Capital Forbearance in the Bank Recovery and Resolution Game*

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Abstract

We analyze the strategic interaction between undercapitalized banks and a supervisor in a recovery and resolution framework in which early recapitalizations can prevent later disorderly failures. Capital forbearance emerges because reputational, political, economic and fiscal costs undermine supervisors' commitment to publicly resolve the banks that miss the request to privately recover. Under a weaker resolution threat, banks' incentives to recover are lower and supervisors may end up having to resolve more banks. When marginal resolution costs steeply increase with the scale of the intervention, private recovery actions become strategic complements, producing too-many-to-resolve equilibria with high forbearance and high systemic costs.

Keywords: bank supervision; bank recapitalization; forbearance.

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1 Introduction

Bank supervisors seek to ensure that banks remain well-capitalized even under adverse economic circumstances. Regular supervisory assessments, stress testing, and prompt corrective action play complementary roles in achieving this goal. Failure to recapitalize banks with weak capital positions may lead at a later stage to disorderly failures, large losses to creditors and deposit insurance schemes, and costly bailouts. Timely intervention is a difficult task, so historical cases of insufficient supervisory intervention (forbearance) are not scarce.

After the Global Financial Crisis, several jurisdictions (e.g. the EU and the UK) adopted new bank recovery and resolution frameworks designed to enhance the prompt corrective action process and reduce the costs to taxpayers. A stylized description of the process goes as follows. Once some banks are assessed as undercapitalized, the supervisor calls for their private recapitalization, either by injecting new equity or undertaking an asset or debt restructuring. Banks that are not recapitalized are then subject to early precautionary public recapitalizations and other interventions such as the activation of bail-in measures. While these tools allow to deal in a timely fashion with troubled banks, their implementation ahead of visible distress face much resistance and is fraught with challenges (Hellwig, 2017).[1]

In this paper, we study the strategic interaction of troubled banks and their supervisor throughout this process. The supervisor prefers troubled banks to recover via a private capital injection (henceforth "recovery"). Shareholders cannot be legally forced to contribute new equity, so the supervisor facing a noncompliant bank has two choices. It can choose to force a public recapitalization, e.g. via a dilutive public capital injection or a bail-in measure (henceforth "resolution"). Alternatively it can forbear, allowing the bank to operate with inadequate capital and hoping for the best. Such a delay in the recognition of insolvency risk may be seen as a form of regulatory gambling, driven by career concerns or political pressure, or as the result of supervisors' aim to spread the costs of dealing with bank trouble over time (e.g. because of fiscal or budgetary frictions). We build a model to analyze how the interaction between banks' private incentives to recover and the supervisor's incentive to forbear gets shaped by a number of factors.

Private incentives to recapitalize are very poor in overleveraged banks because compliance implies a net transfer to creditors or a reduction in the deposit insurance put

[1] Legal and institutional complexity, lack of experience and unanticipated consequences limit the speed and scope of intervention, while full bail-in requirements may have destabilizing effects in systemic crises (see, e.g., Dell'Ariccia et al. (2018a)).

(as in Bhattacharya and Nyborg, 2013). However, a public recapitalization by the supervisor can be even privately costlier due to its dilutive effect on existing equity, as well as restrictions on managerial discretion and compensation that typically accompany supervisory interventions.[2] Reflecting this, in the model we assume that a public recapitalization (resolution) implies higher costs to bank insiders than a private recapitalization (recovery).

In the playing of the recovery and resolution game, the supervisor trades off short-term reputational, political and fiscal costs of early resolution with the costs of potential systemic financial distress in the future. As noticed in previous work, inadequate or delayed supervisory intervention may reflect the desire to hide weak supervisory skills or bad past decisions (Boot and Thakor, 1993; Morrison and White, 2013), or the aim to avoid the disclosure of information that may trigger a panic (Chan and van Wijnbergen, 2017; Walther and White, 2020). Publicly elected officers may be influenced by the fear to lose jobs or elections if too many banks appear to need intervention (Bian et al., 2020).[3] Limited supervisory capacity (Eisenbach at al., 2022) or tax burden smoothing motives can also induce supervisory forbearance.[4] One way or another, if the supervisor trades off some (increasing and convex) short-term costs of resolution with some (increasing and convex) costs of future systemic distress caused by undercapitalized banks, its response may imply not to resolve every non-recoverying bank. Any anticipated forbearance will feed back into banks' incentives to recover.

In the baseline version of the model, distressed banks banks' individual decisions to recover are strategic substitutes, since when more banks restore their insolvency there is less pressure on the supervisor to resolve the remaining ones to limit the potential losses. With a lower resolution intensity, there is a higher chance that an unrecovered

[2]In 2008, the US Treasury intervened in over 600 banks via the Capital Purchase Program (CPP). This included the injection of $125 billion of preferred stock in the largest commercial banks (Veronesi and Zingales, 2010), which was mostly used for recapitalization rather than new lending (Taliaferro, 2021). Banks' eagerness to repay this preferred stock has been interpreted as a desire to avoid the governance restrictions associated with it (Wilson and Wu, 2012; Mücke et al., 2021). The intervention by the ECB of Banca Carige in January 2019 (after it failed to cover a capital shortage identified months earlier) involved the appointment of three administrators and a surveillance committee, and the announcement of a major restructuring (IPOL, 2019).

[3]Short-termism can also be the outcome of failure to transmit soft information due to incentive misalignments, or deliberate resistance to change due to vested interests (Garicano and Rayo, 2016; Dow et al., 2021).

[4]Since fiscal constraints may also trigger systemic runs (Dang et al., 2017), yet another explanation for costs of resolution steeply increasing in the scale of the intervention is the fear of triggering a fiscally-driven systemic run. Authorities pressed by career concerns or political economy considerations may prefer gambling for resurrection rather than incurring the more immediate, certain and sharply rising costs of early resolution.

bank benefits from forbearance. This reduces each troubled bank's marginal incentive to recover.

The underlying strategic interactions lead to either corner equilibria in which troubled banks play pure strategies (e.g., in which no bank recovers) or interior equilibria in which they play mixed strategies, resulting in a fraction of recovering banks. In a typical interior equilibrium of our baseline model, only a fraction of damaged banks opt for recovery and only a fraction of the non-recovered banks are subject to resolution (with the remaining fraction thus benefiting from forbearance). A randomized decision on recovery can be reinterpreted as a choice based on unobservable characteristic that affects their net payoff under each alternative (Harsany, 1973).[5] In our context the marginal preference might be driven by variation in actual leverage, hard to assess given the opaqueness of bank balance sheets.

The comparative statics of the model reveals that a supervisor with a higher cost of resolution may end up being forced to resolve more banks than one with lower costs (while eventually also leaving more banks undercapitalized). This is similar to the outcome in a multi-party brinkmanship game (related to the classic "game of chicken"). The prospect of lower supervisory incentives or capacity to respond to each challenge as the number of challenges rises, encourages lack of compliance and may lead to a breakdown in compliance. Facing a low level of compliance, even a weak authority is compelled to resolve more banks to contain social costs (in our model the systemic costs of future bank defaults), although still producing more forbearance than a stronger authority.[6]

A classic historical example of broad forbearance where delayed intervention deepened distress was the S&L bank default wave in the 1980s (Degennaro and Thompson, 1996). The stress test run in the US in 2009 raised the opposite perception, as authorities signaled their determination to inject capital if necessary.[7] In contrast, some analysts saw forbearance in the EU stress tests run by the Committee of European Banking

[5]Mixed strategy equilibria are commonly used in evolutionary biology to model interactions among animals. While in an evolutionary context the two interpretations (randomization vs. heterogeneity) may have different implications, in our setup the precise interpretation does not affect the implications.

[6]A similar effect may arise when an undermanned police faces greater infringement and is forced into a more extensive enforcement.

[7]While the commitment to such an injection contributed to dissipate investors' doubts about the solvency of major US banks, the stringent design of the Capital Assistance Program (CAP) that accompanied the stress test encouraged undercapitalized banks to raise equity privately. Through the CAP, the Treasury could purchase mandatorily convertible preferred shares while imposing restrictions to the banks. "The shares had onerous terms to encourage institutions to find other sources of capital: they paid dividends of 9 percent, required a halt to dividend payments on other shares, came with limits on executive compensation, and contained warrants that allowed Treasury to purchase additional common stock" (Lawson, 2021).

Supervisors (CEBS) in 2010.[8] Some European banks that ultimately folded were allowed to operate for some time with insufficient capital, possibly because of authorities' limited political or fiscal capacity to intervene.[9]

A distinct result emerges when we extend the analysis to the cases in which the supervisor faces capacity constraints on resolution or highly increasing marginal costs of resolution (e.g. because the short-term political and reputational consequences of early intervention increase steeply with the scale of the early intervention). We find that, as a result of the weak supervisory response, the resolution threat may decline as more banks fail to recapitalize, turning their recovery decisions into strategic complements. In this case, banks can coordinate in low recovery and produce a too-many-to-resolve phenomenon, leading to an equilibrium with high forbearance and severe systemic costs.

In terms of policy insights, our analysis identifies challenges faced by banks' recovery and resolution process in a weak institutional setting. The structural commitment problem faced by the supervisor when using resolution as a threat to induce higher recovery requires granting authorities adequate independence from political interference, a clear mandate to avoid future systemic costs, and sufficient human and financial resources. From this perspective, several features of the Banking Union reform in Europe reflected the need to create an effective and credible recovery and resolution framework, backed by some pooled resources and with a governance less dependent on local fiscal, reputational and political economy considerations than the subsumed national frameworks (ESRB, 2012). However, the reform fell short of granting full fiscal backing to the framework and left many small and medium size banks outside the scope of the single resolution framework (Restoy, 2019).

Another policy insight from our analysis is that, rather than increasing moral hazard as often assumed, the credible commitment to timely (and sufficiently punitive) public equity injections into banks that remain undercapitalized after being called to recover can positively affect private compliance and avoid later systemic costs and bailout losses.

Related theoretical literature. Our paper contributes to the growing literature on bank resolution and is related to several other strands of banking theory. A key novelty of our paper relative to the papers listed below is the analysis of the strategic interactions between the resolution choices of the supervisor and the recovery decisions of multiple

[8]The CEBS estimated in 2010 that "2.5 billion [euro] would have corrected the capital shortfall by the banks that failed the test. Market estimates suggested 300 billion euro, an amount that proved much more accurate" (Onando and Resti, 2011).

[9]On an anecdotal basis, Dexia Bank reported a capital ratio above 10% in July 2011, three months before its collapse.

banks.

In a resolution context, Walther and White (2020) consider a supervisor with discretionary powers to reduce debt overhang through bail-in whose credibility is limited by the risk that a bail-in triggers a bank run; they show that contingent convertible (CoCo) bonds, by providing recapitalization upon publicly observable signals, can implement the optimal intervention. Colliard and Gromb (2018) study a single bank that renegotiates its debt under asymmetric information, in the shadow of a potential government intervention; they show that making the government commit not to interfere can speed up the workout process, improving efficiency. Segura and Vicente (2019) consider bank resolution in a two-country setup where bail-in reduces the (fiscal) costs of a public intervention; they show that a banking union also reduces the overall fiscal distortions but may involve redistribution across countries.

Older studies on late bank interventions focused on closure rules for banks and how to design them to reduce deposit insurance costs (Acharya and Dreyfus, 1989; Allen and Saunders, 1993; Fries et al., 1997). More recently, this literature focused on bailouts, considering supervisors who trade off the cost of supporting troubled banks (which may include fiscal costs as in Philippon and Schnabl, 2013, Shapiro and Skeie, 2015, or reputational costs as in Morrison and White, 2013, and Carletti et al., 2021) with future systemic costs caused by bank weakness (including spillover effects and other bank default costs as emphasized in Bhattacharya et al., 1998).

The issue of credibility and time consistency of public policies (Kydland and Prescott, 1977) has a long tradition in various fields, including monetary policy (Barro and Gordon, 1983) and corporate finance (Chari and Kehoe, 2016). In banking, time inconsistency problems regarding the activation of policy responses such as suspension of convertibility (Ennis and Keister, 2010), the application of bail-in provisions (Keister and Mitkov, 2017), and stress test design (Parlasca, 2022) have been identified as a root cause of bank runs and other inefficient outcomes. In Bernard et al. (2022), where the focus is on the negotiation between the creditors of a single bank and the supervisor on a voluntary bail-in in the presence of network connections between creditors (e.g. other banks), the lack of credibility of the no-bailout threat diminishes the prospect of a bail-in.

The impact of bank resolution policies on the strategic interaction between banks had been examined before by Perotti (2002), Perotti and Suarez (2002), Acharya and Yorulmazer (2007) and Farhi and Tirole (2012) with a focus on ex ante risk taking implications. Merger policy in Perotti and Suarez (2002) rewards the absorbing surviving banks in a way that makes banks' risk taking decisions strategic substitutes. In contrast, bail-out policies and accommodating monetary policy in Acharya and Yorulmazer (2007) and Farhi and Tirole (2012), respectively, are affected by a too-many-to-fail problem which produces strategic complementarity. In our current setup whether banks' recovery decisions are, at the margin, strategic substitutes or complements depends crucially on the degree of convexity of the supervisor's resolution costs in the mass of resolved banks. Quite intuitively, having costs that very steeply increase with the size of the intervention weakens the credibility of the resolution threat when the mass of non-recovering bank is large enough facilitating the existence of a too-many-to-be-resolved equilibrium.

Related empirical literature. The empirical literature that has examined the interaction between distressed banks and their supervisors offers insights and evidence broadly consistent with key assumptions and predictions of our analysis, even though none of the existing contributions can be regarded as a formal test of the empirical implications of our model.

In the modeling of banks' recovery incentives, the assumption that resolution is costlier to bank insiders than recovery plays an essential role. Opposite to descriptions that interpret public capital injections as a blessing for bank insiders, our modeling is consistent with the description in Berger et al. (2021) of these supervisory interventions as a process involving "catch, restrict, and release." Studying the details of public capital injections in the period 2008-2014, they document that supervisors "typically imposed dividend bans, regulatory fees, and other operating restrictions" as well as "board nominations, executive pay limits, and other operating restrictions" on the intervened banks, and only released those constraints after the banks recovered a sound condition.

With data on banks supported by the Capital Purchase Program in the US, Mücke et al. (2021) document the disciplining role of the ability of the government to appoint independent directors to the board of assisted banks in case they missed more than six dividend payment on the injected preferred stock, thus offering indirect evidence of the value attributed to control by bank insiders. In a similar vein, Wilson and Wu (2012) document that the restrictions on executive pay associated with equity injections under the Troubled Asset Relief Program (TARP) incentivized early TARP exit. Beyond losses in term of control rents, the reaction of bank stock valuations to bail-in events both in the EU and in the US (Dell'Ariccia et al., 2018b) and to seasoned equity offerings in a financial crisis context (Chiarella at al, 2019) suggest that the forced recapitalizations of banks in distress have a large dilutive effect on pre-existing equity.

The trade-off between short-term reputational, political, economic or fiscal costs of an early intervention and the longer-term costs of forbearance is key to our modeling of supervisory incentives. Precedents and motivations for this trade-off found in the theoretical literature have already been covered above. Empirically, it has been shown that political concerns play an important role in delaying government interventions on banks. Politicians in power routinely delay bad news about bank solvency (Imai, 2009), and are less likely to inject taxpayers' money in a distressed bank or resolve it before the election year both in emerging economies (Brown and Dinc, 2005) and in advanced economies (Liu and Ngo, 2014; Bian et al, 2020), contributing to systemic costs such as distortions in the allocation of credit (Bian et al., 2020). Other short-term economic considerations (such as local credit supply effects) have also been shown as a source of distortion in the enforcement of capital regulation by national authorities (Gropp at al., 2020), confirming one of the arguments in favor of the creation of a single bank supervisor in the EU. At the other side of the trade-off, several papers provide evidence of the large social costs of bank capital forbearance (see, for instance, Degennaro and Thompson, 1996, Caprio and Klingebiel, 1997, and Cole and White, 2017).

Our model yields predictions consistent with evidence on the relevance of supervisory action effectiveness for bank performance. Agarwal et al. (2014) exploit the rotation in supervisory responsibility between federal and state bank supervisors in US to identify the impact of supervisory toughness (proxied by the assigned supervisory ratings) on bank performance indicators. While the data covers all banks (and not just those facing recapitalization needs), their evidence is consistent with tougher (and possibly more credible) supervisors inducing superior performance among banks (higher capital ratios, quicker repayment of government assistance funds, lower failure rates, etc.).[10] Further analysis suggests that factors making state supervisors typically more lenient include concern about local credit conditions or differences in supervisory resources rather than strictly self-interested motivations (such as revolving doors). The evidence in Berger et al. (2021) about bank resolution in Europe prior to the introduction of the Single Resolution Mechanism suggests that the prospect of restrictions imposed by supervisors during their interventions "encourage better bank behavior" both before and after an intervention, thus reducing bank failure costs.

Consistent with the implications of our result on the too-many-to-resolve problem, Reinhart and Rogoff (2011) show that more fiscally constrained governments face more severe banking crises, although their analysis does not clarify the importance of the recovery and resolution channel for this result. Somewhat more direct is the evidence in

[10] See Hirtle et al. (2020) for complementary evidence of the impact of supervisory attention on bank performance.

Eisenbach et al. (2022), whose structural estimates suggest that expanding the resources of US bank supervisors would promote a large reduction in the probability of bank distress.

One of our comparative statics results implies that bank leverage above a certain threshold has a net negative effect on bank recovery and induces higher forbearance. This prediction is consistent with the documented impact of bank leverage (Mooij et al., 2013) or the growth in bank leverage (Schularick and Taylor, 2012) on the probability and severity of systemic banking crises, although again the existing evidence does not directly tell about the role of the recovery and resolution channel for these results. In other words, there is room for further empirical work that could directly address the challenges involved in the formal testing of our predictions (including having cross-sectional or time-series variation in recovery and resolution costs, in proxies of supervisors' independence, political biases or gambling incentives, and in measures or proxies of the relevant outcome variables in a recovery and resolution setup).

Outline of the paper. The rest of the paper is organized as follows. Section 2 describes the model. Section 3 characterizes the equilibrium of the baseline game between weak banks and the supervisor, discusses its comparative statics, and elaborates on the predictions regarding the effect of bank leverage on equilibrium outcomes. Section 4 discusses the case in which the supervisor faces a too-many-to-resolve problem and conditions when strategic complementarities arise. Section 5 concludes the paper. The Appendix contains all the proofs.

2 The model

We consider a game played between a bank supervisor and some banks damaged by a solvency shock. There are three relevant dates $t = 0, 1, 2$, all agents are risk neutral, and there is a safe asset that pays a zero net rate of return across periods.

2.1 Banks

The banks are owned and managed in the interest of their initial shareholders. Shareholders discount future payoffs at a rate normalized to zero. Banks have outstanding debt liabilities that promise to pay back D at $t = 2$. We assume for the time being that these liabilities are exclusively made of fully insured deposits.

At $t = 0$, a mass ϕ of banks are damaged by a solvency shock. This implies that their assets at $t = 2$ repay $R > D$ with probability $1 - \varepsilon$ (normal state) and $D - s$, with $s \in (0, D]$, with probability ε (adverse state), resulting in bank failure. Failure causes some systemic costs that banks do not internalize. At $t = 0$, parameter s can be interpreted as the capital shortfall in a future adverse scenario and ϕ as the measure of banks detected as undercapitalized in a supervisory review process or stress test.

To prevent their future failure, damaged banks can undertake a *private recovery action* ($r = 1$) at $t = 1$. The most straightforward form of recovery action would be an injection of equity capital s by the bank shareholders (and using it to either reduce D at $t = 1$ or to invest in the safe asset until $t = 2$). Under this action, bank shareholders remain the only residual claimants of the bank and in full control of it, which allows them to extract some non-pecuniary *control rents* Δ (as, e.g., in Grossman and Hart, 1988). Other forms of private recovery can be descriptively different but yield similar payoffs to shareholders.[11] The net expected payoffs of bank shareholders (inclusive of control rents) under a private recovery are

$$\Pi_{r=1} = -s + (1 - \varepsilon)(R + s - D) + \Delta, \tag{1}$$

where, by design, the equity payoffs of a damaged bank in the adverse state equal zero at $t = 2$ and the recapitalization at $t = 1$ is just enough to prevent failure in that state.

If instead, the damaged bank does not recover ($r = 0$), the supervisor can undertake its *resolution* ($i = 1$) or opt for forbearance ($i = 0$). Formally we model resolution as an intervention that results in the forced recapitalization of the unrecovered bank: some outside equity providers (perhaps the supervisor's own resolution fund) receive a share α of the bank's equity in exchange for injecting s at $t = 1$ (again to reduce D at $t = 1$ or invest in the safe asset until $t = 2$). This recapitalization is not a shareholders' bailout because α is set so that the equity injection occurs at terms such that the equity providers break even:[12]

$$\alpha(1 - \varepsilon)(R + s - D) = s. \tag{2}$$

Importantly, resolution implies the dissipation of the control rents Δ of the initial bank

[11] This might include the sale of new fairly-priced equity to outsiders with a structure (dispersed ownership or an allocation of voting rights) that allows the initial shareholders to retain control.

[12] We could also consider resolutions based on converting bail-in debt into equity. For this, the model would include some bail-in debt among the liabilities with total face value D (with the rest being fully insured deposits). A bail-in would then constititute the conversion of bail-in debt with face value s into a fraction α of total equity.

shareholders.[13] Thus, under resolution, the payoffs of an unrecovered bank's initial shareholders are

$$\Pi_{r=0,i=1} = (1-\alpha)(1-\varepsilon)(R+s-D) = (1-\varepsilon)(R-D) - \varepsilon s, \tag{3}$$

which is lower than $\Pi_{r=1}$ by exactly Δ.

Finally, if a bank remains unrecovered ($r = 0$) and unresolved ($i = 0$), its initial shareholders' net expected payoffs are

$$\Pi_{r=0,i=0} = (1-\varepsilon)(R-D) + \Delta, \tag{4}$$

which is larger than $\Pi_{r=1}$ by εs.

Using the no-recovery, no-resolution payoffs $\Pi_{r=0,i=0}$ as a benchmark, the net cost of recovery to the initial owners of a damaged bank are then

$$\Pi_{r=0,i=0} - \Pi_{r=1} = \varepsilon s \equiv c > 0, \tag{5}$$

which reflects an implicit positive transfer from shareholders to debtholders or, with insured deposits, the deposit insurance scheme. Intuitively, shareholders give up their Merton (1977) put on risky bank assets.[14]

Similarly, bank owners' cost of resolution can be expressed as

$$\Pi_{r=0,i=0} - \Pi_{r=0,i=1} = \varepsilon s + \Delta \equiv c + \Delta. \tag{6}$$

Thus, resolution implies the same loss of the Merton put as recovery plus the loss of Δ, which operates as a punishment on bank owners for refusing to recover.

2.2 The supervisor

At $t = 0$, the supervisor identifies the damaged banks through a supervisory review process or stress test exercise and calls them to (voluntarily) recover. At $t = 1$, after observing the measure m of banks that undertook recovery actions ($r = 1$), the supervisor

[13] More generally, Δ might also account for the pecuniary value to the initial shareholders of any further punitive or restrictive action of resolution (e.g., a value of α larger than that implied by (2), replacing the previous managers of the bank or other restrictions on management).

[14] Shareholders' reluctance to recapitalize a levered firm has its roots in long recognized conflicts of interest between shareholders and debtholders (Jensen and Meckling, 1976; Myers, 1977); see Admati et al. (2018) for an interesting restatement.

must decide the measure $n \leq \phi - m$ of banks to resolve ($i = 1$).

Recovered banks cause a zero net cost to the supervisor, while the potential failure at $t = 2$ of the unrecovered banks $\phi - m - n$ implies expected systemic costs $\Lambda(\phi - m - n)$. However, the resolution of unrecovered banks at $t = 1$ also implies reputational, political, economic or fiscal costs to the supervisor, $T(n)$. The supervisor decides on the measure of banks to resolve n so as minimize the overall costs $T(n) + \Lambda(\phi - m - n)$. This implies trading off the expected systemic costs of potential bank failures, Λ, which are decreasing in n, with the resolution costs T, which are increasing in n.

For tractability we derive our baseline results using a simple linear-quadratic specification for both costs which can be microfounded under a fiscal interpretation, that we explain next.[15] Further considerations on the shape of these cost functions will be made in Section 4.

Assume that the supervisor's total opportunity cost of public funds at a certain date or state is determined by

$$G(x, y) = g_0(x + y) + \frac{g_1}{2}(x + y)^2, \tag{7}$$

where g_0 and g_1 are strictly positive parameters, x are exogenous non-bank-related fiscal needs, and y are the fiscal needs associated with either banks' resolution or failure.[16] Reflecting correlation between banks performance and the fiscal position, the exogenous needs x are an intermediate x_i at $t = 1$, a low x_l in the normal state at $t = 2$, and a high x_h in the adverse state at $t = 2$.

Since the public recapitalization of the mass n of resolved banks occurs at financially fair terms, it implies the use of funds $y_i = sn$ at $t = 1$ in exchange for an equity payoff $\alpha(R+s-D)$ from each bank in the normal state at $t = 2$ and a zero payoff in the adverse state. For the equity share α defined by (2), the recapitalization produces a fiscal revenue $y_l = -sn/(1 - \varepsilon)$ in the normal state and zero in the adverse state. Instead, leaving a mass $\phi - m - n$ of damaged banks undercapitalized at $t = 1$ (forbearance) would imply a deposit insurance liability in the adverse state with burden $y_h = s(\phi - m - n)$.

Using (7) and the values y_i, y_l, and y_h obtained above, the total expected fiscal costs

[15]This microfoundation, however, does not preclude a broader interpretation under which both Λ and T include other reputational, political, and economic costs relevant to the supervisor.

[16]Rising marginal costs ($g_1 > 0$) can be due to the cuts in other increasingly valuable public spending, rises in increasingly distortionary taxes or additional issuance of increasingly costly government debt.

over $t = 1$ and $t = 2$ can be written as

$$
\begin{aligned}
C &= k + g_1(x_i - x_l)sn + \frac{2-\varepsilon}{1-\varepsilon}\frac{g_1}{2}s^2n^2 \\
&\quad + \varepsilon(g_0 + g_1x_h)s(\phi - m - n) + \frac{\varepsilon g_1}{2}s^2(\phi - m - n)^2, \qquad (8)
\end{aligned}
$$

where k collects non-bank-related cost components. The second and third terms in (8) reflect the net fiscal costs of resolution:

$$T(n) = \tau_0 n + (\tau_1/2)n^2, \qquad (9)$$

with $\tau_0 \equiv g_1(x_i - x_l)s > 0$ and $\tau_1 \equiv \frac{2-\varepsilon}{1-\varepsilon}g_1s^2 > 0$. Thus, under this microfoundation, $T(n)$ is increasing and convex in n, since resolving more banks increases the marginal opportunity cost of the involved fiscal resources.

The fourth and fifth terms in (8) reflect the fiscal costs from the failure of undercapitalized banks at $t = 2$. The systemic costs would then be:

$$\Lambda(\phi - m - n) = \lambda_0(\phi - m - n) + (\lambda_1/2)(\phi - m - n)^2, \qquad (10)$$

with $\lambda_0 \equiv \varepsilon(g_0 + g_1x_h)s > 0$ and $\lambda_1 \equiv \varepsilon g_1s^2 > 0$. Thus Λ is decreasing and concave in n, since resolution at $t = 1$ reduces the use of (increasingly costly) public funds in the adverse state at $t = 2$.

The opposite dependence of T and Λ with respect to n generates a natural trade-off—an intertemporal cost-smoothing motive—when the supervisor decides on the mass of banks to resolve $n \in [0, \phi - m]$ in order to minimize $T + \Lambda$.

2.3 Sequence of events

The sequence of events is the following:

- At $t = 0$, the supervisor identifies the mass ϕ of damaged banks and calls them to recover.
- At $t = 1$, there are two stages:
  - Stage 1. Damaged banks simultaneously decide whether to recover ($r = 1$) or not ($r = 0$); the resulting measure of recovering banks is m.
  - Stage 2. The supervisor resolves a measure n of the mass $\phi - m$ of unrecovered banks; a measure $\phi - m - n$ of banks remain undercapitalized (forbearance).

- At $t = 2$, aggregate uncertainty realizes; in some bad state the undercapitalized banks fail, causing systemic costs.

Table 1 summarizes the variables and payoffs relevant for the damaged banks and the supervisor in the sequential game played at $t = 1$.

Table 1. Key variables of the recovery and resolution game

Bank-level outcome	Affected mass of banks	Bank owners' per bank cost	Supervisor's overall cost
Recovered	m	c	0
Unrecovered, resolved	n	$c + \Delta$	$T{=}\tau_0 n{+}\frac{\tau_1}{2}n^2$
Unrecovered, unresolved	$\phi{-}m{-}n$	0	$\Lambda{=}\lambda_0(\phi{-}m{-}n){+}\frac{\lambda_1}{2}(\phi{-}m{-}n)^2$

The following assumption allows us to focus the discussion on the most interesting case in which the supervisor finds it worthy to resolve some but not all the unrecovered banks.

Assumption 1. $\lambda_0 + \lambda_1\phi > \tau_0 > \lambda_0$.

Intuitively, if unrecovered banks anticipated that they would be not be resolved, no bank would recover and the mass of potentially failing banks would be ϕ. However, having $\lambda_0 + \lambda_1\phi > \tau_0$ means that, in such a situation, it would pay the supervisor to resolve some banks since that marginal cost of resolving a non-recovering bank at $t = 1$, $dT/dn = \tau_0$, would be lower than the marginal gain from avoiding its failure at $t = 2$, $d\Lambda/dn = -(\lambda_0 + \lambda_1\phi)$. Along a similar logic, having $\tau_0 > \lambda_0$ rules out the possibility of a subgame perfect Nash equilibrium (SPNE) of the game played at $t = 1$ in which all banks privately recover under the implicit threat that any non-recovering bank would be resolved. Intuitively, this is so because when all other banks recover, the marginal cost of resolving a non-recovering bank, $dT/dn = \tau_0$, would not exceed the marginal gain from avoiding its failure at $t = 2$, $d\Lambda/dn = -\lambda_0$.

2.4 No active supervision and full-commitment benchmarks

It is trivial to see that in the absence of a supervisor being able to act on unrecovered banks, no damaged bank would opt for recovery. In other words, damaged banks would simply gamble for survival (in the normal state at $t = 2$), remaining exposed to failure (in the adverse state at $t = 2$) and bearing a cost 0 rather than the recovery cost c. The supervisor's cost in this situation ($m = n = 0$) would be $T + \Lambda = \Lambda = \lambda_0\phi + (\lambda_1/2)\phi^2$. This benchmark provides a lower bound to the cost incurred by banks and an upper bound to the cost incurred by the supervisor in the baseline recovery and resolution game.

Symmetrically, a supervisor ex ante committed to resolve any unrecovered bank would induce all banks to recover. Then the supervisor would incur no cost, $T + \Lambda = 0$, while bank owners would incur a cost c at each damaged bank.

We focus the discussion in the rest of the paper on the case the supervisor has some capacity and incentives to resolve unrecovered banks but is not able to fully commit to resolve every unrecovered bank.

3 Baseline results

This section first characterizes the equilibrium of the sequential game played between the damaged banks and the supervisor. Then we use its comparative statics to analyze the determinants of equilibrium outcomes. The section is closed with a discussion on how targeted resolution (focusing the resolution threat on a subset of damaged banks) might help economize on resolution costs and reduce forbearance.

3.1 Equilibrium

The recovery and resolution game played by the damaged banks and the supervisor at $t = 1$ can be solved by backward induction. In the second stage, the supervisor decides on the measure of banks to resolve n after having observed the mass of banks m that decide to recover in the first stage. The supervisor's reaction function is given by

$$N(m) = \arg \min_{0 \leq n \leq \phi - m} T(n) + \Lambda(\phi - m - n), \tag{11}$$

where T and Λ are specified in (9) and (10).

Solving the first order condition of the implied minimization and taking into account that $N(m)$ must be non-negative, the supervisor's reaction function can be written as

$$N(m) = \max\left\{\frac{\lambda_1(\phi - m) - (\tau_0 - \lambda_0)}{\lambda_1 + \tau_1}, 0\right\}, \tag{12}$$

which is piece-wise linear as depicted in Figure 1. Specifically, $N(m)$ is positive and lower than ϕ at $m = 0$ (by the first inequality in Assumption 1) and decreasing until it reaches a value of zero at $m = \phi - (\tau_0 - \lambda_0)/\lambda_1 < \phi$ (by the second inequality in Assumption 1). In the positive range, the supervisory response $N(m)$ reflects a cost smoothing motive, optimally spreading the pain caused by low recovery across the present (T) and the future (Λ), which involves equalizing the marginal cost of resolving an extra bank $\partial T/\partial n$ to the reduction in the marginal future expected systemic costs $-\partial\Lambda/\partial n$ achieved by doing that.[17]

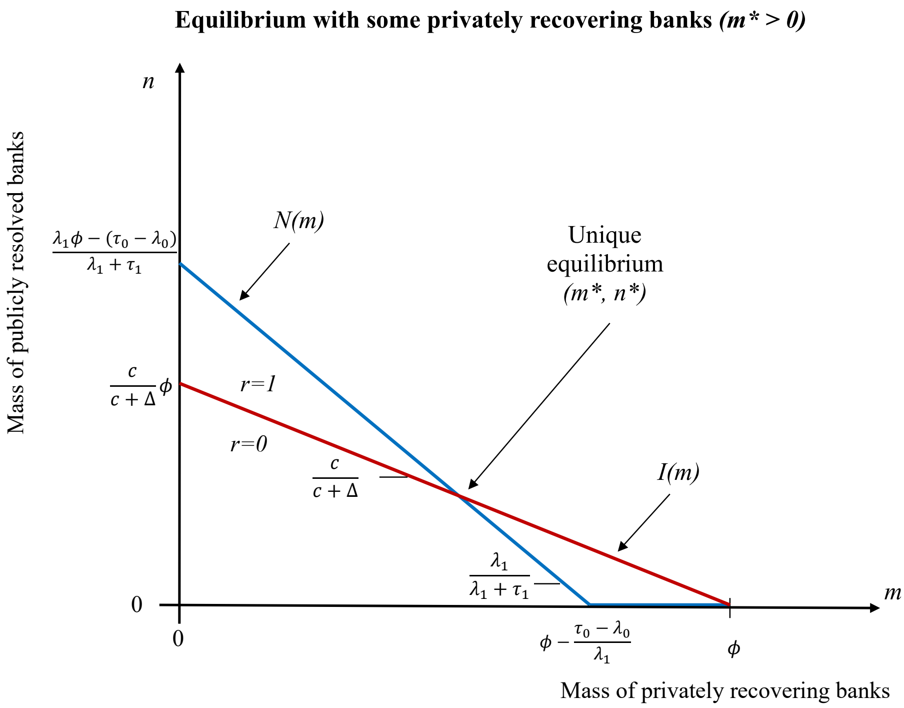

Figure 1: Equilibrium with some recovering banks

In the first stage of the game, banks simultaneously decide whether to incur the costs of recovery ($r = 1$) or not ($r = 0$). Given some expectation about the mass of similarly damaged banks that opt for recovery, m, and the subsequent response of the supervisor, $N(m)$, a damaged bank would be indifferent between the two alternatives if and only if

$$c = \frac{N(m)}{\phi - m}(c + \Delta), \tag{13}$$

where the left hand side (LHS) is bank owners' cost of recovery, c, and the right hand side

[17]In the range where $N(m) = 0$, the cost $\partial T/\partial n$ exceeds the benefit $-\partial\Lambda/\partial n$, so its is optimal not to resolve any bank.

(RHS) is the product of the bank's probability of being subject to resolution, $N(m)/(\phi - m)$, and bank owners' cost in case of resolution, $c + \Delta$. Importantly, for $m < \phi - (\tau_0 - \lambda_0)/\lambda_1$, we have, using (12),

$$d\left(\frac{N(m)}{\phi - m}\right)/dm = -\frac{\tau_0 - \lambda_0}{(\lambda_1 + \tau_1)(\phi - m)^2} < 0, \tag{14}$$

which means that the resolution threat hanging on a marginal unrecovered bank weakens as more banks recover. This leads to the following result.

Lemma 1. *Over the range in which the supervisory resolution threat is not zero (that is, for $m < \phi - (\tau_0 - \lambda_0)/\lambda_1$), banks' individual recovery decisions are strategic substitutes.*

In such a range, if more banks recover, the supervisory resolution threat is weaker because the marginal systemic costs that can be avoided with resolution are lower too. Anticipating a larger probability of benefiting from forbearance if many other banks recover, the marginal individual bank would be more tempted to abstain from recovering, explaining the strategic substitutability between banks' decisions.

To depict banks' indifference condition in the same space as the supervisor's reaction function in Figure 1, we can define

$$I(m) = \frac{c}{c + \Delta}(\phi - m) \tag{15}$$

as the solution of (13) in N. Then, for any $m < \phi$, a damaged bank strictly prefers to undertake its recovery for $n > I(m)$, to remain unrecovered for $n < I(m)$, and is indifferent between the two alternatives for $n = I(m)$. This explains the second line depicted in Figure 1.[18]

Given the strategic substitutability between damaged banks' recovery decisions, the symmetric equilibrium of the game may involve the use of mixed strategies by the banks in the first stage (and a probabilistic threat of resolution on the unrecovered banks in the second stage). The following proposition describes the unique SPNE of the game.

Proposition 1. *The recovery and resolution game has a unique symmetric SPNE with*

[18] We present a case with $I(0) = c/(c+\Delta) < N(0) = (\lambda_1\phi - (\tau_0 - \lambda_0))/(\lambda_1 + \tau_1)$, so that the two curves intersect once on the downward sloping section of $N(m)$. Note that $I(m)$ is downward sloping, with $I(0) = c\phi/(c + \Delta) < \phi$ and $I(\phi) = 0$. Importantly, the point $(\phi, 0)$ does not belong to the indifference line because, for $n = 0$, (13) cannot hold for any m. So $I(m)$ and $N(m)$ do not further intersect at $(\phi, 0)$.

$(m,n) = (m^*, n^*)$, *where*

$$(m^*, n^*) = \begin{cases} \left(\phi - \frac{(\tau_0-\lambda_0)(c+\Delta)}{\lambda_1\Delta-\tau_1 c}, \frac{(\tau_0-\lambda_0)c}{\lambda_1\Delta-\tau_1 c}\right), & \text{if } \Delta > \frac{\tau_1 c}{\lambda_1} \text{ and } \phi > \frac{(\tau_0-\lambda_0)(c+\Delta)}{\lambda_1\Delta-\tau_1 c}, \\ \left(0, \frac{\lambda_1\phi-(\tau_0-\lambda_0)}{\lambda_1+\tau_1}\right), & \text{otherwise.} \end{cases} \quad (16)$$

Thus, depending on the importance of systemic costs relative to private recovery costs and the supervisor's resolution costs, the equilibrium may feature some recoveries, $m^* > 0$, as in Figure 1, or no recoveries at all, $m^* = 0$.[19]

The emergence of the regime with positive recovery ($m^* > 0$) requires a threat of resolution on non-recovering banks strong enough to overcome banks' incentives to gamble for forbearance. Thus, provided that control losses associated with resolution (Δ) are large enough, the relevant conditions hold more easily when the mass of damaged banks (ϕ) is large, the supervisor finds resolution not so costly (τ_1 is low), the systemic costs of bank failure are high (λ_1 is high), and the cost of recovery to bank owners (c) is small. The following corollary summarizes the implications of Proposition 1 for the mass of forborne banks.

Corollary 1. *The level of forbearance implied by the unique symmetric SPNE of the recovery and resolution game is given by*

$$\phi - m^* - n^* = \begin{cases} \frac{(\tau_0-\lambda_0)\Delta}{\lambda_1\Delta-\tau_1 c}, & \text{if } \Delta > \frac{\tau_1 c}{\lambda_1} \text{ and } \phi > \frac{(\tau_0-\lambda_0)(c+\Delta)}{\lambda_1\Delta-\tau_1 c}, \\ \frac{\tau_0-\lambda_0+\tau_1\phi}{\lambda_1+\tau_1}, & \text{otherwise.} \end{cases} \quad (17)$$

In the regime with strictly positive bank compliance ($m^* > 0$), the resolution threat (as measured by $N(m^*)/(\phi - m^*)$) does not change with the mass of damaged banks ϕ. In other words, the rise in ϕ is accommodated with an equal rise in the measure of recovering banks n^*. These outcomes rely on the implicit (and credible) threat that, if the mass of recovering banks grew less than one-to-one with ϕ, the supervisor would resolve the undercapitalized banks with greater intensity and banks would strictly prefer to recover than to be exposed to the risk of resolution.

In the regime where no bank opts for recovery, the mass of resolved banks also grows with ϕ, but less than one-by-one. The rise in the marginal cost of resolution discourages the supervisor from fully offsetting the increase in the mass of damaged banks and

[19]This second regime corresponds to the situation in which $I(0) \geq N(0)$ so that there is no intersection between (the negatively sloped section of) the lines $N(m)$ and $I(m)$.

forbearance rises with ϕ.

3.2 Determinants of forbearance

This section discusses how changes in the various model parameters affect the equilibrium described above. For brevity, we focus on the most interesting regime in which bank recovery m^* is strictly positive.[20] From (16), the necessary and sufficient conditions for having $m^* > 0$ are:

Assumption 2. $\Delta > \frac{\tau_1 c}{\lambda_1}$ *and* $\phi > \hat{\phi} \equiv \frac{(\tau_0 - \lambda_0)(c+\Delta)}{\lambda_1 \Delta - \tau_1 c}$.

The following proposition summarizes the results regarding the impact of model parameters on bank resolution (n^*) and forbearance ($\phi - m^* - n^*$). Its proof also provides details about the effects of the parameters on bank recovery (m^*).

Proposition 2. *The mass of resolved banks and forbearance increase with the supervisor's resolution costs (τ_0 and τ_1) and with banks' recovery costs (c), and decrease with the systemic cost of bank failure (λ_0 and λ_1) and the extra losses that resolution imposes on bank owners (Δ).*

The most intriguing of these results are the effects of the supervisor's resolution cost parameters on the equilibrium level of resolution. A supervisor for whom resolution is more costly imposes a weaker resolution threat on damaged banks, thus discouraging them from recovering and ends up having to resolve more banks. However, this increase in resolution is not enough to compensate for the lower level of recovery so the equilibrium features a higher level of forbearance.[21]

The next proposition uncovers additional results whereby the effects of supervisory resolution costs on the equilibrium mass of resolved banks and forbearance are reinforced when banks' private recovery costs increase.

Proposition 3. *A higher recovery cost c reinforces the effect of the supervisory costs of resolution on resolutions and bank forbearance. Formally, the cross derivatives of both n^* and $\phi - m^* - n^*$ with respect to τ_0 and c (as well as τ_1 and c) are positive.*

[20]In the regime with $m^* = 0$, the mass of resolved banks and the mass of unrecovered, unresolved banks respond exclusively to the costs faced by the supervisor, as shown in the corresponding parts of (1) and (17). The supervisor resolves more (forbears less) when its resolution costs (positively associated with parameters τ_0 and τ_1) are lower, the systemic costs of leaving banks undercapitalized (λ_0 and λ_1) are higher, and the mass of damaged banks (ϕ) is larger.

[21]Intuitively, at the prior level of bank recovery m^*, the resolution threat $N(m^*)$ would be too low to induce an individual bank to recover. Restoring banks' indifference between recovering or not requires a lower level of recovery m^* so that the supervisor responds with more resolution n^*.

These reinforcement effects mean that, ceteris paribus, in a situation in which the supervisory costs of resolution are higher (e.g. because of having supervisors with higher reputational concerns, stronger political biases, greater capture, tighter budgets, higher opportunity costs of public funds, etc.), the same increase in banks' recovery cost (e.g., because of facing a poorer legal protection of investors' rights, a less developed market for seasoned equity offerings, or greater reluctance to give up the valuable Merton put associated with limited liability) would end up producing a higher incidence of resolution and bank forbearance. On the positive side, this result points to the complementarity between reforms that reduce the supervisory cost of resolution (e.g., by introducing effective recovery and resolution legislation) and those that reduce the costs of recovery (e.g., by promoting the use of bonds that can be converted into equity or by increasing the liquidity of the seasoned market for banks' equity).

Finally, we analyze the effects of the parameter s that measures the size of the capital shortfall suffered by damaged banks in the very first description of our model. This shortfall might be interpreted as a reflection of banks' ex ante leverage, as an ex ante better capitalized bank would have a greater loss absorption capacity and require a lower recapitalization to guarantee its solvency at $t = 2$. Under the expressions provided in subsection 2.1, parameter s affects linearly banks' private recapitalization costs c, while it affects linearly the coefficients τ_0 and λ_0 and quadratically the coefficients τ_1 and λ_1 of the supervisor's resolution and systemic costs, respectively. The comparative statics of s uncovers a non-monotonic effect of the shortfall s (or bank leverage) on forbearance:

Proposition 4. *Forbearance is U-shaped related to the capital shortfall s suffered by damaged banks, with a minimum at $\bar{s} = \frac{\Delta(1-\varepsilon)}{2(2-\varepsilon)}$.*

By increasing banks' (linear) costs of recovery the size of the shortfall s has a negative direct effect on m. However s also affects the (quadratic) costs that drive the supervisor's response $N(m)$. On the net, under Assumptions 1 and 2, increasing s increases the supervisor's propensity to resolve the unrecovered banks (the response in n to any m), because the marginal systemic cost of leaving a bank undercapitalized increases more with s than the corresponding marginal resolution cost. This produces a positive indirect effect on banks' recovery decisions. In the proof of the proposition we show that the positive indirect (negative direct) effect dominates when s is below (above) some threshold, producing a hump-shaped relationship between s and the mass of recovering banks m^*. Once m^* declines with s, a point is reached in which the rise in n^* is not enough to avoid the increase in forbearance.[22] The overall pattern is then that forbearance first decreases and then increases with the capital shortfall s. To the extent that s can be reduced by tightening ex ante capital requirements (reducing banks' leverage), this result implies that, when bank leverage is sufficiently large, tightening capital regulation can improve banks' recovery incentives, reduce the need for resolution, and diminish the incidence of forbearance.

3.3 Gaining credibility with targeted resolution

Our recovery and resolution game describes a situation that is likely to arise in a systemic crisis, when a large number of banks are simultaneously discovered to be in trouble and the call for recovery under the threat of resolution cannot be restricted to a subset of the affected banks (e.g. because it is obvious to the public that there are many other affected banks and pretending that some are not in trouble could be considered unlawful). If, on the contrary, it were possible to select a mass $l \leq \phi$ of damaged banks on which to focus the recovery and resolution process, the outcomes of the overall game would be quite different. Clearly the banks not subject to the resolution threat would decide not to recover. But for the banks in the subset of measure l, the indifference condition in (13) would be replaced by

$$c = \frac{N(m)}{l-m}(c+\Delta). \tag{18}$$

This means that the locus of the pairs (m, n) for which the banks in the selected subset are indifferent between recovering or not would now be given by the line

$$I(m;l) = \frac{c}{c+\Delta}(l-m) \tag{19}$$

rather than by (15). This generalization of $I(m)$ implies in terms of Figure 1 that by increasing l the supervisor could effectively shift $I(m)$ inwards, in parallel to the initial $I(m)$.

Thus, under the configurations of parameters for which the baseline game has an interior equilibrium, the supervisor could maximize the recoveries and minimize the need for resolution by setting

$$l^{**} = \phi - \frac{\tau_0 - \lambda_0}{\lambda_1}, \tag{20}$$

[22]Quite intuitively, the threshold $\bar{s}$ above which the size of the shortfall increases forbearance is higher whenever bank owners' extra loss of control rents in case of resolution (Δ) is higher since, other things equal, the aim to avoid this loss encourages bank recovery.

that is, by making the crossing of $I(m;l)$ with the m-axis equal to the point where the slope of $N(m)$ switches from negative to zero. This would induce an equilibrium with $m^{**} = l^{**} > m^*$, $n^{**} = 0 < N(m^*)$, and forbearance equal to $\phi - l^{**} = (\tau_0 - \lambda_0)/\lambda_1 < \phi - m^* - N(m^*)$. Under this approach, the supervisor makes a very effective use of its resolution threat, inducing full recovery among the selected banks without consuming any of its resolution capacity and reaching lower forbearance than in the equilibrium of the baseline game.

In practical terms, this result might rationalize supervisors' practice of announcing that over a specific period their supervision will pay special attention to some specific classes of entities (e.g. entities exposed to vulnerabilities described as "supervisory priorities" in the corresponding period).[23] While this practice may discourage compliance among the entities not subject to special attention, the overall levels of compliance (recovery) may increase and the cost of inducing such a level of compliance (resolution) may decrease.

4 The too-many-to-resolve problem

Under the linear-quadratic specification of the resolution cost functions T and Λ of the baseline model, the supervisor's reaction function $N(m)$ is linear and the conditions on parameters that sustain the interior solution (Assumptions 1 and 2) also imply the strategic substitutability of banks' recovery decisions (Lemma 1). Intuitively, the underlying resolution threat effectively increases with the mass of unrecovered banks so when, other things equal, less banks are expected to recover, the marginal bank is more inclined to recover.

However, richer specifications of the costs T and Λ can modify the shape of $N(m)$, altering the implications. In particular, if $N(m)$ exhibits some degree of concavity, the resolution threat becomes smaller and smaller for lower recoveries m. This may lead the local dominance of the strategic complementarity of banks' recovery decisions (a lowering probability of being resolved when many other banks are simultaneously not recovering). In such a situation, as we will show, the recovery and resolution game may feature multiple equilibria where the interior equilibrium of the baseline model co-exists with an extreme forbearance equilibrium featuring no recoveries.

We explore two variations of the baseline model that exhibit that property. In the

[23]Similar practices are observed among tax authorities when they announce that their supervision (regarding, e.g., income taxes) will pay special attention to specific industries or professions.

first, the choice of $N(m)$ is constrained by the presence of a hard limit n_0 to resolution capacity. When n_0 is sufficiently small, the supervisor's inability to respond to a low level of recoveries with more resolution encourages banks to opt for no recovery under the self-fulfilling expectation of a high forbearance outcome. In the second variation, we show that a qualitatively equivalent taxonomy of equilibrium configurations arises when resolution costs are sufficiently convex, that is, the marginal cost of resolution increases steeply with the mass of banks to resolve.[24]

The explosivity of the marginal costs of resolution could be justified under several of the interpretations about the nature of these costs provided in previous sections. With career-concerned policy makers, the emphasis on the short-term cost of an intervention can respond to a gambling logic. Accordingly, the supervisor would be pressed to avoid the rather imminent or highly likely electoral or career costs of publicly revealing the size of bank trouble (which in some circumstances might feed back into government funding costs or precipitate a panic beyond the directly supported banks) at the cost of a larger exposure to the future and probabilistic systemic cost that the banks left undercapitalized will cause in the adverse scenario.

4.1 The case with a hard limit to resolution capacity

If the supervisor's capacity to resolve banks at $t = 1$ is limited to a maximum mass of n_0 banks, its reaction function becomes

$$N_0(m) = \min\{N(m), n_0\}, \tag{21}$$

where $N(m)$ is the unconstrained reaction function defined in (12). For parameter values and a measure of recovering banks m low enough to make $N(m) > n_0$, the constrained reaction function $N_0(m)$ is no longer sensitive to m. This makes the resolution threat, represented by the probability $n/(\phi - m) = N_0(m)/(\phi - m)$, to be overall increasing (rather than decreasing) in m. Then banks' recovery decisions become *strategic complements* over such range.

Figure 2 depicts a situation in which the baseline game (without a constrained supervisor) features a unique equilibrium (m^*, n^*) with $m^* > 0$ and the newly added capacity constraint is in the range $n_0 \in (n^*, I(0))$. Then the interior equilibrium discussed in reference to Figure 1 coexists with two other SPNE: (i) a corner equilibrium with $m = 0$ and

[24] Conceptually, the first variation can be seen as a limit case of the second, but we start the discussion with it for didactic purposes, as it makes very evident the forces driving the results.

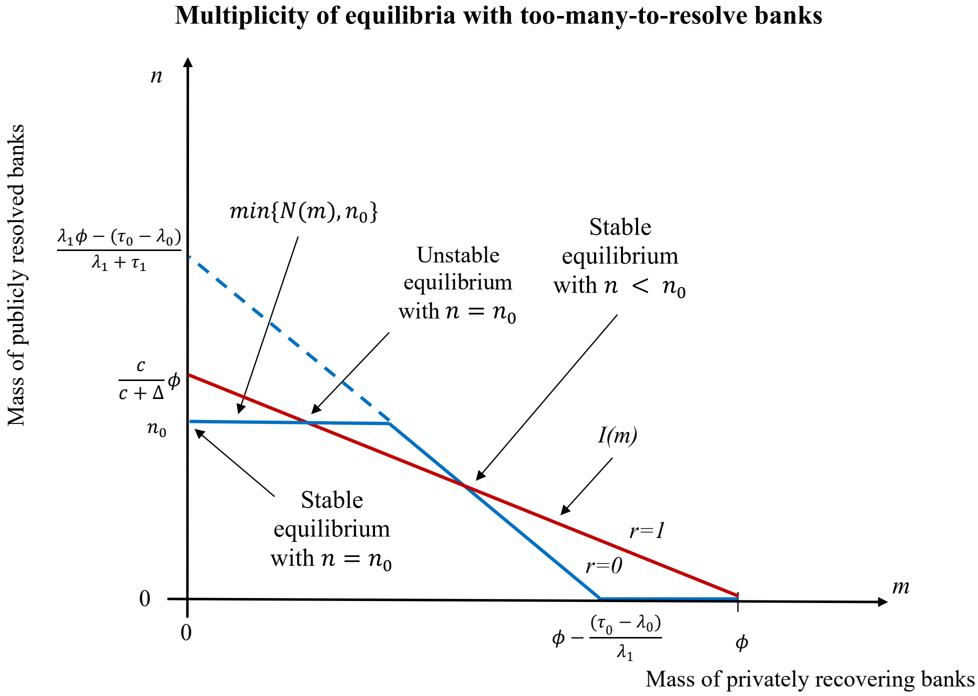

Figure 2: Multiplicity of equilibria with a hard limit on resolution capacity

$n = n_0$, and (ii) a second interior equilibrium with $m = I^{-1}(n_0)$ and $n = n_0$. Importantly, the second interior equilibrium, opposite to the other two, is not stable.[25]

Using Figure 2 as a reference, it is easy to infer the various equilibrium configurations that may emerge depending on the size of n_0 and the equilibrium of the unconstrained game. The following proposition summarizes the conditions under which the second equilibrium with extreme forbearance arises.[26]

Proposition 5. *The presence of a constraint $n_0 < I(0) = c\phi/(c + \Delta)$ to the supervisor's resolution capacity implies the existence of an equilibrium with extreme forbearance. Over*

[25]The second interior equilibrium is not stable in the sense that it is not robust to having an excess mass $\varepsilon \to 0$ of damaged banks arbitrarily deviating to either $r = 0$ or $r = 1$. Shall that happen, all other damaged banks would want to deviate to $r = 0$ or $r = 1$, respectively (evidencing the strategic complementarity between banks' decisions in the neighborhood of this equilibrium). This suggests that the play of the game would converge to the (stable) equilibria with $m = 0$ or $m = m^*$, respectively.

[26]For completeness, the proof of the proposition also discusses the case in which the equilibrium of the unconstrained game features a zero mass of privately recapitalizing banks.

the range $n_0 \in [N(m^*), I(0))$, *with*

$$N(m^*) = \frac{(\tau_0 - \lambda_0)c}{\lambda_1 \Delta - \tau_1 c} > 0, \tag{22}$$

the extreme forbearance equilibrium coexists with the equilibrium of the unconstrained game. For $n_0 \in [0, N(m^*))$, *only the equilibrium with extreme forbearance exists. The multiple equilibria range is increasing in the mass of damaged banks* ϕ.

Thus, the emergence of a too-many-to-resolve problem may turn banks' recovery decisions strategic complements, giving rise to the possibility of a pure strategy equilibrium with no recoveries (and extreme forbearance). For intermediate values of n_0, such equilibrium coexists with the mixed strategy equilibrium of the baseline setup. Intuitively, the larger the mass of damaged banks ϕ, the lower the value of n_0 for which banks' coordination in no-recovery can lead to the exhaustion of resolution capacity and, hence, to the existence of the equilibrium with extreme forbearance.

Similarly to what happens in the unique equilibrium of the baseline unconstrained game when $m^* > 0$ (Proposition 3), private recovery costs and supervisory resolution costs reinforce each other in expanding the range of parameter values for which the equilibrium with extreme forbearance exists. The following proposition states the result.

Proposition 6. *The range of values of the supervisor's capacity constraint* n_0 *for which only the equilibrium with extreme forbearance exists is increasing in the private recovery cost (c) and the supervisor's cost of resolution (*τ_0 *and* τ_1*). Moreover, the effects of these two costs on the length of such range reinforce each other.*

4.2 Explosive resolution costs and strategic complementarity

Next we show that multiple equilibria (including the equilibrium with extreme forbearance) also arise in the less polar scenario in which there is no hard limit to resolution capacity but resolution costs are sufficiently convex. To keep the analysis tractable, we consider a minimal deviation to the baseline model that reformulates the resolution cost in (9) by adding a positive cubic term:

$$T = \tau_0 n + (\tau_1/2)n^2 + (\tau_2/3)n^3. \tag{23}$$

Increasing τ_2 makes this cost more convex and renders the downward sloping section of $N(m)$ more concave.[27]

Akin to (12), the supervisor's reaction function can be written as

$$N(m;\tau_2) = \max\left\{n^+(m;\tau_2), 0\right\}. \tag{24}$$

where $n^+(m,\tau_2)$, which emerges from the relevant first order condition, is decreasing and concave in m for $m < \phi - (\tau_0 - \lambda_0)/\lambda_1$, and more and more concave the larger τ_2 is.[28]

Importantly, in the range where $n^+(m;\tau_2) > 0$, the derivative $d\left(\frac{N(m;\tau_2)}{\phi - m}\right)/dm$ is not necessarily negative (opposite to (14)) and strategic complementarity can locally arise:

Lemma 2. *When resolution costs are sufficiently convex ($\tau_2 > \widehat{\tau_2}$), there exists a critical mass of recovering banks $\widehat{m} < \phi - (\tau_0 - \lambda_0)/\lambda_1$ such that bank recovery decisions are locally strategic substitutes for $m > \widehat{m}$ and strategic complements for $m < \widehat{m}$. Otherwise, bank recovery decisions are locally strategic substitutes for all m.*

When bank resolution costs are marginally increasing at a rate that increases sufficiently with the mass of resolved banks (that is, when $\tau_2 > \widehat{\tau_2}$), the supervisor's reaction function $N(m,\tau_2)$ becomes very concave. Thus, for low values of m, while reductions in the mass of recovering banks get still responded with more resolution, the increase in resolution is not enough to maintain the intensity of the threat of resolution on the non-recovering banks. Thus, if anticipating a sufficiently low mass of recovering banks, further lack of recovery would push the marginal individual bank to be even more inclined to abstain from recovering. This explains the local strategic complementarity between banks' decisions, which opens the possibility of multiple equilibria. In contrast, for high levels of bank recovery, the supervisor faces a low marginal resolution cost and the implicit resolution threat is stronger. So, locally, recovery decisions remain strategic substitutes.

For less convex resolution costs (that is, when $\tau_2 \leq \widehat{\tau_2}$), the marginal response of the supervisor to reductions in the mass of recovering banks is strong enough to maintain (and actually reinforce) the intensity of the resolution threat on the marginal non-recovering bank for all values of m. In this case recovery decisions are globally strategic substitutes

[27]If the explosivity (or growing convexity) affects the systemic cost component Λ of the supervisor's objective function (in the form a cubic term with coefficient λ_2) rather than the resolution costs component T, the supervisor's reaction function becomes convex and banks' recovery decisions are always strategic substitutes. Hence, adding explosivity in Λ does not modify the qualitative nature of the interior equilibrium of the baseline game, but simply increases the recovery m, reduces the resolution n, and reduces the forbearance $\phi - m - n$ associated with it.

[28]See the proof of Lemma 2 for details.

as in the baseline version of the model.

To characterize equilibria in this extended setup,we analyze the possible crossings of the downward sloping section of the supervisory reaction function $n = N(m; \tau_2)$ (see details in the proof of Lemma 2), with banks' indifference line, $n = I(m)$. In the admissible range ($m \geq 0$), $N(m; \tau_2)$ can have one intersection, two intersections or no intersections with the downward sloping line $I(m)$. Figure 3 shows the various equilibrium configurations that may emerge.

Panel A of Figure 3 depicts the situation where the degree of concavity of $N(m; \tau_2)$ is small: it crosses $I(m)$ only once and $N(0; \tau_2) > I(0)$, so the situation is qualitatively identical to that obtained when the baseline model has a unique interior equilibrium (Figure 1). In Panel B, $N(m; \tau_2)$ crosses $I(m)$ twice, with $N(0; \tau_2) \leq I(0)$, and the situation is qualitatively the same as that represented in Figure 2 under a hard limit on resolution capacity. There can be three SPNE: (i) a stable extreme forbearance equilibrium with $m = 0$ and $n = N(0; \tau_2)$ and (ii) two interior equilibria denoted as (m_L, n_L) and (m_H, n_H), of which only the first is stable.

Panel C represents a borderline case in which $N(m; \tau_2)$ is tangent to $I(m)$ rather than crossing it and $N(0; \tau_2) < I(0)$; in this case the only stable SPNE involves extreme forbearance with $m = 0$ and $n = N(0; \tau_2)$. Finally, Panel D describes the case where the concavity of $N(m, \tau_2)$ is so large that it does not cross $I(m)$ on the admissible range, so the unique equilibrium of the recovery and resolution game involves $m = 0$ and $n = N(0; \tau_2)$ (as in Panel C).

The following proposition provides explicit conditions under which each of the stable equilibria emerge.

Proposition 7. *There are critical values $\phi_L(\tau_2)$, $\phi_H(\tau_2)$, $\bar{\tau}_2 > 0$, and $n_L(\tau_2)$, with $0 < \phi_L(\tau_2) \leq \phi_H(\tau_2)$ for $\tau_2 \leq \bar{\tau}_2$, such that the recovery and resolution game (i) features an extreme forbearance equilibrium $(0, n^+(0; \tau_2))$ for $\phi < \phi_L(\tau_2)$ and $\phi \geq \phi_H(\tau_2)$, and (ii) features an interior equilibrium $(\phi - \frac{c+\Delta}{c} n_L(\tau_2), n_L(\tau_2))$ for $\phi \geq \phi_L(\tau_2)$ and $\tau \leq \bar{\tau}_2$. Thus in the region with $\phi \geq \phi_H(\tau_2)$ and $\tau_2 \leq \bar{\tau}_2$, the extreme forbearance equilibrium and the interior equilibrium coexist.*

Figure 4 provides a graphical representation of the results in Proposition 7. It depicts in the space (τ_2, ϕ) the areas in which each of the equilibria can be sustained. The equilibrium with no recoveries and extreme forbearance arises as the only one when the costs of resolution are sufficiently convex ($\tau_2 > \bar{\tau}_2$) and, otherwise, when the mass of damaged banks is sufficiently small ($\phi < \phi_L(\tau_2)$), since in both cases the supervisor regards

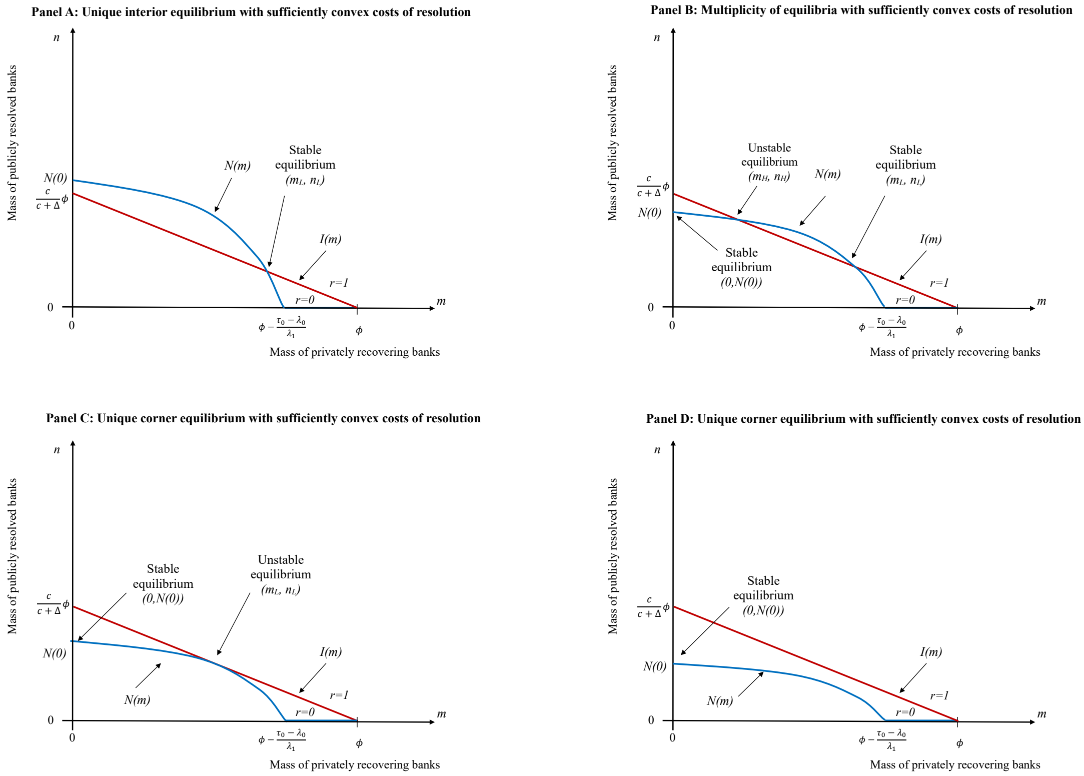

Figure 3: Equilibrium configurations with explosive resolution costs

marginal systemic costs too small relative to marginal resolution costs and imposes a too weak resolution threat on non-recovering banks.[29] The role of the explosivity of resolution costs (τ_2) in causing local strategic complementarity in banks' recovery decisions and the possibility of multiple equilibria is clear in the area with $\tau_2 \leq \bar{\tau}_2$ and $\phi > \bar{\phi}$, where the curve $\phi = \phi_H(\tau_2)$ separates the region where only the interior equilibrium exists from the coexistence region. In the latter, the effective resolution threat on non-recovering banks depends on whether banks coordinate in no recovery (in which case the threat is too soft to provide recovery incentives) or in high enough recovery (in which case the threat is just enough to make any marginal bank indifferent between recovering or not).

A result parallel to Proposition 6 emerges:

Proposition 8. *The ranges of the mass of damaged banks ϕ for which the equilibrium with extreme forbearance exists are increasing in the private recovery cost (c) and the*

[29]Notice that in the limit case with $\tau_2 = 0$, we have $\phi_L(0) = \hat{\phi}$, which is the critical value of the mass of damaged banks below which the baseline model also features an equilibrium with no recoveries and extreme forbearance (Proposition 1).

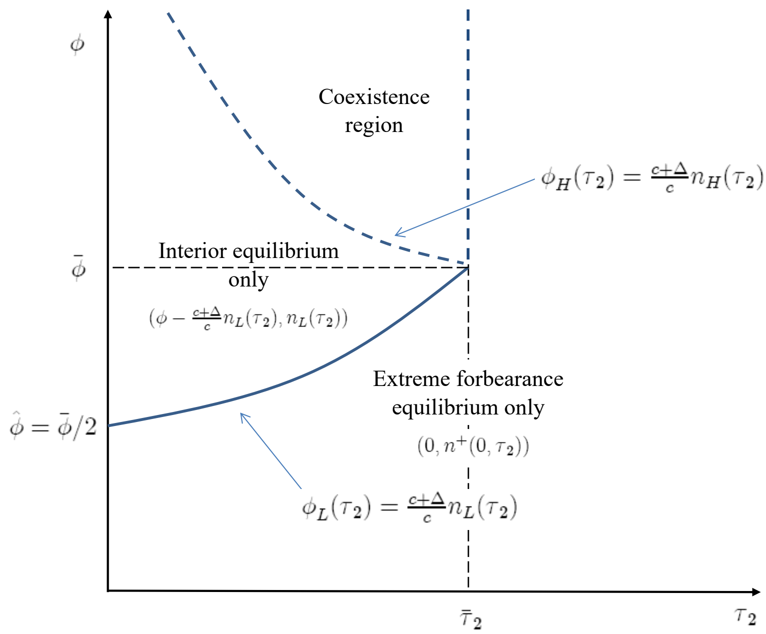

Figure 4: Conditions for multiple equilibria with explosive resolution costs

supervisor's cost of resolution (τ_0 and τ_1). Moreover, the effects of these two costs on the lengths of such ranges reinforce each other.

4.3 Implications for the sovereign-bank nexus

The too-many-to-resolve problem provides a novel channel potentially linking banks to the strength of public finances, thus speaking to the literature on the nexus between sovereign risk and bank risk. Our analysis implies that damaged banks operating under weaker, more shortsighted or more politically biased supervisors (or, alternatively, supervisors subject to tighter capacity or budgetary constraints) face an effectively weaker recovery and resolution framework: one in which the resolution threat is less credible. In an international context, if banks' recovery and resolution remains a national responsibility, differences in the reputation, political cycle, and constraints faced by the relevant authorities may explain differences in the recovery, resolution and forbearance resulting from any given mass of banks in trouble. Softer or weaker supervisors can end up facing extremely low recovery and extremely high forbearance.

In that equilibrium, the economy is exposed to experiencing a high bank failure rate and large systemic costs, both of which can negatively feed back into government finances. These forces identify a novel two-way sovereign-bank nexus. Although the link between banking sector vulnerabilities and the fiscal weakness of the government has been extensively discussed in the literature (Dell'Ariccia et al., 2018a), the mechanism described above is new because the uncovered two-way link (from fiscal capacity to the credibility of the resolution threat and vice versa) does not rely on the combination of government guarantees (or bailout expectations) and banks' holdings of sovereign debt (Acharya et al., 2014; Brunnermeier et al., 2016; Leonello, 2018) but on potential weaknesses of the bank recovery and resolution framework.

5 Concluding remarks

The core of this paper focuses on the interplay between banks and a supervisor after the latter discovers that a significant mass of banks may turn insolvent unless they get properly recapitalized. Our analysis disentangles the strategic interaction between the banks requested to privately recover and a supervisor with the discretionary power to force the required recapitalization using early intervention and resolution tools (such as a precautionary public recapitalization or the forced conversion of bail-in debt into equity). We identify the determinants of the credibility of the supervisory resolution threat and its impact on banks' recovery actions, the resulting levels of resolution and capital forbearance, and the implications for the systemic costs due to the potential failure of the banks that remain undercapitalized. Importantly, we analyze the influence of explosive cost of resolution (which might be due to reputational or political biases or supervisory capacity constraints) on these outcomes.

The analysis of the model produces a number of testable predictions and novel theoretical and policy insights. First, when supervisors face higher reputational, political, economic or fiscal costs of resolution, they end up having to resolve more banks and, yet, exercising more forbearance.

Second, economies characterized by a higher and more steeply increasing marginal costs of supervisory intervention (because of explosive reputational or political costs or hard limits to the availability of relevant resolution resources) are more exposed to suffer from a negative feedback loop whereby the softer threat of resolution discourages banks from recovering, exhausts the supervisor's capacity to respond to lower recovery with sufficiently large increases in resolution, and rises the resulting forbearance and systemic costs. Strategic complementary between banks' recovery decisions can produce multiple equilibria, and lead to the coordination in equilibria with extreme forbearance.

Third, we predict that in economies with highly levered banks, the incidence of resolution of damaged banks as well as the capital forbearance will tend to be higher. In that situation, reinforcing capital regulation can help compensate the weakness of the underlying resolution threat.

On the policy side, the comparative statics of the private costs of recovery and the analysis of the role of leverage suggest that policies facilitating the undertaking of leverage reduction transactions (e.g. by reducing the informational and agency frictions behind equity issuance costs) or reducing the importance of the Merton's put (such as controls on leverage and risk taking by banks) can reduce the need for resolution and the levels of ex post forbearance. While not explicitly considered in our analysis, the ex ante issuance of securities such as contingent convertible debt (CoCos) with high triggers would also help. Intuitively, if their conversion into equity provides for the equivalent to a bank recapitalization without the need to resort on a discretionary decision of bank owners or a large resolution cost to the supervisor, bank incentives to undertake recovery would rise and the need for resolution would diminish.[30]

Finally, the analysis of the too-many-to-resolve problem highlights the importance of supervisors' capacity and credibility to act on non-recovering banks. Key elements of an institutional design conducive to better outcomes include providing the supervisor with independence and a clear mandate to avoid future systemic costs, an internal governance that avoids giving excessive weight to the reputational, political and economic costs of bank resolution, and strong resources, including the financial capacity necessary to back its resolution threats. In a multi-jurisdictional context such as the EU, the delegation of supervision and resolution to union-level authorities might be justified from this perspective.

[30]When the bank is recapitalized through the conversion of CoCos into equity, the initial owners' stake in the bank may also get diluted, both directly and via the reduction in the Merton's put. However, CoCos are designed to not make conversion a discretionary choice of bank owners but happen automatically and imply a lower cost to them than a seasoned equity offering with the same leverage-reduction effect.

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Appendix: Proofs

Proof of Lemma 1 The result follows directly from the discussion provided in the main text.■

Proof of Proposition 1 To prove the results in Proposition 1, it is useful to examine the position of the indifference line $n = I(m)$ relative to the supervisor's reaction function $n = N(m)$.

For $\{\lambda_1/(\lambda_1+\tau_1)-[c/(c+\Delta)]\}\phi-(\tau_0-\lambda_0)/(\lambda_1+\tau_1) > 0$ (or, equivalently, $\Delta > \tau_1 c/\lambda_1$ and $\phi > (\tau_0-\lambda_0)(c+\Delta)/(\lambda_1\Delta-\tau_1 c)$) we have $I(0) < N(0)$, which, given the form of the curves $N(m)$ and $I(m)$, guarantees a single crossing between them in the section of the supervisor's reaction function where $N(m) > 0$. This is the situation depicted in Figure 1. In this case the unique symmetric SPNE of the game involves the values of (m, n) at such intersection, (m^*, n^*). To show that such point is a SPNE, notice that lying on $n = I(m)$ means that bank owners are indifferent between recovering or not. Hence, m^* can be sustained as the result of damaged banks playing an uncorrelated symmetric mixed strategy in which they recover with probability $p^* = m^*/\phi$. Simultaneously, lying also on $n = N(m)$ means that by resolving a mass n^* of the unrecovered banks, the supervisor plays a best response to damaged banks' recovery actions in the first stage. The uniqueness of this equilibrium comes from the fact that, for $m < m^*$, we have $N(m) > I(m)$, which means that banks would prefer $r = 1$, which is incompatible with sustaining $m < \phi$. Instead, for values of $m > m^*$, we have $N(m) < I(m)$, which means that banks would prefer $r = 0$, which is incompatible with sustaining $m > 0$.

For $\{\lambda_1/(\lambda_1+\tau_1)-[c/(c+\Delta)]\}\phi-(\tau_0-\lambda_0)/(\lambda_1+\tau_1) \leq 0$ (or, equivalently, when $\Delta \leq \tau_1 c/\lambda_1$ and/or $\phi \leq (\tau_0-\lambda_0)(c+\Delta)/(\lambda_1\Delta-\tau_1 c)$) we have $I(0) \geq N(0)$, which means that damaged banks' indifference line lies everywhere above the supervisor's reaction function, as depicted in Figure A.1. Then at all points on the supervisor's reaction function banks prefer not to recover. But then the unique symmetric SPNE must involve $m = 0$ and the supervisor's best response to such first stage outcome, that is, $n = N(0) = [\lambda_1\phi-(\tau_0-\lambda_0)]/(\lambda_1+\tau_1)$, as indicated in Figure A.1.■

Proof of Proposition 2 Table A1 summarizes the comparative statics of the equilibrium in which there is a strictly positive mass of privately recovering banks. In the table, signs +, – or = indicate whether increasing the parameter indicated in the first column of the table increases, decreases, or does not change the endogenous variable indicated

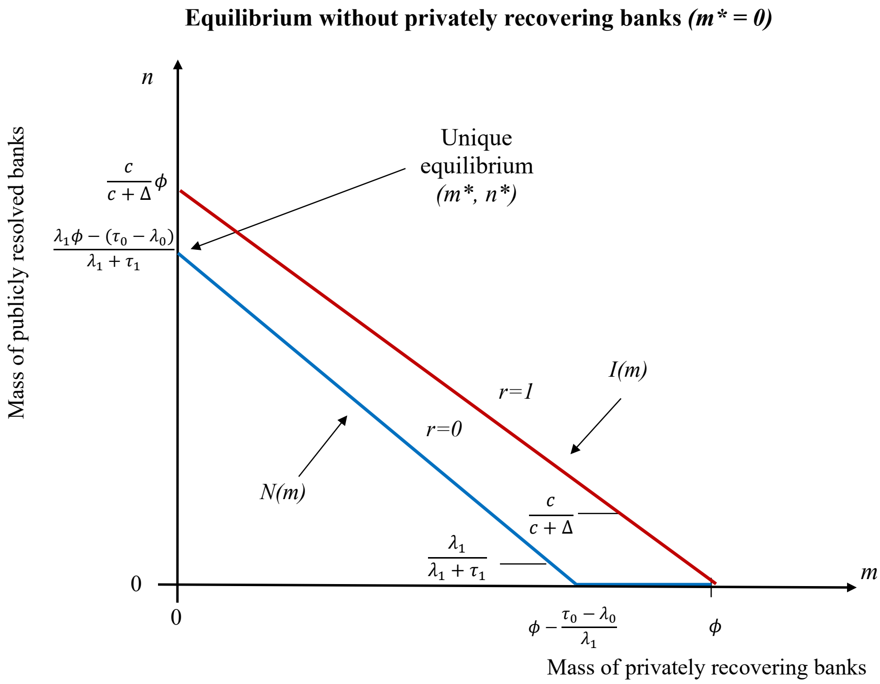

Figure A.1: Equilibrium without recovering banks

in the heading of each column. In a slight abuse of terminology, the table assimilates variations in the difference $\phi-(\tau_0-\lambda_0)(c+\Delta)/(\lambda_1\Delta-\tau_1 c)$ (which needs to be positive for the equilibrium to feature $m^* > 0$) as variations in the "likelihood" of having an equilibrium with a positive level of recoveries ($m^* > 0$). All the results arise immediately from partially differentiating the closed-form expressions of the relevant equilibrium variables.

Table A1. Comparative statics in the regime with $m^* > 0$

Parameter	Likelihood of equilibrium w. $m^* > 0$	Resolution threat $n^*/(\phi – m^*)$	Recapitalized banks: Recovery m^*	Recapitalized banks: Resolution n^*	Capital forbearance $\phi – m^* – n^*$
Damaged banks ϕ	+	=	+	=	=
Systemic cost parameter λ_0	+	=	+	−	−
Systemic cost parameter λ_1	+	=	+	−	−
Resolution cost parameter τ_0	−	=	−	+	+
Resolution cost parameter τ_1	−	=	−	+	+
Private recovery cost c	−	+	−	+	+
Extra loss from resolution Δ	+	−	+	−	−

The effects of changing λ_0, λ_1, c, and Δ go in the natural direction, in the sense that (i) they encourage the player(s) directly suffering the corresponding cost to take actions that reduce the incidence of such cost, and (ii) produce partially offsetting changes in the actions of the opposing player(s). In the case of c and Δ, the direct effects (on m^*) dominate the indirect ones (on n^*) as for the final variation in forbearance $(\phi - m^* - n^*)$. In the case of λ_0 and λ_1, the indirect effect (on m^*) dominates.■

Proof of Proposition 3 First, we find the effect of τ_0 on the mass of resolved banks n^* and on forbearance $\phi - m^* - n^*$ by just deriving in (1) and (17), respectively, using the expressions that correspond to the case with a positive equilibrium mass of recovering banks:

$$\frac{\partial n^*}{\partial \tau_0} = \frac{c}{\lambda_1 \Delta - \tau_1 c} > 0, \tag{25}$$

$$\frac{\partial (\phi - m^* - n^*)}{\partial \tau_0} = \frac{\Delta}{\lambda_1 \Delta - \tau_1 c} > 0. \tag{26}$$

Next, we further derive the above expressions with respect to c:

$$\frac{\partial^2 n^*}{\partial \tau_0 \partial c} = \frac{\lambda_1 \Delta}{(\lambda_1 \Delta - \tau_1 c)^2} > 0, \tag{27}$$

$$\frac{\partial^2 (\phi - m^* - n^*)}{\partial \tau_0 \partial c} = \frac{\tau_1 \Delta}{(\lambda_1 \Delta - \tau_1 c)^2} > 0, \tag{28}$$

whose signs mean that a rise in c reinforces the effects of τ_0 on n^* and $\phi - m^* - n^*$.

Proceeding similarly to find the effects of τ_1 on n^* and $\phi - m^* - n^*$, we obtain:

$$\frac{\partial n^*}{\partial \tau_1} = \frac{(\tau_0 - \lambda_0)c^2}{(\lambda_1 \Delta - \tau_1 c)^2} > 0, \tag{29}$$

$$\frac{\partial(\phi - m^* - n^*)}{\partial \tau_1} = \frac{(\tau_0 - \lambda_0)\Delta c}{(\lambda_1 \Delta - \tau_1 c)^2} > 0, \tag{30}$$

and the cross-derivatives:

$$\frac{\partial^2 n^*}{\partial \tau_1 \partial c} = \frac{2(\tau_0 - \lambda_0)c\lambda_1\Delta}{(\lambda_1 \Delta - \tau_1 c)^3} > 0, \tag{31}$$

$$\frac{\partial^2(\phi - m^* - n^*)}{\partial \tau_1 \partial c} = \frac{2(\tau_0 - \lambda_0)\Delta c^2}{(\lambda_1 \Delta - \tau_1 c)^3} > 0, \tag{32}$$

which imply that a rise in c also reinforces the effects of τ_1 on n^* and $\phi - m^* - n^*$.■

Proof of Proposition 4 Taking into account how coefficients c, τ_0, τ_1, λ_0 and λ_1 in prior expressions depend on the shortfall s according to the details provided in subsection 2.1, we can rewrite the supervisor's reaction function as

$$N(m) = \max\left\{\frac{\tilde{\lambda}_1 s(\phi - m) - (\tilde{\tau}_0 - \tilde{\lambda}_0)}{(\tilde{\lambda}_1 + \tilde{\tau}_1)s}, 0\right\}. \tag{33}$$

where $\tilde{\lambda}_0 = \varepsilon(g_0 + g_1 x_h)$, $\tilde{\lambda}_1 = \varepsilon g_1$, $\tilde{\tau}_0 = g_1(x_i - x_l)$ and $\tilde{\tau}_1 = \frac{2-\varepsilon}{1-\varepsilon} g_1$. An individual bank's indifference condition regarding the choice between recovering or not can be rewritten as

$$I(m) = \frac{\tilde{c}s}{\tilde{c}s + \Delta}(\phi - m). \tag{34}$$

where $\tilde{c} = \varepsilon$. Exploring (33) and (34), it becomes clear that s increases the supervisor's propensity to resolve (that is, shifts its reaction function outwards in a figure similar to Figure 1) and reduces banks' incentives to recover (that is, shifts their indifference condition inwards in a figure similar to Figure 1).

Under Assumption 2, we can rewrite the equilibrium as:

$$m^* = \phi - \frac{(\tilde{\tau}_0 - \tilde{\lambda}_0)(\tilde{c}s + \Delta)}{(\tilde{\lambda}_1 \Delta - \tilde{\tau}_1 \tilde{c}s)s}, \tag{35}$$

$$n^* = \frac{(\tilde{\tau}_0 - \tilde{\lambda}_0)\tilde{c}}{\tilde{\lambda}_1 \Delta - \tilde{\tau}_1 \tilde{c}s} \tag{36}$$

It is immediate to find that

$$\frac{\partial n^*}{\partial s} = \frac{(\tilde{\tau}_0 - \tilde{\lambda}_0)\tilde{\tau}_1 \tilde{c}^2}{(\tilde{\lambda}_1 \Delta - \tilde{\tau}_1 \tilde{c} s)^2} > 0,$$

and also

$$\phi - m^* - n^* = \frac{(\tilde{\tau}_0 - \tilde{\lambda}_0)\Delta}{(\tilde{\lambda}_1 \Delta - \tilde{\tau}_1 \tilde{c} s)s}, \tag{37}$$

which implies

$$\frac{\partial(\phi - m^* - n^*)}{\partial s} = -\frac{(\tilde{\tau}_0 - \tilde{\lambda}_0)\Delta(\Delta\tilde{\lambda}_1 - 2\tilde{\tau}_1 \tilde{c} s)}{(\tilde{\lambda}_1 \Delta - \tilde{\tau}_1 \tilde{c} s)^2 s^2}, \tag{38}$$

which is strictly positive if and only if $s > \bar{s}$ where

$$\bar{s} = \frac{\Delta\tilde{\lambda}_1}{2\tilde{\tau}_1 \tilde{c}} = \frac{\Delta(1-\varepsilon)}{2(2-\varepsilon)}$$

So forbearance is first decreasing and then increasing in s.

To better understand the roots of the non-monotonicity in the effect of leverage on forbearance, notice that

$$\frac{\partial m^*}{\partial s} = -\frac{(\tilde{\tau}_0 - \tilde{\lambda}_0)(\tilde{\tau}_1 \tilde{c}^2 s^2 + 2\tilde{\tau}_1 \Delta \tilde{c} s - \tilde{\lambda}_1 \Delta^2)}{(\tilde{\lambda}_1 \Delta - \tilde{\tau}_1 \tilde{c} s)^2 s^2}, \tag{39}$$

which is strictly positive if and only if

$$\tilde{\tau}_1 \tilde{c}^2 s^2 + 2\tilde{\tau}_1 \Delta \tilde{c} s - \tilde{\lambda}_1 \Delta^2 < 0. \tag{40}$$

The quadratic equation $\tilde{\tau}_1 \tilde{c}^2 s^2 + 2\tilde{\tau}_1 \Delta \tilde{c} s - \tilde{\lambda}_1 \Delta^2 = 0$ has two roots, one of which is negative. The other root is

$$\hat{s} = \frac{\Delta}{\tilde{c}}\left(\sqrt{1 + \frac{\tilde{\lambda}_1}{\tilde{\tau}_1}} - 1\right) = \frac{\Delta}{\varepsilon}\left(\sqrt{1 + \frac{\varepsilon(1-\varepsilon)}{(2-\varepsilon)}} - 1\right) > 0. \tag{41}$$

Thus the inequality (40) is true if and only if $s < \hat{s}$, which means that the mass of recovering banks is first increasing and then decreasing with s, reaching a maximum at $\hat{s}$. Given that the mass of resolved banks n^* is monotonically increasing in s, this inverted U-shaped relationship between s and m^* helps explain the U-shaped relationship between s and forbearance as well as the fact that the latter reaches a minimum at a leverage level $\bar{s}$ strictly larger than $\hat{s}$.■

Proof of Proposition 5 The results follow from the taxonomy of the possibilities that may emerge in the constrained game when the unconstrained game features a positive mass of recovering banks, that is, $m^* > 0$ and, hence, $n^* = N(m^*)$. The situation depicted in Figure 2 corresponds with case 3 in the following list:

1. If $n_0 < N(m^*)$, the constrained game has just one equilibrium with $(m, n) = (0, n_0)$.
2. If $n_0 = N(m^*)$, the constrained game features two equilibria: one with $(m, n) = (0, n_0)$ and the same equilibrium $(m^*, N(m^*))$ as the unconstrained game.
3. If $n_0 \in (N(m^*), (c/(c + \Delta))\phi]$, the constrained game features three equilibria: one with $(m, n) = (0, n_0)$, the same equilibrium $(m^*, N(m^*))$ as the unconstrained game, and a third (unstable) equilibrium with $(I^{-1}(n_0), n_0)$.
4. If $n_0 > (c/(c + \Delta))\phi$, the constrained game has just the same equilibrium as the unconstrained game.

When the unconstrained game features $m^* = 0$ and, hence, $n^* = N(0)$ (as illustrated in Figure A.1), the possibilities that emerge when the supervisor faces a limit n_0 to its resolution capacity are only two:

1. For $n_0 < N(0)$, the constrained game has just one equilibrium with $(m, n) = (0, n_0)$ (which involves larger forbearance than the equilibrium of the unconstrained game).
2. For $n_0 \geq N(0)$, the constrained game has just the same equilibrium $(0, N(0))$ as the unconstrained game.■

Proof of Proposition 6 According to Proposition 5, the range of values of n_0 over which only the equilibrium with extreme forbearance exists is $[0, N(m^*))$. It is immediate to see that $\partial N(m^*)/\partial c > 0$, $\partial N(m^*)/\partial \tau_i > 0$, and $\partial^2 N(m^*)/(\partial \tau_i \partial c) > 0$ for $i = 0, 1$, which proves the result.■

Proof of Lemma 2 Under (23) the first order condition for the minimization of $T + \Lambda$ becomes:

$$\tau_0 + \tau_1 n + \tau_2 n^2 - \lambda_0 - \lambda_1(\phi - m - n) = 0 \tag{42}$$

For given m, the equation (42) has two roots: $n^-(m;\tau_2)$ is always negative and $n^+(m;\tau_2)$ that is strictly positive below the same critical value $m = \phi - (\tau_0 - \lambda_0)/\lambda_1$ as in the piece-wise linear reaction function of the baseline model (see Figure 1):

$$n^+(m;\tau_2) = \frac{-(\tau_1 + \lambda_1) + \sqrt{(\tau_1 + \lambda_1)^2 - 4\tau_2[\tau_0 - \lambda_0 - \lambda_1(\phi - m)]}}{2\tau_2}, \tag{43}$$

When $n^+(m;\tau_2) > 0$, it identifies the supervisor's best response to m; otherwise the best response is zero.

The indifference condition for the damaged bank between recovering or not, is then:

$$c = \frac{N(m;\tau_2)}{\phi - m}(c + \Delta). \tag{44}$$

Importantly, in the range where $n^+(m;\tau_2) > 0$, we have, using (24),

$$d\left(\frac{N(m;\tau_2)}{\phi - m}\right)/dm = \frac{1}{\phi - m}\left[\frac{-\lambda_1}{(\tau_1 + \lambda_1) + 2\tau_2 n^+(m;\tau_2)} + \frac{n^+(m;\tau_2)}{\phi - m}\right], \tag{45}$$

Banks' recovery decision are strategic complements if and only if $d\left(\frac{N(m;\tau_2)}{\phi - m}\right)/dm > 0$, i.e.

$$2\tau_2 n^2 + (\tau_1 + \lambda_1)n - \lambda_1(\phi - m) > 0 \tag{46}$$

or

$$m > \phi - \frac{n(\tau_1 + \lambda_1 + 2\tau_2 n)}{\lambda_1} \equiv \widetilde{m}(n). \tag{47}$$

Note that $\widetilde{m}(n)$ is decreasing and convex in n, with $\widetilde{m}(0) = \phi$ and $\widetilde{m}(\phi) = -\frac{\phi(\tau_1 + 2\tau_2\phi)}{\lambda_1} < 0$, thus $\widetilde{m}(n) = 0$ at some $n < \phi$.

Next, define m as a function of n from (43),

$$\bar{m}(n) = \phi - \frac{\tau_2 n^2 + (\tau_1 + \lambda_1)n + (\tau_0 - \lambda_0)}{\lambda_1} \tag{48}$$

Note that $\bar{m}(n)$ is also decreasing and convex in n, with $\bar{m}(0) = \phi - \frac{\tau_o - \lambda_0}{\lambda_1} > 0$ by Assumption 1 and $\bar{m}(\phi) = -\frac{\phi(\tau_1 + \tau_2\phi)}{\lambda_1} - \frac{\tau_0 - \lambda_0}{\lambda_1} < 0$, thus $\bar{m}(n) = 0$ at some $n < \phi$.

Thus $d\left(\frac{N(m;\tau_2)}{\phi - m}\right)/dm < 0$ for $m = \bar{m}(n)$ when n is sufficiently close to but strictly above 0, so that m is close to but strictly below $\phi - (\tau_0 - \lambda_0)/\lambda_1$. However $d\left(\frac{N(m;\tau_2)}{\phi - m}\right)/dm$ can become positive for smaller values of m; this is the case whenever $\bar{m}(n) > \widetilde{m}(n)$, that is, using (47) and (48), when

$$n > \sqrt{\frac{\tau_0 - \lambda_0}{\tau_2}}. \tag{49}$$

To prove that the local complementarity region exists, we show that $\bar{m}(\sqrt{\frac{\tau_0-\lambda_0}{\tau_2}}) > 0$. Note that $\bar{m}(n) = 0$ if

$$\tau_2 n^2 + (\tau_1 + \lambda_1)n + \tau_0 - \lambda_0 - \lambda_1\phi = 0 \tag{50}$$

that has two roots with only one positive:

$$\bar{n} = \frac{-(\tau_1 + \lambda_1) + \sqrt{(\tau_1 + \lambda_1)^2 - 4\tau_2(\tau_0 - \lambda_0 - \lambda_1\phi)}}{2\tau_2}. \tag{51}$$

Thus, $\bar{m}(\sqrt{\frac{\tau_0-\lambda_0}{\tau_2}}) > 0$ if

$$\sqrt{\frac{\tau_0 - \lambda_0}{\tau_2}} < \frac{-(\tau_1 + \lambda_1) + \sqrt{(\tau_1 + \lambda_1)^2 - 4\tau_2(\tau_0 - \lambda_0 - \lambda_1\phi)}}{2\tau_2}. \tag{52}$$

Squaring both sides, we get

$$\frac{\tau_0 - \lambda_0}{\tau_2} < \frac{2(\tau_1 + \lambda_1)^2 - 4\tau_2(\tau_0 - \lambda_0 - \lambda_1\phi) - 2(\tau_1 + \lambda_1)\sqrt{(\tau_1 + \lambda_1)^2 - 4\tau_2(\tau_0 - \lambda_0 - \lambda_1\phi)}}{4\tau_2^2}. \tag{53}$$

Rearranging the items yields:

$$2(\tau_1 + \lambda_1)^2 - 8\tau_2(\tau_0 - \lambda_0) + 4\tau_2\lambda_1\phi > 2(\tau_1 + \lambda_1)\sqrt{(\tau_1 + \lambda_1)^2 - 4\tau_2(\tau_0 - \lambda_0 - \lambda_1\phi)}. \tag{54}$$

Squaring both sides again and rearranging items, we get

$$\tau_2[2(\tau_0 - \lambda_0) - \lambda_1\phi]^2 - (\tau_1 + \lambda_1)^2(\tau_0 - \lambda_0) > 0 \tag{55}$$

that holds for sufficiently high τ_2:

$$\tau_2 > \widehat{\tau_2} \equiv \frac{(\tau_1 + \lambda_1)^2(\tau_0 - \lambda_0)}{[2(\tau_0 - \lambda_0) - \lambda_1\phi]^2}, \tag{56}$$

where $\widehat{\tau_2} > 0$ under Assumption 1.

Thus, when $\tau_2 > \widehat{\tau_2}$, banks' recovery decision are strategic complements for $n >$

$\sqrt{\frac{\tau_0-\lambda_0}{\tau_2}}$, or equivalently

$$m < \widehat{m} \equiv \bar{m}(\sqrt{\frac{\tau_0-\lambda_0}{\tau_2}}) = \phi - \frac{2(\tau_0-\lambda_0)}{\lambda_1} - \frac{(\tau_1+\lambda_1)\sqrt{\tau_2(\tau_0-\lambda_0)}}{\lambda_1\tau_2}, \tag{57}$$

and strategic substitutes for $m > \widehat{m}$. Instead, when $\tau_2 \leq \widehat{\tau_2}$, banks' recovery decisions are strategic substitutes for all m.■

Proof of Proposition 7 To find the equilibria of the game, we study possible crossings of (15) and (42). Using $n = I(m)$ together with (13) to substitute for m in (42), we obtain that in any interior equilibrium, n would have to solve:

$$\tau_2 n^2 - (\hat{\lambda}_1 - \tau_1)n + (\tau_0 - \lambda_0) = 0, \tag{58}$$

with $\hat{\lambda}_1 \equiv \lambda_1\Delta/c$. In any interior equilibrium, n would have to solve (58), which in some instances features two solutions. In the limit case with $\tau_2 = 0$, (58) yields the same unique candidate interior equilibrium (m^*, n^*) as the baseline linear-quadratic formulation (see (16)). By continuity with the baseline model, if Assumption 2 holds, for sufficiently small values of τ_2, a valid interior solution arbitrarily close to (m^*, n^*) must exit.

The roots of (58) are

$$n_L(\tau_2) = \frac{\lambda_1\Delta/c - \tau_1 - \sqrt{(\lambda_1\Delta/c - \tau_1)^2 - 4\tau_2(\tau_0-\lambda_0)}}{2\tau_2} \tag{59}$$

and

$$n_H(\tau_2) = \frac{\hat{\lambda}_1 - \tau_1 + \sqrt{(\hat{\lambda}_1 - \tau_1)^2 - 4\tau_2(\tau_0-\lambda_0)}}{2\tau_2}. \tag{60}$$

Under Assumption 1, $n_L(\tau_2)$ and $n_H(\tau_2)$ are real if and only if

$$(\hat{\lambda}_1 - \tau_1)^2 - 4\tau_2(\tau_0-\lambda_0) \geq 0 \Leftrightarrow \tau_2 \leq \frac{1}{4}\frac{(\hat{\lambda}_1-\tau_1)^2}{\tau_0-\lambda_0} \equiv \bar{\tau}_2. \tag{61}$$

Intuitively, if τ_2 is too large, $N(m;\tau_2)$ will not cross the indifference condition $I(m)$, as in Panel D of Figure 3, giving rise to a situation similar to that shown in Figure A.1 for the baseline case (see the proof of Proposition 1). In such a case, the only equilibrium involves extreme forbearance, with $m = 0$ and $n = N(0;\tau_2) = n^+(0;\tau_2) \in (0, \phi)$.

The rest of this proof focuses on the case with $\tau_2 \leq \bar{\tau}_2$ where the roots $n_L(\tau_2)$ and

$n_H(\tau_2)$ are real. In this case, under Assumptions 1 and 2, we have

$$0 < n_L(\tau_2) \leq \frac{\hat{\lambda}_1 - \tau_1}{2\tau_2} \leq n_H(\tau_2) < \frac{\hat{\lambda}_1 - \tau_1}{\tau_2}, \tag{62}$$

where the second and third inequalities are strict if and only $\tau_2 < \bar{\tau}_2$. In the limit case with $\tau_2 = \bar{\tau}_2$, the candidate interior solution is unique. Intuitively, $N(m)$ and $I(m)$ are tangent at that point, as in Panel C of Figure 3. The validity of the solution with $n = \bar{n} = \frac{\hat{\lambda}_1 - \tau_1}{2\bar{\tau}_2} = 2\frac{\tau_0 - \lambda_0}{\hat{\lambda}_1 - \tau_1}$ further requires that $\bar{m} = \phi - \frac{c+\Delta}{c}\bar{n} > 0$, that is, a sufficiently large ϕ:

$$\phi > \bar{\phi} = \frac{c + \Delta}{c}\bar{n} = 2\frac{(\tau_0 - \lambda_0)(c + \Delta)}{\lambda_1\Delta - \tau_1 c} = 2\hat{\phi}. \tag{63}$$

The unique candidate interior solution emerging for $\tau_2 = \bar{\tau}_2$ when $\phi > \bar{\phi}$ is, however, unstable in the sense that any perturbation in m would push banks into strictly preferring $r = 0$ to $r = 1$. The only stable equilibrium in this case is still the one with extreme forbearance, that is, with $m = 0$ and $n = n^+(0; \tau_2) \in (0, \phi)$.

Consider now the case with $\tau_2 < \bar{\tau}_2$ but τ_2 arbitrarily close to $\bar{\tau}_2$, which implies that: (i) $n_L(\tau_2) < n_H(\tau_2)$, but $n_L(\tau_2)$ and $n_H(\tau_2)$ are arbitrarily close to each other; (ii) for $\phi > \bar{\phi}$, the two solutions are possibly valid in that $m_i(\tau_2) = \phi - \frac{c+\Delta}{c}n_i(\tau_2) > 0$ for $i = L, H$, like in Panel B of Figure 3. However, the solution $(m_L(\tau_2), n_L(\tau_2))$ is stable, while the solution $(m_H(\tau_2), n_H(\tau_2))$ is unstable. Moreover, a second stable solution with $m = 0$ and extreme forbearance, $(0, n^+(0; \tau_2))$, exists.

As τ_2 declines further down from $\bar{\tau}_2$ towards zero, the solution with $(m_L(\tau_2), n_L(\tau_2))$ remains a valid stable equilibrium for values of ϕ above some threshold lower and lower than $\bar{\phi}$. Specifically, $(m_L(\tau_2), n_L(\tau_2))$ is a valid solution for

$$\phi \geq \phi_L(\tau_2) \equiv \frac{c + \Delta}{c} n_L(\tau_2), \tag{64}$$

which is increasing in τ_2 because $n_L(\tau_2)$ is increasing in τ_2, and reaches the value $\hat{\phi} = \bar{\phi}/2$ when $\tau_2 = 0$.

Instead, as τ_2 declines further down from $\bar{\tau}_2$ towards zero, the solution with $m = 0$ and extreme forbearance $(0, n^+(0; \tau_2))$ only exits for values of ϕ above some threshold higher and higher than $\bar{\phi}$. Specifically, $(0, n^+(0))$ is a valid solution insofar as $n^+(0) \geq n_H(\tau_2)$ (or equivalently $m_H(\tau_2) \geq 0$), which boils down to having

$$\phi \geq \phi_H(\tau_2) \equiv \frac{c + \Delta}{c} n_H(\tau_2), \tag{65}$$

which is decreasing in τ_2 because $n_H(\tau_2)$ is increasing in τ_2, and tends to infinity when τ_2 tends to zero (since $\lim_{\tau_2 \to 0} n_H(\tau_2) = \infty$). Panel A in Figure 3 corresponds to a situation with low enough values of τ_2 and intermediate values of ϕ for which the interior equilibrium exists but the extreme forbearance equilibrium does not. Figure 4 depicts the objects obtained in this proof to characterize graphically the areas in the space of parameters (τ_2, ϕ) where each of the possible equilibrium configurations emerge.■

Proof of Proposition 8 According to Proposition 7, the range of values of ϕ over which the equilibrium with extreme forbearance exists is $\phi < \phi_L(\tau_2)$ and $\phi \geq \phi_H(\tau_2)$. First, consider the effect of private recovery cost of c on $\phi_L(\tau_2)$:

$$\frac{d\phi_L(\tau_2)}{dc} = -\frac{\Delta}{c^2} \cdot n_L(\tau_2) + \frac{\partial n_L(\tau_2)}{\partial c} \cdot \frac{c+\Delta}{c} \tag{66}$$

with

$$\frac{\partial n_L(\tau_2)}{\partial c} = \frac{\lambda_1 \Delta}{c^2} \left(\frac{n_L(\tau_2)}{\sqrt{A}} \right) > 0. \tag{67}$$

where $A \equiv (\hat{\lambda}_1 - \tau_1)^2 - 4\tau_2(\tau_0 - \lambda_0)$. Plugging (67) into (66) yields:

$$\frac{d\phi_L(\tau_2)}{dc} = \frac{n_L(\tau_2)\Delta}{c^2} \left(-1 + \frac{\lambda_1(c+\Delta)}{c\sqrt{A}} \right) \tag{68}$$

which is positive whenever $\lambda_1 > \frac{c\sqrt{A}}{c+\Delta}$. But, since $n_L(\tau_2) = \frac{\hat{\lambda}_1 - \tau_1 - \sqrt{A}}{2\tau_2} > 0$ and $\hat{\lambda}_1 = \lambda_1 \Delta / c$, we have

$$\lambda_1 > \frac{c(\tau_1 + \sqrt{A})}{\Delta} > \frac{c\sqrt{A}}{c+\Delta}, \tag{69}$$

so the condition holds, implying $d\phi_L(\tau_2)/dc > 0$.

The effect of c on $\phi_H(\tau_2)$ can be obtained as

$$\frac{d\phi_H(\tau_2)}{dc} = -\frac{\Delta}{c^2} \cdot n_H(\tau_2) + \frac{\partial n_H(\tau_2)}{\partial c} \cdot \frac{c+\Delta}{c} < 0 \tag{70}$$

since

$$\frac{\partial n_H(\tau_2)}{\partial c} = -\frac{\lambda_1 \Delta}{c^2} \cdot \frac{n_H(\tau_2)}{\sqrt{A}} < 0. \tag{71}$$

Thus, the range of values for which the extreme forbearance equilibrium exist expands ($d\phi_L(\tau_2)/dc > 0$ and $d\phi_H(\tau_2)/dc < 0$) as the private cost of compliance c goes up.

Similarly, the range of values for the extreme forbearance equilibrium expands as the resolution cost parameters τ_0 and τ_1 go up since

$$\frac{d\phi_L(\tau_2)}{d\tau_0} = \frac{c+\Delta}{c\sqrt{A}} > 0, \ \frac{d\phi_H(\tau_2)}{d\tau_0} = -\frac{c+\Delta}{c\sqrt{A}} < 0, \tag{72}$$

$$\frac{d\phi_L(\tau_2)}{d\tau_1} = \frac{n_L(\tau_2)(c+\Delta)}{c\sqrt{A}} > 0, \text{ and } \frac{d\phi_H(\tau_2)}{d\tau_1} = -\frac{n_H(\tau_2)(c+\Delta)}{c\sqrt{A}} < 0. \tag{73}$$

Next, we study the cross effects of the private recovery cost c and the supervisory resolution cost parameters τ_0 and τ_1 on the range of the extreme forbearance equilibrium. First, consider the cross-derivatives with respect to c and τ_0. From (73),

$$\frac{\partial^2\phi_L(\tau_2)}{\partial\tau_0\partial c} = \frac{\Delta}{c^2\sqrt{A}} \cdot \left[-1 + \frac{\lambda_1(c+\Delta)(\hat{\lambda}_1 - \tau_1)}{cA}\right] > 0 \tag{74}$$

where the expression in brackets can be expressed as

$$\frac{\lambda_1(c+\Delta)(\hat{\lambda}_1 - \tau_1)}{cA} - 1 = \frac{(\hat{\lambda}_1 - \tau_1)(\lambda_1 + \tau_1) + 4\tau_2(\tau_0 - \lambda_0)}{A} > 0, \tag{75}$$

while

$$\frac{\partial^2\phi_H(\tau_2)}{\partial\tau_0\partial c} = -\frac{\partial^2\phi_L(\tau_2)}{\partial\tau_0\partial c} < 0. \tag{76}$$

So c and τ_0 reinforce each other in expanding the range of parameters under consideration.

Second, consider the cross-derivatives with respect to c and τ_1. Note that from (72) and (73), $\frac{d\phi_L(\tau_2)}{d\tau_1} = n_L(\tau_2)\frac{d\phi_L(\tau_2)}{d\tau_0}$. Therefore using (74), the corresponding cross-derivatives are:

$$\frac{\partial^2\phi_L(\tau_2)}{\partial\tau_1\partial c} = \frac{\partial n_L(\tau_2)}{\partial c}\frac{\partial\phi_L(\tau_2)}{\partial\tau_0} + \frac{\partial^2\phi_L(\tau_2)}{\partial\tau_0\partial c} \cdot n_L(\tau_2) > 0, \tag{77}$$

and

$$\frac{\partial^2\phi_H(\tau_2)}{\partial\tau_1\partial c} = -\frac{\partial^2\phi_L(\tau_2)}{\partial\tau_1\partial c} < 0, \tag{78}$$

which implies that c and τ_1 also reinforce each other's expansive effect on the range of parameters for which the extreme forbearance equilibrium exists.■