DEBT MATURITY AND THE LIQUIDITY OF SECONDARY DEBT MARKETS

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Abstract

We develop an equilibrium model of debt maturity choice of firms, in the presence of fixed issuance costs in primary debt markets, and an illiquid over-the-counter secondary debt market with search frictions. Liquidity in this market is related to the ratio of buyers to sellers, which is determined in equilibrium via the free entry of buyers. Short maturities improve the bargaining position of debt holders who sell in the secondary market and hence reduce the interest rate that firms need to offer on debt. Long maturities reduce re-issuance costs. The optimally chosen maturity trades off both considerations. Firms individually do not internalize that choosing a longer maturity increases the expected gains from trade in the secondary market, which attracts more buyers, and hence also facilitates the sale of debt issued by other firms. As a result, the laissez-faire equilibrium exhibits inefficiently short maturity choices.

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1 Introduction

Secondary trading in equities takes place mostly on organized exchanges. Equity markets are relatively liquid, and transaction costs are often relatively small. In contrast, secondary trading in the different forms of corporate debt takes place mostly over-the-counter (OTC), meaning that buyers and sellers have to find each other without the help of a matching mechanism provided by an exchange. As a consequence of these institutional differences, corporate debt markets are often less liquid, and transaction costs can be substantial (as documented e.g. by Edwards, Harris, and Piwowar, 2007, for the case of corporate bonds), even though these markets constitute a very important source of financing for firms.[1]

The empirical literature has explored how corporate debt market liquidity co-varies with the characteristics of the issuer as well as the issue. One recurring finding is that a longer remaining time-to-maturity on a debt claim is often associated with lower liquidity in the form of higher transaction costs. In a recent paper, He and Milbradt (2011) provide a possible explanation: They argue that debtholders who need to sell in an OTC market (which they model as a search market with ex-post bargaining) are in a worse bargaining position the longer they are locked-in into their contracts, i.e. the longer the time-to-maturity of their debt is. The worse bargaining position implies a larger discount when selling, and hence higher transaction costs.

In this paper, we take this mechanism and embed it in an equilibrium model in which there is free entry of investors into the secondary market for corporate debt. The liquidity in the secondary market is related to the ratio of buyers to sellers, which is endogenously determined. The expected ratio of buyers to sellers in the secondary market affects the debt maturity decisions of firms. These debt maturity decisions determine how profitable trading in the secondary market is, and in turn affect the ratio of buyers to sellers. Our main result is that in equilibrium, firms' maturities choices are inefficiently short. Although a firm takes into account that an increase in maturity worsens the bargaining position of investors who have to sell its debt and hence raises the required yield, it does not internalize that this also increases the profits of the investors who buy in the secondary market. The presence of more profitable deals available to buyers promotes their entry into the secondary market, which makes it easier for debt holders to liquidate their positions when necessary, and reduces the interest rates demanded by investors on the debt of all firms, at all possible maturities.

From a practical perspective, explaining the differences between private and socially optimal maturity choices is important especially in order to understand the reliance prior to the financial crisis of financial institutions on forms of very short-term debt, which partially motivates recent attempts to regulate maturity choices or the extent of maturity mismatch, e.g. via a Net Stable

[1] For instance, in its Quarterly Review of June 2012, the Bank for International Settlement reports that by December 2011, non-financials had domestic and international debt securities with a combined face value of about 5 trillion USD outstanding, whereas financials had about 15 trillion USD.

Funding Ratio as in the Basel III Accords (Basel Committee on Banking Supervision, 2010). Our results contribute to the discussion by pointing out that part of the reason why financial institutions might not be issuing longer maturity debt is because they believe that at longer maturities, they have to pay a higher liquidity premium and hence face a higher cost of debt finance. Our model highlights that while individually this may be true, if all financial institutions were forced to issue debt instruments with longer maturities, this could create a more liquid secondary market for such instruments, which would reduce liquidity premia. We believe that this applies in particular to the commercial paper market, in which the maturity mismatch created by financial institutions in the run-up to the crisis was particularly severe.

In our continuous-time infinite-horizon model, we have two types of agents with different time preferences. There are *entrepreneurs* who each can set up a firm which undertakes one long-term project that generates perpetual constant cash flows. Entrepreneurs are impatient, i.e. have a high discount rate. There are also *investors*, who in contrast are born patient, although they are subject to random liquidity shocks that make them impatient.

We assume that there is a constant and large inflow of new, patient investors. In order to take advantage of the differences in time preferences between agents, firms can issue debt to investors, which is backed by the future cash flows that the project will generate. Debt holders who become impatient will want to consume, and can attempt to sell to patient investors that do not yet hold debt in the secondary market.

We model this secondary market as a search market in which sellers and buyers are matched according to a constant returns to scale matching function. The intensities at which buyers and sellers get matched with each other depend only on the ratio of buyers to sellers in the secondary market. After a match, the price at which trade is realized is determined through Nash bargaining. The bargaining position of sellers depends on their outside option, which is to refuse to agree on a price and to find another buyer. Importantly, while searching for another buyer, the debt might mature, and the seller would receive the repayment of face value and could consume directly. The shorter the maturity, the more likely this is to happen, and hence the stronger the bargaining position of a seller, and the smaller the discount they face when selling. Investors who buy in the primary market will anticipate this and therefore require lower interest rates for shorter maturities. From the perspective of firms, a shorter maturity therefore implies lower interest payments on debt.

We model the primary debt market in a reduced form manner. We assume that when issuing or when re-issuing in order to roll over, firms can place their debt to investors via a competitive auction, which generates a fixed cost. Everything else being equal, firms will have an incentive to increase maturity in order to decrease the number of times this fixed cost is paid. Although there are types of debt securities, such as corporate bonds, for which a (small) fixed cost of issuance appears to exist (Altinkiliç and Hansen, 2000), more generally, this assumption can also be interpreted as shorthand for other mechanisms that generate a preference for longer maturities, for instance roll-over risk (see e.g. He and Xiong, 2012a).

The maturity decisions of firms trade off the frictions in the primary and secondary debt markets. Intuitively, when agents expect a low ratio of buyers to sellers in the secondary market, a shorter maturity improves the bargaining position of sellers substantially, and hence reduces interest rates required by investors in the primary market by a large amount. Firms then find it optimal to issue short maturity debt. Conversely, when agents expect a high ratio of buyers to sellers in the secondary market, maturity has little effect on the bargaining position of sellers, and hence on interest rates. Firms then find it optimal to issue long maturity debt, which reduces the cost of re-issuing.

Finally, the ratio of buyers to sellers in the secondary market is determined through free entry of buyers to the market. This in turn depends on the gains from trade that stem from the difference in the valuation of debt claims between sellers (who are impatient) and buyers (who are patient). The longer the maturity of debt, the higher the differences in valuations, and hence the more attractive it is for buyers to enter the secondary market.

In equilibrium, agents in our economy correctly anticipate the ratio of buyers to sellers and the maturities in the secondary market, and all decisions are optimal.

Maturities chosen by firms are inefficiently short, because firms do not internalize how their maturity choice affects the ratio of buyers to sellers: When a firm chooses a longer maturity, this increases the gains from trade that can be realized in the secondary market. This attracts more buyers to the market and increases the ratio of buyers to sellers, which reduces the interest rate that all other firms have to pay on their debt. A direct normative conclusion is that a social planner could mandate longer debt maturity and thereby increase liquidity in secondary markets, reduce the cost of debt for all firms, and increase welfare for all agents.

As an extension, we introduce marketmakers into the search market (similar to those in the model of Duffie, Garleanu, and Pedersen (2005)). Marketmakers provide an additional channel for trade and hence speed up trading in the market. This increases welfare, although depending on the bargaining power of the marketmakers, a large proportion of this increase is appropriated by the marketmakers themselves. The bid-ask spread of marketmakers increases with time-to-maturity, due to the effect of maturity on the bargaining position of sellers. Finally, in this extended version of the model the laissez-faire economy also exhibits inefficiently short debt maturity because of the same reasons explained above.

The rest of the paper is organized as follows. Section 2 presents the related literature, both theoretical and empirical. In Section 3 we describe the model. Section 4 discusses the determination of equilibrium, and in Section 5 we discuss its efficiency. We introduce marketmakers into the model in Section 6. Section 7 concludes. All proofs are in the appendix.

2 Related Literature

We first discuss some features of the market for corporate debt claims, and findings of the empirical literature which are relevant for the main mechanism in the model, and then discuss how our paper fits into the theoretical literature.

Secondary trading in corporate debt claims takes place mostly over-the-counter.[2] Most corporate debt claims are traded only very infrequently, at least in comparison to equity claims issued by the same entity. This infrequent trading is apparent for corporate bonds,[3] syndicated loans,[4] as well as for commercial paper,[5] and is indicative of low liquidity. There is evidence that liquidity (or lack thereof) is priced, in corporate bonds,[6] syndicated loans,[7] and commercial paper.[8]

Importantly, time-to-maturity appears to matter for liquidity. For corporate bonds, Edwards, Harris, and Piwowar (2007) and Bao, Pan, and Wang (2011) find that their preferred measure of illiquidity (the estimated transaction price spread and negative price autocovariance respectively) increases with time-to-maturity. For commercial paper, Covitz and Downing (2007) produce no direct evidence that links a measure of illiquidity to time-to-maturity, but do show that yield spreads increase in time-to-maturity. (All of these papers control for credit quality.)

Other important characteristics of debt claims that are related to liquidity, and which our model will not be able to shed light on, are age (measured as time since issuance), and credit risk. For bonds, there is ample evidence that illiquidity increases with age (see e.g. Edwards, Harris, and Piwowar, 2007; Bao, Pan, and Wang, 2011), and that illiquidity increases with credit risk. Conversely, for syndicated loans, there is evidence that illiquidity decreases with credit risk, as

[2]Historically, corporate bonds used to be traded on the NYSE. However, there was a migration towards OTC trading starting in the 1940s (Biais and Green, 2007). As of 2002, only about 5% of all US corporate bonds were still listed on the NYSE (Edwards, Harris, and Piwowar, 2007), and the average trade size on the NYSE is quite small compared to OTC trades (Hong and Warga, 2000).

[3]For instance Edwards, Harris, and Piwowar (2007) report that for their fairly representative dataset, which contains about 12.3 million trades in 22,000 US corporate bonds over 2 years, the median number of trades per day per corporate bond is about one.

[4]Although dollar volumes in the secondary market for the syndicated loan market have exceeded dollar volume in the market for corporate bonds (Bessembinder and Maxwell, 2008), data from the Loan Syndications and Trading Association suggests that most syndicated loans also trade relatively infrequently. The 2008 Loan Market Chronicle reports 63,490 trades on 2,278 facilities over the 3rd Quarter of 2007 (p. 30). By a rough calculation, this suggests an average of 28 trades per quarter per facility, or roughly one every three days.

[5]Covitz and Downing (2007) report that the trades in the secondary market make up only 16% of the total transaction volume in their data (the rest being attributable to primary market transactions).

[6]Longstaff, Mithal, and Neis (2005) document that the non-default related component of yield spreads can be up to around 50% of the total, and suggest that this non-default component is strongly related to measures of illiquidity. Bao, Pan, and Wang (2011) find that higher illiquidity is strongly related to lower prices.

[7]Gupta, Singh, and Zebedee (2008) find that loans with higher expected liquidity can be issued with lower spreads.

[8]Covitz and Downing (2007) argue that higher illiquidity raises yield spreads.

market participants appear to be interested mostly in trading distressed loans (see e.g. Gupta, Singh, and Zebedee, 2008).

Our paper is related to several strands within the theoretical literature. First, it relates to the literature on dynamic capital structure that originates with the seminal paper of Leland (1994). More recently, He and Xiong (2012b) consider the implications of exogenous transaction costs in the secondary market for debt for a Leland (1994)-style bankruptcy decision of shareholders. The most closely related paper to ours is that of He and Milbradt (2011), who extend the set-up of He and Xiong (2012b) by endogenizing transaction costs in the form of discounts that are the result of search and bargaining frictions in the secondary market. The authors study the dynamic feedback between secondary market illiquidity and default risk that arises from the fact that default not only means lower payoffs, but also makes debt claims more illiquid. Our paper shares with theirs the modeling of secondary trading as a search and bargaining process and also the insight for the effect of debt maturity on the bargaining position of sellers. Our model complements their view by examining the implications of joint maturity choices for the entry of buyers into the secondary market, which allows us to study the efficiency properties of the equilibrium and derive normative implications.

In terms of these normative implications, our paper is also related to an emerging literature which considers the effect of aggregate shocks on financial institutions, and finds market failures that generate excessively short maturity structures. Stein (2012) and Segura and Suárez (2012) find that the interaction between pecuniary externalities in the market for funds during liquidity crises and the financial constraints of banks leads to excessive short-term debt issuance. In Farhi and Tirole (2011), the collective expectation of a bailout gives incentives to choose maturities that are too short. These kind of market failures justify potential regulatory intervention that mandates longer maturities. Our model highlights that mandating longer maturities would also increase the liquidity of secondary markets.

From a technical perspective, our paper uses a search-and-matching model with ex-post bargaining. The search approach that we use was initially made popular in a labor market context (Diamond, 1982a,b; Mortensen, 1982; Pissarides, 1985). It has been applied to describing OTC markets by Duffie, Garleanu, and Pedersen (2005); Vayanos and Wang (2007); Vayanos and Weill (2008); Afonso (2011) and others. Our model differs slightly from the typical approach in these papers, in two respects. First, we assume a constant-returns-to-scale (CRS) matching function as opposed to the increasing-returns-to-scale (IRS) matching function used in these papers. Loosely speaking, with an IRS matching function, an additional seller entering the market will make it more attractive for buyers to enter, without making it much harder for other sellers to find buyers. This means that strong positive liquidity externalities are assumed as part of the technology. We assume a CRS matching function in order to highlight that our results do not depend on this technological assumption in order to generate our externality.[9] In this sense, our assumptions are closer to those of Weill (2008) and Lagos and Rocheteau (2007), who also consider a more general matching function. Second, we consider a situation with free entry of buyers, similar to Lagos and Rocheteau (2007).

We focus on a particular motivation for maturity choice. Others are considered in the literature. For example, in the classical banking literature, it is often argued that (forms of) short-term debt can act as a disciplining device (see e.g. Calomiris and Kahn, 1991), or that short-term debt can be used to signal quality (see e.g. Diamond, 1991). More recently, Dangl and Zechner (2006) show that shorter maturities can serve to commit equity holders to reducing leverage after poor performance, Greenwood, Hanson, and Stein (2010) develop a model in which firms choose maturities in response to the maturity choices of government, given a fixed demand by investors for certain maturities, and Brunnermeier and Oehmke (forthcoming) argue that an inability of issuers to commit to a maturity structure can lead to a choice of inefficiently short maturities, as creditors who lend at longer maturities know that their claim on firm value is likely to be diluted ex-post through subsequent issuance at shorter maturities.

3 The Model

Time is continuous and indexed by $t \geq 0$. There are two types of infinitely-lived, utility-maximizing, and risk-neutral agents: Entrepreneurs and investors. There is a set of measure 1 of entrepreneurs. Each entrepreneur has a large endowment of funds, and can set up a firm that can operate one project. The project requires an initial investment of 1 at $t = 0$, and subsequently produces a perpetual cash flow of $x > 0$. Entrepreneurs have discount rate $\rho > 0$.

Each investor is endowed with an equivalent small amount of funds. We normalize this so that a measure 1 of investors has a total endowment of 1.[10] An investor is either *patient* and has a discount rate of 0, or *impatient* and has a discount rate of ρ. Patient investors are subject to (idiosyncratic) liquidity shocks that arrive at Poisson rate θ and are iid across investors. Once hit by the shock, a patient investor irreversibly becomes impatient. At every time t there is a large inflow of patient investors into the economy. Investors can consume their endowment, can store it at a net rate of return of zero, or can buy the debt issued by firms, as described below. Without loss of generality we assume that investors only consume their funds when they are impatient.

Since entrepreneurs attach a higher value to present consumption than patient investors, they may prefer to let the firm finance the investment in the project through issuing debt which is placed with investors. Each firm can have a single debt issue outstanding, with an aggregate face value of

[9] For a discussion of the use of CRS versus IRS matching functions in the labor literature, see e.g. Petrongolo and Pissarides (2001).

[10] For a precise description of the necessary normalizations, see also Appendix B.

1.[11] We assume that maturity is stochastic and arrives at Poisson rate $\delta \geq 0$, chosen by the firm at $t = 0$ and held fixed through time.[12] When a debt issue matures, the repayment of principal is financed via funds raised from re-issuing the maturing debt. Debt also pays a continuous interest rate of r, set in an auction as described below. There is a *primary* and *secondary market* for debt.

In the primary market, firms issue debt at $t = 0$ and then refinance it every time it matures. Debt is placed to investors through an auction in which all investors can freely participate. Investors observe the maturity arrival rate δ of a debt issue, and then submit bids of interest rates r at which they are willing to buy a unit of the debt issue at par.

We assume that firms incur a cost $\kappa > 0$ each time an auction for debt issuance is set up. Because of the assumption of stochastic maturity, firms would be exposed to the risk of having to pay κ at random times in order to re-issue debt. To simplify, we assume that firms can insure against this risk and cover these costs by paying a flow of $\delta\kappa$ per unit of time, equal to the expected issuance cost.[13] As in Dangl and Zechner (2006), debt issuance costs generate a preference for issuing debt with longer maturities that reduce the frequency at which the cost is incurred.

The following two assumptions ensure that when the project is financed with debt its net present value is positive and that debt financing is preferred to financing the project out of the entrepreneur's own funds:

Assumption 1.

$$\frac{x}{\rho} - \kappa > \frac{\theta}{\rho + \theta} \tag{1}$$

Assumption 2.

$$1 - \kappa > \frac{\theta}{\rho + \theta} \tag{2}$$

A debt holder who becomes impatient attaches a lower value to a debt claim than an investor who is still patient. The gains from trade between these two types of agents can be realized in a secondary market, which is subject to search frictions. The debt of all firms trades in the same secondary market. Searching buyers in this market incur a non-pecuniary flow cost of effort $e_B > 0$ per unit of time while they are searching. For simplicity, we assume that sellers incur no such

[11] One can consider a version of the model in which there is a choice as to how much debt to issue. This adds complexity but does not provide important additional insights.

[12] This is for the purpose of analytical tractability, as in Blanchard (1985), Leland (1998), and He and Xiong (2012a).

[13] In a model with deterministic maturity of debt a firm would be able to finance issuance costs by retaining and saving a constant fraction of its cash flow, and this risk would not be present.

cost.[14]

We let $\mu(\alpha_t^S, \alpha_t^B)$ denote the aggregate flow of matches between sellers and buyers, where α_t^S, α_t^B are the measures of sellers and buyers, respectively, in the secondary market at time t. These measures will be endogenously determined in equilibrium. The matching function satisfies $\mu(0, \alpha^B) = \mu(\alpha^S, 0) = 0$, is increasing in both arguments, and has continuous derivatives. In order to highlight that the results derived in the paper do not rely on the strong "thick market externalities" inherent in an increasing returns to scale matching function, we assume that the matching function exhibits constant returns to scale, and let μ be concave and homogeneous of degree one in (α^S, α^B).[15] As long as $\alpha^S > 0, \alpha^B > 0$, we can define $\phi := \frac{\alpha^B}{\alpha^S}$, and then define $\mu_S(\phi) := \mu(\alpha^S, \alpha^B)/\alpha^S = \mu(1, \phi)$ as the rate at which sellers find a counterparty, and $\mu_B(\phi) := \mu(\alpha^S, \alpha^B)/\alpha^B = \mu(\phi^{-1}, 1)$ as the rate at which buyers find a counterparty. We assume that these rates satisfy the following *congestion properties*:

$$\begin{aligned} &\lim_{\phi \to 0} \mu_S(\phi) = 0, \quad \lim_{\phi \to \infty} \mu_S(\phi) = \infty, \\ &\lim_{\phi \to 0} \mu_B(\phi) = \infty, \quad \lim_{\phi \to \infty} \mu_B(\phi) = 0. \end{aligned} \tag{3}$$

These equations simply state that when there are more sellers (buyers) in the market it is more difficult for a seller (buyer) to get matched with a buyer (seller). From the perspective of all agents, the ratio of buyers to sellers ϕ will be a sufficient statistic for describing the state of the secondary market.

When sellers and buyers get matched, they engage in Nash bargaining over the trading price with bargaining powers $\beta, 1 - \beta$, respectively, with $\beta \in (0, 1)$.

To summarize, decisions are as follows: At $t = 0$, firms decide on the maturity intensity δ of their debt. They take this decision based on an expectation of the ratio of buyers to sellers $(\phi_t)_{t \geq 0}$. Then, for every $t \geq 0$ patient investors with funds decide whether to bid in the primary market auctions of any current debt (re-)issue, whether to search to buy in the secondary market, or whether to store their endowment. Impatient investors with funds will consume, and impatient debt holders decide whether to search to sell in the secondary market. These decisions are taken based on the publicly known maturity intensity choices δ of firms and on an expectation of the ratio of buyers to sellers $(\phi_{t'})_{t' \geq t}$.

We focus on steady-state equilibria in which in which all quantities that are determined in equilibrium are constant through time. The equilibrium is characterized by a pair (δ^e, ϕ^e) such that: first, given an expectation of the ratio of buyers to sellers $\phi = \phi^e$, the maturity intensity

[14] We have explored a version of the model in which sellers also incur a search cost. This complicates the analysis substantially but leaves the main result unchanged. The only additional result is that there can also exist equilibria in which there is no trade in secondary markets.

[15] Assuming increasing returns to scale would strengthen the quantitative importance of our normative results.

choices δ^e of firms are optimal, and second, the free entry decisions of investors into both the primary and secondary market are optimal given (δ^e, ϕ^e), which amounts to the condition that investors obtain no rents in any of these markets.

4 Equilibrium

We find the equilibrium of the economy by following a sequence of steps: We first work out how free entry of investors into the primary market determines the interest rate r that firms have to pay on debt as a function of their choice of maturity intensity δ, taking the ratio of buyers to sellers ϕ as given. We then analyze the firm's optimal choice of maturity intensity δ, given ϕ. Finally, we determine the ratio of buyers to sellers ϕ that is compatible with free entry of investors into the secondary market, for a given maturity intensity δ chosen by all firms. Taken together, equilibrium is characterized by the intersection point of two curves in (ϕ, δ)-space.

Throughout this section, we will illustrate the analytical results via a numerical example, for which we use the following parameters: We measure time in years, and choose a cash flow of the projects of $x = 0.9\%$, assume that investors become impatient at rate of θ =1 (i.e. the expected time until becoming impatient is 1 year). We fix the discount rate of impatient investors at ρ=8%. With these numbers, the present value of cash flows of the project, if financed via internal funds, is $x/\rho = 0.009/0.08 = 0.1125$, which is much lower than the initial cost of 1, and hence the net present value (if financing with the entrepreneur's funds) would be negative. In contrast, it will turn out that with debt-financing, the net present value will be positive. We choose a re-issuance costs of κ =1.5 basis points. We pick a simple Cobb-Douglas matching function that satisfies the congestion properties (3):

$$\mu(\alpha^S, \alpha^B) = 10 \left(\alpha^S\right)^{\frac{1}{2}} \left(\alpha^B\right)^{\frac{1}{2}} .$$

We assume equal bargaining power parameters for sellers and buyers, $\beta = 1 - \beta = \frac{1}{2}$, and a flow cost of searching to buy of e_B =1%.

4.1 The interest rate in the primary market

In order to compute the interest rate that is determined in the primary market auctions, we first need to consider the utility that investors derive from holding debt that pays an interest rate of r and has a maturity intensity δ. We use $V_0(r, \delta, \phi)$ and $V_\rho(r, \delta, \phi)$ to denote the utility that a patient and an impatient debt holder obtain, respectively, from holding a unit of the debt issue (r, δ), for a given ratio of buyers to sellers ϕ. Below, we will omit the arguments of V_0 and V_ρ where possible to reduce notational clutter.

Due to the higher discount rate of impatient investors, it is immediately obvious that $V_\rho < V_0$. We normalize the utility that is obtained from consuming an amount of funds equivalent to an investor's endowment to 1.

Patient debt holders do not search to sell in the secondary market, because buyers do not attach a higher utility to holding the debt, and hence there are no potential gains from trade. In contrast, there are gains from trade between impatient debt holder and buyers: Suppose that an impatient debt holder is matched with a patient buyer, and that trade takes place at price P per unit of face value. Then the surplus that the impatient seller obtains is $\Delta_S := P - V_\rho$. The surplus a patient buyer obtains is $\Delta_B := V_0 + 1 - P - V_B$, where we let V_B denote the value for a patient investor who is searching to buy in the secondary market.[16] The total gains from trade are therefore:

$$\Delta_S + \Delta_B = V_0 - V_\rho + 1 - V_B. \tag{4}$$

We will see later that free entry of investors into the (buy side of) the secondary market implies that in equilibrium, $V_B = 1$, and hence total gains from trade in equilibrium are equal to:

$$\Delta_S + \Delta_B = V_0 - V_\rho > 0. \tag{5}$$

We observe that the gains from trade are positive, which confirms that every match will result in a trade, with the price P splitting the surplus according to Nash bargaining,

$$P = \beta V_0 + (1 - \beta) V_\rho, \tag{6}$$

where β and $1-\beta$ are the bargaining power parameters of the seller and buyer, respectively. Since, $P \geq V_\rho$ we see that it is optimal for impatient debt holders to search to sell in the secondary market.

We can now write a system of recursive flow-value equations that V_0 and V_ρ satisfy in steady state:

$$r + \delta(1 - V_0) + \theta(V_\rho - V_0) = 0, \tag{7}$$

$$r + \delta(1 - V_\rho) + \mu_S(\phi)(P - V_\rho) = \rho V_\rho. \tag{8}$$

The first equation states that for a patient investor, the utility flow stemming from the continuous interest payments, the possibility of maturity, and the possibility of becoming impatient, just balance the reduction in utility due to discounting at rate 0 (which is zero). The second equation states that for an impatient investor, the utility flow stemming from the continuous interest payments, the possibility of maturity, and the possibility of locating a buyer in the secondary market and selling at price P, just balance the reduction in utility due to discounting at rate ρ.

Obviously, the utility of a patient debt holder $V_0(r, \delta; \phi)$ is increasing in the interest flow r, and the profits of the firm and hence the utility of the entrepreneur are decreasing in r. There

[16] Note that this surplus includes the term $1 - P$ to account for the part of the buyer's endowment that is left over after paying the price P.

is free entry of patient investors into the primary market auctions, who will compete by bidding successively lower interest rates r, until, in equilibrium,

$$V_0(r, \delta; \phi) = 1. \tag{9}$$

Given the expression for $V_0(r, \delta; \phi)$ that can be derived from equations (7) and (8), using (6), the condition (9) determines the interest rate $r(\delta; \phi)$ that firms have to pay when issuing debt. We summarize this discussion in the following lemma:

Lemma 1. *For a given ratio of buyers to sellers ϕ and maturity intensity choice δ of a firm, the interest rate $r(\delta; \phi)$ that is set in the primary market auctions is given by:*

$$r(\delta; \phi) = \frac{\rho}{\delta + \theta + \rho + \mu_S(\phi)\beta}\theta. \tag{10}$$

The interest rate exceeds 0, the discount rate of patient investors, because bidders require compensation for the utility losses associated with the frictions faced when attempting to sell in the secondary market. They will suffer these losses in case they become impatient before maturity, and need to sell, so that the interest rate can be interpreted as an illiquidity premium. The magnitude of frictions can be indirectly measured via the price discount $1 - P$ that impatient debt holders accept in order to be able to liquidate their position, which can be calculated using equation (7) as

$$1 - P(\delta; \phi) = (1 - \beta)(1 - V_\rho(\delta; \phi)) = (1 - \beta)\frac{r(\delta; \phi)}{\theta} = (1 - \beta)\frac{\rho}{\delta + \theta + \rho + \mu_S(\phi)\beta}. \tag{11}$$

As the ratio of buyers to sellers increases, it becomes easier for sellers to find a buyer. The bargaining position of sellers therefore improves, and hence the price discount and the interest rate decrease. In the limit as $\phi \to \infty$, sellers can find a buyer instantaneously, and the price discount and the interest rate tend to zero. As maturity intensity δ increases, searching sellers are more likely to have their debt mature before they find a buyer, which improves their bargaining position, implying a higher price (and hence a lower price discount), and a lower interest rate, as shown in Figure 1. As liquidity shocks become more frequent (θ increases) the interest rate that investors demand increases because it is more likely that they become impatient before the debt matures. Intuitively, if $\theta \to \infty$, so that investors are never actually patient, the interest rate tends to ρ. However, θ has no effect on the price discount, since the discount is only incurred conditional on already having become impatient. Also, as the bargaining power of sellers increases the price discount and the interest rate decrease.

4.2 The firm's problem

At $t = 0$, firms choose whether or not to invest in the project and how to finance it: either via the entrepreneur's own funds, or via a mix of these and debt financing. If debt is issued, the firm

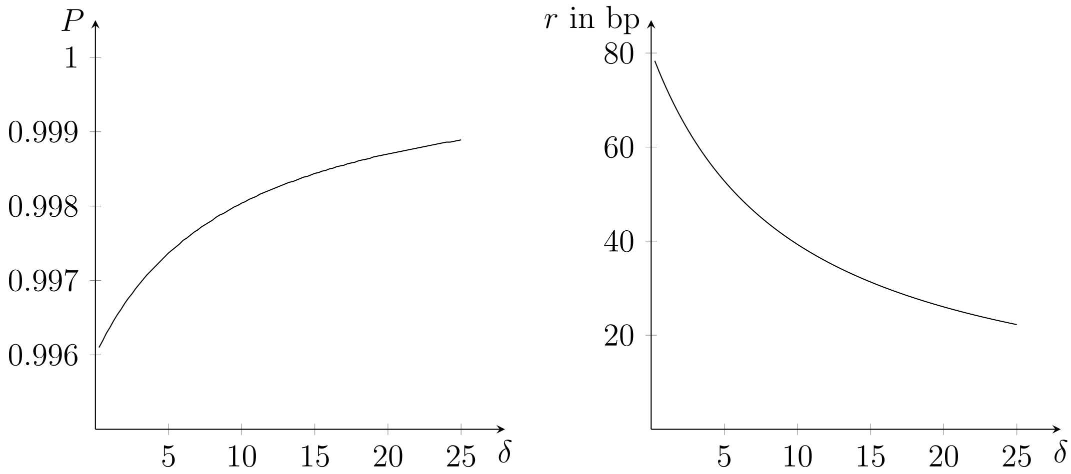

Figure 1: Price in the secondary market, interest rate in the primary market

Interest rate r that the firm has to pay in the primary market and secondary market price P, both as a function of maturity intensity δ. Parameters are as described at the beginning of Section 3, and the ratio of buyers to sellers is set to $\phi = 2.9$.

also needs to decide on the maturity δ. The debt structure of the firm is held fixed through time so that whenever outstanding debt matures the firm sets up a primary market auction in order to raise the funds needed to repay the principal of the maturing debt. The firm anticipates that in order to issue debt at par, it needs to pay an interest flow of $r(\delta;\phi)$ as given by (10).

Under Assumptions 1 and 2, firms will optimally decide to issue debt in order to invest in the project. An entrepreneur consumes the residual cash flows and hence her utility when there is investment and the firm issues debt with maturity δ is:

$$U(\delta, r(\delta;\phi)) = -1 - \kappa + 1 + \int_0^\infty e^{-\rho t}(x - r(\delta;\phi) - \delta\kappa)dt,$$

where the first term is the cost of the investment and the second the cost of the initial debt issuance (which needs to be paid by the entrepreneur). The third accounts for the proceeds from debt issuance. The last term accounts for the discounted value of the net excess cash flows that the firm generates. The expression for U can be rewritten as:

$$U(\delta, r(\delta;\phi)) = \frac{x}{\rho} - \kappa - \frac{r(\delta;\phi) + \delta\kappa}{\rho}. \tag{12}$$

The first term corresponds to the present value that an entrepreneur attaches to the gross cash flows of the project. The second term is the first issuance cost, and the last term represents the discounted cost of servicing the debt for $t > 0$, which includes re-issuance costs as well as interest payments.

The firm's optimal maturity intensity choice can then be written as:

$$\max_{\delta \geq 0} U(\delta, r(\delta;\phi)) \quad \Leftrightarrow \quad \min_{\delta \geq 0} \frac{r(\delta;\phi) + \delta\kappa}{\rho}. \tag{13}$$

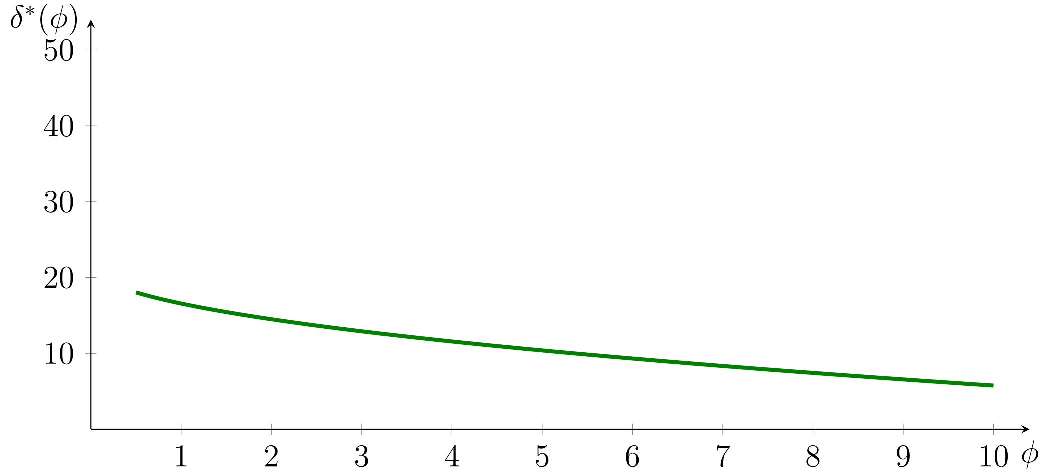

Figure 2: Optimal maturity intensity as function of market liquidity
Optimal maturity intensity $\delta^*(\phi)$ chosen by firms, as a function of the ratio of buyers to sellers ϕ. Parameters are as described at the beginning of Section 3.

We can see that the firm chooses a maturity to minimize the cost of debt service, trading off a higher re-issuance cost against lower required interest rates at higher maturity intensities (i.e. at shorter maturities).

The optimal maturity intensity is described in the following lemma:

Lemma 2. *Under Assumptions 1, 2, it is optimal to invest in the project and to issue debt. In addition, for every ϕ, the firm's problem* (13) *has a unique solution $\delta^*(\phi)$ which is given by:*

$$\delta^*(\phi) = \max\left\{\sqrt{\frac{\rho\theta}{\kappa}} - \theta - \rho - \mu_S(\phi)\beta, 0\right\}.$$

We illustrate how the optimal choice of maturity intensity δ^* varies with the ratio of buyers to sellers ϕ in Figure 2. As buyers become scarce and $\phi \to 0$, the only way in which investors can liquidate an investment is by being repaid at maturity. This makes long maturity debt very expensive for firms and they choose a high maturity intensity (a short expected maturity). As ϕ increases, the maturity of debt becomes less important to investors, since they can more easily liquidate their investment by selling in secondary markets. Hence firms find it optimal to choose a lower maturity intensity (that is, to lengthen the expected maturity), in order to reduce the expected issuance costs. When the ratio of buyers to sellers ϕ becomes sufficiently large, the firm eliminates re-issuance costs completely by setting $\delta = 0$, that is, by issuing perpetual debt.

4.3 Entry into the secondary market

We now consider what ratio of buyers to sellers ϕ is consistent with free entry of buyers into the secondary market, given a choice of maturity intensity δ by firms.

We let $V_B(\delta;\phi)$ denote the utility that a patient investor who enters the secondary market to attempt to buy obtains when firms have chosen a maturity intensity δ, and the current ratio of buyers to sellers is ϕ.

First, we establish that there must be trade after a match: If the patient investor who enters to attempt to buy is matched with a seller, there is trade if and only if the total gains from trade given in equation (4) are positive. If there was no trade following the match, then since searching is costly the entrant would be better off consuming his endowment and not entering, implying that it should be the case that $V_B(\delta;\phi) < 1$. We can see immediately from equation (4) that total gains from trade would be positive in this case, which is a contradiction. Therefore there has to be trade after a match.

Since there is trade after a match, $V_B(\delta;\phi)$ satisfies the following flow-value equation in steady state:

$$-e_B + \mu_B(\phi)(1-\beta)\left(V_0 - V_\rho + 1 - V_B\right) + \theta(1 - V_B) = 0. \tag{14}$$

The equation states that the (dis-)utility flow from the effort cost of searching, the possibility of meeting a seller which leads to trade at a price that gives a fraction $1-\beta$ of the total surplus to the buyer, and the possibility of becoming impatient and having to consume the endowment must just balance the reduction in utility due to discounting at rate 0.

We now turn to possible equilibrium values for $V_B(\delta;\phi)$. Intuitively, if buyers could obtain positive rents in the secondary market, a very large number would enter. This increased competition would have two effects: First, it would become very difficult for any particular buyer to be matched with a seller. Also, it would drive down the profits buyers can obtain if matched with a seller. Both effects would drive down the expected profits to buyers in the market until $V_B \leq 1$. At the same time, if $V_B < 1$, then no buyers should enter. Since there would still be sellers in the market, any buyer that did enter would be matched instantaneously with a seller, and would obtain positive rents. This discussion can be formalized and leads to the following lemma:

Lemma 3. *In equilibrium, free entry ensures that the utility of a searching buyer satisfies* $V_B(\delta;\phi) = 1$.

After substituting $V_B = 1$ into equation (14) and using equation (7), we obtain a *free entry condition* that describes how buyers enter the secondary market, which is summarized in the following lemma:

Lemma 4. *Free entry into the secondary market implies the following* free entry condition*:*

$$e_B = \mu_B(\phi)(1-\beta)\frac{r(\delta;\phi)}{\theta}. \tag{FEC}$$

This equation defines a strictly decreasing function $\phi^{FEC}(\delta)$ *which describes the ratio of buyers to sellers that results from free entry of buyers for each possible choice of* δ *by firms. This function is maximized for* $\delta = 0$*, when it takes a finite value* $\hat{\phi}$*, and tends to zero as* $\delta \to \infty$*.*

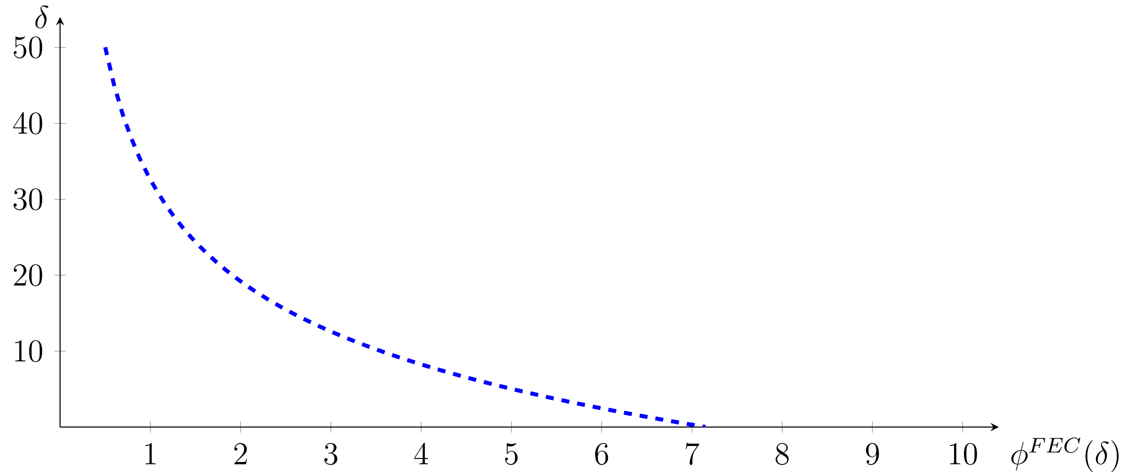

Figure 3: Free entry, maturity intensity, and the ratio of buyers to sellers

The ratio of buyers to sellers produced via free entry of buyers $\phi^{FEC}(\delta)$ as a function of maturity intensity δ chosen by firms (axes reversed to facilitate comparison with Figure 2). Parameters are as described at the beginning of Section 3.

Equivalently, this equation defines a strictly decreasing function $\delta^{FEC}(\phi)$ *which, for each possible ratio of buyers to sellers, describes the maturity intensity choice of firms that will produce this ratio as a result of free entry. This function is defined in the interval* $\phi \in (0, \hat{\phi}]$*, tends to infinity as* $\phi \to 0$ *and is equal to* 0 *at* $\phi = \hat{\phi}$*.*

Figure 3 plots $\phi^{FEC}(\delta)$ (with the axes reversed to facilitate comparison with Figure 2). Using equation (7) we can see that the gains from trade in the secondary market are $1 - V_\rho(\delta;\phi) = \frac{r(\delta;\phi)}{\theta}$. At higher maturity intensities, the bargaining position of sellers improves, and the gains from trade in the secondary market (as well as the interest rate) decrease. This makes entering the market less attractive for buyers, and reduces the ratio of buyers to sellers ϕ. Such a reduction in ϕ has two effects that lead to the reestablishment of the free entry condition (FEC). First, it increases the matching rate $\mu_B(\phi)$ of buyers. Second, it decreases the matching rate $\mu_S(\phi)$ for sellers, meaning that they are in a worse bargaining position when selling, which increases the interest rate paid on debt and the gains from trade in the market. These two effects offset the impact of the increase in maturity intensity, with the end result that $\phi^{FEC}(\delta)$ is a decreasing but not very steep function of δ. Conversely, the inverse function δ^{FEC} is a decreasing and very steep function of δ.

We note that there is a maximum ratio of buyers to sellers of $\phi = \hat{\phi}$ that can be induced via free entry when firms issue perpetual debt ($\delta = 0$). Also, as firms choose maturity intensities that tend to infinity, ϕ tends to zero as the gains from trade in the secondary market vanish and buyers choose not to enter.

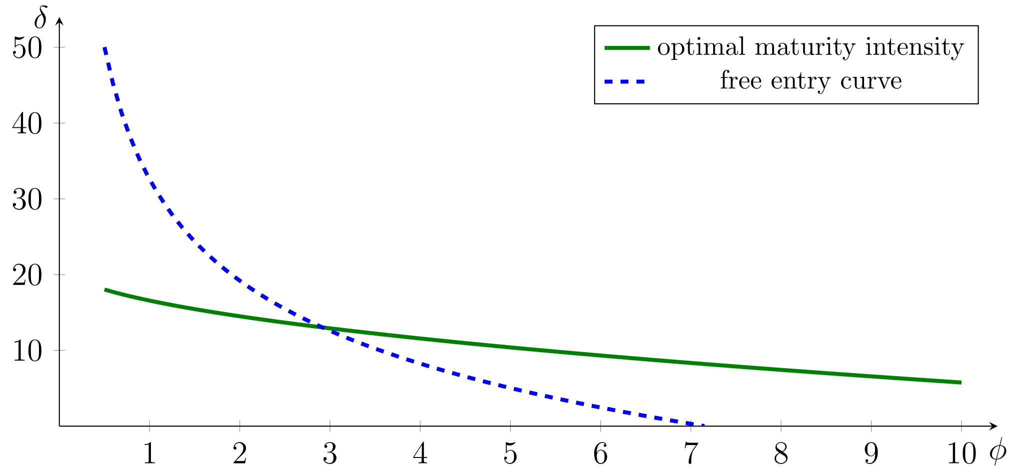

Figure 4: Equilibrium

The optimal maturity intensity $\delta^*(\phi)$ (green solid line) and the free entry curve $\delta^{FEC}(\phi)$ (blue dashed line). The unique steady-state equilibrium (δ^e, ϕ^e) occurs at the intersection of the two curves. Parameters are as described at the beginning of Section 3.

4.4 Equilibrium

Summarizing the discussion in the previous subsections, a steady-state equilibrium can be characterized by the pairs (δ^e, ϕ^e) for which maturity choices are optimal, and for which the free entry condition for buyers into the secondary market is satisfied:

$$\delta^e = \delta^*(\phi^e) \text{ and } \phi^e = \phi^{FEC}(\delta^e).$$

Proposition 1. *There exists a unique steady-state equilibrium (δ^e, ϕ^e) in the economy. Furthermore, if e_B increases, then δ^e increases and ϕ^e decreases. If κ increases, then δ^e decreases and ϕ^e increases.*

The steady-state equilibrium can be described by the intersection of a *maturity choice curve*, and a *free entry curve* as illustrated in Figure 4. Since both curves (seen as functions of ϕ) are decreasing, there could exist multiple intersection points: If firms expect a high ratio of buyers to sellers, they could issue debt with low maturity intensity which generates important gains from trade in the secondary market. This in turn could attract many buyers, and produce the anticipated high ratio of buyers to sellers. Proposition 1, however, states that this kind of self-fulfilling equilibrium does *not* arise in the model. The intuition is that, while the optimal maturity function $\delta^*(\phi)$ depends on the ratio of buyers to sellers ϕ only via the matching intensity of sellers that determines the interest rate set in the primary market, the free entry curve $\delta^{FEC}(\phi)$ depends on ϕ via the matching intensity of sellers as well as that of buyers (as was argued in previous section after Lemma 4). As a consequence, the function $\delta^{FEC}(\phi)$ is more sensitive to changes in ϕ than the function $\delta^*(\phi)$, so that its slope is steeper, and therefore there exists a unique intersection point between the two curves.

We now turn to comparative statics. When the effort cost of searching that buyers incur increases, the free entry curve in Figure 4 shifts to the left: A given maturity intensity will produce less entry, and hence a decrease in the ratio of buyers to sellers. This hurts the sellers. In response to this decrease, firms therefore increase their maturity intensity in order to improve the bargaining position of sellers.

When the issuance cost κ increases, the maturity choice curve in Figure 4 shifts downwards: for a given ratio of buyers to sellers, firms prefer lower maturity intensities to reduce the frequency at which the higher issuance cost is paid. As firms reduce the maturity intensity, the gains from trade in the secondary market increase, which attracts more buyers and hence increases the ratio of buyers to sellers.

We note that given an equilibrium (δ^e, ϕ^e), the steady-state measures of buyers and sellers are uniquely determined, as described in Appendix B.

For the parameters described at the beginning of this section, we have an equilibrium at $(\delta^e \approx 13, \phi^e \approx 2.9)$, implying an (expected) maturity of debt claims of 28 days (just under 1 month), which resembles a maturity one could observe for commercial paper, and a ratio of buyers to sellers of roughly 2.9. At this ratio of buyers to sellers, the rate at which sellers find buyers is about $\mu_S(\phi) \approx 17$, implying an average time-to-trade of about 21 days for sellers, and the rate at which buyers find sellers is about $\mu_B(\phi) \approx 5.9$, implying an average time-to-trade of about 62 days. Both of these are unrealistically high. In Section 6, we will see that the introduction of marketmakers will speed up trading and produce more realistic numbers.

5 Efficiency of equilibrium

It is well known that models of search with ex-post bargaining in general exhibit an inefficient level of entry (see e.g. Pissarides, 1990, chapter 7). In the context of our model let us fix a maturity intensity choice. Then investors who enter the secondary market in order to buy impose a negative externality on other buyers, by making it more difficult for them to be matched with a seller. This externality could lead to an inefficiently high level of entry. At the same time, buyers do not appropriate the whole surplus from a match, and thus they do not have enough incentives to incur the cost of searching, which might lead to an inefficiently low level of entry. The relative importance of the two opposing forces depends on the bargaining power of buyers in the market: when it is high the first dominates and there is excessive entry, when it is low the second dominates and there is insufficient entry. The amount of entry will only be socially efficient for a particular value of the bargaining power of buyers which exactly balances the two effects. At this level of the bargaining power, the price in the secondary market is such that the marginal rates of substitution of the price versus ϕ are equalized across buyers and sellers.[17]

Since this general inefficiency result due to congestion externalities associated with entry decisions are well known in the search literature, we focus in this section on the efficiency properties of our model from a second best perspective. In particular we assume that a Social Planner (SP) can choose the maturity intensity of debt, but cannot influence the entry decisions of investors. The SP chooses the maturity intensity in order to maximize surplus in the economy. Our objective is to understand whether there exist differences between the laissez-faire equilibrium and the welfare maximizing allocation chosen by the SP, and if so, to understand the roots of the discrepancy.

In this scenario, there is still free entry to both the primary and secondary debt markets and, as in the previous section, investors just break even and obtain a utility equal to the utility associated with instantaneously consuming their endowment. The only agents who obtain a surplus are entrepreneurs, and therefore the SP will choose δ in order to maximize their utility. The maximization problem of the SP differs, nevertheless, from the firm's problem in how it takes into account the ratio of buyers to sellers in the secondary market: while firms take ϕ as *given*, the SP *internalizes* the effects of maturity choices on the ratio of buyers to sellers.

More formally, the SP internalizes that a maturity intensity δ induces a level of liquidity $\phi^{FEC}(\delta)$ in the secondary market. We can write the SP's optimization problem in terms of the expression for the utility of entrepreneurs in equation (12) as follows:

$$\max_{\delta \geq 0} U^{SP}(\delta) = U(\delta, r(\delta; \phi^{FEC}(\delta))).$$

Now if the competitive equilibrium (δ^e, ϕ^e) has $\delta^e > 0$, then the first order condition for firms implies that at the equilibrium values (δ^e, ϕ^e),

$$\frac{\partial U}{\partial \delta} + \frac{\partial U}{\partial r}\frac{\partial r}{\partial \delta} = 0,$$

whereas first order condition for the social planner is

$$\frac{dU^{SP}}{d\delta} = \frac{\partial U}{\partial \delta} + \frac{\partial U}{\partial r}\left(\frac{\partial r}{\partial \delta} + \frac{\partial r}{\partial \phi}\frac{d\phi^{FEC}}{d\delta}\right) = 0. \tag{15}$$

We note that $\frac{\partial U}{\partial r} < 0$ or that the utility of entrepreneurs is decreasing in interest rates, that $\frac{\partial r}{\partial \phi} < 0$ or that interest rates are decreasing in the ratio of buyers to sellers, and $\frac{d\phi^{FEC}}{d\delta} < 0$ or that the ratio of buyers to sellers induced by free entry is decreasing in maturity intensity. Hence the first order condition for firms implies that at the equilibrium values (δ^e, ϕ^e),

$$\frac{dU^{SP}(\delta)}{d\delta} < 0, \tag{16}$$

[17] It can be shown that in the context of our model, the first order condition for maximization of welfare with respect to β holds when $\beta = \phi\mu'_B(\phi)/\mu_B(\phi)$, i.e. when β is equal to the elasticity of $\mu_B(\phi)$ with respect to ϕ. This is a very standard condition in the labor-search literature, sometimes referred to as the "Hosios Condition," see Pissarides (1990, chapter 7), or Hosios (1990).

and hence that the equilibrium maturity intensity chosen by firms, δ^e, is too large from the perspective of the SP.

The SP can therefore increase aggregate welfare by reducing maturity intensity. The reason is that any given firm does not internalize that by choosing a smaller maturity intensity, it will increase the gains from trade in the secondary market, which increases the ratio of buyers to sellers and so makes it easier for sellers, including those who want to sell the debt of other firms, to find a buyer. This reduces the interest rates that *all* firms pay. Since investors always break even, the increase in the utility of entrepreneurs brought about by an decrease in maturity intensity is a Pareto improvement.

The local argument above can be extended to a global result which is the main result of the paper:

Proposition 2. *Let (δ^e, ϕ^e) be an equilibrium with $\delta^e > 0$. Then the solution δ^{SP} to the Social Planner's problem satisfies $\delta^{SP} < \delta^e$ and induces $\phi^{SP} > \phi^e$, and it Pareto improves the competitive equilibrium.*

The key implicit assumption that generates the externality is that debt claims with different maturities are all traded in a single secondary search market, in the sense that if there are claims with different maturities being sold in the market, buyers cannot search to be matched only with specific maturities. This means that a single firm who deviates from the equilibrium maturity intensity and chooses $\delta \neq \delta^e$ knows that this deviation will not affect the distribution of maturities available in the market, hence knows that this will not affect entry, and hence will correctly anticipate that this will not affect the ratio of buyers to sellers ϕ^e.

Conversely, consider a situation in which debt claims with different maturities are all traded in different sub-markets, and that buyers can decide in which sub-market they search, and hence can search to be matched only with a specific maturity. In this case, we would have a free entry condition for each sub-market i, and a corresponding ratio of buyers to sellers ϕ_i^e. Suppose that firms who deviate and offer a maturity not yet traded in the market know that this creates a new sub-market. This means that even a single firm which deviates from the equilibrium maturity intensity knows that the ratio of buyers to sellers will be determined by its maturity choice. In this situation firms would internalize the effect of maturity on entry, and on the ratio of buyers to sellers in sub-markets, ϕ_i^e. As a consequence, the maximization problem of the SP would coincide with the one of firms and the laissez-faire equilibrium would exhibit efficient maturity choice.

We note that in a *competitive search* model (or *directed search* model) (Moen, 1997) neither the general inefficiency described above, nor our inefficiency would exist. In such a model, there exist sub-markets, and prices are competitive in the sense that they equalize marginal rates of substitution across buyers and sellers in each sub-market. They key feature in that type of model that would eliminate our externality is the existence of sub-markets, not the competitive pricing.

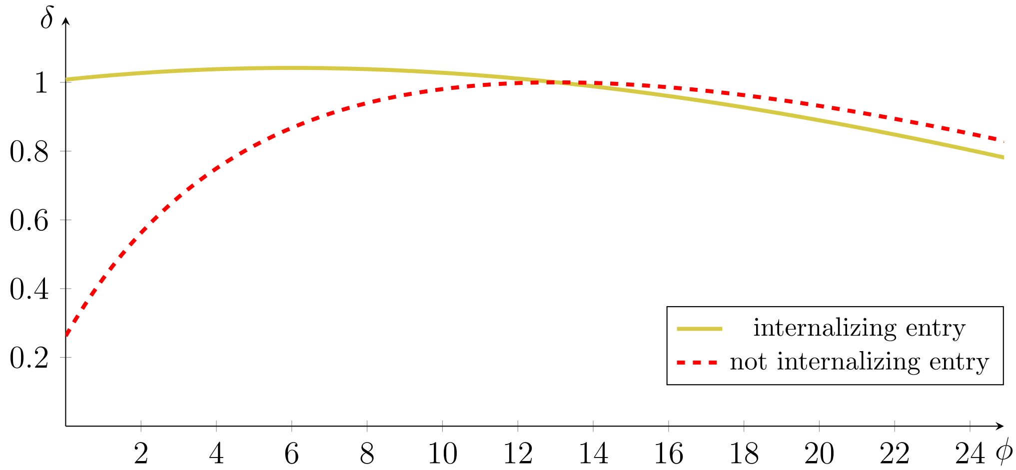

Figure 5: Profits of firms as function of δ
Profits of firms (normalized by profit of firms in the laissez-faire equilibrium), as a function of δ, (a) internalizing the effect of δ on entry and hence $\phi = \phi^{FEC}(\delta)$ (solid yellow line), and (b) not internalizing the effect of δ on entry and hence $\phi = \phi^e$ (red dashed line). Parameters are as described at the beginning of Section 3.

This raises the question as to what extent the assumption of a single secondary market for debt claims (i.e. no sub-markets) is empirically plausible. First, it is clear that the externality cannot operate across debt markets that are clearly distinct. For instance, in practice, maturity decisions on corporate bonds are unlikely to affect the ratio of buyers to sellers in the market for commercial paper or syndicated loans, and hence the maturity decisions on commercial paper and syndicated loans. However, we believe that the externality can operate within one of these markets. From interviews with market practitioners we learnt that there is often not much specialization of traders in terms of maturities. For instance, on corporate bond trading desks (or commercial paper trading desks), there will frequently be one trader assigned to a set of issuers, trading in bonds of all maturities issued by these issuers. Investors who contact traders therefore cannot know ex-ante what specific maturities the traders will be interested in trading. (This contrasts with sovereign bonds, where there are typically several traders assigned to a single large sovereign, where each trader specializes in trading bonds in a certain maturity range. Investors who contact a trader will know ex-ante what range of maturities a trader will trade.) We therefore believe that the metaphor of a single search market is plausible for describing e.g. the corporate bond market in isolation, or the commercial paper market in isolation.

For the parameters chosen at the beginning of Section 3, the socially optimal maturity intensity can be calculated as $\delta^{SP} \approx 6$, implying an (expected) maturity of just above 60 days (again, this more closely resembles a commercial paper maturity rather than a corporate bond maturity). At this maturity intensity, the ratio of buyers to sellers would be about $\phi = 4.7$. This compares to the lower (expected) maturity of about 28 days, and the consequently lower ratio of buyers to sellers of

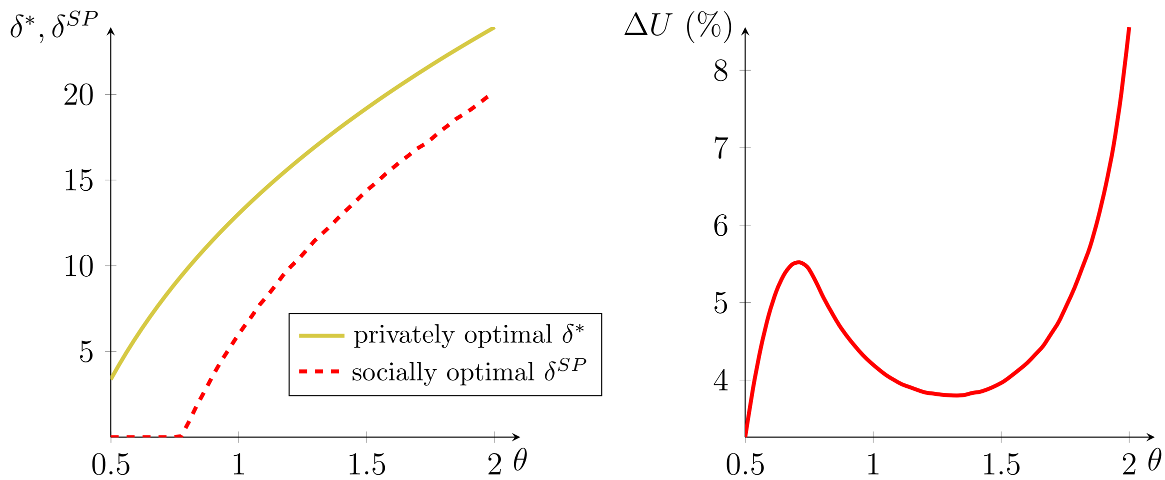

Figure 6: Social gains from regulation and θ

Left panel: Socially optimal maturity intensity δ^{SP} (dashed red line) and privately optimal maturity intensity δ^* (solid yellow line), for different values of θ. Right panel: percentage gain in welfare from implementing the socially optimal maturity intensity, as a function of θ. Parameters are as described at the beginning of Section 3.

about 2.9 that would prevail in a laissez-faire equilibrium as calculated at the end of Subsection 4.4. We illustrate the difference between the socially optimal choice and the privately optimal choice of maturity intensity in Figure 5. In our numerical illustration, we obtain that the maturity intensity chosen by the SP would raise welfare by about 4.2%.

We also calculate the social gains that can arise for different values of θ (the rate at which investors are hit by liquidity shocks), but keeping all other parameters the same, and plot the result in Figure 6. We can see that as the rate at which investors are hit by liquidity shocks increases, the potential to raise welfare also increases (right hand side panel of Figure 6). For instance, if θ were to jump to 2 (i.e. investors are hit by a liquidity shock every 6 months on average), the potential increase in welfare relative to the laissez-faire equilibrium increases to about 8.6%, more than double of the percentage increase in welfare that can be achieved in the baseline case with θ =1. This suggests that there is a particularly strong case for regulating maturities in situations in which investors have a strong preference for liquidity.

6 Marketmakers

Following the set-up of Duffie, Garleanu, and Pedersen (2005), in this section we extend the baseline model in order to incorporate a new class of agents, *marketmakers*, that intermediate between buyers and sellers in the secondary market. Marketmakers will provide an additional channel for trade and hence speed up trading in the market. This increases welfare, although depending on the bargaining power of the marketmakers, a large proportion of this increase is appropriated by the marketmakers themselves. We also describe how the bid-ask spreads of marketmakers relate to maturity intensities.

The set-up is as follows: There exist risk-neutral and utility-maximizing marketmakers, who are not endowed with funds, but have access to a special matching technology in the secondary market, as well as access to an inter-dealer market (described in more detail below). The discount rate of marketmakers is ρ. We deviate from the set-up in the paper of Duffie, Garleanu, and Pedersen (2005) by assuming a constant and large inflow of new marketmakers, who can freely enter the secondary market, and by assuming that marketmakers active in the secondary market incur a flow utility cost $e_M > 0$. These assumptions will play a similar role to our assumptions on free entry of buyers into the secondary market.

We also assume that the special matching technology of marketmakers takes the constant-returns-to-scale form described in Section 3: When marketmakers search in the secondary market, they can be matched with both investors who are searching to buy or sell according to an aggregate matching function $\overline{\mu}(\cdot,\cdot)$ that satisfies the same properties as the function $\mu(\cdot,\cdot)$ that describes the matches between investors in the secondary market. More precisely, if $\alpha^S, \alpha^B, \alpha^M$ are the measures of sellers, buyers and marketmakers searching in the secondary market, matches between sellers and marketmakers occur at a rate of $\overline{\mu}(\alpha^S, \alpha^M)$, and matches between buyers and marketmakers occur at a rate of $\overline{\mu}(\alpha^B, \alpha^M)$. The arrival of matches is independent and thus in a short time interval dt a marketmaker cannot be matched with both a seller and a buyer.

We define $\chi_S = \frac{\alpha^M}{\alpha^S}, \chi_B = \frac{\alpha^M}{\alpha^B}$ as the ratios of marketmakers to sellers and buyers, respectively, and use $\overline{\mu}_S(\chi_S)$ ($\overline{\mu}_M(\chi_S)$) to denote the rate at which a seller (marketmaker) is matched with a marketmaker (seller) and, analogously, use $\overline{\mu}_B(\chi_B)$ ($\overline{\mu}_M(\chi_B)$) to denote the rate at which a buyer (marketmaker) is matched with a marketmaker (buyer). Due to our assumptions on $\overline{\mu}(\cdot,\cdot)$, these matching rates satisfy the standard congestion properties (cf. equation (3)). Note that once we know the ratio of buyers to sellers and the ratio of marketmakers to buyers, we can compute the ratio of marketmakers to sellers ($\chi_S = \phi\chi_B$), such that the variables ϕ and χ_B will be sufficient to determine all the marginal matching intensities.

After a match involving a marketmaker, there is Nash bargaining, where the bargaining power parameter of marketmakers is γ, for both matches with buyers and matches with sellers. As in the set-up of Duffie, Garleanu, and Pedersen (2005), marketmakers also have access to an immediately accessible inter-dealer market in which they can instantly unload the positions which they enter into with investors in the secondary market, and so hold no inventory at any time.

We use B, A to denote the bid and ask prices at which marketmakers are willing to buy from and sell to investors, respectively. We use Q to denote the price in the inter-dealer market. In equilibrium, we will have

$$B \leq Q \leq A.$$

As in the set-up of Duffie, Garleanu, and Pedersen (2005), a marketmaker who is matched with a seller can purchase the unit of debt at the price B in the secondary market using the funds she obtains from instantaneously selling this unit in the inter-dealer market at the price Q. Analogously, a marketmaker who is matched with a buyer can instantaneously purchase a unit of debt in the inter-dealer market at the price Q using the funds she obtains from selling this unit at the price A in the secondary market. Hence, marketmakers do not require their own funds to intermediate.

Taking into account the bargaining powers of the different agents and the equilibrium surpluses to be shared after a match, we can write the following equations that the prices must satisfy, as a function of the values that investors attach to holding the assets:

$$P - V_\rho = \beta(V_0 - V_\rho), \tag{17}$$

$$Q - B = \gamma(Q - V_\rho), \tag{18}$$

$$A - Q = \gamma(V_0 - Q). \tag{19}$$

The first equation is (6) in the baseline model and describes how surplus is split between investors who trade with each other directly when they are matched in the secondary market. The second one describes how the surplus is split between a marketmaker and a seller. In equilibrium, this surplus is the difference between the price Q at which a marketmaker will sell a unit of debt in the inter-dealer market immediately after buying it from a seller, and the valuation that the seller attaches to holding the unit of debt, V_ρ. The third equation describes how surplus is split between a marketmaker and a buyer. Analogously, this surplus is the difference between the valuation that the buyer attaches to holding the unit of debt, V_0, and the price Q at which the marketmaker will buy the claim in the inter-dealer market immediately before selling it on to an investor. The final fourth condition necessary for determining the four prices is that Q must take a value such that the aggregate quantity bought by marketmakers from investors must equal the aggregate quantity sold by marketmakers to investors. This must be the case because marketmakers cannot hold inventory. We will refer to it as the *inter-dealer market clearing* condition.

In order for the inter-dealer market to clear, the equilibrium inter-dealer price Q must lie in the interval $[V_\rho, V_0]$.[18] We describe where in the relevant interval Q is located via a variable λ that we call the inter-dealer price index, defined as follows:

$$\lambda := \lambda \in [0, 1] \text{ such that } Q = \lambda V_0 + (1 - \lambda)V_\rho.$$

We can now use (17) - (19) to write the restrictions on equilibrium prices in terms of V_ρ, V_0, and

[18] If Q lies outside this range, one type of match produces a negative surplus and hence no trade, while the other match produces a positive surplus, and hence trade. Hence marketmakers are only buying from or only selling to investors, which means that the inter-dealer market is not clearing.

λ as

$$
\begin{aligned}
P &= \beta V_0 + (1-\beta)V_\rho, \\
B &= (1-\gamma)\lambda V_0 + (\gamma + (1-\gamma)(1-\lambda))\, V_\rho, \\
A &= (\gamma + (1-\gamma)\lambda)\, V_0 + (1-\gamma)(1-\lambda)V_\rho, \\
Q &= \lambda V_0 + (1-\lambda)V_\rho.
\end{aligned}
$$

To consider inter-dealer market clearing and the determination of λ (or equivalently Q) in more detail, it will be useful to sequentially consider the cases where $\phi < 1, \phi > 1$ or $\phi = 1$.

If $\phi < 1$, there are more sellers than buyers and thus there are more matches between marketmakers and sellers than between marketmakers and buyers. For the inter-dealer market to clear, it must be the case that some of the matches between marketmakers and sellers will not lead to trade, which can happen only if there are no gains from trade associated with these matches, i.e. $\lambda = 0$ and $Q = V_\rho$.

If $\phi > 1$, there are more buyers than sellers and thus there are more matches between marketmakers and buyers than between marketmakers and sellers. For the inter-dealer market to clear, it must be the case that some of the matches between marketmakers and buyers will not lead to trade, which can happen only if there are no gains from trade associated to these matches, i.e. $\lambda = 1$ and $Q = V_0$.

If $\phi = 1$, there are as many buyers as sellers and thus there are as many matches between marketmakers and buyers as between marketmakers and sellers. In this case inter-dealer market clearing on its own is insufficient to pin down λ (or equivalently Q), but the fact that ϕ needs to be equal to 1 provides the necessary additional condition.

In all three cases, the system of recursive flow-value equations that V_0, V_ρ satisfy in steady state is as follows:

$$r + \delta(1 - V_0) + \theta(V_\rho - V_0) = 0 \tag{20}$$

$$r + \delta(1 - V_\rho) + \mu_S(\phi)(P - V_\rho) + \overline{\mu}_S(\chi_S)(B - V_\rho) = \rho V_\rho \tag{21}$$

The first equation is the same as equation (7). The second equation corresponds to equation (8), with the new term $\overline{\mu}_S(\chi_S)(B - V_\rho)$ that accounts for the possibility of locating a marketmaker and selling to him at the bid price B.

Using the conditions on the equilibrium prices P and B, the fact that free entry into the primary market auctions implies $V_0(r, \delta; \phi, \chi_B, \lambda) = 1$ (cf. equation (9)), and that $\chi_S = \phi\chi_B$, we can work out the interest rate as in Section 4.1, and obtain the following result (which is the analogue to Lemma 1):

Lemma 5. *In the model with marketmakers, for a given ratio of buyers to sellers ϕ, a given ratio of marketmakers to buyers χ_B, inter-dealer price index λ, and maturity intensity choice δ of a firm, the interest rate $r(\delta; \phi, \chi_B, \lambda^e)$ that is set in the primary market auctions is given by:*

$$r(\delta; \phi, \chi_B, \lambda) = \frac{\rho}{\delta + \theta + \rho + \mu_S(\phi)\beta + \overline{\mu}_S(\phi\chi_B)(1-\gamma)\lambda}\theta. \tag{22}$$

Comparing to interest rate in the baseline model (equation (10)) we observe that there is an extra term $\overline{\mu}_S(\phi\chi_B)(1-\gamma)\lambda$ in the denominator which reduces the interest rate firms have to pay. This extra term is positive only if $\lambda > 0$, which ensures that the inter-dealer price Q is above V_ρ and thus there are strictly positive gains from trade between sellers and marketmakers, and if $\gamma < 1$, in which case sellers appropriate some of these gains.

We can then follow the same procedure as in the baseline model to work out the optimal maturity intensity choice, and obtain the following result (which is the analogue to Lemma 2):

Lemma 6. *In the model with marketmakers, under Assumptions 1, 2, it is optimal to invest in the project and to issue debt. In addition, for every ϕ, χ_B, λ, the firm's problem* (13) *has a unique solution $\delta^*(\phi, \chi_B, \lambda)$ which is given by:*

$$\delta^*(\phi, \chi_B, \lambda) = \max\left\{\sqrt{\frac{\theta\rho}{\kappa}} - \theta - \rho - \mu_S(\phi)\beta - \overline{\mu}_S(\phi\chi_B)(1-\gamma)\lambda, 0\right\}. \tag{23}$$

Again, as long as $\lambda > 0$ and $\gamma < 1$, firms will take advantage of the presence of marketmakers and the additional possibility they give to debt holders for liquidating their positions, by lowering the maturity intensity δ and reducing the frequency at which the re-issuance cost is paid.

Similarly, a free entry condition for buyers corresponding to that in Lemma 4 can be derived:

Lemma 7. *Free entry into the secondary market implies the following* free entry condition *for buyers:*

$$e_B = [\mu_B(\phi)(1-\beta) + \overline{\mu}_B(\chi_B)(1-\gamma)(1-\lambda)]\,\frac{r(\delta; \phi, \chi_B, \lambda)}{\theta}. \tag{24}$$

Comparing with the free entry condition for buyers in the baseline model (FEC), we observe that there is an additional term, corresponding to the possibility that a searching buyer is matched with a marketmaker. This term is strictly positive as long as $\lambda < 1$ and $\gamma < 1$, in which case the ask price A at which trade with the marketmaker occurs is below the reservation utility V_0 of the buyer. Through this channel, the presence of marketmakers encourages additional entry of buyers.

At the same time, there is a second, more subtle channel through which the presence of marketmakers discourages the entry of buyers. This is easiest to see in the case where $\lambda = 1$ (but $\gamma < 1$), in which buyers do not benefit from being matched with a marketmaker, since the ask price A will then correspond to their reservation utility. However, sellers will benefit from being matched with marketmakers. This means that the interest rate r falls, reducing the gains from trade in the secondary market, which leads to less entry by buyers.

Finally, a free entry condition for marketmakers can be derived, as in the following lemma:

Lemma 8. *Free entry into marketmaking implies the following* free entry condition *for marketmakers:*

$$e_M = (\lambda \overline{\mu}_M(\phi \chi_B) + (1-\lambda)\overline{\mu}_M(\chi_B))\, \gamma \frac{r(\delta; \phi, \chi_B, \lambda)}{\theta}. \tag{25}$$

We can see that in equilibrium, free entry into marketmaking will occur until the expected flow of gains from dealing with both buyers and sellers just compensates for the disutility flow cost e_M.

An equilibrium of the economy in the extended model can be described by a tuple $(\delta^e, \phi^e, \chi_B^e, \lambda^e)$ of maturity intensity choice of firms δ^e, and ratios of buyers to sellers and marketmakers to buyers (ϕ^e, χ_B^e) together with an equilibrium inter-dealer price index $\lambda^e \in [0, 1]$, such that the maturity intensity choice of firms is optimal, the entry decisions of buyers and marketmakers are optimal, and the inter-dealer market clears (which amounts to $\lambda^e = 1$ if $\phi^e > 1$ and $\lambda^e = 0$ if $\phi^e < 1$).

In the model with marketmakers, an equilibrium exists, is unique, and is also inefficient, as summarized in the following two propositions (the analogues to Propositions 1 and 2 in the baseline model):

Proposition 3. *In the model with marketmakers, there exists a unique steady-state equilibrium.*

Proposition 4. *In the model with marketmakers, let $(\delta^e, \phi^e, \chi_B^e, \lambda^e)$ describe an equilibrium with $\delta^e > 0$. Then the solution δ^{SP} to the Social Planner's problem satisfies $\delta^{SP} < \delta^e$.*

When marketmakers have all the bargaining power, i.e. $\gamma = 1$, it can be seen from the above expressions that the equilibria $(\delta^e, \phi^e, \chi_B^e, \lambda^e)$ of this economy correspond to equilibria (δ^e, ϕ^e) of the model with no marketmakers. In this case the presence of marketmakers has no effect on the entry decisions of buyers to the primary or secondary debt market, and no effect on the optimal decisions of firms. The reason is that marketmakers appropriate all of the additional value that they generate through their intermediation, and thus their presence does not affect the decision of other agents. When marketmakers do not have all the bargaining power, i.e. $\gamma < 1$, investors do obtain part of the value created by the alleviation of search frictions. It can be shown that in this case the laissez-faire equilibrium exhibits longer debt maturity than when marketmakers are absent and that firms' profits increase.

Finally, for any given equilibrium, the bid-ask spread $A - B$ of marketmakers can be calculated, as stated in the following lemma:

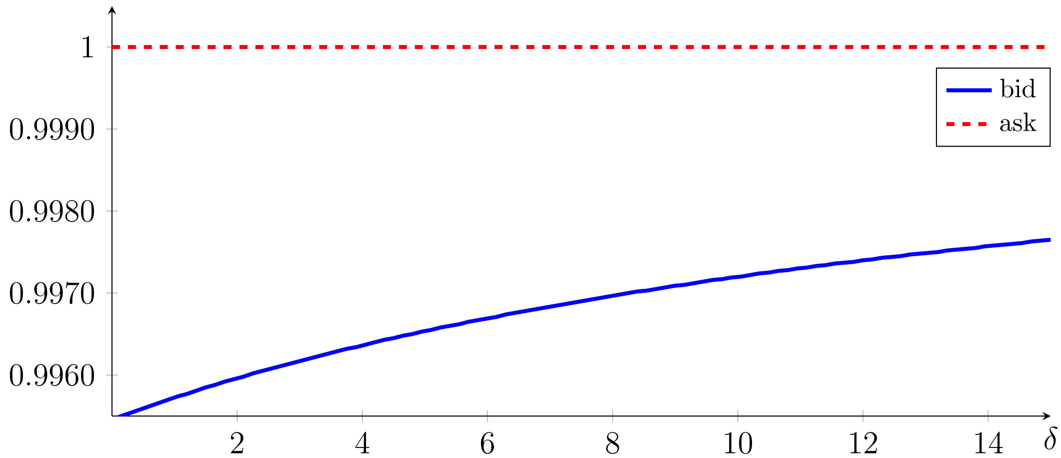

Figure 7: Bid and ask prices

Bid and ask prices as a function of maturity intensity δ (left hand panel) in the equilibrium that results from the parameters for the special marketmaker matching technology as described at the end of Section 6, and other parameters as described at the beginning of Section 3.

Lemma 9. *In the model with marketmakers, the bid-ask spread is given by*

$$A - B = \gamma(V_0 - V_\rho) = \frac{\gamma\rho}{\delta + \theta + \rho + \mu_S(\phi)\beta + \overline{\mu}_S(\phi\chi_B)(1-\gamma)\lambda}. \tag{26}$$

It can be seen that the bid-ask spread is a constant fraction of the gains from trade $V_0 - V_\rho$ that are present in the secondary market, and is increasing in the bargaining power of marketmakers. Also, it is decreasing in the maturity intensity (increasing in the maturity), decreasing in the ratio of buyers to sellers, and decreasing in the ratio of marketmakers to sellers.

We now extend the numerical example used in previous extensions by making assuming the following parameters for the special matching technology of marketmakers: We assume a simple Cobb-Douglas matching function of the type

$$\overline{\mu}(\alpha^i, \alpha^M) = 150\left(\alpha^i\right)^{\frac{1}{2}}\left(\alpha^M\right)^{\frac{1}{2}}, \tag{27}$$

where $i = \{B, S\}$. We note that this makes the matching technology of marketmakers 15 times more efficient, in the sense that for the same measures, marketmakers would obtain 15 times the match rate. We also assume that their flow cost of searching is much higher than the one of buying investor, e_D =56%, and that they have high bargaining power, $\gamma = 0.95$. The high cost of searching may justify both the access to the more efficient matching technology and the high bargaining power after a match as well as the access to the spot inter-dealer market.

With investors now being able to trade not just with each other, but also through marketmakers, it will be easier for investors who become impatient to sell. Firms can therefore afford to choose a higher maturity intensity of $\delta^e \approx 6.5$ in equilibrium, corresponding to an (expected) maturity of about 56 days. In contrast, the equilibrium ratio of buyers to sellers is not really affected, and still equal to about $\phi^e = 2.9$. Since the ratio of buyers to sellers is unchanged, the expected time that it takes for selling investor to find a buying investor is still about 21 days. However, sellers can also find a marketmaker. The combined rate at which sellers find a counterparty (either a buying investor or a marketmaker) can be computed to be about 147, implying a more realistic (expected) time-to-trade of about 2.5 days. This illustrates how the presence of marketmakers can speed up trading.

7 Conclusion

Debt holders who need to sell in an over-the-counter secondary market are in a worse bargaining position the longer they are locked-in into their contracts, i.e. the longer the time-to-maturity of their debt is. This worse bargaining position implies a larger discount when selling, and hence higher transaction costs. Firms can therefore anticipate that they need to offer higher yields on debt with longer maturities, and especially so if the secondary market is very illiquid. But the entry of buyers into the secondary market and hence liquidity is a function of the profits that buyers can obtain in this market, which decreases in the bargaining power of the sellers. We present a model in which which the liquidity of secondary markets for corporate debt, and maturities, are jointly determined in equilibrium, on the basis of this mechanism.

Our main result is that in equilibrium, maturities chosen by firms are inefficiently short. This is because firms do not internalize that an increase in the maturity worsens the bargaining position of sellers, and thus increases the profits that investors can obtain in the secondary market when entering as buyers. When an individual firm increases maturity, this increases expected profits for buyers and hence attracts more buyers to the secondary market, which increases liquidity and reduces the interest rates demanded by investors on the debt of all firms, at all possible maturities.

From a practical perspective, explaining the differences between private and socially optimal maturity choices is important especially in order to understand the reliance prior to the financial crisis of financial institutions on forms of very short-term debt, which partially motivates recent attempts to regulate maturity choices or the extent of maturity mismatch, e.g. via net stable funding ratios as in the Basel III agreements (Basel Committee on Banking Supervision, 2010). Our results contribute to the discussion by pointing out that part of the reason why financial institutions might not be issuing longer maturity debt is because they believe that at longer maturities, they have to pay a higher liquidity premium and hence face a higher cost of debt finance. Our model highlights that while individually this may be true, if all financial institutions were forced to issue debt instruments with longer maturities, this could create a more liquid secondary market for such instruments, which would reduce liquidity premia.

Appendix

A Proofs

Proof of Lemma 1: Substitute (6) into (8), and solve the resulting equation together with (7) for V_0. Apply condition (9) and solve for r to obtain the result. $\square$

Proof of Lemma 2: We note that r is decreasing in ϕ. This fact together with Assumption 2 guarantees that for $\delta = 0$, $1 - \kappa > \frac{r(0;0)+0\cdot\kappa}{\rho} \geq \frac{r(0;\phi)+0\cdot\kappa}{\rho}$, and hence that issuing debt with $\delta = 0$ increases utility. Furthermore, this fact together with Assumption 1 guarantees that $U(0, r(0; \phi)) \geq U(0, r(0; 0) > 0$, and hence that undertaking the project and issuing debt with $\delta = 0$ produces positive utility. Assumption 1 also guarantees that $\frac{x}{\rho} > \frac{x}{\rho} - \kappa > \frac{r(0;0)}{\rho} > \frac{r(0;\phi)}{\rho}$ or $x > r(0; \phi)$, which implies that debt with $\delta = 0$ is feasible (in the sense that the cash flow x is sufficient to cover the cost of debt service). Hence the two assumptions guarantee that investing and issuing debt with $\delta = 0$ is feasible and dominates both (i) investing and not issuing debt, or (ii) not investing at all.

For an optimal choice of δ, call it δ^*, we furthermore have that $U(\delta^*, r(\delta^*; \phi)) \geq U(0, r(0; \phi))$, which implies that $r(\delta^*; \phi) + \delta^*\kappa < r(0; \phi)$. Since debt with $\delta = 0$ is feasible, it therefore must be the case that debt with $\delta = \delta^*$ is feasible.

We now characterize the optimal maturity intensity decisions for given ϕ. Substituting the expression for $r(\delta; \phi)$ in equation (10) into the program (13), we obtain

$$\min_{\delta \geq 0} \frac{\rho}{\delta + \theta + \rho + \mu_S(\phi)\beta} + \delta\kappa$$

The expression describes a function with negative curvature on $(0, +\infty)$ which tends to a positive constant as $\delta \to 0^+$ and to $+\infty$ as $\delta \to +\infty$ and hence has at a unique minimum on $(0, +\infty)$, as characterized by the first order condition

$$-\frac{\rho\theta}{(\delta + \theta + \rho + \mu_S(\phi)\beta)^2} + \kappa = 0$$

which has only one solution that can be non-negative:

$$\delta^\dagger(\phi) = \sqrt{\frac{\rho\theta}{\kappa}} - \theta - \rho - \mu_S(\phi)\beta.$$

Since the choice of δ is constrained to be positive, we can see that U is maximized for

$$\delta^*(\phi) = \max\{\delta^\dagger(\phi), 0\},$$

which concludes the proof. $\square$

Proof of Lemma 3: Consider equation (14). If $V_B(\delta; \phi) > 1$, patient investors would strictly prefer searching to buy over consuming their endowment. Due to the assumption of a large inflow of such investors, this would imply $\alpha^B \to \infty$ and hence $\phi \to \infty$. But since $\lim_{\phi\to\infty} \mu_B(\phi) = 0$, and $\lim_{\phi\to\infty} V_0 - V_\rho = 0$, equation (14) would then imply that $0 = -e_B + \theta(1 - V_B) < 0$, which is a contradiction. (Note that investors who contemplate searching to buy in the secondary market take the interest rate of claims in the market as fixed, and hence the limit of $V_0 - V_\rho$ should be taken for V_0, V_ρ expressed in terms of a fixed r.) On the other hand, if $V_B(\delta;\phi) < 1$ patient investors would strictly prefer consuming their endowment over searching to buy. Therefore it should be the case that $\alpha^B = 0$ and hence that $\phi = 0$. But since $\lim_{\phi\to 0} \mu_B(\phi) = \infty$, while $\lim_{\phi\to 0} V_0(\delta;\phi) - V_\rho(\delta;\phi) = C$ for a positive constant C, equation (14) would imply that $\infty = 0$ which is obviously a contradiction.

Proof of Lemma 4: In order to prove that (FEC) defines a function $\phi^{FEC}(\delta)$, we substitute the expression for $r(\delta;\phi)$ in (10) into the free entry condition in (FEC) and obtain

$$\mu_S(\phi)\beta + \rho + \theta + \delta = \mu_B(\phi)\frac{1-\beta}{e_B}\rho. \tag{28}$$

We note that as $\phi \downarrow 0$ the left hand side tends to a positive constant which is a function of δ, whereas the right hand side tends to ∞. As $\phi \uparrow \infty$, the left hand side tends to infinity, whereas the right hand side tends to 0. Furthermore, from the properties of the matching function, we know that $\mu_S(\phi)$ is continuous and strictly increasing in ϕ (and hence so is the left hand side), and that $\mu_B(\phi)$ is continuous and strictly decreasing in ϕ (and hence so is the right hand side). It therefore follows that for each $\delta \in [0, \infty)$, there exists a unique ϕ that satisfies (28). We denote the function that describes this mapping as $\phi^{FEC}(\delta)$. Its domain is $[0, \infty)$. Using the implicit function theorem, it can be seen that the function is strictly decreasing, implying that it is maximized at $\hat{\phi} := \phi^{FEC}(0)$. We note that since $\lim_{\delta\to\infty} \phi^{FEC}(\delta) = 0$, the function $\phi^{FEC}(\delta)$ has as its image the interval $(0, \hat{\phi}]$. Since $\phi^{FEC}(\delta)$ is strictly decreasing its inverse function $\delta^{FEC}(\phi)$ is well defined, its domain is the interval $(0, \hat{\phi}]$, its image is the interval $[0, \infty)$ and it is strictly decreasing. □

Proof of Proposition 1: We first consider existence, and distinguish between two cases.

First, let us suppose that $\delta^*(\hat{\phi}) = 0$. By definition $\delta^{FEC}(\hat{\phi}) = 0$. Then trivially $\delta^e = 0, \phi^e = \hat{\phi}$ is an equilibrium.

Second, let us suppose that $\delta^*(\hat{\phi}) > 0$. By definition, $\delta^{FEC}(\hat{\phi}) = 0$, and hence $\delta^{FEC}(\hat{\phi}) < \delta^*(\hat{\phi})$. At the same time, $\lim_{\phi\to 0} \delta^{FEC}(\phi) = \infty$, while $\delta^*(0)$ is finite, implying that $\delta^{FEC}(\phi) > \delta^*(\phi)$ for ϕ sufficiently close to zero. By continuity of the two functions $\delta^{FEC}(\phi), \delta^*(\phi)$, there must then exist a pair (δ^e, ϕ^e) such that $\delta^{FEC}(\phi^e) = \delta^*(\phi^e) = \delta^e$. This pair is an equilibrium.

We now prove uniqueness. In order to do so it suffices to prove that

$$\frac{d\delta^{FEC}(\phi)}{d\phi} < \frac{d\delta^*(\phi)}{d\phi} \text{ for all } \phi \in (0, \hat{\phi}]. \tag{29}$$

From the expression for $\delta^*(\phi)$ in Lemma 2, it can be seen that

$$\frac{d\delta^*(\phi)}{d\phi} \geq -\beta\frac{d\mu_S(\phi)}{d\phi}, \quad \forall\phi. \tag{30}$$

From (28), we obtain

$$\frac{d\delta^{FEC}(\phi)}{d\phi} = -\beta\frac{d\mu_S(\phi)}{d\phi} + \frac{d\mu_B(\phi)}{d\phi}\frac{1-\beta}{e_B}\rho, \quad \forall\phi \in (0, \hat{\phi}]. \tag{31}$$

Since $d\mu_S(\phi)/d\phi > 0$ and $\mu_B(\phi)/d\phi < 0$, a direct comparison between equations (30) and (31) leads to the inequality (29).

The proof of the equilibrium effects of changes in the exogenous parameters e_B and κ is straightforward taking into account the intuitive discussion after Proposition 1. It is thus omitted. □

Proof of Proposition 2: For all $\delta > \delta^e$ we have $\phi^{FEC}(\delta) < \phi^{FEC}(\delta^e)$. It follows that for $\delta > \delta^e$,

$$U^{SP}(\delta) = U(\delta, r(\delta; \phi^{FEC}(\delta))) < U(\delta, r(\delta; \phi^{FEC}(\delta^e))) \leq U(\delta^e, r(\delta^e; \phi^{FEC}(\delta^e))) = U^{SP}(\delta^e)$$

where in the first inequality we have used that $U(\delta, r)$ is decreasing in r, which in turn is decreasing in ϕ, and in the second that by the definition of equilibrium, δ^e maximizes firms' objective function for liquidity $\phi^{FEC}(\delta^e)$. Using the inequality $\frac{dU^{SP}(\delta^e)}{d\delta} < 0$ which has been proven in the main text we conclude that:

$$\arg\max_{\delta \geq 0} U^{SP}(\delta) < \delta^e.$$

□

Proof of Lemma 5: Solve (20) and (**??**) for V_0, then find the r that satisfies $V_0(r, \delta; \phi, \chi_B, \lambda) = 1$ to obtain the stated result. □

Proof of Lemma 6: In the model with marketmakers, the interest rate is equal to or lower than the interest rate in the model without market makers. Hence it follows the fact that under Assumptions 1 and 2, it is optimal to invest and to issue debt in the model without market makers (Lemma 2) that it is also optimal to invest and to issue debt here. Finally, with some simple algebra paralleling that in the proof of Lemma 2 (omitted here), the stated expression for $\delta^*(\phi, \chi_B, \lambda)$ can be obtained. □

Proof of Lemma 7: Following the same procedure as for Lemma 4, we note that $V_B = 1$ and hence that

$$\begin{aligned} e_B &= \mu_B(\phi)(V_0 - P) + \overline{\mu}_B(\chi_B)(V_0 - A) \\ &= \mu_B(\phi)(1-\beta)(V_0 - V_\rho) + \overline{\mu}_B(\chi_B)(1-\gamma)(1-\lambda)(V_0 - V_\rho) \\ &= \left[\mu_B(\phi)(1-\beta) + \overline{\mu}_B(\chi_B)(1-\gamma)(1-\lambda)\right]\frac{r(\delta; \phi, \chi_B, \lambda)}{\theta}. \end{aligned} \tag{32}$$

In the first equation we now take into account that buyers can now also be matched with marketmakers, in the second, we use the conditions on P and A, and in the third equation we use the fact that $V_0 = 1$ as well as the definition of r. □.

Proof of Lemma 8: The utility flow to a marketmaker is

$$\overline{\mu}_M(\chi_S)(Q-B)+\overline{\mu}_M(\chi_B)(A-Q)-e_M,$$

reflecting that marketmakers can be matched either with sellers and buyers, and incur the (non-pecuniary) flow cost e_M.

Free entry will occur until in equilibrium this utility flow is zero, implying

$$\begin{aligned} e_M &= \overline{\mu}_M(\chi_S)(Q-B)+\overline{\mu}_M(\chi_B)(A-Q)= \\ e_M &= \overline{\mu}_M(\phi\chi_B)\lambda\gamma(V_0-V_\rho)+\overline{\mu}_M(\chi_B)\gamma(1-\lambda)(V_0-V_\rho)= \\ &= \left(\lambda\overline{\mu}_M(\phi\chi_B)+(1-\lambda)\overline{\mu}_M(\chi_B)\right)\gamma\frac{r(\delta;\phi,\chi_B,\lambda)}{\theta}, \end{aligned} \tag{33}$$

where we have used the conditions on prices and the fact that $\chi_S=\phi\chi_B$, and that $V_0=1$, as well as the relationship between V_0, V_ρ and r. □

Proof of Proposition 3: The liquidity conditions in the secondary market are described by the variables ϕ,χ_B and λ. The idea behind the proof is to reduce this set of variables to a single variable y, and then, as in the baseline model, to characterize equilibria as the intersection in the (y,δ)-space of a curve describing the solution to the firm's problem, and a curve describing free entry.

In order to construct the variable y we have first to make some definitions and manipulate equations (23), (24), and (25).

We define

$$\chi:=\begin{cases}\chi_B \text{ if } \phi\le 1\\ \chi_S \text{ if } \phi>1\end{cases}.$$

Using the variable χ, and the fact that $\phi<1 \implies \lambda=0$, $\phi=1 \implies \lambda\in(0,1)$, and $\phi>1 \implies \lambda=1$, we can re-write the free entry condition for marketmakers (25) as

$$e_M=\overline{\mu}_M(\chi)\gamma\frac{r(\delta;\phi,\chi_B,\lambda)}{\theta}, \tag{34}$$

as well as the free entry condition for buyers. Dividing the free entry condition for buyers by that for marketmakers, we now obtain:

$$\frac{e_B}{e_M}=\frac{\mu_B(\phi)(1-\beta)+\overline{\mu}_B(\chi)(1-\gamma)(1-\lambda)}{\overline{\mu}_M(\chi)\gamma}. \tag{35}$$

It is easy to see that for every ϕ,λ there exists a unique χ such that the equation above is satisfied. Let us denote the function this equation define by $\chi^{FEC}(\phi,\lambda)$. $\chi^{FEC}(\phi,\lambda)$ is continuous for $\phi\in(0,1)$ and $\phi\in(1,\infty)$ and for $\lambda\in[0,1]$, it is increasing in ϕ and is also increasing in λ. In addition:

$$\begin{aligned}
\lim_{\phi\to 1^-}\chi^{FEC}(\phi,\lambda) &= \chi^{FEC}(1,0)=\lim_{\lambda\to 0^+}\chi^{FEC}(1,\lambda),\\
\lim_{\phi\to 1^+}\chi^{FEC}(\phi,\lambda) &= \chi^{FEC}(1,1)=\lim_{\lambda\to 1^-}\chi^{FEC}(1,\lambda),\\
\lim_{\phi\to 0^+}\chi^{FEC}(\phi,\lambda) &= 0,\\
\lim_{\phi\to \infty}\chi^{FEC}(\phi,\lambda) &= \infty.
\end{aligned}$$

We now define:

$$F(y)=\begin{cases}(y,0)\text{ if } y\in(0,1)\\ (1,y-1)\text{ if } y\in[1,2]\\ (y-1,1)\text{ if } y>2\end{cases}$$

The function $\chi^{FEC}(F(y))$ is continuous and strictly increasing in $y\in(0,\infty)$. It also satisfies:

$$\begin{aligned}
\lim_{y\to 0^+}\chi^{FEC}(F(y)) &= 0,\\
\lim_{y\to \infty}\chi^{FEC}(F(y)) &= \infty.
\end{aligned}$$

Let us also define the function:

$$G(y)=\begin{cases}y\text{ if } y\in(0,1)\\ 1\text{ if } y\in[1,2]\\ y-1\text{ if } y>2\end{cases}$$

$G(y)$ is a continuous function.

We can use these definitions to rewrite the optimal maturity intensity choice in equation (23) as

$$\delta^*(y)=\max\left\{\sqrt{\frac{\theta\rho}{\kappa}}-\theta-\rho-\mu_S(G(y))\beta-\big(\mathbf{1}_{y>2}+\mathbf{1}_{y\in[1,2]}(y-1)\big)\,\overline{\mu}_S(\chi^{FEC}(F(y)))(1-\gamma),0\right\},$$

and to rewrite the free entry condition for buyers in equation (24) as

$$\begin{aligned}
\delta^{FEC}(y) &= \frac{\rho}{e_B}\left[\mu_B(G(y))(1-\beta)+\big(\mathbf{1}_{y<1}+\mathbf{1}_{y\in[1,2]}(2-y)\big)\,\overline{\mu}_B(\chi^{FEC}(F(y)))(1-\gamma)\right]- \\
&\quad -\theta-\rho-\mu_S(G(y))\beta-\big(\mathbf{1}_{y>2}+\mathbf{1}_{y\in[1,2]}(y-1)\big)\,\overline{\mu}_S(\chi^{FEC}(F(y)))(1-\gamma).
\end{aligned}\tag{36}$$

Both functions are continuous.

It is easy to convince oneself that the definitions of $G(y)$, $\chi^{FEC}(F(y))$, $\delta^*(y)$ and $\delta^{FEC}(y)$ are such that if (y^e,δ^e) satisfy:

$$\delta^e=\delta^*(y^e)=\delta^{FEC}(y^e),\tag{37}$$

then, if $y^e<1$ the tuple $(\delta^e,\phi^e=G(y^e),\chi_B^e=\chi^{FEC}(F(y^e),\lambda^e=0)$ is an equilibrium, if $y^e>2$ the tuple $(\delta^e,\phi^e=G(y^e),\chi_B^e=G(y^e)\chi^{FEC}(F(y^e),\lambda^e=1)$ is an equilibrium and finally if $y^e\in[1,2]$

the tuple $(\delta^e, \phi^e = 1, \chi_B^e = \chi^{FEC}(F(y^e), \lambda^e = y^e - 1)$ is an equilibrium. Conversely, for every equilibrium $(\delta^e, \phi^e, \chi_B^e, \lambda^e)$, we define $y^e = \phi^e$ if $\phi^e < 1$, define $y^e = 1 + \lambda^e$ if $\phi^e = 1$, and $y^e = \phi^e + 1$ if $\phi^e > 1$. Then (δ^e, y^e) satisfy equation (37).

Therefore, pairs (δ^e, y^e) satisfying equation (37) characterize the equilibria of the model. In order to prove the proposition it suffices to prove that the functions $\delta^*(y), \delta^{FEC}(y)$ have a unique intersection point in the interval $(0, \infty)$. From this point onwards the proof is analogous to the one of Proposition 1.

First, existence is consequence of the continuity of both functions and their behaviour at the limits of the interval $(0, \infty)$:

$$\begin{aligned} \lim_{y \to 0^+} \delta^*(y) &< \lim_{y \to 0^+} \delta^{FEC}(y) = \infty, \\ \lim_{y \to \infty} \delta^*(y) &= 0 > \lim_{y \to \infty} \delta^{FEC}(y) = -\infty. \end{aligned}$$

Second, in order to prove uniqueness it suffices to prove that the inequality

$$\frac{d\delta^{FEC}(y)}{dy} < \frac{d\delta^*(y)}{dy},$$

is satisfied almost everywhere.[19] Comparing the analytical expressions for $\delta^*(y)$ and $\delta^{FEC}(y)$ it suffices to prove that

$$\frac{d}{dy}\left[\mu_B(G(y))(1-\beta) + \left(\mathbf{1}_{y<1} + \mathbf{1}_{y \in [1,2]}(2-y)\right)\overline{\mu}_B(\chi^{FEC}(F(y)))(1-\gamma)\right] < 0. \tag{38}$$

Since $\mu_B(\cdot)$ is decreasing in its argument but $\overline{\mu}_B(\cdot)$ is increasing the sign of the expression above is ambiguous. However, we can rewrite equation (35) as:

$$\mu_B(G(y))(1-\beta) + \left(\mathbf{1}_{y<1} + \mathbf{1}_{y \in [1,2]}(2-y)\right)\overline{\mu}_B(\chi^{FEC}(F(y)))(1-\gamma) = \overline{\mu}_M\left(\chi^{FEC}(F(y))\right)\gamma\frac{e_B}{e_M}$$

and insert the result into the inequality. Since $\chi^{FEC}(F(y))$ is strictly increasing in y and $\overline{\mu}_M(\cdot)$ is strictly decreasing in its argument, it is clear that the inequality holds. □

Proof of Proposition 4: Let us work in the (y, δ)-space defined in the proof of the previous proposition. The equilibrium can be described by a pair (y^e, δ^e). The function $\delta^{FEC}(y)$ defined in equation (36) is strictly decreasing. Let $y^{FEC}(\delta)$ be its inverse function, which is defined for $\delta \geq 0$, strictly decreasing and differentiable except at $\delta = \delta^{FEC}(1)$ and $\delta = \delta^{FEC}(2)$. The SP maximizes:

$$U^{SP}(\delta) = U(\delta; y^{FEC}(\delta)) = \frac{x}{\rho} + (1-\kappa) - \frac{r(\delta; y^{FEC}(\delta)) + \delta\kappa}{\rho},$$

where $r(\delta; y)$ has the following expression:

$$r(\delta; y) = \frac{\theta}{\delta + \theta + \rho + \mu_S(G(y))\beta + \left(\mathbf{1}_{y>2} + \mathbf{1}_{y \in [1,2]}(y-1)\right)\overline{\mu}_S(\chi^{FEC}(F(y)))(1-\gamma)}\rho.$$

[19] The functions are not differentiable at $y = 1, 2$ and at the smallest y for which $\delta^*(y) = 0$.

Taking into account that $G(y)$ is increasing in y and that $\chi^{FEC}(F(y))$ is strictly increasing in y we have that:

$$\frac{\partial r(\delta; y)}{\partial y} < 0 \Leftrightarrow \frac{\partial U(\delta; y)}{\partial y} > 0.$$

From here, we have as in the baseline model that if $\delta^e > 0$ then:

$$\left.\frac{dU^{SP}(\delta)}{d\delta}\right|_{\delta=\delta^e} < 0.$$

The same arguments as in the proof of Proposition 2 where $\phi, \phi^{FEC}(\delta)$ are replaced by $y, y^{FEC}(\delta)$ also lead to:

$$U^{SP}(\delta) < U^{SP}(\delta^e) \text{ for all } \delta > \delta^e.$$

We conclude from the two previous inequalities that:

$$\arg\max_{\delta \geq 0} U^{SP}(\delta) < \delta^e.$$

□

Proof of Lemma 9: Calculate $A - B$ directly from (18) and (19), solve (20) and (**??**) for V_0 and V_ρ and insert to obtain the stated expression. □

B The evolution of the measures of buyers and sellers in the secondary market

In this Appendix we show how the steady state equilibrium pair (δ^e, ϕ^e) uniquely determines the time evolution of the measure of sellers and buyers in the secondary market. Let us denote them as α_t^S, α_t^B, respectively, where the subscript t makes explicit their time dependence.

In the main text, we did not need to be precise about the measures of entrepreneurs, firms, face values, and endowments, because the constant returns to scale assumption means that only the ratios of measures are relevant. Here, however, we need the actual measures, in particular the measure of debt holders in the economy. Here, let $i \in [0,1]$ index firms. Then define the second index $j \in [0,1]$, and let (i,j) index investors. We normalize the aggregate endowment held by a set of investors described by $i \in I, j \in J$ to be of measure $\int_I \int_J didj$. The face value of debt of an individual firm is $\int_{j=0}^{j=1} didj = di$. (In the main text, we describe the face value of debt as being equal to 1, which can be understood as a normalization of the more general description here.) This means that the aggregate face value issued by all firms is of measure $\int_{i=0}^{i=1} di = 1$, as is the set of debt holders in the economy.

The law of motion of α_t^S then is:

$$\begin{aligned} \frac{d\alpha_t^S}{dt} &= \theta(1-\alpha_t^S) - \mu_S(\phi^e)\alpha_t^S - \delta^e\alpha_t^S, \\ \alpha_0^S &= 0. \end{aligned}$$

The first equation describes the sources of time variation in α_t^S. The first term on the right-hand side reflects that a measure $1-\alpha_t^S$ of debt holders are still patient because they have not been hit by a liquidity shock and hence are not trying to sell. These patient debt holders become impatient (and hence become sellers) at rate θ. The second term reflects the rate at which searching sellers are matched, and leave the secondary market. The third term reflects the measure of sellers whose contract matures before they find a counterparty, and who therefore stop searching. The second equation establishes the initial condition for the measure of sellers. The solution of this first order linear differential equation is:

$$\alpha_t^S = \frac{\theta}{\theta + \mu_S(\phi^e) + \delta^e} \left(1 - e^{-(\theta+\mu_S(\phi^e)+\delta^e)t}\right).$$

We observe that α_t^S converges asymptotically to $\frac{\theta}{\theta+\mu_S(\phi^e)+\delta^e}$.

Intuitively, as ϕ^e increases α_t^S strictly decreases for all $t > 0$ as sellers are matched with buyers and leave the market faster. Also, as δ^e increases, α_t^S decreases as sellers recover the principal at maturity and leave the market faster.

Trivially, α_t^B is determined through the equation:

$$\alpha_t^B = \phi^e\alpha_t^S = \frac{\phi^e\theta}{\theta + \mu_S(\phi^e) + \delta^e} \left(1 - e^{-(\theta+\mu_S(\phi^e)+\delta^e)t}\right).$$

We observe that α_t^B also converges asymptotically, to $\frac{\phi^e\theta}{\theta+\mu_S(\phi^e)+\delta^e}$ in this case. Note also that the law of motion of α_t^B can be used to determine the entry flow of new buyers to the secondary market. If we denote this flow by i_t, we have

$$\frac{d\alpha_t^B}{dt} = -\mu_S(\phi^e)\alpha_t^B - \theta\alpha_t^B + i_t,$$

where the first term is the outflow due to matching, the second one the outflow due buyers becoming impatient and the last one is the new inflow i_t of buyers. Using the expression for α_t^B we obtain the equation the inflow will have to satisfy:

$$i_t = \frac{\phi^e\theta}{\theta+\mu_S(\phi^e)+\delta^e}\left(\theta+\mu_S(\phi^e)+\delta^e e^{-(\theta+\mu_S(\phi^e)+\delta^e)t}\right).$$

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