A MEASURE OF RATIONALITY AND WELFARE*

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Abstract. There is evidence showing that individual behavior often deviates from the classical principle of maximization. This evidence raises at least two important questions: (i) how severe the deviations are, and (ii) which method is the best for extracting relevant information from choice behavior for the purposes of welfare analysis. In this paper we address these two questions by proposing an instrument that identifies the closest preference relation to the revealed choices and evaluates the inconsistencies by its associated welfare loss. We call this measure the swaps index.

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1. Introduction

The standard model of individual behavior is based on the maximization principle, in which the alternative chosen by the individual is the one that maximizes a well-behaved preference relation over the menu of available alternatives. The standard model has two main advantages. The first is that it provides a simple, versatile, and powerful account of individual behavior. It is difficult to conceive of a simpler and more operational model with such a high predictive power. Its second main feature is that it suggests the maximized preference relation as a tool for individual welfare analysis. That is, the standard approach allows the policy-maker to reproduce the decisions that, given the chance, the individual would have made of her own volition.

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Over the last decades, however, research has produced increasing amounts of evidence documenting systematic deviations from the notion of rationality implied in the maximization principle. Some phenomena that have attracted a great deal of empirical and theoretical attention and prove difficult, if not impossible, to accommodate within the classical theory of choice are framing effects, menu effects, dependence on reference points, cyclic choice patterns, choice overload effects, etc.[1] The violation in some instances of the maximization principle raises at least two important questions:

Q.1: How severe are the deviations from the classical theory?

Q.2: What is the best way to extract relevant information from the choices of the individual for the purposes of welfare analysis?

By properly addressing Q.1, it would be possible to evaluate how accurately the classical theory of individual behavior describes behavior for certain individuals in specific environments. That is, the focus should not be based on whether or not individuals violate the maximization principle in a given situation, but on how close their behavior is with respect to this benchmark. Moreover, the availability of a reliable tool to assess the distance between actual behavior and behavior consistent with the maximization of a preference relation offers a unique tool to gain a deeper understanding of actual decision-making. This is so because the measure would allow the classification of individuals in "classes of rationality" and hence one may study various issues on the differences between individuals classified in different classes, and the similarities of individuals classified in the same class. This, in turn, may also prove crucial in the development of future choice models, which may take the distribution of the degree of consistency of the individuals as one of their primitives.

By dealing with Q.2, it would be possible to identify, from an external perspective, the good or bad alternatives for the individual even when the behavior of the individual is not fully compatible with the maximization principle. This is of course of prime relevance since welfare analysis is at the core of economics.

In spite of the fact that these two questions are intimately related, the literature has treated them separately. In this paper we offer for the first time a unified treatment of the measurement of rationality and welfare. Relying on standard revealed preference data, we propose an instrument that identifies the closest preference relation to the revealed choices, the welfare ranking, and evaluates the inconsistencies with the

[1]We review the relevant literature in section 2.

preference relation identified on the basis of welfare considerations. We call such an instrument *the swaps index*.

The swaps index evaluates the inconsistency in every observation unexplained by a preference relation by enumerating the number of available alternatives in the menu that are above the chosen one according to the preference relation. That is, it counts in each menu the number of alternatives that need to be swapped with the chosen alternative in order to rationalize the choices of the individual. Then, the swaps index is given by the preference relation that minimizes the total number of swaps in all the observations. The swaps index has several highly attractive characteristics. As mentioned, it offers a unified treatment of welfare analysis and the measurement of rationality. That is, the preference relation identified by the swaps index represents the welfare ranking, and the associated number of swaps required to rationalize all the choices constitutes the measurement of rationality, interpreted as the welfare loss caused by the inconsistency in choices. These considerations make the swaps index unique. To welfare analysis it offers a well-behaved preference relation interpreted as the best approximation to the choices of the individual, and with an associated error term, measured in terms of the welfare loss. To the problem of the measurement of rationality, it is singular in that it evaluates inconsistent behavior in terms of welfare loss, and thus brings into the study of irrationality its welfare implications. Also, it is important to note that the swaps index relies exclusively on the endogenous information arising from the revealed choices, and that, methodologically, it measures rationality and welfare on the basis of how close behavior is to the classical preference maximization model.

Given these considerations, we study the swaps index in detail. First, in section 4.1 we provide a simple example to illustrate the contrast between the treatment that the swaps index gives to the measurement of rationality and the proposals in the literature. The example presents two scenarios that the swaps index treats as diametrically different, by measuring the inconsistency of choice on the basis of welfare considerations, while the approaches that the literature offers may treat them counterintuitively.

In section 4.2 we compare the treatment that the swaps index gives to the analysis of welfare with other suggestions in the literature. This exercise illustrates that the preference relation resulting from the swaps index is not only well behaved, in the sense that it is a linear order and hence its maximization offers a unique alternative in any possible menu of alternatives, but furthermore ranks every pair of alternatives taking into consideration the whole of the choice data, not just those observations where the two alternatives are present. In principle, therefore, the welfare criterion that we suggest provides the policy-maker with a clear guideline, and it endogenizes all the consequences of ranking one alternative over another.

In section 4.3 we show that the computer science literature offers a wealth of techniques that compute the index remarkably well in practice. In particular we establish that the problem of finding the optimal preference relation of the swaps index is equivalent to a well-known problem in the computer science literature, the linear ordering problem (LOP). Hence one can readily use the vast algorithmic literature developed to solve the LOP, for the purpose of computing the optimal preference relation for the swaps index.

We then aim to gain a deeper understanding of the swaps index, by providing, in section 4.4, a complete axiomatic characterization of it. We propose seven properties that any inconsistency index relying exclusively on the endogenous information arising from the choice data should satisfy. We show that these seven properties completely characterize the swaps index. Remarkably, this exercise makes the swaps index the first axiomatically founded inconsistency measure in the literature.

Finally, in section 4.5, we study three generalizations of the swaps index, that use in different ways different sorts of information exogenous to the revealed choices, that on occasions may be available to the analyst, and that may represent useful information in the measurement of rationality and welfare. Importantly, the three proposals share the structure with the swaps index in that they additively evaluate the inconsistency of the data by identifying the preference relation that is closest to observed behavior, and measure the inconsistency of choice on the basis of welfare considerations. The first of the generalizations, which we call the *non-neutral swaps index*, makes use of information on the nature of alternatives, such as their monetary values, or an aggregation of the attributes that define them, etc. The non-neutral swaps index, then, weights differently the different alternatives in the upper contour sets of the inconsistent choices, based on information on the essence of the alternatives. We call the second generalization the *positional swaps index*. The positional swaps index is appropriate when information is available on the cardinal utility values of the alternatives based on their position in the ranking. It then weights an inconsistent choice by the sum of the utility values of the alternatives that have been forgone. This can be interpreted as the total utility loss for the inconsistent choice in that observation. Finally, the last index we propose, the *general weighted index*, represents a broad generalization of all the former ones. General weighted indices are flexible enough to evaluate the inconsistency of choice by weighting each inconsistent observation on the basis of the possible underlying values of the alternatives, the values of the various menus of alternatives, and using particular priors on the plausibility of the different welfare rankings. In section 4.5 we provide the complete characterizations of these three cases, which follow by relaxing some of the properties characterizing the swaps index.

2. Related Literature

This paper relates to several significant strands of literature. First, there is a large empirical literature documenting deviations from the classical model of individual behavior. It is by now well established that individual behavior is often dependent on the framing of the choice situation (Tversky and Kahneman, 1981), exhibits cyclic choice patterns (May, 1954), is influenced by reference points (Thaler, 1980), and is susceptible to various sorts of menu manipulations (Iyengar and Lepper, 2000). The literature has reacted to this evidence by offering theoretical models that, adopting a revealed preference approach, expand the classical notion of rationality to incorporate in various ways stylized accounts of these behavioral phenomena. Some prominent recent examples are Bossert and Sprumont (2003, 2009), Masatlioglu and Ok (2005), Manzini and Mariotti (2007, 2011), Xu and Zhou (2007), Salant and Rubinstein (2008), Masatlioglu and Nakajima (2008), Green and Hojman (2009), Ok, Ortoleva and Riella (2010), and Masatlioglu, Nakajima and Ozbay (2011).

Second, there is a literature on revealed preference tests of the maximization principle, typically run on consumer behavior data, with no axiomatic foundation. The first to propose this type of test was Afriat (1973), which suggested measuring the amount of adjustment required in each budget constraint to avoid any violation of the maximization principle. Then, the inconsistency in Afriat is measured in terms of relative wealth losses. Chalfant and Alston (1988) and Varian (1990) further developed Afriat's approach. Another proposal is to compute the maximal subset of the data that is consistent with the maximization principle. Papers following this approach are Houtman and Maks (1985) and Banker and Maindiratta (1988). In a recent paper, Dean and Martin (2010) provide a powerful algorithm to compute such a maximal subset of the data. Choi, Kariv, Müller and Silverman (2011) apply the measures of Afriat and Houtman-Maks, to test for the actual consistency of individuals in a field experiment. Yet a third approach counts the number of violations of a consistency property detected in the data. In this respect, see the work of Swofford and Whitney (1987) and Famulari (1995). Echenique, Lee and Shum (2010) make use of the monetary structure of budget sets to suggest a version of this notion, the money pump index, that considers the total wealth lost in all revealed cycles. Finally, in a general framework, Gilboa and Schmeidler (2010) rank a given set of theories on their ability to explain a set of data, providing a representation that is a combination of log-likelihood and a measure of complexity.

Lastly, there is a growing number of papers dealing with individual welfare analysis, when the individual's behavior is inconsistent. Bernheim and Rangel (2009) add to the standard choice data the notion of ancillary conditions, or frames. Ancillary conditions are assumed to be observable and may affect individual choice, but are irrelevant in terms of the welfare associated with the chosen alternative. Bernheim and Rangel suggest a welfare preference relation that ranks an alternative as welfare-superior to another only if the latter is never chosen when the former is available. Chambers and Hayashi (2009) characterize an extension of Bernheim and Rangel's model to probabilistic settings. Manzini and Mariotti (2009) offer a critical assessment of Bernheim and Rangel. Rubinstein and Salant (2011) propose the welfare relation that is consistent with a set of preference relations in the sense that all the preference relations in the set could have been generated by the cognitive process distorting that welfare relation. Masatlioglu, Nakajima and Ozbay (2011) suggest a welfare preference based on their limited attention model of decision-making. Green and Hojman (2009) suggest identifying a list of conflicting selves, which, when aggregated, induce the revealed choices, and then using the aggregation rule to make the individual welfare analysis. Finally, Baldiga and Green (2010) analyze the conflict between preference relations in terms of their disagreement on choice. They then use their measures of conflict between preference relations together with Green and Hojman's notion of multiple selves to find the list of multiple selves with the minimal internal conflict that explain a given choice data, and suggest this as a welfare measure.

3. The Swaps Index

We start by formally introducing the fundamental ingredients in our setting. Let X be a finite set of k alternatives. An *observation* is a pair (A, a), where $A \subseteq X$ is a non-empty menu of alternatives and $a \in A$ is the chosen alternative. That is, the observation (A, a) represents the case where the individual confronts the menu A and chooses a. Denote by $\mathcal{O}$ the set of all possible observations. A *collection of observations* is a mapping $f : \mathcal{O} \to \mathbb{Z}_+$, indicating the number of times each observation (A, a) occurs. Note that we allow for the possibility that the same menu A may be in f more than once, with the same or different associated chosen elements. Denote by $\mathcal{F}$ the set of all possible collections of observations.

A preference relation P is a strict linear order on X, that is, an asymmetric, transitive, and connected binary relation. Denote by $\mathcal{P}$ the set of all possible linear orders on X. The collection of observations f is *rationalizable* if every single observation that is registered at least once in f can be explained by the maximization of the same preference relation. Formally, f is rationalizable if there exists a preference relation P such that $m(P, A) = a$ whenever $f(A, a) > 0$, where $m(P, A)$ represents the maximal element in A according to P. Clearly, not every collection of observations is rationalizable. An inconsistency index is a mapping $I : \mathcal{F} \to \mathbb{R}_+$ that measures how inconsistent, or how far away from rationalizability, a collection of observations is.

We are now ready to introduce the prime index we propose in this paper, the *swaps index*. In a nutshell, the swaps index identifies the welfare ranking of the individual, the preference relation that is closest to the revealed choices, and measures the divergence between the welfare ranking and the inconsistent choices by the sizes of the upper contour sets of the inconsistent observations, which can be interpreted in terms of the associated welfare loss.

More concretely, consider an observation (A, a) that is inconsistent with the maximization of a given preference relation P. That is, the chosen element a and the maximal element from A according to P, $m(P, A)$, diverge. This implies that there is a number of alternatives in A that are preferred to the chosen alternative a, according to P, but that are nevertheless ignored by the individual. It is then natural to contemplate that the inconsistency of observation (A, a) with regard to P entails considering the number of alternatives in A that are above the chosen one. These are the alternatives that need to be swapped with the chosen alternative in order to make the choice of a consistent with the maximization of P. Then, the swaps index follows this criterion in the analysis of every single observation composing the data, and finds the preference relation that best fits the collection of all the revealed choices. Namely, the preference P that minimizes the total number of swaps in all the observations:

$$I_S(f) = \min_{P \in \mathcal{P}} \sum_{(A,a)} f(A,a)|\{x \in A : xPa\}|.$$

There are several attractive characteristics of the swaps index that make it unique. Most notably, it provides a joint treatment of inconsistency and welfare analysis. It identifies the closest preference relation to the revealed data, measuring the inconsistency of this in terms of the associated welfare loss. By so doing, it discriminates between the severity of the various inconsistent choices, relying exclusively on the information contained in the choice data, and considers additively every single inconsistent observation. We now turn to the study of various important questions on the swaps index.

4. Discussion

4.1. **The Measurement of Rationality: A Comparison.** We illustrate the treatment that the swaps index gives to the measurement of rationality by way of a simple example, and contrast this with the other proposals in the literature. The example shows that the swaps index, by measuring the inconsistency of choice on the basis of welfare considerations, discriminates sharply between different situations that may be treated counterintuitively by the inconsistency measures offered in the literature.

Consider the set of alternatives $X = \{1, \dots, k\}$, with $k > 1$. Suppose that there is a wealth of observations f showing that the individual is completely consistent with a preference relation P ranking the alternatives as $1P2P \dots Pk$. Assume now that there are two different scenarios. The two scenarios involve the observation of one revealed choice, in addition to the consistent data f. In scenario I we observe that the individual chooses option k from menu X, while in scenario II we observe that the individual chooses option 2 from menu X. Clearly, the additional observation to f generates an inconsistency in both scenarios, since we are assuming that there is sound evidence f indicating that the individual should have chosen option 1 in both scenarios. The question arises, therefore, of how inconsistent these decisions are.

In the eyes of the swaps index, the two scenarios represent markedly different situations. Scenario I involves a large inconsistency, since the individual chooses the worst possible alternative, alternative k, ignoring all the remaining available alternatives, which have in fact been revealed to be better than the selected alternative k. On the other hand, scenario II although it also represents an inconsistency with the maximization principle, to the swaps index, is orders of magnitude lower, since it involves choosing the second best option that is available, option 2. The rationalization of the behavior of the individual in scenario II requires ignoring alternative 1 only, while the case of scenario I requires ignoring every single alternative in X except the chosen one, k. Hence, the swaps index discriminates between the two different scenarios in an intuitive way, offering an unambiguous answer based on the welfare implications of the inconsistent choice.

We now turn to the treatment that the classical proposals in the literature give to the observed inconsistency in the two scenarios. To the best of our knowledge, Afriat (1973) proposes the first method measuring the inconsistency of behavior. In a consumer setting, Afriat suggests measuring the amount of relative wealth adjustment required in each budget constraint to avoid all violations of the maximization principle. The idea is that when a portion of the wealth is considered the budget set shrinks, eliminating some revealed information, and hence some inconsistencies in the data may vanish. Then, the degree of inconsistency of a collection of observations that Afriat proposes is associated with the minimal necessary wealth adjustment that makes all the data consistent with the maximization principle. Therefore, Afriat's judgement of scenarios I and II depends crucially on some external structure, such as the monetary values of the alternatives. The latter, of course, need not necessarily agree with the welfare ranking, and hence may lead to conclusions contradicting the intuitive view that scenario I represents a larger inconsistency. For example, if the monetary value of option k is higher than that of option 2, Afriat would judge scenario II as more inconsistent than scenario I.[2]

Houtman and Maks (1985) propose considering the minimal subset of observations that needs to be eliminated from the data in order to make the remainder rationalizable. The cardinality of this minimal subset to be discarded suggests itself as a measure of inconsistency. In our example, since there is one single inconsistency in both scenarios, Houtman-Maks does not discriminate between them.

[2]Varian (1990) extends Afriat to contemplate different relative wealth adjustments in the different observations, and then consider the necessary aggregated relative wealth adjustments to prevent all the inconsistencies from being revealed. In our example, since there is only one single inconsistency, Afriat and Varian coincide.

Finally, rationality has also been measured by counting the number of times in the data a consistency property, say the Independence of Irrelevant Alternatives, is violated (see, e.g., Swofford and Whitney, 1987; Famulari, 1995). It turns out that the conclusions of this criterion in our example depend on the specific nature of the consistent collection of observations f. Depending on this, the two scenarios may be treated alike, or even scenario II may be regarded as more inconsistent.[3] Relatedly, the judgment of the money pump index of Echenique, Lee and Shum (2010) depends on both the specific nature of f and, as in the case of Afriat, the exogenous information on the monetary values of the alternatives.

4.2. **The Measurement of Welfare: A Comparison.** Let us illustrate our approach to welfare analysis by contrasting it with two pioneering proposals in the literature, Bernheim and Rangel (2009) and Green and Hojman (2009). Interestingly, although these two papers tackle the problem from different angles, they independently suggest the use of the same welfare notion. Let us denote by $\overline{P}$ the Bernheim-Rangel-Green-Hojman preference, defined as $x\overline{P}y$ if and only if there is no observation (A, y) with $x \in A$ such that $f(A, y) > 0$. In other words, x is ranked above y in the welfare ranking $\overline{P}$ if y is never chosen when x is available. Bernheim and Rangel show that whenever every menu A in X is observed at least once, $\overline{P}$ is acyclic, and hence it is consistent with the maximization principle.

We now examine which is the relationship between $\overline{P}$ and the optimal preference relation of the swaps index P^*.[4] It turns out to be the case that the two welfare relations are fundamentally different. That P^* is not contained in $\overline{P}$ follows immediately, since P^* is a linear order, while $\overline{P}$ is incomplete in general. In the other direction, and more importantly, note that while $\overline{P}$ evaluates the ranking of two alternatives x and y by attending exclusively to those menus of alternatives where both x and y are available, to establish the ranking of two alternatives P^* takes into consideration all the observations together. Hence, P^* and $\overline{P}$ may rank two alternative in opposite directions. A simple example shows this point.

[3]Suppose $k = 3$ and $f(X, 1) = f(\{1, 3\}, 1) = f(\{2, 3\}, 2) = 1$ and $f(\{1, 2\}, 1) = m$. It is easy to see that if $m = 1$, scenario I is more inconsistent than scenario II, if $m = 2$ the two scenarios are equally inconsistent, and whenever $m > 2$ scenario II is more inconsistent.

[4]The optimal preference relation P^* is not necesarily unique, but it is easy to show that almost all collections of observations have a unique optimal preference relation as the number of observations grows.

Consider a collection f composed by the following observations: $f(\{x,y\},x) = f(\{y,z\},y) = 3$ and $f(\{x,y,z\},y) = f(\{x,z\},z) = 1$. Clearly, $z\overline{P}x$ since x is never chosen in the presence of z. However, to evaluate the ranking of alternatives x and z, the swaps index considers the whole collection f. Observations $f(\{x,y\},x) = f(\{y,z\},y) = 3$ signify a strong argument for the preference ranking x over y and y over z. This preference implies a mistake in the observations $f(\{x,y,z\},y) = f(\{x,z\},z) = 1$, but rationalizes the more frequent evidence of $f(\{x,y\},x) = f(\{y,z\},y) = 3$. In fact, such a preference is the optimal preference relation P^* for the swaps index, and hence $\overline{P}$ and P^* may follow different directions.

4.3. **Computational Considerations.** We now deal with the question of identifying in practice the closest preference relation to the choice data, and its associated inconsistency level. Given that we have imposed no restriction whatsoever on the nature of the collections of observations, it is not surprising that finding the optimal preference relation may on occasions be computationally complex. Fortunately, in this section we show that we can draw upon existing techniques that address computational problems that are formally equivalent to ours, and that offer good solutions.

Computational considerations are common in the application of the various inconsistency indices that the literature offers. As mentioned above, Houtman and Maks (1984) propose the computation of the maximal subset of the data consistent with the maximization of a preference relation. Importantly, Dean and Martin (2010) establish that the problem studied by Houtman and Maks is equivalent to a well-known problem in the computer science literature, the minimum set covering problem (MSCP), and then show that one can use the wide variety of algorithms that have been designed in the operations research literature to solve the MSCP, for the purpose of computing the maximal subset of the data consistent with the maximization of a preference relation.

Exactly the same strategy can be adopted for the swaps index. Consider another well-known problem in the computer science literature, the linear ordering problem (LOP). The LOP has been related to a wide variety of problems, including various economic problems, particularly to the triangularization of input-output matrices for the study of the hierarchical structures of the productive sectors in an economy.[5] Formally, the (integer) LOP problem over the set of vertices Y, and directed weighted edges connecting all vertices x and y in Y with (integer) cost c_{xy}, consists of finding

[5]See Korte and Oberhofer (1970), Fukui (1986), and Howe (1991).

the linear order over the set of vertices Y that minimizes the total aggregated cost. That is, if we denote by Π the set of all mappings from Y to $\{1, 2, \ldots, |Y|\}$, the LOP involves solving $\arg\min_{\pi\in\Pi} \sum_{\pi(x)<\pi(y)} c_{xy}$. As the following result shows, the LOP and the problem of computing the optimal preference relation for the swaps index are equivalent.[6]

Proposition 1.

(1) *For every $f \in \mathcal{F}$ one can define a LOP with vertices in X, the solution of which provides the optimal preference for the swaps index.*

(2) *For any LOP with vertices in X one can define an $f \in \mathcal{F}$, its optimal preference being the solution to the LOP.*

Intuitively, the linear orders in the LOP are the preference relations in our setting and the cost of having one alternative before the other is the inconsistency that arises from revealed data. Note that the evaluation of the inconsistency associated with having one alternative a over another alternative b is very simple. It is merely the number of observations where the chosen alternative is b but a is present in the menu. Then, the computation of the optimal preference relation requires attending to all the inconsistency values associated with having one alternative over the other, exactly as in the LOP.

Proposition 1 enables the techniques that the literature offers for the solution of the LOP to be used directly in the computation of the optimal preference relation for the swaps index. These techniques involve a good array of algorithms searching for the globally optimal solution.[7] Alternatively, the literature also offers methods that whilst they do not compute the globally optimal solution, are much lighter in terms of the required computational intensity, giving good approximations.[8]

4.4. **A Characterization of the Swaps Index.** In this section we propose seven conditions that shape the treatment that an inconsistency index I may give to different sorts of collections of observations. We then show that the swaps index is characterized by this set of properties. As we will argue below in section 4.5, the first four properties

[6] All the proofs are contained in the Appendix.

[7] See, e.g., Grötschel, Jünger, and Reinelt (1984); see also Chaovalitwongse et al (2010) for a good introduction to the LOP, a review of the relevant algorithmic literature, and the analysis of one such algorithm.

[8] See Brusco, Kohn and Stahl (2008) for a good general introduction and relevant references.

are minimal properties to impose on any inconsistency function, while the appeal of the last three properties may depend on the possible availability of additional external information on the nature of menus, or on the nature of alternatives, or on both. For the time being, when no information other than revealed choices of the individual is assumed to be available, we contend that all seven properties are desirable and that any inconsistency function I should ideally satisfy them.

Rationality (RAT). For every $f \in \mathcal{F}$, $I(f) = 0 \Leftrightarrow f$ is rationalizable.

Rationality imposes that a collection of observations is perfectly consistent if and only if the collection is rationalizable. In line with the maximization principle, Rationality establishes that the minimal inconsistency level of 0 is reached only when every single choice in the collection of observations can be explained by maximizing a preference relation.

In order to introduce our next property, consider first the following definition. Denote by r a rationalizable collection of observations where all the binary menus are observed and by $\mathcal{R}$ the set of all such collections of observations. Clearly, for every $r \in \mathcal{R}$ there exists a unique preference relation, which we denote by P^r, that rationalizes r. Given f and r, we say that f is *r-invariant* if $I(f) = I(f + r)$. That is, we say that f is r-invariant whenever the addition of r to f leaves the inconsistency level associated with f unchanged.

Invariance (INV). For every $f \in \mathcal{F}$, there exists $r \in \mathcal{R}$ such that f is r-invariant.

Invariance implies that every possible collection of observations can be related to some rationalizable collection of observations involving the binary menus of alternatives. No matter the nature of the collection of observations f, there is an r that when added to f does not increase the inconsistency associated with f. In other words, Invariance imposes that any f has at least one sufficiently close rationalizable collection of observations that can be added without consequences for the total inconsistency value. The main intuition here is that for any collection of observations there is always a well defined preference relation that is the closest to it, in the sense that the addition of the associated rationalizable collection does not make the resulting collection more inconsistent. This is a natural requirement when the methodological approach is to measure the data on the basis of the distance from the classical rational model of choice.

Attraction (ATTR). For every $f \in \mathcal{F}$ and every $r \in \mathcal{R}$, there exists a positive integer z such that $f + zr$ is r-invariant.

Consider any collection of observations f and any rationalizable collection r. Attraction establishes that if the structured data r is added sufficiently often to f, extra information confirming the structured data becomes inessential, in the sense of leaving the inconsistency of the data unchanged. Note well that Attraction does not impose that the addition to any f of any r is without consequences, but that there is always a point at which r is sufficiently prevalent for the addition of r once more to be inconsequential.

Separability (SEP). For every $f, g \in \mathcal{F}$, $I(f+g) \geq I(f) + I(g)$, with equality if f and g are r-invariant for some $r \in \mathcal{R}$.

Take any two collections of observations f and g. Separability judges that the sum of the inconsistency of f and that of g can never be greater than the inconsistency associated with the collection of observations resulting from f and g together. To illustrate, suppose that f and g are both rationalizable when taken separately. Clearly, the conjunction of f and g does not need to be rationalizable, and hence arguably the collection of observations $f + g$ can only take the same or a higher inconsistency value than the sum of the inconsistency values of the two collections separately. The same idea applies when either f or g or both are not rationalizable. The sum of f and g can only generate the same or more frictions with the maximization principle, and hence should yield the same or a higher inconsistency value. Further, Separability also states that in the special case where both f and g are related to the same rationalizable collection of observations r, then the inconsistency of $f + g$ should be the same as the sum of the inconsistencies of f and g taken separately. The rationale for this second implication of Separability is that if f and g share a common structure, that is both are r-invariant with the same r, the two collections are analogous enough for them to be aggregated without further consequences for the inconsistency value.

Denote by $\mathbf{1}_{(A,a)}$ the collection uniquely formed by the observation (A, a). We can now introduce our next property.

Ordinal Consistency (OC). For every $\mathbf{1}_{(A,x)} \in \mathcal{F}$ and every $r, r' \in \mathcal{R}$ such that $r(\{a,b\}, a) = r'(\{a,b\}, a)$ for all $a, b \in A$, $I(r + \mathbf{1}_{(A,x)}) = I(r' + \mathbf{1}_{(A,x)})$.

Ordinal Consistency is a weak property on when one can be sure that the addition of one observation, (A, x), to two rationalizable collections of observations, r and r', has no consequences for the evaluation of the inconsistency levels associated with the resulting two collections of observations, $r+\mathbf{1}_{(A,x)}$ and $r'+\mathbf{1}_{(A,x)}$. Whenever the unique preference relations that rationalize r and r', namely P^r and $P^{r'}$, judge every single possible pair of alternatives in the menu A in exactly the same way, that is, for every $a, b \in A$, $aP^r b$ if and only if $aP^{r'} b$, Ordinal Consistency imposes that the inconsistency associated with adding the observation (A, x) to r should be the same as that of adding (A, x) to r'.

Disjoint Composition (DC). For every $f, \mathbf{1}_{(A,x)}, \mathbf{1}_{(B,x)} \in \mathcal{F}$ and every $r \in \mathcal{R}$, if f and $f+\mathbf{1}_{(A,x)}+\mathbf{1}_{(B,x)}$ are r-invariant and $A \cap B = \{x\}$, $I(f+\mathbf{1}_{(A,x)}+\mathbf{1}_{(B,x)}) = I(f+\mathbf{1}_{(A\cup B,x)})$.

Disjoint Composition establishes that under very special circumstances, two observations can be merged into one without affecting the inconsistency level. Take two observations (A, x) and (B, x) that share the same chosen alternative x and nothing else. That is, other than x the two menus A and B are disjoint. Suppose that these two observations are added to a collection f resulting in a collection sharing a common structure represented by the rationalizable collection r. Then, Disjoint Composition implies that the observations (A, x) and (B, x) can be merged into one single observation respecting the choice of x, $(A \cup B, x)$, with no consequences for the value of the inconsistency.

In order to formally introduce our last property, consider the following definition. Given a permutation σ over the set of alternatives X, for any collection of observations f we denote by $\sigma(f)$ the permuted collection of observations such that $\sigma(f)(A, a) = f(\sigma(A), \sigma(a))$.

Neutrality (NEU). For every permutation σ and every $f \in \mathcal{F}$, $I(f) = I(\sigma(f))$.

Neutrality imposes that the inconsistency index should be independent of the names of the alternatives. That is, any relabeling of the alternatives should have no effect on the level of inconsistency.

Theorem 1 states the characterization result.[9]

Theorem 1. *An inconsistency index I satisfies RAT, INV, ATTR, SEP, OC, NEU and DC if and only if it is a positive scalar transformation of the swaps index.*

[9]The seven properties imply that the weight of every alternative, in every menu of alternatives, placed above the chosen one according to any preference relation yields the same positive constant value.

4.5. **Extensions of the Swaps Index.** Theorem 1 provides a complete characterization of the swaps index. One of the features of the swaps index is that it relies exclusively on the endogenous information contained in the revealed choices. This makes the swaps index particularly interesting and amenable to use in applications. On occasions, however, the analyst may have more information and may want to use it in order to assess the consistency of the choices of the individual, and to identify the optimal welfare ranking. For this reason, in this section we explore several extensions. In particular, we suggest three novel indices, that vary in their generality and the use they make of information external to the revealed preferences. Importantly, the three generalizations follow readily by relaxing some of the characterizing properties of the swaps index.

Non-Neutral Swaps Index. We start with a natural extension of the swaps index. Suppose that the analyst has at her disposal additional information on the nature of the alternatives, and that she wants to use it in the assessment of rationality and welfare. In this case, it seems logical to argue that it is not only the number of alternatives in the upper contour set that should be relevant, but the nature of these alternatives. For example, the analyst may consider a metric among the alternatives, say their monetary values or an aggregation of their attributes, and judge each alternative in the upper contour set by its distance from the chosen alternative. This would mean that one could incorporate into the analysis the nature of the alternatives that are endogenously identified as superior to the chosen ones, judged from the point of view of the chosen alternatives. Formally, this would imply the following inconsistency index, which we call the non-neutral swaps index,

$$I_N(f) = \min_{P \in \mathcal{P}} \sum_{(A,a)} f(A,a) \sum_{x \in A : xPa} w_{x,a},$$

where $w_{y,z} > 0$ denotes the weight of the ordered pair formed by alternatives y and z.

Note that with regard to the characterizing properties of Theorem 1, NEU immediately loses its appeal, since now one wishes to treat the alternatives differently, using the exogenous information that is available on them. It turns out that the remaining six properties completely and uniquely characterize the non-neutral swaps index.

Proposition 2. *An inconsistency index I satisfies RAT, INV, ATTR, SEP, OC and DC if and only if it is a non-neutral swaps index.*

Positional Swaps Index. We now suggest another novel inconsistency index, one that uses the endogenous information arising from the revealed preference, but in addition uses some exogenous *cardinal* information. We call it the positional swaps index. Suppose that the analyst has information on the cardinal utility values of the different alternatives, based on their position in the ranking.[10] Then, the positional swaps index evaluates an inconsistent observation by attending to the utility values of the alternatives that have been forgone, judged from the point of view of the chosen alternative. This can be interpreted as the total utility loss for the inconsistent choice in that observation. Then, the positional swaps index is given by the preference relation that minimizes the sum of utility losses. Denote by $\widehat{x}(P)$ the ranking of alternative x in P. Then, an inconsistency index is a positional swaps index if

$$I_P(f) = \min_{P\in\mathcal{P}} \sum_{(A,a)} f(A,a) \sum_{x\in A:xPa} w_{\widehat{x}(P),\widehat{a}(P)},$$

where $w_{i,j} > 0$ denotes the weight associated with the positions i and j.

The main distinctive feature of the positional swaps index is that it incorporates information on the evaluation of alternatives based on their position in the ranking, not based on their nature. Hence, NEU recovers its appeal. At the same time, the incorporation of information on the ranking of the alternatives immediately implies that OC is not satisfied since it completely disregards this type of information. As the following proposition states, the relaxation of OC from the system of properties characterizing the swaps index completely and uniquely characterizes the positional swaps index.

Proposition 3. *An inconsistency index I satisfies RAT, INV, ATTR, SEP, NEU and DC if and only if it is a positional swaps index.*

General Weighted Index. We conclude this section with a broad generalization of the swaps index that may incorporate the sort of information reflected by the non-neutral swaps index, the positional swaps index, and other sorts of information such as priors on the plausibility of the different welfare rankings. We call this index the general weighted index. The basic idea of general weighted indices is to consider

[10] For example, the cardinal utility values may be modeled as the random variables with a common distribution, and then one may sort the realizations in decreasing order of magnitude to focus on the order statistics. The expected value of the order statistics may be taken as the cardinal values of the alternatives based on their ranking, as in Apesteguia, Ballester and Ferrer (2011).

every possible inconsistency between an observation (A, a) and a preference relation P through a weight that may depend on the nature of the menu of alternatives A, the nature of the chosen alternative a, and the preference relation P. Then, for a given collection of observations f, the inconsistency index takes the form of the minimum total inconsistency across all preference relations:

$$I_G(f) = \min_{P \in \mathcal{P}} \sum_{(A,a)} f(A, a) w(P, A, a),$$

where $w(P, A, a) = 0$ if $a = m(P, A)$ and $w(P, A, a) > 0$ otherwise. The generality of the weights $w(P, A, a)$ allows the analyst to consider various types of information on the measurement of inconsistencies.

It turns out that general weighted indices are completely and uniquely characterized by the first four axioms used in Theorem 1.[11]

Proposition 4. *An inconsistency index I satisfies RAT, INV, ATTR and SEP if and only if it is a general weighted index.*

5. Conclusions

In this paper we propose a novel tool that makes a unified treatment of the measurement of rationality and welfare, the swaps index. The swaps index identifies the closest preference relation to the revealed choices, the welfare ranking, and measures its associated inconsistency by enumerating the total number of available alternatives above the chosen ones, the sizes of the respective upper contour sets. The swaps index is unique in measuring rationality in terms of welfare considerations. In addition, it is the only tool in the literature with an axiomatic foundational analysis. With respect to welfare analysis, the swaps index evaluates the welfare ranking of any two alternatives considering the whole collection of observations, and hence internalizes all the consequences of ranking one alternative above another. Moreover, it is unique in that it associates an error term with the welfare ranking.

The swaps index relies exclusively on the endogenous information arising from the choice data. We offer generalizations of the swaps index which share the main features of the swaps index, and which in addition are sensitive to various sorts of exogenous

[11] For completeness, we have also derived the representation arising when only DC is relaxed from the system of seven properties. Furthermore, we have obtained the characterizations of a version of Varian's index and of the Houtman-Maks index, sharing the structure of the characterizations developed in this section. We can provide the details upon request.

information that depending on the application may be available to the analyst, such as information on the nature of the alternatives, on their cardinal utility evaluations, etc.

Appendix A. Proofs

Proof of Proposition 1: For the first part, consider the collection of observations f and define, for every pair of alternatives x and y in X, the weight $c_{xy} = \sum_{(A,a):y=a,x\in A} f(A,a)$. It follows that $\sum_{\pi(x)<\pi(y)} c_{xy} = \sum_{\pi(x)<\pi(y)} \sum_{(A,a):y=a,x\in A} f(A,a)$ $= \sum_{(A,a)} f(A,a)|\{x \in A : \pi(x) < \pi(a)\}|$, and hence solving the LOP provides the optimal preference for the swaps index. To see the second part, consider the LOP given by weights c_{xy}, with $x, y \in X$. Define the collection of observations f given by $f(\{x,y\},y) = c_{xy}$. Since f is defined over binary problems, $\sum_{(A,a)} f(A,a)|\{x \in A :$ $\pi(x) < \pi(a)\}| = \sum_{(\{x,y\},y):\pi(x)<\pi(y)} f(\{x,y\},y) = \sum_{\pi(x)<\pi(y)} c_{xy}$, as desired.■

Proof of Theorem 1: It is easy to see that any positive scalar transformation of the swaps index satisfies the axioms. We prove the converse statement by way of seven lemmas.

Lemma 1. *For every $r \in \mathcal{R}$, $f \in \mathcal{F}$, and $z \in \mathbb{Z}_{++}$, (1) zr is r-invariant, and (2) if f is r-invariant, then $f + zr$ is r-invariant.*

Proof of Lemma 1: Consider any $r \in \mathcal{R}$, $f \in \mathcal{F}$, and $z \in \mathbb{Z}_{++}$. By RAT, we know that $I(zr) = I((z+1)r) = 0$. Hence, zr is r-invariant. This shows the first claim. Now, since zr and $(z+1)r$ are r-invariant, if f is r-invariant SEP implies that $I(f + zr) = I(f) + I(zr)$ and $I(f + (z+1)r) = I(f) + I((z+1)r)$. By RAT, we know that $I(zr) = I((z+1)r) = 0$. Hence, $I(f + zr) = I(f) = I(f + (z+1)r)$ and then $f + zr$ is r-invariant. This shows the second claim.□

Lemma 2. *For every $r \in \mathcal{R}$ and $n \in \mathbb{Z}_{++}$, there exists a positive integer z^r_n such that for every $f \in \mathcal{F}$ with $\sum_{(A,a)} f(A,a) \leq n$, $f + z^r_n r$ is r-invariant.*

Proof of Lemma 2: Take any $r \in \mathcal{R}$ and $n \in \mathbb{Z}_{++}$. Define $z^r_n = \max_{f:\sum_{(A,a)} f(A,a)\leq n} z^r_f$ where z^r_f is the minimal value in $\mathbb{Z}_{++}$ such that $f + z^r_f r$ is r-invariant. Notice that ATTR guarantees that z^r_f always exists and the finiteness of X guarantees that z^r_n is well defined. Then, if $\sum_{(A,a)} f(A,a) \leq n$, it follows that $z^r_n \geq z^r_f$ and by Lemma 1, $f + z^r_n r$ is r-invariant, as desired.□

Lemma 3. *For every $r \in \mathcal{R}$ and $n \in \mathbb{Z}_{++}$, if f is r-invariant and $\sum_{(A,a)} f(A,a) = n$, then $I(f) = \sum_{(A,a)} f(A,a) I(\mathbf{1}_{(A,a)} + z_n^r r)$.*

Proof of Lemma 3: Consider any $r \in \mathcal{R}$ and $n \in \mathbb{Z}_{++}$. Let f be r-invariant with $\sum_{(A,a)} f(A,a) = n$. Then, for any observation (A,a) with $f(A,a) > 0$, the collections of observations $f - \mathbf{1}_{(A,a)}$ and $\mathbf{1}_{(A,a)}$ both have fewer than n observations. By Lemmas 1 and 2, the collections of observations $f - \mathbf{1}_{(A,a)} + (n-1)z_n^r r$ and $\mathbf{1}_{(A,a)} + z_n^r r$ are both r-invariant. Hence, SEP guarantees that $I(f + nz_n^r r) = I(f - \mathbf{1}_{(A,a)} + (n-1)z_n^r r) + I(\mathbf{1}_{(A,a)} + z_n^r r)$. The repeated application of this argument leads to $I(f + nz_n^r r) = \sum_{(A,a)} f(A,a) I(\mathbf{1}_{(A,a)} + z_n^r r)$. Since f is r-invariant, the repeated use of Lemma 1 guarantees that $I(f) = I(f + nz_n^r r)$ and hence, $I(f) = \sum_{(A,a)} f(A,a) I(\mathbf{1}_{(A,a)} + z_n^r r)$, as desired.□

Now define the following weighting function. Given any $P \in \mathcal{P}$, consider the rationalizable collection $r \in \mathcal{R}$ such that $P = P^r$ and define $w(P, A, a) = 0$ if $a = m(P, A)$, and $w(P, A, a) = I(\mathbf{1}_{(A,a)} + z_1^r r)$ otherwise.

Lemma 4. *If f is r-invariant, then $I(f) = \sum_{(A,a)} f(A,a) w(P^r, A, a)$.*

Proof of Lemma 4: Consider a collection of observations f such that f is r-invariant with $\sum_{(A,a)} f(A,a) = n$. By Lemma 3, $I(f) = \sum_{(A,a)} f(A,a) I(\mathbf{1}_{(A,a)} + z_n^r r)$. By RAT, if $a = m(P^r, A)$, then $I(\mathbf{1}_{(A,a)} + z_n^r r) = 0 = w(P^r, A, a)$. Otherwise, by definition of z_1^r, we know that $\mathbf{1}_{(A,a)} + z_1^r r$ is r-invariant. Since by construction $z_1^r \leq z_n^r$, Lemma 1 guarantees that $w(P^r, A, a) = I(\mathbf{1}_{(A,a)} + z_1^r r) = I(\mathbf{1}_{(A,a)} + z_n^r r)$. Hence, $I(f) = \sum_{(A,a)} f(A,a) w(P^r, A, a)$, as desired.□

Lemma 5. *For every $f \in \mathcal{F}$, $P \in \mathcal{P}$, $I(f) \leq \sum_{(A,a)} f(A,a) w(P, A, a)$.*

Proof of Lemma 5: Consider any $f \in \mathcal{F}$ and $P \in \mathcal{P}$. Let r' be such that $P^{r'} = P$. By ATTR there exists $z \in \mathbb{Z}_{++}$ such that $f + zr'$ is r'-invariant. By Lemma 4, $I(f + zr') = \sum_{(A,a)} (f + zr')(A,a) w(P, A, a) = \sum_{(A,a)} f(A,a) w(P, A, a)$. By SEP and RAT, $I(f) = I(f) + I(zr') \leq I(f + zr')$. Hence, $I(f) \leq \sum_{(A,a)} f(A,a) w(P, A, a)$ as desired.□

INV and lemmas 4 and 5 together imply that the inconsistency function I can be represented by $I(f) = \min_{P \in \mathcal{P}} \sum_{(A,a)} f(A,a) w(P, A, a)$.

Lemma 6. *For every $(A,a) \in \mathcal{O}$ and $P \in \mathcal{P}$, $w(P, A, a) = \sum_{x \in A : xPa} w(P, \{x, a\}, a)$.*

Proof of Lemma 6: Recall that we have defined $w(P, A, a) = 0$ if $a = m(P, A)$, and $w(P, A, a) = I(\mathbf{1}_{(A,a)} + z_1^r r)$ otherwise, where $P = P^r$. Then, the repeated application of DC, SEP and Lemma 1 leads to $w(P, A, a) = I(\mathbf{1}_{(A,a)} + |A - 1|z_1^r r) = \sum_{x \in A \setminus \{a\}} I(\mathbf{1}_{(\{x,a\},a)} + z_1^r r)$. By RAT and the definition of the weights, this is equivalent to $w(P, A, a) = \sum_{x \in A: xPa} I(\mathbf{1}_{(\{x,a\},a)} + z_1^r r) = \sum_{x \in A: xPa} w(P, \{x, a\}, a)$.$\square$

Lemma 7. *For every $(A, a) \in \mathcal{O}$ and $P \in \mathcal{P}$, $w(P, A, a) = \Theta|\{x \in A : xPa\}|$, with $\Theta > 0$.*

Proof of Lemma 7: We just need to prove that $w(P, \{x, a\}, a) = w(P', \{y, b\}, b)$ whenever xPa and $yP'b$. Let r and r' be such that $P = P^r$, $P' = P^{r'}$, with $r(\{x, a\}, a) = r'(\{y, b\}, b)$. Also let P_1 be such that xP_1aP_1z for all $z \in X \setminus \{x, a\}$ and let $r_1 \in \mathcal{R}$ denote the collection of observations uniquely rationalized by P_1, with $r(\{x, a\}, a) = r_1(\{x, a\}, a)$. Let P_2 be obtained through a permutation of the elements such that x is permuted into y and a is permuted into b. Let $r_2 \in \mathcal{R}$ be obtained from r_1 by way of the same permutation. Let z be the maximum of the values $z_1^r, z_1^{r'}, z_1^{r^1}$ and $z_1^{r^2}$. By definition of the weights and Lemma 1 we have $w(P, \{x, a\}, a) = I(\mathbf{1}_{(\{x,a\},a)} + z_1^r r) = I(\mathbf{1}_{(\{x,a\},a)} + zr)$. By OC, given that r and r_1 treat alternatives x and a in the same way, we have $I(\mathbf{1}_{(\{x,a\},a)} + zr) = I(\mathbf{1}_{(\{x,a\},a)} + zr_1)$. By NEU, we have $I(\mathbf{1}_{(\{x,a\},a)} + zr_1) = I(\mathbf{1}_{(\{y,b\},b)} + zr_2)$. By OC, given that r' and r_2 treat alternatives y and b in the same way, we have $I(\mathbf{1}_{(\{y,b\},b)} + zr_2) = I(\mathbf{1}_{(\{y,b\},b)} + zr')$. Finally, again by definition of the weights and Lemma 1 we have $I(\mathbf{1}_{(\{y,b\},b)} + zr') = I(\mathbf{1}_{(\{y,b\},b)} + z_1^{r'} r') = w(P', \{y, b\}, b)$, as desired. Given the strict positivity of the weights, the lemma follows.$\square$

The weights in Lemma 7 together with the representation implied by Lemmas 4 and 5 represent a positive scalar transformation of the swaps index.$\blacksquare$

Proof of Proposition 2: It is easy to see that any non-neutral swaps index satisfies the axioms. In order to prove the converse statement, notice that we can use Lemmas 1 to 6 in the proof of Theorem 1. We now show that $w(P, \{x, a\}, a) = w(P', \{x, a\}, a)$ whenever xPa and $xP'a$. Let r and r' be such that $P = P^r$, $P' = P^{r'}$, with $r(\{x, a\}, a) = r'(\{x, a\}, a)$. Let z be the maximum of the values z_1^r and $z_1^{r'}$. By definition of the weights and Lemma 1 we have $w(P, \{x, a\}, a) = I(\mathbf{1}_{(\{x,a\},a)} + z_1^r r) = I(\mathbf{1}_{(\{x,a\},a)} + zr)$ and $w(P', \{x, a\}, a) = I(\mathbf{1}_{(\{x,a\},a)} + z_1^{r'} r') = I(\mathbf{1}_{(\{x,a\},a)} + zr')$. By OC, given that r and r' treat alternatives x and a in the same way, we have $I(\mathbf{1}_{(\{x,a\},a)} + zr) =$

$I(\mathbf{1}_{(\{x,a\},a)} + zr')$ and hence $w(P, \{x, a\}, a) = w(P', \{x, a\}, a)$. This shows that the weights are only dependent on the pair x, a and the proposition follows.■

Proof of Proposition 3: It is easy to see that any positional swaps index satisfies the axioms. In order to prove the converse statement, notice that we can use Lemmas 1 to 6 in the proof of Theorem 1. We now prove that $w(P, \{x, a\}, a) = w(P', \{y, b\}, b)$ whenever x and a (resp. y and b) occupy positions i and j in the preference P (resp. P'). Let r and r' be such that $P = P^r$ and $P' = P^{r'}$. Let z be the maximum of the values z_1^r and $z_1^{r'}$. By definition of the weights and Lemma 1 we have $w(P, \{x, a\}, a) = I(\mathbf{1}_{(\{x,a\},a)} + z_1^r r) = I(\mathbf{1}_{(\{x,a\},a)} + zr)$ and $w(P', \{y, b\}, b) = I(\mathbf{1}_{(\{y,b\},b)} + z_1^{r'} r') = I(\mathbf{1}_{(\{y,b\},b)} + zr')$. A direct application of NEU leads to $I(\mathbf{1}_{(\{x,a\},a)} + zr) = I(\mathbf{1}_{(\{y,b\},b)} + zr')$ and the proposition follows.■

Proof of Proposition 4: The proof follows immediately from Theorem 1.■

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