollover their debt. Since

[36] Parameter π in the above formulation might also capture the costs due to "stigma effects" associated with (early) LLR support. Specifically, stigma might be rationalized as the result of investors becoming massively aware of bank trouble and accelerating the speed of the run (He and Manela 2014). Such acceleration might increase the probability of arriving to a point in which the illiquid assets can no longer be orderly liquidated and authorities must choose between disorderly liquidation or a full bail-out.

the disclosure of information affects the LLR's incentives and those of investors differently, an interesting question for future research would be to investigate which agency or agencies should have authority to gather and disclose information on banks' financial condition in real time.[37]

[37] See Kahn and Santos (2006) for a model in which differences in regulatory agencies' mandates induce agencies to hold information from their counterparts.

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Appendix: Proofs

Proof of Propositions 1-3 These propositions follow directly from simple algebra and the arguments that precede their statement in the main text.■

Proof of Proposition 4 We structure this proof in three parts. First we find expressions for a debtholder's value of not exercising the put option at some $t \in [0, \tau]$ conditional on bank assets being bad and good, respectively. Then, we put together the corresponding unconditional value of not exercising the put at t so as to arrive to (11).

Part I. Value of not exercising the put conditional on assets being bad We can compute this value as the weighted average over two possible courses of events:

1. News arrive prior to date τ. Since news arrival is a Poisson process with intensity λ, the time span to the arrival of (the next) news, say x, follows and exponential distribution with parameter λ. Thus, the probability that news arrive prior to date τ can be computed as $\Pr(x \leq \tau - t)) = 1 - \exp(-\lambda(\tau - t))$. If news about the bad quality of the illiquid assets arrive prior to date τ, the bank ends up liquidated at date τ. Some lucky debtholders will recover D prior to τ and the remaining ones will obtain $Q_b < D$ at liquidation. Since the arrival of the chance to recover D follows a Poisson process with intensity δ, the probability of having a chance to recover D prior to liquidation is $1 - \exp(-\delta(\tau - t))$, so the expected payoff over this course of events can be written as

$$\begin{aligned}
[1 - \exp(-\delta(\tau - t))]\, D + \exp(-\delta(\tau - t))Q_b &= D - \exp(\delta t)\exp(-\delta\tau)(D - Q_b) \\
&= D - \exp(\delta t)[D - C - q_b(1 - C)], \quad (28)
\end{aligned}$$

where the last equality is obtained using (10) for $i = b$.

2. News do not arrive prior to date τ. This happens with probability $\exp(-\lambda(\tau - t))$. When the bank runs out of cash and the quality of its assets remains unknown, the LLR decides to support the bank ($\xi = 1$) if the bank is strong ($\bar{a} > \bar{q}$) and not to support it ($\xi = 0$) if it is weak ($\bar{a} \leq \bar{q}$). So debtholders with the opportunity to exercise their puts prior to date τ will obtain D, while the remaining ones will obtain $\xi D + (1 - \xi)Q_b$, and the expected payoffs over this course of events can be written as

$$\begin{aligned}
&[1 - \exp(-\delta(\tau - t))]\, D + \exp(-\delta(\tau - t))\, [\xi D + (1 - \xi)Q_L] \\
&= D - (1 - \xi)\exp(-\delta(\tau - t))(D - Q_L) \\
&= D - (1 - \xi)\exp(\delta t)[D - C - q_b(1 - C)], \qquad (29)
\end{aligned}$$

where $\exp(-\delta(\tau - t))$ is, as above, the probability of not having the chance to recover D prior to date τ, and we also use (10) to re-express the term in $(D - Q_L)$ in the last equality.

Putting together these results, the value of not exercising the put for a residual debtholder at date t conditional on the illiquid assets being bad can be written as

$$V_t^{ER}(C)_{|i=b} = D - [1 - \xi \exp(-\lambda(\tau - t))]\exp(\delta t)\,[D - C - q_b(1 - C)]\,, \qquad (30)$$

where the term multiplied by ξ captures the contribution of the *subsidy* associated with LLR support in the strong bank case.

Part II. Value of not exercising the put conditional on assets being good The simplest way to obtain an expression for $V_t^{ER}(C)$ conditional on assets being good is also to look at how events may unfold for a typical debtholder who retains her debt at $t = 0$. We can distinguish three mutually exclusive courses of events:

1. The debtholder gets the chance to put her debt and obtain D prior to the arrival of news and prior to the exhaustion of the bank's cash. So the debtholder receives D.

2. The news arrive prior to the debtholder having the opportunity to put her debt and prior to the exhaustion of the bank's cash. So the debtholder obtains B by waiting up to termination, since the crisis self-resolves.

3. The bank runs out of cash prior to the debtholder having the opportunity to put her debt and prior to the arrival of news. So the debtholder obtains $\xi D + (1 - \xi)Q_g$.

Thus, using the fact that the payment associated with the exhaustion of cash will occur at date τ if none of the other relevant events occurs before that date, and the independent nature of the Poisson processes driving the arrival of these events, we can write:

$$\begin{aligned} V_t^{ER}(C)_{|i=g} &= [1 - \exp(-(\delta + \lambda)(\tau - t))] \left(\frac{\delta}{\delta + \lambda} D + \frac{\lambda}{\delta + \lambda} B \right) + \\ & \exp(-(\delta + \lambda)(\tau - t)) \left[\xi D + (1 - \xi)Q_g \right]. \end{aligned} \tag{31}$$

The factors 1–$\exp(-(\delta+\lambda)\tau)$ and $\exp(-(\delta+\lambda)\tau)$ are explained by the fact that if two Poisson processes arrive independently with intensities δ and λ, the arrival of the first of them is a Poisson process with intensity $\delta + \lambda$, and the corresponding span to such an arrival follows an exponential distribution with parameter $\delta + \lambda$. So $\exp(-(\delta + \lambda)\tau)$ is the probability that no first event occurs by date τ and $1 - \exp(-(\delta + \lambda)\tau)$ is the probability that at least one event arrives. The factors $\delta/(\delta + \lambda)$ and $\lambda/(\delta + \lambda)$ describe the probabilities with which the first event is the option to exercise the put and the arrival of (good) news, respectively.

Isolating D and using (10) to write $\exp(-\delta(\tau - t))(D - Q_g)$ as $\exp(\delta t)[D - C - q_b(1 - C)]$, we obtain

$$\begin{aligned} V_t^{ER}(C)_{|i=g} &= D + [1 - \exp(-(\delta + \lambda)(\tau - t))] \frac{\lambda}{\delta + \lambda}(B - D) \\ & -(1 - \xi) \exp(-\lambda(\tau - t)) \exp(\delta t)[D - C - q_g(1 - C)], \end{aligned} \tag{32}$$

which reflects that, conditional on bank assets being good, the residual debtholders at time t do not always end up recovering D during the early run. They gain the additional amount $B - D > 0$ if the good news arrive on time (so that they can wait until termination) and they incur an additional expected loss $\exp(\delta t)[D - C - q_g(1 - C)]$ if the bank is weak ($\xi = 0$) and runs out of cash prior to the revelation of the quality of its assets.

Part III. Unconditional value of not exercising the put in an early run Putting together expressions (30) and (32), we obtain the unconditional value of one unit of residual bank debt during an early run as reported in (11).■

Proof of Proposition 5 Ex ante welfare can be calculated as the expected value of the overall asset returns that the bank generates over all the possible courses of events, which can be described as follows:

1. No shock occurs at $t = 0$. This occurs with probability $1 - \varepsilon$. The bank assets are good and never liquidated. The bank generates returns $C + a_g(1 - C)$.
2. The shock occurs at $t = 0$ and the run starts. This occurs with probability ε.
   (a) The illiquid assets are bad. This happens with (conditional) probability $1 - \mu$.
      i. News arrive prior to date τ. This occurs with (conditional) probability $1 - \exp(-\lambda\tau)$. The bank ends up liquidated, so its overall asset returns are $C + q_b(1 - C)$.
      ii. News do not arrive prior to date τ. This occurs with (conditional) probability $\exp(-\lambda\tau)$. The bank ends up liquidated in the weak bank case ($\xi = 0$) and continued in the strong bank case ($\xi = 1$), so its overall asset returns are $C + [q_b - \xi(q_b - a_b)](1 - C)$.
   (b) The illiquid assets are good. This happens with (unconditional) probability μ.
      i. News arrive prior to date τ. This occurs with (conditional) probability $1 - \exp(-\lambda\tau)$. The bank continues up to termination, so its overall asset returns are $C + a_g(1 - C)$.
      ii. News do not arrive prior to date τ. This occurs with (conditional) probability $\exp(-\lambda\tau)$. The bank ends up liquidated in the weak bank case ($\xi = 0$) and continued in the strong bank case ($\xi = 1$), so its overall asset returns are $C + [q_g + \xi(a_g - q_g)](1 - C)$.

Putting together these payoffs and after some algebra, we obtain the expression reported in (12).■

Proof of Proposition 6 From (13), it is a matter of algebra to check that the first and second derivatives of $W_{-1}^{ER}(C)$ with respect to C can expressed as

$$\frac{dW_{-1}^{ER}(C)}{dC} = -(A_H - 1) + A_L \left(\frac{D - C}{D}\right)^{\lambda/\delta} \left[1 + \frac{\lambda(1 - C)}{\delta(D - C)}\right], \tag{33}$$

$$\frac{d^2W_{-1}^{ER}(C)}{dC^2} = -A_L \left(\frac{D - C}{D}\right)^{\lambda/\delta} \frac{\lambda}{\delta^2(D - C)^2}[\delta(D - 1) + \delta(D - C) + \lambda(1 - C)], \tag{34}$$

where the sign of the first is ambiguous, while the sign of the second is strictly negative. So $W_{-1}^{ER}(C)$ is strictly concave in C. If it is strictly increasing at $C = 0$, i.e.

$$\frac{\lambda}{\delta D} A_L > A_H - A_L - 1, \tag{35}$$

then $W_{-1}^{ER}(C)$ must reach a maximum over the interval $[0, \bar{C}]$ at some point $C^* > 0$. Such point must be unique because $W_{-1}^{ER}(C)$ is strictly concave in C. By the same token, if (35) does not hold, $W_{-1}^{ER}(C)$ reaches its maximum at $C^* = 0$.■

Proof of Proposition 7 Most of the results in this proposition are proven by the arguments already included in the main text, prior to the proposition. It remains to be proven that $TMV_{-1}^{ER}(C)$ is strictly decreasing in C when $\xi = 1$. To see this, let us rewrite the expression in (14) using (13) and $\exp(-\lambda\tau) = ((D - C)/D)^{\lambda/\delta}$:

$$\begin{aligned} TMV_{-1}^{ER}(C) &= C + A_H(1 - C) - A_L \left(\frac{D - C}{D}\right)^{\lambda/\delta} (1 - C) \\ &\quad + \xi\varepsilon \left(\frac{D - C}{D}\right)^{\lambda/\delta} (1 - \mu)[(D - C) - a_b(1 - C)]. \end{aligned} \tag{36}$$

But with $\xi = 1$, we have $A_L = \varepsilon(1 - \mu)(q_b - a_b)$, so the last two terms of the above expression can be grouped together, yielding

$$\begin{aligned} TMV_{-1}^{ER}(C) &= C + A_H(1 - C) + \varepsilon(1 - \mu) \left(\frac{D - C}{D}\right)^{\lambda/\delta} [(D - C) - a_b(1 - C) \\ &\quad -(q_b - a_b)(1 - C)] \\ &= C + A_H(1 - C) + \varepsilon(1 - \mu) \left(\frac{D - C}{D}\right)^{\lambda/\delta} [(D - C) - q_b(1 - C)], \end{aligned} \tag{37}$$

which is strictly decreasing in C since $A_H > 1$ and $q_b < 1$.■

Online Appendix to "Liquidity standards and the value of an informed lender of last resort"

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A The late run equilibrium

We denote a *late run equilibrium* (or LR equilibrium) the subgame perfect Nash equilibrium that begins after the shock arrives at $t = 0$ in which debtholders only start exercising their puts if further news confirm that the bank's illiquid assets are bad. In this equilibrium, the arrival of good news allows the bank to end the crisis with its liquidity untouched.

In the LR equilibrium the situation of the bank only changes when the news come. So, if it is not a profitable deviation for an individual debtholder to exercise her put at $t = 0$, then it will not be a profitable deviation either at any other point before news arrive. But news, on the other hand, arrive in finite time with probability one, revealing the bank to be good with probability μ and bad with probability $1 - \mu$. So debtholders' value of not exercising their put in the late run equilibrium can be written as

$$V_0^{LR}(C) = \mu B + (1 - \mu) [C + q_b(1 - C)], \tag{38}$$

reflecting that debtholders are eventually paid B if the bank is good (recall that there is no discounting) and receive an expected payoff $C + q_b(1 - C)$ if the bank is bad (recall (10)).

Sustaining an equilibrium with late runs requires having $V_0^{LR}(C) \geq D$, so the following proposition can be proven by direct inspection of the relevant expressions.

Proposition 8 *A LR equilibrium exists if and only if $V_0^{LR}(C) \geq D$. Such condition holds when the bank is sufficiently likely to be good. When $V_0^{LR}(0) \geq D$, the LR equilibrium exists even with $C = 0$. When $V_0^{LR}(0) < D \leq V_0^{LR}(\bar{C})$, there is a minimum liquidity standard*

$$\hat{C} = \frac{D - \mu B - (1 - \mu)q_b}{(1 - \mu)(1 - q_b)} \in (0, \bar{C}] \tag{39}$$

such that the LR equilibrium exists if and only if $C \in [\hat{C}, \bar{C}]$.

Proof Evident from the arguments provided above.■

The case with $\hat{C} \in (0, \bar{C}]$ illustrates that the holding of (moderate amounts of) liquidity can facilitate the sustainability of the late run equilibrium. It does so by enhancing the value of the bank when its illiquid assets are bad, which in turn increases debtholders's payoff from waiting for news. In other words, cash reassures debtholders about the value of their stake at the bank and makes them willing to delay the exercise of their option to run.

From an allocational perspective, making the debtholders effectively more patient during a crisis contributes to "buying" the time needed for the arrival of news that, eventually, facilitate an efficient resolution of the crisis, in that the bank with good assets continues and the bank with bad assets is liquidated.[38]

What is the connection between liquidity and LLR support in the LR equilibrium? On the one hand, by facilitating the sustainability of the LR equilibrium, liquidity may contribute to actually make LLR support unneeded on the equilibrium path. On the other, the LLR's willingness to support the bank when its assets are known to be good rules out the possibility of self-fulfilling prophecies that might precipitate the start of a run at date $t = 0$ and lead it not to stop even after news indicating that assets are good.

A.1 Welfare and firm value in the late run equilibrium

Let us first consider the case in which $D \leq V_0^{LR}(\bar{C})$, which means that the LR equilibrium can be sustained by choosing a suitable value of C. And suppose that C is set at a value that indeed sustains the LR equilibrium. How large is the welfare generated by the bank in these circumstances?

We measure the ex ante welfare associated with this equilibrium, $W_{-1}^{LR}(C)$, as the expected value of the overall payoffs generated by the bank from t=–1 onwards, that is, the returns produced by its initial assets across possible states. Using the fact that, in the LR equilibrium, good illiquid assets get continued up to termination, while bad assets get early liquidated, we obtain

$$W_{-1}^{LR}(C) = C + \{[(1-\varepsilon) + \varepsilon\mu]a_g + \varepsilon(1-\mu)q_b\}(1-C) \tag{40}$$

or, in terms of the notation introduced in (13),

$$W_{-1}^{LR}(C) = C + A_H(1-C), \tag{41}$$

where $A_H - 1$ was referred to as the fundamental net present value potentially associated with the bank's investment in illiquid assets. Importantly, $W_{-1}^{LR}(C)$ is linear in C, and strictly decreasing in C if an only if $A_H - 1 > 0$. Therefore:

[38] Strictly speaking, the bank with bad assets continues up to the exhaustion of its cash. Alternatively, we could assume that a resolution authority forces the bank into liquidation as soon as it is learned to be bad. Given the absence of discounting, none of our equations and results would change in such an alternative scenario.

Proposition 9 *If the illiquid assets have strictly positive fundamental net present value at t=–1, and the LR equilibrium is (rightly) anticipated to prevail, it is not socially optimal to set* C *strictly larger than* $\max\{\hat{C}, 0\}$*, where* $\hat{C}$ *is given by (39).*

Proof Proposition 8 implies that sustaining the LR equilibrium requires a minimal C of either 0, if $V_0^{LR}(0) \geq D$, or some $\hat{C} \in (0, \bar{C}]$, if $V_0^{LR}(0) < D \leq V_0^{LR}(\bar{C})$. Assume that the LR equilibrium is (rightly) anticipated to prevail whenever $C \geq \max\{\hat{C}, 0\}$, where $\hat{C}$ is given by (39). However, under $A_H - 1 > 0$, $W_{-1}^{LR}(C)$ is decreasing in C. So setting C strictly larger than $\max\{\hat{C}, 0\}$ would be detrimental to welfare.■

In the LR equilibrium, the LLR never supports the bank, so the full value reflected in $W_{-1}^{LR}(C)$ also constitutes the ex ante *total market value* of the bank in this equilibrium, $TMV_{-1}^{LR}(C)$. This is the object that bank owners aim to maximize when choosing C and selling the bank's debt and equity to investors. Therefore:

Proposition 10 *If the illiquid assets have positive fundamental net present value at t=–1 and the LR equilibrium is (rightly) anticipated to prevail, it is not privately optimal for bank owners to set* C *strictly larger than* $\max\{\hat{C}, 0\}$*, where* $\hat{C}$ *is given by (39).*

Proof Given the absence of subsidies associated with LLR support, we have $TMV_{-1}^{LR}(C) = W_{-1}^{LR}(C)$ and the result follows trivially from the arguments provided in the proof of Proposition 9.■

So, conditional on inducing a LR equilibrium, there appears to be no discrepancy between the private and the social incentives for the choice of C and, hence, no clear rationale for regulatory liquidity standards. Bank owners and the social planner agree that setting $C = \hat{C}$ is the most efficient way to guarantee the existence of the LR equilibrium. However, there might be situations where, even if a LR equilibrium can be sustained with $C = \hat{C} > 0$, bank owners find it privately optimal to set $C < \hat{C}$ and induce the emergence of a different equilibrium (e.g. the ER equilibrium) where the total market value of the bank is larger than $TMV_{-1}^{LR}(\hat{C})$. To discuss this in greater detail, one would need to analyze more systematically the possible coexistence of ER and LR equilibria in our economy, as we do in the next subsection.

A.2 Early run vs. late run equilibria

To analyze the possible coexistence of the ER and LR equilibria, it is useful to start comparing $V_t^{ER}(C)$ with $V_0^{LR}(C)$. To this effect, it is convenient to re-express (11) as

$$\begin{aligned} V_t^{ER}(C) &= \mu\{D + [1 - \exp(-(\delta+\lambda)(\tau - t))]\frac{\lambda}{\delta+\lambda}(B - D)\} \\ &\quad +(1-\mu)\{D - \exp(\delta t)[D - C - q_b(1-C)]\} \\ &\quad -\mu\exp(-\lambda(\tau - t))\exp(\delta t)[D - C - q_g(1-C)] \\ &\quad +\xi\exp(-\lambda(\tau - t))\exp(\delta t)[D - C - \bar{q}(1-C)]. \end{aligned} \tag{42}$$

1. The first term can be compared to the first term in (38): it is smaller. What appears multiplied by μ is lower than B because all the factors that multiply the term $B-D>0$ within the curly brackets are lower than one.

2. The second term can be compared to the second term in (38): it is weakly smaller. Specifically, it is identical for $t=0$ and decreasing in t, so it is strictly smaller for $t \in (0, \tau]$.

3. The third term is negative, while there are no further terms in (38).

4. The fourth term is zero in the weak bank case ($\xi = 0$) and positive (and equal to the expected subsidy associated with LLR support) in the strong bank case ($\xi = 1$).

Therefore:

1. In the weak bank case ($\xi = 0$), we necessarily have $V_t^{ER}(C) < V_0^{LR}(C)$ for all $t \in [0, \tau]$, and hence $V_t^{ER}(C) < D$ for all $t \in [0, \tau]$ whenever $V_0^{LR}(C) < D$. Hence either the ER equilibrium or the LR equilibrium always exists. In fact, in situations with $V_0^{ER}(C) \leq D \leq V_0^{LR}(C)$, the LR and the ER equilibria coexist, due to the self-fulfilling potential of the prophecies (on likelihood that the bank ends up liquidated) attached to the ER equilibrium.

2. In the strong bank case ($\xi = 1$), the fourth term in (42) is a source of ambiguity for the comparison between $V_t^{ER}(C)$ and $V_0^{LR}(C)$. In fact, in this case, the third and fourth terms in (42) can be consolidated into a net positive term:

$$+(1-\mu)\exp(-\lambda(\tau - t))\exp(\delta t)[D - C - q_b(1-C)], \tag{43}$$

   whose comparison with the positive gap between $V_0^{LR}(C)$ and the first two terms of $V_t^{ER}(C)$ is generally ambiguous. In this case, analytical conditions guaranteeing $V_t^{ER}(C) \leq D$ for all $t \in [0, \tau]$ whenever $V_0^{LR}(C) < D$ are convoluted. Yet, numerical examples show that there are parameter values under which this property is preserved, as well as cases in which it is not.

A.3 Taxonomy of equilibria in the strong bank case

To further understand the taxonomy of situations that we may find in the strong bank case, Figure A1 depicts the values of D, $V_0^{LR}(C)$, and $V_t^{ER}(C)$ for all $t \in [0, \tau]$ for a number of examples. The time passed since the possible start of the early run, t, appears on the horizontal axes, while the values of D, $V_0^{LR}(C)$, and $V_t^{ER}(C)$ appear on the horizontal ones. The examples rely on a variation of the strong bank baseline example described in Table 1 of the paper. Specifically, the following parameters are kept fixed throughout the examples that appear in the various panels of Figure A1:

Table A1
Parameter values behind Figure A1

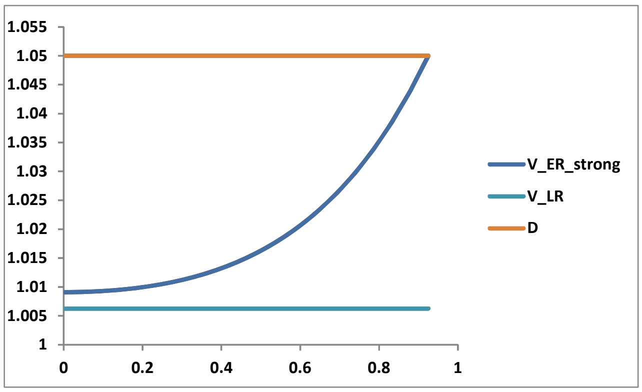
ε	a_g	a_b	q_g	q_b	λ	δ	b	D
0.2	1.2	0.5	0.85	0.75	2.5	0.167	0.0476	1.05

Under all combinations of parameters explored in Figure A1, the bank is strong, i.e. the expected value of its illiquid assets is higher, unconditionally, if continued than if early liquidated, so the previously described complexity regarding the potential taxonomy of equilibria arises. We generate the various panels of Figure A1 by varying the probability of the illiquid assets being good, μ, by rows, and the bank's cash holdings, C, by columns, as indicated under each panel.

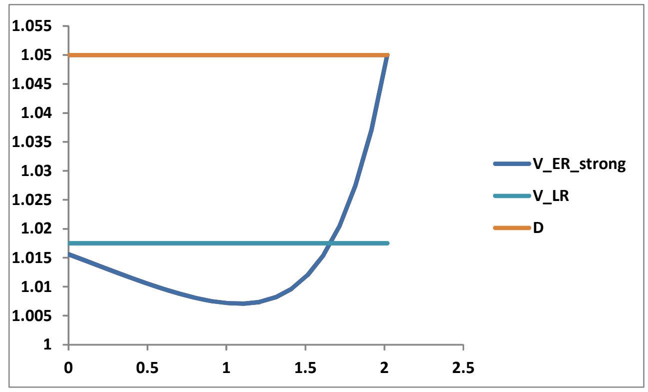
Panel 1 describes a case (with $\mu = 0.7$ and $C = 0.15$) in which the ER equilibrium is sustainable while the LR equilibrium is not. Interestingly, in this case the subsidy linked to LLR support makes $V_t^{ER}(C) > V_0^{LR}(C)$ for all t. Panel 2 shows that holding more liquidity ($C = 0.3$) lengthens the potential duration of the run and modifies the time-profile of $V_t^{ER}(C)$, which now starts below $V_0^{LR}(C)$ but eventually becomes larger than it, but never larger than D. So the ER equilibrium is sustainable, while the LR equilibrium is not.

Panels 3 and 4 illustrate what happens when μ is larger, very close to (but still below) the bound above which the LR equilibrium would become sustainable even with $C = 0.15$. In Panel 3, the ER equilibrium is sustainable while the LR equilibrium is not. In this case, increasing C to 0.3 makes the LR equilibrium sustainable (because $V_0^{LR}(C) > D$), while it turns the ER equilibrium unsustainable (because $V_t^{ER}(C)$ is larger than D at low values of t).

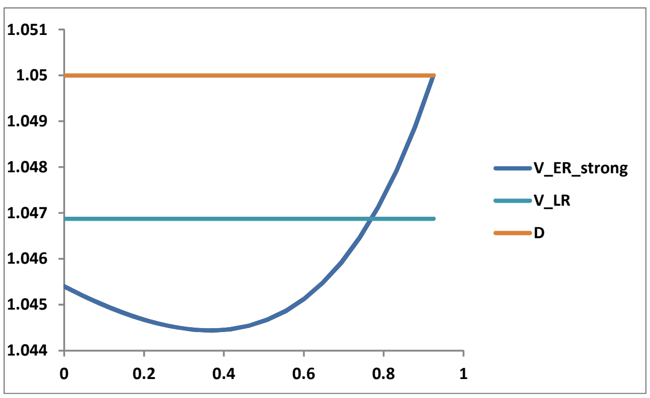
In panels in the bottom row, μ is large enough ($\mu = 0.85$) for the LR equilibrium to be sustainable even with $C = 0.1$ (Panel 5) but with those liquidity holdings the ER equilibrium is also sustainable. In this case, increasing C to 0.2 (Panel 6) makes the ER equilibrium unsustainable, while the LR equilibrium remains sustainable. The reason why the ER ceases to exist is that the additional liquidity holdings reduce the effective net subsidy associated with LLR support in a way that makes $V_t^{ER}(C)$ larger than D at some (low) values of t. This means that a debtholder's best response to anticipating that subsequently debtholders will exercise their put options is no longer to exercise her own option, so the logic sustaining the ER equilibrium unwinds.



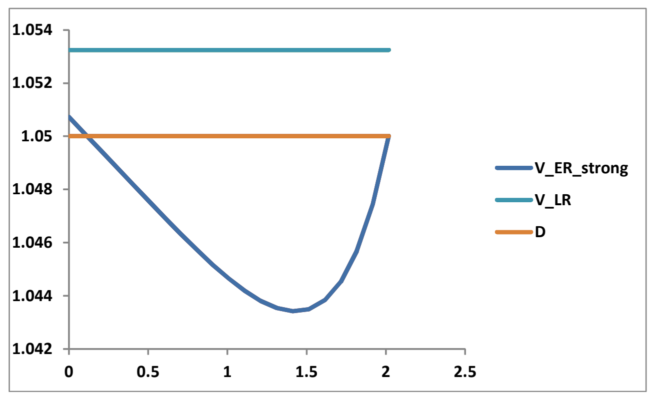
1. (μ, C)=(0.70,0.15). Only ER is an equilibrium



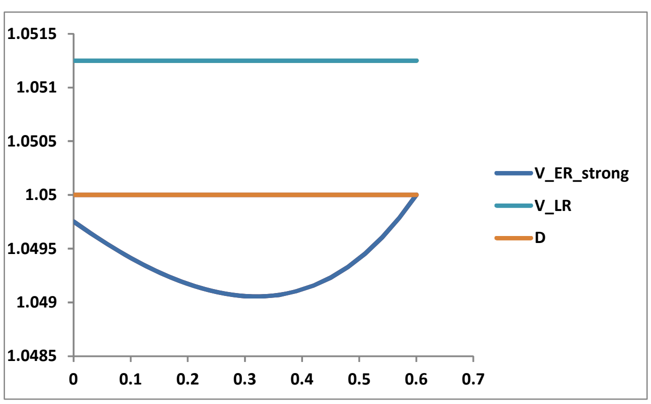
2. (μ, C)=(0.70,0.30). Only ER is an equilibrium



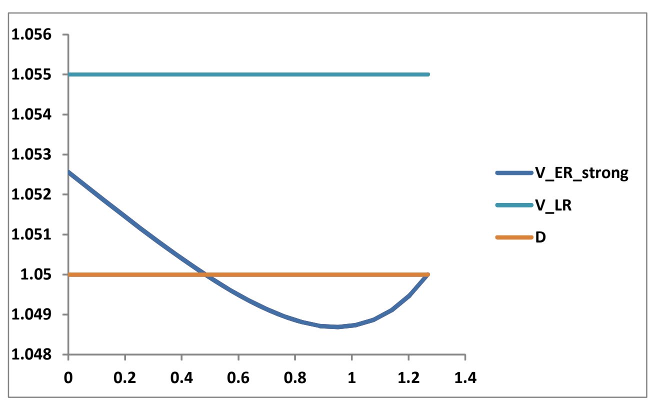
3. (μ, C)=(0.83,0.15). Only ER is an equilibrium



4. (μ, C)=(0.83,0.30). Only LR is an equilibrium



5. (μ, C)=(0.85,0.10). Both LR and ER are equilibria



6. (μ, C)=(0.85,0.20). Only LR is an equilibrium

Figure A1. Early vs. late run equilibria in the strong bank case

B Determinants of optimal liquidity holdings

In principle, the analysis of the determinants of the optimal liquidity standards C^* could be undertaken by performing a standard comparative statics exercise on the first order condition that characterizes C^* when it is interior, $dW_{-1}^{ER}(C)/dC = 0$, where an expression for $dW_{-1}^{ER}(C)/dC$ appears in equation (33) in the paper. However, analytically it is only possible to obtain non-ambiguous signs for the corresponding dC^*/dz for some of the z parameters. The signs (or, in case of ambiguity, interrogation signs) in the following table are obtained by standard use of the Implicit Function Theorem on $dW_{-1}^{ER}(C)/dC = 0$. Detailed derivations are omitted for brevity.

Table B1
Dependence of optimal liquidity holdings on key parameters*

	μ	ε	a_g	a_b	q_g	q_b	b
Strong bank case	−	+	+	−	0	?	0
Weak bank case	?	+	?	0	−	+	0

*For z=δ, λ, D the signs of dC^*/dz are ambiguous.

In the rest of this appendix we explore the numerical examples introduced in subsection 5.2 of the paper. We analyze and discuss how the socially optimal liquidity holdings C^* in those examples vary with some of the key parameters of the model. Rather than realism, the examples seek to understand qualitatively the key forces influencing C^*. The two baseline cases describe in Table 1 of the paper only differ in the probability μ with which illiquid assets remain good if the bank gets hit by the shock at $t = 0$. In the strong bank baseline ($\mu = 0.5$), conditional on being hit by the shock at $t = 0$, illiquid assets have an expected continuation value of 0.85, which is larger than their expected liquidation value of 0.8. In the weak bank baseline ($\mu = 0.25$), these values are 0.675 and 0.775, respectively, so their rank switches. In both cases, the value of δ implies a relatively conservative maturity structure, with an average debt maturity of 6 months, while the value of λ implies a relatively rapid revelation of information, with an expected span of 0.4 months (about 12 days).[39] The probability that the bank is hit by a shock is intendedly high (20% per month) to magnify the importance of the trade-offs and make the qualitative effects more visible.

All panels in Figure B1 (strong bank baseline) and Figure B2 (weak bank baseline) depict, as functions of a different varying parameter, the socially optimal liquidity holdings C^*. The value of such a parameter in the baseline parameterization is indicated by a dashed vertical line. The value (if any) for which moving the parameter further up or down switches from the strong bank case ($\xi = 1$) to the weak bank case ($\xi = 0$) is indicated with a vertical

[39] The corresponding expected time spans can be computed as $1/\delta$ and $1/\lambda$, respectively.

dotted line. As one can see, the baseline parameterization involves an interior value of C^* but changing the parameters often leads to the corner solution with $C^* = 0$.

For a number of parameters, the response of C^* is qualitatively identical across the strong and weak bank cases. Specifically, C^* is decreasing (until it reaches the lower bound of zero) in the continuation value of good assets a_g and, in a extremely mildly manner, the early debt repayment D, while is increasing (once it abandons its lower bound of zero) in the probability ε of the shock hitting the bank at $t = 0$.

The response of C^* to the Poisson rates δ and λ is also similar across the strong and weak bank cases but interestingly non-monotonic. Increasing the rate δ at which debt can be put by investors during a run (which can be interpreted as the result of shortening average debt maturity) or the rate λ at which information about the quality of the illiquid assets arrives during a run first increases but eventually decreases the optimal liquidity holdings. This is because first liquidity becomes more needed (as δ increases) or more effective (as λ increases) as a means to buy time for information to arrive during the run. However, once δ is large enough (relative to λ), the run happens too quickly and liquidity becomes a too costly means to buy information—it requires a too large sacrifice of ex ante profitable investment in the illiquid asset. Similarly, once λ is large enough (relative to δ), liquidity holdings are so effective in providing time to obtain the relevant information that C^* declines in response to an increase in λ.

The remaining parameters affect C^* differently across the strong and weak bank cases. Specifically, the optimal liquidity holding are decreasing in the continuation value of bad assets a_b in the strong bank case, since larger a_b implies a lower type II error when a bad bank receives LLR support, but C^* does not depend on a_b in the weak bank case, since a bank with bad assets never continues in such a scenario (and hence a_b does not enter the relevant welfare calculations). By the symmetric logic, C^* does not depend on the liquidation value of good assets q_g in the strong bank case, but declines with q_g in the weak bank case.

Finally, the sign of the response of C^* to variations in parameters q_b and μ switches depending on whether the bank is supported or not by the LLR in the absence of news. In the strong bank regime, where the uninformed LLR chooses $\xi = 1$, the optimal liquidity holdings are increasing in the liquidation value of bad assets q_b (reflecting the higher value of discovering that the bank is bad on time not to support it) and decreasing in the probability μ that the illiquid assets are good (by exactly the reverse logic). Instead, in the weak bank regime, C^* decreases mildly with q_b (reflecting the larger return overall associated with the investment in illiquid assets and, hence, the large opportunity cost of cash holding) and increases with μ (reflecting the greater value of preventing the mistaken liquidation of good assets that occurs when the uninformed LLR decides $\xi = 0$).

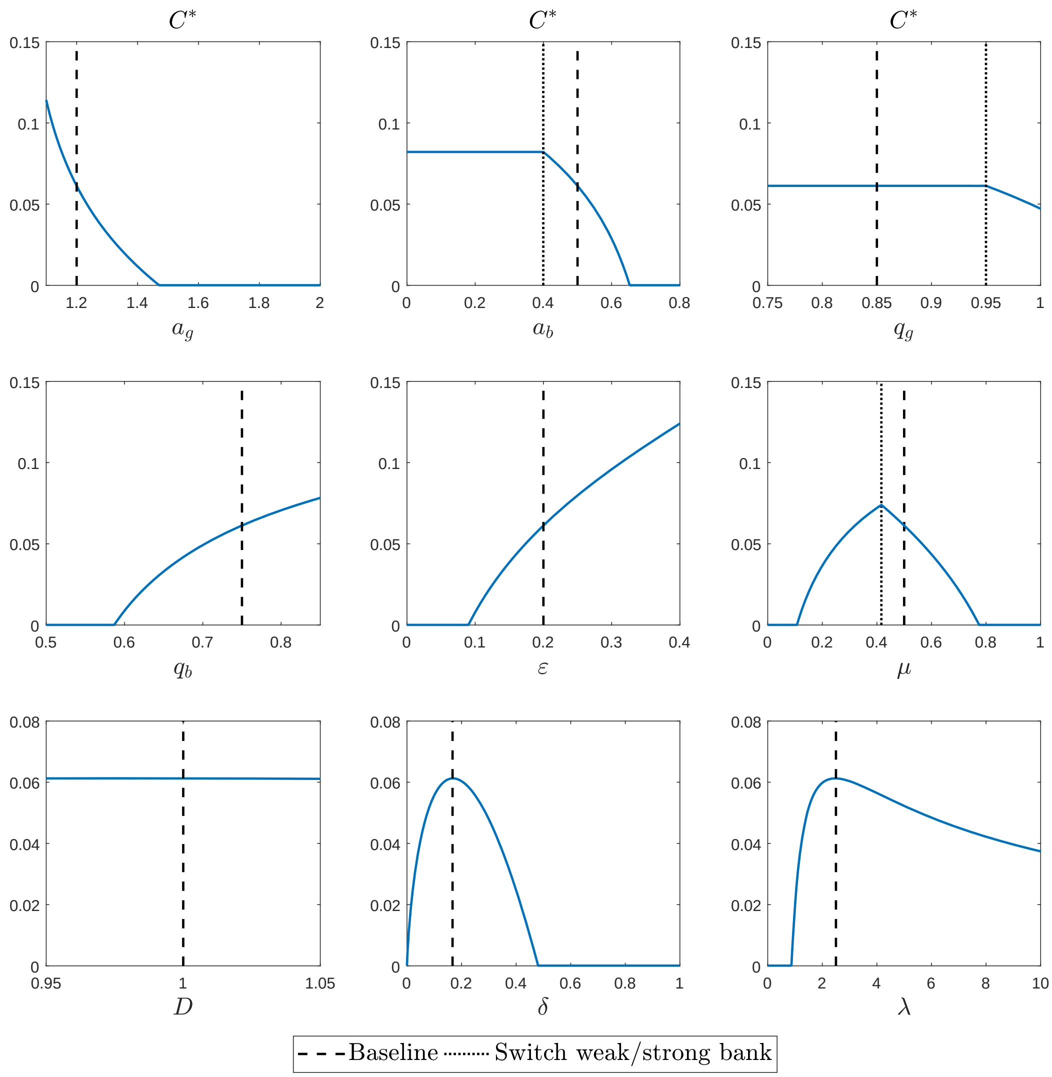


Figure B1 Comparative statics of C^* in the strong bank baseline case

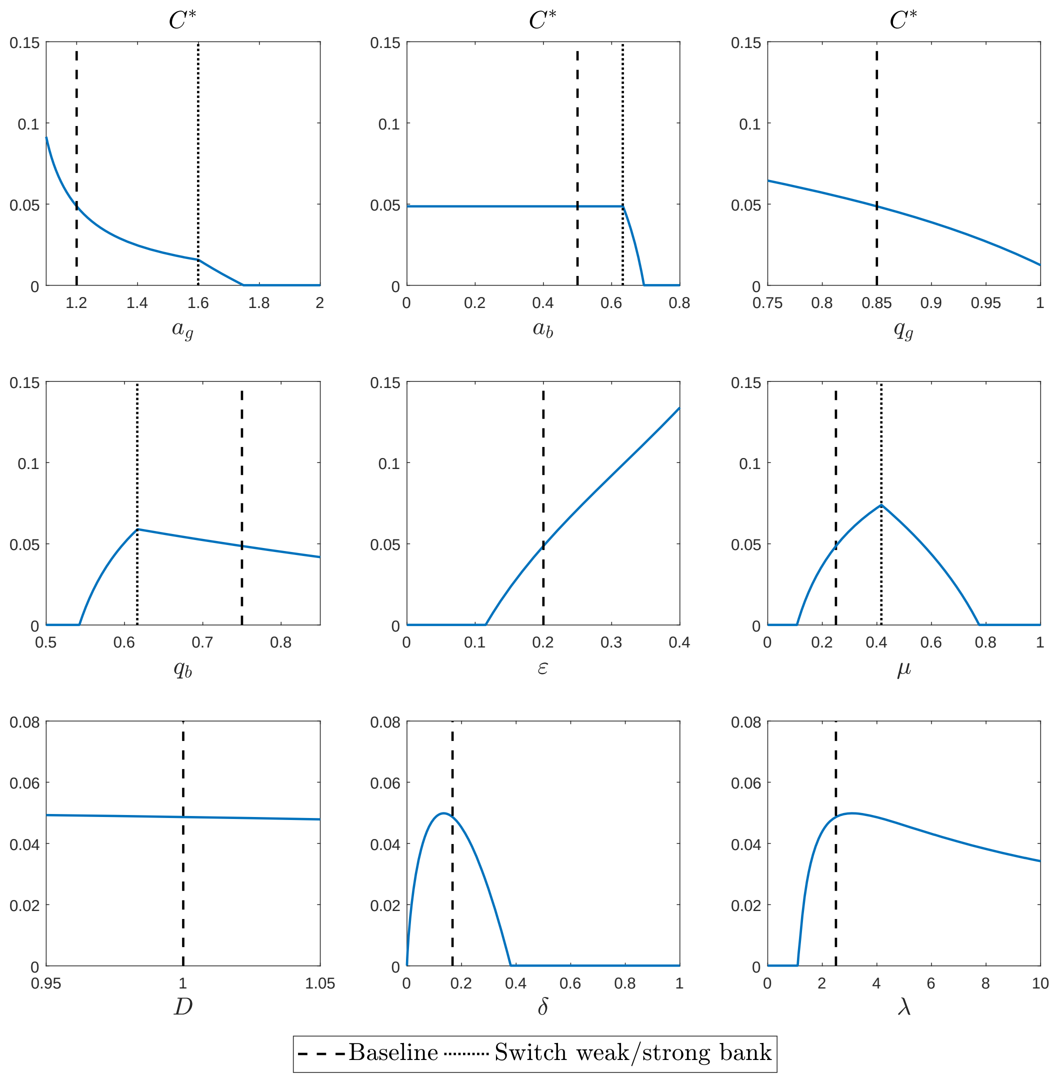


Figure B2 Comparative statics of C^* in the weak bank baseline case

C Additional extensions to our model

In this section, we discuss two additional issues: bankers' incentives to produce information on their condition, and a potential variation of the baseline model in which what needs to be discovered about the bank is not the quality of its assets but the potential systemic importance of its failure.

C.1 Banks' incentives to produce information

In our model, we assume that the arrival of information on the bank's financial condition follows an exogenous Poisson process. As we discussed, the rate λ at which information comes out and the nature of it (whether it is good or bad news) have important implications. For example, the arrival of good news at any time before the bank's cash gets exhausted during an early run eliminates debtholders' incentives to continue exercising their puts, leading the run to an end. In contrast, when the news is bad, debtholders continue exercising their puts and it becomes clear that the LLR will not support the bank once it exhausts its cash. In this context, bank owners are not going to be generally indifferent about whether information gets disclosed, or the rate at which it is disclosed. And this may have implications for the need for supervision to play an active role in the discovery of the relevant information.

To see this, suppose bank owners have the ability to affect the speed at which information gets disclosed during an early run. In the weak bank case, since by default LLR support will not be blindly granted, bank owners will find it advantageous to disclose information about the bank's assets. If the bank is bad, things will not be worse than without the information. But if the bank is good, some extra value can be generated. Social interest in this case is aligned with bank incentives. By contrast, in the strong bank case, bank owners will not be interested in accelerating the production of information. In fact, they will try to delay it, since keeping the LLR "blind" is a way to guarantee its support, and appropriate the corresponding implicit subsidy. So, in this case, involving the LLR in bank supervision or entrusting another agency with the responsibility to produce information about the bank's financial condition (and to share it with the LLR) may be crucial.

C.2 Systemically important banks

One key feature of systemically important banks (SIBs), especially in the absence of a fully effective regime for the recovery and resolution of too-big-to-fail institutions, is the possibility that their early (and disorderly) liquidation causes significant damage to the rest of the financial system or the wider economy (e.g. in the form of fire sale externalities, contagion, etc.). This suggests that for a LLR dealing with a SIB, the trade-offs relevant for deciding whether to grant liquidity support or not might be driven by considerations beyond the fundamental solvency of the bank (or, in model terms, the intrinsic quality of its illiquid assets). One important consideration is the size of the *systemic externalities* that might be avoided by supporting the bank. These externalities increase the social value of allowing the bank to continue in operation after it exhausts its cash, as opposed to pushing it into liquidation.

From the perspective of the LLR, variation in the size of these systemic externalities can play the same role as variation in the quality of the illiquid assets in our model. And, from this viewpoint, we could also assimilate non-SIBs to our weak banks (i.e. banks that, in the absence of further information, would not be supported by the LLR) and SIBs to our strong banks (i.e. banks that, in the absence of further information, would be supported). Hence, it is natural to establish a parallel between the model analyzed in prior sections and a model in which banks in trouble (say, to simplify, with bad illiquid assets) can generate small or large systemic externalities if they fail. In such a setup, liquidity standards would give the LLR time to receive information on the size of the externalities.

A full formal analysis of this alternative framework would require more than a pure relabeling of the objects present in the current model. Parallel to the current setup, the size of the systemic externalities is relevant for the LLR decision and, through it, for debtholders' expectations on whether the bank will be supported or not. But one important difference is that systemic externalities do not directly affect debtholders' payoffs contingent on continuation so their size being large or small cannot be fully assimilated to the value of illiquid assets being high or low in our model. Hence, the details of several equations would change.

Yet, it is safe to conjecture that non-SIBs will have greater incentives than SIBs to choose liquidity holdings close to those that maximize social welfare, since, by a logic similar to the one explored in our model, the subsidies that they will expect to obtain through the support granted by a blind LLR are lower (if any) than the subsidies that SIBs will expect to obtain. As in our analysis above, the socially optimal liquidity standards would have to trade-off gains from increasing the likelihood that the LLR gets informed about the true systemic importance of the bank and the losses from forcing banks to ex ante forgo potentially more profitable uses of funds.