MODEL DESCRIPTION

We describe the model and we focus on the social planner problem of the economy. Section 1 reviews the economy and then discusses the equilibrium conditions of the model. A formal derivation of these conditions appear in Appendix A. Appendices also show how the equilibrium conditions of the model can be expressed in terms of stationary quantities (Appendix B), characterize the steady state equilibrium of the model (Appendix C), linearize equilibrium conditions around the steady state, and discuss how to simulate the economy and to calculate impulse responses (Appendix D).

1 Assumptions

There is one consumption good, the numeraire. Output is produced according to

$$\tilde{Y} = F(\tilde{K},\ \tilde{H}) = \tilde{K}^{\alpha}\tilde{H}^{1-\alpha},$$

where $\tilde{K}$ is the capital stock and $\tilde{H}$ the amount of labor intensive intermediate goods used in production. Labor intensive intermediate goods are produced in *jobs* which consist of firm-worker pairs. A worker can be employed in at most one job where he supplies one unit of labor at an effort cost (in utility terms) c_w. A job with *neutral technology* z produces an amount of intermediate goods equal to $\exp\left(\frac{z}{1-\alpha}\right)$. As in standard vintage models, newly created jobs always embody leading-edge technologies while *old jobs* are incapable of upgrading their previously installed technologies.[1] The idea is that the adoption of new technologies requires the performance of new tasks and workers initially hired to operate specific technologies may not be suitable for their upgrading. Specifically, a job which starts producing at time t operates with a neutral technology z_{it} equal to the economy *leading technology* z_t of that time, while the current period neutral technology of old jobs, z_{it}, remains (in expected value) unchanged:

$$z_{it} = z_{it-1} + \epsilon_{it} \tag{1}$$

where ϵ_{it} is an idiosyncratic shock which is iid normal with standard deviation σ_{ϵ}.[2] The leading edge neutral technology evolves as:

$$z_t = \mu_z + z_{t-1} + \varepsilon_{zt} \tag{2}$$

where ε_{zt} is iid normal with standard deviation σ_z. Hereafter, we will refer to the difference between the leading technology z_t and the job's neutral technology z_{it} as the job *technological gap,* $\tau_{it} \equiv z_t - z_{it}$.

The law of accumulation of capital is $\tilde{K}' = (1-\delta)\tilde{K} + e^{q}\tilde{I}$ where $\tilde{I}$ is the amount of investment expenditures measured in final output and q is the investment specific technology, which evolves according to

$$q_t = \mu_q + q_{t-1} + \varepsilon_{qt} \tag{3}$$

where ε_{qt} is iid normal with standard deviation σ_q.

At every point in time jobs are exogenously destroyed with probability λ. Jobs can also be destroyed when their technological gap is too large relative to an endogenously determined critical threshold τ_t^*. Jobs created at time t starts producing at time $t+1$. Creating new jobs requires the services of recruiters. The cost of creating n new jobs involves a cost in utility terms to recruiters equal to:

$$C(n) = cu^{-\eta_0}n^{\eta_1}, \quad \eta_0, \eta_1 > 0 \tag{4}$$

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[1] See Jovanovic and Lach (1989), Caballero and Hammour (1996), and Aghion and Howitt (1994) for examples of vintage models.

[2] The idiosyncratic shocks ϵ guarantee that the cross-sectional distribution of job technology has no mass points. In turn, this property ensures a smooth transitional dynamics by ruling out the possibility that persistent oscillations occur over the transition path —i.e. the "echo effects" that typically arise in vintage models, see for example Benhabib and Rustichini (1991).

so that unemployment reduces the cost of creating new jobs, as it is standard in search models, see e.g. Pissarides (2000). This formulation embeds others present in the literature. For example, if the matching function has constant returns to scale and the utility cost of posting a vacancy is constant, then $\eta_1 - \eta_0 = 1$. If the cost of posting vacancies is instead increasing in the number of posted vacancies or in the number of newly created jobs as in Caballero and Hammour (1996) and Michelacci and Lopez-Salido (2007), $\eta_1 - \eta_0 > 1$.

The population of workers is constant and normalized to one. We assume that a representative household exists so that workers and recruiters pool their income at the end of the period and choose consumption and effort costs to maximize the sum of the expected utility of the household's members. The instantaneous utility is:

$$\ln \tilde{C} - c_w (1-u) - C(n) \tag{5}$$

where $\tilde{C}$ is aggregate consumption, while u and n denote the unemployment rate and the flow of newly created jobs, respectively. The last two terms in (5) account for the effort cost of working for workers and recruiters, respectively. The household's discount factor is β. The aggregate resource constraint is: $\tilde{Y} = \tilde{I} + \tilde{C}$.

2 Solution

We first review the timing of events within a period. Then we characterize the equilibrium conditions in terms of job destruction, job creation and saving decisions. Finally, we define the job finding and the job separation probability in the model and we briefly discuss how we compute the equilibrium.

2.1 Timing

We adopt the following convention about the timing of events within a period t:

i. Aggregate technology shocks ε_{zt} and ε_{qt} are realized;

ii. Upgrade possibilities materialize for the neutral technology of old jobs;

iii. Old jobs realize whether their job is exogenously destroyed (which occurs with probability λ) and their idiosyncratic shocks ϵ_{it}. New jobs (resulting from matches at time $t-1$) start with neutral technology z_t;

v. Decisions about job destruction, job creation, and investment are taken;

vi. Output is produced, income pooled and consumed. Next period begins.

2.2 Equilibrium conditions

The job destruction decision of firms is characterized by a critical reservation technological gap $\tau_t^* > 0$ such that jobs with higher technological gaps are destroyed. Let $f_t(\tau)$ denote the time-t measure of old jobs which, in case they are kept in operation, would produce with technological gap τ. In the previously described sequence of events, this is the distribution resulting after the events in iii). The distribution of old jobs f_t evolves as

$$f_t(\tau) = (1\text{-}\lambda)\left[\int_{\text{-}\infty}^{\tau^*_{t\text{-}1}} g_\epsilon(i + \mu_z + \varepsilon_{z,t}\text{-}\tau) f_{t\text{-}1}(i)di + g_\epsilon(\mu_z + \varepsilon_{z,t}\text{-}\tau) n_{t\text{-}2}\right], \ \forall \tau \in \mathbb{R}$$

where g_ϵ denotes the density function of the idiosyncratic shock ϵ, which is symmetric around zero. To understand the expression, consider the sequence of events that characterize the evolution of the distribution of old jobs between time $t-1$ and t. At time $t-1$, some old jobs are destroyed while some others remain in operation and produce. Then to obtain the distribution of old jobs at time t one has to take account of the aggregate and idiosyncratic shocks to the job neutral technology that determine the job technological gap, of the probability that jobs are exogenously destroyed and of the inflow of new jobs that start producing at time $t-1$, n_{t-2}, and that will belong to the pool of old jobs at time t. To understand the term in the integral consider a job, which, at time $t-1$, produces with technological gap i. Then, this job will end up with a technological gap τ at the beginning of time t, only if the realization of the idiosyncratic shock ϵ is equal to $\tau - i - \mu_z - \varepsilon_{z,t}$, where $\varepsilon_{z,t}$ is the aggregate shock to the leading neutral technology. Then the measure of jobs with technological gap τ at time t is obtained by integrating over all possible values of technological gap i, which do not lead to job destruction at time $t-1$.

With this notation, unemployment can be expressed as equal to

$$u_t = 1 - \int_{-\infty}^{\tau_t^*} f_t(\tau) d\tau - n_{t-1} \tag{6}$$

since jobs are destroyed when their technology gap is greater than τ_t^* while all newly created jobs are productive. Analogously the efficiency units of labor used in production are given by

$$\tilde{H}_t = \int_{-\infty}^{\tau_t^*} e^{\frac{z_t - \tau}{1-\alpha}} f_t(\tau) d\tau + e^{\frac{z_t}{1-\alpha}} n_{t-1}. \tag{7}$$

To characterize the optimal job creation and job destruction decisions, let $V_t(\tau)$ denote the time-t (social) net value in utils of a job with technological gap τ. Then we show in the Appendix that

$$V_t(\tau) = (1-\alpha) \left(\frac{\tilde{K}_t}{\tilde{H}_t} \right)^{\alpha} e^{\frac{z_t - \tau}{1-\alpha}} \frac{1}{\tilde{C}_t} - c_w - c_1 \eta_0 u_t^{-\eta_0 - 1} n_t^{\eta_1} + J_t(i) \tag{8}$$

where $J_t(i)$ denotes the future net value of a job with technological i. To understand the expression, notice that the first three terms compute the instantaneous net return of the job as the difference between the value of job output in utility terms and the effort cost of working and the marginal value of unemployment. Here unemployment has value because it reduces the cost of creating new jobs. The last term represents instead the expected present value of the job future net surplus, which is equal to

$$J_t(i) \equiv \beta (1-\lambda) E_t \left[\int_{-\infty}^{\tau_{t+1}^*} V_{t+1}(j) g_\epsilon(i + \mu_z + \varepsilon_{z,t+1} - j) dj \right]$$

since jobs are kept in operation only if their technological gap is lower than the critical technological gap τ^* and if no exogenous destruction occurs.

Given (8), the optimal job destruction decision can be expressed as

$$V_t(\tau_t^*) = 0 \tag{9}$$

which determines the critical technological gaps that lead to job destruction, while the optimal number of newly created jobs solves

$$c_0 n_t^{\upsilon} + c_1 \eta_1 n_t^{\eta_1 - 1} u_t^{-\eta_0} = \beta E_t \left(V_{t+1}(0) \right) \tag{10}$$

which equates the marginal cost of creating a new job to its expected future net value. Finally saving decisions are characterized by the well known Euler condition for consumption:

$$\frac{1}{\tilde{C}_t} = \beta e^{q_t} E_t \left(\frac{1}{\tilde{C}_{t+1}} \left[(1-\delta) e^{-q_{t+1}} + \alpha \left(\frac{\tilde{H}_{t+1}}{\tilde{K}_{t+1}} \right)^{1-\alpha} \right] \right). \tag{11}$$

2.3 Job separation and job finding

The fraction of jobs destroyed between time $t-1$ and time t is equal to

$$S_t = \lambda + \frac{\int_{\tau_t^*}^{\infty} f_t(\tau) d\tau}{1-u_{t-1}}$$

while the job finding probability for workers searching between time $t-1$ and time t is given by

$$F_t = \frac{n_{t-1}}{u_{t-1}}$$

With these definitions unemployment evolves as

$$u_t = u_{t-1} + S_t \left(1-u_{t-1}\right) - F_t u_{t-1}.$$

Our economy fluctuates around the stochastic trend given by $X_t \equiv e^{x_t}$, where

$$x_t = \frac{1}{1-\alpha} z_t + \frac{\alpha}{1-\alpha} q_t$$

is a composite index of the neutral and investment-specific technology. To make the environment stationary, we scale quantities by $X_t \equiv e^{x_t}$, see Appendix for details. We then solve the model by log-linearizing the first order conditions around the steady state of the model without aggregate shocks, $\varepsilon_{z,t} = 0$ and $\varepsilon_{q,t+1} = 0$. This yields a system of linear stochastic difference equation that can be solved, for example, with the method proposed by Sims (2002). To characterize the beginning-of-period distribution, f_t, we follow Campbell (1998) and Michelacci and Lopez-Salido (2007) in considering its values at a fixed grid of technological gaps. The following Computational Appendix describes in more details the procedure used.

COMPUTATIONAL APPENDIX

A Derivation of equilibrium conditions

Let $\tilde{K}$, f, n_{-1}, z, and q denote the current capital stock, the beginning of period distribution, the measure of jobs that start producing in this period, the leading edge neutral technology and the investment specific technology, respectively. The social planner problem of our economy can then be written as follows

$$\begin{aligned} \tilde{W}(\tilde{K}, f,\ n_{-1}, z, q) &= \max_{\tilde{C}, n, \tau^*} \ln \tilde{C} - c_w\,(1-u) - c_0 \frac{n^{1+\nu}}{1+\nu} - c_1 u^{-\eta_0} n^{\eta_1} \\ &\quad + \beta E\left[\tilde{W}(\tilde{K}', f', n,\ z', q')\right] \end{aligned} \tag{12}$$

which is subject to the following set of transition equations

$$\begin{aligned} \tilde{K}' &= (1-\delta)\tilde{K} + e^q\left(\tilde{K}^\alpha \tilde{H}^{1-\alpha} - \tilde{C}\right), \\ f'(\tau) &= (1-\lambda)\left[\int_{-\infty}^{\tau^*} g_\epsilon(i + \mu_z + \varepsilon_z' - \tau) f(i)di + g_\epsilon(\mu_z + \varepsilon_z' - \tau) n_{-1}\right] \\ q' &= \mu_q + q + \varepsilon_q', \\ z' &= \mu_z + z + \varepsilon_z', \end{aligned}$$

and to the two identities:

$$\tilde{H} = \int_{-\infty}^{\tau^*} e^{\frac{z-\tau}{1-\alpha}} f(\tau)d\tau + e^{\frac{z}{1-\alpha}} n_{-1} \tag{13}$$

$$u = 1 - \int_{-\infty}^{\tau^*} f(\tau)d\tau - n_{-1} \tag{14}$$

A.1 The Euler equation for consumption

By deriving with respect to $\tilde{C}$ in (12), after taking into account (13) and (14) we obtain:

$$\frac{1}{\tilde{C}} = \beta e^q E\left(\tilde{W}_K'\right) \tag{15}$$

where $\tilde{W}_K'$ denote the partial derivative of the value function of next period with respect to capital. The envelope condition with respect to capital reads as:

$$\tilde{W}_K = \beta E\left(\tilde{W}_K'\right)\left[(1-\delta) + e^q \alpha \left(\frac{\tilde{H}}{\tilde{K}}\right)^{1-\alpha}\right]$$

that, after using (15) to replace $\beta E\left(\tilde{W}_1'\right)$, can be expressed as

$$\tilde{W}_K = \frac{1}{\tilde{C}}\left[(1-\delta)e^{-q} + \alpha\left(\frac{\tilde{H}}{\tilde{K}}\right)^{1-\alpha}\right].$$

After evaluating this derivative in the next period we obtain

$$\tilde{W}_K' = \frac{1}{\tilde{C}'}\left[(1-\delta)e^{-q'} + \alpha\left(\frac{\tilde{H}'}{\tilde{K}'}\right)^{1-\alpha}\right] \tag{16}$$

which substituted into (15) yields

$$\frac{1}{\tilde{C}} = \beta E\left(\frac{1}{\tilde{C}'}\left[(1-\delta)e^{q-q'} + e^{q}\alpha\left(\frac{\tilde{H}'}{\tilde{K}'}\right)^{1-\alpha}\right]\right) \tag{17}$$

This is equation (11) in the paper.

A.2 Destruction

To calculate the first order condition with respect to τ^* notice that by deriving with respect to τ^* in (13) and (14) we obtain:

$$\begin{aligned}\frac{\partial u}{\partial \tau^*} &= -f(\tau^*),\\ \frac{\partial H}{\partial \tau^*} &= e^{\frac{z-\tau^*}{1-\alpha}} f(\tau^*).\end{aligned}$$

After using these two results, deriving with respect to τ^* in (12) yields:

$$(1-\alpha)\left(\frac{\tilde{K}}{\tilde{H}}\right)^{\alpha} e^{\frac{z-\tau^*}{1-\alpha}}\frac{1}{\tilde{C}} - c_w - c_1\eta_0 u^{-\eta_0-1}n^{\eta_1} + J_t(\tau^*) = 0 \tag{18}$$

where

$$J(i) \equiv \beta\,(1-\lambda)\,E_t\left[\int_{-\infty}^{\tau^{*\prime}} V'(j) g_{\epsilon}(i+\mu_z+\varepsilon_z'-j)dj\right]$$

and $V'(i) \equiv \tilde{W}_{f(i)}'$ denotes the net social value of a job with technological distance i in the next period. This is formally defined as the partial derivative of the value function of next period with respect to the density of jobs with technological distance i. Notice that in writing the condition we made use of (15) to replace $\beta E\left(\tilde{W}_1'\right)$.

The envelope condition allows to write

$$V(i) = (1-\alpha)\left(\frac{\tilde{K}}{\tilde{H}}\right)^{\alpha} e^{\frac{z-i}{1-\alpha}}\frac{1}{\tilde{C}} - c_w - c_1\eta_0 u^{-\eta_0-1}n^{\eta_1} + J(i)$$

which is simply equation (8) in the paper. With this notation (18) can simply be expressed as

$$V(\tau^*) = 0,$$

which is equation (9) in the paper.

A.3 Creation

The first order condition with respect to n reads as follows:

$$c_0 n^{\nu} + c_1\eta_1 u^{-\eta_0} n^{\eta_1-1} = \beta E\left(V'(0)\right) \tag{19}$$

where $V'(0) \equiv \tilde{W}'_{n_{-1}}$ is the next period value of a job that produces with technological gap zero (i.e. a newly created job). This is equal to the partial derivative of the value function of next period with respect to the measure of newly created jobs. The envelope condition allows to write

$$V(0) = (1-\alpha)\left(\frac{\tilde{K}}{\tilde{H}}\right)^{\alpha} e^{\frac{z}{1-\alpha}} \frac{1}{\tilde{C}} - c_w - c_1 \eta_0 u^{-\eta_0 - 1} n^{\eta_1} + J(0)$$

which is simply the value of the function V in (8) at τ equal to zero. Thus (19) is equivalent to (10) in the paper.

B Equilibrium definition

One can easily check that the economy evolves around the stochastic trend given by

$$X \equiv e^{\frac{z}{1-\alpha}}\ e^{\frac{\alpha q}{1-\alpha}}$$

To make the environment stationary we defined the following scaled quantities:

$$K_t \equiv \frac{\tilde{K}_t}{e^{\frac{z_t+q_t}{1-\alpha}}},\ H_t = \frac{\tilde{H}_t}{e^{\frac{z_t}{1-\alpha}}},\ \text{ and }\ C_t \equiv \frac{\tilde{C}_t}{e^{\frac{z_t}{1-\alpha}}\ e^{\frac{\alpha q_t}{1-\alpha}}}.$$

Then an equilibrium consists of a stationary tuple

$$(K_t, u_t, H_t, f_t, C_t, V_t, n_t, \tau_t^*, \Delta z_t, \Delta q_t),$$

where f_t and V_t are functions of technological gap while the remaining quantities are scalar, that satisfies the following conditions:

1. The law of motion of capital:

$$K_t = (1-\delta)K_{t-1}e^{-\frac{\mu_z+\mu_q+\varepsilon_{z,t}+\varepsilon_{q,t}}{1-\alpha}} + \left(K_{t-1}^{\alpha}H_{t-1}^{1-\alpha} - C_{t-1}\right)e^{-\frac{\mu_z+\mu_q+\varepsilon_{z,t}+\varepsilon_{q,t}}{1-\alpha}} \tag{20}$$

2. The definition of unemployment:

$$u_t = 1 - \int_{-\infty}^{\tau_t^*} f_t(\tau)d\tau - n_{t-1} \tag{21}$$

3. The definition of the efficiency unit of labor:

$$H_t = \int_{-\infty}^{\tau_t^*} e^{\frac{-\tau}{1-\alpha}} f_t(\tau)d\tau + n_{t-1} \tag{22}$$

4. The law motion of the distribution of technological gaps of old jobs:

$$f_t(\tau) = (1-\lambda)\left[\int_{\text{-}\infty}^{\tau_{t\text{-}1}^*} g_\epsilon(i+\mu_z+\varepsilon_{z,t}-\tau)f_{t\text{-}1}(i)di + g_\epsilon(\mu_z+\varepsilon_{z,t}-\tau)n_{t\text{-}2}\right] \tag{23}$$

5. The Euler equation for consumption:

$$\frac{1}{C_t} = \beta E\left\{\frac{1}{C_{t+1}\ e^{\frac{\mu_z+\mu_q+\varepsilon_{z,t+1}+\varepsilon_{q,t+1}}{1-\alpha}}}\left[(1-\delta)+\alpha\left(\frac{H_{t+1}}{K_{t+1}}\right)^{1-\alpha}\right]\right\} \tag{24}$$

6. The marginal value of jobs at any given technological distance $\tau \leq \tau_t^*$:

$$V_t(\tau) = (1-\alpha)\left(\frac{K_t}{H_t}\right)^{\alpha} e^{\frac{-\tau}{1-\alpha}} \frac{1}{C_t} - c_w - c_1\eta_0 u_t^{-\eta_0-1} n_t^{\eta_1} + J_t(\tau) \tag{25}$$

   where

$$J_t(\tau) \equiv \beta\left(1-\lambda\right) E_t\left[\int_{-\infty}^{\tau_{t+1}^*} V_{t+1}(i) g_{\epsilon}(\tau + \mu_z + \varepsilon_{z,t+1} - i) di\right]$$

7. The optimal job destruction decision: $V_t(\tau_t^*) = 0$ which can be also expressed as

$$(1-\alpha)\left(\frac{K_t}{H_t}\right)^{\alpha} e^{\frac{-\tau_t^*}{1-\alpha}} \frac{1}{C_t} - c_w - c_1\eta_0 u_t^{-\eta_0-1} n_t^{\eta_1} + J_t(\tau_t^*) = 0 \tag{26}$$

8. The optimal number of newly created jobs:

$$c_0 n_t^{\nu} + c_1\eta_1 u_t^{-\eta_0} n_t^{\eta_1-1} = \beta E\left(V_{t+1}(0)\right)$$

9. The laws of motion of z_t and q_t :

$$\begin{aligned} \Delta z_t &= \mu_z + \varepsilon_{z,t} \\ \Delta q_t &= \mu_q + \varepsilon_{q,t} \end{aligned}$$

   where $\varepsilon_{z,t}$ and $\varepsilon_{q,t}$ are iid over time.

C Solving for the steady state

We next characterize the equilibrium conditions of the model in the steady state version o fthe model without aggregate shocks ($\varepsilon_{z,t} = 0$ and $\varepsilon_{q,t} = 0$) and then discuss the algorithm used to compute the equilibrium.

C.1 Equilibrium conditions

The equilibrium conditions of the economy in steady state can be obtained by dropping the time subscripts from the corresponding expressions in the previous Section. A steady state equilibrium is characterized by a tuple

$$(K, u, H, f, C, V, n, \tau^*),$$

where f and V are functions while the remaining quantities are scalar, that satisfies the following conditions:

1. The law of motion of capital:

$$\left[1 - \delta + \left(\frac{H}{K}\right)^{1-\alpha} - e^{\frac{\mu_z+\mu_q}{1-\alpha}}\right] K = C \tag{27}$$

2. The definition of unemployment:

$$u = 1 - \int_{-\infty}^{\tau^*} f(\tau) d\tau - n \tag{28}$$

3. The definition of the efficiency unit of labor:

$$H = \int_{-\infty}^{\tau^*} e^{\frac{-\tau}{1-\alpha}} f(\tau) d\tau + n \tag{29}$$

4. The law motion of the distribution of technological gaps of old jobs:

$$f(\tau) = (1-\lambda)\left[\int_{-\infty}^{\tau^*} g_\epsilon(i + \mu_z - \tau) f(i) di + g_\epsilon(\mu_z - \tau) n\right] \tag{30}$$

5. The Euler equation for consumption:

$$\frac{e^{\frac{\mu_z+\mu_q}{1-\alpha}}}{\beta} = (1-\delta) + \alpha\left(\frac{H}{K}\right)^{1-\alpha} \tag{31}$$

6. The marginal value of jobs at any given technological distance $\tau \leq \tau_t^*$:

$$V(\tau) = (1-\alpha)\left(\frac{K}{H}\right)^{\alpha} e^{\frac{-\tau}{1-\alpha}} \frac{1}{C} - c_w - c_1 \eta_0 u^{-\eta_0 - 1} n^{\eta_1} + J(\tau) \tag{32}$$

where

$$J(\tau) \equiv \beta(1-\lambda) \int_{-\infty}^{\tau^*} V(i) g_\epsilon(\tau + \mu_z - i) di \tag{33}$$

7. The optimal job destruction decision: $V(\tau^*) = 0$ which can be also expressed as

$$(1-\alpha)\left(\frac{K}{H}\right)^{\alpha} e^{\frac{-\tau^*}{1-\alpha}} \frac{1}{C} - c_w - c_1 \eta_0 u^{-\eta_0 - 1} n^{\eta_1} + J(\tau^*) = 0 \tag{34}$$

8. The optimal number of newly created jobs:

$$c_0 n^{\nu} + c_1 \eta_1 u^{-\eta_0} n^{\eta_1 - 1} = \beta V(0) \tag{35}$$

C.2 Algorithm

Notice that given any parameter choice the capital labour ratio is determined by (31) as equal to

$$\frac{K}{H} = \left(\frac{\alpha\beta}{e^{\frac{\mu_z+\mu_q}{1-\alpha}} - (1-\delta)\beta}\right)^{\frac{1}{1-\alpha}} \tag{36}$$

Our algorithm is just a version of value function iteration, which consists of a "guessing", a "checking" and an "updating" part.
Guessing: Guess a function for $J(\tau)$. In practice the function $J(\tau)$ is discretized so that the guess is a vector of order $M_\tau \times 1$.

We then solve the model for a given value of steady state consumption C. We start by guessing a value of unemployment u. Then, we use (35) to solve for n and (34) to solve for τ^*. Then, and given a quadrature approximation (see below), use the previous quantities to determine the value of $f(\tau)$ at I_τ point as a solution of the linear system implied by (30) . We then iterate over u to find the value that solves the resource constraint implied by (28):

$$1 - \int_{-\infty}^{\tau^*} f(\tau) d\tau - n - u = 0$$

Parameter Values						
β:0.99	α:0.36	μ_z:0.0975%	μ_q: 0.8025%	δ: 3.2%	σ_z:0.56%	σ_q:1.3%
λ:3%	η_0:0.66	η_1:4	σ_ϵ: 4.9%	c_w: 0.62	c:408.85	

Table 1: Parameters values used in the baseline specification.

by using a bisecant method. Given this equilibrium value of u we iterate over consumption to find the value of C that solves condition (27) by using a bisecant method:

$$\left[1-\delta+\left(\frac{K}{H}\right)^{-(1-\alpha)}-e^{\frac{\mu_z+\mu_q}{1-\alpha}}\right]\left(\frac{K}{H}\right)H-C=0$$

where $\frac{K}{H}$ is given by (36) and H is given (29).
Updating: Given τ and the new function $V(\tau)$ use (33) to update $J(\tau)$.
Checking: If the updated expression for $J(\tau)$ is equal to the initial guess the algorithm has converged, otherwise construct a new guess for $J(\tau)$ by using the new updated expression.

C.3 Quadrature approximation

To compute integrals with infinite bounds of integration such as $\int_{-\infty}^{\infty} f(x)dx$, we first truncate the range of integration to (x_1, x_M). The abscissas $x_1 < x_2 <....< x_M$ are uniformly distributed over the interval (x_1, x_M) so that each interval has size $(x_M - x_1)/\,(M-1)$. Then the integral is approximated by the weighted sum $\sum_{j=1}^{M} f(x_j)w_j$ where $w_j = (x_M - x_1)/\,(M-1)$ are the integration weights.

In several cases either the lower or the upper bound of the range of integration is truncated and equal to τ^*. We accommodate this by taking into account whether a given point x_j is inside the range of integration and we follows Press et al. (1989) in using a trapezoidal rule to improve the quality of the approximation close to the endpoints. Specifically, we approximate integrals of the type $\int_{-\infty}^{\tau^*} f(x)dx$ with $\sum_{j=1}^{\bar{M}+1} f(x_j)w_j$ where $\bar{M} = \max\{i : x_i \leq \tau^*\}$, $w_j = (x_M - x_1)/\,(M-1)\,, \forall j < \bar{M}$, $w_{\bar{M}} = (1+\bar{w})\cdot 1/2\cdot(x_M - x_1)/\,(M-1)\,$, while $w_{\bar{M}} = \bar{w}/2\cdot(x_M - x_1)/\,(M-1)$ where $\bar{w} = f(x_{\bar{M}})/\left|f(x_{\bar{M}+1}) - f(x_{\bar{M}})\right|$. The weight $\bar{w}$ intends to better approximate the integral by linearly interpolating $x_{\bar{M}}$ and $x_{\bar{M}+1}$ to obtain a more accurate expression for τ^*. The case where the lower bound of the range of integration is truncated is treated analogously. The grid of points used to approximate the integrals is always the finest as available.

C.4 Computational details

In our baseline specification the support for τ given by $[-0.7, 0.3]$. In solving for the steady state we characterize the beginning-of-period distribution at a grid of 100 technological gaps, $I_\tau = 100$, equally spaced on the support for τ. The functions V and J are instead evaluated at a more accurate grid of 1000 technological gaps, $M_\tau = 1000$. The other paramaters are given in the following Table

D Solving the dynamic model

We next discuss how we linearize the equilibrium conditions of the model. Then we implement the Sims (2002) in the context of our model.

D.1 Linearization procedure

We now linearize the equilibrium condition of the model in Section D.

D.1.1 Linearization of dynamics (equation 1)

By using (23), it follows that

$$\begin{aligned}
&\hat{f}_t(\tau) = (1-\lambda)\int_{-\infty}^{\tau^*} g_\epsilon(i+\mu_z-\tau)\hat{f}_{t-1}(i)di \\
&+(1-\lambda)\,g_\epsilon(\tau^*+\mu_z-\tau)f(\tau^*)\hat{\tau}^*_{t-1}+(1-\lambda)\,g_\epsilon(\mu_z-\tau)n\ln\hat{n}_{t-2} \\
&+(1-\lambda)\left[\int_{-\infty}^{\tau^*} g'_\epsilon(i+\mu_z-\tau)f(i)di+g'_\epsilon(\mu_z-\tau)n\right]\varepsilon_{zt}
\end{aligned}$$

D.1.2 Linearization of the law of motion of capital (equation 2)

By using (20), we obtain

$$\begin{aligned}
&e^{\frac{\mu_z+\mu_q}{1-\alpha}}K\ln\hat{K}_t = -C\ln\hat{C}_{t-1}+\left[(1-\delta)K+\alpha K^\alpha H^{1-\alpha}\right]\ln\hat{K}_{t-1} \\
&+(1-\alpha)\,K^\alpha H^{1-\alpha}\ln\hat{H}_{t-1}-\frac{e^{\frac{\mu_z+\mu_q}{1-\alpha}}K}{1-\alpha}\left(\varepsilon_{z,t}+\varepsilon_{q,t}\right)
\end{aligned}$$

D.1.3 Linearization of job destruction decision (equation 3)

Remember that:

$$J_t(\tau^*_t) \equiv \beta\,(1-\lambda)\,E_t\left[\int_{-\infty}^{\tau^*_{t+1}} V_{t+1}(i)g_\epsilon(\tau^*_t+\mu_z+\varepsilon_{z,t+1}-i)di\right].$$

Then by using (26), we obtain that

$$\begin{aligned}
&\left[\beta\,(1-\lambda)\int_{-\infty}^{\tau^*} V(i)g'_\epsilon(\tau^*+\mu_z-i)di-\left(\frac{K}{H}\right)^\alpha e^{\frac{-\tau^*}{1-\alpha}}\frac{1}{C}\right]\hat{\tau}^*_t \\
&+c_1\eta_0u^{-\eta_0-1}n^{\eta_1}\left[(1+\eta_0)\ln\hat{u}_t-\eta_1\ln\hat{n}_t\right] \\
&+(1-\alpha)\left(\frac{K}{H}\right)^\alpha e^{\frac{-\tau^*}{1-\alpha}}\frac{1}{C}\left[-\ln\hat{C}_t+\alpha\ln\hat{K}_t-\alpha\ln\hat{H}_t\right] \\
&+\beta\,(1-\lambda)\int_{-\infty}^{\tau^*} g_\epsilon(\tau^*+\mu_z-i)E_t\left(\hat{V}_{t+1}(i)\right)di = 0
\end{aligned}$$

where we made use of the fact that $E_t(\varepsilon_{zt+1}) = E_t(\varepsilon_{qt+1}) = 0$ and that $V_{t+1}\left(\tau^*_{t+1}\right) = 0$.

D.1.4 Linearization of unemployment (equation 4)

By using (21), it follows that

$$\int_{-\infty}^{\tau^*} \hat{f}_t(i)di + f(\tau^*)\hat{\tau}_t^* + u \ln \hat{u}_t + n \ln \hat{n}_{t-1} = 0$$

D.1.5 Linearization of job creation condition (equation 5)

$$\begin{aligned} &- c_1\eta_1\eta_0 u^{-\eta_0} n^{\eta_1 - 1} \ln \hat{u}_t + \left[c_0 \nu n^{\nu} + c_1\eta_1 \left(\eta_1 - 1\right) u^{-\eta_0} n^{\eta_1 - 1}\right] \ln \hat{n}_t \\ &- \beta E \left(\hat{V}_{t+1}(0)\right) = 0 \end{aligned}$$

D.1.6 Linearization of n_t at $t-1$ (equation 6)

The previous equation evaluated at time $t-1$ can be conveniently expressed as

$$\ln \hat{n}_{t-1} = \ln \hat{n}_{t-1}.$$

D.1.7 Linearization of net value (equation 7)

By using (25), it follows that

$$\begin{aligned} &c_1\eta_0 u^{-\eta_0 - 1} n^{\eta_1} \left[- \left(1 + \eta_0\right) \ln \hat{u}_t + \eta_1 \ln \hat{n}_t\right] \\ &+ \hat{V}_t(\tau) \\ &+ \left(1 - \alpha\right) \left(\frac{K}{H}\right)^{\alpha} e^{\frac{-\tau}{1-\alpha}} \frac{1}{C} \left[\ln \hat{C}_t - \alpha \ln \hat{K}_t + \alpha \ln \hat{H}_t\right] \\ &- \beta \left(1 - \lambda\right) \int_{-\infty}^{\tau^*} g_{\epsilon}(\tau + \mu_z - i) E_t \left(\hat{V}_{t+1}(i)\right) di = 0, \end{aligned}$$

where we made use of the fact that $E_t(\varepsilon_{zt+1}) = E_t(\varepsilon_{qt+1}) = 0$ and that $V_{t+1}\left(\tau_{t+1}^*\right) = 0$.

D.1.8 Linearization of Euler equation for consumption (equation 8)

By deriving in (24), we obtain

$$\begin{aligned} &- \ln \hat{C}_t + E_t \left(\ln \hat{C}_{t+1}\right) \\ &+ \beta e^{-\frac{\mu_z + \mu_q}{1-\alpha}} \alpha \left(1 - \alpha\right) \left(\frac{H}{K}\right)^{1-\alpha} \left[E_t \left(\ln \hat{K}_{t+1}\right) - E_t \left(\ln \hat{H}_{t+1}\right)\right] = 0 \end{aligned}$$

where we made use of the fact that $E_t(\varepsilon_{zt+1}) = E_t(\varepsilon_{qt+1}) = 0$.

D.1.9 Linearization of efficiency unit of labor (equation 9)

$$- \int_{-\infty}^{\tau^*} e^{\frac{-\tau}{1-\alpha}} \hat{f}_t(i)di - e^{\frac{-\tau^*}{1-\alpha}} f(\tau^*)\hat{\tau}_t^* - n \ln \hat{n}_{t-1} + H \ln \hat{H}_t = 0$$

D.2 Implementation of Sims' method

Sims (2002) consider linear rational expectations models written in the form

$$\Gamma_0 y_t = \Gamma_1 y_{t-1} + Co + \Psi z_t + \Pi \eta_t$$

where y_t is the set of variables determined at time t, Co is a vector of constants, z_t is a vector of exogenous shocks while η_t is a vector of expectational errors, $E_{t-1}(\eta_t) = 0$, $\forall t$. The solution of the model takes the form

$$y_t = \Theta_1 y_{t-1} + \Theta_c C + \Theta_z z_t \tag{37}$$

where the last $2j$ columns of Θ_1 are filled with zeros.

In our case $Co = 0$ since we are linearizing around the steady state, while y_t is given by the following vector of dimension $n = 3I_\tau + 10$:

$$y_t = \left[\begin{array}{c}
\underset{I_\tau \times 1}{\hat{f}_t(\tau)} \\ \hdashline
\underset{1\times 1}{\hat{\tau}_t^*} \\ \hdashline
\underset{1\times 1}{\hat{u}_t} \\ \hdashline
\underset{1\times 1}{\ln \hat{n}_t} \\ \hdashline
\underset{1\times 1}{\ln \hat{n}_{t-1}} \\ \hdashline
\underset{I_\tau \times 1}{\hat{V}_t(\tau)} \\ \hdashline
\underset{1\times 1}{\ln \hat{C}_t} \\ \hdashline
\underset{1\times 1}{\ln \hat{K}_t} \\ \hdashline
\underset{1\times 1}{\ln \hat{H}_t} \\ \hdashline
\underset{I_\tau \times 1}{E_t(\hat{V}_{t+1}(\tau))} \\ \hdashline
\underset{1\times 1}{E_t(\ln \hat{C}_{t+1})} \\ \hdashline
\underset{1\times 1}{E_t(\ln \hat{K}_{t+1})} \\ \hdashline
\underset{1\times 1}{E_t(\ln \hat{H}_{t+1})} \\ \hdashline
\end{array}\right] \tag{38}$$

so that the number of expectational errors is given by $r = I_\tau + 3$. Thereafter, we approximate all integrals with quadrature methods by following the procedure detailed in section C.3. The fixed grid of points corresponds to the grid used to characterize the beginning-of-period distribution in the computation of the steady state equilibrium, i.e. $I_\tau = 100$. One can easily see that $\hat{f}_t$ is a predetermined variable, $\hat{V}_t(\tau)$, and $\hat{C}_t$ are jump variables while $\hat{K}_{t+1}$, $\hat{n}_t$ and $\hat{n}_{t-1}$ are redundant in the sense that in principle they could expressed as a function of the remaining variables contained in the vectors y_t and y_{t-1}. This also implies that Γ_0 has no full rank and therefore is not invertible.

D.2.1 The matrix Γ_0

The matrix Γ_0 of dimension $n \times n$ has the following form:

$$
\begin{array}{c|c|c|c|c|c|c|c|c|c|c|c|c}
\underset{I_\tau\times I_\tau}{\mathbf{I}} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} \\ \hline
\underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{\mathbf{D}} & \underset{1\times1}{0} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} \\ \hline
\underset{1\times I_\tau}{0} & \underset{1\times1}{\mathbf{G}} & \underset{1\times1}{\mathbf{H}} & \underset{1\times1}{\mathbf{T}} & \underset{1\times1}{0} & \underset{1\times I_\tau}{0} & \underset{1\times1}{\mathbf{U}} & \underset{1\times1}{\mathbf{V}} & \underset{1\times1}{\mathbf{Z}} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} \\ \hline
\underset{1\times I_\tau}{\mathbf{AA}} & \underset{1\times1}{\mathbf{BB}} & \underset{1\times1}{\mathbf{CC}} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} \\ \hline
\underset{1\times I_\tau}{0} & \underset{1\times1}{\mathbf{DD}} & \underset{1\times1}{\mathbf{EE}} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times I_\tau}{\mathbf{FF}} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} \\ \hline
\underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{\mathbf{GG}} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} \\ \hline
\underset{I_\tau\times I_\tau}{0} & \underset{I_\tau\times1}{0} & \underset{I_\tau\times1}{\mathbf{HH}} & \underset{I_\tau\times1}{\mathbf{LL}} & \underset{I_\tau\times1}{0} & \underset{I_\tau\times I_\tau}{\mathbf{I}} & \underset{I_\tau\times1}{\mathbf{MM}} & \underset{I_\tau\times1}{\mathbf{NN}} & \underset{I_\tau\times1}{\mathbf{OO}} & \underset{I_\tau\times I_\tau}{\mathbf{PP}} & \underset{I_\tau\times1}{0} & \underset{I_\tau\times1}{0} & \underset{I_\tau\times1}{0} \\ \hline
\underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times I_\tau}{0} & \underset{1\times1}{\mathbf{QQ}} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times I_\tau}{0} & \underset{1\times1}{\mathbf{RR}} & \underset{1\times1}{\mathbf{SS}} & \underset{1\times1}{\mathbf{TT}} \\ \hline
\underset{1\times I_\tau}{\mathbf{UU}} & \underset{1\times1}{\mathbf{VV}} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{\mathbf{ZZ}} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{\mathbf{XX}} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} \\ \hline
\underset{I_\tau\times I_\tau}{0} & \underset{I_\tau\times1}{0} & \underset{I_\tau\times1}{0} & \underset{I_\tau\times1}{0} & \underset{I_\tau\times1}{0} & \multicolumn{4}{c|}{} & \underset{I_\tau\times I_\tau}{0} & \underset{I_\tau\times1}{0} & \underset{I_\tau\times1}{0} & \underset{I_\tau\times1}{0} \\ \cline{1-5}\cline{10-13}
\underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \multicolumn{4}{c|}{\underset{r\times r}{\mathbf{I}}} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} \\ \cline{1-5}\cline{10-13}
\underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \multicolumn{4}{c|}{} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} \\ \cline{1-5}\cline{10-13}
\underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \multicolumn{4}{c|}{} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0}
\end{array}
$$

where the sub-matrices denoted with alphabetical letters and a suffix “0” can be recovered from the previous equation while I denotes an identity matrix. Notice that the first 9 blocks of rows correspond to equations 1 to 9, respectively. The remaining three blocks of rows correspond instead to the equations that define the expectational errors.

D.2.2 The matrix Γ_1

The matrix Γ_1 of dimension $n \times n$ has the following form:

$$
\begin{array}{c|c|c|c|c|c|c|c|c|c|c|c|c}
\underset{I_\tau\times I_\tau}{\mathbf{A}} & \underset{I_\tau\times1}{\mathbf{B}} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{\mathbf{C}} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} \\ \hline
\underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times I_\tau}{0} & \underset{1\times1}{\mathbf{D}} & \underset{1\times1}{\mathbf{E}} & \underset{1\times1}{\mathbf{F}} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} \\ \hline
\underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} \\ \hline
\underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} \\ \hline
\underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} \\ \hline
\underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{\mathbf{HH}} & \underset{1\times1}{0} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} \\ \hline
\underset{I_\tau\times I_\tau}{0} & \underset{I_\tau\times1}{0} & \underset{I_\tau\times1}{0} & \underset{I_\tau\times1}{0} & \underset{I_\tau\times1}{0} & \underset{I_\tau\times I_\tau}{0} & \underset{I_\tau\times1}{0} & \underset{I_\tau\times1}{0} & \underset{I_\tau\times1}{0} & \underset{I_\tau\times I_\tau}{0} & \underset{I_\tau\times1}{0} & \underset{I_\tau\times1}{0} & \underset{I_\tau\times1}{0} \\ \hline
\underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} \\ \hline
\underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} \\ \hline
\underset{I_\tau\times I_\tau}{0} & \underset{I_\tau\times1}{0} & \underset{I_\tau\times1}{0} & \underset{I_\tau\times1}{0} & \underset{I_\tau\times1}{0} & \underset{I_\tau\times I_\tau}{0} & \underset{I_\tau\times1}{0} & \underset{I_\tau\times1}{0} & \underset{I_\tau\times1}{0} & \multicolumn{4}{c}{} \\ \cline{1-9}
\underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \multicolumn{4}{c}{\underset{r\times r}{\mathbf{I}}} \\ \cline{1-9}
\underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \multicolumn{4}{c}{} \\ \cline{1-9}
\underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times I_\tau}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \underset{1\times1}{0} & \multicolumn{4}{c}{}
\end{array}
$$

where the sub-matrices denoted with alphabetical letters and a suffix “1” can be recovered from the previous equations .

D.2.3 The matrix Ψ

The matrix Ψ of dimension $n \times 2$ has the following form:

$$\begin{bmatrix} \underset{(I_\tau+1)\times 2}{A^{\boldsymbol{\Psi}}} \\ \cdots\cdots\cdots \\ \underset{(n-I_\tau-1)\times 2}{0} \end{bmatrix}$$

where the sub-matrix $A^{\boldsymbol{\Psi}}$ can be recovered from the previous equations. We keep the convention that the first column refers to the z-shock while the second to the q-shock.

D.3 The matrix Π

The matrix Π of dimension $n \times r$ has the following form:

$$\begin{bmatrix} \underset{(n-r)\times r}{0} \\ \cdots\cdots \\ \underset{r\times r}{\mathbf{I}} \end{bmatrix}.$$

E The ABCD representation

We write the solution of our model in (37) in the following state space form:

$$\begin{aligned} y_{1t} &= Ay_{1t-1} + Bw_t \\ y_{2t} &= Cy_{1t-1} + Dw_t \end{aligned} \tag{39}$$

where the vector y_{1t} is a vector of unobservable state variable given by

$$y_{1t} = \left[\begin{array}{c} \underset{I_\tau \times 1}{\hat{f}_t(\tau)} \\ \hdashline \underset{1\times 1}{\hat{\tau}_t^*} \\ \hdashline \underset{1\times 1}{\hat{u}_t} \\ \hdashline \underset{1\times 1}{\ln \hat{n}_t} \\ \hdashline \underset{1\times 1}{\ln \hat{n}_{t-1}} \\ \hdashline \underset{1\times 1}{\ln \hat{C}_t} \\ \hdashline \underset{1\times 1}{\ln \hat{K}_t} \\ \hdashline \underset{1\times 1}{\ln \hat{H}_t} \\ \hdashline \end{array}\right], \quad y_{1t-1} = \left[\begin{array}{c} \underset{100\times 1}{\hat{f}_{t-1}(\tau)} \\ \hdashline \underset{1\times 1}{\hat{\tau}_{t-1}^*} \\ \hdashline \underset{1\times 1}{\hat{u}_{t-1}} \\ \hdashline \underset{1\times 1}{\ln \hat{n}_{t-1}} \\ \hdashline \underset{1\times 1}{\ln \hat{n}_{t-2}} \\ \hdashline \underset{1\times 1}{\ln \hat{C}_{t-1}} \\ \hdashline \underset{1\times 1}{\ln \hat{K}_{t-1}} \\ \hdashline \underset{1\times 1}{\ln \hat{H}_{t-1}} \\ \hdashline \end{array}\right] \tag{40}$$

which has dimension 107 × 1 in our application. w_t is a column vector of dimension 2 whose first entry corresponds to the neutral technology shock the second to the investment specific technology shock,$w_t = [\epsilon_{zt}|\epsilon_{qt}]$. The vector y_{2t} is a vector of dimension 8×1 that contains the variables in our VAR (with the expeception of inflation)

$$y_{2t} = \left[\begin{array}{c} \Delta q_t \\ \hdashline \Delta y_{nt} \\ \hdashline h_t \\ \hdashline u_t \\ \hdashline s_t \\ \hdashline f_t \\ \hdashline c_t - y_t \\ \hdashline i_t - y_t \\ \hdashline \end{array}\right] \tag{41}$$

which correspond to the rate of growth of q, to the rate of growth of labour productivity, to the logged employment rate (the analogous of hours in our model), to logged unemployment, to the logged separation rate, to the logged finding rate, to the logged consumption-output ratio, to the logged investment-output ratio, respectively.

As a result thematrix A has dimenion 107 × 107, the matrix B has dimension 107 × 2, the matrix C has dimension 8 × 107, the matrix D has dimension 8 × 2.