A Macroeconomic Model with Financially Constrained Producers and Intermediaries *

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Abstract

We propose a model that can simultaneously capture the sharp and persistent drop in macro-economic aggregates and the sharp change in credit spreads observed in the U.S. during the Great Recession. We use the model to evaluate the quantitative effects of macro-prudential policy. The model features borrower-entrepreneurs who produce output financed with long-term debt issued by financial intermediaries and their own equity. Intermediaries fund these loans combining deposits and their own equity. Savers provide funding to banks and to the government. Both entrepreneurs and intermediaries make optimal default decisions. The government issues debt to finance budget deficits and to pay for bank bailouts. Intermediaries are subject to a regulatory capital constraint. Financial recessions, triggered by low aggregate and dispersed idiosyncratic productivity shocks result in financial crises with elevated loan defaults and occasional intermediary insolvencies. Output, balance sheet, and price reactions are substantially more severe and persistent than in non-financial recession. Policies that limit intermediary leverage redistribute wealth from producers to intermediaries and savers. The benefits of lower intermediary leverage for financial and macro-economic stability are offset by the costs from more constrained firms who produce less output.

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1 Introduction

The financial crisis and Great Recession of 2007-09 underscored the importance of the financial system for the broader economy. Borrower default rates, bank insolvencies, government bailouts, and credit spreads all spiked while real interest rates were very low. The disruptions in financial intermediation fed back on the real economy. Consumption, investment, and output all fell substantially and persistently.

These events have caused economists to revisit the role of the financial sector in models of the macro economy. Building on early work that emphasized the importance of endogenous developments in credit markets in amplifying business cycle shocks,[1] a second generation of models has added nonlinear dynamics and a richer financial sector.[2] While a lot of progress has been made in understanding how financial intermediaries affect asset prices and macroeconomic performance, an important remaining challenge is to deliver a quantitatively successful model that can capture the dynamics of financial intermediary capital, asset prices, and the real economy during normal times and credit crises. Such a model requires a government, so that possible crisis responses can be studied, and explicit and implicit government guarantees to the financial sector can be incorporated. Indeed, Central Banks are in search of a model of the financial sector that can be integrated into their existing quantitative macro models. Our paper aims to make progress on this important agenda. It provides a calibrated model that matches key features of the U.S. macroeconomy and asset prices. In addition, it makes three methodological contributions.

First, we separate out the role of producers and banks. The existing literature, as exemplified by the seminal Brunnermeier and Sannikov (2014) paper, combines the roles of financial intermediaries and producers ("experts"). This setup assumes frictionless interaction between banks and borrowers and focuses on the interaction between experts and saving households. It implicitly assumes that financial intermediaries hold equity claims in productive firms. In reality, financial intermediaries make corporate loans and hold corporate bonds which are debt

[1] E.g., Bernanke and Gertler (1989), Bernanke, Gertler, and Gilchrist (1996), Bernanke, Gertler, and Gilchrist (1999), Kiyotaki and Moore (1997), and Gertler and Karadi (2011).

[2] E.g., Brunnermeier and Sannikov (2014), He and Krishnamurthy (2012), He and Krishnamurthy (2013), He and Krishnamurthy (2014), Gârleanu and Pedersen (2011), Adrian and Boyarchenko (2012), Maggiori (2013), Moreira and Savov (2016).

claims.[3] These debt contracts are subject to default risk of the borrowers. Our model has three groups of agents, each with their own balance sheet: savers who lend to intermediaries, entrepreneurs who own the production technology and borrow from intermediaries, and bankers who intermediate between the depositors and entrepreneurs. Intermediaries perform the traditional role of maturity transformation and bear most of the credit risk in the economy. They help to optimally allocate risk across the various agents in the economy. Costly firm bankruptcies endogenously limit the debt capacity of entrepreneurs. In order to discipline banks, we model a Basel-style regulatory capital requirement that limits banks' liabilities at a fraction of their risk-weighted assets. The minimum regulatory capital that banks must hold is a key macroprudential policy parameter.

Our second contribution is to introduce the possibility of default for financial intermediaries. The existing literature is usually cast in continuous time. As the financial sector approaches insolvency, intermediaries reduce risk and prices adjust so that they never go bankrupt. In discrete time, the language of quantitative macroeconomics, the possibility of default of intermediaries cannot be avoided. Far from a technical detail, bank insolvency is an important reality that keeps policy makers up at night. As Reinhart and Rogoff (2009) and Jorda, Schularick, and Taylor (2014) make clear, financial intermediaries frequently become insolvent. When they do, their creditors (mostly depositors) are bailed out by the government. In our model we assume that intermediaries have limited liability and choose to default optimally. When the market value of their assets falls below that of liabilities, the government steps in, liquidates the assets and makes whole their creditors. The banking sector starts afresh the next period with zero wealth. The expectation of a bailout affects banks' risk taking incentives (e.g., Farhi and Tirole (2012)). By allowing for the possibility of bank insolvencies, our model can help explain how a corporate default wave can trigger financial fragility. Vice versa, weak financial balance sheets reduce firms' ability to borrow, invest, and grow.

The third methodological contribution is to endogenize the risk-free interest rate on safe debt. Most models in the intermediary-based macro and asset pricing literature keep the interest rate on safe assets (deposits or government debt) constant, sometimes by virtue of an assumption of risk neutrality of the savers. Once savers are risk averse, a natural assumption

[3] It is well understood that debt-like contracts arise in order to reduce the cost of gathering information and to mitigate principal-agent problems. See for example Dang, Gorton, and Holmstrom (2015).

given that they invest in guaranteed deposits, the dynamics of the model change substantially. In a crisis, intermediaries contract the size of their balance sheet, thereby reducing the supply of safe debt in the economy. Simultaneously, risk averse depositors with strong precautionary savings motives increase their demand for safe assets. As a result, the equilibrium price of safe debt increases substantially. Real interest rates fall sharply. The low cost of debt allows the intermediaries to recapitalize quickly, dampening the effect of the crisis. Put differently, the endogenous price response of safe debt short-circuits the amplification mechanism that arises in a balance sheet recession in partial equilibrium models that hold the interest rate fixed.[4] A partial solution lies in carefully modeling the government side of the model. With counter-cyclical spending and procyclical tax revenues, the government deficit is counter-cyclical. This expands the supply of safe debt in bad times, offsetting the contraction in the supply by the intermediation sector. While rates may still fall in crises, the decline is not as large as it would be without the government sector, and restores the amplification of the balance sheet recession models. Importantly, because the risk averse saver must absorb more debt in bad times, she must reduce spending in high marginal utility states. The ex-ante precautionary savings effect this triggers reduces the unconditional mean interest rate in the economy. While automatic stabilizers in fiscal policy may still be desirable for aggregate welfare, a new insight is that they slow down the recapitalization of banks in a crisis through their general equilibrium effect on the real interest rate.

What results is a rich and quantitatively relevant framework of the interaction between four balance sheets: those of borrower-entrepreneurs, financial intermediaries, saving households, and the government, featuring occasionally binding borrowing constraints for both borrower-entrepreneurs and for intermediaries, and bankruptcy of both borrowers and intermediaries. The model generates amplification whereby aggregate shocks not only directly affect production and investment, but also affect the financial and non-financial sectors' leverage. Tighter financial constraints on banks reduce the availability of credit to firms which hurts investment and output, beyond the effects familiar from standard accelerator models.

[4]One might argue that there are other investors in the market for safe assets whose demand for safe assets may not rise as much because they are less risk averse (maybe institutional investors), but their demand for safe debt would have to be negatively correlated with that of the risk averse savers to offset the effect. Foreigners' demand for U.S. safe debt also increased dramatically in the global financial crisis, further amplifying domestic demand by savers rather than offsetting it.

Our model quantitatively matches the maturity, default risk, and loss-given default of corporate debt. It generates a large and volatile credit spread, again matching the data. The endogenous price of credit risk dynamics amplify the dynamics in the quantity of credit risk. Intermediary wealth fluctuations are behind this resolution of the credit spread puzzle (e.g., Chen (2010)). We use the model to study the differences between regular non-financial recessions and financial recessions, which are recessions that coincide with credit crisis.

Our second main exercise is to investigate the quantitative effects of macro-prudential policies for financial stability, economic growth, economic stability, fiscal stability, and economy-wide welfare. Our model belongs to the class of models where incomplete markets and borrowing constraints create room for macro-prudential policy intervention.[5] We find that while macro-prudential policies improve financial stability and reduce macroeconomic volatility, they also shrink the size of the economy. On net, a reduction in maximum bank leverage has large redistributional consequences shifting wealth from borrowers and savers towards intermediaries. It has modest negative effects on aggregate welfare. Our model offers a quantitative answer to this important policy question.

Our paper provides a state-of-the-art solution technique. The model has two exogenous and persistent sources of aggregate risk. Standard TFP shocks hit the production function. In addition, shocks to the cross-sectional dispersion of idiosyncratic firm productivity govern credit risk. The model also has five endogenous aggregate state variables: the capital stock, corporate debt stock, intermediary net worth, household wealth, and the government debt stock. To solve this complex problem, we provide a nonlinear global solution method, called policy time iteration, which is a variant of the parameterized expectations approach. Policy functions, prices, and Lagrange multipliers are approximated as piecewise linear functions of the exogenous and endogenous state variables. The algorithm solves for a set of nonlinear equations including the Euler equations and the Kuhn-Tucker conditions expressed as equalities.[6]

[5] Other models in this class are Lorenzoni (2008), Mendoza (2010), Korinek (2012), Bianchi and Mendoza (2013), Bianchi and Mendoza (2015), and Guerrieri and Lorenzoni (2015). Farhi and Werning (2016) study macroprudential policy in a model with demand externalities.

[6] One output of this research project will be a set of computer code which will be made publicly available. Discussions with the research department at three different Central Banks indicate that there is a demand for this type of output. Our method improves on existing methods which compute two non-stochastic steady states: one steady state when the constraint never binds and one where it always binds, and then linearizes the solution around both of these states. In this approach, agents inside the model do not take into account the fact that borrowing constraints may become binding in the future due to future shock realizations. As a

The rest of the paper is organized as follows. Section 2 discusses the model setup. Section 3 presents the calibration. Section 4 contains the main results. Section 5 uses the model to study various macro-prudential policies. Section 6 concludes. All model derivations and some details on the calibration are relegated to the appendix.

2 The Model

2.1 Preferences, Technology, Timing

Preferences The model features a government and three groups of households: borrower-entrepreneurs (denoted by superscript B), intermediaries (denoted by superscript I) and savers (denoted by S). Savers are more patient than borrower-entrepreneurs and intermediaries, implying for the discount factors that $\beta_B = \beta_I < \beta_S$. All agents have Epstein-Zin preferences over utility streams $\{u_t^j\}_{t=0}^{\infty}$ with intertemporal elasticity of substitution ν and risk aversion σ.

$$U_t^j = \left\{(1-\beta)\left(u_t^j\right)^{1-1/\nu} + \beta_j \left(\mathrm{E}_t\left[(U_{t+1}^j)^{1-\sigma}\right]\right)^{\frac{1-1/\nu}{1-\sigma}}\right\}^{\frac{1}{1-1/\nu}}, \tag{1}$$

for $j = B, I, S$. Agents derive utility from consumption of the economy's sole good, such that $u_t^j = C_t^j$, for $j = B, I, S$.

Technology Borrower-entrepreneurs own the productive capital stock of the economy and operate its production technology of the form

$$Y_t = Z_t^A \left(K_t\right)^{(1-\alpha)}(Z_t L_t)^{\alpha}, \tag{2}$$

where K_t is capital, L_t is labor, Z_t is labor productivity, and Z_t^A is total factor productivity (TFP). We assume that labor productivity Z_t grows at a deterministic rate μ_G, and TFP fluctuations follow an AR(1) process; Z^A has mean one.

result, the approach ignores agents' precautionary savings motives related to future switches between "regimes" with and without binding constraints. While the piecewise-linear solution may prove sufficiently accurate in some contexts, it remains an open question whether it offers an appropriate solution to models with substantial risk and higher risk aversion, designed to match not only macroeconomic quantities but also asset prices (risk premia). See Guerrieri and Iacoviello (2015) for a nice discussion on these issues.

In addition to the technology for producing consumption goods, borrower-entrepreneurs also have access to a technology that can turn consumption into capital goods subject to adjustment costs.

Borrower-entrepreneurs, intermediaries, and savers are endowed with $\bar{L}^B$, $\bar{L}^I$ and $\bar{L}^S$ units of labor, respectively. We assume that all types of households supply their labor endowment inelastically.

There are two more assets in the economy. One risky long-term bond that borrower-entrepreneurs can issue to intermediaries (corporate loans), and one short-term risk free bond that intermediaries can issue to savers (deposits).

Timing The timing of agents' decisions at the beginning of period t is as follows:

1. Aggregate and idiosyncratic productivity shocks for borrower-entrepreneurs are realized. Production occurs.

2. Intermediaries decide on a bankruptcy policy. In case of a bankruptcy, their financial wealth is set to zero and they incur a utility penalty. At the time of the decision, the magnitude of the penalty is unknown. All agents know its probability distribution, and intermediaries maximize expected utility by specifying a binding decision rule for each possible realization of the penalty.[7]

3. Borrower-entrepreneurs with low idiosyncratic productivity realizations default. Intermediaries assume ownership of bankrupt firms.

4. Intermediaries' utility penalty shock is realized and they follow their bankruptcy decision rule from step 2. In case of intermediary bankruptcy, the government picks up the shortfall in repayments to debt holders (deposit insurance).

5. All agents solve their consumption and portfolio choice problems. Markets clear. All agents consume.

[7]Introducing a random utility penalty is a technical assumption we make for tractability. It makes the value function differentiable and allows us to use our numerical methods which rely on this differentiability. This randomization assumption is common in labor market models (Hansen (1985)). The assumption of making a binding default decision is necessitated in the presence of Epstein-Zin preferences.

Figure 1: Overview of Balance Sheets of Model Agents

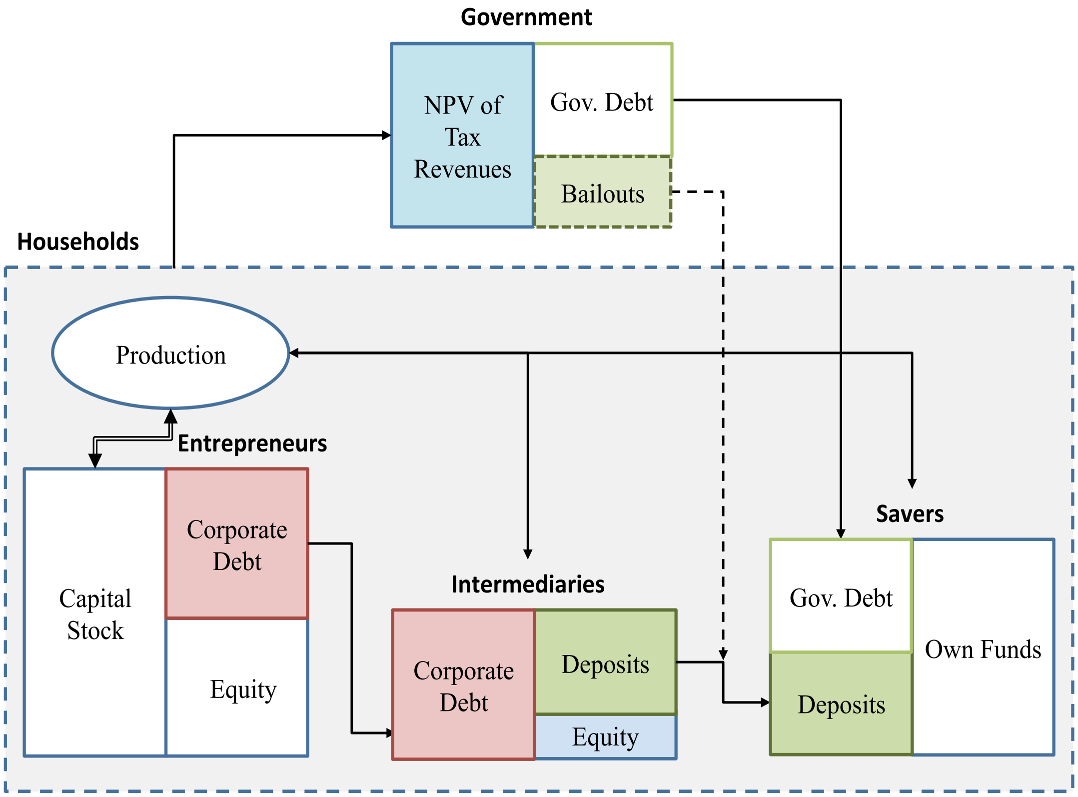


Figure 1 illustrates the balance sheets of the model's agents and their interactions. Each agent's problem depends on the wealth of others; the entire wealth distribution is a state variable. Each agent must forecast how that state variable evolves, including the bankruptcy decisions of borrowers and intermediaries. We now describe each of the three types of household problems and the government problem in detail.

2.2 Borrower-Entrepreneurs' Problem

There is a unit-mass of identical borrower-entrepreneurs indexed by i. The households form a large collective ("family") that provides partial insurance against idiosyncratic shocks.

Each entrepreneur has access to a technology that creates consumption goods $Y_{i,t}$ from capital $K_{i,t}$ and labor $L_{i,t}$. At the beginning of the period, each entrepreneur receives an idiosyncratic productivity shock $\omega_{i,t} \sim F_{\omega,t}$, distributed independently over time. Output depends on aggregate productivity (Z_t^A, Z_t) and idiosyncratic productivity ω_{i_t}:

$$Y_{i,t} = \omega_{i,t} Z_t^A K_{i,t}^{1-\alpha} (Z_t L_{i,t})^\alpha.$$

While each individual entrepreneur manages her own production, the family of borrower-entrepreneurs manages the allocation of production inputs and consumption. Further, the family collectively issues debt to intermediaries. The debt is long-term, modeled as perpetuity bonds. Bond coupon payments decline geometrically, $\{1, \delta, \delta^2, \ldots\}$, where δ captures the duration of the bond. We introduce a "face value" $F = \frac{\theta}{1-\delta}$, a fixed fraction θ of all repayments for each bond issued. Per definition, interest payments are the remainder $\frac{1-\theta}{1-\delta}$.

At the beginning of the period, the family jointly holds K_t^B units of capital, and has A_t^B bonds outstanding. In addition, producers jointly hire their own labor and the labor of intermediaries and savers, denoted by L_t^j, with $j = B, I, S$. As payment each group receives a competitive wage w_t^j per unit of labor. During production, the labor inputs of the three types are combined into aggregate labor:

$$L_t = (L_t^B)^{1-\gamma_S-\gamma_I} (L_t^S)^{\gamma_S} (L_t^I)^{\gamma_I}.$$

Before idiosyncratic productivity shocks are realized, each producer is given the same amount of capital and labor for production, such that $K_{i,t} = K_t^B$ and $L_{i,t} = L_t$. Further, each producer is responsible for repaying the coupon on an equal share of the total debt, $A_{i,t} = A_t^B$.

The individual profit of producer i is therefore given by

$$\pi_{i,t} = \omega_{i,t} Z_t^A (K_t^B)^{1-\alpha} (Z_t L_t)^\alpha - \sum_j w_t^j L_t^j - A_t^B. \tag{3}$$

After production, each producer who achieves a sufficiently high profit, $\pi_{i,t} \geq \underline{\pi}$, returns this profit to the family, where $\underline{\pi}$ is a parameter. Further, capital depreciates during production by fraction δ_K, and individual members with positive profit return the depreciated capital after production. Producers with $\pi_{i,t} < \underline{\pi}$ default on the share of debt they were allocated. The debt is erased, and the intermediary takes ownership of the bankrupt firm, including its share of the capital stock. The intermediary liquidates the bankrupt firms' capital, seizes their output, and pays their wage bill. The remaining funds are the intermediary's recovery value. In return for production, each family member receives the same amount of consumption goods $C_{i,t} = C_t^B$.

From (3), it immediately follows that there exists a cutoff productivity shock

$$\omega_t^* = \frac{\underline{\pi} + \sum_{j=B,I,S} w_t^j L_t^j + A_t^B}{Z_t^A (K_t^B)^{1-\alpha} (Z_t L_t)^\alpha}, \tag{4}$$

such that all entrepreneurs receiving productivity shocks below this cutoff default on their debt.

Using the threshold level ω_t^*, we define $\Omega_A(\omega_t^*)$ to be the fraction of debt *repaid* to lenders and $\Omega_K(\omega_t^*)$ to be the average productivity of the firms that do not default:

$$\Omega_A(\omega_t^*) = \Pr[\omega_{i,t} \geq \omega_t^*], \tag{5}$$

$$\Omega_K(\omega_t^*) = \Pr[\omega_{i,t} \geq \omega_t^*] \; \mathrm{E}[\omega_{i,t} \,|\, \omega_{i,t} \geq \omega_t^*]. \tag{6}$$

After making a coupon payment of 1 per unit of remaining outstanding debt, the amount of outstanding debt declines to $\delta \Omega_A \left(\omega_t^*\right) A_t^B$.

The profit of the producers' business is subject to a corporate profit tax with rate τ_Π^B. The profit for tax purposes is defined as sales revenue net of labor expenses, and capital depreciation and interest payments of non-bankrupt producers:[8]

$$\Pi_t^{B,\tau} = \Omega_K(\omega_t^*) Z_t^A (K_t^B)^{1-\alpha} (Z_t L_t)^\alpha - \Omega_A(\omega_t^*) \left(\sum_j w_t^j L_t^j + \delta_K p_t K_t^B + (1-\theta) A_t^B \right).$$

The fact that interest expenditure $\Omega_A(\omega_t^*)(1-\theta)A_t^B$ and capital depreciation $\delta_K p_t K_t^B$ are deducted from taxable profit creates a "tax shield" and hence a preference for debt funding.

In addition to producing consumption goods, producers jointly create capital goods from consumption goods. In order to create X_t new capital units, the required input of consumption goods is

$$X_t + \Psi(X_t / K_t^B) K_t^B, \tag{7}$$

with adjustment cost function $\Psi(\cdot)$ which satisfies $\Psi''(\cdot) > 0$, $\Psi(\mu_G + \delta_K) = 0$, and $\Psi'(\mu_G + \delta_K) = 0$.

[8] Aggregate producer profit is the integral over the idiosyncratic profit (3) of non-defaulting producers, net of capital depreciation expenses and adding back principal payments θA_t^B which are not tax deductible.

The borrower-entrepreneur family's problem is to choose consumption C_t^B, capital for next period K_{t+1}^B, new debt A_{t+1}^B, investment X_t and labor inputs L_t^j to maximize life-time utility U_t^B in (1), subject to the budget constraint:

$$C_t^B + X_t + \Psi(X_t/K_t^B)K_t^B + \Omega_A(\omega_t^*)A_t^B(1+\delta q_t^m) + p_t K_{t+1}^B + \Omega_A(\omega_t^*)\sum_{j=B,I,S} w_t^j L_t^j + \tau_\Pi^B \Pi_t^{B,\tau}$$
$$\leq \Omega_K(\omega_t^*)Z_t^A(K_t^B)^{1-\alpha}(Z_t L_t)^\alpha + (1-\tau_t^B)w_t^B\bar{L}^B + p_t(X_t + \Omega_A(\omega_t^*)(1-\delta_K)K_t^B) + q_t^m A_{t+1}^B + G_t^{T,B} + O_t^B, \tag{8}$$

and a leverage constraint:

$$FA_{t+1}^B \leq \Phi p_t(1-(1-\tau_\Pi^B)\delta_K)\Omega_A(\omega_t^*)K_t^B. \tag{9}$$

The borrower household uses output, after-tax labor income, sales of old (K_t^B) and newly produced (X_t) capital units, new debt raised ($q_t^m A_{t+1}^B$), where q_t^m is the price of one bond in terms of the consumption good, transfer income from the government ($G_t^{T,B}$), and transfer income from bankruptcy proceedings (O_t^B) to be defined below. These resources are used to pay for consumption, investment including adjustment costs, debt service, new capital purchases, wages, and corporate taxes.

The borrowing constraint in (9) caps the face value of debt at the end of the period, FA_{t+1}^B, to a fraction of the market value of the available capital units after default and depreciation, $p_t(1-(1-\tau_\Pi^B)\delta_K)\Omega_A(\omega_t^*)K_t^B$, where Φ is the maximum leverage ratio. With such a constraint, declines in capital prices (in bad times) tighten borrowing constraints. The constraint (9) imposes a hard upper bound on borrower leverage. In addition, costly defaults of individual borrowers who received bad idiosyncratic shocks, endogenously limit the optimal leverage of borrowers. Borrowers take into account that each marginal unit of debt issued in t increases costly defaults in $t+1$. Therefore, for a high enough maximum leverage ratio Φ, constraint (9) will never be binding.

2.3 Savers

Savers can invest in one-period risk free bonds (deposits and government debt). They inelastically supply their unit of labor $\bar{L}^S$. Entering with wealth W_t^S, the saver's problem is to choose consumption C_t^S and short-term bonds B_t^S to maximize life-time utility U_t^S in (1), subject to the budget constraint:

$$C_t^S + q_t^f B_t^S \leq W_t^S + (1 - \tau_t^S) w_t^S \bar{L}^S + G_t^{T,S} + O_t^S \tag{10}$$

and a short-sale constraints on bond holdings:

$$B_t^S \quad \geq 0. \tag{11}$$

The budget constraint (10) shows that saver uses after-tax labor income, net transfer income, and beginning-of-period wealth to pay for consumption, and purchases of short-term bonds

2.4 Intermediaries

After aggregate and idiosyncratic productivity shocks have been realized, financial intermediaries choose whether or not to declare bankruptcy. Intermediaries who declare bankruptcy have all their assets and liabilities liquidated. They also incur a stochastic utility penalty ρ_t, with $\rho_t \sim F_\rho$, i.i.d. over time and independent of all other shocks. At the time of the bankruptcy decision, intermediaries do not yet know the realization of the bankruptcy penalty. Rather, they have to commit to a bankruptcy decision rule $D(\rho) : \mathbb{R} \to \{0, 1\}$, that specifies the optimal decision for every possible realization of ρ_t. Intermediaries choose $D(\rho)$ to maximize expected utility at the beginning of the period. We conjecture and later verify that the optimal default decision is characterized by a threshold level ρ_t^*, such that intermediaries default for all realizations for which the utility cost is below the threshold. The utility penalty is a computational device to "convexify" the intermediary value function.

After the realization of the penalty, intermediaries execute their bankruptcy choice according to the decision rule. They then face a consumption and portfolio choice problem to be described below. First, while intertemporal preferences are still specified by equation (1), intraperiod utility u_t^j depends on the bankruptcy decision and penalty:

$$u_t^I = \frac{C_t^I}{\exp\left(D(\rho_t)\rho_t\right)}.$$

Intermediaries' portfolio choice consists of loans to borrower-entrepreneurs (A_t^I) and short-term bonds (B_t^I). Loans are modeled as bonds aggregating the debt of the borrowers. The coupon payment on performing loans in the current period is $A_t^I \Omega_A(\omega_t^*)$. For borrower-entrepreneurs that default and enter into foreclosure, the intermediaries repossess their firms, including this period's output, as collateral. Intermediaries must pay the wages owed by the defaulting firms, a senior claim. Payments on defaulted bonds are:

$$M_t = (1-\zeta)\left[(1-\Omega_A(\omega_t^*))(1-\delta_K)p_t K_t^B + (1-\Omega_K(\omega_t^*))(K_t^B)^{1-\alpha}L_t^\alpha\right] - (1-\Omega_A(\omega_t^*))\sum_j w_t^j L_t^j, \tag{12}$$

where ζ is the fraction of capital value and output destroyed in bankruptcy. A fraction η of this joint capital and output loss from bankruptcy is a deadweight loss to society while the remainder is rebated to the households in proportion to their population shares; these are the O_t^i terms in the budget constraints:

$$\sum_{i=B,S,I} O_t^i = (1-\eta)(1-\zeta)\left[(1-\Omega_A(\omega_t^*))(1-\delta_K)p_t K_t^B + (1-\Omega_K(\omega_t^*))(K_t^B)^{1-\alpha}L_t^\alpha\right]$$

This can be interpreted as income payments to the actors involved in bankruptcy cases.

Thus, the total (performing and defaulting) payoff per unit of the bond is $\Omega_A(\omega_t^*) + M_t/A_t^B$. The price per unit of the bond is q_t^m.

In addition, intermediaries can trade in short-term bonds with savers and the government. They are allowed to take a short position in these bonds, using their loans to borrower-entrepreneurs as collateral. Intermediary debt is subject to a leverage constraint:

$$-B_t^I \leq q_t^m \xi A_{t+1}^I. \tag{13}$$

A negative position in the short-term bond is akin to intermediaries issuing deposits. The negative position in the short-term bond must be collateralized by the market value of intermediaries' holdings of long-term loan bonds. The parameter ξ determines how useful loans are as collateral. The constraint (13) is a Basel-style regulatory capital constraint. The parameter ξ is the key macro-prudential policy parameter in the paper.

Denote the wealth (net worth) of an intermediary that did not go into bankruptcy by:

$$W_t^I = \Omega_A(\omega_t^*)(1+\delta q_t^m)A_t^I + M_t + B_{t-1}^I \tag{14}$$

Intermediaries are subject to corporate profit taxes at rate τ_Π^I. Their profit for tax purposes is defined as the net interest income on their loan business:[9]

$$\Pi_t^I = (1-\theta)\Omega_A(\omega_t^*)A_t^I + r_t^f B_{t-1}^I.$$

Intermediaries' also receive after-tax income for supplying their labor to borrower-entrepreneurs, from government transfers, and from bankruptcy transfers. They further need to pay a deposit insurance fee (κ) to the government that is proportional to the amount of short-term bonds they issue. Their budget constraint is:

$$C_t^I + q_t^m A_{t+1}^I + (q_t^f + \mathrm{I}_{\{B_t^I<0\}}\kappa)B_t^I + \tau_\Pi^I \Pi_t^I \leq (1-D(\rho_t))W_t^I + (1-\tau^I)w_t^I \bar{L}^I + G_t^{T,I} + O_t^I. \tag{15}$$

Note that intermediaries only receive wealth W_t^I if they do not declare bankruptcy at the beginning of the period; in case of bankruptcy their wealth is (reset by the government at) zero.

2.5 Government

The actions of the government are determined via fiscal rules: taxation, spending, bailout, and debt issuance policies. Government tax revenues, T_t, are labor income tax, corporate profit tax, and deposit insurance fee receipts:

$$T_t = \sum_{j=B,I,S} \tau_t^j w_t^j L_t^j + \tau_\Pi^B \Pi_t^B + \tau_\Pi^I \Pi_t^I - \mathrm{I}_{\{B_t^I<0\}}\kappa B_t^I$$

[9]We define the risk free interest rate as the yield on risk free bonds, $r_t^f = 1/q_t^f - 1$.

Government expenditures, G_t are the sum of exogenous government spending, G_t^o, transfer spending G_t^T, and financial sector bailouts:

$$G_t = G_t^o + \sum_{j=B,I,S} G_t^{T,j} - D(\rho_t) W_t^I$$

The bailout to the financial sector equals the negative of the financial wealth of intermediaries, W_t^I, in the event of a bankruptcy.

The government issues one-period risk-free debt. Debt repayments and government expenditures are financed by new debt issuance and tax revenues, resulting in the budget constraint:

$$B_{t-1}^G + G_t \leq q_t^f B_t^G + T_t \tag{16}$$

We impose a transversality condition on government debt:

$$\lim_{u \to \infty} \mathrm{E}_t \left[\tilde{\mathcal{M}}_{t,t+u}^S B_{t+u}^G \right] = 0$$

where $\tilde{\mathcal{M}}^S$ is the SDF of the saver.[10] Because of its unique ability to tax, the government can spread out the cost of default waves and financial sector rescue operations over time.

Government policy parameters are $\Theta_t = \left(\tau_t^i, \tau_\Pi^i, G_t^o, G_t^{T,i}, \kappa, \Phi, \xi \right)$. The parameters ϕ in equation (9) and ξ in equation (13) can be thought of as macro-prudential policy tools. One could add the parameters that govern the utility cost of bankruptcy of intermediaries to the set of policy levers, since the government may have some ability to control the fortunes of the financial sector in the event of a bankruptcy.

2.6 Equilibrium

Given a sequence of aggregate productivity shocks $\{Z_t^A\}$, idiosyncratic productivity shocks $\{\omega_{t,i}\}_{i \in B}$, and utility costs of default shocks ρ_t, and given a government policy Θ_t, a competitive equilibrium is an allocation $\{C_t^B, K_{t+1}^B, X_t, A_{t+1}^B, L_t^j\}$ for borrower-entrepreneurs, $\{C_t^S, B_t^S\}$ for

[10] We show below that the risk averse saver is the marginal agent for short-term risk-free debt. In the numerical work below, we keep the ratio of government debt to GDP contained between $\underline{b^G}$ and $\overline{b^G}$ by decreasing taxes linearly when the debt-to-GDP threatens to fall below $\underline{b^G}$ and raising taxes linearly when debt-to-GDP threatens to exceed $\overline{b^G}$.

savers, $\{C_t^I, A_{t+1}^I, B_t^I\}$ for intermediaries, bankruptcy rule $D(\rho_t)$, and a price vector $\{p_t, q_t^m, q_t^f\}$, such that given the prices, borrower-entrepreneurs, savers, and intermediaries maximize lifetime utility subject to their constraints, the government satisfies its budget constraint, and markets clear.

The market clearing conditions are:

1. Risk-free bonds:
$$B_t^G = B_t^S + B_t^I \tag{17}$$

2. Loans: $A_{t+1}^B = A_{t+1}^I$

3. Capital: $K_{t+1}^B = (1-\delta_K)K_t^B + X_t$

4. Labor: $L_t^i = \bar{L}^j$ for all $i = B, I, S$

5. Consumption:

$$\begin{aligned} Y_t =& (C_t^B + C_t^I + C_t^S) + G_t^o + \underbrace{X_t + K_t^B \Psi(X_t/K_t^B)}_{\text{INV}} \\ &+ \underbrace{\eta\zeta\left[(1-\Omega_A(\omega_t^*))(1-\delta_K)p_t K_t^B + (1-\Omega_K(\omega_t^*))(K_t^B)^{1-\alpha} L_t^\alpha\right]}_{\text{DWL}} \end{aligned}$$

The last equation is the economy's resource constraint. It states that total output (GDP) equals the sum of aggregate consumption, discretionary government spending, and investment (INV), and deadweight costs (DWL) incurred when liquidating bankrupt firms, a fraction $\eta\zeta$ of the capital and output of defaulting firms.

2.7 Welfare

In order to compare economies that differ in the policy parameter vector Θ_t, we must take a stance on how to weigh the different agents. We propose a utilitarian social welfare function summing value functions of the agents

$$\mathcal{W}_t(\cdot; \Theta_t) = V_t^B + V_t^S + V_t^I,$$

where the $V^j(\cdot)$ functions are the value functions defined in the appendix. The value functions already incorporate the mass of agents of each type (population shares ℓ^i).[11]

2.8 Model without Intermediation Sector

We also consider a simplified version of the model without intermediaries. We refer to this version as the "consolidated balance sheet" (CBS) model, since it involves merging borrowers and intermediaries into one group of agents, which for simplicity we still call borrowers. In the CBS model, borrowers can directly issue risk-free deposits to savers.[12] Instead of issuing defaultable corporate loans to intermediaries, they now choose holdings B_t^B of risk-free debt subject to the budget constraint:

$$\begin{aligned} C_t^B + X_t + \Psi(X_t/K_t^B)K_t^B + (q_t^f + \mathrm{I}_{\{B_t^B<0\}}\kappa)B_t^B + p_t K_{t+1}^B + \sum_{j=B,I,S} w_t^j L_t^j + \tau_\Pi^B \Pi_t^{B,\tau} \\ \leq Z_t^A (K_t^B)^{1-\alpha}(Z_t L_t)^\alpha + (1-\tau_t^B) w_t^B \bar{L}^B + B_{t-1}^B + p_t(X_t + (1-\delta_K)K_t^B) + G_t^{T,B} + O_t^B, \end{aligned} \tag{18}$$

and are subject to the leverage constraint:

$$-B_t^B \leq \Xi p_t (1-(1-\tau_\Pi^B)\delta_K) K_t^B,$$

where parameter Ξ limits the amount of risk-free debt borrowers can issue. We redefine corporate profits as

$$\Pi_t^{B,\tau} = Z_t^A (K_t^B)^{1-\alpha}(Z_t L_t)^\alpha - \left(\sum_j w_t^j L_t^j + \delta_K p_t K_t^B\right) + r_t^f B_{t-1}^B$$

to reflect the different tax shield. An important difference with the benchmark is that credit frictions associated with corporate debt, and therefore the cross-sectional productivity shocks

[11] Equivalently, we could first express the value functions per capita by scaling them by their population weights, and then calculating a population-weighted average of the per capita value functions.

[12] Given the difference in patience, borrowers will issue debt to savers. We assume that this debt is insured by the government in the same fashion as the intermediaries' debt in the benchmark version of the model. However, for realistic levels of corporate leverage (< 50%), aggregate government bailouts will play no role in the CBS model.

$\omega_{i,t}$, are no longer relevant in the CBS model.

3 Calibration

The model is calibrated at annual frequency. The parameters of the model and their targets are summarized in Table 1.

Aggregate Productivity Labor-augmenting productivity grows at a deterministic rate of μ_G equal to 2.0% per year, in order to match observed average GDP growth of 2.0% per year. Following the macro-economics literature, the TFP process Z_t^A follows an AR(1) in logs with persistence parameter ρ_A and innovation volatility σ^A. Because TFP is persistent, it becomes a state variable. We discretize g_t into a 5-state Markov chain using the Rouwenhorst (1995) method. The procedure chooses the productivity grid points and the transition probabilities between them to match the volatility and persistence of HP-detrended GDP. The latter is endogenously determined but heavily influenced by TFP. Consistent with the model, our measurement of GDP excludes net exports, housing investment, changes in inventories, and government investment. We define the GDP deflator correspondingly. Observed real per capita HP-detrended GDP has a volatility of 2.13% and its persistence is 0.68. The model generates a volatility of 2.24% and a persistence of 0.76.

Idiosyncratic Productivity We calibrate the firm-level productivity risk directly to the micro evidence. We normalize the mean of idiosyncratic productivity at $\mu_\omega = 1$. We let the cross-sectional standard deviation of idiosyncratic productivity shocks $\sigma_{t,\omega}$ follow a 2-state Markov chain. Fluctuations in $\sigma_{t,\omega}$ are the second source of aggregate risk. Fluctuations in $\sigma_{t,\omega}$ govern aggregate corporate credit risk since high levels of $\sigma_{t,\omega}$ cause a larger left tail of low-productivity firms that default. We refer to states with the high value for $\sigma_{t,\omega}$ as *high uncertainty* periods. We set $(\sigma_{L,\omega}, \sigma_{H,\omega}) = (0.095, 0.175)$. The value for $\sigma_{L,\omega}$ targets the unconditional mean corporate default rate. The model-implied average default rate of 2.4% is similar to the data.[13]

[13]We look at two sources of data: corporate loans and corporate bonds. From the Flow of Funds, we obtain delinquency and charge-off rates on Commercial and Industrial loans and Commercial Real Estate loans by U.S. Commercial Banks for the period 1991-2015. The average delinquency rate is 3.1%. The second source of data is Standard & Poors' default rates on publicly-rated corporate bonds for 1981-2014. The average default rate

Table 1: Calibration

Par	Description	Value	Target
		Exogenous Shocks	
μ_G	mean growth	2.0%	Mean rpc GDP gr 53-14 of 2.00%
ρ_A	persistence TFP	0.65	AC(1) HP-detr GDP 53-14 of 0.68
σ_A	innov. vol. TFP	1.9%	Vol HP-detr GDP 53-14 of 2.13%
$\sigma_{\omega,L}$	low uncertainty	0.095	Avg. corporate default rate
$\sigma_{\omega,H}$	high uncertainty	0.175	Avg. IQR firm-level productivity
p^{ω}_{LL}, p^{ω}_{HH}	transition prob	{0.91, 0.80}	Bloom et al. (2012)
		Production, Population, Labor Income Shares	
ψ	marginal adjustment cost	2	Vol. investment-to-GDP ratio 53-14 of 1.23%
α	labor share in prod. fct.	0.71	Labor share of output of 2/3
δ_K	capital depreciation rate	10%	Justiniano, Primiceri, and Tambalotti (2010)
ℓ^i	pop. shares $i \in \{S, B, I\}$	{69,28.3,2.7}%	Population shares SCF 95-13, QCEW 01-15
γ^i	inc. shares $i \in \{S, B, I\}$	{60,37.4,2.6}%	Labor inc. shares SCF 95-13, QCEW 01-15
		Corporate loans	
δ	average life loan pool	0.937	Duration fcn. in App. B.1
θ	principal fraction	0.582	Duration fcn. in App. B.1
ζ	Losses in bankruptcy	0.5	Corporate loan and bond severities 81-15
η	% bankr. loss is DWL	0.2	Bris, Welch, and Zhu (2006)
Φ	maximum LTV ratio	0.45	Vol. of risk free rate 85-14
$\underline{\pi}$	profit default threshold	0.04	FoF non-fin sector leverage 85-14
		Preferences	
$\beta^B = \beta^I$	time discount factor B, I	0.95	FoF fin sector leverage 85-14 of 90%
σ	risk aversion B, I, S	1	Log utility
ν	IES B, I, S	1	Relative volatility of aggr. cons and GDP
β^S	time discount factor S	0.995	Mean risk-free rate 85-14
		Government Policy	
G^o	discr. spending	17.17%	BEA discr. spending to GDP 53-14 of 17.58%
G^T	transfer spending	2.42%	BEA transfer spending to GDP 53-14 of 3.18%
τ	labor income tax rate	28.0%	BEA pers. tax rev. to GDP 53-14 of 17.30%
τ_{Π}	corporate tax rate	21.7%	BEA corp. tax rev. to GDP 53-14 of 3.41%
b_o	cyclicality discr. spending	-2.9	slope log discr. sp./GDP on GDP growth
b_T	cyclicality transfer spending	-27	slope log transfer sp./GDP on GDP growth
b_τ	cyclicality lab. inc. tax	2.6	slope log discr. sp./GDP on GDP growth
κ	deposit insurance fee	0	Deposit insurance fee 97-06
ξ	max. intermediary leverage	0.95	Basel II reg. capital charge for C&I loans
σ_ρ	utility cost bankruptcy	10%	Technical assumption

The high value, $\sigma_{H,\omega}$, is chosen to match the time-series standard deviation of the cross-sectional interquartile range of firm productivity, which is 4.9% according to Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) (their Table 6).

The transition probabilities from the low to the high uncertainty state of 9% and from the high to the low state of 20% are also taken directly from Bloom et al. (2012).[14] The model spends 31% of periods in the high uncertainty regime. Like in Bloom et al., our uncertainty process is independent of the first-moment shocks. About 10% of periods feature both high uncertainty and low TFP realizations. We will refer to those periods as financial recessions or financial crises. Using a long time series for the U.S., Reinhart and Rogoff (2009) find a similar 10% frequency of financial crises.

Production Adjustment costs are quadratic. We set the marginal adjustment cost parameter $\psi = 2$ in order to match the observed volatility of the ratio of investment to GDP, X/Y, of 1.23%. The model generates a value of 1.19%. The adjustment costs are a tiny 0.04% of GDP in the steady state. We set the parameter α in the Cobb-Douglas production function equal to 0.71, which yields an overall labor income share of 66.5%, the standard value in the business cycle literature. We choose δ_K to match an annual depreciation of capital of 10%, a typical value used for example in Justiniano, Primiceri, and Tambalotti (2010).

Population and Labor Income Shares To pin down the population shares of our three different types of households we turn to the Survey of Consumer Finance (SCF). We define savers as those households who hold a low share of their wealth in the form of risky assets. In particular, we compute for each household in the survey the share of assets, net of all real estate, held in stocks or private business equity, considering both direct and indirect holdings of stock. Using this definition of the risky share, we then calculate the fraction of households whose risky share is less than one percent.[15] This amounts to 69% of SCF households. The remaining 31% of households have a large risky asset share. We split them into 28.3% borrowers-entrepreneurs

is 1.5%; 0.1% on investment-grade bonds and 4.1% on high-yield bonds. The model is in between these two values.

[14]They estimate a two-state Markov chain for the cross-sectional standard deviation of establishment-level productivity using annual data for 1972-2010 from the Census of Manufactures and Annual Survey of Manufactures. We annualize their quarterly transition probability matrix.

[15]We use all survey waves from 1995 until 2013 and average across them.

and 2.7% financial intermediaries based on the share of employees that work in the financial sector, defined as "Securities, Investments" and "Credit Intermediation" from the Quarterly Census of Employment and Wages (QCEW), averaged over the longest available sample 2001-2015.

From the same QCEW data, we obtain the wage share for the intermediaries of 2.6%. The labor income share of savers in the SCF is 60%. The income share of the borrower-entrepreneurs is the remaining 37.4%. The income shares determine the Cobb-Douglas parameters γ_I, γ_B, and γ_S. By virtue of the calibration, the model matches basic aspects of the observed income distribution. It also matches the size of the U.S. intermediary sector.[16]

Corporate Loans In the model, a corporate loan is a geometric bond. The issuer of one bond at time t promises to pay 1 at time $t+1$, δ at time $t+2$, δ^2 at time $t+3$, and so on. Given that the present value of all payments $(1/(1-\delta))$ can be thought of as the sum of a principal (share θ) and an interest component (share $1-\theta$), we define the book value of the debt as $F = \theta/(1-\delta)$. This book value of debt is used in the firm's collateral constraint. We set $\delta = 0.937$ and $\theta = 0.582$ ($F = 9.238$) to match the observed duration of corporate bonds. Appendix B.1 contains the details. The model's corporate loans have a duration of 7 years on average.

As in standard trade-off theory, corporate debt enjoys a tax shield but incurs costs of distress. We set the $\zeta = 0.5$ to match the observed average severity rate of 44% on bonds rated by S&P and Moody's rated during 1985-2004. The model produces a similar unconditional loss-given default of 42%. Combined with the average default rate, this LGD number implies a loss rate on corporate loans of 1.1%. Our baseline model generates a modest quantity of corporate default risk, consistent with the data.

A fraction η of the cost of distress to intermediaries is a deadweight loss to the economy. The remainder $1-\eta$ is transfer income that enters in the budget constraint of the agents. We set $\eta = 0.2$ based on evidence in Bris, Welch, and Zhu (2006). The resulting deadweight losses

[16]Intermediaries' labor income is 2.6% of total labor income and 1.73% of GDP pre-tax and 1.25% after-tax. After-tax profits are an additional 2.07% of GDP. Total after-tax intermediary income is 3.32% of GDP in the model. The market value of intermediated assets is 84.6% of GDP. Thus, intermediary income is 3.92% of intermediated assets. Intermediary profits are 2.45% of intermediated assets. Philippon (2015) reports that the cost of financial intermediation has historically been about 2% of intermediated assets.

of default average to 0.54% of GDP in the benchmark model.

Borrowers can obtain a loan with principal value up to a fraction Φ of the market value of their assets. We set the maximum LTV ratio parameter $\Phi = 0.45$. This value is just large enough so that the LTV constraint never binds during expansions (it rarely binds during non-financial recessions). In the simulation of our benchmark model, the borrower's LTV constraint binds in 20% of financial recessions. The LTV constraint limits corporate borrowing as a fraction of the market value of capital. We found that the presence of the capital price (Tobin's q) in the infrequently binding constraint is one of the main amplification mechanisms of the model, causing sharper financial recessions when firms become constrained. The volatility of the risk free rate is a direct indicator of the severity of these episodes in our model, since risk free rates fall sharply during financial recessions. The model produces a risk free rate volatility of 2.34%, in line with data estimates.[17] We note that the volatility of the real interest rate is only 1.1% in the model if we exclude the financial recessions.

We set the profit default threshold to $\underline{\pi} = 0.04$ to target non-financial leverage. The higher this threshold, the more firms will default on average for a given level of firm debt. Since defaults are costly to the borrower family, borrower leverage is decreasing in $\underline{\pi}$. The model generates a ratio of borrower book debt-to-assets of 42%. In the Flow of Funds data, the average ratio of loans and debt securities of the nonfinancial corporate and nonfinancial noncorporate businesses to their non-financial assets is 37%, a slightly lower value.[18]

Preference Parameters Preference parameters affect many equilibrium quantities and prices simultaneously, and are harder to pin down directly by data. In order to highlight the separate roles of intermediaries' and firms' balance sheets, we purposely set the time discount factor and the risk aversion coefficient of borrowers and intermediaries equal. We assume that all households have log utility: $\sigma = \nu = 1$. We set the EIS $\nu = 1$ in order to approximately match the relative volatility of aggregate consumption to that of output. The model currently produces consumption volatility that is slightly lower than in the data. Since a higher EIS leads

[17] To calculate the real rate, we take the nominal one year constant maturity Treasury yield (FRED) and subtract expected inflation over the next 12 months from the Survey of Professional Forecasters for the sample 1985-2014.

[18] For the Flow of Funds leverage data, we use the post-1987 sample. Only in this sample is nonfinancial leverage stationary. Our model certainly misses some reasons for firms to hold more cash (negative debt) such as international tax reasons.

to consumption that is more volatile relative to output, we could use a higher EIS to match this volatility; however, for simplicity, we use log utility.

The subjective time discount factors $\beta_B = \beta_I = 0.95$ target financial sector leverage. The average ratio of total intermediary debt-to-assets for 1985-2014 is 90.7%.[19] The model generates average intermediary debt-to-asset ratio of 91.5% (debt evaluated at book value, assets at market value).

The time discount factor of the saver disproportionately affects the mean of the short-term interest rate. We set $\beta^S = 0.995$ to generate a low average real rate of interest of 2.65%.

Government Parameters To add quantitative realism to the model, we match both the unconditional average and the cyclical properties of discretionary spending, transfer spending, labor income tax revenue, and corporate income tax revenue.

Discretionary and transfer spending as a fraction of GDP are modeled as follows: $G_t^i/Y_t = G^i \exp\{b_i(g_t - \bar{g})\}, i = o, T$. The scalars G^o and G^T are set to match the observed average discretionary spending to GDP of 17.58% in the 1953-2014 NIPA data, and transfer spending to GDP of 3.18%, respectively.[20] We set $b_o = -2.9$ and $b_T = -27$ in order to match the slope in a regression of log spending to GDP on GDP growth and a constant. We match these slopes: -0.81 and -7.57 in the model versus -0.75 and -7.26 in the 1953-2014 data.

Similarly, we model the labor income tax rate as $\tau_t = \tau \exp\{b_\tau(g_t - \bar{g})\}$. We set the tax rate $\tau = 28.0\%$ in order to match observed average income tax revenue to GDP of 17.3%.[21]

[19] Krishnamurthy and Vissing-Jorgensen (2015) identify a group of financial institutions as net suppliers of safe, liquid assets. This group contains U.S. Chartered Commercial Banks and Savings Institutions, Foreign Banking offices in U.S., Bank Holding Companies, Banks in U.S. Affiliated Areas, Credit Unions, Finance Companies, Security Brokers and Dealers, Funding Corporations, Money market mutual funds, GSEs, Agency- and GSE-backed mortgage pools, Issuers of ABS, and REITs. The group of excluded financial institutions are Insurance Companies, other Mutual Funds, Closed-end funds and ETFs, and State, Local, Federal, and Private Pension Funds.

[20] We divide by $\exp\left\{b_i/2\sigma_g^2/(1-\rho_g^2)(b_i-1)\right\}$, a Jensen correction, ensure that average spending means match the targets.

[21] We define income tax revenue as current personal tax receipts (line 3) plus current taxes on production and imports (line 4) minus the net subsidies to government sponsored enterprises (line 30 minus line 19) minus the net government spending to the rest of the world (line 25 + line 26 + line 29 - line 6 - line 9 - line 18). Our logic for adding the last three items to personal tax receipts is as follows. Taxes on production and export mostly consist of federal excise and state and local sales taxes, which are mostly paid by consumers. Net government spending on GSEs consists mostly of housing subsidies received by households which can be treated equivalently as lowering the taxes that households pay. Finally, in the data, some of the domestic GDP is sent abroad in the form of net government expenditures to the rest of the world rather than being consumed domestically. Since

The model generates an average of 18.3%. We set the sensitivity of the tax rate to aggregate productivity growth $b_\tau = 2.6$ to match the observed sensitivity of log income tax revenue to GDP to GDP growth. The regression slope of log income tax revenue to GDP on GDP growth and a constant produces similar pro-cyclicality: 0.98 in the model and 0.70 in the data.

Fourth, we set the corporate tax rate that both financial and non-financial corporations pay to a constant $\tau_\Pi = 21.7\%$ to match observed corporate tax revenues of 3.41% of GDP. The model generates an average of 3.01%. The tax shield of debt and depreciation that firms and banks enjoy in the model substantially reduces the effective tax rate they pay.

The final source of government spending is interest service on the debt, which is endogenous since both quantity and price of government debt are determined in equilibrium. In the data, net interest payments on government debt average to 2.98% of GDP.[22] This number is close to the observed average budget deficit of 3.04% of GDP. We do not aim to match this number since the government cannot run a 3% deficit in perpetuity in the model, lest the debt explodes. In our calibration, the personal and corporate tax revenue is very close to the discretionary and transfer spending; the primary surplus averages 0.5% of GDP. Government debt to GDP averages 55.6% of GDP in a long simulation of the benchmark model. While it fluctuates meaningfully over prolonged periods of time (standard deviation of 46.6%), the government debt to GDP ratio remains stationary.[23]

Macro-prudential Policy We can interpret the intermediary borrowing constraint parameters, ξ, as a regulatory capital constraint set by the government. Under Basel II and III, corporate loans and bonds have a risk weight that depends on their credit quality. For a 40% loss given default, the risk weight on commercial and industrial bank loans with 2.5 year ma-

the model has no foreigners, we reduce personal taxes for this amount, essentially rebating this lost consumption back to domestic agents.

[22]Net interest expenses are interest payments to persons and businesses (line 28) minus income receipts on asses (line 10).

[23]In our numerical work, we guarantee the stationarity of the ratio of government debt to GDP by gradually decreasing personal tax rates τ_t when debt-to-GDP falls below $\underline{b^G} = 0.1$ –the profligacy region– and by gradually increasing personal tax rates when debt-to-GDP exceed $\overline{b^G} = 1.2$ –the austerity region. Specifically, taxes are gradually and smoothly lowered with a convex function until they hit zero at debt to GDP of -0.1. Tax rates are gradually and convexly increased until they hit 60% at a debt-to-GDP ratio of 150%. Our simulations never reach the -10% and +150% debt/GDP states. The simulation spends 25% of the time in the profligacy and 13% of the time in the austerity region. The fraction of time spent in these regions has no effect on the overall resources of the economy.

turity ranges from 13% for AAA, 54% for BBB-, 125% for B+, to 325% for CCC. A blended regulatory capital requirement of 5% (8% times a blended risk weight of 62.5%) seems appropriate. This implies that $\xi = 0.95$. This is the key parameter we vary in or macro-prudential policy experiments.

We set the deposit insurance fee parameter $\kappa = 0$ to reflect the fact that banks were not required to pay any deposit insurance fees between 1997 and 2006.[24] We will explore the alternative value of $\kappa = 0.25\%$.

Utility cost of intermediary bankruptcy The model features a random utility penalty that intermediaries suffer when they default. Because random default is mostly a technical assumption, it is sufficient to have a small penalty at least some of the time. We assume ρ_t is normally distributed with a mean of μ_ρ =1, i.e., a zero utility penalty on average, and a small standard deviation of $\sigma_\rho = 0.10$. The standard deviation of the penalty affects the correlation between negative intermediary wealth and intermediary defaults. The frequency of government bailouts of intermediaries depends on the frequency of credit crises and the endogenous (asset and liability) choices of the intermediaries.

4 Results

Before discussing the main results on macro-prudential policy, we study the behavior of key macro-economic and financial variables. They capture important features of the data and lend credibility to the policy experiments that are to follow. Specifically, we report means and standard deviations from a long simulation of the model (10,000 years), as well as averages conditional on being in a good state (positive TFP growth and low uncertainty, i.e. $\sigma_{\omega,L}$), non-financial recession (negative TFP growth, low uncertainty), and financial recession (negative TFP growth and high uncertainty $\sigma_{\omega,H}$).

[24] FDIC premia were raised after the crisis. Well capitalized banks currently pay 2.5 cents per $100 insured.

Table 2: Unconditional Macroeconomic Quantity Moments

	Data			Model		
	stdev	output corr.	AC	stdev	output corr.	AC
GDP	2.13%	1.00	0.68	2.24%	1.00	0.76
CONS	1.87%	0.91	0.65	1.52%	0.9	0.73
X/Y	1.23%	0.19	0.87	1.19%	0.41	0.36
X/K	0.89%	0.44	0.82	0.75%	0.46	0.30

4.1 Macro Quantities

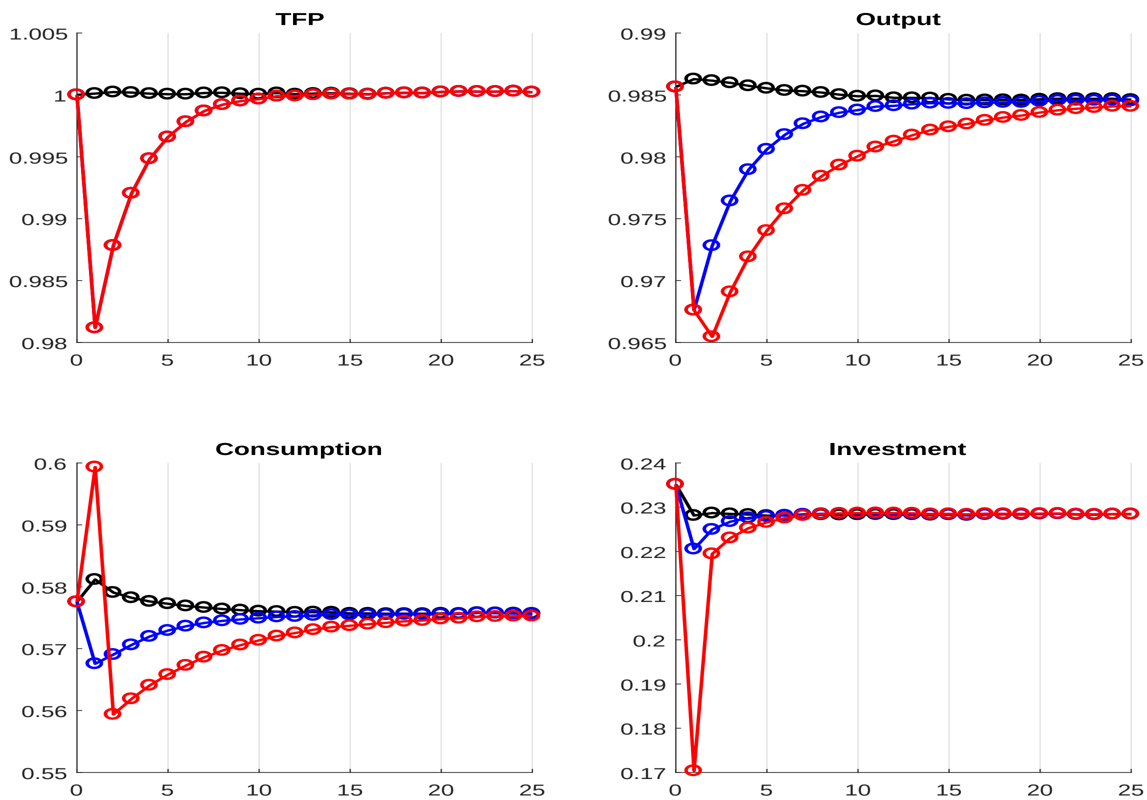
Table 2 reports the standard deviation of aggregate quantities, their correlation with GDP, and their autocorrelation. Moments in the data are computed from HP-detrended series. Moments in the model are deterministically detrended since all time series have a deterministic trend. The model matches the volatility of GDP and aggregate consumption. It also matches the autocorrelation of both series. The model matches the volatility of the investment to GDP ratio (1.19% vs. 1.23%) by virtue of the adjustment cost parameter choice, and delivers an investment rate volatility that approximates the data (0.75% vs. 0.89%). The investment/GDP ratio and investment rate display modest pro-cyclicality in both data and model. Investment rates are insufficiently persistent in the model.[25] As discussed in the calibration section, the model also matches the cyclicality of government spending.

We present impulse-response graphs to explore the behavior of macro-economic quantities conditional on the state of the economy. We start off the model in year 0 in the average TFP state (the middle of the five points on the TFP grid) and in the low uncertainty state ($\sigma_{\omega,L}$). In period 1, the model undergoes a change to the lowest-TFP grid point. In one case (red line), the recession is accompanied by a switch to the high uncertainty state ($\sigma_{\omega,H}$); a financial recession. In the second case, the economy remains in the low uncertainty state; a non-financial recession (blue line). From period 2 onwards, the two exogenous state variables follow their stochastic laws of motion. For comparison, we also show a series that does not undergo any shock in period 1 but where the exogenous states stochastically mean revert from the high-TFP state in period 0 (black line). For each of the three scenarios, we simulate 10,000 sample paths of 25 years and average across them. Figure 2 plots the macro-economic quantities, detrended

[25] As an aside, the investment level is too smooth (1.15% volatility in the model vs. 6.31% in the data) while investment growth is too volatile (10.94% vs. 6.14% in the data).

by their long-term growth rate of 2% per year. The top left panel is for the productivity level Z. By construction, it falls by the same amount in financial and non-financial recessions; a 2% drop. Productivity then gradually mean reverts over the next decade. The black line shows how productivity would have evolved absent a shock in period 1.

Figure 2: Financial vs. Non-financial Recessions: Macro Quantities



The graphs show the average path of the economy through a recession episode which starts at time 1. In period 0, the economy is in the average TFP state. The recession is either accompanied by high uncertainty (high σ_ω), a financial recession plotted in red, or low uncertainty (low σ_ω), a non-financial recession) plotted in blue. From period 2 onwards, the economy evolves according to its regular probability laws. The black line plots the dynamics of the economy absent any shock in period 1. We obtain the three lines via a Monte Carlo simulation of 10,000 paths of 25 periods, and averaging across these paths. **Blue line:** non-financial recession, **Red line:** financial recession, **Black line:** no shocks.

The other three panels show impulse-responses for GDP, consumption, and investment. The percentage drop in GDP is larger than that in productivity and the drop in GDP is larger when the economy is additionally hit by an uncertainty shock (red line) than if it is not (blue line). In financial recessions, the economy suffers from a second period of decline, despite the rebound in productivity. GDP remains lower for longer in a financial recession. The added persistence resembles the slow recovery that typically follows a financial crisis. The bottom right panel shows a 30% drop in investment in financial recessions but only a modest drop in non-financial recessions. The investment rate dynamics track Tobin's q (our variable p) by virtue of the first-order condition for firm investment. Despite the bounce back in period 2, investment remains depressed for a prolonged period of time. Aggregate consumption offsets the initial decline in investment in a financial recession. The low rate of return on savings induces the saver to consume more in a financial crisis. Consumption drops subsequently and remains below the non-financial recession level for the remaining periods.

4.2 Balance Sheet Variables

Next, we turn to the key balance sheet variables in Table 3. The first two columns report the unconditional mean and volatility. The last three columns report conditional averages in expansions, non-financial recessions, and financial recessions, respectively.

Firms The first panel focuses on the borrower-entrepreneurs, the non-financial corporate sector. Rows 1 and 2 display the market value of assets ($p_t K_t^B$) and the market value of liabilities ($q_t^m A_t^B$), both scaled by GDP. Their difference is the market value of firm equity scaled by GDP. Their ratio is the market leverage ratio (row 4). Book leverage, defined as the book value of debt to the book value of assets in row 3, is 43%. Entrepreneurs own more than half of their firms in the form of corporate equity (57%). Market leverage is counter-cyclical while book leverage is pro-cyclical. Firms delever in financial recessions, hence the fall in book leverage, but sharp drops in the market value of assets lead to an increase in market leverage. Indeed, Tobin's q (the variable p in row 16) falls by 3.27% on average from expansions to financial recessions, although this average masks a substantially larger initial drop.

Individual borrowers default when their profits are too low. This is more likely when the cross-sectional distribution of idiosyncratic productivity shocks widens, as the mass of firms with productivity shocks below the threshold ω_t^* increases. The model generates average corporate default and loss rates of 2.85% (row 7) and 1.29% points (row 9), respectively, implying an average loss-given-default rate of 45.4% (row 8). All these numbers are in line with the data.

Default and loss rates are higher in financial recessions (6.6% and 2.97%) than in non-financial recessions and expansions (about 1.1% and 0.5% in both). The model generates the right amount of corporate credit risk, on average, and generates the strong cyclicality in the quantity of risk observed in the data.[26]

Most of the time, firms stay away from the leverage constraint because they are risk averse and take into account the costs of bankruptcy when making their leverage choices. However, borrowers are likely to be constrained in financial recessions (18% of the time, row 6). This occurs because of the fall in the price of collateral. When the constraint binds, firms are forced to cut their borrowing from the financial sector, and cannot pursue the investment projects they would otherwise undertake. Relative to expansions, output falls by 3.8% and investment by 28% in financial recessions, in part due to these binding constraints.

Intermediaries Intermediary market leverage is 93% on average (row 11 of Table 3), matching the data. Intermediaries choose to be so highly levered for a number of reasons. Like the corporate firms, they are impatient and enjoy a tax shield. As the only agent with access to deposits, they alone can earn a large spread (2.03%, row 19) between the short-term deposit rate (2.59%, row 17) and the rate on corporate loans (4.62%, row 18). They bear the interest rate risk associated with the maturity transformation they perform, as well as the credit risk on the loans. Given the low (but realistically calibrated) average loss rate and their relatively low risk aversion, they choose to take up substantial leverage to reach their desired risk-return combination.

Adrian, Boyarchenko, and Shin (2015) show that book leverage is pro-cyclical while market leverage is counter-cyclical both for commercial banks and for broker-dealers. Our model generates this pattern. Market leverage increases from 92.8% in expansions to 93.84% in non-financial recessions, and to 93.19% in financial recessions. Book leverage, in contrast, falls from 92.47% in expansions to 88.79% in financial recessions. Why does market leverage rise in financial recessions? Intermediaries suffer losses on their credit portfolio. The realized excess return on bank assets is -0.08% (row 20). At the same time risk is high and low prices (high

[26] In the 1991 recession, the delinquency rate spiked at 8.2% and the charge-off rate at 2.2%. For the 2007-09 crisis, the respective numbers are 6.8% and 2.7%. These are far above the unconditional averages of 3.1% and 0.7% cited in footnote 13. Similarly, during the 2001 recession, the default rate on high-yield bonds was 9.9%, far above the 1981-2014 average of 4.1%.

Table 3: Balance Sheet Variables and Prices

	Unconditional		**Expansions**	**Non-fin Rec.**	**Fin Rec.**
	mean	stdev	mean	mean	mean
			Borrower		
1. Mkt Val of Capital / Y	1.966	0.038	1.989	1.986	1.920
2. Mkt Val of Corp Debt / Y	0.828	0.035	0.848	0.824	0.787
3. Book val corp debt / Y	0.843	0.021	0.851	0.848	0.825
4. Market corp leverage	42.88%	0.86%	42.81%	42.68%	42.98%
5. Book corp leverage	42.87%	0.72%	43.23%	42.40%	42.03%
6. Fraction leverage constr binds	4.83%	21.44%	0.13%	1.81%	18.21%
7. Default rate	2.85%	2.55%	1.10%	1.15%	6.69%
8. Loss-given-default rate	45.42%	2.13%	46.02%	44.66%	44.19%
9. Loss Rate	1.29%	1.17%	0.51%	0.51%	2.97%
			Intermediary		
10. Mkt fin leverage	93.19%	2.36%	92.81%	93.84%	93.19%
11. Book fin leverage	91.47%	1.88%	92.47%	91.17%	88.79%
12. Fraction leverage constr binds	32.21%	46.73%	0.02%	45.91%	75.52%
13. Bankruptcies	0.00%	0.00%	0.00%	0.00%	0.00%
			Saver		
14. Deposits / Y	0.771	0.031	0.787	0.773	0.732
15. Government Debt / Y	0.879	0.421	0.862	1.013	0.871
			Prices		
16. Tobin's q	1.000	0.015	1.010	0.994	0.978
17. Risk-free rate	2.59%	2.34%	3.42%	3.90%	0.25%
18. Corporate bond rate	4.62%	0.26%	4.46%	4.73%	4.93%
19. Credit spread	2.03%	2.27%	1.04%	0.83%	4.68%
20. Excess return on corp. bonds	0.79%	1.78%	1.34%	0.39%	-0.08%

yields, row 18) of corporate bonds and loans reflect the higher default risk and the higher credit risk premium. This reduces the market value of intermediary assets. A lower value of bank assets tightens their regulatory capital constraint. The intermediary leverage constraint binds in 75.5% of the financial crises compared to 32.2% unconditionally. When binding, intermediaries must reduce liabilities to meet capital requirements in the wake of their credit losses. Given the low cost of deposit funding in a financial crises (0.25%) and the high credit spreads they earn in those states of the world (4.68%), intermediaries would like to raise more deposits and increase corporate lending but their constraint prevents them from doing so.

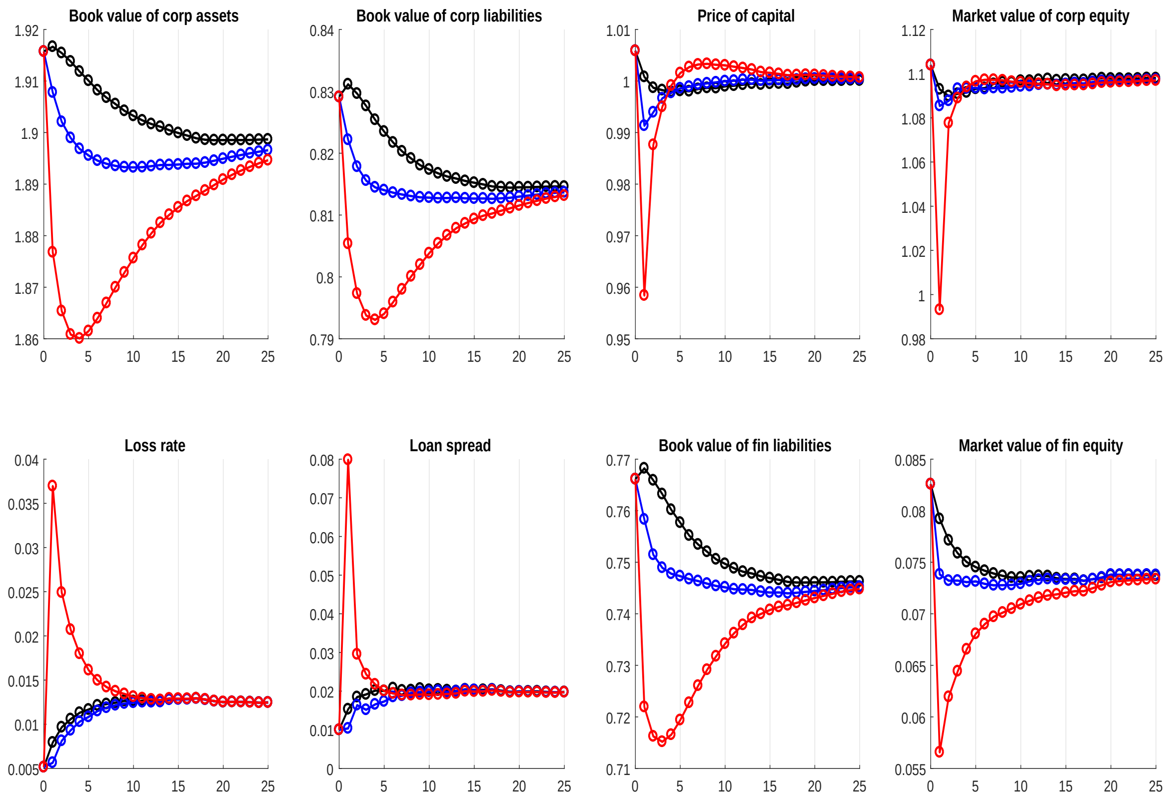
In sharp contrast, intermediaries are only constrained in 46% of the non-financial recessions. The risk free rate rises in such period, to 3.9%, as all agents want to borrow against future income to smooth consumption. At the same time, demand for credit is low due to worse investment opportunities, and the credit spread shrinks. Intermediaries are hardly ever constrained during expansions (.02%). In those periods, investment opportunities are good and intermediaries expand both lending and deposits, while earning their desired rate of return.

The size of the intermediary sector, relative to GDP, shrinks in financial recessions. Both book and market values of intermediary assets shrink about 8% relative to their levels in expansions (rows 2 and 3). At the same time that the value of bank assets shrinks, their liabilities shrink (row 14). Bank liabilities fall from 79% of GDP in expansions to 73% of GDP in financial recessions (row 15). Since GDP itself falls, bank liabilities themselves fall by as much as 12%.

Intermediary net worth, or bank equity, is an important state variable in all intermediary-based models. Intermediary net worth is the difference between the market value of bank assets (row 2) and the value of deposits (row 15). Intermediary equity is 5.67% of GDP unconditionally. It shrinks to 5.4% of GDP in the average financial recession. The reduction in intermediary net worth itself is 15.8%, relative to the unconditional level. The reduction in net worth in financial crises makes intermediaries effectively more risk averse, leading them to charge larger risk premia on new lending. We return to this risk premium effect below. Low net worth hampers the intermediaries' capacity to bear credit risk and do maturity transformation.

In the equilibrium of our model, the intermediation sector as a whole is never insolvent. Even in the absence of systemic intermediary failures in equilibrium, deposit insurance lowers the cost of funding and provides banks with a risk shifting motive vis-a-vis the government. However, as risk averse agents, bank owners are reluctant to hit low net worth states since they imply low consumption and high marginal utility. The balance of these two factors determines the frequency of financial breakdowns.[27].

Figure 3: Financial vs. Non-financial Recessions: Balance Sheet Variables Intermediaries



Blue line: non-financial recession, **Red line:** financial recession, **Black line:** no shocks.

Figure 3 show the impulse-response functions for assets and liabilities of both non-financial firms and banks. The top row reports book values of corporate assets (capital) and liabilities (loans). Corporations shrink ob both sides of their balance sheet during financial recessions. The market value of corporate equity, with both assets and liabilities valued at market prices, drops sharply in financial recessions due to the sharp drop in Tobin's q (top row, right hand side). The book value of corporate liabilities is also the book value of financial assets. In the

[27]Whether or not systemic failures occur depends on both the magnitude of shocks, and the strength of the risk shifting motive. See for example Elenev, Landvoigt, and Van Nieuwerburgh (2016) for a model with a financial intermediary that can choose to hold riskier assets.

third graph on the bottom row, we can see that banks' liabilities (book value) fall by even more than their assets during financial recession; simply put, banks delever during these periods. Because of large losses on their loan portfolio (bottom row, first graphs), banks need to charge large credit spreads (bottom row, second graph). This reduces the demand for loans. Even though financial book leverage declines, market leverage increases in crises, or equivalently, the market value of financial equity declines (bottom row, fourth graph). This is because the market value of liabilities rises more sharply than the market value of assets. Banks slowly rebuild their equity capital (through high spreads and low intermediary household consumption) and expand their loans business as loss rates return to normal levels. In the simulation, this process takes close to 20 years.

Savers Risk averse savers only hold safe debt, provided both by the intermediaries and the government. On average, these two sources of safe assets account for 77% (row 14) and 87% of GDP (row 15). In our model, as in the data, the government's tax revenues are pro-cyclical and its expenses counter-cyclical. However, the small expansion in government debt to GDP more is not enough to offset the reduction in deposits so that the total equilibrium supply of safe short-term assets falls in financial recessions. As a result, the equilibrium real interest rate falls 300 basis points from expansions to financial recessions. In financial recessions, intermediaries want to delever due to their equity losses. This delevering requires savers to increase consumption and reduce savings exactly at a time when their marginal utility of consumption is high. To induce savers to dissave, a large drop in the real interest rate is required. The magnitude of this drop depends on savers' EIS, everything else equal. If savers' EIS is high, they are more willing to increase consumption in crises, and the interest will fall by less. This will shift the cost of crises onto borrowers and intermediaries, who cannot profit from negative real interest rates. If savers' EIS is low, real interest rates become negative and help to recapitalize intermediaries.[28]

Prices Real interest rates on safe debt are 2.59% on average and have a volatility of 2.34% (row 17). Both are very reasonable numbers, especially in a production-based asset pricing

[28]For example, when $\nu_S = 60 >> 1$, the risk-free interest rate volatility approaches zero. Intermediaries no longer benefit from low, even negative interest rates in crises. The absence of cheap funding in crises makes them more reluctant to take on more leverage in the first case. This very high saver EIS world is akin to the continuous time intermediation model of Brunnermeier and Sannikov where risk-free rates are constant. The economy with low EIS behaves quite differently.

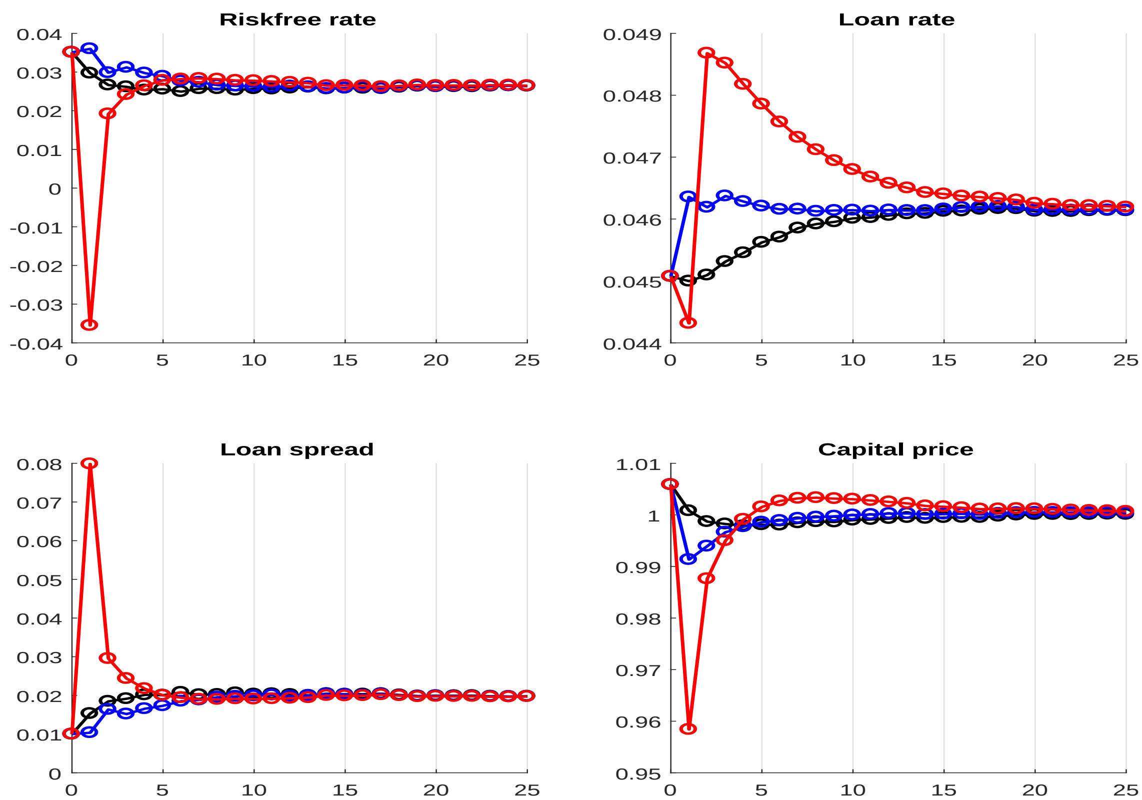
model, and match historical averages. Financial recessions see large declines in collateral values (row 16), negative (excess) returns on bank assets (row 20), high corporate credit spreads (row 19), and low real interest rates (row 17). All of these are important features of real-life financial crises. In contrast, non-financial recessions see only modest reductions in risk-free rates and modest increases in credit spreads.

One important quantitative success of the model is its ability to generate a high unconditional credit spread while matching the observed amount of default risk. The average 2.03% spread is close to the average 2.39% spread in the data.[29] The credit spread is also highly volatile (2.27% standard deviation) and strongly counter-cyclical (four times higher in financial recessions than expansions). The rise in the credit spread in financial recessions to 4.68% reflects not only the increase in the default risk but also an increase in the credit risk premium. The model generates a high and counter-cyclical price of credit risk, which itself comes from the high and counter-cyclical SDF of the intermediary, who is the marginal agent in the corporate loan market. The sharp decline in intermediary wealth is responsible for the sharp increase in the SDF.

Figure 4 shows the impulse-responses in the bust experiment for the interest rates, the credit spread, and the price of capital. In the first period of a financial recession following a boom, the real risk-free rate turns sharply negative and the credit spread blows out to 8%. Financial recessions are periods of high credit risk and credit risk premia, both of which enter in the credit spread. For reasons explained above, the risk free rate plummets to -3.5%. Non-financial recessions see a much smaller risk-free rate and credit spread effect, and a much smaller decline in the price of capital.

[29]We define the credit spread as the difference between the yield on a blended portfolio of investment grade and high yield bonds and the yield on a one-year constant maturity Treasury yield. We use the longest available sample from Barclays U.S. corporate IG and HY bond indices from February 1987 to December 2015. To determine the portfolio weights on the high yield versus investment grade bonds, we use market values of the amounts outstanding, also from Barclays. The average weight on HY is 19.4%. The resulting credit spread has a mean of 3.36% and a volatility of 1.46%. We compare this with a difference measure of the credit spread which takes a 19.4%-80.6% weighted average of the Moody's Aaa and Baa yields and subtracts the one-year CMT rate. Over the same February 1987-December 2015 period, the mean credit spread is 3.05% with a volatility of 1.39%. The second measure of the credit spread has a correlation of 86% with the first one. The advantage of this second measure is that we can compute it back to 1953. The mean spread over the 1953-2015 period is 2.08% with a volatility of 1.55%. While the second credit spread is downward biased, compared to the better first measure, considering a longer sample period would lead us to consider a lower mean credit spread target than 3.36%. For example, we could add the 31 basis point difference between our favorite Barclays measure and the Moody's measure over the post-1987 sample to the full-sample Moody's mean of 2.08% to get to a target mean credit spread for the full 1953-2015 sample of 2.39%.

Figure 4: Financial vs. Non-financial Recessions: Prices



Blue line: non-financial recession **Red line:** financial recession.

4.3 Consumption and Welfare

Table 4 reports the moments of consumption for each agent, as well as each agent's value function, and aggregate welfare. By virtue of being the largest groups of agents, borrowers and savers have the highest consumption shares (relative to GDP). More interesting is consumption growth in the second panel. It reveals that the intermediary has by far the most volatile consumption growth (13%), followed by the saver (3.5%), and the borrower (2.4%). We recall that borrowers and intermediaries have the identical preferences (risk aversion, patience, and IES), so that these differences in consumption volatility arise endogenously from the different roles they perform in the economy (production vs. intermediation). Intermediaries end up absorbing a disproportionate fraction of the aggregate risk in the economy. In financial recessions, all three agents suffer drops in their consumption, but the drop is far larger for intermediaries

Table 4: Consumption and Welfare

	Unconditional		**Expansions**	**Non-fin Rec.**	**Fin Rec.**
	mean	stdev	mean	mean	mean
			Consumption to Output		
Consumption, B	0.229	0.007	0.234	0.226	0.219
Consumption, I	0.018	0.003	0.020	0.017	0.014
Consumption, S	0.329	0.013	0.329	0.315	0.326
			Consumption growth		
Consumption, B	2.00%	2.40%	2.69%	1.58%	0.42%
Consumption, I	2.00%	13.08%	5.36%	0.77%	-6.56%
Consumption, S	2.00%	3.53%	2.16%	0.62%	1.24%
			Welfare		
Aggregate welfare	17.573	0.030	17.583	17.567	17.550
Value function, B	0.3367	0.0034	0.3381	0.3351	0.3340
Value function, I	0.0254	0.0007	0.0259	0.0253	0.0246
Value function, S	17.211	0.032	17.219	17.207	17.191
DWL/GDP	0.008	0.007	0.003	0.003	0.018
			Marginal utility ratios		
log(MU B / MU I)	0.014	0.128	0.082	0.008	-0.134
log(MU B / MU S)	-1.272	0.048	-1.288	-1.305	-1.238
log(MU S / MU I)	1.286	0.142	1.370	1.312	1.104

(-26%) than for borrowers (-6.7%) or savers (-0.8%).

The third panel reports moments related to aggregate welfare. Overall welfare, the population weighted average of the three agents' value functions measured in consumption equivalent units, is highest in expansions and lowest in financial recessions. Given the large difference in discount factors, aggregate welfare calculations are dominated by savers. The welfare difference between expansions and financial recessions for savers and therefore aggregate welfare is small at 0.2%. However, intermediaries have 5.5% lower welfare in financial recessions than in expansions, while borrowers have 1.2% lower welfare.

The last panel reports ratios of marginal utilities between pairs of agents. In a complete markets model with agents whose preference parameters differ, these ratios would differ across pairs of agents but be constant over time. Our model is an incomplete markets model featuring imperfect risk sharing; the marginal utility ratios display high volatility between 5% and 14%. For example, the marginal utility of borrower and intermediary are nearly equalized on average, but the borrower has 8% higher marginal utility than the intermediary in expansions and 13% lower marginal utility in financial recessions.

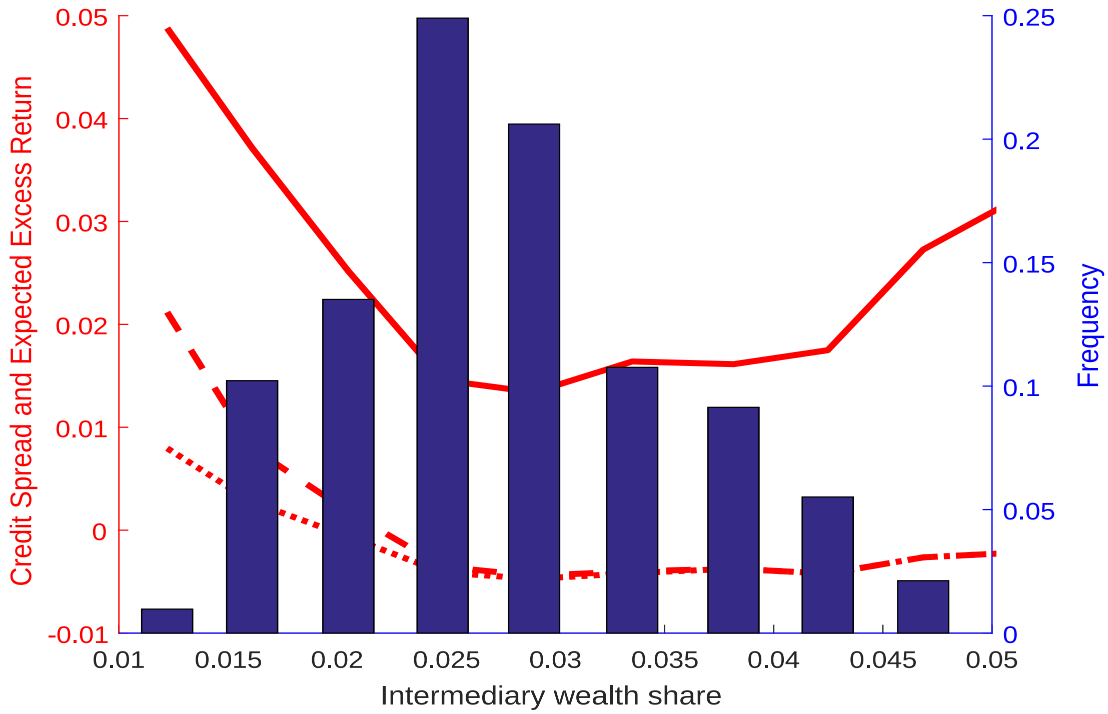
4.4 Credit Spread and Risk Premium

Figure 5 shows the histogram of the intermediary wealth share plotted against three different measures of credit risk compensation earned by intermediaries. The solid red line plots the credit spread, the difference between the yield r_t^m on corporate bonds and the risk-free rate, where we compute the bond yield as $r_t^m = \log\left(\frac{1}{q_t^m} + \delta\right)$.[30] Consistent with the result in He and Krishnamurty (2013), the credit spread is high when the financial intermediary's wealth share is low. Since our model has defaultable debt, the increase in the credit spread reflects both risk-neutral compensation for expected defaults and a credit risk premium.

To shed further light on the source of the high credit spread, we compute the expected excess return (EER) on corporate loans/bonds earned by the intermediary. The EER consists both of the credit risk premium, defined as the (negative) covariance of the intermediary's stochastic discount factor with the corporate bond's excess return, and an additional component that reflects the tightness of the intermediary's leverage constraint. This component arises because the marginal agent in the market for risk-free debt is the saver household, while corporate bonds are priced by the constrained intermediary. The market risk free rate is lower than the "shadow" risk free rate implied by intermediary consumption smoothing. To decompose the expected excess return into the risk premium and constraint tightness components, we plot both the complete EER on corporate bonds (the dashed red line) and the risk premium (the dotted red line). When the intermediary wealth share is relatively high, the leverage constraint is not binding and the EER is equal to the risk premium, which is approximately zero in this region of the state space. Low levels of intermediary wealth result from credit losses, and the lowest levels occur during financial crises. At these times, credit risk increases and the intermediary becomes constrained. In the worst crisis when intermediary wealth gets close to zero, the EER reaches 3 percent. The conventional risk premium (the covariance of intermediary SDF and bonds return) accounts for 1 percentage points of the EER.

[30] This is a simple way of transforming the price of the long-term bond into a yield; however, note that this definition assumes a default-free payment stream $(1, \delta, \delta^2, \ldots)$ occurring in the future.

Figure 5: The Credit Spread and the Financial Intermediary Wealth Share



Solid line: credit spread; **dashed line**: expected excess return; **dotted line**: risk premium (covariance between intermediary SDF and corporate bond excess return). The difference between expected excess return and risk premium is a measure for tightness of the intermediary constraint.

4.5 Effect of Decoupling Firm and Intermediary Balance Sheets

We solve the CBS version of the model, described in section 2.8, to understand how the disaggregation of firm and intermediary balance sheets affects macroeconomic dynamics. We keep all model parameters the same as in the benchmark model. All parameters of intermediation sector and the parameters related to corporate credit frictions are no longer relevant. Since firms directly issue risk-free debt to savers, there is no more corporate default, and the idiosyncratic productivity shocks of firms wash out in the aggregate. This also means that there are no DWL from costly liquidations in the CBS model. We set the parameter governing maximum firm leverage, Ξ, to 0.49, such that the deposit-to-GDP ratio is equal to the benchmark model (87%). Table 5 compares prices and quantities of the CBS model to the benchmark.

Two clear effects emerge from the comparison: the CBS economy is significantly smaller, but also much less volatile. In the CBS model, Tobin's q is 36.5% less volatile (row 2), and this risk free rate is 57.8% less volatile (row 3). Further, the volatilities of output, investment, and consumption all decline by substantially more than their means. Consumption growth volatility of borrowers (who now also include the intermediaries) and savers decline by 52% and 41%, respectively, and the risk sharing between both agents improves, indicated by the much lower volatility of the ratio of the marginal utilities.

GDP and the capital stock are 4% and 14% smaller in the model without intermediary, while the volatility of GDP declines by 7% and that of investment fall by 44%. Even though the capital stock of the benchmark model is larger, the CBS economy achieves higher aggregate consumption, by 0.9%. This is effect is caused by the elimination of DWL from firm bankruptcies, which account for 130% of the consumption increase; in other words, net of the elimination of DWL, the CBS economy generates roughly 0.3% lower consumption than the benchmark.

Table 5: Comparison of Benchmark to Consolidated-Balance-Sheet (CBS) Model

	Bench	**CBS**
	Price Volatility	
2. Tobin's q	0.015	-36.5%
3. Risk-free rate	0.023	-57.8%
	Size	
4. GDP	0.984	-4.2%
5. Capital stock	1.897	-13.9%
6. Consumption, B	0.229	+13.2%
7. Consumption, I	0.018	–
8. Consumption, S	0.329	-2.3%
9. Agg consumption	0.575	+0.9%
	Welfare	
4. Aggregate welfare	17.573	-0.5%
5. Value function, B	0.337	+12.7%
6. Value function, I	0.025	–
7. Value function, S	17.211	-0.6%
8. DWL/GDP	0.008	-100.0%
	Volatility	
9. GDP	0.022	-7.2%
10. Investment	0.015	-43.9%
11. Consumption	0.015	-6.6%
12. Consumption growth, B	0.024	-52.2%
13. Consumption growth, I	0.131	–
14. Consumption growth, S	0.035	-40.9%
15. log (MU B / MU S)	0.048	-52.4%

We draw two conclusions from this analysis: (1) In our calibrated model, the presence of the separate intermediation sector is critical for capturing the volatility of macroeconomic prices and quantities observed in the data. The CBS model with the same fundamental productivity risk is not able to generate large financial recessions, even though a standard financial accelerator effect is present in firms' leverage constraint. (2) The additional volatility and risk caused by the intermediation sector in the benchmark model leads to greater precautionary savings motives, and higher wealth accumulation by borrowers (capital) and savers (risk free debt). In general equilibrium, this effect may lead to higher output and consumption.

5 Macro-prudential Policy

We use our calibrated model to investigate the effects of macro-prudential policy choices. Our main experiment is a variation in the intermediaries' leverage constraint. In the benchmark model, intermediaries can borrow 95 cents against every dollar in assets ($\xi = 0.95$). We explore tighter constraints ($\xi = .70$, $\xi = 0.80$, $\xi = 0.90$), as well as looser constraints ($\xi = .975$). Our second macro-prudential policy experiment is to charge intermediaries $\kappa = 0.025\%$ for deposit insurance, about the present-day level. Tables 6 and 7 show the results. Table 7 reports results in percentage deviation from the benchmark.

Changing maximum intermediary leverage Rows 10 and 11 of Table 6 show that a policy that constrains bank leverage is indeed successful at bringing down that leverage. Banks reduce the size of their assets, both in book and market value terms (rows 2 and 3) and the size of their liabilities (row 14). On net, intermediary equity increases sharply as ξ is lowered. With intermediaries better capitalized, financial fragility falls. With tighter regulation, intermediaries' constraints bind more frequently (row 12). As the economy becomes less risky, intermediaries can be less cautious and exhaust the borrowing capacity more frequently. In sum, tighter regulation leads to a safer intermediary sector, but also to a smaller one.

This increased safety is also reflected in lower corporate default, loss-given-default, and loss rates for non-financial firms as ξ falls (rows 7-9). Firms choose to reduce leverage (rows 4-5) and their LTV constraints rarely bind (row 6). Firms reluctance to undertake more leverage despite the safer environment may be understood from the higher interest rates and spreads

Table 6: Macroprudential Policy

	Benchmark	$\xi = .70$	$\xi = .80$	$\xi = .90$	$\xi = .975$	$\kappa = 0.25\%$
				Borrowers		
1. Mkt value capital/GDP	1.966	1.924	1.950	1.958	1.970	1.957
2. Mkt value corp debt/GDP	0.828	0.606	0.741	0.808	0.835	0.806
3. Book val corp debt/GDP	0.843	0.649	0.785	0.835	0.846	0.835
4. Market corp leverage	0.429	0.337	0.402	0.426	0.429	0.426
5. Book corp leverage	0.429	0.337	0.402	0.426	0.429	0.426
6. Fraction LTV constr binds	4.83%	0.00%	0.13%	3.16%	5.26%	2.91%
7. Default rate	2.85%	1.89%	2.48%	2.79%	2.87%	2.79%
8. Loss-given-default rate	45.42%	29.63%	40.25%	44.57%	45.66%	44.48%
9. Loss Rate	1.29%	0.53%	0.98%	1.24%	1.31%	1.23%
				Intermediaries		
10. Mkt fin leverage	0.932	0.690	0.793	0.886	0.951	0.936
11. Book fin leverage	0.915	0.644	0.749	0.857	0.938	0.904
12. Fraction intermed constr binds	32.21%	90.56%	79.22%	46.71%	22.78%	37.92%
				Savers		
13. Deposits/GDP	0.771	0.418	0.588	0.716	0.794	0.754
14. Government debt/GDP	0.879	0.631	0.731	0.836	0.883	0.655
				Prices		
15. Tobin's q	1.000	1.000	1.000	1.000	1.000	1.000
16. Risk-free rate	2.59%	2.60%	2.56%	2.59%	2.59%	2.58%
17. Corporate bond rate	4.62%	5.16%	5.04%	4.78%	4.56%	4.79%
18. Credit spread	2.03%	2.56%	2.47%	2.19%	1.97%	2.22%
19. Excess return on corp. bonds	0.79%	2.11%	1.55%	1.00%	0.71%	1.03%

they face on debt (rows 18 and 19). When intermediary capacity shrinks (with lower ξ), the reward for providing intermediation services increases. Higher financial sector profitability is evidenced by a higher credit spread in the face of lower loss loan rates.

The smaller banking sector reduces the supply of deposits (row 14). Further, lower non-financial and financial leverage mean that firms and banks take less advantage of their tax shield of debt, increasing government tax revenues and reducing government debt (row 15). The overall reduced supply of safe assets would depress equilibrium interest rates at constant demand. Why then do rates stay roughly constant as ξ is tightened (row 17)? As the economy becomes less volatile, so does saver consumption growth. This reduces saver demand for precautionary savings, offsetting the drop in supply of risk free debt.

Because of the higher profit margin, the intermediary's consumption is higher (row 30 of Table 7). Intermediaries not only experience higher consumption, their consumption also becomes less volatile (row 36). Naturally then, intermediary welfare increases with tighter constraints (row 23). Interestingly (or ironically), it is the agent whose constraint is tightened that benefits the most from the policy change. Intermediaries in the benchmark model could have chosen a much lower leverage ratio. They did not since they did not internalize the positive effect that lower sector-wide intermediary leverage would have had on credit spreads.

The savers' welfare is adversely affected by tighter bank capital regulation (row 24). Their wealth, which is wholly invested in risk free debt, declines substantially, while the return (the risk free rate) stays roughly constant. Hence their consumption falls. Given their large size in the population (69%) and the large weight they receive in aggregate welfare due to their high discount rate, the effect on savers is important for understanding the overall negative welfare effect form tighter macro-prudential policy.

Borrowers' consumption rises weakly (row 29), and their consumption volatility declines (row 36). Therefore, like intermediaries, they are better off from large declines in intermediary leverage (row 22). Risk sharing improves substantially when financial leverage is lower, as witnessed by the reduced volatility in the marginal utility ratios across pairs of agents (rows 39-41).

A first key effect of macro-prudential policies is that they reduce macro-economic volatility. The last panel of Table 7 indeed shows reductions in GDP growth, investment growth, and consumption growth volatility (rows 32, 34, 35).

A second key effect of tighter macro-prudential policy is that the economy's output shrinks (row 26). The capital stock shrinks sizeably (row 27). The reduction in output arises because firms choose to borrow less from a smaller intermediary sector. Debt financing becomes significantly more expensive, eliminating a previously cheap source of financing.

The aggregate welfare effect of tighter bank capital requirements is modest (row 21). Tighter bank capital requirements reduce the risk in the economy but they shrink the total pie. On net, we find that tighter macro-prudential policy has modest welfare losses, on the order of 52-244 basis points consumption equivalent loss, depending on the extent of tightening. Conversely, loosening bank leverage constraints to $\xi = .975$ would increase welfare by 26 basis points.

Increasing cost of deposit insurance The last column of Tables 6 and 7 show the result of an experiment that increases the cost of deposit insurance κ from 0 to 0.25% per unit of deposit.

Table 7: Macroprudential Policy: Macro and Welfare

	Benchmark	$\xi = .70$	$\xi = .80$	$\xi = .90$	$\xi = .975$	$\kappa = 0.25\%$
			Welfare			
21. Aggregate welfare	17.573	-2.44%	-1.54%	-0.52%	+0.26%	-0.04%
22. Value function, B	0.337	+2.22%	+0.55%	+0.02%	+0.00%	+0.49%
23. Value function, I	0.025	+33.81%	+26.03%	+8.96%	-3.71%	-1.07%
24. Value function, S	17.211	-2.59%	-1.63%	-0.55%	+0.27%	-0.05%
25. DWL/GDP	0.008	-35.00%	-13.64%	-2.43%	+0.84%	-2.48%
			Size of the Economy			
26. GDP	0.984	-0.88%	-0.33%	-0.17%	+0.08%	-0.2%
27. Capital stock	1.897	-3.00%	-1.13%	-0.60%	+0.28%	-0.6%
28. Aggr. Consumption	0.575	+0.4%	+0.2%	+0.0%	-0.0%	+0.0%
29. Consumption, B	0.231	+3.0%	+0.8%	+0.0%	+0.0%	+0.5%
30. Consumption, I	0.022	+31.2%	+24.0%	+7.7%	-2.4%	-1.3%
31. Consumption, S	0.333	-3.0%	-1.5%	-0.4%	+0.1%	-0.2%
			Volatilies			
32. GDP	2.24%	-4.26%	-1.77%	-0.59%	+0.32%	-1.3%
33. Risk-free rate	2.34%	-61.8%	-47.2%	-23.6%	+6.9%	-28.2%
34. Investment	1.48%	-37.6%	-19.7%	-7.5%	+2.0%	-11.1%
35. Consumption	1.52%	-5.7%	-7.5%	-3.8%	+1.1%	-3.0%
36. Consumption, B	0.68%	-5.1%	-2.1%	-1.5%	+1.4%	-5.2%
37. Consumption, I	0.28%	-48.9%	-42.7%	-21.8%	+12.9%	-7.5%
38. Consumption, S	1.30%	-21.3%	-13.7%	-6.4%	+1.1%	-5.5%
39. log (MU B / MU S)	0.048	-30.3%	-13.4%	-7.0%	+1.2%	-7.8%
40. log (MU S / MU I)	0.141	-71.4%	-55.0%	-26.7%	+14.2%	-3.0%
41. log (MU B / MU I)	0.128	-73.8%	-71.0%	-35.7%	+17.7%	-9.7%

Column benchmark are moments from a 10,000 period simulation of the benchmark economy. Other columns are percentage changes of these moments relative to the benchmark level in a 10,000 period simulation of the economies with the parameter change indicated by the column header.

This experiment reduces welfare by 4 basis points. Borrowers gain from this experiment, while the other two agents lose. Again, capital and GDP fall, but so does GDP and consumption volatility. The net effect is a redistribution from intermediaries and savers, who bear the "incidence" of the insurance fee, to borrowers.

6 Conclusion

We provide the first calibrated macro-economic model which features intermediaries who extend long-term defaultable loans to firms producing output and raise deposits from risk averse savers, and in which both banks and firms can default. The model incorporates a rich set of fiscal policy rules, including deposit insurance, and endogenizes the risk-free interest rate.

Like in the standard accelerator model, shocks to the macro-economy affect entrepreneurial net worth. Since firm borrowing is constrained by net worth, macro-economic shocks are amplified by tighter borrowing constraints. Unlike the original models, ours features impatient but risk averse and infinitely-lived entrepreneurs. A second financial accelerator arises from explicitly modeling the financial intermediaries' balance sheet as separate from that of the entrepreneur-borrowers and saving households. Intermediaries are subject to regulatory capital constraints. Macro-economic shocks that lead to binding intermediary borrowing constraints amplify the shocks through their direct effect on intermediaries' net worth and the indirect effect on borrowers to whom the intermediaries lend. However, when intermediaries are well enough capitalized to absorb the fundamental shock without constraining the firms, they can dampen the first accelerator mechanism.

We explore the dynamics of quantities and prices in this setting and compare them to U.S. data, with a focus on understanding differences between financial and non-financial recessions. Our main application studies macro-prudential policy and contrasts restrictions on firm leverage to those on bank leverage. While such policies reduce the credit risk and promote macro-economic stability and better risk sharing among the agents, they potentially shrink the size of the economy and may ultimately be welfare-reducing.

Extensions to this model could introduce New Keyenesian elements such as nominal rigidities, monopolistic competition, and monetary policy. Our setting is an interesting one to evaluate the effect of the zero lower bound (ZLB) on nominal short rates. A binding ZLB during a financial crisis would keep the real rate elevated. The ZLB economy would prevent the intermediaries from recapitalizing in a crisis through a negative real deposit rate. Negative real rates mitigate the severity of financial recessions in the current model. The upshot is that a binding ZLB may lead to a severe crisis exactly because it prevents a recapitalization of financial intermediaries.

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A Model Appendix

A.1 Borrower-entrepreneur problem

A.1.1 Technology

The exogenous laws of motion for labor-augmenting productivity Z_t and the TFP level Z_t^A are respectively (lower case letters denote logs):

$$\log Z_t = \log Z_{t-1} + g$$

$$\log Z_t^A = (1-\rho_A)z^A + \rho_A \log Z_{t-1}^A + \epsilon_t^A \quad \epsilon_t^A \sim iid\,\mathcal{N}(0,\sigma^A)$$

Denote $\mu_G = e^g$, and $\mu_{ZA} = e^{z^A + \frac{(\sigma^A)^2}{2(1-\rho_A^2)}}$.

Idiosyncratic productivity of borrower-entrepreneur i at date t is denoted

$$\omega_{i,t} \sim iid\,\mathcal{N}(\mu_\omega, \sigma_{\omega,t}^2),$$

and individual output is

$$Y_{i,t} = \omega_{i,t} Z_t^A K_t^{1-\alpha}(Z_t L_t)^\alpha.$$

Aggregate production is

$$Y_t = \int_\Omega Y_{i,t} dF(\omega_i) = \int_\Omega \omega dF(\omega) Z_t^A K_t^{1-\alpha}(Z_t L_t)^\alpha = \mu_\omega Z_t^A K_t^{1-\alpha}(Z_t L_t)^\alpha.$$

Individual producer profit is

$$\pi_{i,t} = Y_{i,t} - \sum_j w^j L^j - A_t.$$

Therefore, the default cutoff at $\pi_{i,t} = \underline{\pi}$ is

$$\omega_t^* = \frac{\underline{\pi} + \sum_j w_t^j L_t^j + A_t}{Y_t/\mu_\omega}.$$

A.1.2 Preliminaries

We start by defining some preliminaries.

$$\Omega_A(\omega_t^*) = 1 - F_{\omega,t}(\omega_t^*)$$

$$\Omega_K(\omega_t^*) = \int_{\omega_t^*}^\infty \omega dF_{\omega,t}(\omega) = (1 - F_{\omega,t}(\omega_t^*))\mu_\omega + \sigma_{\omega,t}\phi\left(\frac{\omega_t^* - \mu_\omega}{\sigma_{\omega,t}}\right)$$

where $F_{\omega,t}(\cdot)$ is the CDF of $\omega_{i,t}$ with parameters $(\mu_\omega, \sigma^2_{\omega,t})$, and where $\phi()$ is the pdf of a standard normal random variable.

It is useful to compute the derivatives of $\Omega_K(\cdot)$ and $\Omega_A(\cdot)$:

$$\begin{aligned}
\frac{\partial \Omega_K(\omega_t^*)}{\partial \omega_t^*} &= \frac{\partial}{\partial \omega_t^*} \int_{\omega_t^*}^{\infty} \omega f_\omega(\omega) d\omega = -\omega_t^* f_\omega(\omega_t^*), \\
\frac{\partial \Omega_A(\omega_t^*)}{\partial \omega_t^*} &= \frac{\partial}{\partial \omega_t^*} \int_{\omega_t^*}^{\infty} f_\omega(\omega) d\omega = -f_\omega(\omega_t^*),
\end{aligned}$$

where $f_\omega(\cdot)$ is the p.d.f. of a normal distribution with parameters $(\mu_\omega, \sigma^2_\omega)$.

Capital Adjustment Cost Let

$$\Psi(X_t, K_t^B) = \frac{\psi}{2} \left(\frac{X_t}{K_t^B} - (\mu_G + \delta_K) \right)^2 K_t^B.$$

Then partial derivatives are

$$\Psi_X(X_t, K_t^B) = \psi \left(\frac{X_t}{K_t^B} - (\mu_G + \delta_K) \right) \tag{19}$$

$$\Psi_K(X_t, K_t^B) = -\frac{\psi}{2} \left(\left(\frac{X_t}{K_t^B} \right)^2 - (\mu_G + \delta_K)^2 \right) \tag{20}$$

A.1.3 Optimization Problem

We consider the producers's problem in the current period after aggregate TFP and idiosyncratic productivity shocks have been realized, after the intermediary has chosen a default policy, and after the intermediary's random utility penalty is realized. To ensure stationarity of the producer's problem we define the following transformed variables,

$$\left\{ \hat{C}_t^B, \hat{X}_t, \hat{A}_t^B, \hat{K}_t^B, \hat{w}_t^j, \hat{G}_t^{T,B} \right\},$$

where for any variable $v\hat{a}r_t$ denotes division by the current realization of (deterministic, labor-augmenting) productivity Z_t:

$$v\hat{a}r_t = \frac{var_t}{Z_t}.$$

For the choices of capital and debt for the next period we further define

$$\hat{\hat{K}}_{t+1}^B = \frac{K_{t+1}^B}{Z_t}$$

and

$$\hat{\hat{A}}_{t+1}^B = \frac{A_{t+1}^B}{Z_t}.$$

Let $\mathcal{S}_t^B = \left(Z_t^A, \sigma_{\omega,t}, \hat{W}_t^I, \hat{W}_t^S, \hat{B}_{t-1}^G\right)$ represent state variables exogenous to the borrower-entrepreneur's decision.

Then the stationary problem is

$$\hat{V}^B(\hat{K}_t^B, \hat{A}_t^B, \mathcal{S}_t^B) = \max_{\{\hat{C}_t^B, \hat{\hat{K}}_{t+1}^B, 1, \hat{X}_t, \hat{\hat{A}}_{t+1}^B, L_t^j\}} \left\{ (1-\beta_B)\left(\hat{C}_t^B\right)^{1-1/\nu} + \right.$$
$$\left. + \beta_B \mathrm{E}_t \left[\left(e^g \tilde{\hat{V}}^B(e^{-g}\hat{\hat{K}}_{t+1}^B, e^{-g}\hat{\hat{A}}_{t+1}^B, \mathcal{S}_{t+1}^B) \right)^{1-\sigma_B} \right]^{\frac{1-1/\nu}{1-\sigma_B}} \right\}^{\frac{1}{1-1/\nu}}$$

subject to

$$\begin{aligned}
\hat{C}_t^B = &(1-\tau_\Pi^B)\Omega_K(\omega_t^*)\hat{Y}_t/\mu_\omega + (1-\tau_t^B)\hat{w}_t^B \bar{L}^B + \hat{G}_t^{T,B} + p_t[\hat{X}_t + \Omega_A(\omega_t^*)(1-\tilde{\delta}_K)\hat{K}_t^B] \\
&+ q_t^m \hat{\hat{A}}_{t+1}^B - \Omega_A(\omega_t^*)\hat{A}_t^B(1-(1-\theta)\tau_\Pi^B + \delta q_t^m) \\
&- p_t \hat{\hat{K}}_{t+1}^B - \hat{X}_t - \Psi(\hat{X}_t, \hat{K}_t^B) - (1-\tau_\Pi^B)\Omega_A(\omega_t^*) \sum_{j=B,I,S} \hat{w}_t^j L_t^j
\end{aligned} \tag{21}$$

$$F\hat{\hat{A}}_t^B \leq \Phi p_t \Omega_A(\omega_t^*)(1-\tilde{\delta}_K)\hat{K}_t^B, \tag{22}$$

where we have define after-tax depreciation $\hat{\delta}_K = (1-\tau_\Pi^B)\delta_K$.

The continuation value $\tilde{V}^B(\cdot)$ must take into account the default decision of the intermediary at the beginning of next period. We anticipate here and show below that that default decision takes the form of a cutoff rule:

$$\begin{aligned}
\tilde{\hat{V}}^B\left(\hat{K}_t^B, \hat{A}_t^B, \mathcal{S}_t^B\right) =& F_\rho(\rho_t^*)\mathrm{E}_\rho\left[\hat{V}^B(\hat{K}_t^B, \hat{A}_t^B, \mathcal{S}_t^B) \,|\, \rho < \rho_t^*\right] + (1-F_\rho(\rho_t^*))\,\mathrm{E}_\rho\left[\hat{V}^B(\hat{K}_t^B, \hat{A}_t^B, \mathcal{S}_t^B) \,|\, \rho > \rho_t^*\right] \\
=& F_\rho(\rho_t^*)\hat{V}^B(\hat{K}_t^B, \hat{A}_t^B, \mathcal{S}_t^S(\rho_t < \rho_t^*)) + (1-F_\rho(\rho_t^*))\,\hat{V}^B(\hat{K}_t^B, \hat{A}_t^B, \mathcal{S}_t^S(\rho_t > \rho_t^*)),
\end{aligned} \tag{23}$$

where (23) obtains because the expectation terms conditional on realizations of ρ_t and ρ_t^* only differ in the values of the aggregate state variables.

Denote the value function and the partial derivatives of the value function as:

$$\begin{aligned}
\hat{V}_t^B &\equiv \hat{V}(\hat{K}_t^B, \hat{A}_t^B, \mathcal{S}_t^B), \\
\hat{V}_{A,t}^B &\equiv \frac{\partial \hat{V}(\hat{K}_t^B, \hat{A}_t^B, \mathcal{S}_t^B)}{\partial \hat{A}_t^B}, \\
\hat{V}_{K,t}^B &\equiv \frac{\partial \hat{V}(\hat{K}_t^B, \hat{A}_t^B, \mathcal{S}_t^B)}{\partial \hat{K}_t^B}.
\end{aligned}$$

Therefore the marginal values of borrowing and of capital of $\tilde{V}^B(\cdot)$ are:

$$\tilde{\hat{V}}^B_{A,t} = F_\rho(\rho^*_t)\frac{\partial \hat{V}^B(\hat{K}^B_t, \hat{A}^B_t, \mathcal{S}^B_t(\rho_t < \rho^*_t))}{\partial \hat{A}^B_t} + (1 - F_\rho(\rho^*_t))\,\frac{\partial \hat{V}^B(\hat{K}^B_t, \hat{A}^B_t, \mathcal{S}^B_t(\rho_t > \rho^*_t))}{\partial \hat{A}^B_t}$$
$$\tilde{\hat{V}}^B_{K,t} = F_\rho(\rho^*_t)\frac{\partial \hat{V}^B(\hat{K}^B_t, \hat{A}^B_t, \mathcal{S}^B_t(\rho_t < \rho^*_t))}{\partial \hat{K}^B_t} + (1 - F_\rho(\rho^*_t))\,\frac{\partial \hat{V}^B(\hat{K}^B_t, \hat{A}^B_t, \mathcal{S}^B_t(\rho_t > \rho^*_t))}{\partial \hat{K}^B_t}$$

Denote the certainty equivalent of future utility as:

$$CE^B_t = \mathrm{E}_t\left[\left(e^g \tilde{\hat{V}}^B(\hat{K}^B_{t+1}, \hat{A}^B_{t+1}, \mathcal{S}^B_{t+1})\right)^{1-\sigma_B}\right]^{\frac{1}{1-\sigma_B}}.$$

Marginal Cost of Default Before deriving optimality conditions, it is useful to compute the marginal consumption loss due to an increased default threshold ω^*_t

$$\begin{aligned}
\frac{\partial C^B_t}{\partial \omega^*_t} =& \frac{\partial \Omega_K(\omega^*_t)}{\partial \omega^*_t}(1-\tau^B_\Pi)\hat{Y}_t/\mu_\omega \\
&+ \frac{\partial \Omega_A(\omega^*_t)}{\partial \omega^*_t}\left[(1-\tilde{\delta}_K)p_t\hat{K}^B_t - \hat{A}^B_t(1-(1-\theta)\tau^B_\Pi + \delta q^m_t) - (1-\tau^B_\Pi)\sum_j \hat{w}^j_t L^j_t\right] \\
=& -f_\omega(\omega^*_t)\left[(1-\tau^B_\Pi)\omega^*_t\hat{Y}_t/\mu_\omega + (1-\tilde{\delta}_K)p_t\hat{K}^B_t - \hat{A}^B_t(1-(1-\theta)\tau^B_\Pi + \delta q^m_t) - (1-\tau^B_\Pi)\sum_j \hat{w}^j_t L^j_t\right] \\
=& -f_\omega(\omega^*_t)\frac{\hat{Y}_t}{\mu_\omega}\underbrace{\left[\frac{(1-\tilde{\delta}_K)p_t\hat{K}^B_t - \hat{A}^B_t(\theta\tau^B_\Pi + \delta q^m_t)}{\hat{Y}_t/\mu_\omega}\right]}_{=\mathcal{F}_t} \\
=& -f_\omega(\omega^*_t)\frac{\hat{Y}_t}{\mu_\omega}\mathcal{F}_t.
\end{aligned}$$

The function $\mathcal{F}_t$ has an intuitive interpretation as the marginal loss, expressed in consumption units per unit of aggregate output, to producers from an increase in the default threshold. The first term is the loss of capital due to defaulting members. The second term represents gains due to debt erased in foreclosure.

A.1.4 First-order conditions

Loans The FOC for loans $\hat{\hat{A}}^B_{t+1}$ is:

$$\begin{aligned}
&q^m_t\frac{(\hat{u}^B_t)^{1-1/\nu}}{\hat{C}^B_t}(1-\beta_B)(\hat{V}^B_t)^{1/\nu} = \\
&\qquad \lambda^B_t F - \beta_B \mathrm{E}_t[(e^g\tilde{\hat{V}}^B_{t+1})^{-\sigma_B}\tilde{\hat{V}}^B_{A,t+1}](CE^B_t)^{\sigma_B - 1/\nu}(V^B_t)^{1/\nu}
\end{aligned} \qquad (24)$$

where λ_t^B is the Lagrange multiplier on the constraint in (22).

Capital Similarly, the FOC for new capital $\hat{\hat{K}}_{t+1}^B$ is:

$$p_t \frac{(1-\beta_B)(\hat{V}_t^B)^{1/\nu}(\hat{u}_t^B)^{1-1/\nu}}{\hat{C}_t^B} = \beta_B \mathrm{E}_t[(e^g \tilde{\hat{V}}_{t+1}^B)^{-\sigma_B} \tilde{\hat{V}}_{K,t+1}^B](CE_t^B)^{\sigma_B - 1/\nu}(\hat{V}_t^B)^{1/\nu} \tag{25}$$

Investment The FOC for investment $\hat{X}_t$ is:

$$[1+\Psi_X(\hat{X}_t^B, \hat{K}_t^B) - p_t]\frac{(1-\beta_B)(\hat{U}_t^B)^{1-1/\nu}(\hat{V}_t^B)^{1/\nu}}{\hat{C}_t^B} = 0,$$

which simplifies to

$$1+\Psi_X(\hat{X}_t^B, \hat{K}_t^B) = p_t.$$

Labor Inputs Defining $\gamma_B = 1-\gamma_I-\gamma_S$, aggregate labor input is

$$L_t = \prod_{j=B,I,S}(L_t^j)^{\gamma_j}.$$

We further compute

$$\frac{\partial \omega_t^*}{\partial L_t^j} = \mu_\omega \left(\frac{\hat{w}_t^j}{\hat{Y}_t} - \omega_t^* \frac{\mathrm{MPL}_t^j}{\hat{Y}_t} \right),$$

defining the marginal product of labor of type j as

$$\mathrm{MPL}_t^j = \alpha \gamma_j \mu_\omega Z_t^A \frac{L_t}{L_t^j}\left(\frac{\hat{K}_t^B}{L_t}\right)^{1-\alpha}.$$

The FOC for labor input L_t^j is then

$$\frac{(1-\beta_B)(\hat{u}_t^B)^{1-1/\nu}(\hat{V}_t^B)^{1/\nu}}{\hat{C}_t^B}\left[(1-\tau_\Pi^B)\Omega_K(\omega_t^*)\mathrm{MPL}_t^j/\mu_\omega - (1-\tau_\Pi^B)\Omega_A(\omega_t^*)\hat{w}_t^j + \frac{\partial \omega_t^*}{\partial L_t^j}\frac{\partial \hat{C}_t^B}{\partial \omega_t^*}\right] = 0,$$

which yields

$$(1-\tau_\Pi^B)\Omega_K(\omega_t^*)\mathrm{MPL}_t^j/\mu_\omega = (1-\tau_\Pi^B)\Omega_A(\omega_t^*)\hat{w}_t^j + f_\omega(\omega_t^*)\left(\hat{w}_t^j - \omega_t^*\mathrm{MPL}_t^j\right)\mathcal{F}_t. \tag{26}$$

A.1.5 Marginal Values of State Variables and SDF

Loans Taking the derivative of the value function with respect to $\hat{A}^B_t$ gives:

$$
\begin{aligned}
\hat{V}^B_{A,t} &= \left[-\left(1-(1-\theta)\tau^I_\Pi + \delta q^m_t\right)\Omega_A(\omega^*_t) + \frac{\partial \omega^*_t}{\partial \hat{A}^B_t}\frac{\partial \hat{C}^B_t}{\partial \omega^*_t}\right] \frac{(1-\beta_B)(\hat{u}^B_t)^{1-1/\nu}(\hat{V}^B_t)^{1/\nu}}{\hat{C}^B_t} \\
&= -\left[\left(1-(1-\theta)\tau^I_\Pi + \delta q^m_t\right)\Omega_A(\omega^*_t) + f_\omega(\omega^*_t)\mathcal{F}_t\right] \frac{(1-\beta_B)(\hat{u}^B_t)^{1-1/\nu}(\hat{V}^B_t)^{1/\nu}}{\hat{C}^B_t},
\end{aligned} \tag{27}
$$

where we used the fact that $\frac{\partial \omega^*_t}{\partial \hat{A}^B_t} = \frac{1}{\hat{Y}_t/\mu_\omega}$.

Capital Taking the derivative of the value function with respect to $\hat{K}^B_t$ gives:

$$
\begin{aligned}
\hat{V}^B_{K,t} = \Bigg[& p_t\Omega_A(\omega^*_t)\left(1-(1-\tau^B_\Pi)\delta_K\right) + (1-\tau^B_\Pi)(1-\alpha)\Omega_K(\omega^*_t)Z^A_t\left(\frac{\hat{K}^B_t}{L_t}\right)^{-\alpha} - \Psi_K(\hat{X}^B_t,\hat{K}^B_t) + \frac{\partial \hat{C}^B_t}{\partial \omega^*_t}\frac{\partial \omega^*_t}{\partial \hat{K}^B_t} \\
&+ \tilde{\lambda}^B_t \Phi p_t(1-\tilde{\delta}_K)\left(\Omega_A(\omega^*_t) + \hat{K}^B_t \frac{\partial \Omega_A(\omega^*_t)}{\partial \omega^*_t}\frac{\partial \omega^*_t}{\partial \hat{K}^B_t}\right)\Bigg] \frac{(1-\beta_B)(\hat{u}^B_t)^{1-1/\nu}(\hat{V}^B_t)^{1/\nu}}{\hat{C}^B_t}.
\end{aligned}
$$

Taking the derivative

$$
\frac{\partial \omega^*_t}{\partial \hat{K}^B_t} = -\mu_\omega \frac{\omega^*_t}{\hat{Y}_t}(1-\alpha)Z^A_t\left(\frac{\hat{K}^B_t}{L_t}\right)^{-\alpha},
$$

we get

$$
\begin{aligned}
\hat{V}^B_{K,t} = \Bigg\{& p_t\Omega_A(\omega^*_t)(1-\tilde{\delta}_K)\left(1+\Phi\tilde{\lambda}^B_t\right) + (1-\tau^B_\Pi)(1-\alpha)\Omega_K(\omega^*_t)Z^A_t\left(\frac{\hat{K}^B_t}{L_t}\right)^{-\alpha} - \Psi_K(\hat{X}^B_t,\hat{K}^B_t) \\
&+(1-\alpha)f_\omega(\omega^*_t)\omega^*_t\left[Z^A_t\left(\frac{\hat{K}^B_t}{L_t}\right)^{-\alpha}\mathcal{F}_t + \tilde{\lambda}^B_t\Phi p_t(1-\tilde{\delta}_K)\right]\Bigg\} \frac{(1-\beta_B)(\hat{u}^B_t)^{1-1/\nu}(\hat{V}^B_t)^{1/\nu}}{\hat{C}^B_t}.
\end{aligned} \tag{28}
$$

SDF We can define the stochastic discount factor (SDF) from t to $t+1$ of borrowers, conditional on a particular realization of ρ_{t+1} as:

$$
\mathcal{M}^B_{t,t+1}(\rho_{t+1}) = \beta_B e^{-\sigma_B g}\left(\frac{\hat{u}^B_{t+1}(\rho_{t+1})}{\hat{u}^B_t}\right)^{1-1/\nu}\left(\frac{\hat{C}^B_{t+1}(\rho_{t+1})}{\hat{C}^B_t}\right)^{-1}\left(\frac{\hat{V}^B_{t+1}(\rho_{t+1})}{CE^B_t}\right)^{1/\nu}\left(\frac{\tilde{\hat{V}}^B_{t+1}}{CE^B_t}\right)^{-\sigma_B} \tag{29}
$$

Conditional on information at time t, every endogenous random variable at $t+1$ is adapted both to TFP level Z^A_{t+1} and to the event $\mathbb{1}[\rho_{t+1} < \rho^*_{t+1}]$. For any such variable we define the sum

$$
\vec{x}_{t+1} = \mathbb{1}[\rho_{t+1} < \rho^*_{t+1}]\left(x_{t+1} \,|\, \rho_{t+1} < \rho^*_{t+1}\right) + \mathbb{1}[\rho_{t+1} \geq \rho^*_{t+1}]\left(x_{t+1} \,|\, \rho_{t+1} \geq \rho^*_{t+1}\right).
$$

Hence for any payoff x_{t+1} we can write the appropriately discounted payoff as

$$\vec{\mathcal{M}}^B_{t,t+1}\vec{x}_{t+1}.$$

A.1.6 Euler Equations

Loans Recall that $\tilde{V}^B_{A,t+1}$ is a linear combination of $V^B_{A,t+1}$ conditional on ρ_t being below and above the threshold, and with each $V^B_{A,t+1}$ given by equation (27). Substituting in for $\tilde{V}^B_{A,t+1}$ in (24) and using the SDF expression, we get the recursion:

$$q^m_t = \tilde{\lambda}^B_t F + \mathrm{E}_t \left\{ \vec{\mathcal{M}}^B_{t,t+1} \left[\Omega_A(\vec{\omega}^*_{t+1}) \left(1 - (1-\theta)\tau_\Pi + \delta \vec{q}^m_{t+1}\right) + f_\omega(\vec{\omega}^*_{t+1})\vec{\mathcal{F}}_{t+1} \right] \right\}. \tag{30}$$

Capital Likewise, recall that $\tilde{V}^B_{K,t+1}$ is a linear combination of $V^B_{K,t+1}$ conditional on ρ_t being below and above the threshold, and with each $V^B_{K,t+1}$ given by equation (28). Substituting in for $\tilde{V}^B_{K,t+1}$ and using the SDF expression, we get the recursion:

$$\begin{aligned} p_t = \mathrm{E}_t \Bigg[\vec{\mathcal{M}}^B_{t,t+1} \Bigg\{ & \vec{p}_{t+1}\Omega_A(\vec{\omega}^*_{t+1})(1-\tilde{\delta}_K)\left(1 + \Phi\vec{\tilde{\lambda}}^B_{t+1}\right) + (1-\tau_\Pi)(1-\alpha)\Omega_K(\vec{\omega}^*_{t+1})Z^A_{t+1}\left(\frac{\vec{\hat{K}}^B_{t+1}}{\vec{L}_{t+1}}\right)^{-\alpha} \\ & -\Psi_K(\vec{\hat{X}}^B_{t+1}, \vec{\hat{K}}^B_{t+1}) + (1-\alpha)f_\omega(\vec{\omega}^*_{t+1})\vec{\omega}^*_{t+1}\left(Z^A_{t+1}\left(\frac{\vec{\hat{K}}^B_{t+1}}{\vec{L}_{t+1}}\right)^{-\alpha}\vec{\mathcal{F}}_{t+1} + (1-\tilde{\delta}_K)\Phi\vec{\tilde{\lambda}}^B_{t+1}\vec{p}_{t+1}\right)\Bigg\}\Bigg]. \end{aligned} \tag{31}$$

A.2 Intermediaries

A.2.1 Statement of stationary problem

As for borrower-entrepreneurs, we define the following transformed variables for intermediaries:

$$\{\hat{W}^I_t, \hat{C}^I_t, \hat{A}^I_{t+1}, \hat{G}^{T,I}_t, \hat{B}^I_t\}.$$

Denote by $\hat{W}^I_t$ intermediary wealth at the beginning of the period, before their bankruptcy decision. Then wealth after realization of the penalty ρ_t is:

$$\tilde{\hat{W}}^I_t = (1 - D(\rho_t))\hat{W}^I_t,$$

and the effective utility penalty is:

$$\tilde{\rho}_t = D(\rho_t)\rho_t.$$

Let $\mathcal{S}^I_t = \left(Z^A_t, \sigma_{\omega,t}, \hat{K}^B_t, \hat{A}^B_t, \hat{B}^G_{t-1}, \hat{W}^S_t\right)$ denote all other aggregate state variables exogenous to intermediaries.

After the default decision, intermediaries face the following optimization problem over consumption and portfolio composition, formulated to ensure stationarity:

$$\hat{V}^I(\tilde{\hat{W}}_t^I, \tilde{\rho}_t, \mathcal{S}_t^I) = \max_{\hat{C}_t^I, \hat{A}_{t+1}^I, \hat{B}_t^I} \left\{ (1-\beta_I) \left[\frac{\hat{C}_t^I}{e^{\tilde{\rho}_t}} \right]^{1-1/\nu} + \beta_I \mathrm{E}_t \left[\left(e^g \tilde{\hat{V}}^I \left(\hat{W}_{t+1}^I, \mathcal{S}_{t+1}^I \right) \right)^{1-\sigma_R} \right]^{\frac{1-1/\nu}{1-\sigma_R}} \right\}^{\frac{1}{1-1/\nu}} \tag{32}$$

subject to:

$$(1-\tau^I)\hat{w}_t^I \bar{L}^I + \tilde{\hat{W}}_t^I + \hat{G}_t^{T,I} = \hat{C}_t^I + q_t^m \hat{A}_{t+1}^I + (q_t^f + \tau_\Pi^I r_t^f - \kappa I_{\{\hat{B}_t^I<0\}})\hat{B}_t^I, \tag{33}$$

$$\hat{W}_{t+1}^I = e^{-g} \left[\left(\tilde{M}_{t+1} + \Omega_A(\omega_{t+1}^*)\delta q_{t+1}^m \right) \hat{A}_{t+1}^I + \hat{B}_t^I \right], \tag{34}$$

$$\hat{B}_t^I \geq -\xi q_t^m \hat{A}_{t+1}^I, \tag{35}$$

$$\hat{A}_{t+1}^R \geq 0, \tag{36}$$

$$\mathcal{S}_{t+1}^I = h(\mathcal{S}_t^I). \tag{37}$$

For the evolution of intermediary wealth in (34), we have defined the total after-tax payoff per bond

$$\tilde{M}_{t+1} = (1-(1-\theta)\tau_\Pi^I)\Omega_A(\omega_{t+1}^*) + \hat{M}_{t+1}/A_{t+1}^B,$$

where $\hat{M}_{t+1}$ is the total recovery value of bankrupt borrower firms seized by intermediaries, as defined in (12).

The continuation value $\tilde{\hat{V}}^I \left(\hat{W}_{t+1}^I, \mathcal{S}_{t+1}^I \right)$ is the outcome of the optimization problem intermediaries face at the beginning of the following period, i.e., before the decision over the optimal bankruptcy rule. This continuation value function is given by:

$$\tilde{\hat{V}}^I(\hat{W}_t^I, \mathcal{S}_t^I) = \max_{D(\rho)} \mathrm{E}_\rho \left[D(\rho)\hat{V}^I(0, \rho, \mathcal{S}_t^I) + (1-D(\rho))\hat{V}^I(\hat{W}_t^I, 0, \mathcal{S}_t^I) \right] \tag{38}$$

Define the certainty equivalent of future utility as:

$$CE_t^I = \mathrm{E}_t \left[\left(e^g \tilde{\hat{V}}^I \left(\hat{W}_{t+1}^I, \mathcal{S}_{t+1}^I \right) \right)^{1-\sigma_I} \right]^{\frac{1}{1-\sigma_I}}. \tag{39}$$

A.2.2 First-order conditions

Optimal Default Decision The optimization consists of choosing a function $D(\rho) : \mathbb{R} \to \{0, 1\}$ that specifies for each possible realization of the penalty ρ whether or not to default.

Since the value function $\hat{V}^I(W, \rho, \mathcal{S}_t^R)$ defined in (32) is increasing in wealth W and decreasing in the penalty ρ, there will generally exist an optimal threshold penalty ρ^* such that for a given $\hat{W}_t^I$, intermediaries optimally default for all realizations $\rho < \rho^*$. Hence we can equivalently write the optimization problem in (38) as

$$\begin{aligned}\tilde{\hat{V}}^I(\hat{W}_t^I, \mathcal{S}_t^R) =& \max_{\rho^*} \mathrm{E}_\rho \left[\mathbb{1}[\rho < \rho^*]\, \hat{V}^I(0, \rho, \mathcal{S}_t^I) + (1 - \mathbb{1}[\rho < \rho^*])\, \hat{V}^I(\hat{W}_t^I, 0, \mathcal{S}_t^I)\right] \\ =& \max_{\rho^*} F_\rho(\rho^*)\, \mathrm{E}_\rho \left[\hat{V}^I(0, \rho, \mathcal{S}_t^I) \,|\, \rho < \rho^*\right] + (1 - F_\rho(\rho^*))\, \hat{V}^I(\hat{W}_t^I, 0, \mathcal{S}_t^I).\end{aligned}$$

The solution ρ_t^* is characterized by the first-order condition:

$$\hat{V}^I(0, \rho_t^*, \mathcal{S}_t^I) = \hat{V}^I(\hat{W}_t^I, 0, \mathcal{S}_t^I).$$

By defining the partial inverse $\mathcal{F} : (0, \infty) \to (-\infty, \infty)$ of $\hat{V}^I(\cdot)$ in its second argument as

$$\left\{(x, y) : y = \mathcal{F}(x) \Leftrightarrow x = \hat{V}^I(0, y)\right\},$$

we get that

$$\rho_t^* = \mathcal{F}(\hat{V}^I(\hat{W}_t^I, 0, \mathcal{S}_t^I)), \tag{40}$$

and by substituting the solution into (38), we obtain

$$\tilde{\hat{V}}^I(\hat{W}_t^I, \mathcal{S}_t^I) = F_\rho(\rho_t^*)\mathrm{E}_\rho \left[\hat{V}^I(0, \rho, \mathcal{S}_t^I) \,|\, \rho < \rho_t^*\right] + (1 - F_\rho(\rho_t^*))\, \hat{V}^I(\hat{W}_t^I, 0, \mathcal{S}_t^I). \tag{41}$$

Equations (32), (40), and (41) completely characterize the optimization problem of intermediaries.

To compute the optimal bankruptcy threshold ρ_t^*, note that the inverse value function defined in equation (40) is given by:

$$\mathcal{F}(x) = \begin{cases} \log((1 - \beta_I)\hat{C}_t^I) - \frac{1}{1-1/\nu}\log\left(x^{1-1/\nu} - \beta_I (CE_t^I)^{1-1/\nu}\right) & \text{for } \nu > 1 \\ (1 - \beta_I)\log(\hat{C}_t^I) + \beta_I \log(CE_t^I) - \log(x) - (1 - \beta_I) & \text{if } \nu = 1. \end{cases}$$

Optimal Portfolio Choice The first-order condition for the short-term bond position is:

$$\begin{aligned}&(q_t^f + \tau_\Pi r_t^f - \kappa I_{\{\hat{B}_t^I < 0\}})\frac{(1 - \beta_I)(\hat{V}_t^I)^{1/\nu}}{(\hat{C}_t^I)^{1/\nu}} = \\ &\lambda_t^I + \beta_I \mathrm{E}_t[(e^g \tilde{\hat{V}}_{t+1}^I)^{-\sigma_I} \tilde{\hat{V}}_{W,t+1}^I](CE_t^I)^{\sigma_I - 1/\nu}(\hat{V}_t^I)^{1/\nu}\end{aligned} \tag{42}$$

where λ_t^I is the Lagrange multiplier on the borrowing constraint (35).

The first order condition for loans is:

$$\begin{aligned}&(q_t^m + \tau_\Pi r_t^m F)\frac{(1 - \beta_I)(\hat{V}_t^I)^{1/\nu}}{(\hat{C}_t^I)^{1/\nu}} = \lambda_t^I \xi q_t^m + \mu_t^I \\ &+ \beta_I \mathrm{E}_t[(e^g \tilde{\hat{V}}_{t+1}^I)^{-\sigma_I} \tilde{\hat{V}}_{W,t+1}^I \left(\tilde{M}_{t+1} + \delta \Omega_A(\omega_{t+1}^*) q_{t+1}^m\right)](CE_t^I)^{\sigma_I - 1/\nu}(\hat{V}_t^I)^{1/\nu},\end{aligned} \tag{43}$$

where μ_t^I is the Lagrange multiplier on the no-shorting constraint for guaranteed loans (36).

A.2.3 Marginal value of wealth and SDF

Differentiating (41) gives the marginal value of wealth

$$\tilde{\hat{V}}^I_{W,t} = (1 - F_\rho(\rho^*_t)) \frac{\partial \hat{V}^I(\hat{W}^I_t, 0, \mathcal{S}^I_t)}{\partial \hat{W}^I_t},$$

where

$$\frac{\partial \hat{V}^I(\hat{W}^I_t, 0, \mathcal{S}^I_t)}{\partial \hat{W}^I_t} = (\hat{C}^I_t)^{-1/\nu}(1 - \beta_I)(\hat{V}^I(\hat{W}^I_t, 0, \mathcal{S}^I_t))^{1/\nu},$$

The stochastic discount factor of intermediaries conditional on $\rho_{t+1} \geq \rho^*_{t+1}$ is therefore

$$\mathcal{M}^I_{t,t+1} = \beta_I e^{-\sigma_I g} \left(\frac{\hat{V}^I(\hat{W}^I_{t+1}, 0, \mathcal{S}^I_{t+1})}{CE^I_t} \right)^{1/\nu} \left(\frac{\tilde{\hat{V}}^I(\tilde{\hat{W}}^I_{t+1}, \mathcal{S}^I_{t+1})}{CE^I_t} \right)^{-\sigma_I} \left(\frac{\hat{C}^I_{t+1}}{\hat{C}^I_t} \right)^{-1/\nu}.$$

A.2.4 Euler Equations

It is then possible to show that the FOC with respect to $\hat{B}^I_t$ and $\hat{A}^I_{t+1}$, respectively, are:

$$q^f_t = \tilde{\lambda}^I_t + \mathrm{E}_t\left[(1 - F_\rho(\rho^*_{t+1}))\mathcal{M}^I_{t,t+1}\right] + \kappa I_{\{\hat{B}^I_t<0\}} - \tau_\Pi r^f_t, \tag{44}$$

$$q^m_t(1 - \xi\tilde{\lambda}^I_t) = \tilde{\mu}^I_t + \mathrm{E}_t\left[(1 - F_\rho(\rho^*_{t+1}))\mathcal{M}^I_{t,t+1}\left(\tilde{M}_{t+1} + \delta\Omega_A(\omega^*_{t+1})q^m_{t+1}\right)\right]. \tag{45}$$

A.3 Savers

A.3.1 Statement of stationary problem

For savers, we define the following transformed variables:

$$\{\hat{W}^S_t, \hat{C}^S_t, \hat{B}^S_t, \hat{G}^{T,S}_t\}.$$

Let $\mathcal{S}^S_t = \left(Z^A_t, \sigma_{\omega,t}, \hat{K}^B_t, \hat{A}^B_t, \hat{W}^I_t, \hat{B}^G_{t-1}\right)$ be the saver's state vector capturing all exogenous state variables. Scaling by (labor-augmenting) productivity, the stationary problem of the saver – after the intermediary has made default her decision and the utility cost of default is realized – is:

$$\hat{V}^S(\hat{W}^S_t, \mathcal{S}^S_t) = \max_{\{\hat{C}^S_t, \hat{B}^S_t\}} \left\{ (1 - \beta_S)\left[\hat{C}^S_t\right]^{1-1/\nu} + \beta_S \mathrm{E}_t\left[\left(e^g \tilde{\hat{V}}^S(\hat{W}^S_{t+1}, \mathcal{S}^S_{t+1})\right)^{1-\sigma_S}\right]^{\frac{1-1/\nu}{1-\sigma_S}} \right\}^{\frac{1}{1-1/\nu}}$$

subject to

$$\hat{C}_t^S = (1-\tau_t^S)\hat{w}_t^S \bar{L}^S + \hat{G}_t^{T,S} + \hat{W}_t^S - q_t^f \hat{B}_t^S \tag{46}$$
$$\hat{W}_{t+1}^S = e^{-g}\hat{B}_t^S \tag{47}$$
$$\hat{B}_t^S \geq 0 \tag{48}$$
$$\mathcal{S}_{t+1}^S = h(\mathcal{S}_t^S) \tag{49}$$

As before, we will drop the arguments of the value function and denote the marginal value of wealth as:

$$\hat{V}_t^S \equiv \hat{V}_t^S(\hat{W}_t^S, \mathcal{S}_t^S),$$
$$\hat{V}_{W,t}^S \equiv \frac{\partial \hat{V}_t^S(\hat{W}_t^S, \mathcal{S}_t^S)}{\partial \hat{W}_t^S},$$

Denote the certainty equivalent of future utility as:

$$CE_t^S = \mathrm{E}_t\left[\left(e^g \tilde{\hat{V}}^S(\hat{W}_t^S, \mathcal{S}_t^S)\right)^{1-\sigma_S}\right].$$

Like borrower-entrepreneurs, savers must take into account the intermediary's default decisions and the realization of the utility penalty of default. Therefore the marginal value of wealth is:

$$\tilde{\hat{V}}_{W,t}^S = F_\rho(\rho_t^*)\frac{\partial V^S(\hat{W}_t^S, \mathcal{S}_t^S(\rho_t < \rho_t^*))}{\partial \hat{W}_t^S} + (1 - F_\rho(\rho_t^*))\,\frac{\partial V^S(\hat{W}_t^S, \mathcal{S}_t^S(\rho_t > \rho_t^*)}{\partial \hat{W}_t^S}.$$

A.3.2 First-order conditions

The first-order condition for the short-term bond position is:

$$q_t^f(\hat{C}_t^S)^{-1/\nu}(1-\beta_S)(\hat{V}_t^S)^{1/\nu} = \lambda_t^S + \beta_S \mathrm{E}_t[(e^g \tilde{\hat{V}}_{t+1}^S)^{-\sigma_S}\tilde{\hat{V}}_{W,t+1}^S](CE_t^S)^{\sigma_S - 1/\nu}(\hat{V}_t^S)^{1/\nu} \tag{50}$$

where λ_t^S is the Lagrange multiplier on the no-borrowing constraint (48).

A.3.3 Marginal Values of State Variables and SDF

Marginal value of wealth is:

$$\hat{V}_{W,t}^S = (\hat{C}_t^S)^{-1/\nu}(1-\beta_S)(\hat{V}_t^S)^{1/\nu}, \tag{51}$$

and for the continuation value function:

$$\tilde{\hat{V}}_{W,t}^S = F_\rho(\rho_t^*)\frac{\partial V^S(\hat{W}_t^S, \mathcal{S}_t^S(\rho_t < \rho_t^*))}{\partial \hat{W}_t^S} + (1 - F_\rho(\rho_t^*))\,\frac{\partial \hat{V}^S(\hat{W}_t^S, \mathcal{S}_t^S(\rho_t > \rho_t^*)}{\partial \hat{W}_t^S}.$$

Defining the SDF in the same fashion as we did for borrowers, we get:

$$\mathcal{M}^S_{t,t+1}(\rho_{t+1}) = \beta_S e^{-\sigma_S g} \left(\frac{\hat{V}^S_{t+1}(\rho_{t+1})}{CE^S_t} \right)^{1/\nu} \left(\frac{\tilde{\hat{V}}^S_{t+1}}{CE^S_t} \right)^{-\sigma_S} \left(\frac{\hat{C}^S_{t+1}(\rho_{t+1})}{\hat{C}^S_t} \right)^{-1/\nu},$$

and

$$\vec{\mathcal{M}}^S_{t,t+1} = \mathbb{1}[\rho_{t+1} < \rho^*_{t+1}] \left(\mathcal{M}^S_{t,t+1} \,|\, \rho_{t+1} < \rho^*_{t+1} \right) + \mathbb{1}[\rho_{t+1} \geq \rho^*_{t+1}] \left(\mathcal{M}^S_{t,t+1} \,|\, \rho_{t+1} \geq \rho^*_{t+1} \right).$$

A.3.4 Euler Equations

Combining the first-order condition for short-term bonds (50) with the marginal value of wealth, and the SDF, we get the Euler equation for the short-term bond:

$$q^f_t = \tilde{\lambda}^S_t + \mathrm{E}_t \left[\vec{\mathcal{M}}^S_{t,t+1} \right] \tag{52}$$

where $\tilde{\lambda}^S_t$ is the original multiplier λ^S_t divided by the marginal value of wealth.

B Calibration Appendix

B.1 Long-term corporate Bonds

Our model's corporate bonds are geometrically declining perpetuities, and as such have no principal. The issuer of one unit of the bond at time t promises to pay the holder 1 at time $t+1$, δ at time $t+2$, δ^2 at time $t+3$, and so on. Issuers must hold enough capital to collateralize the face value of the bond, given by $F = \frac{\theta}{1-\delta}$, a constant parameter that does not depend on any state variable of the economy. Real life bonds have a finite maturity and a principal payment. They also have a vintage (year of issuance), whereas our bonds combine all vintages in one variable. This appendix explains how to map the geometric bonds in our model into real-world bonds by choosing values for δ and θ.

Our model's corporate loan/bond refers to the entire pool of all outstanding corporate loans/bonds. To proxy for this pool, we use investment-grade and high-yield indices constructed by Bank of America Merill Lynch (BofAML) and Barclays Capital (BarCap). For the BofAML indices[31] we obtain a time series of monthly market values, durations (the sensitivity of prices to interest rates), weighted-average maturity (WAM), and weighted average coupons (WAC) for January 1997 until December 2015. For the BarCap indices[32] we obtain a time series of option-adjusted spreads over the Treasury yield curve.

First, we use market values of the BofAML investment grade and high-yield portfolios to create an aggregate bond index and find its mean WAC c of 5.5% and WAM T of 10 years over our time period. We also add the time series of OAS to the constant maturity treasury rate

[31] Datastream Codes LHYIELD and LHCCORP for investment grade and high-yield corporate bonds, respectively

[32] They are named C0A0 and H0A0 for investment grade and high-yield corporate bonds, respectively.

corresponding to that period's WAM to get a time series of bond yields r_t. Next, we construct a plain vanilla corporate bond with a semiannual coupon and maturity equal to the WAC and WAM of the aggregate bond index, and compute the price for \$1 par of this bond for each yield:

$$P^c(r_t) = \sum_{i=1}^{2T} \frac{c/2}{(1+r_t)^{i/2}} + \frac{1}{(1+r_t)^T}$$

We can write the steady-state price of a geometric bond with parameter δ as

$$P^G(r_t) = \frac{1}{1+r_t}\left[1 + \delta P^G(r_t)\right]$$

Solving for $P^G(y_t)$, we get

$$P^G(r_t) = \frac{1}{1+r_t-\delta}$$

The calibration determines how many units X of the geometric bond with parameter δ one needs to sell to hedge one unit of plain vanilla bond P^c against parallel shifts in interest rates, across the range of historical yields:

$$\min_{\delta,X} \sum_{t=1997.1}^{2015.12} \left[P^c(r_t) - XP^G(r_t;\delta)\right]^2$$

We estimate $\delta = 0.937$ and $X = 12.9$, yielding an average pricing error of only 0.41%. This value for δ implies a time series of durations $D_t = -\frac{1}{P_t^G}\frac{dP_t^G}{dr_t}$ with a mean of 6.84.

To establish a notion of principal for the geometric bond, we compare it to a duration-matched zero-coupon bond i.e. borrowing some amount today (the principal) and repaying it D_t years from now. The principal of this loan is just the price of the corresponding D_t maturity zero-coupon bond $\frac{1}{(1+r_t)^{D_t}}$

We set the "principal" F of one unit of the geometric bond to be some fraction θ of the undiscounted sum of all its cash flows $\frac{\theta}{1-\delta}$, where

$$\theta = \frac{1}{N}\sum_{t=1997.1}^{2015.12} \frac{1}{(1+r_t)^{D_t}}$$

We get $\theta = 0.582$ and $F = 9.18$.

C Computational Solution

The computational solution of the model is implemented using what Judd (1998) calls "time iteration" on the system of equations that characterizes the equilibrium of the economy. We reduce the number of equations to the smallest feasible nonlinear system, i.e. we use linear relationships between equilibrium variables to eliminate these equations, such as budget constraints, or the relationship between the investment ratio and Tobin's q. The remaining system consists of the two Euler equations for borrowers ((30) ,(31)), the three first-order conditions for labor demand ((26), for $j = B, I, S$), the two Euler equations for intermediaries ((44) ,(45)), the Euler equation for savers (52), no-shorting constraints for intermediaries and savers ((36),(48)), the market clearing condition for risk-free debt (17), and the leverage constraints for intermediaries (35) and borrowers (22), respectively.

Policy functions, prices, and Lagrange multipliers are approximated as piecewise linear functions of the exogenous and endogenous state variables. The algorithm solves for a set of nonlinear equations including the Euler equations and the Kuhn-Tucker conditions expressed as equalities. The general solution approach for heterogeneous agent models with incomplete markets and portfolio constraints that we employ in this paper is well described by Kubler and Schmedders (2003). They show that there exist stationary equilibria in this class of models when all exogenous state variables follow Markov chains, as is the case in our model as well.

The procedure consists of the following steps

1. **Define approximating basis for the unknown functions.** The state space consists of

   – two exogenous state variables $[Z_t^A, \sigma_{\omega_t}]$, and

   – five endogenous state variables $[K_t^B, A_t^B, W_t^R, W_t^S, B_t^G]$.

   Denote the sets of values these state variables can take as $\mathcal{S}_x$ and $\mathcal{S}_n$ respectively. The aggregate state space is $\mathcal{S} = \mathcal{S}_x \times \mathcal{S}_n$. There are two sets of unknown functions of the state variables that need to be computed. The first set of unknown functions $\mathcal{C}_P : \mathcal{S} \to \mathcal{P} \subseteq \mathcal{R}^N$ determines the values of endogenous objects specified in the equilibrium definition at every point in the state space. These are the prices, agents' choice variables, and the Lagrange multipliers on the portfolio constraints. There is an equal number of these unknown functions and nonlinear functional equations. The second set of functions $\mathcal{C}_T : \mathcal{S} \times \mathcal{S}_x \to \mathcal{S}_n$ determines the next-period endogenous state as a function of the endogenous state in the current period and the next-period realization of shocks. To approximate the unknown functions, we discretize the state space and use multivariate linear interpolation (splines or polynomials of various orders achieved inferior results due to their lack of global shape preservation). One endogenous state variable can be eliminated for computational purposes since its value is implied by the agents' budget constraints, conditional on any three other state variables. As pointed out by several previous studies such as Kubler and Schmedders (2003), portfolio constraints lead to additional computational challenges since portfolio policies may not be smooth functions of state variables due to occasionally binding constraints. Hence we cluster grid points in areas of the state space where constraints transition from slack to binding, and we test the accuracy of the approximation by computing relative Euler equation errors.

2. **Iteratively solve for the unknown functions.** Given an initial guess $\mathcal{C}^0 = \{\mathcal{C}_P^0, \mathcal{C}_T^0\}$ at each point in the discretized state space compute tomorrow's optimal policies as functions of tomorrow's states. Then, compute expectation terms in the equilibrium conditions

using quadrature methods. Next, solve the system of nonlinear equations for the current-period optimal policies. This defines the value of the next iterate $\mathcal{C}_P^1$ at the given point. Using it, compute the next iterate of the transition approximation $\mathcal{C}_T^1$ at the given point. Once $\mathcal{C}_1$ has been computed at every point on the grid, repeat until convergence in the sup norm.

The system of nonlinear equations at each point in the state space is solved using a standard nonlinear equation solver. Judd, Kubler, and Schmedders (2002) show how Kuhn-Tucker conditions can be rewritten as equality constraints for this purpose. For example, consider the saver's Euler Equation for risk-free bonds and constraint:

$$q_t^f = \tilde{\lambda}_t^S + \mathrm{E}_t\left[\mathcal{M}_{t,t+1}^S\right]$$
$$0 \leq \hat{B}_t^S$$

Now define an auxiliary variable x_t and two functions of this variable, such that $\tilde{\lambda}_t^{S,+} = \max\{0, x_t\}^3$ and $\tilde{\lambda}_t^{S,-} = \max\{0, -x_t\}^3$. Then the two equations above can be written as equalities:

$$q_t^f = \tilde{\lambda}_t^{S,+} + \mathrm{E}_t\left[\mathcal{M}_{t,t+1}^S\right]$$
$$0 = \hat{B}_t^S - \tilde{\lambda}_t^{S,-}$$

The solution variable for the nonlinear equation solver corresponding to the constraint is x_t.

The nonlinear equation solver needs to compute the Jacobian of the system at each step. Numerical central-difference (forward-difference) approximation of the Jacobian can be inaccurate and is computationally costly because it requires $2N+1$ $(N+1)$ evaluations of the system, whereas analytically computed Jacobians are exact and require only one evaluation. We follow Elenev (2017) in pre-computing expectations, which simplifies the nonlinear system such that its Jacobian can be computed analytically. This greatly speeds up calculations.

3. **Simulate the model for many periods using approximated policy functions.** To obtain the quantitative results, we simulate the model for 10,000 periods after a "burn-in" phase of 500 periods. We verify that the simulated time path stays within the bounds of the state space for which the policy functions were computed.

In a long simulation, errors in the nonlinear equations are low. Table 8 reports the median error, the 95^{th} percentile of the error distribution, the 99^{th}, and 100^{th} percentiles.

Table 8: Computational Errors

	Percentile				
	50th	75th	95th	99th	Max
(30)	0.0015	0.0043	0.0165	0.0307	0.0730
(31)	0.0004	0.0007	0.0013	0.0020	0.0056
(26)	0.0001	0.0002	0.0004	0.0005	0.0006
(26)	0.0001	0.0002	0.0004	0.0005	0.0006
(26)	0.0001	0.0002	0.0004	0.0005	0.0006
(44)	0.0251	0.0392	0.0830	0.1516	0.2119
(45)	0.0272	0.0422	0.0878	0.1597	0.2215
(52)	0.0007	0.0011	0.0020	0.0029	0.0082
(35)	0.0009	0.0028	0.0074	0.0117	0.0201
(48)	0.0033	0.0052	0.0066	0.0098	0.0129
(17)	0.0004	0.0006	0.0009	0.0010	0.0013
(36)	0.0001	0.0001	0.0004	0.0006	0.0008
(22)	0.0052	0.0170	0.0343	0.0453	0.0618

The table reports median, 75th percentile, 95th percentile, 99th percentile, and maximum absolute value errors, evaluated at state space points from a 10,000 period simulation of the benchmark model. The 12 equations define policy functions. They are a subset of the 21 equations that define the equilibrium.