Bank Capital Forbearance and Serial Gambling *

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Abstract

We analyze the strategic interaction between undercapitalized banks and a supervisor who may intervene by preventive recapitalization. Supervisory forbearance emerges because limited fiscal and political capacity undermines supervisors' commitment to act preventively. Private incentives to timely recapitalize are lower when supervisors have lower credibility, especially for highly levered banks. In equilibrium supervisors facing higher cost of intervention, who are inclined to tolerate more undercapitalized banks, may end up spending more on public recapitalization. Importantly, when public capacity to intervene is tightly constrained private recapitalization decisions become strategic complements, leading to equilibria with extremely high forbearance and high systemic costs. Anticipating supervisory weakness in the face of widespread distress, banks may choose an ex ante correlated risk strategy, a form of "serial gambling" that undermines the supervisory response.

Keywords: bank supervision; bank recapitalization; forbearance.

JEL Classification: G21, G28

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1 Introduction

Supervisors have the critical task of assessing bank resilience to ensure timely corrective action when required. This process is fraught with challenges. Even when supervisors identify a capital shortfall, they cannot force shareholders to contribute additional capital. Private incentives to recapitalize are particularly poor in overleveraged banks since compliance implies a net transfer to creditors. Since banks' default imposes large costs on the economy, addressing capital shortfalls requires a public backup to control bank gambling and avoid future distress. Yet injecting public funds in private banks as a precautionary measure is unpopular and fiscally costly, especially in difficult economic circumstances. Such a reluctance to intervene decisively will clearly be anticipated and affect ex ante and interim risk incentives. We introduce a conceptual framework able to address two related questions. How do costs and limits on public intervention capacity affect supervisory forbearance? How does a distorted supervisory action affect banks' private recapitalization choices, and ultimately their ex ante risk incentives?

Previous work on supervisory incentives explained an inadequate or delayed intervention as reflecting a desire to hide weak supervisory skills or bad decisions (Boot and Thakor, 1993; Morrison and White, 2013). Supervisory forbearance may also result from the desire to avoid any disclosure that may trigger a panic (Walther and White, 2016; Chan and van Wijnbergen, 2017). We focus on supervisory forbearance driven by limited credibility of the threat of intervention. The prospect of insufficient corrective action undermines banks' incentives to privately recapitalize. To counteract weak private incentives, a supervisor may end intervening more than under stronger supervisory commitment, while also suffering higher fiscal and systemic costs of bank distress. When this mechanism is combined with a hard budget or capacity constraint on public intervention (as during a sovereign debt crisis), banks' private recapitalization decisions become strategic complements and can lead to a self-fulfilling equilibrium with very high forbearance and severe systemic costs.

One historical example of forbearance causing deepened distress by delaying intervention is the S&L bank default wave in the 1980s. Many small banks undermined by rising interest rates were allowed to keep gambling on their mortgage loan portfolios, leading to defaults and massive losses for the US deposit insurance system (Degennaro and Thompson, 1996).

Some analysts also saw an instance of forbearance in the stress test run by the Committee of European Banking Supervisors (CEBS) in 2010.[1] The stress test run by the US authorities in 2009, where many banks were challenged to significantly recapitalize under the threat of partial nationalization, raised the opposite perception. Our interpretation is that a key difference was the existence of a credible commitment to intervene at the necessary scale in case of private inaction.[2] Yet the experiences witnessed in Iceland as well as Greece, Ireland, Portugal and Spain during the Great Recession suggest that a prompt preventive intervention may at times exceed available fiscal capacity. In such circumstances supervisors may be pressed to allow some risky banks to operate with insufficient capital.

Formally, we consider a game between banks and a supervisor with discretionary powers to publicly recapitalize banks in trouble. The game starts when, in the context of a crisis, both banks and the supervisor learn the number of banks in distress.[3] The supervisor interacts with shareholders in the banks subject to debt overhang by first requiring a private recapitalization. The recapitalization solves the debt overhang problem and forces shareholders to give up their limited liability put option (as advocated by Bhattacharya and Nyborg, 2013). Noncompliant banks may be recapitalized publicly. As bank shareholders value control, they prefer not to be intervened by a forced public recapitalization. However, they are aware that the supervisor's decision will balance the systemic costs of bank default against the excess cost of public funds, as well as some political and reputational costs, so eventually may not be willing to intervene all noncompliant banks. Thus, for some banks the supervisor may choose to do nothing (*capital forbearance*).

Capital forbearance implies that some banks remain undercapitalized and shareholders retain control at times of poor incentives, which causes some expected social losses. In the baseline model, distressed banks' individual decisions on whether to privately recapitalize are

[1] The CEBS estimated in 2010 that "2.5 billion [euro] would have corrected the capital shortfall by the banks that failed the test. Market estimates suggested 300 billion euro, an amount that proved much more accurate" (Onando and Resti, 2011). On a more anecdotal basis, Dexia Bank reported a capital ratio above 10% in July 2011, three months before its collapse.

[2] While supervisory forbearance is typically more prevalent in countries with weak fiscal positions, empirical evidence in Bian et al. (2017) points to postponed bank intervention ahead of elections also in Germany.

[3] For simplicity we consider a situation without asymmetries of information in which all troubled banks are identical. We also abstract from the role of market discipline by assuming that bank is funded with fully insured deposits.

strategic substitutes, since when more banks restore their solvency there is less pressure on the supervisor to intervene to limit the potential losses. With fewer public recapitalizations, there is a higher chance that an undercapitalized bank benefits from supervisory forbearance. This reduces each troubled bank's marginal incentive to privately recapitalize.

We are particularly interested in the interaction between private and public recapitalization decisions when the supervisor faces a hard constraint to its recapitalization capacity. The hard constraint may be due to a limited fiscal capacity when banking distress is combined with a sovereign crisis. Such a constraint produces a *too-many-to-recapitalize* context where banks' private recapitalization decisions become strategic complements (similarly to banks' risk-taking decisions in Acharya and Yorulmazer, 2007). Once the constraint binds, a supervisor cannot react to a rise in the number of non-compliant banks by recapitalizing more of them and supervisory forbearance rises quickly, further discouraging private recapitalizations. This gives rise to the possibility of a second high-forbearance equilibrium in which banks coordinate in low recapitalization strategies, producing an exhaustion of the supervisor's intervention capacity, and a deeper systemic crisis. This provides a novel supervisory-based channel for a critical sovereign-bank nexus even without deposit insurance or bank holdings of sovereign debt.

Finally, we study banks' ex ante risk taking in the anticipation of supervisory forbearance. Since having more banks in trouble undermines the supervisory response, banks may prefer to follow risky strategies with correlated outcomes rather than safer or less correlated risky strategies. Banks' ex ante gambling (correlated risk taking) and ex post gambling (reluctance to recapitalize) reinforce each other in a form of endogenous strategic complementarity. A key result is that supervisory and fiscal weakness favor the emergence of "serial gambling."

Our analysis yields predictions consistent with the evidence on the economics and political economy of supervisory interventions and systemic crises. First, a supervisor facing higher political and reputational costs of intervention paradoxically ends up intervening more banks in equilibrium. The intuition is that a higher supervisory cost reduces the intervention threat for each level of private recapitalization. Banks react to this by recapitalizing less privately. Given a low level of compliance, the supervisor is compelled to publicly recapitalize more banks in order to limit the systemic costs of bank default. Yet the increase in preventive intervention does not fully compensate for private non-compliance, thus resulting in greater forbearance and a higher systemic loss. Empirically, it has been shown that political concerns play an important role in delaying government interventions (Brown and Dinç, 2005; Liu and Ngo, 2104; Bian et al., 2017). Relatedly, more captured supervisors encounter higher failure rates among the banks in their jurisdiction (Agarwal et al., 2014).

Second, similar effects occur when the supervisor faces a higher cost of public funds: its threat of intervention is also softer, promoting weaker bank compliance. Despite the rise in publicly intervened banks, more banks are left undercapitalized and their failure causes larger systemic costs. This is consistent with the fact that countries with more fiscally constrained governments face more severe banking crises (Reinhart and Rogoff, 2011).

Third, a rise in banks' leverage has two opposite effects on the equilibrium level of forbearance. On the one hand, leverage directly increases the cost of a private recapitalization, reducing banks' incentives to comply. On the other, it increases the severity of the systemic costs of supervisory inaction and hence the supervisor's propensity to intervene, which in turn pushes banks towards compliance. We find that the indirect effect dominates when leverage is low. Once leverage exceeds a certain level, private inaction grows with it. Eventually, very high leverage increases equilibrium forbearance, thus magnifying the systemic costs due to bank failure. This is consistent with the evidence that advanced economies with a higher growth in bank liabilities (relative to GDP) tend to experience more severe banking crises (Schularick and Taylor, 2012).[4]

Our results have cautionary implications for public policy, allowing for a realistic framing of what may be achieved in a weak institutional setting. The structural commitment problem faced by the supervisor cannot be easily addressed without major institutional changes. Key elements include providing the supervisor with independence and a clear mandate to avoid future systemic costs. However, preventive policy cannot be isolated from political interference as it may need to call upon fiscal capacity to intervene as needed. These elements were certainly important to induce the EU to centralize bank supervision. However, it fell short of providing broader fiscal backing of intervention.

A clear policy indication is that ex ante capital regulation can target key risk factors

[4]See also de Mooij et al. (2013) for direct evidence on the effect of bank leverage on the odds of a systemic crisis.

such as high leverage and risk correlation, which may force forbearance and reinforce poor incentives. Finally, our results on the dramatic consequences of limits to intervention capacity provide a rationale for the *preventive* value of having a strong public backstop for bank resolution. Rather than increasing moral hazard as often assumed, reinforcing the credibility of a timely public recapitalization threat has a strong effect on private compliance and would avoid larger bailout losses.

Related literature. Our paper contributes to the literature on supervisory interventions on banks. The earlier studies focus on closure rules for banks in distress as a tool to reduce deposit insurance liabilities (Acharya and Dreyfus, 1989; Allen and Saunders, 1993; Fries et al., 1997). In these settings private willingness to supply capital is assumed to be exogenous and the supervisor decides on bank closure, which in practice is a rare form of intervention except for very small banks. Our work focuses on recapitalization at an interim stage of bank distress, looking at the interaction between private recapitalization decisions and the prospect of supervisory intervention.

In our model, poor commitment induces an insufficient degree of private recapitalization relative to what is socially optimal. The issue of time inconsistent policy, first analyzed by Kydland and Prescott (1977) has been studied extensively in the literature on monetary policy (Barro and Gordon, 1983). As in our context, the inability to commit leads to an inefficient private response that may be quite costly to reverse. In the banking context, time inconsistency problems regarding policy interventions have been identified as a root cause of bank runs and delayed responses in the analysis of suspension of convertibility (Ennis and Keister, 2010) and the application of bail-in provisions (Keister and Mitkov, 2017).

Our setup incorporates a realistic supervisory trade-off between early intervention costs (including deadweight losses from taxation) and the spillover effects associated with future bank defaults (Bhattacharya et al., 1998). Philippon and Schnabl (2013) consider this trade-off from a normative perspective: characterizing policies that optimally deal with informational asymmetries regarding banks' exposure to debt overhang. Similarly to our setup, the regulator takes into account the deadweight losses of taxation and the negative externality of default. When an undercapitalized bank forgoes profitable lending, it worsens other banks' debt overhang via increased borrowers' defaults. On the other hand, government intervention generates free-riding as well as opportunistic participation. Optimal interventions feature public equity injections with voluntary participation, which leads to self-selection of the weaker banks into the program.

Shapiro and Skeie (2015) also study government interventions in the presence of fiscal costs, in a sequential bailout problem. The supervisor's cost of injecting capital into a bank is private information, so an earlier bailout decision reveals information about future choices. Since there is a trade-off between the cost of a run and the effect of a bailout on moral hazard, a low cost supervisor may choose not to bail out a bad bank in order to signal toughness and reduce subsequent risk taking.

Colliard and Gromb (2017) study how a single bank renegotiates its debt under asymmetric information, in the shadow of a potential government intervention and show how the prospect of such intervention may affect the delay in negotiations. In some circumstances, making the government commit not to interfere speeds up the workout process, improving efficiency. In contrast to these important contributions, our paper abstracts from information asymmetries and puts the emphasis on the interdepency between the decisions of the banks in trouble and between such decisions and the subsequent time-consistent decisions of the supervisor.

Considering a situation of diffused bank distress establishes a connection between our paper and the literature on bailout externalities (Perotti, 2002; Perotti and Suarez, 2002; Acharya and Yorulmazer, 2007; Farhi and Tirole, 2012). In Acharya and Yorulmazer (2007), the ex-post choice of the supervisor to bail out failing banks is affected by a too-many-to-fail problem which gives rise to strategic complementarities in banks' risk decisions: it encourages banks to be more correlated so as to fail together and increase the chance of benefiting from a bailout. In Farhi and Tirole (2012), an accommodating interest rate policy also generates strategic complementarities in banks' risk choices. The too-many-to-fail result resembles conceptually our too-many-to-recapitalize outcome in the presence of a hard limit to the supervisor's public recapitalization capacity. A public commitment to a resolution policy that rewards solvent banks (Perotti and Suarez, 2002) or punishes weak banks (Walther and White, 2016) may positively affect ex ante risk incentives. In the current setup, the lack of commitment to intervene on undercapitalized banks is a source of excessively low private recapitalization, high capital forbearance, and high systemic costs.

Outline of the paper. The rest of the paper is organized as follows. Section 2 describes the model. Section 3 characterizes the equilibrium of the baseline game between weak banks and the supervisor, discusses its comparative statics, and elaborates on the predictions regarding the effect of bank leverage on equilibrium outcomes. Section 4 discusses the case in which the supervisor faces a too-many-to-recapitalize problem. Section 5 extends the analysis to consider the interaction between the too-many-to-recapitalize problem and ex-ante risk taking. Section 6 provides a discussion of the inefficiency derived from the lack of supervisory commitment to intervene and the supervisor's self-interest. Section 7 concludes the paper. The appendices provide a microfoundation of the reduced-form payoff functions used in the model, discuss alternative parametric cases, and contain all the proofs.

2 The model

We consider a game played between a bank supervisor and some banks damaged by a solvency shock. There are three relevant dates $t = 0, 1, 2$, and all agents are risk neutral.

2.1 Banks

At $t = 0$ a mass ϕ of banks are damaged by a solvency shock. The banks are owned and managed in the interest of their initial shareholders. Shareholders discount future payoffs at a rate normalized to zero. Damaged banks, unless recapitalized at $t = 1$, face a significant probability of failing at $t = 2$. Such a failure causes some systemic costs that they do not internalize. Damaged banks can prevent their failure through a recapitalization at $t = 1$.

A *private recapitalization* involves the dilution of the pre-existing equity. It has a net cost c to the initial owners (relative to the non-recapitalization benchmark). This cost reflects an implicit positive transfer to preexisting debtholders, since levered banks give up on a part of their "Merton put" on risky bank assets (see Appendix 1 for a microfoundation).[5]

A *public recapitalization* results from a supervisory intervention. Importantly, we assume

[5] Shareholders' reluctance to recapitalize a levered firm has its roots in long recognized conflicts of interest between shareholders and debtholders (Jensen and Meckling, 1976; Myers, 1977); see Admati et al. (2018) for an interesting restatement.

that it implies the dilution of the pre-existing equity to the extent needed for it to be as financially costly to the initial shareholders as a private recapitalization. In addition, a public recapitalization involves an extra cost $\Delta > 0$ to which we will briefly refer as initial shareholders' (lost) *control rents* (as, e.g., in Grossman and Hart, 1988). This cost includes not only a differential reduction in control rents following a public recapitalization but also losses associated with additional constraints on risk taking or the imposition of a punitive degree of dilution of pre-existing equity. Thus, the overall cost of a public recapitalization to the bank owners is $c + \Delta$, where Δ works as their punishment for refusing to privately recapitalize.

2.2 The supervisor

At $t = 0$, aware of the solvency shock, the supervisor identifies the damaged banks through some supervisory review process or stress test exercise. To prevent the failure of these banks, the supervisor can ask, but not force, each of them to privately recapitalize.

At $t = 1$ each damaged bank can choose to comply ($r = 1$) or not ($r = 0$) with this request. We denote as m the measure of damaged banks that comply. Compliant banks cause a zero net cost to the supervisor.

If a damaged bank refuses to privately recapitalize, the supervisor can still prevent its failure in the bad state by undertaking a public recapitalization. A public recapitalization implies a net cost to the supervisor equal to $\tau_0 + (\tau_1/2)n$ per intervened bank, where n is the overall measure of punished banks, and $\tau_0 > 0$ and $\tau_1 \geq 0$ are parameters. This cost captures both the excess cost of public funding (e.g., if advancing the funds involved in the recapitalization implies a rise in distortionary taxation or the issuance of expensive government debt at $t = 1$) and the political and reputational costs associated with the intervention (including personal losses of a potentially captured supervisor). Thus, the supervisor's overall *early intervention cost* associated with intervening a measure n of banks at $t = 1$ is $\tau_0 n + (\tau_1/2)n^2$.

The alternative course of action on a damaged bank that refuses to privately recapitalize is forbearance. Under this alternative, the bank is likely to fail at $t = 2$ and its owners are left unpunished at $t = 1$. The supervisor then suffers an expected cost of bank failure equal to $\lambda_0 + (\lambda_1/2)(\phi - m - n)$ per forborne bank, where $\lambda_0 > 0$ and $\lambda_1 > 0$ are parameters. Thus, forbearance produces an overall *systemic cost* $\lambda_0(\phi - m - n) + (\lambda_1/2)(\phi - m - n)^2$ whose convexity reflects that the marginal systemic cost associated with bank failure at $t = 2$ is increasing in the measure of failing banks.

The supervisor decides the measure $n \in [0, \phi - m]$ of banks to intervene at $t = 1$ so as to minimize the sum of the expected early intervention cost and the expected systemic cost of its decision. For a microfoundation of the supervisor's payoff functions introduced above, see also Appendix 1.

2.3 Sequence of events

The sequence of events is the following:

- At $t = 0$, the supervisor identifies the mass ϕ of damaged banks and calls them to recapitalize.
- At $t = 1$, there are two stages:
  - Stage 1. Damaged banks simultaneously decide whether to comply ($r = 1$) or not ($r = 0$) with the request; the resulting measure of compliant banks is m.
  - Stage 2. The supervisor intervenes a measure n of the mass $\phi - m$ of non-compliant banks; a measure $\phi - m - n$ of banks are forborne.
- At $t = 2$, aggregate uncertainty realizes; in some bad state the forborne banks fail, causing systemic costs.

The following table summarizes the variables and payoffs relevant for the damaged banks and the supervisor in the sequential game played at $t = 1$.

Table 1. Key variables and payoffs of the game

Bank-level outcome	Affected mass of banks	Per bank cost to bank owners	Overall cost to supervisor
Comply, no intervention	m	c	0
Not comply, intervention	n	$c + \Delta$	$\tau_0 n + (\tau_1/2)n^2$
Not comply, forbearance	$\phi - m - n$	0	$\lambda_0(\phi - m - n) + (\lambda_1/2)(\phi - m - n)^2$

The following assumption allows us to focus the discussion on the most interesting case in which the supervisor finds it worthy to intervene some non-compliant banks but not all of them.

Assumption 1. $\lambda_0 < \tau_0 < \lambda_0 + \lambda_1\phi$.

Intuitively, if non-compliant banks anticipated that they would not be intervened, no bank would comply and the mass of potentially failing banks would be ϕ. But having $\tau_0 < \lambda_0+\lambda_1\phi$ guarantees that in such a situation it would pay the supervisor to intervene some banks since that marginal cost of intervening one bank is τ_0 and the saved marginal cost of letting it undercapitalized is $\lambda_0 + \lambda_1\phi$. As it will become clear below, if this part of Assumption 1 were relaxed, the unique subgame perfect Nash equilibrium (SPNE) of the game between the damaged banks and the supervisor would involve no compliance ($m = 0$) and full forbearance ($n = 0$).

Conversely, $\lambda_0 < \tau_0$ rules out the possibility of having a SPNE in which all banks comply under the implicit threat that any deviant would be intervened. Intuitively, this is so because when all other banks comply, the marginal systemic cost associated with having a deviant, λ_0, would not be enough to justify incurring the marginal cost of its early intervention, τ_0. In Appendix 2 we discuss the case in which this part of Assumption 1 does not hold (as it is still possible that the full compliance equilibrium coexists with equilibria with partial or even no compliance).

2.4 No active supervision and full-commitment benchmarks

It is trivial to see that in the absence of a supervisor being able to act on damaged banks, no damaged bank would privately recapitalize. In other words, damaged banks would simply gamble for survival, remaining exposed to failure at $t = 2$, and bearing a cost 0 (rather than the cost c of a private recapitalization). The supervisor's cost in this situation ($m = n = 0$) would be $\lambda_0\phi + (\lambda_1/2)\phi^2$. This benchmark provides a lower bound to the cost incurred by banks and an upper bound to the cost incurred by the supervisor in the baseline game.

Symmetrically, as further explained in Section 6, a supervisor ex ante committed to publicly recapitalize any non-compliant damaged bank would induce full compliance. Then the supervisor would incur no cost, while bank owners would incur a cost c at each damaged bank.

3 Analysis of the baseline supervisory game

This section first characterizes the equilibrium of the sequential game played between the damaged banks and the supervisor without commitment. Then we discuss the comparative statics of such equilibrium and explore a reparameterization of the model that allows us to account for the (non-monotonic) effects of banks' leverage on capital forbearance.

3.1 Equilibrium

The game played by the damaged banks and the supervisor at $t = 1$ can be solved by backward induction.

In the second stage, the supervisor decides on the measure of banks to publicly recapitalize n after having observed some mass of privately recapitalizing banks m in the first stage. The supervisor's reaction function is given by

$$N(m) = \arg \min_{0 \leq n \leq \phi - m} \tau_0 n + (\tau_1/2)n^2 + \lambda_0(\phi - m - n) + (\lambda_1/2)(\phi - m - n)^2. \tag{1}$$

Solving the first order condition of the implied minimization and taking into account that $N(m)$ must be non-negative, the supervisor's reaction function can be written as

$$N(m) = \max\left\{\frac{\lambda_1(\phi - m) - (\tau_0 - \lambda_0)}{\lambda_1 + \tau_1}, 0\right\}, \tag{2}$$

which is piece-wise linear as depicted in Figure 1. Specifically, $N(m)$ is positive and lower than ϕ at $m = 0$ (by the second inequality in Assumption 1) and decreasing until it reaches a value of zero at $m = \phi - (\tau_0 - \lambda_0)/\lambda_1 < \phi$ (by the first inequality in Assumption 1).

In the first stage of the game, banks simultaneously decide whether to comply ($r = 1$) or not ($r = 0$). Given some expectation about the value of m, a damaged bank would be indifferent between the two alternatives if and only if

$$c = \frac{N(m)}{\phi - m}(c + \Delta), \tag{3}$$

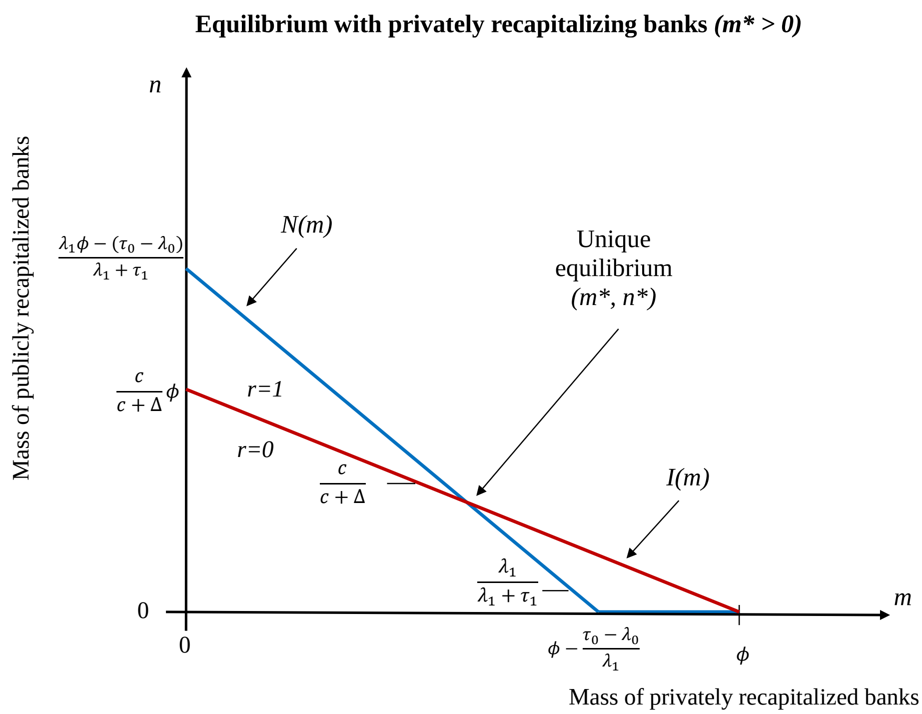

Figure 1: Equilibrium with privately recapitalizing banks

where the left hand side (LHS) is bank owners' cost of compliance, c, and the right hand side (RHS) is the product of the bank's probability of being publicly recapitalized, $N(m)/(\phi-m)$, and bank owners' cost of such intervention, $(c+\Delta)$. Importantly, for $m < \phi - (\tau_0 - \lambda_0)/\lambda_1$, we have, using (2),

$$d\left(\frac{N(m)}{\phi - m}\right)/dm = -\frac{\tau_0 - \lambda_0}{(\lambda_1 + \tau_1)(\phi - m)^2} < 0,$$

which means that the public recapitalization threat hanging on a marginal non-compliant bank weakens as more banks comply. This leads to the following result.

Lemma 1. *Banks' private recapitalization decisions are strategic substitutes over the range in which the public recapitalization threat is not zero (that is, for $m < \phi - (\tau_0 - \lambda_0)/\lambda_1$).*

In such range, if more banks recapitalize privately, the supervisor reduces the public recapitalization threat so banks face a higher chance of benefiting from supervisory forbearance. This boosts a marginal bank's payoff from non-compliance and discourages it from privately recapitalizing, explaining the strategic substitutability between banks' decisions.

To depict banks' indifference condition in the same space as the supervisor's reaction function in Figure 1, define

$$I(m) = \frac{c}{c + \Delta}(\phi - m) \tag{4}$$

as the solution of (3) in n. Then, for any $m < \phi$, a damaged bank strictly prefers to comply for $n > I(m)$, not to comply for $n < I(m)$, and is indifferent between the two alternatives for $n = I(m)$. This explains the second line depicted in Figure 1.[6]

Given the strategic substitutability between damaged banks' decisions to comply, the symmetric equilibrium of the game may involve the use of mixed strategies by the banks in the first stage (and a probabilistic threat of public recapitalization on the non-complying banks in the second stage). The following proposition describes the unique symmetric subgame perfect Nash equilibrium (SPNE) of the game.

Proposition 1. *Under Assumption 1, the game between the damaged banks and the supervisor has a unique symmetric SPNE with* $(m, n) = (m^*, n^*)$*, where*

$$(m^*, n^*) = \begin{cases} \left(\phi - \frac{(\tau_0-\lambda_0)(c+\Delta)}{\lambda_1\Delta-\tau_1 c}, \frac{(\tau_0-\lambda_0)c}{\lambda_1\Delta-\tau_1 c}\right), & \text{if } \left(\frac{\lambda_1}{\lambda_1+\tau_1} - \frac{c}{c+\Delta}\right)\phi - \frac{\tau_0-\lambda_0}{\lambda_1+\tau_1} > 0, \\ \\ \left(0, \frac{\lambda_1\phi-(\tau_0-\lambda_0)}{\lambda_1+\tau_1}\right), & \text{if } \left(\frac{\lambda_1}{\lambda_1+\tau_1} - \frac{c}{c+\Delta}\right)\phi - \frac{\tau_0-\lambda_0}{\lambda_1+\tau_1} \leq 0. \end{cases} \tag{5}$$

Depending on the importance of systemic costs relative to private recapitalization and early intervention costs, the equilibrium may feature some private recapitalizations, $m^* > 0$, as in Figure 1, or no private recapitalizations at all, $m^* = 0$.[7]

The emergence of the regime with positive bank compliance ($m^* > 0$) occurs when banks face a relatively low private cost of compliance and there is a sufficiently high threat of intervention. Banks' cost of compliance are low relative to the cost of risking an intervention whenever bank owners' control rents (Δ) are large and the private recapitalization cost (c) is small. The threat of intervention is high when the supervisor finds early intervention not

[6] We present a case with $I(0) = c/(c + \Delta) < N(0) = (\lambda_1\phi - (\tau_0 - \lambda_0))/(\lambda_1 + \tau_1)$, so that the two curves intersect once on the downward sloping section of $N(m)$. Note that $I(m)$ is downward sloping, with $I(0) = c\phi/(c + \Delta) < \phi$ and $I(\phi) = 0$. Importantly, the point $(\phi, 0)$ does not belong to the indifference line because, for $n = 0$, (3) cannot hold for any m. So $I(m)$ and $N(m)$ do not further intersect at $(\phi, 0)$.

[7] This second regime corresponds to the situation in which $I(0) \geq N(0)$ so that there is no intersection between (the negatively sloped section of) the lines $N(m)$ and $I(m)$.

so costly (τ_1 is low), but the systemic costs of bank failure are sufficiently high (λ_1 is high) and there is a large mass of damaged banks (ϕ is large).[8]

The following corollary summarizes the implications of Proposition 1 for the mass of forborne banks.

Corollary 1. *The level of forbearance implied by the unique symmetric SPNE of the game between the damaged banks and the supervisor is given by*

$$\phi - m^* - n^* = \begin{cases} \frac{(\tau_0-\lambda_0)\Delta}{\lambda_1\Delta-\tau_1 c}, & \textit{if } \left(\frac{\lambda_1}{\lambda_1+\tau_1} - \frac{c}{c+\Delta}\right)\phi - \frac{\tau_0-\lambda_0}{\lambda_1+\tau_1} > 0. \\ \frac{\tau_0-\lambda_0+\tau_1\phi}{\lambda_1+\tau_1}, & \textit{if } \left(\frac{\lambda_1}{\lambda_1+\tau_1} - \frac{c}{c+\Delta}\right)\phi - \frac{\tau_0-\lambda_0}{\lambda_1+\tau_1} \leq 0, \end{cases} \tag{6}$$

In the regime with strictly positive bank compliance ($m^* > 0$), the public recapitalization threat (as measured by $N(m^*)/(\phi - m^*)$) does not change with the mass of damaged banks ϕ. In other words, the rise in ϕ is accommodated with an equal rise in the measure of privately recapitalized banks n^*. These outcomes rely on the implicit (and credible) threat that, if the mass of privately recapitalized banks grew less than one-to-one with ϕ, the supervisor would intervene with greater intensity and banks would strictly prefer to comply than to be exposed to the risk of intervention.

In the regime where no bank complies, the mass of intervened banks grows, but less than one-by-one, with ϕ. The rise in the marginal cost of early intervention discourages the supervisor from offsetting the increase in the mass of damaged banks and forbearance rises with ϕ.

3.2 Comparative statics

This section discusses how changes in the various model parameters affect the equilibrium described above. For brevity, we focus on the most interesting regime in which bank compliance m^* is strictly positive.[9] The following proposition summarizes the results regarding

[8]In fact, if $\lambda_1\phi/(\lambda_1+\tau_1) - (c/(c+\Delta)) \leq 0$, then the regime with $m^* = 0$ prevails irrespectively of the value of ϕ. Otherwise, there is always a large enough value of ϕ above which the equilibrium features $m^* > 0$.

[9]In the regime with $m^* = 0$, the mass of publicly recapitalized banks and the mass of forborne banks respond exclusively to the costs faced by the supervisor, as shown in the corresponding parts of (1) and (6). The supervisor recapitalizes more (forbears less) when its early intervention cost (positively associated with parameters τ_0 and τ_1) is lower, the systemic costs of leaving banks undercapitalized (λ_0 and λ_1) are higher, and the mass of damaged banks (ϕ) is larger.

the impact of model parameters on public recapitalizations (n^*) and capital forbearance ($\phi - m^* - n^*$). Its proof provides also details on the impact of the parameters on n^* and the condition required for having an equilibrium with $m^* > 0$.

Proposition 2. *In the regime with a strictly positive mass of privately recapitalizing banks, the mass of publicly recapitalized banks and capital forbearance increase with the supervisor's intervention costs (τ_0 and τ_1) and with banks' private recapitalization costs (c), and decrease with the systemic cost of bank failure (λ_0 and λ_1) and bank owners' control rents (Δ).*

The most intriguing of these results are the effects of the supervisor's intervention cost parameters on the equilibrium level of supervisory intervention. A supervisor whose interventions are more costly imposes a weaker recapitalization threat on damaged banks, thus discouraging them from complying, and, perhaps paradoxically, ends up intervening more banks. However, this increase in public interventions is not enough to compensate for the lower level of bank compliance so, in the end, such a supervisor exhibits higher capital forbearance.[10]

The next proposition uncovers additional results whereby the effects of intervention costs on public recapitalization and forbearance are reinforced when banks' private recapitalization costs increase.

Proposition 3. *In the regime with a strictly positive mass of privately recapitalizing banks, a higher private recapitalization cost c reinforces the effect of the costs of early supervisory intervention on public recapitalizations and bank capital forbearance. Formally, the cross derivatives of both n^* and $\phi - m^* - n^*$ with respect to either τ_0 and c or τ_1 and c, are all positive.*

These reinforcement effects mean that, ceteris paribus, in a situation in which the costs of supervisory intervention are higher (e.g. because of having supervisors with stronger political biases or more prone to supervisory capture), the same increase in banks' private recapitalization cost (e.g., because of a poor legal protection of investors' rights, a less

[10]Intuitively, at the prior level of bank compliance m^*, the intervention threat $N(m^*)$ would be too low to induce compliance. Restoring banks' indifference requires a lower level of compliance m^* so that the supervisor responds with a higher n^*.

developed market for seasoned equity offerings, or greater reluctance to give up the valuable Merton put associated with limited liability) would end up producing a higher rise in public recapitalizations and bank capital forbearance. On the positive side, this result points to the complementarity between reforms that reduce the social cost of public interventions (e.g., proper recovery and resolution legislation) and those that reduce the costs of private recapilizations (e.g., promoting the use of contingent convertible bonds).

3.3 The effects of bank leverage

In order to analyze the effects of bank leverage, we explicitly relate it to the size of the capital deficits d that damaged banks would have to cover at $t = 1$ to avoid default at $t = 2$. According to this logic, we reparameterize the cost of privately recapitalizing a bank, making it not just c but cd, so that it rises proportionally with the bank's leverage or capital deficit d. Likewise, we make the overall cost of publicly recapitalizing a measure n of banks a quadratic function of not just n but of the capital deficits dn that it has to cover: $\tau_0 dn + (\tau_1/2)(dn)^2$. Finally, we make the systemic costs associated with the likely failure of undercapitalized banks at $t = 2$ a quadratic function of not just $\phi - m - n$ but of the overall shortfall $d(\phi - m - n)$ featured by those banks: $\lambda_0 d(\phi - m - n) + (\lambda_1/2)[d(\phi - m - n)]^2$.

In this extended setup, we can easily reproduce prior results regarding the equilibrium of the game played between the damaged banks and the supervisor. Akin to Proposition 1, the unique SPNE of the game with leverage d is:

$$(m_d^*, n_d^*) = \begin{cases} \left(\phi - \frac{(\tau_0-\lambda_0)(cd+\Delta)}{(\lambda_1\Delta-\tau_1 cd)d}, \frac{(\tau_0-\lambda_0)c}{\lambda_1\Delta-\tau_1 cd}\right), & \text{if } \left(\frac{\lambda_1}{\lambda_1+\tau_1} - \frac{cd}{cd+\Delta}\right)\phi - \frac{(\tau_0-\lambda_0)}{(\lambda_1+\tau_1)d} > 0, \\ \left(0, \frac{\lambda_1\phi d-(\tau_0-\lambda_0)}{(\lambda_1+\tau_1)d}\right), & \text{if } \left(\frac{\lambda_1}{\lambda_1+\tau_1} - \frac{cd}{cd+\Delta}\right)\phi - \frac{(\tau_0-\lambda_0)}{(\lambda_1+\tau_1)d} \leq 0. \end{cases} \tag{7}$$

These expressions allow to examine the impact of leverage on its outcomes, uncovering an interesting non-monotonic effect on capital forbearance. The following proposition summarizes the result.

Proposition 4. *In the regime with a strictly positive mass of privately recapitalizing banks, capital forbearance is U-shaped related to bank leverage, with a minimum at* $\bar{d} = \lambda_1\Delta/(2\tau_1 c)$.

By increasing the cost of a private recapitalization, leverage d reduces banks' incentives to comply. This is a direct effect of leverage on bank compliance. At the same time, leverage increases the systemic cost of leaving banks undercapitalized and, thus, increases the supervisor's propensity to intervene. As a result, public recapitalizations n_d^* rise. This produces an indirect positive effect on banks' compliance, so the overall effect on bank compliance m_d^* is, at first sight, ambiguous. However, in the proof of the proposition we show that the direct (indirect) effect dominates when d is above (below) some threshold, producing a hump-shaped relationship between d and m_d^*. Once m_d^* declines with d, a point is reached in which the rise in n_d^* is not enough to avoid the increase in forbearance. The overall pattern is then that capital forbearance first decreases and then increases with bank leverage.

Quite intuitively, the threshold $\bar{d}$ above which bank leverage increases forbearance is higher whenever bank owners' cost of a public intervention (Δ) and the systemic costs of bank failure (λ_1) are larger, since those encourage directly or indirectly bank compliance. By the same token, the threshold declines with banks' cost of a private recapitalization (c) and the supervisor's cost of intervention (τ_1), which discourage compliance.

4 Adding a limit to supervisory capacity

In this section, we analyze the important implications of introducing an upper limit to the mass of banks that the supervisor is able to early recapitalize. Such constraint might arise for two main reasons.

First, bank interventions require funding, most frequently in the form of an increase in the level of debt of a supervisory agency, if not directly the government. If the authorities in charge have a weak reputation regarding their commitment or capability to intervene at the required level (North, 1993) or the economic climate surrounding the banking crisis is sufficiently adverse, investors may fear that the devised intervention will not preclude the possibility of a fully fledged bank and sovereign crisis in the near future. Afraid of that, they may limit their funding to the agency in charge of bank intervention to an amount that only allows the recapitalization of a mass n_0 of troubled banks.[11]

A second, related possibility is that, if investors are not as well informed as the supervisor

[11] If, on the contrary, institutions are strong, the devised intervention is credible and/or government's financing capacity is plentiful, authorities may face no constraint to their intervention capacity.

on banks' capital needs, the size of the supervisory interventions may signal the severity of a banking crisis (e.g. the size of ϕ in our baseline model). Then it might happen that exceeding a critical level of intervention raises doubts on, e.g., the sufficiency of the deposit insurance fund (or the capacity of the supporting government to repay the insured deposits in full).[12] This might trigger a bank run, in spite of the existence of deposit insurance (Dang et al., 2017). So n_0 can also be interpreted as the maximum level of early supervisory intervention compatible with not triggering a run at $t = 1$.

4.1 The too-many-to-recapitalize problem

If the supervisor's capacity to publicly recapitalize banks at $t = 1$ is limited to a maximum mass of n_0 banks, its reaction function becomes

$$N_0(m) = \min\{N(m), n_0\}, \tag{8}$$

where $N(m)$ is the unconstrained reaction function defined in (2). For parameter values and values of m low enough to make $N(m) > n_0$, the constrained reaction function $N_0(m)$ is no longer sensitive to m. This makes the public recapitalization threat, represented by the probability $n/(\phi - m) = N_0(m)/(\phi - m)$, to be overall increasing (rather than decreasing) in m. Then banks' private recapitalization decisions become *strategic complements* over such range, opening the possibility of having multiple equilibria.

Figure 2 depicts a situation in which the baseline game (without a constrained supervisor) features an equilibrium (m^*, n^*) with $m^* > 0$ and in which the newly added capacity constraint n_0 takes a value in the interval $(n^*, I(0))$. In this situation, the interior equilibrium discussed in reference to Figure 1 coexists with two other SPNE: (i) a corner equilibrium with $m = 0$ and $n = n_0$, and (ii) a second interior equilibrium with $m = I^{-1}(n_0)$ and $n = n_0$. Importantly, the second interior equilibrium, opposite to the other two, is not stable.[13]

[12] This problem can be especially pronounced in banking jurisdictions in which the relevant agencies operate without a credible fiscal backstop.

[13] The second interior equilibrium is not stable in the sense that it is not robust to having an excess mass $\varepsilon \to 0$ of damaged banks arbitrarily deviating to either $r = 0$ or $r = 1$. Shall that happen, all other damaged banks would want to deviate to $r = 0$ or $r = 1$, respectively (evidencing the strategic complementarity between banks' decisions in the neighborhood of this equilibrium). This suggests that the play of the game would converge to the (stable) equilibria with $m = 0$ or $m = m^*$, respectively.

Multiplicity of equilibria with too-many-to-recapitalize banks

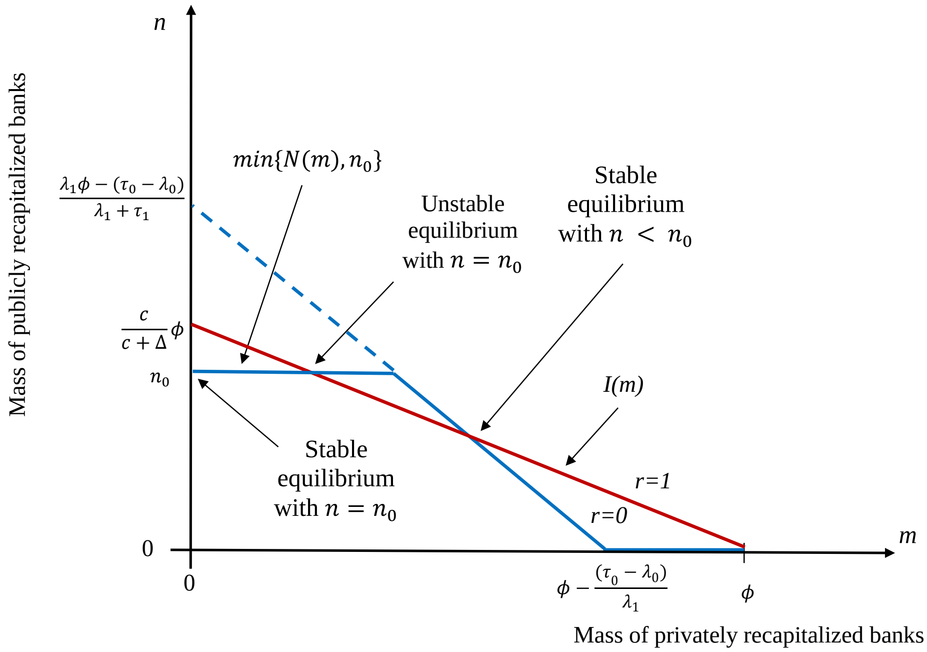

Figure 2: Multiplicity of equilibria with too-many-to-recapitalize banks

Using Figure 2 as a reference, it is easy to infer the various equilibrium configurations that may emerge depending on the size of n_0 and the equilibrium of the unconstrained game. The following proposition summarizes the conditions under which the second equilibrium with extreme forbearance arises.[14]

Proposition 5. *Under circumstances that would otherwise allow to sustain an equilibrium with $m^* > 0$ privately recapitalizing banks, the presence of a constraint $n_0 < I(0) = c\phi/(c+\Delta)$ to the supervisor's public recapitalization capacity implies the existence of an equilibrium with extreme forbearance. Over the range $n_0 \in [N(m^*), I(0))$, with*

$$N(m^*) = \frac{(\tau_0 - \lambda_0)c}{\lambda_1\Delta - \tau_1 c} > 0, \tag{9}$$

the extreme forbearance equilibrium coexists with the equilibrium of the unconstrained game. For $n_0 \in [0, N(m^*))$, only the equilibrium with extreme forbearance exists. The multiple

[14]For completeness, the proof of the proposition also discusses the case in which the equilibrium of the unconstrained game features a zero mass of privately recapitalizing banks.

equilibria range is increasing in the mass of damaged banks ϕ.

Thus, the emergence of a too-many-to-recapitalize problem may turn banks' private recapitalization decisions strategic complements, giving rise to the possibility of a pure strategy equilibrium with no private recapitalizations (and extreme forbearance). For intermediate values of n_0, such equilibrium coexists with the mixed strategy equilibrium of the baseline setup. Intuitively, the larger the mass of damaged banks ϕ, the lower the value of n_0 for which banks' coordination in non-compliance can lead to the exhaustion of intervention capacity and, hence, to the existence of the equilibrium with extreme forbearance.

Similarly to what happens in the unique equilibrium of the baseline unconstrained game when $m^* > 0$ (Proposition 3), private and public recapitalization costs reinforce each other in expanding the range of parameter values for which the equilibrium with extreme forbearance exists. The following proposition states the result.

Proposition 6. *Under circumstances that would otherwise allow to sustain an equilibrium with $m^* > 0$ privately recapitalizing banks, the range of values of the supervisor's capacity constraint n_0 for which only the equilibrium with extreme forbearance exists is increasing in the private recapitalization cost (c) and the supervisor's cost of intervention (τ_0 and τ_1). Moreover, the effects of these two costs on the length of such range reinforce each other.*

4.2 Bank leverage and the too-many-to-recapitalize problem

As in Subsection 3.3, we can reparameterize the model to explicitly consider the effect of bank leverage on the too-many-to-recapitalize problem. As there, we postulate that bank leverage determines the size of the capital deficits d that damaged banks would have to cover by recapitalizing at $t = 1$ and, if not doing it, of the shortfalls emerging at $t = 2$ if they default. The capital deficits affect the costs of private and public recapitalizations and the systemic costs of bank failure in the same fashion as in Subsection 3.3. Importantly, now if the maximum funds available for the supervisor to publicly recapitalize banks at $t = 1$ are K_0, the maximum mass of banks that the supervisor will be able to intervene is $n_0 = K_0/d$. Then the supervisor's reaction function becomes

$$N_0(m) = \min\{N(m), K_0/d\}, \tag{10}$$

implying that, when the capacity constraint binds, the measure of intervened banks decreases in troubled banks' capital deficit d.

In this extended setup, we reproduce prior results regarding the equilibria with too-many-to-recapitalize banks. The following proposition summarizes the effect of bank leverage on the instances and level of extreme forbearance.

Proposition 7. *Under circumstances that without a limit on supervisory intervention would allow to sustain an equilibrium with $m_d^* > 0$ privately recapitalizing banks, bank leverage expands the ranges of values of the supervisor's public recapitalization capacity K_0 for which the equilibrium with extreme forbearance exists and increases the level of extreme supervisory forbearance $\phi - K_0/d$.*

As before, for a given intensity of the intervention threat following no compliance, leverage d decreases bank owners' incentives to privately recapitalize. But opposite to the unconstrained case, the constrained supervisor's intervention threat cannot increase with d. Thus, relative to the case where the supervisor's capacity constraint is not binding, the positive indirect effect of d on compliance disappears. When all other banks are not complying (as it happens in the extreme forbearance equilibrium), a bank's incentive to deviate to privately recapitalizing diminishes with d, explaining the expansion of the parameter region where extreme forbearance arises.

On the other hand, when leverage increases, sustaining the equilibrium with a positive level of compliance requires an increase in the equilibrium intervention threat and thus in the mass of publicly recapitalized banks, $N(m^*) = (\tau_0 - \lambda_0)c/(\lambda_1\Delta - \tau_1 cd)$. This directly rises the critical value of K_0 below which the interior equilibrium cannot be sustained and the equilibrium with extreme forbearance is the only SPNE of the supervisory game.

Finally, since the increase in leverage reduces the maximum number of banks that the supervisor is able to publicly recapitalize, K_0/d, higher leverage rises the level of extreme forbearance, $\phi - K_0/d$.

Additionally to its direct predictive implications (extreme forbearance is more likely in economies with highly levered banks), this proposition points to the potential use of ex ante capital regulation (that is, ex ante reductions in d) as a mitigant of the incidence and severity of capital forbearance.

4.3 Implications for the sovereign-bank nexus

The too-many-to-recapitalize problem provides a novel channel linking banks to the strength of public finances, thus speaking to the literature on the nexus between sovereign risk and bank risk. One of the implications of our analysis is that damaged banks operating in a country with a more fiscally constrained government would face a lower threat of supervisory intervention. As a consequence, their incentives to voluntary restore their solvency (via private recapitalization) also weaken, effectively calling the supervisor to intervene more. However, once the supervisor exhausts the available funds, it can no longer respond by intervening more banks, and an equilibrium with extreme lack of bank compliance and extreme supervisory forbearance emerges.

In such equilibrium, public interventions are not enough to compensate for private inaction and the economy remains exposed to experiencing a high bank failure rate and large future systemic costs, with their obvious impact on future government finances. Although the link between banking sector vulnerabilities and the fiscal weakness of the government has been extensively discussed in the literature (Dell'Ariccia et al., 2018), our model provides a new mechanism where the link (and the possibility of catastrophic outcomes during a crisis) does not come from government guarantees or banks' holdings of sovereign debt (Acharya et al., 2014; Brunnermeier et al., 2016; Leonello, 2018) but from the importance of the supervisor's capacity to undertake precautionary recapitalizations at an early stage and the strategic interaction between the supervised banks.

5 Correlated risk taking and serial gambling

In this section we extend the model to introduce an ex ante risk-taking decision e by individual banks at some date $t = -1$ that affects their probability $1 - e$ of being hit by the solvency shock at $t = 0$. We assume that banks can avoid the exposure to such shock by incurring a cost $(\gamma/2)e^2$, with $\gamma > 0$, at $t = 0$. This cost can be interpreted as the resource cost of prudent risk management that prevents the bank from being damaged by the solvency shock or as the cost of passing up risky investments with upside potential that fail when the solvency shock realizes. Under a symmetric choice of e by a unit mass of identical banks operating at $t = -1$, the measure of banks that will be damaged at $t = 0$ is $\phi = 1 - e$.

Importantly, we assume that the size of the capital deficit suffered by damaged banks at $t = 0$ is increasing in the mass of damaged banks, $d = D(\phi)$, with $D'(\phi) > 0$. This can be justified as the result of some negative fire sale dynamics or negative impact on aggregate economic activity that increases in the mass of damaged banks and feeds back into the size of their capital shortfalls.

To analyze banks' choice of e at $t = -1$ and its subsequent impact on the play of the supervisory game, we can proceed by backwards induction. So the previous sections of the paper provide valid characterizations of the subgame that starts at $t = 0$, for any possible ϕ emerging from banks' decisions at $t = -1$. Thus, the equilibrium of the continuation game differs significantly across the cases in which (i) the supervisor's maximum intervention capacity does not exist or is not binding and (ii) the supervisor's capacity constraint exists and is binding. Further, in the latter scenario, the most interesting insights emerge when the equilibrium of the unconstrained continuation game involves a strictly positive mass of privately recapitalizing banks, say $m^u > 0$. Therefore, we are going to focus the discussion on such a case.

Results in previous sections have shown that, for exogenous values of ϕ and d, the continuation game can feature the interior equilibrium as the unique SPNE, the extreme forbearance equilibrium as the unique SPNE, and the coexistence of both equilibria. In this section, we endogenize ϕ and d as the result of banks' ex ante risk-taking decision e. This enables us to assess how ex ante gambling (via the choice of a low e) and ex post gambling (abstaining from privately recapitalizing the bank when required to do so) interact and get affected by the credibility of the supervisory intervention threat in the continuation game.

5.1 The unconstrained-supervisor equilibrium

Let us consider first the ex ante choice of e when an unconstrained equilibrium with some $m > 0$ is anticipated in the continuation game. The problem of an individual bank at $t = -1$ is:

$$\min_{e\in[0,1]} \frac{\gamma}{2}e^2 + (1-e)\left[\frac{m}{\phi}cd + \frac{n}{\phi}(cd+\Delta)\right], \tag{11}$$

where the first term is the cost of avoiding risk and the second is the probability of being damaged, $1-e$, times the expected costs incurred by a damaged bank in the continuation supervisory game with $m>0$. Importantly, the equilibrium values of m and n in such game (denoted as just m and n to save on notation) are functions of ϕ and d, as described in equation (7). In turn, ϕ and d, in equilibrium, depend on the equilibrium value of e but the atomistic bank decides on its own e taking all these objects as given.

The first order condition for the bank's optimal choice of e implies

$$e=\frac{1}{\gamma}\left[\frac{m}{\phi}cd+\frac{n}{\phi}(cd+\Delta)\right], \tag{12}$$

and is sufficient for optimality provided that such value of e is contained in the interval $[0,1]$. Clearly, e as determined by (12) is always positive, while guaranteeing that it is lower than 1 requires γ to be large enough, as we henceforth assume. Now, imposing $e=1-\phi$ and $d=D(\phi)$ and using the expressions for m and n in (7), equation (12) leads to the following implicit definition of the candidate unconstrained equilibrium mass of damaged banks:

$$\phi_u=1-\frac{c}{\gamma}D(\phi_u). \tag{13}$$

Assuming $\gamma>cD(0)$ (so that γ is indeed large enough) and given that $D'>0$, equation (13) has a unique solution $\phi_u\in(0,1)$.

Such unconstrained solution involves, using (7) and $d=D(\phi_u)$, a mass of publicly recapitalizing banks

$$n_u=\frac{(\tau_0-\lambda_0)c}{\lambda_1\Delta-\tau_1cD(\phi_u)}, \tag{14}$$

and, hence, can be confirmed as a SPNE of the full game insofar as it is compatible with the supervisors' capacity constraint, that is, $n_uD(\phi_u)\leq K_0$, which using (14) can be written as

$$K_0\geq\frac{(\tau_0-\lambda_0)cD(\phi_u)}{\lambda_1\Delta-\tau_1cD(\phi_u)}\equiv K_u, \tag{15}$$

where the RHS is increasing in ϕ_u. So, quite intuitively, the existence of an unconstrained equilibrium of the full game is more likely if the supervisor's capacity to intervene K_0 is large relative the unconstrained fraction of damaged banks ϕ_u (which is independent of K_0).

From (13), the unconstrained equilibrium mass of damaged banks ϕ_u is only affected by the private recapitalization cost c (which reduces it), the cost of avoiding risk γ (which increases it), and the relationship $D(\phi)$. An upward shift in $D(\phi)$ would reduce ϕ_u as banks would try to avoid the larger expected recapitalization costs incurred if they get damaged.

The existence of the unconstrained equilibrium, however, depends also on parameters related with the supervisor's capacity and incentives to intervene. From (15), such existence gets compromised, given ϕ_u, if the intervention capacity K_0 decreases, the intervention cost parameters τ_0 or τ_1 increase, the systemic cost parameter λ_1 decreases (as it makes the supervisor less prone to intervene) or the control rents Δ that bankers lose in case of intervention decrease.

5.2 The constrained-supervisor equilibrium

Consider next the case in which the continuation supervisory game features a constrained supervisor and extreme forbearance.

In such a situation, some pre-determined mass of damaged banks ϕ_c and the implied capital deficit per bank $D(\phi_c)$, determine a mass $n^c = K_0/D(\phi_c)$ of intervened banks such that the supervisor's intervention capacity gets exhausted. Additionally, as implied by arguments in the proof of Proposition 7, sustaining extreme forbearance would also require having

$$K_0 \leq \frac{c\,(D(\phi_c))^2}{cD(\phi_c)+\Delta}\phi_c \tag{16}$$

so that damaged banks have no incentives to privately recapitalize (in terms of Figure 2 this condition is equivalent to the flat portion of the supervisor's reaction function to be below the intercept of banks' indifference condition at the vertical axis).

Assuming for the time being that these conditions are satisfied, the problem of an individual bank at $t=-1$ and, hence, its first order condition are exactly the same as in (11) and (12) but with $m=0$, $n=K_0/D(\phi_c)$, and $d=D(\phi_c)$. So the first order condition leads to

$$e = \frac{1}{\gamma}\frac{K_0}{\phi_c D(\phi_c)}\left(cD(\phi_c)+\Delta\right) \tag{17}$$

and, after imposing $e = 1-\phi_c$, makes the candidate equilibrium value(s) of ϕ_c be given by the solutions to

$$\gamma\phi_c(1-\phi_c) = cK_0 + \frac{K_0\Delta}{D(\phi_c)}. \tag{18}$$

The LHS of this equation is a concave parabola in ϕ_c that takes value zero at both $\phi_c = 0$ and $\phi_c = 1$, and reaches a maximum value of $\gamma/4$ at $\phi_c = 1/2$. The RHS is decreasing in ϕ_c and increasing in K_0. Assuming, for instance, $cK_0 + K_0\Delta/D(0) \leq \gamma/4$ (which again only requires γ to be large enough relative to K_0), one can guarantee the existence of two solutions for (18), $\underline{\phi}$ and $\overline{\phi}$, with $0 < \underline{\phi} < 1/2 < \overline{\phi} < 1$. That is, a candidate extreme forbearance solution with moderate risk taking, $\phi_c = \underline{\phi}$, and a candidate extreme forbearance solution with extreme risk taking, $\phi_c = \overline{\phi}$.

For brevity, we focus the discussion on the existence of the equilibrium with extreme risk taking. From (16), the existence of this equilibrium requires

$$K_0 \leq \frac{c\left(D(\overline{\phi})\right)^2}{cD(\overline{\phi}) + \Delta}\overline{\phi}. \tag{19}$$

However, by the aforementioned properties of (18), the candidate equilibrium mass of damaged banks $\overline{\phi} = \Phi(K_0)$ is decreasing in K_0, with $\Phi(0) = 1$. Together with $D' > 0$, this implies that the RHS of (19) is strictly positive at $K_0 = 0$ and decreasing in K_0 up to some critical supervisory capacity threshold K_c, in which the solution $\overline{\phi} = \Phi(K_0)$ to (18) ceases to exit. So, for $K_0 \in [0, K_c]$, for some $K_c > 0$, the equilibrium with extreme risk taking exists.[15] The equilibrium with extreme risk taking exists (and involves $\overline{\phi} = 1$) for $K_0 = 0$ and, by continuity, it also exists (involving $\overline{\phi} < 1$) for a range $(0, K_c]$ of strictly positive values of K_0.

Intuitively, the prospect of extreme forbearance (which is sustained on the basis of the strategic complementarity between damaged banks' lack of compliance when they anticipate that the supervisor's intervention capacity will be binding) undermines banks' ex ante incentives to avoid being damaged. As the supervisor's capacity to intervene declines, banks' ex ante incentives worsen and the implied mass of damaged banks ϕ increases. This, in turn, increases the severity of the capital shortfalls $D(\phi)$ experienced by the damaged banks and effectively tightens the supervisor's capacity constraint even more, contributing further to the sustainability of the equilibrium with extreme forbearance.

[15] K^c is upper bounded by the value of the RHS of (19) at $\overline{\phi} = 1/2$.

5.3 Coexistence of equilibria in the full game

The prior analysis implies that the low forbearance unconstrained equilibrium with an interior mass ϕ_u of damaged banks (independent of K_0) exists when the supervisor's intervention capacity K_0 is large enough. Furthermore, the extreme forbearance constrained equilibrium with an interior mass $\overline{\phi} = \Phi(K_0)$ of damaged banks (decreasing in K_0) exists when the supervisor's intervention capacity is low enough. The following proposition summarizes these results and establishes two additional findings: (i) both equilibria coexist over some intermediate range of intervention capacity, and (ii) the constrained equilibrium always involves higher ex ante risk taking than the unconstrained one,

Proposition 8. *Consider an extended supervisory game in which banks' ex ante risk taking is endogenous and without a limit on supervisory intervention, an interior equilibrium involving an endogenous mass $\phi_u \in (0,1)$ of damaged banks and a mass $m^u > 0$ of privately recapitalizing banks exists. An unconstrained equilibrium involving $\phi = \phi_u$ and $m = m^u$ exists if and only if $K_0 \geq K_u > 0$. A constrained equilibrium involving $\phi = \Phi(K_0)$ (with $\Phi' < 0$) and $m = 0$ exists if and only if $K_0 \in [0, K_c]$ for some $K_c > K_u$. Moreover, $\Phi(K_c) = \phi_u$, which implies $\Phi(K_0) > \phi_u$ in the coexistence range $K_0 \in [K_u, K_c)$.*

The strategic complementarity between banks' ex ante risk taking decisions in the constrained regime explains the coexistence of the constrained extreme-forbearance equilibrium with the unconstrained moderate-forbearance equilibrium for intermediate values of K_0. In the limit case of $K_0 = K_c$ (as described in Figure 3), the constrained and the unconstrained equilibria coexist and involve the same ex ante risk taking and, thus, the same mass ϕ_u of damaged banks.[16] The reason for the latter is that banks' expected payoffs in the continuation game associated with a mass ϕ_u of damaged banks are identical in the two equilibria.[17] Therefore, they induce exactly the same ex ante risk taking incentives: the ones leading to having a mass ϕ_u of damaged banks.

[16] Figure 3 is the extension of Figure 2 to the case in which the relevant payoffs depend on bank leverage d and such leverege depends on the mass of damaged banks, $d = D(\phi)$. In the figure, we denote $D(\phi_u)$ by D_u.

[17] Both the intersection between the downward slopping sections of $N(m)$ and $I(m)$, which identifies the value of (m, n) in the moderate-forbearance unconstrained equilibrium, and the intersection between the flat portion of $N(m)$ and $I(m)$ with $m = 0$, which identifies the extreme-forbearance constrained equilibrium in this polar case, yield on the indifference curve $I(m)$.

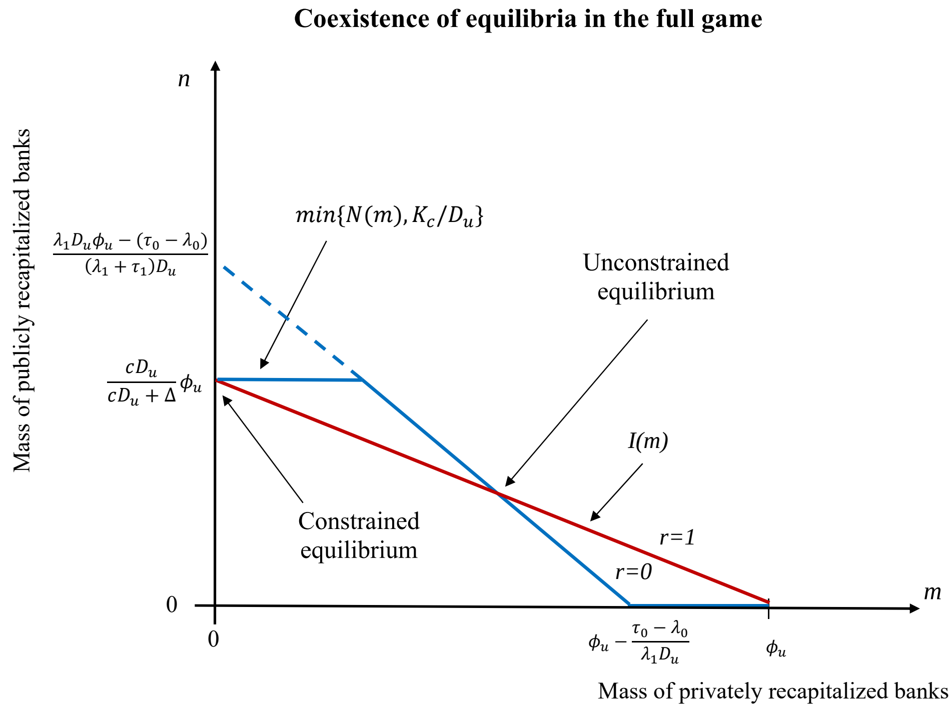

Figure 3: Coexistence of equilibria in the full game

For $K_0 > K_c$, the constrained equilibrium would no longer exist, because the expectation of greater intervention capacity would incentivize banks' to recapitalize. As a result, the only surviving equilibrium of the full game would be the moderate-forbearance unconstrained equilibrium with $\phi = \phi_u$.[18] For $K_0 < K_c$, the constrained equilibrium would still be an equilibrium of the continuation game associated with $\phi = \phi_u$. Banks' payoffs under extreme forbearance would be higher than under the moderate forbearance of the interior equilibrium. In this case, banks anticipate extreme forbearance, and their correlated gambling incentives would increase. Therefore, a mass of damaged banks, $\Phi(K_0)$, is larger than the one featured by the moderate-forbearance equilibrium, ϕ_u.

The coexistence of the low and the extreme forbearance equilibria and higher ex ante risk taking under the latter confirm the interaction between banks' ex ante gambling (higher risk

[18]In such case for the flat portion of the corresponding $N(m)$ would yield above the intercept of the indifference curve. In contrast, for $K_0 < K_c$, the flat portion of $N(m)$ would yield below the intercept of $I(m)$.

taking) and ex post gambling (reluctance to recapitalize). The results reveal that the incentives for both forms of gambling reinforce each other when the credibility of the intervention threat imposed by the supervisor is not large enough.

6 Welfare cost of the lack of supervisory commitment

In this section, we discuss the welfare implication of our results. We define welfare in a utilitarian manner, abstracting from redistributional effects. In our model, the costs incurred by banks' initial owners in case of a private or public recapitalization are redistributive in nature. Those include losses due to the reduction of their net equity-like payoffs (in favor of debtholders or the safety net) when their banks' leverage is reduced and their cash flow and control rights are diluted in favor of new shareholders (possibly including the government). In contrast, the costs taken into account by the supervisor are a mixture of redistributional costs (such as the political and reputational costs of an early intervention) and net deadweight losses (such as the costs of the distortionary taxation necessary to finance an early intervention or the systemic costs of the future failure of undercapitalized banks). Thus welfare in our baseline model can be meaured as

$$W = -(1-\alpha)[\tau_0 n + (\tau_1/2)n^2] - \lambda_0(\phi - m - n) - (\lambda_1/2)(\phi - m - n)^2, \tag{20}$$

where $\alpha \in [0,1]$ is a parameter that accounts for the fraction of the supervisor's early intervention costs that corresponds to political and reputational costs. For $\alpha = 0$, social welfare W is simply equal to the negative of the costs that the supervisor minimizes when deciding on public recapitalizations at $t = 1$. For $\alpha > 0$, the early intervention costs that enter W are smaller than those entering the supervisor's objective function.

6.1 Implementing the first best under commitment

The following proposition characterizes the first best allocation in our baseline supervisory game and the possibility of implementing it if the supervisor can commit to a tough recapitalization rule.

Proposition 9. *If the supervisor could commit to a tough public recapitalization rule, e.g. the rule*

$$\bar{N}(m) = \frac{c+\xi}{c+\Delta}(\phi - m), \tag{21}$$

for any $\xi > 0$, it would uniquely implement the first best allocation in which all banks recapitalize privately ($m = \phi$), no public recapitalizations occur ($n = 0$) and capital forbearance is zero ($\phi - m - n = 0$).

Intuitively, if the public recapitalization rule imposes, for each possible mass of privately recapitalizating banks, a public recapitalization threat strong enough for each individual damaged bank to strictly prefer compliance (recall (3)), then the only Nash equilibrium of the subgame in which banks decide whether to privately recapitalize is one with full compliance, $m = \phi$. The contingency on m of a rule such as the one described in (21) implies $n = 0$ and, trivially, no forbearance, thus pushing welfare W to its maximum possible level of zero.

Of course, the true challenge for welfare maximization in this setup is to overcome the time inconsistency problem that might lead any supervisor or social planner to renege from any such tough rule once at $t = 1$. In theory, writing such rule in a law or charter regulating the behavior of the supervisor (and demanding personal liability to those violating it) might solve the problem. In practice, however, the changing nature of the circumstances in which the supervisor may be called to intervene (or, in model terms, uncertainty about the values of parameters such as ϕ, c, or Δ at the time the law is written) might prevent the usage of rules and leave the supervisor's early intervention decisions subject to discretion.

6.2 Marginal impact of the supervisor's self interest

When the supervisor lacks capacity to commit to a tough public recapitalization rule, the presence of political and reputational costs of early interventions (in other words, having $\alpha > 0$) implies that the supervisor will tend to intervene even less than a benevolent social planner confronted with the same decision would do. The social planner would prefer to impose a tougher public recapitalization threat on banks, leading to an equilibrium with more private recapitalization and less forbearance. Moreover, as stated in the following proposition, in the regime with $m^* > 0$, the equilibrium induced by the planner would also feature less public recapitalizations, thus improving welfare relative to the self-interested supervisor through the reduction in both early intervention costs and systemic costs.

Proposition 10. *Under circumstances that would otherwise allow to sustain an equilibrium with $m^* > 0$ privately recapitalizing banks, if public recapitalizations were decided by a planner that disregards the political and reputational costs of early interventions, private recapitalizations and welfare would increase, while public recapitalizations and forbearance would decrease.*

This result then identifies the marginal distortions due to the supervisor's self-interested concerns about the political and reputational costs of its early interventions. This has implications for the design of the institutions in charge of bank supervision. As in the literature on time inconsistency in the context of, e.g., monetary policy, delegating the relevant decisions to authorities with few political and reputational costs associated with early interventions can be valuable. In fact, our analysis suggests that delegation to authorities attributing lower costs to early public interventions and higher costs to future systemic problems that what an unbiased social planner would do might help. The relative weighting of both costs by the authorities to which the decisions are delegated might be shaped by their own subjective preferences, as well as by the contents of their mandates, their career prospects, and the details of their compensation schemes, identifying an interesting field for future research.[19]

7 Concluding remarks

The core of this paper focuses on the interplay between banks and a supervisor after the latter discovers that a significant mass of banks may turn insolvent unless they get properly recapitalized. Our analysis disentangles the strategic interaction between the banks requested to privately recapitalize and a supervisor with the discretionary power to undertake a public recapitalization (or nationalization) of noncompliant banks. We discuss the determinants of the credibility of the underlying public recapitalization threat and its impact on banks' private recapitalization decisions, the resulting level of supervisory forbearance,

[19]Analyzing the design of an optimal compensation scheme for the supervisor exceeds the scope of this paper since it would require adopting a less reduced-form approach to the elements that enter its payofff function in the current setup.

and its implications for the systemic costs due to bank failure. Importantly, we analyze the influence of leverage and limits to the supervisor's intervention capacity (possibly due to the lack of a fiscal backup) on these outcomes. We also study the interplay between the outcomes of the supervisory game and ex ante risk taking.

The analysis of the model produces a number of testable predictions and novel theoretical and policy insights. First, when supervisors face higher political or reputational costs of early intervention, banks recapitalize less privately and the supervisor ends up recapitalizing more banks publicly. This results in a larger number of undercaptialized banks and therefore higher systemic cost of bank distress.

Second, economies facing a higher cost of public funds or a limit to their fiscal capacity are more exposed to suffer from a negative feedback loop whereby the softer threat of public intervention discourages banks from privately recapitalizing, exhausts the usage of the supervisor's intervention capacity, and rises the resulting forbearance and systemic costs. We uncover a too-many-to-recapitalize problem that can generate a novel dimension of the sovereign-bank nexus. If the prospect of a deep systemic crisis further limits the capacity of the supervisor to early intervene (e.g. via a fiscal channel) and encourages banks to coordinate in high risk, low compliance strategies, the implied self-fulfilling logic can lead to equilibria with extreme levels of forbearance and deeply disturbed government finances.

Third, we identify circumstances in which strategic complementaries between banks' risk taking and their incentives to privately recapitalize, if damaged, produce instances of serial gambling: correlated risk taking followed by undercapitalization and extreme forbearance. Importantly, under a low-credibility intervention threat, banks' ex ante gambling incentives (in the form of correlated risk-taking) and ex post gambling incentives (reluctance to recapitalize) reinforce each other.

Fourth, we predict that the incidence of public recapitalizations of damaged banks will tend to be higher in economies with more highly levered banks, while the overall incidence of capital forbearance will be non-monotonically related to bank leverage: decreasing with it when leverage is small but increasing with it beyond some point.

On the policy side, the comparative statics of the private recapitalization cost and the analysis of the role of leverage suggest that policies reducing the importance of the Merton's put (such as controls on leverage and risk taking by banks) or facilitating the undertaking of leverage reduction transactions (e.g. by reducing the informational and agency frictions behind equity issuance costs) can reduce the need for public intervention on trouble banks and the levels of forbearance. While not explicitly considered in our analysis, the ex ante issuance of securities such as contingent convertible debt (CoCos) that provide for the equivalent to a private recapitalization without the need to resort on the discretionary decision of bank owners (or giving them a cheap alternative to effectively recapitalize their banks) would also help.[20]

Finally, the analysis of the too-many-to-recapitalize problem, the possibility of having a supervisory-based sovereign-bank nexus, and the compounded damage caused by the possibility of "serial gambling" highlight the strong influence of supervisors' commitment and capacity to intervene on equilibrium outcomes. Key elements of an institutional design conducive to better outcomes include providing the supervisor with independence and a clear mandate to avoid future systemic costs, an internal governance that avoids giving excessive weight to the political and reputational costs of intervention, and the access to a credible and sufficiently sizable funding capacity.

[20]When the bank is recapitalized through the conversion of CoCos into equity, the initial owners' stake in the bank may also get diluted, both directly and via the reduction in the Merton's put. However, CoCos might be designed to either not make conversion a discretionary choice of bank owners or imply a lower cost to them than a seasoned equity offering with the same leverage-reduction effect.

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A Appendices

Appendix 1. Microfounding the payoff functions

This appendix provides specific microfoundations for the payoff functions postulated for bank owners and the supervisor in the baseline model. The goal is to show that such reduced-form functions, which help streamline the presentation, can be explicitly connected to a simple structural model in the standard tradition of the banking literature. In particular, the model set up in this appendix is explicit about banks' asset returns and capital structure and, thus, the root of the capital shortfalls that might make banks insolvent in some future states. It is also explicit about the aggregate nature of the risk that can make damaged banks insolvent at the same time and provides a microfoundation for the costs incurred by the supervisor based on the fiscal cost of bank interventions and of honoring deposit guarantees at different dates and states.

Banks' costs

Assume that, absent any private or public recapitalization of damaged banks at $t = 1$, their liabilities at $t = 2$ are fully insured deposits that promise to pay back D, while their assets at that date pay some high return R in a normal state that happens with probability $1-\varepsilon$ and a low return $D-s$, with $s \in (0, D]$, in an adverse state that happens with probability $\varepsilon \in (0, 1)$. From the perspective of $t = 0$, parameter s can then be interpreted as the capital shortfall emerging in a future adverse scenario.

To avoid their insolvency in the adverse state, damaged banks can undertake a recapitalization at $t = 1$ that injects capital s in the bank and leaves it invested in a safe asset with a net return normalized to zero. In a private recapitalization ($r = 1$), the funds s are provided by the bank owners who remain the only residual claimants of the bank and in full control of it (thus extracting some non-pecuniary control rents Δ from it). The net expected payoffs of bank owners (inclusive of control rents) under a private recapitalization are then

$$\Pi_{r=1} = -s + (1 - \varepsilon)(R + s - D) + \Delta,$$

which reflect the fact that, by design, the recapitalization just avoids the insolvency of a damaged bank in the adverse state, making its equity payoffs in such state equal to zero.

If, instead, a damaged bank does not privately recapitalize ($r = 0$) and is not subsequently forced into a public recapitalization ($i = 0$), the net expected payoffs are

$$\Pi_{r=0,i=0} = (1 - \varepsilon)(R - D) + \Delta.$$

The difference

$$\Pi_{r=1} - \Pi_{r=0,i=0} = -\varepsilon s > 0$$

justifies that the cost of a private recapitalization as represented in the baseline model would be $c \equiv \varepsilon s$.

If the damaged bank does not privately recapitalize ($r = 0$) but is subsequently forced into a public recapitalization ($i = 1$), we assume that some outside equity providers (perhaps the supervisor) receive a share α of the equity of the bank in exchange for providing s at

$t = 1$ so as to break even. Since the equity payoffs of the recapitalized bank are only positive (and equal to $R + s - D$) in the normal state, the received share solves

$$\alpha(1-\varepsilon)(R+s-D) = s. \tag{22}$$

In addition, the public intervention implies the dissipation of the control rents Δ so the payoffs of the bank's initial owners become

$$\Pi_{r=0,i=1} = (1-\alpha)(1-\varepsilon)(R+s-D) = (1-\varepsilon)(R-D) - \varepsilon s.$$

Thus the difference

$$\Pi_{r=0,i=1} - \Pi_{r=0,i=0} = -(\varepsilon s + \Delta) > 0,$$

justifies that the cost of a public recapitalization as represented in the baseline model is $c + \Delta \equiv \varepsilon s + \Delta$.

The supervisor's costs

To provide a potential microfoundation for the expressions of the early intervention cost $\tau(n)$ and systemic crisis costs $\lambda(m,n)$ taken into account by the supervisor when deciding on bank intervention, we are going to stick to a purely fiscal interpretation. So we abstract from reputational concerns, political costs, and other social costs associated with financial instability that supervisors might also consider in practice. We assume that the total opportunity costs of public funds faced by the supervisor at a particular date or state are

$$G(x,y) = g_0(x+y) + \frac{g_1}{2}(x+y)^2, \tag{23}$$

where x is some exogenous non-bank-related fiscal burden faced by the supervisor in such date or state and y is the burden imposed by the damaged banks. We also assume that the exogenous fiscal burden is some intermediate x_i at the early intervention date $t = 1$, some low x_l in the normal state at $t = 2$, and some high x_h in the adverse state at $t = 2$, with $x_l < x_i < x_h$.

In an early intervention implying the public recapitalization of n damaged banks, the incurred fiscal cost at $t = 1$ is $y_i = sn$, but in the normal state at $t = 2$ the injected equity yields $\alpha(R + s - D)$, which from the specification in (22), implies a negative fiscal burden $y_l = -sn/(1-\varepsilon)$ in such state and no direct impact on the fiscal burden in the adverse state. On the other hand, leaving a measure $\phi - m - n$ of damaged banks undercapitalized at $t = 1$ implies a contingent deposit insurance liability equal to $y_h = s(\phi - m - n)$ in the adverse state at $t = 2$.

Using (23) and the specified values of y_i, y_l, and y_h, the expected fiscal costs incurred by the supervisor over dates $t = 1$ and $t = 2$ can be expressed as

$$\begin{aligned} C &= k + g_1(x_i - x_l)sn + \frac{2-\varepsilon}{1-\varepsilon}\frac{g_1}{2}s^2n^2 \\ &\quad + \varepsilon(g_0 + g_1x_h)s(\phi - m - n) + \frac{\varepsilon g_1}{2}s^2(\phi - m - n)^2, \end{aligned} \tag{24}$$

where the term k is collects purely non-bank-related cost components. The second and third terms in (24) reflect the net fiscal costs of an early intervention (which, under $x_i > x_l$, are strictly positive for all n) and have the same linear-quadratic structure as postulated in the baseline model. Under this microfoundation we would have

$$\tau_0 \equiv g_1(x_i - x_l)s$$

and

$$\tau_1 \equiv \frac{2-\varepsilon}{1-\varepsilon} g_1 s^2.$$

The fourth and fifth terms in (24) reflect the fiscal costs associated with paying back the deposits of the banks that fail at $t = 2$, which, like the "systemic costs" of the baseline model, are linear-quadratic in the mass $\phi - m - n$ of forborne banks. Under this microfoundation we would have

$$\lambda_0 \equiv \varepsilon(g_0 + g_1 x_h)s$$

and

$$\lambda_1 \equiv \varepsilon g_1 s^2.$$

As discussed in the main body of the paper, the most interesting case in the analysis of the model arises when $\tau_0 > \lambda_0$, in which case the supervisor's reaction function $N(m)$ is piece-wise linear like in Figure 1. From the expressions above, such a case arises if and only if the following condition holds

$$\varepsilon < \frac{g_1(x_i - x_l)}{g_0 + g_1 x_h}, \tag{25}$$

where the right hand side is positive and lower than one. Intuitively, it requires the probability of the adverse state to be low enough relative to the differential fiscal cost of intervening early versus late.

Appendix 2. Relaxing Assumption 1

This appendix provides the solution to our basic model if Assumption 1 is relaxed, that is, when $\lambda_0 \geq \tau_0$. Then the supervisor's reaction function is given by:

$$N(m) = \min\left[\phi - m, \max\left[0, \frac{\lambda_1(\phi - m) + (\lambda_0 - \tau_0)}{\lambda_1 + \tau_1}\right]\right]$$

There are two possible configurations of the supervisor's reaction function. First, if $\lambda_0 - \tau_0 < \tau_1\phi$, the supervisor's reaction function is piece-wise linear. Specifically, $N(m)$ is positive and lower than ϕ at $m = 0$, since $N(m) < \phi - m$ for sufficiently low m. However, we have $N(m) = \phi - m$ for $m \geq \phi - \frac{\lambda_0 - \tau_0}{\tau_1}$.

Banks' decisions to privately recapitalize are strategic complements for low enough m:

$$\frac{\partial\left(\frac{N(m)}{\phi - m}\right)}{\partial m} = \begin{cases} \frac{\lambda_0 - \tau_0}{(\phi - m)^2(\lambda_1 + \tau_1)} > 0, & \text{if } m < \phi - \frac{\lambda_0 - \tau_0}{\tau_1}, \\ 0, & \text{if } m \geq \phi - \frac{\lambda_0 - \tau_0}{\tau_1}. \end{cases}$$

Second, if $\lambda_0 - \tau_0 \geq \tau_1\phi$, the supervisor's reaction function is linear $N(m) = \phi - m$ for all $m \in [0, \phi]$. Therefore, banks' decisions to recapitalize are neither strategic complements, nor substitutes (since $N(m)/(\phi - m) = 1$ is invariant to m).

We solve for equilibrium for each of these two cases. Consider first the case with $\lambda_0 - \tau_0 < \tau_1\phi$. If additionally $\tau_1 c - \lambda_1\Delta > 0$, then for the range $\tau_0 \leq \lambda_0 \leq \tau_0 + \frac{\tau_1 c - \lambda_1\Delta}{c+\Delta}\phi$, $N(0) \leq I(0)$.

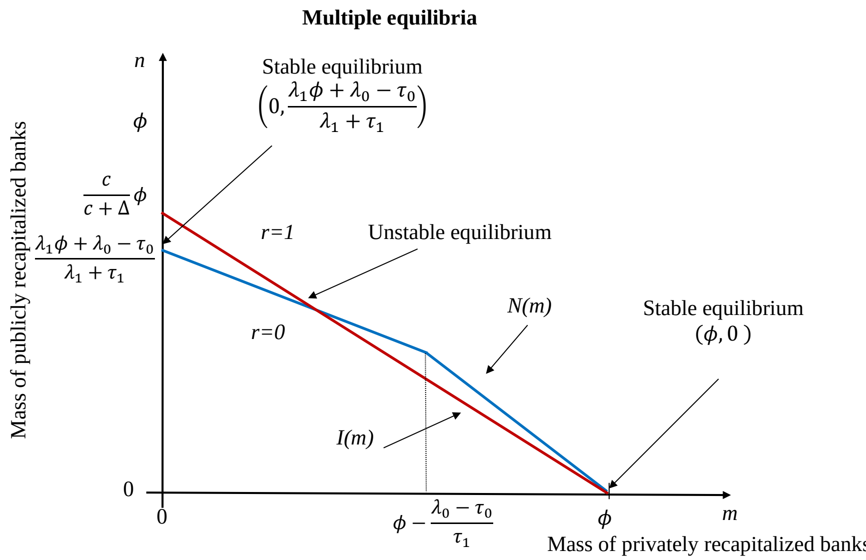

Figure A.1: Multiplicity of equilibria under $0 \leq \lambda_0 - \tau_0 < \tau_1\phi$

Thus, there are two SPNEs as described in Figure A.1:
(i) an equilibrium with $m = \phi$ and $n = 0$,
(ii) an equilibrium with $m = 0$ and $n = \frac{\lambda_1\phi + \lambda_0 - \tau_0}{\lambda_1 + \tau_1}$.

Instead, for $\lambda_0 > \tau_0 + \frac{\tau_1 c - \lambda_1\Delta}{c+\Delta}\phi$, we have $N(0) > I(0)$, so there is a unique SPNE with $m = \phi$ and $n = 0$, as described in Figure A.2.

Consider next the case with $\lambda_0 - \tau_0 \geq \tau_1\phi$. In this case $N(0) > I(0)$ so, similarly to the previous case, there exists a unique SPNE with $m = \phi$ and $n = 0$, as shown in Figure A.3.

Summing up, if $\tau_1 c - \lambda_1\Delta > 0$, and $\lambda_0 \leq \tau_0 + \frac{\tau_1 c - \lambda_1\Delta}{c+\Delta}\phi$, there exist two SPNE equilibria:
(i) an equilibrium with $m = \phi$ and $n = 0$ in which all banks recapitalize privately (and there is no forbearance),
(ii) an equilibrium with $m = 0$ and $n = \frac{\lambda_1\phi + \lambda_0 - \tau_0}{\lambda_1 + \tau_1}$ in which no bank recapitalizes privately (and the level of forbearance is positive).

Otherwise, there exists a unique subgame perfect Nash equilibrium with $m = \phi$ and $n = 0$ in which all banks recapitalize privately (and there is no forbearance).

Appendix 3. Proofs

Proof of Lemma 1

The result follows trivially from the discussion provided in the main text.■

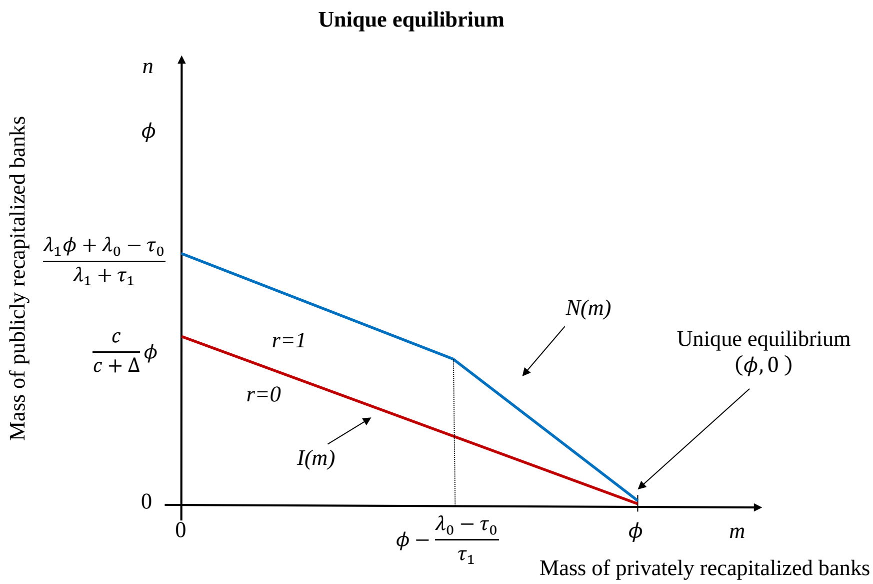

Figure A.2: Unique equilibrium under $0 \leq \lambda_0 - \tau_0 < \tau_1\phi$

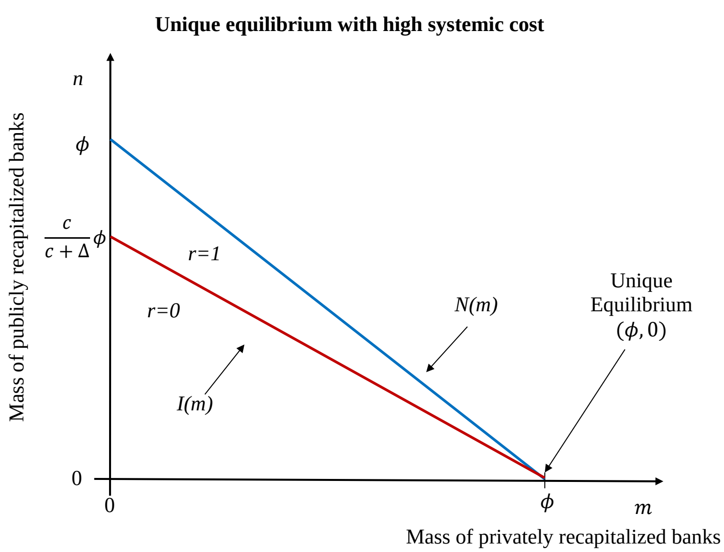

Figure A.3: Unique equilibrium under $\lambda_0 - \tau_0 \geq \tau_1\phi$

Proof of Proposition 1

To prove the results in Proposition 1, it is useful to examine the position of the indifference line $n = I(m)$ relative to the supervisor's reaction function $n = N(m)$.

For $(\lambda_1/(\lambda_1 + \tau_1) - (c/(c + \Delta)))\phi - (\tau_0 - \lambda_0)/(\lambda_1 + \tau_1) \leq 0$, we have $I(0) \geq N(0)$, which means that damaged banks' indifference line lies everywhere above the supervisor's reaction function, as depicted in Figure A.4. Then at all points on the supervisor's reaction function banks prefer not to comply. But then the unique symmetric SPNE must involve $m = 0$ and the supervisor's best response to such first stage outcome, that is, $n = N(0) = (\lambda_1\phi - (\tau_0 - \lambda_0))/(\lambda_1 + \tau_1)$, as indicated in Figure A.4.

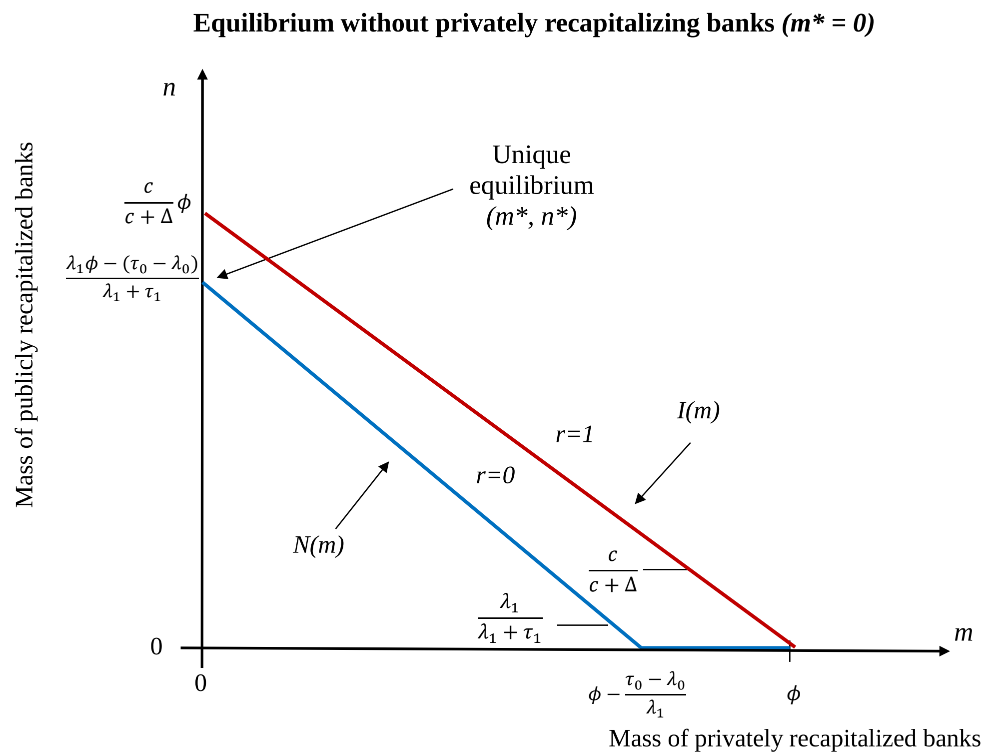

Figure A.4: Equilibrium without privately recapitalizing banks

For $(\lambda_1/(\lambda_1+\tau_1) - (c/(c+\Delta)))\phi - (\tau_0 - \lambda_0)/(\lambda_1+\tau_1) > 0$, we have $I(0) < N(0)$, which, given the form of the curves $N(m)$ and $I(m)$ guarantees a single crossing between them in the section of the supervisor's reaction function where $N(m) > 0$. This situation is depicted in Figure 1. In this case the unique symmetric SPNE of the game involves the values of (m, n) at such intersection, (m^*, n^*). To show that such point is a SPNE, notice that lying on $n = I(m)$ means that bank owners are indifferent between privately recapitalizing or not. Hence, m^* can be sustained as the result of damaged banks playing an uncorrelated symmetric mixed strategy in which they privately recapitalize with probability $p^* = m^*/\phi$. Simultaneously, lying also on $n = N(m)$ means that by publicly recapitalizing a mass n^* of the non-compliant banks, the supervisor plays a best response to damaged banks' actions in the first stage. Finally, the uniqueness of this equilibrium comes from the fact that, for the range $m < m^*$, $N(m) > I(m)$. This means that banks would prefer $r = 1$, which is incompatible with sustaining $m < \phi$, while, for values of $m > m^*$, $N(m) < I(m)$. This means that banks would prefer $r = 0$, which is incompatible with sustaining $m > 0$.■

Proof of Proposition 2

Table A1 summarizes the comparative statics of the equilibrium in which there is a strictly positive mass of privately recapitalizing banks. In the table, signs +, − or = indicate whether increasing the parameter indicated in the first column of the table increases, decreases, or does not change the endogenous variable indicated in the heading of each column. In a slight abuse of terminology, the table assimilates variations in the difference $(\lambda_1/(\lambda_1+\tau_1) - \Delta)\phi - (\tau_0 - \lambda_0)/(\lambda_1+\tau_1)$, which needs to be positive for the equilibrium to feature $m^* > 0$, as variations in the "likelihood" of having an equilibrium with $m^* > 0$. All the results arise immediately from partially differentiating the closed-form expressions of the relevant equilibrium variables.

Table A1. Comparative statics in the regime with $m^* > 0$

Parameter	Likelihood of equilibrium w. $m^* > 0$	Intervention threat $n^*/(\phi\text{–}m^*)$	Recapitalized banks: Privately m^*	Recapitalized banks: Publicly n^*	Forborne banks $\phi\text{–}m^*\text{–}n^*$
Damaged banks ϕ	+	=	+	=	=
Systemic cost λ_0	+	=	+	−	−
Systemic cost λ_1	+	=	+	−	−
Intervention cost τ_0	−	=	−	+	+
Intervention cost τ_1	−	=	−	+	+
Owners' private recap cost c	−	+	−	+	+
Owners' control rent Δ	+	−	+	−	−

The effects of changing λ_0, λ_1, c, and Δ go in the natural direction, in the sense that (i) they encourage the player(s) directly suffering the corresponding cost to take actions that reduce the incidence of such cost, and (ii) produce partially offsetting changes in the actions of the opposing player(s). In the case of c and Δ, the direct effects (on m^*) dominate the indirect ones (on n^*) as for the final variation in forbearance $(\phi - m^* - n^*)$. In the case of λ_0 and λ_1, the indirect effect (on m^*) dominates.■

Proof of Proposition 3

First, we find the effect of τ_0 on the mass of publicly recapitalized banks n^* and on forbearance $\phi - m^* - n^*$ by just deriving in (1) and (6), respectively, using the expressions that correspond to the case in which the equilibrium mass of compliant banks is positive:

$$\frac{\partial n^*}{\partial \tau_0} = \frac{c}{\lambda_1 \Delta - \tau_1 c} > 0, \tag{26}$$

$$\frac{\partial(\phi - m^* - n^*)}{\partial \tau_0} = \frac{\Delta}{\lambda_1 \Delta - \tau_1 c} > 0. \tag{27}$$

Next, we further derive the above expressions with respect to c:

$$\frac{\partial^2 n^*}{\partial \tau_0 \partial c} = \frac{\lambda_1 \Delta}{(\lambda_1 \Delta - \tau_1 c)^2} > 0, \tag{28}$$

$$\frac{\partial^2 (\phi - m^* - n^*)}{\partial \tau_0 \partial c} = \frac{\tau_1 \Delta}{(\lambda_1 \Delta - \tau_1 c)^2} > 0, \tag{29}$$

whose signs mean that a rise in c reinforces the effects of τ_0 on n^* and $\phi - m^* - n^*$.

Proceeding similarly to find the effects of τ_1 on n^* and $\phi - m^* - n^*$, we obtain:

$$\frac{\partial n^*}{\partial \tau_1} = \frac{(\tau_0 - \lambda_0)c^2}{(\lambda_1 \Delta - \tau_1 c)^2} > 0, \tag{30}$$

$$\frac{\partial (\phi - m^* - n^*)}{\partial \tau_1} = \frac{(\tau_0 - \lambda_0)\Delta c}{(\lambda_1 \Delta - \tau_1 c)^2} > 0, \tag{31}$$

and the cross-derivatives:

$$\frac{\partial^2 n^*}{\partial \tau_1 \partial c} = \frac{2(\tau_0 - \lambda_0)c\lambda_1 \Delta}{(\lambda_1 \Delta - \tau_1 c)^3} > 0, \tag{32}$$

$$\frac{\partial^2 (\phi - m^* - n^*)}{\partial \tau_1 \partial c} = \frac{2(\tau_0 - \lambda_0)\Delta c^2}{(\lambda_1 \Delta - \tau_1 c)^3} > 0, \tag{33}$$

which imply that a rise in c also reinforces the effects of τ_1 on n^* and $\phi - m^* - n^*$.■

Proof of Proposition 4

In the extended game, the supervisor's reaction function is defined by

$$N_d(m) = \arg \min_{0 \leq n \leq \phi - m} \quad \tau_0 dn + (\tau_1/2)(dn)^2 + \lambda_0 d(\phi - m - n) + (\lambda_1/2)(d(\phi - m - n))^2, \tag{34}$$

which is solved for

$$N_d(m) = \max \left\{ \frac{\lambda_1 d(\phi - m) - (\tau_0 - \lambda_0)}{(\lambda_1 + \tau_1)d}, 0 \right\}. \tag{35}$$

An individual bank's indifference condition regarding the choice between privately recapitalizing or not is now given by the line $n = I_d(m)$ with

$$I_d(m) = \frac{cd}{cd + \Delta}(\phi - m). \tag{36}$$

Exploring (35) and (36), it becomes clear that leverage d increases the propensity of the supervisor to intervene (that is, would shift its reaction function outwards in a figure similar to Figure 1) and reduces banks' incentives to comply (that is, would shift their indifference condition inwards in a figure similar to Figure 1).

Following the same steps as in the proof of Proposition 1, the unique symmetric SPNE of the extended game is (as in (7)):

$$(m_d^*, n_d^*) = \begin{cases} \left(\phi - \frac{(\tau_0-\lambda_0)(cd+\Delta)}{(\lambda_1\Delta-\tau_1 cd)d}, \frac{(\tau_0-\lambda_0)c}{\lambda_1\Delta-\tau_1 cd}\right), & \text{if } \left(\frac{\lambda_1}{\lambda_1+\tau_1} - \frac{cd}{cd+\Delta}\right)\phi - \frac{(\tau_0-\lambda_0)}{(\lambda_1+\tau_1)d} > 0, \\ \left(0, \frac{\lambda_1\phi d-(\tau_0-\lambda_0)}{(\lambda_1+\tau_1)d}\right), & \text{if } \left(\frac{\lambda_1}{\lambda_1+\tau_1} - \frac{cd}{cd+\Delta}\right)\phi - \frac{(\tau_0-\lambda_0)}{(\lambda_1+\tau_1)d} \leq 0. \end{cases}$$

Focusing on the regime in which the mass of privately recapitalizing banks m_d^* is strictly positive, it is immediate to find that

$$\frac{\partial n_d^*}{\partial d} = \frac{(\tau_0 - \lambda_0)\tau_1 c^2}{(\lambda_1\Delta - \tau_1 cd)^2} > 0,$$

and also

$$\phi - m_d^* - n_d^* = \frac{(\tau_0 - \lambda_0)\Delta}{(\lambda_1\Delta - \tau_1 cd)d},$$

which implies

$$\frac{\partial(\phi - m_d^* - n_d^*)}{\partial d} = -\frac{(\tau_0 - \lambda_0)\Delta(\Delta\lambda_1 - 2\tau_1 cd)}{(\lambda_1\Delta - \tau_1 cd)^2 d^2},$$

which is strictly positive if and only if $d > \bar{d} = \frac{\Delta\lambda_1}{2\tau_1 c}$. So capital forbearance is first increasing and then decreasing in d.

To better understand the roots of the non-monotonicity in the effect of leverage on forbearance, notice that

$$\frac{\partial m_d^*}{\partial d} = -\frac{(\tau_0 - \lambda_0)[\tau_1 c^2 d^2 + 2\tau_1\Delta cd - \lambda_1\Delta^2]}{(\lambda_1\Delta - \tau_1 cd)^2 d^2},$$

which is strictly positive if and only if

$$\tau_1 c^2 d^2 + 2\tau_1\Delta cd - \lambda_1\Delta^2 < 0. \tag{37}$$

The quadratic equation $\tau_1 c^2 d^2 + 2\tau_1\Delta cd - \lambda_1\Delta^2 = 0$ has two roots, one of which is negative. The other root is

$$\hat{d} = \frac{\Delta}{c}\left(\sqrt{1 + \lambda_1/\tau_1} - 1\right) > 0.$$

Thus the inequality (37) is true if and only if $d < \hat{d}$, which means that the mass of privately recapitalizing banks is first increasing and then decreasing with d, reaching a maximum at $\hat{d}$. Given that the mass of publicly recapitalized banks n_d^* is monotonically increasing in d, this inverted U-shaped relationship between d and m_d^* helps explain the U-shaped relationship between d and forbearance as well as the fact that the latter reaches a minimum at a leverage level $\bar{d}$ strictly larger than $\hat{d}$.■

Proof of Proposition 5

The results follow from the taxonomy of the possibilities that may emerge in the constrained game when the unconstrained game features some privately recapitalizing banks, that is, $m^* > 0$ and, hence, $n^* = N(m^*)$. The situation depicted in Figure 2 corresponds with case 3 in the following list:

1. If $n_0 < N(m^*)$, the constrained game has just one equilibrium with $(m, n) = (0, n_0)$.

2. If $n_0 = N(m^*)$, the constrained game features two equilibria: one with $(m, n) = (0, n_0)$ and the same equilibrium $(m^*, N(m^*))$ as the unconstrained game.

3. If $n_0 \in (N(m^*), (c/(c+\Delta))\phi]$, the constrained game features three equilibria: one with $(m, n) = (0, n_0)$, the same equilibrium $(m^*, N(m^*))$ as the unconstrained game, and a third (unstable) one with $(I^{-1}(n_0), n_0)$.

4. If $n_0 > (c/(c + \Delta))\phi$, the constrained game has just the same equilibrium as the unconstrained game.

When the unconstrained game features $m^* = 0$ and, hence, $n^* = N(0)$ (as illustrated in Figure A.4), the possibilities that emerge when the supervisor faces a capacity constraint n_0 are only two:

1. For $n_0 < N(0)$, the constrained game has just one equilibrium with $(m, n) = (0, n_0)$ (which involves larger forbearance than the equilibrium of the unconstrained game).

2. For $n_0 \geq N(0)$, the constrained game has just the same equilibrium $(0, N(0))$ as the unconstrained game.■

Proof of Proposition 6

According to Proposition 5, the range of values of n_0 over which only the equilibrium with extreme forbearance exists is $[0, N(m^*))$. It is immediate to see that $\partial N(m^*)/\partial c > 0$, $\partial N(m^*)/\partial \tau_i > 0$, and $\partial^2 N(m^*)/(\partial \tau_i \partial c) > 0$ for $i = 0, 1$, which proves the result.■

Proof of Proposition 7

From the Proof of Proposition 4, the possibilities that may emerge in the constrained game when the unconstrained game features some privately recapitalizing banks, that is, $m_d^* > 0$ and, hence, $N_d(m_d^*)$ from (7) and (35), include:

1. If $K_0 < dN_d(m_d^*)$, the constrained game has just one equilibrium with $(m, n) = (0, K_0/d)$.

2. If $K_0 = dN_d(m_d^*)$, the constrained game features two equilibria: one with $(m, n) = (0, K_0/d)$ and the same equilibrium (m_d^*, n_d^*) as the unconstrained game.

3. If $K_0 \in (dN_d(m_d^*), (cd^2/(cd+\Delta))\phi]$, the constrained game features three equilibria: one with $(m, n) = (0, K_0/d)$, the same equilibrium (m_d^*, n_d^*) as the unconstrained game, and a third (unstable) one with $(I_d^{-1}(K_0/d), K_0/d)$.

4. If $K_0 > (cd^2/(cd + \Delta))\phi$, the constrained game has just the same equilibrium as the unconstrained game.

Then it is immediate to find that

$$\frac{\partial[dN_d(m_d^*)]}{\partial d} = \frac{(\tau_0 - \lambda_0)\lambda_1\Delta c}{(\lambda_1\Delta - \tau_1 cd)^2} > 0,$$

and also

$$\frac{\partial[(cd^2/(cd + \Delta))\phi]}{\partial d} = \frac{cd\phi(cd + 2\Delta)}{(cd + \Delta)^2} > 0,$$

which implies that the region where the equilibrium with extreme forbearance exists expands with higher leverage, and so does the region where such equilibrium is unique.

Moreover, the level of such extreme forbearance increases in leverage:

$$\frac{\partial(\phi - K_0/d)}{\partial d} = \frac{K_0}{d^2} > 0. \blacksquare$$

Proof of Proposition 8

The results regarding the existence of an unconstrained equilibrium involving $\phi = \phi_u$ for $K_0 \geq K_u > 0$ and a constrained equilibrium involving $\phi = \Phi(K_0)$ (with $\Phi' < 0$) for $K_0 \in [0, K_c]$ are implied by the discussion in the preceding subsections of the main text. It remains to show that $K_c > K_u$ and $\Phi(K_c) = \phi_u$. We will structure the proof in three steps.

Step 1. We first constructively show that there exist a value $\hat{K}$ of K_0 for which the constrained equilibrium exists and involves $\Phi(\hat{K}) = \phi_u$. The existence of such equilibrium requires satisfying (18) and (19), that is,

$$\gamma\phi_u(1 - \phi_u) = c\hat{K} + \frac{\hat{K}\Delta}{D(\phi_u)} \tag{38}$$

and

$$\hat{K} \leq \frac{c\,(D(\phi_u))^2}{cD(\phi_u) + \Delta}\phi_u. \tag{39}$$

But ϕ_u also satisfies banks' FOC in the unconstrained equilibrium, that is, (13), which implies

$$\gamma\phi_u(1 - \phi_u) = cD(\phi_u)\phi_u,$$

and hence allows us to write (38) as

$$cD(\phi_u)\phi_u = c\hat{K} + \frac{\hat{K}\Delta}{D(\phi_u)}$$

or, solving for $\hat{K}$,

$$\hat{K} = \frac{c\,(D(\phi_u))^2}{cD(\phi_u) + \Delta}\phi_u,$$

which satisfies (39) with equality.

Step 2. We next show that $\hat{K}$ is K_u, that is, the highest value of K_0 for which the constrained equilibrium exists. To see this notice that $\Phi(K_0)$ is decreasing, while $D(\phi)$ is increasing in ϕ. So if at $K_0 = \hat{K}$ (39) holds with equality, then (19) will hold with strict inequality for $K_0 < \hat{K}$ and it will not hold for $K_0 > \hat{K}$, because the RHS of (19) increases with K_0 while the left hand side decreases with it. So $K_c = \hat{K}$.

Step 3. We finally show that $K_c > K_u$. We do this with the help of Figure 3, which extends Figure 2 to the case in which the relevant payoffs depend on bank leverage d and such leverage depends on the mass of damaged banks, $d = D(\phi)$. In this figure, where we denote $D(\phi_u)$ by D_u, the intersection of the downward slopping sections of the supervisors' reaction function in the unconstrained game $N(m)$ (blue line) and the indifference condition $I(m)$ (red line) identifies the equilibrium (m_u, n_u) of the unconstrained continuation supervisory game associated with the mass ϕ_u of banks determined in the prior risk-taking stage of the unconstrained game. Notice that the statement of the proposition assumes the existence of this interior SPNE equilibrium of the unconstrained game. In Figure 3 the horizontal portion of the constrained reaction function $N_0(m)$ at the level $n = cD(\phi_u)\phi_u/(cD(\phi_u)+\Delta)$ identifies $K_c/D(\phi_u)$. Clearly, for $K_0 = K_c$, the unconstrained equilibrium is also an equilibrium of the constrained game and one in which the supervisor's capacity constraint holds with slack, that is, $n_u < K_c/D(\phi_u)$. So, by continuity, there is a range of values of $K_0 < K_c$ in the neighborhood of K_c, where both the unconstrained equilibrium (whose outcomes do not depend on K_0) and the constrained equilibrium (with capital forbearance and ex ante risk taking both decreasing in K_0) coexist.■

Proof of Proposition 9

The first best (FB) choice of m and n would emerge from solving

$$\begin{array}{ll} \max_{m\geq 0, n\geq 0} & W, \\ \text{s.t.:} & m + n \leq \phi, \end{array} \tag{40}$$

which, using (20), clearly yields the solution $m^{FB} = \phi$ and $n^{FB} = 0$, under which welfare is $W^{FB} = 0$. In contrast, the outcome of the supervisory game without commitment when the equilibrium features a strictly positive mass of privately recapitalizing banks and $\lambda_0 < (1-\alpha)\tau_0$ is

$$W^* = -\frac{(\tau_0 - \lambda_0)}{\lambda_1\Delta - \tau_1 c} \cdot \left[\frac{(\tau_0 - \lambda_0)[(\tau_1/2)c^2 + (\lambda_1/2)\Delta^2]}{\lambda_1\Delta - \tau_1 c} + \tau_0 c + \lambda_0\Delta\right] < 0,$$

which is obtained by simply plugging in (20) the expressions for m^* and n^* provided in (1).

Now, consider the case in which the supervisor can commit to decide on public recapitalizations using the rule (21), for $\xi \geq 0$. To show that the first best can be implemented, notice that if a damaged bank expects the mass of privately recapitalized banks to be m and the subsequent decision of the supervisor to be $n = \bar{N}(m)$, its owners' payoff from privately recapitalizing would be $-c$, while their expected payoff from not doing so would be

$$-\frac{\bar{N}(m)}{\phi - m}(c + \Delta) = -(c + \xi) < -c,$$

which means that privately recapitalizing is indeed a best response (and strictly so for $\xi > 0$). So an equilibrium with $m = \phi$ and $n = \bar{N}(\phi) = 0$ can be sustained (and for $\xi > 0$ this equilibrium is unique).■

Proof of Proposition 10

Consider the game where public recapitalizations were decided, for each given m, by a social planner who disregards the political and reputational costs of its interventions. Then the supervisor's reaction function is given by:

$$N(m) = \min\left[\phi - m, \max\left[0, \frac{\lambda_1(\phi - m) + (\lambda_0 - \tau_0(1-\alpha))}{\lambda_1 + \tau_1(1-\alpha)}\right]\right]$$

Note that if $\lambda_0 < (1-\alpha)\tau_0$, $N(m) \leq \phi - m$ is not binding, and $N = 0$ for $m < \phi$. Thus, supervisor's reaction function has the shape similar to the one described in Figure 1. Then the equilibrium is given by:

$$(m^{SP}, n^{SP}) = \begin{cases} (\phi - \frac{((1-\alpha)\tau_0 - \lambda_0)(c+\Delta)}{\lambda_1\Delta - (1-\alpha)\tau_1 c}, \frac{((1-\alpha)\tau_0 - \lambda_0)c}{\lambda_1\Delta - (1-\alpha)\tau_1 c}), & \text{if } \left(\frac{\lambda_1}{\lambda_1 + (1-\alpha)\tau_1} - \frac{c}{c+\Delta}\right)\phi - \frac{(1-\alpha)\tau_0 - \lambda_0}{\lambda_1 + (1-\alpha)\tau_1} > 0, \\ \\ (0, \frac{\lambda_1\phi - ((1-\alpha)\tau_0 - \lambda_0)}{\lambda_1 + (1-\alpha)\tau_1}), & \text{if } \left(\frac{\lambda_1}{\lambda_1 + (1-\alpha)\tau_1} - \frac{c}{c+\Delta}\right)\phi - \frac{(1-\alpha)\tau_0 - \lambda_0}{\lambda_1 + (1-\alpha)\tau_1} \leq 0, \end{cases} \tag{41}$$

which implies a level of forbearance given by

$$\phi - m^{SP} - n^{SP} = \begin{cases} \frac{((1-\alpha)\tau_0 - \lambda_0)\Delta}{\lambda_1\Delta - (1-\alpha)\tau_1 c}, & \text{if } \left(\frac{\lambda_1}{\lambda_1 + (1-\alpha)\tau_1} - \frac{c}{(c+\Delta)}\right)\phi - \frac{(1-\alpha)\tau_0 - \lambda_0}{\lambda_1 + (1-\alpha)\tau_1} > 0, \\ \\ \frac{(1-\alpha)(\tau_0 + \tau_1\phi) - \lambda_0}{\lambda_1 + (1-\alpha)\tau_1}, & \text{if } \left(\frac{\lambda_1}{\lambda_1 + (1-\alpha)\tau_1} - \frac{c}{(c+\Delta)}\right)\phi - \frac{(1-\alpha)\tau_0 - \lambda_0}{\lambda_1 + (1-\alpha)\tau_1} \leq 0. \end{cases} \tag{42}$$

And it is immediate to show that in the regime with $m^{SP} > 0$, we have $\partial m^{SP}/\partial\alpha > 0$, $\partial n^{SP}/\partial\alpha < 0$, and $\partial(\phi - m^{SP} - n^{SP})/\partial\alpha < 0$. Moreover, the condition for the prevalence of such regime gets relaxed as α rises. Altogether, this implies that in circumstances in which the equilibrium of the supervisory game features $m^* > 0$, the equilibrium in which the social planner decides on n features $m^{SP} > m^*$, $n^{SP} < n^*$, and $\phi - m^{SP} - n^{SP} < \phi - m^* - n^*$.

However, if $\lambda_0 \geq (1-\alpha)\tau_0$, $N(m) \leq \phi - m$ becomes binding. Similarly to the model solution shown in Appendix 2 when Assumption 1 is relaxed, there may be a region of multiple equilibria. Specifically, if $\tau_1(1-\alpha)c - \lambda_1\Delta > 0$ and $\lambda_0 \leq \tau_0(1-\alpha) + \frac{\tau_1(1-\alpha)c - \lambda_1\Delta}{c+\Delta}\phi$, there exist two SPNE equilibria:

(i) an equilibrium with $m^{SP} = \phi$ and $n^{SP} = 0$ in which all banks recapitalize privately and there is no forbearance;

(ii) an equilibrium with $m^{SP} = 0$ and $n^{SP} = \frac{\lambda_1\phi-(\tau_0(1-\alpha)-\lambda_0)}{\lambda_1+\tau_1(1-\alpha)}$ in which no bank recapitalizes privately and the level of forbearance is $\phi - m^{SP} - n^{SP} = \frac{(1-\alpha)(\tau_0+\phi\tau_1)-\lambda_0}{\lambda_1+\tau_1(1-\alpha)}$.

In contrast, if $\tau_1(1-\alpha)c - \lambda_1\Delta \leq 0$, there exists a unique SPNE where all banks recapitalize privately $m^{SP} = \phi$, $n^{SP} = 0$, and there is no forbearance.

Thus, if the systemic cost of bank failure λ_0 is sufficiently high relative to the social cost of early intervention $(1-\alpha)\tau_0$, it is possible to sustain an equilibrium with full compliance and no forbearance. Such an equilibrium is based on the strength of the implicit recapitalization threat on a marginal non-compliant banks. However, for some parameter values, such equilibrium may coexist with another one with no compliance since if sufficiently many banks lack to comply the intervention threat may actually be weakened.■