Sovereign Debt, Domestic Banks and the Provision of Public Liquidity[*]

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Abstract. This paper explores two mechanisms through which a sovereign default can disrupt the domestic economy via its banking system. First, a sovereign default creates a negative balance-sheet effect on banks, which reduces their ability to raise funds and prevents the flow of resources to productive investments. Second, default undermines internal liquidity as banks replace government securities with less productive investments. I quantify the model using Argentinean data and find that these two mechanisms can generate a deep and persistent fall in output post-default, which accounts for the government's commitment necessary to explain observed levels of external public debt. The balance-sheet effect is more important because it generates a larger output cost of default and a stronger ex-ante commitment for the government. Post-default bailouts of the banking system, although desirable ex-post, are welfare reducing ex-ante since they weaken government's commitment. Imposing a minimum public debt requirement on banks is welfare improving as it enhances commitment by increasing the output cost of default.

Keywords: Sovereign default, public debt, banks, liquidity.

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1. Introduction

Sovereign governments borrow not only from international investors but also from domestic residents. When domestic financial institutions buy bonds issued by their own government, they expose themselves to sovereign risk. A sovereign default will thus deteriorate the defaulting country's financial system.[1] In this context, it becomes important to analyze how a sovereign default can affect the domestic economy and how domestic holdings of public debt can shape the sovereign's incentives to default.

This paper proposes a theory to explore two mechanisms through which a sovereign default can disrupt the domestic economy via its financial system and affect the government's repayment incentives, using a model of endogenous default enriched with a financial sector. A first mechanism is related to banks' balance-sheet exposure to public debt. As argued by existing research, a sovereign default has a negative impact on banks' wealth, which reduces their ability to raise funds and prevents the flow of resources to productive investments. A second and novel effect is related to the liquidity value of public debt. Banks that do not have good investment opportunities invest in public debt to transfer their wealth across time. After a default the domestic supply of public debt is scarce and these banks substitute away from the use of government securities to investments in their less productive projects.

The proposed theory is then quantified using aggregate macroeconomic and banking data for Argentina. The dynamics of the model are consistent with business cycle facts. I quantify the output costs of default and find that the two mechanisms can generate a deep and persistent fall in output of 5.5% in the two years following a default. These endogenous output costs of default are important in inducing government's incentives to repay its debt as they generate all the commitment necessary to explain observed levels of external public debt issuance. Data on aggregate exposure of banks to public debt and on banks' liquidity management before and after default allows me to identify and disentangle the strength of each mechanism. The balance-sheet effect explains 65% of the output cost of default, while the remaining 35% is explained by the liquidity effect.

The model is then used to explore the implications of allowing for post-default bailouts of the banking system and implementing a minimum requirement of public debt holdings in banks.

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[1]More than half of the total public debt in emerging and advanced economies is held by domestic residents. Additionally, more than 10% of banks' net assets are claims on their own government. See, for example, Bolton and Jeanne (2011) and IMF (2010). Reinhart and Rogoff (2011) provide a historical documentation of the joint occurrence of sovereign defaults and banking crises.

Although desirable ex-post, post-default bailouts are welfare reducing ex-ante since they weaken government commitment. On the other hand, imposing a minimum requirement of public debt on banks is welfare improving as it enhances government's commitment by increasing the output cost of default.

The theoretical framework features an economy with heterogenous banks and a government that can issue external and internal public debt and choose to default on it ex-post. Banks can finance projects with idiosyncratic productivity, lend to the government by investing in public debt, or lend to other banks using interbank deposits. The joint analysis of domestic and external debt gives rise to a new insight that is the dual role of sovereign debt. First, public debt is a security that allows the government to transfer aggregate resources across time when the holders of this security are foreign investors. Second, it provides liquidity to the domestic financial system given the presence of financial frictions in the domestic economy that prevent the banking sector from satisfying its demand for liquidity with privately issued securities. Accordingly, the government issues public debt to front-load and smooth consumption for a representative household and at the same time provide public liquidity to the domestic financial system.

A negative *liquidity effect* arises following a sovereign default as the supply of public debt is relatively scarce and its expected return low. Consider a bank with low-productivity investment projects that finds it profitable to invest in public debt. After a default the aggregate supply of public debt is low and so is its return; therefore, this bank will now prefer to make loans to its low-productivity projects. These projects demand labor, which will now be allocated to projects that are, on average, of lower productivity. This in turn, translates into a lower level of aggregate output. The *balance-sheet effect* of default arises due to the presence of a borrowing constraint for banks that links the maximum amount that banks can borrow from each other to their wealth (net worth). Consider now a bank that is invested in public debt and currently has the opportunity to finance high-productivity projects. A sovereign default reduces the wealth of that bank, which in turn limits the amount of credit it can obtain from other banks to finance its projects. This reduces the amount of labor demanded for these projects, thereby reducing the aggregate demand for labor and equilibrium wages. The reduction in wages increases the expected return on projects and induce banks with lower-productivity projects to invest in them. As a result, there is a drop in the average productivity of the economy through a less efficient allocation of labor.

The presence of these effects gives rise to an internal cost of default that the government will take into account when making repayment decisions. The optimal repayment decision entails a trade-off. On the one hand, a sovereign default precipitates an output cost, in this case endogenous, as well as an exogenous cost of a temporary exclusion from external financial markets. On the other hand, by defaulting, the government saves resources from being paid back to foreign investors. The attractiveness of default will thus depend on the composition of the residence of the government's creditors.

Using aggregate macroeconomic and banking data, I quantify the model to match the Argentinean economy for the 1994.Q1-2012.Q4 period that includes the 2001 sovereign default. The model is parametrized to match the exposure of the banking sector to domestic public debt and the historical frequency of sovereign default. The model simulations indicate that the model can generate enough government commitment to sustain observed levels of external public debt issuance. Additionally, the model is able to explain several salient features of emerging markets' business cycles, such as the high variability of consumption, the counter-cyclicality of the trade balance and the negative correlation between output and interest rate spreads. The simulated output dynamics around episodes of sovereign default matches the observed behavior of output in Argentina during the 2001 default, both in terms of the magnitude of the recession and the dynamics of the recovery.

The model is used to perform a series of counterfactual exercises designed to assess the economic relevance of the balance-sheet effect and the liquidity effect in determining post-default output dynamics and in enhancing the ex-ante commitment of the government. I find that the presence of internal output costs, rather than the costs of exclusion from external financial markets, is key in generating the commitment necessary to sustain observed levels of external debt issuance. The counterfactual exercises also indicate that although both channels are economically relevant, the balance-sheet effect is more important as it accounts for a larger fraction of the output cost of default and explains most of the government commitment. Without the balance-sheet effect, the average levels of external public debt issuance would be 66% lower. Without the liquidity effect the average levels of external public debt issuance would be 37% lower. I also find that while the depth of the output cost of default is directly related to the balance-sheet effect, its persistence is more linked to the liquidity effect. The liquidity effect, while less important, makes the slump last longer. The model is also used to analyze how the government's incentives to repay vary with key variables and parameters. I find that the liquidity cost is higher when the period of exclusion from financial markets following a default

is more protracted. In addition, the balance-sheet effect is stronger the higher the exposure of the domestic financial system to public debt, which implies that sovereign risk is negatively related to the stock of domestic debt according to the model's predictions.

I test this last prediction together with other testable implications of the model regarding the conditional co-movements of sovereign spreads with economic activity and public debt levels. To do so, I use quarterly data on GDP, external and domestic public debt and sovereign bond spreads for a panel of fifteen emerging economies for the 1994.Q1-2012.Q4 period. The empirical evidence is consistent with the model's predictions. In particular, I find that: *(i)* sovereign spreads covary negatively with the level of economic activity, *(ii)* spreads covary positively with the level of external public debt, *(iii)* spreads covary negatively with the level of domestic public debt. While the first two findings are consistent with previous empirical studies of sovereign spreads,[2] the last result has not been previously analyzed as it is motivated from this particular model of endogenous default with external and domestic public debt.

The model is also used to study the effects of domestic policies that are targeted to address the government's lack of commitment. First, I study the welfare effects of allowing for a post-default bailout of the banking system. A post-default bailout of the banking system consists of a tax to households for an amount equivalent to the aggregate exposure of the banking system to public debt that is then reimbursed to banks in the form of equity injections. It is designed to eradicate the balance-sheet effect of defaults as banks' wealth is no longer affected by these episodes. The flip side of eliminating a source of internal costs of default is the associated weakening of the government's ex-ante commitment to repay debt. Results indicate that post-default bailouts of the financial system can be desirable ex-post, once the economy is heavily indebted. However, the desirability of this policy is subject to severe time inconsistency as there are significant welfare gains from committing ex-ante not to implement post-default bailouts when the levels of external debt are low. The reason for this last result is that by prohibiting bailouts, the government enhances its commitment and is therefore able to increase its level of external debt, which allows households to enjoy the benefits of larger consumption front-loading.

Second, I study policies that are targeted at increasing the banks' exposure to public debt. These types of policies can have a positive effect on welfare given the presence of a positive externality generated by banks' holdings of public debt. When individual banks solve their portfolio problem, they do not take into account the fact that by investing in public debt they

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[2]See, for example, Edwards (1984) and more recent works of Uribe and Yue (2006) and Maltritz (2012).

increase the cost of default through a stronger balance-sheet effect and enhance the government's commitment to repay its debt. This in turn allows the government to credibly issue higher levels of external debt in equilibrium. I consider the implementation of a minimum requirement of exposure to public debt in every bank. This policy entails a trade-off between higher government commitment and lower levels of output due to a crowding-out of high-productivity investments. I find that welfare is maximized with a minimum public debt requirement of 45% of a bank's net worth which is equivalent to 5% of its total assets.

Related Literature

This paper builds upon the literature on sovereign debt as well as the vast literature on financial frictions. It is most closely related to a rising theoretical and quantitative literature that studies the internal costs of sovereign defaults.

Following the original framework of sovereign defaultable debt developed in Eaton and Gersovitz (1981), a recent body of literature has studied the quantitative dynamics of sovereign debt and sovereign defaults. Arellano (2008) and Aguiar and Gopinath (2006) analyze sovereign debt and business cycle properties in emerging economies. Several studies have extended the framework to study different aspects related to sovereign debt.[3] These papers find that the presence of reputational costs in the form of exclusion from financial markets cannot quantitatively account for observed levels of external borrowing.[4] In particular, they argue that the presence of a domestic cost of default is necessary to reconcile observed levels of external debt with low frequencies of default. This paper sheds light into the nature of those costs by studying the effects of a default on the financial system. Recent theoretical studies depart from the assumption of a representative agent and study the government's incentives to repay when heterogeneous agents hold sovereign debt (for example, Broner and Ventura (2011) and Guembel and Sussman (2009)). As in these papers, the composition of debt by residence of

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[3]For example, Chatterjee and Eyigungor (2012) and Hatchondo and Martinez (2009) analyze the effects of introducing long-term debt into the standard framework whereas Arellano and Ramanarayanan (2012) focuses optimal debt maturity structure. Yue (2010), Benjamin and Wright (2009) and D'Erasmo (2011) study post-default debt renegotiation. Cuadra and Sapriza (2008) analyze the effects of political uncertainty on sovereign debt and spreads.

[4]Several papers analyze the role of reputational costs in generating commitment to repay debt. Bulow and Rogoff (1989) show that under autarky costs, no debt can be sustained in equilibrium if countries are allowed to save after default. This result motivated subsequent research on mechanisms that offset this result (e.g. Hellwig and Lorenzoni (2009), Amador (2012), Broner et al. (2010) and Mengus (2013a)). Aguiar and Amador (2014) provide a survey of recent advances in the literature.

the creditors will be important for the governments incentives to repay. This paper contributes to this literature by providing empirical support to this prediction by analyzing how spreads co-move with the stock of domestic and external public debt using data for a panel of emerging countries.

This paper also relates to the literature that studies the economic effects of financial frictions. The modeling of the financial sector on this paper builds on the quantitative framework developed in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) where financial intermediaries are constrained on the amount they can borrow by their level of wealth. This friction makes the wealth of the aggregate banking sector a relevant variable that will determine the efficiency of the aggregate economy.[5] In this paper, the presence of this friction, coupled with banks' exposure to public debt gives rise to the balance-sheet effect of default. In a related paper, Bocola (2014) explores a similar effect to study the pass-through of sovereign risk to economic activity. This paper departs from Bocola (2014) by introducing an optimizing government that chooses public debt issuance and repayment and analyzing the effect of the cost of default on government's commitment.

The presence of financial frictions also determines the role of sovereign debt as public liquidity. Scheinkman and Weiss (1986), Woodford (1990) and Holmström and Tirole (1998) show that there is room for an active management of public liquidity through the issuance of government securities whenever there is a lack of commitment problem in the private sector that prevents it from satisfying its demand for liquidity with privately issued securities. A strand of the literature has studied different aspects related to the provision of public liquidity.[6] A novel insight of this paper is that the provision of public liquidity can be undermined after a sovereign default and this in turn serves as a commitment device to repay for the government.

This paper also contributes to the theoretical literature on the internal costs of default. Gennaioli et al. (2014), Basu (2009) and Mengus (2013b) provide a theoretical analysis of how

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[5]This is a feature that is present in several papers that study the macroeconomic effects of financial frictions that stem from limited commitment or moral hazard problems. Some classic and more recent references include Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Bernanke et al. (1999), Brunnermeier and Sannikov (2014), He and Krishnamurthy (2012) and Di Tella (2013).

[6]Kiyotaki and Moore (2005) and Kiyotaki and Moore (2012) discuss the role of public liquidity and studies its effect over asset prices. Aiyagari and McGrattan (1998) study how public debt can aliviate financial frictions and crowd-out capital. Shin (2006) and Angeletos et al. (2013) study optimal public debt issuance for provision of liquidity to the domestic economy under the presence of distortionary taxes. Krishnamurthy and Vissing-Jorgensen (2012) provides a quantification of the liquidity value of public debt for the case of US Treasuries.

a sovereign default can weaken the balance-sheet of banks and explore its effect on government's commitment. Brutti (2011) studies the effect of a sovereign default in preventing firms from refinancing investment projects. The contribution of the paper to this literature is the proposal of a new source of internal costs of default that is given by the liquidity effect, as well as the welfare analysis of different government policies.

Finally, the paper is closely related to quantitative studies of sovereign default with effects on the domestic economy. Mendoza and Yue (2012) analyze internal costs of default in the context of a quantitative model of endogenous default. In their model, a sovereign default is assumed to restrict external credit for firms which forces them to substitute imported inputs for domestic ones that are imperfect substitutes, creating a decline in output. This paper focuses on a different aspect that is the effect of default on the banking system. However, its analysis is complementary to theirs as it sheds light into what are the mechanisms that can trigger a decline in credit following a sovereign default. Lastly, Sosa Padilla (2012) considers a closed economy framework to study how a sovereign default can affect domestic credit through the balance-sheet effect. This paper complements his analysis by considering both the balance-sheet effect and the liquidity effect and disentangling their relevance in a model in which public debt can be held domestically or abroad.

Layout

The remaining of the paper is organized as follows. Section 2 presents the model setup and defines a recursive equilibrium in the economy. Section 3 discusses the main mechanisms through which a sovereign default affects the domestic economy (i.e. the balance-sheet effect and the liquidity effect) and analyzes the implications of the model for the government's optimal repayment decisions. Section 4 presents cross-country evidence on spreads and debt that validates the model's predictions regarding the behavior of sovereign spreads. Section 5 discusses the model's calibration, its business cycle properties and provides counterfactual exercises designed to disentangle the economic relevance of the balance-sheet and liquidity effect. Section 6 study domestic policies aimed at addressing the government's lack of commitment problem and finally, section 7 concludes.

2. A Model of Sovereign Debt and a Financial Sector

In this section I formulate a dynamic stochastic general equilibrium model for a small open economy enriched with a financial sector (in the lines of Gertler and Kiyotaki (2010) and Gertler and Karadi (2011)) and a sovereign government that lacks commitment and has access

to external debt markets (as in Eaton and Gersovitz (1981)). In this framework sovereign debt will jointly serve two purposes. First, it will be a security that allows to transfer aggregate resources across time when the holders of this security are foreign investors. Second, sovereign debt will also serve as a provision of liquidity to the domestic financial sector. The role of public debt as liquidity provision given the presence of financial frictions in the banking sector. Particularly, banks will be subject to a borrowing constraint that links the amount of funds they can raise to their net worth.

Households

Each household is composed of a continuum of members that includes bankers and workers. Workers supply a fixed amount of labor in a competitive labor market and return their labor income to the household. Bankers manage a bank and transfer non-negative dividends to the households. Within the household there is perfect consumption insurance. Households are risk averse and their preferences are defined over an infinite stream of non-storable consumption

$$U = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(C_t) \right]$$

where $\beta \in (0, 1)$ is the factor, C_t is consumption in period t and $u(\cdot)$ is increasing and concave. Household members are hand-to-mouth consumers and do not make any savings decision. Let w_t be the wage paid to workers in period t, π_t the dividend payments from bankers and τ_t the lump sum taxes paid to the government, the household budget constraint is given by

$$C_t = w_t + \pi_t - \tau_t \tag{1}$$

where the aggregate labor supply is normalized to one.

Banks

There is a continuum of banks that have access to a constant-returns-to-scale production technology. The technology is stochastic and uses labor l_{t+1} chosen in period t to deliver

$$A_{t+1} z_t l_{t+1}$$

units of consumption in period $t + 1$, where A_{t+1} is an aggregate productivity shock and z_t is an idiosyncratic productivity shock. The aggregate shock is subject to trend shocks

$$A_t = \exp(g_t) A_{t-1}$$

where g_t follows a Markov process with transition probability $f(g_{t+1}, g_t)$ with bounded support. The idiosyncratic shock z_t is known to each banker at period t, and is iid with cummulative

FIGURE 1. Timing of events for a banker
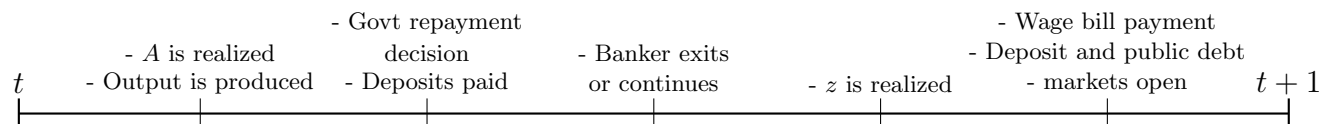
distribution function $G(z)$. Since idiosyncratic shocks are independent across bankers and there is a continumm of bankers, $G(z)$ will also be the realized fraction of bankers with idiosyncratic shock below z.

In order to hire labor, banks need to pay the wage bill $w_t l_{t+1}$ in period t before production takes place. This assumption about the timing gives rise to a need for banks of obtaining credit to produce.

Bankers exit their business with probability $1-\sigma$ each period. When they exit they distribute their accumulated wealth, or net worth, as dividends to the households. The bankers objective is to maximize the expected value of dividends paid to households

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_{0,t+1} \sigma^t (1-\sigma) n_{t+1} \tag{2}$$

where n_t is the banks' net worth in period t (measured in consumption units) and $\Lambda_{t,s} \equiv \beta^s u'(C_s)/u'(C_t)$ is the household stochastic discount factor.

In addition to the production technology, bankers have access to two asset markets: the public debt market and the interbank market. Public debt is a risky one-period security that pays one unit of consumption in the following period if the government repays and zero if the government defaults. Interbank deposits are also risky one-period securities that pay one unit of consumption in the following period, except in the states where there is sovereign default, in which they pay zero. In summary, banks can lend to or borrow from other banks, invest in their production technology by hiring labor and buy public debt. The timeline of events within a period is depicted in Figure (1).

Let $\{l_t, b_t^d, d_t\}$ be the claims on labor, the stock of public debt and the stock of interbank deposits with which a banker comes into period t. Then the amount of consumption goods a banker obtains in a period (net worth) is given by the net repayments on these claims

$$n_t = A_t z_{t-1} l_t + \iota_t (b_t^d + d_t) \tag{3}$$

where $\iota_t \in \{0,1\}$ indicates whether the government defaults or repays its debt in period t, respectively. The net worth that a banker brings into a period, plus the goods he borrows from

other banks (if any), can be used to invest in their productive technology, buy public debt or lend to other banks. This is reflected in the bank's balance-sheet which states that the banks' net worth is equal to the market value of their investments in labor, public debt and interbank deposits. Let q_t^b, q_t^d be the price of public debt and interbank deposits, respectively, then the balance-sheet equation is given by

$$n_t = w_t l_{t+1} + q_t^b b_{t+1}^d + q_t^d d_{t+1}. \tag{4}$$

Note that $d_{t+1} \leq 0$ indicates borrowing from other banks.

The interbank credit market will be subject to a financial friction. In particular, I assume that the amount of borrowing that any banker can raise through interbank loans is capped by a multiple of its own net worth[7]

$$q_t^d d_{t+1} \geq -\kappa n_t. \tag{5}$$

This type of financial friction is commonly used in quantitative models of credit markets. It can be micro-founded by an agency problem in which the banker has the ability to run away with a fraction of his assets and transfer them to their own household.[8] Finally, I also assume that banks cannot take short positions on public debt

$$b_{t+1}(z) \geq 0. \tag{6}$$

The banker's problem is then to choose a sequence $\{l_t, b_t^d, d_t\}_{t=1}^{\infty}$ that maximize (2), subject to the evolution of net worth (3), the balance-sheet definition (4), the limited commitment constraint (5) and the no-short-selling-of-public-debt constraint (6), given an initial level of net worth n_0 and idiosycratic productivity z_0.

Government

The sovereign government faces incomplete markets and issues one-period non-state contingent bonds that pay one unit of consumption next period. These securities can be purchased by domestic banks and/or foreign investors. The government is the only agent that has access

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[7]This assumption alone does not guarantee that the banker will always have enough consumption goods to pay back its deposits. However, it can be shown that by imposing a parametric assumption that bounds the lowest realization of the aggregate productivity $\left(\text{i.e. } \frac{A_{min}}{\mathbb{E}[A]} > \frac{\kappa}{1+\kappa} \right)$ ensures that any banker that borrows will always have enough goods to repay its debt.

[8]For a micro-foundation of this type of financial frictions (and similar variants of it) and an analysis of its role as an accelerator of macroeconomic shocks see, for example, Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Bernanke et al. (1999), Gertler and Kiyotaki (2010) and Gertler and Karadi (2011).

to foreign borrowing from external investors. Foreign investors are risk-neutral and can borrow at a constant risk-free interest rate R.

The government lacks commitment to repay its debt and can ex-post choose to default on its entire stock of public debt. Let B_t^d denote the aggregate stock of domestic public debt (public debt held by domestic banks), B_t^x the stock of external public debt (public debt held by foreign investors) and $B_t = B_t^d + B_t^x$ the stock of total public debt, all due at period t.[9] Then the government budget constraint in states in which it has access to the public debt market is given by

$$q_t^b \left(B_{t+1}^d + B_{t+1}^x \right) + \tau_t = \iota_t \left(B_t^d + B_t^x \right). \tag{7}$$

The government is benevolent and its objective is to maximize expected lifetime utility of the representative household. To do so it chooses the total stock of public debt, lump sum taxes to households and repayment decisions.

If the government chooses to default on its debt it faces an exogenous cost of exclusion from external financial markets for a stochastic number of periods. In particular, if the government defaults it immediately losses access to the market for external credit. Once in financial autarky the government regains access to the external credit market with probability ϕ and, when it does so, it starts with zero external public debt.

While in external financial autarky the government can still issue domestic public debt that can be held by banks. It will do so following a suboptimal exogenous policy of aggregate supply of risk-less domestic public debt such that its equilibrium price is given by $q_t^b = 1/\zeta$ with $\zeta < R$. In other words, during autarky I assume that there will be scarcity of public debt that is reflected in a return of ζ which is assumed to be lower than the international risk-free interest rate. Parametrizing the domestic debt policy during periods of external financial autarky gives enough flexibility to consider different cases. For example, the case of zero domestic debt issuance (which would be consistent with a particularly low value of ζ) would correspond to the extreme case of complete financial autarky from both domestic and external debt markets. Another particular case would be the issuance of non-interest bearing securities (i.e. cash), in which case ζ would be given by the inverse of the gross inflation rate.

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[9]For any variable x define its aggregate counterpart as

$$X \equiv \int x(n,z) d\mathcal{G}(n,z)$$

where $\mathcal{G}(n,z)$ is the endogenous distribution of net-worth and idiosyncratic productivity.

Discussion of Assumptions

This section discusses the assumptions that underlie the setup. Households are agents that do not make active decisions. In particular, they are assumed not to make savings decisions. This assumption is made for simplicity, given that the government, through an active management of lump-sum taxes to households, will be indirectly making the inter-temporal savings decisions for the households.[10]

Banks are assumed to have access to a production technology. The banks in this economy represent a consolidation of the financial and productive sector of the economy. This assumption assigns a direct role of banks in the productive process. The production technology is subject to idiosyncratic productivity shocks, which implies that banks face an idiosyncratic risk that is not insurable. These uninsurable shocks can represent geographic components or specific knowledge of bankers on certain types of industries that are subject to idiosyncratic shocks. It can be shown that this setup is equivalent to a model in which there are firms that operate the production technology in different industries (that face idiosyncratic shocks) and banks can buy claims on firms of a particular industry. This formulation of banks, together with the timing assumption that wages need to be prepaid before production takes place, embeds the idea that domestic credit is key to realize productive projects (as in Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and Brunnermeier and Sannikov (2014)).[11]

The characterization of aggregate productivity shocks as trend shocks -rather than transitory fluctuations around a stable trend- is consistent with recent empirical findings. Aguiar and Gopinath (2007) find that shocks to trend growth are the primary source of fluctuations in emerging markets. Additionally, as shown in Aguiar and Gopinath (2006), the presence of trend shocks in quantitative models of sovereign default help explain high sovereign spreads observed in the data.

The assumption that interbank deposits are not repaid in the state of a sovereign default is done for simplicity. A more standard assumption of risk-less interbank deposits could be adopted and the main theoretical and quantitative results would still carry through. Nevertheless, there is empirical support for this assumption. In the Argentinean default of 2001 several

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[10]This assumption is commonly used in small open economy models in which the government has access to external debt markets (e.g. Mendoza and Yue (2012)). See Wright (2006) for an analysis of an economy with private and public capital flows and sovereign risk.

[11]Additionally, the working capital assumption is often used in business cycles models for emerging economies as they help explain business cycle volatilities and co-movements. See, for example, Neumeyer and Perri (2005) and Uribe and Yue (2006).

private debt contracts were restructured (Barajas et al. (2007)). In addition, as documented in Durbin and Ng (2005), the creditworthiness of firms in a country is usually lower than that of its government (i.e. 'sovereign ceiling').

Implicit in the writing of the government budget constraint (7) is the assumption that the government is not allowed to default selectively on only one type on debt. This assumption is important since, as it will become clear later, the government will have ex-post incentives to default on its external debt and repay its domestic debt. In practice sovereign governments often contain cross-default clauses (see, for example, IMF (2002) and Hatchondo et al. (2012)). These clauses state that a default in any government obligation constitutes a default in the contract containing that clause. Therefore, in practice, a sovereign default on a contract would imply a default on the outstanding stock of public debt. Another relevant assumption is the inability of the government to make transfers (or equity injections) to the banks. The only transfers that the government can make are lump-sum taxes/transfers to households. If the government could inject equity to the banks, it would be able to replicate a selective default on external debt by defaulting on the total public debt and bailing out banks with an equity injection equivalent to the banks' exposure to public debt. In practice, government bailouts of the banking system are occasionally observed in emerging economies. I will relax this assumption in Section 5 and analyze the macroeconomic impact of allowing governments to bail out the banking system after a sovereign default.

Finally, two assumptions are made regarding public debt issuance after default. The exclusion from external financial markets for a stochastic number of periods can be thought of as a reduced form of a punishment from both foreign investors in the context of a dynamic game. This exclusion cost of default, common among Eaton-Gersovitz models, is in line with the empirical evidence from recent emerging market default episodes (see Gelos et al. (2011) and Dias and Richmond (2008)). A less stringent assumption is made regarding domestic public debt issuance in periods of external financial autarky. In these states the stock of public debt will be scarce and this is reflected in its return which is assumed to be lower than the risk-free international interest rate. This assumption reflects a restriction in public debt issuance that the government is able to issue a low stock of public debt due to a potential punishment or loss of confidence in the government's credibility from domestic agents after the default. In this case, a more flexible approach is adopted to obtain a better fit of the data. In particular, the parameter ζ will be disciplined by the data in the calibration section. However, all the

theoretical mechanisms of the model do not rely on this assumption and would still hold under the particular case of complete exclusion from both external and domestic debt markets.

Recursive Equilibrium

This model features, in addition to the government, domestic agents that perform intertemporal optimization problems. In particular, bankers take as given a sequence of expected government policies to make their individual portfolio decisions. The model can admit several equilibria if we allow allocations to depend on past histories.[12] Given that the focus of the paper is the internal costs of default -rather than reputational costs- I will focus on Markov equilibrium in which agents' strategies depend on payoff relevant states.[13] Equilibrium is defined in two steps. First I define a *competitive equilibrium* for a given government policy. Second I define a *Markov perfect equilibrium* as the competitive equilibrium associated to the government policies that are chosen optimally given its time inconsistency problem.

I focus in equilibria in which banks follow cutoff rules to determine their portfolio choices. It will be argued that the unique solution to the banks' problem will be of this type. In particular, denote $\underline{z}$ a threshold level of productivity that is a sufficient statistic of the banks' problem above which banks will invest in their own technology. Additionally, let A_{-1} indicate the level of aggregate productivity in the previous period. The aggregate state of the economy is $\mathbf{s} = (s, h)$ where $s = \left(A_{-1}, g, \underline{z}, B^d, B^x \right)$ and $h \in \{m, a\}$ indicates whether the government has access to external financial markets ($h = m$) or whether it is in external financial autarky ($h = a$). Since I define equilibrium in two steps, the relevant state for the private allocations will be the augmented state $\tilde{\mathbf{s}} = (\mathbf{s}, B', \iota)$ that includes the current government policies.[14]

Additionally, the bank's problem admits a recursive representation (that can be found in Appendix A). The bank's problem will depend on future government policy functions $(\mathcal{B}'(\mathbf{s}), \mathcal{I}(\mathbf{s}))$ and on the law of motion of the aggregate state $\Gamma(\mathbf{s'}, \mathbf{s}, B', \iota)$ which denotes the density function of state $\mathbf{s'}$ conditional on $(\mathbf{s}, B', \iota)$. Denote $v(n, z; \mathbf{s}, B', \iota)$ the value of an individual bank with net worth n, idiosyncratic productivity z in augmented aggregate state $(\mathbf{s}, B', \iota)$ that solves the bank's problem in recursive form.

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[12]See Chari and Kehoe (1993) for an analysis of the set of equilibria in dynamic models of public debt and taxation without commitment.

[13]Other papers that also focus on Markov perfect equilibrium in the context of government with time inconsistency problems are Bianchi and Mendoza (2013) and Krusell et al. (1997).

[14]Current government policies are part of the state for the private allocations since private agents take government policies as given. This will serve as an input when I define the Markov Perfect Equilibrium.

DEFINITION 1. *Given the augmented aggregate state $\tilde{s} = (s, B', \iota)$ and future government policies $\{\iota(s), B'(s)\}$, a competitive equilibrium are household consumption $\{C(\tilde{s})\}$, bank allocations $\{l'(n, z; \tilde{s}), b'^d(n, z; \tilde{s}), d'(n, z; \tilde{s})\}$ and value functions $v(n, z; \tilde{s})$ for all z, dividend payments $\pi(\tilde{s})$, lump-sum taxes $\tau(\tilde{s})$, prices $\{q^d(\tilde{s}), q^b(\tilde{s}), w(\tilde{s})\}$, the joint distribution of bankers $\mathcal{G}(n, z; \tilde{s})$ and the law of motion of the aggregate state $\Gamma(s', s, B', \iota)$ such that:*

(1) *Government policies and taxes satisfy the government budget constraint (7)*

(2) *Given taxes, wages and dividend payments, household consumption is consistent with its budget constraint (1)*

(3) *Given prices, bank allocations and value functions solve the recursive representation of banks' problem (2)-(6)*

(4) *The labor market and the interbank deposit market clear*

$$\int l'(z, n, \tilde{s}) d\mathcal{G}(n, z; \tilde{s}) = 1 \tag{8}$$

$$\int d'(z, n, \tilde{s}) d\mathcal{G}(n, z; \tilde{s}) = 0 \tag{9}$$

(5) *The public debt market clears*

for $h = m$:
$$\int b'^b(z, n, \tilde{s}) d\mathcal{G}(n, z; \tilde{s}) \leq B' \tag{10}$$

$$q^b(s, B') \geq \frac{\mathbb{E}\left[\iota(\tilde{s}')|\tilde{s}\right]}{R} \tag{11}$$

$$\left(\int b'^b(z, n, \tilde{s}) d\mathcal{G}(n, z; \tilde{s}) - B'\right)\left(q^b(s, B') - \frac{\mathbb{E}\left[\iota(\tilde{s}')|\tilde{s}\right]}{R}\right) = 0 \tag{12}$$

for $h = a$:
$$q^b(s) = \frac{1}{\zeta} \tag{13}$$

(6) *The joint distribution of net-worth and productivity evolves according to*

$$\mathcal{G}'(n', z'; \tilde{s}') = \iint_{(n,z):n'=\eta(n,z;\tilde{s},\tilde{s}')} \mathcal{G}(n, z; \tilde{s}) g(z') dn dz$$

where $\eta(\cdot)$ is consistent with the evolution of idiosyncratic net worth given by the bank's allocations and the law of motion of the aggregate state.

(7) *The law of motion of the aggregate state is consistent with current government policies and private allocations, i.e.*

– h' *evolves according to the transition probability*

$$\Pr(h' = m) = \begin{cases} 1 & if\ h = m,\ \iota = 1 \\ 0 & if\ h = m,\ \iota = 0 \\ \phi & if\ h = a \end{cases}$$

– $A = A_{-1}\exp(g)$ *and* g' *evolves according to the conditional density* $f(g', g)$
– $B^{d'}(\tilde{\boldsymbol{s}}) = \int b'^b(z, n, \tilde{\boldsymbol{s}})d\mathcal{G}(n, z; \tilde{\boldsymbol{s}})$, $B^{x'}(\tilde{\boldsymbol{s}}) = B' - B^{d'}(\tilde{\boldsymbol{s}})$ *and the cutoff productivity* $\underline{z}'(\tilde{\boldsymbol{s}})$ *is given by the minimum productivity of a bank that chooses to invest in his own technology*

The way the public debt market clears is nontrivial. For states in which the government is in financial autarky ($h = a$), the government follows an exogenous policy at which public debt is risk-less and its price is given by $1/\zeta$ and the government supplies the necessary securities to satisfy the domestic demand for debt at that price. For states in which the government has access to credit markets ($h = m$), there are two possibilities, as indicated by equations (10)-(12). One possibility is that there is no external debt (equation (10) holds with equality). In this case the equilibrium price of public debt should clear the market domestically and also be such that foreign investors are not willing (or at least indifferent) to buy public debt (inequality (11)). The second case is that there is a positive amount of external public debt. In this case public debt should be priced by foreign investors (equation (11) holds with equality) and the amount of external public debt is determined as the residual between the total stock of public debt issued by the government and the domestic public debt demanded by banks at that price.

We can anticipate that in equilibrium equation (11) will hold with equality as the government has incentives to issue domestic public debt to provide liquidity and also provide a high return on domestic public debt so banks can capitalize more quickly and undo the inefficiency imposed by the limited commitment constraint.[15] For these reasons, even if the government does not want to issue any external debt it will prefer to issue public debt up until the point in which the foreign investor is indifferent between buying or not buying that public debt. It follows that there is no loss in generality to assume that, when issuing debt the government can anticipate what will be the demand for domestic debt at any given price and will be thereby ultimately choosing the stock of external debt. Henceforth government policy functions will be denoted $\{\iota(\mathbf{s}), B^{x'}(\mathbf{s})\}$. Note also that, as is commonly assumed in Eaton-Gersovitz models, we assume

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[15]See section 3 for a discussion on the optimal issuance of public debt for liquidity purposes.

the sovereign faces a pricing curve of public debt for any potential level of external public debt $q^b(s, B^{x\prime})$ and chooses optimally in what point of the curve to issue debt.[16]

In this economy the joint distribution of net worth and idiosyncratic productivity follows and endogenous law of motion. However, since idiosyncratic shocks are assumed to be iid we need not keep track of the entire distribution of net worth across banks but only of the aggregate level of domestic public debt B^d and the threshold productivity $\underline{z}$. Aggregating across banks and using the market clearing condition for deposits and labor we get the evolution of aggregate net worth and dividend payments to households

$$N(\tilde{\mathbf{s}}) = \sigma \left(A \mathbb{E}\left[z | z > \underline{z}\right] + \iota B^d \right) \tag{14}$$

$$\pi(\tilde{\mathbf{s}}) = (1 - \sigma) \left(A \mathbb{E}\left[z | z > \underline{z}\right] + \iota B^d \right). \tag{15}$$

We can now characterize the competitive equilibrium for a given government policy. It can be shown that the individual bank's problem is linear in net worth and its solution will involve corners.[17] Given that the payoffs to interbank deposits and public debt are the same in each possible state it must be the case that $q^b(\tilde{\mathbf{s}}) \geq q^d(\tilde{\mathbf{s}})$ for all states.[18]

The individual banks optimal portfolio choice will depend on their idiosyncratic productivity z and on the prices of public debt and deposits and wages. In the case when the price of deposits equals the price of public debt, the banks with high productivity will choose to borrow in the interbank market up to their constraint and invest the amount borrowed plus all their net worth in their production technology by hiring labor. On the other hand, the banks with low productivity will be indifferent between lending to other banks and investing in public debt.

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[16]The presence of a pricing schedule from which the government can choose is consistent with a sequential borrowing game in which the government announces how many bonds it wants to issue or purchase and then each lender offers the government a price at which he is willing to buy the bonds the government is issuing or to sell the bonds the government wants to purchase. This assumption eliminates a potential source of multiplicity of equilibria, since faced to two points in the pricing curve that yield the same amount borrowed the sovereign will choose the one with lower promised external debt. For a formal discussion of this argument see Lorenzoni and Werning (2013). Calvo (1988) and Cole and Kehoe (2000) also study the existence of multiple equilibria and self-fulfilling crises in the context of sovereign debt models.

[17]This result is due to the fact that the discount factor of the representative household is not affected by the portfolio choices of an individual bank. See Gertler and Kiyotaki (2010), Gertler and Karadi (2011) and Bocola (2014) for similar treatments of the banking sector and Moll (2014) for an example with agents with idiosyncratic productivities.

[18]Otherwise all banks would want to take advantage of the arbitrage opportunity by borrowing from other banks and investing in public debt, but then the interbank market of deposits would not clear.

In the case when the price of deposits is lower than the price of public debt, public debt will not be an attractive asset for any bank since the return on interbank deposits will be higher at any possible state. In this case, banks with low productivity will prefer to lend all their net worth to banks with high productivity and banks with high productivity will choose to borrow in the interbank market up to their constraint and invest the amount borrowed plus all their net worth in their production technology. An illustration of the solution to the banks portfolio problem is depicted in Figure 2.

A formal characterization of the solution is stated in the following proposition. Denote $R^x(\tilde{\mathbf{s}}, \mathbf{s}')$ the realized return of asset x, $\nu(z; \tilde{\mathbf{s}})$ the marginal value of one unit of net worth and $\tilde{\Lambda}(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}') = \Lambda(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')(1 - \sigma + \sigma \mathbb{E}_{z'}[\nu(z; \tilde{\mathbf{s}}')|\tilde{\mathbf{s}}'])$ the augmented stochastic discount factor.[19] Also let $\underline{z}'(\tilde{\mathbf{s}})$ be a threshold productivity level such that the risk-adjusted expected return of investing in the production technology is the same as the risk-adjusted expected return of lending to other banks, i.e. $\mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')R^l(\underline{z}(\tilde{\mathbf{s}}); \tilde{\mathbf{s}}, \tilde{\mathbf{s}}')\right] = \mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')R^d(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')\right]$.

PROPOSITION 1. *(1) For states in which $q^b(\tilde{\mathbf{s}}) = q^d(\tilde{\mathbf{s}})$:*
  – *Banks with $z > \underline{z}'(\tilde{\mathbf{s}})$ will prefer to borrow up to their constraint $q^d(\tilde{\mathbf{s}})d'(z) = -\kappa n(z)$ and invest everything in the productive technology $w(\tilde{\mathbf{s}})l'(z) = (\kappa + 1)n(z)$.*
  – *Banks with $z \leq \underline{z}'(\tilde{\mathbf{s}})$ will be indifferent between borrowing to other banks and investing in public debt: $q^d(\tilde{\mathbf{s}})d'(z) = x \in [0, n(z)]$, and $q^b(\tilde{\mathbf{s}})b'(z) = n(z) - x$.*

(2) For states in which $q^b(\tilde{\mathbf{s}}) > q^d(\tilde{\mathbf{s}})$:
  – *Banks with $z > \underline{z}'(\tilde{\mathbf{s}})$ will prefer to borrow up to their constraint $q^d(\tilde{\mathbf{s}})d'(z) = -\kappa n(z)$ and invest everything in the productive technology $w(\tilde{\mathbf{s}})l'(z) = (\kappa + 1)n(z)$.*
  – *Banks with $z \leq \underline{z}'(\tilde{\mathbf{s}})$ will prefer to lend all their net worth to other banks: $q^d(\tilde{\mathbf{s}})d'(z) = n(z)$.*

Additionally, the value function of bankers is linear in net worth $v(n, z; \tilde{\mathbf{s}}) = \nu(z; \tilde{\mathbf{s}})n$ where

$$\nu(z; \tilde{\mathbf{s}}) = \mathbb{E}\left[\Lambda(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')(1 - \sigma + \sigma\nu(z', \tilde{\mathbf{s}}'))R^d(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')\left[1 + (\kappa + 1)\left(\max\left\{\frac{R^l(z; \tilde{\mathbf{s}}, \tilde{\mathbf{s}}')}{R^d(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')} - 1, 0\right\}\right)\right]\right]$$

(16)

Proof. See Appendix A $\qquad\qquad\square$

FIGURE 2. Solution to Banks' Portfolio Problem
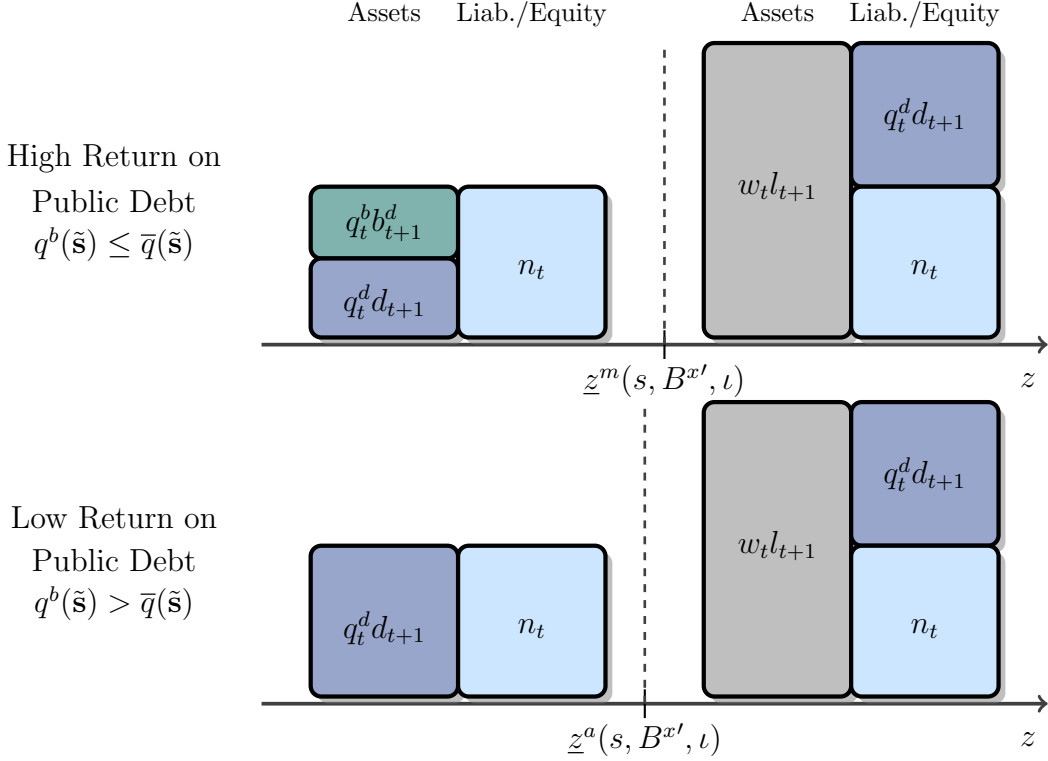
The threshold productivity that determines whether banks will be borrowers or lenders, the equilibrium wage and the price of deposits will vary with the aggregate state of the economy. Some bankers will buy public debt if its risk-adjusted return is higher than the risk-adjusted return of interbank deposits in an equilibrium without public debt. In other words, if public debt provides a lower return than what a bank would obtain from investing in interbank deposits in an equilibrium without public debt, no banker would choose to invest in public debt in equilibrium. It follows that if the government is interested in issuing public debt domestically, it needs to have a sufficiently attractive expected return.

In the states in which there is domestic exposure to public debt (case 1 of Proposition 1) the threshold productivity of the bank that is indifferent between investing in his production technology or investing in public debt (or lending to other banks) is determined by the risk-adjusted return on public debt

$$\mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')A'\right]\frac{\underline{z}'(\tilde{\mathbf{s}})}{w(\tilde{\mathbf{s}})} = \mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')R^b(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')\right]. \tag{17}$$

Higher wages, everything else equal, increase the threshold productivity since it is costlier to hire labor and therefore less profitable to invest in their own technology. The aggregate stock

of domestic public debt is determined as a residual of the net worth of those banks with low productivity that did not lend to other banks[20]

$$q^b(\tilde{\mathbf{s}})B^{d'}(\tilde{\mathbf{s}}) = N(\tilde{\mathbf{s}})\left(G(\underline{z}'(\tilde{\mathbf{s}}))(1+\kappa) - \kappa\right) \tag{18}$$

In states in which banks do not invest in public debt (case 2 of Proposition 1), the threshold productivity of the bank indifferent between lending to other banks and investing in its own technology is determined exclusively by the parameter on the limited commitment constraint[21]

$$\underline{z}'(\tilde{\mathbf{s}}) = G^{-1}\left(\frac{\kappa}{\kappa+1}\right). \tag{19}$$

Finally, the labor market clearing condition is given by

$$(\kappa+1)N(\tilde{\mathbf{s}})\left[1 - G\left(\underline{z}'(\tilde{\mathbf{s}})\right)\right] = w(\tilde{\mathbf{s}}). \tag{20}$$

The demand for labor will depend positively on the aggregate level of banks net worth (which will ultimately determine the volume of interbank lending) and negatively on the fraction of bankers that choose not to invest in their production technology $G\left(\underline{z}(\tilde{\mathbf{s}})\right)$.

The following proposition formalizes the above-mentioned characterization of prices and aggregate allocations in a competitive equilibrium. Let $\overline{q}(\tilde{\mathbf{s}})$ and its associated return a $\overline{R}(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}') = \mathcal{I}(\tilde{\mathbf{s}}')/\overline{q}(\tilde{\mathbf{s}})$ be the price of debt such that its risk-adjusted return is the same as the risk-adjusted return of interbank deposits in an economy without public debt. Formally, $\overline{R}(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')$ satisfies

$$\mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')\overline{R}(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')\right] = \mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')A'\right]\frac{G^{-1}\left(\frac{\kappa}{\kappa+1}\right)}{N(\tilde{\mathbf{s}})}$$

PROPOSITION 2. *For any state equilibrium wages solve (20).*

(1) *For states in which the price of debt is $q^b(\tilde{\mathbf{s}}) < \overline{q}(\tilde{\mathbf{s}})$, the price of deposits is $q^d(\tilde{\mathbf{s}}) = q^b(\tilde{\mathbf{s}})$ and the law of motion for the threshold productivity and aggregate level of domestic debt solve (17)-(18).*

(2) *For states in which $q^b(\tilde{\mathbf{s}}) \geq \overline{q}(\tilde{\mathbf{s}})$, the price of deposits is given by $q^d(\tilde{\mathbf{s}}) = \overline{q}(\tilde{\mathbf{s}})$ and the law of motion for the aggregate level of domestic debt is $B^{d'}(\tilde{\mathbf{s}}) = 0$ and the threshold productivity is given by (19).*

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[20]Note that for the stock of domestic public debt to be non-negative we must have $G(\underline{z}'(\tilde{\mathbf{s}})) \geq \frac{\kappa}{\kappa+1}$. If this condition does not hold, then the equilibrium will be with $B^{d'}(\tilde{\mathbf{s}}) = 0$ and $q^b(\tilde{\mathbf{s}}) > q^d(\tilde{\mathbf{s}})$, as indicated in Proposition 2.

[21]In particular, the aggregate net worth $N(\tilde{\mathbf{s}})$ does not affect the cutoff productivity as it affects scales both the demand and supply of interbank funds in the same way. This case will not happen in equilibrium in the simulations of the model. However, its analysis is useful to understand the role of public debt as a provision of liquidity, which will be discussed in section 3.2.

Proof. See Appendix A □

Given the characterization of the dynamics of aggregate variables in the internal economy we can define the government's problem. Since the government is unable to commit to future policy rules, it chooses its policy rules at any given period taking as given the policy rules that represent future governments' decisions, and a Markov perfect equilibrium is characterized by a fixed point in these policy rules. At this fixed point, the government does not have the incentive to deviate from other government's policy rules, thereby making these rules time-consistent.

Denote $\underline{z}'(s, h; B^{x'}, \iota)$, $B^{d'}(s, h; B^{x'}, \iota)$ and $C(s, h; B^{x'}, \iota)$ be the competitive equilibrium allocations associated to *current government policies* $\{B^{x'}, \iota\}$ and *future government policies* $\{\mathcal{B}^{x'}(\mathbf{s}), \mathcal{I}(\mathbf{s})\}$. These allocations satisfy the conditions stated in Proposition 2.

Given its time inconsistency problem the government will optimally choose current period repayment and external debt issuance to maximize the value function of the representative households given that foreign investors and domestic agents expect future government policies to be $\{\mathcal{B}^{x'}(\mathbf{s}), \mathcal{I}(\mathbf{s})\}$. The value for the government associated to an optimal one-period deviation must solve

$$W(A_{-1}, g, \underline{z}, B^d, B^x) = \max_{\iota \in \{0,1\}} \iota W^m(A_{-1}, g, \underline{z}, B^d, B^x) + (1 - \iota)W^a(A_{-1}, g, \underline{z}) \qquad (21)$$

Where the value of repaying and maintaining access to financial markets is given by

$$W^m(A_{-1}, g, \underline{z}, B^d, B^x) = \max_{B^{x'}} u(C(s, m, B^{x'}, 1)) + \beta \mathbb{E}\left[W(A, g', \underline{z}', B^{d'}, B^{x'}) | s\right] \qquad (22)$$

subject to

$$C(s, m, B^{x'}, 1) = A\mathbb{E}[z | z > \underline{z}] - B^x + q^b(s, B^{x'})B^{x'}$$

$$\underline{z}' = \underline{z}'(s, m; B^{x'}, 1)$$

$$B^{d'} = B^{d'}(s, m; B^{x'}, 1)$$

The consumption equation is given by the resource constraint. The value of defaulting and losing access to financial markets is given by

$$W^a(A_{-1}, g, \underline{z}) = u(C(s, a)) + \beta \mathbb{E}\left[\phi W(A, g', \underline{z}', B^{d'}, 0) + (1 - \phi)W^a(A, g', \underline{z}', B^{d'})\right] \qquad (23)$$

where

$$C(s, a) = A\mathbb{E}[z | z > \underline{z}]$$

$$\underline{z}' = \underline{z}'(s, a; B^{x'}, 0)$$

$$B^{d'} = B^{d'}(s, a; B^{x'}, 0)$$

Note that future government policies affect the government problem as they affect the laws of motion of the threshold productivity and the domestic public debt. The law of motion for $\underline{z}$, B^d are characterized in Proposition 2. Having defined the government problem I define a Markov perfect equilibrium.

DEFINITION 2. *A Markov Perfect equilibrium are aggregate private allocations $\{C(\tilde{s}), \underline{z}'(\tilde{s}), B^{d'}(\tilde{s})\}$, prices $\{q^d(\tilde{s}), q^b(\tilde{s}, B'), w(\tilde{s})\}$, government policy functions $\{B^{x'}(s), \iota(s)\}$ and future government policy functions $\{\mathcal{B}^{x'}(s), \mathcal{I}(s)\}$ such that:*

(1) *Given future government policies, aggregate private allocations and prices are consistent with a competitive equilibrium*

(2) *Given private allocations and expected policies, the government policies solve the government problem (21)-(23)*

(3) *Optimal government policies coincide with future policies $\{\mathcal{B}^{x'}(s), \mathcal{I}(s)\} = \{B^{x'}(s), \iota(s)\}$*

Given the presence of non-stationary aggregate productivity shocks, to solve for equilibrium, I first derive the de-trended version of the banks' and government problems. I do so by normalizing the relevant variables by $A_{-1}\mu_g$. The normalization results in an aggregate productivity level with unconditional average of one. Further details on the de-trended recursive problems are provided in Appendix B. The model is then solved using a global solution that uses projection methods. The competitive equilibrium given any government policy is solved using Euler equation iteration and the government problem is solved using value function iteration methods. A description of the numerical solution algorithm is also provided in Appendix B.

3. DOMESTIC BANKS AND THE INTERNAL COSTS OF SOVEREIGN DEFAULT

When choosing to default on its debt, the sovereign government weighs the benefits from saving resources that are not paid to foreign investors with the costs of default. These costs come in the form of a temporary exclusion from external financial markets and an endogenous output cost that arises as a consequence of reducing the efficiency of the domestic financial system. This section describes the mechanisms that link the government's repayment decisions to the domestic financial system and economic activity.

It will be argued that a sovereign default hits the domestic financial system via two channels: *(i)* a negative balance-sheet channel that undermines the banks ability to raise funds in the interbank market (Section 3.1), and *(ii)* a liquidity cost that comes from the relative scarcity of public debt as a liquid asset with which banks can transfer wealth across time (Section

3.2). Section 3.3 discusses the dynamics of the output costs and its sensitivity to specific state variables and key parameters. Finally, Section 3.4 analyzes the implications of these internal costs of default in terms of the government's incentives to repay its debt and the relationship between sovereign risk and the stock of domestic and external public debt.

3.1. *Balance-sheet Costs of Default*

The banking sector in this economy is subject to a financial friction that prevents resources from flowing within the banking system.[22] Given this friction, the aggregate net worth of the banking sector will be a determinant of how much interbank borrowing can take place. In this context, a sovereign default will hit the banks and prevent credit from flowing to productive investments.[23]

Given a certain exposure of the domestic financial system to domestic public debt, a sovereign default will have a negative impact on the aggregate net worth of banks (equation (14)). Faced with a negative shock to net worth, banks with high productivity levels can raise less resources in the interbank market (compared to what they could have obtained in the case of government repayment) and reduce the levels of investment in their productive technology, thereby reducing aggregate labor demand (equation (20)). Figure 5a illustrates the effect of a decline in net worth caused by a sovereign default in the labor market allocations and banks investment choices. As shown in the left panel the reduction in labor demand lowers equilibrium wages (given the inelastic labor supply). Lower wages have an impact in the optimal portfolio choices of individual banks (right panel). In particular, the banks that used to be indifferent between investing in their own productive technology and investing in public debt (or lending to other banks) will now prefer the former option as the costs of this investment are lower. This reduces the threshold productivity level above which banks prefer to invest in their productive

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[22]There is a vast literature that has studied the macroeconomic effects of financial frictions that stem from limited commitment or moral hazard problems. Some classic and more recent papers include Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Brunnermeier and Sannikov (2014), He and Krishnamurthy (2012) and Di Tella (2013).

[23]A similar mechanism is analyzed in Gennaioli et al. (2014). Bocola (2014) provides a quantitative analysis of this mechanism. In his paper the negative net worth shock lowers the amount that banks can borrow from households via deposits and ends up reducing investment and labor allocations. In my model the negative balance-sheet shock will come from a reduction in interbank lending and less efficient allocation of labor that leads to a lower average productivity.

technology and lowers the average productivity of the aggregate economy generating an output cost.[24]

FIGURE 3. Internal Costs of Sovereign Default

(A) Balance-Sheet Costs of Default
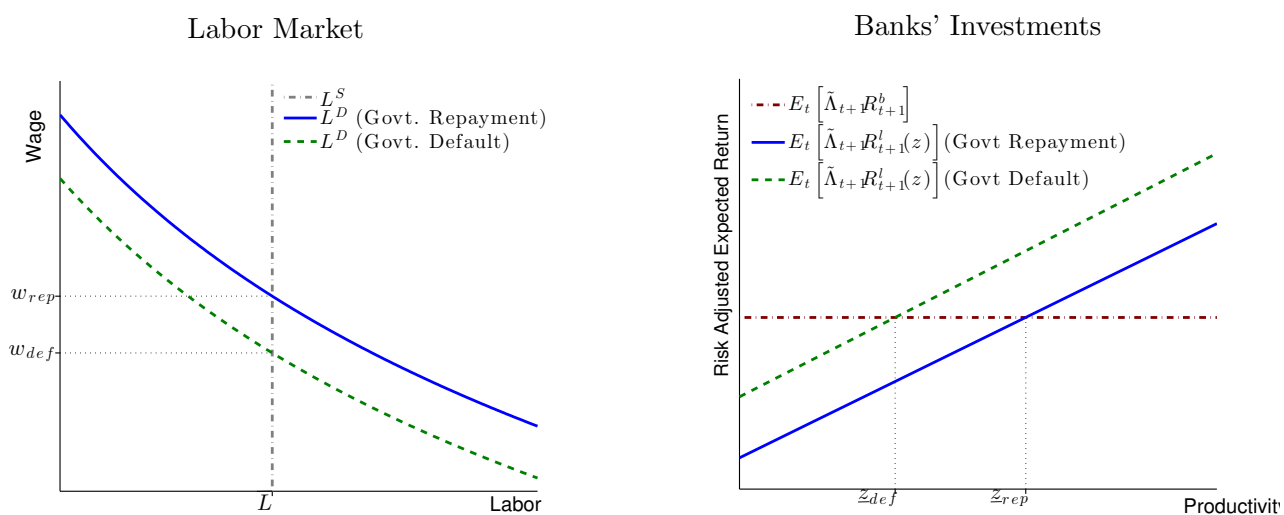
(B) Liquidity Costs of Default
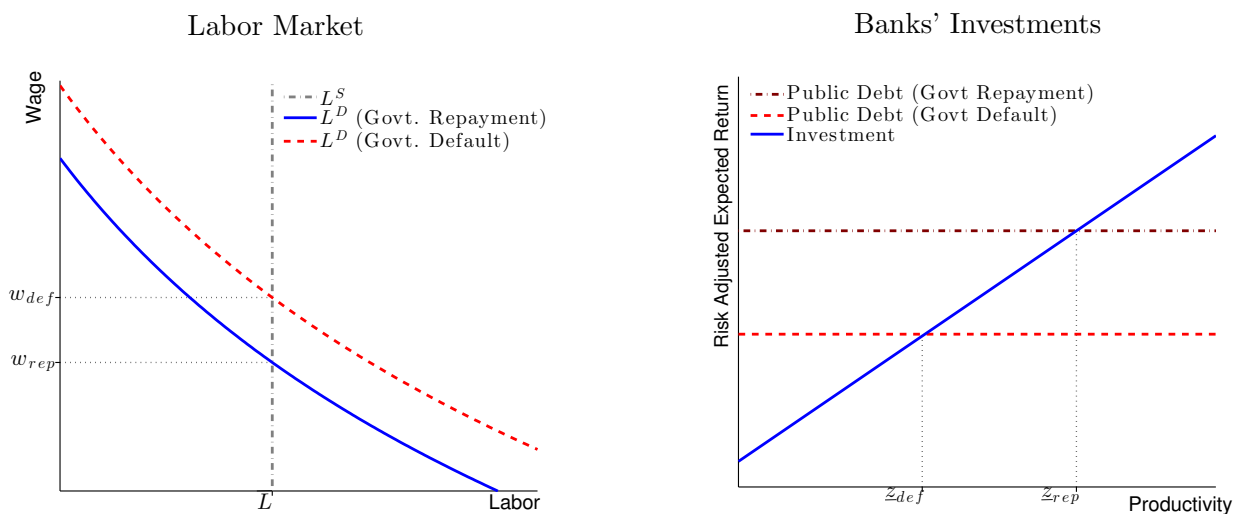
Notes: ...

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[24]In equilibrium the increase in the fraction of banks investing in their own productive technology will have a second order effect by increasing labor demand that only partially offsets the first order effect that is not shown in Figure 5a.

The output cost is due to a composition effect of how labor is allocated to banks with differing productivities.[25] There is no extensive margin through which output is affected as aggregate labor is assumed to be inelastic. It is important to note, however, that the presence of this cost of default does not rely on labor supply being inelastic. If we were to assume an elastic labor supply wages would still decrease and so would aggregate labor, moving further away from the first best level in which the marginal disutility of labor is equated to its expected marginal valuation of the marginal product. In this case the output cost of sovereign default would come from a combination of a composition effect (lower average productivity) and an extensive margin effect (through lower aggregate labor).

3.2. *Liquidity Cost of Default*

As noted at the beginning of Section 2 the role of public debt in this economy is twofold. On the one hand, it is a security that allows for inter-temporal trade for the representative household when the holders of this security are foreign investors. On the other hand, it provides liquidity to the domestic financial system. By liquidity I refer to the availability of instruments that can be used to transfer wealth across periods (Woodford (1990), Holmström and Tirole (1998)). These papers argued that there is room for an active management of public liquidity through the issuance of government securities whenever there is a lack of commitment problem in the private sector that prevents it from satisfying its demand for liquidity with privately issued securities. This way the provision of public liquidity leads to a more efficient functioning of the productive sector.[26]

In this economy individual banks view the availability of public debt as an exogenous technology at which they can transfer resources across time at a given (risky) rate of return. This investment vehicle is particularly attractive for banks with low productivity that cannot obtain high returns by hiring labor and investing in their productive technology. From an aggregate perspective, the availability of public debt provides liquidity value to the domestic economy as it allows low-productivity banks to invest their net worth in an asset with an attractive risk adjusted return while they wait for a high productivity draw in the future. In the absence of

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[25]It could be argued that the negative composition effect can be partially offset by positive effect that is due to the bankruptcy of banks with low-productivities. In this economy, this effect is not present since banks do not go bankrupt (given the limited amount of leverage they can take) and the banks exit their business with an iid probability.

[26]The implicit assumption that these papers have is that, unlike the private sector, the government can commit to agents' future income by enforcing tax payments. In this economy, the government will still be able to provide liquidity to the domestic financial system in spite of its lack of commitment to repay ex-ante.

public debt as an available asset, these banks would have to either lend to other banks at a lower expected return, or invest in their own low-productivity technology. It follows that the liquidity value of public debt is related to its risk-adjusted return. As indicated in proposition 2 for low enough returns, banks will not buy any public debt. As its return increases, it provides a higher liquidity value as it screens away low-productivity banks from investing in their own technology and the average productivity in the economy increases.[27]

The role of public debt as a security that provides liquidity to the domestic economy is due to the presence of financial frictions (i.e. the borrowing constraint) on the banking sector. If financial frictions within the banking system were removed, which would correspond to the particular case of $\kappa = \infty$, then changes in the return of public debt would not have an impact on real allocations. Particularly, in the case of a distribution of idiosyncratic productivities with bounded support, the equilibrium allocations in the friction-less case would correspond to only the bank with the highest productivity investing in his technology and all the other banks lending to this banks and/or investing in public debt. In this context, changes in the return on public debt may affect the return on the different investments but will not affect the real allocation of labor.

Given the efficient screening effect associated to an increase in the return on public debt, the government would be willing to issue sufficient amounts of public debt so that its return is high and it precipitates a more efficient allocation of labor.[28] However, there is a limit on how much liquidity the government can provide. In particular, there will be a return at which foreign investors would be willing to buy public debt. After it reaches this return then issuing public debt in excess would not provide any liquidity domestically since it will be bought by foreign investors with a perfectly elastic demand.

The presence of foreign investors ultimately pricing public debt prevents the price of public debt from reflecting the value of liquidity.[29] However, the availability of this asset will have implications for the price of interbank deposits. Given that the state-contingent payoff of public

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[27]In this case, the assumption of inelastic labor supply is relevant. If labor supply were elastic then the effect of crowding-out low-productivity banks would imply a reduction in labor demand that would in turn lower the equilibrium level of labor. Then the higher average productivity would be compensated by an extensive margin of lower hours.

[28]In the case where the distribution of idiosyncratic productivities has bounded support, the government would want to issue public debt up until the most productive bank is the only bank investing in its technology.

[29]This differs from most models in which public debt provides liquidity. In these models the price of public debt has a liquidity premium (e.g. Holmström and Tirole (1998), Kiyotaki and Moore (2005) and Krishnamurthy and Vissing-Jorgensen (2012)).

debt and interbank deposits is the same, no arbitrage implies that, in the cases in which there is exposure of the domestic banks to public debt, the price of interbank deposits will be equal to the price of public debt.

Given the liquidity value that public debt has domestically, a sovereign default will have associated a negative internal liquidity effect. After a sovereign default the expected return on public debt is now ζ, determined by the exogenous policy, which is lower than its expected return prior to the default. This also impacts the equilibrium price of interbank deposits which will be be higher (and its return lower), as indicated in proposition 2. Figure 3b illustrates the effect of the lower return on public debt due to a sovereign default in the labor market and in banks' investments. As shown in the right panel a lower risk adjusted return of interbank deposits will have an impact on banks' optimal portfolio choices. The banks that used to be indifferent between investing in their own productive technology and investing in public debt (or lending to other banks) will now prefer the former option as both public debt and interbank deposits yield a lower return. This reduces the threshold productivity level above which banks prefer to invest in their productive technology and generates an output cost since labor is now allocated into banks that are, on average, of lower productivity. Additionally, given that a higher share of bankers now prefer to invest in their productive technology, labor demand will increase and so will wages (left panel).[30]

3.3. *Output Cost Dynamics*

This section analyzes the dynamics of the output cost of default. To do so I perform an impulse-response-type of analysis by tracking the endogenous dynamics of output in response to a sovereign default. In the full model default is an endogenous decision that the government takes. However, for the purpose of this exercise we consider an exogenous default (that may be suboptimal) to analyze its impact on output.[31] Several impulse-response exercises are performed

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[30]This last prediction of higher wages after a default is at odds with empirical evidence. Tomz and Wright (2007) show that default episodes tend to be related with low levels of economic activity. Overall, the effect on wages will depend on the combination of the balanc-sheet effect and the liquidity effect, which push in opposite directions.

[31]Specifically, I analyze the effects of a sovereign default starting at the average state of the ergodic set that comes out of simulations computed in Section 5, unless noted otherwise. I simulate the economy 500 times for 20 quarters disregarding the optimal decisions of repayment and compute the evolution of output under repayment and default in the initial state. The simulations are made without aggregate productivity shocks, the only exogenous shocks is the timing of reentry to external financial markets. The average output path from the 500 simulations will reflect an economy that reenters financial markets with probability ϕ.

that differ in the initial exposure of banks to domestic public debt and on the values of the parameter ϕ that determines the length of the exclusion period (both dimensions will be relevant in determining the magnitude and persistence of the effect of a default on economic output). I use the parameters set in the calibration described in Section 5 so that the magnitudes can be appropriately interpreted. The results of the exercise are shown in Figure 4.

FIGURE 4. The Dynamics of the Output Cost of Default

(A) Output Cost under Different Exposure to Domestic Public Debt
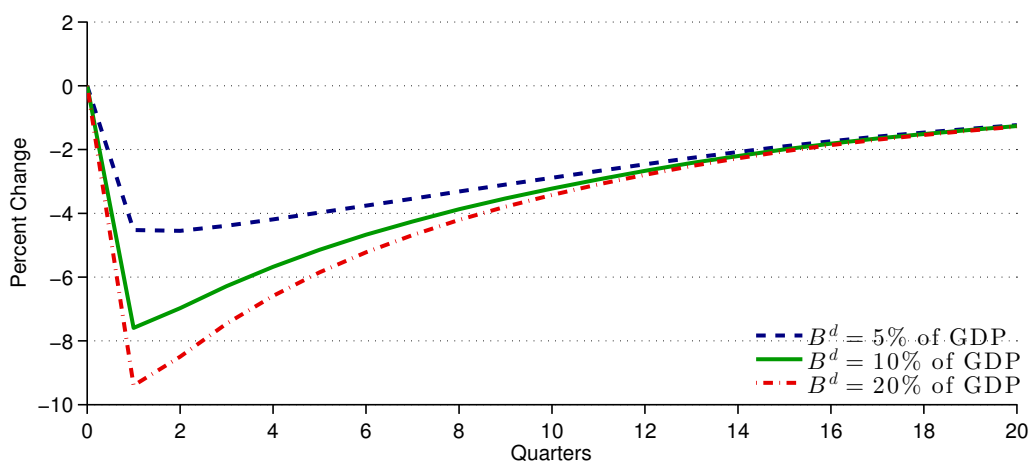
(B) Output Cost under Different Exclusion Periods
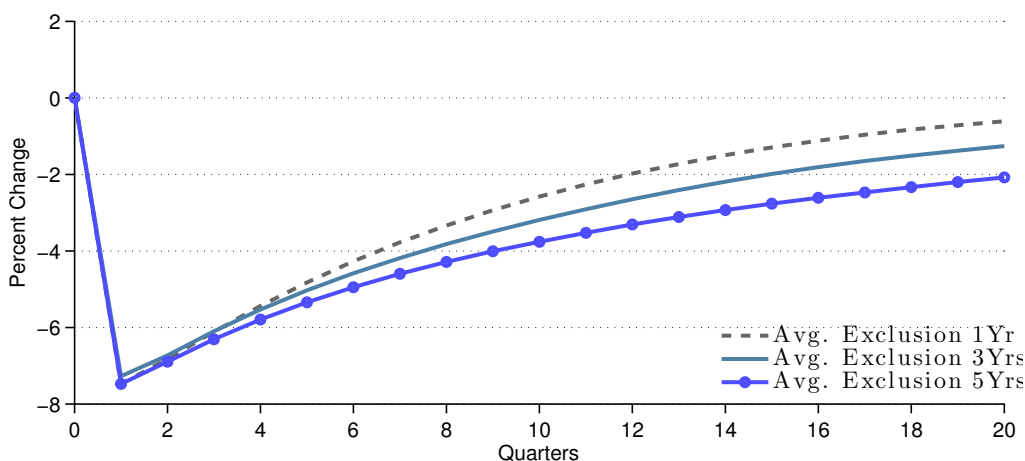
Notes: Graphs plot the percent change of output after a sovereign default with respect to the level of output under no default. Each line is computed by simulating the economy 500 times for 20 quarters under repayment and default in the initial state. The initial state is given by the average state from the ergodic set unless noted otherwise. Simulations are carried out without aggregate productivity shocks, the only exogenous shocks is the timing of reentry to external financial markets.

A sovereign default can precipitate a significant recession that can persist over time. Whereas the depth of the recession is more linked to the strength of the balance-sheet channel, the persistence of the recession is more related to the relevance of the liquidity channel. As shown in panel 7a, the depth of the recession will depend on the exposure of the banking sector to domestic public debt. A sovereign default in the context of a large exposure of banks to public debt will be associated with a larger impact on banks' net worth, a stronger balance-sheet cost and a steeper recession. According to the simulations, if the stock of domestic public debt held by banks at the time of a sovereign default is 5% of GDP, the associated cost is a drop in output of 3.5% on average in the following two years, followed by a slow recovery. If the stock of domestic public debt is 20% the output cost will be around 7.5% in the two years following the default.[32] On the other hand, the persistence of the recession will be linked to the period of government exclusion from external financial markets (panel 7b). If the government is excluded from financial markets for a protracted period of time, the liquidity effect will be persistent even if the banks' net worth recovers and the balance-sheet effect disappears. Results indicate that if the government faces an average exclusion period of a year ($\phi = 0.25$) output will be almost recovered 5 years after the default. If the average exclusion period is 5 years ($\phi = 0.05$), 5 years after the default output would still be 2.1% below what it would be in the absence of a sovereign default. This suggests that the persistence of the output cost of default is determined by the liquidity effect. The balance-sheet effect dissipates more rapidly as banks earn profits on their investments and rebuild their net worth.

3.4. *Output Costs and Sovereign Risk*

The governments' decision to repay or default will depend on the aggregate state of the economy. In particular, when defaulting the government will not only take into account the level of aggregate output $A\mathbb{E}[z|z > \underline{z}]$ and the stock of external public debt B^x (as it is in the standard Eaton-Gersovitz model), but also the stock of domestic public debt B^d as it will affect the magnitude of the output cost of default.

Figure 5 characterizes the repayment and default set and the pricing curve for external debt for the calibrated version of the model. The default set is increasing in B^x. This follows from the fact that the value of repaying is decreasing in B^x (since it implies less aggregate resources available for consumption) and the value of default is independent of B^x. Therefore, if default

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[32]Note that this exercise reflects the output cost that is due to a sovereign default. This cost can differ from the *observed* drop in output at the time of the default since, as we shall see, default tends to occur when there is a negative shock to the exogenous aggregate productivity.

is optimal for a given state with some level of external debt B^x it will still be optimal for a higher level of external debt. On the other hand, the default set is decreasing in the aggregate productivity shock g. This result is less obvious since both the value of repayment and the value of default are increasing in g and is due to the market incompleteness. The intuition behind it is that even if the government faced the same roll-over possibilities in the next period, default is more attractive in low-income times given that the marginal utility of consumption in these states is higher (due to the concavity of the utility function). In addition, investors will internalize this and will offer more stringent roll-over possibilities in low-income times, which will reinforce the attractiveness of default in this states (see, for example, Arellano (2008) for a formal derivation and an explanation of the result).[33] Combining these two results we get that the maximum amount of B^x that will be repaid on equilibrium is increasing in the realization of g, as shown in the left panel of Figure 5a. A corollary of this result is that, given the persistence of the aggregate productivity shock, the government will face a weakly higher price for its debt (for any level of external debt issuance) in a state with a higher realization of g (right panel of Figure 5a). A high realization of g increases the conditional expectation of next period's shock and therefore increases the repayment probability for any given value of $B^{x\prime}$. These features are shared in most Eaton-Gersovitz class of models (e.g. Arellano (2008), Aguiar and Gopinath (2006), Chatterjee and Eyigungor (2012)) and are consistent with the empirical evidence on sovereign defaults (Tomz and Wright (2007)).

A more novel result of this specific model is the role of domestic public debt in affecting the incentives to repay debt and sovereign risk. As shown in Figure 5b, the default set is decreasing in B^d. If default is optimal for a given state with some level of domestic debt B^d it will still be optimal for a lower level of domestic debt. This follows from the fact that while the value of default is independent of B^d, the value of repaying is increasing in B^d (since higher B^d, everything else equal, implies a higher level of banks net worth in the context of debt repayment , a more efficient allocation of resources within the banking sector and a higher average idiosyncratic productivity). In other words, as shown in Figure 7a, a sovereign default in the context of higher B^d will have a larger output cost and, for a given benefit of default, this makes it a less attractive option. Given that on equilibrium the policy function $B^{d\prime}(B^d, \cdot)$ is increasing in B^d (i.e. with a higher exposure to domestic public debt, the banks'

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[33]This result is not shared by models with limited commitment and complete set of contingent claims market (e.g. Kehoe and Levine (1993) and Alvarez and Jermann (2000)). In those models, default never occurs on equilibrium. However, the participation constraint binds in high-endowment states in which the borrower has to pay back debt to the planner.

FIGURE 5. Default Decisions and Sovereign Risk

(A) Aggregate Productivity and External Public Debt

Default Decision

Sovereign Debt Price

(B) Domestic and External Public Debt

Default Decision

Sovereign Debt Price
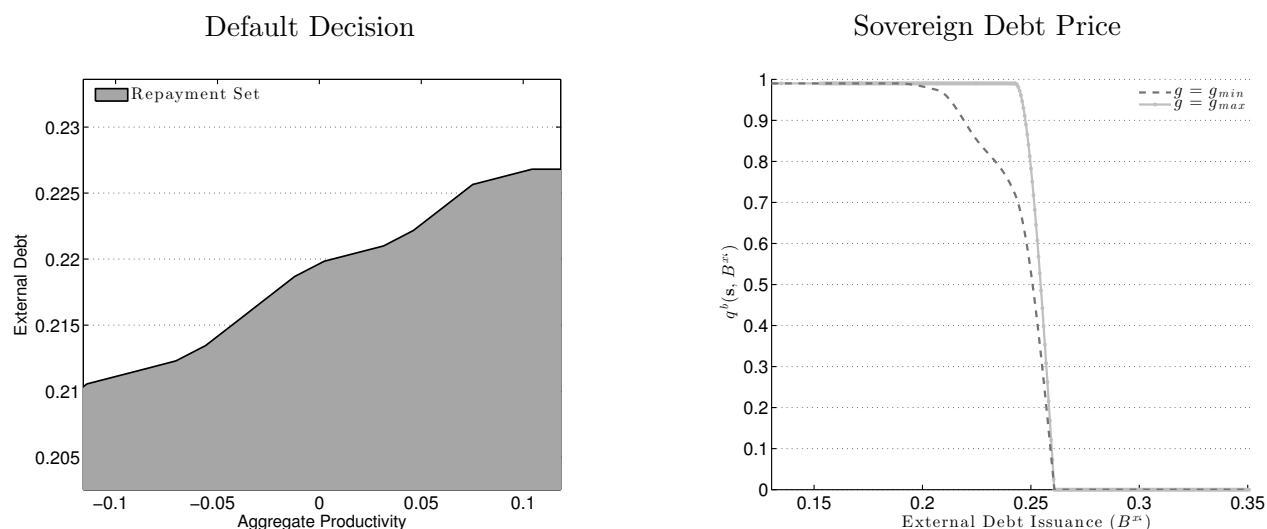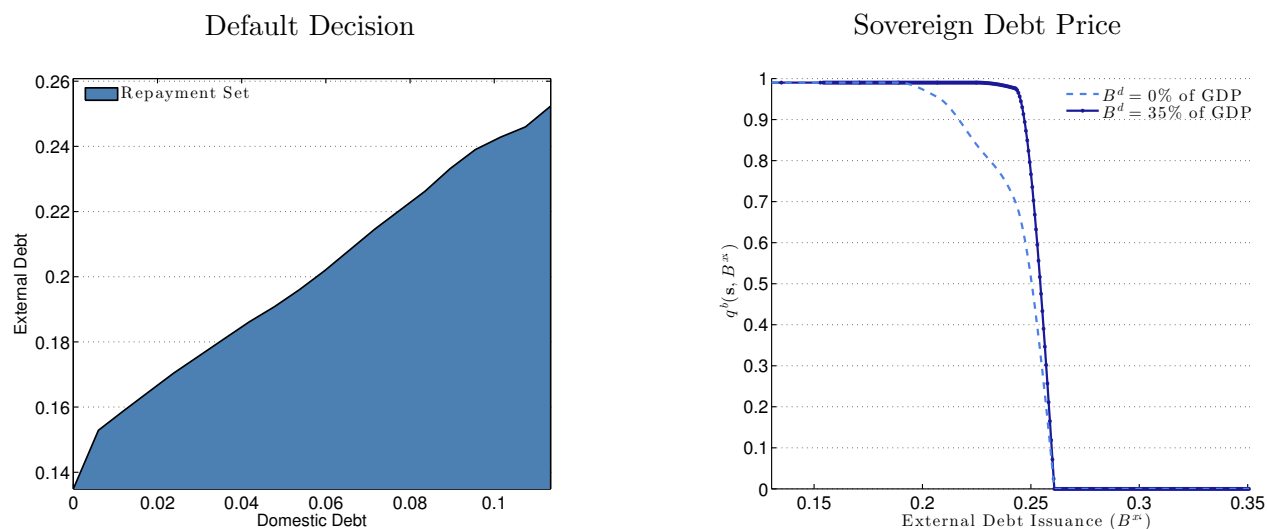
Notes: The omitted states in the figures are fixed at the average levels of the ergodic set. External and domestic debt are expressed as % of annual GDP. g_{max} (g_{min}) correspond to a positive (negative) shock to aggregate productivity of the order of four standard deviations.

aggregate net worth is higher and this allows banks to buy more domestic public), a high stock of domestic public debt today is an indicator of a high stock of domestic public debt next period and therefore of a lower probability of default. This is reflected in the price that foreign investors ask for a given level of external debt issuance. The effect is quantitatively important,

as indicated in the right panel of Figure 5b. When there is no exposure of the banks to domestic public debt the government can issue risk-less external debt up to a little more than 15% of GDP and beyond that level debt rapidly increases its riskiness. On the other hand, when the stock of public debt held in the domestic financial system banks is 35% of GDP the government can issue risk-less external debt up to a little more than 22% of GDP. The next section tests the predictions of the model on what are the determinants of sovereign risk.

4. Cross-Country Evidence on Spreads and Debt

This section conducts an empirical analysis of the testable implications of the model regarding the government's incentives to repay and its relation with sovereign risk. In particular, I investigate the relationship of sovereign spreads with the stock of domestic and external public debt and the level of economic activity.

As predicted by the model, the stock of external and domestic public debt are related to sovereign spreads in opposite directions. An illustrative example is the case of Brazil (Figure 6). As shown in panels 6c and 6b, during the periods of 2000.Q4-2001.Q4 and 2011.Q1-2012.Q2 the Brazilian economy exhibited comparable levels of public debt (66% and 64% of GDP, respectively) and was in a similar phase of the business cycle (economic output was 0.6% and 0.2% above trend, respectively). Nevertheless, sovereign spreads differed substantially in both periods. In 2001 sovereign spreads averaged 800 basis points and in 2011-12 only 250 basis points (panel 6a). A potential explanation for such differences in the level of sovereign risk in both periods is the shift in the composition of public debt. In spite of having similar levels of public debt, while in 2001 27% of the stock of total public debt was external debt, only 5% was external debt during 2011.Q1-2012.Q2 (panel 6c). As argued in the previous section, a higher stock of domestic debt increases the cost of default and a lower stock of external debt reduces the benefits of default, leading to a reduction in sovereign risk.

In order to provide a more comprehensive empirical analysis of the relationship of sovereign spreads with the stock of public debt and economic activity I collected data on these variables for a set of emerging economies. The sample of economies covered in the analysis includes countries that are -or were once included- in J.P. Morgan's Emerging Markets Bond Index Global (EMBIG), subject to the constraint of having sufficient data availability. 15 countries were included in the sample, namely, Argentina, Brazil, Bulgaria, Colombia, Dominican Republic, El Salvador, Hungary, Indonesia, Lithuania, Mexico, Panama, Peru, Philippines, Poland and

FIGURE 6. Public Debt Composition and Sovereign Risk: The Case of Brazil

(A) Sovereign Spreads (in bps)

(B) Economic Activity (Output Gap, in %)

(C) Public Debt and Public Debt Composition
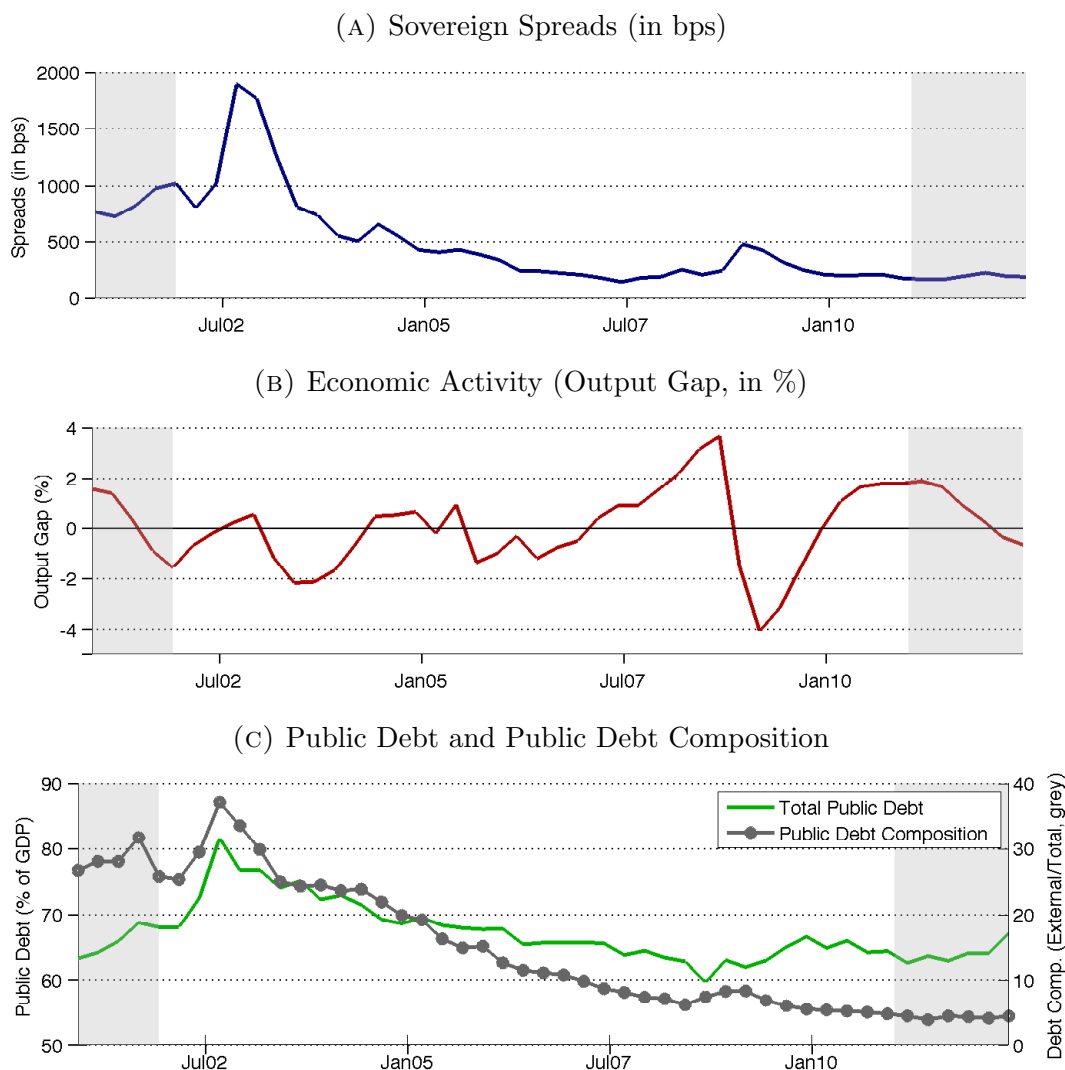
Uruguay.[34] The empirical analysis is carried out with quarterly data. Data on spreads was collected from Datastream and data on GDP and domestic and external public debt was obtained from the IMF and IDB databases. The time period ranges from 1994.Q1 -when EMBIG spreads are initially available- until 2012.Q2. However, most of the observations correspond to the last part of the sample since quarterly data on public debt becomes available only recently for many countries.

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[34]The number of countries is relatively small, with a majority being Latin American countries, given that data on domestic and external public debt on a quarterly frequency is only available for a small set of countries most of them from Latin America.

A bond spread is the excess return of the bond over a risk-free zero-coupon bond (i.e., a US Treasury) of the same maturity. A country's spread is a synthetic measure of the spreads of a representative basket of bonds issued by that country. It is determined in annualized terms and measured in basis points. It reflects the implicit interest rate premium required by investors to be willing to invest in a defaultable bond of that particular country. The stock of domestic and external public debt are measured as a percentage of annual GDP. Finally, output is measured as percentage deviations from trend output.[35]

To assess the conditional co-movements between sovereign spreads and the level of domestic public debt, external public debt and economic activity, a set of panel regressions were estimated. These regressions estimate sovereign spreads as a linear function of output, the stock of external public debt and the stock of domestic public debt as well as time fixed effects and country fixed effects to control for potential systemic shocks to investors' stochastic discount factor and country-specific risk. Table 1 reports the regressions estimates. Column 4 shows the results of the baseline estimation that includes both time fixed effects and country fixed effects. Results indicate that sovereign spreads are negatively related with the level of economic activity. The coefficient on output is negative and significantly different from zero at the 1% level. The coefficient estimate implies that an increase of 1% in output gap is associated with a decrease in sovereign spreads of the order of 41 basis points (as a benchmark, the average level of sovereign spreads in the sample is 410 basis points). The coefficient on external public debt is positive and significantly different from zero at the 1% level suggesting that sovereign spreads are positively related with the level of external public debt. According to the point estimate, an increase in the stock of external public debt from 20% of GDP to 40% of GDP is associated with an increase in spreads of 200 basis points. These results are consistent with the findings of previous empirical studies of sovereign spreads (e.g. Edwards (1984), Maltritz (2012)).

A more novel result is the negative relation between sovereign spreads and the level of domestic public debt. The coefficient on the stock of domestic public debt is negative, significantly different from zero at the 1% level and implies that an increase in the stock of domestic public debt from 20% of GDP to 40% of GDP is associated with an decrease in spreads of 150 basis points.

Regressions 1-3 estimate similar specifications dropping time and country fixed effects one at a time. The coefficient on output remains negative and significant for all specifications. The

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[35]Trend output is computed by applying HP filter to the seasonally adjusted output series.

TABLE 1. Spread Regressions

	(1) Sov. Spread	(2) Sov. Spread	(3) Sov. Spread	(4) Sov. Spread
Output	-39.82***	-47.49***	-23.41**	-42.31***
	(6.731)	(5.071)	(9.686)	(6.446)
External Public Debt	1.716*	9.024***	2.024**	10.00***
	(0.966)	(1.251)	(0.929)	(1.067)
Domestic Public Debt	-0.144	-7.575***	1.118	-7.889***
	(0.925)	(2.128)	(0.864)	(2.154)
Observations	443	443	443	443
Average Spread	409.8	409.8	409.8	409.8
Country Fixed Effects	No	Yes	No	Yes
Quarter Fixed Effects	No	No	Yes	Yes

Notes: Spreads are measured in annualized basis points. Output is measured in percentage point deviations from trend output (HP filter trend). External and domestic public debt are expressed in percentage points of annual GDP.

coefficient on external public debt also remains negative and signifiant at the 1% level under the specification with only country fixed effects and is significant at the 5% and 10% level under the specifications with only quarter fixed effects and no fixed effects, respectively. Finally, the coefficient on domestic public debt remains positive and significant for the specification with country fixed effects and looses significance in the remaining two specifications.

In summary, the results presented in this section show that the levels of sovereign spreads are affected by the level of economic activity, the level of external public debt and domestic public debt in a way that is consistent with the predictions of the model discussed in Section 3.4. In particular, it is shown that: *(i)* spreads covary negatively with the level of economic activity, *(ii)* spreads covary positively with the level of external public debt, *(iii)* spreads covary negatively with the level domestic public debt. This last result has not been previously analyzed by the empirical literature that studies sovereign risk.[36]

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[36]Most of the empirical literature on sovereign spreads investigates the relationship between spreads and the level of public debt or external debt and the level of economic activity. See, for example, Edwards (1984) and more recent works of Uribe and Yue (2006) and Maltritz (2012).

5. Quantitative Analysis of the Model

This section performs a quantitative analysis of the model by calibrating it to the Argentinean economy for the period 1994-2012. I consider the Argentinean economy to be an interesting case for study for three reasons. First, Argentina is an economically relevant emerging economy that is integrated into international financial markets that has already been studied in depth by previous literature on sovereign default.[37] Second, the period of analysis includes one of the largest sovereign defaults on history. In December 2001 the Argentinean government explicitly defaulted on $95 billion of external debt which represented 37% of its GDP. Additionally, by imposing a unfavorable swaps and the conversion of dollars to pesos of its domestic debt it also implicitly defaulted on the outstanding stock of domestic debt at that time.[38] Finally, throughout the entire period of analysis the economy exhibited significant levels of external public debt (23% of annual GDP on average) as well as domestic public debt in held in the banking system (9.3% of annual GDP on average), which makes it a natural candidate for testing a theory that relies on the presence of significant levels of both domestic and external public debt.

5.1. *Calibration*

One period in the model will correspond to one quarter. For the quantitative analysis the following functional forms for the utility function and the distribution of the aggregate and idiosyncratic shocks are used. The instantaneous utility function is assumed to be

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}.$$

Additionally, I assume that idiosyncratic productivity shocks z are distributed Pareto with shape parameter λ (i.e. $G(z) = 1 - z^{-\lambda}$) and that the growth rate of the aggregate productivity is approximated with a log-normal AR(1) process with long run mean μ_g and persistence coefficient $|\rho_g| < 1$, i.e.

$$g_t = (1 - \rho_a)\left(\ln\mu_a - \frac{1}{2}\frac{\sigma_a^2}{1-\rho_a^2}\right) + \rho_a g_{t-1} + \sigma_a \varepsilon_t \qquad \varepsilon_t \sim N(0,1).$$

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[37]Some of papers that provide a quantitative analysis of the Argentinean economy are Arellano (2008), Aguiar and Gopinath (2006), Mendoza and Yue (2012), Chatterjee and Eyigungor (2012), Hatchondo and Martinez (2009).

[38]See Sturzenegger and Zettelmeyer (2008) for an analysis and explanation of the Argentinean sovereign default on external and domestic debt.

The model is parametrized by household specific parameters (β, γ), bank-related parameters $(\sigma, \kappa, \lambda, \mu_a, \rho_a, \sigma_a)$ and government related parameters (R, ϕ, ζ). The model parameter values are summarized in Table 2. The risk aversion coefficient γ is set to 2 and the risk-free interest rate is set to $R = 1.01$, which are standard in quantitative business cycle studies. The reentry probability to external financial markets is set to 0.083 which implies an average period of exclusion of three years which is consistent with the median period of exclusion from international credit markets found in Dias and Richmond (2008) and also in the range of estimates of Gelos et al. (2011).

TABLE 2. Calibrated Parameters

Parameter		Value	Comments
From Literature			
Risk aversion coefficient	γ	2	Standard RBC value
Risk free interest rate	R	1.01	Standard RBC value
Reentry probability	ϕ	0.083	Dias and Richmond (2008)
Shape of idiosyncratic prod. dist.	λ	3.50	Gopinath and Neiman (2011)
From External Data			
Banks LC constraint	κ	7.50	Argentina Banks data
Public debt return in autarky	ζ	0.98	Argentina Banks data
Average growth rate	μ_a	1.01	Argentina GDP data
Calibrated			*Target from data*
Growth rate autocorrelation	ρ_a	0.30	GDP autocorrelation
Std. deviation of growth shocks	σ_a	0.022	GDP volatility
Discount factor	β	0.90	Frequency of default (0.66% quarterly)
Bankers survival probability	σ	0.787	Domestic public debt in banks (9.3% of GDP)

The value of shape of the distribution of idiosyncratic productivity shocks is less straightforward since we need disaggregated data on idiosyncratic productivity. Gopinath and Neiman (2011) use firm level data to study the adjustment in firms' imports in Argentina during the 2002 crisis.[39] I set $\lambda = 3.5$, which generates a standard deviation of productivities of banks that is in line with the cross-sectional dispersion of Argentinean firm productivities estimated in their paper. The parameters of the exogenous process for aggregate productivity were calibrated to match the standard deviation and autocorrelation of detrended GDP in the model as

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[39]I thank Brent Neiman and Gita Gopinath for sharing the moments of their data.

well as the average quarterly growth rate.[40] The corresponding estimated values are $\mu_a = 1.01$, $\rho_a = 0.3$ and $\sigma_a = 0.022$.

The parameter κ in the banks' limited commitment constraint is set to 7.5 to match the average leverage ratio of total net worth to total assets in the banking system of 12% during the sample period.[41] The maximum leverage is also consistent with the bank capital requirements stipulated in Basel-II. However, the targeted leverage ratio is higher than the ones considered in other quantitative studies of the banking system for advanced economies, for example, Gertler and Karadi (2011) and Bocola (2014) use values of leverage ratios in the range of 20-25%.

The return ζ on public debt in periods of exclusion from external financial markets is obtained by analyzing the liquidity management of Argentinean banks before and after the sovereign default of December 2001. During the period of 1995-2001 banks held 6.8% of their assets in liquid government securities. During the period of 2006-2010 after the default, the share of government securities in the banks balance-sheets dropped to 2.7% of total assets (see Figure 10 in Appendix D). Most of this reduction was done at the expense of an increase in banks cash holdings from 3.4% of total assets in 1995-2001 to 15.6% in 2006-2010.[42] Motivated by this fact, I assume that exogenous domestic debt issuance policy after a sovereign default is characterized by a risk-less return on debt that is the same as the return on cash. Therefore I set $\zeta = 0.98$ which is consistent with an average quarterly rate of inflation of 2.5% observed in the sample period.[43]

Finally, the household discount factor β and the exit probability of bankers σ were jointly calibrated to match two moments for the Argentinean economy: a frequency of default of 0.66%

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[40]I adopt this strategy, rather than estimating a an AR(1) process to the quarterly growth rate of seasonally adjusted GDP, since there is an endogenous component to the GDP (in addition to the aggregate productivity) which is given by the average idiosyncratic productivity of banks that are investing in their technology. The calibrated parameters are similar to those obtained with the most standard method.

[41]Historical data on the Argentinean consolidated banking system was obtained from the central bank BCRA. GDP data was obtained from finance ministry MECON.

[42]I exclude the period 2002-2005 that immediately follows the default since during that period the banks had on their balance-sheet government securities that were granted to them as a compensation for the net worth losses that were due to an asymmetric pesification of banks assets and liabilities. For an analysis of the government policies during the 2001 Argentinean financial crisis see Barajas et al. (2007).

[43]The average inflation is computed with INDEC data up until 2007. After then the official measure of inflation stops being trustworthy and the average inflation is taken from taking an average of alternative private measures. See Drenik and Perez (2014) for an explanation of the manipulation of the official CPI figures and for the use of alternative measures of CPI in Argentina for that period.

(which corresponds to five sovereign defaults in the past 188 years) and an average stock of public debt held by banks of 9.3% of annual GDP. The calibrated values were $\beta = 0.9$ and $\sigma = 0.79$. A low discount factor is needed to obtain high levels of external debt issuance and high interest rate spreads. The calibrated value is in the lines of other models of endogenous sovereign default.[44] A sensitivity analysis of the main results to certain key parameters in the model is carried out in Appendix C.

5.2. *Business Cycle Properties of the Model*

This section assesses the model's quantitative performance by comparing moments from the data with moments from the model's ergodic set. To compute the model's moments I simulate the exogenous productivity process g for 5000 periods and trace the evolution of the endogenous states. The moments are computed by eliminating the first 100 observations. The moments from the data were computed for the sample period 1994-2012, excluding the period 2002-2005 in which the Argentinean government was declared on default.[45] The data on National Accounts variables as well as the data on public debt was obtained from MECON. The data on interest rate spreads was obtained from JP Morgan's EMBIG spread which provides a measure of average spreads of a basket of Argentinean sovereign bonds denominated in foreign currency.

Table 3 compares the data moments with the moments generated by the model that were not targeted in the calibration. The model successfully reproduces the observed levels of external public debt issuance (23% of annual GDP on average). This suggests that the presence of an endogenous internal cost of default generated by the balance-sheet effect and the liquidity effect, together with the exogenous punishment of external financial autarky for a random number of periods, are able to generate enough commitment for the government to explain observed levels of external debt issuance.[46] The model underestimates the average levels of sovereign spreads.

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[44]For example, the calibrated value of β is 0.88 in Mendoza and Yue (2012) and 0.8 in Aguiar and Gopinath (2006). These low discount factors are often justified with political economy arguments that make the governments more impatient. Arellano (2008) uses a higher calibrated value for β of 0.953.

[45]Data moments do not change significantly if I include the 2002-2005 period. I exclude this period given that data on spreads correspond to defaulted bonds in arrears.

[46]Previous quantitative models of endogenous default, calibrated for the Argentinean economy, have generated levels of external debt that are, in some cases, lower than observed levels. For example, Arellano (2008) generates levels of external debt of the order of 6% of quarterly GDP and the average level of debt in the model in Aguiar and Gopinath (2006) is 27% of GDP. More recently, Mendoza and Yue (2012) report an average level of external debt-to-annual GDP ratio of 23% and Chatterjee and Eyigungor (2012) use a model of long-term debt that generates levels of external debt of the order of 70% of quarterly GDP.

The average bond spread in the model's simulations is 130 basis points measured on a quarterly basis, which is slightly below the average spread of 194 basis points observed in the data.

TABLE 3. Business Cycle Statistics

Statistic	Data	Baseline Model
Average		
External Debt (% of annual GDP)	23	22
Interest rate spread (quarterly, in bps)	194	131
Volatility		
Public debt	0.24	0.07
Interest rate spread	152	57
Consumption ($\sigma(c)/\sigma(y)$)	1.01	1.32
Trade balance	0.02	0.03
Correlations		
Output - Consumption	0.93	0.74
Output - Trade balance	-0.16	-0.03
Output - Interest rate spread	-0.43	-0.55
Public debt - Interest rate spread	0.13	0.10

Notes: Data moments are computed with quarterly data for the period of 1994.Q1 - 2012Q4 excluding the the post-default period of 2001.Q4- 2005.Q3. Moments from the model are computed from simulating the economy for 5000 periods. Public debt volatility is measured as the coefficient of variation.

The variability of public debt and interest rate spreads is underestimated in the model. The reason is that, given that households are relatively impatient, the government finds optimal to issue the maximum levels of external debt that it can credibly commit to repay (in most state-realizations). As argued by Chatterjee and Eyigungor (2012), these are common features of models of endogenous default that are based on short-term debt (i.e. one-period securities with quarterly periods) and can be improved to better reflect the data moments by using long-term debt.

The model correctly predicts the high co-movement between aggregate consumption and output. Additionally, the model predicts excess volatility of consumption with respect to output although higher in magnitude from what is observed in the data. Two reasons can help understand the excess volatility of consumption relative to output in the model. The first one is that

shocks to aggregate productivity affect the growth rate of productivity. A positive shock to the growth rate of productivity implies a increase in current output and a larger increase in future output. Given this permanent shock to productivity, the optimal reaction of agents (under access to risk-free debt) is be to increase consumption by more than the increase in output.[47] The second reason is that the government faces a endogenous interest rate that is correlated with the business cycle through its relation with the government's incentives of default. As argued in section 3.4, the fact that default is more attractive in states with low realization of the aggregate productivity shock implies that the interest rate that government faces for issuing external debt in low productivity states is higher and this dissuades the government from issuing additional debt to smooth consumption.

The other face of excess volatility of consumption is the counter-cyclicality of trade balance, which is a key feature of emerging markets business cycles. The model yields a negative correlation between trade balance and output, though small in magnitude. As the interest rate increases in bad states sovereign borrow less and net exports increase.

Finally, the model is also consistent with a negative co-movement between interest rate spreads and output and a mild positive correlation between public debt and interest rate spreads. The counter-cyclicality of interest rate spreads can be understood again with the relative attractiveness of default in low productivity states. During bad times the likelihood of default is increases and so do interest rate spreads. The second result is more related to the fact that the total stock of public debt includes external debt and domestic debt which, as was already argued, have different implications for sovereign risk and thus would tend to attenuate any co-movement between the total stock of public debt and interest rate spreads.

5.3. *Output Dynamics Around Default Episodes*

This section studies the dynamics of output in the model around sovereign default episodes and compares it to the data. The dynamics of output in the model are given by the average path of output around the episodes of default identified in the simulations. Figure 7a plots the model's output dynamics compared to the Argentinean output around the default episode of 2001.Q4. Both series are shown as percentage deviations from trend output.[48] One-standard-bands for the model's average are included. The date of the default is set to zero and the window goes from 4 quarters before the default episode to 12 quarters after.

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[47]See, for example, Aguiar and Gopinath (2007) and García-Cicco et al. (2010) for an analysis of business cycle models in emerging economies with permanent and temporary productivity shocks.

[48]Trend output was obtained by applying an HP filter to both the observed and simulated series of output.

FIGURE 7. Output Dynamics Around Default Episodes

(A) The Argentinean Default
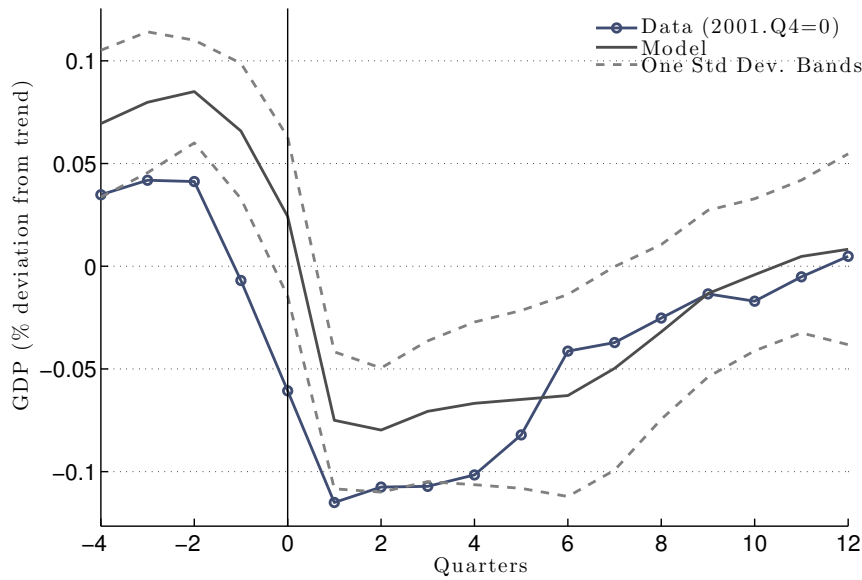
(B) Average Default Episode in Emerging Markets
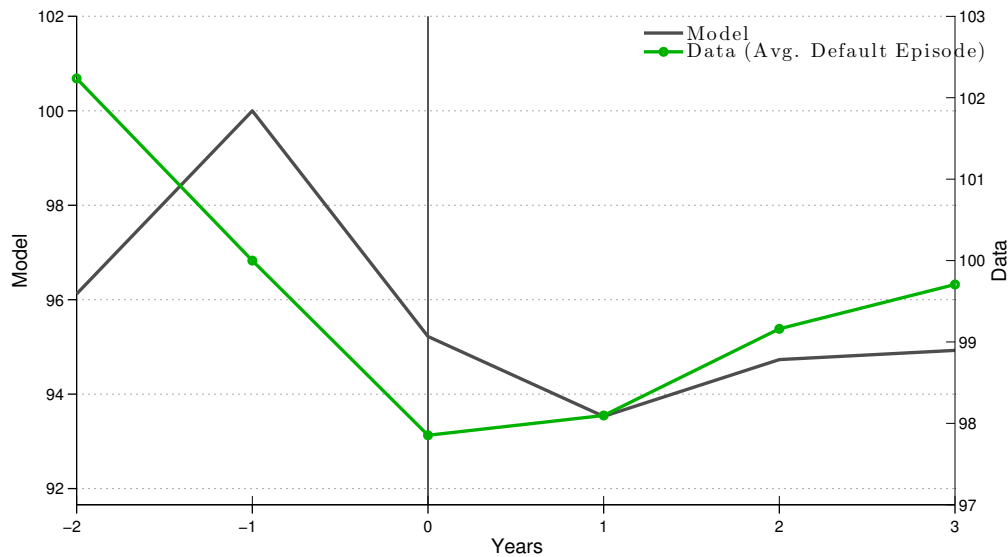
Notes: Model data is obtained from identifying default episodes in simulations and computing the average behavior of output around those episodes. Output around the average default episode in Emerging Markets is the average evolution of output around 41 default episodes identified in Reinhart and Rogoff (2009) for the period of 1980-2005.

The model's output dynamics are similar to the evolution of output during the 2001 default. The overall behavior of output under the simulated default episode replicates the evolution of

output in Argentina both in terms of the magnitude of the fall and the recovery dynamics. In the model, the peak-to-trough fall of output is 15%, the same magnitude as the observed fall in output during the 2001 default. Additionally, the post-default output recovery in the model is consistent with the Argentinean experience. In both cases output starts recovering within a year of the default and three years later recovers its trend level but is still below its pre default levels. The model fails to account for the persistent fall in output prior to the 2001 default. In particular, while Argentinean output started falling three quarters before the default event, output in the model starts decreasing only one quarter before the default event. This fact could be explained with models of long term debt in which an anticipation of the probability of default can be recessionary due to the balance-sheet effect (as in Bocola (2014)). It is important to highlight that the dynamics of output under default episodes were not a target of the calibration.

The fall in output in the model during a default event has two components. Both components lead to a lower productivity.[49] One is the exogenous drop in aggregate productivity that triggers the default. As argued before, default tends to occur in periods of low aggregate productivity, so the observed drop in output will likely include a fall in the aggregate exogenous productivity. In particular, the fall in output in the model prior to the default is due to the fall in aggregate productivity. The second component is the endogenous output cost that comes from the internal costs of default. The following subsection analyzes the latter part of the fall in output.

Figure 7b compares the average output dynamics in the model with the average evolution of output around 41 default episodes in different economies. The default episodes analyzed are those identified in Reinhart and Rogoff (2009) for the period of 1980-2005. The dynamics of output in the model around default episodes are qualitatively similar to the average dynamics of output around cross-country default episodes. Both in the model and in the average default episode, output recovers slowly following a sovereign default and is still below trend in both cases three years after. However, the quantitative behavior is different since the fall in output for the model is higher than the average fall in output for the cross-country default episodes.[50]

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[49]Consistent with the model's implications, Kehoe (2007) argues that most of the drop of output in the Argentinean crisis was due to a fall in TFP. Additionally, Sandleris and Wright (2014) use firm-level data to show that of the fall in TFP in Argentina, most of it was due to labor misallocation.

[50]It would be harder to match the quantitative features of output around defaults for the cross-country episodes given that the model was calibrated to match the Argentinean economy.

5.4. *Disentangling Internal Costs of Sovereign Default*

This section uses the calibrated model to assess the economic impact of the internal costs of sovereign default and disentangle the relevance of the balance-sheet effect and the liquidity effect separately. In particular, I study how these mechanisms quantitatively affect output dynamics and how their presence enhances commitment for the government and allows it to credibly issue positive levels of external debt. To do so I perform counterfactual exercises in which I eliminate these mechanisms one at a time and analyze its impact on key macroeconomic variables.

The balance-sheet effect is eliminated in the model by allowing for post-default government bailouts of the banking system. These bailouts consist of a tax to households for an amount equivalent to the aggregate exposure of the banking system to public debt, that is then reimbursed to banks in the form of equity injections.[51] By implementing such bailout the aggregate net worth will not be hit by a sovereign default and therefore the balance-sheet effect will not be in place. The liquidity cost is eliminated by setting the return on debt in periods of external financial autarky to be the same as the international interest rate $\zeta = R$. This way the banks can still access the same set of assets with same expected returns when they face their portfolio choice problem in states in which the government has access to financial markets and in states in which the government is in external financial autarky.

The first exercise is from an *ex-post* perspective and studies the effect of a sovereign default on aggregate output and disentangles how much of that effect is due to the balance-sheet effect and how much is due to the liquidity effect. To trace the dynamics of output after a sovereign default I use the simulations of the baseline model to identify the states in which it is optimal for the government to default. For these states I compute the dynamics of output under a sovereign default and compare them to the output dynamics that would result if the government repaid its debt and avoided default following the same routine as the one described in section 3.3 for the impulse-response type of exercises. This exercise is designed to analyze how much of the fall in output around a default is due to the default decision and the internal costs of default. Figure 8 shows the average effect of a sovereign default on output for the baseline model in those states where it is optimal for the government to default. A default triggers a drop in output as strong as 7.8% that then recovers gradually and 5 years later is 1.5% below what it would be in the absence of a default. This implies that of the 15% peak-to-trough fall in output in the model around default episodes, half of it is explained by the internal costs (which

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[51]A detailed description of how the bailout is implemented in the model is discussed in section 6.

leaves the other half explained by the fall in exogenous aggregate productivity that triggered the default). In other words, the sovereign default triggers an amplification effect on the contraction of economic activity of approximately 100%. Over the three years following a default, output is on average 5% below what it would be in the absence of default.

Interestingly, the output costs magnitudes are also comparable to the magnitudes of default costs considered in other quantitative models of sovereign default. Mendoza and Yue (2012) find that a shift from imported to domestic inputs in the production function due to a sovereign default generates a drop of 5% on output which then follows an endogenous recovery. Bocola (2014) finds that a typical default event precipitates a decline in output of 3%. It is also comparable to exogenous output costs used to calibrate models of endogenous default. Arellano (2008) calibrates an exogenous asymmetric cost of default that is on average 4% for the entire period of exclusion from financial markets whereas Aguiar and Gopinath (2006) and Yue (2010) consider a proportional drop in output of 2% in the autarky period.

FIGURE 8. Disentangling the Output Costs of Default
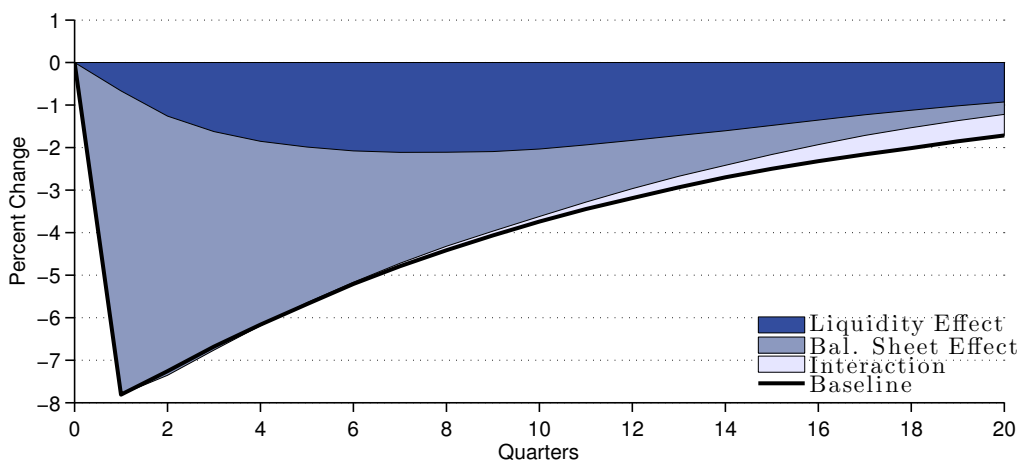
The model is then solved without the balance-sheet effect and without the liquidity effect and the same exercise is performed over the states in which it is optimal to default for the government in the baseline model. The output cost that comes out of the model without balance-sheet effect is attributed to the liquidity effect and, similarly, the output cost that comes out of the model without liquidity effect is attributed to the balance-sheet effect. Finally, the residual of the output cost in the baseline model that exceeds the sum of the output cost under the model without the balance-sheet effect and the model without the liquidity effect is interpreted as the output cost that is due to the interaction between the balance-sheet effect and the liquidity effect.

As shown in Figure 8 of the total output cost of default (defined as the integral of the output cost over the first 20 quarters following a default) 65% of it is explained by the balance-sheet effect, while the remaining 35% is explained by the liquidity effect. The fact that the interaction does not explain any of the output costs suggest that the two effects are additive. Even though the model is non-linear, non-linearities do not play a major role. The relevance of each effect changes as time goes by. The balance-sheet is predominant immediately after the default takes place and it gradually fades away. On the other hand, the liquidity effect is more persistent and explains the most of the output cost in the years 3 to 5 after the sovereign default. As explained in section 3, the balance-sheet effect is associated with an immediate effect that gradually dissipates as banks earn profits on their investments and the aggregate net worth recovers endogenously. The liquidity effect also has an immediate effect that persists over time as the government remains in financial autarky. It follows that the balance-sheet effect will govern the *depth* of the recession, whereas the liquidity effect will determine the persistence of the recession. The liquidity effect, while less important quantitatively, will make the slump last longer. Finally, it is interesting to note that the interaction of these two mechanisms plays no quantitative role in explaining a fraction of the output cost.

It is important to note what are the key aggregate moments that allow us to identify the quantitative relevance of each channel. The source of identification in the quantitative implementation of the model comes from two dimensions of exposure of the banking sector: the aggregate exposure of banks to public debt and the liquidity management of banks before and after default. As argued in section 3.3, the strength of the balance-sheet effect is directly linked to the exposure of banks to public debt. Therefore, the quantitative strength of the balance-sheet effect is guided by observed exposure of banks to public debt. On the other hand, the shift in banks liquidity management from using public debt to investing in cash holdings is informative of the strength of the liquidity effect. A stronger liquidity effect would necessarily be associated with a lower return of public debt in autarky and/or a larger period of exclusion.

The second exercise adopts an *ex-ante* perspective by studying how the presence of both effects affects the government's commitment to repay its debt and its ability to credibly issue external debt in international markets. To assess the relevance of each effect I solve and simulate the model under four different specifications: the baseline model, the model with no balance-sheet effect, the model with no liquidity effect and the model with neither balance-sheet nor liquidity effects. Table 4 reports average statistics of key macroeconomic variables for the different specifications.

TABLE 4. Disentangling Default Costs and Government Commitment

Statistic	Baseline Model	No Liquidity Effect	No Balance Sheet Effect	No Liquidity nor Bal. Sheet Effect
Average External Debt	22.3%	14.0%	7.7%	0.9%
Average Domestic Debt	9.4%	9.0%	9.0%	10.3%
Frequency of Default	0.35%	0.18%	0.36%	0.20%
Output Cost (next 3 yrs)	-5.43%	-3.91%	-1.75%	0.00%
Baseline Simulation States				
Frequency of Default	0.35%	100.00%	100.00%	100.00%

Notes: External and domestic debt are in % of annual GDP. Output cost is the average percentage deviation of output under default with respect to output in absence of default for the following twelve quarters after a default. The last row reports the frequency of default if the government in each specification faced the states that come out of the simulations of the baseline model.

Column 1 shows the average statistics for the baseline model. The average external public debt in the simulations is 22% of GDP and the average domestic public debt is 9.4% of GDP, the last statistic was targeted by the calibration to match its data counterpart. The frequency of default is 0.4%. The output cost reported of 5.4% is obtained by computing the average for all the default episodes in the simulations, of the percentage deviations of output under default with respect to output in the absence of default for the 12 quarters that follow a default.

Column 2 reports the average statistics that correspond to the simulations of the model under no liquidity effect. The absence of the liquidity effect attenuates the output cost of default and makes default a more attractive option ex-post. This in turn reduces ex-ante commitment for the government and undermines its ability to credibly issue external debt. The average external debt under this specification is 14.0% of GDP, which is 37% lower than the average level under the baseline specification. The stock of domestic debt is 9.0% of GDP, comparable to the average level in the baseline model. The frequency of default is 0.18%, lower than in the baseline case. Finally, the average output cost implicit in the states in which default is optimal is 3.9%, lower than the output cost reported in the baseline specification. This result is consistent

with the previous exercise that showed that liquidity costs accounted for approximately 35% of the total output cost and less if we only consider the first three years after a default.[52]

It is important to note that the most informative statistic that captures how removing each of the mechanism affects the government commitment is the level of external debt, rather than the frequency of default or the interest rate spreads. The level of external debt that the government can issue is directly related to its level of ex-ante commitment. Given that households are relatively more impatient than foreign investors, the government optimally chooses to issue levels of external debt that have associated a positive default risk, which is determined by the government's level of commitment.[53] The equilibrium frequency of default, on the other hand, is more related to the steepness of the price schedule of public debt as a function of the choice of external debt issuance and not necessarily to the level of commitment. The government recognizes that an additional unit of debt has an indirect effect on the effective amount of external borrowing $q^b(B^{x\prime})B^{x\prime}$ due to the reduction in the price of debt by the slope of the pricing curve. If the price schedule is steep the government will optimally choose to issue external debt up until the point it starts being risky. If the price schedule decays smoothly the government will choose riskier debt in equilibrium.[54] It follows that how interest rate spreads and the frequency of default change in the different model specifications will depend on how the presence of each mechanism affects the slope of the price curve.

The last row of Table 4 reports the frequency of default if, instead of following the endogenous path of state variables that comes out of each simulation, we feed in the states of the simulations for the baseline specification and compute the optimal repayment decisions for each model specification. A government in an economy that does not experience the liquidity effect after a default would never choose to repay the public debt that was issued by a government in an economy in which the liquidity effect is present. This is due to the fact that the government optimally issues external debt up until the point were it becomes being risky. Once the liquidity

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[52]Nevertheless, it should not be expected that the differences in the output cost reported in Table 4 coincide exactly with the output cost numbers from Figure 8 since the output cost is computed for different default episodes. While in Figure 8 the output cost is computed for states in which it is optimal to default in the baseline specification, in Table 4 the output costs is computed for states in which it is optimal to default in each particular specification.

[53]This implies that the endogenous level of commitment generated in the model is determinant of the level of external debt issuance. In other words, the government rarely chooses to issue low levels of risk-less external debt if they are able to issue additional debt without affecting its default risk.

[54]See Aguiar and Gopinath (2006) for a discussion of the determinants of the steepness of the price schedule in models of endogenous default.

effect is removed, one source of commitment is no longer present and this makes the level of external debt issued under the baseline specification lie entirely in the default set.

Column 3 reports the statistics for the model without the balance-sheet effect. The average stock of external public debt is 7.7% of GDP, which is 65% lower than the average level under the baseline model. This suggests that the balance-sheet effect plays a very important role enhancing the government's commitment and allowing it to issue higher levels of external debt, significantly more than the liquidity effect. This result is consistent with the fact that the larger fraction of the output cost is explained by the balance-sheet effect. Additionally, as shown in Figure 8 most of the output cost of default in the immediate periods following a default are due to the balance-sheet effect. Given that households are impatient, this implies that the part of the output cost that would be more (negatively) valued by the government at the time of making the repayment decision is the part that is due almost entirely to the balance-sheet effect. The frequency of default is 0.36% and the stock of domestic debt is 9.0% of GDP, both in line with the respective values in the baseline model. Additionally, the implicit output cost of default is 1.8% on average for the following three years after a default, which is significantly lower than the 5.4% output cost in the baseline model. Finally, under this specification the government would also optimally choose to default in all the states of the baseline simulations.

The last column of Table 4 shows the statistics for the model under no balance-sheet nor liquidity effects. In this specification the only source of commitment for the government comes from the exogenous exclusion period from financial markets that the government faces after a default. The average external debt that can be sustained with only exclusion costs is only 0.9% of GDP. As previously argued by Arellano (2008) and Aguiar and Gopinath (2006) reputational costs that take the form of exclusion from financial markets cannot account for large amounts (or any amount, in this case) of borrowing since the welfare costs of economic fluctuations are small, as originally noted by Lucas (1987). In addition, the low welfare value of eliminating economic fluctuation is complemented by two facts. First, given that this economy is hit by shocks to the trend productivity, there is a low desire to smooth consumption as shocks are not expected to mean-revert. Second, even if households had some desire to smooth consumption, defaultable debt is not an adequate security for doing this since in low productivity states the pricing curve for external debt falls which limits the amount of consumption smoothing that the household can make. The average level domestic public debt (10.3% of GDP) and the frequency of default (0.2%) are in line with the levels of the baseline specification. The output cost of default is by construction zero since by removing both mechanisms there are no internal

costs of default in the economy. Finally, it is also the case that faced to the states from the baseline simulation, the government will always choose to default on its debt.

In summary, four main conclusions can be drawn from the results presented in this section. First, a sovereign default can trigger a sizable and persistent recession of the order of 5.5% in the two years after the default by affecting the financial system via the balance-sheet effect and the liquidity effect. Second, the presence of the output costs of default are key in inducing government's incentives to repay debt as they account for all of the government's commitment necessary to sustain observed levels of external public debt issuance. Third, the balance-sheet effect is important as it explains 65% of the output cost of default and in its absence the external debt issuance would be 65% lower. Fourth, the liquidity cost, while less important, accounts for 35% of the output cost of default and accounts for the slow recovery of output after default.

6. POLICY ANALYSIS

This section study the effects of domestic policies that are targeted to address the government's problem of lack of commitment. In particular, I analyze two types of policies that differ on the timing of their implementation. Section 6.1 it discusses the possibility of implementing post-default bailouts of the banking system. Section 6.2 considers the implementation of a minimum requirement of exposure to public debt on all banks.

6.1. *Bailouts of the Banking System*

After a sovereign default the banking system suffers a negative hit to its net worth. Given that the bankers are constrained in the ability to raise funds by the level of their net worth, a social planner would be willing to redistribute resources from the unconstrained agents in the economy (households) towards the constrained agents (banks), to ease the borrowing constraint of banks. A post-default bailout of the banking system is a way of implementing such a redistribution. I define a post-default bailout as a lump sum tax to households for an amount equivalent to the aggregate exposure of the banking system to public debt, that is then reimbursed to banks in the form of equity injections. Formally, the bailout is defined as $\tau^b(\mathbf{s})$ where

$$\tau^b(\mathbf{s}) = B^d$$

$$C(\mathbf{s}) = w(\mathbf{s}) + \pi(\mathbf{s}) - \tau(\mathbf{s}) - \tau^b(\mathbf{s})$$

$$N(\mathbf{s}) = \sigma \left(A\mathbb{E}\left[z|z > \underline{z}(\mathbf{s})\right] + \tau^b(\mathbf{s}) \right)$$

where the superscript b indicates bailout. The bailout is assumed to be non-targeted, i.e. equity injections are implemented in a uniform way to all banks rather than on an individual

basis according to each bank's exposure.[55] Such a bailout would replicate the allocations of a selective default in which the government defaults on its external debt and repays its domestic debt. Implementing this operation would eliminate the balance-sheet effect of a sovereign default. At the same time the introduction of this policy will have a negative impact on the ex-ante government's commitment to repay its debt. By eliminating one source of internal costs of default, the government would be more prone to defaulting ex-post and foreign investors would anticipate the change in incentives when offering a pricing schedule for public debt. This in turn forces the government to issue lower levels of external debt at the expense of lower consumption front-loading. Therefore, the introduction of bailouts entails a benefit of being capable of incurring in a default without suffering the internal balance-sheet effects on domestic banks and a cost of facing a lower price of debt. In this section I quantify the welfare effects associated to this policy.

Define the welfare benefit (or cost) of allowing for a technology of post-default bailouts of the banking system conditional on state $\mathbf{s}$, denoted $\delta^b(\mathbf{s})$ as the percent increase in the lifetime consumption stream required by an individual living in the economy in which post-default bailouts are unfeasible in state $\mathbf{s}$ to be as well off as an individual living in an economy with a technology for implementing post-default bailouts. Formally, $\delta(\mathbf{s})$ is implicitly given by

$$\mathbb{E}\left[\sum_{j=0}^{\infty} \beta^t u\left(C_j(1+\delta^b(\mathbf{s}))\right) \middle| \mathbf{s}\right] = W_h^b\left(A_{-1}, g, \underline{z}, B^d, B^x\right)$$

where $W_h^b\left(A_{-1}, g, \underline{z}, B^d, B^x\right)$ denotes the value function in the economy with the technology to implement bailouts in state $\left\{\left(A_{-1}, g, \underline{z}, B^d, B^x\right), h\right\}$. Since $\delta^b(\mathbf{s})$ is state dependent we compute the unconditional average, denoted δ^b, over all the states of the ergodic set.[56] Formally,

$$\delta^b = \sum_{\mathbf{s}} p(\mathbf{s})\delta^b(\mathbf{s})$$

where $p(\mathbf{s})$ is the unconditional probability of state $\mathbf{s}$. Since we are also interested in assessing how the welfare effects of bailouts vary across states, I compute the average permanent change

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[55]With a non-targeted bailout the ex-ante incentives of bankers are not changed as the perceived return on assets remain the same. If, on the contrary, I assumed that bailouts were targeted (i.e. on an individual basis in an amount equivalent to the individual exposures of each bank to public debt), this would create an implicit subsidy to domestic public debt holding that would affect banks' optimal portfolio choices. Since I am interested in isolating the effects of bailouts on the government's incentives I assume the former. See Mengus (2013b) for an analysis of post-default bailouts.

[56]To compute the unconditional average I use the simulation of the baseline model without bailouts to obtain the ergodic set and then take the average of $\delta^b(\mathbf{s})$ over all the path of simulated states.

in consumption *conditional* on those states in which it is optimal to default, which is denoted denoted δ^b_{cond}. Formally,

$$\delta^b_{cond} = \sum_{\mathbf{s}:\iota(\mathbf{s})=0} p(\mathbf{s}|\iota(\mathbf{s})=0)\delta^b(\mathbf{s}|\iota(\mathbf{s})=0)$$

Results are reported in Table 5. For a household living in an economy without the possibility of bailouts it is welfare increasing to allow for bailouts. The representative household living in this economy requires on average an increase of 0.6% percent in consumption every period to be indifferent between living in this economy and living in an economy with the possibility of post-default bailouts. In this case the benefits associated to the redistribution implied in the bailout are immediate and more valuable for the representative household than the costs associated to higher future sovereign risk associated to a lower commitment from the government. In most states of the simulation, the government of the economy with the possibility of bailouts will find it optimal to immediately default on its debt and implement the bailout. The welfare effect is even higher for those states in which it is optimal to default for the government in the economy without bailouts. If we condition on those states, the positive welfare effect of moving to an economy that allows for bailouts is equivalent to a 1.4% increase in lifetime consumption. The reason is that in these states, the benefits of defaulting and not incurring in the balance-sheet effect are higher (by revealed preference).

TABLE 5. Welfare Analysis of Bailouts

Bailout Technology	Unconditional	Conditional on Default
Allowing Bailouts in Economy without Bailouts		
Permanently	0.56%	1.43%
One-time	2.61%	3.52%
Prohibiting Bailouts in Economy with Bailouts		
Permanently	2.39%	-1.27%

Notes: Values are expressed in equivalent change in permanent consumption. The first two rows report the welfare effects of allowing bailouts in the ergodic set of an economy without bailouts. The last row reports the welfare effects of prohibiting bailouts in the ergodic seto of an economy where bailouts are feasible.

To highlight the pure benefit side of post-default bailouts I compute the equivalent change in consumption from an economy without bailouts to an economy in which the government

announces a *credible* one-time bailout which does not entail a higher default risk as foreign investors and domestic banks believe in their announcement. As shown in Table 5, the welfare benefits of implementing this one-time policy are large. The representative household living in an economy without bailouts would be indifferent to live in an economy that implements a one-time bailout if permanent consumption increases by 2.6%. Again, the optimal action to take for the government in the later economy would be to default immediately and implement the one-time bailout. If we condition on those states in which it is optimal to default in the baseline economy without bailouts, the welfare gains are equivalent to an permanent increase in consumption of 3.5%.

A reverse exercise is performed to assess what are the welfare effects of prohibiting the implementation of bailouts in an economy in which bailouts are already feasible. That is, I compute the the percent change in the lifetime consumption stream required by an individual living in the economy in which post-default bailouts are feasible to be as well off as an individual living in an economy in which bailouts are unfeasible. Results are shown in the last column of Table 5. In this case, the welfare effects change direction. That is, an individual living in an economy with bailouts would require an increase of 2.4% in consumption every period to be indifferent between living in this economy and living in an economy where bailouts are unfeasible.[57] This significant welfare gains are related to the possibility of issuing higher levels of external debt (given a higher government commitment) and front-load more consumption. On the other hand, once we condition on states in which it is optimal to default the welfare effects of prohibiting bailouts are negative (1.3% decrease in consumption equivalent units). The reason is that after default the government is in external financial autarky and the benefits of being able to issue higher levels of external debt can only be enjoyed after the government regains access to external financial markets.

Overall, the results presented in Table 5 highlight a time inconsistency problem associated with the implementation of bailouts. In an economy in which investors anticipate that the government can implement bailouts, it is attractive for the same government to tie its own hands and commit to permanently prohibit the implementation of bailouts. This would the align government's incentives to repay its debt and allow for higher levels of external debt in equilibrium and higher consumption front-loading. On the other hand, once the government is

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[57]Note that the magnitude of this welfare effect is significant. The reason behind such a high welfare effects is that by moving into an economy without bailouts would allow to issue roughly 15% of annual GDP worth of external debt (Table 4). Given the calibrated discount factor of $\beta = 0.9$, this would imply a significant welfare improvement just by means of consumption front-loading.

already with high levels of external debt that are consistent with foreign investors internalizing that bailouts are unfeasible, the implementation of bailouts becomes attractive. In this case as the benefits associated to avoiding the balance-sheet effect are more valuable than the costs associated to higher future sovereign risk and lower levels of external debt issuance associated to a lower commitment from the government. A similar time inconsistency problem associated to bailouts has been studied in the context of bankruptcy costs and moral hazard.[58] However, the mechanism behind the time inconsistency problem is different. In that setup, bailouts are not desirable ex-ante since they precipitate excessive risk-taking from banks, and are desirable ex-post given that they avoid bankruptcy costs. In this paper, bailouts are not desirable ex-ante since they weaken government's commitment, and are desirable ex-post given that they avoid the balance-sheet effect through banks.

Whether the implementation of post-default bailouts is politically and/or economically feasible is another interesting question. We have observed in past episodes of default imperfect bailouts of the domestic financial system. For example, in the Argentinean default episode of 2001 the government offered a 'megaswap' in which domestic residents were allowed to swap their public debt (which months later was explicitly subject to default) for iliquid loans with lower face value which were not defaulted on.[59] It has also been argued that bailouts can be costly to implement politically or economically due to a lack of information. In particular, Mengus (2013b) argues that a post-default bailout can be unattractive if the government does not have information on the individual exposure to domestic public debt of all its economic agents.

6.2. *Policies Oriented at Enhancing Banks' Exposure to Public Debt*

This section considers the welfare and economic effects of policies targeted at increasing the banks' exposure to public debt. These type of policies can have a positive effect on welfare given the presence of a positive externality generated by banks' holdings of public debt. When individual banks solve their portfolio problem, they do not take into account the fact that by investing in public debt they enhance the government's commitment to repay its debt by increasing the cost of default. This in turn allows the government to credibly issue higher levels

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[58]Several papers study the effects of government bailouts. Farhi and Tirole (2012) analyze the effects, ex-ante and ex-post of systemic bailouts. Bianchi (2013) provides a quantitative analysis of the optimal bailout policy. Chari and Kehoe (2013) study the optimal regulation when governments cannot commit to not implement bailouts.

[59]The bailout turned out to be imperfect as months later the iliquid bonds were forced to an unfavorable change of currency denomination. See Sturzenegger and Zettelmeyer (2008) for further details on the operation.

of external debt on equilibrium and households to benefit from higher consumption front-loading.[60]

I consider the implementation of a minimum requirement of public debt holdings in every bank. The policy is characterized by the parameter χ that specifies the minimum requirement of public debt as a share of each bank's net worth. The policy introduces the following additional constraint in the bank's problem 24

$$q^b(\tilde{\mathbf{s}})b^d(z) \geq \chi n(z)$$

This constraint will not be binding for low-productivity banks that are indifferent between buying public debt and lending to other banks. However, it will be binding for high-productivity banks since these are forced to allocate part of their asset portfolio in public debt that would otherwise prefer to invest it in their productive technology. A minimum requirement of public debt therefore crowds out investment in productive technology from high-productivity banks. This in turn reduces the demand for aggregate labor, which lowers wages and attracts low-productivity banks to invest in their technology. As a result, the aggregate level of output falls as labor is allocated into technologies with lower productivities on average. The formal derivations of the solution to the banks' problem, as well as the equilibrium threshold productivities, wages and aggregate levels of domestic public debt can be found in Appendix A.

The implementation of a minimum requirement therefore entails a trade-off between lower output due to a lower average productivity and higher external debt issuance due to the enhancement of government commitment. As in the bailout analysis we define the welfare benefit (or cost) of implementing a policy of minimum requirement χ, denoted δ^χ as the unconditional average across all states of the percent change in the lifetime consumption stream required by an individual living in the economy with no minimum requirement of public debt in a given state to be as well off as an individual living in an economy with a minimum public debt requirement χ.[61]

The welfare effects of the implementation of a minimum requirement policy χ are shown in Figure 9. The considered values of χ ranged from 0 to 80% of a bank's net worth. Welfare

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[60]This idea is also explored in contemporaneous work in Chari et al. (2014) that explore conditions under which imposing public debt on financial intermediaries can be optimal.

[61]Formally, $\delta^\chi = \sum_{\mathbf{s}} p(\mathbf{s})\delta^\chi(\mathbf{s})$ where $\delta^\chi(\mathbf{s})$ solves

$$\mathbb{E}\left[\sum_{j=0}^{\infty} \beta^t u\left(C_j(1+\delta^\chi(\mathbf{s}))\right)\bigg|\,\mathbf{s}\right] = W_h^\chi\left(A_{-1}, g, \underline{z}, B^d, B^x\right)$$

where $W_h^\chi\left(A_{-1}, g, \underline{z}, B^d, B^x\right)$ denotes the value function in the economy with policy χ.

is maximized with the implementation of a minimum requirement of 45%, which is equivalent 5.3% of total assets for the high-productivity banks. The representative household living in an economy with no minimum requirement requires on average an increase of 1.7% in consumption every period to be indifferent between living in this economy and living in an economy with a public debt minimum requirement policy of $\chi^* = 45\%$.

FIGURE 9. Welfare Effects of a Minimum Requirement of Public Debt in Banks
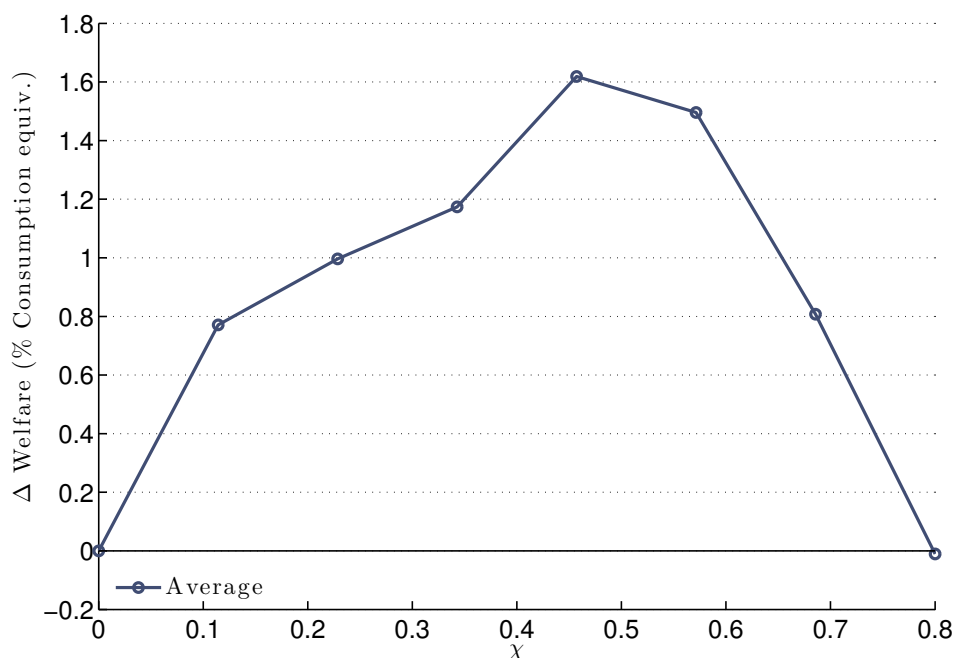
A higher the minimum requirement increases the exposure of public debt in banks which increases the internal costs of default by strengthening the balance-sheet effect. This in turn enhances government commitment and increases the level of external debt in equilibrium. The average level of external public debt in the simulations of an economy with a minimum requirement of public debt in banks of 45% of their net worth is 5% of GDP larger than the average external public debt in the simulations of the baseline economy with no minimum public debt requirement. On the other hand, the implementation of a minimum requirement policy has associated a lower level of output in the ergodic set. In particular, the average level of output in the simulations of an economy with the optimal minimum requirement policy is 1.6% lower than the average level of output in the simulations of the baseline economy with zero minimum requirement of public debt in banks.

7. Conclusion

This paper has developed a dynamic model of endogenous default with heterogeneous banks to explore two mechanisms through which a sovereign default can affect the domestic economy through its banking system. In the model economy the role of public debt is dual. First, it is a security that allows to perform inter-temporal trade when the holders of this security are foreign investors. Second, it provides liquidity to the domestic financial system given the presence of financial frictions in the domestic economy that prevent the banking sector from satisfying its demand for liquidity with privately issued securities.

A negative liquidity effect arises following a sovereign default as the supply of public debt is relatively scare and its return low. This makes banks substitute the use of government securities to transfer wealth across periods for investments in less productive projects. A negative balance-sheet effect of default arises due to a tightening in the banks' borrowing constraint that reduces their ability to raise funds and prevents the flow of resources to productive investments.

By quantifying the model to match the Argentinean economy I find that these two mechanisms can generate a deep and persistent fall in output. Additionally, the presence of an endogenous cost of default is important in aligning the government's incentives to repay. The joint presence of the balance-sheet effect and the liquidity effect can help explain the observed levels of external debt issuance. When disentangling the strength of each effect I find that the balance-sheet effect is more important as it generates a larger output cost of default ex-post and a stronger ex-ante commitment for the government.

Finally, the model is used to explore the welfare and economic effects of post-default bailouts of the banking system and a minimum requirement of public debt in banks. Although highly desirable ex-post, post-default bailouts can be welfare reducing ex-ante as they reduce the output cost of default by removing the balance-sheet effect and therefore weaken the governments commitment. On the other hand, I find that a minimum requirement of public debt on banks of the order of 45% of their net worth maximizes welfare. The existence of an positive optimal minimum-requirement policy is related to the presence of a positive externality in the banks holdings of public debt. When individual banks solve their portfolio problem, they do not take into account the fact that by investing in public debt they enhance the government's commitment to repay its debt by increasing the cost of default. This in turn allows the government to credibly issue higher levels of external debt on equilibrium. A further theoretical exploration of the conditions under which this externality is in place is left for future research.

References

AGUIAR, M. AND M. AMADOR (2014): *Sovereign Debt*, North-Holland, 647–87.

AGUIAR, M. AND G. GOPINATH (2006): "Defaultable Debt, Interest Rates and the Current Account," *Journal of International Economics*, 69, 64–83.

——— (2007): "Emerging Market Business Cycles: The Cycle Is the Trend," *Journal of Political Economy*, 115, 69–102.

AIYAGARI, S. R. AND E. R. MCGRATTAN (1998): "The optimum quantity of debt," *Journal of Monetary Economics*, 42, 447–469.

ALVAREZ, F. AND U. J. JERMANN (2000): "Efficiency, Equilibrium, and Asset Pricing with Risk of Default," *Econometrica*, 68, 775–797.

AMADOR, M. (2012): "Sovereign Debt and the Tragedy of the Commons," Manuscript, Stanford University.

ANGELETOS, G.-M., F. COLLARD, H. DELLAS, AND B. DIBA (2013): "Optimal Public Debt Management and Liquidity Provision," NBER Working Paper No. 18800.

ARELLANO, C. (2008): "Default Risk and Income Fluctuations in Emerging Economies," *The American Economic Review*, 98, 690–712.

ARELLANO, C. AND A. RAMANARAYANAN (2012): "Default and the Maturity Structure in Sovereign Bonds," *Journal of Political Economy*, 120, 187–232.

BARAJAS, A., E. BASCO, V. H. JUAN-RAMON, AND C. QUARRACINO (2007): "Banks During the Argentine Crisis: Were They All Hurt Equally? Did They All Behave Equally?" IMF Staff Papers Vol. 54, No. 4.

BASU, S. (2009): "Sovereign Debt and Domestic Economic Fragility," Manuscript, Massachusetts Institute of Technology.

BENJAMIN, D. AND M. L. WRIGHT (2009): "Recovery Before Redemption: A Theory of Delays in Sovereign Debt Renegotiations," Manuscript, University of California at Los Angeles.

BERNANKE, B. AND M. GERTLER (1989): "Agency Costs, Net Worth, and Business Fluctuations," *The American Economic Review*, 79, 14–31.

BERNANKE, B. S., M. GERTLER, AND S. GILCHRIST (1999): "The Financial Accelerator in a Quantitative Business Cycle Framework," *Handbook of Macroeconomics*, 1, 1341–1393.

BIANCHI, J. (2013): "Efficient Bailouts?" Manuscript, University of Wisconsin Madison.

BIANCHI, J. AND E. G. MENDOZA (2013): "Optimal Time-Consistent Macroprudential Policy," NBER Working Paper No. 19704.

BOCOLA, L. (2014): "The Pass-Through of Sovereign Risk," Manuscript, University of Pennsilvania.

BOLTON, P. AND O. JEANNE (2011): "Sovereign Default Risk and Bank Fragility in Financially Integrated Economies," *IMF Economic Review*, 59, 162–194.

BRONER, F., A. MARTÍN, AND J. VENTURA (2010): "Sovereign Risk and Secondary Markets," *American Economic Review*, 100, 1523–1555.

BRONER, F. AND J. VENTURA (2011): "Globalization and Risk Sharing," *The Review of Economic Studies*, 78, 49–82.

BRUNNERMEIER, M. K. AND Y. SANNIKOV (2014): "A Macroeconomic Model with a Financial Sector," *The American Economic Review*, 104, 379–421.

BRUTTI, F. (2011): "Sovereign Defaults and Liquidity Crises," *Journal of International Economics*, 84, 65–72.

BULOW, J. AND K. ROGOFF (1989): "Sovereign Debt: Is to Forgive to Forget?" *The American Economic Review*, 79, 43–50.

CALVO, G. A. (1988): "Servicing the Public Debt: The Role of Expectations," *The American Economic Review*, 647–661.

CHARI, V. V., A. DOVIS, AND P. J. KEHOE (2014): "On the Optimality of Financial Repression," Manuscript, University of Minnesota.

CHARI, V. V. AND P. J. KEHOE (1993): "Sustainable Plans and Debt," *Journal of Economic Theory*, 61, 230–261.

——— (2013): "Bailouts, Time Inconsistency, and Optimal Regulation," *Federal Reserve Bank of Minneapolis Staff Report 481*.

CHATTERJEE, S. AND B. EYIGUNGOR (2012): "Maturity, Indebtedness, and Default Risk," *The American Economic Review*, 102, 2674–2699.

COLE, H. AND T. KEHOE (2000): "Self-Fulfilling Debt Crises," *The Review of Economic Studies*, 67.

CUADRA, G. AND H. SAPRIZA (2008): "Sovereign Default, Interest Rates and Political Uncertainty in Emerging Markets," *Journal of International Economics*, 76, 78–88.

D'ERASMO, P. (2011): "Government Reputation and Debt Repayment in Emerging Economies," Manuscript, University of Maryland, College Park.

DI TELLA, S. (2013): "Uncertainty Shocks and Balance Sheet Recessions," Manuscript, Stanford University.

DIAS, D. A. AND C. RICHMOND (2008): "Duration of Capital Market Exclusion: Stylized Facts and Determining Factors," Manuscript, UCLA.

DRENIK, A. AND D. J. PEREZ (2014): "Price Setting Under Uncertainty About Inflation," Manuscript, Stanford University.

DURBIN, E. AND D. NG (2005): "The Sovereign Ceiling and Emerging Market Corporate Bond Spreads," *Journal of International Money and Finance*, 24, 631–649.

EATON, J. AND M. GERSOVITZ (1981): "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *The Review of Economic Studies*, 48, 289–309.

EDWARDS, S. (1984): "LDC Foreign Borrowing and Default Risk: An Empirical Investigation, 1976-80," *The American Economic Review*, 74, 726–734.

FARHI, E. AND J. TIROLE (2012): "Collective Moral Hazard, Maturity Mismatch, and Systemic Bailouts," *The American Economic Review*, 102, 60–93.

GARCÍA-CICCO, J., R. PANCRAZI, AND M. URIBE (2010): "Real Business Cycles in Emerging Countries?" *The American Economic Review*, 100, 2510–2531.

GELOS, R. G., R. SAHAY, AND G. SANDLERIS (2011): "Sovereign Borrowing by Developing Countries: What Determines Market Access?" *Journal of International Economics*, 83, 243–254.

GENNAIOLI, N., A. MARTÍN, AND S. ROSSI (2014): "Sovereign Default, Domestic Banks, and Financial Institutions," *The Journal of Finance*, 69, 819–866.

GERTLER, M. AND P. KARADI (2011): "A Model of Unconventional Monetary Policy," *Journal of Monetary Economics*, 58, 17–34.

GERTLER, M. AND N. KIYOTAKI (2010): "Financial Intermediation and Credit Policy in Business Cycle Analysis," *Handbook of Monetary Economics*, 3, 547–599.

GOPINATH, G. AND B. NEIMAN (2011): "Trade Adjustment and Productivity in Large Crises," Manuscript, Harvard University.

GUEMBEL, A. AND O. SUSSMAN (2009): "Sovereign Debt without Default Penalties," *Review of Economic Studies*, 76, 1297–1320.

HATCHONDO, J., L. MARTINEZ, AND C. S. PADILLA (2012): "Debt Dilution and Sovereign Default Risk," Working Paper 10-08, Federal Reserve Bank of Richmond.

HATCHONDO, J. C. AND L. MARTINEZ (2009): "Long-duration Bonds and Sovereign Defaults," *Journal of International Economics*, 79, 117–125.

HE, Z. AND A. KRISHNAMURTHY (2012): "A Model of Capital and Crises," *The Review of Economic Studies*, 79, 735–777.

HELLWIG, C. AND G. LORENZONI (2009): "Bubbles and Self Enforcing Debt," *Econometrica*, 77, 1137–1164.

HOLMSTRÖM, B. AND J. TIROLE (1998): "Private and Public Supply of Liquidity," *Journal of Political Economy*, 106, 1–40.

IMF (2002): "The Design and Effectiveness of Collective Action Clauses," IMF Working Paper.

——— (2010): *Sovereigns, Funding, and Systemic Liquidity*, Global Financial Stability Report.

KEHOE, T. J. (2007): "What can we learn from the 1998-2002 depression in Argentina?" *Great depressions of the twentieth century*.

KEHOE, T. J. AND D. K. LEVINE (1993): "Debt-constrained Asset Markets," *The Review of Economic Studies*, 60, 865–888.

KIYOTAKI, N. AND J. MOORE (1997): "Credit Cycles," *Journal of Political Economy*, 105, 211–248.

——— (2005): "2002 Lawrence R. Klein Lecture Liquidity and Asset Prices*," *International Economic Review*, 46, 317–349.

——— (2012): "Liquidity, Business Cycles, and Monetary Policy," Manuscript, Princeton University.

KRISHNAMURTHY, A. AND A. VISSING-JORGENSEN (2012): "The Aggregate Demand for Treasury Debt," *Journal of Political Economy*, 120, 233–267.

KRUSELL, P., V. QUADRINI, AND J.-V. RÍOS-RULL (1997): "Politico-economic Equilibrium and Economic Growth," *Journal of Economic Dynamics and Control*, 21, 243–272.

LORENZONI, G. AND I. WERNING (2013): "Slow Moving Debt Crises," NBER Working Paper No. 19228.

LUCAS, R. E. (1987): *Models of Business Cycles*, New York: Basil Blackwell.

MALTRITZ, D. (2012): "Determinants of Sovereign Yield Spreads in the Eurozone: A Bayesian Approach," *Journal of International Money and Finance*, 31, 657–672.

MELITZ, M. J. AND S. J. REDDING (2014): "New Trade Models, New Welfare Implications," Manuscript, Harvard University.

MENDOZA, E. G. AND V. Z. YUE (2012): "A General Equilibrium Model of Sovereign Default and Business Cycles," *The Quarterly Journal of Economics*, 127, 889–946.

MENGUS, E. (2013a): "Foreign Debt and Ricardian Equivalence," Manuscript, Toulouse School of Economics.

——— (2013b): "Honoring Sovereign Debt or Bailing Out Domestic Residents: A Theory of Internal Costs of Default," Manuscript, Toulouse School of Economics.

MOLL, B. (2014): "Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation?" Manuscript, Princeton University.

NEUMEYER, P. A. AND F. PERRI (2005): "Business Cycles in Emerging Economies: the Role of Interest Rates," *Journal of Monetary Economics*, 52, 345–380.

REINHART, C. M. AND K. ROGOFF (2009): *This Time Is Different: Eight Centuries of Financial Folly*, Princeton University Press.

REINHART, C. M. AND K. S. ROGOFF (2011): "From Financial Crash to Debt Crisis," *The American Economic Review*, 101, 1676–1706.

SANDLERIS, G. AND M. L. WRIGHT (2014): "The Costs of Financial Crises: Resource Misallocation, Productivity, and Welfare in the 2001 Argentine Crisis," *The Scandinavian Journal of Economics*, 116, 87–127.

SCHEINKMAN, J. A. AND L. WEISS (1986): "Borrowing Constraints and Aggregate Economic Activity," *Econometrica*, 54, 23.

SHIN, Y. (2006): "Ramsey Meets Bewley: Optimal Government Financing with Incomplete Markets," Manuscript, Washington University in St Louis.

SOSA PADILLA, C. (2012): "Sovereign Defaults and Banking Crises," Manuscript, McMaster University.

STURZENEGGER, F. AND J. ZETTELMEYER (2008): "Haircuts: Estimating Investor Losses in Sovereign Debt Restructurings, 1998–2005," *Journal of International Money and Finance*, 27, 780–805.

TAUCHEN, G. AND R. HUSSEY (1991): "Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models," *Econometrica*, 59, 371.

TOMZ, M. AND M. L. J. WRIGHT (2007): "Do Countries Default in "Bad Times" ?" *Journal of the European Economic Association*, 5, 352–360.

URIBE, M. AND V. Z. YUE (2006): "Country Spreads and Emerging Countries: Who Drives Whom?" *Journal of International Economics*, 69, 6–36.

WOODFORD, M. (1990): "Public Debt as Private Liquidity," *The American Economic Review*, 80, 382–388.

WRIGHT, M. L. J. (2006): "Private Capital Flows, Capital Controls, and Default Risk," *Journal of International Economics*, 69, 120–149.

YUE, V. Z. (2010): "Sovereign Default and Debt Renegotiation," *Journal of International Economics*, 80, 176–187.

APPENDIX A. OMITTED PROOFS AND RESULTS

Recursive Representation of Banks' Problem

Let $\tilde{\mathbf{s}}$ denote the augmented aggregate state of the economy. The bank's problem admits the following recursive representation which will depend on future government policies $(\mathcal{B}'(\mathbf{s}), \mathcal{I}(\mathbf{s}))$ and on the law of motion of the aggregate state $\Gamma(\mathbf{s'}, \mathbf{s}, B', \iota)$. Denote $v(n, z; \mathbf{s}, B', \iota)$ the value of an individual bank with net worth n, idiosyncratic productivity z in augmented aggregate state $(\mathbf{s}, B', \iota)$ that solves the bank's problem in recursive form. After knowing his idiosyncratic productivity, a banker faces the following recursive problem

$$v(n, z; \mathbf{s}, B', \iota) = \max_{l' \geq 0, b^{d'} \geq 0, d'} \mathbb{E}\left[\Lambda(\mathbf{s}, \mathbf{s'})\left((1 - \sigma)n' + \sigma v(n', z'; \mathbf{s'}, \mathcal{B}(\mathbf{s'}), \mathcal{I}(\mathbf{s'}))\right)|\mathbf{s}\right] \quad (24)$$

subject to:

$$n(z) = w(\mathbf{s})l'(z) + q^b(\mathbf{s})b^{d'}(z) + q^d(\mathbf{s})d'(z) \quad (25)$$

$$n'(z) = A'zl'(z) + \iota(\mathbf{s'})(b^{d'}(z) + d'(z)) \quad (26)$$

$$q^d(\mathbf{s})d'(z) \geq -\kappa n(z) \quad (27)$$

Proof of Proposition 1

We first conjecture that the value function is linear in net worth, i.e. $v(n, z; \tilde{\mathbf{s}}) = \nu(z; \tilde{\mathbf{s}})n$, then solve the portfolio problem of the banks and finally verify our conjecture. Using equation (25) to substitute away $d'(z)$ we can re-write the recursive problem of the banks as

$$\nu(z; \tilde{\mathbf{s}})n = \max_{l' \geq 0, b^{d'} \geq 0} \mathbb{E}\left[\Lambda(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')\left((1 - \sigma) + \sigma\nu(z'; \tilde{\mathbf{s}}')\right)n'(z)|\tilde{\mathbf{s}}\right] \quad (28)$$

subject to:

$$n'(z) = \left(R^l(z; \tilde{\mathbf{s}}, \tilde{\mathbf{s}}') - R^d(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')\right)w(\tilde{\mathbf{s}})l'(z) + \left(R^b(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}') - R^d(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')\right)q^b(\tilde{\mathbf{s}})b^{d'}(z)$$
$$+ R^d(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')q^d(\tilde{\mathbf{s}})d'(z) \quad (29)$$

$$(1 + \kappa)n \geq w(\tilde{\mathbf{s}})l'(z) + q^b(\tilde{\mathbf{s}})b^{d'}(z) \quad (30)$$

where $R^l(z; \tilde{\mathbf{s}}, \tilde{\mathbf{s}}') \equiv \frac{A'z}{w(\tilde{\mathbf{s}})}$, $R^b(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}') \equiv \frac{\iota(\tilde{\mathbf{s}}')}{q^b(\tilde{\mathbf{s}})}$ and $R^d(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}') \equiv \frac{\iota(\tilde{\mathbf{s}}')}{q^d(\tilde{\mathbf{s}})}$. This problem is linear in $l', b^{d'}$ and its solution involves corners.

If $q^b(\tilde{\mathbf{s}}) = q^d(\tilde{\mathbf{s}})$, the return on deposits and public debt is the same for every state realization and the solution to the portfolio problem depend on the level of z.

    – If $z > \underline{z}'(\tilde{\mathbf{s}})$: $w(\tilde{\mathbf{s}})l'(z) = (1 + \kappa)n$ $q^d(\tilde{\mathbf{s}})d'(z) = -\kappa n$ $q^b(\tilde{\mathbf{s}})b^{d'}(z) = 0$

    – If $z \leq \underline{z}'(\tilde{\mathbf{s}})$: $w(\tilde{\mathbf{s}})l'(z) = 0$ $q^d(\tilde{\mathbf{s}})d'(z) = x \in [0, n(z)]$ $q^b(\tilde{\mathbf{s}})b^{d'}(z) = n(z) - x$

If $q^b(\tilde{\mathbf{s}}) > q^d(\tilde{\mathbf{s}})$, the return on deposits is always greater than the return on public debt. Hence, no bank will invest in public debt and the solution to the portfolio problem is the following.

- If $z > \underline{z}'(\tilde{\mathbf{s}})$: $\quad w(\tilde{\mathbf{s}})l'(z) = (1+\kappa)n \qquad q^d(\tilde{\mathbf{s}})d'(z) = -\kappa n \qquad q^b(\tilde{\mathbf{s}})b^{d'}(z) = 0$
- If $z \le \underline{z}'(\tilde{\mathbf{s}})$: $\quad w(\tilde{\mathbf{s}})l'(z) = 0 \qquad q^d(\tilde{\mathbf{s}})d'(z) = n(z) \qquad q^b(\tilde{\mathbf{s}})b^{d'}(z) = 0$

where $\underline{z}'(\tilde{\mathbf{s}})$ is such that $\mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')R^l(\underline{z}'(\tilde{\mathbf{s}}); \tilde{\mathbf{s}}, \tilde{\mathbf{s}}')\right] = \mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')R^d(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')\right]$.

Now we verify our conjecture of linearity. Substituting the solution to the problem in (28) the level of net worth scales away and we obtain a law of motion for the marginal value of one unit of net worth.

- For $z \le \underline{z}'(\tilde{\mathbf{s}})$:

$$\nu(z; \tilde{\mathbf{s}}) = \mathbb{E}\left[\Lambda(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')\left(1 - \sigma + \sigma\nu(z', \tilde{\mathbf{s}}')\right)R^d(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')\right]$$

- For $z > \underline{z}'(\tilde{\mathbf{s}})$:

$$\nu(z; \tilde{\mathbf{s}}) = \mathbb{E}\left[\Lambda(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')\left(1 - \sigma + \sigma\nu(z', \tilde{\mathbf{s}}')\right)R^d(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')\left[1 + (\kappa + 1)\left(\frac{R^l(z; \tilde{\mathbf{s}}, \tilde{\mathbf{s}}')}{R^d(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')} - 1\right)\right]\right]$$

Proof of Proposition 2

The aggregate demand for labor is determined by the amount of resources that high productivity banks can raise in the interbank deposit market which is given by

$$w(\tilde{\mathbf{s}})L(\tilde{\mathbf{s}}) = \int_{z > \underline{z}(\tilde{\mathbf{s}})} (1+\kappa)n(z)dG(z)$$
$$= N(\tilde{\mathbf{s}})(1+\kappa)\left[1 - G(\underline{z}'(\tilde{\mathbf{s}}))\right]$$

where the second equality uses the independence between the net worth with which banks arrive to the period and the level of idiosyncratic productivity. Given that the aggregate supply of labor is normalized to one and using the market clearing condition we obtain equation (20).

Now we determine the equilibrium in the interbank market. First note that the absence of arbitrage implies that $q^d(\tilde{\mathbf{s}}) \le q^d(\tilde{\mathbf{s}})$. This is shown by contradiction. Suppose $q^d(\tilde{\mathbf{s}}) > q^d(\tilde{\mathbf{s}})$, then any bank, regardless of its productivity, would borrow up to its constraint raising interbank deposits (some of them would use it to invest in their technology, others to buy public debt). This implies that the interbank market for deposits would not clear at that price.

Given that both public debt and interbank deposits have the same payoffs in every state the natural candidate price of deposits is $q^d(\tilde{\mathbf{s}}) = q^d(\tilde{\mathbf{s}})$. However, given that banks are allowed to borrow using interbank deposits but are not allowed to take short positions in public debt it can be the case that $q^d(\tilde{\mathbf{s}}) < q^d(\tilde{\mathbf{s}})$ in equilibrium.

To characterize these two cases I first conjecture that $q^d(\tilde{\mathbf{s}}) = q^d(\tilde{\mathbf{s}})$ and analyze the conditions under which this is indeed the case. In this case the productivity level z that would make a bank indifferent between investing in their own technology and lending to other bank (or buying public debt) is given by

$$\mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')A'\right]\frac{\underline{z}'(\tilde{\mathbf{s}})}{w(\tilde{\mathbf{s}})} = \mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')R^b(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')\right]$$

According to proposition 1 the banks with $z < \underline{z}(\tilde{\mathbf{s}})$ will be indifferent between buying public debt or lending to other banks. Therefore, the volume of interbank lending will be demand-determined and the aggregate demand for public debt will be determined residually

$$q^b(\tilde{\mathbf{s}})B^d(\tilde{\mathbf{s}}) = \int_{z \leq \underline{z}(\tilde{\mathbf{s}})} n(z)dG(z) - \int_{z > \underline{z}(\tilde{\mathbf{s}})} \kappa n(z)dG(z)$$
$$= N(\tilde{\mathbf{s}})\left[G(\underline{z}'(\tilde{\mathbf{s}}))(1 + \kappa) - \kappa\right].$$

This will indeed be an equilibrium if within the banks that are indifferent between buying public debt and lending to other banks there is enough resources to satisfy the demand for interbank lending at that price, or equivalently, if the residual demand for public debt is non-negative. This will be true if and only if

$$G(\underline{z}'(\tilde{\mathbf{s}})) \geq \frac{\kappa}{1 + \kappa}$$

Using the condition for the cutoff productivity we must have

$$\mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')R^b(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')\right] = \mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')A'\right]\frac{\underline{z}(\tilde{\mathbf{s}})}{w(\tilde{\mathbf{s}})}$$
$$\geq \mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')A'\right]\frac{G^{-1}\left(\frac{\kappa}{1+\kappa}\right)}{N(\tilde{\mathbf{s}})}$$

Now conjecture that $q^d(\tilde{\mathbf{s}}) < q^d(\tilde{\mathbf{s}})$. In this case interbank deposits are unambiguously more attractive than public debt and therefore the aggregate demand for public debt will be zero. Using the market clearing condition for the interbank market we can obtain the cutoff productivity for this case

$$\int_{z \leq \underline{z}(\tilde{\mathbf{s}})} n(z)dG(z) - \int_{z > \underline{z}(\tilde{\mathbf{s}})} \kappa n(z)dG(z) = 0$$

which will hold if and only if

$$G(\underline{z}'(\tilde{\mathbf{s}})) = \frac{\kappa}{1 + \kappa}$$

Note that in this case the aggregate level of net worth does not affect the equilibrium cutoff productivity. Finally, to verify that this is an equilibrium the risk-adjusted expected return of the bank with productivity $\underline{z}'(\tilde{\mathbf{s}})$ is greater than the risk-adjusted return on public debt

$$\mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')R^b(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')\right] < \mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')A'\right]\frac{\underline{z}'(\tilde{\mathbf{s}})}{w(\mathbf{s})}$$

$$= \mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')A'\right]\frac{G^{-1}\left(\frac{\kappa}{1+\kappa}\right)}{N(\tilde{\mathbf{s}})}.$$

Competitive Equilibrium with a Minimum Requirement of Public Debt in Banks

The bank's problem with the minimum requirement of public debt is

$$v(n, z; \tilde{\mathbf{s}}) = \max_{l' \geq 0, b^{d'}, d'} \mathbb{E}\left[\Lambda(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')\left((1-\sigma)n' + \sigma v(n', z'; \tilde{\mathbf{s}}')\right)|\tilde{\mathbf{s}}\right] \tag{31}$$

subject to:

$$n(z) = w(\tilde{\mathbf{s}})l'(z) + q^b(\tilde{\mathbf{s}})b^{d'}(z) + q^d(\tilde{\mathbf{s}})d'(z)$$

$$n'(z) = A'zl'(z) + \iota(b^{d'}(z) + d'(z))$$

$$q^d(\tilde{\mathbf{s}})d'(z) \geq -\kappa n(z)$$

$$q^d(\tilde{\mathbf{s}})b^{d'}(z) \geq \chi n(z).$$

Following the same argument as in proposition 1 the solution of this problem in the relevant case of $q^b(\tilde{\mathbf{s}}) = q^d(\tilde{\mathbf{s}})$ is given by

$$w(\tilde{\mathbf{s}})l'(z) = (1 + \kappa - \chi)n \qquad q^d(\tilde{\mathbf{s}})d'(z) = -\kappa n \qquad q^b(\tilde{\mathbf{s}})b^{d'}(z) = \chi n(z), \quad \text{for } z > \underline{z}'(\tilde{\mathbf{s}})$$

$$w(\tilde{\mathbf{s}})l'(z) = 0 \qquad q^d(\tilde{\mathbf{s}})d'(z) = x \in [0, (1-\chi)n(z)] \qquad q^b(\tilde{\mathbf{s}})b^{d'}(z) = n(z) - x, \quad \text{for } z \leq \underline{z}'(\tilde{\mathbf{s}})$$

Using the labor market clearing condition, the indifference condition for the threshold bank and the aggregate demand for domestic public debt we obtain expressions for the wage, threshold productivity and domestic public debt.

$$w(\tilde{\mathbf{s}}) = N(\tilde{\mathbf{s}})(1 + \kappa - \chi)\left[1 - G(\underline{z}'(\tilde{\mathbf{s}}))\right]$$

$$\underline{z}'(\tilde{\mathbf{s}}) = \frac{\mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')R^b(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')\right]}{\mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')A'\right]}w(\tilde{\mathbf{s}})$$

$$q^b(\tilde{\mathbf{s}})B^d(\tilde{\mathbf{s}}) = N(\tilde{\mathbf{s}})\left[G(\underline{z}'(\mathbf{s}))(1 + \kappa - \chi) - \kappa + \chi\right].$$

Appendix B. Numerical Solution

De-trending of the Bank's and Government Problems

First I derive the de-trended recursive banks' problem and then the government problem. The state variables for the banks problem are given by $(n, z; A_{-1}, g, \underline{z}, B^d, B^x)$.[62] The banker's problem is given by

$$v(n_0, z_0; A_{-1}, g_0, \underline{z}_0, B_0^d, B_0^x) = \max_{\{n_t, l_t, b_t^d, d_t\}_{s=1}^\infty} \mathbb{E}_0 \sum_{t=1}^\infty (1-\sigma)\sigma^{t-1}\Lambda_{0,t} n_t \tag{32}$$

subject to

$$n_t = \prod_{s=0}^t R_s^d n_0 + \sum_{s=1}^t \prod_{u=s}^{t-1} R_u^d \left[\left(R_s^l(z) - R_s^d\right) w_{s-1} l_s + \left(R_s^b - R_s^d\right) q_{s-1}^b b_s^d \right] \tag{33}$$

$$q_t^b b_{t+1} \geq \kappa n_t \tag{34}$$

$$b_{t+1}^d \geq 0 \tag{35}$$

Equation (33) is obtained by iterating over the definition of net worth. Now we argue that the constraint set of this maximization problem is homogeneous of degree one in $(n; A_{-1}, B^d, B^x)$. Consider a new initial state given by $(\alpha n, z; \alpha A_{-1}, g, \underline{z}, \alpha B^d, \alpha B^x)$ with $\alpha > 0$. Conjecture that new wages are given by αw_t and that $\Lambda_{0,t}$ is not affected by the change in state. Then given the balance-sheet constraints, it follows that if $\{n_t, l_t, b_t^d, d_t\}_{s=1}^\infty$ is feasible under the initial state, then $\{\alpha n_t, l_t, \alpha b_t^d, \alpha d_t\}_{s=1}^\infty$ is feasible under the new initial state $(\alpha n, z; \alpha A_{-1}, g, \underline{z}, \alpha B^d, \alpha B^x)$ with $\alpha > 0$. Given that the objective function is homogeneous of degree one on n_t it follows that $v(\alpha n, z; \alpha A_{-1}, g, \underline{z}, \alpha B^d, \alpha B^x) = \alpha v(n_0, z_0; A_{-1}, g_0, \underline{z}_0, B_0^d, B_0^x)$.

Now consider the recursive problem of the bank. Consider $\alpha_t = (A_{t-1}\mu_g)^{-1}$ and denote $\hat{x} = (A_{t-1}\mu_g)^{-1}x$ the de-trended version of variable x and $\hat{\mathbf{s}} = (g, \underline{z}, \hat{B}^d, \hat{B}^x)$. Conjecture the that the price of debt is homogeneous of degree zero, i.e. $q^b(\hat{\mathbf{s}}) = q^b(\mathbf{s})$. Then, using the definition of aggregate consumption and the stochastic discount factor it can be shown that

$$\Lambda(\hat{\mathbf{s}}, \hat{\mathbf{s}}') = \Lambda(\mathbf{s}, \mathbf{s}') \exp(g)^{-\gamma} \tag{36}$$

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[62]To simplify notation I consider private allocations to depend only on the aggregate state. This already assumes that private allocations correspond to a Markov equilibrium in which government policies are optimal and depend on the aggregate state $\mathbf{s}$.

Now using the homogeneity of the banks value function we can obtain the de-trended bank's recursive problem

$$v(\hat{n}, z; \hat{\mathbf{s}}) = (A_{-1}\mu_g)^{-1}v(n, z; \mathbf{s})$$

$$= (A_{-1}\mu_g)^{-1} \max_{l' \geq 0, b^{d'} \geq 0, d' \geq -\kappa n/q^d} \mathbb{E}\left[\Lambda(\mathbf{s}, \mathbf{s}')\left((1-\sigma)n' + \sigma v(n', z'; \mathbf{s}')\right)|\mathbf{s}\right]$$

$$= \max_{l' \geq 0, \hat{b}^{d'} \geq 0, \hat{d}' \geq -\kappa \hat{n}/q^d} \mathbb{E}\left[\Lambda(\mathbf{s}, \mathbf{s}')\exp(g)\left((1-\sigma)\hat{n}' + \sigma v(\hat{n}', z'; \hat{\mathbf{s}}')\right)|\mathbf{s}\right]$$

$$= \max_{l' \geq 0, \hat{b}^{d'} \geq 0, \hat{d}' \geq -\kappa \hat{n}/q^d} \mathbb{E}\left[\Lambda(\hat{\mathbf{s}}, \hat{\mathbf{s}}')\exp(g)^{1-\gamma}\left((1-\sigma)\hat{n}' + \sigma v(\hat{n}', z'; \hat{\mathbf{s}}')\right)|\mathbf{s}\right] \qquad (37)$$

where in the third equality I use the definition of $\hat{n}'$ and the homogeneity of the value function, and in the third equality I use equation (36).

Now I derive the de-trended recursive problem for the government. Denote $\Phi(\mathbf{s}_t)$ the budget set of associated to the government problem. Using a similar argument it can be shown that if $(\iota_t, C_t, \underline{z}_{t+1}, B^d_{t+1}, B^x_{t+1}) \in \Phi(\mathbf{s}_t)$ then $(\iota_t, \hat{C}_t, \underline{z}_{t+1}, \hat{B}^d_{t+1}, \hat{B}^x_{t+1}) \in \Phi(\hat{\mathbf{s}}_t)$. Then using homogeneity of degree $1 - \gamma$ of the utility function we can write the recursive problem of the government as

$$W(g, \underline{z}, \hat{B}^d, \hat{B}^x) = \max_{\iota \in \{0,1\}} \iota W^m(g, \underline{z}, \hat{B}^d, \hat{B}^x) + (1-\iota)W^a(g, \underline{z}) \qquad (38)$$

where the value of repayment and keeping access to external financial markets is

$$W^m(g, \underline{z}, \hat{B}^d, \hat{B}^x) = \max_{B^{x'}} u(\hat{C}(m, \hat{s})) + \beta \exp(g)^{1-\gamma}\mathbb{E}\left[W(g', \underline{z}', \hat{B}^{d'}, \hat{B}^{x'})|\hat{s}\right] \qquad (39)$$

subject to

$$\hat{C}(s, m) = \frac{\exp(g)}{\mu_g}\frac{\lambda \underline{z}}{\lambda - 1} - \hat{B}^x + q^b(\hat{s}, \hat{B}^{x'})\hat{B}^{x'}$$

$$\underline{z}' = \underline{z}'\left(\hat{s}, m; \hat{B}^{x'}, \iota\right)$$

$$\hat{B}^{d'} = B^{d'}\left(\hat{s}, m; B^{x'}, \iota\right)$$

and where value of defaulting and loosing access to external financial markets is

$$W^a(g, \underline{z}) = u(\hat{C}(a, \hat{s})) + \beta \exp(g)^{1-\gamma}\mathbb{E}\left[\phi W(g', \underline{z}', \hat{B}^{d'}, 0) + (1-\phi)W^a(g', \underline{z}', \hat{B}^{d'})\right] \qquad (40)$$

where

$$\hat{C}(s, a) = \frac{\exp(g)}{\mu_g}\frac{\lambda \underline{z}}{\lambda - 1}$$

$$\underline{z}' = \underline{z}'\left(\hat{s}, a; \hat{B}^{x'}, \iota\right)$$

$$\hat{B}^{d'} = B^{d'}\left(\hat{s}, a; B^{x'}, \iota\right)$$

Note that the endogenous law of motion of the cutoff productivity and the stock of domestic debt, $\underline{z}\left(\hat{s}, m; \hat{B}^{x\prime}, \iota\right)$ and $\hat{B}^d\left(\hat{s}, m; \hat{B}^{x\prime}, \iota\right)$ correspond to the solution of the competitive equilibrium.

Numerical Algorithm

The model is solved using a global solution that uses projection methods. The competitive equilibrium given any government policy is solved using Euler equation iteration and the government problem is solved using value function iteration methods.

Denote $\hat{x} = \frac{x}{A_{-1}\mu_g}$ the de-trended version of variable x. Let $\hat{\mathbf{s}} = \left\{\left(g, \underline{z}, \hat{B}^d, \hat{B}^x\right), h\right\}$ denote the de-trended aggregate state. First I solve for the set of competitive equilibrium given any *current* government policies $\left\{\hat{B}^{x\prime}, \iota\right\}$, *expected* government policies $\left\{\hat{\mathcal{B}}^{x\prime}(\hat{\mathbf{s}}), \mathcal{I}(\hat{\mathbf{s}})\right\}$ and associated functions of expected consumption and price of public debt $\left\{\hat{\mathcal{C}}(\hat{\mathbf{s}}), q^b(\hat{\mathbf{s}}, \hat{B}^x)\right\}$. This implies solving for equilibrium functions $\left\{\underline{z}'(\hat{\mathbf{s}}; \hat{B}^{x\prime}, \iota), \hat{B}^{d\prime}(\hat{\mathbf{s}}; \hat{B}^x, \iota), \hat{N}(\hat{\mathbf{s}}; \hat{B}^x, \iota), \nu(\hat{\mathbf{s}}; \hat{B}^x, \iota)\right\}$, using the following set of equations

$$\underline{z}'(\hat{\mathbf{s}}; \hat{B}^x, \iota) = \left[(\kappa + 1)\hat{N} \frac{\mathbb{E}\left[\tilde{\Lambda}(\hat{\mathbf{s}}, \hat{\mathbf{s}}') \frac{\mathcal{I}(\hat{\mathbf{s}}')}{q^b(\hat{\mathbf{s}}, \hat{B}^x)}\right]}{\mathbb{E}\left[\tilde{\Lambda}(\hat{\mathbf{s}}, \hat{\mathbf{s}}') \exp(g')\right]}\right]^{\frac{1}{1-\lambda}} \tag{41}$$

$$q^b(\hat{\mathbf{s}}, \hat{B}^x)\hat{B}^{d\prime}(\hat{\mathbf{s}}; \hat{B}^x, \iota) = \hat{N}\left((1 - \underline{z})^{-\lambda}(1 + \kappa) - \kappa\right) \tag{42}$$

$$\hat{N}(\hat{\mathbf{s}}; \hat{B}^x, \iota) = \sigma\left(\frac{\exp(g)}{\mu_g} \frac{\lambda \underline{z}}{\lambda - 1} + \iota \hat{B}^d\right) \tag{43}$$

$$\nu(\hat{\mathbf{s}}; \hat{B}^x, \iota) = \mathbb{E}\left[\tilde{\Lambda}(\hat{\mathbf{s}}, \hat{\mathbf{s}}') \frac{\mathcal{I}(\hat{\mathbf{s}}')}{q^b(\hat{\mathbf{s}}, \hat{B}^x)}\left[1 + \frac{(\kappa + 1)}{\lambda - 1}\underline{z}'(\hat{\mathbf{s}})^{-\lambda}\right]\right] \tag{44}$$

where

$$\tilde{\Lambda}(\hat{\mathbf{s}}, \hat{\mathbf{s}}') = \beta \exp(g)^{1-\gamma}\left(\frac{\hat{\mathcal{C}}(\hat{\mathbf{s}}')}{\hat{C}(\hat{\mathbf{s}})}\right)^{-\gamma}\left(1 - \sigma + \sigma\nu(\hat{\mathbf{s}}')\right) \tag{45}$$

Note that I have used the functional forms used in the calibration to substitute for $u(\cdot), G(\cdot)$ and I have also used the case of $q^d(\mathbf{s}) = q^b(\mathbf{s})$. Additionally, equation (44) comes from solving the expectation over z' in equation (16).

The algorithm to solve for the competitive equilibrium given expected and current government policies follows these steps:

(1) Generate a discrete grid for variable x state space $G_x = x_1, x_2, ...x_{N_x}$, for $x = g, \underline{z}, \hat{B}^d, \hat{B}^x$. I use 10 points for each state variable, except for $\hat{B}^x$ for which I use 15. The total aggregate state space is given by $\mathcal{S} = G_g \times G_{\underline{z}} \times G_{\hat{B}^d} \times G_{\hat{B}^x} \times \{m, a\}$

(2) Feed in some expected government policies $\left\{ \hat{\mathcal{B}}^x(\hat{\mathbf{s}}), \mathcal{I}(\hat{\mathbf{s}}) \right\}$.

(3) Conjecture a functional forms $\mathcal{E}_1(\mathbf{s}, B^{x\prime}, \iota)$ and $\mathcal{E}_2(\mathbf{s}, B^{x\prime}, \iota)$ for all $(\mathbf{s}, B^{x\prime}, \iota) \in \mathcal{S} \times G_{\hat{B}^x} \times \{0, 1\}$, that will be guesses for $\mathbb{E}\left[\tilde{\Lambda}(\hat{\mathbf{s}}, \hat{\mathbf{s}}') \frac{\mathcal{I}(\hat{\mathbf{s}}')}{q^b(\hat{\mathbf{s}}, \hat{B}^x)} \right]$ and $\mathbb{E}\left[\tilde{\Lambda}(\hat{\mathbf{s}}, \hat{\mathbf{s}}') \exp(g') \right]$, respectively.

(4) Solve for $\left\{ \underline{z}'(\hat{\mathbf{s}}), \hat{B}^{d\prime}(\hat{\mathbf{s}}), \hat{N}(\hat{\mathbf{s}}), \nu(\hat{\mathbf{s}}) \right\}$ using (41)-(44). Check whether $\hat{B}^{d\prime}(\hat{\mathbf{s}}) \geq 0$ in every grid point (this ensures that we are under the equilibrium in which $q^d(\mathbf{s}) = q^b(\mathbf{s})$.

(5) Compute $\mathbb{E}\left[\tilde{\Lambda}(\hat{\mathbf{s}}, \hat{\mathbf{s}}') \frac{\mathcal{I}(\hat{\mathbf{s}}')}{q^b(\hat{\mathbf{s}}, \hat{B}^x)} \right]$ and $\mathbb{E}\left[\tilde{\Lambda}(\hat{\mathbf{s}}, \hat{\mathbf{s}}') \exp(g') \right]$ using quadrature methods for computing expectations (Tauchen and Hussey (1991)). For evaluation of the functions outside grid points I use piecewise linear interpolation.

(6) If $\sup_{\mathbf{s}, B^{x\prime}, \iota} \left\| \mathcal{E}_1(\mathbf{s}, B^{x\prime}, \iota) - \mathbb{E}\left[\tilde{\Lambda}(\hat{\mathbf{s}}, \hat{\mathbf{s}}') \frac{\mathcal{I}(\hat{\mathbf{s}}')}{q^b(\hat{\mathbf{s}}, \hat{B}^x)} \right] \right\| < \epsilon$ and $\sup_{\mathbf{s}, B^{x\prime}, \iota} \left\| \mathcal{E}_2(\mathbf{s}, B^{x\prime}, \iota) - \mathbb{E}\left[\tilde{\Lambda}(\hat{\mathbf{s}}, \hat{\mathbf{s}}') \exp(g') \right] \right\| < \epsilon$ then the conjecture is an competitive equilibrium. If not, update (using some dampening) and start again from step two until convergence.

Given the set of competitive equilibria the second part of the algorithm solves for the government problem, given its time inconsistency problem. Following Bianchi and Mendoza (2013), a solution to the Markov Perfect Equilibrium can be found by solving a fixed point between the expected government policies and the optimal one-period deviation policies that solve government problem (38)-(40) in its de-trended recursive representation.

The algorithm to solve for the Markov Perfect equilibrium follows these steps:

(1) Conjecture expected policies $\left\{ \hat{\mathcal{B}}^x(\hat{\mathbf{s}}), \mathcal{I}(\hat{\mathbf{s}}) \right\}$ and a price schedule for public debt $q^b(\hat{\mathbf{s}}, \hat{B}^{x\prime})$ for any $\hat{\mathbf{s}}$ in the previously defined state space $\mathcal{S}$.

(2) Solve for the set of competitive equilibria given any possible current government policy and the conjectured expected government policy. This is done using the first part of the algorithm.

(3) Solve for the recursive government problem (38) - (40). The problem is solved using value function iteration. The value functions are approximated using spline interpolations when the state is not on the grid. The choice of external debt in the maximization problem is done over a finer grid with 200 points to improve accuracy.

(4) Compute $q^b(\mathbf{s}, \hat{B}^{x\prime}) = \mathbb{E}\left[\iota(\hat{\mathbf{s}}') | \hat{\mathbf{s}} \right] / R$ using quadrature methods.

(5) If $\sup_{\mathbf{s}} \| \mathcal{X}(\mathbf{s}) - X(\mathbf{s}) \| < \epsilon$ for $X = B^x, \iota, q^b$ (where $\mathcal{X}$ refers to the expected version of X) then stop. Otherwise update conjectures of expected policies and price of debt (using some dampening parameter) and start from the first step.

Appendix C. Sensitivity Analysis

This section analyzes the sensitivity of the main results to certain key parameters in the model. In particular, I consider the effects of different specifications for the degree of tightness of the limited commitment constraint of banks (captured by parameter κ), the discount factor of households (parameter β) and the dispersion of idiosyncratic bank productivities (captured by the shape of the Pareto distribution of idiosyncratic productivities λ). Results are reported in Table 6. Column 1 shows the main summary statistics for the baseline model.

TABLE 6. Sensitivity Analysis

	(1)	(2)	(3)	(4)
	Baseline	Tighter Banks	Higher Discount	Lower Prod.
Statistic	Model	LC constraint	Factor	Dispersion
		$\kappa = 4.5$	$\beta = 0.96$	$\lambda = 4$
Δ Output (wrt Baseline)	0.0%	-12.0%	-0.2%	-17.4%
External Public Debt	22.3%	22.4%	21.7%	9.1%
Domestic Public Debt	9.4%	9.0%	9.1%	4.5%
Interest Rate Spread	130	107	76	76
Outptut Cost of Default	-5.43%	-5.26%	-5.50%	-2.62%
No Liquidity Effect				
External Public Debt	14.0%	12.6%	11.3%	5.8%
Outptut Cost of Default	-3.91%	-3.64%	-3.53%	-1.58%
No Balance Sheet Effect				
External Public Debt	7.7%	6.7%	8.6%	6.2%
Outptut Cost of Default	-1.75%	-1.88%	-1.87%	-1.68%
Neither Effect				
External Public Debt	0.9%	0.5%	0.7%	1.7%
Outptut Cost of Default	0.00%	0.00%	0.00%	0.00%

Notes: All statistics are averages from each model's simulations. Δ output is the variation in average output with respect to the baseline model. External and domestic debt are in % of annual GDP. Spreads are in basis points. Output cost is the average percentage deviation of output under default with respect to output in absence of default for the following twelve quarters after a default.

Column 2 shows the summary statistics for an alternative specification in which all the parameters of the model are the same as in the baseline case, except for the parameter associated

to the banks' limited commitment constraint which is set to $\kappa = 4.5$. This value is in line with that considered in Gertler and Kiyotaki (2010) and Bocola (2014) which study developed economies. A tighter limited commitment constraint for banks has associated a lower level of output. The average level of output in the simulations of this economy is 12% lower than the average level of output in the simulations of the baseline model. Given a tighter limited commitment constraint banks with high productivities can borrow less from banks with low productivities and can demand less labor. This reduces equilibrium wages and attracts banks with lower productivities to invest in their technology. This in turn reduces the level of output since labor is allocated to technologies that are, on average, of lower productivity. The tighter constraint also increases the liquidity value of public debt given that there is less lending in the interbank market and the availability of public debt helps alleviate the inefficiencies introduced by the limited commitment constraint. Having said this, the variation in the tightness of the limited commitment constraint does not significantly change the economic relevance of the balance-sheet and liquidity effect. The level of external public debt in the model with both effects and the the specifications with only one of them do not change significantly with respect to those in the baseline parametrization.

Column 3 reports the results for an alternative specification of the model with a higher discount factor of $\beta = 0.96$ which is closer to the discount factor considered in standard models of business cycles. A higher discount factor reduces the value of issuing external public debt to front-load consumption. This dissuades the government from issuing external debt in the 'risky' part of the price schedule and reduces the levels of interest rate spreads. The average level of interest rate spread under this parametrization is 76 basis points, lower than 130 basis points in the baseline model. The average level of external and domestic public debt and the output cost of default are similar to those in the baseline model. The relevance of the liquidity effect in generating commitment is higher than in the baseline parametrization. In the model without the balance-sheet effect the average level of external public debt issuance 8.6% compared to 7.7% in the baseline parametrization. The reason is that given the higher discount factor, households value more the output cost of default due to the liquidity effect (that is persistent over time) and this in turns generates more commitment for the government.

Finally, column 4 considers an alternative model parametrization with a lower dispersion in the distribution of idiosyncratic productivity. In particular, I consider the shape of the Pareto distribution of productivities of $\lambda = 4$ which is close to the value used in trade studies

for advanced economies (e.g. Melitz and Redding (2014)). A lower dispersion in the idiosyncratic productivity of banks implies that negative shocks to the financial system translate into shocks of smaller magnitude to output. The reason is that idiosyncratic productivities are more concentrated and hence changes in the composition of banks that are using their production technology will not have large effects on the average productivity and hence on output. This implies a sovereign default will have a smaller effect on output and thus on the government's commitment. Under this specification the average output cost of default in the 3 years that follow a default is 2.6%, compared to 5.4% in the baseline parametrization. Consequently, the average level of external public debt is 9.1% of GDP, less than half of the levels in the baseline parametrization.

Appendix D. Additional Figures

Figure 10. Banks Liquidity Management Before and After Default
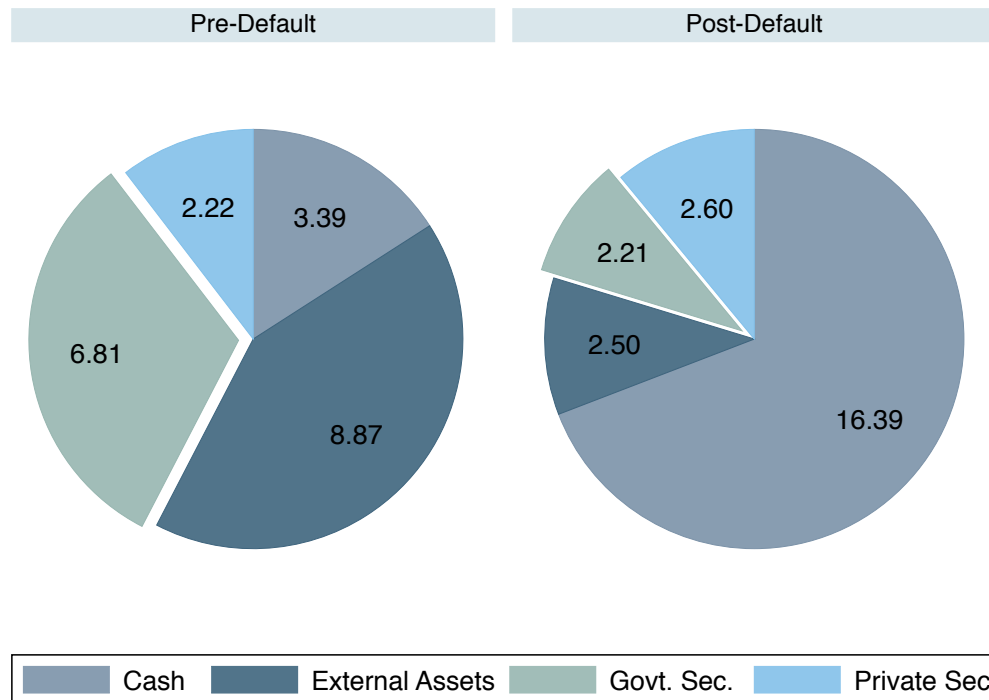
Notes: Values are expressed as % of total assets (therefore they do not add up to 100). Pre-default period corresponds to the average of the period 1995-2000. Post-default period corresponds to the average of the period 2006-2012. The years 2002-05 were excluded since the banks balance-sheet contained government securities that were provided by the government to compensate for net worth losses due to asymmetric currency conversion of assets and liabilities. Data source: BCRA