Dynamic Collective Choice with Endogenous Status Quo*

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Abstract

This paper analyzes an ongoing bargaining situation in which preferences evolve over time and the previous agreement becomes the next status quo, determining the payoffs until a new agreement is reached. In a two-alternative model, we show that the endogeneity of the status quo exacerbates the players' conflict of interest and generates status quo inertia. When players are sufficiently patient, the endogenous status quo can lead the negotiations to a complete gridlock in which legislators never reach an agreement even though their preferences agree arbitrarily often. When players bargain over more than two alternatives, this polarizing effect of the endogenous status quo can be accompanied by a moderating one. However, the latter effect disappears when preferences evolve continuously and players can revise the agreement sufficiently frequently.

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1 Introduction

This paper analyzes a dynamic bargaining situation in which i) there are shocks to the environment that affect individual preferences, and hence call for renegotiation of the past agreements; and ii)

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agreements are determined using an endogenous status quo protocol: the previous agreement stays in place and determines the payoffs until a new agreement is reached. We show that generally the endogenous status quo exacerbates the negotiating parties' conflict of interest, leading to status quo inertia and inefficient bargaining outcomes.

A prominent example of negotiations in a changing environment with an endogenous status quo is legislative bargaining. For instance, legislators' preferences over fiscal policies reflect heterogeneous ideologies and constituencies, but are also affected by shocks such as business cycles, changes in the country's credit rating, or the vagaries of public opinion. At the same time, a vast array of fiscal decisions are taken using the endogenous status quo protocol: once enacted, the law or program continues in effect until further legislative action is taken.[1] Another example is monetary policy making. In many countries, monetary policy is set by a committee that needs to react in a timely manner to the business cycle, but the interest rate stays the same until the committee agrees to change it according to its internal voting rule (see Riboni and Ruge-Murcia 2008).[2]

In a changing environment, the dynamic linkage created by the endogenous status quo presents the negotiating parties with a trade-off between responding to the current shock and securing a favorable position for future bargaining. To illustrate this trade-off and its consequences, consider the case of legislators in the U.S. Congress negotiating the size of mandatory spending. During a contraction, generous deficit spending may be favored by all parties to stimulate short-term economic growth. During a boom, all parties may agree to use the extra tax revenues to bring the public debt under control. In normal times, however, legislators may genuinely disagree on the optimal level of public spending. Anticipating this disagreement, fiscal conservatives may be reluctant to increase public spending during a recession, out of fear that their liberal counterparts will veto a return to fiscal discipline when the economy improves. Similarly, liberals may refuse to lower spending in times of economic growth, out of fear that conservatives will oppose a fiscal expansion when the boom is over. This suggests that with an endogenous status quo, the anticipation of future disagreement exacerbates the conflict of interests between the negotiating parties. As a result, the

[1] For example, about two-thirds of the U.S. federal budget—called mandatory spending—continues year after year by default. Outside of the fiscal sphere, many ideologically charged issues such as immigration, financial regulation, minimum wage, civil liberties, and national security are also affected by shocks (e.g., demographic transitions, financial innovation, national security threats) and are typically regulated by permanent legislation.

[2] Our analysis can shed some light on other dynamic bargaining environments such as the renegotiations of labor or financial contracts, the choice of a firm's manager by the board of directors, trade agreements, and international treaties (e.g., for the World Trade Organization or the European Union).

implemented policy may not be sufficiently responsive to the environment.

To formalize this intuition, we consider the following model. Two players engage in an infinite sequence of choices over two alternatives, called L and R. At the beginning of each bargaining period, one alternative serves as the status quo. If both players agree to move away from the status quo, the new agreement is implemented. Otherwise, the status quo stays in place. In both cases, the implemented agreement determines the players' payoffs in this period and becomes the status quo for the next bargaining period. Players' preferences change over time in a stochastic fashion. We assume that players preferences over the alternatives sometimes disagree, and whenever they do, one player (the rightist) prefers R and the other (the leftist) prefers L.

We show that the endogeneity of the status quo exacerbates the players' conflict of interest. To see why, note that the agreement implemented in a given period affects players' future payoffs only if in the next period players disagree, in which case the status quo stays in place. Conditional on disagreement, however, the rightist player prefers R and the leftist player prefers L. Therefore, the rightist player is willing to vote for R even when R is not currently optimal for her in order to secure R as a status quo for the next period. By the same argument, the leftist player will sacrifice her current payoff to secure L as the next status quo. As a result, players may fail to reach an agreement even when the status quo is Pareto dominated, and hence the status quo stays in place too often, and the bargaining outcome is less responsive to the environment.

These results resonate with the anecdotal evidence of partisanship occurring in the legislative settings, whereby legislators are reluctant to acknowledge any common ground with their political opponents, and are thus unable to reach seemingly beneficial agreements. Partisan behavior is often interpreted as blind allegiance to a party or ideology, but our model shows that similar partisan behavior can be generated by strategic considerations.

Our analysis further shows that players' equilibrium behavior is driven by a vicious cycle in which partisanship and disagreement feed on each other: more partisan players disagree more often; more frequent disagreement increases the importance of securing the favorable status quo; and this further increases partisanship. As a result, the polarizing effect of the endogenous status quo can be quite dramatic. In particular, we show that when players are sufficiently patient, the endogenous status quo can lead the negotiations to a complete gridlock: even if their preferences agree arbitrarily often, players never reach an agreement, and the bargaining outcome is completely unresponsive to the evolution of preferences.

When players bargain over more than two alternatives, the polarizing effect of the endogenous status quo can be accompanied by other equilibrium effects, because with more than two alternatives, there is more than one way to disagree. To see this, consider again the case of legislators bargaining over the budget size. In normal times, conservative and liberal legislators may disagree whether to increase or decrease public spending. As in the case of two alternatives, the anticipation of this type of disagreement makes the legislators more polarized. During a severe economic contraction, however, all parties may want to expand the budget to stimulate the economy, but the liberals typically want to expand it more. If the liberals have the proposal power, the magnitude of the budget expansion they can impose depends on how inadequate the current status quo is. If the status quo budget is excessively low given the economic conditions, they will propose a bigger expansion than the conservatives would like, and the conservatives will have no choice but to accept it. Anticipating this scenario, the conservatives may prefer to have a moderate level of public spending during normal times so as to be able to constrain the liberals with the threat of a veto in case of a severe downturn.

Whether moderation or polarization dominates turns out to depend on the allocation of bargaining power, how rapidly preferences change, and how frequently the negotiating parties can revise the agreement. We provide examples of bargaining environments that result only in polarization, and examples in which the moderating effect dominates. However, we show that if players' preferences evolve continuously and players can bargain sufficiently frequently, the moderating effect disappears. Under these conditions, as in the case of two alternatives, the endogenous status quo leads to partisanship, status quo inertia, and inefficient bargaining outcomes.

Our paper complements the extensive literature on dynamic bargaining with an endogenous status quo. Most of this literature considers environments in which preferences are static, and the bargaining outcome changes over time because the identity of the proposer changes, or because the same proposer seeks the support of a different coalition. In our basic model with two alternatives and unanimity rule, these two channels do not play any role: the allocation of proposal power is irrelevant, and there is only one possible winning coalition. Instead, the main impetus for policy change is the evolution of preferences. In environments with common interest policies, the literature with static preferences (Baron, 1996; Baron and Herron, 2003; and Zapal, 2011a) finds that the endogenous status quo has as a *moderating* effect: players vote for policies that are more moderate than their instantaneous preferences would imply. In contrast, this paper shows that in environments with evolving preferences, the endogenous status quo has a *polarizing* effect: players prefer polices that are more extreme than their instantaneous preferences would imply. Hence, the endogenous status quo has a diametrically different impact on equilibrium behavior in these two environments.

The moderating effect highlighted by the aforementioned literature is similar to the one that occurs in this model in the case of more than two alternatives, because with more than two alternative, the allocation of bargaining power matters in our model as well. Since real world bargaining procedures rarely place binding restrictions on the timing of proposals, however, our results in the limit case in which proposals can be made arbitrarily frequently suggest that the polarizing effect should be of first order importance in actual dynamic bargaining environments.

The paper is organized as follows. First, we review the related literature. In Section 3, we present and solve the basic model with two alternatives. In Section 4 we extend our analysis to more than two alternatives: Section 4.1 discusses the polarization and the moderating effects, Section 4.2 formalizes these effects in a simple environment, and Section 4.3, extends our results to a more general environment.

2 Related literature

The moderating effect that arises in this model (with more than two alternatives), like the moderating effect in Baron (1996), Baron and Herron (2003), and Zapal (2011a), is closely related to the observation initially made by Romer and Rosenthal (1978) that an extreme status quo makes the proposer more powerful, and is thus detrimental to non-proposers. Therefore, with an endogenous status quo, implementing a moderate agreement today can constrain the bargaining power of the next proposer. The same effect is at work in Diermeier and Fong (2011) and Bowen et al. (2012).[3] In our model, the moderating effect can occur even with a monopolistic proposer and the unanimity

[3] Cho (2005), Fong (2006) and Baron et. al. (2012) analyze dynamic models of parliamentary democracy with continuing policies. Baron et. al. (2012) show in a two period model that when the allocation of proposal power is endogenized via elections, the combination of electoral concerns together and the endogenous status quo can actually lead parties to implement policies that are outside the Pareto set. In a dynamic model of elections, Duggan and Forand (2013) show that if the implemented policies affect state transitions, then politicians may implement policies that are subomptimal for the representative voter.

rule, because with large preference shocks, an alternative that is Pareto optimal in a given period can become extreme for next period's preferences.

Kalandrakis (2004, 2010), Bernheim et al (2006), Anesi (2010), Diermeier and Fong (2011), Anesi and Seidmann (2012), Bowen and Zahran (2012), Baron and Bowen (2013), and Richter (2013) analyze distributive policies (using some version of a divide-a-dollar game). These papers focus on the equitability of the division of the surplus, on whether proposals are approved by minimal coalitions, and on whether these coalitions are stable. By virtue of analyzing common interest policies under the unanimity rule, our paper has nothing to say on these issues. Instead, we focus on the responsiveness of policies to the changing environment, and on the phenomenon of partisanship, on which the aforementioned papers are mute. Our findings cannot be mapped in a straightforward way to the distributive setting because with distributive policies, players' preferences do not change over time and are always diametrically opposed.[4]

Even though dynamic bargaining with an endogenous status quo in a stochastic environment is at the center of many economically relevant situations, the existing literature on this topic is scarce. This may be a consequence of the intractability of these games. As Romer and Rosenthal (1978) showed in a static setup with single-peaked preferences, the induced preferences over the status quo are typically not convex, which makes the multi-period extension technically hard to analyze. With a continuum of alternatives and an infinite horizon, the existence of the stationary equilibrium is not guaranteed even under standard preference specifications (see, e.g., Duggan and Kalandrakis 2012).

To the best of our knowledge, only Riboni and Ruge-Murcia (2008), Zapal (2011b), and Duggan and Kalandrakis (2012) make progress on this front. Adding noise to the status quo, Duggan and Kalandrakis (2012) establish the existence of an equilibrium in a very general setting, but the generality of their model does not allow an analytical characterization of the equilibria. Riboni and

[4] Nevertheless, one can draw some qualitative parallels. In Kalandrakis (2004, 2010), the implemented allocations are responsive to the shocks to the proposer power, but Bowen and Zahran (2012), Baron and Bowen (2013), and Richter (2013) find equilibria in which the allocations are completely unresponsive. In contrast, in our basic model, we find that status quo inertia occurs in all equilibria. Battaglini and Palfrey (2012) find experimental evidence of status quo inertia in a divide a dollar setting. Anesi and Seidmann (2012), and Baron and Bowen (2013) show that wasteful allocations can be supported in equilibrium, but these equilibria coexist with non-wasteful equilibria, and they disappear under the unanimity rule. In Kalandrakis (2004, 2010) and Bernheim et al (2006), the implemented allocations are inequitable, which could be interpreted as a form of polarization, but Diermeier and Fong (2011), Anesi and Seidmann (2012), Bowen and Zahran (2012), Baron and Bowen (2013), and Richter (2013) find that more or even perfectly equitable allocations can be implemented.

Ruge-Murcia (2008) analyze a game with quadratic utility functions and a finite state space. They analytically solve a two-period, two-state example, but use numerical solutions for the general model. Zapal (2011b) characterizes an equilibrium in a two-state environment with quadratic preferences and compares it to a different dynamic bargaining protocol. In his setting a constant policy is optimal; hence, the issue of the responsiveness of policies to shocks is irrelevant. Our paper differs from these contributions in that we consider a finite policy space with an arbitrary state space, and our equilibrium characterization allows us to isolate the effect of the endogenous status quo in a transparent way.

Acemoglu, Egorov and Sonin (2008, 2011, 2013) and Bai and Lagunoff (2011) analyze dynamic games in which the policy implemented in a given period affects the allocation of decision power in the next period. They show that this dynamic linkage can lead to inefficient decisions. In this paper, the allocation of decision power is assumed to be exogenous so as to isolate the effect of the endogenous status quo on the evolution of policies. Montagnes (2010) looks at a two-period financial contracting environment in which the current contract serves as the default option in future negotiations. He shows that inefficiencies may arise, and hence both contracting parties may prefer to commit ex ante to ceding a future decision power to avoid these inefficiencies.

Our results on the responsiveness of policies to shocks are related to the political economy literature on growth and the dynamics of welfare policies.[5] In this literature, the current policy affects future preferences (via private or public investment decisions). This dynamic linkage can generate policy persistence. In contrast, in our paper the implemented policy does not affect future preferences, but inertia emerges because today's policy affects players' positions for future negotiations.[6]

Finally, Casella (2005) and Jackson and Sonnenschein (2007) show that linking voting decisions across time allows voters to express their preference intensity, which can be socially beneficial. Our results suggest that the endogenous status quo protocol, despite the pervasiveness of this institution,

[5] See, among others, Glomm and Ravikumar (1995), Krusell and Rios-Rull (1996, 1999), Saint Paul and Verdier (1997), Coate and Morris (1999), Benabou (2000), Saint Paul (2001), Hassler et al. (2003, 2005), Battaglini and Coate (2007, 2008), and Prato (2011).

[6] In Fernandez and Rodrik (1991) and Alesina and Drazen (1991), the distributional uncertainty of policy reforms leads to status quo inertia. In our model, it is not the uncertainty but the evolution of preferences over time that drives the result. Strulovici (2010) shows in a dynamic collective experimentation setting that uncertainty about the identity of future pivotal voters can discourage players from policy experimentation, and hence lead to status quo inertia.

is not an efficient way to elicit preference intensity. Barbera and Jackson (2010) let ex ante identical voters choose the group decision rule after having learned their first-period preferences. Similarly to our paper, in their framework bundling the current and future decision rules generates inefficiencies. But since the dynamic linkage is only between the first period and the subsequent ones, sufficiently patient players always select the optimal voting rule.

3 The $2-$alternative model

3.1 The model

There are two players, l and r, and a set of two alternatives, $X = \{L, R\}$. Time is continuous, but players move only every $\delta > 0$ units of time: at times $t \in \{0, \delta, 2\delta, ...\}$. We call these *bargaining times,* and the $\delta-$long period following each bargaining time is called a *bargaining period.* At each bargaining time t, a status quo $q(t) \in \{L, R\}$ is in place, and players vote simultaneously on which alternative to adopt. If both players vote for the same alternative, this alternative is implemented. If they disagree, this period's status quo $q(t)$ stays in place. The implemented alternative $x(t)$, be it the new agreement or the status quo, determines the players' payoff for the following bargaining period $(t, t+\delta)$ and becomes the status quo for the next bargaining time $t+\delta$.

The flow payoff of player $k \in \{l, r\}$ at instance t from alternative $x \in X$ is denoted by $u_k(\theta(t), x)$, where $\{\theta(t) : t \geq 0\}$ is a continuous-time stochastic Markov process on some arbitrary space Θ. This process captures the dynamic nature of the environment, and we assume that $\theta(t)$ is observed by both players at all times. We assume that $u_k(\theta, x)$ is bounded over Θ and X. Players maximize the expected discounted sum of flow payoffs, and $\rho > 0$ is the discount factor.

We denote the above game by Γ^{en}. In this game, the dynamic linkage across periods comes solely from the endogenous status quo. To characterize its impact on equilibrium behavior and outcomes, we compare Γ^{en} to the game $\Gamma^{ex}(q(t) : t \geq 0)$, which differs from Γ^{en} only in that at every time t, the status quo is exogenously fixed at some $q(t) \in X$, irrespective of players' past actions. The exogenous status quo protocol is arguably the simplest protocol which severs the link between today's agreement and tomorrow's status quo, and is probably the most commonly observed alternative in actual dynamic bargaining environments (see Section 5 for a discussion).

Strategies

As is customary in dynamic voting games with an infinite horizon, we look for stationary equilibria in stage-undominated strategies (henceforth, equilibria) as defined in Baron and Kalai (1993).[7] A stationary strategy for player k in Γ^{ex} or Γ^{en}, denoted by σ_k, maps at each bargaining time t the current state $\theta(t)$ and status quo $q(t)$ into a vote $\sigma_k(\theta(t), q(t)) \in \{L, R\}$. Stage undomination amounts to assuming that in each period, each player votes for the alternative that gives her the greater continuation payoff. It rules out pathological equilibria such as both players always voting for the status quo. Without loss of generality, we assume that when indifferent, a player votes for R.

Additional assumptions and notations

Definition 1 *A payoff function $u : \Theta \times X \to \mathbb{R}$ is more leftist than another payoff function u' (or equivalently, u' is more rightist than u) if for all $\theta \in \Theta$,*

$$u(\theta, R) - u(\theta, L) \leq u'(\theta, R) - u'(\theta, L).$$

Throughout this section, we assume that u_r is more rightist than u_l, and we occasionally refer to player r and l as the rightist and leftist player, respectively. This assumption has a natural interpretation in political economy applications: players can be unambiguously ranked on the ideological spectrum. Note, however, that it imposes no restriction on the preference distribution of a single player, nor on the severity of the conflict of interest between players: both players might prefer the same agreement for an arbitrarily large subset of Θ.

For all $\theta \in \Theta$ and all $x \in X$, define

$$U_k(\theta, x) \doteq E_{\theta(0)=\theta}\left[\int_0^{\delta} (u_k(\theta(t), x)) \rho e^{-\rho t} dt\right], \tag{1}$$

and $U_k(\theta) \doteq U_k(\theta, R) - U_k(\theta, L)$. We assume that for all $\theta \in \Theta$ and all $x \in X$, $U_k(\theta, x)$ is

[7] Stage-undominated stationary equilibria, or variants thereof, are used in almost all of the infinite-horizon models cited in this paper. The only exceptions that we are aware of are Epple and Riordan (1987), and Baron and Ferejohn (1989). Both papers prove results that have the flavor of the folk theorem in repeated games. As shown in Baron and Kalai (1993), stage-undominated stationary equilibria have a focal point property that derives from their simplicity. The stationarity assumption in the legislative sphere can be justified on the grounds that the game is played by a sequence of legislators who are never certain to be reelected. In such cases, the institutional memory required for more sophisticated nonstationary equilibria involving infinitely nested punishment strategies may be inappropriate. See Krehbiel (1991) for a critical discussion of the prevalence of cooperation among legislators.

well-defined. The expression $U_k(\theta)$ measures the relative expected that player k receives from alternative R as compared to L in a bargaining period that starts at state θ. We call $U_k(\theta)$ player k's *current preference,* as the sign of $U_k(\theta)$ determines her preference for the current bargaining period without taking into account the consequences of today's decision on future periods.

Occasionally, we will make the following assumption.

Definition 2 *A payoff profile exhibits strict disagreement if there exist* $p, \bar{u}, \bar{v} > 0$ *such that for all* θ *such that* $0 \leq U_l(\theta) \leq \bar{u}$ *or* $-\bar{u} \leq U_r(\theta) \leq 0$,

$$\Pr\left(\left\{U_l\left(\theta\left(t+\delta\right)\right) < -\bar{v} \text{ and } \bar{v} < U_r\left(\theta\left(t+\delta\right)\right)\right\} | \theta\left(t\right) = \theta\right) \geq p.$$

In words, strict disagreement means that if at some bargaining time, players' current preferences agree, say on R, (i.e., $0 \leq U_l(\theta) \leq U_r(\theta)$), but are close to disagreeing because l's preference is not strong (i.e., $U_l(\theta) \leq \bar{u}$), then at the next bargaining time players' current preferences disagree with positive probability.

To illustrate our results, sometimes we use the following family of payoff profiles:

Definition 3 *A payoff profile is a* constant-bias payoff profile *if* Θ *is a bounded subset of* $\mathbb{R}$, $X \subset \mathbb{R}$ *with* $L < R$, *and for all* k, *all* $\theta \in \Theta$, *and all* $x \in X$, $u_k(\theta, x) = -\left(x - \left(\theta + b_k\right)\right)^2$.

Note that with this specification, l is more leftist than r if and only if $b_l \leq b_r$. Also, a sufficient condition for a constant-bias payoff profile to exhibit strict disagreement is that $b_l < b_r$ and for all $\theta(t)$, conditional on $\theta(t)$, $\theta(t+\delta)$ has full support on a neighborhood of $\theta(t)$.

Comments

A few comments about the modeling assumptions are in order. First, we analyze a two-player game which requires unanimity for changing the status quo, but in Dziuda and Loeper (2012), we show that our results easily extend to an arbitrary number of players with a large class of voting rules. For example, with qualified majority, if players can be ranked from a most leftist to a most rightist, the same two players are pivotal in every decision, so this model is strategically equivalent to the two-player model analyzed in this paper. Second, we assume that at each bargaining time, the outcome is decided by a simultaneous vote, but with two alternatives and two players, most bargaining protocols are equivalent.[8] Finally, we disentangle the evolution of the environment (governed by the process $\{\theta(t) : t \geq 0\}$) from the frequency with which players can revise the agreement (determined by δ). This allows us to study not only situations in which the environment is subject to discontinuous shocks (e.g., when $\theta(t)$ is redrawn at every bargaining time) but also situations in which it evolves smoothly (e.g., when $\theta(t)$ follows a continuous processes). With more than two alternatives, this distinction will be important.

3.2 Equilibrium analysis

As a benchmark, let us first look at the game with an exogenous status quo Γ^{ex}. Consider player k at a bargaining time t at which the state is $\theta(t) = \theta$. Since the bargaining outcome implemented at t has no impact on the subgame starting at the next bargaining time $t + \delta$, player k votes for R if and only if

$$U_k(\theta) \geq 0. \tag{2}$$

Hence, Γ^{ex} has a unique equilibrium in which players simply vote for the best alternative according to their current preferences.

Consider now the same situation in the game with an endogenous status quo Γ^{en}, and let $\sigma = (\sigma_l, \sigma_r)$ be a strategy profile. The agreement x implemented at t affects player k's payoff in the corresponding period $(t, t + \delta)$, and since x becomes the status quo at $t + \delta$, it also affects her continuation value from the subgame that starts at $t + \delta$. Let $W_k^\sigma(\theta, x)$ be the expected value of that continuation game, conditional on $\theta(t) = \theta$ and on σ being played after $t + \delta$, and let $W_k^\sigma(\theta) \doteq W_k^\sigma(\theta, R) - W_k^\sigma(\theta, L)$. One can interpret W_k^σ as the relative expected gain for player k from having R instead of L as the next period's status quo. With a slight abuse of notation, we call W_k^σ *the continuation value of player k*. When no ambiguity arises, we drop the superscript σ.

Stage-undomination requires that at each bargaining time, players vote as if they were pivotal. Hence, in Γ^{en}, if the current state is θ, player k votes for R if and only if

$$U_k(\theta) + e^{-\rho\delta} W_k(\theta) \geq 0. \tag{3}$$

[8] For instance, using standard equilibrium concepts, equilibrium outcomes are the same when players vote simultaneously, sequentially, when they make take-it-or-leave-it offers, or when they make several alternating offers at each bargaining time.

Comparing (2) and (3), one can see that the effect of the endogenous status quo on equilibrium behavior is completely captured by W_k. If $W_k(\theta) = 0$ for all θ, then player k votes according to her current preference as in Γ^{ex}. If $W_k(\theta)$ is positive (negative), then player k votes for R more (less) often than in Γ^{ex}.

The first proposition shows that the endogenous status quo has a *polarizing effect* on players' behavior: it exacerbates their conflict of interest by making the leftist player vote for L and the rightist player vote for R more often than in Γ^{ex}.

Proposition 1 *The game* Γ^{en} *has an equilibrium.*

a. In all equilibria, $W_l(\theta) \leq 0 \leq W_r(\theta)$.

b. Suppose that the payoff profile exhibits strict disagreement in the sense of Definition 2. Then there exists $\bar{w} > 0$ *such that in all equilibria,*

i. for all θ *such that* $U_l(\theta) \leq \bar{w}$, *player* l *votes for* L,

ii. for all θ *such that* $U_r(\theta) \geq -\bar{w}$, *player* r *votes for* R.

Proof. All proofs are in the Appendix. ■

The intuition for Proposition 1 is as follows. The alternative implemented today affects future bargaining outcomes only if players disagree in the next bargaining period. When they disagree, r prefers R while l prefers L. Hence, status quo L gives more option value to player l than status quo R. As a result, player l is willing to vote for L to secure L as the next period's status quo even when alternative R would give her a greater expected payoff in the current period.

This intuition reveals that the direction in which the endogenous status quo biases players' behavior depends only on players' *relative* ideological positions: it depends on which alternative each player prefers when they disagree but not on which alternative they prefer on average.

To understand the implications of Proposition 1 for the equilibrium outcomes, note that under our assumptions, for all $\theta \in \Theta$, $U_l(\theta) \leq U_r(\theta)$. Therefore, from (2), when the status quo is exogenous, players do not reach an agreement when

$$U_l(\theta) < 0 \leq U_r(\theta),$$

while from (3), when the status quo is endogenous, players do not reach an agreement when

$$U_l(\theta) + e^{-\rho\delta} W_l(\theta) < 0 \leq U_r(\theta) + e^{-\rho\delta} W_r(\theta).$$

These inequalities together with Proposition 1 imply the following.

Corollary 1 (Status Quo Inertia) *In any equilibrium of Γ^{en}, the set of states in which players do not reach an agreement is greater in the inclusion sense than in the equilibrium of Γ^{ex}, strictly so when the payoff profile exhibits strict disagreement. In particular, at any bargaining time t, the probability that the status quo stays in place at the next bargaining time is higher in Γ^{en} than in Γ^{ex}.*

The status quo inertia implies that in equilibrium Pareto dominated policies can be implemented. To see this, using the notations of Proposition 1 part (b), suppose that $0 \leq U_l(\theta) \leq \bar{w}$. Then clearly R is Pareto efficient, but if the status quo is L, L stays in place because player l votes for it.

The behavior of the players described in Proposition 1 and Corollary 1 resembles what is commonly referred to as *partisanship.* One dictionary definition for *partisanship* is "a prejudice in favor of a particular cause; a bias." In multiparty systems, this term carries a negative connotation: it refers to those who wholly support their party's policies and are reluctant to acknowledge any common ground with their political opponents. This definition resonates with the players' behavior in our model: players favor policies that are in line with their relative ideology rather than policies that are optimal given their current preferences, which leads to more disagreement and inefficient policies. This model hence shows that when the status quo is endogenous, partisanship can be generated by strategic considerations without assuming any obedience to a doctrine or a group.

To fully understand the strategic underpinning of partisanship in our model, it is important to note that the equilibrium degree of partisanship is the result of a vicious cycle in which players' behaviors feed on themselves. To see this, observe that as player l becomes more partisan for L, status quo L becomes more likely to stay in place, which in turns makes player r more willing to defend her preferred status quo R, and thus makes her more partisan for R.

Because of this strategic complementarity, there can be multiple equilibria. Proposition 2 below shows that the equilibria can be ranked in terms of their degree of partisanship, and the more partisan players are, the worse-off they are.

Proposition 2 *The set of equilibria is a complete lattice in the partisan order defined as follows: for any two strategy profiles σ and σ', σ' is more partisan than σ if for all $\theta \in \Theta$, $W_l^{\sigma'}(\theta) \leq W_l^{\sigma}(\theta) \leq 0 \leq W_r^{\sigma}(\theta) \leq W_r^{\sigma'}(\theta)$. Moreover, if σ and σ' are two equilibria such that σ' is more partisan than σ, then σ Pareto dominates σ'.*

3.3 The determinants of partisanship and gridlock effect

In this section, we investigate the main drivers of the degree of partisanship.

Since players' partisanship depends on their preferences conditional on future disagreement, it should be affected by the degree of their conflict of interest and their patience. The conflict of interest can be captured formally by the following definition: we say that players become more polarized when u_r becomes more rightist and u_l becomes more leftist in the sense of Definition 1. As players become more polarized, their current preferences U_l and U_r disagree more often and more strongly. This means that securing a favorable alternative as a status quo becomes more important, so players have more incentives to vote in a partisan way. Similarly, more patient players are more willing to sacrifice today's payoff to get a more favorable status quo for tomorrow's negotiations, and are thus more partisan. Therefore, with an endogenous status quo, more patient players are less likely to reach an agreement in any given period. In Appendix 6.4, we formally prove that partisanship increases with polarization and patience.[9]

The next proposition shows that the impact of the endogenous status quo on equilibrium behavior can be quite dramatic: there exist equilibria with complete gridlock in which players never reach an agreement, and hence the agreement is totally unresponsive to the evolution of the environment.

Definition 4 *Suppose that at $t = 0$, player k has the choice between implementing R in every period or implementing L in every period. If for any initial state $\theta(0) \in \Theta$, she chooses R (L), then we say that she is* an absolute rightist (leftist).

[9] Because preferences evolve in continuous time, ρ also affects the current preferences, so the comparative statics with respect to ρ requires some qualifications, but this extra complication is of no economic interest; hence, we delegate its careful treatment to the appendix.

Proposition 3 *Let σ^g be the strategy profile in which player l always votes for L and player r always votes for R. Then σ^g is an equilibrium if and only if player r is an absolute rightist and player l is an absolute leftist.*

In Appendix 6.4, we show that players r and l are absolute rightist and leftist, respectively, if and only if they are sufficiently patient and polarized. Note, however, that gridlock can occur for arbitrarily mild conflict of interests. We show in the appendix that with a constant-bias payoff profile (see Definition 3) on a finite state space $\Theta \subset \mathbb{R}$, if $\bar{\theta}$ denotes the expectation of the stationary distribution of $\{\theta(t) : t \geq 0\}$, then players l and r are absolute leftist and rightist, respectively, whenever $\bar{\theta} + b_l < \frac{L+R}{2} < \bar{\theta} + b_r$ and ρ is sufficiently small. As b_l and b_r become arbitrarily close to $\frac{L+R}{2} - \bar{\theta}$, players' current preferences agree arbitrarily often in all periods, but if players are sufficiently patient, there exists an equilibrium in which they always vote for opposite alternatives. Moreover, when players' biases are not too close to each other, gridlock is the unique equilibrium (see Example 1 later in this section).

Note that partisanship and status quo inertia are not driven by the assumption that players can bargain only every δ unit of time. The impact of the bargaining friction δ on equilibrium behavior depends on the fine details of the process $\{\theta(t) : t \geq 0\}$, but partisanship typically does not disappear as players can bargain increasingly often. To see this, observe that the notion of absolute rightist and leftist is independent of δ, so gridlock equilibria can occur even if δ vanishes. Moreover, in the case of a constant-bias payoff profile, if $\{\theta(t) : t \geq 0\}$ is a martingale, then the equilibrium degree of partisanship actually increases as δ decreases (see Proposition 10 in the appendix).

Finally, the following example illustrates the results of this section using a simple parametrization.[10]

Example 1 *Consider a constant-bias payoff profile with $L = -1$, $R = 1$, $b_l = -b_r = 0.24$ and $\Theta = [-2, 2]$. At each instance, $\theta(t)$ either stays constant or is redrawn from a uniform distribution over Θ. The occurrence of draws is distributed as a Poisson process with parameter λ. We assume that $\lambda = 1, \delta = 2,$ and $\rho = 0.15$ (so that the expected discount factor between two draws $\simeq 0.87$).*

The dashed lines in Figure 1 depict the states for which in which L (the lower line) or R (the

[10] The proofs and simulations related to Examples 1 and 2 are available upon request from the authors.

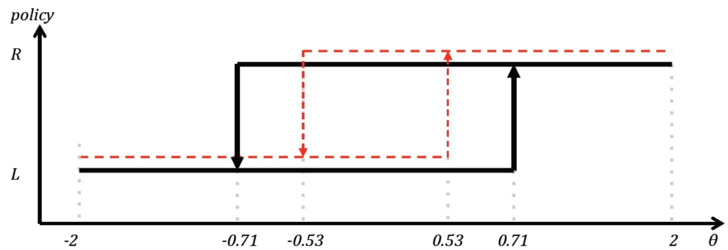

Figure 1: Evolution of policies under exogenous status quo (dashed lines) and in equilibrium (solid lines).

upper line) are Pareto optimal with respect to the current preferences $(U_r(\theta), U_l(\theta))$ *calculated for the assumed parameters.*

One can show that in Γ^{en}*, players use cutoff strategies: there exists a cutoff state* c_k *such that player* k *votes for* R *if and only if* $\theta(t) \geq c_k$*. In equilibrium, the cutoff states are given by* $c_l = -c_r = 0.71$*. Hence, if* $q(t) = L$*, then as long as* $\theta(t) < 0.71$*, player* l *votes for* L*, so* L *stays in place. The agreement switches to* R *only if* $\theta(t) \geq 0.71$*. Similarly, if* $q(t) = R$*, then as long as* $\theta(t) \geq -0.71$*, player* r *votes for* R*, so* R *stays in place. The solid lines in Figure 1 represent the states in which status quo* L *(the lower line) or* R *(the upper line) stay in place. Comparing the solid and the dashed lines one can see that there is status quo inertia, which results in Pareto inferior policies when the state is in* $(-0.71, -0.53)$ *or in* $(0.53, 0.71)$*.*

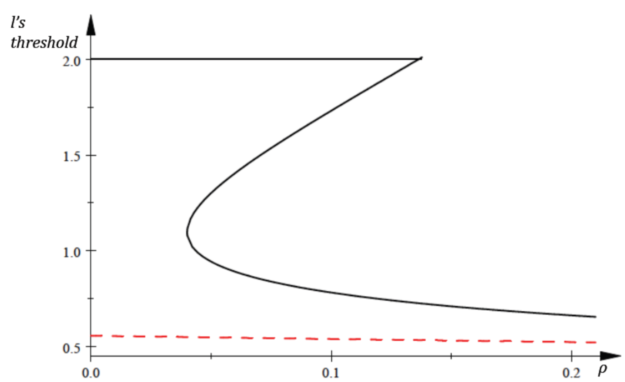

Figure 2: Equilibrium behavior of l as a function of patience.

Figure 2 illustrates how the equilibrium cutoff for player l *changes with patience (the thresholds for* r *are symmetric). The solid curve represents* c_l*, and the dashed curve represents the cutoff in* Γ^{ex} *(*c_l^{ex} *such that* $U_l(c_l^{ex}) = 0$*). As we can see, for large* ρ *as* ρ *decreases,* c_l *increases and the* *distance between c_l and c_l^{ex} increases as well. When $\rho < 0.137$, there are three equilibria, including the gridlock one. As ρ decreases further, only the gridlock equilibrium remains.*

4 $N-$alternative model

4.1 Polarization and moderation

With an arbitrary number of alternatives, it is still true that the status quo matters only when players disagree, so players' equilibrium behavior still depends on which policies they prefer when they disagree. However, players' preferences conditional on disagreement are harder to determine, because there is more than one way to disagree. As in the $2-$alternative model, players might disagree about which alternatives are better than the status quo. In that case, the rightist player r wants to move the agreement to the right of the status quo, while the leftist player l wants to move it to the left. In such configurations, the status quo stays in place, so player r (l) is better-off with a more rightist (leftist) status quo. The anticipation of such disagreements biases player r (l) in favor of more rightist (leftist) policies. This is the same *polarizing effect* as in the case of two alternatives.

However, with more than two alternatives, players may also agree that a certain subset of alternatives is better than the status quo, but disagree about the ranking of these alternatives. Such disagreements can generate incentives that are qualitatively different from the ones generated by the polarizing effect. To see this, consider the case in which players agree on the direction in which the agreement should move, say to the right, but disagree by how much: the rightist player r wants to move the agreement farther to the right than the leftist player l. If l is the proposer, she proposes her bliss point, which is better for r than the status quo, and is therefore accepted. So when l is the proposer, the status quo is irrelevant. If instead r is the proposer, r may be constrained by the threat of a veto from l. As first noted by Romer and Rosenthal (1978), if the status quo is far to the left of l's current bliss point, this threat does not bind, and r can move the agreement as far to the right as she wants. With a more moderate status quo, l's veto power constrains r to move the agreement only slightly to the right. Therefore, in such cases, player l is better-off with a more rightist status quo while player r is better-off with a more leftist status quo. The anticipation of such disagreement biases player l (r) in favor of more rightist (leftist) policies, and therefore moderates the conflict of interests between players. Thus, the endogenous status quo can also have a *moderating effect*.

Note that these two effects have opposite implications in terms of the dynamics of the bargaining outcome. As argued earlier, polarization means that players disagree more often than their current preferences do, and hence the status quo stays in place for a larger set of states than what would be Pareto optimal. That is, polarization leads to status quo inertia. In contrast, moderation means that players agree more often than their current preferences do. Hence it induces players to replace the status quo even when the new agreement is not Pareto improving. That is, moderation leads to status quo instability.

Whether partisanship and status quo inertia still prevail in equilibrium depends on the relative importance of these effects. However, the above discussion suggests two conclusions. First, the allocation of proposal power matters only in the second kind of disagreement. Therefore, the polarizing effect always arises, but whether it is accompanied by another equilibrium effect depends on the allocation of proposal power. In particular, for moderation to occur the player who wants to move farther from the status quo must be the proposer. Second, to generate the second type of disagreement, the preferences must change drastically from one bargaining time to the next: players must agree on a quite leftist agreement at some bargaining time, but prefer sufficiently rightist policies at the next bargaining time. If the preferences do not change too rapidly relative to the frequency at which players can revise the bargaining agreement, such events are unlikely, and the moderating effect should be negligible. In the next section, we formalize these conjectures using the simplest possible environment in which both effects can arise. In Section 4.3, we show in a quite general environment that when preferences evolve smoothly and players can bargain sufficiently frequently, the polarizing effect dominates irrespective of the details of the bargaining protocol.

4.2 An illustration with three alternatives

In this section, we consider an environment in which players bargain over three alternatives, labelled $X = \{L, M, R\}$. As before, players meet every δ amount of time and bargain over which alternative to implement for the next bargaining period. Unlike before, however, with more than two alternatives not all bargaining protocols are equivalent. We restrict attention to proposer/accepter protocols. This class of protocols is defined as follows. At every bargaining time, one designated player proposes an alternative, and the other player announces the set of proposals that she is willing to accept. If the proposal is in the acceptance set, it is implemented, if not, the status quo remains. The identity of the proposer at a bargaining time can depend on the current state and status quo.[11]

The following two protocols will be used to illustrate the role of the proposal power.

$k-$**monopolistic proposer** Player $k \in \{l, r\}$ is the proposer for all $(\theta, q) \in \Theta \times X$.

moderate proposer When the status quo is L, then player l is the proposer, and when the status quo is R, then player r is the proposer (the identity of the proposer for $q = M$ turns out not to matter for our purpose).

The $k-$monopolistic proposer protocol is quite standard in the political economy literature. The moderate proposer protocol is less standard, and is used here mainly as a benchmark. However, one can view it as an approximation of an environment in which the rightist (leftist) player receives more bargaining power after having successfully secured a more rightist (leftist) agreement.

As in the $2-$alternative model, we look at stationary equilibria in stage-undominated strategies. A stationary strategy σ_k for player k maps each pair (θ, q) into a proposal or an acceptance set, depending on whether k is the proposer at (θ, q). The profile of *current preferences* U and *continuation values* W^σ are defined as in Section 3.

As in the case of two alternatives, W^σ completely captures the effect of the endogenous status quo on equilibrium behavior. To see this, suppose that the status quo is L. Under the exogenous status quo, player k may propose or accept M only if $U_k(\theta, M) \geq U_k(\theta, L)$, while under the endogenous status quo, she may do so only if

$$U_k(\theta, M) + e^{-\rho\delta} W_k^\sigma(\theta, M) \geq U_k(\theta, L) + e^{-\rho\delta} W_k^\sigma(\theta, L).$$

Hence, if $W_k^\sigma(\theta, M) \geq (\leq) W_k^\sigma(\theta, L)$, player k is in favor of replacing L with M more (less) often than under the exogenous status quo.

[11] In the case of three alternatives, restricting attention to proposer/accepter protocols is without loss of generality: one can easily show that any deterministic bargaining solution that is individually rational and efficient can be replicated by an equilibrium of a proposer/accepter protocol for an appropriate allocation of proposal power. Moreover, our results can be easily extended to the case of non deterministic protocols.

We make the following assumptions. First, as in the $2-$alternative model, we assume that player r is more rightist than player l. To do so, we assume that $X \subset \mathbb{R}$, with $L < M < R$, and that for all $\theta \in \Theta$ and all $x < y$, $u_l(\theta, y) - u_l(\theta, x) \leq u_r(\theta, y) - u_r(\theta, x)$. Second, we focus on a particular class of equilibria, called *regular,* as defined below.

Definition 5 *A strategy profile σ is* regular *if for all $k \in \{l, r\}$ and all $\theta \in \Theta$, $U_k(\theta, x) + e^{-\rho\delta} W_k^{\sigma}(\theta, x)$ is single-peaked in x.*

Since $U_k(\theta, x) + e^{-\rho\delta} W_k^{\sigma}(\theta, x)$ can be interpreted as player k's intertemporal preferences, an equilibrium is regular if players' intertemporal preferences are single-peaked. As shown in the appendix (step 3 of the proof of Proposition 6), if for all $\theta \in \Theta$, $u_k(\theta, x)$ is strictly concave in x, then all strategy profiles are regular when players are not too patient. Note that the regularity requirement does not exclude in principle any moderating behavior such as players voting for M most of the time. Restricting attention to regular equilibria will allow us to compare in a tractable way the equilibrium behavior in the $2-$ and $3-$alternative model. To formalize this comparison, we use the following notion.

Definition 6 *A profile of continuation values $Z : \Theta \times \{L, M, R\} \to \mathbb{R}^2$ is a concatenation of $2-$alternative equilibria, if $W^{LM}(.) \doteq Z(., M) - Z(., L)$ is an equilibrium profile of value functions for the $2-$alternative model with $X = \{L, M\}$ and $W^{MR}(.) \doteq Z(., R) - Z(., M)$ is an equilibrium profile of value functions for the $2-$alternative model with $X = \{M, R\}$.*

If the equilibrium W^{σ} is a concatenation of $2-$alternative equilibria, then players behave as in the $2-$alternative model in the following sense. The status quo q is replaced by a more rightist (leftist) agreement exactly at the same states at which q is replaced by the next more rightist (leftist) alternative x in the corresponding game with two alteranatives $X = \{q, x\}$ (see Example 2 for a graphical representation).

We are now ready to state our results. The next two propositions formalize the intuition in Section 4.1 that the moderating effect depends on the allocation of proposal power.

Proposition 4 *If σ is a regular equilibrium under the moderate proposer protocol, then the corresponding W^{σ} is a concatenation of $2-$alternative equilibria.*

Proposition 4 states that under the moderate proposer protocol, each player behaves as in a $2-$alternative model, and since in the latter players exhibit partisanship and the agreement exhibits status quo inertia, Proposition 4 implies that the same hold for three alternatives.

To understand Proposition 4, observe that under the moderate proposer protocol, when players agree on the direction of the agreement change but disagree on its magnitude (the second type of disagreement discussed in Section 4.1), it is the player who prefers the more moderate change who is the proposer (hence the name of the protocol). Therefore, in such disagreement states, the proposer is unconstrained by the status quo, and hence the status quo does not matter. As a result, the anticipation of such disagreement does not affect players' behavior, and the moderating effect does not arise.

Consider now the $r-$monopolistic proposer protocol. When $q = R$ and players agree to move the agreement to the left but disagree by how much, then the proposing player r wants a moderate change (to M). Therefore, as in the moderate proposer protocol, it is irrelevant whether $q = R$ or $q = M$: the agreement outcome will be M in either cases. Hence, the anticipation of such disagreements does not generate any moderation in players' choices between R and M. However, in the opposite scenario in which $q = L$ and players agree to change the agreement to the right but disagree by how much, then the proposing player r wants an extreme change (to R). Hence, r prefers the status quo to be L to M, as under the latter status quo, proposal R is vetoed by l. That generates moderation. The following proposition shows indeed that under the $r-$monopolistic proposer protocol, players are less partisan between L and M than in the corresponding $2-$alternative game. Somewhat more surprisingly, when the moderating effect between L and M is sufficiently strong, it can exacerbate the players' partisanship between M and R.[12]

Proposition 5 *Let σ be a regular equilibrium under the $r-$monopolistic proposer protocol. Then there exists a concatenation of $2-$alternative equilibria Z such that for all $\theta \in \Theta$,*

$$\begin{cases} W_l^\sigma(\theta, M) - W_l^\sigma(\theta, L) \geq Z_l(\theta, M) - Z_l(\theta, L), \\ W_r^\sigma(\theta, M) - W_r^\sigma(\theta, L) \leq Z_r(\theta, M) - Z_r(\theta, L), \end{cases}$$

[12] The proof of Proposition 5 reveals that players are more partisan between M and R than in the corresponding $2-$alternative model only if the moderation effect between M and L is so strong that in some states, player r prefers L over M while player l has the opposite preferences. Example 2 shows that this does not happen in a simple environment.

and

$$\begin{cases} W_l^\sigma(\theta, R) - W_l^\sigma(\theta, M) \leq Z_l(\theta, R) - Z_l(\theta, M), \\ W_r^\sigma(\theta, R) - W_r^\sigma(\theta, M) \geq Z_r(\theta, R) - Z_r(\theta, M). \end{cases}$$

Proposition 5 shows that the moderating effect can occur, but is silent on the relative strength of the two effects. Hence, players may still be partisan when it comes to the choice between L and M, just less so than under the moderate proposer protocol. The example below demonstrates, however, that the moderating effect can dominate, in which case we have status quo instability instead of status quo inertia.

Example 2 *Consider a constant-bias preference profile with three alternatives,* $L = -1, M = 0$ *and* $R = 1$*,* $\rho = 0.1$*, and the remaining parameters as in Example 1. The dashed lines in Figure 3 depict the set of states for which each alternative is Pareto optimal with respect to the current preferences* $(U_r(\theta, x), U_l(\theta, x))$.

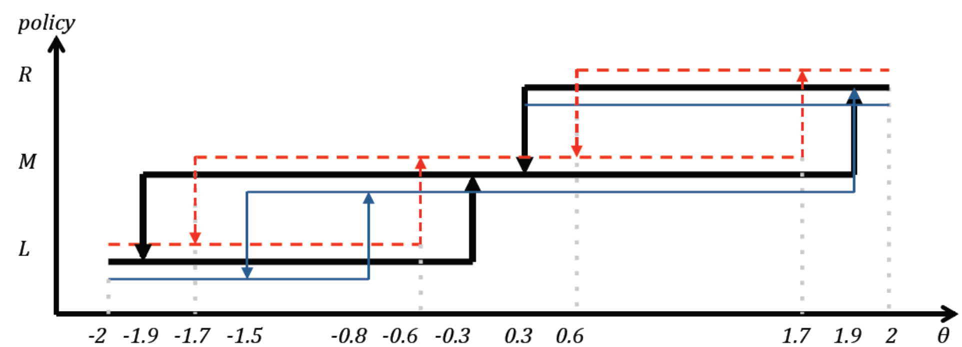

Figure 3: Evolution of policies under the exogenous status quo (dashed lines), moderate proposer (thick lines), and $r-$monopolistic proposer (thin lines).

One can calculate that in the (unique) equilibrium of the $2-$*alternative model with* $X = \{L, M\}$: M *replaces* L *only if* $\theta > -0.3$ *and* L *replaces* M *only if* $\theta < -1.9$*. In the equilibrium of the* $2-$*alternative model with* $X = \{M, R\}$, R *replaces* M *only if* $\theta > 1.9$ *and* L *replaces* M *only if* $\theta < 0.3$*. Proposition 4 implies that in the model with* $X = \{L, M, R\}$ *under the moderate proposer protocol, the agreement change on the equilibrium path will follow exactly the same pattern. The thick solid lines in Figure 3 represent the states in which* $q = L$ *(the lower line),* $q = M$ *(the middle line), and* $q = R$ *(the upper line) stay in place in this equilibrium, and the arrows point to the policies that replace the status quo.*

The thin solid lines in Figure 3 represent the equilibrium for the $r-$monopolistic proposer protocol. Consistent with Proposition 5 (see footnote 12), the transition between R and M occurs exactly for the same set of states as for the moderate proposer protocol. However, if $q \in \{L, M\}$, then the status quo remains in place for a set of states which is not only smaller than the corresponding set of states for the moderate proposer protocol, but also smaller than the set of states at which these alternatives are Pareto optimal. As a result, for relatively low states, there is status quo instability.

In Section 4.1, we argued that when players can bargain sufficiently frequently and their preferences evolve smoothly, the moderating effect should vanish. The following proposition formalizes this intuition in the case in which players have single-peaked preferences and are not too patient. Under these simplifying assumptions, for δ sufficiently small the moderating effect completely disappears irrespective of the allocation of proposal power, and (except possibly at $t = 0$) players behave as in the $2-$alternative model. That is, a status quo is replaced by an adjacent alternative exactly in the same states in which it would be replaced in the two-alternative model with only these two policies.

Proposition 6 *Suppose that Θ is compact, that $u_k(\theta, x)$ is continuous in θ and strictly concave in x, and that there exists $K > 0$ such that with probability 1, $u_k(\theta(t), x)$ is $K-$Lipschitz continuous in t. Then exists $\bar{\delta} > 0$ such that for all $\delta < \bar{\delta}$, for all ρ sufficiently large, for any bargaining protocol and any equilibrium, there exists a concatenation of $2-$alternative equilibria Z such that for almost all realizations of $\{\theta(t) : t \geq 0\}$, for all $t > 0$, given $q(t)$, the outcome $x(t)$ is the same as it would be if players behaved as if their continuation value were Z.*

It is worth noting that this proposition is proven in the appendix for an arbitrary finite number of alternatives.

4.3 Status quo inertia in a general $N-$alternative model

Proposition 6 shows that when players can revise the status quo sufficiently frequently, only the polarizing effect arises, and thus status quo inertia occurs. This finding is independent of the bargaining protocol used by the players. However, this sharp result is established under several simplifying assumptions, namely single-peaked preferences, a strong notion of continuity, and impatient players.

Characterizing all equilibria in general dynamic bargaining games with an endogenous status quo is a task that is known to be elusive (see, e.g., Duggan and Kalandrakis 2012) mainly because of the richness of the strategic effects. For the same reason, in many settings, the qualitative impact of the endogenous status quo turns out to be quite sensitive to the parameters of the models.[13] Similarly in our model, when one ventures beyond a simple setting as the one analyzed in Section 4.2, many competing effects can arise, which precludes an exhaustive equilibrium analysis. In this section we show, however, that when preferences evolve smoothly and the bargaining frictions vanish, the inertial effect of the endogenous status quo holds across all equilibria independently of the space of alternatives, the bargaining protocol, the preference distribution, and patience.

We consider an environment in which a set P of players bargain over a finite set X of alternatives. As before, players' preferences evolve in continuous time according to an arbitrary Markov process $\{\theta(t) : t \geq 0\}$ and players bargain every δ unit of time. Players' current preferences U_k^δ are derived from their flow payoff as in (1), but we now use a superscript δ to emphasize the dependence on δ. To characterize outcomes as δ vanishes, we assume that current preferences have a limit as δ tends to 0 : for all $\theta \in \Theta$ and all $x \in X$, $\lim_{\delta \to 0} \frac{U_k^\delta(\theta,x)}{1-e^{-\rho\delta}}$ exists; we denote that limit $\hat{u}_k(\theta, x)$.[14] To capture the idea that preferences evolve smoothly, we assume that Θ is endowed with a topology such that for all $\theta \in \Theta$ and all neighborhoods B of θ,

$$\Pr(\theta(t+\delta) \notin B | \theta(t) = \theta) = o(\delta). \tag{4}$$

For instance, if Θ is a Euclidean space with the standard topology, this continuity requirement is satisfied for Brownian motions, but not for processes with discontinuous shocks such as the one used in Examples 1 and 2.[15] In what follows, all the primitives of the model except δ are held constant.

Fix a Markovian bargaining protocol. Together with a strategy profile it defines a mapping from $\Theta \times X$ into X which maps the current state and status quo (θ, q) into a bargaining outcome. We

[13] For instance, in the redistributive setting with majoritarian bargaining, the equilibrium allocation and its dynamics can vary substantially depending on the choice of the equilibrium, the degree of risk aversion, or the time horizon of players. Compare for instance Bernheim et al. (2006) and Diemeier and Fong (2011), or Kalandrakis (2004) and Bowen and Zahran (2012).

[14] Note that $\frac{U_k^\delta(\theta,x)}{1-e^{-\rho\delta}}$ is the expected average discounted flow payoff for $t \in (0, \delta)$, so for most processes $\{\theta(t) : t \geq 0\}$, its limit $\hat{u}_k(\theta, x)$ is simply equal to the flowpayoff $u_k(\theta, x)$.

[15] To see this, observe that if $\{\theta(t) : t \geq 0\}$ is a Wiener process, and if F denotes the c.d.f. of the normal distribution,

denote this mapping by ϕ, and we call it a bargaining outcome function. The endogeneity of the status quo implies that at each bargaining time t, the status quo at $t+\delta$ is given by $\phi(\theta(t), q(t))$. Note that ϕ uniquely pins down the continuation value W^δ as follows: for all $k \in P$, all $\theta \in \Theta$, and all $x \in X$,

$$W_k^\delta(\theta, x) = E_{\theta(0)=\theta}\left[U_k^\delta(\theta(\delta), \phi(\theta(\delta), x)) + e^{-\rho\delta} W_k^\delta(\theta(\delta), \phi(\theta(\delta), x))\right]. \tag{5}$$

Instead of fully specifying the bargaining protocol and the equilibrium concept, we impose equilibrium conditions directly on ϕ. For a given δ, we say that a bargaining outcome function ϕ is an equilibrium if the bargaining outcome always gives a greater continuation payoff than the status quo to both players: for all $k \in P$, all $\theta \in \Theta$, and all $x \in X$,

$$U_k^\delta(\theta, \phi(\theta, x)) + e^{-\rho\delta} W_k^\delta(\theta, \phi(\theta, x)) \geq U_k^\delta(\theta, x) + e^{-\rho\delta} W_k^\delta(\theta, x).$$

We further assume that players reach agreements that they themselves would not want to revise immediately: for all $\theta \in \Theta$ and all $x \in X$, $\phi(\theta, \phi(\theta, x)) = \phi(\theta, x)$ (for most bargaining protocols, this is implied by stage-undomination). Hence, the only assumptions we implicitly impose on the bargaining protocol and players' behavior is that bargaining is voluntary, immediate, and the status quo is endogenous. These conditions encompass a large class of dynamic bargaining games (see Acemoglu, Egorov and Sonin 2013 for a related equilibrium notion). Note that ϕ is assumed to be deterministic, but this is only for notational simplicity.

To characterize the limit behavior as bargaining frictions disappear, we use the following notion of convergence: a sequence of bargaining outcome functions $(\phi_n)_{n\in\mathbb{N}}$ converges at (θ, q) to x if there

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then

$$\begin{aligned}
\Pr_{\theta(t)=\theta}(|\theta(t+\delta) - \theta(t)| > c|) &= 2\left(1 - F\left(\frac{c}{\delta}\right)\right) \\
&= \frac{\delta}{c}\int_{\frac{c}{\delta}}^{+\infty} \frac{c}{\delta} dF(x)\, dx \\
&\leq \frac{\delta}{c}\int_{\frac{c}{\delta}}^{+\infty} x dF(x)\, dx.
\end{aligned}$$

The left-hand side of the above inequality is a $o(\delta)$ since $\int_a^{+\infty} x dF(x)\, dx \to 0$ as $a \to +\infty$.

If instead $\{\theta(t) : t \geq 0\}$ is piecewise constant with stochastic jumps drawn from a Poisson process with parameter λ, as in Examples 1 and 2, then $|\theta(t+\delta) - \theta(t)| > c$ whenever $\theta(t)$ is redrawn between t and $t+\delta$. The probability of that event is $1 - e^{-\lambda\delta} \sim \lambda\delta$, which is not a $o(\delta)$.

exists a neighborhood B of θ such that for n sufficiently large, for almost all $\zeta \in B$, $\phi_n(\zeta, q) = x$.[16]

Proposition 7 *Let $(\delta_n)_{n\in\mathbb{N}}$ be such that $\delta_n \to 0$, and let $(\phi_n)_{n\in\mathbb{N}}$ be a sequence of bargaining outcome functions such that for all $n \in \mathbb{N}$, ϕ_n is an equilibrium outcome function for δ_n. For all $\theta \in \Theta$ and all $q, x \in X$, if $(\phi_n)_{n\in\mathbb{N}}$ converges at (θ, q) to x, then for all $k \in P$, $\hat{u}_k(\theta, x) \geq \hat{u}_k(\theta, q)$.*

Proposition 7 states that when the negotiating parties can revise the bargaining agreement sufficiently frequently, the status quo q can be replaced by x only if x Pareto improves on q in terms of the current preferences. Note that with an exogenous status quo, the status quo q is replaced exactly when an alternative x Pareto improves on q in terms of the current preferences. Hence, Proposition 7 implies that if the endogenous status quo generates a distortion of the agreement dynamics relative to the exogenous status quo, this distortion must take the form of status quo inertia.

The intuition for Proposition 7 is fairly simple. Consider player k deciding whether to agree to replace q with x in some bargaining time t. If the other players agree to such a change, then since preferences evolve smoothly, a small instant later they are likely to still agree to it. Hence, if q gives a higher current payoff than x to player k, player k prefers to oppose the change in t and reconsider it an instant latter.

Note that the continuity assumption stated in (4) precisely captures how fast the process can change so as to guarantee that the above intuition is correct. To see this, observe that in the case of the discontinuous jump model used in Examples 1 and 2, with the standard Euclidean topology, for a small neighborhood B of θ, $\Pr(\theta(t+\delta) \notin B | \theta(t) = \theta) \sim \lambda\delta$ (see footnote 15). In that case, numerical simulations show that as $\delta \to 0$, the equilibrium in Example 2 still exhibits status quo instability.

The generality of the environment considered in Proposition 7 does not allow us to quantify the inertial effect of the endogenous status quo. However, as argued in Section 3.3, this effect should not vanish as δ tends to 0. To see this, suppose by contradiction that there is no strict status quo inertia as $\delta \to 0$. From Proposition 7, this implies that in the limit, a status quo q ceases

[16] Observe that convergence is guaranteed for almost all (θ, q) (i.e., possibly except for a set of types of measure 0) for instance when the equilibria are in cutoff strategies as in Example 1 and 2. More precisely, if $\Theta = \mathbb{R}$ is endowed with the standard topology, and if $(\phi_n)_{n\in\mathbb{N}}$ is such that for all $q, x \in X$, $\{\theta \in \Theta : \phi_n(\theta, q) = x\}$ is an interval, then the bounds of the intervals must converge for some subsequence of $(\phi_n)_{n\in\mathbb{N}}$, so according to our convergence criterion, $(\phi_n)_{n\in\mathbb{N}}$ converges at all interior points of the limit intervals.

to be stable exactly at the state θ at which it becomes Pareto dominated (in terms of current preferences) by the new bargaining outcome x. If players do not have identical current preferences, at this cut-off state θ, one player k strictly prefers x to q while the other player k' is indifferent. If right after having reached θ, the state moves back into the region in which player k' prefers q to x, by continuity, player k still prefers x to q, so the bargaining outcome does not switch back to q. Anticipating such disagreements, player k' should veto a change from q to x at θ, and hence strict status quo inertia should occur.

5 Concluding remarks

Negotiations in a changing environment with an endogenous status quo are at the center of many economically relevant situations. They present the negotiating parties with a fundamental trade-off between responding adequately to the current environment and securing a favorable bargaining position for the future. In this paper, we show that this trade-off has a detrimental impact on the efficiency of bargaining outcomes and their responsiveness to political and economic shocks.

The natural and most common alternative to the endogenous status quo protocol is the exogenous status quo protocol.[17] For instance, in the U.S. budget process, federal spending is divided into two categories. One—called mandatory spending—continues year after year by default. The other one—called discretionary spending—requires annual appropriation bills, which means that the status quo is exogenously fixed at zero.[18] In the legislative sphere, the exogenous status quo is also implemented in the form of automatic sunset provisions: clauses that specify a duration after which an act expires, unless further legislative action is taken.[19]

[17] For example, the permanent provisions of the Agricultural Adjustment Act of 1938 and the Agriculture Act of 1949 serve as a fixed status quo for U.S. farm bills (Kwan 2009). Also, bilateral international agreements implicitely have an exogenous status quo of no agreement because either country can unilaterally opt out. See Lowi (1969), Weaver (1985, 1988), Hird (1991), and Gersen (2007) for more detailed studies of the ongoing and temporary nature of the laws enacted by the U.S. congress.

[18] Mandatory spending, also called direct spending, consists almost entirely of entitlement programs such as Social Security benefits, Medicare and Medicaid. Discretionary spending includes the budgets of most federal agencies (e.g., defense, national parks) and pork barrel projects. Mandatory and discretionary spending currently represents about two thirds and one third of the federal budget, respectively.

[19] One example of automatic sunset provisions is the sunset legislation in twenty-four U.S. states that requires automatic termination of a state agency, board, commission, or committee (see The Book of the States, 2011, Council of State Governments). See Kearney (1990) for more on the use of sunset provisions by U.S. state legislatures. In the U.S., automatic sunset clauses are less common at the federal level, although there have been attempts to introduce them systematically in Congress (the Federal Sunset Act). In the budget process, the Byrd rule is equivalent to imposing an automatic sunset clause on any provision that increases the deficit and that does not garner a filibuster-proof majority. In a similar spirit, in 2007, the Liberal Democratic Party in Australia proposed an automatic sunset

In light of our results, it is natural to ask how these two simple protocols compare in terms of welfare. Clearly, the partisanship generated by the endogenous status quo is detrimental to welfare as Pareto-dominated alternatives are implemented with positive probability. As a result, the exogenous status quo may dominate. In the legislative bargaining context, this provides a rationale for sunset provisions. Sunset provisions have usually been advocated to improve parliamentary control of executive agencies, or to evaluate the efficiency of new laws. The rationale suggested by this paper has a more strategic flavor: sunset provisions sever the link between today's agreement and tomorrow's status quo, which mitigates conflicts of interest among legislators, and makes policies more responsive to the environment.

However, automatic sunset provisions are bound to be beneficial only if the reversion point they use is socially optimal. This, however, may be hard to implement if the optimal reversion point is not constant over time. Since under the endogenous status quo, the status quo is determined by the equilibrium behavior, and hence evolves, the endogenous status quo may dominate.[20]

In light of the above, an interesting extension of our paper would be to allow for sunset provisions to emerge endogenously. One can shed light on this issue by considering an extension in which in every period the players first vote on whether the policy should be subject to a sunset clause or whether it should become the future status quo, and only then they vote on the policy. We leave this for future research.

provision in all legislation that does not get the support of a 75 percent parliamentary supermajority. In Canada, any law that overrides the Canadian Charter of Rights and Freedoms (section 33) has an automatic 5-year sunset. In Germany, all emergency legislations have an automatic sunset of six months.

[20] An interested reader is referred to Example 1 in Dziuda and Loeper (2012) which shows an example of an environment in which the endogenous status quo may dominate.

6 Appendix

Section 6.1 establishes some notation, and Section 6.2 proves lemmas that will be used throughout the appendix.

6.1 Notations

We denote by m the bound of $|u_k(\theta, x)|$ over all $k \in \{l, r\}$, $\theta \in \Theta$, and $x \in X$. For all $\theta \in \Theta$, P_θ denotes the probability distribution of the random variable $\theta(\delta)$, conditional on $\theta(0) = \theta$. Let $\mathcal{F}$ be the set of mappings f_k from Θ into $[-m, m]$ such that $f_k(\theta(\delta))$ is $P_\theta-$integrable, and let $f = (f_l, f_r)$ be an arbitrary pair of such mappings.

Denote

$$\begin{cases} D(f) = \{\theta \in \Theta : f_l(\theta) < 0 \text{ and } f_r(\theta) \geq 0\} \\ D'(f) = \{\theta \in \Theta : f_l(\theta) \geq 0 \text{ and } f_r(\theta) < 0\} \end{cases}. \tag{6}$$

We define the mappings V and Ω from $\mathcal{F}^2$ into itself as follows: for all $f \in \mathcal{F}^2$, all $k \in \{l, r\}$, and all $\theta \in \Theta$,

$$V_k(f_k, \theta) \mathrel{\hat{=}} U_k(\theta) + e^{-\rho\delta} f_k(\theta), \tag{7}$$

and

$$\Omega_k(f)(\theta) = \int_{\zeta \in D(f) \cup D'(f)} f_k(\zeta)\, dP_\theta(\zeta). \tag{8}$$

When f_k is equal to some continuation value W_k^σ for some strategy profile σ, then $V_k(W^\sigma, \theta)$ is simply the relative gain for player k of implementing alternative R instead of L at some bargaining time t in the game Γ^{en}, conditional on $\theta(t) = \theta$ and conditional on σ being played thereafter. Then, $\Omega_k(V_k(f_k))(\theta)$ can be interpreted as the expectation of that relative gain in period $t + \delta$ conditional on $\theta(t)$ and on players disagreeing at $t + \delta$.

We denote by $(\leq, \geq)$ the partial order on $\mathcal{F}^2$ defined as follows: for all f and f', $f'(\leq, \geq) f$ if for all $\theta \in \Theta$, $f_l'(\theta) \leq f_l(\theta)$ and $f_r'(\theta) \geq f_r(\theta)$. This order on payoff profiles corresponds to the partisanship order on strategy profiles defined in Proposition 2.

6.2 Preliminary Lemmas

Lemma 1 *Let T be a $P_\theta-$measurable set, and let $f_k \in \mathcal{F}$ be such that for all $\theta \in \Theta$, using the notation introduced in (7),*

$$f_k(\theta) \geq \int_{\zeta \in T} V_k(f_k, \zeta)\, dP_\theta(\zeta).$$

If $U_k(\theta) \geq 0$ for all $\theta \in T$, then for all $\theta \in \Theta$, $f_k(\theta) \geq 0$. The symmetric implication holds with the opposite inequalities.

Proof. Let $f^* = \inf_{\theta \in \Theta} f_k(\theta)$ and let $(\theta_n)_{n \in \mathbb{N}}$ be a sequence of states such that $f_k(\theta_n) \to f^*$. Under our assumptions, using (7), for all $n \in \mathbb{N}$,

$$\begin{aligned} f_k(\theta_n) &\geq \int_{\zeta \in T} V_k(f_k, \zeta)\, dP_{\theta_n}(\zeta) \\ &\geq \int_{\zeta \in T} U_k(\zeta)\, dP_{\theta_n}(\zeta) + e^{-\rho\delta} f^* P_{\theta_n}(T). \end{aligned} \tag{9}$$

Since $\left|\int_{\zeta \in T} U_k(\zeta)\, dP_{\theta_n}(\zeta)\right| \leq m$ and $|P_{\theta_n}(T)| \leq 1$, we can assume w.l.o.g. that these sequences converge to some u and p, respectively. Necessarily, $p \leq 1$, and since $U_k(\theta) \geq 0$, $u \geq 0$. Taking the limit in (9) and rearranging terms, we obtain $f^* \geq u/\left(1 - pe^{-\rho\delta}\right) \geq 0$, as needed. The second part is proved analogously by using a sequence $(\theta_n)_{n \in \mathbb{N}}$ such that $f_k(\theta_n) \to \sup_{\theta \in \Theta} f_k(\theta)$. ■

Lemma 2 *The map Ω is isotone for the order $(\leq, \geq)$.*

Proof. For all $f, f' \in \mathcal{F}^2$,

$$\begin{aligned} \Omega_r(f', \theta) - \Omega_r(f, \theta) &= \int_{D(f')} f'_r(\zeta)\, dP_\theta(\zeta) - \int_{D(f)} f_r(\zeta)\, dP_\theta(\zeta) \\ &\quad + \int_{D'(f')} f'_r(\zeta)\, dP_\theta(\zeta) - \int_{D'(f)} f_r(\zeta)\, dP_\theta(\zeta) \end{aligned}$$

Let $A_r(f, f', \theta)$ and $B_r(f, f', \theta)$ denote the expression on the first and second line, respectively, of the right-hand side of the above equation.

Suppose that $f' (\leq, \geq) f$. Then from (6), $D(f) \subset D(f')$, so

$$\begin{aligned} A_r\left(f, f', \theta\right) &= \int_{D(f')} f'_r(\zeta)\, dP_\theta(\zeta) - \int_{D(f)} f'_r(\zeta)\, dP_\theta(\zeta) + \int_{D(f)} f'_r(\zeta)\, dP_\theta(\zeta) - \int_{D(f)} f_r(\zeta)\, dP_\theta(\zeta) \\ &= \int_{D(f')\setminus D(f)} f'_r(\zeta)\, dP_\theta(\zeta) + \int_{D(f)} \left(f'_r(\zeta) - f_r(\zeta)\right) dP_\theta(\zeta). \end{aligned}$$

From (6), f'_r is positive on $D(f')$, so $\int'_{D(f')\setminus D(f)} f'_r(\zeta)\, dP_\theta(\zeta)$ is positive. Since $f'_r \geq f_r$, $\int_{D(f)} (f'_r(\zeta) - f_r(\zeta))\, dP_\theta(\zeta)$ is positive. Therefore, $A_r(f, f', \theta)$ is positive.

From (6), if $f' (\leq, \geq) f$, $D'(f') \subset D'(f)$, so

$$\begin{aligned} B_r\left(f, f', \theta\right) &= \int_{D'(f')} f'_r(\zeta)\, dP_\theta(\zeta) - \int_{D'(f')} f_r(\zeta)\, dP_\theta(\zeta) + \int_{D'(f')} f_r(\zeta)\, dP_\theta(\zeta) - \int_{D'(f)} f_r(\zeta)\, dP_\theta(\zeta) \\ &= \int_{D'(f')} \left(f'_r(\zeta) - f_r(\zeta)\right) dP_\theta(\zeta) - \int_{D'(f)\setminus D'(f')} f_r(\zeta)\, dP_\theta(\zeta). \end{aligned}$$

From (6), f_r is negative on $D'(f)$, so $\left(-\int_{D'(f)\setminus D'(f')} f_r(\zeta)\, dP_\theta(\zeta)\right)$ is positive. Since $f'_r \geq f_r$, then $\int_{D'(f')} (f'_r(\zeta) - f_r(\zeta))\, dP_\theta(\zeta)$ is positive. Therefore, $B_r(f, f', \theta)$ is positive, and $\Omega_r(f', \theta) \geq \Omega_r(f, \theta)$. A symmetric argument for player l completes the proof. ■

Lemma 3 *A strategy profile σ is an equilibrium of the $2-$alternative model if and only if the corresponding continuation value W^σ is a fixed point of the map $f \to \Omega(V(f))$.*

Proof. Let σ be an equilibrium profile of strategies, let W^σ be the corresponding continuation value as defined in Section 3.2, let t be a bargaining period, and let $x(t)$ be the alternative implemented at t. This alternative $x(t)$ matters in the next period $t+\delta$ only when players disagree. Using Condition (3) and the notations introduced in (6), players disagree when $\theta(t+\delta) \in D(V(W^\sigma)) \cup D'(V(W^\sigma))$. For any such state $\theta(t+\delta)$, the difference in continuation value from $t+\delta$ onwards for player k between $x(t) = R$ and $x(t) = L$ is simply

$$U_k(\theta(t+\delta)) + e^{-\rho\delta} W^\sigma_k(\theta(t+\delta)).$$

Conditional on $\theta(t)$, the expectation of this difference at t is given by

$$W^\sigma_k(\theta(t)) = \int_{\zeta \in D(V(W^\sigma)) \cup D'(V(W^\sigma))} \left[U_k(\zeta) + e^{-\rho\delta} W^\sigma_k(\zeta)\right] dP_{\theta(t)}(\zeta).$$

Using the notations introduced in (7) and (8), the above equation can be rewritten as $W_k^\sigma(\theta(t)) = \Omega_k(V(W^\sigma), \theta(t))$, as needed.

Reciprocally, let $W^o \in \mathcal{F}^2$ be such that $W^o = \Omega_k(V(W^o))$. Consider the strategy profile σ such that each player k votes for R if and only if $V_k(W^o, \theta) \geq 0$. By construction of σ, players disagree in state θ if and only if $\theta \in D(V(W^o)) \cup D'(V(W^o))$, in which case the status quo stays in place. Therefore, W_k^σ satisfies

$$\begin{aligned} W_k^\sigma(\theta) &= \int_{\zeta \in D(V(W^o)) \cup D'(V(W^o))} \left[U_k(\zeta) + e^{-\rho\delta} W_k^\sigma(\zeta)\right] dP_\theta(\zeta) \\ &= \int_{\zeta \in D(V(W^o)) \cup D'(V(W^o))} V_k(W_k^\sigma, \zeta)\, dP_\theta(\zeta). \end{aligned} \tag{10}$$

Since $W^o = \Omega_k(V(W^o))$, (8) implies that W_k^o is also a solution to (10). From (7), the right hand-side of (10) is an $e^{-\rho\delta}$-contraction in W_k^δ for the sup norm. Therefore, (10) has a unique solution, which implies that $W_k^o = W_k^\sigma$. Together with the definition of σ, this shows that under σ, player k votes for R if and only if $V_k(W_k^\sigma, \theta) \geq 0$. This means that player k use stage undominated strategies, so σ is an equilibrium. ■

6.3 Proofs for Section 3.2

Proof of Proposition 1. Equilibrium existence follows from Proposition 2 and the fact that a complete lattice is non empty.

Part (a). Let W be an equilibrium continuation function. From Lemma 3, $W = \Omega(V(W))$, so $W_k = \int_{\zeta \in D(V(W)) \cup D'(V(W))} V_k(W_k, \zeta)\, dP_\theta(\zeta)$. Hence, we can write

$$W_r - W_l = \int_{\zeta \in D(V(W)) \cup D'(V(W))} (V_r(W_r, \zeta) - V_l(W_l, \zeta))\, dP_\theta(\zeta).$$

Setting $f_k = W_r - W_l$ in Lemma 1 and using the fact that $U_r(\theta) - U_l(\theta) \geq 0$ for all $\theta \in \Theta$, we obtain that $W_r(\theta) - W_l(\theta) \geq 0$. Substituting this inequality and $U_r(\theta) \geq U_l(\theta)$ in (7), we obtain $V_r(W_r, \theta) \geq V_l(W_l, \theta)$. From (6), this implies that $D'(V(W)) = \emptyset$. Hence, $W_k = \int_{\zeta: V_l(W_l, \zeta) < 0 \text{ and } V_r(W_r, \zeta) \geq 0} V_k(W_k, \zeta)\, dP_\theta(\zeta)$, which implies that $W(\leq, \geq) 0$, as needed.

Part (b). Let W be an equilibrium continuation function. From Part (a) above, we have that for all $\theta \in \Theta$, $W_l(\theta) = \int_{D(V(W))} V_l(W_l, \zeta)\, dP_\theta(\zeta)$.From (6), $V_l(W_l, \zeta) \leq 0$ for all $\zeta \in D(V(W))$.

Therefore, for all $\Delta \subseteq D(V(W))$ and all $\theta \in \Theta$,

$$W_l(\theta) \leq \int_\Delta V_l(W_l, \zeta)\, dP_\theta(\zeta). \tag{11}$$

From Part (a), $W(\leq, \geq) 0$, so one can see from (6) that for all $v > 0$, $\{\theta \in \Theta : U_l(\theta) < -v \;\&\; U_r(\theta) \geq v\} \subseteq D(V(W))$. Therefore, (11) implies that

$$\begin{aligned} W_l(\theta) &\leq \int_{\zeta : U_l(\zeta) < -v \;\&\; U_r(\zeta) \geq v} V_l(W_l, \zeta)\, dP_\theta(\zeta) \\ &< \int_{\zeta : U_l(\zeta) < -v \;\&\; U_r(\zeta) \geq v} \left(-v + e^{-\rho\delta} W_l(\zeta)\right) dP_\theta(\zeta) \\ &\leq -v \Pr_{\theta(0)=\theta}\left(U_l(\theta(\delta)) < -v \;\&\; U_r(\theta(\delta)) \geq v\right) \end{aligned} \tag{12}$$

Using the notations in Definition 2, let $\theta \in \Theta$ be such that $0 \leq U_l(\theta) \leq \bar{u}$. Since the payoff profile exhibits strict disagreement, then $\Pr_{\theta(0)=\theta}\left(U_l(\theta(\delta)) < -\bar{v} \;\&\; U_r(\theta(\delta)) \geq \bar{v}\right) \geq p$, so (12) implies that $W_l(\theta) < -\bar{v}p$. Let $\bar{w} \doteq \min\left(\bar{u}, e^{-\rho\delta}\bar{v}p\right)$. The latter inequality implies that for all θ such that $0 \leq U_l(\theta) \leq \bar{w}$,

$$V_l(W_l, \theta) = U_l(\theta) + e^{-\rho\delta} W_l(\theta) < \bar{w} - e^{-\rho\delta}\bar{v}p \leq 0,$$

and hence l votes for L. A symmetric reasoning holds for player r. ■

Proof of Proposition 2. Note that $\mathcal{F}^2$ is a complete lattice for $(\leq, \geq)$. One can easily see from (7) that the map $f \to V(f)$ is isotone for the order $(\leq, \geq)$. Together with Lemma 2, this shows that the map $f \to \Omega(V(f))$ is isotone. Tarski's fixed point theorem implies then that the set of fixed points of $f \to \Omega(V(f))$ is a complete lattice for the order $(\leq, \geq)$. The result follows then from Lemma 3, and the fact that the order $(\leq, \geq)$ on continuation value functions corresponds to the partisan order on strategy profiles as defined in Proposition 2.

To show the Pareto ranking properties, let σ and $\hat{\sigma}$ be two equilibria such that $W^{\hat{\sigma}}(\leq, \geq) W^{\sigma}$. We will show that σ is Pareto superior to $\hat{\sigma}$ by constructing a series of "one-bargaining time deviations" that lead from σ to $\hat{\sigma}$ such that all deviations are payoff reducing. Suppose that both players "deviate" from σ to $\hat{\sigma}$ only at the first bargaining time. Since $W^{\hat{\sigma}}(\leq, \geq) W^{\sigma}$, from Proposition 1, players disagree more often under $\hat{\sigma}$ than under σ, so the only way in which this deviation can change the outcome at $t = 0$ is if under σ a agreement change is implemented while under $\hat{\sigma}$ the status quo remains. Suppose w.l.o.g. that $q(0) = R$, and hence L is implemented under σ while R stays in place under $\hat{\sigma}$. Since players play their equilibrium strategy σ in the subgame starting at $t = \delta$, the net effect of the deviation to $\hat{\sigma}$ for player k is $U_k(\theta(0)) + e^{-\rho\delta} W_k^{\sigma}(\theta(0))$. By assumption, both players vote for L under σ (only then the change from $q(0) = R$ can occur under σ), so $U_k(\theta(0)) + e^{-\rho\delta} W_k^{\sigma}(\theta(0))$ must be negative for both $k \in \{l, r\}$, which implies that the deviation decreases the payoff of both players.

To conclude the argument, consider the strategy profile in which players play $\hat{\sigma}$ at $t = 0$ and σ afterwards, and let players deviate from σ to $\hat{\sigma}$ also at $t = \delta$. The same reasoning as above shows that this deviation decreases the payoffs of both players irrespective of the status quo distribution at the beginning of the second bargaining time. By induction on the number of times at which players deviate from σ to $\hat{\sigma}$, the proposition follows. ■

6.4 Formalization of the discussion and proofs from Section 3.3

Proposition 8 *Let (u_l', u_r') be more polarized than (u_l, u_r) (see the definition of polarization in Section 3.3), and let W' and W be the profile of continuation values at the Pareto best (or worst) equilibrium for (u_l', u_r') and (u_l, u_r), respectively. Then*

$$W_l'(\theta) \leq W_l(\theta) \leq 0 \leq W_r(\theta) \leq W_r'(\theta).$$

Proof. Let U and U' be the profile of current preferences derived from the flowpayoffs (u_l, u_r) and (u_l', u_r'), respectively. Under our assumptions, $U'(\leq, \geq) U$. If V and V' refer to the maps defined in (7) corresponding to U and U', respectively, then for all $f \in \mathcal{F}^2$, $V'(f)(\leq, \geq) V(f)$. Together with Lemma 2, this implies that for all $f \in \mathcal{F}^2$, $\Omega(V'(f))(\leq, \geq)\Omega(V(f))$. Since $f \to \Omega(V(f))$ and $f \to \Omega(V'(f))$ are isotone in f for $(\leq, \geq)$ (see the proof of Proposition 2), Corollary 1 in Villas-Boas (1997) implies that their respective smallest (or greatest) fixed points W and W' for the order $(\leq, \geq)$ are such that $W'(\leq, \geq) W$. Proposition 8 follows then immediately from Lemma 3, and from the fact that the order $(\leq, \geq)$ on continuation value functions coincides with the Pareto order on strategy profiles (see Proposition 2). ■

To analyze the impact of patience on partisanship, note first that since preferences evolve in continuous time, ρ affects the way players trade-off payoffs not only across periods but also within periods. The effect of patience on the trade-off within periods can depend on the fine details of the process $\{\theta(t) : t \geq 0\}$. For simplicity, we consider the standard case in which flow payoffs are constant within each bargaining period. This allows us to isolate the effect of patience on the trade-off across periods.[21] In that case, one can rewrite the equilibrium conditions (3) in terms of flow payoff as follows: with an exogenous status quo, player k votes for R if and only if $u_k(\theta, R) - u_k(\theta, L) \geq 0$, while with an endogenous status quo, she votes for R if and only if $u_k(\theta, R) - u_k(\theta, L) + e^{-\rho\delta} \frac{W_k(\theta)}{1-e^{-\rho\delta}} \geq 0$. So in the former case, ρ has no effect on equilibrium behavior, while in the latter case, its effect is captured by $e^{-\rho\delta} \frac{W_k(\theta)}{1-e^{-\rho\delta}}$. The following proposition shows that players disagree more often because they care more about the future—this direct effect corresponds to an increase in $e^{-\rho\delta}$—but also because they expect more partisanship in the future—this strategic effect corresponds to an increase in $\left|\frac{W_k(\theta)}{1-e^{-\rho\delta}}\right|$.

Proposition 9 *Suppose flow payoffs are constant within bargaining periods. Let $\rho > \rho' > 0$, and let W and W' be the profiles of continuation values at the Pareto best (or worst) equilibria for ρ and ρ', respectively, then*

$$\frac{W_l'(\theta)}{1 - e^{-\rho'\delta}} \leq \frac{W_l(\theta)}{1 - e^{-\rho\delta}} \leq 0 \leq \frac{W_r(\theta)}{1 - e^{-\rho\delta}} \leq \frac{W_r'(\theta)}{1 - e^{-\rho'\delta}}.$$

Proof. Consider the map $\hat{V}$ from $\mathcal{F}^2$ into itself defined as follows: for all $f \in \mathcal{F}^2$, all $k \in \{l, r\}$, and all $\theta \in \Theta$,

$$\hat{V}_k(f, \theta) = \frac{U_k(\theta)}{1 - e^{-\rho\delta}} + e^{-\rho\delta} f_k(\theta). \tag{13}$$

One can easily adapt Lemma 3 to show that W is an equilibrium continuation value if and only if $W / \left(1 - e^{-\rho\delta}\right)$ is a fixed point of the mapping $f \to \Omega\left(\hat{V}(f)\right)$, and from Proposition 1, we can restrict attention to the fixed points of that mapping in $\Pi^+ = \left\{f \in \mathcal{F}^2 : f(\leq, \geq) 0\right\}$. Clearly $\hat{V}$ is isotone in f for the order $(\leq, \geq)$. Now let $\hat{V}$ and $\hat{V}'$ refer to the mappings defined in (13) for ρ and ρ', where $\rho \geq \rho' > 0$. Note that under our assumptions, $U_k(\theta) / \left(1 - e^{-\rho\delta}\right) = u_k(\theta, R) - u_k(\theta, L) = U_k'(\theta) / \left(1 - e^{-\rho'\delta}\right)$. Using (13), this implies that for all $f \in \Pi^+$, $\hat{V}'(f)(\leq, \geq)\hat{V}(f)$, and from Lemma 2, $\Omega\left(\hat{V}'(f)\right)(\leq, \geq)\Omega\left(\hat{V}(f)\right)$. The same monotonicity argument as in the proof

[21] The careful reader will note that this process is not a homogeneous Markov process, but one can easily check that $\{U_k(\theta(t)) : t = 0, \delta, 2\delta, ...\}$ is a (discrete time) homogenous Markov process on Θ, and all our results in which δ is fixed hold unchanged in that case.

of Proposition 8 applied to the restriction of $f \to \Omega\left(\hat{V}(f)\right)$ on Π^{+} implies then that if W and W' are the continuation value of the Pareto best (or worst) equilibria for ρ and ρ', respectively, then $W'/\left(1-e^{-\rho'\delta}\right)(\leq,\geq)W/\left(1-e^{-\rho\delta}\right)$. ■

The next proposition derives comparative statics w.r.t. δ for a constant bias specification in which $\{\theta(t): t\geq 0\}$ is a martingale. In that case, $U_k(\theta) = (u_k(\theta,R) - u_k(\theta,L))\left(1-e^{-\rho\delta}\right)$ so as in the above case in which $\{\theta(t): t\geq 0\}$ is constant within bargaining periods, the relevant measure of partisanship is $e^{-\rho\delta}\frac{W(\theta)}{1-e^{-\rho\delta}}$.

Proposition 10 *Consider a constant bias payoff profile (see Definition 3) such that $\{\theta(t): t\geq 0\}$ is a martingale. Let $\delta > \delta' > 0$, and let W and W' be the profiles of continuation values at the Pareto best (or worst) equilibria for δ and δ', respectively, then*

$$\frac{W_l'(\theta)}{1-e^{-\rho\delta'}} \leq \frac{W_l(\theta)}{1-e^{-\rho\delta}} \leq 0 \leq \frac{W_r(\theta)}{1-e^{-\rho\delta}} \leq \frac{W_r'(\theta)}{1-e^{-\rho\delta'}}.$$

Proof. Since $\Theta \subset [-m,m]$ for some $m>0$, $E_{\theta(0)}\left[\int_0^\delta |\theta(t)|\rho e^{-\rho t}dt\right] \leq m$ and $\int_0^\delta E_{\theta(0)}[|\theta(t)|]\rho e^{-\rho t}dt \leq m$, so Fubini's theorem together with the martingale property imply that

$$E_{\theta(0)}\left[\int_0^\delta \theta(t)\rho e^{-\rho t}dt\right] = \int_0^\infty E_{\theta(0)}[\theta(t)]\rho e^{-\rho t}dt = \left(1-e^{-\rho\delta}\right)\theta,$$

so

$$\begin{aligned}
U_k(\theta) &= E_{\theta(0)=\theta}\left[\int_0^\delta \left((L-(\theta+b_k))^2 - (R-(\theta+b_k))^2\right)\rho e^{-\rho t}dt\right] \\
&= E_{\theta(0)=\theta}\left[\int_0^\delta 2(R-L)\left(\theta(t)+b_k-\frac{L+R}{2}\right)\rho e^{-\rho t}dt\right] \\
&= 2(R-L)\left(\theta+b_k-\frac{L+R}{2}\right)\left(1-e^{-\rho\delta}\right) \\
&= (u_k(\theta,R)-u_k(\theta,L))\left(1-e^{-\rho\delta}\right).
\end{aligned}$$

Therefore, as in the proof of Proposition 9, we have that $U_k(\theta)/\left(1-e^{-\rho\delta}\right)$ is independent of δ. Moreover, if we define $\hat{V}_k(f,\theta)$ as in the proof of Proposition 9, one can see that $\hat{V}_k(f,\theta)$ depends on ρ and δ in the same way. Hence, the same argument implies that partisanship increases as δ decreases. ■

The following proposition proves that, as claimed in the main text, that players l and r are absolute leftist and rightist, respectively, if and only if they are sufficiently polarized and patient.

Proposition 11 *If player k is an absolute leftist (rightist) for some u_k and some ρ, then she is also an absolute leftist (rightist) for any u'_k that is more leftist (rightist) than u_k and any ρ' such that $\rho' \leq \rho$.*

Proof. The comparative statics with respect to u_k is straightforward and omitted for brevity.

Suppose that player with u_k is an absolute leftist for some ρ, and let $\rho' \leq \rho$. To show that k is an absolute leftist for ρ', consider

$$\Phi\left(\rho'\right) \doteq E_{\theta(0)}\left[\int_0^\infty \frac{e^{(\rho-\rho')t}}{\rho-\rho'} E_{\theta(t)}\left[\int_t^\infty \left(u_k\left(\theta\left(\tau\right), R\right) - u_k\left(\theta\left(\tau\right), L\right)\right) e^{-\rho\tau} d\tau\right] dt\right].$$

Since u_k is an absolute leftist for ρ, and since $\theta(t)$ is Markov, for all t and all realization of $\theta(t)$, $E_{\theta(t)}\left[\int_t^\infty \left(u_k\left(\theta\left(\tau\right), R\right) - u_k\left(\theta\left(\tau\right), L\right)\right) e^{-\rho\tau} d\tau\right] \leq 0$. This together with the fact that $\frac{e^{(\rho-\rho')t}}{\rho-\rho'} \geq 0$ implies that $\Phi\left(\rho'\right) \leq 0$. Using the law of iterated expectation and inverting the order of integration, we get

$$\begin{aligned} 0 \geq \Phi\left(\rho'\right) &= E_{\theta(0)}\left[\int_0^\infty \left(u_k\left(\theta\left(\tau\right), R\right) - u_k\left(\theta\left(\tau\right), L\right)\right) e^{-\rho\tau} \left(\int_0^\tau \frac{e^{(\rho-\rho')t}}{\rho-\rho'} dt\right) d\tau\right] \\ &= E_{\theta(0)=\theta}\left[\int_0^\infty \left(u_k\left(\theta\left(\tau\right), R\right) - u_k\left(\theta\left(\tau\right), L\right)\right) e^{-\rho'\tau} d\tau\right]. \end{aligned}$$

The above inequality means that k is an absolute leftist at ρ'. The proof when k is an absolute rightist is analogous. ■

Proof of Proposition 3. Suppose σ_l^g is not a best response for player l to σ_r^g. From the one-step-deviation principle, this is true if and only if for some $\theta \in \Theta$, conditional on $\theta(0) = \theta$, player r is better-off voting for R than for L at $t = 0$, given that he expects σ^g to be played in all future periods. By definition of σ^g, this is equivalent to saying that player r is better-off with alternative R in all periods, than with alternative L in all periods. Hence, we have shown that σ_l^g is not a best response for player l to σ_r^g if and only if l is not an absolute leftist. A symmetric argument for player l completes the proof of Proposition 3. ■

Proposition 12 *Consider a constant bias payoff profile (see Definition 3) on a finite Θ whose sta-*

tionary distribution has mean $\bar{\theta}$.[22] *Then player l and r are absolute rightist and leftist, respectively, whenever $b_l < \frac{L+R}{2} - \bar{\theta} < b_r$ and ρ is sufficiently small.*

Proof. Since $\Theta \subset [-m, m]$ for some $m > 0$, $E_{\theta(0)}\left[\int_0^\infty |\theta(t)| \rho e^{-\rho t} dt\right] \leq m$ and $\int_0^\infty E_{\theta(0)}[|\theta(t)|] \rho e^{-\rho t} dt \leq m$, so Fubini's theorem yields

$$E_{\theta(0)}\left[\int_0^\infty \theta(t) \rho e^{-\rho t} dt\right] = \int_0^\infty E_{\theta(0)}[\theta(t)] \rho e^{-\rho t} dt.$$

Therefore

$$\begin{aligned}
\left| E_{\theta(0)}\left[\int_0^\infty \theta(t) \rho e^{-\rho t} dt\right] - \bar{\theta} \right| &\leq \int_0^{\frac{1}{\sqrt{\rho}}} \left| E_{\theta(0)}[\theta(t)] - \bar{\theta} \right| \rho e^{-\rho t} dt + \int_{\frac{1}{\sqrt{\rho}}}^\infty \left| E_{\theta(0)}[\theta(t)] - \bar{\theta} \right| \rho e^{-\rho t} dt \\
&= 2m \int_0^{\frac{1}{\sqrt{\rho}}} \rho e^{-\rho t} dt + \sup_{s \geq \frac{1}{\sqrt{\rho}}} \left| E_{\theta(0)}[\theta(t)] - \bar{\theta} \right| \int_{\frac{1}{\sqrt{\rho}}}^\infty \rho e^{-\rho t} dt \\
&= 2m \left(1 - e^{-\sqrt{\rho}}\right) + \left(\sup_{t \geq \frac{1}{\sqrt{\rho}}} \left| E_{\theta(0)}[\theta(t)] - \bar{\theta} \right| \right) e^{-\sqrt{\rho}} \to_{\rho \to 0} 0.
\end{aligned}$$

Since $\{\theta(t) : t \geq 0\}$ has an invariant distribution with mean $\bar{\theta}$, for all $\theta(0) \in \Theta$, $\limsup_{t\to\infty} \left| E_{\theta(0)}[\theta(t)] - \bar{\theta} \right| = 0$, so the above equation shows that $E_{\theta(0)}\left[\int_0^\infty \theta(t) \rho e^{-\rho t} dt\right] \to \bar{\theta}$ as $\rho \to 0$. Hence,

$$\begin{aligned}
E_{\theta(0)}\left[\int_0^\infty (u_k(\theta(t), R) - u_k(\theta(t), L)) \rho e^{-\rho t} dt\right] &= E_{\theta(0)}\left[\int_0^\infty 2(R-L)\left(\theta(t) + b_k - \frac{L+R}{2}\right) \rho e^{-\rho t} dt\right] \\
&= 2(R-L)\left(\int_0^\infty E_{\theta(0)}[\theta(t)] \rho e^{-\rho t} dt + b_k - \frac{L+R}{2}\right) \\
&\to 2(R-L)\left(\bar{\theta} + b_k - \frac{L+R}{2}\right).
\end{aligned}$$

Therefore, if $b_l < \frac{L+R}{2} - \bar{\theta} < b_r$, then for ρ sufficiently small, for all $\theta(0) \in \Theta$, the left-hand side of the above equation is positive for $k = r$ and negative for $k = l$, as needed. ■

6.5 Proofs for Section 4

Proof of Proposition 4. Let W be the continuation value function of a regular equilibrium. For all $k \in \{l, r\}$ and all $\theta \in \Theta$, let $\succ_k^{W,\theta}$ denote the intertemporal preferences of player k derived from

[22] The existence of a stationary distribution is guaranteed whenever $\{\theta(t) : t \geq 0\}$ is irreducible and positive recurrent, and whenever it exists, it is always unique.

the intertemporal utility function $x \to U_k(\theta, x) + e^{-\rho\delta} W_k(\theta, x)$, with a tie breaking rule that when $x > y$ and $U_k(\theta, x) + e^{-\rho\delta} W_k(\theta, x) = U_k(\theta, y) + e^{-\rho\delta} W_k(\theta, y)$, then $x \succ_k^{W,\theta} y$. This tie breaking rule amounts to assume that when indifferent, players favor the more rightist alternative, but the same proofs would hold for any other tie breaking rule. Since W is regular, $\succ_k^{W,\theta}$ can be only of one of the four types: $L \succ_k^{W,\theta} M \succ_k^{W,\theta} R$, $M \succ_k^{W,\theta} L \succ_k^{W,\theta} R$, $M \succ_k^{W,\theta} R \succ_k^{W,\theta} L$, or $R \succ_k^{W,\theta} M \succ_k^{W,\theta} L$. Using stage undomination, one can map any pair of such rankings and a status quo into the equilibrium bargaining outcome. In the following table, we represent such a mapping. The cell corresponding to some preference profile $\left(\succ_l^{W,\theta}, \succ_r^{W,\theta}\right)$ contains a triplet (x, y, z) which is interpreted as follows: x, y, and z are the bargaining outcome under the moderate proposer protocol when the intertemporal ranking is $\left(\succ_l^{W,\theta}, \succ_r^{W,\theta}\right)$ and the status quo is L, M, and R, respectively.

	$L \succ_r^{W,\theta} M \succ_r^{W,\theta} R$	$M \succ_r^{W,\theta} L \succ_r^{W,\theta} R$	$M \succ_r^{W,\theta} R \succ_r^{W,\theta} L$	$R \succ_r^{W,\theta} M \succ_r^{W,\theta} L$
$L \succ_l^{W,\theta} M \succ_l^{W,\theta} R$	(L, L, L)	(L, M, M)	(L, M, M)	(L, M, R)
$M \succ_l^{W,\theta} L \succ_l^{W,\theta} R$	(L, M, L)	(M, M, M)	(M, M, M)	(M, M, R)
$M \succ_l^{W,\theta} R \succ_l^{W,\theta} L$	(L, M, M)	(M, M, M)	(M, M, M)	(M, M, R)
$R \succ_l^{W,\theta} M \succ_l^{W,\theta} L$	(L, M, R)	(M, M, R)	(R, M, R)	(R, R, R)

(14)

A close inspection of (14) shows that the bargaining outcome depends on whether the status quo is M or R if and only if one of the following three (mutually exclusive) configurations arises: (i) $M \succ_l^{W,\theta} R$ and $R \succ_r^{W,\theta} M$, (ii) $R \succ_l^{W,\theta} M$ and $M \succ_r^{W,\theta} R$, or (iii) $M \succ_l^{W,\theta} L \succ_l^{W,\theta} R$ and $L \succ_r^{W,\theta} M \succ_r^{W,\theta} R$. In cases (i) and (ii), the status quo stays in place. In case (iii), the outcome is M for $q = M$, and L for $q = R$ (case (iii) is the configuration whose anticipation can lead to the moderating effect, see section 4.1). Therefore,

$$\begin{aligned} & W_k(\theta, R) - W_k(\theta, M) \qquad (15) \\ = & \int_{\substack{\zeta: M \succ_l^{W,\zeta} R \& R \succ_r^{W,\zeta} M \\ \text{or } R \succ_l^{W,\zeta} M \& M \succ_r^{W,\zeta} R}} \left[U_k(\zeta, R) - U_k(\zeta, M) + e^{-\rho\delta}\left(W_k(\theta, R) - W_k(\theta, M)\right)\right] dP_\theta(\zeta) \\ & + \int_{\zeta: M \succ_l^{W,\zeta} L \succ_l^{W,\zeta} R \& L \succ_r^{W,\zeta} M \succ_r^{W,\zeta} R} \left[U_k(\zeta, L) - U_k(\zeta, M) + e^{-\rho\delta}\left(W_k(\theta, L) - W_k(\theta, M)\right)\right] dP_\theta(\zeta). \end{aligned}$$

By definition of $\succ_k^{W,\theta}$, the integrand of the second term on the right-hand side of (15) is positive on its integration set for $k=r$, and negative for $k=l$. Hence, subtracting (15) for $k=l$ from (15) for $k=r$, we obtain

$$\begin{aligned}&W_r(\theta,R)-W_r(\theta,M)-(W_l(\theta,R)-W_l(\theta,M))\\ \geq\ &\int_{\substack{\zeta:M\succ_l^{W,\zeta}R\&R\succ_r^{W,\zeta}M\\ \text{or } R\succ_l^{W,\zeta}M\&M\succ_r^{W,\zeta}R}}\left[\begin{array}{c}U_r(\zeta,R)-U_r(\zeta,M)-(U_l(\zeta,R)-U_l(\zeta,M))\\ +e^{-\rho\delta}((W_r(\theta,R)-W_r(\theta,M))-(W_l(\theta,R)-W_l(\theta,M)))\end{array}\right]dP_\theta(\zeta).\end{aligned}$$

Under our assumptions, for all $\theta\in\theta$, $U_r(\theta,R)-U_r(\theta,M)-(U_l(\theta,R)-U_l(\theta,M))\geq 0$. Hence, Lemma 1 applied to the above inequality implies that for all $\theta\in\Theta$, $W_r(\theta,R)-W_r(\theta,M)\geq W_l(\theta,R)-W_l(\theta,M)$. By definition of $\succ_k^{W,\theta}$, the latter two inequalities imply that for all $\theta\in\Theta$, $R\succ_l^{W,\theta}M\Rightarrow R\succ_r^{W,\theta}M$. Observe that under the moderate proposer protocol, alternatives M and R play the same role for player l and r, respectively, than alternatives M and L do for player r and l, respectively. Therefore, a symmetric argument on $W_k(\theta,L)-W_k(\theta,M)$ shows that $L\succ_r^{W,\theta}M\Rightarrow L\succ_l^{W,\theta}M$. Therefore, the preference profile $M\succ_l^{W,\theta}L\succ_l^{W,\theta}R$ and $L\succ_r^{W,\theta}M\succ_r^{W,\theta}R$ cannot occur. Hence (15) can be simplified as follows:

$$\begin{aligned}&W_k(\theta,R)-W_k(\theta,M)\\ =\ &\int_{\substack{\zeta:M\succ_l^{W,\zeta}R\&R\succ_r^{W,\zeta}M\\ \text{or } R\succ_l^{W,\zeta}M\&M\succ_r^{W,\zeta}R}}\left[U_k(\zeta,R)-U_k(\zeta,M)+e^{-\rho\delta}(W_k(\theta,R)-W_k(\theta,M))\right]dP_\theta(\zeta)\\ =\ &\Omega_k(V(W(R)-W(M)),\theta).\end{aligned}$$

Together with Lemma 3, the above equation shows that $W(R)-W(M)$ must be an equilibrium value function of the 2−alternative model for $X=\{M,R\}$. The proof for $W(M)-W(L)$ is analogous. ∎

Proof of Propositon 5. Using the notation $\succ_k^{W,\theta}$ introduced in the proof of Proposition 4, for the $r-$monopolistic proposer protocol, the table analogous to (14) is as follows:

	$L \succ_r^{W,\theta} M \succ_r^{W,\theta} R$	$M \succ_r^{W,\theta} L \succ_r^{W,\theta} R$	$M \succ_r^{W,\theta} R \succ_r^{W,\theta} L$	$R \succ_r^{W,\theta} M \succ_r^{W,\theta} L$
$L \succ_l^{W,\theta} M \succ_l^{W,\theta} R$	(L, L, L)	(L, M, M)	(L, M, M)	(L, M, R)
$M \succ_l^{W,\theta} L \succ_l^{W,\theta} R$	(L, M, L)	(M, M, M)	(M, M, M)	(M, M, R)
$M \succ_l^{W,\theta} R \succ_l^{W,\theta} L$	(L, M, M)	(M, M, M)	(M, M, M)	(R, M, R)
$R \succ_l^{W,\theta} M \succ_l^{W,\theta} L$	(L, M, R)	(M, M, R)	(M, M, R)	(R, R, R)

(16)

A close inspection of (16) shows that the bargaining outcome depends on whether the status quo is M or L if and only if $\left(\succ_l^{W,\theta}, \succ_r^{W,\theta}\right)$ are such that one of the following three configurations arises: (i) $L \succ_l^{W,\theta} M$ and $M \succ_r^{W,\theta} L$, (ii) $M \succ_l^{W,\theta} L$ and $L \succ_r^{W,\theta} M$, or (iii) $M \succ_l^{W,\theta} R \succ_l^{W,\theta} L$ and $R \succ_r^{W,\theta} M \succ_r^{W,\theta} L$. In cases (i) and (ii), the status quo stays in place. In case (iii), the outcome is M for $q = M$, and L for $q = R$ (case (iii) is the configuration whose anticipation can lead to the moderating effect, see section 4.1). Therefore, for all $\theta \in \Theta$,

$$\begin{aligned}
& W_k(\theta, M) - W_k(\theta, L) \qquad (17) \\
= & \int_{\substack{\zeta : L \succ_l^{W,\zeta} M \& M \succ_r^{W,\zeta} L \\ \text{or } M \succ_l^{W,\zeta} L \& L \succ_r^{W,\zeta} M}} \left[U_k(\zeta, M) - U_k(\zeta, L) + e^{-\rho\delta}\left(W_k(\zeta, M) - W_k(\zeta, L)\right)\right] dP_\theta(\zeta) \\
& + \int_{\zeta : M \succ_l^{W,\zeta} R \succ_l^{W,\zeta} L \& R \succ_r^{W,\zeta} M \succ_r^{W,\zeta} L} \left[U_k(\zeta, M) - U_k(\zeta, R) + e^{-\rho\delta}\left(W_k(\zeta, M) - W_k(\zeta, R)\right)\right] dP_\theta(\zeta).
\end{aligned}$$

By definition of $\succ_k^{W,\theta}$, the integrand of the second term on the right-hand-side of (17) is positive on its integration set for $k = l$, and negative for $k = r$, and the first term on the right-hand-side of (17) is equal to $\Omega_k(V(W(M) - W(L)), \theta)$. Therefore, (17) implies that

$$\Omega(V(W(M) - W(L)))\,(\leq, \geq)\, W(M) - W(L).$$

Let $\Pi^{L,M} = \left\{f \in \mathcal{F}^2 : f\,(\leq, \geq)\, W(M) - W(L)\right\}$. The above inequality together with Lemma 2 shows that $\Omega\left(\Pi^{L,M}\right) \subset \Pi^{L,M}$. Since $\Pi^{L,M}$ is a complete lattice, Tarski's fixed point theorem implies that Ω has a fixed point $W^{L,M}$ in $\Pi^{L,M}$. By construction, $W^{L,M}\,(\leq, \geq)\, W(M) - W(L)$, and from lemma 3, $W^{L,M}$ is an equilibrium continuation value of the $2-$alternative model with $X = \{L, M\}$, as needed.

A close inspection of (14) shows that the impact of changing the status quo from M to R has the same effect on the bargaining outcome under the $r-$monopolistic proposer as under the moderate proposer protocol. Therefore, the same argument as in the proof of Proposition 4 shows that for all $\theta \in \Theta$,

$$\begin{aligned} W_k(\theta, R) - W_k(\theta, M) \quad = \quad & \Omega_k(V(W(R) - W(M)), \theta) \qquad (18) \\ & + \int_{\substack{\zeta: M \succ_l^{W,\zeta} L \succ_r^{W,\zeta} R \\ \& L \succ_l^{W,\zeta} M \succ_r^{W,\zeta} R}} \left[U_k(\zeta, L) - U_k(\zeta, M) + e^{-\rho\delta}(W_k(\theta, L) - W_k(\theta, M)) \right] dP_\theta(\zeta). \end{aligned}$$

and that the integrand on the right-hand side of (18) is positive on its integration set for $k = r$, and negative for $k = l$. Therefore, (18) implies that

$$W(R) - W(M)(\leq, \geq)\, \Omega(V(W(R) - W(M))).$$

Let $\Pi^{M,R} = \left\{ f \in \mathcal{F}^2 : W(R) - W(M)(\leq, \geq) f \right\}$. The above inequality together with Lemma 2 shows that $\Omega\left(\Pi^{M,R}\right) \subset \Pi^{M,R}$. Since $\Pi^{M,R}$ is a complete lattice, Ω has a fixed point $W^{M,R}$ in $\Pi^{M,R}$. By construction, $W(R) - W(M)(\leq, \geq) W^{M,R}$, and from lemma 3, $W^{M,R}$ is the equilibrium continuation value of the $2-$alternative model with $X = \{L, M\}$, as needed. ■

Proof of Proposition 6. The proof proceeds in steps as follows. In steps 1-3, we show that when δ is small and players are sufficiently patient, then the intertemporal preferences of the players are single-peaked (that is, all strategy profiles are regular). Steps 4, 5, and 6 establish that if x_n is selected at t, then at $(t + \delta)$ and at $(t + 2\delta)$ it can be replaced only by x_{n-1} or only by x_{n+1}. Step 7 shows that in certain states the equilibrium value function coincides with the value function from a two-alternative model. Step 8 completes the proof.

Let K be the Lipschitz constant for u. Let $\left(\mathcal{F}^2\right)^X$ denote a set of functions mapping X into $\mathcal{F}^2$, where $\mathcal{F}$ is defined as in Section 6.1. For all $f \in \left(\mathcal{F}^2\right)^X$ and all $\theta \in \Theta$, we define an intertemporal preference profile $\succ^{f,\theta}$as in the proof of Proposition 4.

Step 1: There exists $\bar{\delta} > 0$ such that for all $\delta \leq \bar{\delta}$, all $\theta \in \Theta$, and all $x, y, z \in X$, if $x < y < z$

or $x > y > z$, then

$$\begin{cases} (i)\ \ u_k(\theta, x) \geq u_k(\theta, y) - 2\delta \Rightarrow u_k(\theta, x) > u_k(\theta, z) + (4K+2)\,\delta, \\ (ii)\ \ u_k(\theta, x) \geq u_k(\theta, z) - 2\delta \Rightarrow u_k(\theta, y) > u_k(\theta, z) + (4K+2)\,\delta. \end{cases}$$

Proof: Suppose (i) is false. Then there exists θ^n, x^n, y^n, z^n and $\delta^n \to 0$ such that

$$u_k(\theta, x^n) \leq u_k(\theta, z^n) + (4K+2)\,\delta^n \text{ and } u_k(\theta, x^n) \geq u_k(\theta, y^n) - 2\delta^n.$$

Since X is finite and Θ is compact, we can assume that x^n, y^n, and z^n are constant and $\theta^n \to \theta$. Taking the limit in the above inequality, we have that

$$u_k(\theta, x) \leq u_k(\theta, z) \text{ and } u_k(\theta, x) \geq u_k(\theta, y)\,,$$

which is impossible since $u_k(\theta, .)$ is strictly concave.

We show (ii) by contraposition. Suppose $u_k(\theta, y) \leq u_k(\theta, z) + (4K+2)\,\delta$, then $u_k(\theta, z) - u_k(\theta, y) \geq -2\delta$. From point (i), this implies that $u_k(\theta, z) - u_k(\theta, x) > (4K+2)\,\delta$, so $u_k(\theta, z) - u_k(\theta, x) > 2\delta$, as needed.

Step 2: For all $\delta < \bar{\delta}$, there exists $\bar{\rho}$ such that for all $\rho \geq \bar{\rho}$, all $k \in \{l, r\}$, all $f_k \in (\mathcal{F})^X$, all $\theta \in \Theta$, and all $x \in X$,

$$\left| U_k(\theta, x) + e^{-\rho\delta} f_k(\theta, x) - u_k(\theta, x) \right| < \delta.$$

Proof: Using the definition of $U_k(\theta, x)$, the Lipschitz continuity of $u_k(\theta(s), x)$ in s, and the boundedness of $f_k(\theta, x)$ and $u_k(\theta, x)$ by m, we obtain that

$$\begin{aligned} & \left| U_k(\theta, x) + e^{-\rho\delta} f_k(\theta, x) - u_k(\theta, x) \right| \\ = & \left| E_{\theta(0)=\theta} \left[\int_0^\delta \left(u_k(\theta(t), x) - u_k(\theta, x) \right) \rho e^{-\rho t} dt \right] + e^{-\rho\delta} \left(f_k(\theta, x) - u_k(\theta, x) \right) \right| \\ \leq & \int_0^\infty K t \rho e^{-\rho t} dt + e^{-\rho\delta} 2m = \frac{K}{\rho} + e^{-\rho\delta} m \to_{\rho\to\infty} 0. \end{aligned}$$

Step 3: (All strategy profiles are regular, i.e., $\succ_k^{f,\theta}$ is single peaked) For all $\delta > 0$, all $\rho \geq \bar{\rho}$, all $f \in \left(\mathcal{F}^2\right)^X$, all $k \in \{l, r\}$, all $\theta \in \Theta$, all $x, y, z \in X$ such that $x < y < z$ or $x > y > z$,

$$x \succ_k^{f,\theta} y \Rightarrow x \succ_k^{f,\theta} z.$$

Proof: Suppose $x \succ_k^{f,\theta} y$. From step 2, this implies that $u_k(\theta, x) > u_k(\theta, y) - 2\delta$, and then step 1 (i) implies that $u_k(\theta, x) > u_k(\theta, z) + 2\delta$. Using step 2 again, this implies that $x \succ_k^{f,\theta} z$.

In what follows, we fix $\delta \leq \bar{\delta}$ and $\rho \geq \bar{\rho}$, and W denotes an equilibrium profile of continuation values for some bargaining protocol for these parameters.

Step 4: For all $f, g \in \left(\mathcal{F}^2\right)^X$, all $k \in \{l, r\}$, all $\theta \in \Theta$, and all $x, y, z \in X$ such that $x < y < z$ or $x > y > z$, with probability 1,

$$\begin{cases} (i) \;\; x \succ_k^{f,\theta(t)} y \Rightarrow x \succ_k^{g,\theta(t+\delta)} z, \\ (ii) \;\; x \succ_k^{f,\theta(t)} z \Rightarrow y \succ_k^{g,\theta(t+\delta)} z. \end{cases}$$

Proof: As shown in the proof of step 3, $x \succ_k^{f,\theta(t)} y$ implies that $u_k(\theta(t), x) \geq u_k(\theta(t), z) + 2K\delta + 2\delta$. Since $u_k(\theta(s), w)$ is $K-$Lipschitz continuous in s with probability 1 for all $w \in X$, the latter inequality implies in turn that $u_k(\theta(t+\delta), x) \geq u_k(\theta(t+\delta), z) + 2\delta$. The implication (i) follows then from step 2. The implication (ii) is established in the same way by using step 1 (ii) instead of 1 (i).

For all $n = 1, ..., N-1$, and all $f \in \left(\mathcal{F}^2\right)^X$, let Θ_f^n be the set of states such that if $\theta(t) \in \Theta_f^n$ then with probability 1, at $t + \delta$, x_n can be $\succ^{f,\theta(t+\delta)}$ $-$Pareto dominated only by x_{n+1}, and x_{n+1} can be $\succ^{f,\theta(t+\delta)}$ $-$Pareto dominated only by x_n.

Step 5: If $x_n \succ_k^{f,\theta} x_{n+1}$ and $x_{n+1} \succ_{k'}^{f,\theta} x_n$ for some $k, k' \in \{l, r\}$, then $\theta \in \Theta_f^n$.

Proof: Using step 4 (i) (with $x = x_n, y = x_{n+1}$ and $f = g$), $x_n \succ_k^{f,\theta(t)} x_{n+1}$ implies that $x_n \succ_k^{f,\theta(t+\delta)} z$ for all $z > x_{n+1}$, and by the single-peakendess of $\succ_k^{f,\theta t}$ (step 3), $x_{n+1} \succ_k^{f,\theta(t+\delta)} z$ for all such z. This shows that x_n and x_{n+1} cannot be Pareto dominated by $z > x_{n+1}$. A similar argument shows that x_n and x_{n+1} cannot be Pareto dominated by $z < x_n$.

Step 6: For all $t \geq 0$ and all $n \in \{1, ..., N\}$, if in equilibrium $q(t+\delta) = x_n$, then with probability 1, one of the following happens: (i) only $x_{n+1} \succ^{W,\theta(t+\delta)}$ $-$Pareto dominates x_n and $\theta(t+\delta) \in \Theta_W^n$, (ii) only $x_{n-1} \succ^{W,\theta(t+\delta)}$ $-$Pareto dominates x_n and $\theta(t+\delta) \in \Theta_W^{n-1}$, and (iii) x_n

is $\succ^{W,\theta(t+\delta)}$ $-$Pareto optimal and $\theta(t+\delta) \in \Theta_W^n \cup \Theta_W^{n-1}$.

Proof: In equilibrium, $q(t+\delta) = x_n$ only if $x(t) = x_n$, so x_n must be $\succ^{W,\theta(t)}$ $-$Pareto optimal: $x_n \succ_k^{W,\theta(t)} x_{n+1}$ and $x_n \succ_{k'}^{W,\theta(t)} x_{n-1}$ for some $k, k' \in \{l, r\}$. Step 4 (i) implies then that with probability 1, $x_n \succ_k^{W,\theta(t+\delta)} z$ for all $z < x_{n-1}$, and $x_n \succ_{k'}^{W,\theta(t+\delta)} z$ for all $z > x_{n+1}$, so only x_{n-1} or x_{n+1} can $\succ^{W,\theta(t+\delta)}$ $-$Pareto dominate x_n. From step 3, $\succ_k^{W,\theta(t+\delta)}$ is strictly single-peaked, so x_n cannot be $\succ^{W,\theta(t+\delta)}$ $-$Pareto dominated by x_{n-1} and x_{n+1} at the same time. Suppose then that x_{n+1} does. By the preceding argument applied to x_{n+1}, we know that at $t+2\delta$, x_{n+1} can be only $\succ^{W,\theta(t+2\delta)}$ $-$Pareto dominated by x_n or by x_{n+2}. Hence, to show that $\theta(t+\delta) \in \Theta_W^n$, it remains to show that x_{n+1} is not $\succ^{W,\theta(t+2\delta)}$ $-$Pareto dominated by x_{n+2}. To see this, observe that since x_n is $\succ^{W,\theta(t)}$ $-$Pareto optimal, $x_n \succ_k^{W,\theta(t)} x_{n+1}$ for some k, so from step 2, $u_k(\theta(t), x_n) > u_k(\theta(t), x_{n+1}) - 2\delta$. Step 1 implies then that $u_k(\theta(t), x_n) > u_k(\theta(t), x_{n+2}) + (4K+2)\delta$, and therefore, $u_k(\theta(t+2\delta), x_n) > u_k(\theta(t+2\delta), x_{n+2}) + 2\delta$. Step 2 implies then that $x_n \succ_k^{W,\theta(t+2\delta)} x_{n+2}$, as needed. A similar argument shows that if x_{n-1} $\succ^{W,\theta(t+\delta)}$ $-$Pareto dominates x_n, then case (ii) holds. If neither x_{n+1} nor x_{n-1} $\succ^{W,\theta(t+\delta)}$ $-$Pareto dominates x_n, then case (iii) follows directly from step 5.

Step 7: For all $n \in \{1, ..., N-1\}$, let V^n the mapping defined in (7) for $U^n(\theta) \doteq U_k(\theta, x_{n+1}) - U_k(\theta, x_n)$. There exists $W^n \in \mathcal{F}^2$ satisfying $W^n = \Omega(V^n(W^n))$ (hence, from Lemma 3, W^n is an equilibrium of a 2$-$alternative model with $X = \{x_n, x_{n+1}\}$) such that for all $\theta \in \Theta_W^n$, $W(\theta, x_{n+1}) - W(\theta, x_n) = W^n(\theta)$.

Proof: Suppose $\theta(t) \in \Theta_W^n$ and consider player k comparing her continuation value from $q(t+\delta) = x_n$ and $q(t+\delta) = x_{n+1}$. By definition of Θ_W^n, either of this alternatives can be $\succ^{W,\theta(t+\delta)}$ $-$Pareto dominated only by x_{n+1} or x_n. Hence, at $t+\delta$ either both players agree on how to rank x_n and x_{n+1} and the identity of $q(t+\delta)$ does not matter, or $q(t+\delta)$ stays in place. The latter happens when $x_n \succ_k^{W,\theta(t+\delta)} x_{n+1}$ and $x_{n+1} \succ_{k'}^{W,\theta(t+\delta)} x_n$ for $k \neq k'$. Therefore, for all $k \in \{l, r\}$ and all $\theta \in \Theta_W^n$,

$$W_k(\theta, x_{n+1}) - W_k(\theta, x_n) = \int_{\substack{\zeta \in \Theta : x_n \succ_l^{W,\zeta} x_{n+1} \& x_{n+1} \succ_r^{W,\zeta} x_n \\ \text{or } x_{n+1} \succ_l^{W,\zeta} x_n \& x_n \succ_r^{W,\zeta} x_{n+1}}} \left[U_k^n(\zeta) + e^{-\rho\delta} \left(W_k(\zeta, x_{n+1}) - W_k(\zeta, x_n) \right) \right] dP_\theta(\zeta). \tag{19}$$

For all $k \in \{l, r\}$ and all $\theta \in \Theta$, define

$$W_k^n(\theta) \mathrel{\hat=} \int_{\substack{\zeta\in\Theta: x_n \succ_l^{W,\zeta} x_{n+1} \& x_{n+1} \succ_r^{W,\zeta} x_n \\ \text{or } x_{n+1}\succ_l^{W,\zeta} x_n \& x_n \succ_r^{W,\zeta} x_{n+1}}} \left[U_k^n(\zeta) + e^{-\rho\delta}\left(W_k(\zeta, x_{n+1}) - W_k(\zeta, x_n)\right)\right] dP_\theta(\zeta). \quad (20)$$

By construction of W^n, for all $\theta \in \Theta^n$, $W_k^n(\theta) = W_k(\theta, x_{n+1}) - W_k(\theta, x_n)$. From step 5, the domain of integration of the right-hand side of (20) is included in Θ^n, so for all $\theta \in \Theta$,

$$W_k^n(\theta) = \int_{\substack{\zeta\in\Theta: x_n \succ_l^{W,\zeta} x_{n+1} \& x_{n+1} \succ_r^{W,\zeta} x_n \\ \text{or } x_{n+1}\succ_l^{W,\zeta} x_n \& x_n \succ_r^{W,\zeta} x_{n+1}}} \left[U_k^n(\zeta) + e^{-\rho\delta} W_k^n(\zeta)\right] dP_\theta(\zeta).$$

To show that $W^n = \Omega(V^n(W^n))$ (recall that Ω is defined in 8), it remains to show that

$$\left\{\theta \in \Theta : x_n \succ_l^{W,\theta} x_{n+1} \& x_{n+1} \succ_r^{W,\theta} x_n \text{ or } x_{n+1} \succ_l^{W,\theta} x_n \& x_n \succ_r^{W,\theta} x_{n+1}\right\} = D(V^n(W^n)) \cup D'(V^n(W^n)), \quad (21)$$

where D and D' are defined in (6). By step 5, for any θ that belongs to one side of (21) we have $\theta \in \Theta_W^n$, and hence by what precedes, $W^n(\theta) = W(\theta, x_{n+1}) - W(\theta, x_n)$, which in turn implies that θ belongs to the other side.

Step 8: Let $Z \in (\mathcal{F}^2)^X$ be such that for all $n \in \{1, ..., N-1\}$, all $k \in \{l, r\}$, and all $\theta \in \Theta$, $Z(\theta, x_{n+1}) - Z(\theta, x_n) = W^n(\theta)$ (where W^n is as defined in step 7, so Z is a concatenation of 2−alternative equilibria). Then for all $t > \delta$, the equilibrium outcome $x(t)$ is the same as if players vote using continuation values Z.

Proof: Suppose that $x(t) = x_n = q(t+\delta)$ for some $n \in \{1, ..., N\}$. From step 6, there are only three possible cases, labelled A, B, and C below.

Case A: x_{n+1} is the only alternative that $\succ^{W,\theta(t+\delta)}$ −Pareto dominates x_n, and $\theta(t+\delta) \in \Theta_W^n$. So from Step 7, $W^n(\theta) = W(\theta, x_{n+1}) - W(\theta, x_n)$. Then by definition of Z, x_{n+1} also $\succ^{Z,\theta(t+\delta)}$ −Pareto dominates x_n. Step 3 implies then that x_n is not $\succ^{Z,\theta(t+\delta)}$ −Pareto dominated by any $z < x_n$. Since the equilibrium outcome in t was x_n, x_n is not $\succ^{W,\theta(t)}$ −Pareto dominated by x_{n+1}, so from step 4, x_n is not $\succ^{Z,\theta(t+\delta)}$ −Pareto dominated by any $z > x_{n+1}$. Hence, we have shown that x_{n+1} is also the only alternative that $\succ^{Z,\theta(t+\delta)}$ −Pareto dominates x_n, so x_{n+1} is also the bargaining outcome in $t+\delta$ if players vote as if their continuation value is Z.

Case B: x_{n-1} is the only alternative that $\succ^{W,\theta(t+\delta)}$ −Pareto dominates x_n, and $\theta(t+\delta) \in \Theta_W^n$.

The proof follows the same steps as in Case A.

Case C: x_n is $\succ^{W,\theta(t+\delta)}$ $-$Pareto undominated, and $\theta(t+\delta) \in \Theta_W^n \cup \Theta_W^{n-1}$. So from Step 7, $W^n(\theta) = W(\theta, x_{n+1}) - W(\theta, x_n)$ and $W^{n-1}(\theta) = W(\theta, x_n) - W(\theta, x_{n-1})$. Then by definition of Z, we have that x_n also $\succ^{Z,\theta(t+\delta)}$ $-$Pareto dominates x_{n+1} and x_{n-1}. Since $\succ^{Z,\theta(t+\delta)}$ is single-peaked (step 3), x_n must be $\succ^{Z,\theta(t+\delta)}$ $-$Pareto optimal, and hence x_n is also the bargaining outcome in $t+\delta$ if players vote as if their continuation value is Z. ■

Proof of Proposition 7. Suppose $(\phi_n)_{n\in\mathbb{N}}$ converges at (θ, q) to x. Then by definition, there exists a neighborhood B of θ such that for n sufficiently large, for all $\zeta \in B$, $\phi_n(\zeta, q) = x$. Using the equilibrium requirement that x replaces q only if it yields a greater continuation payoff to all players and the definition of $W_k^{\delta_n}$, we have that for all $k \in P$,

$$\begin{aligned} U_k^{\delta_n}(\theta, x) - U_k^{\delta_n}(\theta, q) &\geq -e^{-\rho\delta_n}\left(W_k^{\delta_n}(\theta, x) - W_k^{\delta_n}(\theta, q)\right) \qquad (22) \\ &= -e^{-\rho\delta_n}\int_{\zeta\in\Theta}\left(\begin{array}{c} U_k^{\delta_n}(\zeta, \phi(\zeta, x)) + e^{-\rho\delta_n}W_k^{\delta_n}(\theta, \phi_n(\zeta, x)) \\ -\left(U_k^{\delta_n}(\zeta, \phi(\zeta, q)) + e^{-\rho\delta_n}W_k^{\delta_n}(\theta, \phi_n(\zeta, q))\right)\end{array}\right) dP_\theta(\zeta). \end{aligned}$$

Since $\phi(\theta, q) = x$, the equilibrium requirement that $\phi(\theta, \phi(\theta, q)) = \phi(\theta, q)$ implies that for all $\zeta \in B$, $\phi_n(\zeta, x) = x = \phi_n(\zeta, q)$. Substituting the latter equality in (22), we obtain

$$U_k^{\delta_n}(\theta, x) - U_k^{\delta_n}(\theta, q) \geq -e^{-\rho\delta_n}\int_{\zeta\in\Theta\setminus B}\left(\begin{array}{c} U_k^{\delta_n}(\zeta, \phi(\zeta, x)) + e^{-\rho\delta_n}W_k^{\delta_n}(\theta, \phi_n(\zeta, x)) \\ -\left(U_k^{\delta_n}(\zeta, \phi(\zeta, q)) + e^{-\rho\delta_n}W_k^{\delta_n}(\theta, \phi_n(\zeta, q))\right)\end{array}\right) dP_\theta(\zeta).$$

Since flow payoffs are bounded by m, the absolute value of the integrand in the right-hand side of the above inequality is bounded by $2m$. Therefore, the above inequality implies that

$$\frac{U_k^{\delta_n}(\theta, x) - U_k^{\delta_n}(\theta, q)}{1 - e^{-\rho\delta_n}} \geq -\frac{e^{-\rho\delta_n}}{1 - e^{-\rho\delta_n}} 2m \Pr(\theta(\delta_n) \notin B | \theta(0) = 0).$$

Under our assumption, the left-hand side of the above inequality tends to $\hat{u}_k(\theta, x) - \hat{u}_k(\theta, q)$. Moreover, $\Pr(\theta(\delta) \notin B | \theta(0) = 0) = o(\delta_n)$ and $\frac{e^{-\rho\delta_n}}{1-e^{-\rho\delta_n}} \sim \frac{1}{\rho\delta_n}$, so the right-hand side of the above inequality tends to 0. ■

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