

Search-Based Endogenous Illiquidity and the Macroeconomy

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Abstract

We endogenize asset liquidity in a dynamic general equilibrium model with search frictions on asset markets. In the model, asset liquidity is tantamount to the ease of issuance and resaleability of private financial claims, which is driven by investors' participation on the search market. Limited funding ability of private claims creates a role for liquid assets, such as government bonds or fiat money, to ease funding constraints. We show that liquidity and asset prices can positively co-move. When the capacity of the asset market to channel funds to entrepreneurs deteriorates, the hedging value of liquid assets increases. Our model is thus able to match the liquidity hoarding observed during recessions.

Keywords: endogenous asset liquidity; search frictions

classification: E22; E44; G11; G12

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1 Introduction

Illiquidity of privately issued financial assets arises from impediments to their issuance and later transactions. Empirical evidence points to procyclical variation in the market liquidity of a wide range of financial assets.¹ The view that asset liquidity dries up during recessions has been further reinforced by the 2007-2009 financial crisis, when illiquidity problems were most pronounced for commercial paper and asset-backed securities.²

Illiquid primary or secondary equity and debt markets reduce firms' ability to finance investment, which creates a role for liquid assets, such as fiat money or government bonds. These liquid assets provide insurance against funding constraints as they can be readily used for financing purposes at any time.³ When funding constraints tighten in recessions, firms tend to rebalance their portfolios towards such liquid assets - a phenomenon referred to as "flight to liquidity". Variations in asset liquidity and the idea of liquidity hoarding as a hedging device against funding constraints goes back to [Keynes \(1936\)](#) and [Tobin \(1969\)](#). Nevertheless, the link between asset liquidity and aggregate fluctuations is often ignored in state-of-the-art dynamic general equilibrium models.

We propose a framework in which *endogenous* variation in asset liquidity interacts with macroeconomic conditions. To this end, we incorporate a search market for financial assets into an almost-standard real business cycle model. Search frictions give rise to asset illiquidity both on primary markets (issuance of new assets) and secondary markets (liquidation of existing assets). Asset liquidity can be measured by the endogenous fraction of new or existing assets that can be sold. The search market structure in our model is a stand-in for financial intermediation via markets or banks, both of which involve a costly matching process between capital providers and seekers.

The model shows how a drop in investor participation in the search market simultaneously reduces asset liquidity, tightens funding constraints, and decreases asset prices, which dampens real investment and production. Our central contributions are (i) to demonstrate that endogenizing liquidity is essential to generate co-movement between asset liquidity and

¹Studies by [Huberman and Halka \(2001\)](#), [Chordia, Roll, and Subrahmanyam \(2001\)](#), [Chordia, Sarkar, and Subrahmanyam \(2005\)](#) and [Naes, Skjeltorp, and Odegaard \(2011\)](#) assert that market liquidity is procyclical and highly correlated across asset classes such as bonds and stocks in the US. This observation implies that common factors drive liquidity.

²[Dick-Nielsen, Feldhütter, and Lando \(2012\)](#) identify a break in the market liquidity of corporate bonds at the onset of the sub-prime crisis. The liquidity component of spreads of all but AAA rated bonds increased and turnover rates declined, making refinancing on market more difficult. Commercial paper (CP), which is largely traded on a search market with dealers as match-makers, experienced large illiquidity in recessions reported by [Anderson and Gascon \(2009\)](#). In addition, money market mutual funds, the main investors in the CP market, shifted to highly liquid and secure government securities. Finally, [Gorton and Metrick \(2012\)](#) show that the repo market has registered strongly increasing haircuts during the crisis.

³In fact, U.S. nonfinancial firms only fund 35% of fixed investment through financial markets, of which 76% through debt and equity issuance and 24% through portfolio liquidations ([Ajello, 2012](#)).

asset prices; and (ii) to show that shocks to the cost of financial intermediation are an important source of flight to liquidity and business cycles.

Consider an economy where privately issued financial claims are backed by cash flow from physical capital used for production. There is a continuum of households whose members are temporarily separated during periods. Some become workers, others entrepreneurs. Only the latter have access to investment opportunities for capital goods creation. All household members are endowed with a portfolio of liquid assets (money)⁴ and private claims, which we interpret as a catch-all for privately issued assets such as corporate bonds and equity.

To finance investment, entrepreneurs exploit all available modes of funding: They issue new financial claims to their investment projects and liquidate their existing asset portfolio. Money is readily available for financing purposes and hence commands a liquidity premium. Private claims (both new and old) are only partially liquid, because they are traded on the search market. Participation in the search market is costly for both buyers and sellers. Individual buyers and sellers are matched by an intermediary who determines the transaction price by maximizing the total surplus, similar to the bargaining process in the labor search literature (e.g., [Diamond \(1982\)](#), [Mortensen and Pissarides \(1994\)](#), and [Shimer \(2005\)](#)).

Asset liquidity is measured by the fraction of private claims that can be sold or resold on this market in a given period. In addition, existing assets are effectively liquidated at a cost below the transaction price. Due to the limited funding from asset markets, entrepreneurs are financing constrained and cannot fund the first-best level of investment. The household will thus hold a portfolio of illiquid private claims and money. Such linkage between asset liquidity, financing constraints, and investment is similar to [Shleifer and Vishny \(1992\)](#).

This structure intends to emulate the features of over-the-counter (OTC) markets, in which a large fraction of corporate bonds, asset-backed securities, and private equity is traded. Participation costs in these markets arise from information acquisition as well as brokerage and settlement services from dealers and market makers.⁵ Alternatively, our framework can also be interpreted as a reduced-form approach towards modeling bank-based financial intermediation. In particular, the search market structure captures the costly matching process between savers (investors) and the corporate sector through financial intermediaries.

We consider two types of exogenous shocks: an aggregate productivity shock and a symmetric shock to the participation costs of buyers and sellers, which we interpret as an “intermediation cost shock”.

Negative aggregate productivity (TFP) shocks decrease the return to capital, make in-

⁴For simplicity, we consider all government-issued assets as money. Our framework could easily be extended to general interest bearing liquid assets as illustrated in the model section.

⁵See, e.g., [Dick-Nielsen, Feldhütter, and Lando \(2012\)](#), [Bao, Pan, and Wang \(2011\)](#), [Anderson and Gascon \(2009\)](#).

vestment into capital goods less attractive, and, hence crowd out investors from the search market. Negative intermediation cost shocks, on the other hand, make investment into liquid assets more attractive to hedge future investment. This reduces the incentive for investors to post costly buy orders on the search market. In either case, the fall in demand on the asset market exceeds that of supply (under some regularity conditions), such that sellers have a lower chance of encountering a buyer. Hence, the sales rate - or liquidity - of financial claims drops. Because a lower sales rate implies that entrepreneurs need to retain a larger equity stake in new investment projects, their financing constraints tighten and the option of breaking off negotiations becomes less valuable. Entrepreneurs are thus willing to accept a lower transaction price. In the aggregate, lower asset liquidity and prices restrict the funding available to entrepreneurs and, thereby, reduce real investment.

While both shocks generate procyclical asset liquidity and prices, only intermediation cost shocks induce a pronounced flight to liquidity. In the case of persistent adverse TFP shocks, investors have a weaker incentive to hedge against future investment, because of lower current and future returns to capital. Adverse intermediation cost shocks, however, do not deteriorate the quality of investment itself either today or tomorrow. Investors thus value the hedging service from liquid assets more strongly. Therefore, asset price movements are stronger. Intermediation cost shocks thus allow the model to match the volatility of asset prices and liquidity hoarding and their co-movement with GDP in the data.

To our knowledge, we are the first to incorporate endogenous asset liquidity in a dynamic macroeconomic model in a tractable way and to explore the feedback effects between asset liquidity and the real economy.⁶ Notice that [Kiyotaki and Moore \(2012\)](#) (henceforth, KM) demonstrate how *exogenous* asset market liquidity interact with aggregate fluctuations, in which firms can only sell an *exogenous* fraction of private claims to finance new investment. In contrast, money or government bonds can be fully sold and thus provide a liquidity service. When the resaleable fraction of private claims falls, financing constraints tighten and agents shift to money. As a result, real investment and production fall, and a recession begins.

Nevertheless, as pointed out by [Shi \(2012\)](#), exogenous liquidity variations leads to counterfactual asset price dynamics: A negative shock to asset saleability reduces the supply of financial assets, while demand remains relatively stable since the quality of investment projects is unaffected by liquidity shocks. The negative supply shock induces an persistent asset price boom that is at odds with the data. This counterfactual highlights the need to model asset liquidity endogenously, as we do in this paper.

Literature Review. Following KM, we model liquidity differences between private

⁶A recent study by [Yang \(2013\)](#) also considers endogenous asset liquidity. The difference is that we model liquid and illiquid assets together and the corresponding portfolio choice simultaneously.

claims and government-issued assets. As highlighted by them, the irrelevance result of [Wallace \(1981\)](#) on the neutrality of central banks’ portfolios no longer holds in such a setting. In fact, open market operations that change the composition of liquid and illiquid assets in agents’ portfolios have real effects. [Del Negro, Eggertsson, Ferrero, and Kiyotaki \(2011\)](#) analyze such “unconventional policy” after an exogenous fall in liquidity in an extended version of KM’s model. With standard monetary policy constrained by the “Zero Lower Bound”, liquidity injections effectively dampen the liquidity shortfall and stabilize the economy.⁷

Highlighted by [Shi \(2012\)](#), negative exogenous liquidity shocks lead to counterfactual asset price boom. We thus build a model with endogenous liquidity. The search literature provides a natural theory of endogenous liquidity as in [Lagos and Rocheteau \(2009\)](#) and has been applied to a wide range of markets such as housing⁸, OTC markets for asset-backed securities, corporate bonds, federal funds, private equity, etc.⁹ This line of research shows that search frictions can explain substantial variation in a wide range of measures of asset market liquidity (e.g., bid-ask spreads, trade volume, and trading delays).

Work by [Den Haan, Ramey, and Watson \(2003\)](#), [Wasmer and Weil \(2004\)](#), and [Petrosky-Nadeau and Wasmer \(2013\)](#) has emphasized the role of search and matching frictions in credit markets and their impact on aggregate dynamics,¹⁰ but does not study asset prices and portfolio rebalancing. We highlight that search frictions have important implications for asset saleability and prices, portfolio compositions, and hence on the real economy.

An alternative approach to endogenizing liquidity uses information frictions. [Eisfeldt \(2004\)](#) develops a partial equilibrium model with adverse selection in asset markets, in which investment and trading volume are amplified if asset liquidity endogenously varies with productivity. Dynamic adverse selection, asset prices, and trading delays are analyzed in an endowment economy framework by [Guerrieri and Shimer \(2012\)](#). While endogenizing asset liquidity, these studies do not consider the feedback effects of fluctuations in liquidity on production and employment. A notable exception is [Kurlat \(2013\)](#), who extends KM with endogenous resaleability through adverse selection but neglecting the role of liquid assets. In [Eisfeldt and Rampini \(2009\)](#) firms need to accumulate liquid funds in order to finance investment opportunities. While the supply of liquid assets affects investment, secondary markets for asset sales are shut off as an alternative means of financing. In contrast to these contributions, we jointly model endogenous liquidity on primary and secondary asset markets, the role of liquid assets as the lubricant of investment financing, and asset liquidity’s

⁷More generally, [Kara and Sin \(2013\)](#) show that market liquidity frictions induce a trade-off between output and inflation stabilization off the ZLB that can be attenuated by quantitative easing measures.

⁸See e.g. [Wheaton \(1990\)](#) and [Ungerer \(2012\)](#).

⁹See e.g. [Duffie, Gârleanu, and Pedersen \(2005, 2007\)](#); [Ashcraft and Duffie \(2007\)](#); [Feldhutter \(2011\)](#).

¹⁰As shown in [Beaubrun-Diant and Tripier \(2013\)](#), search frictions also help explain salient business cycle features of bank lending relationships, such as countercyclical net interest margins and loan separation rates.

feedback effects on business cycles.¹¹

Our framework also differs along important dimensions from search-theoretic models of money such as [Lagos and Wright \(2005\)](#) and [Rocheteau and Wright \(2005\)](#). In this literature, money has a transaction function in anonymous search markets. Recent extensions include privately created liquid assets such as claims to capital ([Lagos and Rocheteau, 2008](#)) or bank-deposits ([Williamson, 2012](#)) as media of exchange. Our framework rather emphasizes the role of financial assets - both public and private - as stores of value, i.e. money and equity claims are used for financing purposes. Moreover, our approach is able to generate endogenous variation in asset liquidity and the associated premia, because private claims are subject to search frictions themselves, rather than serving to overcome such frictions on other markets. These differences notwithstanding, a common tenet is that liquid assets play an important role in economic transactions by relaxing deep financial frictions.

By studying intermediation cost shocks which affect asset market liquidity, we also complement the literature on financial shocks. Recent contributions by [Jermann and Quadrini \(2012\)](#), [Christiano, Motto, and Rostagno \(2014\)](#), and [Jaccard \(2013\)](#) identify financial shocks as an important source of business cycle fluctuations. Our approach shows how such shocks may be endogenously amplified within financial markets.

2 The Environment

This model is a variant of a standard real business cycle (RBC) model. Time is discrete and infinite ($t = 0, 1, 2, \dots$). The economy has three sectors: final goods producers, households (with entrepreneurs and workers), and financial intermediaries. Final goods producers generate output by renting capital and labor from households.¹² Financial intermediaries facilitate asset transaction, and there are search frictions afflicting the purchase and sale of financial assets issued by previous and current entrepreneurs. In addition, liquid government-issued assets can be traded on a spot market. To abstract from government policies, we model liquid assets as non-interest bearing money. We focus on equilibrium in which this intrinsically worthless asset is valued for its liquidity service and accepted by all market participant.¹³

¹¹Asset illiquidity may further interact with financing constraints to induce delays in asset sales as in [Cui \(2013\)](#). This interaction prolongs shocks to the financing conditions of the private sectors and results in countercyclical productivity dispersion.

¹²They rebate profits back to households. In equilibrium, profits are zero because of perfect competitions.

¹³The derivation with interest-bearing government bonds and taxation is available upon request.

2.1 Final Goods Producers

Competitive firms rent aggregate capital stock K_t and hire aggregate labor N_t from households to produce output (general consumption goods) according to

$$Y_t = e^{z_{a,t}} F(K_t, N_t),$$

where $F(K_t, N_t) = K_t^\alpha N_t^{1-\alpha}$, $\alpha \in (0, 1)$, and $z_{a,t}$ measures exogenous aggregate productivity. The profit-maximizing rental rate and wage rate are thus

$$r_t = e^{z_{a,t}} F_K(K_t, N_t), \quad w_t = e^{z_{a,t}} F_N(K_t, N_t). \quad (1)$$

2.2 Households

Households are comprised of entrepreneurs (who invest in new capital) and workers (who save). Workers earn wages by supplying labor. Entrepreneurs do not work, but only they have investment opportunities. They issue new claims and/or sell existing claims to finance new investment, to the extent possible. Claims that are not money need to be issued or resold through intermediation with a search technology. These claims are illiquid in the sense that for every unit of capital put on sale only a fraction $\phi_{u,t}$ are sold. Financial frictions are thus represented by the fact that entrepreneurs have to retain $(1 - \phi_{u,t})$ of new investment which can be put up for sale in the same period as it is being incurred.

2.2.1 A Representative Household

At the beginning of t , the aggregate productivity and the unit cost of trading private claims are realized. A representative household specifies policy rules for its members, who receive *equal shares* of assets accumulated from previous periods. Then, they receive a shock that determines their type, which is idiosyncratic across members and through time. With a probability χ a member becomes an entrepreneur (called type u), and with a probability $(1 - \chi)$ a worker (called type v).¹⁴ By the law of large numbers, each household thus consists of a fraction χ of entrepreneurs and a fraction $(1 - \chi)$ of workers. Both groups are temporarily separated during each period and there is no consumption insurance between them.

In the middle of t , final goods producers rent capital and labor from households to produce consumption goods and the payoffs from private claims are thus realized. At the

¹⁴Following the notation in the labor search literature, we denote workers as type v and entrepreneurs as type u members. The underlying logic is that workers post purchase orders on the search market which are akin to “vacant” asset positions, while entrepreneurs post assets for sale which are in a sense “unemployed” when lying idle on their balance sheets.

same time, household members trade liquid assets on a competitive market in exchange for consumption goods. For equity, entrepreneurs put assets on sale and workers put buying quotes of assets, both through financial intermediaries. Then, these intermediaries match potential buyers and sellers and intermediate a transaction price. Entrepreneurs then invest in physical capital, after which workers and entrepreneurs consume.

At the end of t , members come together again to share their accumulated assets. All members hence enter the next period again with an equal share of their household's assets.¹⁵

Preferences. The household objective is to maximize

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} [U(c_{u,t+s}, c_{v,t+s}) - (1 - \chi)h(n_{t+s})], \quad (2)$$

where $\beta \in (0, 1)$ is the discount factor, $U(c_{u,t}, c_{v,t}) = \chi u(c_{u,t}) + (1 - \chi)u(c_{v,t})$ is the total utility derived from consumption by entrepreneurs ($c_{u,t}$) and workers ($c_{v,t}$). $u(\cdot)$ is a standard strictly increasing and concave utility function, and $h(\cdot)$ captures the dis-utility derived from labor supply n_t . \mathbb{E}_t is the expectation operator conditional on information at time t .

Balance Sheet. Physical capital (K_t) is owned by households and rented to final goods producers such that capital earns a return. There is a claim to the future return of every unit of capital, which household members can either retain or offer for sale to outside investors. We normalize equity by capital stock, equity depreciates with capital stock at the same rate (denoted by δ). These claims, if successfully sold, can be sold at unit price q_t (determined by the intermediation).

In addition, households could hold money with nominal price level P_t . Hence, at the onset of period t , households own a portfolio of liquid assets, equity claims on other households' return on capital, and own physical capital. These assets are financed by net worth plus equity claims issued against their own physical capital. The financing structure gives rise to the beginning-of-period balance sheet in Table 1.

Table 1: Household's Balance Sheet

liquid assets	B_t/P_t	equity issued	$q_t S_t^I$
other's equity	$q_t S_t^O$		
capital stock	$q_t K_t$	net worth	$q_t S_t + B_t/P_t$

Since new claims and old claims are both traded on the search market, we only need to

¹⁵The representative household with temporarily separated agents has been introduced in Lucas (1990) and applied to the KM framework in Shi (2012) and Del Negro, Eggertsson, Ferrero, and Kiyotaki (2011).

keep track of net equity, defined as

$$S_t = \underbrace{S_t^O}_{\text{equity claims on others' capital}} + \underbrace{K_t - S_t^I}_{\text{unissued capital stock}}.$$

2.2.2 Individual Members

Let s_{jt} and b_{jt} be net equity and money for a typical household member j . Then, the net equity evolves according to

$$s_{j,t+1} = (1 - \delta)s_{j,t} + i_{j,t} - m_{j,t}, \quad (3)$$

where $i_{j,t}$ is investment into capital goods, and $m_{j,t}$ corresponds to asset sales. Let $c_{j,t}$ and $n_{j,t}$ denote consumption and labor supply, respectively.

Workers flow-of-funds. The household delegates equity purchases on the search market to workers, because they do not have investment opportunities ($i_{v,t} = 0$). Therefore, workers $j = v$ post asset positions v_t to acquire new or old equity at unit cost κ_v . On the search market, each posted position is filled with a probability $\phi_{v,t} \in [0, 1]$, and an individual buyer expects to purchase an amount $m_{v,t} = -\phi_{v,t}v_t$. Notice that a worker's flow-of-funds constraint reads

$$c_{v,t} + \kappa_v v_t + \frac{b_{v,t+1}}{P_t} = w_t n_{v,t} + r_t s_{v,t} - q_t \phi_{v,t} v_t + \frac{b_{v,t}}{P_t}, \quad (4)$$

where labor income and the return on equity and money are used to finance consumption, search costs, and the new accumulation of equity claims and money. To simplify, we define the effective purchasing price per equity as

$$q_{v,t} \equiv q_t + \frac{\kappa_v}{\phi_{v,t}}, \quad (5)$$

where q captures the transaction price and $\frac{\kappa_v}{\phi_v}$ represents search costs per transaction (scaled by the probability of encountering a seller ϕ_v). By using (3) and $m_{v,t} = \phi_{v,t}v_t$, the flow-of-funds constraint (4) becomes

$$c_{v,t} + q_{v,t} s_{v,t+1} + \frac{b_{v,t+1}}{P_t} = w_t n_{v,t} + r_t s_{v,t} + (1 - \delta) q_{v,t} s_{v,t} + \frac{b_{v,t}}{P_t}. \quad (6)$$

Entrepreneurs' flow-of-funds. Entrepreneurs $j = u$ decide how many assets u_t to put up for sale at unit cost κ_u in order to finance new investment ($i_{u,t} > 0$). These assets include existing equity claims on other households' capital stock and their own unissued capital stock

(in total $s_{u,t}$), plus claims on new investment, $i_{u,t}$. Then, the amount of private financial claims that are up for sale is bounded from above by the existing stock of equity and the volume of new investment, $u_t \leq (1 - \delta) s_{u,t} + i_{u,t}$. Offers are matched with a buyer with probability $\phi_{u,t} \in [0, 1]$. Therefore, an individual entrepreneur expects to sell $m_{u,t} = \phi_{u,t} u_t$. Notice that the returns on equity and money are used to finance consumption, search costs, and the accumulation of equity (with new investment taken into account) and money. The flow-of-funds constraint can thus be written as

$$c_{u,t} + i_{u,t} + \kappa_u u_t + \frac{b_{u,t+1}}{P_t} = r_t s_{u,t} + q_t \phi_{u,t} u_t + \frac{b_{u,t}}{P_t}, \quad (7)$$

We define the effective selling price of unit financial asset as

$$q_{u,t} \equiv q_t - \frac{\kappa_u}{\phi_{u,t}}. \quad (8)$$

When $\kappa_u > 0$, the effective selling price is below the transaction price. Hence, not only entrepreneurs face constraints in issuance and reselling equity, they also face a price reduction in liquidation. These two have important implications on the degree of financing constraints and households' portfolio choices, similar to [Shleifer and Vishny \(1992\)](#). Together with (3) and $m_{u,t} = \phi_{u,t} u_t$, the flow-of-funds constraint (7) becomes

$$c_{u,t} + i_{u,t} + q_{u,t} [s_{u,t+1} - i_{u,t} - (1 - \delta) s_{u,t}] + \frac{b_{u,t+1}}{P_t} = r_t s_{u,t} + \frac{b_{u,t}}{P_t}. \quad (9)$$

Further, it is helpful to substitute out new investment by defining $e_{u,t} \in [0, 1]$, which denotes the fraction of total assets that entrepreneurs put on sale

$$u_t = e_{u,t} [(1 - \delta) s_{u,t} + i_{u,t}].$$

Then, we express flow-of-funds constraint (9) as

$$c_{u,t} + q_{r,t} s_{u,t+1} + \frac{b_{u,t+1}}{P_t} = r_t s_{u,t} + [e_{u,t} \phi_{u,t} q_{u,t} + (1 - e_{u,t} \phi_{u,t}) q_{r,t}] (1 - \delta) s_{u,t} + \frac{b_{u,t}}{P_t}, \quad (10)$$

$$\text{where } q_{r,t} \equiv \frac{1 - e_{u,t} \phi_{u,t} q_{u,t}}{1 - e_{u,t} \phi_{u,t}}. \quad (11)$$

The left-hand side (LHS) of (10) captures entrepreneurs' spending on consumptions and accumulations of equity and money, while the right-hand side (RHS) represents entrepreneurial (total) *net-worth* including rental income from capital claims, the value of existing equity claims, and the real value of money. Note that a fraction $e_{u,t} \phi_{u,t}$ is saleable and, hence, val-

ued at $q_{u,t}$, while a fraction $(1 - e_{u,t}\phi_{u,t})$ is retained and valued at $q_{r,t}$, which is the effective replacement cost of existing assets. To see this, notice that entrepreneurs can sell a fraction $e_u\phi_{u,t}$ of their financial assets at price $q_{u,t}$. For every unit of new investment, they will accordingly need to make a down-payment $(1 - e_{u,t}\phi_{u,t}q_{u,t})$ and retain a fraction $(1 - e_{u,t}\phi_{u,t})$ as inside equity. With this interpretation, if entrepreneurs replace existing assets by new assets issued against investment, $q_{r,t}$ is indeed the effective replacement cost.¹⁶

Because of the comparison with worker's budget constraint, (10) involves gross investment. New investment can be backed out from $s_{u,t+1} = (1 - e_{u,t}\phi_{u,t})(s_{u,t} + i_t)$ and (10). Formally,

$$i_{u,t} = \frac{\left[(r_t + e_{u,t}\phi_{u,t}q_{u,t}(1 - \delta)) s_{u,t} + \frac{b_{u,t}}{P_t} \right] - c_{u,t}}{1 - e_{u,t}\phi_{u,t}q_{u,t}}. \quad (12)$$

which says that entrepreneurs' liquid net-worth net of consumption can be levered at $(1 - e_u\phi_u q_u)^{-1}$ to invest in new capital.

2.2.3 A Household's Problem

Aggregation. Recall that $j \in \{u, v\}$ indicates workers and entrepreneurs, respectively. We define aggregate type-specific variables as $X_{u,t} \equiv \chi x_{u,t}$ and $X_{v,t} \equiv (1 - \chi) x_{v,t}$. Household-wide variables is the aggregation of workers' and entrepreneurs' quantities, i.e., $X_t = X_{v,t} + X_{u,t}$. For example, aggregate consumption is the sum of consumption of workers and entrepreneurs, i.e., $C_t = C_{v,t} + C_{u,t}$.

For simplicity, we switch to recursive notation, i.e., let x and x' denote x_t and x_{t+1} . Since all household members equally divide the assets accumulated before, $S_u = \chi S$, $S_v = (1 - \chi)S$, $B_u = \chi B$, and $B_v = (1 - \chi)B$. Because entrepreneurs do not work, we also have that $N = N_v$. Given these simplifications, individual budget constraints (6) and (10) aggregate to¹⁷

$$C_v + q_v S'_v + \frac{B'_v}{P} = wN + [r + q_v(1 - \delta)](1 - \chi)S + (1 - \chi)\frac{B}{P}. \quad (13)$$

$$C_u + q_r S'_u + \frac{B'_u}{P} = [r + [\phi_u q_u + (1 - \phi_u)q_r](1 - \delta)]\chi S + \chi\frac{B}{P}, \quad (14)$$

Note that every entrepreneur chooses the same $e = e_u$, total investment can be aggregated

¹⁶This down-payment price captures the effect of search costs on equity accumulation: higher search costs decrease the effective sales price, which increases the down-payment that in turn depresses equity accumulation. Therefore, the entrepreneurs' ability to leverage will be lower if search costs are higher.

¹⁷Notice that we implicitly impose that $S'_v \geq (1 - \delta)(1 - \chi)S$ such that workers in a household are always buyers. Such condition is satisfied in our later numerical analysis because we focus on shocks that will not push workers to sell assets to smooth consumption. Aggregation takes into account type-specific transactions on the search market and evolutions of equity.

from (12) to

$$I = \frac{\chi [(r + e\phi_u q_u (1 - \delta)) K + \frac{B}{P}] - C_u}{1 - e\phi_u q_u}. \quad (15)$$

A Household's Problem. Let $J(S, B; \Gamma)$ be the value of the representative household with equity claims S and money B , given the collection of aggregate state variables Γ whose evolution is taken as given by the household.¹⁸ Since at the end of the period workers and entrepreneurs reunite to share their stocks of equity and money, we have

$$S' = S'_v + S'_u, \quad B' = B'_v + B'_u. \quad (16)$$

Then, the value satisfies the following Bellman equation,

Problem 1:

$$J(S, B; \Gamma) = \max_{\{e, N, C_u, C_v, S'_u, S'_v, B'_v\}} \chi u \left(\frac{C_u}{\chi} \right) + (1 - \chi) \left[u \left(\frac{C_v}{1 - \chi} \right) - h \left(\frac{N}{1 - \chi} \right) \right] + \beta \mathbb{E}_\Gamma [J(S', B'; \Gamma')]$$

s.t. (13), (14), and (16).

2.3 Search, Matching, and Asset Price

Search and Matching. Matching between buyers and sellers of private claims is handled by zero-profit intermediaries owned by the household. The implicit assumption is that it is extremely costly for individual buyers to meet sellers, and vice versa. Financial intermediations provide specialized services to find counterparts, and later on screening and monitoring. The cost of matching buyers and sellers are paid through the search costs, i.e., κ_u and κ_v . Note that we do not distinguish financial institutions (e.g., banks) and dealers in financial markets in our model. They are both modeled as the financial sector with costly matching technology which intermediates the asset price. A detail discussion between these two types of agents and related economic consequence can be found in [De-Fiore and Uhlig \(2011\)](#).

Buyers post total asset positions $V = \phi_v^{-1} [S'_v - (1 - \delta)S_v]$ that are to be filled. Sellers put their new and old assets on sale, offering $U = e[(1 - \delta)\chi S + I]$. After V and U are determined, the number of aggregate matches M is determined by intermediations' matching technology

$$M(V, U) = \xi V^{1-\eta} U^\eta,$$

¹⁸Once we proceed to the equilibrium definition, $\Gamma \equiv (K, B; z_a, z_\kappa)$ where K is the total capital stock, B is the total amount of money circulated, z_a is total factor productivity in final goods production, and z_κ is an intermediation cost shock in the search market. The exogenous stochastic processes for z_a and z_κ are specified in the numerical examples in Section 4.

where $\eta \in (0, 1)$ is the elasticity of matches with respect to assets on sale, and ξ measures the matching efficiency.

Defining θ as the ratio of vacant asset positions V to assets on sale U , we have

$$\theta \equiv \frac{V}{U}, \quad \phi_v \equiv \frac{M}{V} = \xi\theta^{-\eta}, \quad \phi_u \equiv \frac{M}{U} = \xi\theta^{1-\eta}, \quad (17)$$

where ϕ_v captures the probability of a buyer meeting a seller for each unit of asset positions posted, and ϕ_u the probability of a seller meeting a buyer for each unit of assets put on sale. Recall that ϕ_u also represents the fraction of financial assets that can be sold *ex post* in a given period. Therefore, we refer to ϕ_u as asset saleability or liquidity.

Notice that θ expresses the search market tightness from a buyer's perspective. A larger θ indicates that buyers have difficulty in finding appropriate investment opportunities on the search market, such that U are relatively small compared to V . Lastly, noticing that $\phi_v^{-1}\phi_u = \theta$, we can link the relationship between ϕ_v and ϕ_u as

$$\phi_v = \xi^{\frac{1}{1-\eta}} \phi_u^{\frac{\eta}{\eta-1}}. \quad (18)$$

Asset Prices. Once a unit of offered assets is matched to a vacancy position, intermediaries offer a price q to both party. Since intermediaries makes zero profits and are owned by the households, they seek to maximize the total surplus of each trade by bargaining on behalf of each side of the trade. Notice that the amount of matched assets $m_{j,t}$ is predetermined at the point of bargaining. Therefore, buyers and sellers interact at the margin $m_{j,t}$, i.e., the match surplus for both buyers and sellers is the respective marginal value of an additional transaction.

Denote by J^v and J^u the value of individual workers and entrepreneurs from the point of view of the household. In consumption goods unit, a buyer's surplus amounts to

$$-J_m^v = -q + \beta \mathbb{E}_\Gamma \left[\frac{J_S(S', B'; \Gamma')}{u'(c_v)} \right].$$

Intuitively, if the deal is agreed the buyer sacrifices q today but gains the household value of one more unit of assets tomorrow (normalized by the marginal utility of a worker's consumption).¹⁹ Similarly, the sellers' surplus is the marginal value to the household of an additional

¹⁹Note that search market participation costs are already sunk at the bargaining stage. However, search costs are not ignored since households take them into account when determining optimal asset posting decisions by workers and entrepreneurs.

match for entrepreneurs

$$J_m^u = q - \frac{1}{e\phi_u} + \beta \left(\frac{1}{e\phi_u} - 1 \right) \mathbb{E}_\Gamma \left[\frac{J_S(S', B'; \Gamma')}{u'(c_u)} \right],$$

which says that the seller gains $(q - e^{-1}\phi_u^{-1})$ today plus a continuation value from a successful match. The contemporary surplus reflects that entrepreneurs earn the bargaining price q , but spend $e^{-1}\phi_u^{-1}$ resources per additional match on new investment projects. The evolution of entrepreneurs' equity position can be expressed as the difference between offered and sold assets (i.e., $s'_u = u - m_u = (e^{-1}\phi_u^{-1} - 1)m_u$). Entrepreneurs retain a fraction $(e^{-1}\phi_u^{-1} - 1)$ for each unit of successful matches as inside equity, which is brought back to the household. Therefore, the continuation value of a match consists of the marginal value of future assets to the household multiplied by this factor (normalized by the marginal utility of an entrepreneur's consumption).

Note that all members within the groups of buyers and sellers are homogeneous, such that the type-specific valuations are identical in all matched pairs. We consider the case in which the transaction price q is determined by surplus division between buyers and sellers. That is, intermediaries set a price q to maximize

$$\max_q \{ (J_m^u)^\omega (-J_m^v)^{1-\omega} \} \quad (19)$$

where $\omega \in (0, 1)$ is the fraction of the surplus that goes to sellers. Notice that this setup is similar to bilateral (generalized) Nash bargaining between buyers and sellers over the match surplus. In the bilateral bargaining case, ω is the bargaining power of sellers. In this sense, our price setting by intermediations is similar to the wage determining process in [Ravn \(2008\)](#) and [Ebell \(2011\)](#), where individual workers come to bargain on behalf of their respective household.

2.4 Recursive Competitive Equilibrium

We close the model by defining the recursive competitive equilibrium. It is mainly a collection of conditions in the previous discussion.

Definition 1:

The recursive competitive equilibrium is a mapping $K \rightarrow K'$, with associated consumption, investment, labor, and portfolio choices $\{C_v, C_u, N, e, I, S'_v, S'_u, B'_v, B'_u\}$, asset liquidity $\{\phi_u, \phi_v\}$, and a collection of prices $\{P, q_v, q_u, q_r, w, r\}$, given exogenous evolutions of aggregate productivity z_a and search costs (κ_v, κ_u) , such that

1. *final goods producers' optimality conditions in (1) hold;*
2. $S_v = (1 - \chi)S$, $S_u = \chi S$, $B_v = (1 - \chi)B$, and $B_u = \chi B$. *Given prices, the policy functions solve the representative household's problem (Problem 1), the household budget constraint (13) and (14), and aggregate investment in (15);*
3. *market clearing conditions hold, i.e.,*

(a) *the capital market clears: $K' = (1 - \delta)K + I$ and $K = S$;*

(b) *the search market clears: (18) holds, q solves (19), and the effective prices are*

$$q_v = q + \frac{\kappa_v}{\phi_v}, \quad q_u = q - \frac{\kappa_u}{\phi_u}, \quad q_r = \frac{1 - e\phi_u q_u}{1 - e\phi_u};$$

(c) *the market for liquid assets clears: $B' = B$;*

To verify that Walras' Law is satisfied, notice that the investment equation and the household budget constraint resemble the entrepreneurs' and workers' budget constraints (13) and (14). These two constraints imply the aggregate resource constraint

$$C + I + \kappa_v V + \kappa_u U = e^{z\alpha} K^\alpha N^{1-\alpha}, \quad (20)$$

where U and V are again the total number of assets on sale and asset positions to be filled.

3 Equilibrium Characterization

We mainly focus on the interesting equilibrium with positive participation costs on both sides, i.e., $\kappa_v > 0$ and $\kappa_u > 0$. The limiting case when costs become zero follows in the end. Notice that the cost of participation may be so large that households find it better to stay internal financing and search market is not active.

We restrict our attention to the economy in which search market is active. That is, the replacement costs $q_r \leq 1$. Using the definition of q_r , we know that effective selling price $q_u \geq 1$. Then, the effective buying price is strictly greater than 1 (note: $\kappa_v > 0$ and $\kappa_u > 0$). Thus, $q_v > q > q_u \geq 1 > q_r$.²⁰ Compared to workers who value equity at price q_v , the

²⁰As shown in Corollary 1, in a frictionless economy with costless search market participation the capital price approaches $q_t = 1$. In this case, the internal equals the external cost of creating capital goods, such that capital production yields zero profits and financial constraints cease to exist. Empirically, the capital price captures Tobin's q , which ranges between 1.1 and 1.21 in the U.S. economy, i.e. well above 1. For this empirically relevant case, capital production is profitable, which reflects financial constraints of firms. During recessions q_t typically falls and erodes firms' net worth, which tightens financing constraints further.

price of equity is strictly cheaper on the view of entrepreneurs. Therefore, the household will prompt entrepreneurs to spend whatever net worth they are not consuming on creating new equity. Entrepreneurs thus sell as many existing equity claims as possible and do not invest into money, i.e., $e = 1$ (or $u = (1 - \delta)s + i$) and $B'_u = 0$.

To ensure $q_r \leq 1$, we restrict exogenous parameters. To see this, first define

$$\gamma \equiv \frac{\omega}{1 - \omega} \frac{\kappa_v}{\kappa_u}$$

and let the steady state value of κ_v , κ_u , and γ be $\bar{\kappa}_v$, $\bar{\kappa}_u$, and $\bar{\gamma}$. Formally,

Lemma 1:

Suppose $\kappa_v > 0$ and $\kappa_u > 0$. If the following parameter restriction is satisfied

$$\frac{\beta^{-1} - (1 - \chi)}{\chi} \geq \frac{\bar{\kappa}_v \left(\frac{\beta^{-1} - (1 - \chi)}{\chi} \right)^\eta}{\xi \bar{\gamma}^\eta} + \frac{\bar{\kappa}_u \left(\frac{\beta^{-1} - (1 - \chi)}{\chi} \right)^{\eta-1}}{\xi \bar{\gamma}^{\eta-1}} + 1, \quad (\text{A1})$$

then $q_u \geq 1$, $q_r < 1$ in the neighborhood around steady state.

Proof. See the Appendix C.1. □

As an illustration, if we further restrict $\bar{\kappa}_v = \bar{\kappa}_u = \kappa$, the above restriction implies an upper bound for the search costs κ . (A1) then directly implies that cost of participation should not be too large.

3.1 Households' Portfolio Choice

To reduce the number of prices, we define the ratio of effective buying price and effective replacement cost:

$$\text{where } \rho \equiv \frac{q_v}{q_r}. \quad (21)$$

By using the types' budget constraints (13) and (14) to substitute out C_u and C_v in Problem 1, and using $e = 1$ and $B'_u = 0$, we know that a household's optimal choice can be reduced to the set $\{N, S'_u, S'_v, B'_v\}$. The first-order condition for labor is²¹

$$u'(c_v) w = \mu. \quad (22)$$

This result is because firms are leveraged, such that the contraction in their funding base due to the negative shock to net worth is strongly amplified.

²¹As in a portfolio choice problem, the corresponding first-order conditions are also sufficient due to the concavity of the objective function.

The first-order conditions for S'_u and S'_v are

$$u'(c_u) q_r = \beta \mathbb{E}_\Gamma [J_S(S', B'; \Gamma)], \quad u'(c_v) q_v = \beta \mathbb{E}_\Gamma [J_S(S', B'; \Gamma)],$$

from which we learn that

$$u'(c_u) = \rho u'(c_v). \quad (23)$$

ρ is inversely related to risk-sharing among workers and entrepreneurs. When $\rho = 1$, search frictions disappear and entrepreneurs are not financing constrained (see Corollary 1). In this case, (23) naturally implies $c_u = c_v$, i.e., perfect consumption risk-sharing among household members. In an economy where the search market structure imposes financing frictions, we have $\rho > 1$. Therefore, $c_u < c_v$ and the risk-sharing capacity of the household decreases in ρ . Finally, the optimality condition for money holdings B'_v is

$$u'(c_v) \frac{1}{P} = \beta \mathbb{E}_\Gamma [J_B(S', B'; \Gamma)].$$

We derive asset pricing formulae for equity and money. Using the envelope condition and noticing that $\phi_u q_u + (1 - \phi_u) q_r = 1$,²²

$$\begin{aligned} J_S &= u'(c_u) \chi [r + 1 - \delta] + u'(c_v) (1 - \chi) [r + q_v (1 - \delta)] \\ &= u'(c_v) [(\chi \rho + 1 - \chi) r + (1 - \delta) (\chi \rho + (1 - \chi) q_v)], \end{aligned}$$

together with the first-order condition for equity S'_v we obtain

$$\mathbb{E}_\Gamma \left[\frac{\beta u'(C'_v) (\chi \rho' + (1 - \chi)) r' + (1 - \delta) (\chi \rho' + (1 - \chi) q'_v)}{u'(C_v) q_v} \right] = 1, \quad (24)$$

where the second term in the expectations operator captures the internal return on equity from the perspective of the household. Similarly, we can derive another asset pricing formula for money by applying the envelope condition again

$$\mathbb{E}_\Gamma \left[\frac{\beta u'(C'_v) \chi \rho' + 1 - \chi}{u'(C_v) \pi'} \right] = 1, \quad (25)$$

where the second term in the expectations operator is the internal return on money from

²²One could interpret $\phi_u q_u + (1 - \phi_u) q_r = 1$ in the following way. The fully resaleable fraction of existing equity worth $\phi_u q_u$, while the non-resaleable fraction of existing equity is $1 - \phi_u q_u$ which is the net-worth paid by the entrepreneurs.

the perspective of the household, and inflation is defined as

$$\pi' \equiv \frac{P'}{P}.$$

In the steady state, condition (25) implies that $[\chi\rho + 1 - \chi]\pi^{-1} = \beta^{-1}$. Because $\bar{\pi} = 1$, we have

$$\bar{\rho} = \chi^{-1}[\beta^{-1} - (1 - \chi)] > 1.$$

As a result, the real interest rate π^{-1} will be lower than the time preference rate β^{-1} .²³ This fact shows that money provide a liquidity service and, accordingly, carries a liquidity premium, which is easiest to be seen in the steady state:

Proposition 1:

Suppose (A1) holds. In the neighborhood around steady state, money provides a liquidity service. The steady state liquidity premium amounts to

$$\Delta_B \equiv [\chi\bar{\rho} + (1 - \chi) - 1] \frac{1}{\bar{\pi}} = \frac{(\bar{\rho} - 1)\chi}{\bar{\pi}} = \beta^{-1} - 1 > 0.$$

To illustrate, when $\rho > 1$, liquidity frictions matter and entrepreneurs are financing constrained. An additional unit of money then relaxes entrepreneurs' constraints by increasing their net-worth, which allows them to leverage their investment or, equivalently, their future equity position. Once we solved the endogenous asset price, we will show in Corollary 1 that in the limiting case ($\rho \rightarrow 1$) equity can be sold frictionlessly and money loses its liquidity value. The asset pricing formulae then collapse to standard Euler equations in a RBC model. However, if the search market is not frictionless, the liquidity premium is not zero and may vary substantially over time. The liquid asset share in households' portfolio composition will also shift over time.

3.2 The Bargained Asset Price

Asset price is set to maximize the total surplus of buyers and sellers. Assuming an interior solution,²⁴ the sufficient and necessary first-order condition yields

$$\frac{\omega}{u'(c_u)(q - \phi_u^{-1}) + (\phi_u^{-1} - 1)\beta\mathbb{E}_\Gamma J_S(S', B'; \Gamma')} = \frac{1 - \omega}{-u'(c_v)q + \beta\mathbb{E}_\Gamma J_S(S', B'; \Gamma')}.$$

²³Though we focus on fiat money such that $P' = P$ in the steady state (and $\pi^{-1} = 1 < \beta^{-1}$), one can easily imagine an economy where the government steps in and may run inflation or deflation.

²⁴We will discuss the corner solution when costs of participation go to zero.

By using the household's optimality condition for asset holdings, $u'(c_v) q_v = \beta \mathbb{E}_\Gamma J_S(S', B'; \Gamma')$, and the risk-sharing condition, $u'(c_u) = \rho u'(c_v)$, we can derive an analytical solution for the asset price, stated in the following proposition:

Lemma 2:

Suppose (A1) holds. When both $\kappa_v > 0$ and $\kappa_u > 0$, the asset price solution simplifies to

$$\rho = \frac{\omega}{1 - \omega} \frac{\kappa_v}{\kappa_u} \theta, \quad (26)$$

which can be solved for q as a function of saleability ϕ_u ²⁵

$$q = \frac{\gamma \left(1 + \frac{\kappa_u}{\omega}\right) \phi_u - \kappa_v}{\xi^{\frac{1}{1-\eta}} \phi_u^{\frac{\eta}{\eta-1}} \left[1 + \left(\gamma (\xi^{-1} \phi_u)^{\frac{1}{1-\eta}} - 1\right) \phi_u\right]}. \quad (27)$$

Proof. See Appendix C.2. □

Proposition 2 is the main result of our model, which links equity price and the costs of intermediation. The financial frictions lead to a price of equity that is above the cost of capital (which is one). Entrepreneurs would like thus to issue more claims, but the frictions prevent them to further doing so which keeps the price of equity higher than cost of capital. Note that if financial frictions are completely removed, asset price will fall to the cost of capital. Hence, the central question is what happens to the equity price when financial frictions change mildly, instead of comparing an economy with financial frictions to one without such frictions at all.

To illustrate how asset price responds to relatively mild changes in the financial frictions, we focus on the steady state and leave equilibrium dynamics in numerical examples. On the one hand, (26) implies that a higher κ_v decreases asset price since the demand side finds it more costly and participates less; On the other hand, an increase of κ_u in (26) will push up asset price as it makes even more costly for entrepreneurs to issue and supply equity (more financing constrained) such that equity price will be even higher than the cost of capital. The net-effects depends on the parameters.

For discussion simplicity, we restrict attention to the case that the ratio κ_v and κ_u is kept the same. Recall in steady state $\bar{\rho} = \chi^{-1}[\beta^{-1} - (1 - \chi)]$, which is independent of search costs. Further, ϕ_u and ϕ_v are also independent of search costs from (17) and (26). Then, asset price drops when intermediation costs are higher.

²⁵An intermediate step in the derivation, which is used for simplification later, is $q = \frac{\rho \left(1 + \frac{\kappa_u}{\omega}\right) - \frac{\kappa_v}{\phi_u}}{1 + (\rho - 1) \phi_u}$.

Proposition 2:

Suppose (A1) holds and the ratio of $\bar{\kappa}_v/\bar{\kappa}_u = g$ is fixed. If the following condition is satisfied

$$\xi \left[\frac{\beta^{-1} - (1 - \chi)}{\chi} \right]^{1-\eta} < (1 - \omega)^\eta \omega^{1-\eta} g^{1-\eta},$$

then in steady state asset price q drops when $\bar{\kappa}_v$ and $\bar{\kappa}_u$ increase.

Proof. See the Appendix C.3. □

The above condition is more likely to hold when g is higher. That is, when the cost of buying is relatively higher than the cost of selling, the demand side is more sensitive to the participation costs' increase. Higher intermediation costs reduce more demand than supply, which depresses asset price. Such endogenous changes of asset demand and supply are the key insight from an endogenous search market.

In our model, endogenizing asset liquidity gives rise to a non-trivial relationship between q and ϕ_u . Similar to the discussion above, if ϕ_u is small enough, the asset price also falls together with a drop of liquidity ϕ_u .

Proposition 3:

q correlates positively with asset saleability ϕ_u (i.e. $\frac{\partial q}{\partial \phi_u} > 0$) and negatively with the purchase rate ϕ_v (i.e. $\frac{\partial q}{\partial \phi_v} < 0$), if

$$\phi_u < \left[\frac{\eta}{1-\eta} + \left(1 + \frac{\kappa_u}{\omega} \right) \right] \left[\frac{\eta}{1-\eta} + 2\gamma(\xi^{-1}\phi_u)^{\frac{1}{1-\eta}} - 1 \right]^{-1}$$

When $\eta = 0.5$, the above sufficient condition simplifies to $\phi_u < \sqrt[3]{(1 + \frac{\kappa_u}{2\omega})\gamma^{-1}\xi^2}$.

Proof. See Appendix C.4. □

Intuitively, the drop in saleability implies that a larger share of investment needs to be financed out of entrepreneurs' own funds. On the one hand, this tightens the contemporaneous financing constraints of entrepreneurs. The threat point for entrepreneurs of breaking off negotiations over an additional asset sale and self-financing at the margin becomes less attractive. Entrepreneurs are thus more willing to accept a lower bargaining price. On the other hand, retaining a larger fraction of equity stakes also implies that entrepreneurs return more assets to the household, which relaxes the funding constraints of future generations of entrepreneurs. This effect supports the threat point, such that entrepreneurs ask for a higher transaction price in a successful match. Thus, a trade-off emerges between current and future funding constraints.

Proposition 3 shows that the contemporaneous effect dominates as long as the sales rate is small enough, because current financial constraints bind strongly. If financial frictions are sufficiently tight, entrepreneurs will have to accept a lower price when the demand side is less willing to participate. Our model can thus generate simultaneous decreases in asset liquidity and the asset price through the simultaneous reaction of *supply and demand*.

In contrast, an exogenous drop in asset saleability, such as in KM and Shi (2012), acts like a negative supply shock on the asset market: The decline in saleability translates into a tighter financing constraint for entrepreneurs and less supply of financial claims on the asset market. However, the productivity of capital is not affected by the shock such that asset demand does not fall much. The dominating supply contraction triggers an asset prices boom - a counter-factual phenomenon in recessions. Although this effect is still present in our framework, there are competing forces from the demand side that outweigh the supply contraction under the condition of Proposition 3.

Frictionless limiting case. When there are no search costs for either buyers or sellers, i.e. $\kappa_v = 0$ and $\kappa_u = 0$, the search market price will go to $q = 1$, because there is no asset supply shortage. In this case, money loses its liquidity premium and the economy collapses to the RBC framework. Households' Euler equation becomes the standard ones in a RBC framework since $\rho = 1$. In summary,

Corollary 1:

When $\kappa_u \rightarrow 0$ and $\kappa_v \rightarrow 0$, $q_v \rightarrow q_u \rightarrow q \rightarrow 1$ and $\rho \rightarrow 1$.

Proof. See Appendix C.5. □

To back out asset liquidity and saleability ϕ_u when $\kappa_u \rightarrow 0$ and $\kappa_v \rightarrow 0$, one could solve consumption and investment first in the relevant RBC model because of $q = 1$ and perfect consumption risk-sharing. Then, ϕ_u can be backed out by the aggregate investment in (15).²⁶

4 Numerical Examples

We obtained some analytical results in previous discussion. This section uses numerical tools to illustrate system dynamics. As in a standard specification, we consider an AR(1) process for aggregate productivity, i.e.,

$$z'_a = \rho_a z_a + \epsilon'_a,$$

²⁶When $\kappa_u = 0$ but $\kappa_v > 0$, we have similar outcome, except that households always spend a fix fraction of resources to purchase equity. When $\kappa_v = 0$ but $\kappa_u > 0$, the household will not participate in the search market. This is because when $\kappa_v = 0$ the bargain price $q = 1$ which implies that $q_u < 1$ (given $\kappa_u > 0$). That is, (A1) is violated when $\kappa_v = 0$ and $\kappa_u > 0$.

with i.i.d. $\epsilon'_a \sim N(0, \sigma_a^2)$. We further introduce a symmetric shock to the cost of financial intermediation, which in our asset search framework corresponds to an increase in these participation costs. We let

$$\kappa_u = e^{z_\kappa} \bar{\kappa}_u, \quad \kappa_v = e^{z_\kappa} \bar{\kappa}_v,$$

where z_κ follows an AR(1) process

$$z'_\kappa = \rho_\kappa z_\kappa + \epsilon'_\kappa,$$

with i.i.d. $\epsilon'_\kappa \sim N(0, \sigma_\kappa^2)$. Rather than affecting the production frontier of the economy, this shock simply impairs the capacity of the search market to intermediate funds between workers and entrepreneurs and, thus, unfolds its effects solely through asset prices and liquidity.

Using the model as a laboratory, we compare the different equilibrium responses to aggregate productivity shocks and intermediation cost shocks. In particular, we compare to the quarterly data with the model's prediction of liquidity ratio and asset price, together with standard macro variables. Liquidity ratio is defined as the total amount of nominal liquid assets PB (essentially money and government bonds, see Appendix A) circulated in the U.S. over total assets $PB + PqK$. For asset price, we use Wilshire 5000 price full cap index from 1971Q1-2013Q4. We use 1971Q1-2013Q4 GDP data, which is the sum of real private consumption and real private fixed investment, since we do not model government.

We use HP filter (coefficient 1600) to detrend all time series. We found that liquidity ratio negatively correlated with GDP (correlation -0.58) while asset price positively correlated with GDP (correlation 0.50). These two correlations suggest that asset price drops in recessions and there is portfolio rebalance towards liquid assets at the same time, though there are some other noises that make the correlations imperfect.

4.1 Calibration

We set utility function as a standard CRRA one, i.e., $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ and disutility for labor as $h(n) = \mu n$.²⁷ Parameters β , σ , and δ are chosen exogenously and are similar to standard macro model calibration. We choose α and μ to target the investment-to-GDP ratio and working hours (Table 2). Note that the parameter χ can be interpreted as the fraction of firms which adjust capital in a period. According to [Doms and Dunne \(1998\)](#), the annual fraction is 0.20 which translates to $\chi = 0.054$ quarterly.

There are five search-market related parameters $\{\xi, \bar{\kappa}_v, \bar{\kappa}_u, \eta, \omega\}$. Due to the constant returns to scale matching technology on the search market, ξ and η are not independent.

²⁷The reason for such disutility function is to solve steady state easily. A more complicated function would not change the main results (available upon request)

Table 2: Baseline calibration

	Parameter	Baseline Value	Target/Source
<i>Preferences and Production Technology</i>			
Household discount factor	β	0.9850	Exogenous
Relative risk aversion	σ	2	Exogenous
Utility weight on leisure	μ	3.8995	Working time: 33%
Mass of entrepreneurs	χ	0.0540	Doms and Dunne (1998)
Depreciation rate of capital	δ	0.0280	Exogenous
Capital share of output	α	0.3214	Investment-to-GDP ratio: 18.0%
<i>Search and Matching</i>			
Supply sensitivity of matching	η	0.5000	Exogenous
Matching efficiency	ξ	0.6100	Saleability $\phi_u = 0.2800$
Buyer search costs	$\bar{\kappa}_v$	0.1108	Tobins $q = 1.1500$
Seller search costs	$\bar{\kappa}_u$	0.0147	Cost of Intermediation-to-investment 0.1000
Bargaining weight of sellers	ω	0.4463	$B/(B + PqK) = 0.0953$

Notes: The model is calibrated to quarterly frequency. Standard errors of estimated parameters are in brackets.

Without loss of generality, we set $\eta = 0.5$ and calibrate ξ . We are then left with four independent parameters, which we calibrate to match four targets. Tobin’s q ranges from 1.1 to 1.21 in the U.S. economy according to Compustat data. We target $q = 1.15$. The liquidity ratio is around 10% on average from 1971Q1-2013Q4, which can calibrate ω . US flow-of-funds data shows that ϕ_u is approximately 0.28,²⁸ and we target $\phi_u = 0.28$. Finally, we target the total cost of intermediation to be 10% of total investment, in-line with the findings in the cost of initial public offering (IPO) (e.g., [Chen and Ritter \(2000\)](#)).²⁹

We set the persistence and the size of shocks to target the volatility (0.02) and 1st order correlation (0.89) of GDP’s cyclical components. By using only productivity shocks, we have

$$\rho_a = 0.88, \sigma_a = 0.008.$$

By using only shocks to intermediation costs, the exercise gives

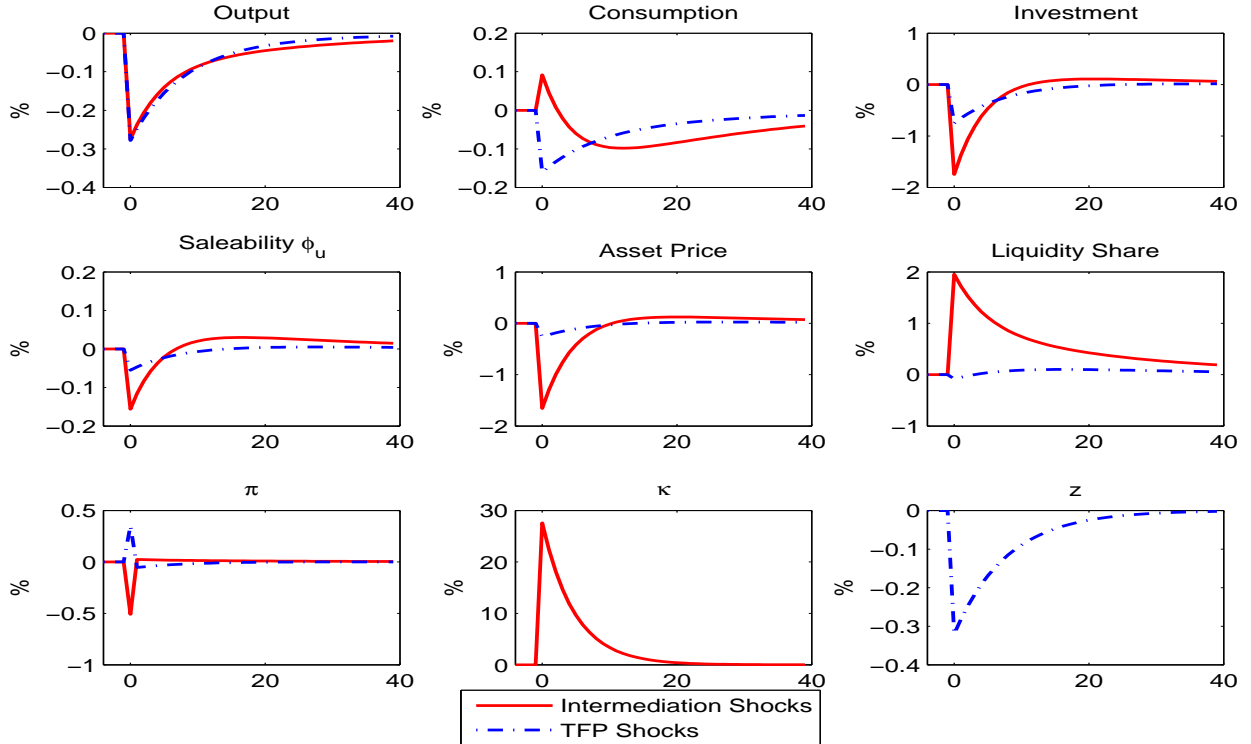
$$\rho_\kappa = 0.81, \sigma_\kappa = 0.69.$$

We use these parameters to show two numerical simulations in the following. By design, these two shocks will generate very similar aggregate output dynamics. The focus will be the different paths of other variables.

²⁸[Nezafat and Slavik \(2010\)](#) use the US flow-of-funds data for non-financial firms to estimate the stochastic process of ϕ_u . Interpreting ϕ_u as the ratio of funds raised in the market to fixed investment, they find that the mean of ϕ_u is 0.28.

²⁹The share of financial industry is about 8% on average of GDP according to [Philippon \(2013\)](#). However, the measure of financial industry includes insurance industry and intermediation activities for consumer loans. Therefore, we choose to direct target investment related activities. Note that our share of “financial industry” is only about 2% of total GDP, which is a conservative calibration about the effects of cost of financial intermediation.

Figure 1: Impulse responses (as percentage deviations from steady state values) after intermediation shocks and aggregate productivity (TFP) shocks at time 0.



4.2 Equilibrium Responses to Shocks

Adverse aggregate productivity shocks. Suppose a shock hit at time 0 (see z dynamics in Figure 1). This shock depresses the rental rate of capital and its value to the household. Search for investment into entrepreneurs is less attractive and the amount of purchase orders from workers drops. The demand-driven fall is reflected in the sharp drop in asset saleability ϕ_u . This endogenous decline of asset liquidity amplifies the initial shock in two ways: (1) it reduces the quantity of assets that entrepreneurs are able to sell; (2) the bargaining price, i.e. private assets' resale value falls - though only modestly - in line with our analytical result in Proposition 3. Both effects constrain entrepreneurs and thus tighten their financing constraints. As a result, investment falls.

In principle, money's liquidity service becomes more valuable to households when private claims' liquidity declines. However, in the case of a persistent TFP shock lower expected returns to capital make future investment less attractive. This effect works against the incentive to hedge against asset illiquidity for future investment. Which effect dominates depends on the calibration and is thus an empirical question. In our calibration, the profitability of investment projects falls sufficiently for the liquidity ratio to drop. This fact can also be

reflected on inflation $\pi = P/P_{-1}$, which indicates that nominal price P increases and gradually declines to the steady state. To the extent that total factor productivity reverts back to the steady state while asset liquidity is still subdued, hedging becomes more attractive which explains the relatively fast recovery of the liquidity ratio.

Intermediation shocks. Suppose a shock hit at time 0 (see κ dynamics in Figure 1). By construction, higher search costs generate the same output dynamics. The shocks immediately depress the effective sales price of private claims, while raising their effective purchase price. In addition, higher search costs bind resources. Both the substitution and income effect induce households to adjust their portfolios. Realizing that search market participation is more costly now and later, households seek to reduce their exposure to private financial claims. On the supply side, though less investment can be issued, financing-constrained entrepreneurs still want to sell as many assets as possible in order to take full advantage of profitable investment opportunities. Asset demand on the search market thus shrinks relative to asset supply, which depresses asset resaleability while improving the purchase rate.

Since the sharp drop in asset liquidity tightens entrepreneurs financing constraints substantially, the threat point of abandoning the bargaining process with a potential buyer worsens. Entrepreneurs as sellers are willing to accept a lower price. The bargaining price thus falls strongly and amplifies the initial shock by depressing entrepreneurs' net worth further. This effect is mirrored in a significant decline of investment activity, the impact response of which is about six times stronger than that of output.

While the intermediation cost shock depresses the demand for and liquidity of private assets, it substantially increases the hedging value of money. To see this, note that future investment remains profitable since the productivity of capital is not affected by the shock. To take advantage of future investment opportunities, households seek to hedge against the persistent illiquidity of private claims by expanding their liquidity holdings. This motive consequently drives up the liquidity ratio.

4.3 Discussion

The equilibrium dynamics suggest two key results. (1) In order to reconcile declining asset liquidity with falling asset prices, liquidity must be an endogenous phenomenon. In other words, it must be a consequence, rather than a cause of economic disturbances. (2) Both standard productivity and genuine search market shocks affect the hedging value of liquid assets. However, only the latter unambiguously implies a negative co-movement between liquidity ratio and aggregate output.

4.3.1 Cycle Statistics

Some key business cycle statistics of the model in comparison to the data are reported in Table 3, where only aggregate productivity shocks are considered. Our main targets are consumption, investment, asset price, and liquidity share. As in a usual RBC model, consumption and investment volatility, autocorrelation, and correlation with GDP are in-line with data. However, the liquidity ratio and asset price move too little in the model. Besides, compared to the data, liquidity ratio in the model does not negatively co-move with GDP enough, while asset price move too closely with GDP in the model.

Table 3: Cycle statistics with only aggregate productivity shocks

Variable x	Relative volatility $\frac{\sigma_x}{\sigma_y}$		Correlation $\rho(x, y)$		1st auto-correlation	
	Data	Model	Data	Model	Data	Model
Output	0.02	0.02	1.00	1.00	0.89	0.89
Consumption	0.44	0.67	0.88	0.98	0.80	0.92
Investment	3.45	2.43	0.96	0.98	0.85	0.87
Liquidity Ratio	3.44	0.87	-0.58	-0.27	0.89	0.98
Asset Price	5.23	0.79	0.50	0.88	0.81	0.84

Note: The volatility of output (y) is reported as is. The relative volatilities and correlations of other variables are measured against y .

Table 4 shows the relevant statistics when there are only intermediation shocks. Unlike the economy with only productivity shocks, the volatility of liquidity ratio and asset price are much higher in the economy with only intermediation shocks. The volatility is closer to the data (though liquidity ratio fluctuates more and asset price fluctuates less than the data). The liquidity ratio in the model successfully generate countercyclical movements in liquidity ratio, mimicking the liquidity hoarding in recessions. Note that since the data indicates a negative correlation around -0.60, intermediation shocks alone predicts too much correlation.

Table 4: Cycle statistics with only intermediation shocks

Variable x	Relative volatility $\frac{\sigma_x}{\sigma_y}$		Correlation $\rho(x, y)$		1st auto-correlation	
	Data	Model	Data	Model	Data	Model
Output	0.02	0.02	1.00	1.00	0.89	0.89
Consumption	0.44	0.79	0.88	0.44	0.80	0.97
Investment	3.45	4.61	0.96	0.78	0.85	0.78
Liquidity Ratio	3.44	7.85	-0.58	-0.96	0.89	0.91
Asset Price	5.23	4.36	0.50	0.75	0.81	0.78

Note: The volatility of output (y) is reported as is. The relative volatilities and correlations of other variables are measured against y .

4.3.2 Shocks with Different Persistence

As a comparison, we vary the persistence of the two aggregate shocks, replacing ρ_a or ρ_κ by a more persistent number (0.90) or a less persistent number (0.80). These exercises are to illustrate the persistence effects of shocks on portfolio rebalance. The different persistence could also affect macro variables directly (through production) and indirectly (through investment).

The qualitative difference is small (Figures 2 and 3), but different persistence do change the speed and magnitude of macro variables, liquidity shares, and asset prices in the two experiments. The key reason is again how valuable is the hedging value provided by liquid assets for future investment.

For example, if low aggregate productivity is perceived to be more persistent in the future, hedging value of liquid assets will be depressed longer. The liquidity share drops to a slightly lower value than the baseline and takes longer time to come back. In contrast, when intermediation shocks are perceived to more persistent, liquidity share takes longer time to come back, together with a deeper jump in the beginning. Note that liquidity share is always above the steady state value regardless of persistence.

Figure 2: Impulse responses (as percentage deviations from steady state values) after aggregate productivity shocks with different persistence ($\rho_a = 0.90$, $\rho_a = 0.80$, and the baseline).

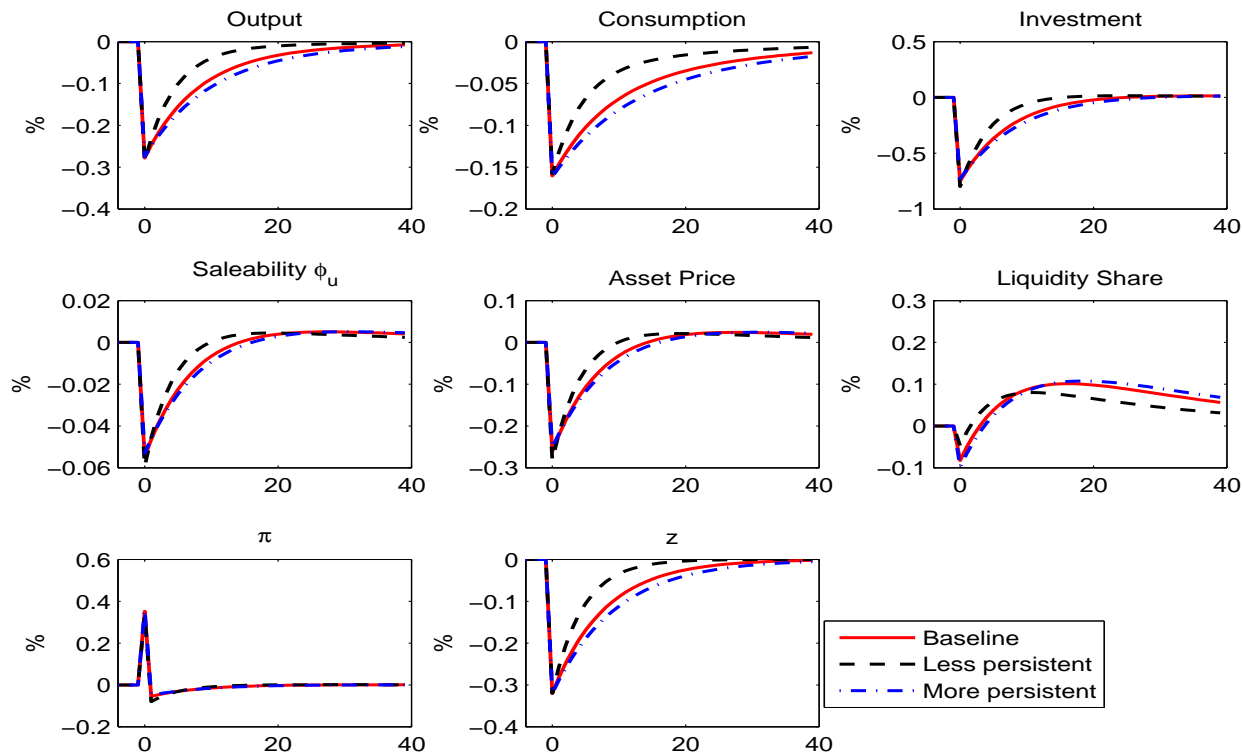
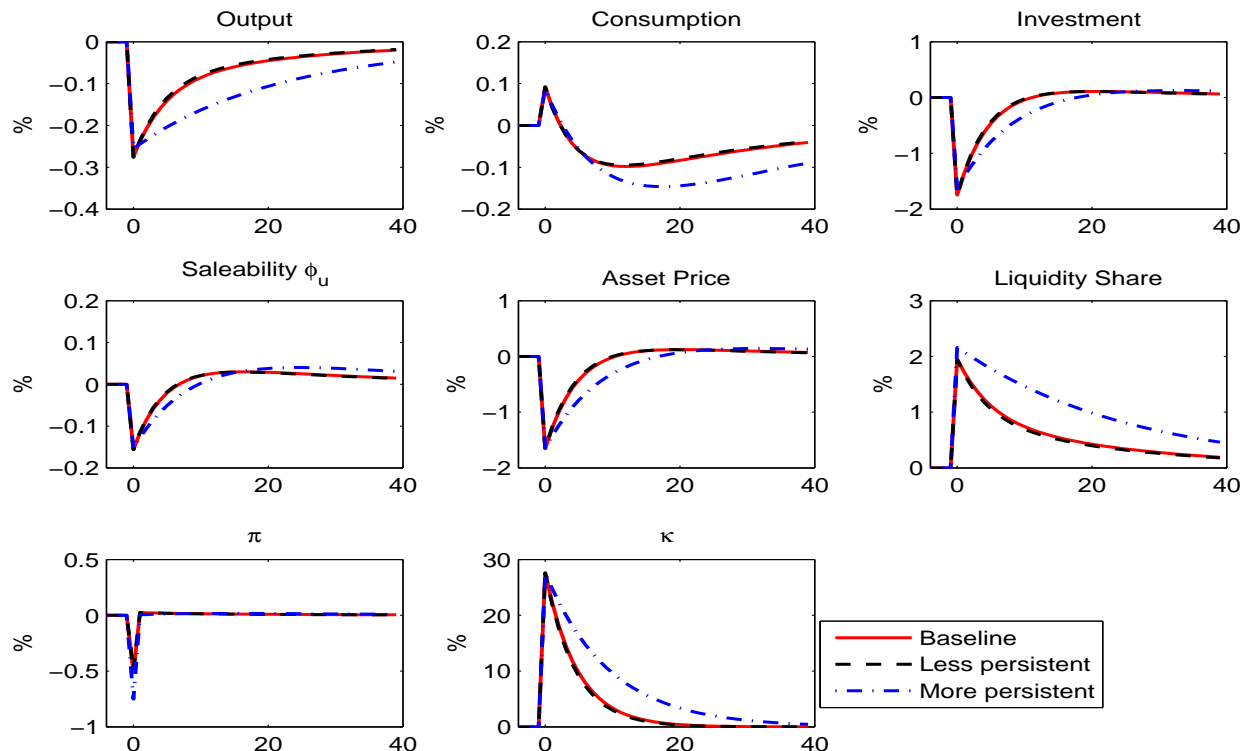


Figure 3: Impulse responses (as percentage deviations from steady state values) after intermediation shocks with different persistence ($\rho_\kappa = 0.90$, $\rho_\kappa = 0.80$, and the baseline).

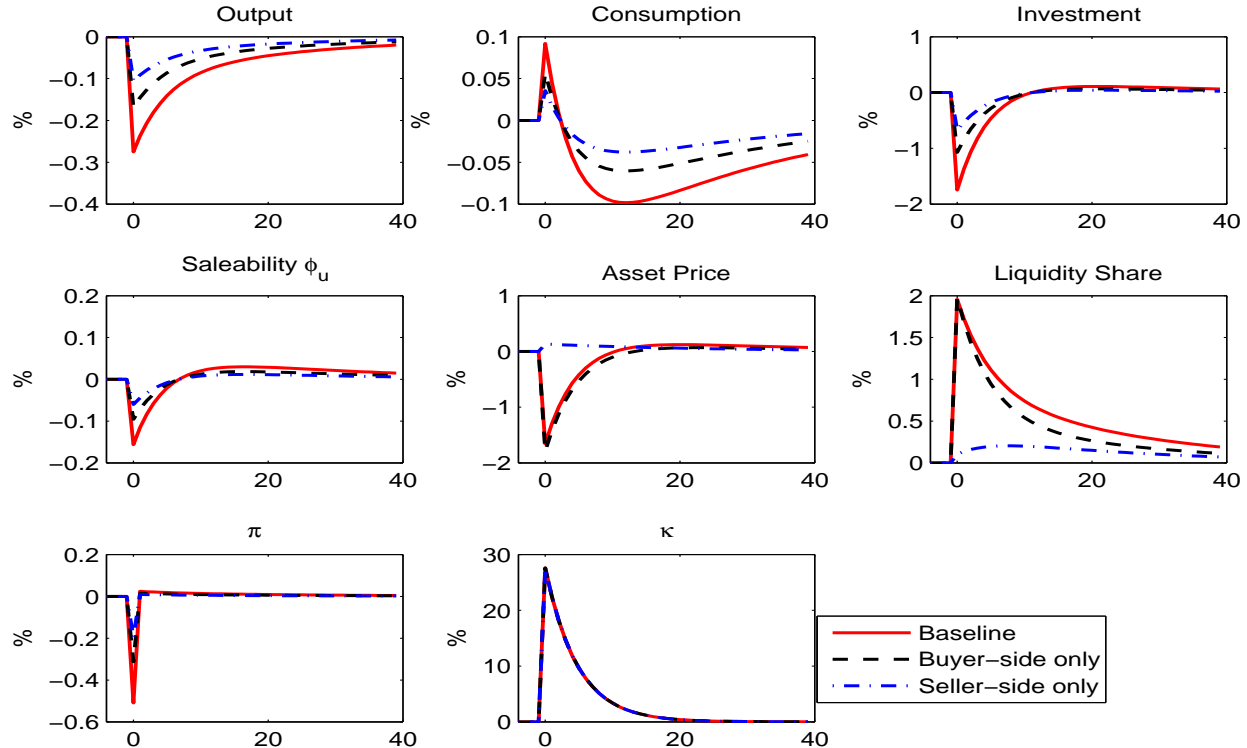


4.3.3 Non-symmetric Shocks

Finally, note that the shocks to financial intermediation affect both sides of the market. This feature leads us to examine the equilibrium responses to shocks to only one side of the participants. If the shocks only affect buyers' side (Figure 4), the dynamics is similar to the baseline impulse responses although the magnitude is smaller. If the shocks only affect sellers' side, macro variable dynamics and the tightness of financing constraints react similarly, but clearly asset price and liquidity share are different. Asset price boom and there is not much flight to liquidity, which contributes to very mild increase in liquidity share.

Importantly, this equilibrium response after shocks to only sellers' side is very similar to the result in [Kiyotaki and Moore \(2012\)](#) and [Shi \(2012\)](#). In their studies, an exogenous tightened liquidity constraint (a reduction of ϕ_u) leads to a boom of asset price. Both new and old assets become harder to sell and the supply of assets is depressed significantly. Demand for private claims thus drop. But the drop of demand cannot compensate the reduction of supply, which pushes up asset price. Our endogenous liquidity mechanism shows that to overturn this asset price anomaly, disturbances in financial sectors need to strongly affect demand side. One should also notice the amplification mechanism built-in: anticipating

Figure 4: Impulse responses (as percentage deviations from steady state values) after intermediation shocks to both buyers and sellers, only buyers, and only sellers.



persistent drop of demand, both issuance and resale will be hard for a while, which leads to a further reduction of demand today.³⁰

Shi (2012) suggests that aggregate productivity shocks are necessary to overturn the asset price anomaly generated by exogenous liquidity shocks. Our simulation shows that this does not have to be the case, provided that liquidity is modeled endogenously. Further more, the cost shocks that leads to endogenous variation in asset liquidity are different from aggregate productivity shocks, since it affects mostly the final investment settlement. To see this, recall the goods market clearing condition (20)

$$C + I + \kappa_v V + \kappa_u U = Y.$$

³⁰The above discussion leads to a final check of the endogenous liquidity mechanism. An alternative way of thinking financial disturbance is to shock the matching function itself. More specifically, we shock the matching efficiency ξ in order to check whether an efficiency problem generated from the financial sector could lead to drop of asset price and liquidity. This line of reasoning is very similar to the productivity shocks in a standard RBC exercise. But these shocks affect the financial sector itself (instead of to the goods producer sector). An adverse efficiency shock, for example because of excess-borrowing and later contagious bank run, makes the financial sector functioning less as before. Nevertheless, the answer is negative. When matching technology is worse, the dominant force is still the supply of capital in which asset price will increase. For the sake of space, the detail simulation and comparison is available upon request.

Aggregate productivity shocks affect the right-hand side, while intermediation cost shocks affect $\kappa_v V + \kappa_u U$ on the left-hand side. One can thus interpret the cost of intermediation shocks as a particular investment specific technology shocks (e.g., Fisher (2006) and Primiceri, Justiniano, and Tambalotti (2010)).

5 Conclusion

We endogenize asset liquidity in a macroeconomic model with search frictions. Endogenous fluctuation of asset liquidity may be triggered by shocks that affect asset demand and supply on the search market either directly (intermediation cost shock), or indirectly (productivity shock). By tightening entrepreneurs' financing constraints, they feed into investment, consumption and output. Interpreting liquidity as asset saleability, we show that asset prices can co-move with liquidity. The *endogenous* nature of asset liquidity is key to match this pro-cyclicality, as *exogenous* liquidity shocks would act as negative supply shocks on the asset market and lead to higher asset prices in recessions.

We also show that the liquidity service provided by intrinsically worthless government-issued assets, such as money, is higher when financing constraints bind tightly. As a result, shocks to the cost of financial intermediation increase the hedging value of liquid assets, enabling our model to replicate the “flight to liquidity” or countercyclical ratio of liquid assets relative to GDP observed in the U.S. data.

Our search framework can be interpreted as a model of market-based financial intermediation. It can, however, also be seen as a short-cut to model bank-based financial intermediation: financial intermediaries help channel funds from investors to suitable creditors in need of outside funding, a process which resembles a matching process. Adding further texture by explicitly accounting for intermediaries' balance sheets would open interesting interactions between liquidity cycles and financial sector leverage and maturity transformation.

Regarding government interventions, our framework suggests that, as in KM, open market operations in the form of asset purchase programs can have real effects by easing liquidity frictions. However, there might be potential crowding-out effects on the private market participants once we incorporate endogenous liquidity frictions. Future research could focus on the optimal design of conventional and unconventional monetary as well as fiscal policy measures in the presence of illiquid asset markets.

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Appendices

A Data

The measure of liquid assets B_t consists of all liabilities of the federal government circulated inside the US economy. To obtain the measure, we use U.S. flow-of-funds data. In particular, we use Treasury securities, net of saving bonds (for financing World War II), net of holdings by the monetary authority and the rest of the world, plus reserves and vault cash of depository institutions with the monetary authority, plus checkable deposits and currency net of the monetary authority's liabilities due to the rest of the world and due to the federal government. This measure is similar to the one in [Del Negro, Eggertsson, Ferrero, and Kiyotaki \(2011\)](#), but is cleaned from liquid assets held by agents outside the US.

Following again [Del Negro, Eggertsson, Ferrero, and Kiyotaki \(2011\)](#), PqK measure the value of capital in the economy. We use the balance sheet of households, the non-corporate and the corporate sectors to obtain the market value of aggregate capital. On households' side, we add real estate, equipment and software of non-profit organizations, and consumer durables. As for the non-corporate sector, we add real estate, equipment and software and inventories. As for the corporate sector, we obtain the market value of the capital stock by summing the market value of equity and liabilities net of financial assets. Finally, we subtract from the market value of capital the government credit market instruments, TARP, and trade receivables.

B Equilibrium Conditions

B.1 Recursive Competitive Equilibrium

Notice that $C = C_v + C_u$, such that ³¹

$$C_v = \rho_v C, \quad C_u = \rho_u C,$$

where

$$\rho_v \equiv \frac{1 - \chi}{1 - \chi + \rho^{-1/\sigma} \chi}, \quad \rho_u \equiv \frac{\chi}{\rho^{1/\sigma} (1 - \chi) + \chi}.$$

We use ρ_v and ρ_u in the subsequent analysis. We change the recursive equilibrium slightly by using $\rho = \frac{q_v}{q_r}$ (instead of using q_r and q_u), defining real liquidity $L = \frac{B}{P_{-1}}$, and adding aggregate output Y . Given the aggregate state variables $\Gamma = (K; z_a, z_\kappa)$, we solve the equilibrium system

$$(K', L, C, I, N, Y, \rho, \rho_u, \rho_v, \phi_u, \phi_v, q_v, q, r, w, \pi)$$

together with the exogenous laws of motion of (z_a, z_κ) , i.e., $z'_a = \rho_a z_a + \epsilon'_a$ and $z'_\kappa = \rho_\kappa z_\kappa + \epsilon'_\kappa$. To solve for these 16 endogenous variables, we use the following 16 equations:

³¹Using the utility function $u(c_j) = \frac{c_j^{1-\sigma} - 1}{1-\sigma}$ in (23) and noting that $C = C_v + C_u$, we obtain $C_v = \rho_v C$ and $C_u = \rho_u C$, where $\rho_v \equiv \frac{1-\chi}{1-\chi+\rho^{-1/\sigma}\chi}$ and $\rho_u \equiv \frac{\chi}{\rho^{1/\sigma}(1-\chi)+\chi}$.

1. The representative household's optimality conditions:

$$\left(\frac{\rho_v C}{1-\chi}\right)^{-\sigma} w = \mu, \rho_v \equiv \frac{1-\chi}{1-\chi+\rho^{-1/\sigma}\chi}, \quad \rho_u \equiv \frac{\chi}{\rho^{1/\sigma}(1-\chi)+\chi}$$

$$1 = \beta \mathbb{E}_\Gamma \left[\left(\frac{\rho'_v C'}{\rho_v C}\right)^{-\sigma} [\chi \rho' + 1 - \chi] \frac{1}{\pi'} \right] \quad (28)$$

$$1 = \beta \mathbb{E}_\Gamma \left[\left(\frac{\rho'_v C'}{\rho_v C}\right)^{-\sigma} \frac{(\chi \rho' + 1 - \chi) r' + (1 - \delta)(\chi \rho' + (1 - \chi) q'_v)}{q_v} \right] \quad (29)$$

$$I = \frac{\chi \left[\left(r + \left(\phi_u q_v - \left[\frac{(1-\omega)\rho}{\omega} + 1 \right] \kappa_u \right) (1 - \delta) \right) K + \frac{L}{\pi} \right] - \rho_u C}{1 - \phi_u q_v + \left[\frac{(1-\omega)\rho}{\omega} + 1 \right] \kappa_u} \quad (30)$$

2. Final goods producers:

$$r = e^{z_a} F_K(K, N), \quad w = e^{z_a} F_N(K, N), \quad Y = e^{z_a} F(K, N) \quad (31)$$

3. Market clearing:

(a) Consumption goods

$$(\rho_v + \rho \rho_u) C + q_v K' + L' = wN + [(\chi \rho + (1 - \chi)) r + (1 - \delta)(\chi \rho + (1 - \chi) q_v)] K + [\chi \rho + (1 - \chi)] \frac{L}{\pi}$$

(b) Capital $K' = (1 - \delta)K + I$

(c) Search market (note: $\gamma \equiv \frac{\omega \kappa_v}{1-\omega \kappa_u}$, $\kappa_u = e^{z_\kappa} \bar{\kappa}_u$, $\kappa_v = e^{z_\kappa} \bar{\kappa}_v$)

$$\phi_u = \xi (\gamma^{-1} \rho)^{1-\eta}, \quad \phi_v = \xi^{\frac{1}{1-\eta}} \phi_u^{\frac{\eta}{\eta-1}}$$

$$q_v = \frac{\rho \left[1 + \kappa_u + \rho \frac{(1-\omega)\kappa_u}{\omega} \right]}{1 + (\rho - 1) \phi_u}, \quad q = q_v - \frac{\kappa_v}{\phi_v}$$

(d) Liquid assets (note: $L' = \frac{B'}{P}$, $\pi' = \frac{P'}{P}$) $L' = \frac{L}{\pi}$.

B.2 Steady State

In the deterministic steady state, any variable $X = X'$. With a slight abuse of notation, we denote the steady state of X as X itself in this section. First notice that $z_a = 0$, $z_\kappa = 0$ such that $\kappa_u = \bar{\kappa}_u$, $\kappa_v = \bar{\kappa}_v$.

We can now solve for all prices analytically. Market clearing for liquid assets implies

$\pi = 1$. Next, we use (28) to obtain

$$\rho = \chi^{-1} [\beta^{-1} - (1 - \chi)], \quad \rho_v \equiv \frac{1 - \chi}{1 - \chi + \rho^{-1/\sigma} \chi}, \quad \rho_u \equiv \frac{\chi}{\rho^{1/\sigma} (1 - \chi) + \chi}$$

This directly implies

$$\phi_u = \xi (\gamma^{-1} \rho)^{1-\eta}, \quad q_v = \frac{\rho \left[1 + \kappa_u + \rho \frac{(1-\omega)\kappa_u}{\omega} \right]}{1 + (\rho - 1) \phi_u}$$

From (29) and (31) we have

$$r = \frac{\frac{q_v}{\beta} - (1 - \delta) (\chi \rho + (1 - \chi) q_v)}{\chi \rho + 1 - \chi}, \quad w = (1 - \alpha) \left(\frac{r}{\alpha} \right)^{\frac{\alpha}{\alpha-1}}, \quad C = \left(\frac{w}{\mu} \right)^{1/\sigma} \frac{1 - \chi}{\rho_v}.$$

Now, we express labor supply N as a function of K

$$N = \left(\frac{r}{\alpha} \right)^{\frac{1}{1-\alpha}} K.$$

Investment $I = \delta K$ and real liquidity can be rewritten as a function of K using (30)

$$L = \chi^{-1} \{ \rho_u C + [\delta - \chi r - \phi_u q_u (\delta + \chi (1 - \delta))] K \}.$$

Since N and L are both linear in K , we solve K from the household's budget constraint

$$K = \frac{(\rho_v + \rho_u) C}{\frac{(1-\alpha)}{\alpha} r + A_K + (\rho - 1) [\delta - \chi r - \phi_u q_u (\delta + \chi (1 - \delta))]},$$

where $A_K = (\chi \rho + 1 - \chi) r + (1 - \delta) (\chi \rho + (1 - \chi) q_v) - q_v$.

C Proofs

C.1 Lemma 1

We use a guess-and-verify strategy. Suppose $q_u \geq 1$, then the search market for private claims is active and we seek the parameter restriction that yields $q_u \geq 1$. Using asset price derived in Lemma 2, $q = \frac{\rho(1 + \frac{\kappa_u}{\omega}) - \frac{\kappa_v}{\phi_v}}{1 + (\rho - 1)\phi_u}$, the selling price $q_u = q - \frac{\kappa_u}{\phi_u}$ becomes

$$q_u = \frac{\rho(1 + \frac{\kappa_u}{\omega}) - \frac{\kappa_v}{\phi_v} - \frac{\kappa_u}{\phi_u} - (\rho - 1)\kappa_u}{1 + (\rho - 1)\phi_u}.$$

Therefore, $q_u \geq 1$ is equivalent to

$$\rho(1 + \frac{\kappa_u}{\omega}) - (\rho - 1)(\kappa_u + \phi_u) \geq 1 + \frac{\kappa_v}{\phi_v} + \frac{\kappa_u}{\phi_u},$$

or

$$\rho\left(1 + \frac{\kappa_u}{\omega} - \kappa_u - \phi_u\right) + \kappa_u + \phi_u \geq 1 + \frac{\kappa_v}{\phi_v} + \frac{\kappa_u}{\phi_u}.$$

Using again the asset price solution $\rho = \frac{\omega}{1-\omega} \frac{\kappa_v}{\kappa_u} \frac{\phi_u}{\phi_v}$, one can simplify the above inequality to

$$(1 - \phi_u)\left(\rho - 1 - \frac{\kappa_v}{\phi_v} - \frac{\kappa_u}{\phi_u}\right) \geq 0.$$

Since $\phi_u \in [0, 1]$, we have

$$\rho \geq 1 + \frac{\kappa_v}{\phi_v} + \frac{\kappa_u}{\phi_u} = 1 + \frac{\kappa_v}{\xi(\gamma^{-1}\rho)^{-\eta}} + \frac{\kappa_u}{\xi(\gamma^{-1}\rho)^{1-\eta}},$$

where the last equality uses the fact that $\phi_u = \xi\theta^{1-\eta}$ and $\rho = \gamma\theta$. Finally, notice that from the steady state derivation, ρ can be expressed as $\rho = \chi^{-1}[\beta^{-1} - (1 - \chi)]$, we obtain the condition stated in the lemma. \square

C.2 Lemma 2

We first simplify the first-order condition associated with the bargaining solution to

$$\frac{\omega}{\rho\left(q - \frac{1}{\phi_u}\right) + \frac{1-\phi_u}{\phi_u}q_v} = \frac{1-\omega}{q_v - q},$$

by using $u'(c_v)q_v = \beta\mathbb{E}_\Gamma J_S(S', B'; \Gamma')$ and $u'(c_u) = \rho u'(c_v)$. Then

$$\omega \frac{\kappa_v}{\phi_v} = (1 - \omega) \left[\rho \left(q - \frac{1}{\phi_u} \right) + \frac{1 - \phi_u q_u}{\phi_u} \frac{(1 - \phi_u) q_v}{1 - \phi_u q_u} \right],$$

which can be further simplified to

$$\omega \frac{\kappa_v}{\phi_v} = (1 - \omega) \rho (q - q_u),$$

by realizing that $\rho \equiv \frac{q_v}{q_r} = \frac{(1-\phi_u)q_v}{1-\phi_u q_u}$. Solving the above equation for ρ yields

$$\rho = \frac{\omega}{1-\omega} \frac{\kappa_v}{\kappa_u} \frac{\phi_u}{\phi_v} = \gamma\theta,$$

which is (26).

One can further express q in terms of ρ or ϕ_u . Using the above expression along with the definition of ρ

$$\rho \equiv \frac{q_v}{q_r} = \frac{(1 - \phi_u) q_v}{1 - \phi_u q_u} = \frac{(1 - \phi_u) \left(q + \frac{\kappa_v}{\phi_v} \right)}{1 - \phi_u q + \kappa_u},$$

we can express q as

$$q = \frac{\rho(1 + \kappa_u) - (1 - \phi_u) \frac{\kappa_v}{\phi_v}}{1 + (\rho - 1) \phi_u} = \frac{\rho \left(1 + \frac{\kappa_u}{\omega}\right) - \frac{\kappa_v}{\phi_v}}{1 + (\rho - 1) \phi_u},$$

where the last line uses (26) again. Realizing that $\phi_v = \xi(\gamma^{-1}\rho)^{-\eta}$ and $\phi_u = \xi(\gamma^{-1}\rho)^{1-\eta}$ we can rewrite q as a function of ρ

$$q = \frac{\rho^{1-\eta} \left(1 + \frac{\kappa_u}{\omega}\right) - \kappa_v \gamma^{-\eta} \xi^{-1}}{\rho^{-\eta} [1 + (\rho - 1) \xi(\gamma^{-1}\rho)^{1-\eta}]},$$

or, equivalently as a function of ϕ_u

$$q = \frac{\gamma^{1-\eta} \xi^{-1} \phi_u \left(1 + \frac{\kappa_u}{\omega}\right) - \kappa_v \gamma^{-\eta} \xi^{-1}}{\gamma^{-\eta} (\xi^{-1} \phi_u)^{\frac{\eta}{\eta-1}} \left[1 + \left(\gamma(\xi^{-1} \phi_u)^{\frac{1}{1-\eta}} - 1\right) \phi_u\right]} = \frac{\gamma \phi_u \left(1 + \frac{\kappa_u}{\omega}\right) - \kappa_v}{\xi^{\frac{1}{1-\eta}} \phi_u^{\frac{\eta}{\eta-1}} \left[1 + \left(\gamma(\xi^{-1} \phi_u)^{\frac{1}{1-\eta}} - 1\right) \phi_u\right]}.$$

□

C.3 Proposition 2

Notice that in steady state, ρ , ϕ_u , and ϕ_v are functions of parameters that are independent of search costs κ_v and κ_u , when $\kappa_v/\kappa_u = g$ is fixed (such that $\gamma = \frac{\omega}{1-\omega} \frac{\kappa_v}{\kappa_u}$ is also fixed). Therefore, $\frac{\partial q}{\partial \kappa_v} > 0$ is equivalent to

$$\frac{\gamma}{\omega g} \phi_u - 1 < 0.$$

Using the definition for γ and g , we have

$$\phi_u < 1 - \omega.$$

Since in the steady state $\rho = \chi^{-1}(\beta^{-1} - (1 - \chi))$ and $\phi_u = \xi(\gamma^{-1}\rho)^{1-\eta}$, the above inequality is equivalent to

$$\xi [\chi^{-1}(\beta^{-1} - (1 - \chi))]^{1-\eta} < (1 - \omega)^\eta \omega^{1-\eta} g^{1-\eta}.$$

C.4 Proposition 3

By differentiating the asset price from (27) with respect to ϕ_u , we get

$$\begin{aligned} & \frac{\partial q}{\partial \phi_u} \left[\xi^{\frac{1}{1-\eta}} \phi_u^{\frac{\eta}{\eta-1}} [1 + (\rho - 1) \phi_u] \right] \\ &= \gamma \left(1 + \frac{\kappa_u}{\omega}\right) - q \frac{\partial}{\partial \phi_u} \left[\xi^{\frac{1}{1-\eta}} \phi_u^{\frac{\eta}{\eta-1}} \left[1 + \left(\gamma(\xi^{-1} \phi_u)^{\frac{1}{1-\eta}} - 1\right) \phi_u\right] \right], \end{aligned} \quad (32)$$

where

$$\frac{\partial}{\partial \phi_u} \left[\xi^{\frac{1}{1-\eta}} \phi_u^{\frac{\eta}{1-\eta}} \left[1 + \left(\gamma (\xi^{-1} \phi_u)^{\frac{1}{1-\eta}} - 1 \right) \phi_u \right] \right] = \rho^{-1} \gamma \left[\phi_u \left(2\rho - \frac{1-2\eta}{1-\eta} \right) - \frac{\eta}{1-\eta} \right].$$

Note that $2\rho - \frac{1-2\eta}{1-\eta} = \frac{\eta}{1-\eta} + 2\rho - 1$. A necessary and sufficient condition for $\frac{\partial q}{\partial \phi_u} > 0$ is for the RHS of (32) to be non-negative. This is the case, whenever

$$\phi_u < \left[\frac{\eta}{1-\eta} + \left(1 + \frac{\kappa_u}{\omega} \right) \frac{\rho}{q} \right] \left[\frac{\eta}{1-\eta} + 2\rho - 1 \right]^{-1}.$$

This condition requires ϕ_u to be small enough for the asset price and asset liquidity to correlate positively. Replacing ρ and notice that $\rho/q < 1$, a sufficient condition is

$$\phi_u < \left[\frac{\eta}{1-\eta} + \left(1 + \frac{\kappa_u}{\omega} \right) \right] \left[\frac{\eta}{1-\eta} + 2\gamma (\xi^{-1} \phi_u)^2 - 1 \right]^{-1}.$$

When $\eta = 0.5$, $\frac{\eta}{\eta-1} = 1$ and the sufficient condition becomes $\phi_u < \sqrt[3]{\frac{\xi^2(1+\frac{\kappa_u}{2\omega})}{\gamma}}$.

Note that $\frac{\partial q}{\partial \phi_u} > 0$ implies $\frac{\partial q}{\partial \phi_v} < 0$, because $\frac{\partial q}{\partial \phi_v} = \frac{\partial q}{\partial \phi_u} \frac{\partial \phi_u}{\partial \phi_v}$ and

$$\frac{\partial \phi_u}{\partial \phi_v} = \frac{\eta-1}{\eta} \xi^{\frac{1}{\eta}} \phi_v^{-\frac{1}{\eta}} < 0.$$

Hence, the same parameter restriction that ensures $\frac{\partial q}{\partial \phi_u} > 0$ also ensures $\frac{\partial q}{\partial \phi_v} < 0$. \square

C.5 Corollary 1

Notice that the surplus from entrepreneurs are

$$J_m^u = u'(c_u) \left(q - \frac{1}{\phi_u} \right) + \frac{1-\phi_u}{\phi_u} \beta \mathbb{E}_\Gamma [J_S(S', B'; \Gamma')] = u'(c_u) \left[\rho \left(q - \frac{1}{\phi_u} \right) + \frac{(1-\phi_u)q_v}{\phi_u} \right].$$

Since $\frac{(1-\phi_u)q_v}{\phi_u} = \frac{1-\phi_u q_u}{\phi_u} \frac{(1-\phi_u)q_v}{1-\phi_u q_u} = \frac{1-\phi_u q_u}{\phi_u} \rho$, the term in the bracket is equal to $\rho(q - q_u)$. When $\kappa_u \rightarrow 0$, the surplus of the entrepreneurs goes to zero because $q_u \rightarrow q$. In this case, intermediation will set price $q = 1$ in order to maximize workers' surplus. Further, when $\kappa_v \rightarrow 0$, we thus know that $q_v \rightarrow q \rightarrow q_u \rightarrow 1$ and $\rho \rightarrow 1$.