Borrowing from employees: Wage dynamics with financial constraints*

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Abstract

We analyze how the financial conditions of the firm affect the compensation structure of workers, the size of the firm, and its dynamics. Firms that are financially constrained offer long-term wage contracts characterized by an increasing wage profile, that is, they pay lower wages today in exchange of higher future wages, effectively borrowing from their employees. Because constrained firms also operate at a suboptimal scale, which then increases gradually over time, we have that younger and smaller firms grow faster and pay lower wages.

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1 Introduction

The empirical evidence suggests that firms differ in the wages they pay. Brown and Medoff (1989) document a positive association between wages and the size of the employer. Krueger and Summers (1988) find persistent differences in wages across industries. Abowd and Kramarz (2000) document that employers’ characteristics account for a relevant part of the dispersion of wages. These findings are often regarded as suggesting the existence of some form of labor market imperfections. In this paper we show that financial market imperfections can also contribute to generate a dependence of wages from the characteristics of the employers.

Our interest in understanding the impact of financial factors on wages is motivated by a set of regularities relating the financial conditions of the firm to its size, dynamics and the wages it pays. As discussed in Cooley and Quadrini (2001), the view that emerges from the financial literature is that small and fast growing firms face tighter financial constraints, either in the form of lower ability to raise funds or in the form of higher cost of funds. It also appears that financially distressed firms pay lower wages. See Nickell and Nicolitsas (1999), Hanka (1998) and Blanchflower, Oswald, and Garrett (1990).

We study a model in which workers are ex-ante identical and the labor market is perfectly competitive. However, firms may face different financial conditions along the life-cycle. If the investment that the firm can finance with external investors is limited—that is, the firm is financially constrained—it will find optimal to sign long-
term contracts with their employees, offering wage profiles that are increasing with the tenure of the worker. By paying lower wages today in exchange of higher future wages, the firm *implicitly* borrows its employees. Overtime, the reinvestment of profits relaxes the financial constraints and the firm grows in size. Thus, the model predicts that wages increase with the age and size of the firm. At the same time, because constrained firms grow in size, we also have that growing firms pay lower wages.

The ability of the firm to borrow from workers beyond what it can borrow from external investors is made possible by a form of implicit “collateral” available only to workers. In the event of repudiation, external investors can punish the firm only by confiscating its physical assets, which represent the only collateral that the firm can use to raise funds in financial markets. Workers, instead, can also punish the firm by withdrawing their effort and quitting. If the replacement of workers is costly—due to recruiting costs, training expenses and the loss of job specific human capital—workers have a credible punishment tool in the event of repudiation that is not available to investors. This can be used to sustain the long-term wage contracts between the firm and its workers.

There is both direct and indirect evidence that firms do borrow from their employees. In some cases, firms borrow explicitly from their workers.¹ In others, the

¹An example is Energy Services Group International, an energy-services engineering and construction company in Williamsburg, VA. The company got a major new contract from an electric utility in Florida but it could not persuade banks to lend any more money. Only employees came forward with investments that ranged from $200 to $74,000 in exchange of promissory notes. See Inc. Magazine,
borrowing is implicit in the compensation structure offered to employees, as in our model. For example, the widespread use of stock options and/or stock grants to ordinary workers—whose effort, when individually considered, is likely to have a negligible effect on the overall value of the firm—can hardly be justified as a way to provide better incentives to workers. This view is also expressed in Hall and Murphy (2003). Most likely, with the use of stock options, firms delay the cash compensation of their employees and effectively borrow from them. In accordance with this interpretation, Core and Guay (2001) provide direct evidence that the use of stock options is more common among financially constrained firms.

The plan of the paper is as follows. The next section describes the model and Section 3 characterizes the wage policy and dynamics of the firm. Section 4 discusses the implementation of the long-term wage contract and Section 5 outlines possible extensions for future research.

2 The model

Consider a risk-neutral infinitely lived entrepreneur with initial wealth $a_0$ and with lifetime utility $\sum_{t=0}^{\infty} \beta^t c_t$, where $\beta$ is the intertemporal discount factor and $c_t$ is consumption.

The entrepreneur has the managerial skills to run an investment project that generates revenues $y = A \cdot N$. The variable $N$ denotes the number of hired workers and

$A$ is a constant. The project is subject to the capacity constraint $N \leq \bar{N}$.

The employment of each worker requires two types of fixed investment: fungible investment, $\kappa_f$, and worker-specific investment, $\kappa_w$. The first type of investment, $\kappa_f$, has an external value and can be resold at no cost. The second type, $\kappa_w$, represents the cost incurred by the firm in recruiting and training a new worker for the specific job. The worker-specific investment is lost if the worker quits or is fired. As we will see later, this second component of the investment plays a central role for the non-renegotiation of the wage contract. Let $\kappa \equiv \kappa_f + \kappa_w$ denote the total per-worker investment. Then the total capital accumulated at the end of time $t$ by a firm created at time zero is $\kappa \sum_{\tau=0}^{t} n_\tau$, where $n_\tau$ is the number of workers hired at time $\tau$ (and start producing at time $\tau + 1$). The output produced by the firm at $t + 1$ is $A \sum_{\tau=0}^{t} n_\tau$.

The investment $\kappa$ necessary to employ a worker creates a financial need for the firm. Part of the funds are raised from financial markets where there is a continuum of atomistic investors supplying funds at the fixed rate $r$. Using the renegotiation idea of Hart and Moore (1994) and Kiyotaki and Moore (1997), the entrepreneur can borrow only the amount that can be collateralized. In case of liquidation, investors can seize only the fungible capital $\kappa_f$. Since the collateral must also guarantee the interests on the loan, the firm can borrow at most $\bar{\kappa}_f = \kappa_f/(1+r)$ per each worker. The borrowing limit, then, can be written as $b_t \leq \bar{\kappa}_f \sum_{\tau=0}^{t} n_\tau$, where $b_t$ denotes the debt level contracted at time $t$. We assume that $1/(1+r) \geq \beta$. This guarantees that external financing is never dominated by internal financing.
Workers are infinitely lived with lifetime utility \( \sum_{t=0}^{\infty} \beta^t U(c_t) \), where \( \beta \) is the discount factor, and the function \( U(c_t) \) is strictly increasing and strictly concave. For simplicity we assume that workers cannot save and/or borrow. Therefore, consumption is simply equal to their wages.

When new workers are hired, the firm signs long-term contracts that specify the whole sequence of wages paid to each individual worker. Because the labor market is competitive, the initial lifetime utility is equal to the market-clearing level offered to all newly hired workers. This value, which is exogenous in the model, is also the reservation value for the worker, that is, the value that the worker would earn by re-entering the labor market. We denote the reservation value by \( q_{\text{res}} \).

3 The firm’s problem and dynamic properties

We will start characterizing the optimization problem under the assumption that the firm and the workers commit not to renegotiate the long-term wage contracts in future periods. After characterizing the optimal solution with commitment, we will discuss in the next section the conditions under which the long-term contracts are free from renegotiation.

Let \( \{w_{t,t+j}\}_{j=1}^{\infty} \) be the sequence of wages that the firm promises to the workers hired at time \( t \). Here \( w_{t,t+j} \) denotes the wage paid at time \( t+j \) to workers hired at time \( t \). The total wage payments at time \( t+1 \) are \( \sum_{\tau=0}^{t} n_\tau w_{\tau,t+1} \). Let \( a_t \) denote the net worth of the firm at the end of period \( t \)—that is, after production and after the payment of
wages and interests. The firm’s net worth, $a_t$, plus the debt, $b_t$, is equal to the sum of
the firm’s capital, $\kappa \sum_{\tau=0}^{t} n_{\tau}$, and the dividends, $d_t$. Thus, $d_t = a_t + b_t - \kappa \sum_{\tau=0}^{t} n_{\tau}$.

Given the initial assets $a_0$, the firm maximizes the discounted value of the entre-
preneur’s consumption. Consumption is equal to the dividends since the entrepreneur
is at least as impatient as the market, that is, $\beta \leq 1/(1 + r)$. Thus, at time zero, the
firm chooses the whole sequence of debt, employment and wages to solve the problem:

$$V(a_0) = \max_{\{b_t, n_t, \{w_{t,t+j}\}_{j=1}^{\infty}\}} \sum_{t=0}^{\infty} \beta^t d_t$$

subject to

$$d_t = a_t + b_t - \kappa \sum_{\tau=0}^{t} n_{\tau} \geq 0,$$ \hspace{1cm} (2)

$$b_t \leq \bar{\kappa} \sum_{\tau=0}^{t} n_{\tau},$$ \hspace{1cm} (3)

$$\sum_{j=1}^{\infty} \beta^j U(w_{t,t+j}) \geq q_{res},$$ \hspace{1cm} (4)

$$a_{t+1} = (\kappa + A) \sum_{\tau=0}^{t} n_{\tau} - \sum_{\tau=0}^{t} n_{\tau} w_{\tau,t+1} - (1 + r)b_t,$$ \hspace{1cm} (5)

with $a_0$ given. Equation (2) defines the dividends, that are constrained to be non-
negative because consumption cannot be negative. Constraint (3) imposes the bor-
rowing limit and (4) is the worker’s participation constraint. This imposes that the
lifetime utility generated with the sequence of wages offered to a new hired worker,
cannot be smaller than its reservation value $q_{res}$. Finally, constraint (5) defines the
law of motion for the end-of-period net worth.

Let $\gamma_t$, $\mu_t$ and $\lambda_t n_t$ be the lagrange multipliers associated with the constraints (2),
(3) and (4), respectively. The first order conditions with respect to $b_t$ (debt contracted in period $t$) and $w_{\tau,t}$ (wage paid at time $t$ to workers hired at time $\tau$) are:

$$\mu_t = 1 + \gamma_t - \beta(1 + r)(1 + \gamma_{t+1})$$

(6)

and

$$\lambda_{\tau} U_c(w_{\tau,t}) = 1 + \gamma_t,$$

(7)

respectively. The variable $\mu_t$ is the value of an additional unit of debt by relaxing the borrowing limit, and $1 + \gamma_t$ is the value of an additional unit of internal funds. If the non-negativity constraint on dividends is binding, $\gamma_t$ is positive. Therefore, one additional unit of internal funds has a value greater than 1.

Condition (6) says that the value of one additional unit of debt, $\mu_t$, is equal to the firm’s valuation of cash flows generated by this additional debt. By borrowing more, the firm gets immediately one unit of funds or cash flow. This has a value of $1 + \gamma_t$. In the next period, however, the firm has to repay $1 + r$, which will be valued at $1 + \gamma_{t+1}$. Therefore, the current value of the debt repayment is $\beta(1 + r)(1 + \gamma_{t+1})$.

To interpret (7), notice that the variable $\lambda_{\tau}$ is the marginal cost to the firm of providing one unit of utility to a worker hired at time $\tau$, while $U_c$ denotes the marginal utility of consumption. This implies that the term $\lambda_{\tau} U_c(w_{\tau,t})$ is the marginal cost of reducing wages. Therefore, condition (7) says that the optimal wage policy of the firm is such that the marginal cost of reducing wages at time $t$ (and increasing the current cash flows) is equal to the marginal value of internal funds. In other words, the firm
borrows from a worker until the cost of this type of borrowing is equal to the marginal value of internal funds.

3.1 Firm dynamics and wage structure

Equations (6) and (7) allow us to characterize some basic properties of the dynamics and wage structure of the firm. Let’s observe first that, if the initial wealth of the entrepreneur $a_0$ is small, the firm is initially unable to finance the investment necessary to employ the optimal number of employees $N$. Since the firm is financially constrained, we have that the non-negativity constraint on dividends (2) and the borrowing limit (3) are both binding, and therefore, $\mu_t > 0$ and $\gamma_t > 0$. Equation (6) then implies that $\gamma_t$ declines over time until it approaches zero.\footnote{This property can be clearly seen in the special case in which $\beta(1 + r) = 1$. In this case equation (6) reads as $\mu_t = \gamma_t - \gamma_{t+1}$, which says that $\gamma_t$ decreases over time when $\mu_t$ is strictly positive. Although the decreasing pattern of $\gamma_t$ can be seen clearly only in this special case, this property holds more generally also in the case in which $\beta(1 + r) < 1$.} As the firm retains its earnings and accumulates internal wealth, the firm hires more workers and eventually it reaches the optimal scale $N$. Thus, we have the following property:

\textbf{P1: Financially constrained firms are younger, smaller and grow faster.}

To consider how the financial conditions of the firm affect the dynamics of wages, consider equation (7). The multiplier $\gamma_t$ captures the tightness of financial constraints and depends on the firm’s net worth. If $a_t$ is small, the financial needs of the firm are
high, which imply that the value of an extra unit of internal funds is also high. As the firm retains earnings, its assets increase over time and the variable $\gamma_t$ decreases monotonically to zero. Equation (7) then shows that the wages paid to each cohort of workers increase over time until $\gamma_t = 0$. We summarize this property as follows:

**P2:** The wages paid to each cohort of workers increase over time until the firm becomes unconstrained.

Another important property is that, within the same firm, workers with longer employer tenure receive higher wages than more recent hires. To show this, consider again condition (7). As observed above, the declining value of $\gamma_t$ implies that wages increase over time. This also implies that the lifetime utility of workers increases. Because all workers start with the same utility level, $q_{res}$, new hires have an initial utility that is smaller than older workers. But then, since condition (7) also implies that the ratio of marginal utilities for any two cohorts of workers remains constant, we must have that new hires receive lower wages than older workers at any point in time. In summary:

**P3:** Within the same firm, senior workers earn higher wages than junior workers.

Figure 1 shows the above properties with a numerical example in which the utility function takes the log-form $U(c_t) = \log(c_t)$, and the parameters values are as follows: $r = 0.03$, $\beta = 0.934$, $q_{res} = U(0.6)/(1 - \beta)$, $A = 1$, $\bar{N} = 1,000$, $\kappa = 2.8$, $\kappa_f/\kappa = 0.3$, $\kappa_f/\kappa = 0.3$. 
and $a_0$ is such that the initial size of the firm is 10 percent the maximum scale. This is obtained by setting $a_0 = 196$.

Figure 1: Employment dynamics and wage patterns over age and size.

The first panel of Figure 1 plots the employment dynamics. The firm starts with an initial employment of 100 workers and then gradually grows over time until it reaches the optimal scale $N = 1,000$. The transition takes place in 11 periods. The second panel plots the wage profile of the first cohort of workers (those hired at time 0) and the initial wage paid to newly hired workers. The wage profile of the first cohort of workers (continuous line) is increasing until the firm reaches the unconstrained status. The dashed line is the wage earned by the newly hired cohort of workers. As the firm gets closer to the optimal scale, it offers higher initial wages, and therefore, the wage
profile of more recent workers is less steep overall.

The third panel plots the average wage paid by the firm as a function of its age and the fourth panel the average wage as a function of its size (measured by the number of employees). The average wage increases with the size and age of the firm. This is a direct consequence of the fact that, when the firm is young and constrained, it operates at a suboptimal scale and offers an increasing profile of wages.

Especially important for the overall dynamics of wages is the concavity of the workers’ utility $U(c_t)$ and the initial assets $a_0$. To analyze the effects of differences in the degree of concavity, consider the utility function $U(c_t) = (c_t^{1-\sigma} - 1)/(1 - \sigma)$ and increase the degree of concavity by increasing $\sigma$. As shown in the first panel of Figure 2, the lower is the concavity and the stronger is the size dependence of wages. Clearly, with a smaller $\sigma$ the worker is more willing to accept a non-flat consumption profile and it becomes cheaper for the firm to borrow from workers. In the extreme case in which $\sigma = 0$ (linear utility), the financing premium required by the workers is zero and the firm will borrow as much as possible from them. In this case the firm would pay zero wages until it can operate at the optimal scale.

The initial assets of the firm also play an important role. For given $\overline{N}$, smaller values of $a_0$ imply tighter financial constraints, that is, the firm is constrained to start with a smaller scale. This also implies that the firm has a greater incentive to rely on its wage policy to finance its growth. As a result, the firm pays smaller initial wages as shown in the second panel of Figure 2.
We can summarize the above properties as follows:

**P4:** *The initial wages are lower when the firm faces tighter constraints (smaller initial assets) and the worker utility is less concave (cheaper cost of borrowing).*

### 4 Contract enforceability

The analysis conducted in the previous section was based on the assumption that firms and workers commit to the long-term wage contract. In this section we consider the
conditions under which this contract is free from renegotiation. For limitation of space we only provide an intuitive discussion. For the detailed analysis see Michelacci and Quadrini (2004).

As we have seen in the previous section, the wage profile is increasing or constant. This implies that at any point in time the worker’s lifetime utility is greater or equal than his or her reservation value $q_{res}$. Therefore, the worker will never renege the contract. This also explains why in the firm’s problem (1) we have not imposed this constraint.

The renegotiation problem for the firm is more complex. The increasing profile of wages implies that the firm does have an incentive to renege its promises. In fact, the firm could always replace an existing worker with a new hire, to whom it will offer a contract with initial lifetime utility $q_{res}$. Because this initial utility is smaller than the utility promised to an existing worker, the firm would gain by replacing the worker. To prevent this possibility, workers must have some direct or indirect mechanism to punish the firm in case it reneges the contract. This mechanism is provided by the worker-specific investment $\kappa_w$ which is lost if the worker quits the firm. It can be shown that, if $\kappa_w$ is sufficiently large, the optimal contract can be sustained by trigger strategies in which the worker provides effort as long as the firm pays the contracted wages, while he or she will quit if the firm reneges its promises. In a way, the worker-specific investment acts as a collateral for the worker’s credit toward the firm, which is not available to external investors.
Of course, there is a limit to the amount of credit that the firm can get from workers. In particular, if the initial wealth $a_0$ is small and the worker’s utility is not very concave, then the profile of wages is very steep. This implies that at some point the firm will have an incentive to renege. In this case the characterization of the optimal contract is more complex because it requires the imposition of additional constraints. However, it is still true that the wage profile is initially increasing. The difference is that the increasing pattern may stop before the firm operates at the optimal scale. In other words, the promised wages cannot be too high if these promises have to be guaranteed by the sunk investment $\kappa_w$. When the value of the sunk investment is insufficient, the firm may have to rely on some other implementation mechanism to borrow from workers. The grants of assets, such as stock options, may serve this purpose.

5 Extensions, future research and conclusion

In this paper we have studied how the financial condition of the firm affects its wage policy. In the (static) perfectly competitive labor market model, wage differences just reflect differences in workers’ abilities. In reality, firms’ characteristics account for a relevant part of the observed dispersion in earnings. This is usually taken as evidence of labor market imperfections. In our model workers are (ex-ante) identical and the labor market is perfectly competitive. Yet, due to financial market imperfections, workers’ earnings also depend on the characteristics of their employers. In particular, wages tend to increase with the age and size of the firm.
As reviewed in Brown and Medoff (1989) and Oi and Idson (1999), the positive relation between firm’s size and wages is a robust empirical regularity. A closer inspection of our results, however, reveals that our simple model in unable to fully capture this regularity. In our model, in fact, the impact of firm size on wages is not independent from the impact of firm age. In the data, instead, the size of the firm has a positive effect on wages even after controlling for its age. See Brown and Medoff (2003).

This conclusion, however, is only a consequence of the assumption that all firms run the same identical technology. If we allow firms to have different technologies—for instance, they have different employment capacities \( \overline{N} \)—and/or they start with different initial assets \( a_0 \), then in a cross section of firms, size will have an independent effect on wages. In Michelacci and Quadrini (2004) we consider a more general model in which firms differ along these dimensions. We also allow for firms entry, exit and worker turnover. Using that model we study whether financial frictions can explain, at least in part, the firm size-wage relation. Preliminary results point out that financial markets imperfections contribute significantly to explaining this well-known empirical regularity.
References


