FROM BASEL I TO BASEL II:
AN ANALYSIS OF THE THREE PILLARS

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Abstract

This paper presents a dynamic model of banking supervision to analyze the impact of each of Basell II three pillars on banks' risk taking. We extend previous literature providing an analysis of ratings-based supervisory policies. In Pillar 2 (supervisory review) the supervisor audits more frequently low rated banks and restricts their dividend payments in order to build capital. In Pillar 3 (market discipline) the supervisor reduces the level of deposit insurance coverage compelling not-fully insured depositors to adjust interest rates contingent on the bank's external rating. We also analyze the risk sensitiveness of Pillar 1 (capital requirements) concluding that all three Pillars reduce banks' risk taking incentives.

JEL Codes: G12, G21, G28, E58.
Keywords: Capital requirements, market discipline, ratings-based policies, risk taking decisions, supervisory review.

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1 Introduction

Basel I, the framework of minimum capital standards introduced in 1988 by the Basel Committee on Banking Supervision (BCBS, 1988), was designed to increase the safety and soundness of the international banking system and to set a level playing field for banking regulation. For such enterprise, it was equipped with just a minimum capital requirements rule. Although praised for achieving its initial goals, it has been bitterly criticized because the low risk sensitiveness of its capital requirements may lead to greater risk taking and regulatory capital arbitrage practices by banks (cf. BCBS, 1999, and Jones, 2000). Basel II, its successor (BCBS, 2004), relies on *three pillars* (minimum capital requirements, supervisory review, and market discipline) to attain the safety and soundness of the financial system.

We study the effects of each pillar of Basel II on banks' risk taking incentives. Using a continuous time model we apply a technique to analytically solve systems of ordinary differential equations, which are required to analyze **rating-based supervisory policies**. Such policies constitute the essence of Pillars 2 and 3, the less precise and researched part of the New Accord. This represents an important step forward in the continuous time banking literature started by Merton (1978), which does not incorporate any regulatory or market-based policy *discriminating banks according to their rating*.

Specifically, we make the rate at which the supervisor audits banks a decreasing function of their rating, we consider restrictions in the dividend payments to low rated banks and, finally, we study the effect of making (not-fully insured) deposit interest rates contingent on the banks' external rating. We find that both the risk sensitivity of the capital rule (Pillar 1) and the principles underlying Pillars 2 and 3 (tighter examination, control and exposure to the market to lower rated banks) will reduce banks' risk taking incentives.

Bank's assets, funded by capital and deposits, follow a geometric Brownian motion characterized by a volatility (or risk) level. A bank's risk and asset levels are unobservable except to equityholders. In order to verify that banks hold enough capital levels, banking supervisors audit them to find out their risk and asset levels. Equityholders choose the risk level taking into account the different regulatory measures, knowing that if their capital falls below the required one they will be closed in case of an audit. The risk level can be either low or high.

In this setting, we interpret the risk level as the banks' underlying propensity, bias, or predisposition towards risk, rather than particular risk choices for specific projects or investments. A bank choosing high risk in our setting would take, on average, riskier investments than a bank choosing low risk.

The chosen risk level depends on the banks' initial asset value (relative to capital). In particular, we show that there exists a unique asset level, referred to as *gambling threshold*, such that banks with higher (lower) initial asset levels choose low (high) risk. A regulatory measure is said to reduce banks' risk taking incentives if it reduces the gambling threshold. In other words, banks with low capital relative to assets (or, equivalently, high debt relative to assets) will choose high risk. For a given distribution of the banks' financial situation (i.e. capital/asset ratios), a lower gambling threshold reduces the number of risky banks.

The capital requirements rule, contained both in Basel I and in Pillar 1 of Basel II, requires banks to hold a minimum capital level as a function of their risk level. In a risk sensitive capital rule the higher the assets risk the higher the fraction of those assets that has to be funded with capital. Although Basel I already incorporates some limited degree of risk sensitivity, Pillar 1 of Basel II significantly increases the risk sensitivity of the capital rule.

We show that *the higher the risk sensitivity of the capital rule and the supervisor audit frequency the lower banks' risk taking incentives*. However, *the effectiveness of risk sensitivity and audit frequency are negatively related*: an increase in the risk sensitivity of capital regulations will reduce more banks' risk taking incentives the lower the frequency with which they are audited, and vice versa.

Basel II puts most of its attention in Pillar 1, describing the different approaches to compute minimum capital requirements, and although it outlines the principles and objectives of the two other Pillars it is not very precise about their implementation, which seems left to national supervisors' discretion. We put forward different ways to enforce Pillars 2 and 3, characterized by a different treatment to banks according to their ratings by both the supervisor and the market (depositors in our case).

For **Pillar 2**, supervisory review, we propose a rating-based audit frequency and a rating-based dividend restrictions policy which resemble the *Prompt Corrective Action* (PCA) provisions introduced by US banking authorities in 1991 through the FDIC Improvement Act, *FDICIA*.[1] We consider three rating categories for banks: undercapitalized, low rated and high rated. At each audit, the supervisor determines the bank's rating according to its capital and risk levels. Banks whose capital level is lower than the required by the capital rule are considered undercapitalized and closed. Low rated banks will, compared to high rated banks, be subject to tighter audit frequencies and dividend restrictions (in order to build capital). Our results show that *a rating-based audit frequency is less expensive and more effective than a constant one in reducing banks' risk taking incentives. Dividend restrictions to low rated banks also reduce banks' risk levels.* These theoretical results are in line with the empirical evidence provided by Aggarwal and Jacques (2001) showing that the implementation of the rating-based regulatory measures included in FDICIA brought significant reductions in banks' risk levels.

For **Pillar 3**, market discipline, we consider external rating agencies which, like the supervisor, audit the bank and, unlike the supervisor, make a public statement about the bank's rating. Not fully insured depositors, using published ratings, require

[1] PCA is a framework for supervisory actions based on the capital level of the bank, considering five capital levels (from well capitalized to critically undercapitalized). Such supervisory actions include, among others, restrictions in capital distributions and management fees, capital restoration, close monitoring, and restriction on activities. See Comptroller of the Currency (1993) and Benston and Kaufman (1997).

higher interest rates the lower the bank's rating and the deposit insurance coverage. We show that *the informational role played by rating agencies and the discipline enforced by uninsured depositors allows the supervisor to reduce banks' risk taking incentives by reducing the fraction of insured deposits.*

This paper is part of a line of research applying continuous time models for banking supervision, started by Merton (1978). In this literature, the bank's asset value follows a continuous process, generally lognormally distributed with given expected return, risk and payout ratio. The supervisor audit frequency is considered constant, i.e. independent of the bank's financial situation. In this framework, Merton derives the actuarially fair deposit insurance premium, considering a constant risk level and an exogenous and costless dividend/recapitalization policy. Subsequent papers keep the main characteristics of this standard model and relax some of its assumptions, mainly endogenizing and making dynamic and costly dividend/recapitalization and risk choices.

Milne and Whalley (2001) extend the Merton model allowing the bank to (costlessly and dynamically) choose dividend and risk levels. Banks that are found with a capital level below the required by regulators are given the option to (costly) recapitalize and avoid closure. Bhattacharya et al. (2002) analyze the supervisor's choice of the asset level below which the bank is considered undercapitalized and closed, in order to eliminate banks' risk taking incentives. Peura (2003) assumes that banks' capital levels are perfectly observable by the supervisor and study optimal dividend and recapitalization decisions when dividends can be implemented instantaneously but capital issuance is costly. Keppo and Peura (2005) consider the case where recapitalization is not only costly, but it has a time delay between the moment it is decided and the moment it comes into place.

Dangl and Lehar (2004) extend the standard model to the case where the bank can dynamically decide its risk level, but subject to a cost. The model is applied to analyze the impact of a risk sensitive capital requirements rule on banks' risk taking decisions. Decamps, Rochet and Roger (2004) assume banks' cash flows are perfectly observable by the supervisor and the market and propose a market discipline mechanism where banks are obliged to issue a certain level of subordinated debt with infinite maturity but renewed at stochastic dates.

None of the above papers analyze regulatory or market measures where banks are treated differently according to their ratings. As such, when solving for the value of a given claim (equity, debt, ...) they deal with an ordinary differential equation (ODE). Considering rating-based schemes requires solving systems of ODEs instead. The technique we use for solving systems of ODEs allows for a direct and simple way of analyzing rating-based policies such as rating-based audit frequency, rating-based dividend restrictions, rating-based deposit rates, and others.

In order to focus on our contribution and keep the tractability of the model, we abstract from building a model incorporating all extensions proposed in the literature. We build upon the standard model where the dividend/recapitalization rule is exogenous and the risk level is kept constant through time. Banks, however, choose their initial risk level.[2] *The framework we propose to analyze rating-based regulations can handle any of the extensions the literature has propose regarding risk and dividend decisions (endogenous, dynamic and costly). Moreover, although we analyze three particular rating-based measures, the model can be used to analyze any other rating-based regulation.* Our aim is to compare Basel I and Basel II and their implications for the risk taking incentives in the banking industry. We do not study which capital accord, or which combination of the different regulatory and supervisory measures analyzed, achieves higher social welfare, for which one needs to propose a social welfare function. We leave the analysis of these questions for further research.

The paper is organized as follows. Section 2 presents the model and Section 3 the parameter values. Section 4 analyzes the impact of Basel I capital rule on banks'

[2] Continuous time models which analyze optimal dividend/recapitalization decisions can be found, among others, in Milne and Robertson (1994), Asmussen, Højgaard and Taksar (1998), Fan and Sundaresan (2000), Milne and Whalley (2001), Peura (2003), and Keppo and Peura (2005). Ross (1997), Højgaard and Taksar (1998) and Choulli, Taksar and Zhou (2000) analyze both optimal dynamic risk switching and dividend policies.

decisions in order to use the results as a benchmark to study, in Section 5, the effects of each of the three Pillars of Basel II. Section 6 offers some concluding remarks. The Appendix contains all the mathematical tools and procedures for solving the model.

2 Model

Consider a bank which, at each time t in which it is open, has an asset size A_t funded by deposits and capital. Deposits, whose size is constant and normalized to 1, receive a continuous interest rate d and are initially fully insured. Accounting capital, used for regulatory purposes, is measured as the difference between assets and deposits, $A_t - 1$. The market value of capital corresponds to equity value.[3]

The bank is owned by risk neutral equityholders who enjoy limited liability. We assume that there are neither intermediation costs nor any deposit insurance premium.

The bank's asset value A_t follows a geometric Brownian motion:

$$\frac{dA_t}{A_t} = (\mu - \delta)\, dt + \sigma dW_t, \tag{1}$$

where expected return μ, payout ratio δ, and volatility σ are constant. W_t is a standard Brownian motion: its increments are independent and normally distributed, and represent the random component of the assets' growth. Such randomness is controlled by the risk parameter σ. The non-random part of the assets' growth is given by the total expected return μ minus the fraction δ of assets paid out to security holders.

As long as it is open, the bank has available at any time t an amount of cash δA_t to distribute among equityholders and depositors. Depositors receive a continuous rate d; the rest, $\delta A_t - d$, is distributed to equityholders as dividend payment. $\delta A_t - d$

[3] With random asset values, the assumption of constant deposits accounts for exogenous fluctuations in the deposit volume (which would reduce the asset value) but not for endogenous withdrawals of deposits. Later on, we reduce the level of deposit insurance and depositors renegotiate the deposit rate according to the bank's probability and costs of closure. The flexibility in fixing the deposit rate makes deposit withdrawals irrational.

is negative for values of A_t below the critical level d/δ.[4] Although equityholders do not endogenously control dividend payments when the bank is open, they have the option to voluntarily close the bank at any time. Such closure does not involve any cost or final payment, i.e. they give up to depositors all rights over the bank's assets and future cash flows. A_E denotes the (optimally chosen) asset level such that as soon as A_t goes below A_E equityholders close the bank. Obviously, $A_E \leq d/\delta$.

The bank's asset A_t and risk σ levels are observed by equityholders continuously. Any other agent can only observe those levels auditing the bank. Initially we assume that only the supervisor audits the bank. Audit times are stochastic and follow a Poisson distribution characterized by an, initially constant, intensity parameter λ.[5]

Equityholders are required to hold a minimum capital level, function of the assets' risk σ, in order to keep control over the bank. If, in an audit, the supervisor discovers the bank not complying with the capital requirements rule, it takes control of the bank from its equityholders. Otherwise, no action is taken. From the viewpoint of equityholders, intervention is equivalent to closure, which is the term we will use hereafter. Therefore, the bank can be closed by the supervisor or voluntarily by its equityholders.

The capital requirements rule both under Basel I and (Pillar 1 of) Basel II is

$$\frac{\text{Capital}}{\text{Risk Weighted Assets}} \geq \rho, \tag{2}$$

where ρ is the minimum required capital ratio. The only part of the previous rule which changes from Basel I to Basel II is the denominator, Risk Weighted Assets RWA, computed as the volume of assets in the bank's portfolio weighted by their risk level.

Equityholders have two risk levels to choose from: low $\underline{\sigma}$ and high $\overline{\sigma}$, where $\underline{\sigma} < \overline{\sigma}$. The risk sensitivity of the capital requirements rule (2) comes from the risk sensitivity

[4] A negative $\delta A_t - d$ can be enforced either by equityholders directly injecting money into the bank or raising money by issuing new shares. Issuing new shares does not reduce the total value of equity, but the value of each individual share (which is referred to as dilution of equity).

[5] λ is the rate of audits per unit of time and represents the mean and variance of the number of audits per unit of time. Our parametrization shall consider the year as the time unit.

of the risk weighted assets. The general form of RWA is assumed to be:

$$RWA\,(A,\sigma,\theta)=\begin{cases}(1+\theta)\,A & \text{if } \sigma=\overline{\sigma},\\ (1-\theta)\,A & \text{if } \sigma=\underline{\sigma},\end{cases} \tag{3}$$

where $\theta \geq 0$ is the risk sensitivity parameter of the capital rule: the higher θ the more risk sensitiveness.[6] A risk insensitive capital requirements rule ($\theta = 0$) assigns the same risk weight to assets independently of their risk.

Since the volume of deposits is constant, the capital rule (2) can be mapped into a minimum asset level $A_S\,(\sigma,\theta)$ such that if $A_t < A_S\,(\sigma,\theta)$ the bank does not satisfy it:

$$A_S\,(\sigma,\theta)=\begin{cases}[1-\rho\,(1+\theta)]^{-1} & \text{if } \sigma=\overline{\sigma},\\ [1-\rho\,(1-\theta)]^{-1} & \text{if } \sigma=\underline{\sigma}.\end{cases}$$

Let $\tau > 0$ denote the time at which the bank is closed. τ is the minimum between two stopping times, τ_E and τ_S, representing the closure time at which equityholders and the supervisor close the bank respectively:

$$\begin{aligned}\tau &= \min\{\tau_S,\tau_E\}\,,\\ \tau_S &= \inf\{t>0 \mid A_t < A_S\,(\sigma,\theta) \text{ and } t \text{ is an audit time}\}\,,\\ \tau_E &= \inf\{t>0 \mid A_t \leq A_E\}\,.\end{aligned}$$

Given a risk level σ, equityholders choose $A_E\,(\sigma)$ to maximize the value of equity. We can express equity value at $t = 0$ as a function of the bank's initial asset A_0 as follows:

$$V\,(\sigma \mid A_0) = \max_{A_E(\sigma)} E_0\left[\int_0^{\tau} e^{-rs}\,(\delta A_s - d)\,ds \mid A_0\right]. \tag{4}$$

The expected value is taken under the risk neutral measure and, as a consequence, the discount rate coincides with the risk free rate r.

[6] Other specifications for RWA, e.g. concave or convex functions of σ, could have been considered.

We compute $V(\sigma \mid A_0)$ for each risk level $\sigma \in \{\underline{\sigma}, \overline{\sigma}\}$ and, given the initial asset level A_0, equityholders choose the risk level σ^* which maximizes equity value:

$$\sigma^*(A_0) = \arg \max_{\sigma \in \{\underline{\sigma}, \overline{\sigma}\}} V(\sigma \mid A_0). \tag{5}$$

The chosen risk level σ^* and the corresponding asset value $A_E(\sigma^*)$ are decided at $t = 0$ and kept constant over the life of the bank.

Before moving on to solve for equity (4) the reader might find useful to review Appendix A, which outlines the general process for computing the value of all claims used throughout the paper (equity, audit costs and not fully insured deposits). The general procedure consists on (i) expressing the dynamics of the claim's value as an ODE (or system of ODEs), (ii) finding the theoretical solution of such an ODE and, finally, (iii) solving for the desired value using a proper set of boundary conditions and the particular values of model parameters. The final outcome of such a process consists on an exactly identified system of equations (i.e. the number of equations equals the number of unknowns.)

2.1 Equity value

We transform (4) into a single ODE and solve it using a set of boundary conditions. The ODE characterizes the dynamics of equity value in the next time interval dt. To simplify notation, we drop the subindex t of the bank asset level A_t and, since equity is solved for each risk level σ separately and for a fixed risk sensitivity parameter θ, we denote equity value (4) by $V(A)$ (or simply V), the risk-dependent supervisor closure level $A_S(\sigma, \theta)$ by A_S, and the voluntary closure level $A_E(\sigma)$ by A_E.

Assuming $A_E < A_S$, which (given our parametrization) holds for all cases analyzed in the paper, and following Appendix A, the value of equity V for a given risk level σ follows the ODE

$$\begin{array}{llr} \Gamma_V(A) + \delta A - d & = 0 & \text{for } A \geq A_S, \\ \Gamma_V(A) + \delta A - d - \lambda V(A) & = 0 & \text{for } A < A_S, \end{array} \tag{6}$$

where

$$\Gamma_V(A) = (1/2)\,\sigma^2 A^2 V''(A) + (r - \delta)\, A V'(A) - r V(A). \tag{7}$$

V' and V'' denote the first and second derivatives of the equity V with respect to asset value A. Due to the fact that we are using risk neutrality, the total expected return μ does not appear in (7), being replaced by the risk free rate r.[7]

Since equityholders receive (or pay if negative) $\delta A - d$ as long as the bank is open, this term appears in both parts of (6). When the bank satisfies the minimum capital requirements rule ($A \geq A_S$) it will not be closed even if an audit takes place. In contrast, if $A < A_S$ and the supervisor audits the bank, he will close it and its equityholders will be expropriated of their equity. This is represented by the term $-\lambda V(A)$ in the second part of the ODE; λ represents the probability of an audit taking place and $V(A)$ the loss for equityholders.[8]

Following Appendix A, the general solution of (6) can be written as:

$$V(A) = \begin{cases} V_b(A) = K_{1,b} A^{\beta_{1,b}} + K_{2,b} A^{\beta_{2,b}} + A - d/r & \text{for } A \geq A_S, \\ V_a(A) = K_{1,a} A^{\beta_{1,a}} + K_{2,a} A^{\beta_{2,a}} + \frac{\delta A}{\lambda + \delta} - \frac{d}{r + \lambda} & \text{for } A < A_S, \end{cases}$$

where

$$\begin{aligned} \beta_{1,b} &= \sigma^{-2}[(\sigma^2/2 - r + \delta) + \sqrt{(\sigma^2/2 - r + \delta)^2 + 2r\sigma^2}] > 1, \\ \beta_{2,b} &= \sigma^{-2}[(\sigma^2/2 - r + \delta) + \sqrt{(\sigma^2/2 - r + \delta)^2 + 2r\sigma^2}] < 0, \\ \beta_{1,a} &= \sigma^{-2}[(\sigma^2/2 - r + \delta) + \sqrt{(\sigma^2/2 - r + \delta)^2 + 2(r + \lambda)\sigma^2}] > 1, \\ \beta_{2,a} &= \sigma^{-2}[(\sigma^2/2 - r + \delta) - \sqrt{(\sigma^2/2 - r + \delta)^2 + 2(r + \lambda)\sigma^2}] < 0. \end{aligned} \tag{8}$$

The unknown constants $K_{1,b}$, $K_{2,b}$, $K_{1,a}$ and $K_{2,a}$ and the asset level A_E at which equityholders decide to close the bank are determined by the following boundary conditions:

[7] See, among others, Dixit and Pindyck (1994, Section 4.3) for details.

[8] The probability of an audit taking place over the next time interval dt is λdt, provided $dt \to 0$.

1. Ruling out speculative bubbles, the following condition must hold

$$\lim_{A\to\infty} V_b\left(A_0\right) = E_0\left[\int_0^\infty e^{-rt}\left(\delta A_t - d\right)dt \mid A_0\right].$$

If assets tend to infinity, the probability of closure goes to zero. In that case, the value of equity (expected discounted future dividends) is just $A_0 - d/r$.[9]

2. At A_S we have the traditional value-matching and smooth-pasting conditions $V_b\left(A_S\right) = V_a\left(A_S\right)$ and $V_b'\left(A_S\right) = V_a'\left(A_S\right)$. These conditions guarantee continuity and smoothness of the equity value at A_S.

3. At A_E we have a value-matching condition and a first order condition (which guarantees its optimality): $V_a\left(A_E\right) = 0$ and $V_a'\left(A_E\right) = 0$.

Since the first boundary condition implies $K_{1,b} = 0$, we have to solve a system of four unknowns ($K_{2,b}$, $K_{1,a}$, $K_{2,a}$ and A_E) and four equations.

2.2 Risk taking incentives: Gambling Threshold

Solving for the value of equity for both risk levels and computing the optimal risk level (5), the results show that for any audit frequency λ, there exists a unique initial asset level such that for initial asset values A_0 lower (higher) than it equityholders prefer high (low) risk. This result holds for all cases considered in the paper. Such a unique asset value, called *gambling threshold* and denoted by A_σ, represents a measure of the banks' incentives to take high risks:

$$\sigma^*\left(A_0\right) = \begin{cases} \overline{\sigma} & \text{if } A_0 \leq A_\sigma, \\ \underline{\sigma} & \text{if } A_0 > A_\sigma. \end{cases}$$

The lower the gambling threshold A_σ the less incentives banks have to take high risks: the lower has to be the initial asset value of a bank for equityholders to prefer high risk level.

[9] See Merton (1978, condition 7d) and Naqvi (2003, condition 7).

3 Parameter values

Table 1 contains the parameter values used in the paper. We use Bhattacharya et al. (2002) calibration exercise, which uses data on commercial banks over the period 1989-98, to set the values of the risk free interest rate r, payout ratio δ and high and low risk levels, $\underline{\sigma}$ and $\overline{\sigma}$. Initially, deposits are fully insured and the deposit rate d equals the risk free rate r.

Basel II does not explicitly state any particular capital ratio ρ below which the bank should be closed. We assume, as in Dangl and Lehar (2004), that such level is the minimum capital required by the Basel Accords, 8%. Any other choice will not qualitatively change our results.[10]

Parameter		Value (%)
r	Risk free interest rate	5
δ	Payout ratio	4.2
$\underline{\sigma}$	Low risk level	10
$\overline{\sigma}$	High risk level	20
d	Deposits' interest rate	5
ρ	Minimum capital ratio	8

Table 1. Parameter values.

4 Basel I: Capital requirements

Although Basel I already includes some limited risk sensitivity, one of the main contributions of Basel II is to make the capital requirements rule more risk sensitive. In order to simplify the presentation, we focus on the main differences between the two accords and consider a *stylized version of Basel I* characterized by a risk insensitive capital rule ($\theta = 0$), a constant audit frequency λ, and by absence of market discipline (assuming that deposits are fully insured). Moreover, we assume that the asset weight is 100%, which holds if all the claims in the bank's portfolio correspond, for example, to corporate lending.

[10] FDICIA explicitly states that a bank will be closed if its capital ratio falls below 2% (Comptroller of the Currency, 1993).

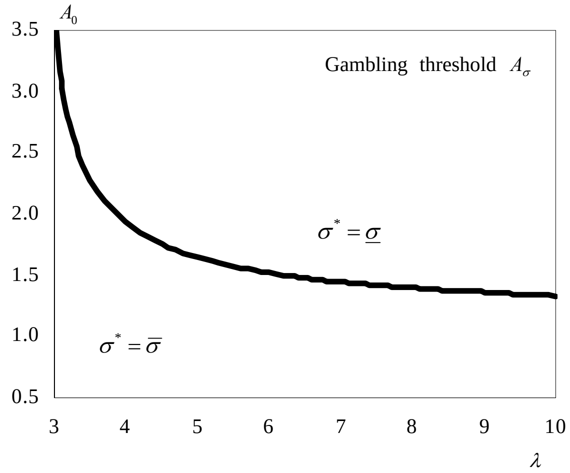

Figure 1: Gambling threshold A_σ as a function of the supervisor (constant) audit frequency λ for a risk insensitive capital requirements rule ($\theta = 0$).

Figure (1) represents the gambling threshold A_σ as a function of the supervisor audit frequency λ when the capital requirements rule is risk insensitive. *The higher the audit frequency λ the lower the gambling threshold A_σ.*

Although not discussed here, it can be shown that the gambling threshold A_σ is a positive function of both risk levels $\underline{\sigma}$ and $\overline{\sigma}$, and a negative function of the minimum capital requirements ρ.

5 Basel II

To clearly discern the effect of the policies proposed below, we introduce and analyze each of them assuming the rest are not in place. In particular, these policies are: a risk sensitive capital requirements rule (Pillar 1), a rating-based audit frequency (Pillar 2), rating-based dividend restrictions (Pillar 2), and rating-based deposit rates (Pillar 3).

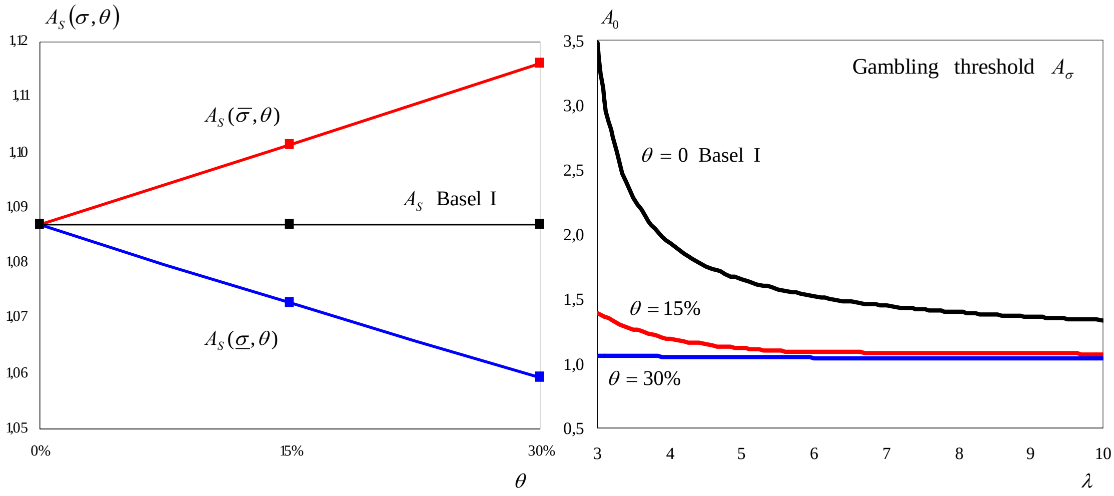

Figure 2: Supervisor closure level $A_S(\sigma, \theta)$ as a function of the risk sensitivity parameter θ, and gambling threshold A_σ implied by three different risk sensitivity parameters θ as a function of the supervisor audit frequency λ.

5.1 Pillar 1: Minimum capital requirements

> "*All of us have strongly wished for greater risk sensitivity.* The lack of differentiation of risk in the original Capital Accord was heavily criticized by banks and observers."
>
> Crockett (2002), General Manager of the Bank of International Settlements. Emphasis added.

Basel II capital requirements rule is more risk-sensitive than Basel I rule. In Basel I we assigned assets a 100% risk weight independently of the risk level ($\theta = 0$); in Basel II low risk $\underline{\sigma}$ (high risk $\overline{\sigma}$) assets will receive a risk weight lower (higher) than 100% ($\theta > 0$).

The left panel of Figure (2) shows the supervisor closure level $A_S(\sigma, \theta)$ as a function of the risk sensitivity parameter θ for low and high risk levels, and the right panel shows the gambling threshold A_σ implied by three different risk sensitivity parameters θ as a function of the supervisor audit frequency λ. The higher the risk sensitivity θ the lower the gambling threshold A_σ and therefore banks' risk taking incentives.

Additionally, the lower the audit frequency λ the higher the decrease of the gambling threshold A_σ when we increase the risk sensitivity parameter θ. Therefore, a more risk sensitive capital rule will reduce more banks' risk taking incentives in countries where the supervisor audit frequency is low. In the same way, a higher audit frequency will be more effective in reducing banks' risk taking incentives the lower the sensitivity of the capital rule. Thus, *although capital requirements and frequency of supervision are complementary tools to deter banks from taking high risk levels, the effectiveness of each measure is a decreasing function of the other's.*

5.2 Pillar 2: Supervisory review process

Pillar 2 seeks to strengthen and reinforce the role that national supervisors play to guarantee the effectiveness of the accord. The BCBS seems, through Pillar 2, to head the supervisory review process to a more risk-sensitive role; the same idea that US banking authorities introduced in 1991 through FDICIA.

5.2.1 Rating-based audit frequency

> "Accordingly, supervisors may wish to adopt an approach to *focus more intensely on those banks whose risk profile* or operational experience *warrants such attention.*"
>
> BCBS (2004, paragraph 722). Emphasis added.

It seems reasonable to, as Pillar 2 suggests, make the audit frequency a function of the banks' financial health: the better the financial situation of a bank in the last audit the lower will be the audit frequency with which it is audited.

The financial situation of a bank is proxied by its rating. We consider a rating system where a bank can either be under, low or high rated. A bank is undercapitalized if it not satisfies the capital requirements rule when it is audited, i.e. $A < A_S$, in which case it is closed by the supervisor. A bank is high rated if, at the audit time, the value of its assets A falls above a certain (risk-dependent) level $A_R(\sigma) > A_S$.

Otherwise, $A \in [A_S, A_R(\sigma)]$, the bank is considered low rated.[11] The bank's rating is reviewed at each audit.

We define $A_R(\sigma)$ as

$$A_R(\sigma) = A_S + 2\sigma, \tag{9}$$

for $\sigma \in \{\underline{\sigma}, \overline{\sigma}\}$. Since A_R depends positively on the risk level σ, a high risk bank is required a higher asset value than a low risk bank in order to be considered high rated, $A_R(\overline{\sigma}) > A_R(\underline{\sigma})$. Thus the rating of the bank is a measure of its capital and risk levels.[12]

For a given risk level σ, $A_{E,H}(\sigma)$ and $A_{E,L}(\sigma)$ represent, respectively, the levels at which equityholders decide to close voluntarily a high and a low rated bank. To simplify notation, when solving the differential equations, we shall drop the references to σ from $A_R(\sigma)$, $A_{E,H}(\sigma)$ and $A_{E,L}(\sigma)$, as well as when we refer to any claim (equity, audit costs and debt) values.

We propose the following rating-based audit frequency:

$$\lambda = \begin{cases} \lambda_H & \text{for a high rated bank,} \\ \lambda_L & \text{for a low rated bank,} \end{cases}$$

where $\lambda_L > \lambda_H$: *low rated banks are more frequently audited.*

Any rating-based policy, and this in particular, makes the value of any claim dependent on the bank's current rating. Moreover, the value of such a claim for a low rated bank depends on its value for a high rated bank, and vice versa, because a bank can switch rating level at any time. As a consequence, the value of the claim has to be expressed as a system of ODEs rather than, as before, as a single ODE. This holds for all the proposed rating-based measures analyzed next under Pillars 2 and 3.

[11] Since, without loss of generality, we assume a risk insensitive capital rule to analyze Pillars 2 and 3, A_S is no longer a function of neither risk σ nor the sensitivity parameter θ.

[12] The qualitative results of the different ratings-based regulations analyzed throughout the paper do not qualitatively vary if A_R is independent of the risk level σ.

Assume $\max\{A_{E,H}, A_{E,L}\} < A_S$, which holds in all cases analyzed, and the following notation for the value of equity:

$$\begin{cases} H(A) & \text{for a high rated bank,} \\ L(A) & \text{for a low rated bank.} \end{cases} \tag{10}$$

Equity value is characterized by the following system of ODEs:

$$\begin{array}{lll} \Gamma_H(A) + \delta A - d & = 0 & \text{for } A > A_R, \\ \Gamma_H(A) + \delta A - d - \lambda_H(H(A) - L(A)) & = 0 & \text{for } A \in [A_S, A_R], \\ \Gamma_H(A) + \delta A - d - \lambda_H H(A) & = 0 & \text{for } A < A_S, \end{array} \tag{11}$$

for H, and

$$\begin{array}{lll} \Gamma_L(A) + \delta A - d - \lambda_L(L(A) - H(A)) & = 0 & \text{for } A > A_R, \\ \Gamma_L(A) + \delta A - d & = 0 & \text{for } A \in [A_S, A_R], \\ \Gamma_L(A) + \delta A - d - \lambda_L L(A) & = 0 & \text{for } A < A_S, \end{array} \tag{12}$$

for L, where

$$\Gamma_H(A) = (1/2)\sigma^2 A^2 H''(A) + (r-\delta) AH'(A) - rH(A), \tag{13}$$

$$\Gamma_L(A) = (1/2)\sigma^2 A^2 L''(A) + (r-\delta) AL'(A) - rL(A). \tag{14}$$

H', L', H'' and L'' denote the first and second derivatives of L and H with respect to A.

Imagine a high rated bank (ODE 11). If $A > A_R$, the bank will remain high rated whether there is an audit or not. If $A \in [A_S, A_R]$ and an audit takes place, the bank is downgraded to low rated and the value of equity switches from $H(A)$ to $L(A)$. This happens with probability λ_H and is represented by $-\lambda_H(H(A) - L(A))$. If $A < A_S$ and there is an audit the supervisor closes the bank and equityholders are expropriated, which is represented by $-\lambda_H H(A)$. The same intuition lies behind the ODE (12) characterizing a low rated bank equity value L.

The function L appears in the non-homogeneous part of the ODE (11) governing the dynamics of H for $A \in (A_S, A_R)$ and the function H appears in the non-homogeneous part of the ODE (12) governing the dynamics of L for $A > A_R$. The *key to solve the previous system of ODEs analytically* is that the intervals in which L

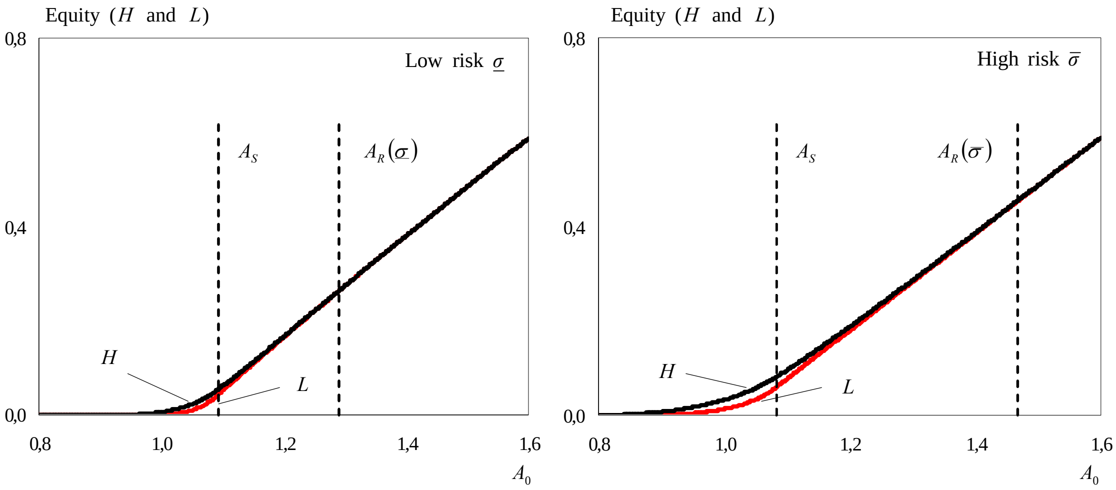

Figure 3: Value of equity for a low (L) and high (H) rated bank, for low $\underline{\sigma}$ and high $\overline{\sigma}$ risk levels, as a function of the initial asset value A_0. We consider a risk insensitive closure rule ($\theta = 0$) and a rating-based audit frequency $\{\lambda_H = 2, \lambda_L = 6\}$.

appears in the ODE of H and H appears in the ODE of L are disjoint. To solve for H in the interval $A \in (A_S, A_R)$ we substitute L by its solution from (12) in the same interval, which does not depend explicitly on H. The same argument is applied to solve the ODE characterizing the dynamics of L.

Due to its length and mathematical complexity the solution of the system of ODEs (11) and (12) is included in Appendix B in order to focus here in policy implications for Basel II.

Figure (3) shows the value of equity for a low L and high H rated bank, for low $\underline{\sigma}$ (left panel) and high $\overline{\sigma}$ (right panel) risk levels, as a function of the bank's initial asset value A_0. We consider a risk insensitive closure rule ($\theta = 0$) and a rating-based audit frequency $\{\lambda_H = 2, \lambda_L = 6\}$.

Equity value is higher for high rated banks because they are subject to a lower audit frequency. The difference between the values of equity for high H and low L rated banks is higher for asset levels around the supervisor closure level A_S. When the asset value increases, the value of equity for a low rated bank L converges to the value of equity of a high rated bank H, because the higher the asset value the

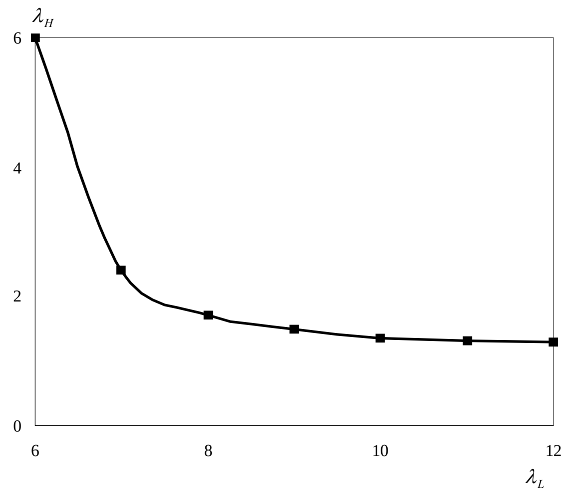

Figure 4: Combinations of low and high rated banks audit frequencies, λ_H and λ_L, which imply a gambling theshold $A_\sigma = 1.52$.

higher the probability of the bank being found high rated by the supervisor in the next audit. In the same way, as the asset value decreases, the value of equity for a high rated bank H converges to the value of equity of a low rated bank L. The loss in equity value when a bank is downgraded from high to low rated, $H - L$, is higher the higher the risk level.

Since banks' audits are costly, auditing a low rated bank with the same frequency than a high rated bank is a waste of resources. The advantage of a rating-based audit frequency over a constant audit frequency is precisely the savings in audit costs for high rated banks, which can be used to increase the audit frequency of low rated banks. With the same resources than with a constant audit frequency λ, a rating-based audit frequency with $\lambda_H < \lambda < \lambda_L$ allows the supervisor to achieve a lower gambling threshold. Alternatively, a rating-based frequency achieves the same gambling threshold with lower audit costs. We analyze this last case next.

We compare the following two cases: (i) a constant audit frequency $\lambda = 6$, which implies a gambling threshold $A_\sigma = 1.52$, and (ii) a rating-based audit frequency implying the same gambling threshold. In order to compute the gambling threshold when a rating-based policy is in place, we consider that equity value for asset values

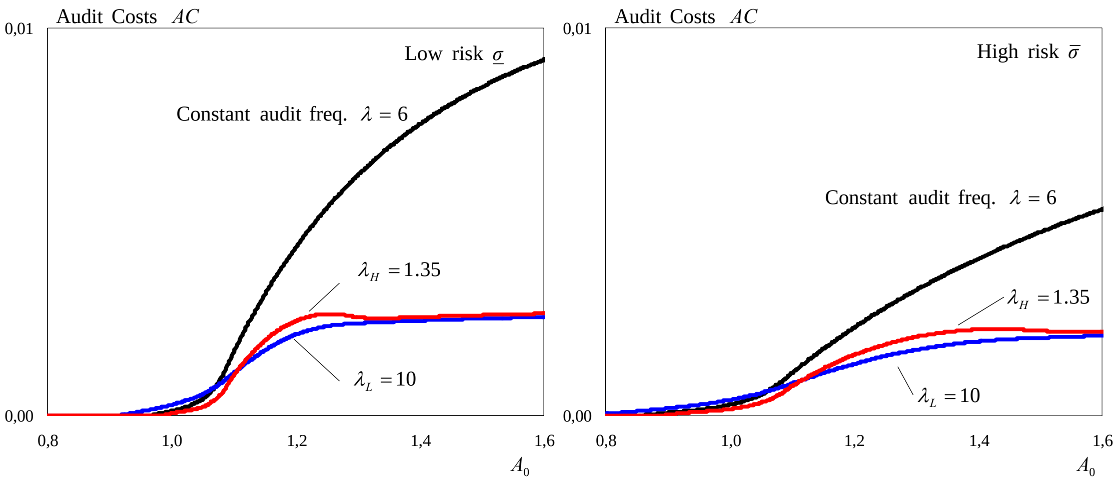

Figure 5: Audit costs AC as a function of the bank's asset value A_0 for low (left panel) and high (right panel) risk levels and for the two cases analyzed, both implying a gambling threshold $A_\sigma = 1.52$: (i) constant audit frequency $\lambda = 6$ and (ii) rating-based audit frequency $\{\lambda_H = 1.35, \lambda_L = 10\}$. Cost per audit and unit of deposits $\xi = 0.01\%$. Risk insensitive capital rule ($\theta = 0$).

A_0 lower (higher) than A_R is equal to the value of equity of a low (high) rated bank L (H). In other words, when equityholders decide their risk levels at $t = 0$, those with an asset level higher than A_R act as if they were high rated, and those with an asset level below A_R act as if they were low rated.[13]

Figure (4) represents combinations of low and high rated banks audit frequencies, λ_H and λ_L, which imply a gambling threshold $A_\sigma = 1.52$.

We assume, as in Merton (1978), a constant cost ξ per audit and unit of deposits, and compute (assuming risk neutrality) the value of the supervisor audit costs AC as a function of the banks' initial asset value A_0. Audit costs are the expected discounted value of the cost of future audits. Once a bank is closed the supervisor stops auditing it and audit costs become zero. Appendix C solves for the value of audit costs AC for constant and rating-based audit frequencies.

[13] This assumption is reasonable assuming that at $t = 0$ a bank does not have a rating assigned because it has not been audited so far. We proceed in the same way for the rest of rating-based measures analyzed later on.

Figure (5) represents audit costs AC as a function of the bank's asset value A_0 for low and high risk levels, for a constant audit frequency $\lambda = 6$ and a rating-based audit frequency $\{\lambda_H = 1.35, \lambda_L = 10\}$, both implying the same gambling threshold A_σ. Without loss of generality we fix $\xi = 0.01\%$. *The costs of the constant audit frequency are*, except for very low asset values, always *higher*; the higher the higher the asset value A.

5.2.2 Rating based dividend restrictions policy

> "*Supervisors should seek to intervene at an early stage to prevent capital from falling below the minimum levels required* to support the risk characteristics of a particular bank and should require rapid remedial action if capital is not maintained or restored. - ... - These actions may include the *monitoring of the bank*; *restricting the payment of dividends*; ..."
>
> Fourth key principle of supervisory review, BCBS (2004). Emphasis added.

We have already analyzed the policy concerning the *monitoring of the bank* through the audit frequency. Additionally, the model allows us to study is the action concerning *restricting the payment of dividends.*

We need to make a further assumption: the supervisor is able to monitor the payout ratio continuously. We consider a dividend restrictions policy where the supervisor requires low rated banks to reduce their payout ratio from δ to $\hat{\delta} = (1 - f)\,\delta$, for $f \in [0, 1]$. The higher f the higher dividend payment restrictions.[14]

Equity value is solved similarly to the rating-based audit frequency case, but substituting the unrestricted payout ratio δ by the restricted payout ratio $\hat{\delta}$ in the ODE which characterized a low rated bank equity value (12), and considering a constant audit frequency. Appendix D presents the system of ODEs for the value of equity in the rating-based dividend restrictions case.

[14] We can assume that the punishment to low capitalized banks' managers for not enforcing $\hat{\delta}$ is high enough as to make them prefer to always comply with the dividend restriction. Alternatively, the supervisor would need to audit low capitalized banks continuously, i.e. $\lambda_L \to \infty$ in terms of the ratings-based audit frequency.

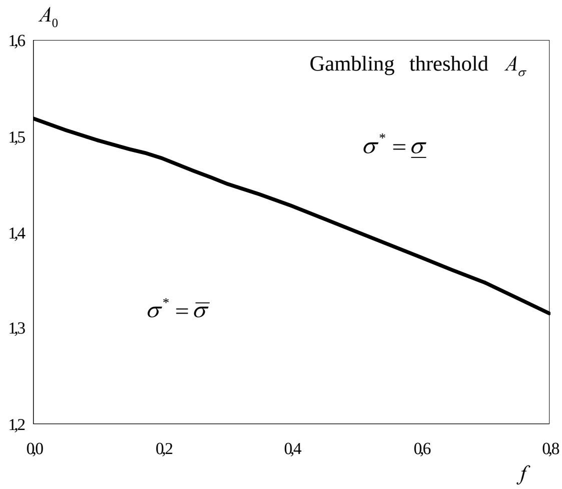

Figure 6: Gambling threshold A_σ as a function of the dividend restrictions parameter f. Constant audit frequency $\lambda = 6$ and risk insensitive capital rule ($\theta = 0$).

We assume a constant audit frequency $\lambda = 6$ and a risk insensitive capital requirements rule ($\theta = 0$). Figure (6) represents the gambling threshold A_σ as a function of the dividend restrictions parameter f. *The tougher the dividend restrictions the lower* the gambling threshold and, as a consequence, *banks' risk taking incentives.*

5.3 Pillar 3: Market discipline

"The purpose of pillar three - market discipline - is to complement the minimum capital requirements (Pillar 1) and the supervisory review process (Pillar 2). The Committee aims to *encourage market discipline* by developing a set of disclosure requirements which will *allow market participants to assess key pieces of information on* the scope of application, capital, risk exposures, risk assessment processes, and hence *the capital adequacy of the institution.*"

BCBS (2004, paragraph 809). Emphasis added.

Following the recommendations of Basel II, we propose a market discipline mechanism in which (not fully insured risk neutral) depositors receive timely information about the bank's financial situation and renegotiate the deposit rate with the bank accordingly.

Apart from the supervisor, there exist rating agencies which audit the bank and make public the results of their audits. Rating agencies audit times are assumed stochastically distributed as a Poisson random variable with intensity λ_R, independent of the supervisor audit times. Depositors only receive information about the bank's financial situation at rating agencies audit times, whereas the supervisor receives information about the bank's risk and asset levels not only when he performs bank audits but also at rating agencies audit times (because rating agencies reveal their findings to the market). Therefore, rating agencies auditing activity increases the effective audit frequency of the supervisor from λ to $\lambda + \lambda_R$.[15]

Any effective market discipline mechanism has to satisfy three requirements. First, it has to ensure depositors receive timely information about the financial situation of the bank, which in our case is provided by rating agencies. Second, it has to ensure depositors have incentives to use that information, which requires deposits not to be fully insured. Finally, the market discipline mechanism has to affect the behavior of banks: we assume the deposit rate d is renegotiated after each rating agency audit.

Let i denote the fraction of insured deposits, set by the supervisor. If the bank is closed, either voluntarily by equityholders or by the supervisor, equityholders are expropriated and a fraction $1 - \phi$ of the bank's asset value is lost. ϕ is the recovery rate and it is assumed 0.9.

The definition of high and low rated banks used hereafter takes into account depositors information and not the supervisor information as in Pillar 2. A bank will be low (high) rated if its asset value at the last rating agency audit was lower (higher) than A_R.

[15] Alternatively to the role of rating agencies in providing information to the market, Basel II explicitly suggests that supervisors can make some or all of the information in regulatory reports publicly available (BCBS, 2004, paragraph 811). Any of the two cases would fit our market discipline mechanism. Furthermore, one can consider the situation in which banks themselves disclaim their ratings to the market.

Depositors receive a continuous rate d as long as the bank is open and, in case of closure, they receive either the liquidation value of the bank ϕA or the amount of insured deposits i, whichever higher. For each fraction of uninsured deposits i and risk level σ, the deposit rate d depends on the bank's rating:

$$d = \begin{cases} d_H(i,\sigma) & \text{for a high rated bank,} \\ d_L(i,\sigma) & \text{for a low rated bank.} \end{cases}$$

Assuming constant audit frequencies, λ and λ_R, and a risk insensitive capital requirements rule ($\theta = 0$), the value of equity for low and high rated banks, denoted by (10), is given by the system of ODEs:

$$\begin{array}{lll} \Gamma_H(A) + \delta A - d_H(i,\sigma) & = 0 & \text{for } A > A_R, \\ \Gamma_H(A) + \delta A - d_H(i,\sigma) - \lambda_R(H(A) - L(A)) & = 0 & \text{for } A \in [A_S, A_R], \\ \Gamma_H(A) + \delta A - d_H(i,\sigma) - (\lambda + \lambda_R) H(A) & = 0 & \text{for } A < A_S, \end{array} \quad (15)$$

for H, and

$$\begin{array}{lll} \Gamma_L(A) + \delta A - d_L(i,\sigma) - \lambda_R(L(A) - H(A)) & = 0 & \text{for } A > A_R, \\ \Gamma_L(A) + \delta A - d_L(i,\sigma) & = 0 & \text{for } A \in [A_S, A_R], \\ \Gamma_L(A) + \delta A - d_L(i,\sigma) - (\lambda + \lambda_R) L(A) & = 0 & \text{for } A < A_S, \end{array} \quad (16)$$

for L. $\Gamma_H(A)$ and $\Gamma_L(A)$ are given by (13) and (14) respectively. We solve for the value of equity in the same way as for the two previously analyzed rating-based policies. Again, the previous system of ODEs represents the case where $\max\{A_{E,H}, A_{E,L}\} < A_S$.[16]

If a bank is high rated, ODE (15), equityholders receive $\delta A - d_H(i,\sigma)$ independently of the banks' asset value A. If the asset value remains above A_R the bank's rating does not change whether there is a rating agency audit or not. If the asset value A moves into the interval $[A_S, A_R]$ the bank would not be closed if there is a supervisor audit; but if there is a rating agency audit its rating goes down to low rated. This is represented by $-\lambda_R(H(A) - L(A))$ in the second part of (15).

[16] For illustration purposes, Appendix E.1 includes the system of ODEs for the particular case in which $A_{E,H} < A_S < A_{E,L}$. It turns out that for the parameter values used $A_{E,H} < A_{E,L} < A_S$ always hold. However, due to the fact that deposit rates could be now significantly higher than the risk free rate r (when the fraction of insured deposits i is low), when solving for the value of equity one also has to consider cases in which $\max\{A_{E,H}, A_{E,L}\} \geq A_C$, always assuming $A_{E,H} < A_{E,L}$.

Finally, if the asset value goes below the supervisor closure level A_S the bank will be closed if an audit, carried out either by the supervisor or by the rating agency, takes place. This is represented by the term $-(\lambda + \lambda_R) H(A)$. $\lambda + \lambda_R$ represents the probability of an audit taking place and $H(A)$ the loss suffered by equityholders. The same intuition lies behind the ODE (16) for the value of equity L of a low rated bank.

Two further normalizations are considered (which do not qualitatively affect our results). First, depositors require a high rated bank a deposit rate equal to the risk free rate, $d_H(i, \sigma) = r$. Second, the deposit rate required to a low rated bank $d_L(i, \sigma)$ guarantees that the market value of deposits of a low rated bank is equal to its face value 1 for an asset value midway between A_S and A_R. Since rating agencies only report the rating of the bank, it seems appropriate to assume that for fixing the deposit rate $d_L(i, \sigma)$ depositors use the average of the asset values which characterize a low rated bank.[17] Appendix E.2 solves for the market value of deposits.

Given the risk level $\sigma \in \{\underline{\sigma}, \overline{\sigma}\}$, fraction of insured deposits i, and an initial value for $d_L(i, \sigma)$ we solve for the value of equity. Then, using the implied equityholders' closure levels $A_{E,H}$ and $A_{E,L}$, we compute the market value of deposits of a low rated bank at $A = (A_R + A_S)/2$. If the market value of deposits is higher (lower) than the face value of deposits, we reduce (increase) the uninsured deposit rate $d_L(i, \sigma)$, and calculate again the value of equity and equityholders' closure levels $A_{E,H}$ and $A_{E,L}$. We iterate this procedure until we find the level of $d_L(i, \sigma)$ for which the market value of deposits at $A = (A_R + A_S)/2$ is equal to the face value of deposits.

We assume constant audit frequencies $\lambda = \lambda_R = 3$ for both the supervisor and the rating agencies and a risk insensitive capital requirements rule. The left panel of Figure (7) shows low rated banks' deposit rate $d_L(i, \sigma)$, for high $\overline{\sigma}$ and low $\underline{\sigma}$ risk levels, as a function of the fraction of insured deposits i. Low rated banks' deposit rate $d_L(i, \sigma)$ is: (i) always higher than high rated banks' deposit rate $d_H(i, \sigma)$, (ii)

[17] Strictly speaking, we assume that depositors also know the risk level of the bank. Since it is constant one can assume rating agencies announce it in the first audit.

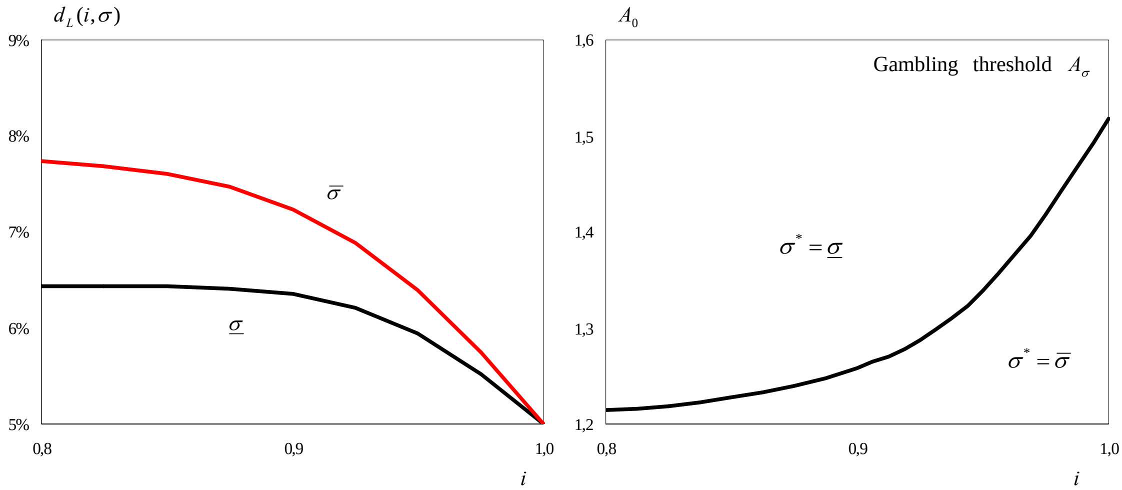

Figure 7: The left panel shows low rated banks' deposit rate $d_L(i, \sigma)$, for high $\underline{\sigma}$ and low $\overline{\sigma}$ risk levels, as a function of the fraction of insured deposits i. The right panel represents the gambling threshold A_σ as a function of the fraction of insured deposits i. Constant audit frequencies $\lambda = \lambda_R = 3$, and risk insensitive capital requirements rule ($\theta = 0$).

an increasing function of the bank's risk level σ, and (iii) a decreasing function of the fraction of insured deposits i.[18]

The right panel of Figure (7) represents the gambling threshold A_σ as a function of the fraction of insured deposits i. *Market discipline, understood as a lower deposit insurance coverage i, reduces the gambling threshold and therefore banks' risk taking incentives.*

[18]When the fraction of insured deposits i is low enough as to make the bank's liquidation value ϕA higher than insured deposits i, depositors always receive the bank's liquidation value ϕA if the bank is closed. Thus, for low enough values of i, low capitalized banks' deposit rate d_L remains constant.

6 Conclusion

Basel II represents an increase in the risk sensitivity of banking regulation and supervision and, as a consequence, reduces banks' risk taking incentives and supervision costs.

We have broken down the analysis of Basel II into the three Pillars of which it is composed: minimum capital requirements, supervisory review process and market discipline. Though analyzed separately in order to reduce the complexity of the model, it would be straightforward to consider a model in which all the policies analyzed are implemented at the same time.

Although the supervisor audit frequency is an effective tool to induce prudent behavior in banks, audit costs concerns show that a rating-based audit frequency is more effective and less expensive than a constant audit frequency. The other measure analyzed within Pillar 2, restricting dividend payments to low rated banks, also reduces banks' risk taking incentives. Both rating-based measures resemble the gradualism in supervisory actions introduced by US banking authorities in 1991 through the FDIC Improvement Act, FDICIA. Given our results, national supervisors might find appropriate to borrow from the Prompt Corrective Action provisions (PCA) of FDICIA when using the discretion granted by Pillar 2. Notice that while rating-based actions are mandatory for supervisors in FDICIA, they are just suggestions in Basel II.

Finally, we propose a market discipline mechanism for Pillar 3 in which the supervisor reduces the deposit insurance coverage, compelling depositors to require banks a deposit rate based on the bank's financial situation, about which they learn through rating agencies. Banking regulations are intended to monitor and control the banks' risks; Pillar 3 transfers part of such task to depositors themselves. The effectiveness of market discipline to reduce banks' risk taking incentives relies on the validity of the model's assumptions: (i) timely and reliable information about the banks' financial health must be disclosed to the market (either by rating agencies as in our model, by supervisors or by the banks themselves), (ii) depositors must be (and feel) compelled to use such information to discriminate among banks through deposit rates, and (iii) they need to have enough bargaining power to make such discrimination effective. Although the BCBS maintains a line of work to promote effective information disclosure (BCBS, 2000), an effective market discipline mechanism requires advances in the other two points through policies targeting depositors' (rather than banks') risk awareness and risk control.

Rating-based supervisory and regulatory measures reduce banks' risk taking incentives, and this reduction is higher the higher the rating levels considered. With a discrete rating system we are throwing away useful information, because we are transforming a continuous variable measuring the bank's financial situation (asset or capital level) into a discrete variable (the bank's rating). In the limit, we would have a continuous rating system, with a different treatment for each capital or asset level. In fact, increasing the number and precision of rating levels used by rating agencies would benefit market discipline because depositors would receive higher quality information about the banks' financial situation.

However, collapsing the information about the financial situation of the bank into a discrete system of ratings helps in the implementation of the rating-based policies and in the diffusion of information to the market. Rating systems are discrete in real life and to exploit the empirical applications of our framework the discretization is required. There is a trade-off between the effectiveness of rating-based policies and the ability to apply them and inform the market about the bank's financial situation which is reflected on the number of ratings. The optimal number of ratings and the cut-off capital levels differentiating them requires an objective function for the supervisor and a measure of the costs and benefits of the previous trade-off.

The proposed framework allows one to analyze any rating-based measure, and therefore can be used to evaluate any new (or existing) one. In particular, although not discussed here and neither included in the Basel Accords, the model allows to consider risk-based deposit insurance premiums, revising the deposit insurance premiums charged to each bank after each bank audit depending on the bank's rating level. A rating-based deposit insurance premium policy will be equivalent to the rating-based market discipline mechanism analyzed in Pillar 3, in which the deposit insurer would adjust the bank's deposit insurance premium after each audit, in order to control the risks for the deposit insurance fund derived from the possibility of the bank's closure. Additionally, the model allows to extend the regulatory measure considered by Suarez (1994) in which the supervisor restricts the maximum risk level a bank can take. In our framework such a measure could be made rating-dependent: banks with higher ratings would be allowed to take higher maximum risks.

Appendix

A General procedure for computing the value of a claim

This Appendix is intended as a general introduction to the claims' value solving process using ordinary differential equations (ODEs) when no rating-based policy is in place. Rather than solving single ODEs, when rating-based measures are used (Pillars 2 and 3) computing the claims' value requires solving systems of ODEs, which will be explained as they appear. The procedure for solving such systems correspond to the one outlined here for solving ODEs.

Consider a given claim on the bank's assets and denote as ω the continuous payment received by its holders at any time previous to the bank's closure, as γ the payment they receive when the bank is closed, and as η the payment they receive whenever an audit takes place. ω, γ and η may depend on the asset level and can be positive or negative. Table 2 contains the values of ω, γ and η for the different claims used throughout the paper.

Claim	ω	γ	η
Equity	$\delta A - d$	0	0
Audit costs	0	0	ξ
(Not-fully insured) Deposits	d	$\max\{\phi A, \pi\}$	0

Table 2. Values of ω, γ and η for several claims.

Under risk neutrality and if τ denotes the time at which the bank is closed, the value of such a claim at time $t < \tau$ for a risk level σ and audit frequency λ is given, as a function of the asset level A_t, by

$$J(A_t) = E\left[\int_0^\tau e^{-rs}\left(\omega(A_s) + \lambda\eta(A_s)\right)ds + \gamma(A_\tau)e^{-r\tau} \mid A_t\right].$$

Dropping the subindex t of the asset value A_t, it can be shown (cf. Dixit and Pindyck, 1994, Chp. 5) that the previous expectation can be transformed into the following ODE

$$\begin{array}{lll} \Gamma(A) + \omega(A) + \lambda\eta(A) & = 0 & \text{for } A \geq A_S, \\ \Gamma(A) + \omega(A) + \lambda\eta(A) + \lambda\left(\gamma(A) - J(A)\right) & = 0 & \text{for } A < A_S. \end{array}$$

A_S denotes the asset value below which, if audited, a bank is closed by the supervisor. $\Gamma(A)$ is common to all claims and given by

$$\Gamma(A) = (1/2)\,\sigma^2 A^2 J''(A) + (r - \delta)\,AJ'(A) - rJ(A),$$

where J, J' and J'' represent the value of the claim for any value of A, and its first and second partial derivatives with respect to A.

Once we have the ODE characterizing the dynamics of a claim, the second step is to find its solution.

A.1 Solution of an ODE

Consider an asset whose value J depends on the asset A and satisfies the ODE

$$xA^2 J''(A) + yAJ'(A) - zJ(A) + j(A) = 0, \tag{A1}$$

where $j(\cdot)$ is not a function of J.

First, we find the general solution of the homogeneous part of (A1):

$$xA^2 J''(A) + yAJ'(A) - zJ(A) = 0. \tag{A2}$$

Since the second-order homogeneous differential equation is linear in the dependent variable J and its derivatives, its general solution can be expressed as a linear combination of any two independent solutions (cf. Dixit and Pindyck, 1994, Chp. 5.2). If we try the function KA^{β}, it satisfies (A2) provided β is a root of the quadratic equation

$$x\beta(\beta - 1) + y\beta - z = 0.$$

The two roots are

$$\begin{aligned} \beta_1 &= \tfrac{1}{2x}[(x-y) + \sqrt{(x-y)^2 + 4xz}] > 1, \\ \beta_2 &= \tfrac{1}{2x}[(x-y) - \sqrt{(x-y)^2 + 4xz}] < 0. \end{aligned}$$

Thus, the general solution of (A1) can be written as

$$J(A) = K_1 A^{\beta_1} + K_2 A^{\beta_2}, \tag{A3}$$

for (unknown) constants K_1 and K_2.

Next, we add a term $h(A)$ to the solution of the homogeneous part (A3) to account for the function $j(A)$ in (A1):

$$J(A) = K_1 A^{\beta_1} + K_2 A^{\beta_2} + h(A). \tag{A4}$$

Lemma 1 *It can be shown, by substitution, that, for an ODE like (A1) with* $j(A) = \sum_i w_i A^{v_i}$, $h(A)$ *is given by*

$$h(A) = \sum_i \frac{w_i A^{v_i}}{z - xv_i(v_i - 1) - yv_i}.$$

Once the theoretical form (A4) is obtained, a proper set of boundary conditions is used in each case to solve for the unknown constants K_1 and K_2 in order to compute the value of the claim for the model parameters.

B Rating-based audit frequency: equity value

Using the results in Appendix A, the system of ODEs (11) and (12) imply the following solutions for the value of equity (10) of high and low rated banks (which we explain below):

$$H = \begin{cases} H_c = K_{1,c}A^{\beta_{1,c}} + K_{2,c}A^{\beta_{2,c}} + A - d/r & \text{for } A > A_R, \\ H_b = K_{1,b}A^{\beta_{1,b}} + K_{2,b}A^{\beta_{2,b}} + S_{H_b} & \text{for } A \in [A_S, A_R], \\ H_a = K_{1,a}A^{\beta_{1,a}} + K_{2,a}A^{\beta_{2,a}} + \frac{\delta A}{\lambda_H + \delta} - \frac{d}{r + \lambda_H} & \text{for } A < A_S, \end{cases} \tag{B1}$$

and

$$L = \begin{cases} L_c = G_{1,c}A^{\eta_{1,c}} + G_{2,c}A^{\eta_{2,c}} + S_{L_c} & \text{for } A > A_R, \\ L_b = G_{1,b}A^{\eta_{1,b}} + G_{2,b}A^{\eta_{2,b}} + A - d/r & \text{for } A \in [A_S, A_R], \\ L_a = G_{1,a}A^{\eta_{1,a}} + G_{2,a}A^{\eta_{2,a}} + \frac{\delta A}{\lambda_L + \delta} - \frac{d}{r + \lambda_L} & \text{for } A < A_S, \end{cases} \tag{B2}$$

where S_{H_b}, S_{L_c}, $\beta_{1,a}$, $\beta_{1,b}$, $\beta_{1,c}$, $\beta_{2,a}$, $\beta_{2,b}$, $\beta_{2,c}$, $\eta_{1,a}$, $\eta_{1,b}$, $\eta_{1,c}$, $\eta_{2,a}$, $\eta_{2,b}$ and $\eta_{2,c}$ are functions of the parameters. $K_{1,a}$, $K_{1,b}$, $K_{1,c}$, $K_{2,a}$, $K_{2,b}$, $K_{2,c}$, $G_{1,a}$, $G_{1,b}$, $G_{1,c}$, $G_{2,a}$, $G_{2,b}$ and $G_{2,c}$ are a set of unknown constants whose values, together to that of $A_{E,L}$ and $A_{E,H}$, have to be found to characterize the value of equity for each rating and asset level.

For a high rated bank H_a and H_c in (B1) are computed from the corresponding ODEs in (11) for $A < A_S$ and $A > A_R$ respectively using the techniques described in Appendix A for solving single ODEs. The reason is that for those particular asset intervals the ODEs do not explicitly contain the equity value L of a low rated bank (because there is no possibility of a rating switch from high to low rated). The same applies for the solution of L_a and L_b in (B2). $\beta_{1,a}$, $\beta_{1,c}$, $\beta_{2,a}$, $\beta_{2,c}$, $\eta_{1,a}$, $\eta_{1,b}$, $\eta_{2,a}$ and $\eta_{2,b}$ are computed along H_a, H_c, L_a and L_b and are given below.

To solve for H_b and S_{H_b} consider the specification of the ODE (11) for $A \in [A_S, A_R]$ and denote the value of equity for a low and high rated bank in that particular interval by H_b and L_b respectively:

$$\Gamma_{H_b}(A) + (\delta A - d) - \lambda_H (H_b(A) - L_b(A)) = 0, \tag{B3}$$

where

$$\Gamma_{H_b}(A) = \frac{1}{2}\sigma^2 A^2 H_b''(A) + (r - \delta) A H_b'(A) - r H_b(A).$$

For $A \in [A_S, A_R]$ the ODE (12) for a low rated bank does not explicitly depend on the value H of a high rated bank and, as a consequence L_b does not depend on H. Therefore, L_b does not belong to the homogeneous part of the ODE (B3), but will be used to solve its particular solution.

On the one hand, the homogeneous part of (B3), given by

$$\begin{aligned} \Gamma_{H_b}(A) - \lambda_H H_b(A) &= 0, \\ \frac{1}{2}\sigma^2 A^2 H_b''(A) + (r - \delta) A H_b'(A) - (r + \lambda_H) H_b(A) &= 0, \end{aligned} \tag{B4}$$

has the general solution

$$K_{1,b}A^{\beta_{1,b}} + K_{2,b}A^{\beta_{2,b}}, \tag{B5}$$

where $\beta_{1,b}$ and $\beta_{2,b}$ are computed by substituting (B5) into (B4) as in Appendix A.

On the other hand, the non-homogeneous part of (B3) is given by

$$H_b(A) = (\delta A - d) + \lambda_H L_b(A). \tag{B6}$$

Substituting L_b into (B6) and rearranging we obtain

$$H_b(A) = A(\delta + \lambda_H) - d(1 + \lambda_H/r) + \lambda_H G_{1,b}A^{\eta_{1,b}} + \lambda_H G_{2,b}A^{\eta_{2,b}}.$$

S_{H_b} is the particular solution we add to (B5) to compute H_b accounting for the non-homogeneous part H_b of the ODE (B3). According to Lemma 1:

$$\begin{aligned} S_{H_b} &= \frac{A(\delta+\lambda_H)}{\lambda_H+\delta} - \frac{d(1+\lambda_H/r)}{r+\lambda_H} + \frac{\lambda_H G_{1,b}A^{\eta_{1,b}}}{(r+\lambda_H) - \frac{1}{2}\sigma^2\eta_{1,b}\left(\eta_{1,b}-1\right) - (r-\delta)\eta_{1,b}} \\ &+ \frac{\lambda_H G_{2,b}A^{\eta_{2,b}}}{(r+\lambda_H) - \frac{1}{2}\sigma^2\eta_{2,b}\left(\eta_{2,b}-1\right) - (r-\delta)\eta_{2,b}}. \end{aligned}$$

Proceeding in the same way we solve for L_c, S_{L_c}, $\eta_{1,c}$ and $\eta_{2,c}$. S_{L_c} is given by:

$$\begin{aligned} S_{L_c} &= \frac{A(\delta+\lambda_L)}{\lambda_L+\delta} - \frac{d(1+\lambda_L/r)}{r+\lambda_L} + \frac{\lambda_L K_{1,c}A^{\beta_{1,c}}}{(r+\lambda_L) - \frac{1}{2}\sigma^2\beta_{1,c}\left(\beta_{1,c}-1\right) - (r-\delta)\beta_{1,c}} \\ &+ \frac{\lambda_L K_{2,c}A^{\beta_{2,c}}}{(r+\lambda_L) - \frac{1}{2}\sigma^2\beta_{2,c}\left(\beta_{2,c}-1\right) - (r-\delta)\beta_{2,c}}. \end{aligned}$$

Finally, it can be shown that $\beta_{1,a}$, $\beta_{1,b}$, $\beta_{1,c}$, $\beta_{2,a}$, $\beta_{2,b}$, $\beta_{2,c}$, $\eta_{1,a}$, $\eta_{1,b}$, $\eta_{1,c}$, $\eta_{2,a}$, $\eta_{2,b}$ and $\eta_{2,c}$ are given by:

$$\begin{aligned} \beta_{1,c} = \eta_{1,b} &= \sigma^{-2}[\left(\tfrac{\sigma^2}{2} - r + \delta\right) + \sqrt{\left(\tfrac{\sigma^2}{2} - r + \delta\right)^2 + 2r\sigma^2}] > 1, \\ \beta_{2,c} = \eta_{2,b} &= \sigma^{-2}[\left(\tfrac{\sigma^2}{2} - r + \delta\right) - \sqrt{\left(\tfrac{\sigma^2}{2} - r + \delta\right)^2 + 2r\sigma^2}] < 0, \\ \beta_{1,b} = \beta_{1,a} &= \sigma^{-2}[\left(\tfrac{\sigma^2}{2} - r + \delta\right) + \sqrt{\left(\tfrac{\sigma^2}{2} - r + \delta\right)^2 + 2(r+\lambda_H)\sigma^2}] > 1, \\ \beta_{2,b} = \beta_{2,a} &= \sigma^{-2}[\left(\tfrac{\sigma^2}{2} - r + \delta\right) - \sqrt{\left(\tfrac{\sigma^2}{2} - r + \delta\right)^2 + 2(r+\lambda_H)\sigma^2}] < 0, \\ \eta_{1,c} = \eta_{1,a} &= \sigma^{-2}[\left(\tfrac{\sigma^2}{2} - r + \delta\right) + \sqrt{\left(\tfrac{\sigma^2}{2} - r + \delta\right)^2 + 2(r+\lambda_L)\sigma^2}] > 1, \\ \eta_{2,c} = \eta_{2,a} &= \sigma^{-2}[\left(\tfrac{\sigma^2}{2} - r + \delta\right) - \sqrt{\left(\tfrac{\sigma^2}{2} - r + \delta\right)^2 + 2(r+\lambda_L)\sigma^2}] < 0. \end{aligned} \tag{B7}$$

To solve for the value of equity we have to find 14 unknowns ($K_{1,c}$, $K_{2,c}$, $K_{1,b}$, $K_{2,b}$, $K_{1,a}$, $K_{2,a}$, $G_{1,c}$, $G_{2,c}$, $G_{1,b}$, $G_{2,b}$, $G_{1,a}$, $G_{2,a}$, $A_{E,H}$ and $A_{E,L}$) for which we use the following 14 boundary conditions:

$$
\begin{aligned}
\lim_{A\to\infty} H_c(A)/A &= 1 \Rightarrow K_{1,c} = 0, && \text{(B8)}\\
\lim_{A\to\infty} L_c(A)/A &= 1 \Rightarrow G_{1,c} = 0, && \text{(B9)}\\
H_c(A_R) &= H_b(A_R), && \text{(B10)}\\
H_c'(A_R) &= H_b'(A_R), && \text{(B11)}\\
L_c(A_R) &= L_b(A_R), && \text{(B12)}\\
L_c'(A_R) &= L_b'(A_R), && \text{(B13)}\\
H_b(A_S) &= H_a(A_S), && \text{(B14)}\\
H_b'(A_S) &= H_a'(A_S), && \text{(B15)}\\
L_b(A_S) &= L_a(A_S), && \text{(B16)}\\
L_b'(A_S) &= L_a'(A_S), && \text{(B17)}\\
H_a(A_{E,H}) &= 0, && \text{(B18)}\\
H_a'(A_{E,H}) &= 0, && \text{(B19)}\\
L_a(A_{E,L}) &= 0, && \text{(B20)}\\
L_a'(A_{E,L}) &= 0. && \text{(B21)}
\end{aligned}
$$

C Audit costs

When computing audit costs we take as given equityholders' closure levels: A_E in the constant audit frequency case and $A_{E,L}$ and $A_{E,H}$ for low and high rated banks respectively in the rating-based case.

C.1 Constant audit frequency

Assuming a cost ξ per audit and $A_E < A_S$, audit costs AC follow the ODE

$$
\begin{array}{lll}
\Gamma_{AC}(A) + \lambda\xi & = 0 & \text{for } A \geq A_S,\\
\Gamma_{AC}(A) + \lambda(\xi - AC(A)) & = 0 & \text{for } A < A_S,
\end{array}
$$

where $\Gamma_{AC}(A)$ is given by (7), substituting V by AC. According to Appendix A

$$
AC = \left\{ \begin{array}{ll}
AC_b = K_{1,b}A^{\beta_{1,b}} + K_{2,b}A^{\beta_{2,b}} + \lambda\xi/r & \text{for } A \geq A_S,\\
AC_a = K_{1,a}A^{\beta_{1,a}} + K_{2,a}A^{\beta_{2,a}} + \frac{\lambda\xi}{r+\lambda} & \text{for } A < A_S.
\end{array} \right.
$$

$\beta_{1,b}$, $\beta_{2,b}$, $\beta_{1,a}$ and $\beta_{2,a}$ are given by (8). We need to find four unknowns ($K_{1,a}$, $K_{2,a}$, $K_{1,b}$ and $K_{2,b}$), for which we use the following boundary conditions:

$$
\begin{aligned}
\lim_{A\to\infty} AC_b(A) &= \lambda\xi/r \Rightarrow K_{1,b} = 0,\\
AC_b(A_S) &= AC_a(A_S),\\
AC_b'(A_S) &= AC_a'(A_S),\\
AC_a(A_E) &= 0.
\end{aligned}
$$

C.2 rating-based audit frequency

Consider the notation (10) for audit costs AC as a function of the bank's asset value A. Assuming $\max\{A_{E,H}, A_{E,L}\} < A_S$ and following the arguments employed to explain the system of ODEs (11) and (12) which characterize the value of equity for a rating based audit frequency, one obtains the following system of ODEs for audit costs AC :

$$\begin{array}{lll} \Gamma_H(A) + \lambda_H \xi & = 0 & \text{for } A > A_R, \\ \Gamma_H(A) + \lambda_H(\xi - H(A) + L(A)) & = 0 & \text{for } A \in [A_S, A_R], \\ \Gamma_H(A) + \lambda_H(\xi - H(A)) & = 0 & \text{for } A < A_S, \end{array} \tag{C1}$$

for H, and

$$\begin{array}{lll} \Gamma_L(A) + \lambda_L(\xi - L(A) + H(A)) & = 0 & \text{for } A > A_R, \\ \Gamma_L(A) + \lambda_L \xi & = 0 & \text{for } A \in [A_S, A_R], \\ \Gamma_L(A) + \lambda_L(\xi - L(A)) & = 0 & \text{for } A < A_S, \end{array} \tag{C2}$$

for L, where $\Gamma_H(A)$ and $\Gamma_L(A)$ are given by (13) and (14) respectively.

Using the methodology outlined in Appendix B for solving systems of ODEs, it can be shown that the system (C1) and (C2) implies the following solutions for the value of audit costs AC for high and low rated banks:

$$H = \begin{cases} H_c = K_{1,c}A^{\beta_{1,c}} + K_{2,c}A^{\beta_{2,c}} + \lambda_H \xi / r & \text{for } A > A_R, \\ H_b = K_{1,b}A^{\beta_{1,b}} + K_{2,b}A^{\beta_{2,b}} + S_{H_b} & \text{for } A \in [A_S, A_R], \\ H_a = K_{1,a}A^{\beta_{1,a}} + K_{2,a}A^{\beta_{2,a}} + \frac{\lambda_H \xi}{r + \lambda_H} & \text{for } A < A_S, \end{cases}$$

and

$$L = \begin{cases} L_c = G_{1,c}A^{\eta_{1,c}} + G_{2,c}A^{\eta_{2,c}} + S_{L_c} & \text{for } A > A_R, \\ L_b = G_{1,b}A^{\eta_{1,b}} + G_{2,b}A^{\eta_{2,b}} + \lambda_L \xi / r & \text{for } A \in [A_S, A_R], \\ L_a = G_{1,a}A^{\eta_{1,a}} + G_{2,a}A^{\eta_{2,a}} + \frac{\lambda_L \xi}{r + \lambda_L} & \text{for } A < A_S, \end{cases}$$

where S_{H_b} and S_{L_c} are computed using Lemma 1 and $\beta_{1,a}$, $\beta_{1,b}$, $\beta_{1,c}$, $\beta_{2,a}$, $\beta_{2,b}$, $\beta_{2,c}$, $\eta_{1,a}$, $\eta_{1,b}$, $\eta_{1,c}$, $\eta_{2,a}$, $\eta_{2,b}$ and $\eta_{2,c}$ are given by (B7).

The solution of such a system generates 12 unknowns ($K_{1,c}$, $K_{2,c}$, $K_{1,b}$, $K_{2,b}$, $K_{1,a}$, $K_{2,a}$, $G_{1,c}$, $G_{2,c}$, $G_{1,b}$, $G_{2,b}$, $G_{1,a}$, $G_{2,a}$) which can be solved using the boundary conditions

$$\begin{aligned} \lim_{A\to\infty} H(A)/A &= \lambda_H \xi / r \Rightarrow K_{1,c} = 0, \\ \lim_{A\to\infty} L(A)/A &= \lambda_L \xi / r \Rightarrow G_{1,c} = 0, \end{aligned}$$

plus (B10), (B11), (B12), (B13), (B14), (B15), (B16), (B17), (B18), and (B20).

D rating-based dividend restrictions: equity value

Consider a constant audit frequency, the notation (10) for equity and assume $\max\{A_{E,H}, A_{E,L}\} < A_S$. The system of ODEs for equity value in the rating-based dividend restrictions case is given by:

$$\begin{array}{lll}\Gamma_H(A) + \delta A - d & = 0 & \text{for } A > A_R, \\ \Gamma_H(A) + \delta A - d - \lambda(H(A) - L(A)) & = 0 & \text{for } A \in [A_S, A_R], \\ \Gamma_H(A) + \delta A - d - \lambda H(A) & = 0 & \text{for } A < A_S,\end{array} \tag{D1}$$

for H, and

$$\begin{array}{lll}\Gamma_L(A) + \hat{\delta} A - d - \lambda(L(A) - H(A)) & = 0 & \text{for } A > A_R, \\ \Gamma_L(A) + \hat{\delta} A - d & = 0 & \text{for } A \in [A_S, A_R], \\ \Gamma_L(A) + \hat{\delta} A - d - \lambda L(A) & = 0 & \text{for } A < A_S,\end{array} \tag{D2}$$

for L, where $\hat{\delta} = (1-f)\,\delta$. $\Gamma_H(A)$ is given by (13), and $\Gamma_L(A)$ is given by:

$$\Gamma_L(A) = (1/2)\,\sigma^2 A^2 L''(A) + \left(r - \hat{\delta}\right) A L'(A) - rL(A).$$

The general solution of (D1) and (D2) is computed following the procedure described in Appendix B to solve the system of ODEs (11) and (12), and using exactly the same boundary conditions (B8) to (B21).

E rating-based market discipline mechanism

E.1 Equity

Assuming $A_{E,H} < A_S < A_{E,L}$, the value of equity for low and high rated banks, denoted by (10), is given by the system of ODEs:

$$\begin{array}{lll}\Gamma_H(A) + \delta A - d_H(i,\sigma) & = 0 & \text{for } A > A_R \\ \Gamma_H(A) + \delta A - d_H(i,\sigma) - \lambda_R(H(A) - L(A)) & = 0 & \text{for } A \in [A_{E,L}, A_R], \\ \Gamma_H(A) + \delta A - d_H(i,\sigma) - \lambda_R H(A) & = 0 & \text{for } A \in [A_S, A_{E,L}], \\ \Gamma_H(A) + \delta A - d_H(i,\sigma) - (\lambda + \lambda_R) H(A) & = 0 & \text{for } A < A_S,\end{array}$$

for H, and

$$\begin{array}{lll}\Gamma_L(A) + \delta A - d_L(i,\sigma) - \lambda_R(L(A) - H(A)) & = 0 & \text{for } A > A_R, \\ \Gamma_L(A) + \delta A - d_L(i,\sigma) & = 0 & \text{for } A \leq A_R,\end{array}$$

for L, where $\Gamma_H(A)$ and $\Gamma_L(A)$ are given by (13) and (14) respectively.

In contrast with the case in which $A_{E,H} < A_{E,L} < A_S$, when $A_S < A_{E,L}$ if a high rated bank with an asset A in the interval $[A_S, A_{E,L}]$ is audited by a rating agency, it is downgraded to low rated and closed straight away by its equityholders ($A \leq A_{E,L}$). Since $A_S < A_{E,L}$, a low rated bank will never be closed by the supervisor but by its equityholders.

E.2 Market value of deposits

Consider that H and L represent, respectively, the value of deposits for a high and low rated bank respectively, and that $A_{E,H}$ and $A_{E,L}$ represent the levels at which the equityholders decide to voluntarily close the bank (taken as given to compute the market value of deposits).

If the bank is closed with assets A, depositors receive $\max\{\phi A, i\}$. To write the ODEs for H and L one has to consider the relative position of the asset levels i/ϕ, $A_{E,H}$, $A_{E,L}$ and A_S, which is determined (i) in the equityholders' maximization problem, which yields $A_{E,H}$ and $A_{E,L}$, (ii) by the percentage of insured deposits i, and (iii) by the recovery rate ϕ. Assuming $A_{E,H} < A_{E,L}$, there are 12 possible different orderings of the previous quantities to be considered. If a bank is closed with an asset level A above (below) i/ϕ its depositors receive a final payment of ϕA (i).

For illustration purposes we include here the system of ODEs for the market value of deposits in two different cases: $A_{E,H} < A_{E,L} < i/\phi < A_S$. and $A_{E,H} < i/\phi < A_S < A_{E,L}$. In all cases, we solve for H and L in the same way than for the value of equity with a rating-based audit frequency, using the corresponding boundary conditions.

In case $A_{E,H} < A_{E,L} < i/\phi < A_S$ the ODEs for H and L are:

$$\begin{aligned}
&\Gamma_H(A) + d_H(i,\sigma) &&= 0 &&\text{for } A > A_R,\\
&\Gamma_H(A) + d_H(i,\sigma) + \lambda_R(L(A) - H(A)) &&= 0 &&\text{for } A \in [A_S, A_R],\\
&\Gamma_H(A) + d_H(i,\sigma) + (\lambda + \lambda_R)(\phi A - H(A)) &&= 0 &&\text{for } A \in (i/\phi, A_S),\\
&\Gamma_H(A) + d_H(i,\sigma) + (\lambda + \lambda_R)(i - H(A)) &&= 0 &&\text{for } A \leq i/\phi,
\end{aligned}$$

for H, and

$$\begin{aligned}
&\Gamma_L(A) + d_L(i,\sigma) + \lambda_R(H(A) - L(A)) &&= 0 &&\text{for } A > A_R,\\
&\Gamma_L(A) + d_L(i,\sigma) &&= 0 &&\text{for } A \in [A_S, A_R],\\
&\Gamma_L(A) + d_L(i,\sigma) + (\lambda + \lambda_R)(\phi A - L(A)) &&= 0 &&\text{for } A \in (i/\phi, A_S),\\
&\Gamma_L(A) + d_L(i,\sigma) + (\lambda + \lambda_R)(i - L(A)) &&= 0 &&\text{for } A \leq i/\phi,
\end{aligned}$$

for L. $\Gamma_H(A)$ and $\Gamma_L(A)$ are given by (13) and (14) respectively.

In case $A_{E,H} < i/\phi < A_S < A_{E,L}$ the ODEs for H and L are:

$$\begin{aligned}
&\Gamma_H(A) + d_H(i,\sigma) &&= 0 &&\text{for } A > A_R,\\
&\Gamma_H(A) + d_H(i,\sigma) + \lambda_R(L(A) - H(A)) &&= 0 &&\text{for } A \in [A_{E,L}, A_R],\\
&\Gamma_H(A) + d_H(i,\sigma) + \lambda_R(\phi A - H(A)) &&= 0 &&\text{for } A \in [A_S, A_{E,L}],\\
&\Gamma_H(A) + d_H(i,\sigma) + (\lambda + \lambda_R)(\phi A - H(A)) &&= 0 &&\text{for } A \in [i/\phi, A_S],\\
&\Gamma_H(A) + d_H(i,\sigma) + (\lambda + \lambda_R)(i - H(A)) &&= 0 &&\text{for } A < i/\phi,
\end{aligned}$$

for H, and

$$\begin{aligned}
&\Gamma_L(A) + d_L(i,\sigma) + \lambda_R(H(A) - L(A)) &&= 0 &&\text{for } A > A_R,\\
&\Gamma_L(A) + d_L(i,\sigma) &&= 0 &&\text{for } A \leq A_R,
\end{aligned}$$

for L.

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