

# Locating inside the Salop circle: Demand rotations in a micro-founded model<sup>1</sup>

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## Abstract

We present a micro-founded model of design that leads to simple demand rotations. We present simple sufficient conditions that determine when design should be extreme (fully standardized or fully tailored) or rather take intermediate positions.

## 1 Introduction

Firms constantly make decisions about pricing their goods, but also concerning the kind of goods to produce. Even though some choices of product characteristics may be costless to the firm, they are non-trivial decisions since making a product more attractive to some consumers may make it less attractive to others. This leads to interesting strategic trade-offs between the pricing and design decision.

The influential paper by Johnson and Myatt (2006) on design starts exactly on this premise.<sup>2</sup> They model design as a decision that rotates demand. The possible design choices lead to *ordered* rotations. By focusing on a particular ordering of the possible demand curves, they show that profits are quasi-convex in design so that design decisions are always extreme. They also provide micro-foundations (based on information-provision and also choice of characteristics) that lead to

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<sup>2</sup>See also Anderson and Renault (2006) and Lewis and Sappington (1994) among others.

designs as a family of ordered demand-rotations with the appropriate property on the direction of the ordering.

Here, we provide an intuitive micro-foundation in which demand rotations arise naturally. However, our approach is flexible in that it provides conditions under which the demand curves that arise follow the Johnson and Myatt (2006) ordering, or can lead to the opposite ordering. In addition, we show that although the direction of the order of rotation is a sufficient condition for quasi-convexity of profits in design, it is not necessary. We provide simple sufficient conditions in our framework for rotation ordering and therefore for quasi-convexity of profits in design.

While our model may be of independent interest, this is a useful contribution in the context of some debate about the realism and generality on the ordering that is considered in Johnson and Myatt (2006). In particular, Tirole and Weyl (2010) argue that quasi-convex profits may be more the exception than the rule, and that thus the conditions in Johnson and Myatt must be restrictive. In fact, Tirole and Weyl assume that profits are always maximized at an interior. In our setting we can also provide conditions for when this is to be expected.

We introduce a framework where a monopolist decides on price and design. Design has two components, one is a horizontal decision of which particular type of consumer to target, while the other component captures the extent the product is specifically tailored to this type of consumer or is more broadly suitable for other types. Tailoring implies a trade-off for the firm. Increased tailoring implies a higher valuation by those consumers being targeted by the firm but a lower valuation for those with different tastes. This is key to our analysis, and differentiates us from other studies that consider other strategic decisions that have a vertical flavor, such as quality, which is liked by all consumers.<sup>3</sup>

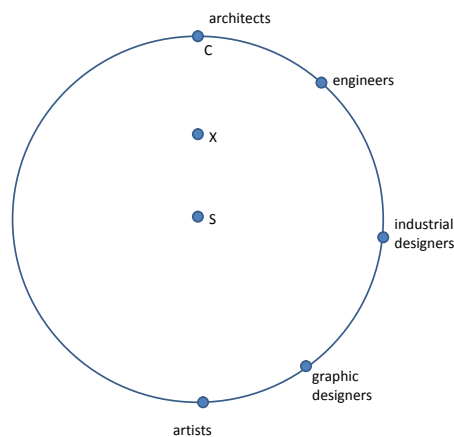
Specifically, we build on the classic Salop (1979) circle model of product location, with consumers uniformly distributed on the perimeter of a circle. Our point of departure is to allow the monopolist to locate on the interior of the circle. While locating a product along an arc captures the usual notion of horizontal differentiation, locating it into the interior of the circle captures the idea that the product is not as tailored as if it was further towards the perimeter. Accordingly, the center of the circle corresponds to a fully standardized product which is equally valued by all consumers. Assumptions about the travel costs along radials into the interior of the circle or along

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<sup>3</sup>For instance, Ungern-Sternberg (1988) and Hendel and De Figueiredo (1997) study a strategic choice that they refer as design, but which is a straight vertical quality decision.

arcs characterize the ordering of different designs and determine whether or not profits are quasi-convex in design, as we show in Section 3.

As an illustrative example, consider a software company introducing a graphic design application. The firm needs to decide the kind of consumer-base it wants to target. The firm may target artists, architects, engineers, industrial or graphic designers. The following diagram demonstrates how our framework captures key trade-offs in the firm's choice.



Illustrative product design choice for a developer of a graphic tool

As an example, the firm could build a fully generic or standard software tool intended for all tastes, labelled by  $S$  in the graph; a partially customized software for architects, such as  $X$  on the graph; or a fully customized one  $C$ . Obviously architects most preferred product would be  $C$ . It is also natural to assume that artists prefer product  $S$  to  $C$ ; otherwise locating inside the circle is irrelevant. Under these conditions, some consumers may also prefer  $C$  or  $S$  and some others will prefer  $X$ . In such a setting, and depending on different aspects such as the intensity of preferences of the different consumer types, their relative abundance etc., it may be very well the case that the monopolist may choose any of the three designs. Our framework allows us to analyze such a choice.

## 2 Model

Consider a ring of outer-radius 1 and inner-radius  $B$  with  $B > 0$ .<sup>4</sup> Consider a mass of  $2\pi$  consumers uniformly distributed on the outer circle. There is a monopolist with a constant per unit marginal cost  $mc$ , which can locate anywhere within the ring. If the firm locates right where a consumer is, she values the product  $V$ . Otherwise, she needs to incur some travel costs in order to reach the firm. She must first travel along a radius towards the center of the ring and, only then, travel along the arc. If she travels a distance  $y$  along the radius and  $x$  along the arc, the travel costs are assumed to be  $c(y) + x$  with  $c'(\cdot) > 0$ . That is, we assume linear unit travel costs along the arc, but allow any increasing shape for the cost of travelling along an arc. This is all illustrated in the figure below:

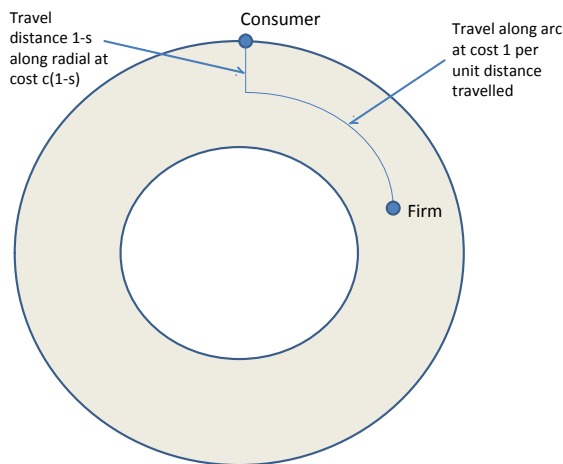


Fig 2: Design and consumer travel costs

Without loss of generality the firm is located at angle 0. Thus, the location decision boils down to choosing how far inside the ring it wants to be, which we capture by  $s \in [B, 1]$ . Locating at  $s = 1$  corresponds to a fully tailored design in which the firm aims for a niche consumer base. Similarly  $s = B$  is the most general design possible. While still aiming to a particular type of consumer it remains as broad as possible.

If a monopolist chooses a price  $p$  and a design  $s$ , the marginal consumers who are indifferent

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<sup>4</sup>The software example presented in the introduction would correspond to a  $B = 0$ . For minor technical reasons, the algebra in this paper is written for  $B > 0$ . The results for the  $B = 0$  case can be analyzed as the limit case as  $B$  tends to 0.

between purchasing or not are located at angles  $x$  and  $-x$ , where  $x$  satisfies:

$$V - c(1 - s) - sx - p = 0,$$

Thus, the demand for a monopoly who chooses price  $p$  and design  $s$  is:

$$q(p, s) = \max(0, \min(2\pi, \frac{2}{s}(V - c(1 - s) - p)))$$

### 3 Analysis

We consider the optimal design choice for a monopolist who can choose both design and price. For simplicity, we assume that optimal choices lead to a demand that is interior (which holds if  $V$  is sufficiently high).<sup>5</sup> In this case the demand function simplifies to:

$$q(p, s) = \frac{2}{s}(V - c(1 - s) - p)$$

and the monopolist's problem is to choose  $s$  and  $p$  in order to maximize:

$$\Pi(p, s) = \frac{2}{s}[V - c(1 - s) - p](p - mc). \quad (1)$$

Note first that the (inverse) demand function  $q(p, s)$  is linear and that, the higher  $s$ , i.e. the nichier the design, the steeper the slope of demand and the higher the intercept with the price axis. Thus, any two designs result in demands that cross only once (rotation point). As a result the family of demands for different designs constitute a family of rotations according to Definition 1 in Johnson and Myatt (2006). Further, Johnson and Myatt show (Proposition 1) that imposing that the rotation price decreases with  $s$  is a sufficient condition to guarantee an extreme optimal design decision. This allows us to prove the following:

**Proposition 1** *The rotation price is decreasing with  $s$  if and only if the cost of travelling along a radial  $c(\cdot)$  is concave. As a consequence, if  $c(\cdot)$  is concave the firm optimally chooses an extremal design.*

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<sup>5</sup>The same qualitative results are obtained for the case in which it is optimal to serve the whole circle.

**Proof.** Let  $p_{ij}$  denote the price at the rotation point between designs  $i$  and  $j$ . By definition of a rotation point, at price  $p_{ij}$  demands with designs  $i$  and  $j$  have to be equal. Thus,

$$\frac{2\pi}{i} [V - c(1 - i) - p_{ij}] = \frac{2\pi}{j} [V - c(1 - j) - p_{ij}] \Rightarrow p_{ij} = \frac{V(i - j) - ic(1 - j) + jc(1 - i)}{i - j}$$

Now, consider any three designs  $x > y > z$ . The ordering condition requires that price is decreasing in  $s$ , that is,  $p_{yz} > p_{xy}$ , which, using the expression above, implies that

$$\begin{aligned} \frac{V(y - z) - yc(1 - z) + zc(1 - y)}{y - z} &> \frac{V(x - y) - xc(1 - y) + yc(1 - x)}{x - y} \Leftrightarrow \\ (x - z)c(1 - y) - (y - z)c(1 - x) - (x - y)c(1 - z) &> 0 \Leftrightarrow \\ c(1 - y) &> + \frac{(y - z)}{(x - z)}c(1 - x) + \frac{(x - y)}{(x - z)}c(1 - z) \end{aligned}$$

Given that  $y \in (z, x)$  there exists a  $t \in (0, 1)$  such that  $y = tx + (1 - t)z$ . Thus, we can rewrite the previous expression as

$$c(t(1 - x) + (1 - t)(1 - y)) > tc(1 - x) + (1 - t)c(1 - z)$$

which is the condition for  $c(\cdot)$  to be concave. ■

We provide some intuition for this result. Consider the different demand curves that are traced out as the monopolist chooses different designs. A concave travel cost  $c(\cdot)$  ensures that as the monopolist moves from niche designs that induce steep demand functions to flatter broad designs the drop-off in the price intercept is not too severe. In particular, the upper envelope of what can be achieved by the family of demand rotations is traced out by the most niche and the most broad designs. Thus, the monopolist chooses one of these two designs.

Note that the converse to Proposition 1 is not true. That is, if  $c(\cdot)$  is convex, it is not guaranteed that the optimal design is going to be an interior one. In other words, firm profits can be quasi-convex in design even if the cost of travelling along a radial is convex. This can be easily perceived by explicitly looking at the first order conditions for the profit maximization program described in (1).

The first order condition with respect to price, yields that at the optimum

$$p^* = \frac{V - c(1 - s) + mc}{2}, \quad (2)$$

and it is easily verified that the second order condition is satisfied. Substituting the optimal price in (1) we obtain

$$\Pi(s) = \frac{2}{s} \left( \frac{V - c(1 - s) - mc}{2} \right)^2 \quad (3)$$

and writing the first order condition with respect to the design  $s$ , yields:

$$c'(1 - s^*) = \frac{V - c(1 - s^*) - mc}{2s^*}. \quad (4)$$

The second derivative of the firm's objective function (3) with respect to  $s$  is given by:

$$\frac{(V - c(1 - s) - mc)^2}{s^3} - \frac{V - c(1 - s) - mc}{s^2} c'(1 - s) - \frac{V - c(1 - s) - mc}{s} c''(1 - s) + \frac{(c'(1 - s))^2}{s} \quad (5)$$

At the singular point, we can substitute for  $c'(1 - s)$  from (4) to rewrite the second order condition as

$$-\frac{V - c(1 - s^*) - mc}{s^*} c''(1 - s^*) + \frac{(V - c(1 - s^*) - mc)^2}{4(s^*)^3}. \quad (6)$$

Since, at a maximal price, the price must be above the marginal cost then  $\frac{V - c(1 - s^*) - mc}{2} > 0$ , it is straight-forward that a concave  $c(\cdot)$  ensures a positive second order condition and, thus, an extreme optimal design. Note also that a slightly convex  $c(\cdot)$  would also deliver extreme design decisions.

However, the condition suggests that if the cost  $c(\cdot)$  is sufficiently convex then an interior optimal design should be expected. In terms of the upper-envelope of the induced demand curves, assuming strictly convex costs  $c(\cdot)$  implies that each design contributes a point on the upper envelope. Whether or not an intermediate design is optimal, depends on how fast the "price-intercept" falls. If radial transport costs are sufficiently convex, the fall is rapid enough to ensure an intermediate optimal design.

In general, (6) may not be trivial to verify especially since it relies on determining the solution to (4). We can, however, establish more elementary conditions that are sufficient to guarantee that an interior design is optimal.

**Proposition 2** *An interior optimal design arises if*

$$2Bc'(1 - B) + c(1 - B) > V - mc > 2c'(0)$$

**Proof.** The firm necessarily prefers an interior solution if the objective function (3) satisfies  $\Pi'(1) < 0$  and  $\Pi'(B) > 0$ . Given that

$$\Pi'(s) = -\frac{(V - c(1 - s) - mc)^2}{2s^2} + \frac{c'(1 - s)}{s} (V - c(1 - s) - mc)$$

we have that

$$\begin{aligned} \Pi'(1) < 0 &\Leftrightarrow V - mc > 2c'(0) \\ \Pi'(B) > 0 &\Leftrightarrow 2Bc'(1 - B) + c(1 - B) > V - mc \end{aligned}$$

which concludes the proof. ■

Essentially, in order for a solution to be interior one needs that the cost function  $c(\cdot)$  to be sufficiently flat at  $y = 0$  and steep enough at  $y = 1 - B$ . While these two conditions may not be always satisfied, they are interesting for two reasons. First, they are simple to check and interpret, and second, they do not impose any particular functional behavior in the interior of the domain, in particular whether the function needs to be concave or convex.

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