Incomplete Wage Posting

Claudio Michelacci and Javier Suarez

Centro de Estudios Monetarios y Financieros (CEMFI)

We consider a directed search model in which workers differ in productivity. Productivity becomes observable to firms after assessing their workers on the job, but it is not verifiable. Firms with vacancies choose between posting a noncontingent wage and leaving wages subject to bargaining with the worker. Under wage bargaining, firms cannot optimize the trade-off between paying higher wages and having a larger probability of filling vacancies. But wage bargaining makes wages increasing in worker productivity and so may allow firms to attract better workers into the vacancy. When workers’ heterogeneity is large and bargaining powers come close to satisfying Hosios’s rule, firms opt for bargaining. Yet, equilibria with bargaining fail to maximize aggregate net income and sometimes are not constrained Pareto optimal.

I. Introduction

While some companies are committed to rather transparent and well-defined compensation structures, others leave starting salaries and other important components of pay subject to negotiation with each worker. The importance of these negotiations is indirectly evidenced by the abundance of popular literature and Web sites that advise workers on

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how to negotiate better pay for their jobs. However, the use of bargaining seems paradoxical in light of labor models in the directed search tradition, including Montgomery (1991), Peters (1991), Moen (1997), Acemoglu and Shimer (1999a, 1999b), Shi (2001, 2002), and Shimer (2005), which predict that firms should post all the terms of the employment contract in advance. By doing so, firms can efficiently optimize the trade-off between paying higher wages to their workers and filling vacancies with higher probability. In contrast, leaving wages subject to ex post bargaining tends to lead to either too high wages or too few applicants relative to the level that would maximize the value of a vacancy. These search inefficiencies related to bargaining always arise, unless the distribution of bargaining power satisfies the well-known condition first derived by Hosios (1990).

In this paper we argue that firms may choose to subject wages to bargaining because, in the jargon of contract theory, posted wage structures are bound to be incomplete (see Hart 1995). The idea is that there are differences in workers’ productivity that can be subjectively assessed by managers and supervisors after observing the workers on the job, but these differences cannot be verified by courts, making it impossible to enforce compensation contracts explicitly contingent on them. In this context, an advantage of negotiating wages once the firm assesses the worker on the job is that the resulting wage ends up tailored to the worker’s productivity even if courts cannot verify productivity. Thus bargained wages can be particularly attractive to relatively more productive workers, and firms may offer such wages in order to improve the composition of their pool of job applicants. By the same token, firms that compete by offering a posted wage may be left with relatively less productive workers, thus suffering an adverse selection problem.

This paper shows that the adverse selection problem associated with wage posting and the search inefficiencies related to wage bargaining produce a nontrivial trade-off for firms deciding the wage-setting mechanism attached to their jobs. We consider a directed search model as in Acemoglu and Shimer (1999a), where firms create vacancies and
announce whether they will pay a given wage or set the wage through bargaining after assessing the worker on the job. We find that, in equilibrium, firms opt for wage posting when bargaining powers are far from satisfying Hosios’s rule and the heterogeneity in workers’ productivity is small. Conversely, wage bargaining emerges when the search inefficiencies induced by bargaining are mild or when workers’ productivity is so dispersed that the adverse selection problem of wage posting is severe. Interestingly, when both the search inefficiencies related to bargaining and the adverse selection problem of wage posting are mild, the market gets segmented: some firms set wages through bargaining and attract the most productive workers, whereas others post a wage and draw the least productive workers.

Equilibria with and without bargaining may coexist, since firms’ posting decisions produce externalities on other firms in the market: the composition of the pool of applicants for vacancies with a posted wage worsens if sufficiently many firms offer to bargain their wages. This reduces the profitability of wage posting and reinforces the sustainability of bargaining in equilibrium.

Relative to wage posting, bargaining redistributes income from low- to high-productivity workers, but it produces search inefficiencies that generally entail either excessive vacancies or excessive unemployment. Thus wage bargaining always induces a reduction in aggregate net income.\(^4\) Moreover, when there are multiple equilibria, the equilibrium without bargaining is Pareto dominant.

Our analysis contributes to the directed search literature by noting that, if worker productivity is unverifiable, wage posting suffers from adverse selection and wage bargaining may arise despite its associated search inefficiencies.\(^5\) Ellingsen and Rosen (2003) and Camera and Delacroix (2004) use random search models with unverifiable worker heterogeneity to analyze firms’ choice between bargaining and posting. In these models, however, the wage-setting mechanism plays no role in attracting workers to vacancies or in tackling search inefficiencies.\(^6\) Our analysis also relates to the work of McAfee (1993) and Peters (1997), where buyers have heterogeneous private valuations of the exchanged good and sellers can publicly post a pricing mechanism for the good. These papers show that second-price sealed-bid auctions are sellers’ preferred pricing mechanism.\(^6\) Taken literally, those auctions imply that each vacancy is bought out by the winning worker, who be-

\(^4\) Aggregate income net of job creation costs is the standard social welfare measure used in the labor search literature (see, e.g., Pissarides 2000; Shimer and Smith 2001).

\(^5\) Bester (1993) makes a related point in a product market model in which sellers choose the noncontractible quality of their product, which the buyers observe after meeting the seller; he shows that bargaining may better motivate sellers to choose high quality.

\(^6\) For an analysis of the role of auctions in the labor market, see Shimer (1999).
comes the residual claimant of the future output. However, when output is not verifiable, as we assume, auctions may not be feasible if workers are wealth constrained or output goes directly to the employer. And auctions may be suboptimal if workers are risk-averse or if having the employer as a residual claimant is convenient for, say, incentive reasons (see Hart and Moore 1990).

In our model, firms observe their workers’ productivity after the matching process is completed. If workers’ productivity could be assessed earlier in the recruiting process, firms could rank their applicants, and even vacancies with a posted wage would deliver greater utility to high-productivity workers than to low-productivity ones. However, the findings in Lang and Dickens (1993) and Lang, Manove, and Dickens (2005) suggest that ranking (and the resulting nondegenerate distribution of posted wages) would discriminate across worker types very differently from fully contingent hiring and compensation policies. We conjecture that such a setup would still leave room for the emergence of wage bargaining (and its coexistence with wage posting) for reasons akin to those explored in this paper.

The rest of the paper is organized as follows. In Section II we describe the model. Section III defines our notion of equilibrium and provides some important preliminary results. Section IV characterizes the various possible equilibrium regimes. In Section V we compare the various regimes in terms of social welfare. The conclusions appear in Section VI. The Appendix contains all proofs.

II. The Model

We consider a static labor market with a unit mass of workers and free entry of firms. Firms and workers are risk-neutral and maximize their expected net income. Each firm can create a vacancy at a cost \( c > 0 \). There are two types of workers, \( i = 0, 1 \), in proportions \( 1 - \mu \) and \( \mu \), respectively. The labor services of a low-productivity worker \( (i = 0) \) produce output \( y_0 > c \) when delivered to a firm with an open vacancy. In the same circumstances, the labor services of a high-productivity worker \( (i = 1) \) produce \( y_1 > y_0 \). For simplicity we assume that workers earn no income if unemployed and incur no direct cost in searching for a job.

The process whereby workers and firms match is subject to search.

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7 The human resources literature acknowledges that the scope for screening job applicants is limited. Arvey and Campion (1982) review the vast evidence about the limited reliability of job interviews. Baron and Kreps (1999) question the validity of usual psychometric tests (pp. 352–53) and claim that substantial information about workers’ productivity gets disclosed in probation periods (p. 345).

8 In a related note (available at http://www.cemfi.es/~michela/note.pdf) we investigate this claim in more detail.
frictions. Firms can costlessly announce their vacancies among all workers, but workers have limited capacities to submit job applications and to coordinate their decisions. Specifically, each worker can apply for at most one vacancy. Workers first choose the vacancy announcement they prefer (possibly using a mixed strategy) and then send an application by uniformly randomizing over the firms making the announcement. Thus some firms will receive several applications, whereas others receive none. Firms with several applicants choose one to whom they make a job offer. For informational problems that we explain below, firms are unable to distinguish applicants’ types before the match, so firms with multiple applicants choose one at random. This produces a match.

Both firms and workers face uncertainty about whether they will be successfully matched, and firms also face uncertainty about the productivity of the worker with whom they match. To model the matching uncertainty, we assume that the probability that a firm making a given announcement gets matched is \( Q(n) \), which depends on the vacancy’s expected number of applicants, \( n \). The analogous matching probability for a worker is \( P(n) \). More general reduced-form functions have been used before by Acemoglu and Shimer (1999a, 1999b). The function \( Q(n) \) could also be motivated as the probability implied by a standard matching function à la Pissarides (2000). Matching functions appear in the directed search models of Moen (1997), Acemoglu (2001), and Mortensen and Wright (2002).

We assume that these functions are twice continuously differentiable, with \( Q'(n) > 0 \) and \( P'(n) < 0 \), and \( P(n) = Q(n)/n \), where the last requirement guarantees aggregate consistency between the number of filled vacancies and the number of employed workers.

Each worker knows his own productivity type from the start, whereas firms learn that type after matching with the worker. We assume that third parties (say, courts) can verify only labor delivery and wage payments. This guarantees that contracts specifying that “worker A will receive a wage \( w \) in exchange for the delivery of his labor to firm B” are enforceable. However, workers’ type, their output, the structure of the bargaining between a firm and a worker, the outcome of the matching process (i.e., whether a match has occurred), and workers’ application decisions are assumed to be unverifiable. Unverifiability means that courts cannot ascertain those types, outcomes, processes, and decisions at a reasonable cost, and it implies that they cannot enforce rules or provisions concerning them. In this setup, we assume that firms can choose either announcing a wage for “whoever is hired” (wage posting)
or announcing that the wage will be set through bargaining between the firm and the worker once they match (wage bargaining).\textsuperscript{12}

If a firm posts a wage \( x \in \mathbb{R}_+ \), the matching between the firm and a worker of type \( i \) yields a compensation \( x \) to the worker and a profit \( y_i - x \) to the firm. If the firm has announced bargaining, which we represent as the posting of \( x_\infty \), then the worker and the firm bargain on the wage at a point at which the outside options of the worker (becoming unemployed) and the firm (leaving the vacancy empty) are worth zero. The surplus to be split is \( y_i \), which, after the match, is observable to both of them. As in many papers in the random search tradition, we postulate a generalized Nash bargaining solution, implying that the worker and the firm receive \( \beta y_i \) and \( (1 - \beta)y_i \), respectively, where \( \beta \in (0, 1) \) represents the worker’s bargaining power. Therefore, bargaining implies a higher wage for high-productivity workers than for low-productivity ones.

We make the following assumptions.

**Assumption 1.** \( \lim_{n \to \infty} Q(n) = \lim_{n \to 0} P(n) = 1. \)

**Assumption 2.** The elasticity of \( Q(n) \) with respect to \( n \), \( \epsilon_d(n) = Q'(n)/Q(n) \), is weakly decreasing.

**Assumption 3.** \( \lim_{n \to \infty} \epsilon_d(n) < 1 - (c/y_0). \)

Assumptions 1 and 3 help to guarantee the existence of equilibrium.\textsuperscript{13} Assumption 2 guarantees that the number of posted wages is unique.\textsuperscript{14}

To facilitate the diagrammatic representation of equilibria, we describe workers’ demand for a vacancy whose expected number of applicants is \( n \) through the variable \( d = 1/P(n) = n/Q(n) \in [1, \infty) \) (the inverse of the workers’ employment probability). Because \( d \) is a strictly increasing transformation of \( n \), there is a strictly increasing function \( n = N(d) \) that allows us to reexpress firms’ and workers’ probabilities of getting matched as \( q(d) = Q(N(d)) \) and \( p(d) = P(N(d)) \), respectively. Similarly, the elasticity of \( Q(n) \) with respect to \( n \) can be written as

\[ \eta(d) = \epsilon_d(N(d)) = Q'(N(d))d, \quad (1) \]

which is decreasing in \( d \), by assumption 2.

\textsuperscript{12} Wage posting can be seen as a firm’s announcement that constrains the bargaining. It is the strongest form of commitment and an easily enforceable one. In principle the firm may want to constrain the bargaining set in a less stringent manner, e.g., by announcing a minimum or a maximum wage for the job. Under Nash bargaining, one can show that announcing a minimum wage is irrelevant: firms can always mimic its effects by either announcing a posted wage or offering to bargain. Maximum wage policies are instead useful, but only when workers’ bargaining power is high. The possibility of committing to a maximum wage tends to reinforce firms’ preference for bargaining and to reduce their incentive to post a wage.

\textsuperscript{13} In particular, assumption 3 ensures that even if all workers had low productivity, creating some vacancies would be socially profitable.

\textsuperscript{14} The urn-ball process mentioned in n. 9 satisfies assumptions 1 and 2.
III. Equilibrium

We can distinguish two stages in the operation of the labor market. In the first stage, firms simultaneously decide whether to create a vacancy at a cost $c$ and, in that case, which announcement $x \in X = \mathbb{R}_+ \cup \{x_0\}$ to post. The resulting set of announcements $X^*$ and the measure $\nu(x)$ of firms posting each $x \in X^*$ are then observed by all workers. In the second stage, workers simultaneously decide (possibly using a mixed strategy) their preferred announcement and submit an application by uniformly randomizing over the firms posting it. For each $x \in X^*$ these actions imply a demand $d(x)$ and an expected proportion of high-productivity applicants $\gamma(x)$ for the corresponding vacancies.

A. Definition of Equilibrium

Allocations are described by the set $X^*$ and the functions $\nu(x)$, $d(x)$, and $\gamma(x)$. Each allocation uniquely determines the expected utility attained by each worker type, $U_i$ with $i = 0, 1$. Workers’ application decisions are based on rational forecasts of $d(x)$ for each announcement $x \in X^*$. Firms’ entry decisions are based on rational predictions of $d(x)$ and $\gamma(x)$ for each possible announcement $x \in X$ (including announcements not made by any firm). This requires extending the domain of these functions to the whole set $X$. Since workers and firms are infinitesimal, they formulate their decisions taking the functions $d(x)$ and $\gamma(x)$ as given.\(^\text{15}\)

For simplicity the productivity after a successful match of a worker who fills a vacancy with an announcement $x \in X$ will be denoted by the random variable $\tilde{y}(x) \in [y_0, y_1]$. Analogously his wage is the random variable

$$\tilde{w}(x) = \begin{cases} x & \text{if } x \in \mathbb{R}_+, \\ B\tilde{y}(x) & \text{if } x = x_0. \end{cases}$$

(2)

So a firm’s net profit from creating a vacancy and announcing $x$ can be expressed as

$$V(x) = q(d(x))E_{\gamma(x)}[\tilde{y}(x) - \tilde{w}(x)] - c,$$

(3)

where the subscript in the expectation operator denotes the (possibly degenerate) probability that the hired worker is of the high-productivity type.

\(^{15}\) This assumption is standard in the directed search tradition. For announcements made by a zero mass of firms, the implicit assumption is that all workers of a given type choose the announcement to which they reply by randomizing over all vacancies that give them the same utility.
Definition. An equilibrium is an allocation \( \{X^*, v(x), d(x), \gamma(x), U_0, U_1\} \) that satisfies the following conditions:

1. Firms’ optimization and free entry:
   \[ V(x) = 0 \geq V(x') \quad \text{for all } x \in X^* \text{ and } x' \in X. \]

2. Workers’ optimization: For all \( x \in X \) and \( i = 0, 1 \),
   \[ U_i \geq \frac{E[\hat{v}(x)]}{d(x)} \quad \text{and} \quad [1 - i - \gamma(x)]N(d(x))\left[U_i - \frac{E[\hat{v}(x)]}{d(x)}\right] = 0. \]

3. Aggregate consistency: For \( i = 0, 1 \),
   \[ \sum_{x \in X^*} [1 - i - \gamma(x)]N(d(x))v(x) = 1 - i - \mu. \]

4. Balanced expectations: If \( U_0 = U_i \), then \( \gamma(x) = \mu \) for all \( x \in \mathbb{R}_+ \setminus X^* \).

Condition 1 requires that posted announcements maximize firms’ profits and firms’ profits are zero. Condition 2 establishes that workers apply for the vacancies that maximize their utility and imposes the constraint that a vacancy can attract applicants of a given type only if it maximizes their utility. Condition 3 guarantees that the masses of applicants of a given type add up to the exogenous total mass of workers of that type. Under conditions 1–3, it is possible to uniquely determine the composition of the pool of applicants for any posted announcement except for out-of-equilibrium wages posted when low- and high-productivity applicants obtain the same utility, \( U_0 = U_i \). Condition 4 rules out equilibria with arbitrary out-of-equilibrium beliefs based on this indeterminacy. It requires that, if both worker types are indifferent between an out-of-equilibrium posted wage \( x \) and some equilibrium announcement, firms will expect \( x \) to attract a proportion \( \mu \) of high-productivity applicants.

B. Some Intermediate Results

The following results greatly simplify the analysis.

Lemma 1.
1. A posted wage \( x \) attracts a demand \( d(x) = \max\{1, x/U_0\} \).
2. Wage bargaining attracts a demand \( d(x, \omega) = \max\{1, \beta x_1/U_i\} \).
3. Vacancies offering a posted wage \( x > U_0 \) feature \( \gamma(x) = 0 \) if \( U_i > U_0 \) and \( \gamma(x) = \mu \) if \( U_i = U_0 \).
4. The set of equilibrium announcements \( X^* \) can include at most one posted wage.
5. When \( x \in X^* \), posted wages can attract only low-productivity applicants.

In equilibrium the demand for each vacancy will adjust so as to guarantee that the applying workers obtain their equilibrium utility. Posted wages give the same utility to both types of applicants. However, bargained wages are increasing in the worker’s productivity, so they can give higher utility to high-productivity workers than to low-productivity ones. Thus bargained wages attract high-productivity workers first, and low-productivity workers are the first to be attracted to vacancies with a posted wage. This explains why demands for vacancies can be related to the equilibrium utilities of low- and high-productivity workers (parts 1 and 2 of the lemma). Moreover, vacancies offering a posted wage render a utility of at most \( U_0 \); thus high-productivity workers never opt for wage posting when \( U_1 > U_0 \) (part 3). The wage-invariant compositions implied by part 3 together with the properties of the \( q(\cdot) \) function under assumption 2 make firms’ profits a strictly quasi-concave function of the posted wage; therefore, conditional on posting a wage, the solution to the firms’ optimization problem is unique (part 4). Finally, it turns out that the presence of wage bargaining in equilibrium implies \( U_1 > U_0 \), which, by part 3, makes high-productivity workers not interested in posted wages (part 5).

Lemma 1 already shows the disadvantage of wage posting vis-à-vis bargaining in terms of attracting high-productivity applicants. This adverse selection problem plays an important role in the analysis below.

IV. Characterization of Equilibria

Lemma 1 leaves us with three possible equilibrium regimes: (i) pure posting, where all firms post a wage, say \( w_p \); (ii) pure bargaining, where all firms announce bargaining; and (iii) a mixed regime, where some firms post a wage, say \( w_m \), and others announce bargaining. Actually, depending on the application decisions of low-productivity workers, we can distinguish two cases within the mixed regime: (a) semiseparation, where low-productivity workers are indifferent between the two existing types of vacancies; and (b) full separation, where they strictly prefer the vacancies with a posted wage. The emergence of one equilibrium or another is driven by the interaction between the adverse selection prob-

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\[^{16}\text{This differs from the model in Burdett and Mortensen (1998), where the equilibrium distribution of posted wages is continuous. In those models, firms can hire an unlimited number of workers, and the continuum of equilibrium wages results from the trade-off between raising the number of workers and reducing the wage per worker. In directed search models, posted wages directly affect firms’ hiring probability } q(\cdot), \text{ whose properties, under assumption 2, ensure the strict quasi concavity of firms’ profits, which rules out multiplicity.}\]
lem of wage posting and the standard search inefficiencies related to bargaining.

A. When Does Each Equilibrium Arise?

The critical value $\beta^* = \eta(d_p)$, where $d_p$ is the demand for a vacancy in the pure posting equilibrium, plays a central role in the next proposition. This is the value of the workers’ bargaining power that satisfies the well-known condition first derived by Hosios (1990), under which the search inefficiencies related to bargaining are nil (at least in a pure bargaining equilibrium). The larger the distance between $\beta$ and $\beta^*$, the greater the importance of search inefficiencies. The following proposition characterizes when each equilibrium arises.

**Proposition 1.** There are some critical values with $0 < p < \beta^* < p' < f' < 1$ and $f < s < b < \beta^* < b' < s' < f'$ such that pure posting is an equilibrium for values of the workers’ bargaining power $\beta \in (p, p')]$: pure bargaining is an equilibrium for $\beta \in [b, b')$: semiseparation is an equilibrium for $\beta \in [s, b) \cup (b', s']$: and full separation is an equilibrium for $\beta \in [f, s) \cup (s', f')$. Thus an equilibrium always exists and equilibria with and without bargaining coexist for some values of $\beta$. In general, $p (p')$ cannot be compared with $s$ and $b (s' and b')$.

For values of the workers’ bargaining power $\beta$ at or in the proximity of $\beta^*$, pure bargaining is an equilibrium and pure posting is not. When $\beta$ moves sufficiently farther away from $\beta^*$, pure bargaining ceases to be an equilibrium, giving way first to semiseparation and then to full separation, which are equilibria in which vacancies offering bargained wages and posted wages coexist. For sufficiently extreme values of $\beta$, equilibria involving bargaining cease to exist. When search inefficiencies are sufficiently severe, pure posting is the unique equilibrium. At some intermediate levels of the search inefficiencies, pure posting and one of the equilibria with bargaining coexist. The possibility of multiple equilibria is due to a negative externality that wage bargaining imposes on the firms that post a wage. By attracting all high-productivity workers, the presence of a positive mass of firms that bargain their wages causes a discrete fall in the profitability of posting a wage, relative to the situation in which no positive mass of firms opt for bargaining (recall lemma 1).

We can further illustrate proposition 1 using some diagrams. We focus on the cases of pure posting, pure bargaining, and full separation equilibria. From (3), the net profit from a vacancy with demand $d$ and a posted wage $w$ can be written as $V_i = q(d)[E_x(\gamma) - w] - c$, where $\gamma$ indicates the probability of hiring a high-productivity worker (given by part 3 of lemma 1). Part 4 of lemma 1 implies that iso-profit curves are increasing and concave in the $(d, w)$ plane. Higher levels of profits are
reached by moving southeast as well as by increasing $\gamma$. Workers' indifference curves have the form $w/d = U$ and so are rays from the origin with slope $U$. By parts 1 and 2 of lemma 1, the demands for vacancies with posted and bargained wages always lie on the indifference curves of levels $U_0$ and $U_1$, respectively. An optimal posted wage corresponds to the unique tangency between the relevant iso-profit curve and the ray of slope $U_0$.

In a pure posting equilibrium, we have $U_0 = U_1 = U_\gamma$ where $U_\gamma$ can be pinned down by noting that, by free entry, the equilibrium must lie on the iso-profit curve of level zero for $\gamma = \mu$ (point $A$ in fig. 1). For this to be an equilibrium, firms must not profit by offering to bargain. Such a deviation would attract high-productivity workers only and would get a demand $d(x_{\infty}) = \beta \gamma_1 / U_1$, corresponding to the point on the workers' equilibrium indifference line at which the wage is $\beta \gamma_1$. In figure 1, pure posting is an equilibrium if $\beta \gamma_1$ is either smaller than the wage at point $B$ or larger than the wage at point $C$. Intuitively, for a very low $\beta$ the deviation is not profitable because it would attract too few high-productivity workers, whereas for a very high $\beta$ the deviation is not profitable because hiring a worker would be too expensive.
The equilibrium demand for vacancies at firms that offer to bargain in a pure bargaining equilibrium is \( d_0 \), which is the demand on the iso-profit curve of level zero for \( \gamma = \mu \) and a wage \( \beta E_s(\tilde{v}) \) (i.e., at point \( A \) in fig. 2). For this to be an equilibrium, no firm must profit by posting a wage. Since wage posting would attract just low-productivity workers, pure bargaining is an equilibrium if the iso-profit curve of level zero for \( \gamma = 0 \) lies entirely below the ray with slope \( \gamma \) (as \( \gamma \) does in fig. 2). When \( \beta \) becomes close to either zero or one, \( \gamma \) falls because either wages are too low (for low \( \beta \)) or demand is too high (for high \( \beta \)). Eventually \( \gamma \) becomes low enough to cross the curve. Intuitively, low-productivity workers are so “cheap” to attract that a firm can profit by posting a wage, in which case pure bargaining ceases to be an equilibrium.

In a full separation equilibrium, the demand and wage of the firms that post a wage correspond to the point at which the iso-profit curve of level zero for \( \gamma = 0 \) is tangent to the ray from the origin that defines \( \gamma \) (point \( A \) in fig. 3). The demand and the expected wage corresponding to vacancies with bargained wages are given by the point on the iso-profit curve of level zero for \( \gamma = 1 \) at which the wage is \( \beta y_1 \) (point \( B \))
in the figure). Full separation is an equilibrium if (i) low-productivity workers do not prefer to negotiate their wage and (ii) high-productivity workers do not prefer a posted wage. In figure 3 both conditions are satisfied, since point $C$ lies below the $U_0$ ray and point $B$ is such that $U_1 > U_0$. Intuitively, mixed equilibria require that $\beta$ be neither too close to satisfy the Hosios condition—because otherwise bargaining would be so appealing to the low-productivity types that condition i would fail—nor too close to zero or one—because otherwise bargaining would no longer appeal to the high-productivity types and condition ii would fail.

B. Further Comparative Statics

In the next proposition we characterize the effects of a mean-preserving increase in the dispersion of workers' productivity (i.e., an increase in $y_1$ and a decrease in $y_0$ that keeps $E_\omega(\bar{y})$ constant).

Proposition 2. A mean-preserving increase in the dispersion of workers' productivity contracts the region in which pure posting is an equilibrium and expands the region in which pure bargaining and, more generally, equilibria with bargaining emerge.
Intuitively, increasing the dispersion in workers’ productivity amounts to worsening the adverse selection problem of wage posting. After this change, in a candidate pure posting equilibrium, a deviation to bargaining becomes more profitable because it involves a greater improvement in the expected productivity of the applicants. Similarly, in equilibria with bargaining, posting a wage now involves a greater fall in the expected productivity of the applicants (relative to that under wage bargaining) and, hence, is less likely to be dominant. So with greater productivity dispersion, the pure posting equilibrium is more difficult to sustain and bargaining equilibria are more likely to arise.

Next, we consider an increase in the proportion of high-productivity workers, $m$.

**Proposition 3.** Increasing the proportion of high-productivity workers expands the pure posting and pure bargaining regions, contracts the semiseparation region, and leaves the full separation region unaffected.

As the proportion of high-productivity workers in the population increases, sustaining pooling equilibria such as pure posting and pure bargaining becomes easier: the expected productivity of the pool of applicants improves, and the expected productivity of the applicants attracted by deviating from the candidate equilibrium remains unchanged. Thus deviations to bargaining from pure posting and to posting from pure bargaining become less profitable.

In the model, wage inequality is always greater in equilibria with bargaining than under pure posting. Proposition 2 implies that a small increase in the dispersion of workers’ productivity can lead to a sharp increase in wage inequality if the equilibrium shifts from pure posting to one of the equilibria with bargaining. A similar change can occur if high-productivity workers become scarcer in the labor force, since proposition 3 implies that the overall bargaining region (covering pure bargaining, semiseparation, and full separation equilibria) does not change with $m$ but the pure posting region shrinks when $m$ falls.

**V. Efficiency**

In this section we compare the social welfare of the various possible equilibria. Following the literature, we first identify social welfare with the sum of all firms’ and workers’ net income, which, since firms’ equilibrium profits are zero, can be expressed as the weighted sum of the equilibrium utilities of each worker type, $\mu U_i + (1 - \mu) U_w$. Bargaining can affect social welfare through two different margins. First, the standard search inefficiencies make aggregate net income fall, because they lead to either excessive vacancies (when $\beta < \beta^*$) or excessive unemployment (when $\beta > \beta^*$). Second, bargaining can promote (full or par-
tial) separation of low- and high-productivity workers, as in our mixed regimes. This can be a source of efficiency gains since it can help better reproduce the first-best allocation: with verifiable worker types, a social planner would set up two submarkets, one for high- and one for low-productivity workers, with different demand. The next proposition shows that the net effect is unambiguous.

**Proposition 4.** The pure posting allocation generates larger aggregate net income than any of the equilibria with bargaining.

In the pure bargaining and pure posting allocations, the market is not segmented. When search inefficiencies are absent ($\beta = \beta^*$), the pure bargaining allocation (which is an equilibrium in such a case) yields the same net income as the pure posting allocation. But as search inefficiencies increase, the welfare associated with the pure posting allocation remains constant, and the welfare generated by pure bargaining decreases. As search inefficiencies keep increasing, pure bargaining ceases to be an equilibrium, and first semiseparation and eventually full separation become equilibria (recall proposition 1), but aggregate welfare continuously declines. This does not mean that mixed regime allocations cannot dominate the pure posting allocation in terms of social welfare; they can for sufficiently low search inefficiencies. But the semi-separation and full separation equilibria can be sustained only for sufficiently large search inefficiencies, because otherwise bargaining is so attractive that low-productivity workers no longer accept a posted wage. Thus whenever bargaining emerges in equilibrium, its net welfare contribution is always negative.

Interestingly, the welfare losses induced by search inefficiencies can be so large that not only low- but also high-productivity workers are better off in a pure posting equilibrium than in an alternative equilibrium with bargaining. Indeed, we can prove the following proposition.

**Proposition 5.** Whenever the pure posting equilibrium and an equilibrium with bargaining coexist, the former is Pareto dominant.

The reason is that coexistence occurs only when search inefficiencies are sufficiently severe (recall proposition 1), so severe that either low wages (if $\beta < \beta^*$) or a high unemployment probability (if $\beta > \beta^*$) makes high-productivity workers worse off than in the pure posting equilibrium.

**VI. Conclusions**

We have presented a tractable directed search model in which firms compete for heterogeneous workers by announcing the wage-setting

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17 For example, full separation and the first-best allocation coincide if the Hosios condition holds in the bargaining segment of the market.
mechanism associated with their vacancies. Productivity becomes observable to firms after assessing their workers on the job, but it is not verifiable. So wages bargained after a match can be tailored to a worker’s productivity, whereas posted wages cannot. Firms’ decisions are driven by the trade-off between the possibility of using bargaining as a means to attract better applicants to a vacancy and the standard search inefficiencies that bargaining involves. A key prediction of our analysis is that more productive workers bargain their wages, whereas less productive workers accept a posted wage within the same industry, at least for some parameter values. Other predictions are that the incidence of wage bargaining should be higher in jobs and occupations in which the dispersion in workers’ productivity is more pronounced and workers’ skills are scarcer.

The evidence about these predictions is so far indirect. The challenge is to identify variation in wage-setting practices in the data. Unionized firms (as well as firms and agencies that belong to the government sector) are more likely to be constrained to offer wage structures that satisfy the “same job, same pay” principle (see Freeman and Medoff 1984) and, hence, forced to post wages. If this is the case, we would expect them to attract lower-productivity workers than firms in the same industry that can opt for bargaining. Indeed, the evidence in Hirsch (1991) and Booth (1995) suggests that unionized firms have lower productivity. Other studies have analyzed firms’ propensity to reward workers through promotions, bonuses, and permanent pay raises on the basis of subjective performance evaluation—practices likely to lead firms and workers to negotiate on pay. MacLeod and Parent (1999) find that these practices are pervasive in jobs that involve a large number of tasks (where the dispersion in workers’ productivity is likely to be larger) and more common among professionals, technicians, managers, and administrators (i.e., in occupations in which workers’ skills arguably have larger effects on productivity).

Another interesting implication of the analysis is that wage inequality is always greater in equilibria with bargaining than in a pure posting equilibrium. This suggests that changes in the wage-setting regime may explain time and cross-sectional variation in wage inequality. If the dispersion in workers’ productivity increases or if high-productivity workers become relatively scarcer, the labor market may switch from a pure posting equilibrium to one of the equilibria with bargaining, producing an increase in wage inequality. Processes like these might have contributed to the rise in wage inequality observed in the United States over the 1980s and 1990s (see, e.g., Juhn, Murphy, and Pierce 1993). The information technology revolution may have required skills that were (at least temporarily) scarce and dispersed. Some casual evidence suggests that firms’ compensation practices have indeed changed in a di-
rection consistent with this interpretation. From a cross-country perspective, the prevalence of predetermined wage structures in Japan (discussed by, among others, Klein [1992] and Baron and Kreps [1999]) suggests that Japanese firms are closer than U.S. firms to the compensation practices that characterize our pure posting equilibrium. The low level of wage inequality in Japan relative to the United States (see OECD 1996, table 3.1) is consistent with this interpretation.

Appendix

Proof of Lemma 1

Consider any given \( X^* \) and \( v(x) \). It is easy to see that workers’ optimization (equilibrium condition 2) and equation (2) necessarily imply that

\[
\frac{y_{i}}{y_{i}} U_{i} \leq U_{n} \leq U_{i}. \tag{A1}
\]

Notice that \( d(x) = 1 \) means that workers do not apply for vacancies that announce \( x \), whereas by the second part of equilibrium condition 2, \( d(x) > 1 \) requires \( E_i[\tilde{w}(x)] / d(x) = U_i \) for at least one of the worker types.

Part 1: Consider first a vacancy with \( x \in \mathbb{R}_+ \) and \( x \leq U_{i} \). In this case we necessarily have \( d(x) = 1 \), since the alternative \( d(x) > 1 \) would imply \( x / d(x) < U_{i} \), which would contradict equilibrium condition 2. Now, for \( x \in \mathbb{R}_+ \) and \( x > U_{i} \), we can prove by contradiction that \( d(x) = x / U_{i} \). In fact, if \( d(x) < x / U_{i} \), condition 2 would be violated for \( i = 0 \). If \( d(x) > x / U_{i} > 1 \), (A1) implies \( U_i \geq U_{i} > x / d(x) \), which, given condition 2, contradicts \( d(x) > 1 \).

Part 2: If \( \beta_{y_{i}} \leq U_{i} \), then (A1) and condition 2 immediately imply that \( d(x_{y_{i}}) = 1 \). If \( \beta_{y_{i}} > U_{i} \), we can prove by contradiction that \( d(x_{y_{i}}) = \beta_{y_{i}} / U_{i} \). In fact, if \( d(x_{y_{i}}) < \beta_{y_{i}} / U_{i} \), the first part of condition 2 would be violated for \( i = 1 \). If \( d(x_{y_{i}}) > \beta_{y_{i}} / U_{i} > 1 \), (A1) would also imply \( d(x_{y_{i}}) > \beta_{y_{i}} / U_{i} \), which, given condition 2, would contradict \( d(x_{y_{i}}) > 1 \).

Part 3: If \( x > U_{i} \), \( E_i[\tilde{w}(x)] = E_i[\tilde{w}(x)] = x \), by (2), and \( d(x) = x / U_{i} > 1 \), by part 1 of this lemma. Then if \( U_i > U_{i} \), condition 2 for \( i = 1 \) implies that no high-productivity worker will apply for such vacancies, \( \gamma(x) = 0 \). If \( U_i = U_{i} \), \( d(x) = x / U_{i} \) implies that both worker types might be willing to apply. If \( x \in \mathbb{R}_- \cap X^* \), the result that \( \gamma(x) = \mu \) can be proved by contradiction. Suppose first that \( \gamma(x) < \mu \). Then part 1 of this lemma and condition 4 would imply that an alternative announcement \( x' \in \mathbb{R}_- \setminus X^* \), arbitrarily close to \( x \), would feature \( \gamma(x') = \mu \) and thus, thanks to the discrete improvement in the composition of the pool of applicants, yield \( V(x') > V(x) \), which contradicts condition 1. Suppose alternatively that \( \gamma(x) > \mu \). Then condition 3 for \( i = 1 \) implies that there must be at least one other announcement \( x' \in X^* \) with

\[\]
utility. The posted wage must solve

$$\gamma(x) < \mu.$$ Moreover, \(x^*\) cannot be \(x^2\), since part 2 of this lemma and the second part of condition 2 for \(i = 0\) imply \(\gamma(x) = 1\) when \(U_i = U\). But if we have \(x^* \in \mathbb{R}_+\), with \(\gamma(x^*) < \mu\) in the set of equilibrium announcements, then the same contradiction that we showed in the previous subcase arises.

Part 4: Part 1 of this lemma implies \(d(x) = x/U\) for \(x > U_i\), and part 3 implies \(\gamma(x) = \gamma\) for all \(x \in \mathbb{R}_+\). From here, if we particularize (3) for \(x > U_i\) and derive with respect to \(x \in \mathbb{R}_+\), we obtain

$$V' = q(d(x))\left(E_u(y) - x\right) \frac{1}{U_0} - q(d(x)).$$

Now, if we substitute \(x/d(x)\) for \(U_0\) and use (1) to write \(q'(d)\) as

$$\frac{\eta(d)}{1 - \eta(d)} \cdot \frac{q(d)}{d},$$

we can obtain

$$V'(x) = \frac{q(d(x))}{1 - \eta(d)} \left[ \eta(d(x)) \left( \frac{E_u(y) - x}{x} - 1 \right) \right].$$ (A2)

where the term out of the brackets is always positive. The sign of the term in brackets is generally ambiguous, but such a term is decreasing in \(x\) since \(\eta(d)\) is decreasing in \(d\) by assumption 2 and \(d(x)\) is increasing in \(x\) by part 1 of this lemma. Thus, as \(x\) increases over the range \(x > U_i\), the sign of \(V'(x)\) can shift from positive to negative at most once, which implies quasi concavity. Moreover, \(\lim_{x \to \infty} V(x) = -\infty\) and \(\lim_{x \to \infty} V(x) = -\infty\). Thus, over the range \(x > U_i\), \(V(x)\) can be either monotonically decreasing or first increasing and then decreasing. In the first case, no posted wage can yield zero profits, so the stated result is true. In the second case, there is a unique critical point satisfying \(V'(x) = 0\) that maximizes \(V(x)\). This is the candidate to belong to \(X^*\).

Part 5: We prove that \(x_0 \in X^*\) implies that \(U_0 < U_i\). Then the result emerges as a corollary of part 3 of this lemma. If \(X^*\) includes only wage bargaining, \(U_0 < U_i\) follows immediately from the fact that \(\beta_{y_0} > \beta_{y_0};\) see (2). If \(X^*\) also includes a posted wage, part 4 of this lemma implies that this must be unique, say \(w \in \mathbb{R}_+\). In this case, having \(U_0 = U_i\) would lead to a contradiction. If \(U_0 = U_i\), part 3 of this lemma would imply \(\gamma(w) = \mu\) and then \(\gamma(x) = \mu\) by condition 3. But then condition 2 would imply that \(\beta_{y_0}/d(x) = U_0\) and \(\beta_{y_1}/d(x) = U_i\), which contradicts \(U_0 = U_i\), since \(y_0 < y_1\). QED

**Proof of Proposition 1**

We prove this result by examining the necessary and sufficient conditions for the existence of each of the possible equilibrium regimes.

i. Pure posting: In a pure posting equilibrium, all firms with vacancies post a wage \(w_p\), get a demand \(d_p\), and attract high-productivity applicants in the same proportion \(\mu\) as they exist in the labor force. Both worker types attain the same utility \(U_p = w_p/d_p\). The posted wage must solve

$$w_p = \arg \max_{x \in \mathbb{R}_+} q \left( \frac{x}{U_p} \right) \left( E_u(y) - x \right) - c, \tag{X1}$$

where we use part 1 of lemma 1 and adopt the convention that \(q(d) = 0\) when \(d < 0\). Moreover, the firms’ zero-profit condition implies \(w_p > U_p\). Then \(w_p\) satisfies
a first-order condition, which, from \((A2)\), can be written as
\[
w_p = \eta(d_p)E_s(\tilde{y}). \tag{A3}\]
So firms’ zero-profit condition simplifies to
\[
q(d_p)[1 - \eta(d_p)]E_s(\tilde{y}) = \epsilon, \tag{A4}
\]
which uniquely determines \(d_p\) and, recursively, \(w_p\) and \(U_p\). Pure posting is an equilibrium if \(V(x_o) \leq 0\). Given part 2 of lemma 1, \(d(x_o) = \max (1, \beta y_1/U_p)\). Moreover, \(\gamma(x_o) = 1\) since with this \(d(x_o)\) low-productivity workers would attain an expected income lower than \(U_p\) if they applied for this vacancy. Thus the condition \(V(x_o) \leq 0\) reads as
\[
q\left(\frac{\beta y_1}{U_p}\right) (1 - \beta) y_1 - \epsilon \leq 0.
\]
Given that \((A3)\) implies \(U_p = \eta(d_p)E_s(\tilde{y})/d_o\) this condition can be rewritten as
\[
P(\beta) = (1 - \beta) q\left(\frac{\beta y_1 d_p}{\eta(d_p)E_s(\tilde{y})}\right) \leq \frac{\epsilon}{y_1}, \tag{A5}
\]
The function \(P(\beta)\) can be shown to be nonnegative and quasi-concave and to reach a maximum at \(\hat{\beta} \leq \beta^* = \eta(d_p)\). Moreover, \(\hat{\beta} = \beta^*\) and \(P(\beta^*) = c/y_1\) in the limit case in which \(\mu = 1\). Finally, \(P(\beta)\) is decreasing in \(\mu\). Thus, as \(\mu\) decreases, \(P(\beta)\) shifts upward and gives rise to an interval \((\mu, \mu') \subseteq (0, 1)\) of values of the bargaining power \(\hat{\beta}\) in which \(P(\hat{\beta}) > c/y_1\) (see fig. A1). Out of that range, (A5) holds and, hence, pure posting is an equilibrium. Since \(\hat{\beta} \leq \beta^*\) and \(\beta^*\) is an increasing function of \(\mu\), we also have \(\hat{\beta} \leq \beta^* \leq \beta^*|_{\mu=1}\), which implies that \(\beta^* \in (\mu, \mu')\).

ii. Pure bargaining: In a pure bargaining equilibrium, all firms announce bargaining, get a demand \(d_o\), and attract high-productivity applicants in the same proportion \(\mu\) as they exist in the labor force. Given that bargained wages amount to a fraction \(\hat{\beta}\) of workers’ output, the utility of high-productivity workers, \(U_{b_1} = \beta y_1/d_o\), is greater than that of low-productivity workers, \(U_{b_0} = \beta y_0/d_o\). Firms’ zero-profit condition is
\[
q(d_o)(1 - \beta)E_s(\tilde{y}) = \epsilon, \tag{A6}
\]
which uniquely determines \(d_o\). Firms’ optimization requires \(V(x) \leq 0\) for all \(x \in \mathbb{R}_+\). The best wage that a firm can post is
\[
w' = \arg \max_{x \in \mathbb{R}_+} \left(\frac{x}{U_{b_0}}\right) (y_o - x) - \epsilon,
\]
which is always larger than \(U_{b_0}\) (since \(U_{b_0} < y_o\)). From (A2), the corresponding first-order condition becomes
\[
w' = \eta(d') y_o, \tag{A7}
\]
where \(d' = w'/U_{b_0}\) is the demand that \(w'\) would generate according to part 1 of lemma 1. But since \(U_{b_0} = \beta y_0/d_o\) we can rewrite (A7) as
\[
\beta d' = \eta(d') d_o, \tag{A8}
\]
which uniquely determines \(d'\), since \(d_o\) is already determined by (A6). With this notation, the condition for the absence of a profitable deviation,
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Fig. A1.—The prevalence of pure posting

\[ q(w'U_m)(y_0 - w') \leq \epsilon, \] is equivalent to

\[ B(\beta) = \frac{1 - \eta(d')q(d')}{y_0} \leq \frac{\epsilon}{y_0}. \] (A9)

The function \( B(\beta) \) can be shown to be nonnegative and quasi-convex and to reach a minimum at \( \beta^* = \eta(d_0) \). Moreover, \( B(\beta^*) = c/y_0 \) in the limit case in which \( \mu = 0 \), and \( B(\beta) \) is decreasing in \( \mu \). Thus, as \( \mu \) increases, \( B(\beta) \) shifts downward and gives rise to a range \([b, b'] \subseteq (0, 1)\) of values of \( \beta \) that contains \( \beta^* \) and in which (A9) holds (see fig. A2). Finally, before we proceed, notice that (A4) and (A6) imply that when 

\[ d_p = d_b \] when \( \beta = \beta^* \), which will be used several times below.

iii. Mixed regimes: In a mixed regime, some firms post a wage \( w_{m^*} \), receive a demand \( d_{m^*} \), and, given part 5 in lemma 1, attract just low-productivity workers, \( \gamma(w_{m^*}) = 0 \). The remaining firms announce bargaining and receive a demand \( d_{b^*} \). Thus low- and high-productivity workers obtain utility \( U_{m^*} = w_{m^*}/d_{m^*} \) and \( U_{b^*} = \beta y_1/d_{b^*} \), respectively. To determine \( w_{m^*} \) we can use (A2) to impose the first-order condition that yields

\[ w_{m^*} = \eta(d_{m^*})y_0, \]

and the zero-profit condition reads as

\[ q(d_{m^*})(1 - \eta(d_{m^*}))y_0 = \epsilon, \] (A10)

which uniquely determines \( d_{m^*} \). Two equilibrium variables remain to be fixed:
the demand for the vacancies offering bargained wages, \( d_{a1} \), and the proportion of high-productivity workers among their applicants, say \( \gamma \), which must lie in the interval \((\mu, 1]\) by condition 3. The zero-profit condition for the firms announcing bargaining requires

\[
q(d_{a1})(1 - \beta)E(\gamma) = c, \quad (A11)
\]

where \( \gamma \in (\mu, 1] \) must be compatible with workers’ optimization. Then two mutually exclusive possibilities arise.

a. Semiseparation: In this case we must have

\[
U_{a1} = \frac{\beta y_0}{d_{a1}} < \frac{\beta y_1}{d_{a1}}, \quad (A12)
\]

and the value \( \gamma \) that solves (A11) for \( d_{a1} = \beta y_0 / U_{a0} \) must lie in the interval \((\mu, 1]\).

b. Full separation: In this case we must have

\[
\frac{\beta y_0}{d_{a1}} < U_{a0} \leq \frac{\beta y_1}{d_{a1}}, \quad (A13)
\]

where \( d_{a1} \) solves (A11) for \( \gamma = 1 \).

In a semiseparation equilibrium, firms that announce bargaining behave like firms in the pure bargaining equilibrium of an “artificial” economy in which the proportion of high-productivity workers is some (endogenously determined)
\[ \gamma \in (\mu, 1) \] rather than \( \mu \). Since the firms that post the wage \( w \) must also break even, condition (A9) for the existence of a pure bargaining equilibrium must hold with equality in the artificial economy. Then we can identify the values of \( \beta \) for which a semiseparation equilibrium exists by varying the proportion of high-productivity workers in the interval \((\mu, 1]\) and then considering the intersections between the horizontal line \( c/\gamma_0 \) and the graph of \( B(\beta) \) for the corresponding artificial economy. The values of \( \beta \) that lead to semiseparation equilibria with \( \gamma = 1 \) correspond to the left and right intersections between the horizontal line \( c/\gamma_0 \) and the graph of \( B(\beta) \) (denoted as \( s \) and \( s' \) in fig. A2). Conversely, the pure bargaining equilibria that emerge with \( \beta = b \) and \( \beta = b' \) can be seen as degenerated semiseparation equilibria with \( \gamma = \mu \). Since increasing \( \mu \) shifts \( B(\beta) \) downward, it is clear that \( s < b \) and \( b' < s' \), and we can conclude that a semiseparation equilibrium exists for all \( \beta \in (s, b) \cup (b', s') \).

To characterize the region in which full separation is an equilibrium, we need to check condition (A13). For \( \beta = s, s' \), the semiseparation equilibrium involves \( \gamma = 1 \). But (A13) fails to hold just because \( U_{\alpha_1} = \beta \gamma_0/d_{\alpha_1} \), and so the first inequality holds with equality. However, the value of \( d_{\alpha_1} \) determined by (A11) for \( \gamma = 1 \) varies with \( \beta \) in such a way that \( \beta \gamma_0/d_{\alpha_1} \) is a quasi-concave function of \( \beta \) with a maximum at \( \beta |_{\alpha_1} = [s, s'] \). In contrast, \( U_{\alpha_1} \) is determined in the posting segment of the market and, thus, is independent of \( \beta \). Therefore, for values of \( \beta \) right below \( s \) or right above \( s' \), we have \( \beta \gamma_0/d_{\alpha_1} < U_{\alpha_1} \) (whereas, by continuity, we still have \( U_{\alpha_1} = \beta \gamma_0/d_{\alpha_1} \)). Thus a full separation equilibrium exists. However, as \( \beta \) moves toward the extremes, \( U_{\alpha_1} = \beta \gamma_1/d_{\alpha_1} \) becomes closer and eventually equal to \( U_{\alpha_1} \). In particular, we get to \( \beta \gamma_1/d_{\alpha_1} = U_{\alpha_1} \) when, in the artificial economy with \( \mu = 0 \), the condition (A5) for the existence of pure posting holds with equality. Graphically, this occurs at the intersections \( \bar{\beta} = \bar{f} \) and \( \bar{\beta} = \bar{f}' \) between the horizontal line \( c/\gamma_1 \) and the graph of \( P(\beta)_|_{\alpha_1} \) (see fig. A2).

Finally, notice that since \( P(\beta) \) shifts downward as \( \mu \) increases, we have that \( f < \bar{f} \) and \( \bar{f}' < f' \). So the full range \([0, 1]\) of possible values of \( \beta \) is covered by one equilibrium or another, and actually there is coexistence between pure posting and one of the equilibria with bargaining for values of \( \beta \) in the intervals \([f, \bar{f}]\) and \([\bar{f}', f']\). QED

Proof of Proposition 2

Consider the experiment of increasing \( y_1 \) and decreasing \( y_0 \) without changing workers’ average productivity \( E_y(\bar{\gamma}) \). It follows from (A4) that \( d_1 \) remains unchanged. Thus, in inequality (A5) for the existence of pure posting, the left-hand side rises and the right-hand side falls, so the condition is less likely to hold. In terms of figure A1, \( P(\beta) \) shifts upward and the line \( c/\gamma_1 \) shifts downward, so the interval \((p, p')\) expands. Also, it follows from (A6) that \( d_i \) remains unchanged. Thus, in (A9), the left-hand side remains constant and the right-hand side increases, so the condition for the existence of pure bargaining is more likely to hold. In terms of figure A2, \( B(\beta) \) remains unchanged and the line \( c/\gamma_0 \) moves upward, so the interval \((b, b')\) expands. For similar reasons, the thresholds \( s \) and \( f \) move toward the left, and \( s' \) and \( f' \) move toward the right. QED

Proof of Proposition 3

In terms of figures A1 and A2, increasing \( \mu \) shifts down the graphs of both \( P(\beta) \) and \( B(\beta) \). So the interval \((p, p')\) contracts and the interval \((b, b')\) expands,
which immediately means that pure posting and pure bargaining are sustainable over larger sets of values of $\beta$. On the other hand, the thresholds $f, f', s,$ and $s'$ remain unaffected since the graphs of $P(\beta)$ and $B(\beta)$ in the artificial economies with $\mu = 0$ and $\mu = 1$ do not vary with $\mu$. Thus the ranges of values of $\beta$ in which full separation is an equilibrium are unchanged, and the ranges in which semiseparation emerges shrink because of the expansion of the interval $(b, b')$. QED

**Proof of Proposition 4**

Let $W_j$ denote the social welfare levels attained at the various possible allocations: pure posting ($j = PP$), pure bargaining ($j = PB$), semiseparation ($j = SS$), and full separation ($j = FS$). Consider the function $G(\beta, \mu)$, which yields $W_{\text{eq}}$ for different values of $\beta$ and $\mu$:

\[ G(\beta, \mu) = \mu U_{\text{eq}} + (1 - \mu)U_{\text{eq}} = \frac{\beta E_\mu(\gamma)}{d_\mu}, \tag{A14} \]

where $d_\mu$ is implicitly determined by (A6). This function can be shown to be quasi-concave in $\beta$ and to reach a maximum at $\beta = \beta^* = \eta(d_\mu)$. Such a maximum corresponds to point $A$ in figure A3. When $\beta = \beta^*$, the pure posting and pure bargaining allocations coincide in terms of both the average wage paid and workers’ demand (see the proof of proposition 1). Thus $W_{\text{pp}} = W_{\text{eq}}$ at $\beta = \beta^*$. But the pure posting allocation and then $W_{\text{pp}}$ are independent of $\beta$, and $W_{\text{pp}}$ falls as $\beta$ moves away from $\beta^*$. So we have $W_{\text{pp}} > W_{\text{eq}}$ for all $\beta \neq \beta^*$.

In the semiseparation regime, workers’ utilities, and thus social welfare $W_{\text{ss}}$,
are independent of $\beta$ because $U_{w_0}$ is determined in the posting segment of the labor market, and $U_{w_0} = \frac{y_t}{y_0}U_{w_0}$ by (A12). We have already noticed that the semiseparation allocation degenerates into a pure bargaining allocation at the critical values $\tilde{\beta} = b, b'$. Hence, $W_{s} = G(b, \mu) = G(b', \mu)$ for all $\tilde{\beta}$ (see fig. A3).

Finally, in the full separation regime, the bargaining segment of the labor market attracts all the high-productivity workers, and it functions like pure bargaining in an economy with $\mu = 1$ so $U_{w_0} = G(\tilde{\beta}, 1)$. The posting segment of the market instead attracts all the low-productivity workers, and it functions like pure posting in an economy with $\mu = 0$ so $U_{w_0} = \max_{\gamma} G(\tilde{\beta}, 0)$. Thus $W_{s} = G(b, 1) + (1 - \mu) \max_{\gamma} G(\tilde{\beta}, 0)$. Importantly, at $\tilde{\beta} = s, s'$ we have $W_{s} = W_{s'}$, since the semiseparation allocation involves $\gamma(x_0) = 1$. Moreover, since $G(\tilde{\beta}, 1)$ is strictly quasi-concave in $\tilde{\beta}$ and reaches a maximum at $\beta^{*}\vert_{s} = (s, s')$, it follows that $W_{s} < W_{s'}$ for all $\tilde{\beta} \in [f, s] \cup (s', f']$.

Given proposition 1, the solid sections of the curves depicted in figure A3 identify the values of $\beta$ for which the corresponding allocation can be sustained as an equilibrium. The result stated in the proposition follows. QED

Proof of Proposition 5

Since $U_{t} \geq U_{0}$, it is enough to show that when pure posting coexists with either full separation, semiseparation, or pure bargaining, high-productivity workers are not better off in the equilibrium with bargaining than in a pure posting equilibrium. Recall that pure posting is an equilibrium if and only if

$$g(d)(1 - \beta)\gamma_{1} \leq \epsilon_{1}, \quad (A15)$$

where $d = \max (1, \beta_{y_{1}}/U_{j})$ from part 2 in lemma 1, and $U_{j}$ is workers’ utility in a pure posting equilibrium. In a bargaining equilibrium, high-productivity workers earn $U_{i} = \beta y_{1}/d_{i}$, where $d_{i}$ solves

$$g(d_{i})(1 - \beta)E_{t}(\gamma_{1}) = \epsilon_{1}, \quad (A16)$$

and $\gamma = \mu$ in pure bargaining, $\gamma \in (\mu, 1]$ in semiseparation, and $\gamma = 1$ in full separation. Clearly, (A15) and (A16) imply that $d_{i} \geq d \geq d_{j}$ for all $\gamma \in [\mu, 1]$. But then it follows from the definition of $d$ that $d_{i} \geq d_{y_{1}}/U_{j}$, and so $U_{i} \geq U_{j} = \beta y_{1}/d_{i}$ for all $\gamma \in [\mu, 1]$. QED

References


