To sell or to borrow?

A Theory of Bank Liquidity Management*

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Abstract

This paper studies banks' decision whether to borrow from the interbank market or to sell assets in order to cover liquidity shortage in presence of credit risk. The following trade-off arises. On the one hand, tradable assets decrease the cost of liquidity management. On the other hand, uncertainty about credit risk of tradable assets might spread from the secondary market to the interbank market, lead to liquidity shortages and socially inefficient bank failures. The paper shows that liquidity injections and liquidity requirements are effective in eliminating liquidity shortages and the asset purchases are not. The paper explains how collapse of markets for securitized assets contributed to the distress of the interbank markets in August 2007. The paper argues also why the interbank markets during the 2007-2009 crisis did not freeze despite uncertainty about banks' quality.

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One of the most significant features of modern banks is their ability to turn individual illiquid loans into tradable securities and use them as source of liquidity (directly or through repo transactions and asset-backed commercial paper that rely on such securities). At the same time these banks still use cash reserves and interbank markets as sources of liquidity. On the one hand, reliance on tradable securities reduces banks' cost of liquidity management. On the other hand, as the 2007-09 financial crisis showed reliance on such securities exposed the whole financial system to shocks to credit risk embedded in these securities. Banks that relied on such securities became the center of events in August 2007, when markets for securitized assets showed signs of stress, which immediately spread to interbank markets hampering banks' ability to manage their liquidity. This paper provides a theory of bank liquidity management that captures the above mentioned trade-off and derives novel policy implications. To our best knowledge, this paper is the first one to model transmission of shocks between two markets crucial for modern banking systems: the market for bank assets, critical for smooth functioning of the originate-to-distribute model of banking, and the interbank market, critical for banks' liquidity management and monetary policy transmission.

We analyze the banks' choice between cash reserves, interbank borrowing and asset sales in case of exposure to liquidity and credit risk. After each bank allocates its endowment between a risky asset and cash reserves, it receives a more precise but private signal about the risk of its asset and learns its liquidity need. Next, each illiquid bank decides how to cope with its liquidity need: use its cash reserves, borrow on the interbank market or sell (at least some of) its asset at a secondary market to outside investors. Liquidity is then reallocated between illiquid banks, liquid banks and outside investors using the interbank and secondary markets.

This very simple and generic setup á la Diamond-Dybvig (1983) generates powerful results. First, asymmetric information about risk of banks' assets implies that the banks' preference for liquidity sources depends on the risk of their asset. As in Myers and Majluf (1984) banks with safer assets (safer banks) prefer to borrow rather than sell their assets (shock to the credit risk reduces expected return on banks' asset).[1] Although borrowing on the interbank market also entails an adverse selection cost, the safer banks prefer to borrow, because they can retain their

[1] First, we assume that risk of assets maps directly to risk of banks. Second, credit risk materializes as an increase in expected probability of default of the asset.

valuable assets instead of selling them at an adverse selection discount. The adverse selection on the secondary market is worsened by the fact that also riskier but liquid banks are willing to sell, further diminishing the expected quality of assets sold. The safer banks' preference to borrow is crucial for understanding the vulnerability of relying on secondary and interbank markets when managing liquidity. This vulnerability materializes in a transmission of adverse selection from the secondary to the interbank market.

Next, as long as the share of the riskier banks is sufficiently low, liquidity transfer between the illiquid and liquid banks as well as investors purchasing assets is uninterrupted. For given cash reserves in the banking system, an equilibrium has two notable features. First, the average riskiness of borrowing banks is lower than the one of the selling banks. For a given amount of cash reserves held by banks, in general there is not enough interbank loans for all banks to borrow. In such a case, the safer illiquid banks borrow and the riskier illiquid banks go to the secondary market, because they are more willing to sell than the safer banks.[2] Second, as cash reserves in the banking system increase and the interbank loans become more abundant, the equilibrium interbank loan rate might increase and the equilibrium price of the asset decreases. The reason is that higher cash reserves increase supply of interbank loans inviting the safer of previously selling banks to the interbank market. Because of increasing riskiness of borrowing and selling banks, the equilibrium price of the asset falls and the equilibrium interbank loan rate might increase despite of the fact that the interbank loans become more abundant.[3]

Once share of the riskier banks is sufficiently high, the liquidity transfer breaks down and banks fail due to illiquidity. Due to decrease in the expected value of the sold asset, its price becomes so low that the illiquid banks cannot cover their liquidity shortfall just by selling the asset. Hence, all illiquid banks have to borrow on the interbank market. However, this means there is not enough liquidity for all banks to borrow.[4] Because the interbank loan rate is capped by the illiquid banks'

[2] In a model with continuous types, the additional effect is that the riskiest banks also hope to get subsidized on the secondary market, given that there will be a positive mass of banks with different risk selling.

[3] In the base-line version we assume away a possibility of fire-sales on the secondary market.

[4] The equilibrium is similar to credit rationing because not all of the banks can get loans. The precise structure of an equilibrium depends on the assumption how loans are allocated (see Keeton (1979)). In the baseline model we assume that banks that borrow get the loan amount equal to its liquidity shortfall and the rest of banks sell their assets. We discuss also the case in which the amount of loan is rationed, implying that all banks have to sell at least some of its asset to obtain the rest of needed cash.

capacity to borrow, the supply of loans is backward bending. Because the illiquid banks cannot obtain enough liquidity from the secondary market, their demand for interbank loans is inelastic in the interbank loan rate. In turn, the interbank market clears for the maximum loan rate for which the illiquid banks are indifferent between borrowing and defaulting. Hence, only some of the banks obtains loans, and the banks that are unable to obtain loans default. It has to be noted that liquidity shortage on the interbank market occurs despite of the assumption that supply of liquidity on the secondary market is unlimited. The driver of liquidity shortage on the interbank market is solely adverse selection on the secondary market due to safer banks' preference to borrow. Indeed, our results hold for parameters such that the illiquid banks would become liquid by selling if *all* safe and risky illiquid banks sold (despite of the fact that the riskier but liquid banks would be selling at the same time).

At the time when the banks decide how much cash to hold, they trade off the cost of holding cash, which is a lower investment in the long-term asset, with the benefit of holding cash, which is twofold. A liquid bank can earn a positive net return on interbank lending (a speculative motive). An illiquid bank can lower its borrowing need (a precautionary motive). We obtain two results. First, the banks choose higher cash reserves as the expected return on the long-term asset decreases. Second, because the banks do not internalize their failures, they might choose such cash reserves for which liquidity shortage on the interbank market and bank failures arise.

Our model provides several policy implications. First, our paper stresses the need for a central bank to understand the reason for an intervention (see also Bolton, Santos and Scheinkman (2011)). In cases when banks' choices of cash reserves are such that there is no liquidity shortage, policy interventions are unnecessary, because liquidity transfer is not interrupted. This concerns, specifically, liquidity injections triggered in reality by increases in the interbank loan rates. As argued above cash hoarding by banks might result in an increasing interbank loan rate (and falling asset prices) due to increase in borrowing by riskier banks that has nothing to do with scarcity of liquidity. In such a case liquidity injections prompted by an increase in the interbank loan rate might lead to its further increase, because more interbank loans are available for riskier banks. Of course, liquidity injections that are massive enough will eventually bring down the loan rate, however, without any change in welfare. In addition, liquidity injections on the interbank market lead to falling asset prices, because increase in supply of interbank loans attracts away the safer of the riskier banks from the secondary market. Falling prices might further increase a perception that there are liquidity problems, whereas in fact there are none.

Once the banks choose such cash reserves for which liquidity shortage occur, socially inefficient bank defaults due to illiquidity occur. We study an impact of three common policy tools. First, liquidity injections on the interbank market eliminate liquidity shortage and bank failures, because they occur due to scarce liquidity. Usually liquidity injections have been advocated as a measure to prevent illiquid financial intermediaries from selling assets that might lead to disruptive fire sales (see Sarkar (2009) for a discussion). In our paper, liquidity injections are necessary, because adverse selection on the secondary market materializes as illiquidity on the interbank market. Second, asset purchases cannot eliminate liquidity shortage, because the collapse of liquidity transfer on the secondary market is due to adverse selection (this result holds even when there are fire sales of bank assets). Third, liquidity requirements eliminate liquidity shortage, because the banks could be required to hold such cash reserves that explicitly prevent entering liquidity shortage region (see Perotti and Suarez (2012) for a model discussing other aspects of liquidity regulation).

Our model is well suited for thinking about the effect of uncertainty surrounding quality of banks assets on their liquidity management. First, our paper aims at explaining how stress in the secondary markets for bank assets transmits to the interbank markets. This helps us to understand the conditions for effectiveness of different policy tools in addressing liquidity shortages. Second, we view our paper as a model of acute liquidity shocks, during which access to liquidity sources other than cash, interbank lending, asset sales and central bank is impossible. In that sense, our model serves well to depict the 2007-09 financial crisis, which was triggered by an increase of uncertainty about riskiness of the mortgage-backed securities owned by banks.

Existing empirical evidence provides some support for our interpretation of events in August 2007. First, as Acharya, Afonso and Kovner (2013) show the U.S. banks exposed to the shock at the ABCP market increase their interbank borrowing in the first two weeks of August 2007. This is consistent with our argument about how an acute shock from a secondary market may transmit to the interbank market. Second, Kuo, Skeie, Youle, and Vickrey (2013) point out that contrary to the conventional wisdom the term interbank markets for which the counterparty risk is an important determinant of borrowing costs did not freeze during the 2007-2009 crisis (see also Afonso, Kovner and Schoar (2011) for similar evidence on the Fed Funds market). Specifically, the volume actually increased right after the start of the crisis in August 2007 despite of the jump in the spread between the 1- and 3-month LIBOR and OIS (Figures 1 and 2). This supports our argument that the interbank market served as an important source of liquidity during an acute shock originating from the secondary market.[5] The jump in the volume might have been also facilitated by the Federal Reserve's liquidity injections, which occurred immediately after the start of the crisis.[6]

Key theoretical novelty of our paper is that we model explicitly the interbank and secondary markets. We show how coexistence of secondary and interbank markets under asymmetric information about credit risk might impair liquidity distribution. To be more precise, we provide conditions under which adverse selection on the secondary market due to existence of interbank market might materialize as illiquidity on the interbank market. Because safer banks choose interbank market, they leave secondary market vulnerable to even small changes in the asset quality. Once asset quality worsens so that the banks cannot become liquid by selling, all illiquid banks want to borrow leading to liquidity shortage on the interbank market and bank failures. If the worsening of asset quality is not sufficiently strong, the secondary and interbank market might exhibit signs of stress (elevated loan rates and depressed prices), but liqiudity transfer might still be uninterrupted. The reason is that purchasing liquidity on the interbank market entails a lower adverse selection cost than doing so on the secondary market.

Literature Review. The paper is related to an extensive literature on bank liquidity and shares many common features with other papers. Our contribution is to model the efficiency trade-off that arises from banks reliance on the secondary and interbank markets at the same time:

[5] Acharya, Afonso and Kovner (2013) discuss also banks' strategies to deal with the ABCP shock in a longer horizon.

[6] Brunetti, di Filippo and Harris (2011) show interestingly that liqiudity injection by the European Central Bank were ineffective in the fall 2007.

tradable securities, on the one hand, improve efficiency by lowering reliance on cash and interbank market, but, on the other hand, they might lead to vulnerabilities as the recent crisis have shown. Because the coexistence of the secondary and interbank markets is crucial for individual banks and the financial system as a whole, our paper bridges a gap between two strands of literature on bank liqiudity that are concerned with only one of these two markets at a time.

The first strand of literature concerns liquidity provision through interbank markets. The paper closest to ours is the work of Freixas, Martin and Skeie (2011) who provide a model of an interbank market which is affected by uncertainty about distribution of liquidity shocks. Their model captures variation in banks' liqiudity needs during the 2007-2009 crisis. They provide a theoretical justification for the Federal Reserve's interest rate management and an explanation how it contributed to the stability of Fed Funds market. Our work complements their work by emphasizing an effect of uncertainty about quality of banks' tradable assets on the liquidity distribution through the interbank markets. We provide conditions justifying liqiudity injections in case of shocks to fundamentals and also provide a reason for why the interbank markets did not freeze.

Freixas and Holthausen (2005), in the context of cross-border interbank markets, and Heider, Hoerova and Holthausen (2009), in the context of recent crisis, also examine role of asymmetric information about banks' quality on the interbank market. Both papers show how an interbank market can freeze, when the highest quality banks stop borrowing due to severe adverse selection. The interbank market does not freeze in our model, because we model an acute stress event, in which the highest quality banks experience the lowest adverse selection cost of purchasing liquidity at the interbank markets.

Allen and Gale (2000) and Freixas, Parigi and Rochet (2000) study contagion through interbank markets. Rather than vulnerability of interbank networks, our paper can be viewed as a model of contagion through different markets to which the banks are exposed.

Our paper is also related to papers focused on liquidity hoarding on the interbank markets. Acharya and Skeie (2011) show how moral hazard due to too high leverage impairs banks' ability to roll over debt, increases liquidity hoarding and decreases interbank lending. Ashcraft, McAndrews and Skeie (2011) provide another model of precautionary liquidity hoarding against liquidity shocks. Gale and Yorulmazer (forthcoming) study liquidity hoarding in a model combining speculative and precautionary motives.

The second strand of literature studies vulnerability of liquidity provision through secondary markets for banks' assets. The two closest papers to ours are Malherbe (forthcoming) and Bolton, Santos and Scheinkman (2011), which both analyze the effect of asymmetric information about quality of banks' assets on liquidity distribution in the banking system. Malherbe (forthcoming) models self-fulfilling collapse of liquidity provision in an Akerlof (1970) spirit. Bolton, Santos and Scheinkman (2011) are interested in the timing of asset sales on secondary markets. The common feature of these two papers with ours is the source of adverse selection on the secondary markets: uncertainty whether the selling banks trade due to illiquidity or quality reasons. Using the same source of adverse selection, Brunnermeier and Pedersen (2009) show how secondary markets dry up, when funding necessary to keep the secondary market liquid depends on the liqiudity of the very same market.

Other related papers study the impact of fire sales on intermediaries' liqiudity. Fecht (2004) using the Diamond (1997)-extension of Diamond and Dybvig (1983) shows how fire sales impact banks linked by a common secondary market. Martin, Skeie and von Thadden (2012) show how reliance on securitization might increase probability of bank runs when the investors from whom banks borrow value banks collateral very low.

The remainder of the paper is organized as follows. Section 1 describes the setup. In Section 2 and 3, we derive optimal bank behavior and equilibria for given cash reserves. Section 4 and 5 describe ex ante equilibria and welfare. Section 6 discusses policy implications. The Appendix contains proofs of the results.

1 Setup

There are three dates, t=0,1,2, and one period. At t=0 each bank decides how to split one unit of its endowment between an asset and cash reserves (called also cash or reserves) in order to maximize its return at t=2. The endowment belongs to the bank, i.e., we abstract from any debt except the interbank debt incurred at t=1 (this simplifies algebra considerably without affecting the results). At t=1 each bank receives two signals about the return structure of its asset and its liquidity need. After signals are revealed, the interbank market for banks' loans and secondary market for banks' asset open. At t=2 the asset's returns are realized and payments are made.

The banks. At t=0 there is a continuum of mass one of identical banks. Each bank can invest fraction $\lambda \in [0; 1]$ of endowment in cash that returns 0 in net terms and $1 - \lambda$ in the asset.[7]

At t=1 each bank receives a private signal about the return structure of its asset. With probability q the bank learns that the asset is good and returns $R > 1$ at t=2 with certainty (we call such a bank a "good bank"). With probability $1 - q$ the bank learns that the asset is bad ("bad bank") and has the following return structure: it pays R at t=2 with probability $p > 0$ and 0 otherwise, where $pR < 1$. It holds that $[q + (1 - q)p]\, R > 1$.

At t=1 each bank receives also a private signal about its liquidity need. We model the liquidity need (shock) as a need to pay an amount $d < 1$ (e.g. Rochet (2004), Tirole (2011)). If the bank does not pay d it goes out of business. We call banks hit by the liquidity need illiquid, and the other banks liquid. With probability π the bank is liquid, and with $1 - \pi$ illiquid. Although unmodelled we envision d as a need to repay withdrawing depositors or to fund drawdowns on credit facilities. We could have used an approach similar to Freixas and Holthausen (2005) to model explicitly bank-specific liquidity shocks in a Diamond-Dybvig (1983) setup, but it would not add any additional insight and explicit modelling of deposits would make the algebra more complicated.

Finally, we assume that shocks to the asset's returns and liquidity are independent. That way we can focus on the impact of shock to the asset returns on banks' liquidity choices.

The interbank and secondary market. We impose several assumptions on how both markets function. Interbank market is modelled as in Freixas and Holthausen (2005) who also model asymmetric information about asset returns: interbank lending is unsecured, diversified, anonymous, and competitive with banks acting as price takers. Diversified interbank loans make default risk on interbank loans tractable and allow for anonymous lending, because each lender

[7] Alternatively to model a short-term liquidity management, we could assume that the bank starts out with a unit of an asset and has some spare liquidity and decides what fraction of this spare liqudity to preserve and to "consume" (see Gale and Yorulmazer (2011)).

will be exposed to average risk on the credit market.

In the secondary market the banks can sell their assets only to outside investors such as pension and hedge funds etc. (see Bolton, Santos and Scheinkman (2011)). These outside investors are competitive and are able to absorb any amount of assets that appear on the market as in Malherbe (forthcoming). This "deep pockets" assumption allows only for cash-in-market pricing on the interbank market. Hence, it implies that banks will not be willing to buy assets because the expected return on asset purchases will be the same as on cash storage and equal to 1, whereas the cash-in-the-market effect on interbank market will lead to a return on interbank loans that is generally higher than 1. An interesting extension in a model of short-term/acute liquidity shocks would be to introduce cash-in-the-market effect (fire sales) on the secondary market too. Although fire sales on the secondary market would allows us to model asset purchases by banks, it would not add any additional insights beyond our main results. However, we will discuss the impact of fire sales on certain results whenever it might matter.

Although our modelling of interbank and secondary markets is very crude, the only crucial assumption is that banks do not have access to additional sources of liquidity at t=1 beyond cash reserves, interbank borrowing and asset sales (we discuss central bank's liquidity injections in section on policy implications). Lack of additional sources of liquidity *coupled* with adverse selection will induce cash-in-the-market effect on the interbank that is responsible for our main results.[8] However, our assumption is not restrictive, because we think of our model as a discussion of an acute liquidity shock, which is impossible to accommodate with funding sources other than three assumed here. Other options such as raising retail deposits, long-term debt and equity take time, so we implicitly assume that the cost of raising them in a short period of time is infinite.

2 Banks at t=1

Because we solve the model backwards, we start at t=1. For exposition purpose we characterize optimal behavior of liquid and illiquid banks in this section and present the equilibria at t=1 in

[8] If there is no adverse selection, liquidity is not a problem because banks can always sell their project for its fair price.

the next.

At t=1 there are four types of banks: good and liquid (GL), bad and liquid (BL), good and illiquid (GI), and bad and illiquid (BI). Illiquid banks decide how to cope with the liquidity shock. They can use their cash reserves, borrow on the interbank market and/or sell at least part of the asset. Once they have cash in excess of liquidity shortfall they can lend it out too. Liquid banks decide how much to lend in the interbank market and how much of its asset to sell (borrowing from another bank is not profitable because the only investment opportunity is to lend it back to other banks). Each bank takes the interbank loan rate R_D, the expected fraction $\widehat{p}$ of banks repaying their interbank loans at t=2, and the asset price P as given when deciding what to do. All four types of banks face the same R_D and P due to asymmetric information about the risk of their asset and their liquidity needs. Moreover, we concentrate on the case in which the cash reserves from t=0 are not higher than the liquidity need, $\lambda \leq d$, because holding $\lambda > d$ would never be optimal when storage of cash yields 0 in net terms. We discuss separately the optimal choices of liquid and illiquid banks because of differences in behavior of these two types of banks.

2.1 Liquid banks

Because the GL and BL banks differ only in the probability of success of their asset, p_i, we can write down a decision problem for a BL bank. The results for a GL banks obtain after we substitute p with 1. A liquid bank chooses amount of interbank lending l and amount of its asset to sell S according to the following program:

$$\max_{l,S} p[(1-\lambda-S)R + (SP+\lambda-l) + \widehat{p}R_D l] + (1-p)[(SP+\lambda-l) + \widehat{p}R_D l]$$

$$\text{s.t. } S \in [0; 1-\lambda], \; l \in [0; SP+\lambda].$$

The first line is liquid bank's expected return at t=2. The first term is the return in case the asset succeeds with probability p. $(1-\lambda-S)R$ is the return on the asset after selling S units. $SP+\lambda-l$ is the excess cash left after the bank carries λ from $t=0$, receives SP from selling S of the asset at a price P and lends l at the interbank market. $\widehat{p}R_D l$ is the expected return on the interbank loans at t=2. The return on the interbank loans is deterministic because we assumed that each lending bank has a diversified interbank loan portfolio. The second term is the return in the case the asset does not pay, which comprises of the excess cash and return on the interbank loans. In the second line there are two constraints under which the bank maximizes its expected return. The first constraint represents the amount of the asset available for sale. The second constraint limits the amount the bank can lend on the interbank market. The highest amount of lending equals to the sum of cash carried from t=0, λ, and cash raised from selling S of the asset at price P at t=1.

We can prove the following Lemma.

Lemma 1: *The liquid bank's lending decision is:* $l = SP + \lambda$ *for* $\widehat{p}R_D > 1$, $l \in [0; SP + \lambda]$ *for* $\widehat{p}R_D = 1$, $l = 0$ *for* $\widehat{p}R_D < 1$. *The liquid bank's selling decision is:* $S = 1 - \lambda$ *for* $P \max[\widehat{p}R_D; 1] > p_i R$, $S \in [0; 1 - \lambda]$ *for* $P \max[\widehat{p}R_D; 1] = p_i R$, $S = 0$ *for* $P \max[\widehat{p}R_D; 1] < p_i R$, *where* $p_i = p$ *for the BL bank and* $p_i = 1$ *for the GL bank.*

Proof: in the appendix.

The main observation from Lemma 1 is that the GL bank is less willing to sell than BL bank. The reason is that the GL banks would have to sell their high-value asset for the same price as the BL banks their low-value asset (from the above result w56e can see that the good bank sells if $P \max[\widehat{p}R_D; 1] > R$ and the bad one if $P \max[\widehat{p}R_D; 1] > pR$).

While deriving the above result, without loss of generality we concentrated on the case where $P \in [pR; R)$ and assumed that the bad bank for $P = pR$ will sell all of its asset. Observe that P can never reach R in equilibrium because for any $P \geq pR$ the bad banks will always sell, and can never be less than pR under our assumption of deep pockets.

2.2 Illiquid banks

Again we do not have to distinguish explicitly between the good and bad bank and can study a general program of return maximization of an illiquid bank. A program for the illiquid bank differs from the one for a liquid bank for several reasons. First, the liquidity need d has to be taken into account in the expression for the bank's expected return and in the constraint on the amount of cash the illiquid bank needs to have. Second, the illiquid bank can borrow an amount b (we use here the convention that lending is described by $b < 0$). Third, the illiquid bank's options to raise cash to cop with the liquidity shock depend on the anticipated equilibrium price P. If P is such that $P(1-\lambda)+\lambda > d$, then the illiquid bank can raise enough cash to pay d just by using its cash reserves from t=0 and selling all of its assets. Hence, the illiquid bank can cope with the liquidity shock either by borrowing or selling. However, if $P(1-\lambda)+\lambda < d$, the bank has to borrow, because it cannot raise enough cash to pay d only by selling the asset.

Because P determines the illiquid bank's options to raise cash, it also affects its program for return maximization. When the bank anticipates that $P(1-\lambda)+\lambda > d$ its program reads:

$$\max_{b,S} \begin{cases} p[(1-\lambda-S)R+(SP+\lambda+b-d)-\widehat{p}R_D b]+(1-p_i)[(SP+\lambda+b-d)-\widehat{p}R_D b], \text{ if } b<0, \\ p[(1-\lambda-S)R+(SP+\lambda+b-d)-R_D b]+(1-p_i)\max\left[0;(SP+\lambda+b-d)-R_D b\right], \text{ if } b \geq 0. \end{cases}$$

$$\text{s.t. } S \in [0;1-\lambda], SP+\lambda+b \geq d.$$

If the illiquid bank lends ($b < 0$), its expected return (the first line of the above program) is analogous to the one for the liquid bank with the difference that the excess cash is diminished by d. If the illiquid bank borrows ($b > 0$ and we have included the case $b = 0$ here too) and raise enough cash to repay d ($SP+\lambda+b \geq d$), the expected return (the second line) differs from the case with $b < 0$, because (i) the return on interbank lending $\widehat{p}R_D$ has to be substituted with a cost of interbank borrowing R_D, and (ii) the illiquid bank can default on its interbank loan in case its asset pays 0 (as represented by the max-operator). The above expression for the expected return takes into account that the illiquid bank would never borrow and lend at the same time, because lending is never more profitable than borrowing ($\widehat{p}R_D \leq R_D$). The second constraint in the above program guarantees that the bank raises enough cash. Observe that because of $P(1-\lambda)+\lambda > d$ the bank will always generate a positive expected return. This does not need to be the case when $P(1-\lambda)+\lambda < d$. Hence, the illiquid bank's expected return reduces to the expression:

$$\max_{b,S} p\max[0;(1-\lambda-S)R+(SP+\lambda+b-d)-R_D b]+(1-p_i)\max\left[0;(SP+\lambda+b-d)-R_D b\right] \text{ for } b \geq 0,$$

because the bank has to borrow and may default if the anticipated cost of borrowing is too high (in case $P(1-\lambda)+\lambda > d$ the bank could always sell instead of borrow and generate positive return).

The following lemma reports optimal choices taken by an illiquid bank for $\widehat{p}R_D \geq 1$ (we do not report the results for $\widehat{p}R_D < 1$ because they are not relevant in equilibrium).

Lemma 2: *Denote $p_i = p$ for the BL bank and $p_i = 1$ for the GL bank. If $P(1-\lambda)+\lambda > d$, the result is as follows. An illiquid bank with $p_i > \widehat{p}$ pays d in the following way. For $R_D < \frac{R}{P}$ the bank uses all of its cash reserves λ, borrows $d-\lambda$ and does not sell. For $R_D \in \left(\frac{R}{P}; \frac{p_i}{\widehat{p}}\frac{R}{P}\right)$ the bank uses all of its cash reserves λ, sells only $\frac{d-\lambda}{P}$ and does not borrow. For $R_D > \frac{p_i}{\widehat{p}}\frac{R}{P}$ the bank sells all of its asset $1-\lambda$ and lends out the remaining cash after paying d. An illiquid bank with $p_i \leq \widehat{p}$ pays d in the following way. For $R_D < \frac{R}{P+\left(\frac{\widehat{p}}{p_i}-1\right)\left(P-\frac{d-\lambda}{1-\lambda}\right)}$ the bank uses all of its cash reserves λ, borrows $d-\lambda$ and does not sell. For $R_D > \frac{R}{P+\left(\frac{\widehat{p}}{p_i}-1\right)\left(P-\frac{d-\lambda}{1-\lambda}\right)}$ the sells all of its asset $1-\lambda$ and lends out the remaining cash after paying d. The bank with riskier asset ($p_i < \widehat{p}$) is more willing to sell than the bank with safer asset ($p_i > \widehat{p}$).*

If $P(1-\lambda)+\lambda \leq d$ the result is as follows. If R_D is such that after borrowing $d-\lambda$ an illiquid bank is solvent with probability p_i at t=2, then the bank uses all of its cash reserves λ, borrows $d-\lambda$ and does not sell. If R_D and P are such that the bank is always bankrupt at t=2, the bank is indifferent between borrowing and selling.

Proof: in the appendix.

The first result is that no illiquid bank carries positive cash reserves till t=2. Those banks that do not lend use all of its cash from t=0, because it is cheaper than borrowing and selling. Then they borrow and sell just enough to cover the remaining liquidity shortfall $d-\lambda$ The bank that lends sells all and lends out all remaining cash after paying d because the return on lending is generally higher than return on storing cash.

For sufficiently high price ($P(1-\lambda)+\lambda > d$) the illiquid bank can either borrow or sell to cope with its liquidity shock. The higher the loan rate, the more attractive is selling. Again the bad bank is more willing to sell (and sell all of its asset) due to adverse selection.[9]

For sufficiently low price ($P(1-\lambda)+\lambda < d$) and loan rate such that the bank can repay its loan

[9] In our setup in which the illiquid banks can lend there is an additional effect that makes selling more profitable for the bad bank. The reason is that the bad bank's loan portfolio would be less risky than the bank itself ($p < \widehat{p}$) making lending and selling more attractive for such a bank than keeping its asset. This effect is however not responsible for results in Lemma 2, which go through without lending too.

at t=2, the bank prefers to borrow only because it is the only way to achieve positive return. For any other R_D and P the illiquid bank will always be insolvent at t=2 and, therefore, is indifferent between borrowing or selling at t=1.

3 Equilibria at t=1

To determine equilibria at t=1 we use the concept of perfect Bayesian equilibrium. As seen in Lemma 2 the illiquid bank's choice of how to cope with its liquidity shock depends on the anticipated relative cost of acquiring liquidity on the interbank and secondary market.[10] However, because there is adverse selection on both markets this relative cost depends also on the beliefs of investors and all banks about the GI and BI banks' choices and market clearing conditions. In an equilibrium the illiquid banks' choices have to be optimal and consistent with other banks' and investors' beliefs, these beliefs have to be consistent with banks' choices as well as the markets clear (the last condition is relevant only for the interbank market given the perfectly elastic demand for bank assets from investors).

We use also the Cho-Kreps intuitive criterion to impose some discipline on the off-equilibrium beliefs and eliminate multiple equilibria (see proof of Proposition 1 for more details). The intuitive criterion will eliminate an equilibrium, in which at t=1 the illiquid banks do not borrow due to some pessimistic beliefs about quality of borrowing banks. Alternatively, we could eliminate such an equilibrium by introducing banks that use only interbank market to manage their liquidity (i.e. traditional banks whose loans may not be tradeable). That way we would pin down an interbank loan rate on the equilibrium path and eliminate possibility that banks with tradable assets do not borrow.

In order to simplify the exposition of the results, we split the discussion of the equilibria at t=1 into two sections. In Section 3.1 we discuss the case in which the equilibrium price P^* is such that $P^*(1-\lambda)+\lambda > d$ and there is no liquidity shortage on the interbank market. In Section 3.2 we discuss the opposite case, which also includes $P^*(1-\lambda)+\lambda = d$. In both sections we discuss

[10] In that sense our model is similar to the one proposed by Freixas and Holthausen (2005), where illiquid banks also have two options: borrow on the domestic or foreign interbank market.

only the equilibrium choices of the illiquid banks, because they are the object of our interest.

3.1 Equilibria without liquidity shortage

Proposition 1: *Suppose that parameters of the model are such that there are equilibria in which* $P^* (1 - \lambda) + \lambda > d$*. There exist* λ_1 *and* λ_2 *such that* $\lambda_1 \leq \lambda_2 < d$ *for which the following results obtain. If* $\lambda_1 > 0$*, for* $\lambda \in [0; \lambda_1)$ *the GI banks are indifferent between borrowing and selling, and the BI banks sell. For* $\lambda = \lambda_1 > 0$ *the GI banks borrow and the BI banks sell. If* $\lambda_2 > 0$*, for* $\lambda \in (\max[0; \lambda_1]; \lambda_2)$ *the GI banks borrow and the BI banks are indifferent between borrowing and selling. For* $\lambda \in [\max[0; \lambda_2]; d)$ *the GI and BI borrow.*

Proof: in the appendix.

Proposition 1 highlights that the riskiness of borrowing and selling banks increases with cash reserves from t=0. For very low cash reserves ($\lambda \in [0; \lambda_1)$), only some of the GI banks borrow and the rest of the illiquid banks sell. As the cash reserves increase ($\lambda \in [\lambda_1; \lambda_2)$), all GI banks prefer to borrow and are followed by some of the bad banks to the interbank market. Finally for the sufficiently high cash reserves ($\lambda \in [\lambda_2; d)$), the banks' riskiness in both markets is the highest, with all banks borrowing and only the bad illiquid banks willing to sell.

The result that the riskiness of borrowing and selling banks increases with cash reserves from t=0 is driven by interplay of the "cash-in-the-market" effect and adverse selection on the interbank market. On the one hand, adverse selection attracts the BI banks to the interbank market, because the GI banks are always more willing to borrow, and, therefore, the BI banks can get subsidized by the GI banks on the interbank market. On the other hand, supply of interbank loans increases with cash reserves that the banks carry from t=0. If the supply of interbank loans is scarce (λ is sufficiently low) some illiquid banks cannot obtain loans and are forced to sell (the "cash-in-the-market" effect). Because the banks that are more willing to sell are the BI banks, they are the first ones to drop out of the interbank market when supply of loans becomes scarce (as λ falls below λ_2). Eventually, for very low supply of interbank loans (λ falls below λ_1) the GI banks are also forced to sell. Vice versa, as interbank loans are more abundant (λ increases above λ_1), the GI banks are the first one to switch from the secondary to interbank market. Next, with increasing

λ more of the BI banks can join the GI banks and contaminate the interbank market. See also discussion of case with continuous types later in the paper.

To close up the discussion of equilibria at t=1 we provide monotonicity results for equilibrium price, loan rate and total lending volume as functions of cash reserves λ.

Lemma 3: Suppose that parameters of the model are such that there are equilibria in which $P^* (1 - \lambda) + \lambda > d$.

1. $P^* > pR$ is decreasing for $\lambda \in [0; \lambda_1)$ and equals pR for $\lambda \in [\lambda_1; d)$.

2. R_D^* is increasing for $\lambda \in [0; \lambda_1)$, can be increasing or decreasing for $\lambda \in (\lambda_1; \lambda_2)$, and is constant and reaches its minimum for $\lambda \in (\lambda_2; d)$. For $\lambda = \lambda_1$ and $\lambda = \lambda_2$ R_D^* is indeterminate.

3. Total lending volume is decreasing for $\lambda \in [0; \lambda_1]$, can be non-monotonic (first increasing and then decreasing) for $\lambda \in (\lambda_1; \lambda_2)$, and decreasing for $\lambda \in [\lambda_2; d)$.

Proof: Proofs are straightforward.

Equilibrium price P^* of the asset decreases with cash reserves, because the average riskiness of the assets sold on the secondary market increases with λ for two reasons. First, as supply of interbank loans increases with λ more banks can borrow. Because the first banks to borrow and leave the secondary market are the GI, the average riskiness of the assets sold on the secondary market increases. Second, as λ increases, the liquidity shortfall $d - \lambda$ decreases. Because the GI banks sell only $\frac{d-\lambda}{P}$ and the BI banks sell $1 - \lambda$ the share of the good asset sold decreases with λ more rapidly than the share of the bad asset leading to increase in the average riskiness of the asset (see also Malherbe (forthcoming) for a similar effect). The increase in the average riskiness occurs until all GI banks withdraw from the secondary market (for $\lambda \in [\lambda_1; d)$).

The changes in the equilibrium loan rate R_D^* reflect the interplay between adverse selection and "cash-in-the-market" effect. On the one hand, the equilibrium loan rate increases with cash reserves, because more of the BI banks can borrow and the loan rate reflects increasing risk of lending. On the other hand, the loan rate decreases with λ, because supply of interbank loans becomes more abundant. For $\lambda = \lambda_1$ *and* $\lambda = \lambda_2$ R_D^* is indeterminate, because the demand and supply of interbank loans are inelastic and the market clears for any loan rate that makes the liquid banks lend and the bad illiquid banks borrow (see also Freixas, Martin and Skeie (2011) for a similar result). This indeterminacy seizes to exist in a model with continuous types.

Total lending volume is generally decreasing in λ because each illiquid bank's loan demand $d-\lambda$ decreases. However, for $\lambda \in (\lambda_1;\lambda_2)$ total lending volume can be here non-monotonic. The reason is that increasing share of BI banks borrow increasing the overall loan demand.

3.2 Equilibrium with liquidity shortage

Now we turn to the case where selling all of the asset cannot generate enough cash to pay d, i.e., the equilibrium price is such that $P^*(1-\lambda)+\lambda<d$. This case obtains in our setup if[11]

$$pR(1-\lambda_1)+\lambda_1<d \Leftrightarrow d>\frac{qpR}{qpR+\pi(1-pR)}. \tag{1}$$

First, (1) means that the BI banks are insolvent if they have too little cash carried from t=1. Second, (1) implies that for some λ around λ_1 banks will not be able to cope with liquidity shocks by selling. More precisely, for some λ the equilibrium in which some GI and all BI banks sell and the rest of GI borrows as well as the equilibrium in which some of the BI banks sell and the rest of banks borrow do not exist. Formally, the region of λ for which this happens is derived by solving $P^*(1-\lambda)+\lambda=d$, where P^* is the equilibrium price obtained while proving Proposition 1.[12] Graphic interpretation is depicted in Fig. 3. Intuitively, the liquidity shortage occurs when the liquidity need d is higher or the probability of the asset being good q is lower, all else equal.

The main implication of (1) is the equilibrium liquidity shortage on the interbank market

[11] In a setup with continuous types $pR<d$ is sufficint for this case to exist.

[12] Situation that for some P $d>\lambda+(1-\lambda)P$ would occur for λ in the interval for which a lower bound comes from $d=\lambda+(1-\lambda)P$ and the upper bound comes from lower of λ_0 and λ such that $d=\lambda+(1-\lambda)pR$. The lower bound is

$$\lambda_{LB}=\frac{(\overline{p}R+d(-2+\overline{p}R-\pi(R-1)))-\sqrt{4d(\overline{p}R-1)(d(1-\pi)+R\pi-\overline{p}R)+(\overline{p}R+d(-2+\overline{p}R-\pi(R-1)))^2}}{2(\overline{p}R-1)}$$

and the upper bound is

$$\lambda_{UB}=\min\left[\lambda_0;\frac{d-pR}{1-pR}\right].$$

Of course, d might be so high that rationing will occur for all λ. This happens when λ_{LB} turns negative, which happens for

$$d>\frac{R(\overline{p}-\pi)}{1-\pi}.$$

whenever the equilibrium price P^* is such that $P^*(1-\lambda)+\lambda \leq d$ and cash reserves are such that all illiquid banks cannot obtain loans ($\lambda < \lambda_2$). To see this we need to show that there is no equilibrium loan rate for which the interbank market clears. First, from Lemma 2 we know that for $P^*(1-\lambda)+\lambda < d$ each bank borrows the full amount, $d-\lambda$, for *any* loan rate R_D for which it can repay the interbank loan at t=2 and is indifferent between borrowing and selling for any R_D for which it cannot repay the loan at t=2. This implies that demand for interbank loans never decreases with loan rate in contrast to the case when an illiquid banks can switch to selling if the loan rate is too high. Second, the liquid banks supply loans only if (i) the loan rate allows to break even given the anticipated probability of loan repayment, and (ii) if the loan rate is sufficiently low so that the illiquid banks can repay their loans at t=2. Hence, the supply of loans is not monotonic in the loan rate. Never-decreasing demand and non-monotonic supply together with the fact that there is not enough liquidity on the interbank market for all illiquid banks ($\lambda < \lambda_2$) imply that there is no loan rate for which, supply of loans is not sufficient for all banks. Hence, the equilibrium loan rate is the highest loan rate for which the illiquid banks are indifferent between borrowing and defaulting. The interbank market clears trivially because for such an equilibrium loan rate the illiquid banks are indifferent for loans (see Fig. 4). We call such an outcome: equilibrium with liquidity shortage.[13]

The precise structure of an equilibrium with liquidity shortage depends on the assumption about how the interbank loans are allocated (see Keeton (1979)). For the time being we assume banks that borrow receive the full loan amount $d-\lambda$ and that the types of borrowing banks are chosen randomly. The banks that are left without loans will sell all of its asset, given that they cannot borrow and they have nothing to lose. Later we discuss the case in which all illiquid banks borrow, but the amount of loan is rationed, implying that the banks have to sell at least some of its asset to obtain the rest of needed cash.

Proposition 2: *Suppose that parameters of the model are such that there are equilibria in which* $P^*(1-\lambda)+\lambda \leq d$. *There exist* λ_{UB}, λ_{FR} *and* λ_{LB} *such that* $\lambda_{UB} \leq \lambda_{FR} \leq \lambda_{LB}$ *for which the following equilibria obtain. If* $\lambda_{UB} > 0$, *for* $\lambda \in (\max[0; \lambda_{FR}]; \lambda_{UB}]$ *it holds* $P^*(1-\lambda)+\lambda < d$.

[13] Such an equilibrium is very similar to crei rationing, where some of the banks are left without loans. However, here the interbank clears trivially for a loan rate for which the banks are indifferent.

In such a case a share of illiquid banks obtains loans and the rest sells all of its assets and fails. If $\lambda_{FR} > 0$*, for* $\lambda \in [\max[0; \lambda_{LB}]; \lambda_{FR})$ *it holds* $P^* (1 - \lambda) + \lambda = d$*. In such a case there are no failures and all illiquid banks are indifferent between borrowing and selling.*

In general, liquidity shortage occurs for intermediate cash reserves. The reason is the decreasing share of good asset on the secondary market as cash reserves increase. For low λ this share is high keeping the price sufficiently high to preserve $P^* (1 - \lambda) + \lambda > d$ in equilibrium. For sufficiently high λ the average riskiness of sold asset falls so low that in equilibrium the banks cannot sell to cope with their liquidity shock and liquidity shortage occurs. Once, the amount of λ increases to such a point that $d - \lambda$ is low, the liquidity shortage stops, because either even for the lowest possible price the banks can cover $d - \lambda$ or there is enough interbank loans for all illiquid banks to borrow.

Given that we have only two types of illiquid banks there is a region in which $P^* (1 - \lambda) + \lambda = d$. In such a case the illiquid banks are indifferent between borrowing or selling and each bank can pay d. The reason is that for $\lambda \in [\max[0; \lambda_{LB}]; \lambda_{FR})$ the price that would occur under liquidity shortage with failures is so high that the illiquid banks could pay d by selling all. But then more of the borrowing BI banks would prefer to sell again, leading to a price declines again. Hence, this gives a rise to an equilibrium in which the BI banks have to be indifferent between selling and borrowing for $P^* (1 - \lambda) + \lambda = d$.

Surprising feature of our model is that bank failures occur due to existence of the interbank market (despite of unlimited supply of liquidity on the secondary market). Indeed, without the interbank market all GI banks would have to sell and the price of the asset would be always so high that the illiquid banks could pay d for any $\lambda \in [0; d]$. With the interbank market at least some of the GI banks can borrow, which increases the average riskiness of sold asset. As λ increases the riskiness increase and the price of the asset fall to such a point that none of the illiquid banks cannot pay d by selling. Hence, all illiquid banks want to borrow leading to liquidity shortage on the interbank market and bank failures. The effect of the interbank market can be also seen in (1), which implies that an increase in the share of liquid banks π lowers the threshold for which the liquidity shortage occurs. The reason is that higher π reduces the share of the GI banks that need to sell depressing the price of the asset.

3.3 Social welfare at t=1

Social welfare is the sum of profits of all four types of banks at t=1 and the sum of payments due to the liquidity shock d, $d(1-\pi)$. Although details are left unmodelled we assume that the liquidity shock d arises either due to withdrawing depositors or borrowers drawing on their lines of credit. That way the liquidity shock is welfare neutral in case all illiquid banks can pay d, i.e., we avoid a pitfall where it would be socially beneficial to close the illiquid banks and sell their assets without making the payment d.

Lemma 4: *Social welfare at t=1 in equilibria in which $P^*(1-\lambda)+\lambda \geq d$ is equal to the expected value of the asset and cash kept by all banks, $(1-\lambda)(q+(1-q)p)R+\lambda$.Social welfare at t=1 in equilibria in which $P^*(1-\lambda)+\lambda < d$ is lower than $(1-\lambda)(q+(1-q)p)R+\lambda$.*

In equilibria with $P^*(1-\lambda)+\lambda \geq d$ the highest possible social welfare at t=1 is achieved, because the full value of banking is reached. First, all illiquid banks obtain liquidity to pay d and none of them fails. Second, transfer of liquidity on the interbank market is welfare-neutral, because it just redistributes cash reserves and rents associated with this redistribution. Third, assets sales are also welfare-neutral, because the investors can generate the full value from the asset and pay the expected return on the asset.

In equilibria with $P^*(1-\lambda)+\lambda < d$ the highest possible social welfare at t=1 for a given λ cannot be achieved for two reasons. First, because some illiquid banks fail despite of being ready to borrow, the banks lending on the interbank market extract rents only from the illiquid bank that do not fail. In turn, the lending banks achieve lower returns for a given λ than they would had no illiquid banks failed. Second, because some of the illiquid banks fail, payments of d from these banks is lost.

4 Equilibria at t=0

At t=0 the banks maximize their expected t=2-return by choosing λ consistent with an equilibrium that they anticipate will arise at t=1 (see e.g. Freixas and Holthausen (2005)). The following result summarizes the optimal choice of λ at t=0.

Proposition 3: *When the parameters are such that there is no liquidity shortage at t=1, there exist such $\underline{R}$, $\overline{R}$ and $\widetilde{R}$ that at t=0 the banks choose $\lambda = \lambda_2$ for $R \in \left(\frac{1-(1-q)pR}{q}; \underline{R}\right]$, some $\lambda \in (\lambda_1; \lambda_2)$ for $R \in \left(\underline{R}; \overline{R}\right)$, $\lambda = \lambda_1$ for $R \geq \overline{R}$, and $\lambda = 0$ for $R \geq \widetilde{R}$. When the parameters are such that there might be liquidity shortage at t=1 for some λ, for some intermediate R and high d the banks choose at t=0 such λ that liquidity shortage at t=1 occurs.*

Proof: In the appendix.

Proposition 3 shows that the bank's choice of liquidity depends on the profitability of the asset (see Fig. 5). The less profitable the asset is the more cash the bank is willing to carry from t=0. At t=0 the bank faces the following trade-off. On the one hand, higher cash reserves reduce the expected return at t=2 because the bank invests less in the long-term asset. On the other hand, the bank anticipates that cash reserves might be valuable at t=1 for two reasons. If the bank is liquid, cash might be used to earn positive net return on interbank lending. If the bank is illiquid, cash reduces the need to acquire costly cash through either interbank market or selling (if the bank becomes GI-type). Hence, as the long-term asset becomes less profitable, the bank invests more in cash at t=0.

In consequence, the banks at t=0 would also choose such λ that later at t=1 they might fail due to liquidity shortage. The reason is that for some intermediate R (and d is such that liquidity shortage might occur) the profitability of the long-term asset is such that it is perfectly rational for the banks to choose cash reserves for which they might fail at t=1.

5 Welfare at t=0

The analysis up till now reveals that banks' choice of cash reserves at t=0 is not socially efficient in general. This happens for two reasons. First, the socially efficient choice of cash is 0 when it does not lead to liquidity shortage. Such a choice maximizes social welfare at t=0, because the banks invest all of its endowment in the long-term asset that is more socially valuable than cash. However, the banks choose positive cash reserves, because cash has private value for them. This value comes from using the cash to earn a return on interbank lending or to lower the cost of interbank borrowing. Second, the banks might choose cash reserves such that liquidity shortage occurs. The reason is that they do not internalize the social cost of their failure.

The coexistence of secondary and interbank markets under asymmetric information is the main driver behind the two above mentioned reasons for the inefficient choice of cash reserves by banks in our model. First, positive choice of cash by banks at t=0 occurs, because the existence of the interbank market, which redistributes fixed amount of cash reserves, gives rise to "cash-in-the-market" effect under adverse selection. Adverse selection pushes the GI banks from the secondary market to the interbank market making it more valuable for the BI banks to borrow. In turn, liquidity becomes scarce and makes cash valuable to banks. Had the interbank market not existed or had there been no adverse selection, cash would have no private value for banks.[14]

Second, existence of the interbank market impairs functioning of the secondary market as a source of liquidity leading to liquidity shortage. Because the GI banks prefer to borrow and they do not internalize the effect of their choice on the price and the loan rate, the price of the asset due to adverse selection might fall so low that the secondary market might not provide enough liquidity to cope with severe liquidity shocks. This impairment of the secondary market causes all illiquid banks to enter the interbank market leading to liquidity shortage and bank failures. Hence, despite of the "deep-pockets" assumption on the secondary market the banks might still end up being illiquid and insolvent.

The straightforward policy implication would be to ban from the interbank market the banks

[14] See Malherbe (forthcoming) that even without the interbank market there might be inefficient cash hoarding at t=0 under adverse selection on the seconadry market. He shows that cash hoarding at t=0 is endogenous to the anticipated motive for selling at t=1. If the banks are anticipated to sell because of lack of liqiudity, the asset price will be high and cash hoarding is not valuable (hence choice of $\lambda = 0$ is efficient). If the banks are anticipated to sell due to asset quality issues, the asset price will be low making inefficient cash hoarding privately valuable. The reason why cash hoarding is endogenous is that the banks do not internalize the effect of their choice on the price of the asset. We have the similar effect here as well. However, this effect occurs only when the banks find it optimal to choose λ such that only some of the GI banks can borrow. Hence, it arises due to the binary nature of the model and seizes to exist with continuous types.

that are able to manage their liquidity using tradable assets.[15] However, such a prescription is not realistic and also too far reaching given that our model does not include inefficiencies stemming from the secondary market such as fire sales. Modelling fire sales would introduce a trade-off between the efficiency loss coming from the interbank and secondary market. However, it would also make liquidity shortage phenomenon more likely given that fire sales would further lower the banks' ability to raise cash on the secondary market. For the sake of exposition of the most important results we do not model the effect of fire sales and just discuss it in more length later.

In what follows we assume that instead of a welfare-maximizing social planner there exists an authority such as central bank that cares only about bank failures due to banks' decisions. The reason is that our policy implications would go through in a more elaborate model in which interbank market would be socially beneficial.

6 Policy Implications

In this section we discuss several policy implications, i.e., tools that are at the central bank's disposal to address the liquidity shortage that leads to bank failures and inefficiency loss when compared with a case in which the banks would not fail. Hence, we are only interested in situations in which private banks' choices lead to liquidity shortage equilibrium in which some banks fail. Observe that we do not attempt at this point to compare different tools using social welfare as criterion because with $\lambda = 0$ being the only socially optimal choice it is not interesting. For the time being we only assess whether and how the studied policy tools can eliminate the liquidity shortage equilibrium.

6.1 Central bank's liquidity management on the interbank market

Liquidity shortage on the interbank market occurs because there is not enough liquidity to accommodate all illiquid banks that want to borrow given the equilibrium price of the asset is so low that selling all of the asset is not be enough to cover the liquidity shortfall. Hence, liquidity short-

[15] Observe that imposing $\lambda = 0$ can be an alternative to a ban from the interbank market *only* if d is so small that there is no credit rationing for $\lambda = 0$.

age can be tackled at t=1 by addressing one of two issues: insufficient liquidity on the interbank market or insufficient price on the secondary market.

Both above mentioned issues could be addressed by the central bank's liquidity management. The first issue, insufficient amount of liquidity on the interbank market, could be addressed by simply injecting enough liquidity on the interbank market for all illiquid banks to borrow. In the proof of Proposition 2 we saw that the amount of failing banks ν is simply a difference between all illiquid banks demanding loans, $1-\pi$, and the amount of loans that could be made (given by the ratio of total supply of loans made by all liquid banks, $\pi\left[\lambda+(1-q)(1-\lambda)P_R\right]$, where the BL banks sell its asset at equilibrium price P_R to lend, and the liquidity shortfall that each illiquid bank has, $d-\lambda$):

$$\nu = 1-\pi-\frac{\pi\left[\lambda+(1-q)(1-\lambda)P_R\right]}{d-\lambda}>0.$$

Hence, the central bank could reduce ν to zero by lending a sufficient amount of cash reserves c on the interbank market at a new interbank loan rate (see Freixas, Martin and Skeie (2011) for a similar mechanism of liquidity injections to stabilize interest rate on the interbank market). Formally, c would increase the total supply of loans in the above expression for ν. Moreover, in a new equilibrium with all illiquid banks borrowing the new price of the asset would be pR, because only the BL banks would sell. Hence, the sufficient amount of cash injected into the interbank market by the central bank would be given by an equation that equalizes the amount of failing banks to zero under the new price:

$$1-\pi-\frac{\pi\left[\lambda+(1-q)(1-\lambda)pR\right]+c}{d-\lambda}=0$$

or

$$c=(1-(1-q)\pi pR)(\lambda_2-\lambda)>0.$$

The sign obtains because liquidity shortage equilibrium arises for $\lambda<\min\left[\lambda_2;\lambda_{UB}\right]$ and $1>(1-q)\pi pR$ given our assumptions on parameters. It can be easily shown that such an intervention raises social welfare for a given λ to the highest possible $(1-\lambda)(q+(1-q)p)R+\lambda$.

Alternatively, in our model the central bank could address the insufficient price on the secondary market by draining the liquidity from the interbank market (we discuss asset purchases in the next section). To see this, one has to realize that the low price on the secondary market is due to the GI banks preferring to borrow. As Proposition 2 shows an equilibrium in which the banks can survive by selling arises for λ lower than the ones for which liquidity shortage arises. Hence, the central bank should drain a sufficient amount of liquidity by borrowing on the interbank market. That way it would force more of the GI banks to go to the secondary market. In turn, the average riskiness of sold assets would decreases pushing the price upwards to the point where the banks could sell to obtain sufficient liquidity to pay d. We do not advocate this alternative, because its effect is not robust to the existence of fire sales on the secondary market. Draining liquidity from the interbank market would contribute to fire sales, because more banks would be forced to sell. Hence, liquidity shortage could be even worsened given that the secondary market issues would not be alleviated.

We need to note also that central bank's liquidity management is ineffective outside of the liquidity shortage region in raising the social welfare at t=1. Of course the liquidity management by the central bank will affect the equilibrium price, loan rate, interbank market volume and the fraction of interbank loans to be repaid. However, otherwise, it will only contribute to welfare transfer between the central banks and the banks. This shows that an elevated interbank loan rate might not be yet a signal to an intervention that would be valuable. More interestingly, given a non-monotonicity of the loan rate for some regions of λ as shown in Lemma 3 it can be that the loan rate even increases after the injections because riskier banks enter the interbank market (despite of falling liquidity premium). Similarly, sufficiently low injections might even increase the total lending volume because more risky banks can borrow. The result on the ineffectiveness of the liquidity injections serves as a note of caution and provides the conditions for which the liquidity injections might be effective and also warranted from the efficiency perspective.

6.2 Asset purchases

In our model we can show the ineffectiveness of asset purchases in avoiding liquidity shortage equilibrium. A central bank that poses the same knowledge about the quality of assets as the investors on the secondary market cannot mitigate liquidity shortage equilibrium at t=1 by buying up assets. The reason is that too low a price that arises in such an equilibrium is due to adverse selection and not due to scarce liquidity on the secondary market.

The result on the ineffectiveness of asset purchases shows the limits of such policy to tackle the liquidity issues such as liquidity shortage. This result gains on importance, because it is also robust to introduction of fire sales on the secondary markets. Given that in an extended model the liquidity shortage on the interbank market would arise due to adverse selection and fire sales on the secondary market, asset purchases could only address the price decline due to fire sales and would not eliminate liquidity shortage due to adverse selection. In fact, asset purchases eliminating fire sales could even lead to further price decline due to adverse selection. The reason is that an increase in liquidity on the secondary market could attract more of the riskier banks to the secondary market (who are more willing to sell than the GI banks). However, an increase in a share of riskier banks would put the downward pressure on the price due to falling asset quality and still trap in the banking system in liquidity shortage equilibrium.

This ineffectiveness of asset purchases in liquidity shortage equilibrium has to be contrasted with liquidity injection in the interbank market whose efficacy does not depend on asymmetric information about riskiness of banks (and asset quality). Although liquidity injections on the interbank are in a narrow sense a similar action to asset purchases (they increase the supply of liquidity), their effect differs due to different motives for which the illiquid banks use these markets to purchase liquidity. In an adverse selection environment the interbank market is a preferable source of liquidity for good banks, hence its collapse is not due to too much adverse selection but to too little liquidity, whereas the reason for the collapse of the secondary market (collapse in the sense of not being able to provide liquidity) is the opposite.

The impossibility of eliminating liquidity shortage at t=1 eliminates the possibility that the central bank can credibly commit to asset purchases at t=1 as a means of guaranteeing a sufficiently high price. To use asset purchases to guarantee the price would be an empty threat, because the banks understand that the asset purchases could only eliminate only the fire sales but not the price declines due to adverse selections.

6.3 Liquidity requirements

The source of liquidity shortage at t=1 is a bank's choice of liquidity at t=0. For that reason, it is interesting to take a look at liquidity requirements that a bank regulator such as a central bank could impose at t=0. As assumed above a liquidity requirement would be targeted at eliminating the liquidity shortage equilibria. Hence, in our setup the bank's choice of liquidity at t=0 could be constrained in the following way: the bank could take λ such that $\lambda \leq \max[0; \lambda_{FR}]$ and $\lambda \geq \min[\lambda_2; \lambda_{UB}]$. Here the liquidity requirements would avoid bank failures due to too low a price of the asset.

It has to be noted, however, that the precise role of the liquidity requirements in avoiding bank failures depends on the assumed form of loan allocation. Indeed, when we assume that all banks can borrow but the loan amount is lower than $d - \lambda$, then sufficiently low price is *only* a necessary condition for bank failures. The reason is that all illiquid banks can borrow a part of their liquidity shortfall and obtain the rest by selling a portion of their asset. Hence, in some cases even if the price of the asset is too low for the banks to obtain all of the needed liquidity on the secondary market, it is still sufficiently high to obtain some of it and to survive the liquidity shock. Only when the banks have little cash, bank failures occur, because the banks have to borrow so much that they cannot repay their interbank loans at t=2. Hence, the liquidity requirement would be in form of $\lambda \geq \underline{\lambda}$, where $\underline{\lambda}$ would guarantee that the banks can repay their interbank loans at t=2.

7 Conclusion

The paper provides a generic model of liqiudity provision augmented by existence of a secondary market for banks' assets. We show how uncertainty about quality of bank assets lead to banks' inability to become liqiud by selling their assets and forces them to borrow, resulting in liqiudity shortage on the interbank market and socially inefficient bank failures. The effectiveness of policy implications in affecting welfare depends strongly on the state of the interbank markets. When the decrease in banks' quality is not sufficiently high, the interbank loan rate might incraese and asset prices fall, but liqiudity distribution is not interrupted, leading to welfare-neutral interventions.

Only when the decrease in banks' quality is sufficiently high, bank failures due to illiquidity occur. In such a case liqiudity injections eliminate bank failures and asset purchases do not. Ex-ante liquidity requirements can eliminate liquidity shortage on the interbank markets too.

The paper provides a novel explanation for a transmission of a shock from the secondary market for bank assets to the interbank markets. As such the paper can be viewed as an interpretation of events in August 2007. We also argue why, contrary to conventional wisdom, the interbank markets did not freeze despite collapse of securitization markets. We argue that in case of an acute stress the interbank markets provide outside liquidity at the lowest adverse selection cost.

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8 Proofs

Proof of Lemma 1

After rewriting the bank's expected return the decision problem reads:

$$\max_{l,S}(1-\lambda-S)pR+(SP+\lambda-l)+\widehat{p}R_D l,\text{ s.t. } S\in[0;1-\lambda],\ l\in[0;SP+\lambda]\,.$$

Because the bank's expected return is linear in l, the bank lends all of its cash, $l=SP+\lambda$, if $\widehat{p}R_D>1$. After inserting $l=SP+\lambda$ into the bank's expected return we get

$$(1-\lambda)pR+\widehat{p}R_D\lambda+(P\widehat{p}R_D-pR)S.$$

Hence, $S = 1 - \lambda$, for $P\widehat{p}R_D > p_iR_i$, be indifferent for $P\widehat{p}R_D = p_iR_i$, and keep all of it otherwise. If $\widehat{p}R_D \leq 1$, the bank is either indifferent between lending or not lending, or does not lend at all. The effect on the objective function is the same because the terms with l in the objective function vanish and the objective function reads:

$$(1 - \lambda)pR + \lambda + (P - pR)S.$$

Hence, the bank will sell all of its asset, $S = 1 - \lambda$, for $P > p_iR_i$, be indifferent for $P = p_iR_i$, and keep all of it otherwise. Hence, using $\max\left[\widehat{p}R_D; 1\right]$ we can summarize the rule for selling as in the Result 1.■

Proof of Lemma 2

The proof is pretty straightforward after realization that at optimum the illiquid bank will always exhaust the constraint $SP + \lambda + b \geq d$. If this constraint is slack then the bank either borrowed too much or sold too much or didn't lend all of its cash reserves. Given that all this is more expensive than storage (or at least cost is the same as storage), the illiquid bank prefers to exhaust this constraint. Hence, it has to hold that $SP + \lambda + b = d$.

For $P(1 - \lambda) + \lambda > d$ it is easy to see that each illiquid bank taking P, R_D and $\widehat{p}$ as given will have three options to cover the liquidity shortfall: $b = d - \lambda$ and $S = 0$, $b = 0$ and $S = \frac{d-\lambda}{P}$, and $b = -\left[(1 - \lambda)P + \lambda - d\right]$ and $S = 1 - \lambda$. Hence, for each of these solutions we can compare the value of the objective function and obtain the Result 2.

For $P(1 - \lambda) + \lambda \leq d$ the decision problem for $b < 0$ reads:

$$\max_{b,S} \begin{cases} p[(1 - \lambda - S)R + (SP + \lambda + b - d) - R_Db], \text{ if } (1 - \lambda - S)R + (SP + \lambda + b - d) - R_Db \geq 0 \\ 0, \text{ otherwise} \end{cases}$$

$$\text{s.t. } S \in [0; 1 - \lambda], SP + \lambda + b = d.$$

The objective function takes into account that the necessary condition for the bank to survives at t=2 with a positive probability is that

$$(1 - \lambda - S)R + (SP + \lambda + b - d) - R_D b \geq 0. \qquad (2)$$

The crucial part of the proof is to realize that the bank will survive at t=2 only if there is such S and b satisfying

$$SP + \lambda + b = d \qquad (3)$$

for which (2) holds. Otherwise, there is no borrowing b under which the bank can survive at t=2, implying the bank is indifferent between borrowing and selling. Hence, first, we will show under which conditions the bank is able to borrow and survive at t=2 with some positive probability, and second, we will show that under these condition it is optimal to borrow only.

First, we look for conditions under which the bank can survive at t=2 when borrowing at t=1. Observe that if the both binding (2) and (3) are rewritten for b as a function of S they are decreasing in S. The reason is that $P < R$ and $R_D > 1$ (the latter for lending to be possible). Then for $S = 1 - \lambda$ binding (2) delivers a negative b and (3) a positive b, meaning that: (i) for $S = 1 - \lambda$ the bank cannot survive, and (ii), in order to survive, for $S = 0$ b for the binding (2) cannot be lower than for (3) (i.e. the bank is able to repay the loan equal to its liquidity shortfall when its asset pays). From (ii) follows that for $S = 0$ and b from (3) (2) boils down to $R(1 - \lambda) \geq R_D(d - \lambda)$, which is the condition under which the bank can borrow at $t = 1$ and survive at t=2 with some positive probability.

Second, using $SP + \lambda + b = d$ implies that the above program becomes

$$\max_{S \geq 0} \begin{cases} p[(1-\lambda)R - R_D(d-\lambda) + S(PR_D - R)], \text{ if } (1-\lambda)R - R_D(d-\lambda) + S(PR_D - R) \geq 0 \\ 0, \text{ otherwise} \end{cases}$$

Now, observe that $P(1 - \lambda) + \lambda < d$ and $R(1 - \lambda) \geq R_D(d - \lambda)$ imply $PR_D < R$, meaning that the above expected return is decreasing in S whenever R_D is such that the bank can survive at t=2 after borrowing at t=1. This proves our claim that each bank would like borrow rather than sell when R_D is such that the bank is solvent at t=2.

Proof of Proposition 1

We postpone a discussion of the multiple equilibria to the end of the proof and concentrate here on the equilibria in which the interbank market is active, i.e., the banks and the investors expect the GI banks to borrow. We construct equilibria in turns. We start with an equilibrium in which all illiquid banks borrow. Then we proceed to equilibrium in which the BI banks are indifferent. Finally we discuss the equilibrium in which all BI banks sell and GI banks are indifferent between selling and borrowing.

Denote as P^*, R_D^* and $\widehat{p}^*$ the equilibrium values of price, loan rate and the expected fraction of borrowing banks repaying their loans at t=2, and assume that $\lambda + P^*(1-\lambda) > d$. We construct the equilibria in the following way. We first assume a certain equilibrium, which requires to assume certain optimal choices on the side of banks. Next, we derive P^*, R_D^* and $\widehat{p}^*$ consistent with the stipulated equilibrium. Finally, we check if the assumed optimal choice of the banks are indeed optimal under the derived P^*, R_D^* and $\widehat{p}^*$ (the step simply requires if the conditions from Results 1 and 2 relating to the assumed optimal choice are satisfied for the derived P^*, R_D^* and $\widehat{p}^*$).

The share of the banks that repay their loans equals to the expected probability of success of assets held by the borrowing banks, $\widehat{p}$. The reason is that holding cash reserves till t=2 that keep the bank afloat when its asset does not repay will not be optimal when the bank will find it optimal to borrow.

Equilibrium in which the all illiquid banks borrow Let's assume that we have an equilibrium in which all illiquid banks borrow. Such an equilibrium requires that the liquid banks supply enough liquidity for all illiquid banks. Total demand for interbank loans is $(1-\pi)(d-\lambda)$, where each of the illiquid banks, whose fraction in the population of all banks is $1-\pi$,demands a loan equal to its liquidity shortfall $d-\lambda$. Total supply of interbank loans is $\pi q\lambda + \pi(1-q)(\lambda + P(1-\lambda))$, which is the sum of cash reserves provided by all GL banks, $\pi q\lambda$, and by all BL banks, $\pi(1-q)(\lambda + P^*(1-\lambda))$ who provide their cash reserves from t=0 λ and sell all of its asset $(1-\lambda)$ for a price P^*. Observe that an assumption that the GI banks borrow implies that the GL banks do not sell (from Result 2 the GI banks borrow if $R > PR_D$, which implies that $R > PR_D \geq P\widehat{p}R_D$ and per Result 1 that the GL do not sell). Given that only the bad banks sell, then the equilibrium price is $P^* = pR$. Hence, there is enough liquidity on the interbank market for all illiquid banks, when total supply is not lower than total demand:

$$\pi \left[q\lambda + (1-q)(\lambda + P^*(1-\lambda)) \right] \geq (1-\pi)(d-\lambda),$$

or (using $P^* = pR$)

$$\lambda \geq \frac{(1-\pi) d - (1-q)\pi p R}{1-(1-q)\pi p R} \equiv \lambda_2.$$

We split the discussion into two cases: $\lambda > \lambda_2$ and $\lambda = \lambda_2$.

For $\lambda > \lambda_2$ the interbank market clears only if the lending banks are indifferent between lending or storing cash, because there is excess supply of liquidity on the interbank market. Hence, the expected return on lending in equilibrium $\widehat{p}^* R_D^*$ has to equal 1. Given that in equilibrium all illiquid banks borrow, the expected fraction of borrowing banks repaying their loans is $\widehat{p}^* = q + (1-q)p$ implying an equilibrium loan rate $R_D^* = (q + (1-q)p)^{-1}$. Using the conditions from Result 1 we confirm that for stipulated P^*, $\widehat{p}^*$ and R_D^* the liquid banks are indifferent between lending and cash storage, the GL does not sell (because $P^* \widehat{p}^* R_D^* = pR < R$) and the BL sells all of its asset (because $P^* \widehat{p}^* R_D^* = pR \geq pR$). Using the Result 2 we also confirm that borrowing is optimal for the GI and BI banks (for the GI banks it holds that $R > P^* \widehat{p}^* R_D^* = \frac{pR}{\widehat{p}^*}$, because $p < \widehat{p}^*$, and we know that once the GI prefer to borrow, the same holds for BI).

For $\lambda = \lambda_2$ the supply and demand for loans are equal. Hence, the equilibrium loan rate is indeterminate because both supply and demand are inelastic for certain ranges of R_D as in Freixas, Martin and Skeie (2011). The indeterminacy of the loan rate has identical roots as in Freixas, Martin and Skeie (2011): the supply and demand of liquidity is inelastic, because the liquid banks will lend out for any return that pays at least as storage nut is not higher than a loan rate that would cause illiquid banks to default at t=2 and illiquid banks will pay any loan rate up not higher than the one that makes them choose to sell. The liquid banks lend for any R_D such that lending in expectation is at least as profitable as cash storage, $(q + (1-q)p) R_D \geq 1$, but not higher than it would cause default of a borrowing banks, $R_D \leq \frac{1-\lambda}{d-\lambda} R$. Because the BI banks are the first one to withdraw from the interbank market, so the upper bound on the loan rate for which all illiquid banks borrow is given by the BI banks' indifference condition from the Result 2, i.e., $R_D \leq \frac{R}{P^*+\left(\frac{\widehat{p}^*}{p}-1\right)\left(P^*-\frac{d-\lambda}{1-\lambda}\right)}$, where P^* and $\widehat{p}^*$ are the same as in the previous case. Some basic algebra shows that the upper bound on R_D for which all illiquid banks borrow is lower than the upper bound fro which the liquid banks lend, $\frac{R}{P^*+\left(\frac{\widehat{p}^*}{p}-1\right)\left(P^*-\frac{d-\lambda}{1-\lambda}\right)} < \frac{1-\lambda}{d-\lambda}R \Leftrightarrow \lambda + P^*(1-\lambda) > d$. Hence, the interbank market clears for any $R_D^* \in \left[\frac{1}{\widehat{p}^*}; \frac{R}{P^*+\left(\frac{\widehat{p}^*}{p}-1\right)\left(P^*-\frac{d-\lambda}{1-\lambda}\right)}\right]$. It is again easy to check that P^*, R_D^* and $\widehat{p}^*$ satisfy the conditions from Result 1 and 2 for which the stipulated banks' choices are optimal.

Equilibrium in which the BI banks are indifferent Again we first assume that such an equilibrium exists: the BI banks are indifferent between selling and borrowing. From Result 2 and the preceding analysis we know also that in such an equilibrium the GI still borrow. As we showed above that GL banks do not sell. Hence, the equilibrium price is still $P^* = pR$.

Denote as σ the fraction of the BI banks that sell. Then in an equilibrium with the BI banks being indifferent the following conditions have to be fulfilled:

$$\begin{aligned} & \pi\left[q\lambda + (1-q)(\lambda + P^*(1-\lambda))\right] + (1-\pi)(1-q)\sigma^*(\lambda + P^*(1-\lambda) - d) \\ = \; & (1-\pi)(q + (1-q)(1-\sigma^*))(d-\lambda) \end{aligned} \tag{4}$$

$$\widehat{p}^* = \frac{q}{q+(1-q)(1-\sigma^*)} + \frac{(1-q)(1-\sigma^*)}{q+(1-q)(1-\sigma^*)}p \tag{5}$$

$$R_D^* = \frac{R}{P^* + \left(\frac{\widehat{p}^*}{p}-1\right)\left(P^* - \frac{d-\lambda}{1-\lambda}\right)} \tag{6}$$

(4) is the interbank market clearing condition. The left hand side of (4) is the supply of interbank loans. It differs from the case $\lambda \geq \lambda_2$ by the term $(1-\pi)(1-q)\sigma^*(\lambda + P^*(1-\lambda) - d)$, which represents the supply of cash by equilibrium fraction σ^* of the BI banks that sell all of its asset, cover their liquidity shortfall and lend out the rest of the cash on the interbank market. The right hand side of (4) is the demand for loans. It differs from the case $\lambda \geq \lambda_2$ because this time only

the fraction $q+(1-q)(1-\sigma)$ of illiquid banks borrows. (5) is the expected fraction of borrowing banks that repay their loans at t=2. This time it take into account that only fraction $(1-\sigma)$ of the BI banks borrows. (6) is the equilibrium loan rate that makes the BI banks indifferent between borrowing and selling, which is given by indifference condition for these banks from the Result 2. (4)-(6) provide all the equilibrium variables. With some algebra we can see that (6) satisfies $\widehat{p}R_D \geq 1$, meaning that the lending is profitable. Moreover, from Result 2 we know that once the BI banks are indifferent between borrowing and selling, then the GI banks prefer to borrow, and this implies that GL banks prefer to keep their asset and only lend. The requirement that the BL banks sell and lend is trivially satisfied.

Solving (4) for σ^* delivers the equilibrium fraction of the BI banks that sell, σ^*:

$$\sigma^* = \frac{(1-\pi)\,d - \pi\,(1-q)\,P^* - \lambda\,(1-(1-q)\,\pi P^*)}{(1-\pi)\,(1-q)\,(1-\lambda)\,P^*}$$

Hence, the equilibrium obtains for $\sigma^* \in (0;1)$. It can be easy verified that $\sigma^* > 0$ for $\lambda < \lambda_2$ and $\sigma^* < 1$ for $\lambda > \lambda_1 \equiv \frac{(1-\pi)d-(1-q)pR}{1-(1-q)pR}$.

For $\lambda = \lambda_1$ the interbank market clears because the amount of loans supplied by all liquid banks and selling BI banks is the same as demand for loans from all GI banks. Hence, the market clears for such loan rates that the GI banks want to borrow and the BI banks want to sell, $R_D^* \in \left[\frac{R}{P^*+\left(\frac{1}{p}-1\right)\left(P^*-\frac{d-\lambda}{1-\lambda}\right)}; \frac{1}{p}\right]$, where $\widehat{p}^* = 1$ and $P^* = pR$.

Equilibrium with all BI banks selling For $\lambda \in [0;\lambda_1)$ the supply of loans from all liquid and BI banks is lower than the demand for loans from all GI banks. Hence, the market clearing is achieved when some of the GI banks decide to sell. Because only some of the GI banks will borrow we get that $\widehat{p}^* = 1$. Then the Result 2 the GI bank is indifferent between all three options listed in the Result 2, what implies that in equilibrium it holds that $R = P^*R_D^*$. In addition, $R = P^*R_D^*$ implies that the GL banks are also indifferent between keeping and selling all. Denote γ_S the fraction of the GI banks selling only $\frac{d-\lambda}{P}$ and γ_A the fraction of the GI banks selling all, and γ the fraction of the GL banks selling all. Then the equilibrium conditions are

$$\begin{aligned} & \pi\left[q\left[(1-\gamma)\lambda+\gamma\left(\lambda+P^*(1-\lambda)\right)\right]+(1-q)\left(\lambda+P^*(1-\lambda)\right)\right] \\ & +(1-\pi)(1-q)\left(\lambda+P^*(1-\lambda)-d\right) \\ & +(1-\pi)q\gamma_A\left(\lambda+P^*(1-\lambda)-d\right) \\ = & \ (1-\pi)q\left(1-\gamma_S-\gamma_A\right)(d-\lambda) \end{aligned} \tag{7}$$

$$\begin{aligned} P^* = & \frac{\pi q\gamma(1-\lambda)+(1-\pi)q\left[\gamma_S\frac{d-\lambda}{P}+\gamma_A(1-\lambda)\right]}{(1-q)(1-\lambda)+\pi q\gamma(1-\lambda)+(1-\pi)q\left[\gamma_S\frac{d-\lambda}{P}+\gamma_A(1-\lambda)\right]}R \\ & +\frac{(1-q)(1-\lambda)}{(1-q)(1-\lambda)+\pi q\gamma(1-\lambda)+(1-\pi)q\left[\gamma_S\frac{d-\lambda}{P}+\gamma_A(1-\lambda)\right]}pR, \end{aligned} \tag{8}$$

$$R_D^* = \frac{R}{P^*} \tag{9}$$

(7) is the market clearing condition for the interbank market taking into account that fraction $1-\gamma$ of the GL banks lends only cash reserves, fraction γ of the GL banks sells all and lends, all bad banks sell, fraction γ_A of the GI banks lends out what is left after selling all of their assets and covering the shortfall $d-\lambda$, and only the fraction $1-\gamma_S-\gamma_A$ of the GI banks borrows. (8) is the equilibrium price paid by the competitive investors who take into account that all bad banks sell all of their assets, a fraction γ of the GL banks sells all, a fraction γ_S of the GI banks sell only $\frac{d-\lambda}{P}$ and fraction γ_A sell all. (9) is the loan rate that guarantees that the GI banks are indifferent between borrowing and selling and the GL banks between keeping and selling all. Moreover, it is easy to check (**??**) to see that the BI banks prefer to sell all and lend out under (9). Despite the fact that there are more unknowns that equations P^* is uniquely determined by (7)-(9),

$$P^* = \frac{R(d(1-\pi)-\lambda)}{d(1-\pi)-\lambda+(1-p)(1-q)(1-\lambda)R},$$

implying the same for R_D^*.

Finally, we can show that there is another perfect Bayesian equilibrium in which all GI banks sell and the interbank market is inactive. In such a case the investors set expectations in such a way that the equilibrium price P^e is such that

$$P^e = \frac{(1-\pi)\, q\frac{d-\lambda}{P^e}}{(1-q)(1-\lambda) + (1-\pi)\, q\frac{d-\lambda}{P^e}} R + \frac{(1-q)(1-\lambda)}{(1-q)(1-\lambda) + (1-\pi)\, q\frac{d-\lambda}{P^e}} pR, \tag{10}$$

where it is taken into account that all bad banks sell all of their assets and the GI banks sell only the amount they need to cover the shortfall $d-\lambda$. Hence, the aggregate amount of bad assets on the market is $(1-q)(1-\lambda)$ and of good assets $(1-\pi)\, q\frac{d-\lambda}{P}$. The only condition that has to be checked is the condition for the GI banks. The off-equilibrium loan rate on the interbank market is $1/p$ which reflects the most pessimistic belief of the banks that have cash to lend that the borrowing bank is of bad type. Hence, using (**??**) combined with $R_D = \frac{1}{p}$ and $\widehat{p} = p$ we get that the GI banks prefer to sell $\frac{d-\lambda}{P}$ if $R_D P = \frac{P}{p} > R \Leftrightarrow \lambda \in [0; \lambda_{ME})$ (it is easy to see that the GI banks will never sell all to lend as $R > \frac{\widehat{p}}{p_G} P R_D = \frac{p}{1} P \frac{1}{p} = P$).

However, the intuitive criterion will eliminate such an equilibrium. Assume that off-equilibrium beliefs are such that a bank that wants to borrow is a GI bank. Hence, a loan rate would be 1 for such a bank. Given that it is obvious that a GI bank would like to deviate (prefer to borrow rather than sell), because it does not incur any adverse selection cost of borrowing. In fact, we can show that even for such a loan rate the BI banks prefer to stick to the on-equilibrium strategy of selling. After some tedious algebra one can show that the BI banks' payoff from selling for P^e, $P^e(1-\lambda) + \lambda - d$, is not lower than the payoff from deviating to borrowing for $R_D = 1$, $p\,(R\,(1-\lambda) - (d-\lambda))$. In fact, we can show that

$$P^e(1-\lambda) + \lambda - d \geq p\,(R\,(1-\lambda) - (d-\lambda))$$

is equivalent to $\lambda \leq d$, which proves our claim, i.e., intuitive criterion kills the only-selling equilibrium, because we can find such "reasonable" off-equilibrium beliefs, for which the GI banks want to deviate from the equilibrium and the BI banks do not.

■

Proof of Proposition 2

The share of those going bankrupt is the pool of all illiquid $1-\pi$ after those who get loans, whose number is just given by the total supply of loans divided by the loan amount. Hence the share is $\nu = 1 - \pi - \frac{\pi[\lambda+(1-q)(1-\lambda)P_R]}{d-\lambda}$. Moreover, the bankrupt banks are indifferent between selling or not, and we assume that they dump all of their assets on the market (alternatively, we could have assumed that they are taken over and acutely a regulator liquidates these assets to cover the needed repayment of depositors without waiting on how these assets play out). Hence, the equilibrium price is given by

$$P_R = \frac{\nu q (1-\lambda) + p [\nu (1-q)(1-\lambda) + \pi (1-q)(1-\lambda)]}{\nu (1-\lambda) + \pi (1-q)(1-\lambda)} R. \tag{11}$$

The equilibrium loan rate is just $R_{D,R} = \frac{1-\lambda}{d-\lambda} R$, which is always higher than the minimum needed for the lenders to break even $(\overline{p})^{-1}$ (observe that $\frac{1-\lambda}{d-\lambda} > 1$).

This equilibrium exists for λ such that $P_R (1-\lambda) + \lambda < d$, hence there is again an interval $\lambda \in (\lambda_{FR}; \lambda_{UB}]$, where $\lambda_{FR} > \lambda_{LB}$.

Observe here that the defaults here are basically assumed by the form of loan allocation. ν is also default rate here.

Again we have a mixed strategy region for the same reason as above. If γ_R is the share of BI that borrows, then the number of loans $n = \frac{\pi[\lambda+(1-q)(1-\lambda)P_R]}{d-\lambda}$ is split between $\frac{q}{q+(1-q)\gamma_R}$ of GI and $\frac{(1-q)\gamma_R}{q+(1-q)\gamma_R}$ of BI. Hence the γ_R is given by the following price equation

$$P_R = \frac{\left[q(1-\pi) - \frac{q}{q+(1-q)\gamma_R} n\right](1-\lambda) + p\left[\left[(1-q)(1-\pi) - \frac{(1-q)\gamma_R}{q+(1-q)\gamma_R} n\right](1-\lambda) + \pi(1-q)(1-\lambda)\right]}{\left[q(1-\pi) - \frac{q}{q+(1-q)\gamma_R} n\right](1-\lambda) + \left[(1-q)(1-\pi) - \frac{(1-q)\gamma_R}{q+(1-q)\gamma_R} n\right](1-\lambda) + \pi(1-q)(1-\lambda)} R \text{ and } P \tag{12}$$

It can be shown that γ_R is 0 for $\lambda = \lambda_{LB}$ and 1 for $\lambda = \lambda_{FR}$, which is to be expected. In the mixed strategy stuff there is no defaults, as the price is exactly the one at which the banks can cover their liquidity shortfall.

Proof of Lemma 4

For the case $P^*(1-\lambda)+\lambda > d$ the expressions for social welfare are just the sum of profits for each type of bank for λ and the equilibrium variables R_D^*, P^*, and $\widehat{p}^*$ as well as the payments made by illiquid banks due to toe liquidity shock, $d(1-\pi)$. Hence, social welfare at t=1 is $SW_1 = \Pi(\lambda; R_D^*; P^*; \widehat{p}^*) + d(1-\pi)$ for $R_D = R_D^*$, $P = P^*$ and $\widehat{p} = \widehat{p}^*$, where (we denote this sum as $\Pi(\lambda; R_D^*; P^*; \widehat{p}^*)$

$$\Pi(\lambda; R_D; P; \widehat{p}) = \begin{cases} \pi q\left[(1-\gamma)\left[R(1-\lambda)+\widehat{p}R_D\lambda\right]+\gamma\widehat{p}R_D\left((1-\lambda)P+\lambda\right)\right]+ \\ \pi(1-q)\widehat{p}R_D\left((1-\lambda)P+\lambda\right)+ \\ (1-\pi)q\left[(1-\gamma_S-\gamma_L)\left(R(1-\lambda)-R_D(d-\lambda)\right)+\gamma_S R\left(1-\lambda-\frac{d-\lambda}{P}\right)\right]+ \\ +(1-\pi)q\gamma_L\widehat{p}R_D\left((1-\lambda)P+\lambda-d\right)+(1-\pi)(1-q)\widehat{p}R_D\left((1-\lambda)P+\lambda-d\right), \text{ for } \lambda \\ \\ \pi\left[q\left[R(1-\lambda)+\widehat{p}R_D\lambda\right]+(1-q)\widehat{p}R_D\left((1-\lambda)P+\lambda\right)\right]+ \\ (1-\pi)\left[q\left[R(1-\lambda)-R_D(d-\lambda)\right]+(1-q)\widehat{p}R_D\left((1-\lambda)P+\lambda-d\right)\right], \text{ for } \lambda=\lambda_1 \\ \\ \pi\left[q\left[R(1-\lambda)+\widehat{p}R_D\lambda\right]+(1-q)\widehat{p}R_D\left((1-\lambda)P+\lambda\right)\right]+ \\ (1-\pi)\left[q\left[R(1-\lambda)-R_D(d-\lambda)\right]+(1-q)(1-\beta)p[(1-\lambda)R-R_D(d-\lambda)]\right]+ \\ (1-\pi)\beta(1-q)\widehat{p}R_D\left((1-\lambda)P+\lambda-d\right), \text{ for } \lambda\in(\lambda_1;\lambda_2) \\ \\ \pi\left[q\left[R(1-\lambda)+\widehat{p}R_D\lambda\right]+(1-q)\widehat{p}R_D\left((1-\lambda)P+\lambda\right)\right]+ \\ (1-\pi)\left[q\left[R(1-\lambda)-R_D(d-\lambda)\right]+(1-q)p[(1-\lambda)R-R_D(d-\lambda)]\right], \text{ for } \lambda\in[\lambda_2;d] \end{cases} \tag{13}$$

After some algebra one can show that for $\lambda \in [0; d]$, $R_D = R_D^*$, $P = P^*$ and $\widehat{p} = \widehat{p}^*$

$$\Pi(\lambda; R_D^*; P^*; \widehat{p}^*) = (1-\lambda)(q+(1-q)p)R + \lambda - d(1-\pi).$$

Hence, $SW_1 = (1-\lambda)(q+(1-q)p)R + \lambda$.

For $P^*(1-\lambda)+\lambda=d$ we can show after some algebra that

$$\begin{aligned} SW_1 &= \Pi\left(\lambda;R_D^*;P^*;\widehat{p}^*\right)+d\left(1-\pi\right) \\ &= \pi\left[q\left[R\left(1-\lambda\right)+\widehat{p}^*R_D^*\lambda\right]+\left(1-q\right)\widehat{p}^*R_D^*\left(\left(1-\lambda\right)P^*+\lambda\right)\right]+d\left(1-\pi\right) \\ &= \left(1-\lambda\right)\left(q+\left(1-q\right)p\right)R+\lambda, \end{aligned} \tag{14}$$

where $R_D^*=\frac{1-\lambda}{d-\lambda}R$, $P^*=\frac{d-\lambda}{1-\lambda}$ and $\widehat{p}^*=\frac{q}{q+(1-q)\gamma_R}+\frac{(1-q)\gamma_R}{q+(1-q)\gamma_R}p$ and γ_R is as given by (12).

For $P^*(1-\lambda)+\lambda<d$ we have that

$$\begin{aligned} SW_1 &= \Pi\left(\lambda;R_D^*;P^*;\widehat{p}^*\right)+d\left[\left(1-\pi\right)-\nu\right] \\ &= \pi\left[q\left[R\left(1-\lambda\right)+\widehat{p}^*R_D^*\lambda\right]+\left(1-q\right)\widehat{p}^*R_D^*\left(\left(1-\lambda\right)P^*+\lambda\right)\right]+d\left[\left(1-\pi\right)-\nu\right] \end{aligned} \tag{15}$$

where $R_D^*=R_{D,R}=\frac{1-\lambda}{d-\lambda}R$, $P^*=P_R$ as given by (11), $\widehat{p}^*=q+(1-q)p$ and $\nu=1-\pi-\frac{\pi[\lambda+(1-q)(1-\lambda)P_R]}{d-\lambda}$. Then it easy to show that SW_1 given by (15) is lower than $(1-\lambda)(q+(1-q)p)R+\lambda$. We can show it just by comparing the second line in (14) that expresses $(1-\lambda)(q+(1-q)p)R+\lambda$ with the second line in (15). Both expressions differ due to different $\widehat{p}^*$, P^* and the fraction of d that is repaid. First, the profits in (15) have to be lower than in (14) because the equilibrium fraction of loans repaid and the equilibrium price are lower in the case where $P^*(1-\lambda)+\lambda<d$. Second, because $\nu>0$ less banks repay further reducing welfare.

Proof of Proposition 3

At t=0 each bank chooses optimal λ anticipating its and other banks' optimal behavior as one of 4 possible types of banks at t=1 as a function of λ as determined in the Result 3. Hence, using the notation that the banks' expected profit as a function of λ, P, R_D and $\widehat{p}$ is $\Pi(\lambda;R_D;P;\widehat{p})$ the bank solves the following problem taking as given P, R_D and $\widehat{p}$:

$$\max_{\lambda\in[0;d]}\Pi\left(\lambda;R_D;P;\widehat{p}\right),$$

where the functional form of $\Pi(\lambda; R_D; P; \widehat{p})$ depends on the anticipated equilibrium. Hence, for any λ such that $P^* (1-\lambda) + \lambda > d$ $\Pi(\lambda; R_D; P; \widehat{p})$ would be given by (13), for $P^* (1-\lambda) + \lambda = d$ $\Pi(\lambda; R_D; P; \widehat{p})$ would be given by the corresponding expression in (14), and for $P^* (1-\lambda) + \lambda < d$ $\Pi(\lambda; R_D; P; \widehat{p})$ would be given by (15).

First, observe that at t=0 a bank will choose λ at most λ_2, because taking more than λ_2 would be waste of resources. For any $\lambda > \lambda_2$ there would be excess supply of liquidity on the interbank market.

For $\lambda = \lambda_2$ in equilibrium R_D at t=0 has to be such that the first order condition with respect to λ holds with equality. Otherwise, R_D would be such that the bank would either prefer to set λ smaller or bigger than λ_2. The first order condition is in such a case is given by deriving the fourth expression in (13) for λ :

$$\pi \left[q(-R + \widehat{p} R_D) + (1-q) \widehat{p} R_D (-P+1) \right] + (1-\pi) \left[q(-R + R_D) + (1-q) p (-R + R_D) \right].$$

Given that in equilibrium for $\lambda = \lambda_2$ the banks expect that $\widehat{p}^* = q + (1-q)p$ and $P^* = pR$ the ex-ante equilibrium R_D^* at $t = 0$ is $\frac{qR_G + (1-\pi)(1-q)pR}{(q+(1-q)p)(1-\pi(1-q)pR)}$. Under such a loan rate the anticipated banks' choice of $\lambda = \lambda_2$ is optimal if $R_D^* = \frac{qR_G + (1-\pi)(1-q)pR}{(q+(1-q)p)(1-\pi(1-q)pR)}$ is in the interval $\left[\frac{1}{q+(1-q)p}; \frac{1}{[q+(1-q)p] - \frac{q(1-p)}{pR} \frac{d-\lambda_2}{1-\lambda_2}} \right]$ as determined in the proof of the Result 3 (i.e. the lending banks break even and the BI banks borrow). We can see that is always holds that $\frac{qR_G + (1-\pi)(1-q)pR}{(q+(1-q)p)(1-\pi(1-q)pR)} > \frac{1}{q+(1-q)p}$ (which is equivalent to $(q + (1-q)p) R > 1$) and that $\frac{[q+(1-\pi)(1-q)p]R}{(q+(1-q)p)(1-\pi(1-q)pR)} \leq \frac{1}{[q+(1-q)p] - \frac{q(1-p)}{pR} \frac{d-\lambda_2}{1-\lambda_2}} \Leftrightarrow$ $R \leq \underline{R}$, where $\underline{R}$ is given by solving the last inequality for R with equality sign. Hence, the bank chooses $\lambda = \lambda_2$ for $R \leq \underline{R}$.

For $\lambda \in (\lambda_1; \lambda_2)$ we have a similar procedure. The only difference is that this time equilibrium values of P, R_D, σ and $\widehat{p}$ are pinned down at t=1 too. The result is an interior solution for λ given by five equations: the binding first order condition, and equations defining $P^* = pR$, R_D^*, σ^* and $\widehat{p}^*$ as functions of λ as given by (4), (5), and (6). These five equations deliver the equilibrium values of λ, P, R_D, σ and $\widehat{p}$. Apart from P all values are complicated objects, so we refrain from providing them (the reader is more than welcome to ask the author for the Mathematica code that offers the solutions). The solutions constitute an equilibrium if the equilibrium value of λ is between $(\lambda_1; \lambda_2)$. This occurs for $R \in (\underline{R}; \overline{R})$, where $\overline{R}$ is the solution of an equation in which λ_1 is equal to the chosen λ^*. Hence, the bank chooses optimally some $\lambda \in (\lambda_1; \lambda_2)$ for $R \in (\underline{R}; \overline{R})$.

For $\lambda = \lambda_1$ again we apply the same procedure. The bank at t=0 is indifferent between any choice of λ for $R_D = \frac{qR}{1-(1-q)pR}$, which has to be in the interval $\left[\frac{1}{1-\frac{1-p}{pR}\frac{d-\lambda_1}{1-\lambda_1}}; \frac{1}{p}\right]$. We have that it always holds that $\frac{qR_G}{1-(1-q)pR} \leq \frac{1}{p}$, and that $\frac{qR_G}{1-(1-q)pR} \geq \frac{1}{1-\frac{1-p}{pR}\frac{d-\lambda_1}{1-\lambda_1}} \Leftrightarrow R \geq \overline{R}$. Hence, the bank chooses optimal $\lambda = \lambda_1$ for any $R \geq \overline{R}$.

For $\lambda \in [0; \lambda_1)$ (this interval is non-empty if $d > \frac{p(1-q)R}{1-\pi}$) the matters get a little bit more complicated, because P and R_D have to be determined jointly (this was not the case in other cases where $P = pR$). The derivative of the objective function with respect to λ reads

$$(R_D - R)\, q\, (1 - \pi\gamma - (1-\pi)(\gamma_S + \gamma_A)) +$$
$$(1-P)\left[R_D\,(1 - q + \pi q \gamma + (1-\pi)\, q \gamma_S) + \frac{R}{P}(1-\pi)\, q\gamma_S\right].$$

Using $R_D = \frac{R}{P}$ which has to hold if the GI banks are indifferent between borrowing and selling at t=1 the above condition boils down to

$$\frac{R}{P}(1-P).$$

First, if the highest possible price at t=1, which obtains for $\lambda = 0$ (because the equilibrium price at t=1 is decreasing in λ and reaches pR for $\lambda = \lambda_1$), is lower than 1, then the bank will never choose any $\lambda \in [0; \lambda_1)$, because the objective function is increasing in λ for all $\lambda \in [0; \lambda_1)$. This occurs for the price (taken from Result 5) $P = \frac{R(d(1-\pi)-\lambda)}{d(1-\pi)-\lambda+(R-pR)(1-q)(1-\lambda)}$ for $\lambda = 0$, i.e., $P(\lambda = 0) = \frac{Rd(1-\pi)}{d(1-\pi)+(R-pR)(1-q)}$, that is lower than 1. That holds for any R if $d \in \left(\frac{p(1-q)R}{1-\pi}; \frac{(1-p)(1-q)R}{1-\pi}\right]$ and $p < \frac{1}{2}$ or for $R \in \left(\frac{1}{q+(1-q)p}; \widetilde{R}\right)$ if $d > \max\left\{\frac{p(1-q)R}{1-\pi}; \frac{(1-p)(1-q)R}{1-\pi}\right\}$, where $\widetilde{R} \equiv \frac{d(1-\pi)}{d(1-\pi)-(1-q)(1-p)}$ and it holds that $(q + (1-q)\,p)\,\widetilde{R} > 1$.

Second, when $R \geq \widetilde{R}$ and $d > \max\left\{\frac{p(1-q)R}{1-\pi}; \frac{(1-p)(1-q)R}{1-\pi}\right\}$, then we can have multiple equilibria. One equilibrium is $\lambda^* = 0$, because then the equilibrium price is higher than 1 and choice of $\lambda = 0$ is consistent with that price because the first order condition is negative. Moreover, we can always find such γ, γ_S and γ_S such that the interbank market clears. It is also possible that the bank decides to choose $\lambda^* \geq \lambda_1$. In such a case the equilibrium price is pR and the first order condition is positive, which is consistent with choice of $\lambda \geq \lambda_1$. Again the interbank market would clear. Observe that although $P^* = 1$ nullifies the first order condition it is an unstable equilibrium. The reason is that if we take any arbitrarily small perturbation from $P^* = 1$ the bank would prefer to set either $\lambda = 0$ or $\lambda \geq \lambda_1$, given that the equilibrium loan rate would adjust and the interbank market would always clear.

When parameters are such that liquidity shortage might occur, i.e., $d > \frac{qpR}{qpR+\pi(1-pR)}$, we need to check first whether the bank can still obtain the optimal choice without the liquidity shortage.

First, the equilibrium in which all GI borrow and all BI sell for $\lambda^* = \lambda_1$ is not feasible, because it is ruled out by the condition guaranteeing liquidity shortage, (1).

Second, $\lambda^* = \lambda_2$ is always feasible, because the illiquid banks do not rely on the secondary market. As long as the neighboring equilibrium, in which some of the BI banks prefer to sell exists ($\lambda_2 > \lambda_{UB}$), the parameters for which the equilibrium with all banks borrowing exists are the same as in the case without liquidity shortage. These parameters change once $\lambda_2 \leq \lambda_{UB}$. Now, because in the neighboring equilibrium all banks earn 0, they stick to borrowing until their profits are zero. That means the equilibrium loan rate, $\frac{[q+(1-\pi)(1-q)p]R}{(q+(1-q)p)(1-\pi(1-q)pR)}$, is not higher than the loan rate is such that their profits are zero, $\frac{1-\lambda_2}{d-\lambda_2}R$.

Third, the choice of λ^* such that all GI and only some BI borrow is still optimal as long as its feasible, $\lambda_2 > \lambda_{UB}$, and it belongs in the interval $(\lambda_{UB}; \lambda_2)$. Indeed one can find such parameters for which the latter interval is not empty.

Fourth, the equilibrium in which only some of the GI banks borrow for $\lambda^* = 0$ exists only if $\lambda_{LB} > 0$. The threshold for optimality of this equilibrium stays the same.

Now we need to find optimal choice of λ such that liquidity shortage might occur.

First, the full liquidity shortage occurs for $\lambda \in [\max[0; \lambda_{FR}]; \min[\lambda_2; \lambda_{UB}])$. When the bank anticipates that for its choice of λ liquidity shortage would occur it maximizes the following expression for the expected return at t=2 (adapted from (15)):

$$\pi \left[q \left[R (1-\lambda) + \widehat{p} R_D \lambda \right] + (1-q) \widehat{p} R_D \left((1-\lambda) P + \lambda \right) \right].$$

The first order condition equalized to zero together with equilibrium values of $R_D^* = R_{D,R} = \frac{1-\lambda}{d-\lambda}R$,

$P^* = P_R$ as given by (11), and $\widehat{p}^* = q + (1-q)\,p$ gives an interior solution for optimal λ. Then this optimal solution has to be in the interval $\lambda \in [\max[0; \lambda_{FR}]\,; \min[\lambda_2; \lambda_{UB}])$ for it to be optimal. This solution is quite a complicated object again but indeed numerical examples can show that there is one.

Second, the banks can choose such λ that the equilibrium price is just sufficient to stay afloat for $\lambda \in [\max[0; \lambda_{LB}]\,; \min[\lambda_2; \lambda_{FR}])$. When the bank anticipates that for its choice of λ liquidity shortage would occur it maximizes the following expression for the expected return at t=2 (adapted from (15)):

$$\pi\left[q\left[R\left(1-\lambda\right)+\widehat{p}R_D\lambda\right]+\left(1-q\right)\widehat{p}R_D\left(\left(1-\lambda\right)P+\lambda\right)\right].$$

The first order condition equalized to zero together with equilibrium values of $R_D^* = \frac{1-\lambda}{d-\lambda}R$, $P^* = \frac{d-\lambda}{1-\lambda}$ and $\widehat{p}^* = \frac{q}{q+(1-q)\gamma_R} + \frac{(1-q)\gamma_R}{q+(1-q)\gamma_R}p$ and γ_R is as given by (12). Then this optimal solution has to be in the interval $\lambda \in [\max[0; \lambda_{LB}]\,; \min[\lambda_2; \lambda_{FR}])$ for it to be optimal. This solution is quite a complicated object again but indeed numerical examples can show that it.

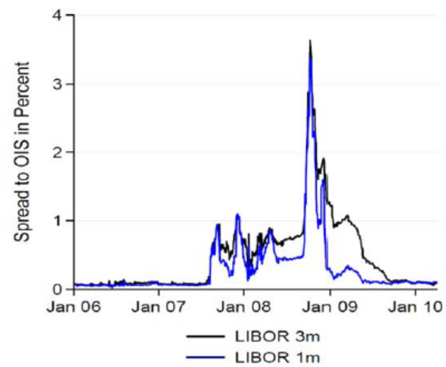

Figure 1: Source: Kuo, Skeie, Youle, and Vickrey (2013). This figure (Figure 1 in Kuo, Skeie, Youle, and Vickrey (2013)) depicts the spread between the 1- and 3-month Libor and OIS.

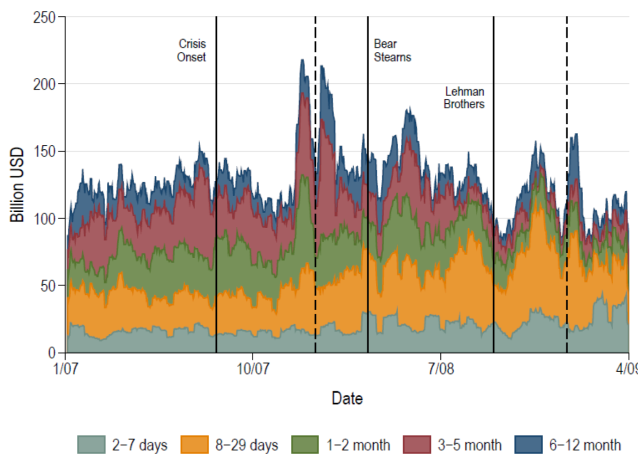

Figure 2: Source: Kuo, Skeie, Youle, and Vickrey (2013). This figure (Figure 5 in Kuo, Skeie, Youle, and Vickrey (2013)) depicts maturity-weighted volume of term interbank loans orginiated between January 2007 and March 2009.

Figures

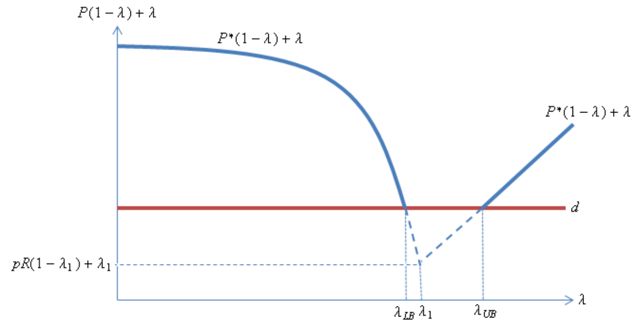

Figure 3: The blue curve above the red d-line is the sum of cash reserves from t=0, λ, and cash generated from selling all of the asset, $P^* (1 - \lambda)$, under the equilibrium price P^* from Proposition 1. The non-monotonicity of the blue curve obtains because of the nature of P^* derived in Lemma 3. The dotted curve below the d-line would be the same sum generated by the equilibrium price P^* if the banks were able to cover the liquidity shortfall by selling all. As can be seen from the figure for $\lambda \in [\lambda_{UB}; \lambda_{LB}]$ the total cash after the sales is not enough to cover the liquidity shortfall for the equilibrium prices derived Proposition 1.

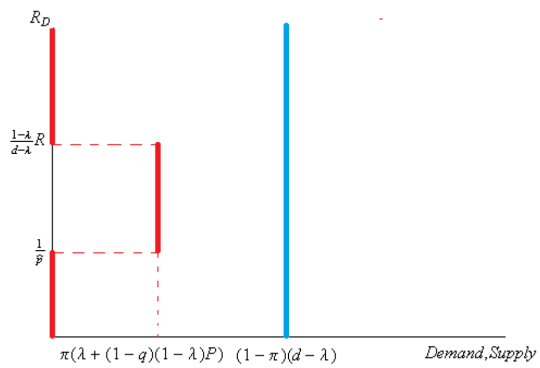

Figure 4: The red line is supply of loans. The blue line is demand for loans, which is set equal to $d - \lambda$ for any $R_D \geq \frac{1-\lambda}{d-\lambda} R$ for exposition purposes.

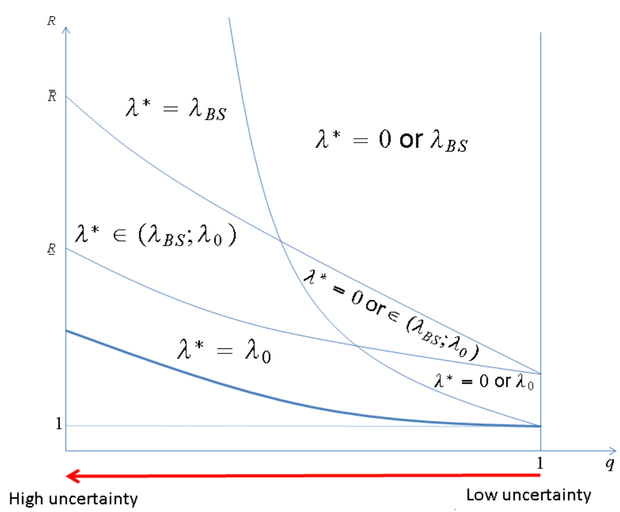

Figure 5: Proposition 3 in case in which there is no credit rationing for any λ.