Search-Based Endogenous Illiquidity and the Macroeconomy*

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Abstract

We endogenize asset liquidity in a dynamic general equilibrium model with search frictions on asset markets. In the model, asset liquidity is tantamount to the ease of issuance and resaleability of private financial claims, which is driven by investors' participation on the search market. Limited funding ability of private claims creates a role for liquid assets, such as government bonds or fiat money, to ease funding constraints. We show that liquidity and asset prices positively co-move. When the capacity of the asset market to channel funds to entrepreneurs deteriorates, the hedging value of liquid assets increases. Our model is thus able to match the liquidity hoarding observed during recessions.

Keywords: endogenous asset liquidity; search frictions

classification: E22; E44; E58.

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1 Introduction

Illiquidity of privately issued financial assets arises from impediments to their issuance and later transactions. Empirical evidence points to procyclical variation in the market liquidity of a wide range of financial assets.[1] The view that asset liquidity dries up during recessions has been further reinforced by the 2007-2009 financial crisis, when illiquidity problems were most pronounced for commercial paper and asset-backed securities.[2]

Illiquid primary or secondary equity and debt markets reduce firms' ability to finance investment, which creates a role for liquid assets, such as fiat money or government bonds. These liquid assets provide insurance against funding constraints as they can be readily used for financing purposes at any time.[3] When funding constraints tighten in recessions, firms tend to rebalance their portfolios towards such liquid assets - a phenomenon referred to as "flight to liquidity". Variations in asset liquidity and the idea of liquidity hoarding as a hedging device against funding constraints goes back to Keynes (1936) and Tobin (1969). Nevertheless, the link between asset liquidity and aggregate fluctuations is often ignored in state-of-the-art dynamic general equilibrium models.

We propose a framework in which *endogenous* variation in asset liquidity interacts with macroeconomic conditions. To this end, we incorporate a search market for financial assets into an almost-standard real business cycle model. Search frictions give rise to asset illiquidity both on primary markets (issuance of new assets) and secondary markets (liquidation of existing assets). Asset liquidity can be measured by the endogenous fraction of new or existing assets that can be sold and the price impact of such trades. The search market structure in our model is a stand-in for financial intermediation via markets or banks, both of which involve a costly matching process between capital providers and seekers.

The model shows how a drop in investor participation in the search market simultaneously tightens funding constraints and decreases asset prices, which dampens real investment and production. Our central contributions are (i) to demonstrate both analytically and numerically that endogenizing liquidity is essential to generate co-movement between asset

[1] Studies by Huberman and Halka (2001), Chordia, Roll, and Subrahmanyam (2001), Chordia, Sarkar, and Subrahmanyam (2005) and Naes et al. (2011) assert that market liquidity is procyclical and highly correlated across asset classes such as bonds and stocks in the US. This observation implies that common factors drive liquidity.

[2] Dick-Nielsen, Feldhütter, and Lando (2012) identify a break in the market liquidity of corporate bonds at the onset of the sub-prime crisis. The liquidity component of spreads of all but AAA rated bonds increased and turnover rates declined, making refinancing on market more difficult. Commercial paper (CP), which is largely traded on a search market with dealers as match-makers, experienced large illiquidity in recessions reported by Anderson and Gascon (2009). In addition, money market mutual funds, the main investors in the CP market, shifted to highly liquid and secure government securities. Finally, Gorton and Metrick (2012) show that the repo market has registered strongly increasing haircuts during the crisis.

[3] In fact, U.S. nonfinancial firms only fund 35% of fixed investment through financial markets, of which 76% through debt and equity issuance and 24% through portfolio liquidations (Ajello, 2012).

liquidity and asset prices; and (ii) to show that shocks to the cost of financial intermediation are an important source of flight to liquidity and business cycles.

Consider an economy where privately issued financial claims are backed by cash flow from physical capital used for production. There is a continuum of households whose members are temporarily separated during periods. Some become workers, others entrepreneurs. Only the latter have access to investment opportunities for capital good creation. All household members are endowed with a portfolio of liquid assets (money)[4] and private claims, which we interpret as a catch-all for privately issued assets such as corporate bonds and equity.

To finance investment, entrepreneurs exploit all available modes of funding: They issue new financial claims to their investment projects and liquidate their existing asset portfolio. Money is readily available for financing purposes and hence commands a liquidity premium. Private claims (both new and old) are only partially liquid, because they are traded on the search market. Asset liquidity is measured by the fraction of private claims that can be sold or resold on this market in a given period and the price impact of transactions. Due to the limited funding from asset markets, entrepreneurs are financing constrained and cannot fund the first-best level of investment.

Participation in the search market is costly for both buyers and sellers. Individual buyers and sellers are matched by an intermediary who determines the transaction price by maximizing the total match surplus, similar to the bargaining process in the labor search literature (e.g., Diamond (1982), Mortensen and Pissarides (1994), and Shimer (2005)). This structure intends to emulate the features of over-the-counter (OTC) markets, in which a large fraction of corporate bonds, asset-backed securities, and private equity is traded. Participation costs in these markets arise from information acquisition as well as brokerage and settlement services provided by dealers and market makers.[5] Alternatively, our framework can also be interpreted as a reduced-form approach towards modeling bank-based financial intermediation. In particular, the search market structure captures the costly matching process between savers (investors) and the corporate sector through financial intermediaries.

We consider two types of exogenous shocks: an aggregate productivity shock and a symmetric shock to the market participation costs of buyers and sellers, which we interpret as an "intermediation cost shock".

Negative aggregate productivity (TFP) shocks decrease the return to capital, make investment into capital goods less attractive, and, hence crowd out investors from the search market. Negative intermediation cost shocks, on the other hand, make investment into liquid

[4]For simplicity, we consider all government-issued assets as money. Our framework could easily be extended to general interest bearing liquid assets as illustrated in the model section.

[5]See, for example, Dick-Nielsen, Feldhütter, and Lando (2012), Bao, Pan, and Wang (2011), Anderson and Gascon (2009).

assets more attractive. This reduces the incentive for investors to post costly buy orders on the search market. In either case, the fall in demand on the asset market exceeds that of supply (under some regularity conditions), such that sellers have a lower chance of encountering a buyer. Hence, the sales rate - or liquidity - of financial claims drops. Because a lower sales rate implies that entrepreneurs need to retain a larger equity stake in new investment projects, their financing constraints tighten and the option of breaking off negotiations becomes less valuable. Entrepreneurs are thus willing to accept a lower transaction price. In the aggregate, lower asset liquidity and prices restrict the funding available to entrepreneurs and, thereby, reduce real investment.

While both shocks generate procyclical asset liquidity and prices, only intermediation cost shocks induce a pronounced flight to liquidity. In the case of persistent adverse TFP shocks, investors have a weaker incentive to hedge against future illiquidity of private claims, because of lower current and future returns to capital. Adverse intermediation cost shocks, however, do not deteriorate the quality of investment itself either today or tomorrow. Investors thus value the hedging service provided by liquid assets more strongly. Therefore, asset price movements are stronger. Intermediation cost shocks thus allow the model to match the the volatility of asset prices and liquidity hoarding and their co-movement with GDP in the data.

To our knowledge, we are the first to incorporate endogenous asset liquidity in a dynamic macroeconomic model in a tractable way and to explore the feedback effects between asset liquidity and the real economy.[6] Notice that Kiyotaki and Moore (2012) (henceforth, KM) demonstrate how *exogenous* asset market liquidity interact with aggregate fluctuations, in which firms can only sell an *exogenous* fraction of private claims to finance new investment. In contrast, money or government bonds can be fully sold and thus provide a liquidity service. When the resaleable fraction of private claims falls, financing constraints tighten and agents shift to money. As a result, real investment and production fall, and a recession begins.

However, as pointed out by Shi (2012), exogenous liquidity variations leads to counterfactual asset price dynamics: A negative shock to asset resaleability reduces the supply of financial assets, while demand remains relatively stable since the quality of investment projects is unaffected by liquidity shocks. The negative supply shock induces an persistent asset price boom that is at odds with the data. This counterfactual highlights the need to model asset liquidity endogenously, as we do in this paper.

Relationship to the Literature. Following KM, we model liquidity differences between private claims and government-issued assets. As highlighted by them, the irrelevance

[6]A recent study by Yang (2013) also considers endogenous asset liquidity. The difference is that we model liquid and illiquid assets together and the corresponding portfolio choice simultaneously.

result of Wallace (1981) on the neutrality of central banks' portfolios no longer holds in such a setting. In fact, open market operations that change the composition of liquid and illiquid assets in agents' portfolios have real effects. Del Negro, Eggertsson, Ferrero, and Kiyotaki (2011) analyze such "unconventional policy" after an exogenous fall in liquidity in an extended version of KM's model. With standard monetary policy constrained by the "Zero Lower Bound", liquidity injections effectively dampen the liquidity shortfall and stabilize the economy.[7]

Highlighted by Shi (2012), negative exogenous liquidity shocks lead to counterfactual asset price boom. We thus build a model with endogenous liquidity. The search literature provides a natural theory of endogenous liquidity as in Lagos and Rocheteau (2009) and has been applied to a wide range of markets such as housing[8], OTC markets for asset-backed securities, corporate bonds, federal funds, private equity, etc.[9] This line of research shows that search frictions can explain substantial variation in a wide range of measures of asset market liquidity (e.g.,bid-ask spreads, trade volume, and trading delays).

Work by Haan, Ramey, and Watson (2003), Wasmer and Weil (2004), and Petrosky-Nadeau and Wasmer (2013) has emphasized the role of search and matching frictions in credit markets and their impact on aggregate dynamics,[10] but does not study asset prices and portfolio rebalancing. In contrast, we highlight that search frictions have important implications for asset saleability and prices, portfolio compositions, and hence on the real economy.

An alternative approach to endogenizing liquidity uses information frictions. Eisfeldt (2004) develops a partial equilibrium model with adverse selection in asset markets, in which investment and trading volume are amplified if asset liquidity endogenously varies with productivity. Dynamic adverse selection, asset prices, and trading delays are analyzed in an endowment economy framework by Guerrieri and Shimer (2012). While endogenizing asset liquidity, these studies do not consider the feedback effects of fluctuations in liquidity on production and employment. A notable exception is Kurlat (2013), who extends KM with endogenous resaleability through adverse selection but neglecting the role of liquid assets. In Eisfeldt and Rampini (2009) firms need to accumulate liquid funds in order to finance investment opportunities. While the supply of liquid assets affects investment, secondary markets for asset sales are shut off as an alternative means of financing. In contrast to these contributions, we jointly model endogenous liquidity on primary and secondary asset

[7]More generally, Kara and Sin (2013) show that market liquidity frictions induce a trade-off between output and inflation stabilization off the ZLB that can be attenuated by quantitative easing measures.

[8]See e.g. Wheaton (1990) and Ungerer (2012).

[9]See e.g. Duffie, Gârleanu, and Pedersen (2005, 2007); Ashcraft and Duffie (2007); Feldhutter (2011).

[10]As shown in Beaubrun-Diant and Tripier (2013), search frictions also help explain salient business cycle features of bank lending relationships, such as countercyclical net interest margins and loan separation rates.

markets, the role of liquid assets as the lubricant of investment financing, and asset liquidity's feedback effects on business cycles.[11]

Our framework also differs along important dimensions from search-theoretic models of money such as Lagos and Wright (2005) and Rocheteau and Wright (2005). In this literature, money has a transaction function in anonymous search markets. Recent extensions include privately created liquid assets such as claims to capital (Lagos and Rocheteau, 2008) or bank-deposits (Williamson, 2012) as media of exchange. Our framework rather emphasizes the role of financial assets - both public and private - as stores of value, i.e. money and equity claims are used for financing purposes. Moreover, our approach is able to generate endogenous variation in asset liquidity and the associated premia, because private claims are subject to search frictions themselves, rather than serving to overcome such frictions on other markets. These differences notwithstanding, a common tenet is that liquid assets play an important role in economic transactions by relaxing deep financial frictions.

By studying intermediation cost shocks which affect asset market liquidity, we also compliment the literature on financial shocks. Recent contributions by Jermann and Quadrini (2012), Christiano, Motto, and Rostagno (2014), and Jaccard (2013) identify financial shocks as an important source of business cycle fluctuations. Our approach shows how such shocks may be endogenously amplified within financial markets.

2 The Environment

This model is a variant of a standard real business cycle (RBC) model. Time is discrete and infinite ($t = 0, 1, 2, ...$). The economy has three sectors: final goods producers, households (with entrepreneurs and workers), and financial intermediaries. Final goods producers generate output by renting capital and labor from households.[12] Financial intermediaries faciliate asset trasaction, and there are search frictions afflicting the purchase and sale of financial assets issued by previous and current entrepreneurs. In addition, liquid government-issued assets can be traded on a spot market. To abstract from government policies, we model liquid assets as non-interest bearing money. We focus on an equilibrium in which this intrinsically worthless asset is valued for its liquidity service and hence accepted by all market participant.[13]

[11] Asset illiquidity may further interact with financing constraints to induce delays in asset sales as in Cui (2013). This interaction prolongs shocks to the financing conditions of the private sectors and results in countercyclical productivity dispersion.

[12] They rebate profits back to households. In equilibrium, profits are zero because of perfect competitions.

[13] The derivation of the model with interest-bearing government bonds and taxation is available upon request.

2.1 Final Goods Producers

Competitive firms rent aggregate capital stock K_t and hire aggregate labor N_t from households to produce output (general consumption goods) according to

$$Y_t = e^{z_{a,t}} F\left(K_t, N_t\right),$$

where $F\left(K_t, N_t\right) = K_t^{\alpha} N_t^{1-\alpha}$, $\alpha \in (0,1)$, $z_{a,t}$ measures exogenous aggregate productivity. The profit-maximizing rental rate and wage rate are thus

$$r_t = e^{z_{a,t}} F_K(K_t, N_t), \quad w_t = e^{z_{a,t}} F_N(K_t, N_t). \tag{1}$$

2.2 Households

Before going to the details, we illustrate the basic household problem. Households are comprised of entrepreneurs (who invest in new capital) and workers (who save). Workers earn wages by supplying labor. Entrepreneurs do not work, but only they have investment opportunities. They issue new claims and/or sell existing claims to finance new investment, to the extent possible. Claims that are not money need to be issued or resold through intermediation with a search technology. These claims are illiquid in the sense that for every unit of capital put on sale only a fraction $\phi_{u,t}$ are sold. Financial frictions are thus represented by the fact that entrepreneurs have to retain $(1 - \phi_{u,t})$ of new investment which can be put up for sale in the same period as it is being incurred.

2.2.1 A Representative Household

At the beginning of t, the aggregate productivity and the unit cost of trading private claims are realized. A representative household specifies policy rules for its members, who receive equal shares of assets accumulated from previous periods. Then, they receive a shock that determines their type, which is idiosyncratic across members and through time. With a probability χ a member becomes an entrepreneur (called type u), and with a probability $(1-\chi)$ a worker (called type v).[14] By the law of large numbers, each household thus consists of a fraction χ of entrepreneurs and a fraction $(1-\chi)$ of workers. Both groups are temporarily separated during each period and there is no consumption insurance between them.

In the middle of t, final goods producers rent capital and labor from households to produce

[14]Following the notation in the labor search literature, we denote workers as type v and entrepreneurs as type u members. The underlying logic is that workers post purchase orders on the search market which are akin to "vacant" asset positions, while entrepreneurs post assets for sale which are in a sense "unemployed" when lying idle on their balance sheets.

consumption goods and the payoffs from private claims are thus realized. At the same time, household members trade liquid assets on a competitive market in exchange for consumption goods. Entrepreneurs put assets on sale and workers put buying quotes of assets, both through financial intermediations. Then, financial intermediation match potnential buyers and sellers and intermediate a transaction price. Entrepreneurs then invest in physical capital, after which workers and entrepreneurs consume.

At the end of t, members come together again to share their accumulated assets. All members hence enter the next period with an equal share of their household's assets.[15]

Preferences. The household objective is to maximize

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} \left[U(c_{u,t+s}, c_{v,t+s}) - (1-\chi) h(n_{t+s}) \right], \tag{2}$$

where $U(c_{u,t}, c_{v,t}) = \chi u(c_{u,t}) + (1-\chi) u(c_{v,t})$ is the total utility derived from consumption by entrepreneurs ($c_{u,t}$) and workers ($c_{v,t}$). $u(.)$ is a standard strictly increasing and concave utility function, and $h(.)$ captures the dis-utility derived from labor supply n_t. $\mathbb{E}_t$ is the expectation operator conditional on information at time t.

Balance Sheet. Physical capital (K_t) is owned by households and rented to final goods producers such that capital earns a return. There is a claim to the future return of every unit of capital, which household members can either retain or offer for sale to outside investors. These claims, if successfully sold, can be sold at unit price q_t (determined by the intermediation). In addition, households could hold money with nominal price level P_t. Hence, at the onset of period t, households own a portfolio of liquid assets, equity claims on other households' return on capital, and own physical capital. These assets are financed by net worth plus equity claims issued against their own physical capital. Since new claims and old claims are both traded on the search market, we only need to keep track of net equity, defined as

$$S_t = \underbrace{S_t^O}_{\text{equity claims on others' capital}} + \underbrace{K_t - S_t^I}_{\text{unmortgaged capital}} .$$

Since we normalize equity by capital stock, equity depreciates with capital stock at the same rate (denoted by δ). The financing structure gives rise to the beginning-of-period balance sheet in Table 1.

[15] The representative household with temporarily separated agents has been introduced in Lucas (1990) and applied to the KM framework in Shi (2012) and Del Negro, Eggertsson, Ferrero, and Kiyotaki (2011).

Table 1: Household's Balance Sheet

other's equity	$q_t S_t^O$		
capital stock	$q_t K_t$	net worth	$q_t S_t + B_t/P_t$

2.2.2 Individual Members

For simplicity, at the beginning of period t, household members are given *equal share* of equity and money from the household. Let s_{jt} and b_{jt} be net equity and money for a typical household member j. Then, the net equity evolves according to

$$s_{j,t+1} = (1-\delta)s_{j,t} + i_{j,t} - m_{j,t}, \tag{3}$$

where $i_{j,t}$ is investment into capital goods, and $m_{j,t}$ corresponds to asset sales. Let $c_{j,t}$ and $n_{j,t}$ denote consumption and labor supply, respectively.

Workers flow-of-funds. The household delegates equity purchases on the search market to workers, because they do not have investment opportunities ($i_{v,t} = 0$). Therefore, workers $j = v$ post asset positions v_t to acquire new or old equity at unit cost κ_v. On the search market, each posted position is filled with a probability $\phi_{v,t} \in [0,1]$, and an individual buyer expects to purchase an amount $m_{v,t} = -\phi_{v,t} v_t$. Notice that a worker's flow-of-funds constraint reads

$$c_{v,t} + \kappa_v v_t + \frac{b_{v,t+1}}{P_t} = w_t n_{v,t} + r_t s_{v,t} - q_t \phi_{v,t} v_t + \frac{b_{v,t}}{P_t}, \tag{4}$$

where labor income and the return on equity and money are used to finance consumption, search costs, and the new accumulation of equity claims and money. To simplify, we define the effective purchasing price of per equity as

$$q_{v,t} \equiv q_t + \frac{\kappa_v}{\phi_{v,t}}, \tag{5}$$

where q captures the transaction price and $\frac{\kappa_v}{\phi_v}$ represents search costs per transaction (scaled by the probability of encountering a seller ϕ_v). By using (3) and $m_{v,t} = \phi_{v,t} v_t$, the flow-of-funds constraint (4) becomes

$$c_{v,t} + q_{v,t} s_{v,t+1} + \frac{b_{v,t+1}}{P_t} \quad = \quad w_t n_{v,t} + r_t s_{v,t} + (1-\delta) q_{v,t} s_{v,t} + \frac{b_{v,t}}{P_t}. \tag{6}$$

Entrepreneurs' flow-of-funds. Rather than purchasing equity claims on the search market, entrepreneurs $j = u$ decide how many assets u_t to put up for sale at unit cost κ_u in order to finance new investment ($i_{u,t} > 0$). These assets include existing equity claims on other households' capital stock and their own unissueld capital stock (in total $s_{u,t}$), plus claims on new investment, $i_{u,t}$. Then, the amount of private financial claims that are up for sale is bounded from above by the existing stock of equity and the volume of new investment, $u_t \leq (1-\delta)\, s_{u,t} + i_{u,t}$. Offers are matched with a buyer with probability $\phi_{u,t} \in [0,1]$. Therefore, an individual entrepreneur expects to sell $m_{u,t} = \phi_{u,t} u_t$. Notice that the returns on equity and money are used to finance consumption, search costs, and the accumulation of equity (with new investment taken into account) and new holding of money. The flow-of-funds constraint can thus be written as

$$c_{u,t} + i_{u,t} + \kappa_u u_t + \frac{b_{u,t+1}}{P_t} = r_t s_{u,t} + q_t \phi_{u,t} u_t + \frac{b_{u,t}}{P_t}, \tag{7}$$

If we define the effective selling price of unit financial asset as

$$q_{u,t} \equiv q_t - \frac{\kappa_u}{\phi_{u,t}}, \tag{8}$$

and by using (3) and $m_{u,t} = \phi_{u,t} u_t$, then the flow-of-funds constraint (7) becomes

$$c_{u,t} + i_{u,t} + q_{u,t}\left[s_{u,t+1} - i_{u,t} - (1-\delta)s_{u,t}\right] + \frac{b_{u,t+1}}{P_t} \quad = \quad r_t s_{u,t} + \frac{b_{u,t}}{P_t}. \tag{9}$$

Further, It is helpful to substitute out new investment by defining $e_{u,t}$, where $e_{u,t} \in [0,1]$ denotes the fraction of total assets that entrepreneurs put on sale

$$u_t = e_{u,t}[(1-\delta)s_{u,t} + i_{u,t}].$$

Then, we can express flow-of-funds constraint (9) as

$$c_{u,t} + q_{r,t} s_{u,t+1} + \frac{b_{u,t+1}}{P_t} = r_t s_{u,t} + \left[e_{u,t}\phi_{u,t} q_{u,t} + (1 - e_{u,t}\phi_{u,t}) q_{r,t}\right](1-\delta)s_{u,t} + \frac{b_{u,t}}{P_t}, \tag{10}$$

$$\text{where} \quad q_{r,t} \equiv \frac{1 - e_{u,t}\phi_{u,t} q_{u,t}}{1 - e_{u,t}\phi_{u,t}}. \tag{11}$$

The left-hand side (LHS) of (10) captures entrepreneurs' spending on consumptions and accumulations of equity and money, while the right-hand side (RHS) represents entrepreneurial (total) *net-worth* including rental income from capital claims, real value of money, and the value of existing claims. Note that a fraction $e_{u,t}\phi_{u,t}$ is saleable and, hence, valued at $q_{u,t}$, while a fraction $(1 - e_{u,t}\phi_{u,t})$ is retained and valued at $q_{r,t}$, which is the effective replacement cost of existing assets. To see this, notice that entrepreneurs can sell a fraction $e_u\phi_{u,t}$ of their financial assets at price $q_{u,t}$. For every unit of new investment, they will accordingly need to make a down-payment $(1 - e_{u,t}\phi_{u,t}q_{u,t})$ and retain a fraction $(1 - e_{u,t}\phi_{u,t})$ as inside equity. With this interpretation, if entrepreneurs replace existing assets by new assets issued against investment, $q_{r,t}$ is indeed the effective replacement cost only.[16]

Because of the comparison with worker's budget constraint, (10) involves gross investment. New investment can be backed out from $s_{u,t+1} = (1 - e_{u,t}\phi_{u,t})(s_{u,t} + i_t)$ and (10)

$$i_{u,t} = \frac{\left[(r_t + e_{u,t}\phi_{u,t}q_{u,t}(1-\delta))\, s_{u,t} + \frac{b_{u,t}}{P_t}\right] - c_{u,t}}{1 - e_{u,t}\phi_{u,t}q_{u,t}}. \tag{12}$$

which says that entrepreneurs' liquid net-worth net of consumption can be levered at $(1 - e_u\phi_u q_u)^{-1}$ to invest in new capital.

2.2.3 A Household's Problem

Aggregation. Recall that $j \in \{u, v\}$ indicates workers and entrepreneurs, respectively. We define aggregate type-specific variables as $X_{u,t} \equiv \chi x_{u,t}$ and $X_{v,t} \equiv (1-\chi)\, x_{v,t}$. Household-wide variables is the aggregation of workers' and entrepreneurs' quantities, i.e., $X_t = X_{v,t} + X_{u,t}$. For example, aggregate consumption is the sum of aggregate consumption of workers and entrepreneurs, i.e., $C_t = C_{v,t} + C_{u,t}$.

For simplicity, we switch to recursive notation, i.e., let x and x' denote x_t and x_{t+1}. Note that all household members equally divide the assets accumulated before, $S_u = \chi S$, $S_v = (1-\chi)S$, $B_u = \chi B$, $B_v = (1-\chi)B$. Since entrepreneurs do not work, we also have that $N = N_v$. Given these simplifications, individual budget constraints (6) and (10) aggregate to[17]

$$C_v + q_v S_v' + \frac{B_v'}{P} = wN + \left[r + q_v(1-\delta)\right](1-\chi)\, S + (1-\chi)\,\frac{B}{P}. \tag{13}$$

$$C_u + q_r S_u' + \frac{B_u'}{P} = \left[r + \left[\phi_u q_u + (1-\phi_u)\, q_r\right](1-\delta)\right]\chi S + \chi\frac{B}{P}, \tag{14}$$

Since every entrepreneur chooses the same $e = e_u$, total investment can be aggregated from

[16] This down-payment price captures the effect of search costs on equity accumulation: higher search costs decrease the effective sales price, which increases the down-payment that in turn depresses equity accumulation. Therefore, the entrepreneurs' ability to leverage will be lower if search costs are higher.

[17] Notice that we implicitly impose that $S_v' \geq (1-\delta)(1-\chi)S$ such that workers in a household are always buyers. Such condition is satisfied in our later numerical analysis because we focus on shocks that will not push workers to sell assets to smooth consumption. Aggregation takes into account type-specific transactions on the search market and evolutions of equity.

(12) as

$$I = \frac{\chi\left[\left(r + e\phi_u q_u \left(1-\delta\right)\right) K + \frac{B}{P}\right] - C_u}{1 - e\phi_u q_u}. \tag{15}$$

A Household's Problem. Let $J\left(S, B; \Gamma\right)$ be the value of the representative household with equity claims S and money B, given the collection of aggregate state variables Γ whose evolution is taken as given by the household.[18] Since at the end of the period workers and entrepreneurs reunite to share their stocks of equity and money, we have

$$S' = S'_v + S'_u, \quad B' = B'_v + B'_u. \tag{16}$$

Then, the value satisfies the following Bellman equation,

Problem 1:

$$J(S, B; \Gamma) = \max_{\{e, N, C_u, C_v, S'_u, S'_v, B'_v\}} \chi u\left(\frac{C_u}{\chi}\right) + (1-\chi)\left[u\left(\frac{C_v}{1-\chi}\right) - h\left(\frac{N}{1-\chi}\right)\right] + \beta \mathbb{E}_\Gamma\left[J(S', B'; \Gamma')\right]$$

$$s.t. \quad (13), \quad (14), \quad and\ (16).$$

2.3 Search, Matching, and Asset Price

Search and Matching. Matching between buyers and sellers of private claims is handled by zero-profit intermediaries owned by the household. The implicity assumption is that it is extremely costly for individual buyers to meet sellers, and vice versa. Financial intermediations provide specialized services to find counterparties, such as screening and mornitoring. The cost of matching buyers and sellers are paid through the search costs, i.e., κ_u and κ_v. Note that we do not distinguish financial institutions (e.g., banks) and dealers in financial markets in our model. They are both modeled as the financial sector with costly matching technology which intermediates the asset price for private claims. A detail discussion between these two types of agents and related economic consequence can be found in **?**.

Buyers post total asset positions $V = \phi_v^{-1}\left[S'_v - (1-\delta)S_v\right]$ that are to be filled. Sellers put their new and old assets on sale, offering $U = e[(1-\delta)\chi S + I]$. After V and U are determined, the number of aggregate matches M is determined by intermediations' matching technology

$$M(V, U) = \xi V^{1-\eta} U^{\eta},$$

[18] Once we proceed to the equilibrium definition, $\Gamma \equiv (K, B; z_a, z_\kappa)$ where K is the total capital stock, B is the total amount of money circulated, z_a is total factor productivity in final goods production, and z_κ is an intermediation cost shock in the search market. The exogenous stochastic processes for z_a and z_κ are specified in the numerical examples in Section 4.

where $\eta \in (0,1)$ is the elasticity of matches with respect to assets on sale, and ξ measures matching efficiency.

Defining θ as the ratio of vacant asset positions V to assets on sale U, we have

$$\theta \equiv \frac{V}{U}, \quad \phi_v \equiv \frac{M}{V} = \xi\theta^{-\eta}, \quad \phi_u \equiv \frac{M}{U} = \xi\theta^{1-\eta}, \tag{17}$$

where ϕ_v captures the probability of a buyer meeting a seller for each unit of asset positions posted, and ϕ_u the probability of a seller meeting a buyer for each unit of assets put on sale. Recall that ϕ_u also represents the fraction of financial assets that can be sold *ex post* in a given period. Therefore, we refer to ϕ_u as asset saleability or liquidity.

Notice that θ expresses the search market tightness from a buyer's perspective. A larger θ indicates that buyers have difficulty in finding appropriate investment opportunities on the search market, such that U are relatively small compared to V. Lastly, noticing that $\phi_v^{-1}\phi_u = \theta$, we can pin down the relationship between ϕ_v and ϕ_u as

$$\phi_v = \xi^{\frac{1}{1-\eta}} \phi_u^{\frac{\eta}{\eta-1}}. \tag{18}$$

Asset Prices. Once a unit of offered assets is matched to a vacancy position, intermediation offer a price q to both party. Since intermediations makes zero profits and are owned by the households, they seek to maximize the total surplus of each trade. Notice that the amount of matched assets $m_{j,t}$ is predetermined at the point of bargaining. Therefore, buyers and sellers interact at the margin $m_{j,t}$, i.e., the match surplus for both buyers and sellers is the respective marginal value of an additional transaction.

Denote by J^v and J^u the value of individual workers and entrepreneurs from the point of view of the household. In consumption goods unit, a buyer's surplus amounts to

$$-J^v_m = -q + \beta \mathbb{E}_\Gamma \left[\frac{J_S(S', B'; \Gamma')}{u'(c_v)} \right].$$

Intuitively, if the deal is agreed the buyer sacrifices q today but gains the household value of one more unit of assets tomorrow (normalized by the marginal utility of workers' consumption).[19] Similarly, the sellers' surplus is the marginal value to the household of an additional match for entrepreneurs

$$J^u_m = q - \frac{1}{e\phi_u} + \beta \left(\frac{1}{e\phi_u} - 1 \right) \mathbb{E}_\Gamma \left[\frac{J_S(S', B'; \Gamma')}{u'(c_u)} \right],$$

[19] Note that search market participation costs are already sunk at the bargaining stage. However, search costs are not ignored since households take them into account when determining optimal asset posting decisions by workers and entrepreneurs.

which says that the seller gains $(q - e^{-1}\phi_u^{-1})$ today plus a continuation value from a successful match. The contemporary surplus reflects that entrepreneurs earn the bargaining price q , but spend $e^{-1}\phi_u^{-1}$ resources per additional match on new investment projects. The evolution of entrepreneurs' equity position can be expressed as the difference between offered and sold assets, i.e. $s_u' = u - m_u = (e^{-1}\phi_u^{-1} - 1)\, m_u$. Thus, entrepreneurs retain a fraction $(e^{-1}\phi_u^{-1} - 1)$ for each unit of successful matches as inside equity, which is brought back to the household. Therefore, the continuation value of a match consists of the marginal value of future assets to the household multiplied by this factor (normalized by the marginal utility of entrepreneurs' consumption).

Note that all members within the groups of buyers and sellers are homogeneous, such that the type-specific valuations are identical in all matched pairs. We consider the case in which the transaction price q is determined surplus division between buyers and sellers, i.e., intermediations set a price q to maximize

$$\max_q \{ (J_m^u)^{\omega} (-J_m^v)^{1-\omega} \} \tag{19}$$

where $\omega \in (0, 1)$ is the division of surplus that goes to sellers. Notice that this set-up is similar to bilateral (generalized) Nash bargaining between buyers and sellers over the match surplus. In the bargaining case, ω is the bargaining power of sellers. In this sense, our price setting by intermediations is similar to the wage determining process in Ravn (2008) and Ebell (2011), where individual workers come to bargain on behalf of their respective household.

2.4 Recursive Competitive Equilibrium

We close the model by defining the recursive competitive equilibrium. It is mainly a collection of conditions in the previous discussion. Finally, in our subsequent numerical analysis, we refer to *investment* as the sum of I, $\kappa_v V$, and $\kappa_u U$.[20]

Definition 1:
The recursive competitive equilibrium is a mapping $K \to K'$, with associated consumption, investment, labor, and portfolio choices $\{C_v, C_u, N, e, I, B'\}$, asset liquidity $\{\phi_u, \phi_v\}$, a

[20] To verify that Walras' Law is satisfied, notice that the investment equation and the household budget constraint resemble the entrepreneurs' and workers' budget constraints (13) and (14). These two constraints imply that the aggregate resource constraint is

$$C + I + \kappa_v V + \kappa_u U = e^{z_\alpha} K^{\alpha} N^{1-\alpha}, \tag{20}$$

where U and V are the total number of assets on sale and asset positions to be filled.

collection of prices $\{P, q_v, q_u, q_r, w, r\}$, *given exogenous evolutions of aggregate productivity* z_a *and search costs* (κ_v, κ_u) *such that*

1. *final goods producers' optimality conditions in* (1) *hold;*

2. *given prices, the policy functions solve the representative household's problem (Problem 1), the household budget constraint* (13) *and* (14)*, and investment is given by* (15)*;*

3. *market clearing conditions hold, i.e.,*

   (a) *the capital market clears:* $K' = (1-\delta)K + I$ *and* $K = S$*;*

   (b) *the search market clears:* (18) *holds,* q *solves* (19)*, and the effective prices are*

$$q_v = q + \frac{\kappa_v}{\phi_v}, \quad q_u = q - \frac{\kappa_u}{\phi_u}, \quad q_r = \frac{1 - e\phi_u q_u}{1 - e\phi_u};$$

   (c) *the market for liquid assets clears:* $B' = B$*;*

3 Equilibrium Characterization

We mainly focus on the interesting equilibrium with positive participation costs on both sides, i.e., $\kappa_v > 0$ and $\kappa_u > 0$. The limiting case when either one of the cost become zero follows in the end.

Notice that the cost of participation may be so large that households find it better to stay internal financing and search market is not active. We restrict our attention to the economy in which search market is active. That is, the replacement costs $q_r \leq 1$. Otherwise, internal costs of investing are cheaper than issuance.

Using the definition of q_r, we know that effective selling price $q_u \geq 1$. Then, the effective buying price is strictly greater than 1 (note: $\kappa_v > 0$ and $\kappa_u > 0$). Thus, $q_v > q > q_u \geq 1 > q_r$.[21] Compared to workers who value equity at price q_v, the price of equity is strictly cheaper on the view of entrepreneurs. Therefore, the household will prompt entrepreneurs to spend whatever net worth they are not consuming on creating new equity. Entrepreneurs

[21] As shown in Corollary 1, in a frictionless economy with costless search market participation the capital price approaches $q_t = 1$. In this case, the internal equals the external cost of creating capital goods, such that capital production yields zero profits and financial constraints cease to exist. Empirically, the capital price captures Tobin's q, which ranges between 1.1 and 1.21 in the U.S. economy, i.e. well above 1. For this empirically relevant case, capital production is profitable, which reflects financial constraints of firms. During recessions q_t typically falls and erodes firms' net worth, which tightens financing constraints further. This is because firms are leveraged, such that the contraction in their funding base due to the negative shock to net worth is strongly amplified.

thus sell as many existing equity claims as possible and do not invest into money, i.e., $e = 1$ (or $u = (1-\delta)s + i$) and $B'_u = 0$. To ensure $q_r \leq 1$, we should give conditions of exogenous parameters. Define

$$\gamma \equiv \frac{\omega}{1-\omega}\frac{\kappa_v}{\kappa_u}$$

and let the steady state value of κ_v, κ_u, and γ be $\bar{\kappa}_v$, $\bar{\kappa}_u$, and $\bar{\gamma}$. Then,

Lemma 1:

Suppose $\kappa_v > 0$ and $\kappa_u > 0$. If the following parameter restriction is satisfied

$$\frac{\beta^{-1} - (1-\chi)}{\chi} \geq \frac{\bar{\kappa}_v(\frac{\beta^{-1}-(1-\chi)}{\chi})^{\eta}}{\xi\bar{\gamma}^{\eta}} + \frac{\bar{\kappa}_u(\frac{\beta^{-1}-(1-\chi)}{\chi})^{\eta-1}}{\xi\bar{\gamma}^{\eta-1}} + 1 \tag{A1}$$

then $q_u \geq 1$, $q_r < 1$ in the neigborhood around steady state.

Proof. See the Appendix. □

As an illustration, if we further restrict $\bar{\kappa}_v = \bar{\kappa}_u = \kappa$, the above restriction implies a upper bound for the search costs κ. Therefore, (A1) implies that cost of participation should not be too large. To reduce the number of prices, we define the ratio of effective buying price and effective replacement cost:

$$\text{where} \quad \rho \equiv \frac{q_v}{q_r}. \tag{21}$$

3.1 Households' Portfolio Choice

By using the types' budget constraints (13) and (14) to substitute out C_u and C_v in Problem 1, and using $e = 1$, $B'_u = 0$, a household's optimal choice can be reduced to the set $\{N, S'_u, S'_v, B'_v\}$. The first-order condition for labor is[22]

$$u'(c_v)\, w = \mu. \tag{22}$$

The first-order conditions for S'_u and S'_v are

$$u'(c_u)\, q_r = \beta\mathbb{E}_\Gamma\left[J_S(S', B'; \Gamma')\right], \quad u'(c_v)\, q_v = \beta\mathbb{E}_\Gamma\left[J_S(S', B'; \Gamma')\right],$$

from which we learn that

$$u'(c_u) = \rho u'(c_v) \tag{23}$$

[22]As in a portfolio choice problem, the corresponding first-order-conditions are also sufficient due to the concavity of the objective function.

ρ is inversely related to risk-sharing among workers and entrepreneurs. When $\rho = 1$, search frictions disappear and entrepreneurs are not financing constrained (see Corollary 1). In this case, (23) naturally implies $c_u = c_v$, i.e., perfect consumption risk-sharing among household members. In an economy where the search market structure imposes financing frictions, we have $\rho > 1$. Therefore, $c_u < c_v$ and the risk-sharing capacity of the household decreases in ρ. Finally, the optimality condition for money holdings B'_v is

$$u'(c_v)\frac{1}{P} = \beta\mathbb{E}_\Gamma\left[J_B(S', B'; \Gamma')\right].$$

It is instructive to derive asset pricing formulae for equity and money corresponding to the optimality conditions. Using the envelope condition and noticing that $\phi_u q_u + (1-\phi_u) q_r = 1$,[23]

$$\begin{aligned} J_S &= u'(c_u)\chi[r+1-\delta] + u'(c_v)(1-\chi)[r + q_v(1-\delta)] \\ &= u'(c_v)\left[(\chi\rho + 1 - \chi) r + (1-\delta)(\chi\rho + (1-\chi) q_v)\right], \end{aligned}$$

together with the first-order condition for equity S'_v we obtain

$$\mathbb{E}_\Gamma\left[\frac{\beta u'(C'_v)}{u'(C_v)} \frac{(\chi\rho' + (1-\chi)) r' + (1-\delta)(\chi\rho' + (1-\chi) q'_v)}{q_v}\right] = 1, \tag{24}$$

where the second term in the expectations operator captures the internal return on equity from the perspective of the household. Similarly, we can derive another asset pricing formula for money by applying the envelope condition again

$$\mathbb{E}_\Gamma\left[\frac{\beta u'(C'_v)}{u'(C_v)} \frac{\chi\rho + 1 - \chi}{\pi'}\right] = 1, \tag{25}$$

where the second term in the expectations operator is the internal return on money from the perspective of the household and inflation is defined as

$$\pi' \equiv \frac{P'}{P}.$$

In the steady state, condition (25) implies that $[\chi\rho + 1 - \chi]\pi^{-1} = \beta^{-1}$. If $\rho > 1$, the real interest rate π^{-1} will be lower than the time preference rate β^{-1}.[24] This fact shows that money may provide a liquidity service and, accordingly, carries a liquidity premium, which

[23] One could intrepretate $\phi_u q_u + (1-\phi_u) q_r = 1$ in the following way. The fully resaleable fraction of existing equity worth $\phi_u q_u$, while the non-resaleable fraction of existing equity is $1 - \phi_u q_u$ which is the net-worth paid by the entrepreneurs.

[24] Though we focus on fiat money such that $P' = P$ in the steady state (and $\pi^{-1} = 1 < \beta^{-1}$), one can easily imagine an economy where the government steps in and may run inflation or deflation.

is easiest to be seen in the steady state:

Proposition 1:
Suppose (A1) holds, then in the neigborhood around steady state, money provides a liquidity service. The steady state liquidity premium amounts to

$$\Delta_B \equiv [\chi\rho + (1-\chi) - 1]\,\frac{1}{\pi} = \frac{(\rho-1)\,\chi}{\pi} > 0.$$

To illustrate, when $\rho > 1$, liquidity frictions bind and entrepreneurs are financing constrained. An additional unit of money then relaxes entrepreneurs' constraints by increasing their net-worth, which allows them to leverage their investment or, equivalently, their future equity position. Once we solved the endogenous asset price, we will show in Corollary 1 that in the limiting case ($\rho \to 1$) equity can be sold frictionlessly and money loses its liquidity value also off the steady state. The asset pricing formulae then collapse to standard Euler equations in a RBC model. However, if the search market is not frictionless, the liquidity premium is not zero and may vary substantially through time.

3.2 The Bargained Asset Price

Asset price is set to maximize the total surplus of buyers and sellers. Assuming an interior solution (and we will discuss the corner solution), the sufficient and necessary first-order condition yields

$$\frac{\omega}{u'(c_u)\left(q-\phi_u^{-1}\right)+\left(\phi_u^{-1}-1\right)\beta\mathbb{E}_\Gamma J_S\left(S',B';\Gamma'\right)} = \frac{1-\omega}{-u'(c_v)q+\beta\mathbb{E}_\Gamma J_S\left(S',B';\Gamma'\right)}.$$

By using the household's optimality condition for asset holdings, $u'\left(c_v\right)q_v = \beta\mathbb{E}_\Gamma J_S\left(S',B';\Gamma'\right)$, and the risk-sharing condition, $u'(c_u) = \rho u'(c_v)$, we can derive an analytical solution for the bargaining price, stated in the following proposition:

Lemma 2:
Suppose (A1). When both $\kappa_v > 0$ and $\kappa_u > 0$, the search market bargaining solution simplifies to

$$\rho = \frac{\omega}{1-\omega}\frac{\kappa_v}{\kappa_u}\theta, \tag{26}$$

which can be solved for q as a function of resaleability ϕ_u[25]

$$q = \frac{\gamma\left(1+\frac{\kappa_u}{\omega}\right)\phi_u - \kappa_v}{\xi^{\frac{1}{1-\eta}}\phi_u^{\frac{\eta}{\eta-1}}\left[1+\left(\gamma(\xi^{-1}\phi_u)^{\frac{1}{1-\eta}}-1\right)\phi_u\right]}. \tag{27}$$

Proof. See Appendix C.2. □

Proposition 2 is the main result of our model. The financial frictions lead to a price of equity that is above the cost of capital (which is one). Entrepreneurs would like thus to issue more claims, but the frictions prevent them to further doing so which keeps the price of equity higher than than the (internal) price of capital. Note that if financial frictions are completely removed, asset price will fall to the cost of capital. Hence, the central question is what happens to the equity price when financial frictions change mildly, instead of comparing an economy with financial frictions to one without such frictions at all.

To illustrate how asset price responds to the relatively mild changes in the financial frictions, we focus on the steady state. From money's first-order condition (25), $\rho = \chi^{-1}[\beta^{-1} - (1-\chi)]$. Therefore, ϕ_u and ϕ_v are independent of search costs. On the one hand, (26) implies that a higher κ_v decreases asset price since demand sides find it more costly and participate less; On the other hand, an increase of κ_u in (26) will push up asset price as it makes even more costly for entrepreneurs to issue and supply equity (more financing constrained) such that equity price will be even higher than the cost of capital. The net-effects depends on the parameters. For discussion simplicity, we restrict attention to the case that κ_v and κ_u increase by the same fraction. Then,

Proposition 2:

Suppose (A1) holds and the ratio of $\kappa_v/\kappa_u = g$ is fixed. If the following condition is satisfied

$$\xi\left[g\frac{\beta^{-1}-(1-\chi)}{\chi}\right]^{1-\eta} < (1-\omega)^{\eta}\omega^{1-\eta}$$

then in steady state asset price q drops when κ_v and κ_u increase.

Proof. See the Appendix. □

The above condition is more likely to hold when g is small. That is, when cost of buying is relatively smaller than the cost of selling, demand sides are more sensitive to the participation costs' increase. When cost of buying is high, the buyers participation will be already low in steady state. Such endogenous changes of asset demand and supply are the key insight

[25] An intermediate step in the derivation, which is used for simplification later, is $q = \frac{\rho\left(1+\frac{\kappa_u}{\omega}\right)-\frac{\kappa_v}{\phi_v}}{1+(\rho-1)\phi_u}$.

from an endogenous search market, compared to exogenous financing constraints illustrated in Kiyotaki and Moore (2012) and Shi (2012). In our model, endogenizing asset liquidity gives rise to a non-trivial relationship between q and ϕ_u. Similar to the discussion above, if ϕ_u is small enough, the asset price also falls together with a drop of liquidity ϕ_u.

Proposition 3:
q correlates positively with asset resaleability ϕ_u (i.e. $\frac{\partial q}{\partial \phi_u} > 0$) and negatively with the purchase rate ϕ_v (i.e. $\frac{\partial q}{\partial \phi_v} < 0$), if

$$\phi_u < \left[\frac{\eta}{1-\eta} + \left(1 + \frac{\kappa_u}{\omega}\right)\right]\left[\frac{\eta}{1-\eta} + 2\gamma(\xi^{-1}\phi_u)^{\frac{1}{1-\eta}} - 1\right]^{-1}$$

When $\eta = 0.5$, the above sufficient condition simplifies to

$$\phi_u < \frac{-1 + \sqrt{1 + 8\xi^{-2}\gamma(3 + \kappa_u/\omega)}}{4\gamma\xi^{-2}} \equiv \bar{\phi}_u$$

Proof. See Appendix C.4. □

Intuitively, the drop in saleability implies that a larger share of investment needs to be financed out of entrepreneurs' own funds. On the one hand, this tightens the contemporaneous financing constraints of bargaining entrepreneurs. The threat point for entrepreneurs of breaking off negotiations over an additional asset sale and self-financing at the margin becomes less attractive. Entrepreneurs are thus more willing to accept a smaller bargaining price. On the other hand, retaining a larger fraction of equity stakes also implies that entrepreneurs return more assets to the household, which relaxes the funding constraints of future generations of entrepreneurs. This effect supports the threat point, such that entrepreneurs ask for a higher transaction price in a successful match. Thus, a trade-off emerges between current and future funding constraints.

Proposition 3 shows that the contemporaneous effect dominates as long as the sales rate is small enough, because current financial constraints bind strongly. Provided that financial frictions are sufficiently tight, our model can thus generate simultaneous decreases in asset liquidity and the asset price through the simultaneous reaction of supply *and* demand.

In contrast, an exogenous drop in asset saleability, such as in KM and Shi (2012), acts like a negative supply shock on the asset market: The decline in saleability translates into a tighter financing constraint for entrepreneurs and less supply of financial claims on the asset market. However, the productivity of capital is not affected by the shock such that asset demand does not fall much. The dominating supply contraction triggers an asset prices boom - a counter-factual phenomenon in recessions. Although this effect is still present in our framework, there are competing forces from the demand side that usually outweigh the supply contraction as argued before.

Frictionless limiting case. When there are no search costs for either buyers or sellers, i.e. $\kappa_v = 0$ and $\kappa_u = 0$, the search market price will go to $q = 1$, because there is no asset supply shortage. In this case, money loses its liquidity premium and the economy collapses to the RBC framework. Households' Euler equation becomes the standard ones in a RBC framework because $\rho = 1$. When $\kappa_u = 0$ but $\kappa_v > 0$, we have similar outcome, except that households always spend a fix fraction of resources to purchase equity. When $\kappa_v = 0$ but $\kappa_u > 0$, the household will not participate in the search market. This is because when $\kappa_v = 0$ the bargain price $q = 1$ which implies that $q_u < 1$ (given $\kappa_u > 0$). That is, (A1) is violated in this case.

Corollary 1:
When $\kappa_u \to 0$, or both $\kappa_u \to 0$ and $\kappa_v \to 0$

1. $q_v \to q_u \to q \to 1$ *and* $\rho \to 1$
2. $\theta \to \frac{1-\omega}{\omega}$, $\phi_u \to \xi \left(\frac{1-\omega}{\omega}\right)^{1-\eta}$, *and* $\phi_v \to \xi \left(\frac{1-\omega}{\omega}\right)^{-\eta}$.

Proof. See Appendix C.5. □

4 Numerical Examples

This section uses numerical tools to illustrate system dynamics. As in standard specification, we consider an AR(1) process for aggregate productivity, i.e.,

$$z_a' = \rho_a z_a + \epsilon_a,$$

with i.i.d. $\epsilon_a \sim N(0, \sigma_a^2)$. We further introduce a symmetric shock to the cost of financial intermediation, which in our asset search framework corresponds to an increase in these participation costs. We let

$$\kappa_u = e^{z_\kappa}\bar{\kappa}_u, \quad \kappa_v = e^{z_\kappa}\bar{\kappa}_v,$$

where z_κ follows an AR(1) process and

$$z_\kappa' = \rho_\kappa z_\kappa + \epsilon_\kappa',$$

with i.i.d. $\epsilon_\kappa' \sim N\left(0, \sigma_\kappa^2\right)$. Rather than affecting the production frontier of the economy, this shock simply impairs the capacity of the search market to intermediate funds between workers and entrepreneurs and, thus, unfolds its effects solely through asset prices and liquidity.

4.1 Calibration

We set utility function as a standard CRRA one, i.e., $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ and disutility for labor as $h(n) = \mu n$.[26] Parameters β, σ, and δ are chosen exogenously and are similar to standard macro models. We target quarterly frequency and choose α and μ to target the investment-to-GDP ratio and working hours (see Table 2). Note that the parameter χ can be interpreted as the fraction of firms which adjust capital in a period. According to Doms and Dunne (1998), the annual fraction is 0.20 which translates to $\chi = 0.054$ in quarterly frequency.

Table 2: Baseline calibration

	Parameter	Baseline Value	Target/Source
Preferences and Production Technology			
Household discount factor	β	0.9850	Exogenous
Relative risk aversion	σ	2	Exogenous
Utility weight on leisure	μ	3.8995	Working time: 33%
Mass of entrepreneurs	χ	0.0540	Doms and Dunne (1998)
Depreciation rate of capital	δ	0.0280	Exogenous
Capital share of output	α	0.3214	Investment-to-GDP ratio: 18.0%
Search and Matching			
Supply sensitivity of matching	η	0.5000	Exogenous
Matching efficiency	ξ	0.6100	Saleability $\phi_u = 0.2800$
Buyer search costs	$\bar{\kappa}_v$	0.1108	Tobins $q = 1.1500$
Seller search costs	$\bar{\kappa}_u$	0.0147	Cost of Intermedation-to-investment 0.1000
Bargaining weight of sellers	ω	0.4463	$B/(B + PqK) = 0.0953$

Notes: The model is calibrated to quarterly frequency. Standard errors of estimated parameters are in brackets.

There are five search-market related parameters $\{\xi, \bar{\kappa}_v, \bar{\kappa}_u, \eta, \omega\}$. Due to the constant returns to scale matching technology on the search market, ξ and η are not independent. Without loss of generality, we set $\eta = 0.5$ and calibrate ξ. We are then left with four independent parameters, which we calibrate to match four targets. Tobin's q ranges from 1.1 to 1.21 in the U.S. economy according to Compustat data. We target $q = 1.15$. The liquidity ratio, defined as the total amount of nominal liquid assets PB (essentially money and government bonds, see details in Appendix A) circulated in the U.S. over total assets $PB + PqK$, is around 10% on average over the sample periods. **?** use the US Flow-of-Funds data for non-financial firms to estimate the stochastic process of ϕ_u. Interpreting ϕ_u as the ratio of funds raised in the market to fixed investment, they find that the mean of is 0.28. Therefore, we set $\phi_u = 0.28$. Finally, we target the total cost of intermediation to be 10% of total investment, in-line with the findings in the cost of initial public offering (IPO) (STILL

[26]The reason for such disutility function is to solve steady state easily. A more complicated function would not change the main results (available upon request)

A REFERNCE NEEDED).[27]

Using the model as a laboratory, we set the persistence and the size of shocks to target the volatility (0.02) and 1st order correlation (0.90) of GDP's cyclical components. We use GDP data from 1971Q1-2013Q4 and take the HP filterred (with coefficient 1600) cyclical components. Note that GDP is the sum of real private consumption and real private fixed investment. By using only productivity shocks, the exercise gives

$$\rho_a = 0.88, \sigma_a = 0.008.$$

By using only shocks to intermediation costs, the exercise gives

$$\rho_\kappa = 0.81, \sigma_\kappa = 0.69.$$

We now compare the different equilibrium responses to aggregate productivity shocks and intermediation cost shocks. In particular, we compare to the data with the model's prediction of liquidity ratio and asset price (together with standard macro variables). For asset price, we use Wilshire 5000 price full cap index from 1971Q1-2013Q4.

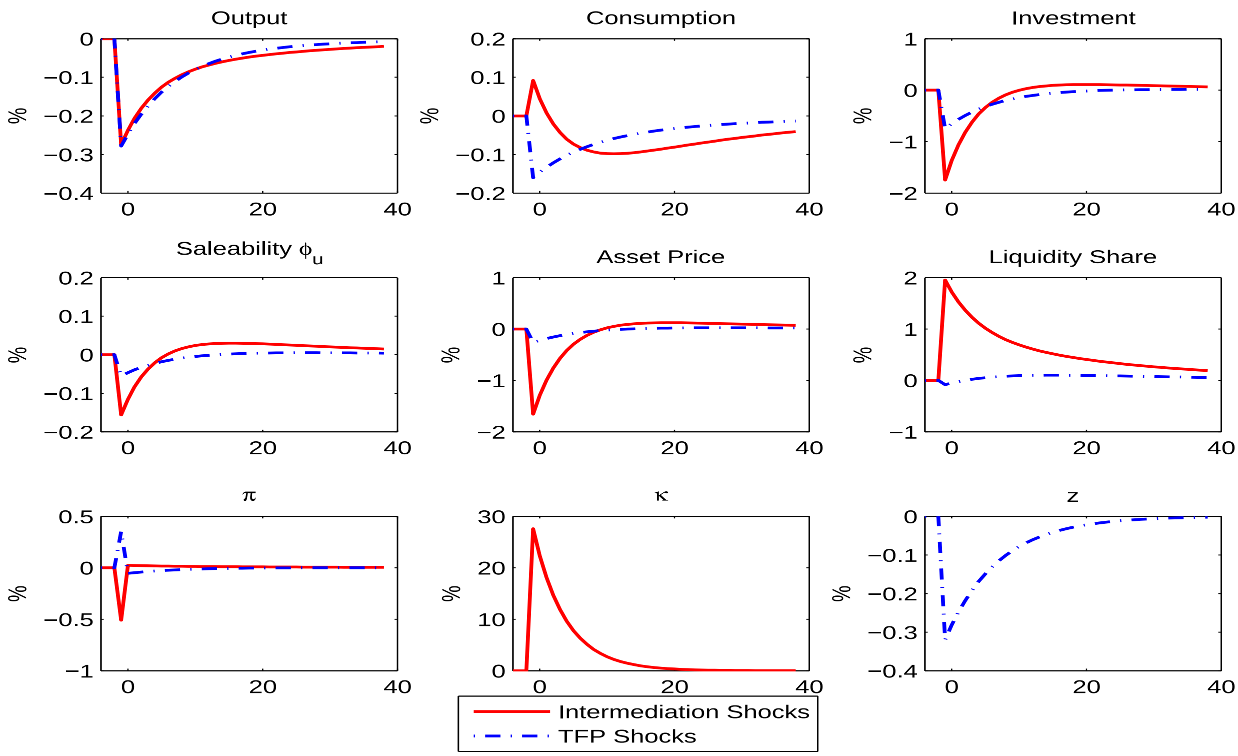
4.2 Equilibrium Responses to Shocks

Adverse aggregate productivity shocks. This shock depresses the rental rate of capital and its value to the household. Accordingly, the total match surplus. This makes search for investment into entrepreneurs less attractive and the amount of purchase orders from workers drops. The demand-driven fall is reflected in the sharp drop in asset resaleability ϕ_u (Figure 1). This endogenous decline of asset liquidity amplifies the initial shock in two ways: (1) it reduces the quantity of assets that entrepreneurs are able to sell; (2) the bargaining price, i.e. private assets' resale value falls - though only modestly - in line with our analytical result in Proposition 3. Both effects constrain the flow of liquid funds to entrepreneurs and thus tighten their financing constraints. As a result, investment falls strongly.

In principle, money's liquidity service becomes more valuable to households when private claims' liquidity declines. However, in the case of a persistent TFP shock lower expected returns to capital make future investment less attractive. This effect works against the incentive to hedge against asset illiquidity. Which effect dominates depends on the calibration and is thus an empirical question. In our calibration, the profitability of investment projects falls sufficiently for the liquidity ratio to drop. To the extent that total factor productivity

[27] The share of financial industry is about 8% on average of GDP according to Philippon (2013). However, the measure of financial industry includes insurance industry and financial intermediation activities for consumer loans. Therefore, we choose to direct target investment related activities.

Figure 1: Impulse responses after financial shocks and productivity shocks



reverts back to the steady state while asset liquidity is still subdued, hedging becomes more attractive which explains the relatively fast recovery of the liquidity ratio.

Financial shocks. By construction, higher search costs generate the same dynamics of output. The shocks immediately depress the effective sales price of private claims, while raising their effective purchase price. In addition, higher search costs bind resources. Both the substitution and the income effect induce households to adjust their portfolios. Realizing that search market participation is more costly now and in the future, households seek to reduce their exposure to private financial claims. On the supply side, though less investment can be issued, financing-constrained entrepreneurs still want to sell as many assets as possible in order to take full advantage of profitable investment opportunities. Asset demand on the search market thus shrinks relative to asset supply, which depresses asset resaleability while improving the purchase rate (see Figure 1).

Since the sharp drop in asset liquidity tightens entrepreneurs financing constraints substantially, the threat point of abandoning the bargaining process with a potential buyer worsens. Entrepreneurs as sellers are willing to accept a lower price. The bargaining price thus falls strongly and amplifies the initial shock by depressing entrepreneurs' net worth further. This effect is mirrored in a significant decline of investment activity, the impact response of which is about six times stronger than that of output.

While the intermediation cost shock depresses the demand for and liquidity of private assets, it substantially increases the hedging value of money. To see this, note that future investment remains profitable since the productivity of capital is not affected by the shock. To take advantage of future investment opportunities, households seek to hedge against the persistent illiquidity of private claims by expanding their liquidity holdings. This motive consequently drives up the liquidity ratio in line with the data.

Remark. In our framework, the liquidity (resaleability) of financial assets is endogenously generated through the features of the search market. In the absence of search frictions, these liquidity effects would not occur after adverse shocks. In a RBC world, negative TFP shocks primarily affect the demand for capital goods and thus reduce the optimal level of investment. In addition to this effect, entrepreneurs are financing constrained in our model, which strongly amplifies the response of investment as argued above.

4.3 Discussion

The dynamic behavior of our economy suggests two key results. (1) In order to reconcile declining asset liquidity with falling asset prices, liquidity must be an endogenous phenomenon. In other words, it must be a consequence, rather than a cause of economic disturbances. (2) Both standard productivity and genuine search market shocks affect the hedging value of liquid assets. However, only the latter unambiguously implies a negative co-movement between liquidity ratio and aggregate output. As a comparison, we first vary the persistence of the two aggregate shocks, replacing ρ_a or ρ_κ by a more persistent number (0.90) or a less persistent number (0.80).

The qualitative difference is small (Figures 4 and 3), but different persistence do change the speed and magnitude of liquidity share in the two experiments. The reason is again how hedging by using liquid assets is valuable for future investment. For example, if low aggregate productivity is perceived to be more persistent in the future, hedging value of liquid assets will be depressed longer. The liquidity share drops to a slightly lower value than the baseline and takes longer time to come back. In contrast, liquidity share takes longer time to come back when persistence increases, together with a higher jump in the beginning.

Second, we compare business cycle statistics from the model to the data. Some key business cycle statistics of the model in comparison to the data are reported in Table 3, where only aggregate productivity shocks are considered. Our main targets are consumption, investment, asset price, and liquidity share. As in a usual RBC model, consumption and investment volatility, autocorrelation, and correlation with GDP are in-line with data. However, the liquidity ratio and asset price move too little compared to the data. Besides,

Figure 2: Impulse responses after productivity shocks with different persistence ($\rho_a = 0.90$ or $\rho_a = 0.80$ compared to the baseline).

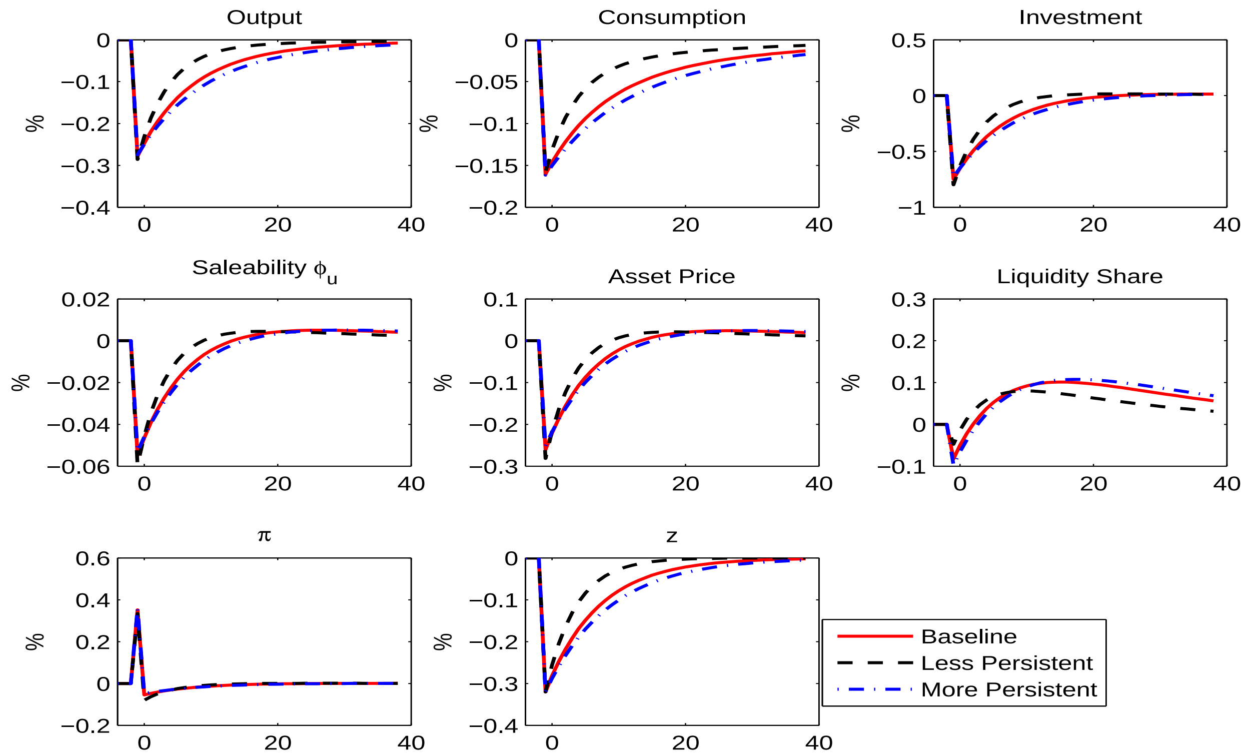


Table 3: Cycle statistics with only aggregate productivity shocks

	Relative volatility $\frac{\sigma_x}{\sigma_y}$		Correlation $\rho(x, y)$		1st auto-correlation	
Variable x	Data	Model	Data	Model	Data	Model
Output	0.02	0.02	1.00	1.00	0.89	0.89
Consumption	0.44	0.67	0.88	0.98	0.80	0.92
Investment	3.45	2.43	0.96	0.98	0.85	0.87
Liquidity Ratio	3.44	0.87	-0.58	-0.26	0.89	0.98
Asset Price	5.23	0.79	0.50	0.88	0.81	0.84

Note: The volatility of output (y) is reported as is. The relative volatilities and correlations of other variables are measured against y.

liquidity ratio does not negatively co-move with GDP as in the data, while asset price move too closely with GDP in the model.

Table 4 shows the relevant statistics when there are only financial shocks. Compared to the economy with only productivity shocks, the volatility of liquidity ratio and asset price are much higher in the economy with only intermediation shocks. The volatility is closer to the data, though liquidity ratio fluctuates more and asset price fluctuate less than the data. The liquidity ratio in the model successfully generate countercyclical movements in liquidity ratio, mimicking the liquidity hoarding in recessions. Note that since the data indicates a

Figure 3: Impulse responses after financial shocks with different persistence ($\rho_\kappa = 0.90$ or $\rho_\kappa = 0.80$ compared to the baseline).

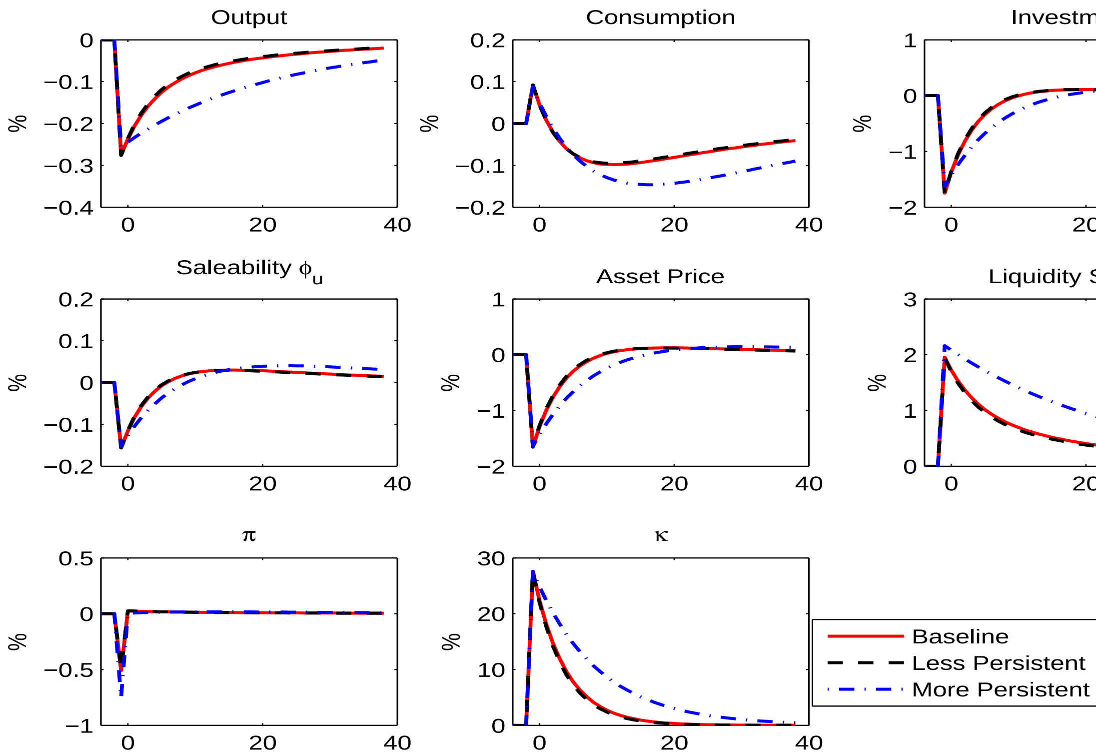


Table 4: Cycle statistics with only financial shocks

	Relative volatility $\frac{\sigma_x}{\sigma_y}$		Correlation $\rho(x, y)$		1st auto-correlation	
Variable x	Data	Model	Data	Model	Data	Model
Output	0.02	0.02	1.00	1.00	0.89	0.89
Consumption	0.44	0.79	0.88	0.44	0.80	0.97
Investment	3.45	4.61	0.96	0.78	0.85	0.78
Liquidity Ratio	3.44	7.85	-0.58	-1.00	0.89	0.91
Asset Price	5.23	4.36	0.50	0.75	0.81	0.78

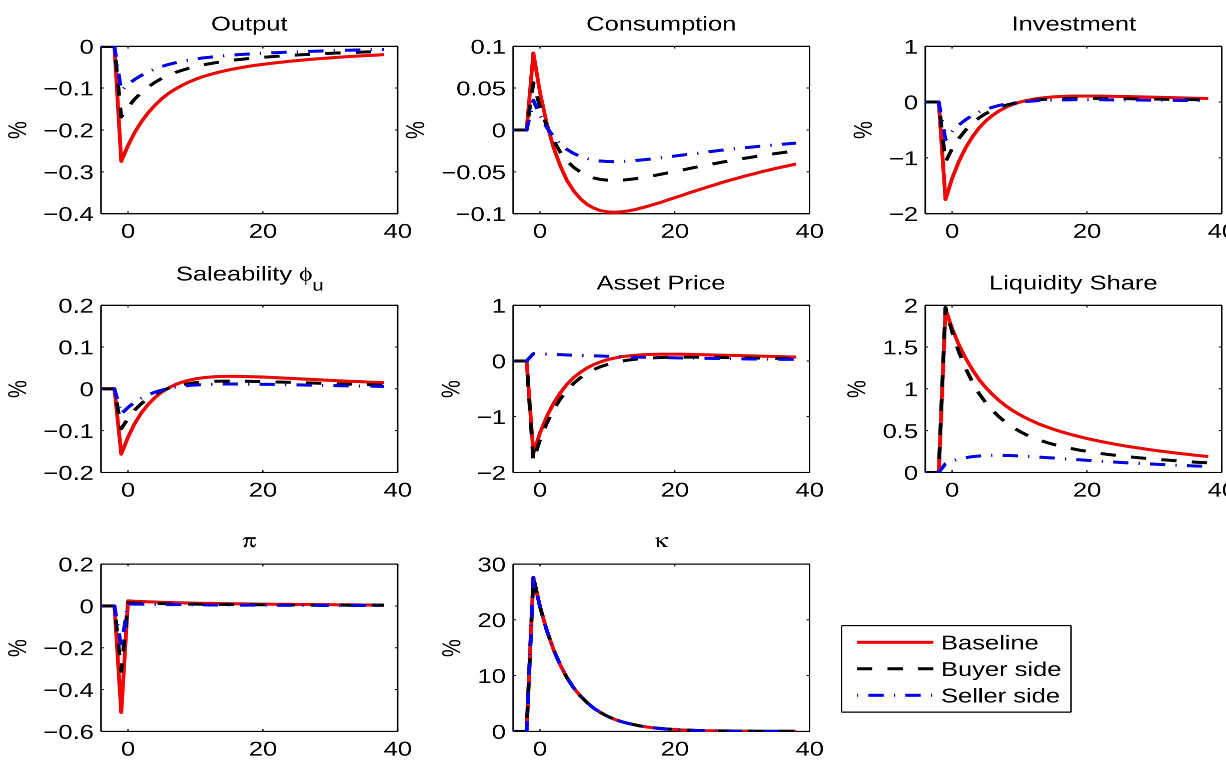
Note: The volatility of output (y) is reported as is. The relative volatilities and correlations of other variables are measured against y.

negative correlation around -0.60, financial shocks alone predicts too much correlation.

Finally, note that the shocks to financial intermediation affect both sides of the market. This feature leads us to examine the equilibrium responses after shocks to only one side of the participants. If the shocks only affect buyers' side, the dynamics is similar to the baseline impulse responses although the magnitude is smaller. If the shocks only affect sellers'side, quantity dynamics and the tightness of financing constraints react similarly, but clearly asset price and liquidity share are different. Asset price boom, which contributes to very mild increase in liquidity share.

Importantly, this equilibrium response after shocks to only seller side is very similar to the result in Shi (2012) and Kiyotaki and Moore (2012). In their studies, an exogenous tightened liquidity constraint (a reduction of ϕ_u) leads to a boom of asset price. Both new and old assets become harder to sell and supply of assets is depressed significantly. Also, demand for private claims thus drop. But the drop of demand cannot compensate the reduction of supply, which pushes up asset price. Our endogenous liquidity mechanism shows that to overturn this asset price anomaly, disturbances in financial sectors need to affect demand side strong enough. One should also notice the amplification mechanism built-in: anticipating persistent drop of demand, both issuance and resale will be hard for a while, which leads to a further reduction of demand today.[28]

Figure 4: Impulse responses after productivity shocks with different persistence



[28]The above discussion leads to a final check of the endogenous liquidity mechanism. An alternative way of thinking financial disturbance is to shock to the matching function itself. More specifically, we shock the matching efficiency ξ in order to check whether a problem generated from the financial sector could lead to drop of asset price and liquidity. This line of reasoning is very similar to the productivity shocks in standard RBC exercise, but productivity shocks to the financial sector itself (instead of to the goods producer sector). An adverse efficiency shock, for example because of excess-borrowing and later contagious bank run, makes the financial sector functioning less as before. Nevertheless, the answer is negative. When matching technology is worse, the dominant force is still the supply of capital in which asset price will increase. For the sake of space, the detail simulation and comparison is available upon request.

5 Conclusion

We endogenize asset liquidity in a macroeconomic model with search frictions. Endogenous fluctuation of asset liquidity may be triggered by shocks that affect asset demand and supply on the search market either directly (intermediation cost shock), or indirectly (productivity shock). By tightening entrepreneurs' financing constraints, they feed into investment, consumption and output. Interpreting liquidity as asset resaleability, we show that asset prices co-move with liquidity in a reasonably calibrated model. The *endogenous* nature of asset liquidity is key to match this pro-cyclicality, as *exogenous* liquidity shocks would act as negative supply shocks on the asset market and lead to higher asset prices in recessions.

We also show that the liquidity service provided by intrinsically worthless government-issued assets, such as money, is higher when financing constraints bind tightly. As a result, shocks to the cost of financial intermediation increase the hedging value of liquid assets, enabling our model to replicate the "flight to liquidity" or countercyclical ratio of liquid assets relative to GDP observed in the U.S. data.

Our search framework can be interpreted as a model of market-based financial intermediation. It can, however, also be seen as a short-cut to model bank-based financial intermediation: financial intermediaries help channel funds from investors to suitable creditors in need of outside funding, a process which resembles a matching process. Adding further texture by explicitly accounting for intermediaries' balance sheets would open interesting interactions between liquidity cycles and financial sector leverage and maturity transformation.

Regarding government interventions, our framework suggests that, as in KM, open market operations in the form of asset purchase programs can have real effects by easing liquidity frictions. However, we highlight that with endogenous liquidity such policies need to be carefully designed, because of potential crowding-out effects on the private market participants. Future research could focus on the optimal design of conventional and unconventional monetary as well as fiscal policy measures in the presence of illiquid asset markets.

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A Data

The measure of liquid assets B_t consists of all liabilities of the federal government circulated inside the US economy. To obtain the measure, we use U.S. flow-of-funds data. In particular, we use Treasury securities, net of saving bonds (for financing World War II), net of holdings by the monetary authority and the rest of the world, plus reserves and vault cash of depositary institutions with the monetary authority, plus checkeable deposits and currency net of the monetary authority's liabilities due to the rest of the world and due to the federal government. This measure is similar to the one in Del Negro, Eggertsson, Ferrero, and Kiyotaki (2011), but is cleaned from liquid assets held by agents outside the US.

Following again Del Negro, Eggertsson, Ferrero, and Kiyotaki (2011), PqK measure the value of capital in the economy. We use the balance sheet of households, the non-corporate and the corporate sectors to obtain the market value of aggregate capital. On households' side, we add real estate, equipment and software of non-profit organizations, and consumer durables. As for the non-corporate sector, we add real estate, equipment and software and inventories. As for the corporate sector, we obtain the market value of the capital stock by summing the market value of equity and liabilities net of financial assets. Finally, we subtract from the market value of capital the government credit market instruments, TARP, and trade receivables.

B Equilibrium Conditions

B.1 Recursive Competitive Equilibrium

Notice that $C = C_v + C_u$, such that [29]

$$C_v = \rho_v C, \quad C_u = \rho_u C,$$

where

$$\rho_v \equiv \frac{1-\chi}{1-\chi+\rho^{-1/\sigma}\chi}, \quad \rho_u \equiv \frac{\chi}{\rho^{1/\sigma}(1-\chi)+\chi}.$$

We use ρ_v and ρ_u in the subsequent analysis. We change the recursive equilibrium slightly by using $\rho = \frac{q_v}{q_r}$ (instead of using q_r and q_u) , defining real liquidity $L = \frac{B}{P_{-1}}$, and finallyadding aggregate output Y. Given the aggregate state variables $\Gamma = (K; z_a, z_\kappa)$, we can solve the equilibrium system

$$(K', L, C, I, N, Y, \rho, \rho_u, \rho_v, \phi_u, \phi_v, q_v, q, r, w, \pi)$$

together with the exogenous laws of motion of (z_a,z_κ), i.e., $z'_a = \rho_a z_a + \epsilon'_a$ and $z'_\kappa = \rho_\kappa z_\kappa + \epsilon'_\kappa$. To solve for these 16 endogenous variables, we use the following 16 equations:

[29]Using the utility function $u(c_j) = \frac{c_j^{1-\sigma}-1}{1-\sigma}$ in (23) and noting that $C = C_v + C_u$, we obtain $C_v = \rho_v C$ and $C_u = \rho_u C$, where $\rho_v \equiv \frac{1-\chi}{1-\chi+\rho^{-1/\sigma}\chi}$ and $\rho_u \equiv \frac{\chi}{\rho^{1/\sigma}(1-\chi)+\chi}$.

1. The representative household's optimality conditions:

$$\left(\frac{\rho_v C}{1-\chi}\right)^{-\sigma} w = \mu$$

$$\rho_v \equiv \frac{1-\chi}{1-\chi+\rho^{-1/\sigma}\chi}, \quad \rho_u \equiv \frac{\chi}{\rho^{1/\sigma}(1-\chi)+\chi}$$

$$1 = \beta \mathbb{E}_\Gamma \left[\left(\frac{\rho_v' C'}{\rho_v C}\right)^{-\sigma} [\chi\rho' + 1 - \chi] \frac{1}{\pi'}\right] \tag{28}$$

$$1 = \beta \mathbb{E}_\Gamma \left[\left(\frac{\rho_v' C'}{\rho_v C}\right)^{-\sigma} \frac{(\chi\rho' + 1 - \chi)\, r' + (1-\delta)\,(\chi\rho' + (1-\chi)\, q_v')}{q_v}\right] \tag{29}$$

$$I = \frac{\chi\left[\left(r + \left(\phi_u q_v - \left[\frac{(1-\omega)\rho}{\omega} + 1\right]\kappa_u\right)(1-\delta)\right) K + \frac{L}{\pi}\right] - \rho_u C}{1 - \phi_u q_v + \left[\frac{(1-\omega)\rho}{\omega} + 1\right]\kappa_u} \tag{30}$$

2. Final goods producers:

$$r = e^{z_a} F_K(K,N), \quad w = e^{z_a} F_N(K,N), \quad Y = e^{z_a} F(K,N) \tag{31}$$

3. Market clearing:

   (a) Consumption goods

$$\begin{aligned}(\rho_v + \rho\rho_u)\, C + q_v K' + L' &= wN + [(\chi\rho + (1-\chi))\, r + (1-\delta)\,(\chi\rho + (1-\chi)\, q_v)]\, K \\ &\quad + [\chi\rho + (1-\chi)]\, \frac{L}{\pi}\end{aligned}$$

   (b) Capital

$$K' = (1-\delta)K + I$$

   (c) Search market (note: $\gamma \equiv \frac{\omega}{1-\omega}\frac{\kappa_v}{\kappa_u}$, $\kappa_u = e^{z_\kappa}\bar{\kappa}_u$, $\kappa_v = e^{z_\kappa}\bar{\kappa}_v$)

$$\phi_u = \xi\left(\gamma^{-1}\rho\right)^{1-\eta}, \phi_v = \xi^{\frac{1}{1-\eta}} \phi_u^{\frac{\eta}{\eta-1}}$$

$$q_v = \frac{\rho\left[1 + \kappa_u + \rho\frac{(1-\omega)\kappa_u}{\omega}\right]}{1 + (\rho - 1)\,\phi_u}, \quad q = q_v - \frac{\kappa_v}{\phi_v}$$

   (d) Liquid assets (note: $L' \equiv \frac{B'}{P}$, $\pi' \equiv \frac{P'}{P}$)

$$L' = \frac{L}{\pi}$$

B.2 Steady State

In deterministic steady state, any variable $X = X'$. With a slight abuse of notation, we denote the steady state of X as X itself in this section. First notice that

$$z_a = 0, \quad z_\kappa = 0$$

such that $\kappa_u = \bar{\kappa}_u$, $\kappa_v = \bar{\kappa}_v$. We can now solve for all prices analytically. Market clearing for liquid assets implies

$$\pi = 1$$

Next, we use (28) to obtain

$$\rho = \chi^{-1}\left[\beta^{-1} - (1-\chi)\right], \quad \rho_v \equiv \frac{1-\chi}{1-\chi+\rho^{-1/\sigma}\chi}, \quad \rho_u \equiv \frac{\chi}{\rho^{1/\sigma}(1-\chi)+\chi}$$

This directly implies

$$\phi_u = \xi\left(\gamma^{-1}\rho\right)^{1-\eta}, \quad q_v = \frac{\rho\left[1+\kappa_u+\rho\frac{(1-\omega)\kappa_u}{\omega}\right]}{1+(\rho-1)\phi_u}$$

From (29) and (31) we have

$$r = \frac{\frac{q_v}{\beta} - (1-\delta)(\chi\rho+(1-\chi)q_v)}{\chi\rho+1-\chi}, \quad w = (1-\alpha)\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}, \quad C = \left(\frac{w}{\mu}\right)^{1/\sigma}\frac{1-\chi}{\rho_v}.$$

Now, we express labor supply N as a function of K

$$N = \left(\frac{r}{\alpha}\right)^{\frac{1}{1-\alpha}} K.$$

Notice that investment $I = \delta K$ and real liquidity can be rewritten as a function of K using (30)

$$L = \chi^{-1}\left\{\rho_u C + \left[\delta - \chi r - \phi_u q_u\left(\delta+\chi(1-\delta)\right)\right]K\right\}.$$

Since N and L are both linear in K, we solve K from the household's budget constraint

$$K = \frac{(\rho_v+\rho_u)C}{\frac{(1-\alpha)}{\alpha}r + A_K + (\rho-1)\left[\delta-\chi r-\phi_u q\left[\delta+\chi(1-\delta)\right]\right]},$$

where $A_K = (\chi\rho+1-\chi)r + (1-\delta)(\chi\rho+(1-\chi)q_v) - q_v$.

C Proofs

C.1 Lemma 1

We use a guess-and-verify strategy. Suppose $q_u \geq 1$, then the search market for private claims is active and we seek the parameter restriction that yields $q_u \geq 1$. We can use the asset price derived in Lemma 2, $q = \frac{\rho(1+\frac{\kappa_u}{\omega})-(1-\phi_u)\frac{\kappa_v}{\phi_v}}{1+(\rho-1)\phi_u}$. Then, the selling price $q_u = q - \frac{\kappa_u}{\phi_u}$

becomes

$$q_u = \frac{\rho(1+\frac{\kappa_u}{\omega}) - \frac{\kappa_v}{\phi_v} - \frac{\kappa_u}{\phi_u} + (\rho-1)\kappa_u}{1+(\rho-1)\phi_u}$$

Therefore, $q_u \geq 1$ is equivalent to

$$\rho(1+\frac{\kappa_u}{\omega}) + (\rho-1)(\kappa_u - \phi_u) \geq 1 + \frac{\kappa_v}{\phi_v} + \frac{\kappa_u}{\phi_u}$$

or

$$\rho(1+\frac{\kappa_u}{\omega} - \kappa_u - \phi_u) + \kappa_u + \phi_u \geq 1 + \frac{\kappa_v}{\phi_v} + \frac{\kappa_u}{\phi_u}$$

Using again the asset price solution $\rho = \frac{\omega}{1-\omega}\frac{\kappa_v}{\kappa_u}\frac{\phi_u}{\phi_v}$, one can simplify the above inequality to

$$(1-\phi_u)(\rho - 1 - \frac{\kappa_v}{\phi_v} - \frac{\kappa_u}{\phi_u}) \geq 0$$

Since $\phi_u \in [0,1]$, we have

$$\rho \geq 1 + \frac{\kappa_v}{\phi_v} + \frac{\kappa_u}{\phi_u} = 1 + \frac{\kappa_v}{\xi\left(\gamma^{-1}\rho\right)^{-\eta}} + \frac{\kappa_u}{\xi(\gamma^{-1}\rho)^{1-\eta}},$$

where the last equality uses the fact that $\phi_u = \xi\theta^{1-\eta}$ and $\rho = \gamma\theta$. Finnaly, notice that from the steady state derviation, ρ can be expressed as $\rho = \chi^{-1}\left[\beta^{-1} - (1-\chi)\right]$, we obtain the condition stated in the lemma. □

C.2 Lemma 2

We first simplify the first-order condition associated with the Nash bargaining solution to

$$\frac{\omega}{\rho\left[q - \frac{1}{\phi_u}\right] + \frac{1-\phi_u}{\phi_u}q_v} = \frac{1-\omega}{q_v - q},$$

by using $u'\left(c_v\right)q_v = \beta\mathbb{E}_\Gamma J_S\left(S', B'; \Gamma'\right)$ and $u'\left(c_u\right) = \rho u'(c_v)$. Then

$$\omega\frac{\kappa_v}{\phi_v} = (1-\omega)\left[\rho\left(q - \frac{1}{\phi_u}\right) + \frac{1-\phi_u q_u}{\phi_u}\frac{\left(1-\phi_u\right)q_v}{1-\phi_u q_u}\right],$$

which can be further simplified to

$$\omega\frac{\kappa_v}{\phi_v} = \left(1-\omega\right)\rho\left(q - q_u\right),$$

by realizing that $\rho \equiv \frac{q_v}{q_r} = \frac{(1-\phi_u)q_v}{1-\phi_u q_u}$. Solving the above equation for ρ yields

$$\rho = \frac{\omega}{1-\omega}\frac{\kappa_v}{\kappa_u}\frac{\phi_u}{\phi_v} = \gamma\theta,$$

which is the expression in (26). One can further express the asset price q in terms of ρ or ϕ_u. Using the above expression along with the definition of ρ

$$\rho \equiv \frac{q_v}{q_r} = \frac{(1-\phi_u)\,q_v}{1-\phi_u q_u} = \frac{(1-\phi_u)\left(q+\frac{\kappa_v}{\phi_v}\right)}{1-\phi_u q+\kappa_u},$$

we can solve for the bargaining price q as

$$q = \frac{\rho\,(1+\kappa_u)-(1-\phi_u)\,\frac{\kappa_v}{\phi_v}}{1+(\rho-1)\,\phi_u} = \frac{\rho\left(1+\frac{\kappa_u}{\omega}\right)-\frac{\kappa_v}{\phi_v}}{1+(\rho-1)\,\phi_u}$$

where the last line uses (26) again. Realizing that $\phi_v = \xi\,(\gamma^{-1}\rho)^{-\eta}$ and $\phi_u = \xi\,(\gamma^{-1}\rho)^{1-\eta}$ we can rewrite the asset price as a function of ρ

$$q = \frac{\rho^{1-\eta}\left(1+\frac{\kappa_u}{\omega}\right)-\kappa_v\gamma^{-\eta}\xi^{-1}}{\rho^{-\eta}\left[1+(\rho-1)\,\xi\,(\gamma^{-1}\rho)^{1-\eta}\right]},$$

or, equivalently, ϕ_u

$$\begin{aligned} q &= \frac{\gamma^{1-\eta}\xi^{-1}\phi_u\left(1+\frac{\kappa_u}{\omega}\right)-\kappa_v\gamma^{-\eta}\xi^{-1}}{\gamma^{-\eta}(\xi^{-1}\phi_u)^{\frac{\eta}{\eta-1}}\left[1+\left(\gamma(\xi^{-1}\phi_u)^{\frac{1}{1-\eta}}-1\right)\phi_u\right]} \\ &= \frac{\gamma\phi_u\left(1+\frac{\kappa_u}{\omega}\right)-\kappa_v}{\xi^{\frac{1}{1-\eta}}\phi_u^{\frac{\eta}{\eta-1}}\left[1+\left(\gamma(\xi^{-1}\phi_u)^{\frac{1}{1-\eta}}-1\right)\phi_u\right]}. \end{aligned}$$

□

C.3 Proposition 2

Notice that in steady state, ρ, ϕ_u, and ϕ_v are functions of parameters that are indepdent of search costs κ_v and κ_u. When κ_v/κ_u is fixed, $\gamma = \frac{\omega}{1-\omega}\frac{\kappa_v}{\kappa_u}$ is fixed as well. Therefore, $\frac{\partial q}{\partial \kappa_v} > 0$ is equivalent to

$$\frac{\gamma g}{\omega}\phi_u - 1 < 0$$

Using the definition for γ and g, one has

$$\phi_u < 1-\omega$$

Since in the steady state $\rho = \chi^{-1}\left(\beta^{-1}-(1-\chi)\right)$ and $\phi_u = \xi\,(\gamma^{-1}\rho)^{1-\eta}$, the above inequality is equivalent to

$$\xi\left[\chi^{-1}g\left(\beta^{-1}-(1-\chi)\right)\right]^{1-\eta} < (1-\omega)^{\eta}\omega^{1-\eta}$$

C.4 Proposition 3

By differentiating the asset price from (27) with respect to ϕ_u, we get

$$
\begin{aligned}
&\frac{\partial q}{\partial \phi_u}\left[\xi^{\frac{1}{1-\eta}}\phi_u^{\frac{\eta}{\eta-1}}\left[1+(\rho-1)\,\phi_u\right]\right] \\
&=\gamma\left(1+\frac{\kappa_u}{\omega}\right)-q\frac{\partial}{\partial \phi_u}\left[\xi^{\frac{1}{1-\eta}}\phi_u^{\frac{\eta}{\eta-1}}\left[1+\left(\gamma(\xi^{-1}\phi_u)^{\frac{1}{1-\eta}}-1\right)\phi_u\right]\right], \qquad (32)
\end{aligned}
$$

where

$$
\frac{\partial}{\partial \phi_u}\left[\xi^{\frac{1}{1-\eta}}\phi_u^{\frac{\eta}{\eta-1}}\left[1+\left(\gamma(\xi^{-1}\phi_u)^{\frac{1}{1-\eta}}-1\right)\phi_u\right]\right]=\rho^{-1}\gamma\left[\phi_u\left(2\rho-\frac{1-2\eta}{1-\eta}\right)-\frac{\eta}{1-\eta}\right].
$$

Note that $2\rho-\frac{1-2\eta}{1-\eta}=\frac{\eta}{1-\eta}+2\rho-1$. A necessary and sufficient condition for $\frac{\partial q}{\partial \phi_u}>0$ is for the RHS of (32) to be non-negative. This is the case, whenever

$$
\phi_u \;<\; \left[\frac{\eta}{1-\eta}+\left(1+\frac{\kappa_u}{\omega}\right)\frac{\rho}{q}\right]\left[\frac{\eta}{1-\eta}+2\rho-1\right]^{-1}
$$

This condition requires ϕ_u to be small enough for the asset price and asset liquidity to correlate positively. When $\eta=0.5$, a sufficient condition is that

$$
\phi_u<\left[1+\left(1+\frac{\kappa_u}{\omega}\right)\right]\left[1+2\gamma(\xi^{-1}\phi_u)^2-1\right]^{-1}
$$

by using $\rho/q>1$ and noticing the relationship between ρ and ϕ_u. Such inequality with quadratic term of ϕ_u yields

$$
\phi_u<\frac{-1+\sqrt{1+8\xi^{-2}\gamma(3+\kappa_u/\omega)}}{4\gamma\xi^{-2}}\equiv\bar{\phi}_u
$$

Note that $\frac{\partial q}{\partial \phi_u}>0$ implies $\frac{\partial q}{\partial \phi_v}<0$, because $\frac{\partial q}{\partial \phi_v}=\frac{\partial q}{\partial \phi_u}\frac{\partial \phi_u}{\partial \phi_v}$ and

$$
\frac{\partial \phi_u}{\partial \phi_v}=\frac{\eta-1}{\eta}\xi^{\frac{1}{\eta}}\phi_v^{-\frac{1}{\eta}}<0.
$$

Hence, the same parameter restriction that ensures $\frac{\partial q}{\partial \phi_u}>0$ also ensures $\frac{\partial q}{\partial \phi_v}<0$. □

C.5 Corollary 1

Suppose that $\kappa_v\to 0$ and $\kappa_u\to 0$. Then $q_v\to q$ and the excess value of an additional match for the buyers goes to

$$
-J_m^v=-u'(c_v)q+\beta\mathbb{E}_\Gamma\left[J_S\left(S',B';\Gamma'\right)\right]=u'\left(c_v\right)\left(q_v-q\right)\to 0
$$

Hence, buyers become indifferent between trading and not trading at the margin. For trading not to break down, the sellers must also have zero excess value. To see this, consider a negative excess value for sellers. They would then be better off not to engage in asset sales at the margin, such that trading would break down. Suppose, in contrast, that sellers' excess value is positive. Since in the absence of search costs buyers are indifferent between trading and not trading at *any* price, sellers can improve by demanding a price that maximizes their welfare from an additional match; i.e. sellers would choose a q such that the excess value goes to zero, $J_m^u \to 0$, and they become indifferent with respect to an additional trade. Therefore,

$$J_m^u = u'(c_u)\left(q - \frac{1}{\phi_u}\right) + \frac{1-\phi_u}{\phi_u}\beta\mathbb{E}_\Gamma\left[J_S(S', B'; \Gamma')\right] = u'(c_v)\left[\rho\left(q - \frac{1}{\phi_u}\right) + \frac{1-\phi_u}{\phi_u}\right] \to 0$$

which shows that in the limit

$$\rho\left(q - \frac{1}{\phi_u}\right) + \frac{1-\phi_u}{\phi_u} = 0.$$

By rearranging we get

$$\rho = \frac{1-\phi_u}{1-\phi_u q}. \tag{33}$$

Remember that when $\kappa_v \to 0$ and $\kappa_v \to 0$, the limit of $\rho \equiv \frac{q_v}{q_r}$ simplifies to

$$\rho = \frac{(1-\phi_u)\, q}{1-\phi_u q}. \tag{34}$$

Equating conditions (33) and (34) then yields $q \to 1$, which implies $\rho \to 1$. Furthermore, we have that $\gamma \to \frac{\omega}{1-\omega}$, such that

$$\theta = \frac{\rho}{\gamma} \to \frac{1-\omega}{\omega}, \quad \phi_u \to \xi\left(\frac{1-\omega}{\omega}\right)^{1-\eta}, \quad \phi_v \to \xi\left(\frac{1-\omega}{\omega}\right)^{-\eta}$$

□