Capital Allocation and Productivity in South Europe*

Gita Gopinath
Harvard University and NBER

Şebnem Kalemli-Özcan
University of Maryland, NBER, and CEPR

Loukas Karabarbounis
Chicago Booth, FRB Minneapolis, and NBER

Carolina Villegas-Sanchez
ESADE – Universitat Ramon Llull

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Abstract

Following the introduction of the euro in 1999, countries in the South experienced large capital inflows and low productivity. We use data for manufacturing firms in Spain to document a significant increase in the dispersion of the return to capital across firms, a stable dispersion of the return to labor across firms, and a significant increase in productivity losses from misallocation over time. We develop a model of heterogeneous firms facing financial frictions and investment adjustment costs. The model generates cross-sectional and time-series patterns in size, productivity, capital returns, investment, and debt consistent with those observed in production and balance sheet data. We illustrate how the decline in the real interest rate, often attributed to the euro convergence process, leads to a decline in sectoral total factor productivity as capital inflows are misallocated toward firms that have higher net worth but are not necessarily more productive. We conclude by showing that similar trends in dispersion and productivity losses are observed in Italy and Portugal but not in Germany, France, and Norway.

JEL-Codes: D24, E22, F41, O16, O47.

Keywords: Misallocation, Productivity, Dispersion, Capital Flows, Europe.

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1 Introduction

Following the introduction of the euro, so-called imbalances emerged across countries in Europe. Countries in the South received large capital inflows. During this period productivity diverged, with countries in the South experiencing slower productivity growth than other European countries. Economists and policymakers often conjecture that the decline in productivity resulted from a misallocation of resources across firms or sectors in the South.

This paper has two goals. First, we bring empirical evidence to bear on the question of how the misallocation of resources across firms evolves over time. Between 1999 and 2012, we document a significant increase in the dispersion of the return to capital and a deterioration in the efficiency of resource allocation across Spanish manufacturing firms. Second, we develop a model with firm heterogeneity, financial frictions, and investment adjustment costs to shed light on these trends. We demonstrate how a decline in the real interest rate increases the dispersion of the return to capital and generates lower total factor productivity (TFP) as capital inflows are directed to less productive firms operating within relatively underdeveloped financial markets.

Our paper contributes to the literatures of misallocation and financial frictions. Pioneered by Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), the misallocation literature documents large differences in the efficiency of factor allocation across countries and the potential for these differences to explain observed TFP differences. But so far there is little systematic evidence on the dynamics of misallocation within countries. Models with financial frictions, such as Kiyotaki and Moore (1997), have natural implications for the dynamics of capital misallocation at the micro level. Despite this, there exists no empirical work that attempts to relate capital misallocation at the micro level to firm-level financial decisions and to the aggregate implications of financial frictions. Our work aims to fill these gaps in the literature.

To answer these questions, we use a firm-level dataset from ORBIS-AMADEUS that covers manufacturing firms in Spain between 1999 and 2012. Our data cover roughly 75 percent of the manufacturing economic activity reported in Eurostat (which, in turn, uses Census sources). Further, the share of economic activity accounted for by small and medium sized firms in our data is representative of that in Eurostat. Unlike datasets from Census sources, our data contain information on both production and balance sheet variables. This makes it possible to relate real economic outcomes to financial decisions at the firm level in a large and representative sample of firms.

We begin our analysis by documenting the evolution of misallocation measures within four-digit level manufacturing industries. First, we report trends in the dispersion of the return to capital, as measured by the log marginal revenue product of capital (MRPK), and the return to labor, as measured by the log marginal revenue product of labor (MRPL). As emphasized by Hsieh and Klenow (2009), an increase in the dispersion of a factor's return across firms could reflect increasing barriers to the efficient allocation of resources and be associated with a loss in TFP at the aggregate level. We document an increase in the dispersion of the MRPK in Spain in the pre-crisis period between 1999 and 2007 that further accelerated in the post-crisis period between 2008 and 2012. By contrast, the dispersion of the MRPL does not show any significant trend throughout this period. Second, we document a significant increase in the loss in TFP due to misallocation. Third, we show that the cross-sectional correlation between capital and firm productivity decreased over time. This suggests that capital inflows were increasingly directed toward less productive firms over time.

To interpret these facts and evaluate the potential link to financial variables and the implications for sectoral TFP, we develop a parsimonious small open economy model with heterogeneous firms, financial frictions, and investment adjustment costs. Firms compete in a monopolistically competitive environment and employ capital and labor to produce manufacturing varieties. They are heterogeneous in terms of their permanent productivity and also face transitory idiosyncratic productivity shocks. Firms save in a risk-free bond to smooth consumption over time and invest to accumulate physical capital. Financial frictions take the form of borrowing constraints that depend on firm size. Smaller firms do not have access to credit, whereas larger firms are able to borrow in order to finance investment and consumption. The three model elements that generate dispersion of the MRPK across firms are borrowing constraints, a risky time-to-build technology of capital accumulation, and investment adjustment costs.

Given a stochastic process for firm productivity estimated directly from the data, we parameterize the financial friction and the adjustment cost technology such that the model matches the empirically observed positive relationship between firm capital growth and either productivity or net worth using within-firm variation. After parameterizing the model using only these two moments, we compare the model to the data using a series of additional moments that are not targeted during the parameterization. We show that the model generates within-firm and cross-sectional patterns that match patterns observed in the microdata in terms of variables such as firm size, productivity, MRPK, capital, net worth, and leverage. These patterns allow us to establish the link between capital misallocation at the micro level and firm-level production and financial decisions.

Similar to the experience in Spain following the transition to and adoption of the euro, we illustrate how a decline in the real interest rate generates transitional dynamics characterized by an inflow of capital, an increase in MRPK dispersion across firms, and a decline in sectoral TFP. In our model firms with higher net worth are willing to pay the adjustment cost and increase their investment in response to the decline in the cost of capital. For these unconstrained firms, the real interest rate drop generates a decline in their MRPK. On the other hand, firms that happen to have lower net worth despite being potentially more productive delay their adjustment until they can internally accumulate sufficient funds. These firms do not experience a commensurate decline in their MRPK. Therefore, the dispersion of the MRPK between financially unconstrained and constrained firms increases. Capital flows into the sector, but not necessarily to the most productive firms, which generates a decline in sectoral TFP.

To corroborate the mechanism generated by the model, we present direct evidence showing that firms with higher initial net worth accumulated more capital and debt during the pre-crisis period conditional on their initial idiosyncratic productivity. Further, we demonstrate that industries relying more heavily on external finance, as measured by Rajan and Zingales (1998), experienced larger increases in their MRPK dispersion and larger TFP losses from misallocation before the crisis. We illustrate the robustness of our conclusions to extensions of the model that consider endogenous entry and exit, heterogeneity in labor distortions across firms, and overhead labor. We also illustrate that alternative narratives of the pre-crisis period, such as a relaxation of borrowing constraints or transitional dynamics that arise purely from investment adjustment costs, do not generate the patterns observed in the aggregate data. Additionally, we show that the increase in the dispersion of the MRPK in the pre-crisis period cannot be explained by changes in the stochastic process governing firm productivity. During this period, we actually document a decline in the dispersion of productivity shocks across firms.

The post-crisis dynamics are characterized by even larger increases in the dispersion of the MRPK, declines in TFP, and capital flow reversals. It is often argued that a financial shock, expressed as a tightening of the borrowing constraint, plays an important role in explaining the post-crisis dynamics in the South. In the model, a financial shock that forces firms to deleverage is consistent with declining TFP and capital. However, the large increase in the dispersion of the MRPK in the data suggests an additional role for uncertainty shocks at the micro level. Indeed, we document that idiosyncratic shocks become significantly more dispersed across firms during the post-crisis period.

In the final part of the paper, we extend our empirical analysis to Italy (1999-2012), Portugal (2006-2012), Germany (2006-2012), France (2000-2012), and Norway (2004-2012). With the exception of Germany, our coverage in all countries is high and averages from roughly 60 to more than 90 percent of the coverage observed in Eurostat. For all countries, the sample appears to be representative in terms of the contribution of small and medium sized firms to manufacturing economic activity.

We find interesting parallels between Spain, Italy, and Portugal. As in Spain, there is a trend increase in MRPK dispersion in Italy before the crisis and a significant acceleration of this trend in the post-crisis period. Portugal also experiences an increase in MRPK dispersion during its sample period that spans mainly the post-crisis years. By contrast, MRPK dispersion is relatively stable in Germany, France, and Norway throughout their samples. Further, we show that the dispersion of the MRPL does not exhibit significant trends in any country in the sample. Finally, we find significant trends in the loss in TFP due to misallocation in some samples in Italy and Portugal, but do not find such trends in Germany, France, and Norway.

Related Literature. Our paper contributes to a recent body of work that studies the dynamics of dispersion and misallocation. Oberfield (2013) and Sandleris and Wright (2014) document the evolution of misallocation during crises periods in Chile and Argentina respectively. Larrain and Stumpner (2013) document changes in resource allocation in several Eastern European countries during financial market liberalization episodes. Bartelsman, Haltiwanger, and Scarpetta (2013) examine the cross-country and time-series variation of the covariance between productivity and size as a measure of resource allocation. Kehrig (2015) presents evidence for a countercyclical dispersion of (revenue) productivity in U.S. manufacturing.

Asker, Collard-Wexler, and De Loecker (2014) show how risky time-to-build technologies and investment adjustment costs can rationalize dispersion of firm-level revenue productivity. Following their observation, our model allows for the possibility that increases in the dispersion of firm-level outcomes are driven by changes in second moments of the stochastic process governing idiosyncratic productivity. Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) demonstrate that increases in the dispersion of plant-level productivity shocks is an important feature of recessions in the United States.

Banerjee and Duflo (2005) discuss how capital misallocation can arise from credit constraints. An earlier attempt to link productivity and financial frictions to capital flows in an open economy is Mendoza (2010). Recently, several papers have endogenized TFP as a function of financial frictions in dynamic models (Midrigan and Xu, 2014; Moll, 2014; Buera and Moll, 2015). A typical prediction of these models is that a financial liberalization episode is associated with capital inflows, a better allocation of resources across firms, and an increase in TFP (see, for instance, Buera, Kaboski, and Shin, 2011; Midrigan and Xu, 2014). This shock, however, does not match the experience of countries in South Europe where TFP declined.

One important difference between our paper and these papers is that we focus on transitional dynamics generated by a decline in the real interest rate. Contrary to a financial liberalization shock, the decline in the real interest rate generates an inflow of capital and a decline in TFP in the short run of our model. Misallocation increases along the transitional dynamics, as financial frictions and adjustment costs prevent some productive firms from increasing their capital.[1]

[1]Buera and Shin (2011) study episodes of capital outflows and higher TFP in the open economy. They attribute capital outflows from higher TFP countries to economic reforms that remove idiosyncratic distortions.

The problems associated with large current account deficits and declining productivity in the euro area were flagged early on by Blanchard (2007) for the case of Portugal. Reis (2013) argues that large capital inflows were allocated to new and inefficient firms, worsening the allocation of capital in Portugal in the 2000s. Benigno and Fornaro (2014) alternatively suggest that the decline in aggregate productivity resulted from a shift in resources from the traded sector, which is the source of endogenous productivity growth, to the non-traded sector following the consumption boom that accompanied the increase in capital inflows. In contemporaneous work, Dias, Marques, and Richmond (2014) and Garcia-Santana, Moral-Benito, Pijoan-Mas, and Ramos (2015) present descriptive statistics on trends in resource allocation within sectors, including construction and services, for Portugal (1996-2011) and Spain (1995-2007) respectively.

2 Description of the Data

Our data come from the ORBIS database. The database is compiled by the Bureau van Dijk Electronic Publishing (BvD). ORBIS is an umbrella product that provides firm-level data for many countries worldwide. Administrative data at the firm level are initially collected by local Chambers of Commerce and, in turn, relayed to BvD through roughly 40 different information providers including official business registers. Given our paper's focus, we also use the AMADEUS dataset which is the European subset of ORBIS. One advantage of focusing on European countries is that company reporting is regulatory.

The dataset has financial accounting information from detailed harmonized balance sheets, income statements, and profit or loss accounts of firms. Roughly 99 percent of companies in the dataset are private. This crucially differentiates our data from other datasets commonly used in the literature such as Compustat for the United States, Compustat Global, and Worldscope that mainly contain information on large listed companies.

Our analysis focuses on the manufacturing sector for which challenges related to the estimation of the production function are less severe than in other sectors. In the countries that we examine, the manufacturing sector accounts for roughly 20 to 30 percent of aggregate employment and value added. The ORBIS database allows us to classify industries in the manufacturing sector according to their four-digit NACE 2 industry classification.[2]

A well-known problem in ORBIS-AMADEUS is that, while the number of unique firm identifiers matches the number in official data sources, key variables, such as employment and materials, are missing once the data are downloaded. There are several reasons for this. Private firms are not required to report materials. Additionally, employment is not reported as a balance sheet item but in memo lines. Less often, there can be other missing variables such as capital or assets. Variables are not always reported consistently throughout time in a particular disk or in a web download, either from the BvD or the Wharton Research Data Services (WRDS) website. BvD has a policy by which firms that do not report during a certain period are automatically deleted from their later vintage products creating an artificial survival bias in the sample. An additional issue that researchers face is that any online download (BvD or WRDS) will cap the amount of firms that can be downloaded in a given period of time. This cap translates into missing observations in the actual download job instead of termination of the download job.

We follow a comprehensive data collection process to try and address these problems and maximize the coverage of firms and variables for our six countries over time. Broadly, our strategy is to merge data available in historical disks instead of downloading historical data at once from the WRDS website. We rely on two BvD products, ORBIS and AMADEUS. These products have been developed independently and, therefore, they follow different rules regarding the companies and years that should be included. AMADEUS provides data for at most 10 recent years for the same company while ORBIS only reports data for up to 5 recent years. In addition, AMADEUS drops firms from the database if they did not report any information during the last 5 years while ORBIS keeps the information for these companies as long as they

[2]Industry classifications changed from the NACE 1.1 revision to the NACE 2 revision in 2008. To match industry classifications, we start from the official Eurostat correspondence table that maps NACE 1.1 codes to NACE 2 codes. Often there is no one-to-one match between industries in the official correspondence table. When multiple NACE 2 codes are matched to a given NACE 1.1 code, we map the NACE 1.1 code to the first NACE 2 code provided in the official table. In many cases the first code is the most closely related industry to the one in NACE 1.1 classification. As an example, consider the NACE 1.1 code "10.20: Mining and agglomeration of lignite." This code is matched to three NACE 2 codes: "5.2: Mining of lignite," "9.90: Support activities for other mining and quarrying," and "19.20: Manufacture of refined petroleum products." We match "10.20: Mining and agglomeration of lignite" to "5.20: Mining of lignite." Finally, when industries are completely missing from the official correspondence tables, we manually match codes by reading the descriptions of the codes.

Table 1: Coverage in ORBIS-AMADEUS Relative to Eurostat (SBS): Spain Manufacturing

	Employment	Wage Bill	Gross Output
1999	0.56	0.69	0.75
2000	0.58	0.71	0.76
2001	0.61	0.73	0.77
2002	0.65	0.75	0.79
2003	0.65	0.74	0.78
2004	0.66	0.75	0.78
2005	0.66	0.74	0.77
2006	0.67	0.74	0.77
2007	0.67	0.74	0.77
2008	0.65	0.72	0.72
2009	0.71	0.72	0.75
2010	0.68	0.73	0.74
2011	0.69	0.74	0.75
2012	0.65	0.71	0.72

are active. We merge data across several vintages of these two products (ORBIS disk 2005, ORBIS disk 2009, ORBIS disk 2013, AMADEUS online 2010 from WRDS, and AMADEUS disk 2014).[3]

Finally, it is sometimes the case that information is updated over time and the value of variables that was not available in early disks is made available in later vintages. Additionally, because of reporting lags the coverage in the latest years of a certain disk can be poor. To maximize the number of firms in the sample and the coverage of variables we merge across all products using a unique firm identifier and we update information missing in early vintages by the value provided in later vintages. An issue when merging data across disks is that there can be changes in firm identifiers over time. We use a table with official identifiers changes provided by BvD to address this issue.

Table 1 summarizes the coverage in our data for Spain. In Section 7 we additionally present

[3]For example, consider a company that files information with BvD for the last time in year 2007. However, suppose that BvD has information from the Business Registry that this company is still active. In AMADEUS disk 2013 this company will not be included in the database. However, information for the period 2002-2007 for this company will still be available in ORBIS disk 2013.

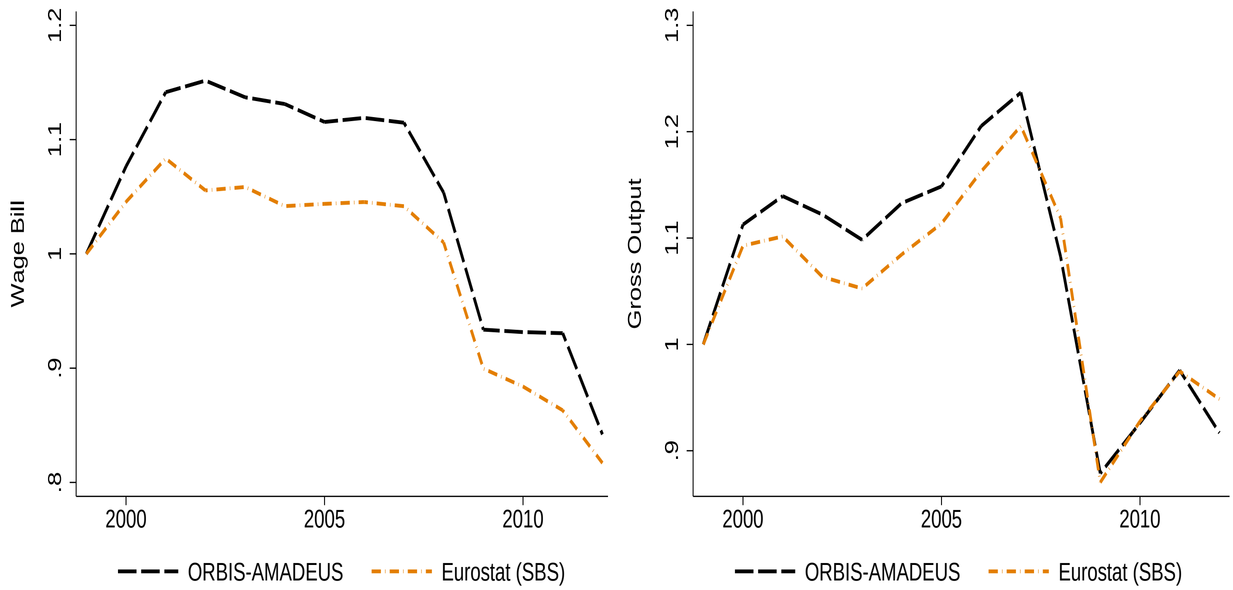


Figure 1: Aggregates in ORBIS-AMADEUS and Eurostat (SBS)

the coverage for Italy, Portugal, Germany, France, and Norway. The columns in the table represent the ratio of aggregate employment, wage bill, and gross output recorded in our sample relative to the same object in Eurostat as reported by its Structural Business Statistics (SBS). The data in Eurostat are from Census sources and so they represent the universe of firms. The coverage statistics we report are conservative because we drop observations with missing, zero, or negative values for gross output, wage bill, capital stock, and materials, that is the variables necessary for computing productivity at the firm level.[4] As Table 1 shows the coverage in our sample is consistently high and averages roughly 75 percent for the wage bill and gross output and typically more than 65 percent for employment.[5]

Figure 1 plots the aggregate real wage bill and the aggregate real gross output in our ORBIS-AMADEUS dataset. It compares these aggregates to the same aggregates as recorded by Eurostat. Except for the wage bill in the first two years of the sample, these series track each other closely.

Table 2 presents the share of economic activity accounted for by firms belonging in three

[4]Appendix A provides a detailed description of the process we follow to clean the data and presents summary statistics of the main variables used in our analysis.

[5]A difference between our sample and Eurostat is that we do not have data on the self-employed. While this has little impact on our coverage of the wage bill and gross output relative to Eurostat, it matters more for employment for which the coverage is somewhat lower.

Table 2: Share of Total Manufacturing Economic Activity By Size Class in Spain (2006)

		Employment	Wage Bill	Gross Output
ORBIS-AMADEUS	1-19 employees	0.24	0.19	0.14
	20-249 employees	0.50	0.47	0.42
	250+ employees	0.26	0.34	0.45
Eurostat (SBS)	0-19 employees	0.31	0.20	0.14
	20-249 employees	0.43	0.43	0.38
	250+ employees	0.26	0.37	0.49

size categories in 2006.[6] Each column presents a different measure of economic activity, namely employment, wage bill, and gross output. The first three rows report statistics from ORBIS-AMADEUS and the next three from Eurostat. The entries in the table denote the fraction of total economic activity accounted for by firms belonging to each size class. For example, in our data from ORBIS-AMADEUS, firms with 1-19 employees account for 19 percent of the total wage bill, firms with 20-249 employees account for 47 percent of the total wage bill, and firms with 250 or more employees account for 34 percent of the total wage bill. The corresponding numbers provided by Eurostat's SBS are 20, 43, and 37 percent.

Our sample is mainly composed of small and medium sized firms that account for a significant fraction of economic activity in Europe and the majority of economic activity in the South. Table 2 illustrates that our sample is broadly representative in terms of contributions of small and medium sized firms to manufacturing employment, wage bill, and gross output. This feature is an important difference of our paper relative to the literature that works with both financial and real variables at the firm level. Most of this literature focuses on listed firms that account for less than 1 percent of the observations in our sample.

[6]The share of economic activity by size category in our sample relative to Eurostat is relatively stable over time. We show year 2006 in Table 2 for comparability with our analyses of other countries below that also start in 2006. The sum of entries across rows within each panel and source may not add up to one because of rounding.

3 Dispersion and Misallocation Facts

In this section we document the evolution of measures of dispersion and misallocation for the manufacturing sector in Spain. We build our measurements on the framework developed by Hsieh and Klenow (2009). We consider an industry s at time t populated by a large number N_{st} of monopolistically competitive firms. We define industries in the data by their four-digit industry classification.

Total industry output is given by a CES production function:

$$Y_{st} = \left[\sum_{i=1}^{N_{st}} D_{ist} \left(y_{ist} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{1}$$

where y_{ist} denotes firm i's real output, D_{ist} denotes demand for firm i's variety, and ε denotes the elasticity of substitution between varieties. We denote by p_{ist} the price of variety i and by P_{st} the price of industry output Y_{st}. Firms face an isoelastic demand for their output given by $y_{ist} = \left(p_{ist}/P_{st} \right)^{-\varepsilon} \left(D_{ist} \right)^{\varepsilon} Y_{st}$.

Firms' output is given by a Cobb-Douglas production function:

$$y_{ist} = A_{ist} k_{ist}^{\alpha} \ell_{ist}^{1-\alpha}, \tag{2}$$

where k_{ist} is capital, ℓ_{ist} is labor, A_{ist} is physical productivity, and α is the elasticity of output with respect to capital. Throughout our analysis we set $\alpha = 0.35$. Our dispersion measures are not affected by the assumption that α is homogeneous across industries because these measures use within-industry variation of firm outcomes.

We measure firm nominal value added, $p_{ist} y_{ist}$, as the difference between gross output (operating revenue) and materials. We measure real output, y_{ist}, as nominal value added divided by an output price deflator. Given that we do not observe prices at the firm level, we use gross output price deflators from Eurostat at the two-digit industry level. We measure the labor input, ℓ_{ist}, with a firm's wage bill deflated by the same industry price deflator. We use the wage bill instead of employment as our measure of ℓ_{ist} to control for differences in the quality of the workforce across firms. We measure the capital stock, k_{ist}, with the book value of fixed assets and deflate this value with the price of investment goods.[7] In fixed assets we include both tangible and intangible fixed assets.[8]

Denoting the inverse demand function by $p(y_{ist})$, firms choose their price, capital, and labor to maximize their profits:

$$\max_{p_{ist},k_{ist},\ell_{ist}} \Pi_{ist} = (1-\tau^y_{ist})\, p(y_{ist})y_{ist} - \left(1+\tau^k_{ist}\right)(r_t+\delta_{st})\, k_{ist} - w_{st}\ell_{ist}, \tag{3}$$

where w_{st} denotes the wage, r_t denotes the real interest rate, δ_{st} denotes the depreciation rate, τ^y_{ist} denotes a firm-specific wedge that distorts output decisions, and τ^k_{ist} denotes a firm-specific wedge that distorts capital relative to labor decisions. For now we treat wedges as exogenous and endogenize them later in the model of Section 4.

The first-order conditions with respect to labor and capital are given by:

$$\text{MRPL}_{ist} := \left(\frac{1-\alpha}{\mu}\right)\left(\frac{p_{ist}y_{ist}}{\ell_{ist}}\right) = \left(\frac{1}{1-\tau^y_{ist}}\right) w_{st}, \tag{4}$$

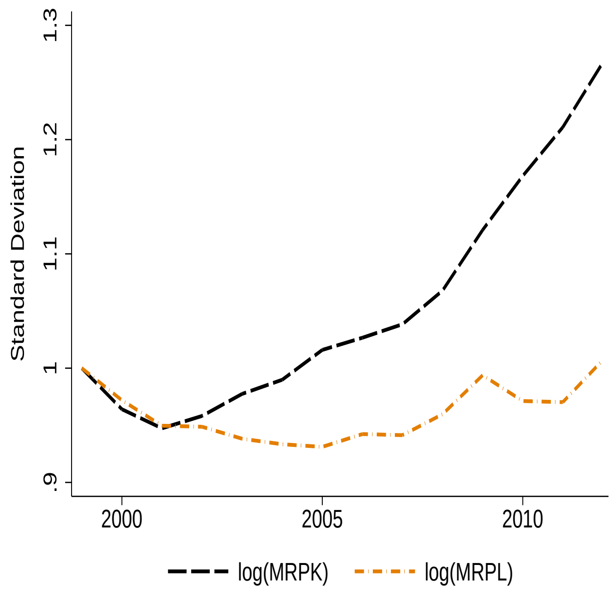
$$\text{MRPK}_{ist} := \left(\frac{\alpha}{\mu}\right)\left(\frac{p_{ist}y_{ist}}{k_{ist}}\right) = \left(\frac{1+\tau^k_{ist}}{1-\tau^y_{ist}}\right)(r_t+\delta_{st}), \tag{5}$$

where $\mu = \varepsilon/(\varepsilon-1)$ denotes the constant markup of price over marginal cost. Equation (4) states that firms set the marginal revenue product of labor (MRPL) equal to the wage times the wedge $1/\left(1-\tau^y_{ist}\right)$. Similarly, in equation (5) firms equate the marginal revenue product of capital (MRPK) to the cost of capital times the wedge $\left(1+\tau^k_{ist}\right)/\left(1-\tau^y_{ist}\right)$. With Cobb-Douglas production function, the marginal revenue product of each factor is proportional to the factor's revenue-based productivity.

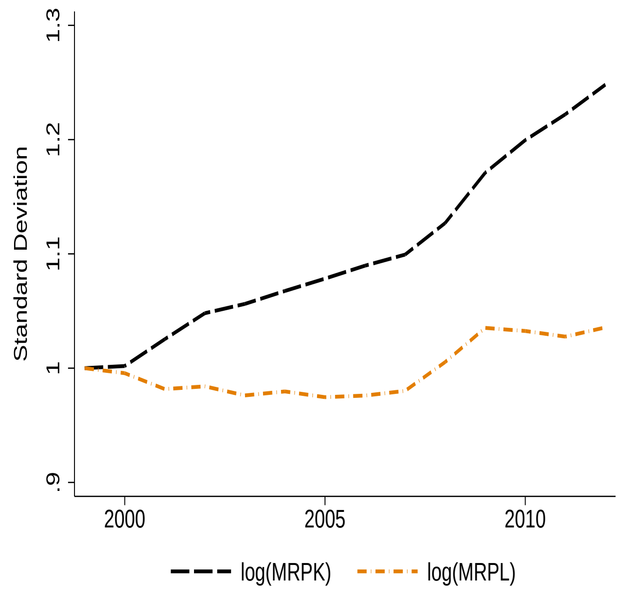
Following the terminology used in Foster, Haltiwanger, and Syverson (2008) and Hsieh and Klenow (2009), we define the revenue-based total factor productivity (TFPR) at the firm level

[7] Deflating fixed assets matters for our results only through our measures of capital and TFP at the aggregate level. We choose to deflate the book value of fixed assets because in this paper we are interested in measuring changes (rather than levels) of capital and TFP. Changes in book values across two years reflect to a large extent purchases of investment goods valued at current prices. We use country-specific prices of investment from the World Development Indicators to deflate the book value of fixed assets, as we do not have industry-specific price of investment goods for the whole sample period.

[8] Our results do not change in any meaningful way if we measure k_{ist} with the book value of tangible fixed assets with one exception. In 2007 there was a change in the accounting system in Spain and leasing items that until 2007 had been part of intangible fixed assets were from 2008 included under tangible fixed assets. If we measure k_{ist} with tangible fixed assets, we observe an important discontinuity in some of our dispersion measures in Spain between 2007 and 2008 that is entirely driven by this accounting convention.



(a) Permanent Sample



(b) Full Sample

Figure 2: Evolution of MRPK and MRPL Dispersion

as the product of price p_{ist} times physical productivity A_{ist}:

$$\text{TFPR}_{ist} := p_{ist}A_{ist} = \frac{p_{ist}y_{ist}}{k_{ist}^{\alpha}\ell_{ist}^{1-\alpha}} = \mu \left(\frac{\text{MRPK}_{ist}}{\alpha}\right)^{\alpha} \left(\frac{\text{MRPL}_{ist}}{1-\alpha}\right)^{1-\alpha}. \tag{6}$$

Firms with higher output distortions τ_{ist}^{y} or higher capital distortions τ_{ist}^{k} have higher marginal revenue products and, as equation (6) shows, a higher TFPR_{ist}.

In this economy, resources are allocated optimally when all firms face the same (or no) distortions in output ($\tau_{ist}^{y} = \tau_{st}^{y}$) and capital markets ($\tau_{ist}^{k} = \tau_{st}^{k}$). In that case, more factors are allocated to firms with higher productivity A_{ist} or higher demand D_{ist}, but there is no dispersion of the returns to factors, that is the MRPL and the MRPK are equalized across firms.[9] On the other hand, the existence of idiosyncratic distortions, τ_{ist}^{y} and τ_{ist}^{k}, leads to a dispersion of marginal revenue products and a lower sectoral TFP.

In Figure 2 we present the evolution of the dispersion of the log (MRPK) and log (MRPL) in Spain. To better visualize the relative changes over time, we normalize these measures to 1 in the first sample year. The left panel is based on the subset of firms that are continuously present in our data. We call this subset of firms the "permanent sample." The right panel is

[9]Without idiosyncratic distortions, $\text{TFPR}_{ist} = p_{ist}A_{ist}$ is equalized across firms since p_{ist} is inversely proportional to physical productivity A_{ist} and does not depend on demand D_{ist}. This also implies that capital-labor ratios are equalized across firms.

based on the "full sample" of firms. The full sample includes firms that enter or exit from the sample in various years and, therefore, comes closer to matching the coverage of firms observed in Eurostat.[10]

The time series of the dispersion measures are computed in two steps. First, we calculate a given dispersion measure across firms i in a given industry s and year t. Second, for each year we calculate dispersion for the manufacturing sector as the weighted average of dispersions across industries s. Each industry is given a time-invariant weight equal to its average share in manufacturing value added. We always use the same weights when aggregating across industries. Therefore, all of our estimates reflect purely variation within four-digit industries over time.

Figure 2 shows a large increase in the standard deviation of $\log(\text{MRPK})$ over time. With the exception of the first two years in the permanent sample, we always observe increases in the dispersion of the $\log(\text{MRPK})$. The increase in the dispersion of the $\log(\text{MRPK})$ accelerates during the post-crisis period between 2008 and 2012. We emphasize that we do not observe similar trends in the standard deviation of $\log(\text{MRPL})$. The striking difference between the evolution of the two dispersion measures argues against the importance of changing distortions that affect both capital and labor at the same time. For example, this finding is not consistent with heterogeneity in price markups driving trends in dispersion because such an explanation would cause similar changes to the dispersion of both the $\log(\text{MRPK})$ and the $\log(\text{MRPL})$.[11] Finally, we note that while we use standard deviations of logs to represent dispersion, all of our results are similar when we measure dispersion with either the 90-10 or the 75-25 ratio.

Under a Cobb-Douglas production function, an increasing dispersion of the $\log(\text{MRPK})$ together with stable dispersion of the $\log(\text{MRPL})$ implies that the covariance between $\log(\text{TFPR})$ and $\log(k/\ell)$ across firms is decreasing over time. To see this point, write:

$$\text{Var}(\text{mrpk}) = \text{Var}(\text{tfpr}) + (1-\alpha)^2 \text{Var}\left(\log\left(\frac{k}{\ell}\right)\right) - 2(1-\alpha)\text{Cov}\left(\text{tfpr}, \log\left(\frac{k}{\ell}\right)\right), \tag{7}$$

[10] From Eurostat, we calculate that in 2000 the entry rate among firms with at least one employee is 6.5 percent. The entry rate declines over time and stabilizes at around 2 to 3 percent after 2010. Our permanent sample of firms differs from the full sample both because of real entry and exit and because firms with missing reporting in at least one year are excluded from the permanent sample but are included in the full sample during years with non-missing reporting. See Appendix A for more details on the construction of the two samples.

[11] The relationship between markups and misallocation has been recently the focus of papers such as Fernald and Neiman (2011) and Peters (2013).

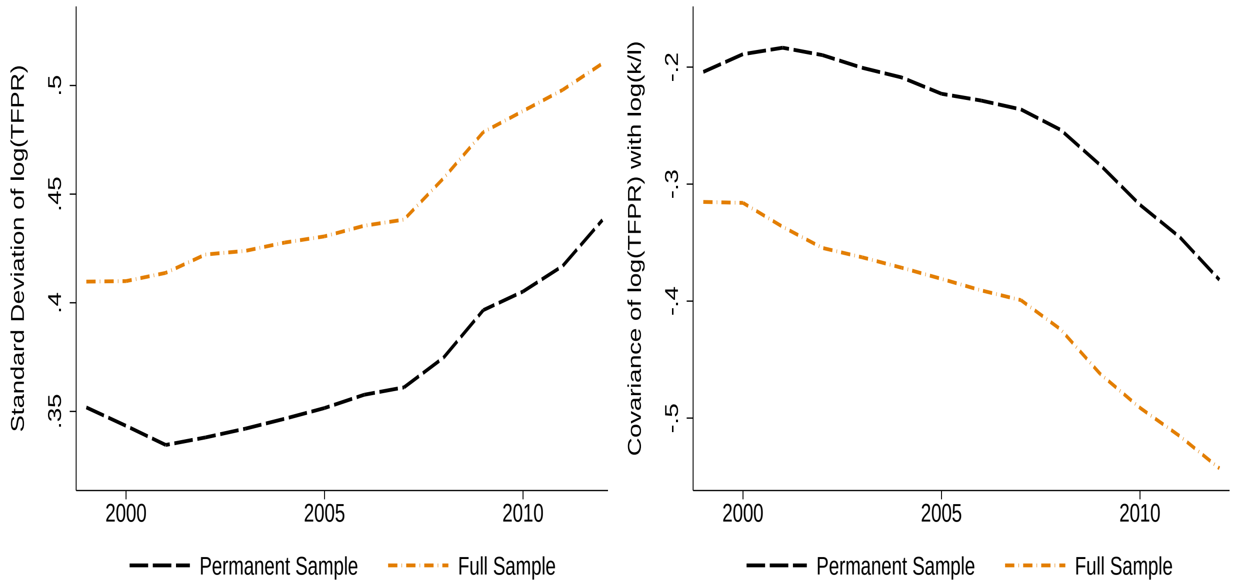


Figure 3: TFPR Moments

$$\mathrm{Var}\left(\mathrm{mrpl}\right)=\mathrm{Var}\left(\mathrm{tfpr}\right)+\alpha^{2}\mathrm{Var}\left(\log\left(\frac{k}{\ell}\right)\right)+2\alpha\mathrm{Cov}\left(\mathrm{tfpr},\log\left(\frac{k}{\ell}\right)\right), \tag{8}$$

where we define $\mathrm{mrpk}=\log\left(\mathrm{MRPK}\right)$, $\mathrm{mrpl}=\log\left(\mathrm{MRPL}\right)$, and $\mathrm{tfpr}=\log\left(\mathrm{TFPR}\right)$. Figure 3 confirms that the dispersion of tfpr is increasing over time and that the covariance between tfpr and $\log(k/\ell)$ is decreasing over time. The variance of the log capital-labor ratio (the second term) is also increasing over time.

We now discuss measures of productivity and misallocation. Total factor productivity at the industry level is defined as the wedge between industry output and an aggregator of industry inputs, $\mathrm{TFP}_{st}:=Y_{st}/(K_{st}^{\alpha}L_{st}^{1-\alpha})$, where $K_{st}=\sum_i k_{ist}$ is industry capital and $L_{st}=\sum_i \ell_{ist}$ is industry labor. We can write TFP as:[12]

$$\mathrm{TFP}_{st}=\frac{Y_{st}}{K_{st}^{\alpha}L_{st}^{1-\alpha}}=\frac{\overline{\mathrm{TFPR}}_{st}}{P_{st}}=\left[\sum_i\left(\underbrace{\left(D_{ist}\right)^{\frac{\varepsilon}{\varepsilon-1}}A_{ist}}_{Z_{ist}}\frac{\overline{\mathrm{TFPR}}_{st}}{\mathrm{TFPR}_{ist}}\right)^{\varepsilon-1}\right]^{\frac{1}{\varepsilon-1}}. \tag{9}$$

We note that for our results it is appropriate to only track a combination of demand and productivity at the firm level. From now on we call "firm productivity," $Z_{ist}=\left(D_{ist}\right)^{\frac{\varepsilon}{\varepsilon-1}}A_{ist}$, a

[12] To derive equation (9), we substitute into the definition of TFP the industry price index $P_{st}=\left(\sum_i\left(D_{ist}\right)^{\varepsilon}\left(p_{ist}\right)^{1-\varepsilon}\right)^{1/(1-\varepsilon)}$, firms' prices $p_{ist}=\mathrm{TFPR}_{ist}/A_{ist}$, and an industry-level TFPR measure, $\overline{\mathrm{TFPR}}_{st}=P_{st}Y_{st}/(K_{st}^{\alpha}L_{st}^{1-\alpha})$. Equation (9) is similar to the one derived in Hsieh and Klenow (2009), except for the fact that we also allow for idiosyncratic demand D_{ist}.

combination of firm productivity and demand.

To derive a measure that maps the allocation of resources to TFP performance, we follow Hsieh and Klenow (2009) and define the "efficient" level of TFP as the TFP level we would observe in the first-best allocation in which there is no dispersion of the MRPK, MRPL, and TFPR across firms. Plugging $\text{TFPR}_{ist} = \overline{\text{TFPR}}_{st}$ into equation (9), we see that the efficient level of TFP is given by $\text{TFP}^e_{st} = \left[\sum_i Z^{\varepsilon-1}_{ist}\right]^{\frac{1}{\varepsilon-1}}$. The difference in $\log(\text{TFP})$ arising from misallocation, $\Lambda_{st} = \log(\text{TFP}_{st}) - \log(\text{TFP}^e_{st})$, can be expressed as:

$$\begin{aligned}\Lambda_{st} &= \frac{1}{\varepsilon-1}\left[\log\left(\mathbb{E}_i Z^{\varepsilon-1}_{ist}\mathbb{E}_i\left(\frac{\overline{\text{TFPR}}}{\text{TFPR}_{ist}}\right)^{\varepsilon-1} + \text{Cov}_i\left(Z^{\varepsilon-1}_{ist}, \left(\frac{\overline{\text{TFPR}}}{\text{TFPR}_{ist}}\right)^{\varepsilon-1}\right)\right)\right] \\ &\quad -\frac{1}{\varepsilon-1}\log\left(\mathbb{E}_i Z^{\varepsilon-1}_{ist}\right). \end{aligned} \tag{10}$$

To construct this measure of misallocation, we need estimates of Z_{ist}. Employing the structural assumptions on demand and production used to arrive at equation (10), we estimate firm productivity as:[13]

$$\tilde{Z}_{ist} = \left(\frac{(P_{st}Y_{st})^{-\frac{1}{\varepsilon-1}}}{P_{st}}\right)\left(\frac{(p_{ist}y_{ist})^{\frac{\varepsilon}{\varepsilon-1}}}{k^{\alpha}_{ist}\ell^{1-\alpha}_{ist}}\right), \tag{11}$$

where $p_{ist}y_{ist}$ denotes firm nominal value added and $P_{st}Y_{st} = \sum_i p_{ist}y_{ist}$ denotes industry nominal value added.

Figure 4 plots changes relative to 1999 in the difference in $\log(\text{TFP})$ relative to its efficient level. We use an elasticity of substitution between varieties equal to $\varepsilon = 3$. As with our measures of dispersion, we first estimate the difference Λ_{st} within every industry s and then use the same time-invariant weights to aggregate across industries. Between 1999 and 2007, we document declines in TFP relative to its efficient level of roughly 3 percentage points in the permanent sample and 7 percentage points in the full sample. By the end of the sample in 2012, we observe declines in TFP relative to its efficient level of roughly 7 percentage points in the permanent sample and 12 percentage points in the full sample.[14]

[13]To derive equation (11), first use the production function to write $\tilde{Z}_{ist} = A_{ist}D^{\frac{\varepsilon}{\varepsilon-1}}_{ist} = D^{\frac{\varepsilon}{\varepsilon-1}}_{ist}y_{ist}/\left(k^{\alpha}_{ist}\ell^{1-\alpha}_{ist}\right)$. Then, from the demand function substitute in $D^{\frac{\varepsilon}{\varepsilon-1}}_{ist} = (p_{ist}/P_{st})^{\frac{\varepsilon}{\varepsilon-1}}(y_{ist}/Y_{st})^{\frac{1}{\varepsilon-1}}$.

[14]The 1999 level of the difference Λ_{st} is roughly -0.21 in the permanent sample and -0.28 in the full sample. We also note that for an elasticity $\varepsilon = 5$ we obtain declines of roughly 4 and 10 percentage points for the permanent and the full sample between 1999 and 2007 and declines of roughly 13 and 19 percentage points between 1999 and 2012. For an elasticity $\varepsilon = 5$, the 1999 level of Λ_{st} is roughly -0.36 in the permanent sample and -0.46 in the full sample.

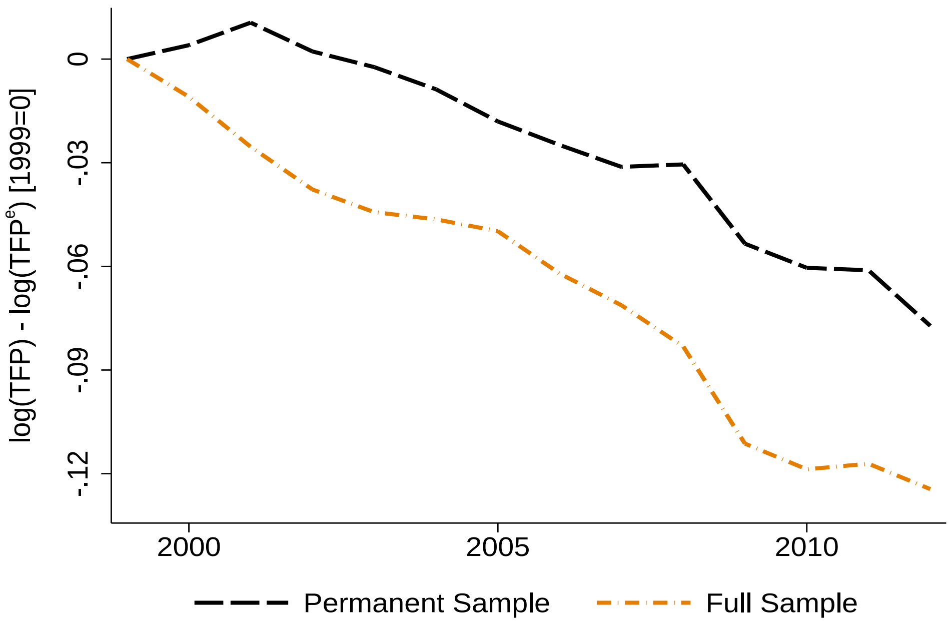
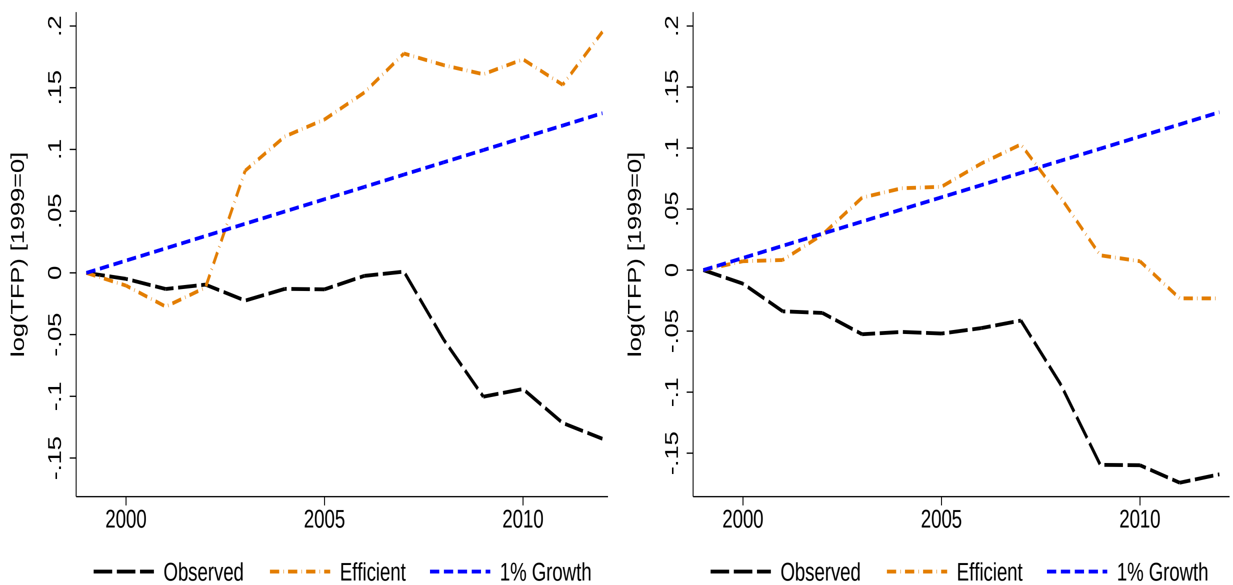


Figure 4: Evolution of log (TFP) Relative to Efficient Level



(a) Permanent Sample

(b) Full Sample

Figure 5: Evolution of Observed log (TFP) Relative to Benchmarks

In Figure 5 we plot changes in manufacturing log(TFP) in the data. We measure log(TFP) for each industry as $\log(\text{TFP}_{st}) = \log\left(\sum_i y_{ist}\right) - \alpha \log\left(K_{st}\right) - (1-\alpha)\log\left(L_{st}\right)$ and use the same time-invariant weights to aggregate across industries s. Manufacturing TFP could be changing over time for reasons other than changes in the allocation of resources (for example, labor hoarding, capital utilization, entry, and technological change). We, therefore, compare observed log (TFP) in the data to two baseline log (TFP) paths. The first path is the efficient path implied by the model, $\log\left(\mathrm{TFP}^e_{st}\right) = \left(\frac{1}{\varepsilon-1}\right)\left(\log\left(N_{st}\right) + \log\left(\mathbb{E}_i \tilde{Z}^{\varepsilon-1}_{ist}\right)\right)$. The second path corresponds to a hypothetical scenario in which TFP grows at a constant rate of one percent per year. Figure 5 documents that observed log (TFP) lies below both baseline paths. Our loss measures in Figure 4 suggest that an increase in the misallocation of resources across firms is related to the observed lower productivity performance relative to these benchmarks.[15]

To explain the joint trends in MRPK dispersion and TFP losses due to misallocation, our model relates a decline in the real interest rate to inflows of capital that are directed to some less productive firms. We now present some first evidence supporting this narrative. It is useful to express the dispersion of the log (MRPK) in terms of dispersions in firm log productivity and log capital and the covariance between these two:

$$\mathrm{Var}_i\left(\log \mathrm{MRPK}_{ist}\right) = \gamma_1 \mathrm{Var}_i\left(\log Z_{ist}\right) + \gamma_2 \mathrm{Var}_i\left(\log k_{ist}\right) - \gamma_3 \mathrm{Cov}_i\left(\log Z_{ist}, \log k_{ist}\right), \tag{12}$$

for some positive coefficients γ's.[16] Loosely, equation (12) says that we expect an increase in the dispersion of the log (MRPK) if capital becomes more dispersed across firms for reasons unrelated to their underlying productivity. More formally, holding constant $\mathrm{Var}_i\left(\log Z_{ist}\right)$, an increase in $\mathrm{Var}_i\left(\log k_{ist}\right)$ or a decrease in $\mathrm{Cov}_i\left(\log Z_{ist}, \log k_{ist}\right)$ is associated with higher $\mathrm{Var}_i\left(\log \mathrm{MRPK}_{ist}\right)$.

The left panel of Figure 6 plots an increasing cross-sectional dispersion of capital over time. The right panel shows the unconditional correlation between firm productivity (as estimated by $\tilde{Z}_{ist}$) and capital in the cross section of firms. In general, more productive firms invest more in capital. However, the correlation between productivity and capital declines significantly over

[15]The path of model-based TFP, as constructed in the last part of equation (9), does not in general coincide with the path of "Observed" TFP in Figure 5. We make use of the CES aggregator to move from the definition of TFP as a wedge between output and an aggregator of inputs to the last part of equation (9). The divergence between the two series is a measurement issue because "Observed" TFP does not use the CES aggregator or the price index. We use Figure 5 to only show that a measure of TFP in the data lies below some benchmarks and do not wish to make any quantitative statements about allocative efficiency based on this figure. Finally, we note that in Figure 5 the larger increase in $\log\left(\mathrm{TFP}^e_{st}\right)$ in the permanent sample relative to the full sample is explained by the fact that the latter includes new entrants that typically have lower productivity.

[16]The coefficients are given by $\gamma_1 = \left(\frac{\varepsilon-1}{1+\alpha(\varepsilon-1)}\right)^2$, $\gamma_2 = \left(\frac{1}{1+\alpha(\varepsilon-1)}\right)^2$, and $\gamma_3 = \frac{2(\varepsilon-1)}{(1+\alpha(\varepsilon-1))^2}$. Equation (12) is derived by substituting the solution for labor ℓ_{ist} into the definition of MRPK and treating the choice of k_{ist} as given. In our model we justify treating k_{ist} as a predetermined variable with a standard time-to-build technology.

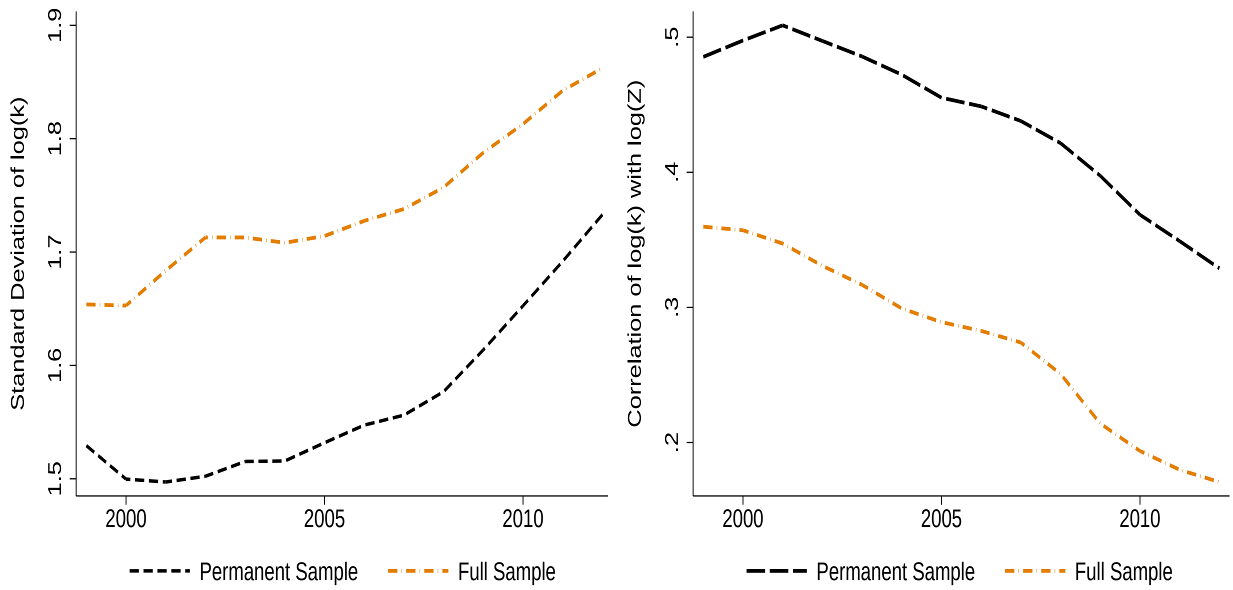


Figure 6: Log Capital Moments

time. This fact suggests that inflows of capital may have been allocated inefficiently to less productive firms.[17]

4 Model of Firm Dispersion, TFP, and Capital Flows

We consider an infinite-horizon, discrete time $t = 0, 1, 2, ...$, small open economy populated by a large number of $i = 1, ..., N$ heterogeneous firms. Firms produce differentiated varieties of manufacturing products. The three key elements of the model that generate dispersion of the MRPK across firms are borrowing constraints that depend on firm size, risky time-to-build technology of capital accumulation, and investment adjustment costs. By contrast, in our baseline model, there is no MRPL dispersion across firms. Also, firms do not face entry and exit decisions. We consider these margins in extensions of the baseline model.

4.1 Firms' Problem

Firms produce output with a Cobb-Douglas production function $y_{it} = Z_{it}k_{it}^{\alpha}\ell_{it}^{1-\alpha}$, where Z_{it} is firm productivity, k_{it} is the capital stock, and ℓ_{it} is labor. Labor is hired in a competitive labor

[17] We present the correlation between log productivity and log capital to make the interpretation of the figure clearer. We emphasize that the covariance between log productivity and log capital is similarly decreasing. The $\text{Var}_i(\log Z_{ist})$ is decreasing until 2007 and then it increases in the post-crisis period.

market at an exogenous wage w_t. Varieties of manufacturing goods are supplied monopolistically to the global market. Each firm faces a downward sloping demand function for its product, $y_{it} = p_{it}^{-\varepsilon}$, where p_{it} is the price of the differentiated product and ε is the absolute value of the elasticity of demand. We denote by $\mu = \varepsilon/(\varepsilon - 1)$ the markup of price over marginal cost.[18]

Firms can save in a risk-free bond traded in the international credit market at an exogenous real interest rate r_t. Denoting by β the discount factor, firms choose consumption of tradeables c_{it}, debt b_{it+1}, investment x_{it}, labor ℓ_{it}, and the price p_{it} of their output to maximize the present discounted value of utility flows:

$$\max_{\{c_{it}, b_{it+1}, x_{it}, \ell_{it}, p_{it}\}_{t=0}^{\infty}} \mathbb{E}_{i0} \sum_{t=0}^{\infty} \beta^t U(c_{it}), \tag{13}$$

where the utility function is given by $U(c_{it}) = \left(c_{it}^{1-\gamma} - 1 \right) / (1 - \gamma)$. This maximization problem is subject to the sequence of budget constraints:

$$c_{it} + x_{it} + (1 + r_t) b_{it} + \frac{\psi \left(k_{it+1} - k_{it} \right)^2}{2 k_{it}} = p_{it} y_{it} - w_t l_{it} + b_{it+1}, \tag{14}$$

and the capital accumulation equation:

$$k_{it+1} = (1 - \delta) k_{it} + x_{it}, \tag{15}$$

where δ denotes the depreciation rate of capital. Firms are subject to quadratic adjustment costs. The parameter ψ controls the magnitude of these costs.

Firms own the capital stock and augment it through investment. This setup differs from the setup in Hsieh and Klenow (2009) where firms rent capital in a static model. We do not adopt the convenient assumption in Moll (2014), Midrigan and Xu (2014), and Buera and Moll (2015) that exogenous shocks during period $t+1$ are known at the end of t before capital and borrowing decisions are made for $t + 1$. This timing assumption effectively renders the choice of capital static and generates an equivalence with the environment in Hsieh and Klenow (2009). Instead, in our model firms face idiosyncratic investment risk which makes capital and debt imperfect

[18] We normalize both the sectoral price index and idiosyncratic demand to one in the demand function $y_{it} = p_{it}^{-\varepsilon}$. It is appropriate to abstract from the determination of the sectoral price index because manufacturing in a small open economy accounts for a small fraction of global manufacturing production. For most results in this paper it is necessary to only track a combination of idiosyncratic productivity and demand. Similarly to our analysis in Section 3, we call this combination "firm productivity" and denote it by Z_{it}.

substitutes in firms' problem. Risk in capital accumulation is an additional force generating MRPK dispersion across firms in our model.

Borrowing possibilities differ between small and large firms. This could be because some large and politically connected firms obtain better deals from banks and can access finance more easily. Alternatively, a model in which small firms are more likely to be credit rationed would yield such a heterogeneity.[19] Without writing such models explicitly, here we simply assume that firms with installed physical capital below some threshold κ_t cannot borrow. Firms with physical capital above the threshold κ_t can access the credit market and can borrow up to a value that equals their installed capital stock. We write the borrowing constraint as:

$$b_{it+1} \leq \begin{cases} k_{it+1}, & \text{if } k_{it+1} > \kappa_t \\ 0, & \text{if } k_{it+1} \leq \kappa_t \end{cases}. \tag{16}$$

We write firm productivity Z_{it} as the product of an aggregate effect Z_t^A, an idiosyncratic permanent effect z_i^P, and an idiosyncratic transitory effect z_{it}^T:

$$Z_{it} = Z_t^A z_i^P \exp\left(z_{it}^T\right). \tag{17}$$

We denote by ν the standard deviation of permanent productivity across firms. Idiosyncratic transitory productivity follows an AR(1) process in logs:

$$z_{it}^T = -\frac{\sigma_t^2}{2(1+\rho)} + \rho z_{it-1}^T + \sigma_t u_{it}^z, \quad \text{with} \quad u_{it}^z \sim \mathrm{N}(0,1). \tag{18}$$

In equation (18), ρ parameterizes the persistence of the process and σ_t denotes the standard deviation of idiosyncratic productivity shocks u_{it}^z. We allow σ_t to potentially vary over time to capture uncertainty shocks at the micro level. The constant term in equation (18) guarantees that the mean of transitory productivity, $\mathbb{E}\exp\left(z_{it}^T\right)$, does not change as we vary ρ and σ_t.

We define firm net worth in period t as $a_{it} := k_{it} - b_{it} \geq 0$. Using primes to denote next-period variables and denoting by $\mathbf{X}$ the vector of exogenous aggregate shocks, we now use net

[19] Berger and Udell (1988) argue that small and young firms have lower access to finance because informational constraints cause investors to perceive them as more risky. Khwaja and Mian (2005) show that politically connected firms receive preferential treatment from government banks. Johnson and Mitton (2003) present evidence that ties market values of firms to political connections and favoritism. In a European Central Bank (2013) survey, small and medium sized firms were more likely than larger firms to mention access to finance as one of their most pressing problems.

worth to rewrite firm's problem in recursive form as:

$$V\left(a, k, z^P, z^T, \mathbf{X}\right) = \max_{a', k', \ell, p} \left\{U(c) + \beta \mathbb{E} V\left(a', k', z^P, (z^T)', \mathbf{X}'\right)\right\}, \tag{19}$$

subject to the budget constraint:

$$c + a' + \frac{\psi\left(k' - k\right)^2}{2k} = p(y)y - w\ell - (r + \delta)k + (1 + r)a, \tag{20}$$

the borrowing constraint:

$$k' \leq \begin{cases} \infty, & \text{if } k' > \kappa \\ a', & \text{if } k' \leq \kappa \end{cases}, \tag{21}$$

the production function $y = Zk^{\alpha}\ell^{1-\alpha}$ and the demand function $y = p^{-\varepsilon}$.

The reformulation of the borrowing constraint in equation (21) shows that small firms cannot install capital beyond their net worth, whereas large firms do not face such a constraint in their capital accumulation. While in general firms have an incentive to increase their capital in order to relax their borrowing constraint, in the initial equilibrium of our model (capturing the period before 1995) the high real interest rate implies that the optimal capital stock is lower than κ for all firms. Given that the productivity process has a mean reverting component, some firms will initially be financially constrained. As the real interest rate declines along the transitional dynamics of our model, some firms increase their capital beyond the threshold κ and become permanently unconstrained.

4.2 Parameterization

We use the Wooldridge (2009) extension of the Levinsohn and Petrin (2003) methodology to estimate firm productivity and denote this estimate by $\hat{Z}_{ist}$.[20] In the estimation, we allow the elasticities of value added with respect to inputs to vary at the two-digit industry level. We discuss our estimates in more detail in Appendix B. Here we note that we estimate reasonable

[20] Olley and Pakes (1996) and Levinsohn and Petrin (2003) use a two-step method to estimate production functions in which investment and intermediate inputs respectively proxy for unobserved productivity. Ackerberg, Caves, and Frazer (2006) highlight that if a variable input (e.g. labor) is chosen as a function of unobserved productivity, then the coefficient on the variable input is not identified. Wooldridge (2009) suggests a generalized method of moments estimation to overcome some limitations of these previous methods, including correcting for the simultaneous determination of inputs and productivity, relaxing constant returns to scale, and robustness to the Ackerberg, Caves, and Frazer (2006) critique.

Table 3: Baseline Parameters

ψ	κ	γ	ε	α	δ	β	r	w	Z^A	ρ	σ	ν
3.10	4.20	3.00	3.00	0.35	0.06	0.92	0.06	1.00	1.00	0.59	0.13	0.33

elasticities, with their sum ranging from 0.75 to 0.91. Our estimate $\hat{Z}_{ist}$ uncovers a combination of idiosyncratic productivity and demand as we do not separately observe firm prices.[21]

We estimate the productivity process at the firm level using the regression:

$$\log\left(\hat{Z}_{ist}\right) = d_i + d_{st} + \rho \log\left(\hat{Z}_{ist-1}\right) + u^z_{ist}, \tag{22}$$

where d_i denotes the firm permanent effect and d_{st} denotes a four-digit industry-year fixed effect. We calibrate ρ and σ using regression (22). Based on the results of this regression we set $\rho = 0.59$.[22] We use the cross-sectional standard deviation of residuals u^z_{ist} from regression (22) to calibrate σ. The value of $\sigma = 0.13$ corresponds to the average standard deviation over time.

The permanent component of productivity is drawn from the following distribution:

$$z^P_i = \begin{cases} 1+\nu, & \text{with probability } 1/2 \\ 1-\nu, & \text{with probability } 1/2 \end{cases}, \tag{23}$$

We choose the standard deviation of the permanent component $\nu = 0.33$ such that, together with our estimated $\rho = 0.59$ and $\sigma = 0.13$, the model generates a standard deviation of $\log(Z_{it})$ equal to 0.38. The latter is the corresponding standard deviation of $\log\left(\hat{Z}_{it}\right)$ in the data.

Table 3 summarizes the parameters of the model. We start the economy in an initial equilibrium in which the real interest rate is at a high level $r = 0.06$. Most parameters are standard

[21]For this reason our elasticities are more appropriately defined as revenue elasticities. The correlation between $\log\left(\hat{Z}_{ist}\right)$ and $\log\left(\tilde{Z}_{ist}\right)$, which was defined in equation (11), in the cross section of firms ranges between 0.8 and 0.9 and is stable over time. Unless otherwise noted, from now on we always use $\log\left(\hat{Z}_{ist}\right)$ to construct moments in the data.

[22]Including firm fixed effects in a regression with a lagged dependent variable and a short time series leads to a downward bias in the estimated persistence of a process. When we estimate the AR(1) process in equation (22) we obtain an estimated persistence parameter of 0.46. Therefore, we set $\rho = 0.59$ such that, in model-generated data of 14 sample periods, the estimated persistence parameter equals 0.46. To maximize the length of the time series, all our estimates related to the productivity process are obtained from the permanent sample of firms between 1999 and 2012.

and, therefore, here we discuss only the adjustment cost parameter ψ and the threshold parameter κ in the borrowing constraint (16). We choose these two parameters to match the response of firm capital growth to productivity and net worth using with-firm variation. Specifically, in the data we regress:

$$\frac{k_{ist+1} - k_{ist}}{k_{ist}} = d_i + d_{st} + \beta_z \log\left(Z_{ist}\right) + \beta_a \log(a_{ist}) + \beta_k \log(k_{ist}) + u^k_{ist}, \tag{24}$$

where d_i denotes a firm fixed effect and d_{st} denotes a four-digit industry-year fixed effect. We vary the two parameters ψ and κ such that, in response to the transitional dynamics generated by our model between 1999 and 2007 following the decline in the real interest rate from $r = 0.06$ to $r = 0.00$, a similar regression with simulated data produces estimated coefficients that equal $\beta_z = 0.10$ and $\beta_a = 0.09$. We discuss in more detail these regressions in Section 5.

5 Firm-Level Implications of the Model

In this section we discuss firms' optimal policies and compare micro-level outcomes in the model to the data from Spain.

5.1 Labor, Prices, and Capital

We first solve for labor ℓ and prices p for a given state vector $\left(a, k, z^P, z^T, \mathbf{X}\right)$.[23] Given that capital is predetermined at some level k, at the beginning of each period firms face decreasing returns to scale with respect to the variable input ℓ. Therefore, the marginal cost MC is increasing in the scale of production:

$$\mathrm{MC} = \left(\frac{1}{Z}\right)\left(\frac{w}{1-\alpha}\right)\left(\frac{\ell}{k}\right)^{\alpha}. \tag{25}$$

Combining the first-order condition for labor, $(1-\alpha)py/\ell = \mu w$, with the demand function for output, the production function, and the expression for the marginal cost, we obtain labor demand:

$$\ell = Z^{\frac{\varepsilon-1}{1+\alpha(\varepsilon-1)}} \mu^{\frac{-\varepsilon}{1+\alpha(\varepsilon-1)}} \left(\frac{w}{1-\alpha}\right)^{\frac{-\varepsilon}{1+\alpha(\varepsilon-1)}} k^{\frac{\alpha(\varepsilon-1)}{1+\alpha(\varepsilon-1)}}. \tag{26}$$

[23]Given decisions for ℓ and p, we then iterate on the Bellman equation (19) to obtain the optimal policy for next period's net worth a' and capital k'. We solve the model with standard value function iteration methods. We discretize permanent productivity, transitory productivity, net worth, and capital into 2, 5, 60, and 60 points respectively. We have examined the robustness of our conclusions to alternative grid sizes.

Labor is increasing in capital k and productivity Z. The labor allocation is undistorted because the marginal revenue product of labor is equalized across firms, $\text{MRPL} := ((1-\alpha)/\mu)(py/\ell) = w, \forall\big(a, k, z^P, z^T, \mathbf{X}\big)$. We motivate this feature of the model with the fact that we do not observe trends in the dispersion of the MRPL in the data. Below, we extend our model to allow for a constant MRPL dispersion over time.

The price of each differentiated variety equals $p = \mu\text{MC}$. Equations (25) and (26) demonstrate a negative relationship between capital k and the marginal cost of production MC. Given that firms charge a constant markup μ over their marginal cost, high k firms have lower prices p. Similarly, high productivity Z firms have lower marginal cost and lower price.

In general, the MRPK is not equalized across firms. We define $\text{MRPK} := (\alpha/\mu)(py/k) := (1+\tau^k)(r+\delta)$, where τ^k denotes the percent deviation of the MRPK from the frictionless cost of capital $r+\delta$. To illustrate the sources of MRPK dispersion in our model, denote by χ the multiplier on the borrowing constraint (16) and by $\text{AC} = (\psi/2)(k'-k)^2/k$ the adjustment cost technology and consider the first-order condition with respect to capital for a firm characterized by some state vector $\big(a, k, z^P, z^T, \mathbf{X}\big)$:[24]

$$\mathbb{E}\left[\frac{\beta U'(c')}{U'(c)}\right]\left[\text{MRPK}' - (r'+\delta) - \frac{\partial \text{AC}'}{\partial k'}\right] = \frac{\chi}{U'(c)} + \frac{\partial \text{AC}}{\partial k'}. \tag{27}$$

In the absence of borrowing constraints, risk in capital accumulation, and investment adjustment costs, there would be no dispersion of the MRPK across firms. More productive firms would choose higher capital stocks but would lower their price p one-to-one with their productivity Z, leading to an equalization of the MRPK across firms. Under these assumptions, equation (27) simplifies to $\text{MRPK} = r+\delta$ for all firms $\big(a, k, z^P, z^T, \mathbf{X}\big)$.

By contrast, binding borrowing constraints, risk in capital accumulation, and investment adjustment costs introduce dispersion of the MRPK across firms. Binding borrowing constraints are captured by a positive multiplier χ in equation (27). Adjustment costs are captured by the derivatives of the adjustment cost function AC and AC$'$. Finally, similar to the analysis of Asker, Collard-Wexler, and De Loecker (2014), a capital stock determined in some previous

[24]Equation (27) holds only at points of differentiability. We solve the model using discrete state space methods and use this equation only for illustrative reasons in this section.

period may not be optimal ex-post, that is after productivity is realized. As a result, part of the dispersion of the MRPK across firms would also arise in an undistorted economy in which the capital stock is chosen under uncertainty and becomes productive in next period. Equation (27) shows that, even in the absence of borrowing constraints and adjustment costs, the MRPK does not in general equal the frictionless cost of capital $r + \delta$. Capital is chosen to equalize the expected value of the product of the stochastic discount factor with the gap between MRPK$'$ and $r' + \delta$.

5.2 Investment, Debt, Productivity, and Net Worth Within Firms

We present results from two regressions that use within-firm variation. The first is the capital growth regression shown in equation (24) and the second is a similar regression but with the change in (net) debt to capital ratio on the left-hand side. The choice of regressors is motivated by our model in which productivity, net worth, and capital are state variables summarizing firm capital and debt decisions. The first two regressors resemble sales and cash flow, commonly used by the finance literature in investment regressions. In Appendix C we report such regressions and document the similarity with the results reported in this section.

We measure firm net worth a in the data as the difference between the book value of total assets and total liabilities and deflate this difference with the industry output price deflators previously described in Section 3. We measure (net) debt b with the book value of current liabilities minus cash holdings and also deflate this difference with the same price deflators. Short-term debt is our preferred measure of debt because our model abstracts from a maturity choice of debt and savings in long-term assets.

Regressions in the data include firm fixed effects and industry-year fixed effects and cover the period between 1999 and 2007. The regressions in the model cover the same period using simulated data from the transitional dynamics of our model in response to an unexpected and permanent decline in the real interest rate from $r = 0.06$ to $r = 0.00$ that takes place in 1995. As we show below, the decline in the real interest rate generates trends in dispersion and misallocation similar to those documented in the data in Section 3. Regressions in the model

Table 4: Firm-Level Investment and Debt Decisions: Model vs. Data (1999-2007)

		Model				**Sample**	
						Permanent	Full
		(1)	(2)	(3)	(4)	(5)	(6)
	Adjustment Cost ψ	0.0	0.0	3.1	3.1		
	Borrowing Threshold κ	0.0	4.2	0.0	4.2		
$(k'-k)/k$	$\log Z$	1.16	1.53	0.13	0.10	0.10	0.11
	$\log a$	0.02	0.02	0.01	0.09	0.09	0.09
	$\log k$	-0.99	-1.13	-0.22	-0.16	-0.46	-0.63
$(b'-b)/k$	$\log Z$	1.03	1.35	0.02	-0.09	-0.38	-0.48
	$\log a$	0.03	0.03	0.01	0.22	0.15	0.14
	$\log k$	-1.02	-1.12	-0.28	-0.15	-0.34	-0.54

also include firm and year fixed effects.

To understand our calibration strategy, in column 1 of Table 4 we begin with regressions in a model without adjustment costs ($\psi = 0.0$) and no financial frictions ($\kappa = 0.0$). In the first panel, an increase in log productivity in the current period, $\log Z$, is associated with a strong increase in the (net) investment rate, $(k' - k)/k$. Firms invest more in response to a higher $\log Z$ because productivity is a persistent process and firms expect a higher marginal product of capital in period $t + 1$. Given the lack of adjustment costs, capital is not a persistent process. A coefficient of roughly minus one on log lagged capital when the left-hand side variable is capital growth implies that log capital is not very sensitive to its past value. Finally, with these parameters, net worth is not a significant determinant of capital growth.

The second panel shows that debt increases strongly following an increase in $\log Z$. The change in firm debt, $b' - b$, equals the difference between the flow of investment and the flow of saving. An increase in $\log Z$ increases saving because firms desire to smooth consumption in response to transitory productivity shocks. However, given the lack of adjustment costs and financial frictions, investment increases more than saving in response to an increase in $\log Z$.

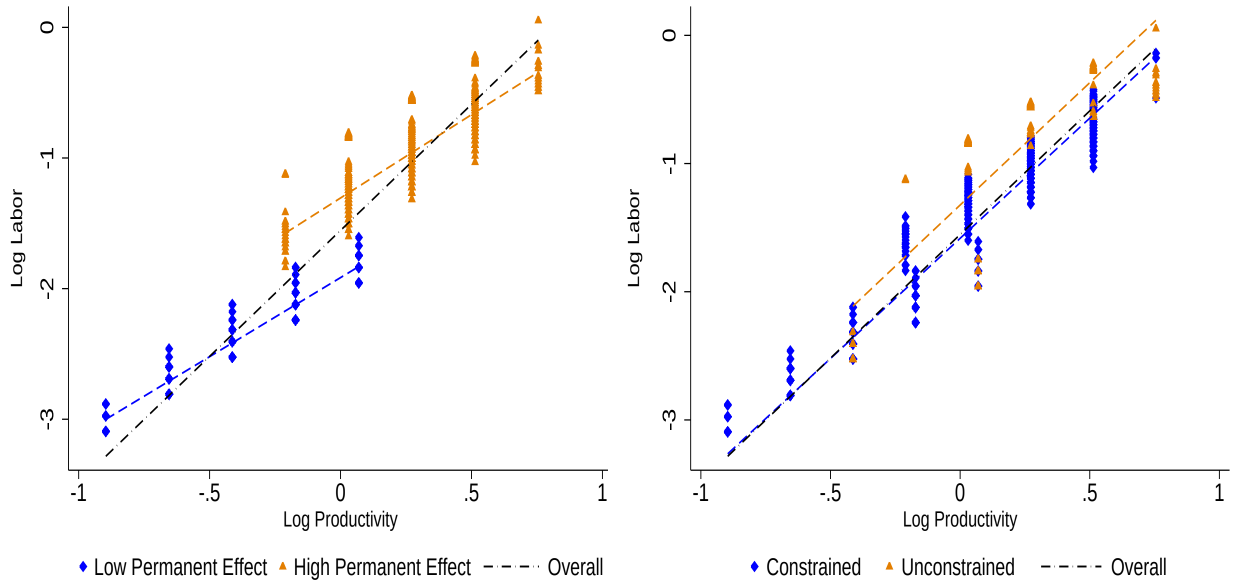
The firm behavior implied by the model with $\psi = 0.0$ and $\kappa = 0.0$ is at odds with the data. In columns 5 and 6 of Table 4 we see that $\log Z$ is positively related to firm capital growth, but this relationship is an order of magnitude smaller than predicted by the model without frictions. Additionally, changes in debt are actually negatively related to firm productivity. Finally, in the data an increase in net worth is associated with higher capital growth and borrowing and capital is a more persistent process than implied by the frictionless model. Given the large sample size (more than 100,000 observations in the permanent sample and 400,000 observations in the full sample), all coefficients in columns 5 and 6 are statistically significant at levels below 1 percent. We present the standard errors of our estimates in Appendix C.

Columns 2 to 4 show how the estimated coefficients in the model change progressively as we introduce adjustment costs (increasing ψ from 0.0 to 3.1) and financial frictions (increasing κ from 0.0 to 4.2). A positive value of κ without adjustment costs does not generate an important role for net worth. Adjustment costs ameliorate the responsiveness of capital growth to productivity, but without financial frictions they cannot explain the significance of net worth for capital growth. In column 4, we choose $\psi = 3.1$ and $\kappa = 4.2$ to match the responsiveness of capital growth to within-firm variations in productivity and net worth as observed in the permanent sample. We note that with these parameters, the model matches the observed negative correlation between changes in debt and productivity using within-firm variation.[25]

5.3 Size, Productivity, MRPK, Net Worth, and Leverage Across Firms

We now discuss cross sectional implications of our model in terms of variables such as firm size, productivity, MRPK, net worth, and leverage that are not targeted during the parameterization of the model. We use our firm-level dataset from Spain between 1999 and 2007 to

[25]Over the transitional dynamics generated by our model between 1999 and 2007, the mean adjustment cost equals 6.5 percent of value added conditional on adjusting the capital stock and the mean frequency of adjustment is 25 percent. The value of 6.5 percent lies within the range of estimates that Bachmann, Caballero, and Engel (2013) report, with their preferred estimate being 3.6 percent and the majority of other estimates from the literature exceeding 10 percent. The threshold level of capital $\kappa = 4.2$ implies that only firms with a high permanent component z^P potentially overcome their borrowing constraint. The value of $\kappa = 4.2$ equals 2.3 times the mean capital stock of high z^P firms and equals 12.5 times the mean capital stock of low z^P firms over the transitional dynamics of the model. The value of $\kappa = 4.2$ corresponds to roughly 3,400,000 euros in 2005 prices.



(a) By Permanent Productivity

(b) By Constraint Status

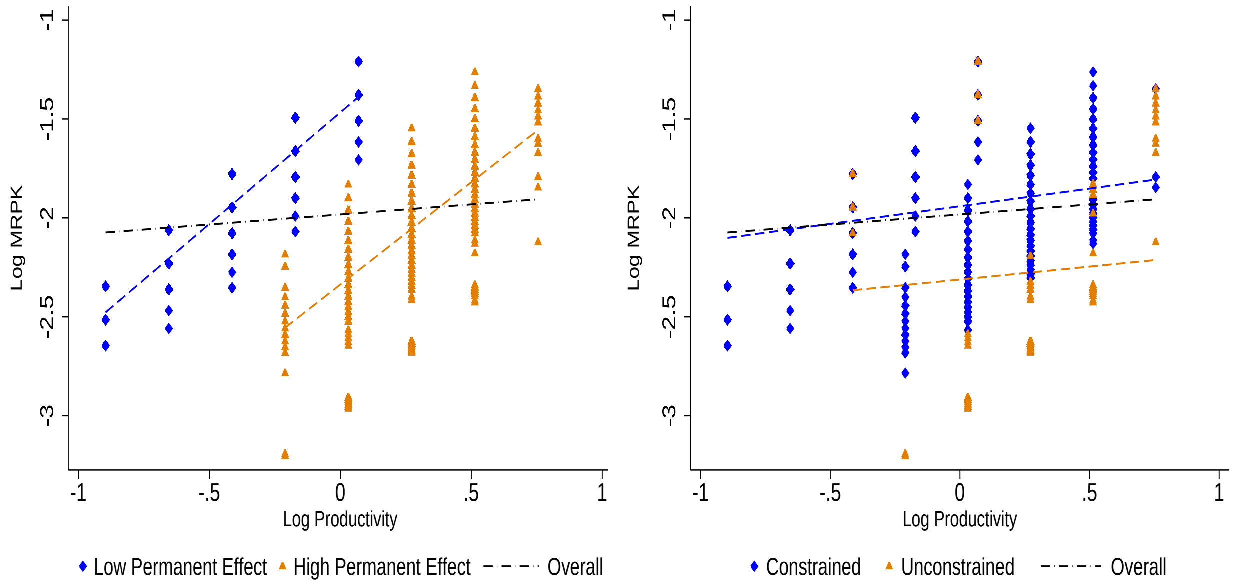
Figure 7: Firm Size and Productivity in the Model

set a benchmark for the model. As before, simulated data from the model are generated along the transitional dynamics between 1999 and 2007 in response to an unexpected and permanent decline in the real interest rate from $r = 0.06$ to $r = 0.00$ that takes place in 1995.

Figure 7 plots firm size (as measured by log labor) against firm log productivity, $\log Z$, in the cross section of firms in our model. In the left panel, firms are differentiated with respect to their permanent productivity z^P, with blue diamonds representing low productivity firms and dark orange triangles representing high productivity firms. In the right panel, firms are differentiated according to whether their borrowing constraint in equation (16) binds, with blue diamonds representing constrained firms and dark orange triangles representing unconstrained firms. Consistently with equation (26), both panels show that more productive firms are in general larger.[26] As shown in the two panels, the relationship between productivity and size is stable across firms with different permanent productivity and different constraints. Overall, there is a strong relationship between log productivity and log labor in the model, with a correlation of 0.97. The corresponding correlation in the permanent sample is 0.65.

Figure 8 plots $\log(\text{MRPK})$ against log productivity, $\log Z$. Two model elements lead to a

[26] By inspection of equation (26) we see that the lack of perfect correlation between log labor and log productivity in the model is explained by the less than perfect correlation between log capital and log productivity.



(a) By Permanent Productivity

(b) By Constraint Status

Figure 8: MRPK and Firm Productivity in the Model

positive correlation. The first is the risky time-to-build technology. As an example, consider two firms that start with the same state vector $\left(a, k, z^P, z^T, \mathbf{X}\right)$ in some period and, therefore, choose the same capital for next period k'. If in next period one of these firms receives a higher productivity shock, then that firm would have higher revenues, MRPK, and TFPR ex-post. The second element is the borrowing constraint. Constrained firms with higher productivity shocks have higher return to capital than constrained firms with lower productivity shocks.

The correlation between $\log(\text{MRPK})$ and $\log Z$ is 0.13 in the model. This is close to the data in which we find a correlation of 0.03. The left panel of Figure 8 helps understand the positive but low correlation generated by the model. Within the set of firms with the same permanent productivity z^P, there is a strong correlation between $\log(\text{MRPK})$ and $\log Z$, reflecting transitory productivity shocks in an environment with time-to-build technology and a borrowing constraint. However, across firms with different permanent productivity, the correlation between $\log(\text{MRPK})$ and $\log Z$ weakens significantly. This is because, in response to the decline in the real interest rate, some firms with high permanent productivity z^P accumulate capital higher than κ, become financially unconstrained, and tend to have lower MRPK.

In the baseline model there is no MRPL dispersion across firms and, therefore, $\log(\text{MRPK})$

Table 5: Summary Statistics in the Cross Section of Firms (1999-2007)

	Statistic	Model	Permanent Sample	Full Sample
Dispersion	Std (log ℓ)	0.78	1.13	1.21
	Std (log k)	0.87	1.52	1.70
	Std (log MRPK)	0.30	0.88	1.12
Productivity	Corr (log Z, log MRPK)	0.13	0.03	0.05
	Corr (log Z, log ℓ)	0.96	0.65	0.58
	Corr (log Z, ℓ/L)	0.91	0.54	0.48
	Corr (log Z, log k)	0.82	0.62	0.52
	Corr (log Z, k/K)	0.66	0.53	0.44
	Corr (log Z, log (k/ℓ))	-0.13	0.22	0.16
MRPK	Corr (log MRPK, log ℓ)	-0.13	-0.03	0.01
	Corr (log MRPK, ℓ/L)	-0.19	-0.05	-0.03
	Corr (log MRPK, log k)	-0.46	-0.62	-0.68
	Corr (log MRPK, k/K)	-0.57	-0.31	-0.28
	Corr (log MRPK, log (k/ℓ))	-1.00	-0.95	-0.96
Financial	Corr (log Z, log a)	0.81	0.75	0.65
	Corr (log MRPK, log a)	-0.20	-0.14	-0.14
	Coefficient of b/k on log k	0.14	0.15	0.23

and log (TFPR) are perfectly correlated. For this reason, statements about the covariation of log (MRPK) with various firm-level outcomes in the model carry over immediately to log (TFPR). To compare the baseline model to the data, we focus on the behavior of the MRPK because this is the key object for understanding the intuitions generated by the model. When we extend our model to allow for MRPL dispersion, we will also discuss TFPR separately from MRPK.[27]

Table 5 presents various summary statistics in the data and the model. We construct summary statistics in the data in a similar manner to the dispersion and misallocation measures presented in Section 3. We first calculate statistics across firms within each industry s and then use the same time-invariant weights to average these statistics across industries in any given year. Summary statistics both in data and the model are averaged between 1999 and 2007.

The first panel shows that the model produces a standard deviation of log labor which

[27]The focus on the MRPK (instead of the TFPR) has the additional advantage that the data moments along which we evaluate the model are insensitive to the value of the elasticity α in the production function.

represents roughly 69 percent of the dispersion observed in the permanent sample. The standard deviation of log capital represents roughly 57 percent of the dispersion observed in the data. The dispersion of the log (MRPK) generated by the model is smaller and represents roughly 34 percent of the dispersion observed in the permanent sample. Below we discuss several extensions to the model that can generate higher levels of labor, capital, and MRPK dispersion without changing either the key mechanisms that govern dispersion and misallocation patterns in the model or the evolution of aggregates in response to the decline in the real interest rate.

The second panel of the table shows correlations of variables with log productivity. As discussed in Figure 7, the model successfully replicates the positive and high correlation between log productivity and firm size (measured either by log labor or log capital). The model also matches the positive and high correlation between firm log productivity and share in sectoral economic activity (measured either by labor or capital).[28] Additionally, as discussed in Figure 8, the model matches the low but positive correlation between firm log productivity and log (MRPK). Both in the model and in the data, the correlation between firm log productivity and the log capital-labor ratio is much weaker than the correlations between firm log productivity and either labor or capital. However, this correlation is positive (and low) in the data while negative (and close to zero) in the model because the model produces a stronger correlation between productivity and labor than observed in the data.

The third panel presents correlations between log (MRPK) and various firm-level outcomes. An important prediction of the model is that moving from physical to revenue-based measures of productivity (such as TFPR and MRPK) lowers the correlations between productivity and firm size as measured either by the log of either factor of production or by the share of either factor in sectoral aggregates. In the model, log (MRPK) is negatively correlated with firm size because smaller firms are more likely to be financially constrained. This negative correlation is also observed in the data.[29] Finally, the model also matches the strong negative correlation

[28]The correlations between log productivity and firm share in economic activity resemble the measures of resource allocation emphasized by Bartelsman, Haltiwanger, and Scarpetta (2013). Following Olley and Pakes (1996), these authors decompose aggregate productivity measures into an unweighted average of firm productivity and the covariance between firm size and firm productivity.

[29]Hsieh and Olken (2014) argue that smaller firms in India, Indonesia, and Mexico have lower average product of capital than larger firms. We do not find this pattern in Spain. In Appendix H we show that a positive correlation

between the log capital-labor ratio and the log (MRPK) observed in the data.

The last panel of Table 5 presents cross-sectional correlations between financial and real variables. The model successfully replicates the observed positive cross-sectional correlation between firm log productivity and firm log net worth. In our model, firms with higher net worth tend to be less constrained and tend to have a lower return to capital. The model is, therefore, successful in matching the negative correlation between log (MRPK) and log net worth observed in the data. Finally, in the last row of the table we present the cross-sectional relationship between leverage b/k and size (as measured by log capital). We find a positive relationship between leverage and size in the model, which follows from the assumption embedded in the constraint (16) that firms with higher capital can relax their borrowing constraint. The last two columns of Table 5 show that there also exists a positive relationship between leverage and log capital in the data.[30] We obtain a similar result in the data when we use other measures of firm size such as the wage bill.

6 Macroeconomic Implications

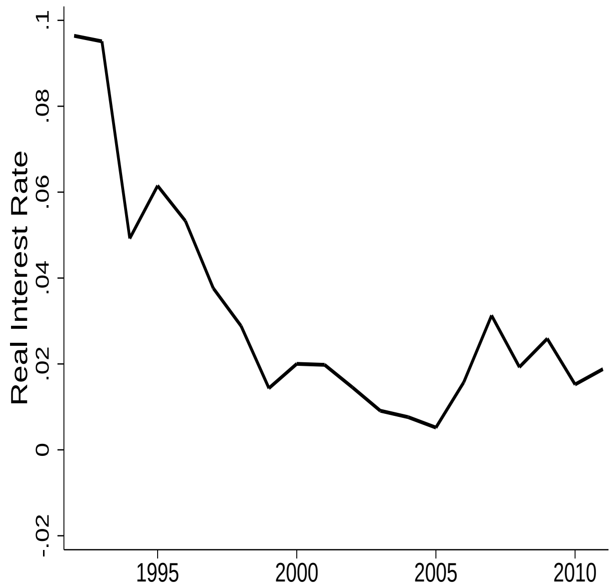
Having documented the success of the model to match several aspects of firm-level behavior, we now turn to the model's aggregate implications.

6.1 Real Interest Rate Decline

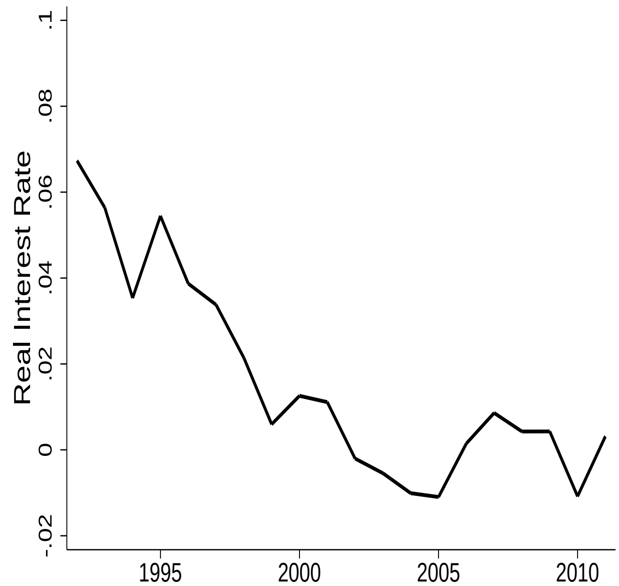
We associate trends in dispersion, misallocation, and capital flows to the secular decline in the real interest rate r_t. Figure 9 presents the evolution of r_t since the early 1990s. In the left panel, r_t is the difference between the nominal corporate lending rate to non-financial firms and next year's expected inflation. The lending rate comes from Eurostat and refers to loans with size less than one million euros that mature within one year. Expected inflation is given by the fitted values from an estimated AR(1) process for inflation. In the right panel, r_t is the real interest

between MRPK and labor can be obtained in an environment with both financial frictions and labor wedges.

[30] Since our model does not consider the distinction between short and long term liabilities or assets, the regressions in the data control for the difference between long-term liabilities and assets. Additionally, cross-sectional regressions control for firm age. We define firm age in period t as t minus the date of incorporation plus one. Firm age is a firm-specific linear time trend and, therefore, is absorbed by the firm fixed effect in regressions that use within-firm variation over time.



(a) Eurostat (up to 1 year)



(b) IMF (3-month)

Figure 9: Evolution of Real Interest Rate in Spain

rate from IMF (2014), defined as the difference between the 3 month nominal government bond yield and expected inflation. Both series decrease significantly during the 1990s and stabilize at a permanently lower level in the 2000s.

6.1.1 Real Interest Rates and Misallocation: An Illustrative Example

The main experiment in the model considers an unexpected and permanent decline in r_t from 6 percent to 0 percent in 1995. Before showing aggregate responses, we first present a simple example that illustrates the mechanism generating misallocation in our model in response to the decline in r_t. Figure 10 depicts outcomes for two firms following the decline in r_t. Period $t = 0$ corresponds to the year when r_t declines permanently. The initial conditions for these two firms are drawn from the stochastic steady state of the model.[31]

The drop in the real interest rate increases desired investment for both firms. The two firms have the same productivity in all periods. The firms, however, differ in their initial net worth and debt. Firm A has initially higher net worth and is financially unconstrained ($b_{A0} < 0$ and

[31]We start the economy in a stochastic steady state, defined as an equilibrium of the model in which aggregate shocks are constant over time. In the stochastic steady state, firms are hit by idiosyncratic productivity shocks and change their production, savings, and investment decisions over time. Aggregate variables and the distribution of firms over states are stationary over time.

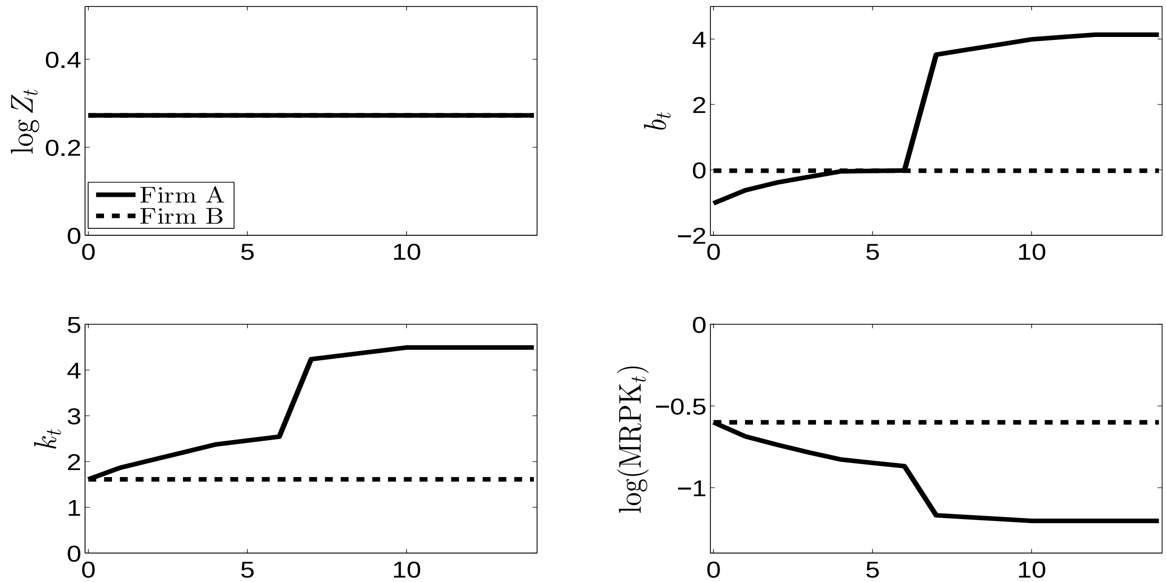


Figure 10: Real Interest Rates and Misallocation: An Illustrative Example

$k_{A0} < a_{A0}$). Following the decline in the real interest rate, firm A is willing to pay for the adjustment cost in order to increase its capital stock. In the first few periods capital growth is financed by internal savings. In period $t = 7$, firm A finds it optimal to increase its capital above the threshold κ. As the borrowing constraint is lifted, firm A uses the inflow of debt to finance an even higher level of capital. The decline in the real interest rate causes a decline in the firm's MRPK.

Firm B has initially lower net worth and is financially constrained ($b_{B0} = 0$ and $k_{B0} = a_{B0}$). This firm also desires to increase its capital. However, the lack of sufficient funds prevents the firm from doing so.[32] This financially constrained firm does not experience changes in its MRPK and, therefore, the dispersion of the MRPK between the two firms increases.

This example illustrates how a decline in the real interest rate in an environment with financial frictions and adjustment costs can cause capital inflows to be misallocated. Misallocation here means that capital is entirely flowing into one firm despite both firms being equally productive. It is not crucial that both firms are equally productive. We would obtain the same result if firm A experienced a few negative productivity shocks along its transition.

[32] Firm B will start accumulating internal funds to overcome its constraint as soon as it receives a positive productivity shock. In the new stationary equilibrium of our model, firms with high permanent productivity z^P become unconstrained and resources are efficiently allocated within the set of high permanent productivity firms.

Table 6: Capital Growth and Initial Net Worth: Model vs. Data

		Model				**Sample**	
						Permanent	Full
		(1)	(2)	(3)	(4)	(5)	(6)
	Adjustment Cost ψ	0.0	0.0	3.1	3.1		
	Borrowing Threshold κ	0.0	4.2	0.0	4.2		
$(k_{07}-k_{99})/k_{99}$	$\log Z_{99}$	0.15	0.40	0.42	0.31	1.14	1.49
	$\log a_{99}$	-0.03	-0.19	0.00	0.12	0.17	0.15
	$\log k_{99}$	-0.14	-0.17	-0.12	-0.12	-0.96	-1.11
$(b_{07}-b_{99})/k_{99}$	$\log Z_{99}$	0.01	0.05	0.30	-0.06	1.12	1.47
	$\log a_{99}$	0.02	-0.07	0.03	0.35	0.20	0.11
	$\log k_{99}$	-0.24	-0.09	-0.11	-0.09	-0.86	-0.98

6.1.2 Was Capital Misallocated to Higher Net Worth Firms?

Following the logic of the example with the two firms, our model generates misallocation because capital inflows are directed to firms that are not necessarily more productive but instead have higher initial net worth. We now provide direct evidence that supports this mechanism.

The first panel of Table 6 shows the cross-sectional relationship between capital growth over 1999 and 2007, $(k_{07}-k_{99})/k_{99}$, and initial net worth, $\log a_{99}$, conditional on initial firm productivity, $\log Z_{99}$, and capital, $\log k_{99}$. Column 4 of the table shows that our model (with $\psi = 3.1$ and $\kappa = 4.2$) implies that firms with higher initial net worth invest more in capital, conditional on their initial productivity and capital. Further, the second panel of the table shows that in our model firms with higher initial net worth finance their capital accumulation by borrowing more. The first three columns show that models without adjustment costs or financial frictions do not generate these patterns in general.

The last two columns of the table confirm that, in the data, firms with higher initial net worth borrowed more and invested more than firms with similar initial productivity but lower

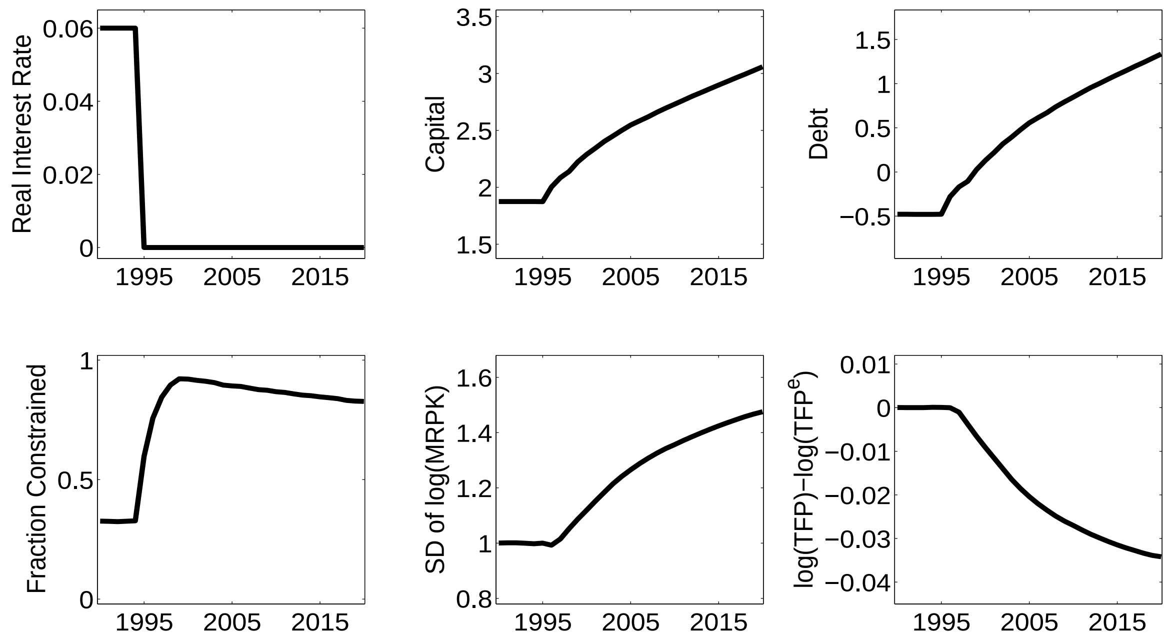


Figure 11: Decline in the Real Interest Rate: Baseline Model

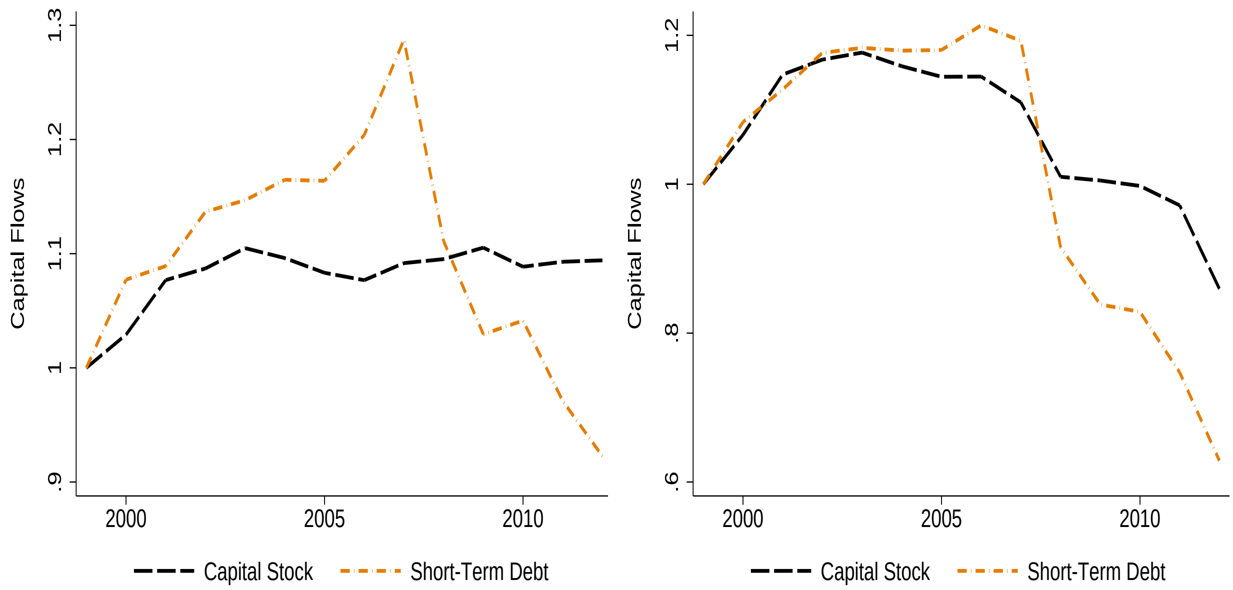
initial net worth. All coefficients in columns 5 and 6 are statistically significant at levels below 1 percent (see Appendix D). The regressions in the data also include industry fixed effects.

6.1.3 Aggregate Impulse Responses

Figure 11 shows the evolution of aggregates for the sector in response to the decline in the real interest rate r_t. As documented in Section 6.1.2, both in the data and in the model, wealthier firms are more likely to finance capital accumulation. As these firms grow, they eventually overcome their borrowing constraint and accumulate debt. Capital is not allocated to its most efficient use because some productive but financially constrained firms do not grow in the short run. In response to a decline in r_t, the model generates capital inflows, an increase in the dispersion of the $\log(\text{MRPK})$, and a decline in $\log(\text{TFP})$ relative to its efficient level.[33]

These predictions match the experience of Spain in the first years following the introduction of the euro. In Section 3 we documented increases in the dispersion of the $\log(\text{MRPK})$ and

[33] In all model impulses we will normalize $\log(\text{TFP}) - \log(\text{TFP}^e)$ to zero before the shock and, therefore, in the last panel we measure the percentage point change in TFP relative to its efficient level. Efficient log total factor productivity, $\log(\text{TFP}^e)$, does not change in the model unless there is a change in the underlying distribution of firm productivity. We also note that, contrary to TFP, labor productivity increases following the decline in the real interest rate. Labor productivity, $Y_t/L_t = \text{TFP}_t\,(K_t/L_t)^\alpha$, increases because capital deepening dominates the decline in TFP.



(a) Permanent Sample

(b) Full Sample

Figure 12: Capital Flows in the Manufacturing Sector

declines in log (TFP) relative to its efficient level. In Figure 6 we further documented that the increase in the dispersion of log capital was associated with a declining correlation between log capital and log productivity. We obtain a similar prediction in the model.[34] Finally, Figure 12 plots the evolution of capital flows to the manufacturing sector in Spain from our dataset. Similar to the transitional dynamics generated by the model, in the data we observe an increase in aggregate capital in the first few periods after the introduction of the euro. In line with the prediction of the model, capital growth is financed by accumulation of short-term debt. We discuss the decline in debt during the post-crisis period in Section 6.6.

In our model with a size-dependent borrowing constraint both financial frictions and adjustment costs are important in generating these patterns. As we show in Appendix D, in the absence of adjustment costs, firms with a high permanent productivity component z^P increase significantly their capital stock and overcome instantaneously their borrowing constraints. Such a model would generate an increase in the dispersion of the log (MRPK) but a negligible decline in log (TFP). Appendix D also shows that the model with only investment adjustment costs and no financial frictions does not generate significant changes either in log (MRPK) dispersion

[34] In the model, the standard deviation of $\log k$ increases from roughly 0.82 before the shock to 0.95 by the end of sample period and the correlation between $\log k$ and $\log Z$ declines from roughly 0.87 to 0.79.

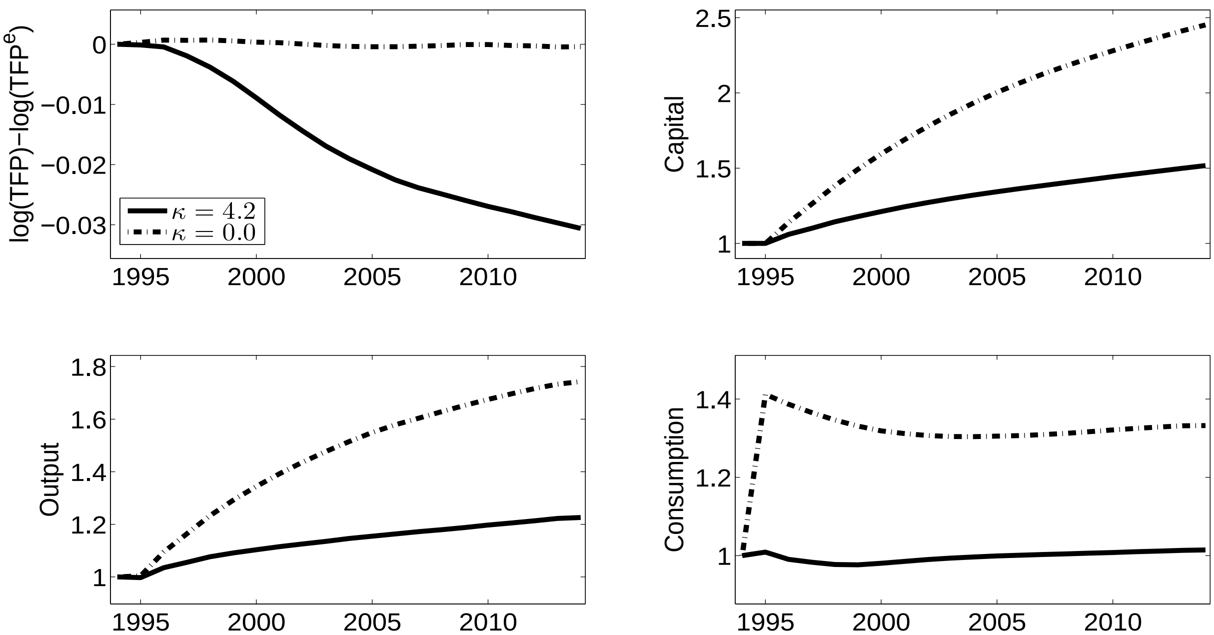


Figure 13: Impact of Misallocation on Aggregate Dynamics

or in log (TFP).

6.1.4 Impact of Misallocation on Aggregate Dynamics

The inflow of capital in our model is associated with a deterioration in the allocation of resources across firms. We now ask what is the additional impact of this deterioration on aggregate dynamics following the decline in the real interest rate. To answer this question, we compare impulse responses in the baseline model with financial frictions to the impulses generated by a model without financial frictions. This comparison allows us to isolate the effect of financial frictions on aggregate dynamics, holding constant the other two factors that generate MRPK dispersion (risky time-to-build capital accumulation and investment adjustment costs).

Figure 13 shows that the impulse responses in the baseline model with financial frictions (labeled by $\kappa = 4.2$ in the figure) differ significantly from the impulses generated by a model without financial frictions (labeled by $\kappa = 0.0$ in the figure), holding constant all other parameters at the baseline values shown in Table 3. Output and capital grow by significantly less in the model with financial frictions. We also note an important difference in the initial growth of aggregate consumption. Consumption grows by substantially more in the model without financial frictions because permanent income grows by more in this model than in the model with financial frictions. We conclude that, in response to the decline in the real interest rate, there is an important quantitative effect of misallocation due to financial frictions on the transitional dynamics of aggregate variables.

6.1.5 External Financial Dependence, Dispersion, and Productivity

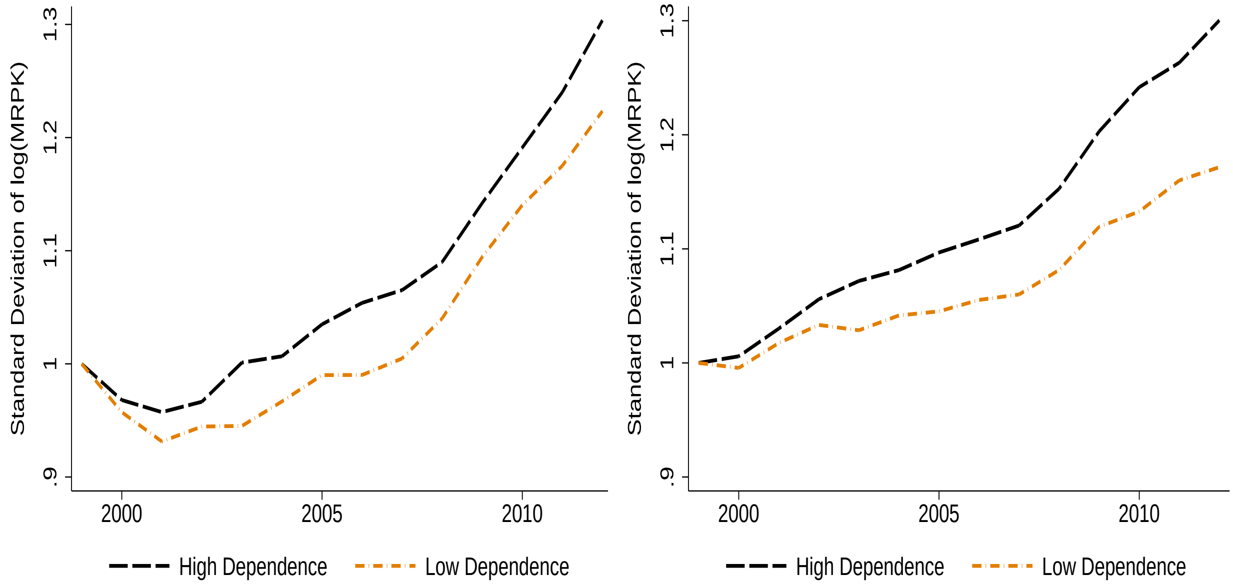
In line with Spain's experience, the model generates an increase in MRPK dispersion and a decline in TFP in response to the decline in the real interest rate. The key mechanism leading to these patterns is that the decline in the cost of capital in an environment with financial frictions (and adjustment costs) causes capital to flow to some wealthy but not necessarily productive firms. A natural implication of this narrative is that increases in MRPK dispersion and declines in TFP should be stronger among industries that depend more heavily on external finance. In this section we show that this is indeed the case.

We show trends in dispersion and misallocation for two groups of industries. The groups are differentiated according to their external financial dependence as measured by Rajan and Zingales (1998) for U.S. firms from Compustat at the two digits. We classify industries as "high dependence" if their measure of dependence is higher than the median dependence and as "low dependence" if their measure of dependence is lower than the median dependence. We use the same time-invariant weights as in the rest of our analysis to aggregate industries in each group.

Figure 14 shows that high dependence industries experienced larger increases in the dispersion of the $\log(\text{MRPK})$ between 1999 and 2007 than low dependence industries. Figure 15 shows trends in the difference of $\log(\text{TFP})$ from its efficient level for the two groups of industries. We observe that high dependence industries experienced larger declines in $\log(\text{TFP})$ relative to the efficient level. All facts hold for both the permanent and the full sample of firms.

6.1.6 Robustness to Specific Model Features and Comparison to Other Models

The directional response of the dispersion of the $\log(\text{MRPK})$ to various shocks is a general feature of models with financial frictions and not an artifact of specific features of our model. In Appendix E we consider a simpler model without a size-dependent borrowing constraint, time-



(a) Permanent Sample (b) Full Sample

Figure 14: Evolution of MRPK Dispersion and External Financial Dependence

(a) Permanent Sample (b) Full Sample

Figure 15: Evolution of log (TFP) Relative to Efficient Level and External Financial Dependence

to-build capital accumulation technology, and adjustment costs. Specifically, the constraint takes the form $b_{it+1} \leq \tilde{\theta}_t k_{it+1}$ for all firms or, equivalently and using recursive notation, $k \leq \theta a$ for $\theta = 1/(1-\tilde{\theta})$. This model is closer to the environment considered by Midrigan and Xu (2014), Moll (2014), and Buera and Moll (2015) in which firms face a financial constraint of the form $k \leq \theta a$ and there is perfect foresight about next period's productivity.

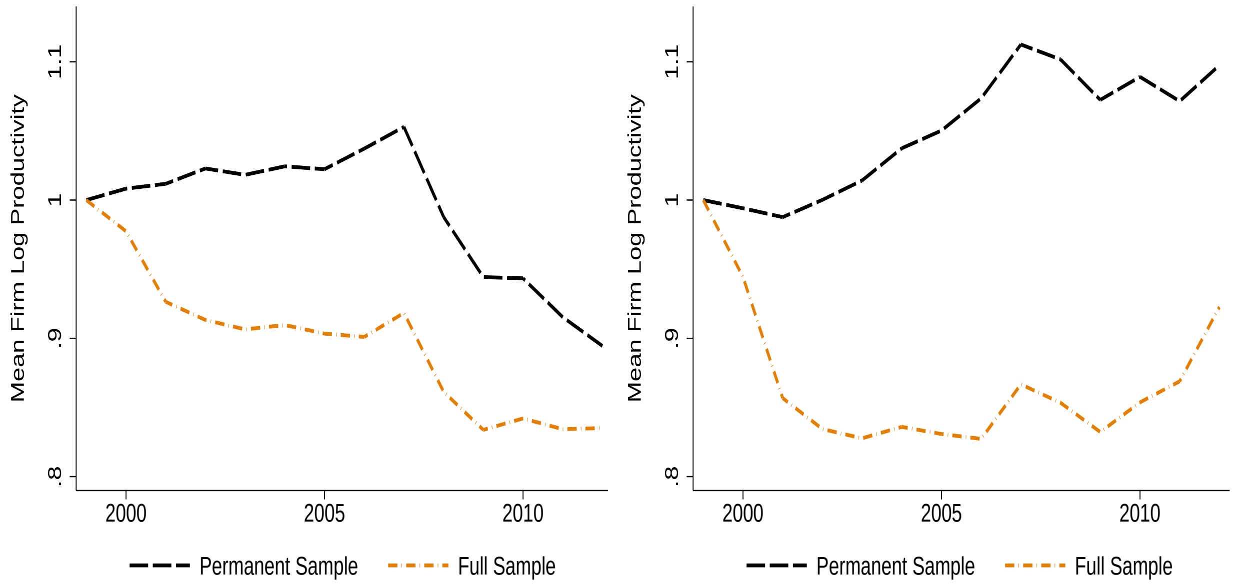
Appendix E derives closed-form solutions within this simpler environment for the response of the dispersion of the log (MRPK) to various shocks. We show that, unless all firms are either constrained or unconstrained, the dispersion of the log (MRPK) increases when: (i) the cost of capital decreases; (ii) financial frictions increase; (iii) exogenous aggregate productivity or demand increase. We stress that all responses have the same sign as the responses generated by our richer model.

In Appendix F we consider a decline in the real interest rate in the full model with a risky time-to-build technology of capital accumulation and adjustment costs, but with a financial constraint of the form $k' \leq \theta a'$. Similar to our model with a size-dependent borrowing constraint, we show that log (TFP) declines when the real interest rate falls. However, for similarly calibrated models that target the responsiveness of capital growth to within-firm variations in productivity and net worth, the increase in the dispersion of the log (MRPK) and the decline in log (TFP) are much weaker in the model with a financial constraint of the form $k' \leq \theta a'$.

Additionally, the two models differ with respect to the cross-sectional moments discussed in Section 5.3. As in the data, the model with a size-dependent borrowing constraint generates a positive but low correlation between log (MRPK) and log productivity and a negative correlation between log (MRPK) and size. However, in the model with a financial constraint of the form $k' \leq \theta a'$ both correlations are positive and high. The two models differ with respect to both aggregate responses and cross-sectional patterns because with a size-dependent borrowing constraint some firms experience significant growth and overcome permanently their borrowing constraint in response to the decline in the real interest rate. We, therefore, obtain a greater dispersion of MRPK across firms and a negative cross-sectional correlation between size and MRPK.

6.2 Real Interest Rate Decline With Endogenous Entry and Exit

In this section we examine the robustness of our results to endogenous entry and exit. We motivate this extension with Figure 16 that presents the evolution of mean log productivity in our sample. We show mean firm productivity both for the measure $\log \hat{Z}$ estimated with the



(a) $\log \hat{Z}$ Measure

(b) $\log \tilde{Z}$ Measure

Figure 16: Evolution of Mean Log Productivity

Wooldridge (2009) extension of the Levinsohn and Petrin (2003) methodology and the measure $\log \tilde{Z}$ defined in equation (11). The figure shows that in the full sample of firms mean log productivity declines significantly relative to the permanent sample. This suggests that less productive firms have entered into the sample over time.

Here we just describe the main elements of the model with entry and exit and leave for Appendix G the more detailed presentation. At any given point of time, firms can operate either in manufacturing or produce in the outside sector. We think of the outside sector as a sector that uses capital less intensively than manufacturing (for instance, home production). Starting in manufacturing, a firm decides whether in next period it will continue to operate in manufacturing or sell its capital and exit to the outside sector. Firms starting in the outside sector decide whether to enter into manufacturing in next period or continue operations in the outside sector. Firms entering in manufacturing incur a cost that is increasing and convex in the scale of production. We calibrate the cost such that, for $\psi = 3.1$ and $\kappa = 4.2$, the model replicates the responsiveness of capital growth to within-firm variations in productivity and net worth as documented in the full sample of firms in Table 4. All other parameters are fixed at the values shown in Table 3 for our baseline model.

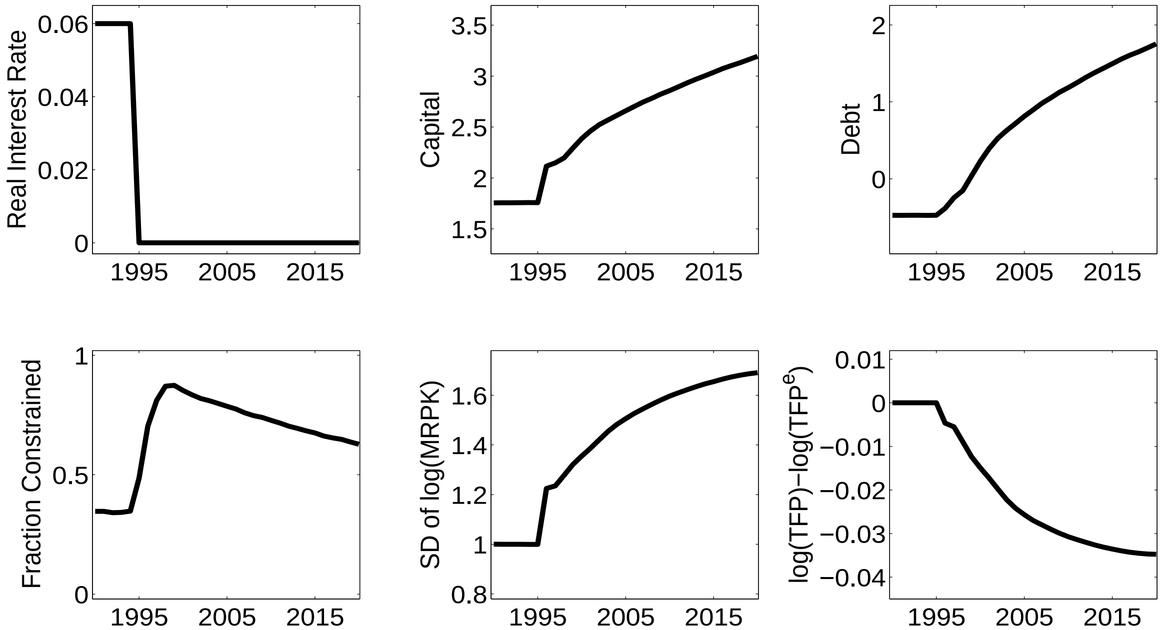


Figure 17: Decline in the Real Interest Rate: Model With Entry and Exit

Figure 17 shows impulses in response to the decline in the real interest rate r_t in the model with endogenous entry and exit. Before the decline in r_t, less productive firms select to produce in the outside sector. The drop in r_t causes a decline in production costs in manufacturing relative to the outside sector. As a result, the decline in r_t incentivizes some of the less productive firms previously operating in the outside sector to enter in manufacturing. Since these firms are less productive on average, mean log productivity of firms operating in manufacturing drops in the post-shock period relative to the pre-shock period. As shown in the figure, this model generally produces similar responses as our baseline model, with MRPK dispersion increasing over time and $\log(\text{TFP})$ declining relative to its efficient level.

6.3 Real Interest Rate Decline With MRPL Dispersion

This section extends the baseline model to allow for MRPL dispersion across firms. We begin our analysis with a model of MRPL dispersion arising from exogenous labor wedges. The labor wedge τ takes the form of a proportional tax that firms pay on their compensation to labor. Thus, if w is the wage and ℓ is labor, the after-tax compensation to labor equals $(1+\tau)wl$. We rebate the tax revenue $\tau w\ell$ lump-sum to each firm and, as a result, taxes affect firm behavior

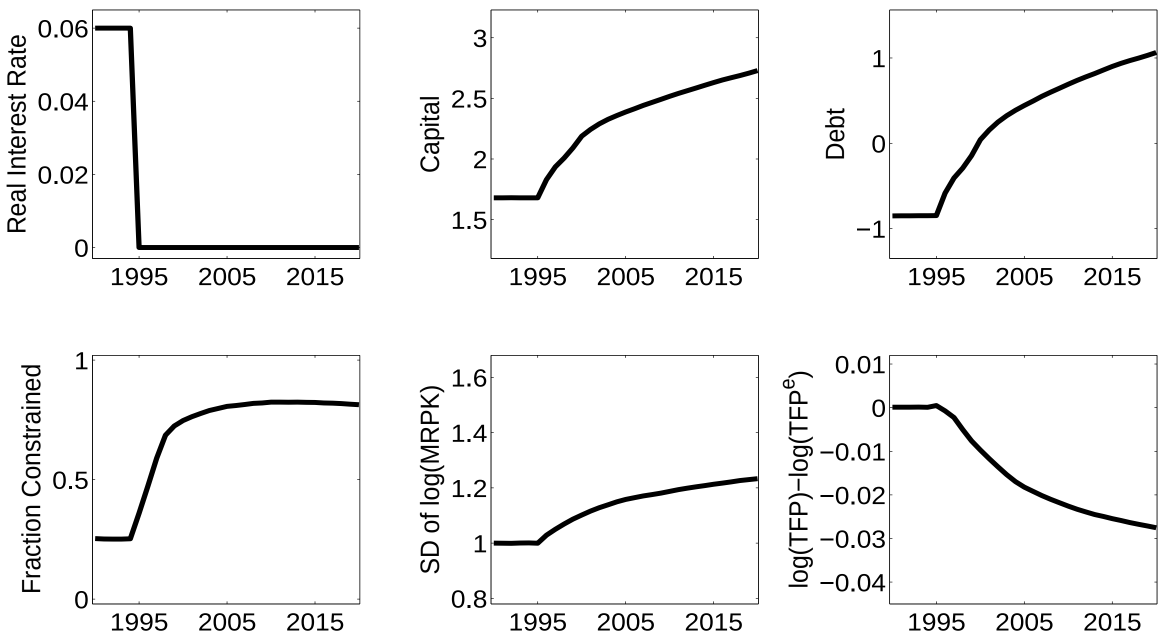


Figure 18: Decline in the Real Interest Rate: Model With Exogenous MRPL Dispersion

only through production decisions. All other elements of the model are the same as in our baseline model without MRPL dispersion.

The labor wedge is heterogeneous across firms and can take two values, $\tau = -\bar{\tau}$ and $\tau = +\bar{\tau}$. It follows an exogenous first-order Markov process $\pi(\tau'|\tau)$ that is independent of firm productivity and takes the values $\pi\left(\tau' = \bar{\tau}|\tau = \bar{\tau}\right) = \pi\left(\tau' = -\bar{\tau}|\tau = -\bar{\tau}\right) = \pi_\tau$. Motivated by the facts documented in Section 3, the process $\pi(\tau'|\tau)$ is independent of calendar time and, as a result, MRPL dispersion is constant in the model.

We calibrate the values of $\bar{\tau}$ and π_τ to match two moments estimated from the permanent sample of firms. First, the standard deviation of $\log\left(\text{MRPL}\right)$ equals 0.30. Second, the first-order autocorrelation coefficient of $\log\left(\text{MRPL}\right)$ estimated from a regression with firm and industry-year fixed effects equals 0.48. With 14 sample periods, we obtain the values $\bar{\tau} = 0.29$ and $\pi_\tau = 0.81$. Given the stochastic process of the MRPL, we calibrate again ψ and κ such that the model replicates the responsiveness of capital growth to within-firm variations in productivity and net worth in the permanent sample of firms in Table 4. We find that $\psi = 3.2$ and $\kappa = 3.8$. All other parameters are fixed at the values shown in Table 3 for our baseline model.

Figure 18 presents impulses in response to the decline in real interest rate in the model with exogenous and constant MRPL dispersion. We stress that the impulses look quite similar to the impulses in Figure 11 for the baseline economy without MRPL dispersion. The somewhat smaller increase in MRPK dispersion and smaller decline in TFP are explained by the fact that there is more uninsurable risk in the model with stochastic labor wedges. Thus, firms accumulate more precautionary savings before the shock and are somewhat less likely to be constrained both before and after the decline in the real interest rate. In Appendix H we compare the model with exogenous MRPL dispersion to the baseline model without MRPL dispersion with respect to the cross-sectional moments discussed in Section 5.3.

In the model of Bartelsman, Haltiwanger, and Scarpetta (2013), overhead labor endogenously generates MRPL dispersion across firms. Such a model would, however, imply changes in MRPL dispersion over time in response to shocks. We, therefore, started with the simpler approach of specifying exogenous labor wedges at the firm level and assumed that the dispersion of these wedges is constant over time. In Appendix I we develop the model with overhead labor. We show that the aggregate impulses generated by the model with overhead labor in response to the decline in the real interest rate are almost identical to the impulses generated by the baseline model without MRPL dispersion. This is because overhead labor does not interact in a significant quantitative way with firm investment and debt decisions.[35]

6.4 Easing of Borrowing Constraints

It is often conjectured that countries in the South received large capital inflows following a financial liberalization associated with the adoption of the euro. We now evaluate the implications of such a development through the lens of our baseline model. A financial liberalization episode in our model, modeled as a decline in the borrowing threshold κ_t, is associated with an increase in borrowing that allows previously constrained firms to increase their capital accumulation. Therefore, this shock captures the common view that the adoption of the euro was associated

[35] In our baseline model we obtain a lower level of cross-sectional dispersion of capital and especially of MRPK relative to the data. In Appendix J we describe a model with higher levels of capital and MRPK dispersion based on the idea that there is an unmeasured input that enters additively with capital in production. The idea of an unmeasured input is similar to overhead labor, with the difference being that the input is added instead of subtracted from capital. We show that this model implies quite similar changes in aggregate variables in response to the decline in the real interest rate as our baseline model. We, therefore, argue that the relatively low level of dispersion in our baseline model is not so important about the main messages that emerge from our analysis.

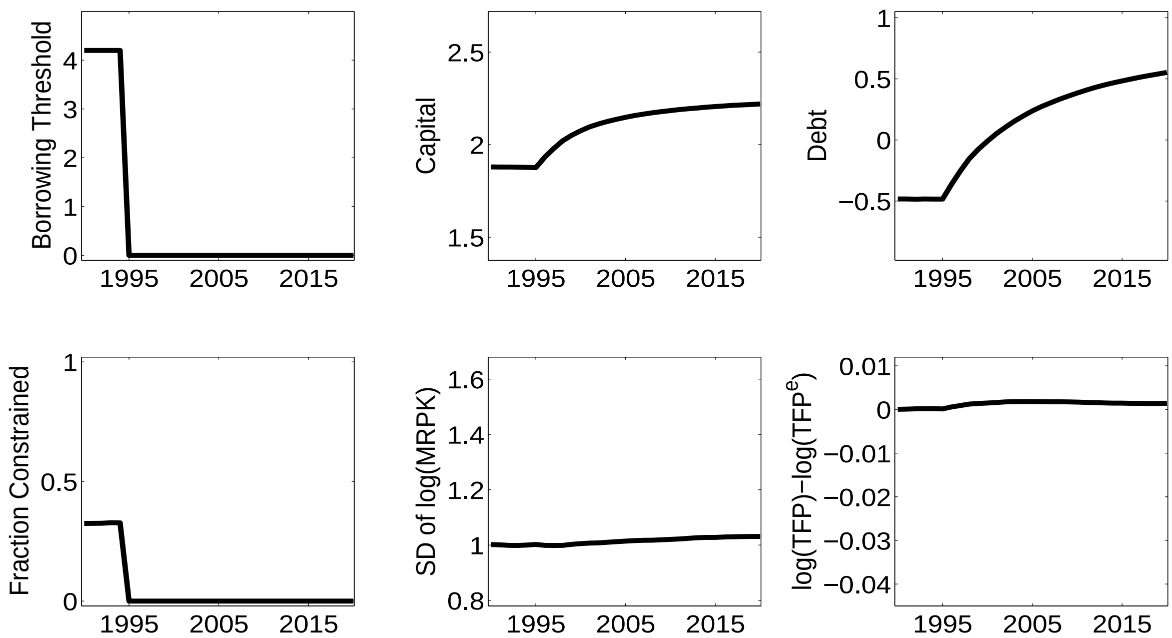


Figure 19: Decline in Borrowing Threshold (With Adjustment Costs)

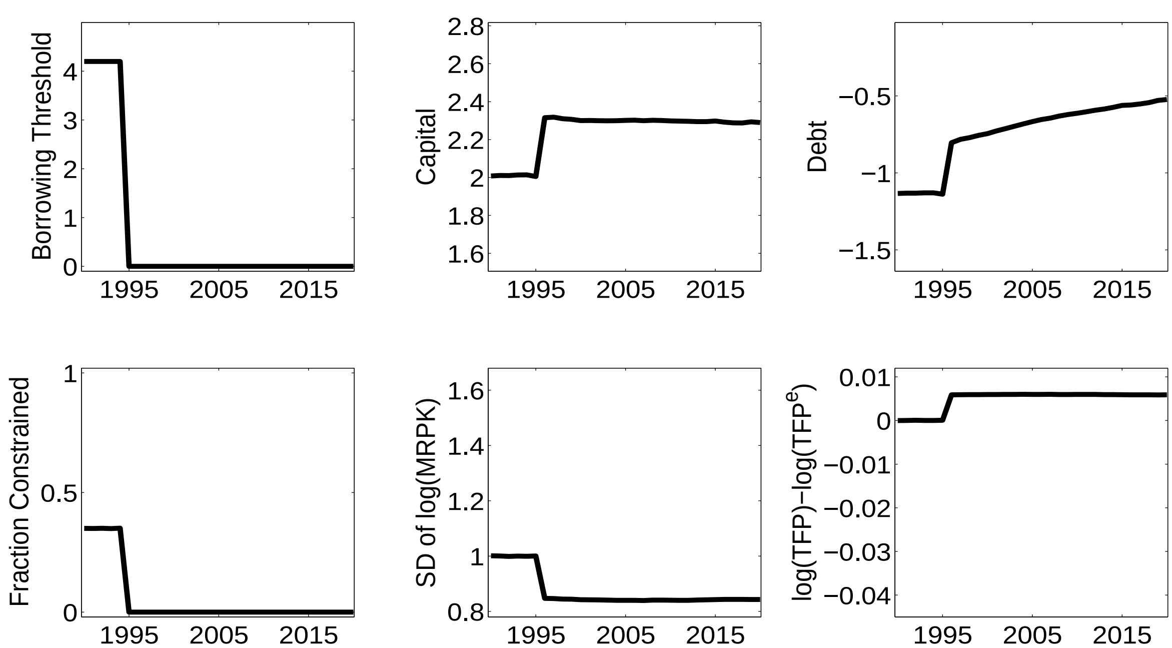


Figure 20: Decline in Borrowing Threshold (Without Adjustment Costs)

with capital inflows to the South.

Figure 19 presents impulses in response to an unexpected and permanent decline in κ_t using the baseline parameters.[36] The decline in κ_t generates a more efficient allocation of resources and a small increase in $\log(\text{TFP})$. This contradicts the key fact that capital inflows in Spain were accompanied by a decline in $\log(\text{TFP})$ relative to its efficient level.[37]

Under our baseline parameterization with adjustment costs, the dispersion of the $\log(\text{MRPK})$ increases in response to the decline in κ_t. In Figure 20 we show transitional dynamics in response to a decline in κ_t in a model without adjustment costs ($\psi = 0$). Here, both $\log(\text{TFP})$ increases and the dispersion of the $\log(\text{MRPK})$ decreases. Consistently with this prediction, Appendix E shows analytically that an increase in θ generates a decline in the dispersion of the $\log(\text{MRPK})$ in a model with a financial constraint of the form $k \leq \theta a$ and no adjustment costs.

The prediction that financial liberalization episodes are associated with increasing productivity is common in models with financial frictions (see, for instance, Buera, Kaboski, and Shin, 2011; Midrigan and Xu, 2014).[38] While we do not deny that such a financial liberalization may have taken place, our empirical and theoretical results imply that the decline in the real interest rate is more important for understanding the evolution of productivity in Spain in the first few years after the adoption of the euro.

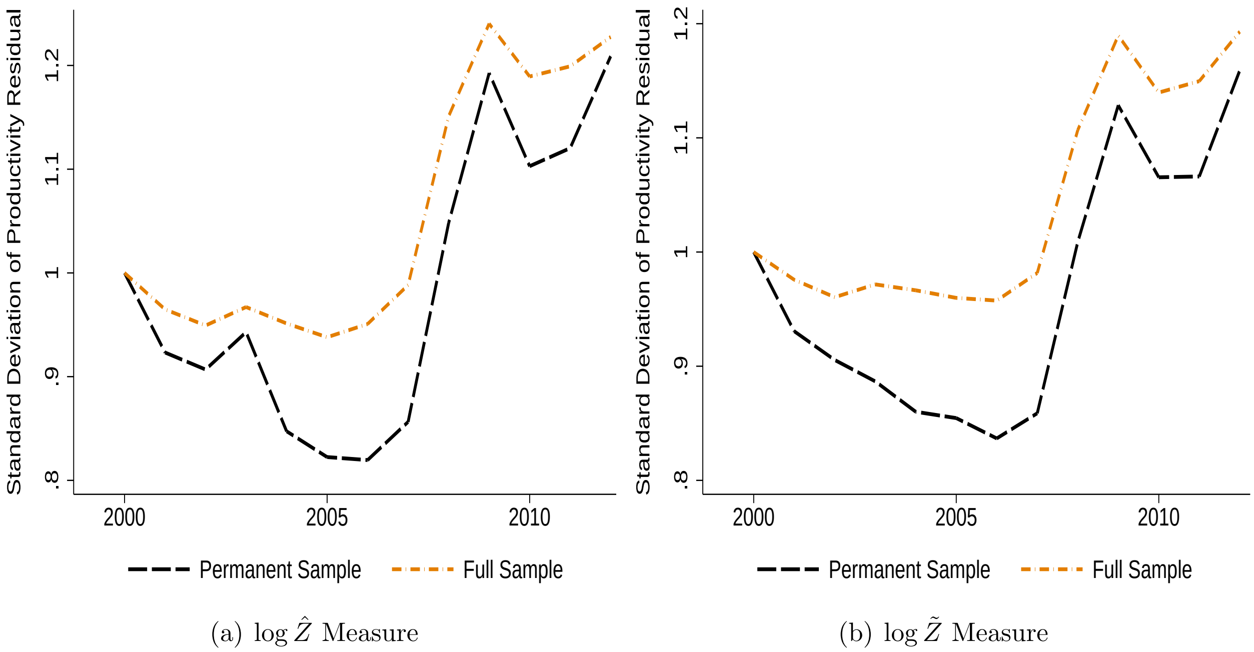
6.5 Changes in the Productivity Process

Can changes in the process governing firm productivity explain Spain's experience? Figure 21 presents the evolution of the standard deviation of productivity shocks across firms σ_t in

[36] For this experiment and the experiment of a decrease in σ_t described in Section 6.5 we do not recalibrate the model. This is because in both cases we simply wish to make the qualitative point that the directional responses of dispersion and misallocation to these shocks in the model differ from the directional responses observed in the aggregate data.

[37] Consistently with the findings of Moll (2014) and Midrigan and Xu (2014), in our model the TFP loss due to financial frictions in the stochastic steady state is small because firm productivity is a persistent process. Our baseline experiment in Section 6.1 is a decline in the real interest rate in a model with financial frictions, rather than a change in financial frictions per se. The decline in the real interest rate generates a loss in TFP along the transitional dynamics that is substantially larger than the steady state TFP loss. Additionally, in Section 6.1.4 we show that there is a substantial impact of financial frictions on the transitional dynamics of aggregate variables in response to a decline in the real interest rate.

[38] We point out that this prediction is tied to the assumption that the relaxation of borrowing constraints is homogeneous across firms. One could generate a decline in $\log(\text{TFP})$ by assuming that the decline in κ_t is larger for low productivity firms.



(a) $\log \hat{Z}$ Measure

(b) $\log \tilde{Z}$ Measure

Figure 21: Evolution of Dispersion of Productivity Residuals

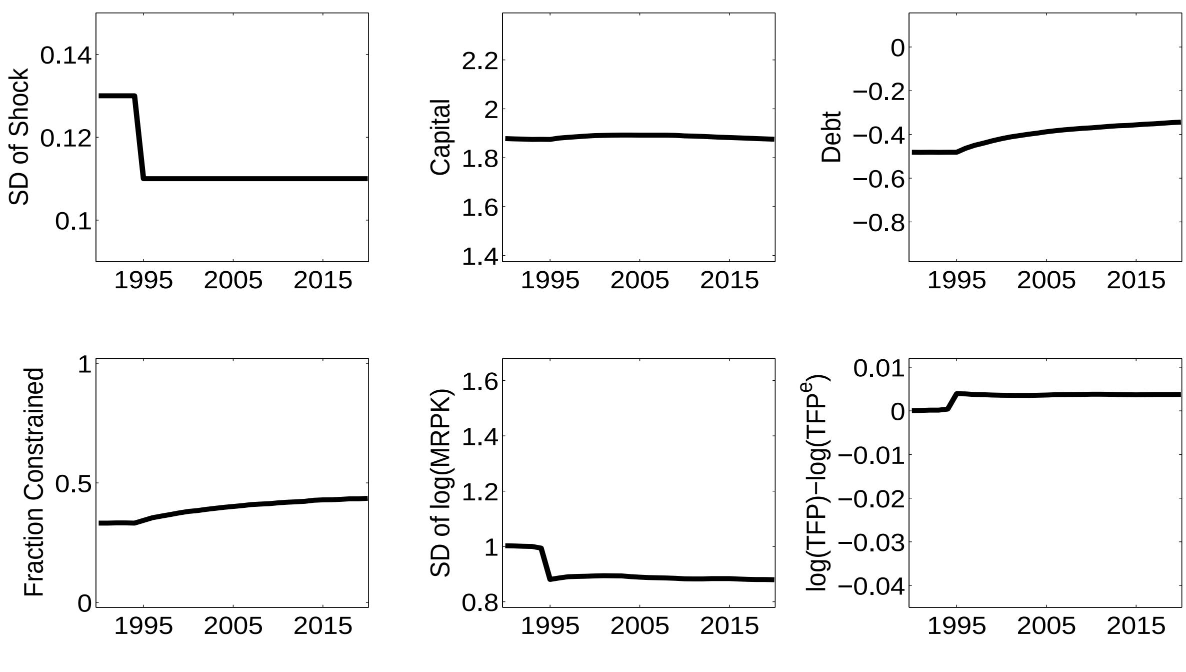


Figure 22: Decline in Dispersion of Productivity Shocks

the data. To obtain idiosyncratic productivity shocks, we follow Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) and estimate the firm-level AR(1) process shown in equation (22). The left panel uses the measure $\log \hat{Z}$ estimated with the Wooldridge (2009) extension of the Levinsohn and Petrin (2003) methodology and right panel uses the measure $\log \tilde{Z}$ defined in equation (11). Before the crisis, we find a decreasing dispersion of productivity shocks in the permanent sample and a relatively stable dispersion in the full sample. After the crisis, we document a sharp increase in the dispersion of productivity shocks.

Figure 22 shows impulses in response to an unexpected and permanent decrease in the standard deviation of productivity shocks σ_t within our baseline model. In line with the analysis of Asker, Collard-Wexler, and De Loecker (2014), a lower dispersion of productivity across firms leads to lower observed MRPK dispersion. Additionally, a decline in σ_t is associated with an increase in $\log(\text{TFP})$ relative to its efficient level. We conclude that changes in σ_t cannot explain the dynamics of dispersion, productivity, and capital flows before the crisis. Below, however, we highlight that the increase in the dispersion of idiosyncratic productivity shocks plays an important role for understanding the post-crisis period.

6.6 Post-Crisis Period: Deleveraging and Uncertainty Shocks

In Section 3 we documented an acceleration of the increase in the dispersion of the $\log(\text{MRPK})$ in the post-crisis period and a continuation of the decline in $\log(\text{TFP})$ relative to its efficient level. Further, Figure 12 has shown a sudden reversal of capital flows during the post-crisis period. We now discuss the role of deleveraging and uncertainty shocks in accounting for these facts. For the experiments in this section, we use our baseline model with the parameters shown in Table 3.

First, consider the role of a deleveraging shock. We modify firms' borrowing constraint to:

$$k' \leq \begin{cases} a'/(1-\tilde{\theta}) & \text{if } k' > \kappa \\ a', & \text{if } k' \leq \kappa \end{cases}, \tag{28}$$

where $\tilde{\theta} \in [0,1]$ corresponds to the fraction of installed capital that can be used as a collateral for borrowing. Our previous borrowing constraint in equation (21) is nested by this specification for the value of $\tilde{\theta} = 1$. We modify the borrowing constraint to be able to capture a deleveraging shock among firms that borrowed heavily in the pre-crisis period. We model the deleveraging shock with an unexpected and permanent decline in $\tilde{\theta}$ in 2008. The decline in $\tilde{\theta}$ implies that large firms that had overcome their borrowing constraints by 2008 now potentially find themselves constrained and are forced to reduce their leverage. We note that firms with installed capital below κ are not affected by this shock as they are not able to borrow to begin with.

In Figure 23 we present aggregate impulses in response to the decline in $\tilde{\theta}$. We introduce the shock in 2008 as the economy is still transitioning in response to the decline in the real interest rate in 1995. The model generates a significant further decline in $\log(\text{TFP})$ relative to its efficient level and a sudden reversal of capital and debt accumulation. However, the shock slows down high growing firms and, as a result, the dispersion of the MRPK declines.[39]

Figure 21 documented an increase in the dispersion of productivity shocks across firms in the post-crisis period. Figure 24 considers the role of this uncertainty shock in the model. The rise in σ_t causes a large increase in the dispersion of the $\log(\text{MRPK})$ and a significant decline in $\log(\text{TFP})$ relative to its efficient level.[40] However, the shock does not result in a significant reversal of capital flows.

To summarize, both a deleveraging shock and an uncertainty shock are consistent with further declines in $\log(\text{TFP})$ relative to its efficient level in the post-crisis period. Additionally, the deleveraging shock generates a capital flows reversal. However, it does not generate a significant increase in MRPK dispersion. By contrast, the uncertainty shock generates a sharp increase in MRPK dispersion but it does not generate capital flows reversals. We conclude that a combination of deleveraging and uncertainty shocks are jointly important in understanding the post-crisis dynamics in Spain characterized by TFP declines, MRPK dispersion increases,

[39] Large firms in the model have accumulated substantial debt by 2008 because the decline in the real interest rate causes a tilt of consumption toward the present. Some of these firms are forced to default and shut down production in response to the decline in $\tilde{\theta}$. We assume that these firms move permanently to an outside sector.

[40] The shock is consistent with the decline in mean log productivity documented in Figure 16 for the post-crisis period. We note that the efficient level of $\log(\text{TFP})$ increases when σ_t increases. Observed $\log(\text{TFP})$ increases slightly upon impact and then declines, explaining the increasing gap between the two productivity measures in the last panel of Figure 24. We have also considered a simultaneous increase in σ_t and decrease in aggregate productivity Z_t^A such that the efficient level of $\log(\text{TFP})$ remains constant. Such a combination of shocks causes a larger decline in observed $\log(\text{TFP})$ and generates very similar transitional dynamics to the dynamics shown in Figure 24.

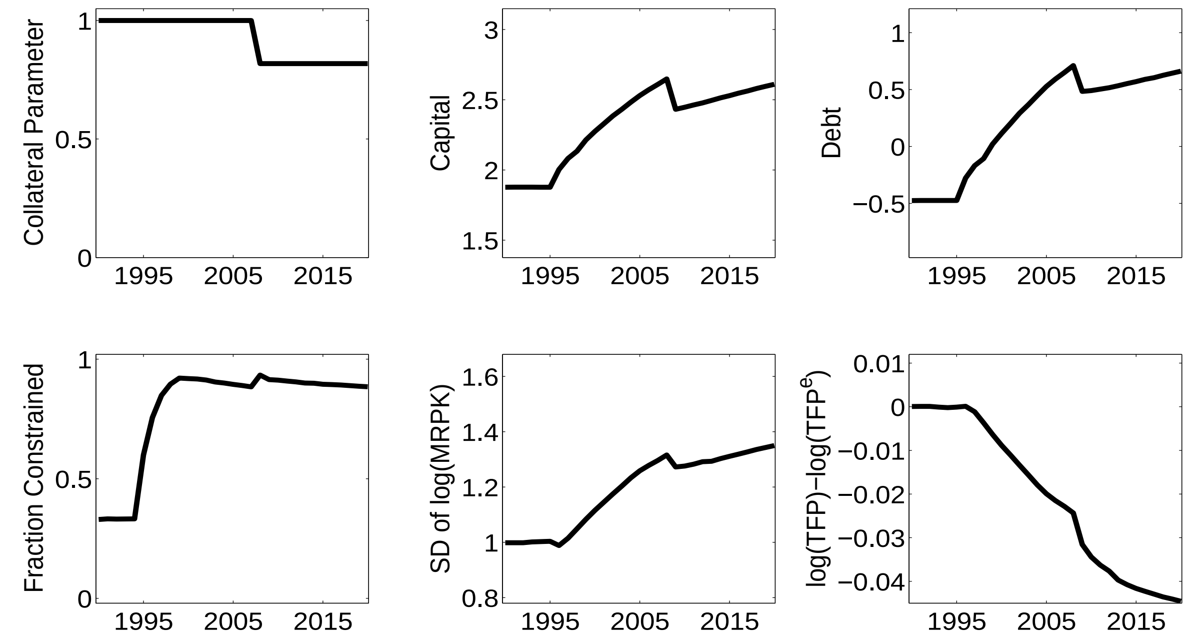


Figure 23: Decline in the Real Interest Rate and Tightening of Borrowing Constraint

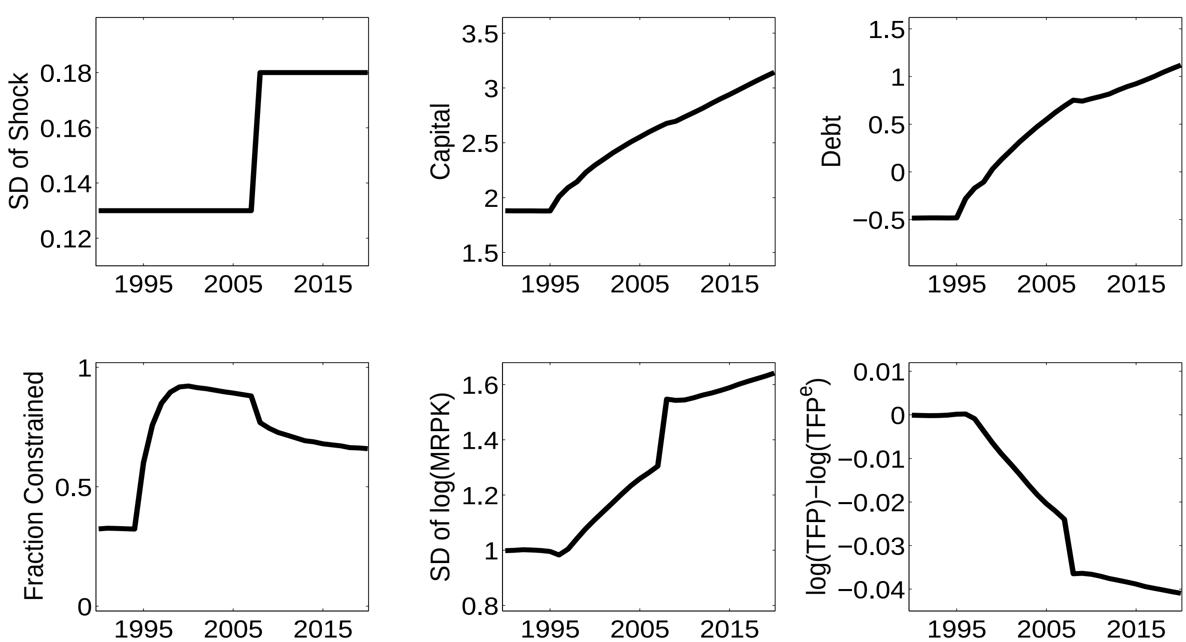


Figure 24: Decline in the Real Interest Rate and Increase in Dispersion of Shocks

Table 7: Coverage in Wage Bill Relative to Eurostat (Manufacturing Industries)

	Spain	Italy	Portugal	Germany	France	Norway
1999	0.69	0.59				
2000	0.71	0.63			0.70	
2001	0.73	0.62			0.72	
2002	0.75	0.69			0.75	
2003	0.74	0.68			0.73	
2004	0.75	0.71			0.71	0.66
2005	0.74	0.72			0.71	0.67
2006	0.74	0.73	0.91	0.34	0.72	0.71
2007	0.74	0.73	0.94	0.34	0.73	0.73
2008	0.72	0.84	0.97	0.28	N/A	0.65
2009	0.72	0.81	0.96	0.28	0.71	0.85
2010	0.73	0.83	0.96	0.30	0.73	0.82
2011	0.74	0.86	0.97	0.28	0.75	0.82
2012	0.71	0.85	0.96	0.25	0.73	0.87

and capital flows reversals.

7 Evidence From Other Euro Countries

In this section we extend parts of our empirical analyses to Italy (1999-2012), Portugal (2006-2012), Germany (2006-2012), France (2000-2012), and Norway (2004-2012). To preview our results, countries in the South share some similar trends in the MRPK dispersion and the TFP loss due to misallocation. By contrast, these trends differ significantly in the North.

Table 7 presents coverage statistics for the wage bill relative to the wage bill reported in Eurostat's SBS for all countries. The coverage is high and averages from roughly 60 to more than 90 percent of the coverage observed in Eurostat. The exception is Germany, for which we have roughly one-third of the wage bill starting in 2006. The entry for France in 2008 is missing because of a missing observation in Eurostat. Generally, we obtain slightly higher coverage when we calculate similar statistics based on gross output and somewhat lower coverage when we calculate similar statistics based on employment.

Table 8: Share of Total Manufacturing Economic Activity By Size Class (2006)

		Spain	Italy	Portugal	Germany	France	Norway
		Employment					
ORBIS-AMADEUS	1-19 employees	0.24	0.13	0.25	0.05	0.10	0.18
	20-249 employees	0.50	0.55	0.53	0.32	0.35	0.47
	250+ employees	0.26	0.32	0.22	0.63	0.56	0.35
Eurostat (SBS)	0-19 employees	0.31	0.40	0.32	0.15	0.19	0.20
	20-249 employees	0.43	0.38	0.49	0.32	0.34	0.42
	250+ employees	0.26	0.22	0.19	0.53	0.47	0.38
		Wage Bill					
ORBIS-AMADEUS	1-19 employees	0.19	0.11	0.18	0.01	0.08	0.14
	20-249 employees	0.47	0.53	0.50	0.33	0.30	0.43
	250+ employees	0.34	0.36	0.32	0.67	0.61	0.43
Eurostat (SBS)	0-19 employees	0.20	0.22	0.21	0.07	0.14	0.15
	20-249 employees	0.43	0.44	0.49	0.26	0.31	0.41
	250+ employees	0.37	0.34	0.30	0.67	0.55	0.44
		Gross Output					
ORBIS-AMADEUS	1-19 employees	0.14	0.12	0.12	0.06	0.05	0.11
	20-249 employees	0.42	0.49	0.43	0.27	0.23	0.40
	250+ employees	0.45	0.40	0.46	0.67	0.72	0.49
Eurostat (SBS)	0-19 employees	0.14	0.20	0.14	0.06	0.09	0.13
	20-249 employees	0.38	0.41	0.42	0.22	0.27	0.36
	250+ employees	0.49	0.39	0.43	0.72	0.64	0.51

Table 8 reports the share of economic activity accounted for by firms belonging in three size categories in 2006 for all countries in our sample. Each panel presents a different measure of economic activity, namely employment, wage bill, and gross output. Within each panel, the first three rows report the measures from ORBIS-AMADEUS and the next three rows report the measures from Eurostat. As with the case of Spain, Table 8 illustrates that our sample for other countries is also broadly representative in terms of contributions of small and medium sized firms to economic activity.

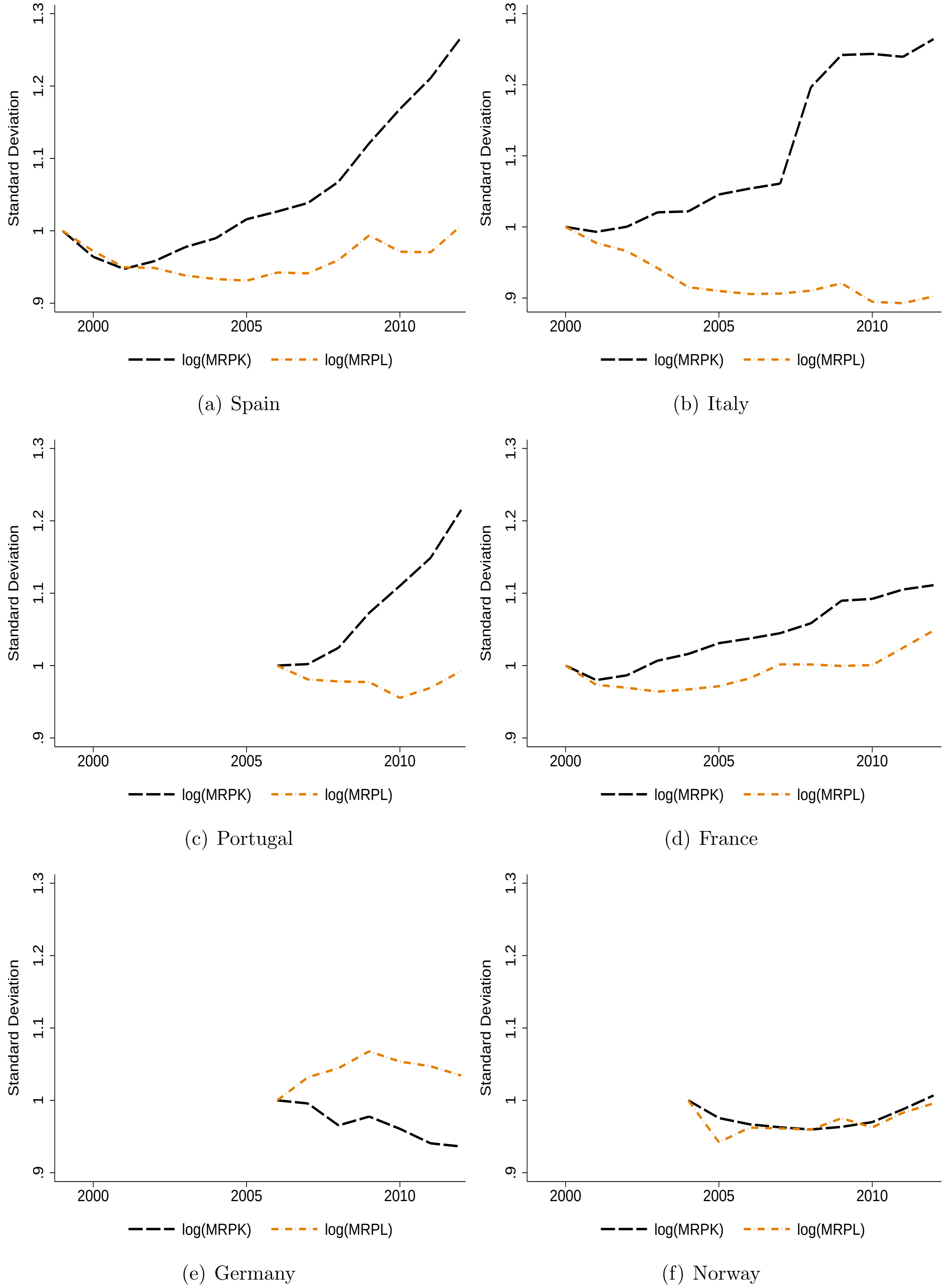
Figures 25 and 26 present the evolution of the standard deviation of log (MRPK) and log (MRPL) for each country in the permanent sample and the full sample respectively. As before, we calculate the standard deviation for the manufacturing sector as the weighted-average of the standard deviations within each four-digit industry. The figures show a significant increase in the standard deviation of log (MRPK) in Spain and Italy before the crisis. During the same period, France experienced a smaller increase. We document significant increases in the dispersion of the log (MRPK) in all countries of the South during and after the crisis. By contrast, we do not observe such trends in the North. Additionally, we do not see significant changes in the dispersion of the log (MRPL) in any country in our sample. This holds both during the pre-crisis period and during the post-crisis period.

Figure 27 plots the evolution of the loss in TFP due to misallocation, previously defined in equation (10). Similarly to Spain, we observe significant declines in log (TFP) in Italy's full sample throughout the period, in Italy's permanent sample during the crisis, and in Portugal's permanent sample that mostly covers the crisis period. We do not observe trend declines in Germany, France, or Norway.

8 Conclusions

The aim of this paper is to explain the joint dynamics of capital flows, dispersion of factor returns, and productivity in South Europe following the adoption of the euro. The first contribution of our work is to bring empirical evidence on the dynamics of misallocation over time. Employing a large and representative sample of Spanish manufacturing firms, we document a significant increase in MRPK dispersion over time and a decline in TFP relative to its efficient level. We also show that capital inflows were increasingly directed to less productive firms. Interestingly, we do not find an important role for a changing dispersion of labor distortions.

Our second contribution is to empirically link patterns of capital misallocation at the micro level to firm-level financial decisions and to the macroeconomic implications of financial fric-



(a) Spain

(b) Italy

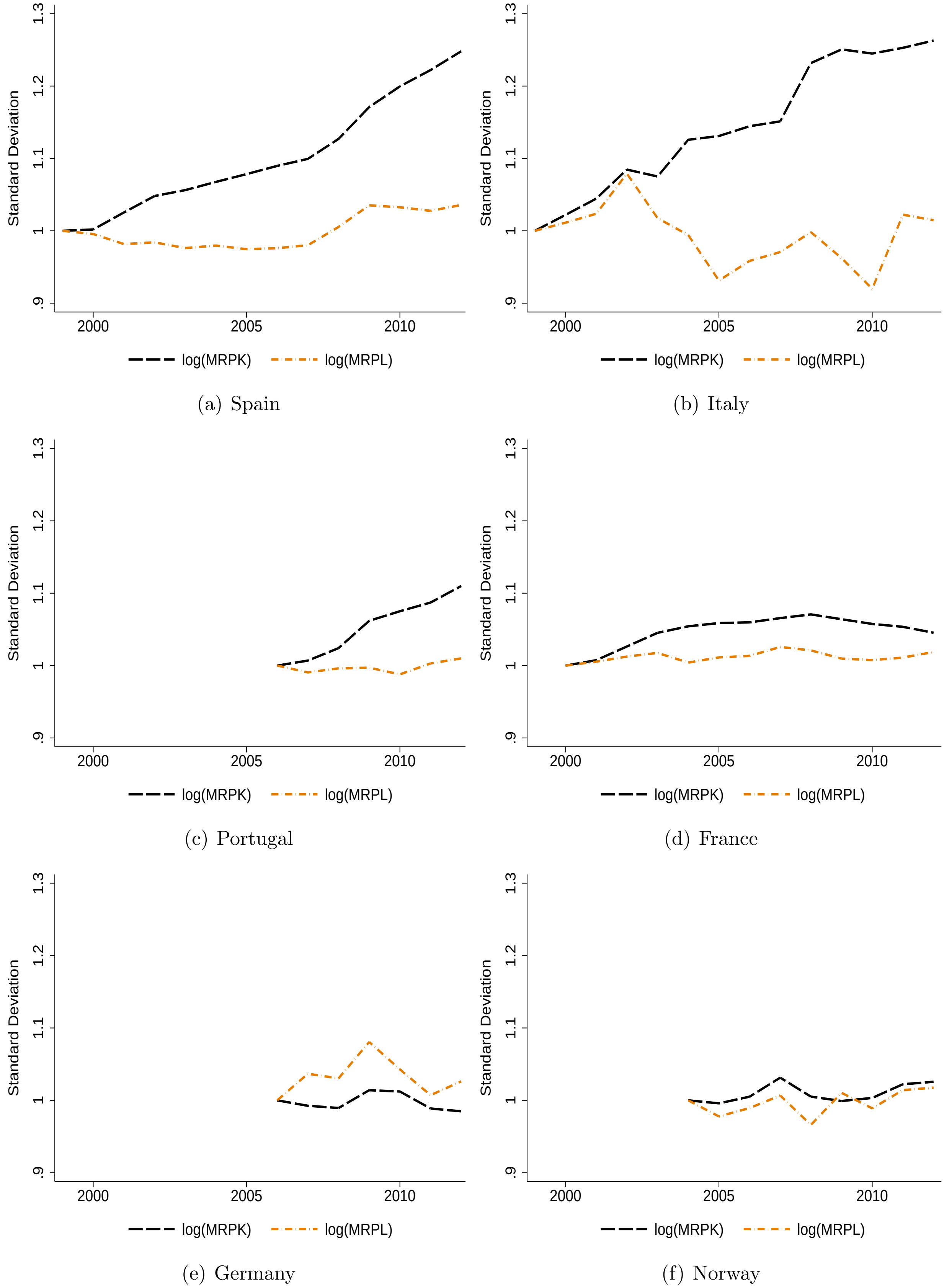
(c) Portugal

(d) France

(e) Germany

(f) Norway

Figure 25: Evolution of MRPK and MRPL Dispersion in Permanent Sample



(a) Spain

(b) Italy

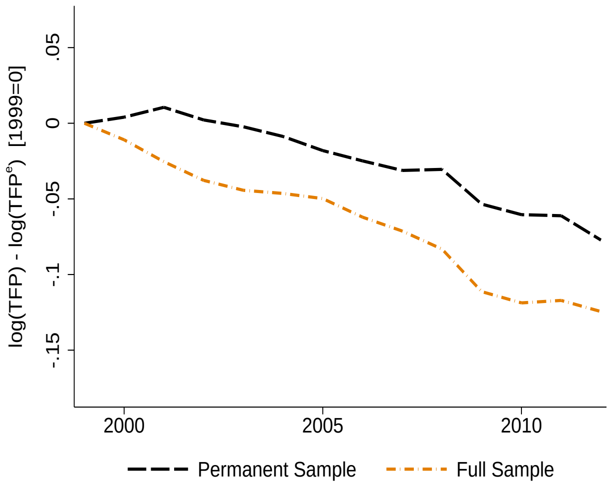
(c) Portugal

(d) France

(e) Germany

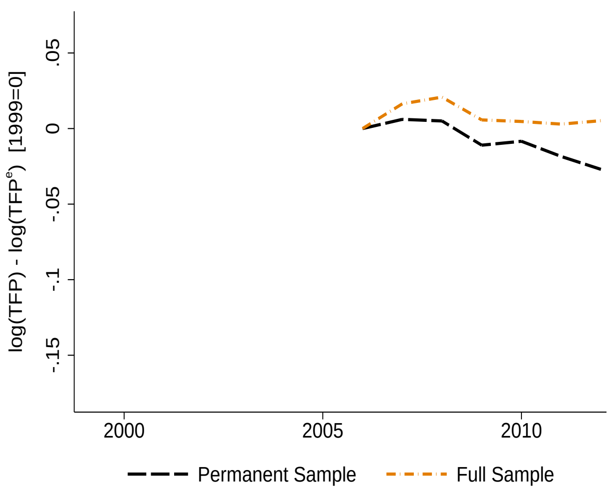
(f) Norway

Figure 26: Evolution of MRPK and MRPL Dispersion in Full Sample

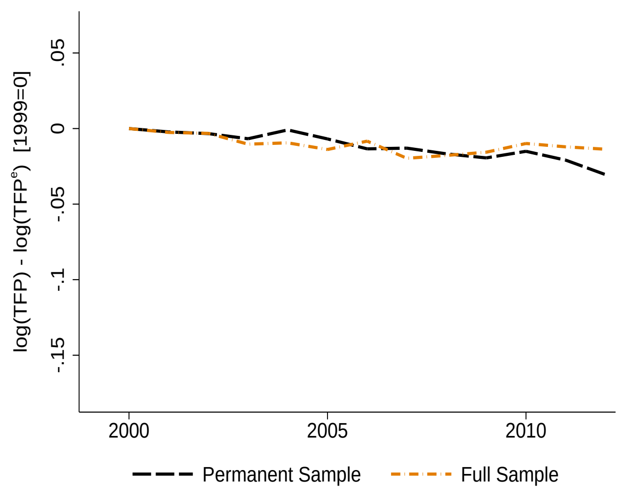


(a) Spain

(b) Italy

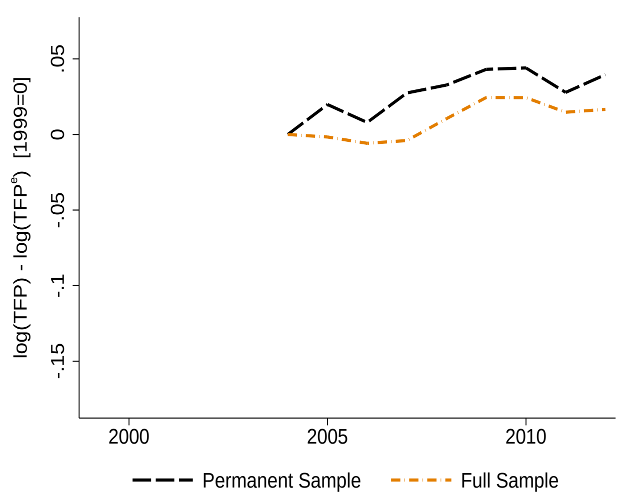


(c) Portugal



(d) France

(e) Germany



(f) Norway

Figure 27: Evolution of log (TFP) Relative to Efficient Level

tions. We have developed a model with heterogeneous firms, financial frictions, and investment adjustment costs that matches closely various moments estimated from production and balance sheet data. Using this calibrated model, we illustrate how the decline in the real interest rate generates transitional dynamics that are similar to the dynamics of dispersion, productivity, and capital flows observed in the data during the pre-crisis period. We also discuss the role of deleveraging and micro-level uncertainty shocks during the post-crisis period.

Finally, we have documented that trends in the dispersion of the return to capital and in productivity losses from misallocation differ significantly between countries in the South and countries in the North part of the euro area. We find these differences suggestive, given that firms in the South are likely to operate in less developed financial markets. However, a more complete analysis of the differences and sources of the discrepancies between the South and the North remains a promising avenue for future research.

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Capital Allocation and Productivity in South Europe

Online Appendix

Gita Gopinath Sebnem Kalemli-Ozcan Loukas Karabarbounis Carolina Villegas-Sanchez

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A Data Cleaning and Summary Statistics

Our dataset combines firm-level information across different BvD products (ORBIS disk 2005, ORBIS disk 2009, ORBIS disk 2013, AMADEUS online 2010 from WRDS, and AMADEUS disk 2014). We work only with unconsolidated accounts. We clean the data in four steps. First, we clean the data of basic reporting mistakes. Second, we verify the internal consistency of balance sheet information. The first two steps are implemented at the level of the total economy. Third, we do a more specific quality control on variables of interest for firms in the manufacturing sector. Finally, we winsorize variables.

A.1 Cleaning of Basic Reporting Mistakes

We implement the following steps to correct for basic reporting mistakes:

1. We drop firm-year observations that have missing information on total assets and operating revenues and sales and employment.

2. We drop firms if total assets are negative in any year, or if employment is negative or greater than 2 millions in any year, or if sales are negative in any year, or if tangible fixed assets are negative in any year.

3. We drop firm-year observations with missing, zero, or negative values for materials, operating revenue, and total assets.

4. We drop firm-year observations with missing information regarding their industry of activity.

A.2 Internal Consistency of Balance Sheet Information

We check the internal consistency of the balance sheet data by comparing the sum of variables belonging to some aggregate to their respective aggregate. We construct the following ratios:

1. The sum of tangible fixed assets, intangible fixed assets, and other fixed assets as a ratio of total fixed assets.

2. The sum of stocks, debtors, and other current assets as a ratio of total current assets.

3. The sum of fixed assets and current assets as a ratio of total assets.

4. The sum of capital and other shareholder funds as a ratio of total shareholder funds.

5. The sum of long term debt and other non-current liabilities as a ratio of total non-current liabilities.

6. The sum of loans, creditors, and other current liabilities as a ratio of total current liabilities.

7. The sum of non-current liabilities, current liabilities, and shareholder funds as a ratio of the variable that reports the sum of shareholder funds and total liabilities.

After we construct these ratios, we estimate their distribution for each country separately. We then exclude from the analysis extreme values by dropping observations that are below the 0.1 percentile or above the 99.9 percentile of the distribution of ratios.

A.3 Further Quality Checks for Manufacturing Firms

After the implementation of the basic cleaning steps in the total economy sample we turn to examine the quality of the variables for firms in the manufacturing sector used in our analysis. At each stage, we provide the number of dropped observations for the Spanish sample. We start with 1,127,566 observations that correspond to 149,779 firms in the Spanish manufacturing sector.

1. **Age.** We construct the variable "age" of the firm as the difference between the year of the balance sheet information and the year of incorporation of the firm plus one. We drop

firms that report dates of incorporation that imply non-positive age values. This step reduces the observations in our sample by 35.

2. **Liabilities.** As opposed to listed firms, non-listed firms do not report a separate variable "Liabilities." For these firms we construct liabilities as the difference between the sum of shareholder funds and liabilities ("SHFUNDLIAB") and shareholder funds or equity ("SHFUNDS"). We drop observations with negative or zero values. This step reduces the observations in our sample by 1,374.

   We could also have computed liabilities as the sum of current liabilities and non-current liabilities. However, we find that there are more missing observations if we follow this approach. Nevertheless, for those observations with non-missing information we compare the value of liabilities constructed as the difference between SHFUNDLIAB and SHFUNDS and the value of liabilities constructed as the sum of current and non-current liabilities. We look at the ratio of the first measure relative to the second measure. Due to rounding differences the ratio is not always exactly equal to one and so we remove only firm-year observations for which this ratio is greater than 1.1 or lower than 0.9. This step reduces the observations in our sample by 1,349.

   We drop firm-year observations with negative values for current liabilities, non-current liabilities, current assets, loans, creditors, other current liabilities, and long term debt. This step reduces the observations in our sample by 40. Finally, we drop observations for which long term debt exceeds total liabilities. This step reduces the observations in our sample by 44.

3. **Net Worth.** We construct net worth as the difference between total assets ("TOTASSTS") and total liabilities. This variable should be equal to the variable SHFUNDS provided by the BvD. We drop observations that violate this identity. This step reduces the observations in our sample by 32.

4. **Wage Bill.** We drop firm-year observations with missing, zero, or negative values for the wage bill. This step reduces the observations in our sample by 20,571.

5. **Capital Stock.** We construct our measure of the capital stock as the sum of tangible fixed assets and intangible fixed assets and, therefore, we drop observations with negative values for intangible fixed assets. This step reduces the observations in our sample by 2,176. We drop observations with missing or zero values for tangible fixed assets. This step reduces the observations in our sample by 42,744. We drop firm-year observations when the ratio of tangible fixed assets to total assets is greater than one. This step reduces the observations in our sample by 4,921. We drop firm-year observations with negative depreciation values. This step reduces the observations in our sample by 1.

6. **Capital-Labor Ratio.** Next, we examine the quality of the capital to the wage bill variable. We first drop firms if in any year they have a capital to wage bill ratio in the bottom 0.1 percent of the distribution. This step reduces the observations in our sample by 5,801. After we remove the very high extreme values of this ratio there is a very positively skewed distribution of the ratio and, therefore, we drop observations with ratios higher than the 99.9 or lower than the 0.1 percentile. This step reduces the observations in our sample by 1,836.

7. **Equity.** We drop observations with negative SHFUNDS (equity or shareholders funds). This step reduces the observations in our sample by 123,208. We drop observations in the bottom 0.1 percentile in the ratio of other shareholders funds (that includes items such as reserve capital and minority interests) to TOTASSTS. This step reduces the observations in our sample by 925.

8. **Leverage Ratios.** We calculate the ratios of tangible fixed assets to shareholder funds and the ratio of total assets to shareholder funds and drop extreme values in the bottom 0.1 or top 99.9 percentile of the distribution of ratios. This step reduces the observations in our sample by 3,555.

9. **Value Added.** We construct value added as the difference between operating revenue and materials and drop negative values. This step reduces the observations in our sample by 3,966. We construct the ratio of wage bill to value added and drop extreme values in

the bottom 1 percentile or the top 99 percentile. This step reduces the observations in our sample by 18,362. In this case we choose the 1 and 99 percentiles as thresholds to drop variables because the value of the ratio at the 99 percentile exceeds 1. In addition, we drop firm-year observations if the ratio is greater than 1.1. This step reduces the observations in our sample by 11,629.

The final sample for Spain has 884,997 firm-year observations, corresponding to 124,993 firms in the manufacturing sector. This is what we call the "full sample" in our analysis. The "permanent sample" is a subset of the full sample, consisting of firms with identifiers that are observed continuously for all years between 1999 and 2012. The permanent sample has 193,452 observations, corresponding to 13,818 firms.

A.4 Winsorization

We winsorize at the 1 and the 99 percentile variables such as value added, tangible fixed assets, wage bill, operating revenue, materials, total assets, shareholder funds, fixed assets, the sum of tangible and intangible fixed assets (capital), other fixed assets, and total liabilities. We winsorize at the 1 and the 99 percentile all of our estimated firm productivity variables and the productivity residuals from an AR(1) process used to construct our uncertainty measures. Similarly, we winsorize at the 1 and the 99 percentile net worth, cash flow to total assets, and sales to total assets. In addition, we winsorize at the 0.1 and 99.9 percentile the MRPK and the MRPL before calculating our dispersion measures to make our dispersion measures less sensitive to outliers. Finally, we winsorize at the 2 and the 98 percentile the net investment to lagged capital ratio used in our regressions because this ratio has a very long right tail.

A.5 Summary Statistics

Table A.1 presents summary statistics for all countries in our dataset. Except for employment, all entries in the table are in millions of euros. Value added, wage bill, total assets, and total liabilities are deflated with gross output price indices at the two digits industry level with a base year of 2005. For France and Norway we do not have these price deflators at the two

Table A.1: Summary Statistics of Selected Variables

		Permanent Sample		Full Sample	
Country	Statistic	Mean	Standard Deviation	Mean	Standard Deviation
Spain	Value Added	1.23	3.00	2.16	4.08
	Employment	24.87	138.73	42.07	234.69
	Wage Bill	0.54	1.19	0.91	1.58
	Capital Stock	0.89	2.47	1.47	3.20
	Total Assets	2.66	7.10	4.52	9.32
	Total Liabilities	1.54	4.12	2.40	5.22
Italy	Value Added	2.73	5.31	4.76	6.81
	Employment	36.12	171.25	55.75	156.40
	Wage Bill	0.86	1.62	1.49	2.07
	Capital Stock	1.36	3.17	2.35	4.04
	Total Assets	5.31	11.4	9.10	14.30
	Total Liabilities	3.73	7.75	5.95	9.51
Portugal	Value Added	0.75	1.91	3.54	8.50
	Employment	22.83	71.51	39.19	152.09
	Wage Bill	0.30	0.65	1.60	3.51
	Capital Stock	0.51	1.49	1.39	4.37
	Total Assets	1.65	4.43	5.03	14.70
	Total Liabilities	1.00	2.65	3.18	9.27
Germany	Value Added	18.90	38.80	37.60	52.20
	Employment	183.47	554.55	320.62	620.60
	Wage Bill	7.39	14.40	14.40	19.20
	Capital Stock	6.41	15.80	12.80	21.60
	Total Assets	26.40	64.30	53.50	87.80
	Total Liabilities	16.40	40.80	32.40	55.80
France	Value Added	2.53	7.46	3.33	8.47
	Employment	39.51	399.07	48.43	280.92
	Wage Bill	0.95	2.52	1.26	2.85
	Capital Stock	0.66	2.40	0.85	2.73
	Total Assets	3.07	10.20	4.04	11.60
	Total Liabilities	1.80	6.04	2.24	6.71
Norway	Value Added	2.71	7.38	0.98	2.17
	Employment	29.76	122.32	28.34	76.94
	Wage Bill	1.26	3.11	0.38	0.74
	Capital Stock	1.08	3.80	0.64	1.63
	Total Assets	3.95	12.90	2.09	4.93
	Total Liabilities	2.56	8.27	1.21	2.88

digits and, therefore, we deflate with the price index for total manufacturing. The capital stock is the sum of tangible and intangible fixed assets and is deflated with the economy-wide price of investment goods. For each year, we first calculate means and standard deviations without weighting across all firms and industries. Entries in the table denote the means and standard deviations averaged across all years in each country.

B Production Function Estimates

In this appendix we discuss estimates of the production function. We estimate the production function separately for each two-digit industry s:

$$\log y_{it} = d_t(s) + \beta^{\ell}(s) \log \ell_{it} + \beta^k(s) \log k_{it} + \log Z_{it} + \epsilon_{it}, \tag{A.1}$$

where $d_t(s)$ is a time fixed effect, y_{it} denotes nominal value added divided by the two-digit output price deflator, ℓ_{it} denotes the wage bill divided by the same output price deflator, and k_{it} denotes the (book) value of fixed assets divided by the aggregate price of investment goods. In equation (A.1), $\beta^{\ell}(s)$ denotes the elasticity of value added with respect to labor and $\beta^k(s)$ denotes the elasticity of value added with respect to capital. These elasticities vary at 24 industries defined by their two-digit industry classification. Our estimation uses the methodology developed in Wooldridge (2009) and we refer the reader to his paper for details of the estimation process. Given our estimated elasticities $\hat{\beta}^{\ell}(s)$ and $\hat{\beta}^k(s)$, we then calculate firm (log) productivity as $\log Z_{it} = \log y_{it} - \hat{\beta}^{\ell}(s) \log \ell_{it} - \hat{\beta}^k(s) \log k_{it}$.

In Table A.2 we present summary statistics for the sum of the elasticities $\hat{\beta}^l(s) + \hat{\beta}^k(s)$ estimated from regression (A.1) separately in each country. Our estimates look reasonable as the sum of elasticities is close to 0.80. Because we do not observe prices at the firm level, these elasticities are more appropriately defined as revenue elasticities. In the presence of markups, these estimates are lower bounds for the true elasticities in the production function. With a constant returns to scale production function, we would estimate a sum of elasticities equal to 0.80 when the markup equals 20 percent.

The summary statistics in Table A.2 exclude industries for which at least one of the coefficients estimated with the Wooldridge (2009) extension of the Levinsohn and Petrin (2003)

Table A.2: Summary Statistics of Production Function Estimation

Sum of Elasticities	Spain	Italy	Portugal	Germany	France	Norway
Mean	0.80	0.71	0.79	0.85	0.82	0.76
Median	0.80	0.70	0.77	0.84	0.83	0.76
Max	0.91	0.81	0.87	1.03	0.90	0.90
Min	0.75	0.59	0.72	0.74	0.62	0.58
Standard Deviation	0.04	0.04	0.05	0.07	0.06	0.07

procedure results in a zero, negative, or missing value. Across 6 countries for which we separately estimate elasticities at the two-digit industry level we have few such industries (2 in Spain, 4 in Italy, 6 in Portugal, 3 in Germany, 2 in France, and 5 in Norway). Typically, these industries have a very small number of firms and account for a negligible fraction of total manufacturing activity. Therefore, we do not drop them from our analysis.

C Comparison of Regressions With Finance Literature

Table A.3 compares the investment and debt regressions using our regressors to similar regressions but with regressors more commonly used by the finance literature. All regressions include firm fixed effects and industry-year fixed effects. The regressors that we used in the main text are motivated by our theory in which productivity, net worth, and capital are the state variables that summarize firm capital and debt decisions.

As the table shows, using the sales to capital ratio instead of productivity and the cash flow to capital ratio instead of log net worth leads to highly similar results. With one exception, all coefficient signs are the same across the two types of regressions. All coefficients except for the coefficient on the cash flow to capital ratio in the debt regression in the full sample are statistically significant at the 1 percent level.

Table A.3: Firm-Level Investment and Debt Decisions in the Data (Spain, 1999-2007)

Dependent Variable	Regressors	Permanent Sample	Full Sample
$(k' - k)/k$	$\log Z$	0.10***	0.11***
		(0.011)	(0.007)
	$\log a$	0.09***	0.09***
		(0.006)	(0.003)
	$\log k$	-0.46***	-0.63***
		(0.005)	(0.003)
$(k' - k)/k$	$\log(\text{Sales}/k)$	0.13***	0.14***
		(0.008)	(0.005)
	Cash Flow$/k$	0.04***	0.05***
		(0.007)	(0.003)
	$\log k$	-0.31***	-0.47***
		(0.008)	(0.005)
$(b' - b)/k$	$\log Z$	-0.38***	-0.48***
		(0.027)	(0.022)
	$\log a$	0.15***	0.14***
		(0.013)	(0.009)
	$\log k$	-0.34***	-0.54***
		(0.011)	(0.010)
$(b' - b)/k$	$\log(\text{Sales}/k)$	-0.45***	-0.49***
		(0.020)	(0.014)
	Cash Flow$/k$	0.07***	0.00
		(0.020)	(0.013)
	$\log k$	-0.66***	-0.91***
		(0.019)	(0.015)

D Further Results in the Baseline Model

In Table A.4 we present the standard errors in the regressions described in Section 6.1.2. All coefficient estimates are statistically significant at the 1 percent level.

Next, we elaborate on our baseline results in Section 6.1.3 and present aggregate impulses in response to the decline in r_t as a function of the adjustment cost parameter ψ and the borrowing threshold κ. For convenience, we repeat the baseline case with $\psi = 3.1$ and $\kappa = 4.1$ in Figure A.1.

Table A.4: Capital Growth and Initial Net Worth

Dependent Variable	Regressors	Permanent Sample	Full Sample
$(k_{07} - k_{99})/k_{99}$	$\log Z_{99}$	1.14***	1.49***
		(0.08)	(0.06)
	$\log a_{99}$	0.17***	0.15***
		(0.03)	(0.02)
	$\log k_{99}$	-0.96***	-1.11***
		(0.03)	(0.03)
$(b_{07} - b_{99})/k_{99}$	$\log Z_{99}$	1.12***	1.47***
		(0.11)	(0.10)
	$\log a_{99}$	0.20***	0.11***
		(0.04)	(0.03)
	$\log k_{99}$	-0.86***	-0.98***
		(0.05)	(0.04)

Figure A.2 shows impulses in response to the decline in r_t in a model without adjustment costs and financial frictions (corresponding to $\psi = 0.0$ and $\kappa = 0.0$). In this model, capital and debt are growing but there is little change in the dispersion of the $\log(\text{MRPK})$ or in $\log(\text{TFP})$. Figure A.3 shows that the model with only adjustment costs and no financial frictions ($\psi = 3.1$ and $\kappa = 0.0$) also does not yield significant changes in the dispersion of the $\log(\text{MRPK})$ or in $\log(\text{TFP})$. Finally, in Figure A.4 we show the model with financial frictions and no adjustment costs ($\psi = 0.0$ and $\kappa = 4.2$). In this model we obtain a significant increase in the dispersion of the MRPK but a very small decline in $\log(\text{TFP})$. In the absence of adjustment costs, firms with a high permanent productivity component z^P increase significantly their capital stock and overcome instantaneously their borrowing constraints.

E Dispersion of the MRPK in a Simpler Model

In this appendix we use a simpler model to derive in closed-form the response of the dispersion of the $\log(\text{MRPK})$ to various shocks. In this simpler model we show that the dispersion of the MRPK increases when: (i) the cost of capital decreases; (ii) financial frictions increase; (iii) exogenous aggregate productivity or demand increase. All responses have the same sign as the

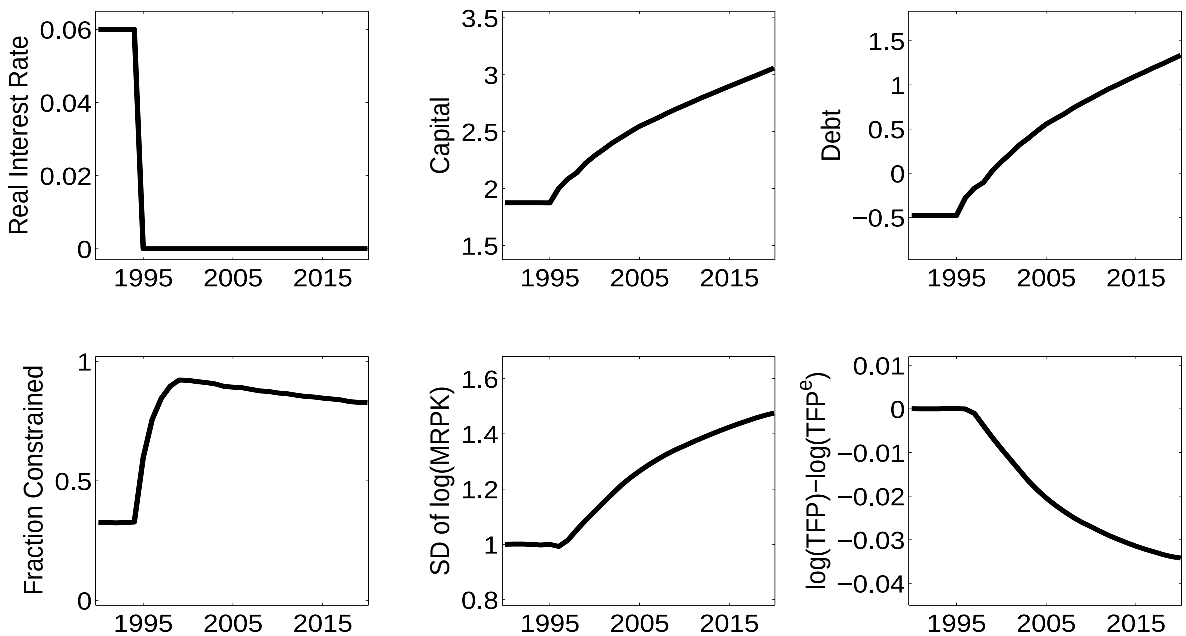


Figure A.1: Decline in the Real Interest Rate ($\psi = 3.1$ and $\kappa = 4.2$)

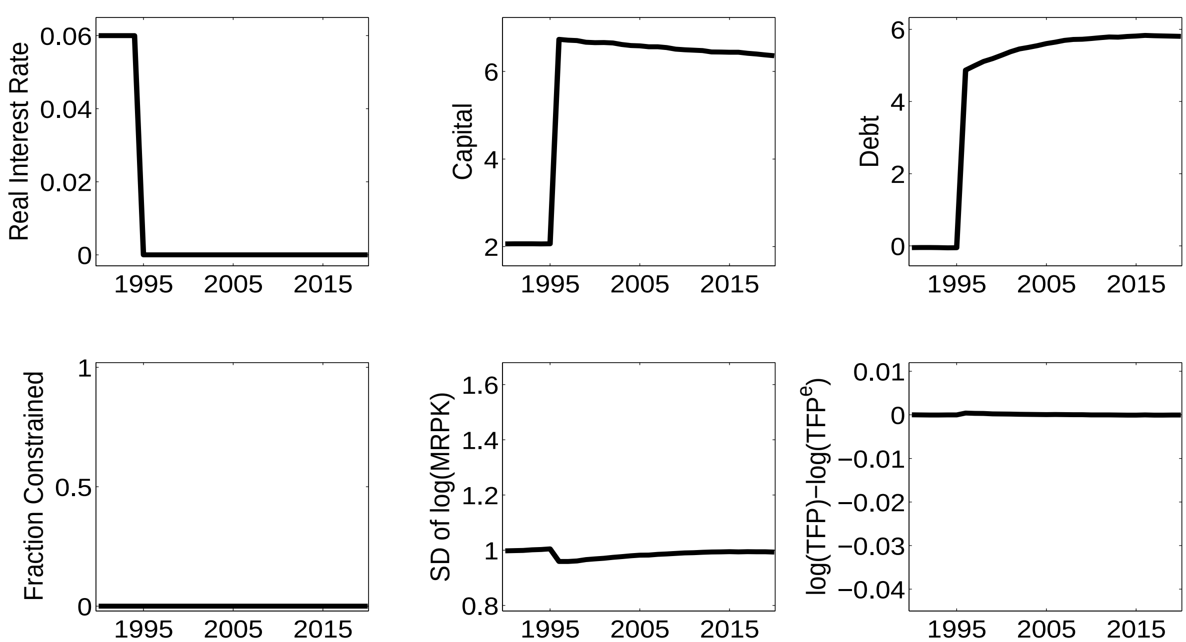


Figure A.2: Decline in the Real Interest Rate ($\psi = 0.0$ and $\kappa = 0.0$)

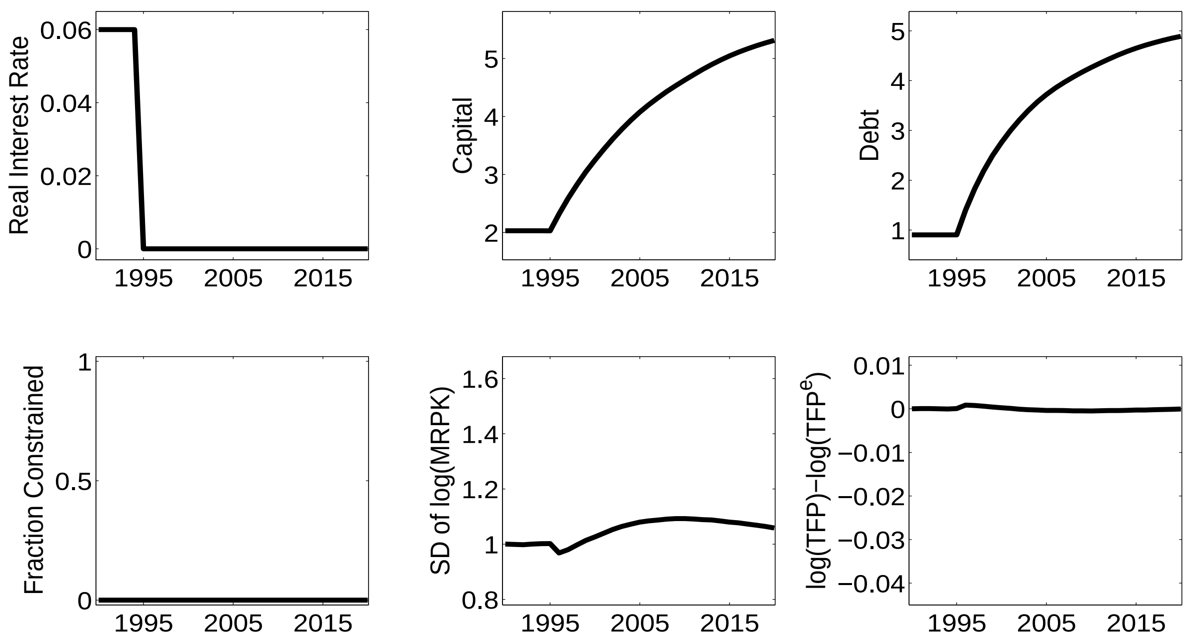


Figure A.3: Decline in the Real Interest Rate ($\psi = 3.1$ and $\kappa = 0.0$)

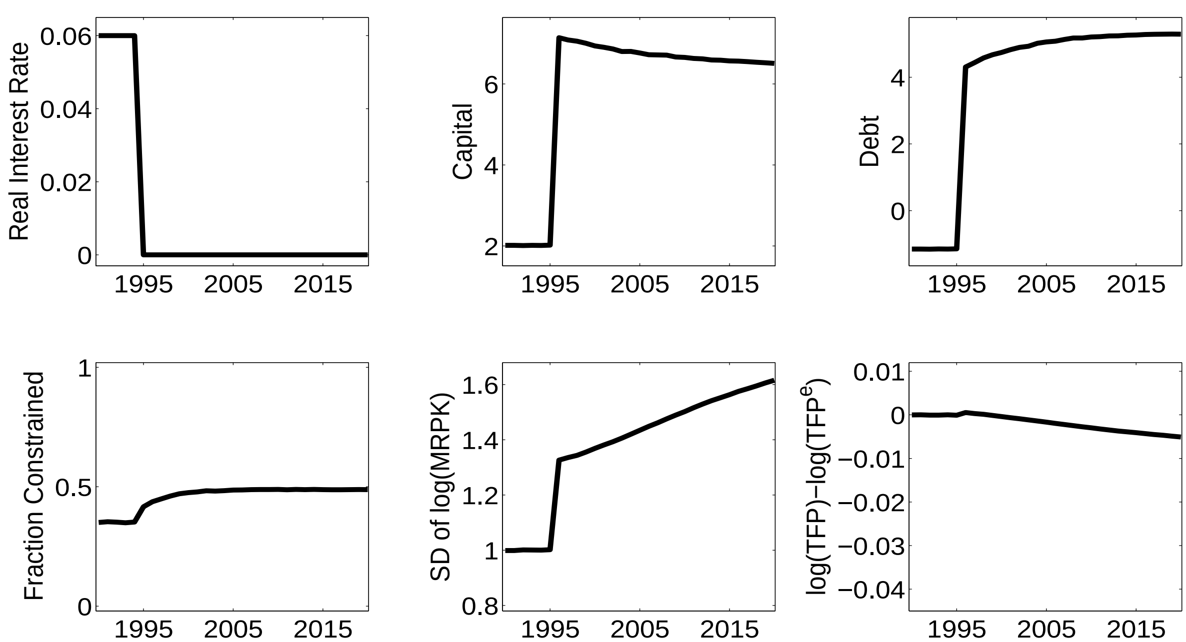


Figure A.4: Decline in the Real Interest Rate ($\psi = 0.0$ and $\kappa = 4.2$)

responses generated by our richer model.

The environment is close to one considered by Midrigan and Xu (2014), Moll (2014), and Buera and Moll (2015). Similar to these papers, we assume that firms maximize the discounted present value of utility flows under perfect foresight about next-period's productivity. This assumption implies that debt and capital are perfect substitutes and effectively renders the choice of capital a static decision. A firm's budget constraint is:

$$c + a' = \pi(Z^A z, k) + (1 + r)a - Rk. \tag{A.2}$$

where c is consumption, a is assets, π is profits, r is the interest rate, k is capital, and $R = r + \delta$ denotes the cost of capital.

The reduced-form profit function is given by:

$$\pi(Z^A z, k) = \frac{Z^A z}{\eta} k^\eta, \tag{A.3}$$

where Z^A is the aggregate component of productivity and z denotes the idiosyncratic component of productivity (which lumps together both the transitory and the permanent component of idiosyncratic productivity). While for simplicity we call it productivity, the product $Z^A z$ represents a reduced-form for productivity, demand, and wages. The concavity of the profit function, $\eta < 1$, can reflect a combination of decreasing returns to scale and a downward sloping demand for a firm's product.[1] The marginal revenue product of capital is:

$$\text{MRPK} = Z^A z k^{\eta-1}. \tag{A.4}$$

Following Midrigan and Xu (2014), Moll (2014), and Buera and Moll (2015), we specify the borrowing constraint as:

$$k \leq \theta a, \tag{A.5}$$

where the parameter $\theta \geq 1$ captures the degree of financial frictions. A lower θ denotes more severe financial frictions. When $\theta = 1$, firms cannot borrow and have to self-finance capital accumulation. When $\theta \to \infty$, there are no financial frictions in capital accumulation.

[1]The assumption that $\eta < 1$ is an important difference between our model in this section and some of the previous literature. If the profit function was linear, then firm size would be pinned down by the financial constraint. Below we show that when all firms are constrained in the initial equilibrium, small changes in R, θ, or Z^A do not affect MRPK dispersion.

Unconstrained firms equalize the MRPK to the cost of capital $R = r + \delta$. The unconstrained level of capital is:

$$k^* = \left(\frac{Z^A z}{R}\right)^{\frac{1}{1-\eta}}, \tag{A.6}$$

and so capital is given by $k = \min\{k^*, \theta a\}$. Firms with productivity z above some threshold Z^* are constrained and can finance capital only equal to θa. The cutoff productivity level is given by:

$$Z^* = (\theta a)^{1-\eta} \frac{R}{Z^A}. \tag{A.7}$$

We denote the joint distribution of productivity and net worth at any particular point in time by $G(a, z)$. We denote the probability density function of productivity z conditional on assets a by $f(z|a)$, the cumulative density function of z conditional on a by $F(z|a)$, and the marginal probability density function of a by $g(a)$. We denote by z_L and z_H the lowest and highest levels of productivity.

The goal is to solve for changes in the variance of the log (MRPK) in response to changes the cost of capital R, financial frictions θ, and aggregate productivity or demand Z^A. Our solutions should be understood as the first period of an impulse response. We note that assets a are predetermined at the period of the shock, which allows us to treat their distribution as given.

As a preliminary step for our comparative statics we calculate the following quantities:

$$\log(\text{MRPK}) = \log(Z^A z) - (1-\eta)\log(k) = \begin{cases} \log(R), & \text{if } z \le Z^* \\ \log\left(Z^A z \theta^{\eta-1} a^{\eta-1}\right), & \text{if } z > Z^* \end{cases}, \tag{A.8}$$

$$\mathbb{E}\log(\text{MRPK}) = \int_a \left[\int_{z_L}^{Z^*} \log(R) f(z|a)dz + \int_{Z^*}^{z_H} \log\left(Z^A z \theta^{\eta-1} a^{\eta-1}\right) f(z|a)dz\right] g(a)da, \tag{A.9}$$

$$(\log(\text{MRPK}))^2 = \left(\log(Z^A z) - (1-\eta)\log(k)\right)^2 = \begin{cases} (\log(R))^2, & \text{if } z \le Z^* \\ \left(\log\left(Z^A z \theta^{\eta-1} a^{\eta-1}\right)\right)^2, & \text{if } z > Z^* \end{cases}, \tag{A.10}$$

$$\mathbb{E}\left[(\log(\text{MRPK}))^2\right] = \int_a \left[\int_{z_L}^{Z^*} (\log(R))^2 f(z|a)dz + \int_{Z^*}^{z_H} \left(\log\left(Z^A z\theta^{\eta-1}a^{\eta-1}\right)\right)^2 f(z|a)dz\right] g(a)da. \tag{A.11}$$

We use these expectations to calculate the response of the variance $\text{Var}(\log(\text{MRPK}))$ to any shock X:

$$\frac{\partial \text{Var}(\log(\text{MRPK}))}{\partial X} = \frac{\partial \mathbb{E}\left[(\log(\text{MRPK}))^2\right]}{\partial X} - 2\left[\mathbb{E}\log(\text{MRPK})\right]\frac{\partial\left[\mathbb{E}\log(\text{MRPK})\right]}{\partial X}. \tag{A.12}$$

E.1 Changes in the Cost of Capital

We consider how small changes in R impact the dispersion of the MRPK. Using Leibniz's rule we obtain:

$$\frac{\partial\left[\mathbb{E}\log(\text{MRPK})\right]}{\partial R} = \int_a \left[\frac{F(Z^*|a)}{R} + \log(R)f(Z^*|a)\frac{\partial Z^*}{\partial R} - \log\left(Z^A Z^*\theta^{\eta-1}a^{\eta-1}\right) f(Z^*|a)\frac{\partial Z^*}{\partial R}\right] g(a)da.$$

Note that the two last terms in the integral cancel out because at the cutoff Z^* we have $R = Z^A Z^* \theta^{\eta-1} a^{\eta-1}$. Therefore:

$$\frac{\partial\left[\mathbb{E}\log(\text{MRPK})\right]}{\partial R} = \frac{1}{R}\int_a F(Z^*|a)\, g(a)da. \tag{A.13}$$

Similarly:

$$\frac{\partial \mathbb{E}\left[(\log(\text{MRPK}))^2\right]}{\partial R} = \frac{2\log(R)}{R}\int_a F(Z^*|a)\, g(a)da. \tag{A.14}$$

Plugging (A.13) and (A.14) into (A.12) we obtain:

$$\frac{\partial \text{Var}(\log(\text{MRPK}))}{\partial R} = \left(\frac{2}{R}\right)\left(\log R - \mathbb{E}\log(\text{MRPK})\right)\int_a F(Z^*|a)\, g(a)da \le 0. \tag{A.15}$$

The variance is weakly decreasing in R because $\log R \le \mathbb{E}\log(\text{MRPK})$ and $F(Z^*|a) \ge 0$ at the initial point of differentiation. Note that the variance does not change in the limiting cases of no firm being initially constrained (i.e. $\log R = \mathbb{E}\log(\text{MRPK})$) or all firms being initially constrained (i.e. $F(Z^*|a) = 0$). Finally, we note that locally R does not affect dispersion through the cutoff Z^*. This assumes that there is a smooth distribution of z conditional on a and that there are no mass points.

E.2 Changes in Financial Frictions

We consider how small changes in θ impact the dispersion of the MRPK. We obtain:

$$\frac{\partial\left[\mathbb{E}\log\left(\mathrm{MRPK}\right)\right]}{\partial\theta} = \int_a \left[\log(R) f(Z^*|a)\frac{\partial Z^*}{\partial\theta} - \log\left(Z^A Z^* \theta^{\eta-1} a^{\eta-1}\right) f(Z^*|a)\frac{\partial Z^*}{\partial\theta} + \left(\frac{\eta-1}{\theta}\right)\int_{Z^*}^{z_H} f(z)dz\right] g(a)da.$$

Note that the two first terms in the integral cancel out because at the cutoff Z^* we have $R = Z^A Z^* \theta^{\eta-1} a^{\eta-1}$. Therefore:

$$\frac{\partial\left[\mathbb{E}\log\left(\mathrm{MRPK}\right)\right]}{\partial\theta} = \left(\frac{\eta-1}{\theta}\right)\int_a \int_{Z^*}^{z_H} f(z)dzg(a)da = \left(\frac{\eta-1}{\theta}\right)\int_a \left(1 - F\left(Z^*|a\right)\right) g(a)da. \tag{A.16}$$

We also have:

$$\begin{aligned}\frac{\partial\mathbb{E}\left[\left(\log\left(\mathrm{MRPK}\right)\right)^2\right]}{\partial\theta} = &\int_a \left[\left(\log(R)\right)^2 f(Z^*|a)\frac{\partial Z^*}{\partial\theta} - \left(\log\left(Z^A Z^* \theta^{\eta-1} a^{\eta-1}\right)\right)^2 f(Z^*|a)\frac{\partial Z^*}{\partial\theta}\right] g(a)da \\ &+ \int_a \left[\int_{Z^*}^{z_H} \frac{2(\eta-1)\log\left(Z^A z \theta^{\eta-1} a^{\eta-1}\right)}{\theta} f(z)dz\right] g(a)da.\end{aligned}$$

The first two terms cancel out and therefore this derivative can be simplified to:

$$\frac{\partial\mathbb{E}\left[\left(\log\left(\mathrm{MRPK}\right)\right)^2\right]}{\partial\theta} = \left(\frac{2(\eta-1)}{\theta}\right)\int_a \int_{Z^*}^{z_H} \log\left(Z^A z \theta^{\eta-1} a^{\eta-1}\right) f(z)dzg(a)da.$$

or

$$\frac{\partial\mathbb{E}\left[\left(\log\left(\mathrm{MRPK}\right)\right)^2\right]}{\partial\theta} = \left(\frac{2(\eta-1)}{\theta}\right)\int_a \mathbb{E}\left(\log\left(\mathrm{MRPK}\right)|z > Z^*, a\right)\left(1 - F\left(Z^*|a\right)\right) g(a)da. \tag{A.17}$$

Plugging (A.16) and (A.17) into (A.12) we finally obtain:

$$\frac{\partial\mathrm{Var}\left(\log\left(\mathrm{MRPK}\right)\right)}{\partial\theta} = \left(\frac{2(\eta-1)}{\theta}\right)\int_a \left[\mathbb{E}\left(\log\left(\mathrm{MRPK}\right)|z > Z^*, a\right) - \mathbb{E}\log\left(\mathrm{MRPK}|a\right)\right]\left(1 - F\left(Z^*|a\right)\right) g(a)da \leq 0. \tag{A.18}$$

The bracket is weakly positive because the expected marginal revenue product of capital is higher conditional on productivity being above Z^*. Given that $\eta < 1$, the derivative of the variance is weakly negative.

E.3 Changes in Aggregate Productivity or Demand

We consider how small changes in Z^A impact the dispersion of the MRPK. We obtain:

$$\frac{\partial\left[\mathbb{E}\log\left(\mathrm{MRPK}\right)\right]}{\partial Z^A} = \int_a \left[\log(R) f(Z^*|a)\frac{\partial Z^*}{\partial Z^A} - \log\left(Z^A Z^* \theta^{\eta-1} a^{\eta-1}\right) f(Z^*|a)\frac{\partial Z^*}{\partial Z^A} + \left(\frac{1}{Z^A}\right)\int_{Z^*}^{z_H} f(z)dz\right] g(a)da.$$

Note that the two first terms in the integral cancel out because at the cutoff Z^* we have $R = Z^A Z^* \theta^{\eta-1} a^{\eta-1}$. Therefore:

$$\frac{\partial \left[\mathbb{E} \log (\text{MRPK})\right]}{\partial Z^A} = \left(\frac{1}{Z^A}\right) \int_a \int_{Z^*}^{z_H} f(z) dz g(a) da = \left(\frac{1}{Z^A}\right) \int_a \left(1 - F\left(Z^*|a\right)\right) g(a) da. \quad \text{(A.19)}$$

We also have:

$$\begin{aligned} \frac{\partial \mathbb{E}\left[(\log (\text{MRPK}))^2\right]}{\partial Z^A} &= \int_a \left[(\log(R))^2 f(Z^*|a) \frac{\partial Z^*}{\partial Z^A} - \left(\log \left(Z^A Z^* \theta^{\eta-1} a^{\eta-1}\right)\right)^2 f(Z^*|a) \frac{\partial Z^*}{\partial Z^A}\right] g(a) da \\ &+ \int_a \left[\int_{Z^*}^{z_H} \frac{2 \log \left(Z^A z \theta^{\eta-1} a^{\eta-1}\right)}{Z^A} f(z) dz\right] g(a) da. \end{aligned}$$

The first two terms cancel out and therefore this derivative can be simplified to:

$$\frac{\partial \mathbb{E}\left[(\log (\text{MRPK}))^2\right]}{\partial Z^A} = \left(\frac{2}{Z^A}\right) \int_a \int_{Z^*}^{z_H} \log \left(Z^A z \theta^{\eta-1} a^{\eta-1}\right) f(z) dz g(a) da.$$

or

$$\frac{\partial \mathbb{E}\left[(\log (\text{MRPK}))^2\right]}{\partial Z^A} = \left(\frac{2}{Z^A}\right) \int_a \mathbb{E}\left(\log (\text{MRPK}) | z > Z^*, a\right) \left(1 - F\left(Z^*|a\right)\right) g(a) da. \quad \text{(A.20)}$$

Plugging (A.19) and (A.20) into (A.12) we finally obtain:

$$\frac{\partial \text{Var}\left(\log (\text{MRPK})\right)}{\partial Z^A} = \left(\frac{2}{Z^A}\right) \int_a \left[\mathbb{E}\left(\log (\text{MRPK}) | z > Z^*, a\right) - \mathbb{E} \log (\text{MRPK}|a)\right] \left(1 - F\left(Z^*|a\right)\right) g(a) da \geq 0. \quad \text{(A.21)}$$

The bracket is weakly positive because the expected marginal revenue product of capital is higher conditional on productivity being above Z^*. Therefore, the derivative of the variance is weakly positive.

F A Financial Constraint of the Form $k' \leq \theta a'$

In Appendix E we provided analytical solutions for the immediate impact of various shocks on MRPK dispersion within a simplified version of our model with perfect foresight about next period's productivity, no adjustment costs, and a financial constraint of the form $k \leq \theta a$. We now simulate our full model with a risky time-to-build technology of capital accumulation and adjustment costs under the financial constraint $k' \leq \theta a'$. So, the only difference relative to the baseline model is that we replace the size-dependent borrowing constraint in equation (21) with the constraint $k' \leq \theta a'$.

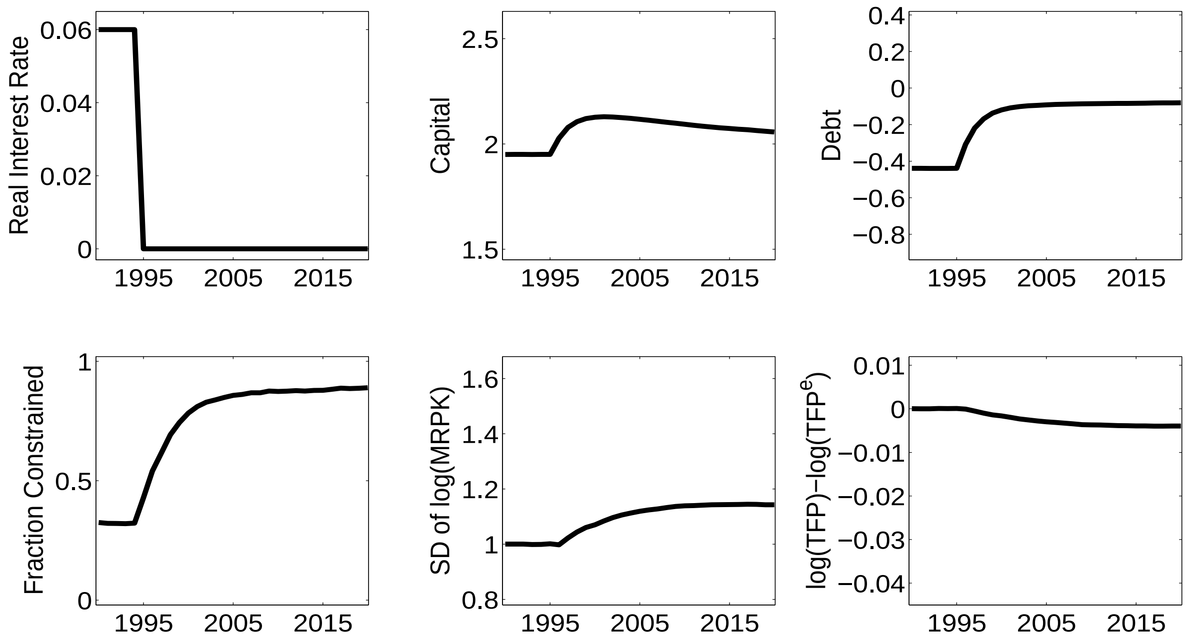


Figure A.5: Decline in the Real Interest Rate ($\psi = 3.1$ and $\theta = 1.0$)

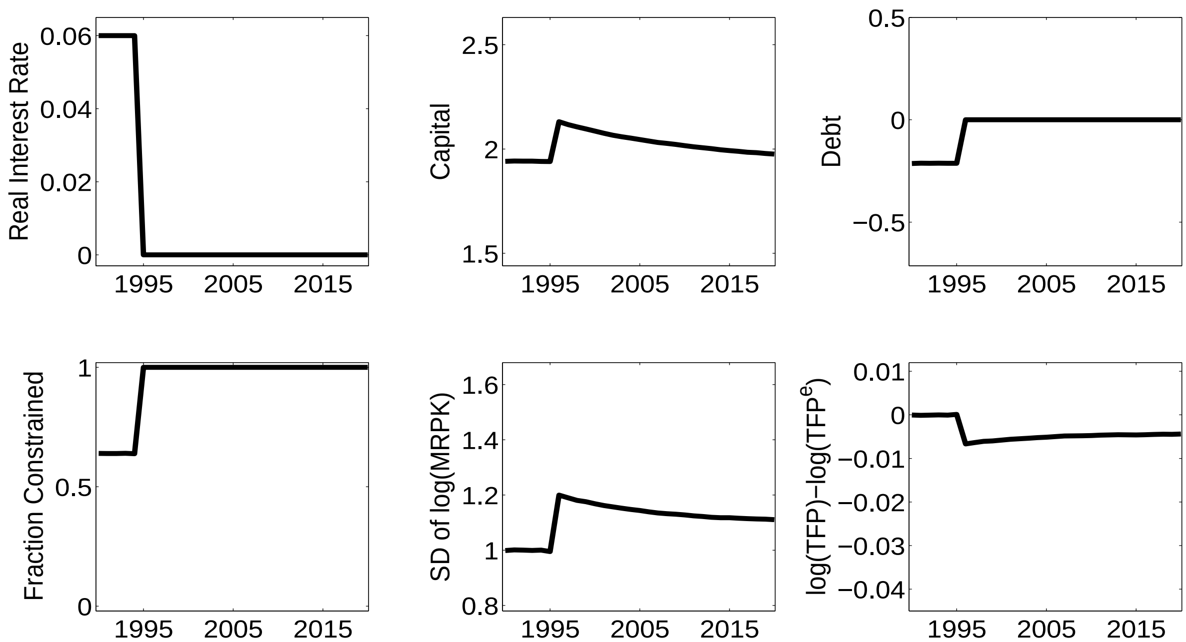


Figure A.6: Decline in the Real Interest Rate ($\psi = 0.0$ and $\theta = 1.0$)

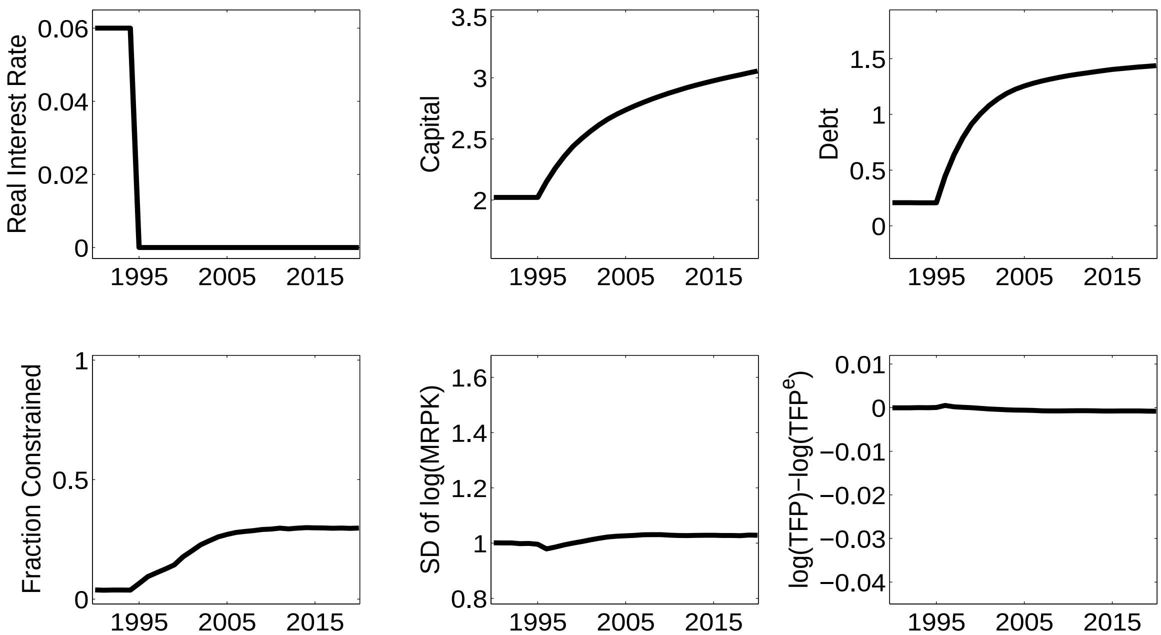


Figure A.7: Decline in the Real Interest Rate ($\psi = 6.5$ and $\theta = 2.2$)

In Figures A.5 and A.6 we present aggregate impulses in response to a decline in the real interest rate from 6 to 0 percent. Figure A.5 uses the adjustment cost parameter $\psi = 3.1$ calibrated from our baseline model and sets $\theta = 1$ which implies that no firm in the economy can borrow. This parameterization is useful because it guarantees that the model with the financial constraint of the form $k' \leq \theta a'$ shares exactly the same initial equilibrium with the model with a size-dependent borrowing constraint. Figure A.6 shuts down adjustment costs ($\psi = 0.0$) and still uses $\theta = 1$. All other parameters are fixed to the values shown in Table 3 for the baseline model. The point of these figures is to show that, in response to the decline in the real interest rate, the model with the alternative financial constraint also generates an increase in MRPK dispersion and a decline in TFP.

Next, we calibrate the model with the financial constraint $k' \leq \theta a'$ in a similar manner to our baseline model with a size-dependent borrowing constraint. Specifically, we set $\psi = 6.5$ and $\theta = 2.2$ to match the responsiveness of firm capital growth to within-firm changes in productivity and net worth. These responses are captured by the coefficients $\beta_z = 0.10$ and $\beta_a = 0.09$ in regression (24) for the permanent sample of firms. In Figure A.7 we present impulses in response to the decline in the real interest rate for this calibrated model. We find

Table A.5: Summary Statistics in the Cross Section of Firms (1999-2007)

		Model		**Sample**	
	Statistic	Baseline	$k' \leq \theta a'$	Permanent	Full
Dispersion	Std $(\log \ell)$	0.78	0.70	1.13	1.21
	Std $(\log k)$	0.87	0.64	1.52	1.70
	Std $(\log \text{MRPK})$	0.30	0.22	0.88	1.12
Productivity	Corr $(\log Z, \log \text{MRPK})$	0.13	0.57	0.03	0.05
	Corr $(\log Z, \log \ell)$	0.96	0.98	0.65	0.58
	Corr $(\log Z, \ell/L)$	0.91	0.96	0.54	0.48
	Corr $(\log Z, \log k)$	0.82	0.87	0.62	0.52
	Corr $(\log Z, k/K)$	0.66	0.87	0.53	0.44
	Corr $(\log Z, \log (k/\ell))$	-0.13	-0.57	0.22	0.16
MRPK	Corr $(\log \text{MRPK}, \log \ell)$	-0.13	0.40	-0.03	0.01
	Corr $(\log \text{MRPK}, \ell/L)$	-0.19	0.43	-0.05	-0.03
	Corr $(\log \text{MRPK}, \log k)$	-0.46	0.09	-0.62	-0.68
	Corr $(\log \text{MRPK}, k/K)$	-0.57	0.10	-0.31	-0.28
	Corr $(\log \text{MRPK}, \log (k/\ell))$	-1.00	-1.00	-0.95	-0.96
Financial	Corr $(\log Z, \log a)$	0.81	0.85	0.75	0.65
	Corr $(\log \text{MRPK}, \log a)$	-0.20	0.09	-0.14	-0.14
	Coefficient of b/k on $\log k$	0.14	0.01	0.15	0.23

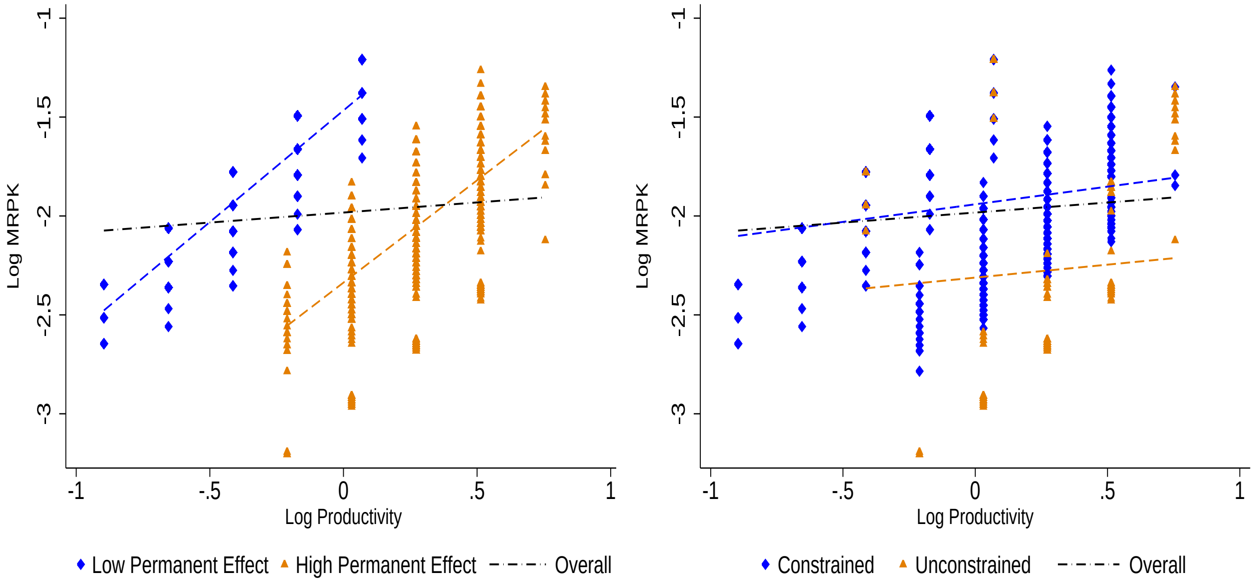
that the model generates a small increase in MRPK dispersion and a negligible decline in TFP. Intuitively, our calibration implies that very few firms are initially constrained before the shock hits. Similar to the analysis in Appendix E, we expect the response of dispersion and TFP to be the smallest when initially all firms are either unconstrained or constrained.

Table A.5 repeats the analysis underlying Table 5 in the main text and compares the model with a financial constraint of the form $k' \leq \theta a'$ to our baseline model with a size-dependent borrowing constraint with respect to various second moments. A key difference between the two models is that the model with the financial constraint $k' \leq \theta a'$ does not generate a negative correlation between measures of revenue-based productivity, such as log (MRPK), and measures of size, such as labor and capital. Additionally, the model with the financial constraint $k' \leq \theta a'$ produces a much stronger correlation between log productivity and log (MRPK) than the model with a size-dependent borrowing constraint.

We now provide intuition for these differences. Figures A.8 and A.9 plot the cross-sectional relationship between log productivity, $\log Z$, and $\log(\text{MRPK})$ in the two models. As we discussed in the main text, within the set of firms with the same permanent productivity z^P, there is a strong correlation between $\log(\text{MRPK})$ and $\log Z$, reflecting transitory productivity shocks in an environment with time-to-build technology and a borrowing constraint. This holds both in the model with a size-dependent borrowing constraint and in the model with a financial constraint of the form $k' \leq \theta a'$.

In response to the decline in the real interest rate, the model with a size-dependent borrowing constraint generates a high dispersion of capital across firms with different permanent productivity because high z^P firms increase significantly their capital to overcome permanently the borrowing constraint. Permanent differences in capital increase significantly the variation of MRPK across firms with different z^P components, leading to a low overall correlation between $\log Z$ and $\log(\text{MRPK})$. By contrast, in the model with a financial constraint of the form $k' \leq \theta a'$ there is no additional incentive to increase capital because the borrowing constraint does not depend on size. This greatly reduces capital and MRPK differences across firms with different z^P components. Therefore, the overall correlation between $\log Z$ and $\log(\text{MRPK})$ is high in the model with a financial constraint of the form $k' \leq \theta a'$.

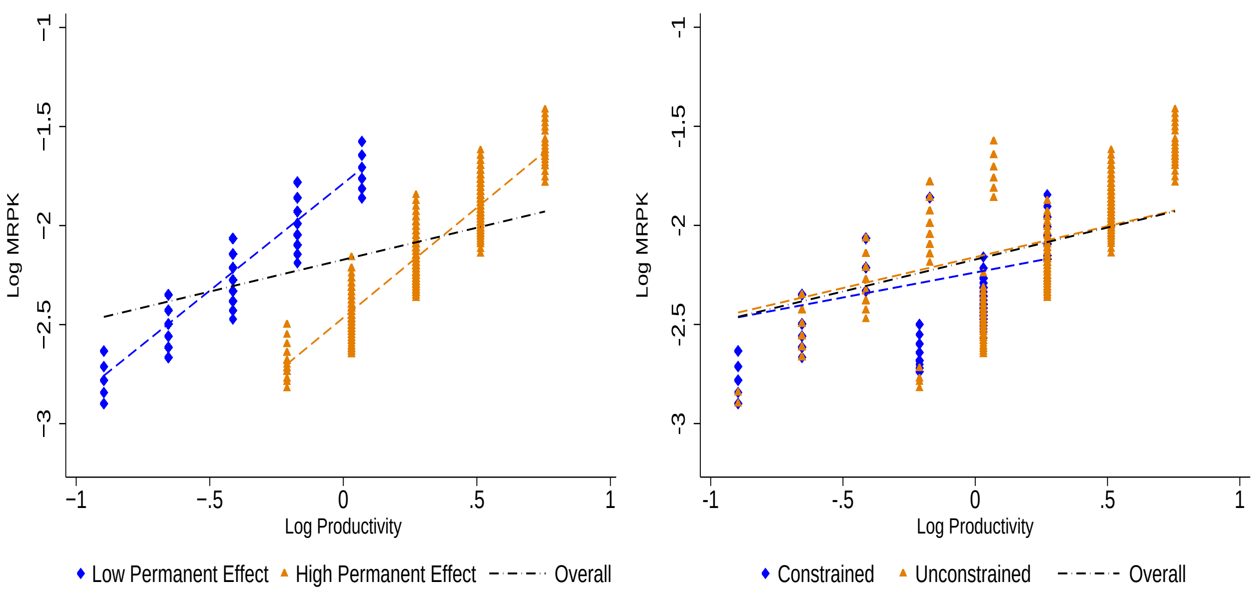
Figures A.10 and A.11 plot the cross-sectional relationship between size (measured with log labor) and $\log(\text{MRPK})$ in the two models. The model with a financial constraint of the form $k' \leq \theta a'$ generates a positive and high correlation between size and $\log(\text{MRPK})$. The intuition is similar to the intuition described above for the relationship between productivity and $\log(\text{MRPK})$, with the time-to-build technology and financial frictions leading to a positive and high correlation between size and $\log(\text{MRPK})$. By contrast, the model with a size-dependent borrowing constraint generates a negative correlation between size and $\log(\text{MRPK})$. This key difference between the two models emerges because, with a size-dependent borrowing constraint, the decline in the real interest rate incentivizes some firms to grow and permanently overcome their borrowing constraint. As a result, in the model with a size-dependent borrowing constraint,



(a) By Permanent Productivity

(b) By Constraint Status

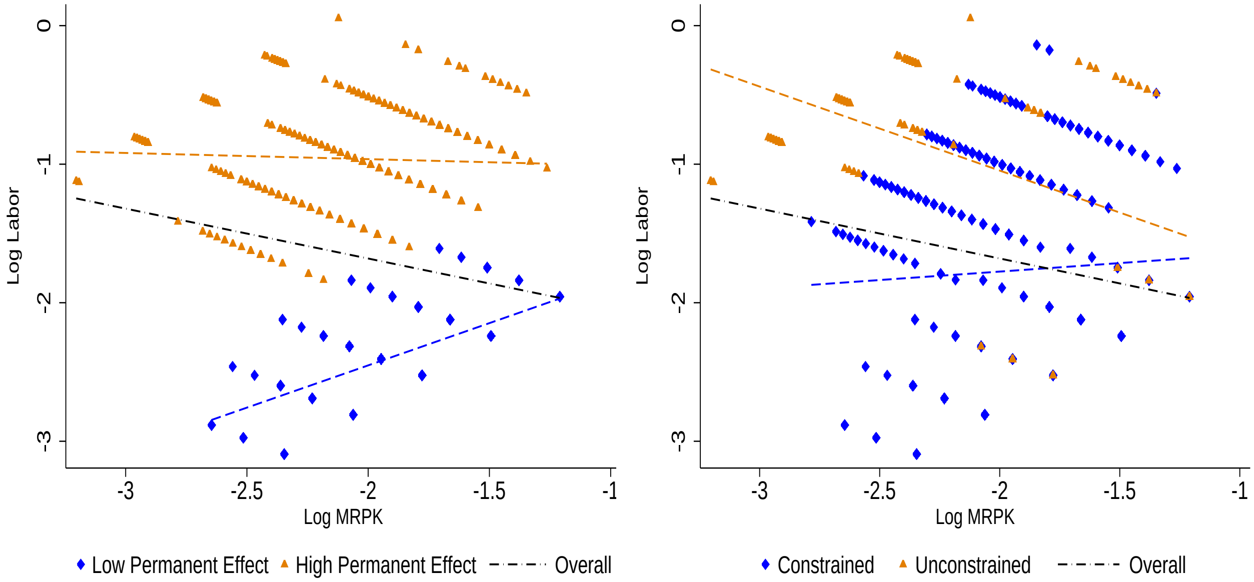
Figure A.8: MRPK and Productivity in Model With Size-Dependent Borrowing Constraint



(a) By Permanent Productivity

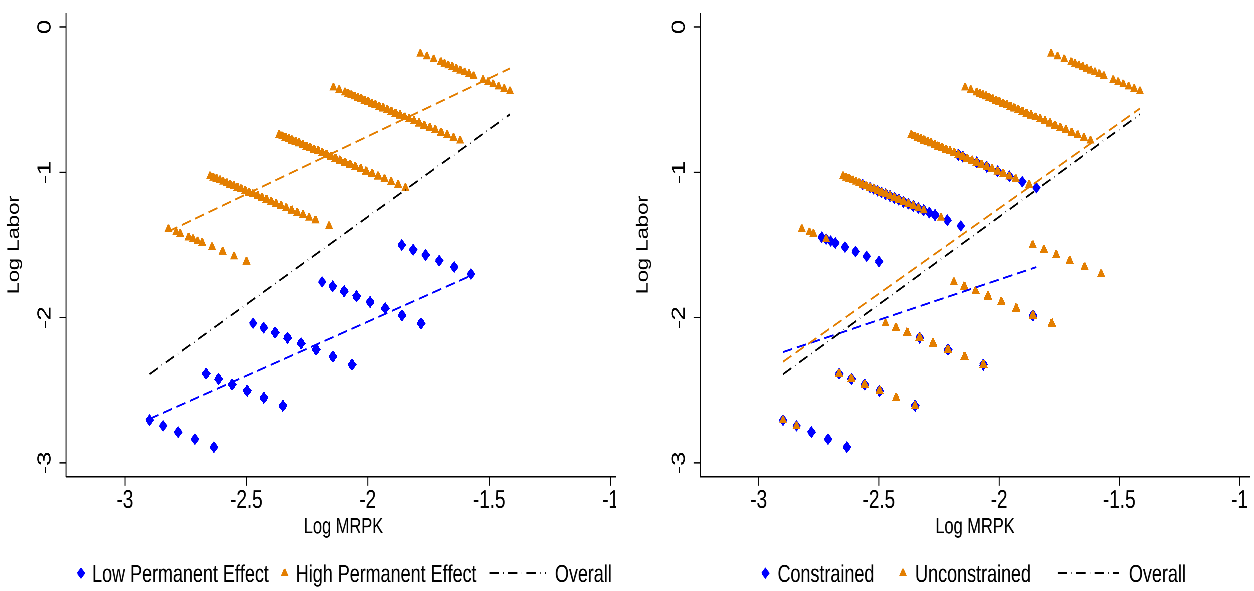
(b) By Constraint Status

Figure A.9: MRPK and Productivity in Model With Financial Constraint $k' \leq \theta a'$



(a) By Permanent Productivity (b) By Constraint Status

Figure A.10: MRPK and Size in Model With Size-Dependent Borrowing Constraint



(a) By Permanent Productivity (b) By Constraint Status

Figure A.11: MRPK and Size in Model With Financial Constraint $k' \leq \theta a'$

larger firms tend to be unconstrained and tend to have a lower return to capital.

G Endogenous Entry and Exit

In this appendix we describe the model with endogenous entry and exit. Let $m_{it} = 0$ denote a firm that operates in the outside sector and let $m_{it} = 1$ denote a firm that produces in manufacturing. The period t status of a firm is a state variable and the period $t+1$ status of a firm is a choice variable. We write the budget constraint of a firm as a function of its state in period t and its entry decision in period $t+1$.

1. When $m_{it} = 1$ and $m_{it+1} = 1$, the budget constraint is:

$$c_{it} + k_{it+1} + (1+r_t)b_{it} + \frac{\psi\left(k_{it+1} - k_{it}\right)^2}{2k_{it}} = \pi_{it} + (1-\delta)k_{it} + b_{it+1}, \tag{A.22}$$

   where $\pi_{it} = p_{it}y_{it} - w_t\ell_{it}$ denotes revenues less compensation to labor.

2. When $m_{it} = 1$ and $m_{it+1} = 0$, the budget constraint is:

$$c_{it} + (1+r_t)b_{it} = \pi_{it} + (1-\delta)k_{it} + b_{it+1}. \tag{A.23}$$

   Firms that operate in manufacturing and decide to exit are assumed to sell their capital ($k_{it+1} = 0$) without incurring an exit cost.

3. When $m_{it} = 0$ and $m_{it+1} = 0$, the budget constraint is:

$$c_{it} + (1+r_t)b_{it} = h_t + b_{it+1}, \tag{A.24}$$

   where h_t denotes the income of firms operating in the outside sector.

4. When $m_{it} = 0$ and $m_{it+1} = 1$, the budget constraint is:

$$c_{it} + k_{it+1} + (1+r_t)b_{it} = h_t - \zeta(k_{it+1}) + b_{it+1}, \tag{A.25}$$

   where $\zeta(k_{it+1})$ denotes an entry cost. We assume that entry costs are an increasing function of the capital stock upon entry.

We now write the problem of a firm in recursive form in the model with endogenous entry and exit. The Bellman equation is:

$$V\left(a, k, m, z^P, z^T, \mathbf{X}\right) = \max_{a', k', m', \ell, p} \left\{ U(c) + \beta \mathbb{E} V\left(a', k', m', z^P, (z^T)', \mathbf{X}'\right) \right\}, \tag{A.26}$$

subject to the budget constraint:

$$c + a' = m\left(\pi - (r+\delta)k - m'\frac{\psi\left(k'-k\right)^2}{2k}\right) + (1-m)\left(h - m'\zeta(k')\right) + (1+r)a, \tag{A.27}$$

where $\pi = p(y)y - wl$ and $y = Zk^{\alpha}\ell^{1-\alpha} = p^{-\varepsilon}$.

In our numerical simulations we work with the quadratic cost $\zeta_{it} = \bar{\zeta} k_{it+1}^2$. We set $\bar{\zeta} = 0.30$ and $h_t = 0.08$. We choose these parameters such that the model replicates the responsiveness of capital growth to within-firm variations in productivity and net worth as observed in the full sample in Table 4.

H Exogenous MRPL Dispersion

Table A.6 repeats the analysis underlying Table 5 in the main text. With this table, we compare the model with exogenous MRPL dispersion to both our baseline model without MRPL dispersion and to the data with respect to various second moments.

We stress three main differences between the model with exogenous MRPL dispersion and the model without MRPL dispersion. First, the model with exogenous MRPL dispersion generates a higher dispersion of firm size (as captured by log labor) and a higher dispersion of log (TFPR) than the baseline model. Second, in the model with exogenous MRPL dispersion there is a weaker correlation between firm log productivity, log Z, and either log labor or the share of firm labor in sectoral labor. This happens because variations of labor across firms in the model with exogenous MRPL dispersion partly reflect variations of the labor wedge. Given that the labor wedge is uncorrelated with firm productivity, the unconditional correlation between firm labor or share in sectoral labor and firm productivity becomes weaker.

The third important difference between the two models is that the model with exogenous MRPL dispersion does not generate the negative correlation between log (MRPK) and either log labor or the share of firm labor in sectoral labor observed in the data. The baseline model

Table A.6: Summary Statistics in the Cross Section of Firms (1999-2007)

		Model		**Sample**	
	Statistic	Baseline	MRPL Dispersion	Permanent	Full
Dispersion	Std (log ℓ)	0.78	0.91	1.13	1.21
	Std (log MRPL)	0.00	0.30	0.30	0.33
	Std (log k)	0.87	0.79	1.52	1.70
	Std (log MRPK)	0.30	0.39	0.88	1.12
	Std (log TFPR)	0.10	0.16	0.35	0.42
Productivity	Corr (log Z, log TFPR)	0.13	0.22	0.46	0.43
	Corr (log Z, log MRPK)	0.13	0.25	0.03	0.05
	Corr (log Z, log ℓ)	0.96	0.78	0.65	0.58
	Corr (log Z, ℓ/L)	0.91	0.70	0.54	0.48
	Corr (log Z, log k)	0.82	0.78	0.62	0.52
	Corr (log Z, k/K)	0.66	0.65	0.53	0.44
	Corr (log Z, log (k/ℓ))	-0.13	-0.16	0.22	0.16
TFPR	Corr (log TFPR, log ℓ)	-0.13	-0.39	0.02	-0.01
	Corr (log TFPR, ℓ/L)	-0.19	-0.38	0.01	0.01
	Corr (log TFPR, log k)	-0.46	-0.24	-0.38	-0.50
	Corr (log TFPR, k/K)	-0.57	-0.28	-0.14	-0.16
	Corr (log TFPR, log (k/ℓ))	-1.00	0.28	-0.60	-0.69
MRPK	Corr (log MRPK, log ℓ)	-0.13	0.38	-0.03	0.01
	Corr (log MRPK, ℓ/L)	-0.19	0.32	-0.05	-0.03
	Corr (log MRPK, log k)	-0.46	-0.28	-0.62	-0.68
	Corr (log MRPK, k/K)	-0.57	-0.33	-0.31	-0.28
	Corr (log MRPK, log (k/ℓ))	-1.00	-0.92	-0.95	-0.96
MRPL	Corr (log MRPL, log ℓ)	—	-0.58	0.31	0.34
	Corr (log MRPL, ℓ/L)	—	-0.53	0.20	0.22
	Corr (log MRPL, log k)	—	0.01	0.33	0.34
	Corr (log MRPL, k/K)	—	0.00	0.22	0.22
	Corr (log MRPL, log (k/ℓ))	—	0.86	0.18	0.15
Financial	Corr (log Z, log a)	0.81	0.70	0.75	0.65
	Corr (log TFPR, log a)	-0.20	-0.05	0.07	0.00
	Corr (log MRPK, log a)	-0.20	-0.07	-0.14	-0.14
	Corr (log MRPL, log a)	—	0.01	0.45	0.44
	Coefficient of b/k on log k	0.14	0.24	0.15	0.23

without MRPL dispersion generates a negative correlation because smaller firms are more likely to be constrained. In the model with exogenous MRPL dispersion, an increase in the labor wedge τ causes both firm labor and MRPK to decrease (the latter decreases because k is predetermined and revenues decrease). This tends to increase the overall correlation between the two variables in the cross section of firms. We also note that, conditional on a labor wedge shock, labor and $\log(\text{MRPL})$ are negatively correlated in the model. However, in the data this correlation is positive.

I Overhead Labor

A model with overhead labor, such as the one developed by Bartelsman, Haltiwanger, and Scarpetta (2013), can match the observed positive correlation between firm size and measured $\log(\text{MRPL})$. Consider the production function:

$$y_{it} = Z_{it} k_{it}^{\alpha} \left(\ell_{it} - \phi_{\ell}\right)^{1-\alpha}, \tag{A.28}$$

where ϕ_{ℓ} denotes overhead labor. With this production function, all firms equalize the true marginal revenue product of labor to the common wage. However, the *measured* marginal revenue product of labor varies across firms. To see this, we write:

$$\text{MRPL}_{it} := \left(\frac{1-\alpha}{\mu}\right)\left(\frac{p_{it} y_{it}}{\ell_{it}}\right) = \left(1 - \frac{\phi_{\ell}}{\ell_{it}}\right) w_t. \tag{A.29}$$

Firms with higher labor also have higher measured MRPL.

Next, we calibrate and simulate the model with overhead labor.[2] The economic environment is similar to our baseline model with the only exception that we use the production function with overhead labor in equation (A.28) instead of the Cobb-Douglas production function. We calibrate jointly the adjustment cost parameter ψ, the borrowing threshold κ, and overhead labor ϕ_{ℓ} to match three moments. As before, the two moments are the responsiveness of capital growth to within-firm variations in productivity and net worth as observed in the permanent

[2] In parallel to the model with exogenous labor taxes, we rebate $\phi_{\ell} w$ back to each firm. This allows us to make consistent comparisons between the model with overhead labor and our baseline model without MRPL dispersion. In the absence of this rebate some small firms would eventually be forced to shut down which, in turn, would change the distribution of productivity across firms in the model.

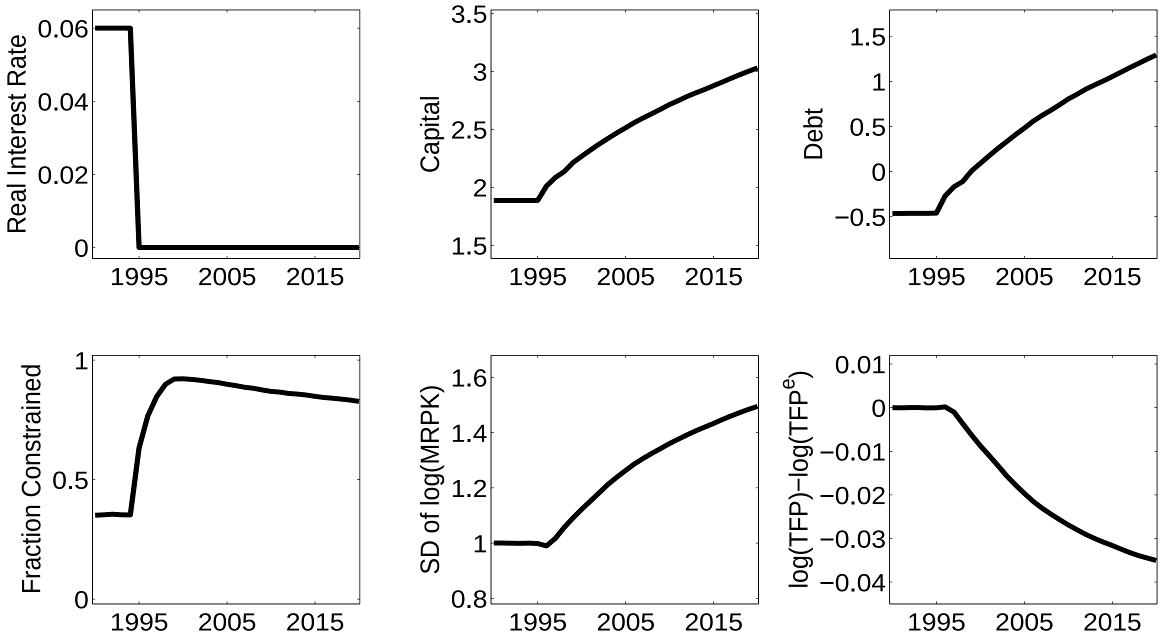


Figure A.12: Decline in the Real Interest Rate: Model With Overhead Labor

sample of firms in Table 4. The third moment is the standard deviation of log (MRPL) which in the data equals 0.30. We find that $\psi = 3.0$, $\kappa = 4.3$, and $\phi_\ell = 0.11$. All other parameters are fixed to the values shown in Table 3 for the baseline model.

In Figure A.12 we present impulses in response to a decline in the real interest rate in the model with overhead labor. We note that the impulses are almost identical to those in the baseline model presented in Figure 11.[3] This is not surprising because our calibrated values of $\psi = 3.0$ and $\kappa = 4.3$ are very close to the calibrated values $\psi = 3.1$ and $\kappa = 4.2$ in the baseline model. Overhead labor does not interact in an important quantitative way with firm investment and debt decisions as captured by the regressions that use within-firm variation in Table 4.

In Table A.7 we compare the model with overhead labor to our baseline model without MRPL dispersion and to the data with respect to various second moments. Consistent with the logic that overhead labor does not interact quantitatively with investment and debt decisions, various moments related to leverage, net worth, capital, and MRPK are similar between the model with overhead labor and the baseline model without MRPL dispersion. While the model with

[3] We define aggregate total factor productivity as $\text{TFP}_t := Y_t / \left(K_t^\alpha \left(L_t - \phi_\ell N_t \right)^{1-\alpha} \right)$, where $\phi_\ell N_t$ denotes total overhead labor in the economy. That is, we do not allow overhead labor to artificially bias measured TFP in the model.

Table A.7: Summary Statistics in the Cross Section of Firms (1999-2007)

		Model		**Sample**	
	Statistic	Baseline	Overhead Labor	Permanent	Full
Dispersion	Std ($\log \ell$)	0.78	0.49	1.13	1.21
	Std ($\log$ MRPL)	0.00	0.30	0.30	0.33
	Std ($\log k$)	0.87	0.87	1.52	1.70
	Std ($\log$ MRPK)	0.30	0.29	0.88	1.12
	Std ($\log$ TFPR)	0.10	0.21	0.35	0.42
Productivity	Corr ($\log Z, \log$ TFPR)	0.13	0.95	0.46	0.43
	Corr ($\log Z, \log$ MRPK)	0.13	0.12	0.03	0.05
	Corr ($\log Z, \log \ell$)	0.96	0.95	0.65	0.58
	Corr ($\log Z, \ell/L$)	0.91	0.91	0.54	0.48
	Corr ($\log Z, \log k$)	0.82	0.82	0.62	0.52
	Corr ($\log Z, k/K$)	0.66	0.66	0.53	0.44
	Corr ($\log Z, \log (k/\ell)$)	-0.13	0.57	0.22	0.16
TFPR	Corr ($\log$ TFPR, $\log \ell$)	-0.13	0.81	0.02	-0.01
	Corr ($\log$ TFPR, ℓ/L)	-0.19	0.74	0.01	0.01
	Corr ($\log$ TFPR, $\log k$)	-0.46	0.61	-0.38	-0.50
	Corr ($\log$ TFPR, k/K)	-0.57	0.42	-0.14	-0.16
	Corr ($\log$ TFPR, $\log (k/\ell)$)	-1.00	0.32	-0.60	-0.69
MRPK	Corr ($\log$ MRPK, $\log \ell$)	-0.13	-0.17	-0.03	0.01
	Corr ($\log$ MRPK, ℓ/L)	-0.19	-0.19	-0.05	-0.03
	Corr ($\log$ MRPK, $\log k$)	-0.46	-0.46	-0.62	-0.68
	Corr ($\log$ MRPK, k/K)	-0.57	-0.56	-0.31	-0.28
	Corr ($\log$ MRPK, $\log (k/\ell)$)	-1.00	-0.74	-0.95	-0.96
MRPL	Corr ($\log$ MRPL, $\log \ell$)	—	0.97	0.31	0.34
	Corr ($\log$ MRPL, ℓ/L)	—	0.91	0.20	0.22
	Corr ($\log$ MRPL, $\log k$)	—	0.92	0.33	0.34
	Corr ($\log$ MRPL, k/K)	—	0.76	0.22	0.22
	Corr ($\log$ MRPL, $\log (k/\ell)$)	—	0.74	0.18	0.15
Financial	Corr ($\log Z, \log a$)	0.81	0.81	0.75	0.65
	Corr ($\log$ TFPR, $\log a$)	-0.20	0.69	0.07	0.00
	Corr ($\log$ MRPK, $\log a$)	-0.20	-0.23	-0.14	-0.14
	Corr ($\log$ MRPL, $\log a$)	—	0.86	0.45	0.44
	Coefficient of b/k on $\log k$	0.14	0.13	0.15	0.23

overhead labor generates a positive correlation between log (MRPL) and size in the cross section of firms, there are two important discrepancies relative to the data. First, overhead labor reduces the log labor dispersion across firms. This happens because less productive firms that would otherwise optimally choose to be small are forced to hire more labor than the overhead. Second, the model with overhead labor generates a strong positive correlation between log (TFPR) and firm size as measured either by labor or capital. In our data for Spain, however, this correlation is close to zero or negative.

J Unmeasured Inputs and Higher MRPK Dispersion

In our baseline model we obtained a lower level of cross-sectional dispersion of capital and especially of MRPK relative to the data. In this appendix we describe a model with an unmeasured input that allows us to rationalize a higher level of capital and MRPK dispersion. We argue that such a modification does not change significantly our main results.

Consider the production function:

$$y_{it} = Z_{it} \left(k_{it} + \phi_k\right)^{\alpha} \ell_{it}^{1-\alpha}, \tag{A.30}$$

where ϕ_k denotes some unmeasured input that enters additively with capital in production. This input could represent some form of intangible capital that is not well measured in the data. Note that the production function (A.30) is similar to the production function in equation (A.28) in the model with overhead labor, with the difference being that in the former we add ϕ_k to capital whereas in the latter we subtract ϕ_ℓ from labor.[4] Similarly to our baseline model, we calibrate values of $\psi = 3.2$, $\kappa = 3.7$, and $\phi_k = 0.30$ to match the responsiveness of capital growth to productivity and net worth using within-firm variation. All other parameters are set at their baseline values shown in Table 3.

Table A.8 repeats the analysis underlying Table 5 in the main text and compares the model with the unmeasured input to our baseline model and to the data along various second moments

[4] Similarly to the model with overhead labor, we tax lump-sum each firm an amount equal to $(r_t + \delta)\phi_k$. Also, we define aggregate total factor productivity as $\text{TFP}_t := Y_t / \left((K_t + \phi_k N_t)^{\alpha} L_t^{1-\alpha} \right)$, where $\phi_k N_t$ denotes the total unmeasured input in the economy. That is, we do not allow this input to artificially bias measured TFP in the model.

Table A.8: Summary Statistics in the Cross Section of Firms (1999-2007)

	Statistic	**Model** Baseline	**Model** Unmeasured Input	**Sample** Permanent	**Sample** Full
Dispersion	Std ($\log \ell$)	0.78	0.74	1.13	1.21
	Std ($\log k$)	0.87	1.37	1.52	1.70
	Std ($\log$ MRPK)	0.30	0.71	0.88	1.12
Productivity	Corr ($\log Z, \log$ MRPK)	0.13	-0.63	0.03	0.05
	Corr ($\log Z, \log \ell$)	0.96	0.97	0.65	0.58
	Corr ($\log Z, \ell/L$)	0.91	0.92	0.54	0.48
	Corr ($\log Z, \log k$)	0.82	0.85	0.62	0.52
	Corr ($\log Z, k/K$)	0.66	0.68	0.53	0.44
	Corr ($\log Z, \log (k/\ell)$)	-0.13	0.63	0.22	0.16
MRPK	Corr ($\log$ MRPK, $\log \ell$)	-0.13	-0.79	-0.03	0.01
	Corr ($\log$ MRPK, ℓ/L)	-0.19	-0.77	-0.05	-0.03
	Corr ($\log$ MRPK, $\log k$)	-0.46	-0.94	-0.62	-0.68
	Corr ($\log$ MRPK, k/K)	-0.57	-0.85	-0.31	-0.28
	Corr ($\log$ MRPK, $\log (k/\ell)$)	-1.00	-1.00	-0.95	-0.96
Financial	Corr ($\log Z, \log a$)	0.81	0.82	0.75	0.65
	Corr ($\log$ MRPK, $\log a$)	-0.20	-0.79	-0.14	-0.14
	Coefficient of b/k on $\log k$	0.14	0.43	0.15	0.23

in the cross section of firms. There are two key differences between the two models. First, as shown in the first panel of the table, the model with the unmeasured input comes much closer than the baseline model in matching the level of capital and MRPK dispersion observed in the data.

The second important difference between the two models is that the model with the unmeasured input generates a more negative correlation between MRPK and measures of size or productivity across firms. To understand this point, we write the *true* MRPK as:

$$\overline{\text{MRPK}}_{it} := \left(\frac{\alpha}{\mu}\right)\left(\frac{p_{it}y_{it}}{k_{it}+\phi_k}\right) = \left(1+\tau_{it}^k\right)(r_t+\delta). \tag{A.31}$$

where τ_{it}^k denotes the percent deviation of the true MRPK from the frictionless cost of capital $r_t+\delta$. As in our baseline analysis, the wedge τ_{it}^k arises because of a binding borrowing constraint,

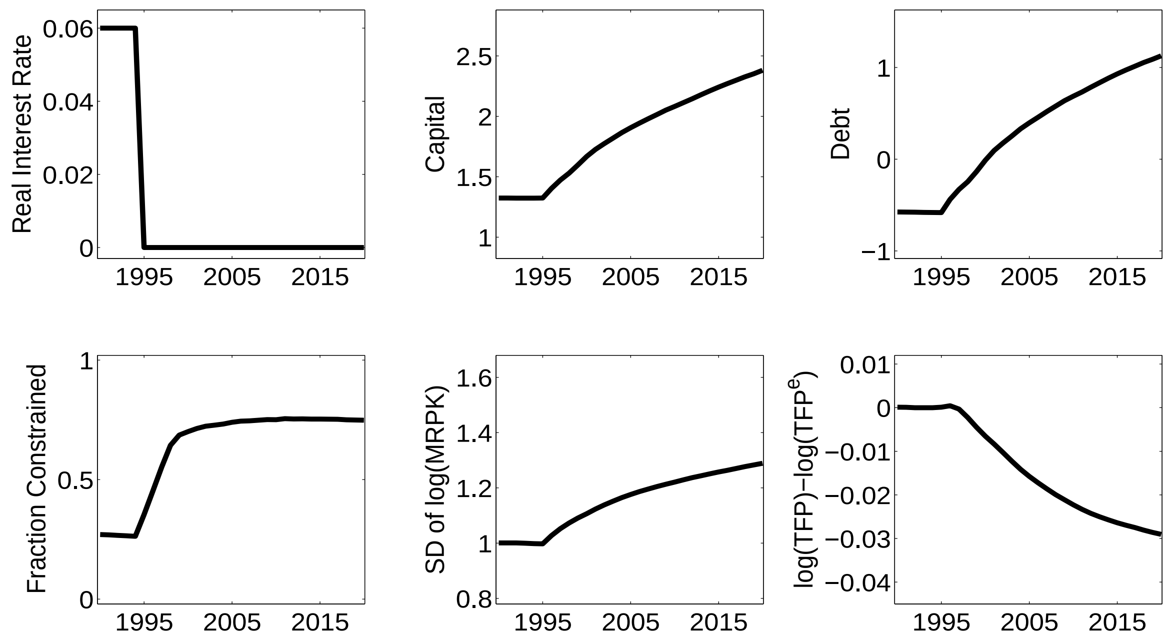


Figure A.13: Decline in the Real Interest Rate: Model With Unmeasured Input

a risky time-to-build technology of capital accumulation, and investment adjustment costs.

Next, consider the *measured* MRPK:

$$\text{MRPK}_{it} := \left(\frac{\alpha}{\mu}\right)\left(\frac{p_{it}y_{it}}{k_{it}}\right) = \left(1+\frac{\phi_k}{k_{it}}\right)\overline{\text{MRPK}}_{it} = \left(1+\frac{\phi_k}{k_{it}}\right)\left(1+\tau_{it}^k\right)(r_t+\delta). \qquad \text{(A.32)}$$

Equation (A.32) shows that ϕ_k introduces an additional wedge between the frictionless cost of capital and the measured MRPK. Firms with higher capital k_{it} will tend to have lower measured MRPK. The existence of this additional wedge explains why the model with the unmeasured input generates more negative cross-sectional correlations between MRPK and either productivity or size.

In Figure A.13 we present impulses in response to the decline in the real interest rate in the model with the unmeasured input. The impulses look quite similar to the impulses generated by our baseline model. We, therefore, argue that the relatively low level of capital and MRPK dispersion generated by the baseline model is not crucial for the main results that emerge from our analysis.