

# A New Look at Uncertainty Shocks: Imperfect Information and Misallocation

Tatsuro Senga\*

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Abstract

Uncertainty faced by individual firms appears to be heterogeneous. In this paper, I construct new empirical measures of firm-level uncertainty using data from the I/B/E/S and Compustat. These new measures reveal persistent differences in the degree of uncertainty facing individual firms not reflected by existing measures. Consistent with existing measures, I find that the average level of uncertainty across firms is countercyclical, and that it rose sharply at the start of the Great Recession. I next develop a heterogeneous firm model with Bayesian learning and uncertainty shocks to study the aggregate implications of my new empirical findings. My model establishes a close link between the rise in firms' uncertainty at the start of a recession and the slow pace of subsequent recovery. These results are obtained in an environment that embeds Jovanovic's (1982) model of learning in a setting where each firm gradually learns about its own productivity, and each occasionally experiences a shock forcing it to start learning afresh. Firms differ in their information; more informed firms have lower posterior variances in beliefs. An uncertainty shock is a rise in the probability that any given firm will lose its information. When calibrated to reproduce the level and cyclicity of my leading measure of firm-level uncertainty, the model generates a prolonged recession followed by anemic recovery in response to an uncertainty shock. When confronted with a rise in firm-level uncertainty consistent with advent of the Great Recession, it explains 79 percent of the observed decline in GDP and 89 percent of the fall in investment.

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\* Please find the latest version at <http://www.tatsuro-senga.net/uploads/3/4/5/5/3455091/uim.pdf>

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# 1 Introduction

*“Subjective uncertainty is about the “unknown unknowns”. When, as today, the unknown unknowns dominate, and the economic environment is so complex as to appear nearly incomprehensible, the result is extreme prudence, [...], on the part of investors, consumers and firms.”* Olivier Blanchard (2012)

How large is the role of increased uncertainty in driving economic downturns? Is there a link between a rise in firm-level uncertainty and the subsequent pace of economic recovery? To explore these questions, I construct new empirical measures of firm-level uncertainty, and I show that the degree of uncertainty facing individual firms is heterogeneous and the average level of uncertainty, as well as its dispersion, across firms is countercyclical. To account for these regularities, I develop a heterogeneous firm model that incorporates Bayesian learning at the firm level with uncertainty shocks. My model can explain my new empirical findings well. A calibrated version of my model produces 79% of the observed decline in GDP and 89% of the drop in investment seen in the last U.S. recession. In addition, the model predicts that such a sharp economic downturn is followed by a slow recovery. The half-life of the impulse response of output is 6 years.

A defining feature of this paper is that the uncertainty faced by firms not only varies over time but also varies across firms. That is, the conditional variance of idiosyncratic shocks is heterogeneous among firms. To study the implications of time-varying uncertainty in this environment, I adopt a dynamic Bayesian inference approach. The more common approach in the uncertainty shock literature, following the seminal work of Bloom (2009), has been to study stochastic volatility models. I break with this tradition primarily because stochastic volatility models cannot deliver the heterogeneous uncertainty evident in the microeconomic data.<sup>1</sup> By contrast, heterogeneous uncertainty naturally arises in my model, where I integrate Jovanovic’s (1982) model of learning into an otherwise standard heterogeneous firm business cycle framework. Firms are heterogeneous in both productivity and their confidence about that productivity; more informed firms have lower posterior variances of their beliefs. Bayesian learning implies two different firms can have the same posterior mean while differ-

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<sup>1</sup>In stochastic volatility models, there is full information and all agents know the true distribution of shocks that they face, including its volatility, which varies over time. In uncertain times, the volatility that every agent faces rises equally.

ing in their posterior variances. Hence, uncertainty differs across firms. A second appealing feature of the model is the fact that the recession in response to an uncertainty shock is not followed by a sharp recovery, as happens in existing stochastic-volatility based uncertainty shock models.<sup>2</sup> Instead, Bayesian learning with heterogeneous uncertainty drives a slow economic recovery as firms gradually regain information and confidence. Moreover, these results requires no additional rigidity or frictions. In the absence of labor and capital adjustment costs, uncertainty shocks still cause recessions.<sup>3</sup>

The new set of empirical measures of firm-level uncertainty are based on a panel dataset constructed from the Institutional Brokers' Estimate System (I/B/E/S) and Compustat. By merging these two datasets, I construct an annual panel of firms' ex-ante earnings forecasts by market analysts and ex-post-realized forecast errors. Appealing features of the dataset include: (1) it is disaggregated at the firm level, thereby allowing me to examine the cross-sectional characteristics of firm-level uncertainty, (2) it contains ex-ante information on earnings forecasts, which is arguably better suited than ex-post information for gauging the degree of uncertainty individual firms face, and (3) the result obtained can be fairly directly mapped into a neoclassical model. In particular, I transform earnings data into return on capital data and use the latter to calibrate the model.

The new firm-level measures of uncertainty uncover the following new facts. First, the degree of uncertainty surrounding individual firms differs across firms; for example, Apple's measure of uncertainty was much lower than Ford's during the Great Recession in 2009, and vice versa during the dot-com recession in 2001.<sup>4</sup> Second, the first and second moments of the distribution of firm-level uncertainty measures are countercyclical. Specifically, the median, mean, and cross-sectional dispersion are all negatively correlated with GDP growth rates. Third, these measures of uncertainty are positively correlated with other measures of uncertainty that are commonly used in the literature, including stock price volatility-based and balance sheet-based measures.

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<sup>2</sup>See, for example, Bloom (2009), Bloom et al. (2014), and Bachmann and Bayer (2013). See also the discussion of Bachmann et al. (2013).

<sup>3</sup>The large body of literature about the relation between uncertainty and investment studies the real options effect in models with adjustment costs, as in Bertola and Caballero (1994), Dixit and Pindyck (1994), Abel and Eberly (1996) and Caballero and Engel (1999).

<sup>4</sup>This can be seen in figure A1, A2 and A3, which present uncertainty measures constructed from stock price volatilities and EPS forecasts of market analysts.

In light of the evidence above, I propose a new model that features heterogeneous uncertainty, and I study its role in propagating aggregate shocks. My model builds on a standard heterogeneous firm business cycle model, but I deviate from the standard model in three ways. First, idiosyncratic productivity has two components: an i.i.d. transitory component around a base component. These components cannot be observed separately, and therefore each firm must learn the true value of its base component in a Bayesian way.<sup>5</sup> Second, the base component is randomly reset. When this occurs, a new base productivity is drawn from a common distribution known to all firms. Though a firm knows when its base has been reset, it does not know its new realization; thus it must restart the process of learning its value. Otherwise, the firm maintains its current base component and continues its learning. In this way, I integrate learning into a model of heterogeneous firms that are subject to persistent shocks to idiosyncratic productivity, as in Hopenhayn (1992). Third, I assume that the common reset probability of firms' base productivity components is a stochastic, two-state Markov process. When the reset probability is high, many firms draw new base productivities, which leads to a larger variance of TFP growth rates across the distribution of firms. Thus, an uncertainty shock is associated with a rise in the variance of firm-level TFP growth rates. In this way, this model is consistent with an important empirical observation documented in previous work regarding uncertainty shocks (Bloom et al. (2014)). The rise in the reset probability also implies that many firms lose information and restart learning. This additional effect increases the population share of firms that have large conditional variance of idiosyncratic shocks.

My main findings are as follow. First, the model produces rapid downturns and slow recoveries in aggregate variables following uncertainty shocks. Second, the inclusion of uncertainty shocks alongside conventional aggregate productivity shocks allows the model to reproduce the negative correlation between hours and labor productivity, consistent with data.<sup>6</sup> Third, aggregate productivity shocks deliver responses quite similar to those in conventional equilibrium business cycle models, and these shocks remain an important source of fluctuations in my model. For this reason, the model delivers familiar second moments for

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<sup>5</sup>Bernanke (1983) develops a single-firm, partial equilibrium model with dynamic Bayesian inference specifications to study short-term fluctuations of irreversible investment under time-varying option values.

<sup>6</sup>Takahashi (2014) reproduces the negative correlation between hours worked and average labor productivity with a heterogeneous households model. Empirical evidence on the negative correlation between hours and average labor productivity is documented by Ohanian and Raffo (2011).

the cyclical component of aggregate quantities.

The recession following an uncertainty shock in my model stems from two effects, one uncertainty and one distributional. The uncertainty effect arises as all firms anticipate a higher reset probability, implying an increased likelihood of large changes in their productivities. Given one-period time to build capital, this leads them to change their target levels of capital. Firms that believe their current base component is higher than the unconditional mean reduce their capital targets. On the other hand, firms that believe their current base productivity is low relative to the unconditional mean raise their capital targets. Given that the distribution of the base component of productivity is symmetric and the production function has decreasing returns to scale, the net impact on aggregate investment is negative. This effect is immediately reversed when the shock ends, which on its own would deliver a quick expansion led by pent-up investment demand. However, this fails to spur aggregate investment because of the offsetting impact of the distributional effect. As an unusually large number of firms experience a reset of their base components, the economy becomes increasingly populated by uninformed firms as the uncertainty shock persists. These firms, having lost their information, must restart their learning. In early stages of the learning process, each firm puts more weight on the prior mean rather than the mean of its observations; the posterior variance is large. Unless a firm is fully informed about its base component, its capital stock is either excessive or insufficient relative to the full information efficient levels consistent with the true value of its base component and the interest rate. Thus, there is a misallocation of resources arising from over- or undercapacity. In particular, since uninformed firms are cautious and have low confidence, while their population share rises over an uncertainty driven recession, aggregate investment, employment, and GDP fall. This cannot be quickly reversed when the uncertainty shock ends, as it takes time for the distribution of firms to recover their knowledge about their productivity. Thus, the negative impact of an uncertainty shock persists beyond the shock itself.

At the start of a recession, the uncertainty and distributional effects reinforce each other, and this leads to a rapid drop in investment, GDP and other aggregate variables. However, in the recovery phase, the uncertainty and distributional effects offset each other. Their relative strengths must be quantitatively assessed. In my calibrated model, the distributional effect dominates. This leads to a sluggish recovery, a finding that stands in sharp contrast to other models in this literature.

I explore the U.S. Great Recession using my model economy. When I choose the size of two aggregate shocks to replicate the rise in the measure of uncertainty as well as the decline in measured total factor productivity, I find that the model explains 79% of the decline in GDP and 89% of the fall in investment. A massive and rapid drop in aggregates is followed by a slow recovery; the half-life of the impulse response of output is 6 years.

**Related Literature** The idea that links uncertainty to business cycles and especially to the slow rate of recovery after slumps, dates back to Keynes (1936) and has been formulated by Bernanke (1983) in partial equilibrium.<sup>7</sup> In the recent equilibrium business cycle literature, the seminal contribution of Bloom (2009) studies a business cycle model where individual firms face time-varying volatility shocks to their own productivities. He shows that uncertainty shocks, defined as a shock to the variance of the idiosyncratic productivity process, generate bust-boom cycles. A rise in stochastic volatility, in a setting where firms face nonlinear costs of factor adjustment, deters investment as firms adopt a “wait and see” policy to the shock. In this class of models with exogenous shocks to volatility, the aggregate effects tend to be short-lived.<sup>8</sup> However, Bachmann et al. (2013) argue that the quick recovery following the “wait-and-see” effect is not consistent with U.S. data. In particular, they document persistent and prolonged dynamics following a rise in their measure of uncertainty. I contribute to this literature by developing a tight link between uncertainty at the start of a recession and the gradualism of subsequent recovery.

In recent years, there has been an increased interest in uncertainty and learning over the business cycle.<sup>9</sup> For example, Fajgelbaum et al. (2013) show a mechanism by which recessions increase uncertainty in a model of irreversible investment. Saijo (2013) builds a model with nominal rigidities and proposes a mechanism for endogenous fluctuations in uncertainty.

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<sup>7</sup>As stated in The General Theory, Ch. 22, “*it might be possible to achieve a recovery without the elapse of any considerable interval of time [...]. But, in fact, this is not usually the case [...]. It is the return of confidence, to speak in ordinary language, which is so insusceptible to control in an economy of individualistic capitalism. This is the aspect of the slump which bankers and business men have been right in emphasizing...*”

<sup>8</sup>See also Gilchrist, Sim, and Zakrajsek (2010), Arellano, Bai, and Kehoe (2012), Christiano, Motto, Rostagno (2010), and Fernandez-Villaverde et al. (2013), and Vavra (2014), among others.

<sup>9</sup>There are papers that examine economic environments where agents learn from market outcomes. For example, Amador and Weill (2010), Van Nieuwerburgh and Veldkamp (2006), and Caplin and Leahy (1993) study the relation between the flow of information and economic activity in models without uncertainty shocks.

Both papers analyze fluctuations in the amount of information available to agents. In recessions, economic activity contracts, and this reduces the flow of information and increases uncertainty. Neither this feedback nor real and nominal rigidities are necessary in my model for uncertainty shocks to produce recessions. Furthermore, unlike these papers, my model has time-varying distribution of firms, which is part of the aggregate state. Following uncertainty shocks, my model delivers endogenous fluctuations in TFP through changes in the misallocation of capital and labor, leading to a sluggish economic recovery in the presence of learning.

My paper is also related and complementary to existing papers that study the role of the allocation of resources across heterogeneous agents and its impact on aggregate productivity (e.g. Restuccia and Rogerson (2008)). Hsieh and Klenow (2009) argue that misallocation of resources have a substantial impact on aggregate TFP in India and China. More recently, the role of financial frictions generating capital misallocation and its aggregate implications has been studied in several quantitative environments (Khan and Thomas (2013), Buera and Moll (2013), Buera et al. (2011)). Instead of financial frictions, my paper studies the role of information frictions in causing a loss in aggregate productivity through the misallocation of resources. David et al. (2013) also study misallocation in a model of learning at the firm level. However, my paper looks at the implications of misallocation over business cycles, while they focus on a stationary equilibrium.

I also contribute to the empirical literature on uncertainty. The literature has been developing several proxies, ranging from the volatility of GDP or stock prices to disagreement and forecast errors in survey data, as uncertainty is difficult to identify. For example, Leahy and Whited (1996) construct a measure of uncertainty from the volatility of stock returns for individual firms. Guiso and Parigi (1999) use survey data on demand forecasts by Italian firms to infer the level of uncertainty facing these individual firms. Bond et al. (2005) consider several measures including volatility in monthly consensus earnings forecasts, the variance of forecast errors for consensus forecasts, and the dispersion in earnings forecasts across market analysts. To estimate the impact of uncertainty on investment, they use panel data and look at the average cross-sectional distribution of firms over uncertainty and the investment behavior of individual firms, rather than cyclical properties as in Bloom et al. (2014), Kehrig (2011) and Vavra (2014). In this paper, I use data on earnings forecasts by individual analysts as in Bond (2005), however I examine not only the average cross-sectional distribution but

also the cyclical changes of this uncertainty measure. In line with Bachmann et al. (2013) who use survey data from the IFO Business Climate Survey, which asks forecasters about their own future prospects rather than about macroeconomic variables such as GDP, to extensively study various measures of uncertainty, I also use forecast disagreement to measure uncertainty.

My model builds on Jovanovic's (1982) learning model which has been applied to study a broad range of topics such as the disparate response of heterogeneous firms to aggregate shocks (Lee and Weinberg (2003) and Alti (2003)) and the differential sensitivity of product switching behavior among exporters learning about their demand (Timoshenko (2013)).

The rest of the paper is organized as follows. Section 2 reports empirical results. In Section 3, the model of heterogeneous firms with Bayesian learning is developed. Section 4 describes the calibration of this model which matches a variety of micro level moments as well as a set of aggregate moments commonly targeted in quantitative macroeconomic models. Section 5 presents my quantitative results, both in stationary equilibrium and in the presence of aggregate shocks. Section 6 examines my model's response following shocks designed to emulate the Great Recession. Section 7 concludes.

## **2 Empirical Facts**

In this section, I first build an annual panel data of firms' ex-ante earnings forecast dispersion and ex-post forecast errors using data from the I/B/E/S and Compustat. I then use this panel to construct new empirical measures of firm-level uncertainty. These new measures reveal persistent differences in the degree of uncertainty facing individual firms. Consistent with existing measures, these new measures show that the average level of uncertainty across firms is countercyclical. In particular, there was a sharp rise at the start of the Great Recession.

### **2.1 Forecast Dispersion, Forecast Errors and Uncertainty**

The uncertainty measures constructed here involve survey data. The data contains information about ex-ante forecast dispersion among forecasters, ex-post forecast errors, and



the cross-sectional dispersion of balance-sheet items.<sup>10</sup> Among many papers that use survey data to measure uncertainty, Zarnowitz and Lambros (1987) show a positive relationship between forecast dispersion and uncertainty using data from the U.S. Survey of Professional Forecasters. Using the Livingston survey, Bomberger and Fraser (1981) and Bomberger (1996) also find a positive association between forecast dispersion and uncertainty.<sup>11</sup> While these papers use data on disagreement among forecasters about macroeconomic variables, there are several papers that examine measures of uncertainty constructed from survey data on individual forecasters' future prospects. For example, Bachmann et al. (2013) use micro data from the IFO Business Climate Survey and show a strong positive correlation between forecast dispersion and forecast errors. I follow the literature in using earnings forecast dispersion among analysts as a proxy for uncertainty as in Johnson (2004), Bond et al. (2005), and Janunts (2010).

## 2.2 Data and Sample Selection

This paper uses two main data sources to construct a panel dataset of firms. First, I use the Institutional Brokers' Estimate System (I/B/E/S). The I/B/E/S dataset contains a point forecast of earnings per share (EPS) by an individual analyst, together with actual earnings records, dating back to 1976 for U.S. stocks. For each firm, a researcher can calculate the cross-analyst dispersion of forecasts about earnings at any given date. By comparing these forecasts with the actual earnings of each firm, one can also calculate forecast errors. This way, I can construct a panel data of firms that contains both ex-ante and ex-post measures of uncertainty. Second, I use Compustat data. By merging the I/B/E/S data with Compustat data, I add accounting fundamentals data for each firm. Since earnings data is

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<sup>10</sup>Ideal data would be managers' subjective distribution for future events. See Guiso and Parigi (1999) for cross-sectional survey data for Italian firms involving the subjective probability distribution of future demand.

<sup>11</sup>On the other hand, there are studies exploring an issue for using disagreement as a proxy for uncertainty. Rich and Tracy (2010) find little evidence in support of using disagreement for measuring uncertainty from their analysis of forecasts about inflation from the Survey of Professional Forecasters managed by the Federal Reserve Bank of Philadelphia. Boero, Smith and Wallis (2012) also argue that underlying macroeconomic conditions influence the usefulness of disagreement as a proxy using data from the Bank of England Survey of External Forecasters. While these papers examine disagreement among forecasters about aggregate variables such as GDP and inflation, my paper is different in using disagreement among forecasters for individual firms.

not directly mapped into my model, I implement a data transformation that can be used for the quantitative analysis in this paper. In particular, earnings data from the I/B/E/S data alongside capital stock data from the Compustat data allows me to construct a return on capital data set. The result is a panel containing forecast dispersion and forecast errors about the return on capital for individual firms.

### Forecast dispersion

Analyst  $j$  releases a point forecast of earnings per share for firm  $i$  during a year  $t$ ,  $feps_{ijt}$ . Earnings forecasts are transformed into return on capital forecasts by using data on the number of outstanding shares during year  $t$ ,  $sharenum_{it}$ , and capital stock data at the end of the previous year  $t - 1$ ,  $cap_{it-1}$ :

$$froc_{ijt} = \frac{(feps_{ijt} * sharenum_{it})}{cap_{it-1}}. \quad (1)$$

Having calculated the forecast dispersion in terms of the return on capital for each firm  $i$  during year  $t$ , I define the following ex-ante measure of uncertainty:

$$fdis_{it} = \frac{\text{standard deviation of } froc_{ijt}}{fmed_{it}}. \quad (2)$$

This is the cross-analyst standard deviation of return on capital forecasts divided by the median forecast,  $fmed_{it}$ .

### Forecast errors

With the median forecast and realized return on capital for each firm  $i$  during year  $t$ ,  $roc_{it}$ , I define forecast errors as follow.

$$ferror_{it} = roc_{it} - fmed_{it}. \quad (3)$$

## 2.3 Moments: forecast dispersion and actual performance

Table 1 reports basic statistics of the panel dataset. The sample is an unbalanced panel, which includes firms appearing for at least 30 years between 1976 and 2012, consisting of 10,466 firm-year observations across 302 firms.<sup>12</sup>

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<sup>12</sup>By including only firms that appear for 30 years, I eliminate cyclical frequency in sampling.

First, forecast dispersion varies across firms. This is evidence of heterogeneity in the extent of uncertainty faced by individual firms. Second, forecast errors also exhibit cross-sectional dispersion. Third, the return on capital is more serially correlated than the investment rate.<sup>13</sup> In the next section, I construct time series indices of firm-level uncertainty from this panel data and show the cyclical properties of these measures.

Table 1 - Basic Statistics of the Panel Dataset

|                     | Mean | Standard deviation | Serial correlation | 25%    | Median | 75%  |
|---------------------|------|--------------------|--------------------|--------|--------|------|
| Return on capital   | 0.19 | 0.51               | 0.29               | 0.06   | 0.12   | 0.23 |
| Investment rate     | 0.15 | 0.42               | 0.04               | 0.08   | 0.12   | 0.18 |
| Forecast dispersion | 0.04 | 0.53               | 0.15               | 0.002  | 0.004  | 0.01 |
| Forecast error      | 0.04 | 0.79               | 0.006              | 0.0004 | 0.003  | 0.02 |

Notes: The table above shows the cross-sectional moments of the panel dataset. Return on capital is calculated as earnings (= Street earnings per share (EPS) multiplied by the number of outstanding shares) divided by capital stock (= the sum of Property, Plant, and Equipment and inventory). Investment rate is defined as capital expenditure divided by capital stock. Forecast dispersion is the cross-analyst standard deviation of return on capital forecasts normalized by the median value of forecasts. Forecast error is calculated as the gap between realised return on capital and the median value of forecasts. The panel dataset is constructed by merging data from both the IBES and Compustat, resulting in an unbalanced panel that contains firms appearing for at least 30 years between 1975 and 2012, consisting of 10,466 data with 302 firms.

## 2.4 Cyclical properties of uncertainty at the firm level

To construct empirical measures of uncertainty from the panel dataset, I define the following variables for each year  $t$ : (1)  $roc\_sd$  as the standard deviation of  $roc_{it}$ , (2)  $fdis\_med$  as the median of  $fdis_{it}$ , (3)  $fdis\_mean$  as the mean of  $fdis_{it}$ , (4)  $fdis\_sd$  as the standard

<sup>13</sup>The moments of investment rates shown in the table is in line with what other studies on micro level investment find. See, for example, Cooper and Haltiwanger (2006) for plant-level investment moments.

deviation of  $fdis_{it}$ , (5)  $fdis\_iqr$  as the interquartile range of  $fdis_{it}$ , (6)  $ferror\_med$  as the median of  $ferror_{it}$ . Further, I take (7)  $dis\_BOS$  from Bachmann et al. (2013) and (8)  $sd\_TFP$  from Bloom et al. (2014). Specifically,  $dis\_BOS$  is the forecast disagreement index from Bachmann et al. (2013).  $sd\_TFP$  is the cross-sectional standard deviation of TFP shocks among U.S. establishments from Bloom et al. (2014).

Table 2 shows a correlation matrix between uncertainty measures, together with the GDP growth rates and the mean of return on capital across firms for each year,  $roc\_mean$ . First, the listed uncertainty measures, ( $roc\_sd$ ,  $fdis\_med$ ,  $fdis\_mean$ ,  $fdis\_sd$ ,  $fdis\_iqr$ , and  $ferror\_med$ ), are negatively correlated with GDP growth rates, ranging between -0.33 and -0.48. Second, the correlation between forecast dispersion-based measures ( $fdis\_med$ ,  $fdis\_mean$ ,  $fdis\_sd$ , and  $fdis\_iqr$ ) and  $ferror\_med$  is strongly positive, ranging from 0.40 to 0.59, consistent with Bachmann et al. (2013). My forecast-based uncertainty measures are also positively correlated with the commonly used existing uncertainty measures in the literature including  $dis\_BOS$  and  $sd\_TFP$ .

To sum up, by merging the I/B/E/S data and the Compustat data, I construct an annual panel of firms' ex-ante earnings forecasts by market analysts, which is my preferred measure of uncertainty, and ex-post-realized forecast errors. I then document the following stylized facts. First, the degree of uncertainty facing individual firms is heterogeneous; the cross-sectional standard deviation of forecast dispersion across firms is 0.53, with a serial correlation of 0.15. Second, the first and second moments of the distribution of the measure are countercyclical; the correlation with GDP growth rates are negative for the mean (-0.48) and the standard deviation (-0.38). Thus, forecast dispersion and its variance both fall with higher GDP growth. Finally, my measures of uncertainty are positively correlated with other common measures in the literature, including stock price volatility-based and balance sheet-based measures.

Table 2 - Correlation between Uncertainty Measures

|            | roc_mean | roc_sd | fdis_med | fdis_mean | fdis_sd | fdis_iqr | ferror_med | dis_BOS | sd_TFP | GDP  |
|------------|----------|--------|----------|-----------|---------|----------|------------|---------|--------|------|
| roc_mean   | 1.00     |        |          |           |         |          |            |         |        |      |
| roc_sd     | 0.69     | 1.00   |          |           |         |          |            |         |        |      |
| fdis_med   | -0.47    | 0.17   | 1.00     |           |         |          |            |         |        |      |
| fdis_mean  | -0.14    | 0.45   | 0.79     | 1.00      |         |          |            |         |        |      |
| fdis_sd    | -0.05    | 0.40   | 0.60     | 0.91      | 1.00    |          |            |         |        |      |
| fdis_iqr   | -0.13    | 0.48   | 0.86     | 0.88      | 0.67    | 1.00     |            |         |        |      |
| ferror_med | 0.14     | 0.52   | 0.40     | 0.59      | 0.51    | 0.59     | 1.00       |         |        |      |
| dis_BOS    | 0.21     | 0.24   | 0.13     | 0.21      | 0.19    | 0.23     | 0.32       | 1.00    |        |      |
| sd_TFP     | 0.35     | 0.71   | 0.36     | 0.56      | 0.51    | 0.54     | 0.57       | 0.19    | 1.00   |      |
| GDP        | -0.06    | -0.51  | -0.48    | -0.43     | -0.38   | -0.48    | -0.33      | -0.33   | -0.48  | 1.00 |

Notes: This table is the correlation matrix for various measures of uncertainty. Return on capital is calculated as earnings (= Street earnings per share (EPS) multiplied by the number of outstanding shares) divided by capital stock (= the sum of Property, Plant, and Equipment and inventory). Forecast dispersion is the cross-analysts standard deviation of earnings forecasts normalized by the median value of earning forecasts. roc\_mean is calculated for each year as the cross-sectional mean of return on capital for the panel dataset and roc\_sd is the cross-sectional standard deviation for the same data. Using the forecast dispersion data, for each year, fdis\_med, fdis\_mean, fdis\_sd and fdis\_iqr are calculated as the cross-sectional median, mean, standard deviation and interquartile range, respectively. ferror\_med is the cross-sectional median of forecast errors, defined as the realised return on capital minus the median of forecasts. dis-BOS is the forecast disagreement index from Bachman et al. (2013). sd\_TFP is the cross-sectional standard deviation of TFP shocks for the U.S. establishment from Bloom et al. (2014). GDP is the GDP growth rates.

### 3 Model

Below, I take a standard equilibrium business cycle model with heterogeneous firms and extend it as follows. First, I assume that firms' idiosyncratic productivity has both a base and a temporary component, and these two components cannot be observed separately. The temporary component is i.i.d. while the base component is persistent and, as such, relevant for firms' investment decisions. Firms learn about their base components over time, by observing their total productivity and updating their beliefs as in Jovanovic (1982). Second, each firm is subject to exogenous shocks to this base component. In each period, a firm retains its current base component with probability  $1 - \pi$ , but loses the current level and draws a new one with probability  $\pi$ . The new base component is drawn from a time-invariant

distribution and independent of last period's productivity level. Whenever a firm draws a new base component, it must restart the learning process. Third, I assume that  $\pi$  is time-varying. A rise in uncertainty in this model happens when  $\pi$  is high, which implies that an unusually large number of firms change their productivity level and begin the process of learning anew.

### 3.1 Production, learning

The model economy is perfectly competitive and has an infinite horizon. There are a large number of competitive firms producing a homogenous good. Each firm uses capital stock  $k$ , and labor  $n$ , via an increasing and concave production function,

$$y = z\varepsilon F(k, n), \tag{4}$$

where  $F(k, n) = (k^\alpha n^{1-\alpha})^\nu$ , with  $0 < \alpha < 1$  and  $0 < \nu < 1$ .

There are two productivity terms in the production function, one aggregate,  $z$ , and one idiosyncratic,  $\varepsilon$ .  $z$  represents an exogenous stochastic total factor productivity common across all firms:  $z \in \{z_1, \dots, z_{N_z}\}$ , where  $\Pr(z' = z_m \mid z = z_l) \equiv \pi_{lm}^z \geq 0$ , and  $\sum_{m=1}^{N_z} \pi_{lm}^z = 1$  for each  $l = 1, \dots, N_z$ . For the firm-specific idiosyncratic counterpart, I assume that  $\varepsilon$  is the sum of two components: a persistent one,  $\theta$ , and a transitory one,  $a$ ;

$$\varepsilon = \theta + a. \tag{5}$$

The base component of firm specific productivity,  $\theta$ , changes infrequently and the timing of such changes, though not their value, is known to the firm. As noted above, with probability  $1 - \pi$ , the current base component is maintained. With probability  $\pi$ , the current base component is lost and a new value is drawn. This is independent of the firm's state. The transitory component,  $a$ , is independently and identically distributed over time. The distribution of both  $\theta$  and  $a$  are known to all firms:  $\theta \sim N(\bar{\theta}, \sigma_\theta^2)$  and  $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ .

Firms observe  $\varepsilon$ , but  $\theta$  and  $a$  are not observed separately. Firms can extract information about their  $\theta$  by accumulating observations of  $\varepsilon$ . While these observations are affected by the i.i.d. draws of  $a$  every period, repeatedly observing  $\varepsilon$ , firms learn about their  $\theta$ .

We formalize this learning process as follows. Consider a firm with  $\bar{\varepsilon}$  —the mean of the observations of idiosyncratic shocks  $\varepsilon_i$  for  $i = 1, \dots, t$ , where  $t$  is the number of observations.

To form a belief about their base component  $\theta$ ,  $(\bar{\varepsilon}, t)$  is sufficient information. Therefore, a firm with  $(\bar{\varepsilon}, t)$  infers the posterior distribution:  $\theta \sim N(A, B)$  with

$$A = \frac{\sigma_a^2}{\sigma_a^2 + t\sigma_\theta^2}\bar{\theta} + \frac{t\sigma_\theta^2}{\sigma_a^2 + t\sigma_\theta^2}\bar{\varepsilon} \quad (6)$$

$$B = \frac{\sigma_a^2\sigma_\theta^2}{\sigma_a^2 + t\sigma_\theta^2} \quad (7)$$

where  $\bar{\varepsilon} = (\sum_{i=1}^t \varepsilon)/t$  and  $t$  is the number of observations. Each period after observing  $\varepsilon$ , the posterior distribution of  $\theta$  is updated, and over time it converges to the true value of  $\theta$  as  $t$  becomes large enough.

### 3.2 Distribution of firms

The exogenous aggregate state is summarized by  $s = (z, \pi)$ . In addition, a non-trivial, time-varying distribution of firms is a part of the aggregate state. As shown in the last section, firms form expectations over their productivity next period. Starting with the last period when their base component is reset, firms observe their productivity over time, and the mean of these observations and the number of observations are a part of each firm's state. This number of observations corresponds to the time-since-reset. Thus, firms at the beginning of each period are identified by the mean of their observations of idiosyncratic shocks,  $\bar{\varepsilon}$ , the number of these observations,  $t$ , and their current productivity draw,  $\varepsilon$ , alongside their predetermined capital stock,  $k$ . I summarize the distribution of firms over  $(\bar{\varepsilon}, t, \varepsilon, k)$  using the probability measure  $\mu$  defined on the Borel algebra,  $\mathcal{S}$ , generated by the open subsets of the product space,  $\mathcal{S} = \mathbb{R}_+ \times \mathbb{Z} \times \mathbb{R}_+ \times \mathbb{R}_+$ .

Given the distribution of firms, the aggregate state of the economy is fully summarized by  $(s, \mu)$ , and the distribution of firms evolves over time according to a mapping,  $\Gamma$ , from the current aggregate state;  $\mu' = \Gamma(s, \mu)$ .

### 3.3 Firm's problem

Firms solve the following problem given their firm-level state together with the aggregate state. The problem consists of choosing the capital stock for the following period,  $k'$ , and the labor input for current period,  $n$ . Let  $V(\bar{\varepsilon}, t, \varepsilon, k; s, \mu)$  be the value function of a firm,

$$\begin{aligned}
V(\bar{\varepsilon}, t, \varepsilon, k; s, \mu) &= \max_{n, k'} \left[ z\varepsilon(k^\alpha n^{1-\alpha})^\nu - \omega n + (1 - \delta)k - k' \right. \\
&\quad \left. + (1 - \pi)E_{s'|s}d(s', s, \mu) E_{\bar{\varepsilon}'|\bar{\varepsilon}, t}V(\bar{\varepsilon}', t + 1, \varepsilon', k'; s', \mu') \right. \\
&\quad \left. + \pi E_{s'|s}d(s', s, \mu) E_{\varepsilon'}V\left(\frac{\bar{\theta} + \varepsilon'}{2}, 2, \varepsilon', k'; s', \mu'\right) \right] \tag{8}
\end{aligned}$$

$$\text{subject to} \quad : \quad \bar{\varepsilon}' = \frac{t\bar{\varepsilon} + \varepsilon'}{t + 1}, \tag{9}$$

$$\text{and} \quad : \quad \mu' = \Gamma(s, \mu). \tag{10}$$

Each firm's profits are its output less wage payments and investment. With probability  $1 - \pi$ , the current base component is maintained and hence their expectation over  $\varepsilon'$  and thus  $\bar{\varepsilon}'$  are conditional on  $(\bar{\varepsilon}, t)$ . Furthermore, they discount next period's value by the state contingent discount factor,  $d(s', s, \mu)$ . With probability  $\pi$ , the current base component is lost and a new one is drawn, independent of the current state. In the first period after any reset of the base component, firms take an average of the mean value of  $\bar{\theta}$  and the first draw of  $\varepsilon'$ . The state contingent discount factor is determined by households decision rules as explained below.

### 3.4 Households

There is a large number of identical households in this economy, formally a unit measure. Households choose consumption, supply labor, and hold their wealth in firm shares to maximize lifetime expected utility as follows.



$$V^h(\lambda; s, \mu) = \max_{c, n^h, \phi', \lambda'} \left[ U(c, 1 - n^h) + \beta E_{s'|s} V^h(\lambda'; s', \mu') \right] \quad (11)$$

$$\begin{aligned} \text{subject to:} \quad & c + \int_{\mathbf{S}} \rho_1(\bar{\varepsilon}', t + 1, \varepsilon', k'; s, \mu) \lambda'(d[\bar{\varepsilon}' \times t + 1 \times \varepsilon' \times k']) \leq \\ & w(s, \mu) n^h + \int_{\mathbf{S}} \rho_0(\bar{\varepsilon}, t, \varepsilon, k; s, \mu) \lambda(d[\bar{\varepsilon} \times t \times \varepsilon \times k]) \end{aligned} \quad (12)$$

$$: \quad \mu' = \Gamma(s, \mu) \quad (13)$$

Households hold one-period shares in firms, which is denoted by the measure  $\lambda$ . Given the prices—the real wage,  $w(s, \mu)$ , and the prices of shares,  $\rho_0(\bar{\varepsilon}, t, \varepsilon, k; s, \mu)$  and  $\rho_1(\bar{\varepsilon}', t + 1, \varepsilon', k'; s, \mu)$ , households choose their current consumption,  $c$ , hours worked,  $n^h$ , and the numbers of new shares,  $\lambda'(d[\bar{\varepsilon}' \times t + 1 \times \varepsilon' \times k'])$ .

Let  $C^h(\lambda; s, \mu)$  and  $N^h(\lambda; s, \mu)$  represent the household decision rules for consumption, hours worked, and let  $\Lambda^h(\bar{\varepsilon}', t + 1, \varepsilon', k'; \lambda; s, \mu)$  be the household decision rule for shares purchased in firms that will begin the next period with  $(\bar{\varepsilon}', t + 1, \varepsilon', k')$ .

### 3.5 Recursive equilibrium

A recursive competitive equilibrium is a set of functions

$$\begin{aligned} \text{prices} & : (\omega, d, \rho_0, \rho_1) \\ \text{quantities} & : (N, K, C, N^h, \Lambda^h) \\ \text{values} & : (V, V^h) \end{aligned}$$

that solve firm and household problems and clear the markets for assets, labor, and output:

1.  $V$  satisfies (5) - (7), and  $(N, K)$  are the associated policy functions for firms.
2.  $V^h$  satisfies (8) - (10), and  $(C, N^h, \Lambda^h)$  are the associated policy functions for households.
3.  $\Lambda^h(\bar{\varepsilon}, t, \varepsilon, k, \mu; s, \mu) = \mu(\bar{\varepsilon}, t, \varepsilon, k)$  for each  $(\bar{\varepsilon}, t, \varepsilon, k) \in \mathcal{S}$ .

4. The labor and goods market clear.

$$N^h(\mu; s, \mu) = \int_{\mathbf{s}} [N(\bar{\varepsilon}, t, \varepsilon, k)] \cdot \mu(d[\bar{\varepsilon} \times t \times \varepsilon \times k])$$

$$C(\mu; s, \mu) = \int_{\mathbf{s}} [z\varepsilon F(k, N(\bar{\varepsilon}, t, \varepsilon, k)) - (K(k, b, \varepsilon; z, \mu) - (1 - \delta)k)] \cdot \mu(d[\bar{\varepsilon} \times t \times \varepsilon \times k])$$

5. the resulting individual decision rules for firms and households are consistent with the aggregate law of motion,  $\Gamma$ , where  $\Gamma$  defines the mapping from  $\mu$  to  $\mu'$ .

Using  $C(s, \mu)$  and  $N(s, \mu)$  to describe the market-clearing values of household consumption and hours worked, it is straightforward to show that market-clearing requires that (a) the real wage equal the household marginal rate of substitution between leisure and consumption:

$$w(s, \mu) = D_2U\left(C(s, \mu), 1 - N(s, \mu)\right) / D_1U\left(C(s, \mu), 1 - N(s, \mu)\right),$$

that (b) firms' state-contingent discount factors are consistent with the household marginal rate of substitution between consumption across states:

$$d(s', s, \mu) = \beta D_1U\left(C(s', \mu'), 1 - N(s', \mu')\right) / D_1U\left(C(s, \mu), 1 - N(s, \mu)\right).$$

## 4 Solution and calibration

In this section, I present my solution of the model and calibration strategy to match both micro and macro data. I solve the model using a non-linear method, which involves value function iterations over the state space described in the model section. In this paper, there is a non-trivial time-varying distribution of firms, which is the part of the aggregate state in this economy.<sup>14</sup> Following aggregate shocks, the model economy's response is expected to be highly non-linear as each firm responds to shocks to a different degree or even in a different direction, and therefore it is not straightforward to track the dynamics of the

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<sup>14</sup>Terry (2014) compares a variety of alternative approaches to solve heterogeneous firm models with aggregate uncertainty.

distribution of firms. To tackle these problems, I take the Krusell Smith (1997) approach that is implemented by Khan and Thomas (2003, 2008) in a heterogeneous firm model, and I further explore an extension that captures highly non-linear dynamics in aggregates similar to Khan and Thomas (2013).

## 4.1 Functional forms and stochastic processes

I assume that the representative household's period utility is  $u(c, L) = \log c + \eta L$ , as in the models of indivisible labor (e.g. Hansen (1985), Rogerson (1988)). As seen in the previous sections, I assume that each heterogeneous firm undertakes production via Cobb-Douglas production function:  $z\varepsilon(k^\alpha n^{1-\alpha})^\nu$ , where  $\alpha$  determines capital and labor's share of income and  $\nu$  governs returns to scale in this economy. For aggregate and idiosyncratic productivity processes:  $z$  and  $\varepsilon = \theta + a$ , I assume

$$\log z' = \rho_z \log z + \eta'_z \quad \text{with } \eta'_z \sim N\left(0, \sigma_{\eta_z}^2\right) \quad \text{and} \quad (14)$$

$$\varepsilon = \theta + a$$

$$: \theta \sim N(\bar{\theta}, \sigma_\theta^2) \quad \text{and}$$

$$: a \sim N(0, \sigma_a^2). \quad (15)$$

$\bar{\theta}$  is the mean and  $\sigma_\theta^2$  is the variance of the base component of idiosyncratic productivity, and  $\sigma_a^2$  is the variance of the temporary component of idiosyncratic productivity.

For time-varying  $\pi$ , I assume that  $\pi$  follows a two-state Markov chain with  $\pi_L$  and  $\pi_H$ . The transition matrix is  $\Pi = \begin{bmatrix} \rho_L & 1 - \rho_L \\ 1 - \rho_H & \rho_H \end{bmatrix}$ .

### Common parameters

I calibrate the following five parameters against aggregate moments for the U.S. economy: (1)  $1 - \alpha$ : labor's income share, (2)  $\nu$ : returns to scale, (3)  $\beta$ : the household discount factor, (4)  $\delta$ : the depreciation rate and (5)  $\eta$ : the leisure preference. First, I set  $\nu$  to imply an average private capital-to-output ratio of 2.55, given the value of  $1 - \alpha$  determining the average labor share of income at 0.6. Next, the depreciation rate,  $\delta$ , is taken so that the model matches an average investment-to-capital ratio at 0.08. The preference parameter,  $\eta$ , is set to imply an

average hours worked of one-third. Finally, I set the household discount factor to match an average real interest rate of 4 percent as in Gomme, Ravikumar and Rupert (2011).<sup>15</sup>

### **Firm-level parameters and aggregate shocks**

Given the common parameters calibrated as above, I jointly calibrate the following firm-level parameters, and then I set the parameters that govern exogenous aggregate shock processes. First, (1) the mean of base components of idiosyncratic productivity,  $\bar{\theta}$ , (2) the variance of base components of idiosyncratic productivity,  $\sigma_{\theta}^2$ , (3) the variance of temporary components of idiosyncratic productivity,  $\sigma_a^2$ , and (4) the steady state level of the reset probability,  $\pi_L$ , are calibrated to match the following panel data: the mean (0.19) and serial correlation (0.29) of the return on capital as well as the mean of forecast dispersions, `fdis_mean` (0.04), and the mean of firms' investment rates (0.15). Second, I calibrate the process for the two aggregate shocks as follows. I set the high reset probability,  $\pi_H$ , to reproduce the size of changes in forecast dispersions between low- to high-uncertainty periods in the panel data (74%).<sup>16</sup> The transition probabilities are estimated to match the transition patterns between low- to high-uncertainty periods during the same years in the panel data. Finally, the stochastic process of aggregate productivity is calibrated by setting  $\rho_z$  to 0.852 as in Khan and Thomas (2013) and setting  $\sigma_{\eta_z}$  to reproduce an unconditional standard deviation of output at 1.97.<sup>17</sup> All parameters and targets are summarized in Table 3.

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<sup>15</sup>The average private capital-to-output ratio and the average investment-to-capital ratio are calculated from the U.S. National Income and Product Accounts Tables and Fixed Assets Accounts Tables from 1976 to 2012.

<sup>16</sup>I apply a linear trend for `fdis_mean` over the sample periods between 1976 and 2012. Low-uncertainty periods correspond to years of `fdis_mean` below the trend, and high -uncertainty periods correspond to years of `fdis_mean` above the trend.

<sup>17</sup>The standard deviation of the HP-filtered (with smoothing parameter 100) log series of real GDP between 1976 and 2012 is targeted.

## 5 Results

### 5.1 Steady state

#### Two-sided capital misallocation

Imperfect information about total factor productivity across firms causes a misallocation of capital and labor. This pattern of misallocation is distinct from that which appears with financial frictions such as lending subject to default risk (Khan et al. (2014)). Firms operating with imperfect information deviate from the optimal allocation of resources and exhibit both over- as well as undercapacity.

When a firm believes its base productivity is higher (lower) than the prior, its capital stock tends to be lower (higher) than the full information efficient level. This undercapacity (overcapacity) persists over time until the posterior mean converges to the true base productivity and the posterior variance approaches 0. Overall, unless firms are fully informed about their base components, their capital stock is either excessive or insufficient relative to the efficient level consistent with the true value of its base component and the interest rate. The longer it takes a firm to learn, the more severe the resource misallocation problem. Figure 4 shows capital choices of firms as a function of the mean observations of productivity,  $\bar{\epsilon}$ , and time-since-reset  $t$ . The figure illustrates two-sided misallocation. For high  $\bar{\epsilon}$ ,  $k$  choice is lower than its full information counterpart and there is undercapacity. By contrast, for  $\bar{\epsilon}$  close to 0,  $k$  is slightly higher than it would be under full information, when  $t$  is close to 1. This is overcapacity. The small distortion apparent for  $\bar{\epsilon}$  low relative to the downward distortion when  $\bar{\epsilon}$  is high is the result of decreasing returns to scale. It foreshadows the rise in misallocation that will follow an uncertainty shock.

Figure 5 provides an example of learning and capital accumulation patterns by one individual firm. It compares my full model with gradual learning (the left panels) with an otherwise identical full information model where  $\theta$  is always known (the right panels). The top two panels show the true base component, overall idiosyncratic productivity, and the capital stock. The bottom two panels plot the firm's idiosyncratic productivity growth rate and investment rate. In the benchmark model with learning, the firm slowly adjusts its capital stock following a resetting of its base productivity. For example, between period 27 and 31, the firm gradually scales up its capital stock following a rise in its long-run produc-

tivity. Over this episode, its capital stock is inefficiently low. In contrast, in the model with full information, the firm quickly adjusts its capital stock during the same episode.

As the left panels of figure 5 show, every time base components are reset and there is a change in long-run productivity. Firms adjust their capital stocks slowly, so there is a misallocation of capital. These occasional resets of base productivity lead to the non-trivial distribution of firms shown in Figure 6. As shown in the figure, capital across firms that have recently experienced a resetting of their base productivity tends to be highly concentrated as these firms place a large weight on the common prior. Over time, as the conditional posterior variance falls, we see firms adopting clearly different capital paths. The time-varying reset probabilities have important cyclical implications as they change the shape of the distribution of firms over time and thus the degree of misallocation of capital and labor. In the next section, we explore business cycles in this environment.

## 5.2 Business cycles

Table 4 presents the business cycle moments for a 1,500-period unconditional simulation with both aggregate productivity shocks and uncertainty shocks. Some of the features of the model business cycle are summarized as follows. First, most second-moment statistics generated from the simulation are standard and familiar when evaluated against the business cycle literature. Specifically, consumption, investment and hours comove with output. Consumption is less volatile than output, while investment is more volatile than output, and indeed more than its empirical counterpart. Second, this combined uncertainty shock and aggregate productivity shock simulation allows the model to resolve two puzzles in business cycle research. Specifically, the low correlation between output and labor productivity and the negative correlation between hours worked and labor productivity are reproduced, 0.124 and  $-0.311$ , respectively.

Table 4 - Business Cycle Moments

|                          | Y    | C    | I     | N    | Y/N  | K    |
|--------------------------|------|------|-------|------|------|------|
| Standard deviation (s.d) | 1.87 | 0.88 | 12.67 | 1.46 | 0.49 | 1.34 |
| Relative s.d. to Y       | 1.00 | 0.47 | 6.76  | 0.78 | 0.26 | 0.72 |
| Correlation with Y       | 1.00 | 0.60 | 0.91  | 0.92 | 0.12 | 0.08 |

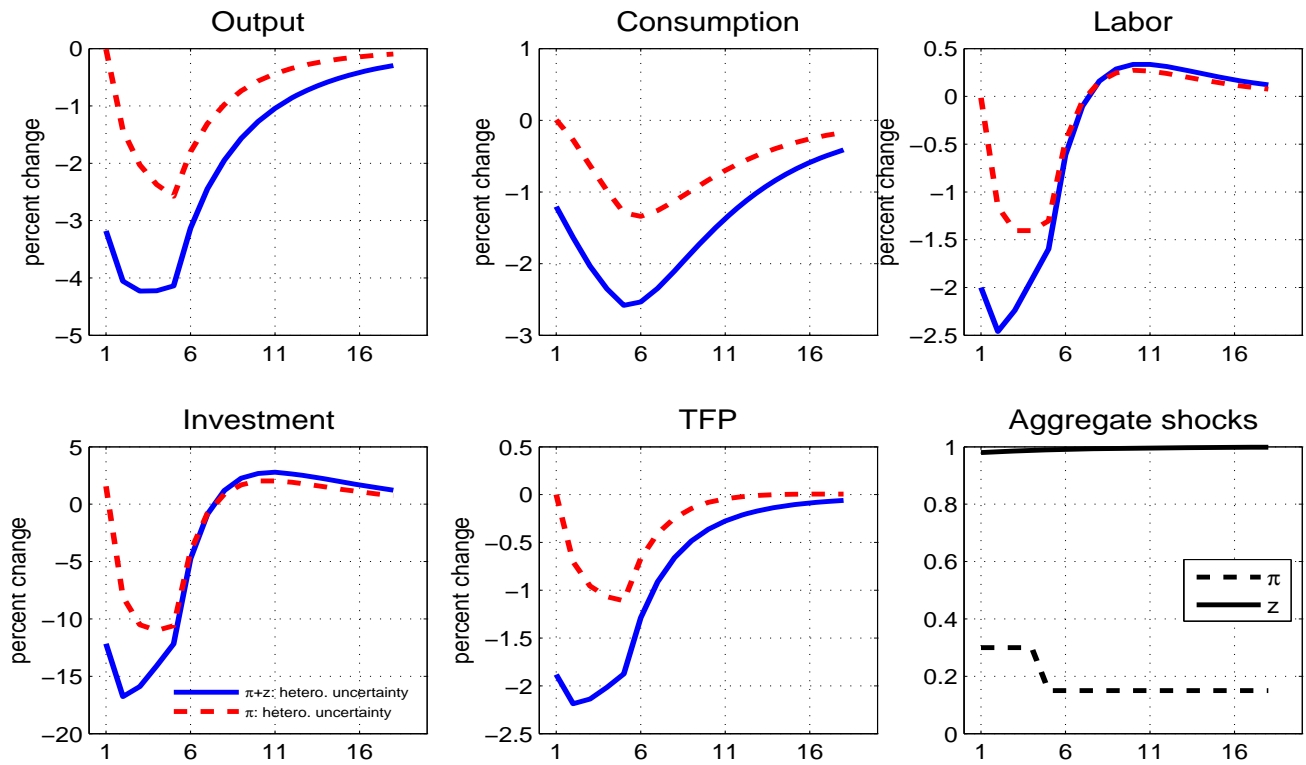
Notes: The table above presents business cycle moments from a 1,500-period unconditional simulation. All series are HP-filtered in logs with smoothing parameter 100. The first row reports the standard deviation of the HP-filtered log series. The second row reports the relative size of the standard deviation of the HP-filtered log series to the standard deviation of output series. The third row reports the contemporaneous correlation with output series.

## 6 The Great Recession simulation

In this section, I explore the mechanism that propagates uncertainty shocks in the model. Towards this, I study an impulse response following shocks to both uncertainty and aggregate TFP. The uncertainty shock is set to the average level of my measure, `fdis_mean`, observed in 2008 and 2009. Given this first shock, the TFP shock is chosen to reproduce the overall change in the measured Solow residual over the Great Recession. As the uncertainty shock on its own reduces measured TFP in the model, the overall fall in TFP exceeds that implied by the aggregate productivity shock alone. Figure 7 plots the dynamics of the model economy. The dashed line is the response to only the uncertainty shock. The solid line includes the aggregate productivity shock.

Table 5 compares the size of the recession between the model and data. In the data, the size of the recession is measured by the percentage change in each variable from the peak to the trough, 2007Q4 to 2009Q2. First, the recession in this model with both the uncertainty shock and aggregate productivity shock can reproduce much of the recession seen in the data. Specifically, GDP falls by 4.42%, which is 79% of its fall in the data. Investment falls by 16.94%, which is 89% when compared to the data. Second, the uncertainty shock alone

Figure 7 - The Great Recession simulations with/without aggregate TFP shocks



Notes: Each panel except the lower right one plots the aggregate economy's responses to both aggregate TFP shocks and uncertainty shocks, plotted by a blue line and labeled as  $\pi+z$ : hetero. uncertainty, the model's responses to only uncertainty shocks, plotted by a red dashed line and labeled as  $\pi$ : hetero. uncertainty. The aggregate shocks are plotted in the lower right panel: aggregate TFP shocks (two percentage drop at the initial period) and uncertainty shocks (the high  $\pi$  for four periods and the low  $\pi$  in the rest of the periods).



reduces measured TFP by 1.11%, which is 51% of the observed reduction and GDP by 2.57%, which 46% of the total fall explained using both shocks.

Table 5 - Peak-to-trough Drops for the Great Recession and Model

|                                                          | GDP  | Investment | TFP  |
|----------------------------------------------------------|------|------------|------|
| data                                                     | 5.59 | 18.98      | 2.18 |
| model (uncertainty shock + aggregate productivity shock) | 4.42 | 16.94      | 2.18 |
| model (uncertainty shock)                                | 2.57 | 10.99      | 1.11 |

Notes: The peak-to-trough drops are calculated with log deviations from 07Q4 to 09Q2, detrended using the HP filter with parameter 1600. GDP and investment series are taken from BEA GDP Tables. Measured TFP is the Solow Residual series.

In the following sections, I further investigate how uncertainty shocks produce recessions in the model. As a first step, I explain the difference between the mechanism through which conventional uncertainty shocks in stochastic volatility models operate and the distinct mechanism in my model. I then focus on the role of Bayesian learning in shaping aggregate fluctuations. This involves two different models: one with learning and one without learning. The latter has firms learning their new base component of productivity immediately after any reset. This will reveal an important mechanism through which the process of learning prolongs recessions in the model.

### **Relationship with conventional uncertainty shocks**

The conventional framework used in business cycle studies of uncertainty shocks assumes stochastic volatility where the variance of a stochastic processes is allowed to be time-varying. In Bloom (2009), for example, shocks to the variance of productivity innovations. In stochastic volatility models, a shock to the variance can lead to a recession. One mechanism that has been emphasized by the literature is the real option value associated with factor adjustment in the presence of nonlinear adjustment costs.

For any given firm, there are two effects that work in opposite directions following a shock to the variance of productivity innovations. On the one hand, the firm might increase its

investment due to Jensen's inequality effects (Oi-Hartman- Abel effects).<sup>18</sup> This is because the optimal choice of capital is convex in productivity. On the other hand, the firm might pause its investment completely and wait for the resolution of uncertainty. With more volatile productivity shocks, the value of an option to wait to see future outcomes in the following period increases. Therefore, the firm may undertake no investment. In economies with heterogeneous firms, some firms pause investment while other firms increase investment, depending on their levels of individual productivity. Quantitative studies in the literature have shown that the latter effect dominates and the aggregate economy falls into recession with higher uncertainty.

### **Recessions with Bayesian uncertainty shocks**

One important point to the above argument is the mean-preserving feature of shocks. In order for mean-preserving uncertainty shocks to be able to drive recessions, adjustment costs are required. However, they are unnecessary if uncertainty shocks have no mean-preserving features. Suppose a shock to the variance of productivity moves mean productivity. This would be possible if the shock asymmetrically widens the distribution of productivity, pushing one tail of the distribution more than the other tail. Without adjustment costs, in the single firm example, the firm increases investment for the case with an upward shift in the mean. On the other hand, when a shock makes the distribution of productivity more left-skewed by pushing the lower tail more than the higher tail, this could lead to recessions if negative effects due to the lower mean of the distribution dominate positive effects due to the Oi-Hartman-Abel effect.

In the Bayesian uncertainty shock framework in this paper, a rise in the reset probability has disparate impacts across firms: some firms experience negative shocks while other firms have positive shocks to the base component of their productivity. Firms take expectations about their future productivity levels by looking at two different distributions of productivity simultaneously: one is their own posterior distribution that has been updated by learning, and the other is the unconditional distribution that is the common prior known to all firms. Facing a higher reset probability, firms put more weight on the unconditional distribution than their posterior distribution. Since the variance of the unconditional distribution is larger

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<sup>18</sup>The positive impact of uncertainty shocks on investment is known as the Oi-Hartman-Abel effect (Oi (1961), Hartman (1972), Abel (1983)).

than that of the posterior, firms effectively infer a larger variance of their future productivity distribution. The direction of the shift in the posterior mean depends on the current posterior mean. For a firm with a current posterior that is higher than the prior mean, a higher reset probability leads to a larger variance of productivity shocks and a fall in its mean. This leads it to scale down its capital stock. In contrast, for a firm whose current posterior is lower than the prior mean, a higher reset probability results in a larger variance of productivity shocks with an upward shift in the mean, inducing it to increase its capital stock. Nonetheless, these two effects do not offset each other. As the production function exhibits decreasing returns to scale and the productivity distribution of firms is symmetric around its mean, the net impact on aggregate investment is negative, and the economy enters a recession despite the absence of adjustment costs. This is confirmed by Figure 7, which shows the benchmark model producing a recession even without an aggregate productivity shock.

## 6.1 The role of learning

In figure 8, I show the model economy's response in the Great Recession simulation for two cases: the benchmark model with imperfect information and a model without imperfect information. This second model (homogeneous uncertainty) has an observable base component and is otherwise identical to the benchmark. As argued, uncertainty shocks that asymmetrically widen the distribution of shocks perceived by firms can cause recessions without adjustment costs. Figure 8 shows that this happens in both models regardless of learning. However, there is a sharp difference between these two models in how the recovery occurs. While investment, labor, and measured TFP series overshoot in the model without learning, the benchmark model with learning eliminates this rapid recovery. Instead, the benchmark model with learning exhibits a gradual recovery following recession.<sup>19</sup>

To gauge the importance of imperfect information following uncertainty shocks, I decompose the impact of uncertainty shocks into the two effects explained below. I first categorize firms into cohorts by their time-since-reset, and I look at the average investment for each cohort. This allows me to see the disaggregated investment response of firms. Furthermore, by

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<sup>19</sup>This pattern in the model without learning is not influenced by the first moment shock. To isolate the impact of uncertainty shocks, Figure 9 shows the aggregate dynamics of the two models in response to only uncertainty shocks.

comparing the mass of firms for each cohort, I can trace changes in the distribution of firms throughout the recession. This will prove useful for understanding the mechanism behind the rapid drops and slow recoveries in my model. Figure 10 and 11 show this exercise for the start of the recession and the recovery separately.

### 6.1.1 A steep recession

When firms anticipate a higher reset probability for their base components, an uncertainty effect leads them to change their target levels of capital. As argued above, for firms whose current posterior is higher than the prior mean, the higher reset probability implies a larger variance for productivity shocks and fall in their mean, leading to downward adjustment of capital stocks. On the other hand, for firms whose current posterior mean is lower than the prior, uncertainty shocks imply a larger variance of productivity shocks with an upward shift in the mean, resulting in an upward adjustment of capital stocks. Given that the distribution of the base component of productivity is symmetric, and the production function has decreasing returns to scale, the downward adjustment of capital stocks for the top 50% of firms tends to dominate the opposing force from the bottom 50% of firms. Thus investment falls for cohorts 4 to 20. This shifts the average investment curve from the dashed to the bold line.<sup>20</sup> Coupled with the steady-state distribution of mass of firms in each cohort, aggregate investment falls in general equilibrium.

The right panel of Figure 10 highlights the distributional effects in the following period. While uncertainty effects are now less pronounced and investment has largely reversed (see Figure 12), there is a large inflow of firms into cohort 1 relative to the pre-recession level. Since the average investment level of cohort 1 is low, this shift of the firm distribution leads to a further drop in aggregate investment.

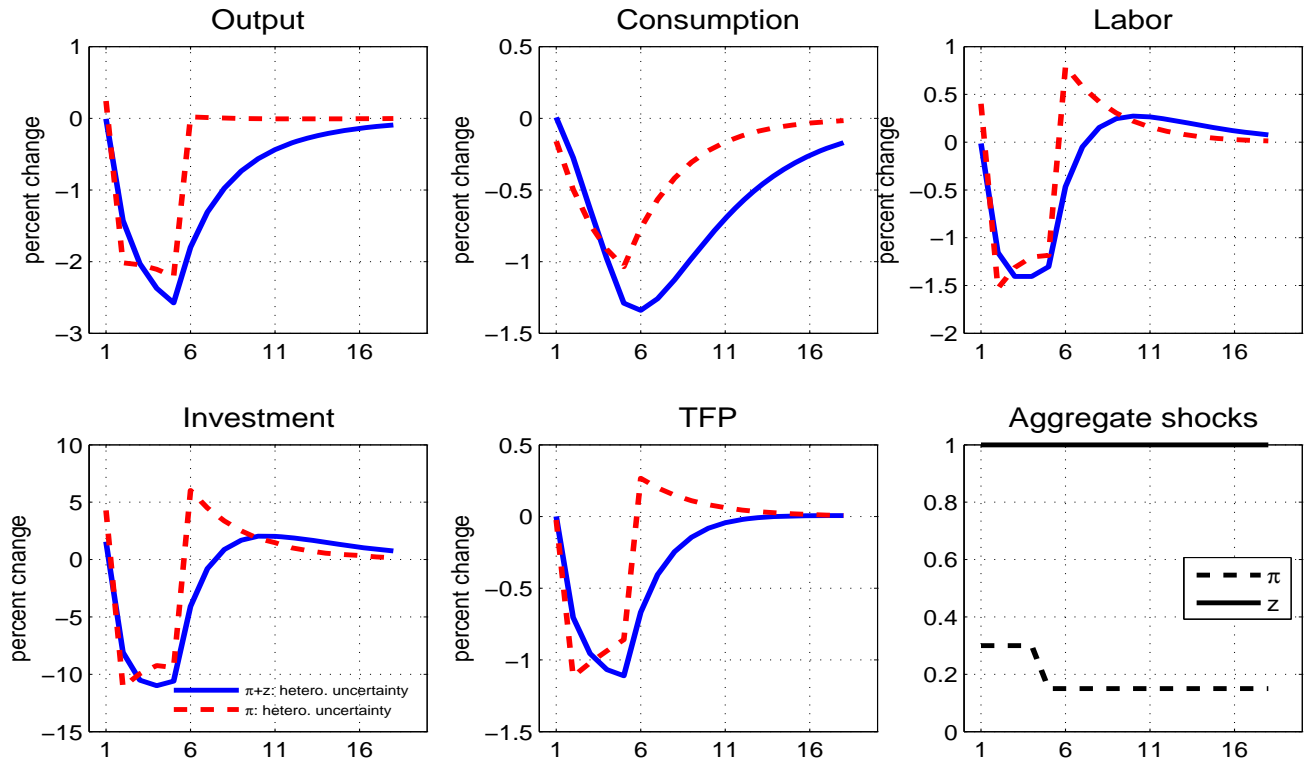
### 6.1.2 A slow recovery

In this subsection, I examine how imperfect information eliminates an overshoot of investment. After the shock ends, firms raise their expectation of maintaining their current

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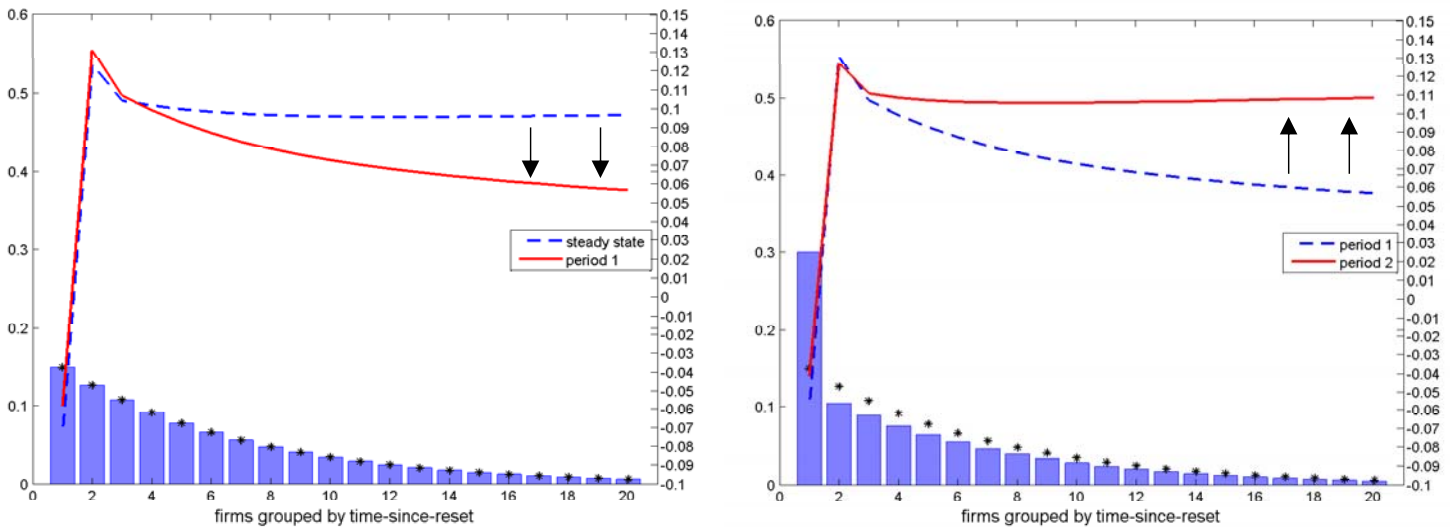
<sup>20</sup>This is easier to see in Figure 12 in which each line represents the percent change in investment from the steady state. The rise in the average investment in cohort 1 to 3 is a general equilibrium effect. As shown in Figure 14, these rises are not present in partial equilibrium.

Figure 9 - Uncertainty shock simulations with/without imperfect information



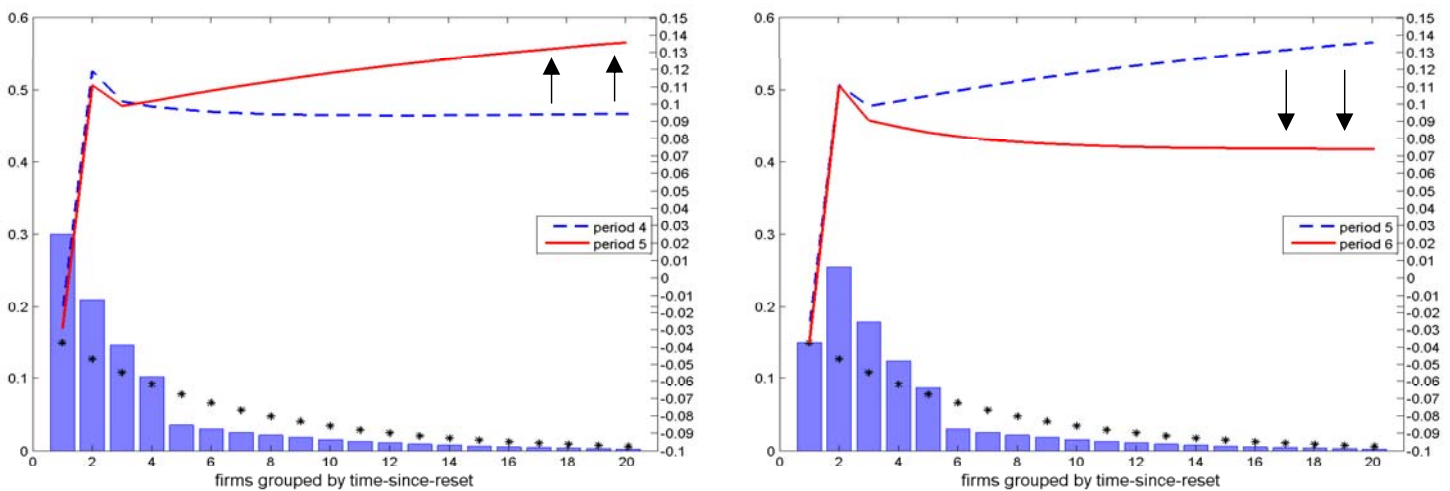
Notes: Each panel except the lower right one plots the aggregate economy's responses to uncertainty shocks, plotted by a blue line and labeled as  $\pi+z$ : *hetero. uncertainty*, the responses of a model without imperfect information to uncertainty shocks, plotted by a red line and labeled as  $\pi$ : *homo. uncertainty*. The aggregate shocks are plotted in the lower right panel: no aggregate TFP shocks and uncertainty shocks (the high  $\pi$  for four periods and the low  $\pi$  in the rest of the periods).

Figure 10 - Dynamics of firm distribution and investment onset of recessions



Notes: Each bin represents the mass of firms in each cohort grouped by the time-since-reset of their base component (left axis). Recall a larger time-since-reset implies that firms are more informed about their productivity levels. Each line plots the average level of investment for each cohort (right axis). Each dot shows the steady-state mass of firms in each time-since-reset bin.

Figure 11 - Dynamics of firm distribution and investmet at the recovery phase



Notes: Each bin represents the mass of firms in each cohort grouped by the time-since-reset of their base component (left axis). Recall a larger time-since-reset implies that firms are more informed about their productivity levels. Each line plots the average level of investment for each cohort (right axis). Each dot shows the steady-state mass of firms in each time-since-reset bin.

level of productivity. Now, when firms believe that their base component is higher than the mean they are more confident in raising their scale of production. If firms believe that their base component is lower than the mean they reduce the scale of their production scale. For the reasons explained in the previous subsection, the pent-up demand of firms with higher productivity shifts the average investment curve from the dashed to the bold line. As may be seen in the left panel of Figure 11, this pent-up investment demand effect is strong in cohorts with large time-since-reset. Firms in lower cohorts have less accurate information and this makes them cautious.

Pent-up investment demand is stronger for firms with more accurate information. As the uncertainty shock ends, the mass of firms in such cohorts is small compared to the pre-recession level. While the fraction of firms that experience a reset of their base component in each period returns to its pre-recession level, many firms have already experienced a shock and they remain in cohorts 1-4 with low time-since-reset. As information is inaccurate in these cohorts, aggregate investment is not pushed up by pent-up demand even after the shock ends.

Imperfect information not only eliminates an overshooting of investment but also slows the pace of recovery afterwards. The right panel of Figure 11 explains how the model economy slowly recovers to its pre-recession level in the periods after the uncertainty shock. The key mechanism is misallocation. To achieve an efficient level of capital stock, firms need to have accurate information about their productivity. Thus, misallocation of capital and labor is more severe among cohorts with smaller time-since-reset. As the figure shows, the mass of firms within cohorts 2 and 3 is larger than in the steady-state, while that in cohort 4 and beyond is smaller than before the recession. As time goes by, the mass of firms in cohort 2 and 3 will gradually fill up the gap the size of mass in cohort 5 and further. Due to a slow-moving distribution of firms related to learning, the negative aggregate effect from misallocation persists until the distribution of information in the economy eventually returns to match that in steady state.

## 7 Conclusion

This paper develops a heterogeneous firm model that incorporates Bayesian learning at the firm level with uncertainty shocks, establishing a close link between the rise in firms'

uncertainty at the start of a recession and the slow pace of subsequent recovery. Uncertainty shocks drive recessions through two effects. First, an uncertainty effect appears as all firms anticipate a higher likelihood of large changes in their productivity. Thereafter, as these changes appear, there is a distributional effect as the average conditional variance of firms' posteriors rises. As the distribution of firms over information becomes more concentrated around low levels of information about productivity, there is a fall in aggregate investment. At the start of the recession, the uncertainty and distributional effects reinforce each other, whilst the uncertainty and distributional effects offset each other during the subsequent recovery.

Uncertainty shocks operate in an environment with dynamic Bayesian inference, rather than stochastic volatility as in the existing literature. Both ex-ante and ex-post uncertainty periodically rise at a subset of firms, consistent with data. Uncertainty shocks in aggregate increase this set and drive economic downturns.

This approach to modeling uncertainty may be useful in other applications. For example, while this model has a very simple hiring and firing decision, economists have emphasized jobless recoveries and the mechanism proposed in this paper may offer important insights that link these to a rise in firms' uncertainty. Further, there have been attempts to link financial markets and aggregate fluctuations since the recent financial crisis. As stated by Bernanke (2008), *"The crisis we face in the financial markets has many novel aspects, [...] at the root of the problem is a loss of confidence by investors and the public in the strength of key financial institutions and markets."* The liquidity crisis we have seen may be interpreted as arising from a loss of confidence among investors. Researchers may find it useful to examine model environments with type of time-varying uncertainty proposed here so as to study the link between a deterioration of trust in financial markets and recessions. Senga (2013) takes a first step in this direction.



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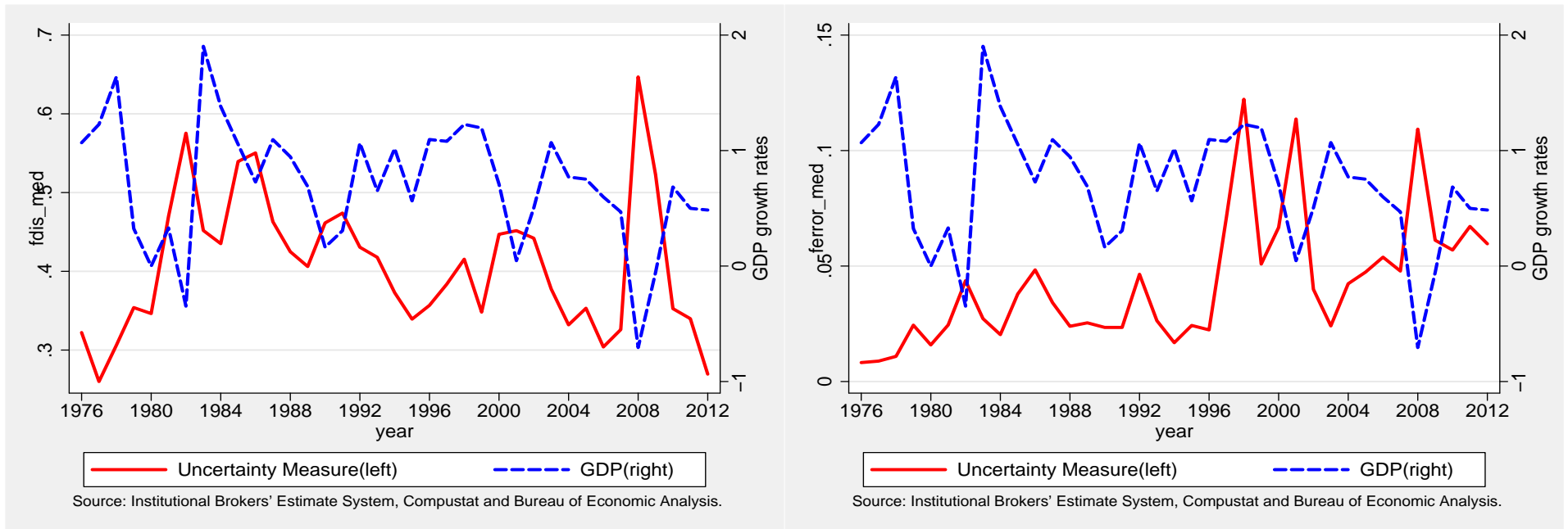
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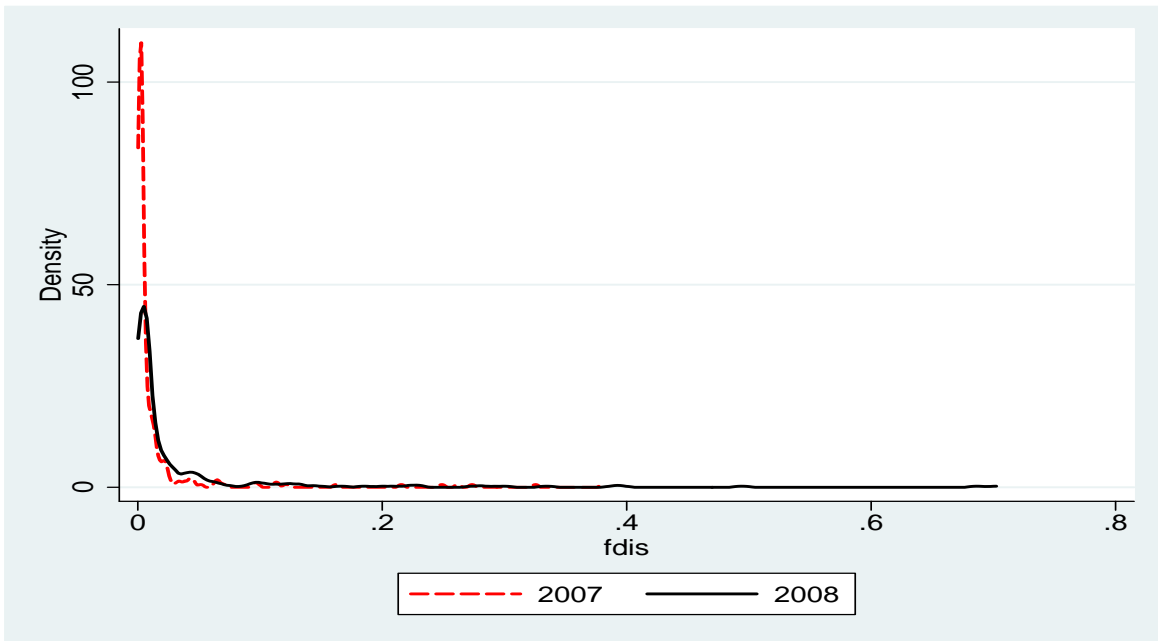
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Figure 1 - Uncertainty Measures



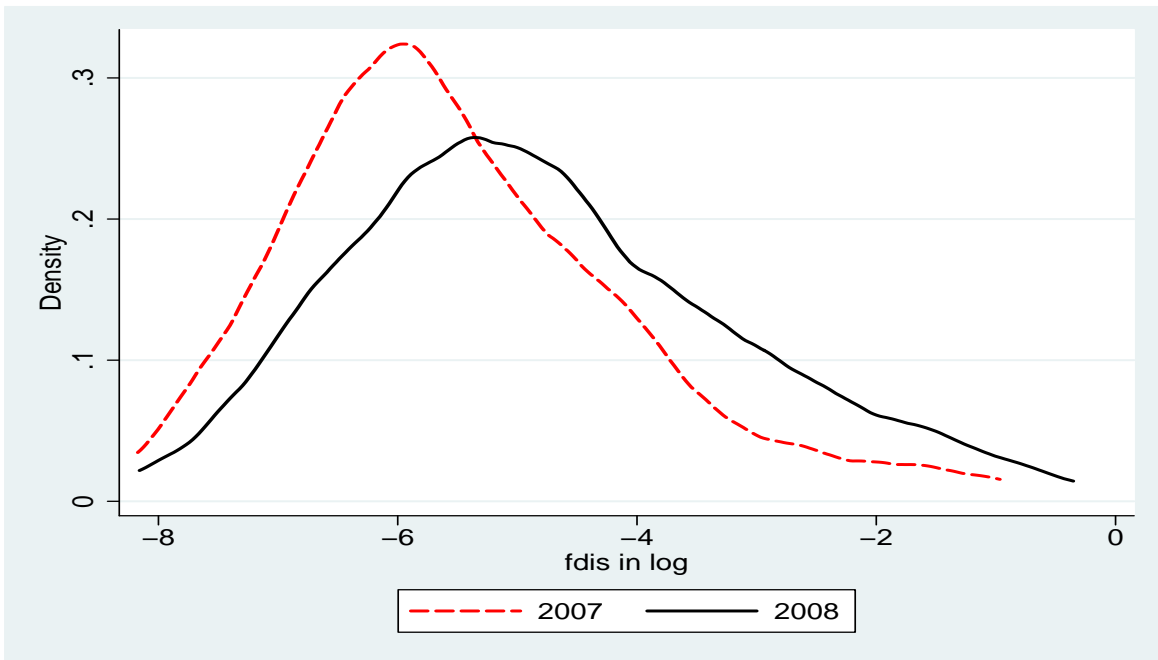
Notes: The left panel shows annual time series of `fdis_median`: the median of forecasts dispersions, calculated as cross-analyst standard deviations of forecasts of return on capital. The right panel exhibits annual time series of `ferror_med`: the median of forecast errors, calculated as the difference between realised return on capital and the median forecasts.

Figure 2 - Distribution of return on capital forecasts



Notes: The figure plots the distribution of the uncertainty measure (in level), calculated as the cross-analyst standard deviations of forecasts of return on capital normalized by the median forecast, for 2007 (dash, red) and 2008 (solid, black) respectively.

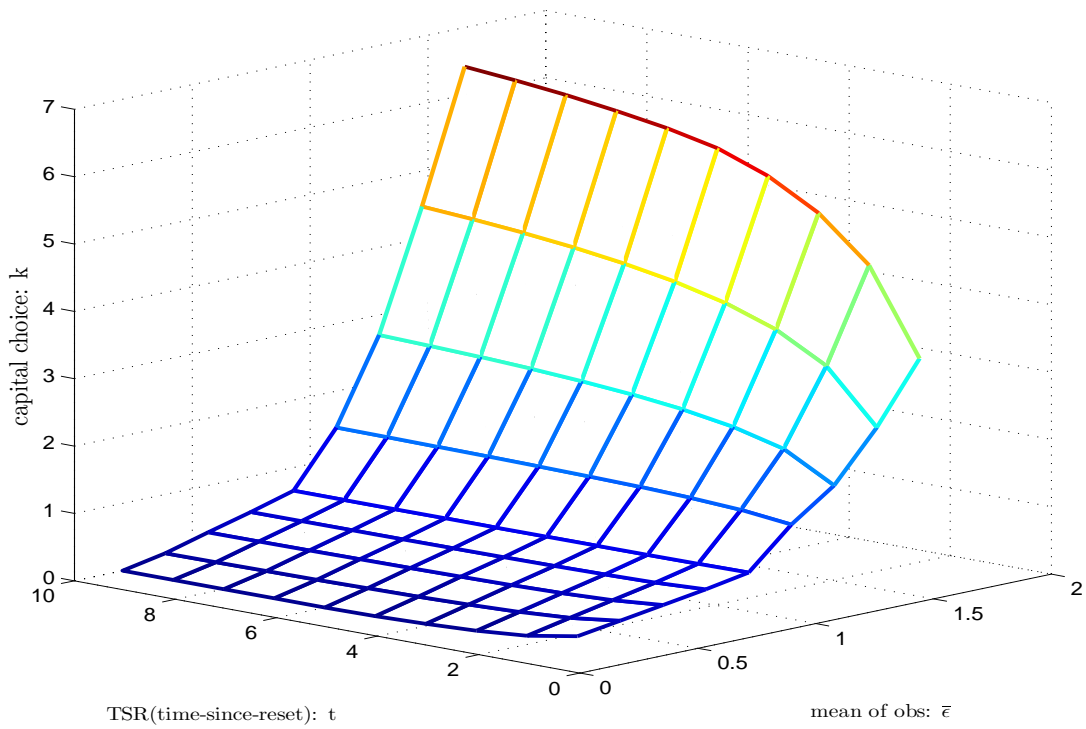
Figure 3 - Distribution of return on capital forecasts (log-transformed)



Notes: The figure plots the distribution of the uncertainty measure (log-transformed), calculated as the cross-analyst standard deviations of forecasts of return on capital normalized by the median forecast, for 2007 (dash, red) and 2008 (solid, black) respectively.

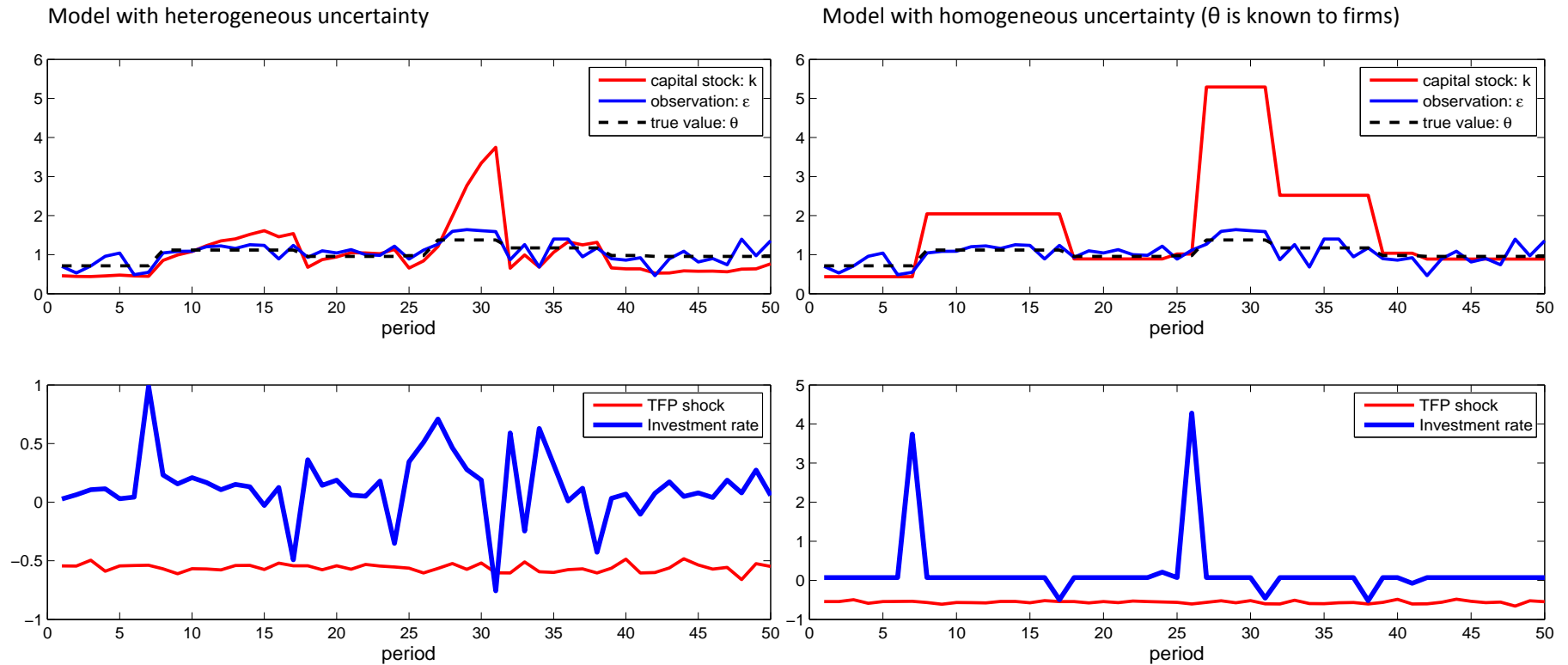


Figure 4 - Capital choice



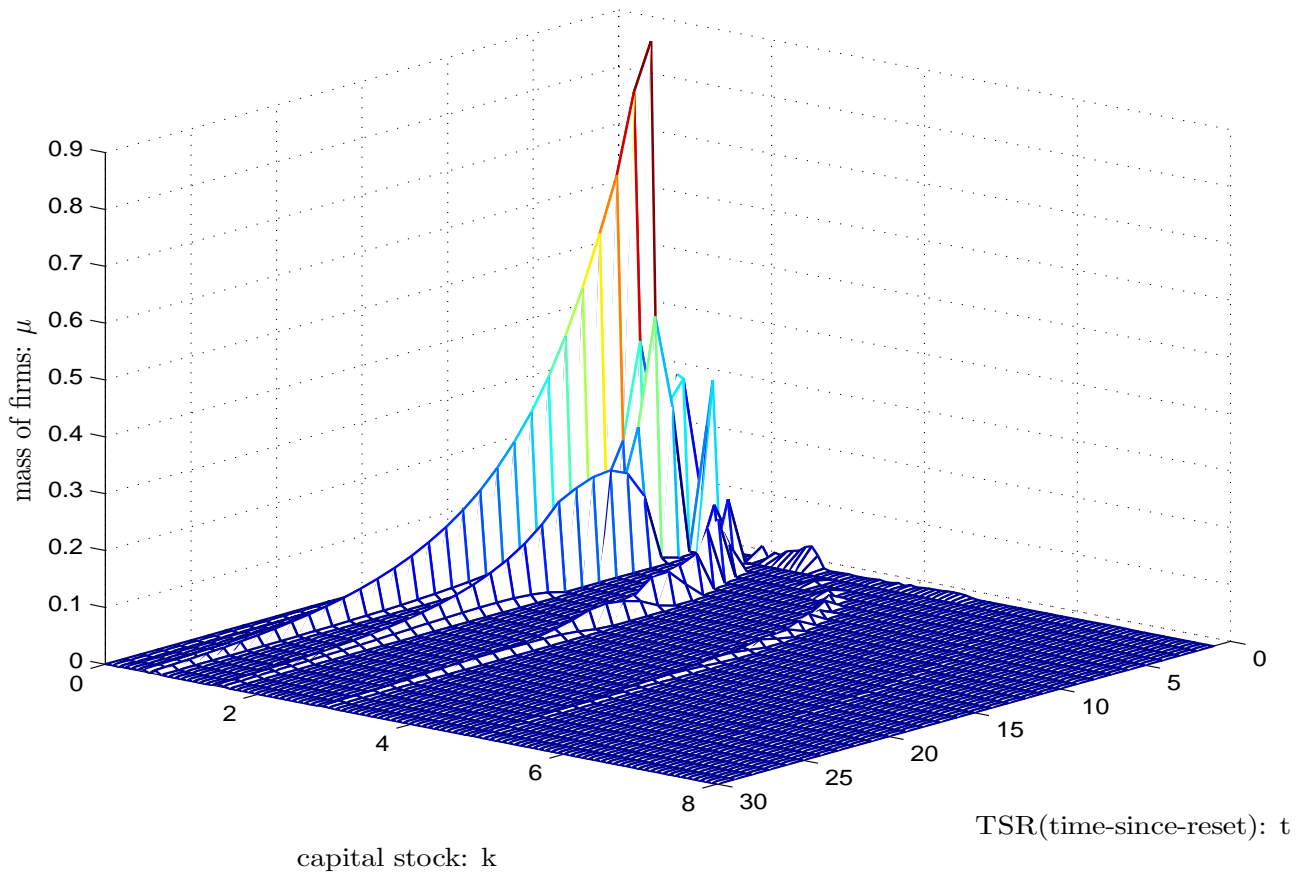
Notes: The figure shows each firm's capital choice as a function of the mean observation of idiosyncratic productivity and time-since-reset (TSR), from the model at steady state.

Figure 5 - Simulation results



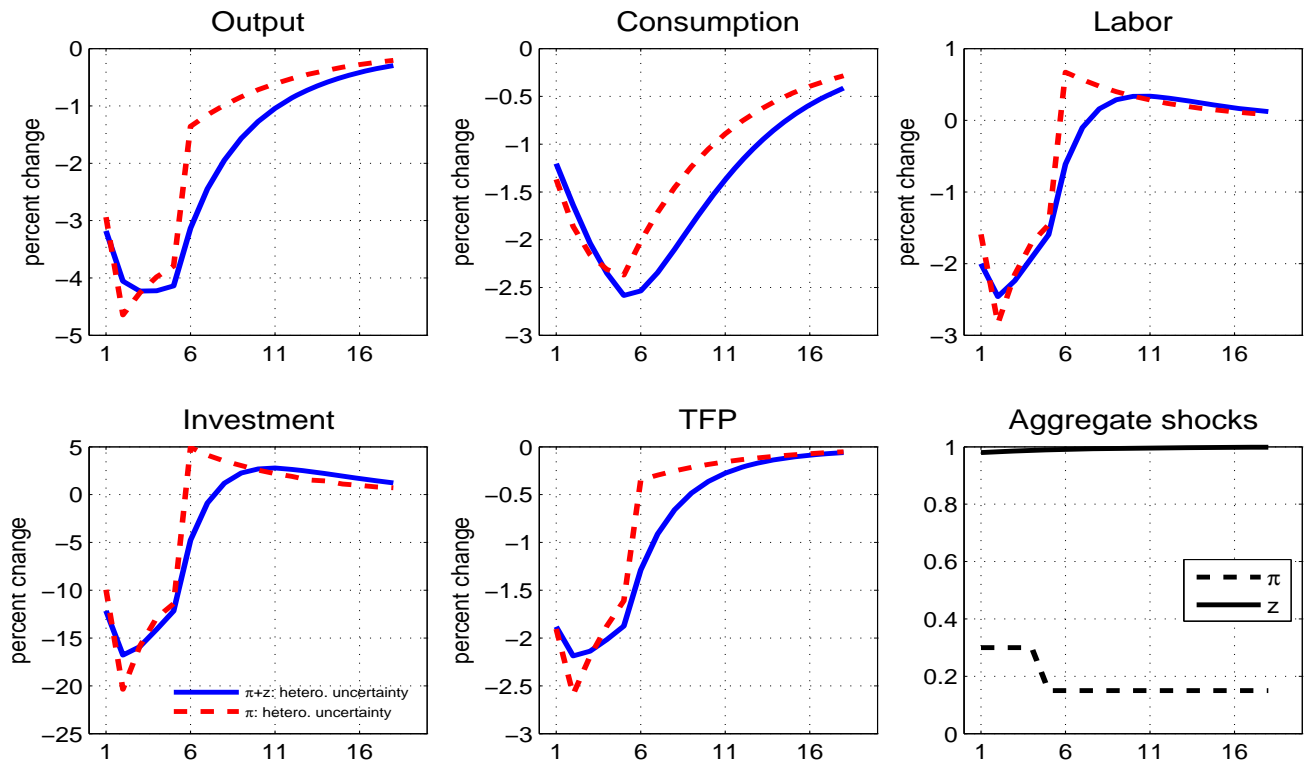
Notes: The figure plots the patterns of the behavior of firms throughout a simulation without aggregate shocks. A 50 period simulation result for one firm is taken from an entire simulation (2,000 firms for 1,500 periods). The upper panel shows a series of capital stock, idiosyncratic productivity (observed), and the base component (unobserved), respectively, in levels. The lower panel shows a series of TFP growth rates and investment rates in percentages.

Figure 6 - Stationary distribution of firms



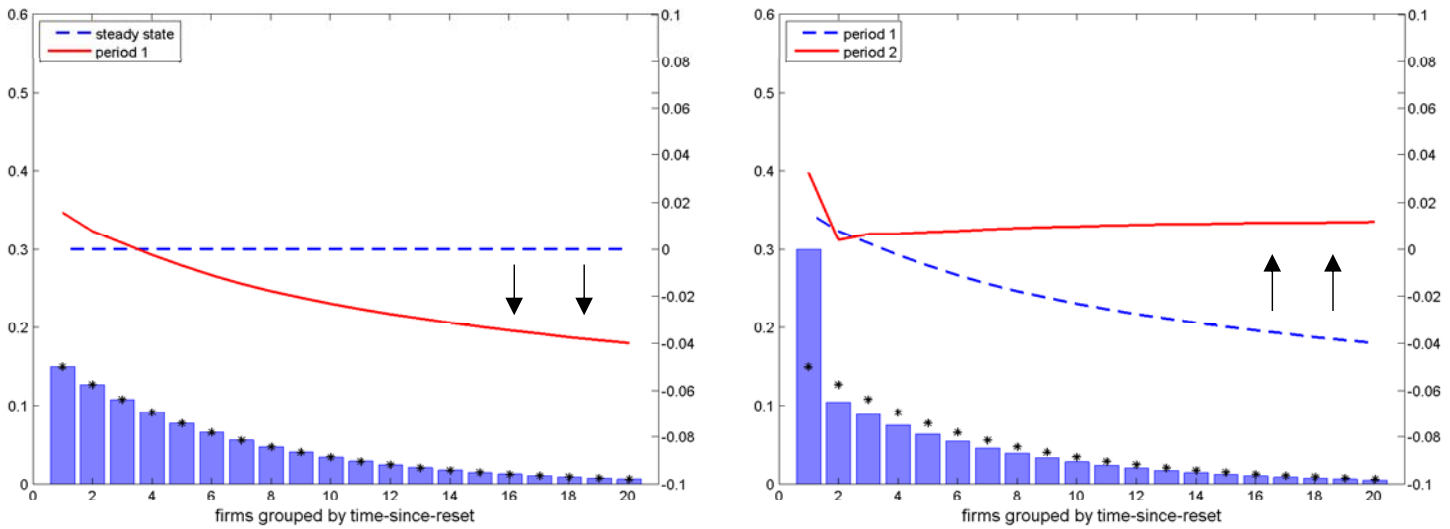
Notes: The figure shows the stationary distribution of firms over capital stock and time-since-reset (TSR).

Figure 8 - The Great Recession simulations with/without imperfect information



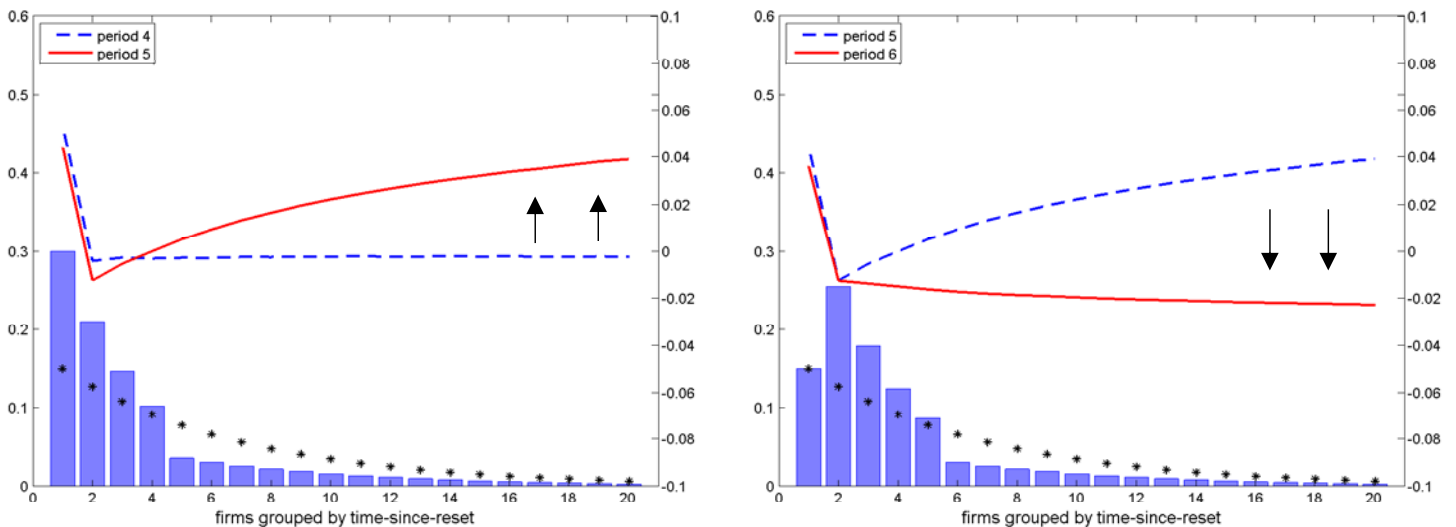
Notes: Each panel except the lower right one plots the aggregate economy's responses to both aggregate TFP shocks and uncertainty shocks, plotted by a blue line and labeled as  $\pi+z$ : *hetero. uncertainty*, the responses of a model without imperfect information, plotted by a red line and labeled as  $\pi$ : *homo. uncertainty*. The aggregate shocks are plotted in the lower right panel: no aggregate TFP shocks and uncertainty shocks (the high  $\pi$  for four periods and the low  $\pi$  in the rest of the periods).

Figure 12 - Dynamics of firm distribution and investment (% change from SS) onset of recessions



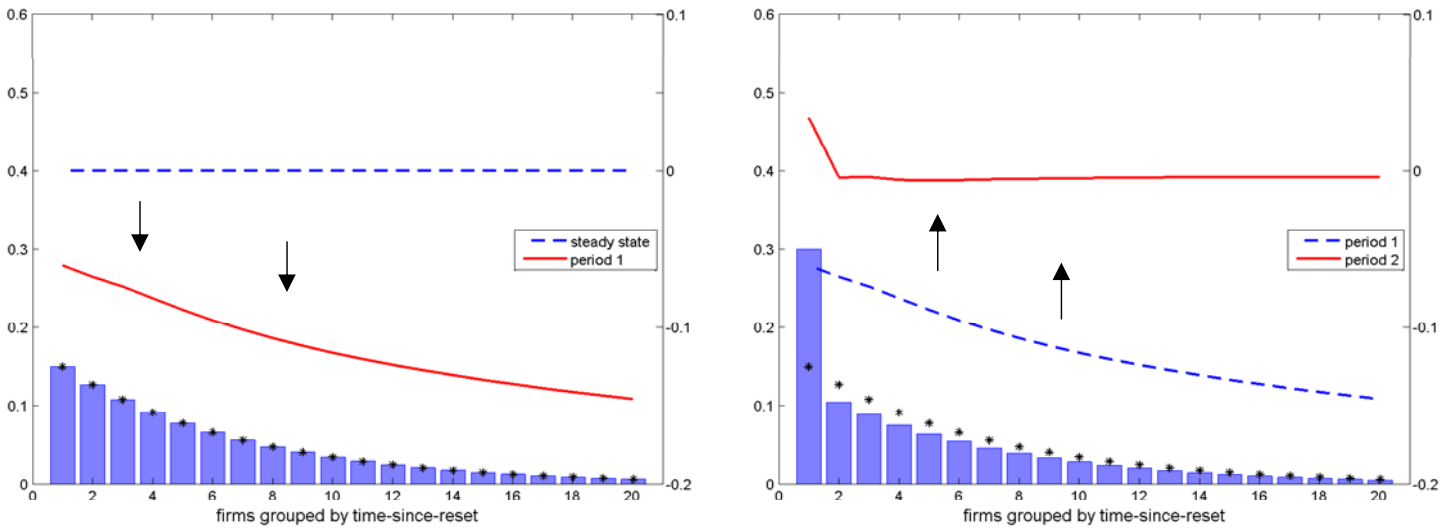
Notes: Each bin represents the mass of firms in each cohort grouped by the time-since-reset of their base component (left axis). Recall a larger time-since-reset implies that firms are more informed about their productivity levels. Each line plots the average percentage change in investment relative to the steady state for each cohort (right axis). Each dot shows the steady-state mass of firms in each time-since-reset bin.

Figure 13 - Dynamics of firm distribution and investment (% change from SS) at the recovery phase



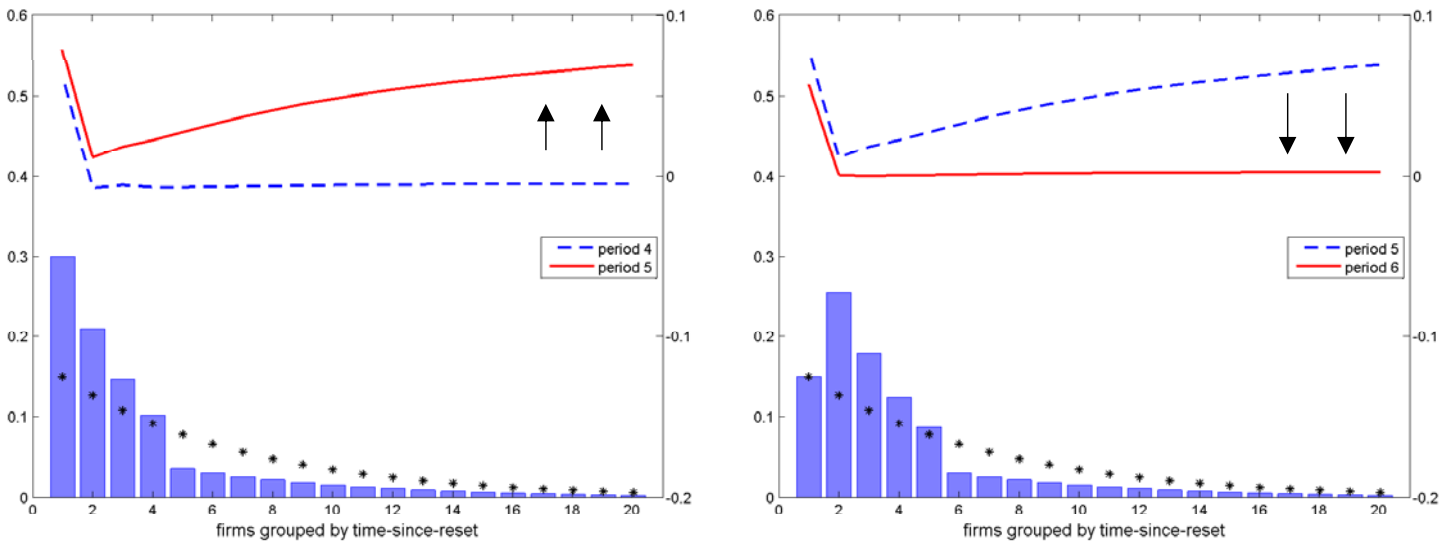
Notes: Each bin represents the mass of firms in each cohort grouped by the time-since-reset of their base component (left axis). Recall a larger time-since-reset implies that firms are more informed about their productivity levels. Each line plots the average percentage change in investment relative to the steady state for each cohort (right axis). Each dot shows the steady-state mass of firms in each time-since-reset bin.

Figure 14 - Dynamics of the distribution onset of recessions (partial equilibrium)



Notes: Each bin represents the mass of firms in each cohort grouped by the time-since-reset of their base component (left axis). Recall a larger time-since-reset implies that firms are more informed about their productivity levels. Each line plots the average percentage change in investment relative to the steady state for each cohort (right axis). Each dot shows the steady-state mass of firms in each time-since-reset bin.

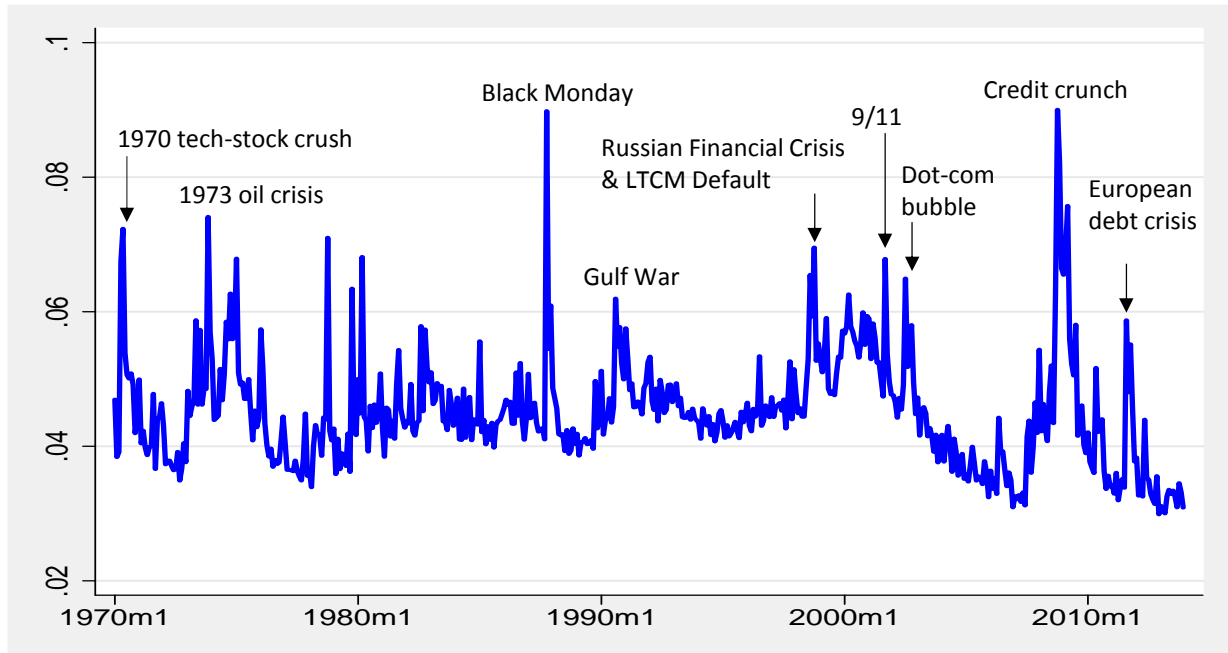
Figure 15 - Dynamics of the distribution at the recovery phase (partial equilibrium)



Notes: Each bin represents the mass of firms in each cohort grouped by the time-since-reset of their base component (left axis). Recall a larger time-since-reset implies that firms are more informed about their productivity levels. Each line plots the average percentage change in investment relative to the steady state for each cohort (right axis). Each dot shows the steady-state mass of firms in each time-since-reset bin.

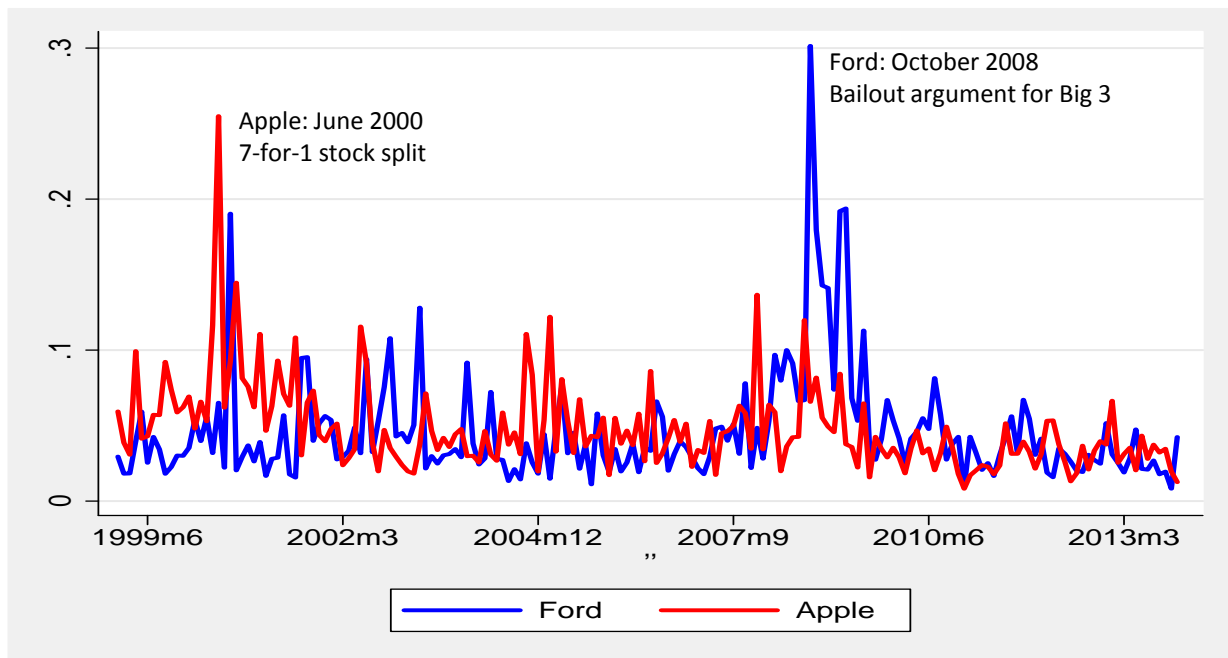
# Appendix

Figure A1 - Aggregated uncertainty measure



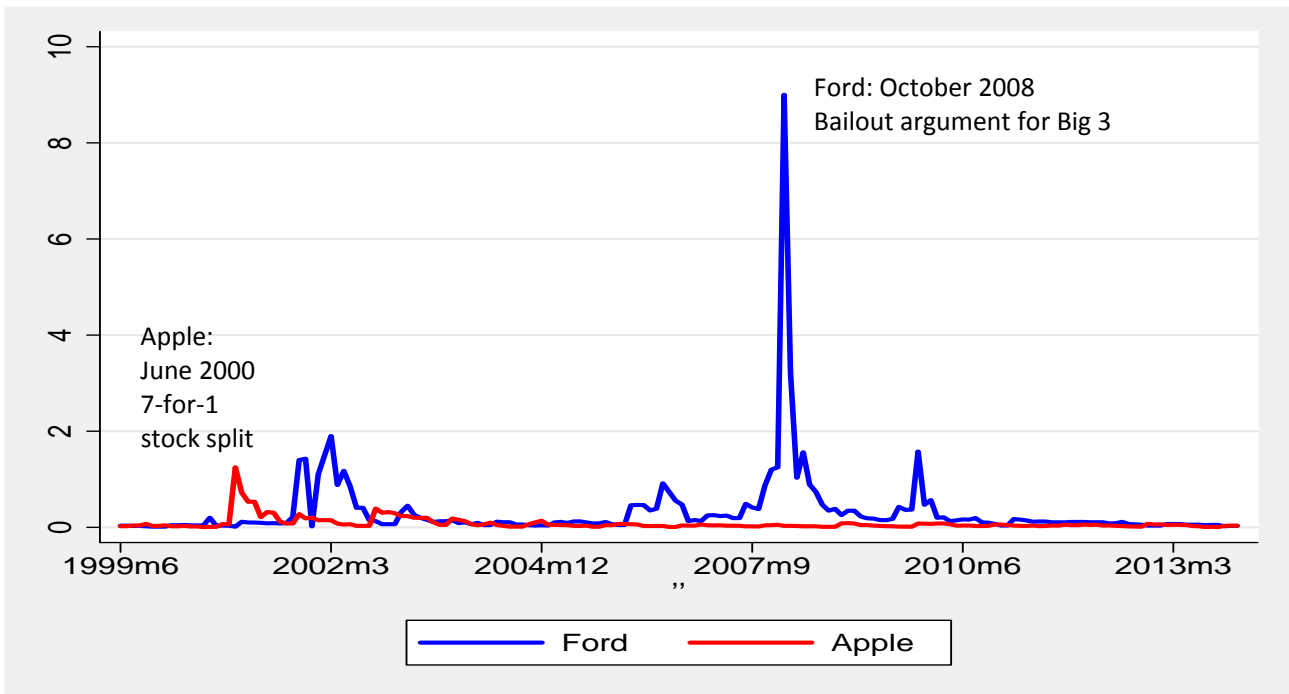
Notes: This figure plots the monthly U.S. stock price volatility, calculated as the mean of the volatility of prices of individual stocks that is normalised by the mean price during the same month. Data is taken from the Center for Research in Security Prices (CRSP).

Figure A2 - Disaggregated uncertainty measure: Ford Motor Company and Apple Inc.



Notes: This figure plots the monthly stock price volatility for Ford Motor Co. and Apple Inc., using data from the Center for Research in Security Prices (CRSP).

Figure A3 - Disaggregated uncertainty measure: Ford Motor Company and Apple Inc.



Notes: This figure shows the monthly EPS forecasts dispersions for Ford Motor Co. and Apple Inc., calculated as the cross-analyst standard deviation of EPS forecasts that is normalised by the median forecast. Data is taken from the I/B/E/S (Institutional Broker's Estimate System).