

# Long-Term Finance and Investment with Frictional Asset Markets

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## Abstract

This paper develops a theory of investment and maturity choices and studies its implications for the macroeconomy. The novel ingredient is an explicit secondary market with trading frictions which leads to a liquidity spread which increases with maturity and generates an upward sloping yield curve. As a result, trading frictions induce firms to borrow and invest at shorter horizons than in a frictionless benchmark. Economies with more severe frictions exhibit a steeper yield curve which further affects maturity and investment choices of firms. A model calibrated to match cross-country moments suggests that reductions in trading frictions—a new channel of financial development—can promote economic development. A policy intervention with government-backed financial intermediaries in the secondary market can improve liquidity and reduce the cost of long-term finance which promotes investment in longer-term projects and generates substantial welfare gains.

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# 1 Introduction

Firms in emerging economies tend to borrow and invest at shorter maturities compared to those in advanced countries which may have adverse effects on the aggregate economy. The lower prevalence of long-term financing and investment is thought of by many as a contributor to poor aggregate performance. As a result, there is a policy debate on how to stimulate long-term finance.<sup>1</sup> This paper contributes to this literature and policy debate by developing a theoretical framework of maturity choice and its aggregate effects. It builds on a key feature of capital markets: both in advanced and emerging economies, corporate debt markets exhibit trading frictions. The main result is that more severe frictions make long-term finance relatively more expensive, inducing firms to borrow and invest at shorter horizons with detrimental effects on productivity and the aggregate economy.

The central mechanism of the paper is that a long-maturity asset will trade in the secondary market more times than a shorter one. Hence, the lack of liquidity in secondary markets—a severe trading friction—affects more long- than short-maturity assets generating two important results for the yield curve. First, the liquidity spread increases with maturity. Second, economies with less liquid secondary markets have a steeper yield curve and firms invest at shorter horizons and lower productivity projects. A calibrated model matches key features of the yield curve in US and Argentina and predicts about one-half of the differences in maturity and output between these countries. Finally, a policy intervention with subsidized financial intermediaries can improve liquidity, stimulate long-term finance, and induce investment at longer horizons generating substantial welfare gains.

The modeling framework combines a fairly standard production economy with an over-the-counter (OTC) secondary market for debt as in [Duffie et al. \(2005\)](#). To finance investment, firms borrow in the primary market and choose the maturity structure of their liabilities. When deciding the gestation period of their projects, firms balance the trade-off between projects of higher productivity and more expensive financing due to an upward sloping yield curve.

Liquidity in secondary markets shapes interest rates in two ways. First, the liquidity spread is increasing in maturity. Liquidity needs shocks hit debt holders which cause them to become potential sellers. However, trading frictions prevent them from immediately sell the asset as

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<sup>1</sup>For empirical evidence see [Demirgüç-Kunt and Maksimovic \(1998\)](#) and [Levine \(2005\)](#), among others. Some policy concerns are expressed in [World Economic Forum \(2011\)](#); [European Commission \(2013\)](#); [OECD \(2013\)](#); [Group of Thirty \(2013\)](#); [World Bank \(2015\)](#).

they need to search for a counterpart and bargain over the terms of trade. Alternatively, the maturity of the asset also provides liquidity to debt holders. Hence, gains from trade in the secondary market increase with the maturity of the asset which delivers an upward sloping yield curve. Second, liquidity is more important for long-term assets in the sense that a reduction of the trading friction not only reduces the liquidity spread for all maturities but it also lowers the slope of the yield curve. Therefore, improvements in liquidity generate a flattening of the yield curve.

Decentralized asset markets affect investment costs which propagate to the real economy. Long-maturity projects have high returns but need financing for a prolonged horizon. Hence, if the yield curve is upward sloping, short-term projects are relatively more attractive than longer ones. As a result, variations in trading frictions change the steepness of the yield curve and distort financial and investment choices which affect the aggregate economy.

Free entry to the secondary market determines the equilibrium liquidity. To evaluate variations in trading frictions, i.e., financial development, we consider changes in the entry cost to the secondary market. When there is a lower search cost, more buyers are willing to enter the market, and this increases liquidity in equilibrium. Therefore, financial development generates more liquid markets in which liquidity spreads diminish for all maturities, but in particular for long-term debt and induces firms to invest in more profitable and high productivity longer-term projects. This result suggests that the empirical evidence that firms in emerging economies borrow at short maturities can be the result of a substitution effect between maturity and liquidity of secondary markets.

Estimates of the non-default component of credit spreads are abundant for the US but scarce for emerging economies. One option to identify the slope of the yield curve is to compare credit spreads of the same firm issuing two bonds on the same day but at different maturities. This estimation only needs data on credit spreads and can be implemented in emerging economies. Estimation for the US and Argentina find that credit spreads increase more with maturity in Argentina than in the US. Then, a calibration of the model to the US target the slope of the yield curve among other standard moments. Next, validation exercises show that the model does a reasonable job of matching the level of the liquidity spread in the US which was not a target of the calibration. Counterfactual exercises that introduce the trading frictions of Argentina—measure with the estimate of the slope of the yield curve—in the US economy explain about one-half of the maturity and output differences between Argentina and US. Although the model is stylized and tractable, the quantitative results suggest that the theory captures important features of corporate debt markets.

The presence of frictional asset markets suggests that a policy intervention may increase the liquidity of financial markets and improve credit conditions for the corporate sector which can

generate benefits for the real economy. Based on existing policies like Government-Sponsored Enterprises (i.e., Fannie Mae and Freddie Mac) or large-scale asset purchases (i.e., quantitative easing), the paper evaluates one intervention that subsidizes financial intermediaries in the secondary market, named *Government-Sponsored Intermediaries* (GSIs). The government has four instruments: the size of the intervention, the prices at which the government-sponsored dealers buy and sell from private investors, and a distortionary tax rate to finance the costs of GSIs. Under the optimal policy, government intermediaries buy at higher prices than in private meetings to provide more gains from trade to private sellers which alleviates trading friction. On the other hand, government agents sell securities at a lower price than in private meetings to stimulate the entry of potential buyers to the market. The optimal policy increases liquidity which flattens the yield curve and stimulates the use of long-term finance. Quantitatively, this policy generates an increase in maturity of five months for an economy like the US and eight months for an economy with a less-developed financial system as in Argentina.

The main results are robust both quantitatively and qualitatively to several extensions. First, in the benchmark model firms can borrow only at the beginning of the project, so the maturity of the project matches the maturity of financing by assumption. An extension of the model allows entrepreneurs to rollover short-term contracts to finance long-term projects with a fixed cost of issuance. Quantitatively, the effect of a change in liquidity on the choice of projects is similar both with and without rollover opportunities regardless the value of the issuance cost. Second, in the benchmark model, there is a single secondary market for assets of different maturities. The paper also considers a specification in which buyers direct themselves to markets segmented by maturity. The main takeaway is that even though the market tightness (defined as the ratio of sellers-to-buyers) for short-term debt increases, the tightness for long-term assets remains similar to the benchmark model with a single market. As a result, the yield curve is similar to the curve for the benchmark economy.

**Related literature** This paper contributes to the literature on maturity choice by proposing a novel channel based on trading frictions in the secondary market which generates an upward sloping yield curve. In the canonical models of [Diamond \(1991\)](#) and [Leland and Toft \(1996\)](#) frictions between lenders and borrowers shape maturity choices while in this paper the friction is within lenders in financial markets. More broadly, this paper provides a different perspective on how financial development can influence aggregate outcomes, the subject of a large body of work (e.g., [Greenwood et al., 2010](#); [Buera et al., 2011](#); [Moll, 2014](#); [Midrigan and Xu, 2014](#); [Cole et al., 2016](#), among others). All these papers focus on contracting frictions between lenders and borrowers and interpret financial development as a reduction of that friction. Instead, this paper considers trading frictions within lenders in financial markets in which financial development is

an increase in the liquidity of the market and focuses on the choice of maturity which is absent in previous analyses.

This paper is also related to the literature on OTC markets following the seminal work of [Duffie et al. \(2005\)](#). Some papers applied the theory to corporate bonds markets (e.g., [Chen et al., 2012](#); [He and Milbradt, 2014](#); [Chen et al., 2017](#)), while others consider the interaction between primary and secondary markets ([Bruche and Segura, 2017](#); [Arseneau et al., 2017](#); [Bethune et al., 2017a](#)). In particular, the financial structure is a hybrid between [He and Milbradt \(2014\)](#) and [Bruche and Segura \(2017\)](#). On the one hand, [He and Milbradt \(2014\)](#) studies the interaction between liquidity and default and abstracts from the effects on maturity. It applies the model to the US corporate debt market in a framework with exogenous elements such as output, maturity, and liquidity (in the sense that meeting intensities are not an equilibrium outcome). In contrast, this paper studies the interaction between liquidity and maturity to understand cross-country differences, and, for this application and the policy analysis, is key to have output, maturity, and liquidity as equilibrium outcomes. On the other hand, [Bruche and Segura \(2017\)](#) applies the theory to US commercial paper in a framework with endogenous maturity and liquidity. However, it assumes exogenous profits, maturity is stochastic (i.e., arrives at a Poisson rate rather than at a deterministic date) and does not provide quantitative evaluation nor policy analysis, which are important contributions of this paper.

Many papers studied the term structure of interest rates through the lens of the consumption-based capital asset pricing model (see [Gürkaynak and Wright, 2012](#), for a recent review). Those papers extend the expectation hypothesis framework which posits that long-term interest rates are expectations of future average short-term rates. This paper is closer to a classic idea present in undergraduate textbooks (e.g., [Mishkin, 2015](#)) that attributes the shape of the yield curve to liquidity considerations. [Geromichalos et al. \(2016\)](#) propose a monetary-search model, with assets of two maturities, to rationalize the yield curve. The contribution of this paper is to present a model with a continuum of maturities and study both theoretically and quantitatively the implications for maturity choices of the corporate sector, the effects on the real economy, and the role of policy interventions.

The rest of the paper is organized as follows. [Section 2](#) presents the model and [Section 3](#) characterizes the equilibrium. [Section 4](#) presents the quantitative results. Next, [Section 5](#) performs policy analysis. [Section 6](#) extends the model in several dimensions. Finally, [Section 7](#) concludes. Proofs and additional results are gathered in the Appendices.

## 2 Theory

Time is continuous, starts at  $t = 0$ , and goes on forever. The economy is populated by agents in the production and the financial sector. Firms in the production sector choose investment projects from a menu of opportunities such that the return increases with the duration of investment. To finance the projects, firms borrow from the financial sector with debt issuances. These securities are traded in an over-the-counter (OTC) secondary market by members of the financial sector.

### 2.1 Production sector

Every period a measure  $\mu^0$  of identical entrepreneurs choose a new project from a menu of potential investments differentiated by the life-cycle of returns. To guide the quantitative analysis, we consider the following microfoundation based on a simple production model in which firms invest in productivity. However, the qualitative results hold for a large class of models such that there are back-loaded projects that require long-term financing.

**Life-cycle** Two empirical facts motivate the assumptions about the life-cycle of projects. First, small firms grow faster than large firms (e.g., [Akcigit and Kerr, 2017](#)). Second, small firms are more financially constrained, and in particular for research and development ([Midrigan and Xu, 2014](#); [Itenberg, 2015](#)). To capture these facts in a stylized model, we make a stark assumption and divide the life-cycle of firms in two stages: (i) *investment* for  $t \leq \tau$ , and (ii) *production* for  $t > \tau$ . A newborn firm chooses  $\tau$ , the age at which she starts production. Therefore, a new firm has a menu of potential projects summarized by  $\tau \geq 0$ . With Poisson arrival rate  $\delta$  the firm is hit by an exit shock and the value of the project goes to zero.<sup>2</sup>

**Investment stage** A young firm invests in research and development (R&D) to improve its productivity. Let  $z(t)$  be the productivity of the firm such that  $z(0) = 0$  and  $\dot{z}(t) = \zeta$  for  $t \leq \tau$ . At maturity  $\tau$  productivity is  $z(\tau) = \zeta\tau$ . The flow cost of investment is  $\kappa$  per unit of time doing R&D. We label  $z$  as productivity, but it can be broadly interpreted as any factor of production that takes time to build such as the quality of the product, capital, or demand accumulation, among many others.

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<sup>2</sup>To reduce notation assume that this process has the same intensity for firms in the investment and production stages. However, it is simple to consider two different processes.

**Production stage** At age  $\tau$  the firm stops doing R&D and starts the production phase with a production technology that is linear in productivity  $y = z$ .<sup>3</sup> The net present value of a firm that spent  $\tau$  periods doing R&D is  $F(\tau) = z(\tau) \int_0^\infty e^{-(\rho+\delta)t} dt$  where  $\rho$  is the discount factor. Let  $A = \frac{\zeta}{\rho+\delta}$  so  $F(\tau) = A\tau$ . All firms are identical, choose the same maturity  $\tau$  and have the same productivity  $z$ . Hence, in a steady state equilibrium there is a measure  $\mu^F(\tau) = \frac{\mu^0 e^{-\delta\tau}}{\delta}$  of identical firms that produce aggregate output equal to  $Y(\tau) = \mu^F(\tau)z(\tau)$ .

Both qualitative and quantitative results do not depend on this specific production model. Qualitative results also hold under a general return function  $F(\tau)$  such that it is increasing and concave in  $\tau$ . Appendix B shows that a similar return function arise in models of the quality ladder or a model of time-to-build capital. Quantitative results depend on the parametric forms and the estimation. Section 6.3 considers an alternative production function that combines productivity  $z$  and labor  $l$  to produce output with technology  $y = z^{1-\sigma}l^\sigma$ . We repeat the main quantitative analysis under this specification and show that results are relatively similar regardless of the value of the labor share  $\sigma$ .

**Borrowing** Firms do not have internal funds and need to borrow. They can borrow only at the beginning of the project, i.e., they match the maturity of the project and the debt. This assumption helps us to obtain a sharper characterization of the equilibrium and captures the idea that it might be costly or risky to borrow short-term to finance long-term projects. In fact, empirical evidence shows that firms tend to match the maturity of assets and liabilities.<sup>4</sup> Section 6.1 relaxes this assumption and allow firms to issue short-term debt to finance long-term projects. In that extension we assume a fixed cost of issuance and find similar results than in the benchmark model.

Corporate bonds have no coupon, default arrival rate  $\delta$ , and face value one. The firm deposits the proceeds from the issuance in a bank account with risk-free rate  $\rho$  and withdraws  $\kappa$  per unit of time for investment. Hence, a firm needs initial funds equal to  $I(\tau) = \kappa \frac{1-e^{-\rho\tau}}{\rho}$  to invest for  $\tau$  periods. The firm takes the price of a bond with maturity  $\tau$  and liquidity  $\lambda$ ,  $P(\tau, \lambda)$ , as given and chooses maturity  $\tau$  and issuances  $B$  to maximize its value<sup>5</sup>

$$\max_{\tau, B} e^{-(\rho+\delta)\tau} (F(\tau) - B) \quad \text{s.t. } BP(\tau, \lambda) = I(\tau). \quad (1)$$

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<sup>3</sup>Assume that a young firm does not produce. However, results are similar if a young firm produces and use internal funds for investment while  $\kappa$  are the external funds needed for investment.

<sup>4</sup>See [Stohs and Mauer \(1996\)](#); [Guedes and Opler \(1996\)](#); [Frederick C. Scherr \(2001\)](#); [Dew-Becker \(2012\)](#) for the US and [Schiantarelli and Sembenelli \(1997\)](#); [Schiantarelli and Srivastava \(1997\)](#); [Schiantarelli and Jaramillo \(2002\)](#) for international evidence. [Myers \(1977\)](#) provides a theory for maturity matching.

<sup>5</sup>Section 2.3 defines liquidity  $\lambda$  as the intensity at which a seller finds a buyer, an equilibrium variable. It is a sufficient statistic to capture trading frictions in the secondary market.

## 2.2 Primary market

In the primary market firms from the production sector issue corporate bonds and lenders from the financial sector buy those securities. Assume that there are no frictions in this market and there is a large mass of potential lenders willing to buy the assets. Free entry to the primary market implies that the price of the bond is equal to the value of holding the asset for a lender

$$P(\tau, \lambda) = D^H(\tau; \lambda), \quad (2)$$

where  $D^H(\tau, \lambda)$  is the value of holding a bond with time-to-maturity  $\tau$  when the liquidity of the secondary market is  $\lambda$ .

## 2.3 Secondary market

The financial sector trades corporate bonds in an OTC market as in [Duffie et al. \(2005\)](#). An agent of the financial sector can have either zero or one unit of the asset.<sup>6</sup> An agent without the asset can pay a search cost  $c$ , enter into the secondary market, and search for a counterpart. There is a large measure of potential enters which implies a free-entry condition to the market.

An agent can buy the asset either in primary or secondary markets, but always starts as a *high valuation* agent. However, he faces an idiosyncratic liquidity risk of becoming *low valuation*. With Poisson intensity  $\eta$  a high valuation agent becomes low valuation which implies that he has to pay a holding cost  $h$  per unit of time.<sup>7</sup> This idiosyncratic risk generates differences in valuations which causes motives for trade in the secondary market. Note that asset holders are heterogeneous in two dimensions. First, they can be either high or low valuation. Second, they hold assets with time-to-maturity  $y \in [0, \tau]$ . Let  $\mu^H(y)$  and  $\mu^L(y)$  denote the measure of high- and low-valuation agents holding an asset of time-to-maturity  $y$ , respectively.

Sellers in the secondary market are all the low-valuation agents. There is random matching in this market—assets of different time-to-maturity trade in the same market—so the total mass of sellers is  $\mu^S = \int_0^\tau \mu^L(y) dy$ . [Section 6.4](#) extend the model to consider markets segmented by maturity and shows that the results are similar to the model with a single secondary market. On the other hand, a measure  $\mu^B$  of buyers are agents without an asset searching in the secondary market. Assume a constant-returns-to-scale matching function between buyers and sellers  $M(\mu^S, \mu^B) = A(\mu^S)^\alpha (\mu^B)^{1-\alpha}$ , and define the market tightness as the ratio of sellers-to-buyers,  $\theta = \frac{\mu^S}{\mu^B}$ .

<sup>6</sup>This portfolio restriction is common in the literature because it simplifies the tractability of the model.

<sup>7</sup>The modeling assumptions about high- and low-valuation agents is standard in the literature (e.g., [Duffie et al., 2005](#); [He and Milbradt, 2014](#)). A low investor may have (i) high discounting (ii) high financing costs; (iii) hedging reasons; (iv) tax disadvantage, or (v) lower personal use of the asset.



A seller finds a counterpart at rate  $\lambda = A\theta^{\alpha-1}$  and a buyer finds a counterpart at rate  $\beta = A\theta^\alpha$ . Upon a match, with probability  $\frac{\mu^L(y)}{\mu^S}$  a buyer meets with a seller of an asset with time-to-maturity  $y$ . It is useful to define the liquidity of the secondary market as  $\lambda$  because it is the key object through which the secondary market feeds back into the primary market and affect the borrower's problem.<sup>8</sup>

Let  $P^S(y; \lambda)$  be the price in the secondary market for an asset of time-to-maturity  $y$  and assume a Nash Bargaining protocol in which  $\gamma$  is the bargaining power of the seller

$$P^S(y) = \arg \max_{P^S(y)} (P^S(y; \lambda) - D^L(y; \lambda))^\gamma (D^H(y; \lambda) - P^S(y; \lambda))^{(1-\gamma)}. \quad (3)$$

$D^H(y; \lambda)$  is the value of holding an asset for a high-valuation agent—the buyer—and  $D^L(y; \lambda)$  is the value of holding an asset for a low-valuation agent—the seller.

The value of search in the secondary market,  $D^S(\lambda)$ , is

$$\rho D^S(\lambda) = -c + \beta \int_0^\tau \frac{\mu^L(y)}{\mu^S} (D^H(y; \lambda) - D^S(\lambda) - P^S(y; \lambda)) dy. \quad (4)$$

The discounted value of search is equal to the search cost and the expected gains from trade. With intensity  $\beta \frac{\mu^L(y)}{\mu^S}$  a buyer matches with a seller of a bond with time-to-maturity  $y$ , and gains from trade are  $D^H(y; \lambda) - D^S(\lambda) - P^S(y; \lambda)$ . The buyer becomes high valuation agent, with value  $D^H(y; \lambda)$  and pays the price  $P^S(y; \lambda)$ . Free entry in the secondary market implies that, in equilibrium,  $D^S(\lambda) = 0$ .

## 2.4 Equilibrium

Definition 1 states the steady-state equilibrium.

**Definition 1.** *A steady-state equilibrium is a selling intensity  $\lambda$ , debt's maturity  $\tau$ , prices in the primary market  $P(y; \lambda)$ , prices in the secondary market  $P^S(y; \lambda)$  and measures  $\mu^H(y)$  and  $\mu^L(y)$  such that:*

1. *Firms in the corporate sector solve (1);*
2. *Free entry in the primary market solve (2);*
3.  *$P^S(y, \lambda)$  solves the Nash Bargaining problem (3);*
4.  *$D^S(\lambda) = 0$  solves free entry in the secondary market (4).*

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<sup>8</sup>Note that  $\beta = A \left(\frac{\lambda}{A}\right)^{\frac{\alpha}{\alpha-1}}$ . It is equivalent to define functions depending on the market tightness  $\theta$ , but it is easier to derive the intuition of the results thinking in the space of the selling intensity  $\lambda$ .

### 3 Equilibrium characterization

This section fully characterizes the solution of the model. First, we solve for the distribution of agents in the financial sector. Then, the main results show how the liquidity of the secondary market affects prices in the primary market, interest rates, and maturity choices. Next, a fixed point between the maturity choice  $\tau$  and liquidity  $\lambda$  characterizes the equilibrium of the model. Finally, counterfactual exercises examine the effects of financial development.

#### 3.1 Lenders

Buyers from primary and secondary markets start as high valuation. Over time, some agents receive liquidity shocks while others trade in the secondary market. The law of motions for the measure of high- and low-valuation agents are

$$\dot{\mu}^H(y) = (\eta + \delta) \mu^H(y) - \beta \frac{\mu^L(y)}{\mu^S} \mu^B \quad (5)$$

$$\dot{\mu}^L(y) = -\eta \mu^H(y) + (\delta + \lambda) \mu^L(y) \quad (6)$$

with boundary conditions  $\mu^H(\tau) = \mu^0$  and  $\mu^L(\tau) = 0$ . Equations (5) and (6) show that as we move closer to maturity (lower  $y$ ) a fraction  $\eta$  of high-valuation agent become low-valuation agents and a fraction  $\delta$  of both types of agents are holding an asset that is hit by a default shock. Moreover, a measure  $\beta \frac{\mu^L(y)}{\mu^S} \mu^B$  of buyers find a counterpart in the secondary market and become high-valuation agents. Finally, a measure  $\lambda$  of low-valuation agents are able to sell in the secondary market. Lemma 1 characterizes the steady-state distribution of financiers.

**Lemma 1.** *The distribution of financiers is given by*

$$\mu^H(y) = \frac{\mu^0 \eta}{\eta + \lambda} \left( e^{\delta(y-\tau)} \frac{\lambda}{\eta} + e^{(\eta+\lambda+\delta)(y-\tau)} \right) \quad (7)$$

$$\mu^L(y) = \frac{\mu^0 \eta}{\eta + \lambda} \left( e^{\delta(y-\tau)} - e^{(\eta+\lambda+\delta)(y-\tau)} \right) \quad (8)$$

When the secondary market is well-functioning—the selling intensity,  $\lambda$ , is relatively high—the mass of low valuation agents  $\mu^L(y)$  is small. However, when  $\lambda$  diminishes, the secondary market is more illiquid and the mass of low valuation agents increases.<sup>9</sup> Note that  $\mu^L(y)$  enters in the free-entry condition for the secondary market (4), so we need to use (8) to solve for the equilibrium liquidity.

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<sup>9</sup>It is straightforward to show that the measure of low valuation agents is decreasing in the meeting intensity.

**Private valuations** The values for high- and low-valuation agents holding an asset are

$$\rho D^H(y; \lambda) = -\frac{\partial D^H(y; \lambda)}{\partial y} + \eta (D^L(y; \lambda) - D^H(y; \lambda)) + \delta (0 - D^H(y; \lambda)), \quad (9)$$

$$\rho D^L(y; \lambda) = -h - \frac{\partial D^L(y; \lambda)}{\partial y} + \lambda (P^S(y; \lambda) - D^L(y; \lambda)) + \delta (0 - D^L(y; \lambda)). \quad (10)$$

At maturity both types of investors receive the face value of the asset which implies the boundary conditions  $D^H(0; \lambda) = D^L(0; \lambda) = 1$ . Equation (9) defines the value of high-valuation agents. The left-hand side is the required return from holding the bond. The first term on the right-hand-side represents the change in value due to being closer to maturity. The second term captures the liquidity shocks that transform the investor into a low-valuation agent, which occurs at intensity  $\eta$ . The third term captures the risk of default of the bond. Equation (10) captures the value of low valuation agents and follows a similar intuition as the previous equation. A low-valuation investor incurs a holding cost  $h$ , and with intensity  $\lambda$  the investor meets a counterpart and sells his bond at price  $P^S(y; \lambda)$ .

**Price in secondary market** The price in the secondary market,  $P^S(y; \lambda)$ , is the solution of the Nash Bargaining problem between the seller and the buyer in (3)

$$P^S(y; \lambda) = D^L(y; \lambda) + \gamma (D^H(y; \lambda) - D^L(y; \lambda)). \quad (11)$$

The gains from trade are  $D^H(y; \lambda) - D^L(y; \lambda)$  and the seller gets a fraction  $\gamma$  of them.

### 3.2 Effects of liquidity on the primary market

The central result of the paper is the characterization of how the liquidity of the secondary market feeds back on prices in the primary market. Proposition 1 solves Equations (9) and (10) using the equilibrium price in the secondary market (11) and characterizes the price in the primary market.

**Proposition 1.** *The price in the primary market is*

$$P(\tau, \lambda) = e^{-(\rho+\delta)\tau} - \mathcal{L}(\tau, \lambda), \quad (12)$$

the illiquidity cost  $\mathcal{L}(\tau, \lambda)$  is

$$\mathcal{L}(\tau, \lambda) = h \frac{\eta}{\eta + \lambda\gamma} \int_0^\tau e^{-(\rho+\delta)y} (1 - e^{-(\eta+\lambda\gamma)y}) dy. \quad (13)$$

The illiquidity cost satisfies the following properties:

1.  $\mathcal{L}(\tau, \lambda)$  is positive.

2. Sensitivity with respect to liquidity shocks  $\eta$ :

(a) If there are no liquidity shocks,  $\eta = 0$ , then  $\mathcal{L}(\tau, \lambda) = 0$ ;

(b) If  $\eta \rightarrow \infty$  (i.e., always has to pay the cost  $h$ ) then

$$\lim_{\eta \rightarrow \infty} \mathcal{L}(\tau, \lambda) = h \frac{1 - e^{-(\rho+\delta)\tau}}{\rho + \delta}.$$

3. Sensitivity with respect to maturity  $\tau$ :

(a)  $\mathcal{L}(\tau, \lambda)$  is increasing in  $\tau$ ;

(b)  $\mathcal{L}(\tau, \lambda)$  has a finite limit,  $\lim_{\tau \rightarrow \infty} \mathcal{L}(\tau, \lambda) = h \frac{\eta}{(\rho+\delta)(\rho+\delta+\eta+\gamma\lambda)}$ .

4. Sensitivity with respect to liquidity  $\lambda$ :

(a)  $\mathcal{L}(\tau, \lambda)$  is decreasing in  $\lambda$ ;

(b) If there are no secondary markets,  $\lambda = 0$ , the liquidity term only represents the expected holding costs, i.e.,

$$\mathcal{L}(\tau, 0) = h \int_0^\tau e^{-(\rho+\lambda^D)y} (1 - e^{-\eta y}) dy;$$

(c) If secondary markets are totally liquid (i.e.,  $\lambda \rightarrow \infty$ ) then  $\mathcal{L}(\tau, \lambda) = 0$ .

5. Liquidity is more important for long-term assets:  $\frac{\partial^2 \mathcal{L}(\tau, \lambda)}{\partial \tau \lambda} \leq 0$ .

Proposition 1 shows that we can decompose the price in the primary market  $P(\tau, \lambda)$  in two terms. The first component represents the *frictionless* solution: the value of a promise to pay one unit in  $\tau$  periods when the discount rate is  $\rho$  and the default intensity is  $\delta$ . Note that absent the second term, the expectation hypothesis holds: long-term interest rates are equivalent to the average of short-term rates. The second term,  $\mathcal{L}(\tau, \lambda)$ , represents the *illiquidity cost*. When this term is different from zero the expectation hypothesis does not hold and borrowing at longer horizons becomes more expensive than the average of short-term rates.

The illiquidity cost captures the expected discounted time that the holder of the asset is low valuation and has to pay the holding cost. If there are no secondary markets the illiquidity cost is equivalent to the holding cost  $h$  times the expected discounted time between the stopping time in which the agent receives the idiosyncratic shock—which occurs at intensity  $\eta$ —and

maturity. On the other hand, if there are no frictions in the secondary market, upon a shock the agent can sell the asset instantaneously and recover the fundamental value which implies that the illiquidity cost would be equal to zero. Hence, both how easy is to sell in the secondary market, captured by  $\lambda$ , and how much of the gains from trade a seller can retain, measured by  $\gamma$ , shape the illiquidity cost. Let  $s^H(y)$  and  $s^L(y)$  be the adjusted probabilities that a security of age  $y$  is held by high and low valuations, respectively.<sup>10</sup> This is an adjusted probability because the transition takes into account the bargaining power

$$\begin{aligned}\dot{s}^H(y) &= -\eta s^H(y) + \lambda \gamma s^L(y) \\ \dot{s}^L(y) &= \eta s^H(y) - \lambda \gamma s^L(y)\end{aligned}$$

with initial condition  $s^H(0) = 1$  and  $s^L(0) = 0$ . Then, the illiquidity cost is

$$\mathcal{L}(\tau, \lambda) = h \int_0^\tau e^{-(\rho+\delta)y} s^L(y) dy.$$

Therefore, the illiquidity cost is the expected discounted time that the holder of the asset is low valuation.

Proposition 1 establishes several results about the illiquidity cost. The first two properties show that  $\mathcal{L}$  is positive, if there are no idiosyncratic shocks  $\mathcal{L}$  is equal to zero, and if the agent always has to pay the holding cost then the illiquidity cost is equal to the net present value of paying  $h$ . More interestingly, the left panel Figure 1 summarizes how maturity and liquidity affect the illiquidity cost. First, the illiquidity cost is increasing in maturity. Longer securities spend more time in the market which increases the expected time that they are held by low-valuation agents. Second, the illiquidity cost is decreasing in liquidity  $\lambda$ . If the liquidity of the secondary market increases, the holders of that security spend less time paying the holding cost which implies a lower illiquidity cost.

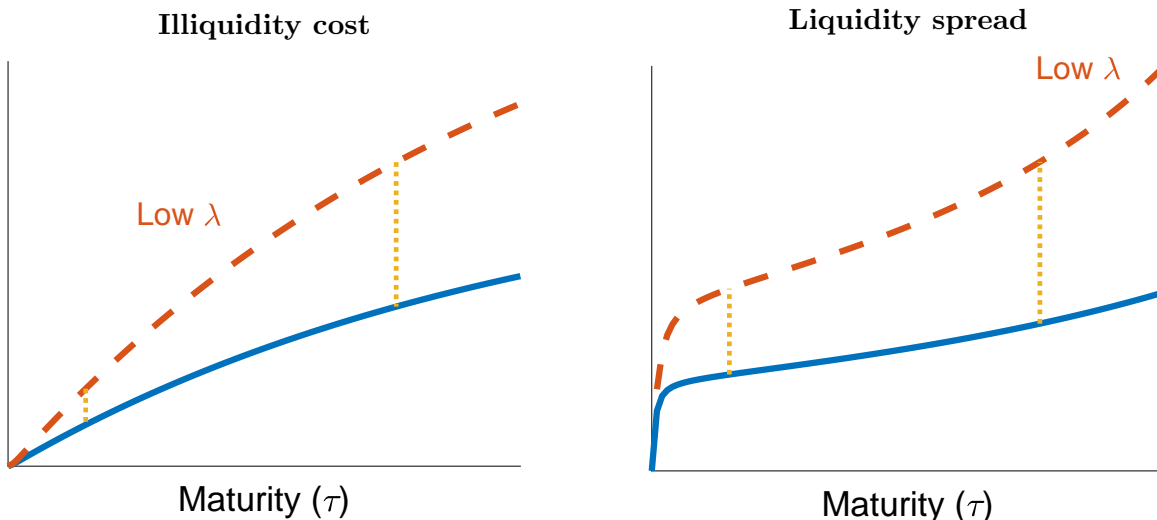
The central result is that the trading friction is more important for long-term assets, i.e., the cross partial derivative of the illiquidity cost to maturity and liquidity is negative:  $\frac{\partial^2 \mathcal{L}(\tau, \lambda)}{\partial \tau \lambda} < 0$ . An investor that wants to exit a financial position can either sell in the secondary market or wait until maturity. Hence, the role of the secondary market is more important for an agent holding a longer-term asset. As a consequence, a reduction of the trading friction benefits more long- than short-term securities. This result highlights the importance of decentralized asset trading to study long-term finance.

Inspection of equation (13) reveals that the product of  $\lambda$  times  $\gamma$  captures the feedback of

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<sup>10</sup> Note that  $y$  is the age of the asset, not the time to maturity. Also, note that it is absent of default, as we include the default rate  $\delta$  in the discount factor.

Figure 1: Illiquidity cost and spread.



*Note: Illiquidity cost and liquidity spread are increasing in maturity and decreasing in liquidity. Parameter values are discussed in Section 4.*

secondary market's liquidity to prices in primary markets, a standard result in the literature. The first term,  $\lambda$ , is the selling intensity in the secondary market—an equilibrium object. If  $\lambda$  increases, it becomes easier to sell in the secondary market. The second term,  $\gamma$ , is the bargaining power of the seller in the secondary market—a parameter. When  $\gamma$  increases, sellers keep a larger fraction of the gains from trade. Therefore,  $\lambda$  captures the feedback of the friction in the secondary market into prices in the primary market and investment choices.

**Yield curve** For the quantitative analysis on Section 4 is useful to transform the price in the primary market into an interest rate schedule. Define  $r(\tau, \lambda)$  as the compound interest rate that solves  $P(\tau, \lambda) = e^{-r(\tau, \lambda)\tau}$ , that is the value of the asset conditional on it being held to maturity without default or trading. The interest rate for a bond of maturity  $\tau$  is

$$r(\tau, \lambda) = \rho + \delta + \frac{1}{\tau} \log \left( \frac{1}{1 - e^{(\rho + \delta)\tau} \mathcal{L}(\tau, \lambda)} \right). \quad (14)$$

The first term,  $\rho$ , is the risk-free rate, while the remaining two terms capture the credit spread. Decompose the spread into a default and a liquidity component, following He and Milbradt (2014). Consider a marginal investor with no idiosyncratic liquidity risk. Such investor requires an interest rate equal to  $\rho + \delta$ . Therefore, the credit spread due to default is  $\delta$ . Finally, define the credit spread due to liquidity by subtracting the default component. Hence, the third term

corresponds to the liquidity spread

$$cs^{liq}(\tau, \lambda) = \frac{1}{\tau} \log \left( \frac{1}{1 - e^{(\rho+\delta)\tau} \mathcal{L}(\tau, \lambda)} \right), \quad (15)$$

and  $r(\tau, \lambda) = \rho + cs^{def} + cs^{liq}(\tau, \lambda)$ . Note that in this model the term premium—the difference between long- and short-term rates—is equivalent to the credit spread due to liquidity. Hence, variations in interest rates across maturities are only explained by differences in the liquidity component of the security. All assets are traded in the same secondary market, however the liquidity spread varies with maturity as the importance of the secondary market is different across assets. Section 4 exploits this observation to identify the liquidity component in the data. Lemma 2 describes the properties of the liquidity spread.

**Lemma 2.** *The liquidity spread  $cs^{liq}(\tau, \lambda)$  is:*

1. *Increasing in maturity  $\frac{\partial cs^{liq}(\tau, \lambda)}{\partial \tau} \geq 0$ ;*
2. *Decreasing in liquidity  $\frac{\partial cs^{liq}(\tau, \lambda)}{\partial \lambda} \leq 0$ ;*
3. *Increasing in the default intensity  $\frac{\partial cs^{liq}(\tau, \lambda)}{\partial \delta} \geq 0$ .*

Lemma 2 establish important properties about the liquidity spread. First, it is increasing in maturity and decreasing in liquidity. The right panel of Figure 1 shows the interest rate as a function of maturity for two levels of liquidity. Note that an increase in  $\lambda$  affects more the long-end of the yield curve. These results are also present in the illiquidity cost  $\mathcal{L}(\tau, \lambda)$  and the yield curve preserves the properties.

Lemma 2 also shows that there is a feedback-loop between default and liquidity; the liquidity spread is increasing in the default intensity  $\delta$ . Note that  $\delta$  has the same role as the discount factor  $\rho$ . An increase in the discount factor decreases the value of illiquidity at maturity which increases the liquidity spread. Section 6.2 extends the analysis and study how changes in default interact with maturity choices.

The liquidity spread increases with maturity while the default spread is constant. This result holds exactly in this model but it is likely to hold in a large class of models for the following reasons. On the one hand, an aggregate default shock affects the value of the asset for all agents and is independent of the time-to-maturity. In particular, in this model the value is equal to zero. On the other hand, an idiosyncratic liquidity shock does not affect the value of the asset for other potential buyers nor the value that the investor recovers at maturity. However, the closer is the maturity of the asset, the lower is the cost associated to hold the security due to the frictions in the secondary market. Hence, in more sophisticated models this result might not hold exactly, but these forces indicate that the liquidity component will still

affect the term structure while the default component can generate either upward or downward sloping yield curves.<sup>11</sup>

**Bid-ask spreads** Define the proportional bid-ask spread as the gains from trade normalized by the mid-price

$$BA(y; \lambda) = \frac{D^H(y; \lambda) - D^L(y; \lambda)}{1/2(D^H(y; \lambda) + D^L(y; \lambda))}.$$

Lemma 3 shows that the proportional bid-ask spread is increasing in maturity. An asset of longer maturity has larger gains from trade, and as a result the bid-ask spread increases with maturity. Importantly, all the predictions in Lemmas 2 and 3 are consistent with the empirical evidence (see, for example, Edwards et al., 2007).

**Lemma 3.** *The proportional bid-ask spread is increasing in maturity.*

### 3.3 Entry to the secondary market

The free-entry condition to the secondary market characterizes liquidity  $\lambda$  as a function of the maturity at issuance of the bonds,  $\tau$ . Replace the equilibrium price in the secondary market (11) in the free-entry condition (4) so

$$c = \beta(1 - \gamma) \int_0^\tau \frac{\mu^L(y)}{\mu^S} (D^H(y; \lambda) - D^L(y; \lambda)) dy. \quad (16)$$

Proposition 2 analyze the free-entry condition (16). It describes how the selling intensity changes with the maturity at issuance,  $\lambda(\tau)$ .

**Proposition 2.**  $\lambda(\tau)$  is increasing in  $\tau$  and  $\lambda(\tau) : \mathbb{R}_+ \mapsto [0, \bar{\lambda}]$ .

When assets are of zero maturity there are no gains from trade which implies no entry in secondary markets and a selling intensity equal to zero,  $\lambda(0) = 0$ . Gains from trade are increasing in  $\tau$ , which implies that there are more incentives to enter in the secondary market as  $\tau$  increases. Hence,  $\lambda(\tau)$  is increasing. When  $\tau$  goes to infinity the gains from trade are bounded which implies that  $\lambda$  converges to a finite number. Section 3.5 solves the equilibrium between lenders and borrowers in which  $\lambda(\tau)$  represents the *lenders curve*.

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<sup>11</sup>Indeed, in traditional models of corporate default (e.g., Merton, 1974) the change in the term premium with respect to a change in maturity can be either positive or negative.



### 3.4 Optimal maturity

The solution of the firm's problem (1) is characterized by the following trade-offs

$$\frac{\partial F(\tau)}{\partial \tau} = (\rho + \delta) F(\tau) + \frac{\partial I(\tau)}{\partial \tau} e^{r(\tau, \lambda)\tau} + I(\tau) e^{r(\tau, \lambda)\tau} c_{S^{liq}}(\tau, \lambda) (1 + \epsilon_{c_{S^{liq}, \tau}}(\tau, \lambda)), \quad (17)$$

where  $\epsilon_{c_{S^{liq}, \tau}}(\tau, \lambda)$  is the elasticity of the liquidity spread to maturity

$$\epsilon_{c_{S^{liq}, \tau}}(\tau, \lambda) = \frac{\partial c_{S^{liq}}(\tau, \lambda)}{\partial \tau} \frac{\tau}{c_{S^{liq}}(\tau, \lambda)}.$$

Consider a marginal increase in  $\tau$ . The left-hand side of Equation (17) represents the benefits of operating a firm with higher productivity and the right-hand side captures three associated costs. First, a project in which returns are more back-loaded requires more time to become profitable. This implies a higher time-discount on future profits. Second, a larger firm requires more investment. Note that even without financial frictions (i.e. constant interest rates,  $r(\tau) = \rho + \delta$ ) we have an interior solution for  $\tau$ .<sup>12</sup> Intuitively, there is an interior solution because as the firm chooses a larger  $\tau$  it is both more costly and it takes more time to complete.

The third term of Equation (17) captures the effect of the financial cost. First, as maturity increases, the firm has to pay the liquidity spread for a longer period. Second, Lemma 2 shows that the liquidity spread increases with maturity which is captured by the elasticity  $\epsilon_{c_{S^{liq}, \tau}}$ . These two forces induce the firm to choose a shorter maturity than in the frictionless economy. Let  $\tau(\lambda)$  be the optimal maturity when the selling intensity is equal to  $\lambda$ .

**Proposition 3.** *The optimal maturity is increasing in the liquidity of the secondary market and  $\tau(\lambda) : [0, \bar{\lambda}] \mapsto [\underline{\tau}, \bar{\tau}]$  with  $0 \leq \underline{\tau} \leq \bar{\tau} < \infty$ .*

The optimal maturity increases with the liquidity of the secondary market. By Lemma 2, the liquidity spread is lower when secondary markets are more liquid. This implies that it is cheaper to borrow and in particular at longer horizons. Hence, when  $\lambda$  increases firms choose projects of longer maturity.

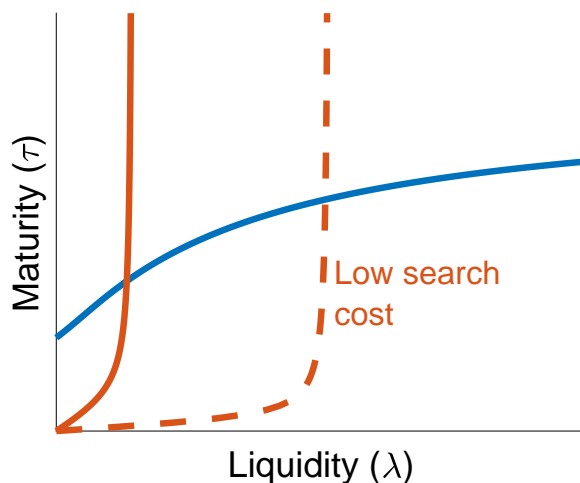
### 3.5 Equilibrium

The equilibrium is a fixed point between maturity and liquidity. On the one hand, firms take the liquidity of the secondary market as given and choose maturity—Equation (17)—which delivers a curve  $\tau(\lambda)$ . On the other hand, agents in the financial sector take the maturity of assets as given and the free-entry condition to the secondary market—Equation (16)—delivers  $\lambda(\tau)$ . An

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<sup>12</sup>Note that this is also true even if there is no default,  $\delta = 0$ .

Figure 2: **Equilibrium characterization and financial development.**



*Note: Equilibrium characterization and financial development. The parameter values are discussed in Section 4.*

equilibrium is  $(\tau^*, \lambda^*)$  such that  $\tau(\lambda(\tau^*)) = \tau^*$ . proposition 4 states that an equilibrium exists. The proof follows directly from Propositions 2 and 3. Figure 2 shows a the characterization of the equilibrium in the space of liquidity and maturity. The solid red and blue curves show the lenders and borrowers locus, respectively. The intersection of these curves characterizes the equilibrium of the economy.

**Proposition 4.** *A steady-state search and matching equilibrium always exists.*

### 3.6 Financial development

Financial development are improvements on the efficiency of secondary markets, which, in the model, correspond to reductions in the search cost  $c$ .<sup>13</sup> On the one hand, the optimal maturity, characterized by the curve  $\tau(\lambda)$ , depends on the equilibrium  $\lambda$ , but not directly on  $c$ , which implies that the curve  $\tau(\lambda)$  is independent of  $c$ . On the other hand, the curve  $\lambda(\tau)$  depends directly on  $c$ . For a given maturity, a lower search cost induces more potential buyers to enter the market, which reduces the tightness of the secondary market, and increases the liquidity. As a result, the curve moves to the right. The red-dashed curve in Figure 2 shows the new locus for the equilibrium condition under financial development: both liquidity and maturity increase. This exercise shows that financial development—an increase in the efficiency of secondary markets—causes an increase of debt maturity. In the next section we evaluate this

<sup>13</sup>Improvements on the efficiency of the matching technology generate similar results.

mechanism quantitatively and show that the liquidity of the secondary market can generate quantitatively large movements in maturity choices.

What is *financial development*? First, there is a literal interpretation as the technology to execute trades. In developed markets, there are clearing houses such as *Euroclear* or *Clearstream* while in emerging economies the time to execute a trade is delayed by technological constraints. For example, in Argentina investors liquidate securities in one place, but make payments in a different bank.<sup>14</sup> Second, a broader interpretation is to think about the participants in the market. In developed financial markets, there are large and active mutual funds which are agents that trade more frequently than the rest of market’s participants. Because these funds are either small or inexistent in emerging economies, this could imply lower liquidity. Finally, [Bethune et al. \(2017b\)](#) shows that private information about the valuation of the security creates informational rents and can reduce trading. Hence, their model predicts that markets will be more illiquid when there are larger informational rents which might be the case for developing countries, for example, due to weaker credit bureaus.

## 4 Quantitative analysis

This section presents a quantitative evaluation of the theory. The most difficult task is to measure the non-default component of credit spreads for emerging economies in which data is scarce and standard methods used in the US cannot be applied. We present an estimation of the non-default component of credit spreads that only relies on credit spread data, apply it to the US and Argentina, and show that the non-default component of credit spread is larger in the emerging economy and firms borrow at shorter maturities. Next we use the estimation for the US to discipline the calibration of the model and exploit additional measurements as validation exercises. Finally, the estimation for Argentina allow us to do counterfactual analysis. Overall, credit frictions explain about 50% of maturity differences between the US and Argentina and have detrimental effects for productivity and the real economy.

### 4.1 Empirical estimate of liquidity spread

We collect data on credit spreads for the US and Argentina. For the US consider all corporate debt issuances in 2017 in Mergent Fixed Income Securities Database (FISD) and keep corporate bonds of domestic borrowers, in local currency and with fixed interest rate. Credit spreads are the difference between the interest rate and the Treasury rate with the same maturity. The

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<sup>14</sup>Recently in Argentina, BYMA—the local exchange market—is trying to unify these operations to increase the liquidity of the market.

Table 1: **Credit spread data: Summary statistics for US and Argentina**

	US	Argentina
<i>Initial sample</i>		
Issuances	598	153
Firms	389	54
Maturity		
Mean	10.3	2.6
Median	8.1	2.0
Coefficient of variation	0.8	0.5
Credit spread (median)	140	397
<i>Matched sample</i>		
Observations	171	35
Firms	93	15
Maturity (median)	7.1	2.3
Maturity short (median)	3.1	1.5
Maturity difference (median)	5.0	1.5

top panel of Table 1 describes the sample. There are 598 issuances of 389 firms with a median maturity of 8.1 years and a median credit spread of 140 basis points.

For Argentina consider all the active corporate bonds in August 2017 in the domestic market (MAE) and keep issuances in local currency, with 100% amortization, and interest rate as a spread on the Badlar rate (which is the reference short-term rate in Argentina).<sup>15</sup> The top panel of Table 1 describes the data. There are 153 issuances of 54 firms with a median maturity of 2 years and an average credit spread of 397 basis points.

**Firm fixed effects** Let  $cs_{i,t,k}$  be the credit spread of issuance  $k$  of firm  $i$  on day  $t$  and let  $m_{i,t,k}$  be the maturity of that issuance. Start with a simple specification for each country in which credit spreads depend on a firm fixed effect and maturity

$$cs_{i,t,k} = \gamma_i + \beta m_{i,t,k} + \epsilon_{i,t,k}. \tag{18}$$

Equation (14) establish that in the model the default component of credit spread is independent of maturity so the firm fixed effect captures the risk of default. On the other hand, the liquidity spread increases with maturity so  $\beta$  measures the marginal effect of an increase in maturity on the liquidity spread.<sup>16</sup>

The first and second columns of Table 2 show the estimated coefficients suggesting that

<sup>15</sup>These are floating interest rates bonds with a fixed spread so the credit spread is just the spread on the Badlar rate because non-arbitrage implies that agents can swap the variable Badlar rate for a fixed rate.

<sup>16</sup>Note that as the data is for 2017 we can abstract from a time fixed effect. The next regression addresses this concern more seriously.

Argentina is more illiquid than the US. An additional year of maturity increases credit spreads by 2.6 and 29.7 basis points in the US and Argentina, respectively.

**Credit spread differential** To control for changes in default risk and market’s conditions more seriously consider the set of firms that issue two or more bonds in the same day. The second panel of Table 1 describe the matched sample. For the US there are 171 pairs of observations (i.e., 342 issuances) of 93 firms. For each pair of bonds, the short issuance has a median of 3.1 years, while the spread on maturity—i.e., the difference between the short and the long issuance—is of 5 years. For Argentina we have 35 pairs (i.e., 70 issuances) for 15 firms. The short issuance has a median maturity of 1.5 years, and the long issuance increase the maturity by 1.5 years.

Let  $\Delta cs_{i,t,k_1,k_2}$  and  $\Delta m_{i,t,k_1,k_2}$  be the increase in credit spread and maturity for firm  $i$  in day  $t$  issuing bonds  $k_1$  and  $k_2$  such that  $m_{i,t,k_1} < m_{i,t,k_2}$ . Estimate the following specification

$$\Delta cs_{i,t,k_1,k_2} = \beta \Delta m_{i,t,k_1,k_2} + \epsilon_{i,t,k_1,k_2}. \quad (19)$$

The short maturity issuance controls for the default risk while the coefficient  $\beta$  captures how a marginal increase in maturity affects the liquidity spread.

The third and fourth columns of Table 2 report the estimation and suggest that Argentina is more illiquid than the US. For the quantitative evaluation is useful to summarize these regressions with the effect at the median maturity. For the US, maturity increases from 3.1 to 8.1 years and the estimated regression implies an increase of the credit spread of 15 basis points. On the other hand, in Argentina an increase from 1.5 to 3 years increases the credit spread by 62 basis points. Note that these moments capture the slope of the credit spread yield curve but not the level. The next section uses the moment for the US to calibrate the model. Then, validation exercises show that the level of the liquidity spread predicted by the model is consistent with additional estimates for the US. Finally, the estimation for Argentina disciplines counterfactual exercises.

## 4.2 Calibration for the US

We match moments from the US corporate debt market. Some parameters can be calibrated “externally”, while others must be calibrated “internally” from the simulation of the model. Table 3 summarize the parameter values and the targeted moments.

**Liquidity** Target the slope of the liquidity spread from Table 2. The median maturity for the short and long issuances are 3.1 and 8 years, respectively. Hence, target the increase in

Table 2: **Credit spreads: US and Argentina**

	Firm fixed effect		Difference	
	US	Argentina	US	Argentina
Maturity	2.570*** (0.135)	29.68** (12.62)	2.983*** (0.201)	41.42*** (3.182)
R-squared	0.419	0.126	0.564	0.833
Observations	598	153	171	35
Number of firms	389	54	93	15
Median maturity	8.1	2.0	7.1	2.3

the liquidity spread between these two maturities to be 15 basis points, i.e.,  $cs^{liq}(8.1, \lambda) - cs^{liq}(3.1, \lambda) = 15$ . The expected time to sell for corporate bonds in the US is about two weeks (i.e.,  $\lambda = 26$ , see [He and Milbradt \(2014\)](#)). These two moments discipline the search cost  $c$  and the holding cost  $h$ . Next, set the intensity of the liquidity shocks,  $\eta$ , to match an annual turnover rate of 57% ([He and Milbradt, 2014](#); [Chen et al., 2017](#)).

**Maturity** Normalize the measure of new firms  $\mu^0 = 1$ . For the maturity of corporate bonds target a debt maturity at issuance of 8.1 years (Table 1). By inspection of equation (17) note that we can normalize  $\kappa = 1$  and internally calibrate  $\zeta$  to match the target on maturity.<sup>17</sup>

**Matching** For the secondary market assume a constant-return-to-scale Cobb-Douglas matching function with efficiency parameter normalized to one, and symmetric loadings on sellers and buyers. Further assume symmetric bargaining power,  $\gamma = 0.5$ .

Finally, the discount factor is  $\rho = 0.02$  ([He and Milbradt, 2014](#)) and default rate is  $\delta = 0.03$  to match the default rate of speculative grade firms ([Moody, 2015](#)).

Table 3 shows that the model is able to match the four target moments. Even though the four parameters are jointly chosen to match the four target moments, we can derive some intuition on the identification of the parameters. Appendix D discusses how the moments provide useful information to calibrate the parameters.

### 4.3 Evidence and validation exercises

The predictions of the model are consistent with several dimensions not directly targeted in the calibration.

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<sup>17</sup>Recall that  $F(\tau) = A\tau$  and  $A = \frac{\zeta}{\rho + \delta}$ .

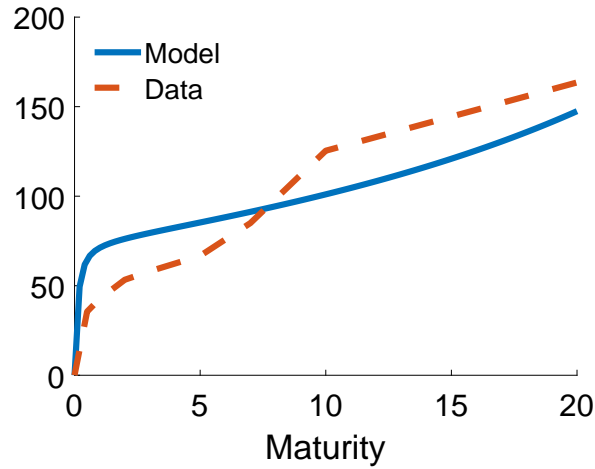
Table 3: Parameters and moments.

Parameter		Value	Target/source	Model	Data
<i>Financial sector</i>					
Intensity of liquidity shocks	$\eta$	0.58	Turnover rate	0.57	0.57
Search cost	$c$	0.0002	Slope liquidity	15	15
Holding cost	$h$	0.17	Expected time to sell	2.00	2.00
<i>Production sector</i>					
$F(\tau) = A\tau$	$A$	2.55	Maturity	8.08	8.08
$I(\tau) = \kappa \frac{1-e^{-\rho\tau}}{\rho}$	$\kappa$	1.00	Normalization		
<i>Matching</i>					
Share of sellers	$\alpha$	0.50	Normalization		
Bargaininw power of sellers	$\gamma$	0.50	Normalization		
<i>Others</i>					
Discount factor	$\rho$	0.02	He Milbradt (2014)		
Default rate	$\delta$	0.03	Moodys (2015)		

**Yield curve** For the US there is extensive additional empirical evidence about the liquidity spread that provides information about the level of the liquidity spread. For example, it is well known that there is a positive spread between the yields on treasuries and corporate bonds even after controlling for default, colloquially known as the *convenience yield* (e.g., [Krishnamurthy and Vissing-Jorgensen, 2012](#)). Moreover, there is evidence that this spread increases with maturity. For example, [Gilchrist and Zakrajšek \(2012\)](#) finds that credit spreads are increasing in maturity after controlling for distance to default and bond-specific characteristics (amount outstanding, coupon rate, callable, industry fixed effects, and credit ratings fixed effects). To assess the level of the liquidity spread compute the difference between the yield curve for two set of assets which coincide in their cash flow but differ on their liquidity. For the liquid asset consider the zero-coupon yield curve for treasuries which is an asset without liquidity problems. For the illiquid asset use the zero coupon yield curve for the high-quality market of corporate debt. These securities only include bonds with rating above A such that we can abstract from default risk but are known for its secondary market’s illiquidity. Appendix F shows that expected credit losses are quite small for these securities and provides additional details. The red dotted line on Figure 3 shows that the spread between Treasuries and corporate bonds is positive and increases with maturity. For example, at the maturity of 8 years the level of the liquidity spread is 125 basis points.

The calibration target the slope of th liquidity spread estimated from FISD. Figure 3 shows that the model also predicts the level of the liquidity spread yield curve at different maturities which was not a target of the calibration. For example, the level at a maturity of 8.1 years is 95 basis points. Hence, the calibration of the model is consistent both with the level and the

Figure 3: **Liquidity spread: Data versus Model.**



*Note: Liquidity spread in the data is computed as the spread between treasury and corporate yield curves. See text for details.*

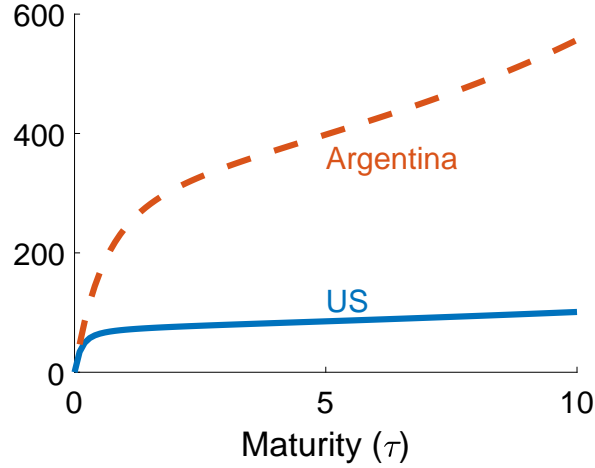
slope of the liquidity spread yield curve.

**Evidence from credit default swaps and international issuances** Empirical evidence is in line with the main mechanism of the paper: when financial markets are more liquid, firms choose to issue at longer maturities. First, [Saretto and Tookes \(2013\)](#) compares issuance of firms with and without credit default swaps (CDS) and argues that securities of companies with CDS trade in more liquid financial markets. The paper finds that firms with CDS increase the maturity between 0.68 and 1.79 years relative to those without CDS. Second, [Cortina Lorente et al. \(2016\)](#) studies firms in emerging economies issuing bonds both in domestic and in foreign markets. The paper finds that maturity increases by 1.6 years for foreign issuances relative to domestic ones. This evidence also corroborates the mechanism of the paper, as foreign markets are more liquid than domestic ones. Hence, the empirical evidence supports that companies issue at longer maturities when secondary markets are more liquid.

**Cross-sectional implications for the US** The second mechanism of the paper argues that when the term premium increases, firms choose to invest in shorter-term projects. [Dew-Becker \(2012\)](#) use data from the US and concludes that when the term premium increases the duration of investment decreases. [Yamarthy \(2016\)](#) also finds that when firms shift their long-term debt ratio to longer maturities, profitability and investment rates are higher. Finally, [Foley-Fisher et al. \(2016\)](#) use cross-sectional variation to show that companies with more dependence on long-term debt benefit more when the yield curve flattens.



Figure 4: **Liquidity spread: US and Argentina**



*Note: Liquidity spread in the model calibrated for the US and Argentina.*

#### 4.4 Counterfactual: Argentina

This exercise provides a quantitative evaluation of the importance of trading frictions for maturity choices and the real economy. Ideally, one would adjust all the parameters and re-estimate the model for different countries. However, due to data availability, adopt the following strategy. First, keep all the parameters except the search cost at the values calibrated for the US. Second, change the search cost to match the slope of the liquidity spread in Argentina and evaluate the effects on maturity choices and the real economy. Hence, this exercise evaluates the effects of bringing the financial system of Argentina to the US.

Section 4.1 shows that in Argentina the median maturities for the short and long issuances are 1.5 and 3 years, respectively. Hence, target an increase in the liquidity spread between these two points to be 62 basis points, i.e.,  $cs^{liq}(3, \lambda) - cs^{liq}(1.5, \lambda) = 62$ . Figure 4 shows the estimated liquidity spread yield curve for the US and Argentina. The spread for Argentina is both higher and increases more with maturity than for the US.

Proposition 3 shows that firms borrow at shorter maturities in economies with lower liquidity. The calibration for the US and Argentina allow us to quantitatively assess the mechanism. Table 4 shows that firms in an economy like the US but with the financial system of Argentina would borrow at a maturity of 5.2 years while the median maturity in Argentina is 2 years. Therefore, trading frictions explain about one-half of the maturity differences between US and Argentina.

**Real economy** When search costs increase long-term finance becomes more expensive and firms tilt their maturity choices toward the short-end. On the real side of the economy, this

implies that entrepreneurs invest in shorter-term projects which have lower productivity and, therefore, lower aggregate output. Table 4 shows that aggregate output reduces by 30% under the Argentinean financial system while in the data it is about 60% lower. Hence, trading frictions can account for about one-half of the differences in output between US and Argentina due to changes in the duration of investment and aggregate productivity.

The effects on the real economy depend on assumptions about the production technology. Section 6.3 considers an alternative production model that combines labor with productivity and show that for different parameter values the change in the liquidity spread generates a large effect on aggregate output (between 20% and 26%).

Table 4: **Counterfactual: US and Argentina**

	US		Argentina	
	Data	Model	Data	Model
<i>Liquidity (bps)</i>				
Increase 3.1 - 8.1 years	15	15	207	140
Increase 1.5 - 3.0 years	4	5	62	62
Liquidity spread	125	95		405
Maturity (years)	8.1	8.1	2.0	5.2
Output	1.0	1.0	0.4	0.7

**Alternative measures of liquidity** The counterfactual exercise relies on the estimate for Argentina. Appendix E considers an alternative empirical strategy. First, the model is calibrated for the US and liquidity is disciplined by the difference between Treasuries and the high quality market of corporate debt. Next, the net interest margin (a common measure of intermediation in the literature) disciplines the non-default component of credit spreads across countries. Overall, counterfactual exercises support the same results than the estimation for Argentina: trading frictions are an important driver of maturity choices across countries.

## 5 Policy intervention

While it is outside the scope of this paper to fully characterize the optimal policy, we explore the effects of a government intervention designed to increase the liquidity of financial markets which is call *Government-sponsored intermediaries* (GSIs). The intervention consists of government agents acting as intermediaries in the secondary market that buy and sell bonds at different prices than in private bilateral meetings. The government charges a distortionary profit tax on the corporate sector to finance the policy. We interpret this exercise as a lower bound for the effects of government interventions. In particular, assume that the government cannot avoid

the search frictions or holding costs but government agents face the same constraints as private agents. However, public agents can participate in secondary markets and take different actions than the private sector.

This intervention has some similitudes with financial policies in the US and in emerging economies. First, Government-sponsored enterprises (GSEs) such as Freddie Mac and Fannie Mae, are institutions intended to improve the credit to households (see [Fieldhouse et al., 2017](#), for a recent review of this organizations). The proposed policy is similar to GSEs but targeting the efficiency of credit to the corporate sector instead of households. Second, during the 2008 financial crisis, the central bank perform large-scale asset purchased (known as QE1, QE2, and QE3). [Gertler et al. \(2013\)](#) argues that this policy was effective due to limits to arbitrage in private intermediation. In a similar way, GSIs will intermediate assets in financial markets.

There are some examples of policies implemented or proposed in emerging economies that also share several features of GSIs. First, the “priority-sector lending” in India requires banks to lend at least 40% of their net credit to the “priority sector” and also establishes specific targets for different sub-sectors (which includes agriculture, and small-scale industry among others).<sup>18</sup> And interesting feature of this policy is that banks have to trade to meet all the specific targets, and, therefore, increase the liquidity of the secondary market of assets in the priority sector (a similar motive for trade as in the Fed Funds Market in the US, see [Afonso and Lagos \(2015\)](#)). One difference between “priority-sector lending” and GSIs is that the policy in India is for short-term debt (usually below one year) while this paper argues that liquidity is relatively more important for long-term assets. Hence, we propose similar policies but for the long-end of the market.

There is another example on policy proposals in Brazil. The private capital markets association (Anbima) launched a project to facilitate long-term financing. Some of the measures aim to increase the secondary market liquidity by the creation of a “Liquidity Improvement Fund” in which private agents manage public resources to act as market makers which are very similar to the proposed GSIs.<sup>19</sup>

## 5.1 Government-sponsored intermediaries

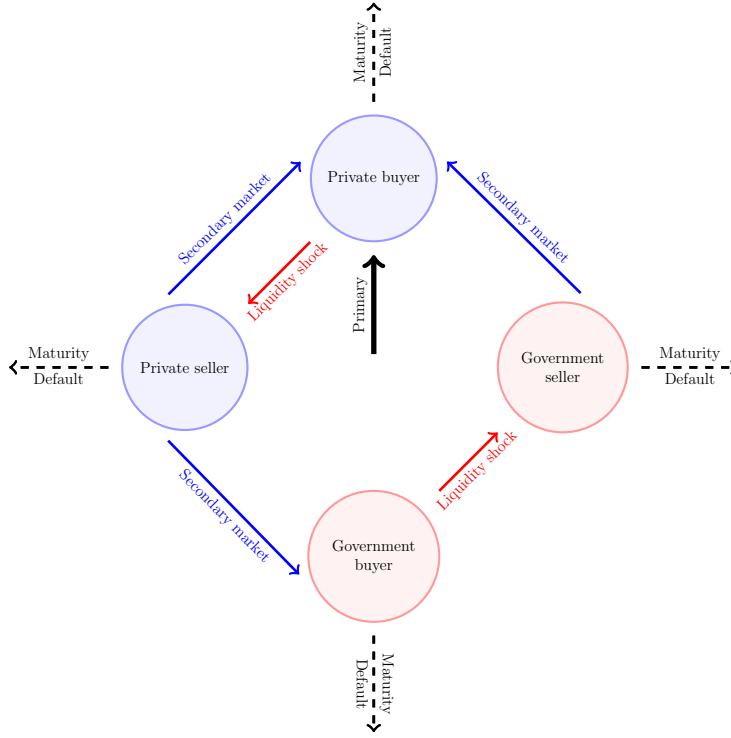
The government agency intermediates assets in the secondary market to improve the liquidity of financial markets. Government agents are subject to the same idiosyncratic risk of holding costs

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<sup>18</sup>For details see [Banerjee and Duflo \(2014\)](#) and Reserve Bank of India master circular of July 2015, “Priority Sector Lending- Targets and Classification” available at [https://rbi.org.in/scripts/bs\\_viewmascirculardetails.aspx?id=9857](https://rbi.org.in/scripts/bs_viewmascirculardetails.aspx?id=9857). Importantly, most of the banks in India are in the public sector, hence they could be interpreted as the government agents in the model.

<sup>19</sup>See [Park \(2012\)](#) for further details.

Figure 5: Schematic representation of GSIs.



as private agents so they act both as buyers and sellers in the secondary market.<sup>20</sup> One possible interpretation for this idiosyncratic needs for public agents can be balance sheet requirements such as the priority sector lending in India discussed before. However, the government can choose different prices than those charged in private meetings. If they buy (sell) at a high (low) price they will run a deficit which is financed by distortionary taxes on the corporate sector.

Figure 5 shows a schematic representation of the model with GSIs. Note that private sellers can now sell both to private and government agents. Moreover, private buyers can now buy in the secondary market from either private or government sellers. In this section, we describe the key features of the model with GSIs while Appendix C.1 contains additional details.

The government has four instruments: the size of GSIs, prices for buying and selling for their trading agents, and the corporate tax rate. The objective is to maximize aggregate steady-state welfare subject to running a balanced budget and equilibrium conditions. Recall that both primary and secondary financial markets are competitive—i.e., participants make zero profits in expectation. However, the production sector—i.e., the borrowers—have positive profits in equilibrium. Hence, we define the objective of the government to maximize the value of the corporate sector.

<sup>20</sup>The formulation of government agents is similar to Aiyagari et al. (1996).

**Matching** There is random matching between sellers and buyers. In the benchmark policy we assume that both government and private agents have the same efficiency to find a counterpart. However, for robustness exercises of Section 5.4, consider a general formulation in which government and private agents may differ in the efficiency to find a counterpart. Let  $e^{i,j}$  be the efficiency for  $i = p, g$  (private and government agents, respectively) of  $j = b, s$  (buy and sell, respectively). In the benchmark model we assume that  $e^{i,j} = 1$  for all  $i, j$ .

The total mass of sellers,  $\mu^s$ , is composed by private and government agents. Private sellers are  $\mu^{p,s} = \int_0^\tau e^{p,s} \mu^{p,l}(y) dy$  where  $\mu^{p,l}(y)$  is the measure of low-valuation private agents holding an asset and willing to sell. Similarly, government sellers are  $\mu^{g,s} = \int_0^\tau e^{g,s} \mu^{l,g}(y) dy$ , where  $\mu^{l,g}(y)$  is the measure of low-valuation government agents holding an asset and willing to sell. The total measure of buyers has private buyers  $\mu^{p,b}$ , determined by a free-entry condition, and government buyers  $\mu^{g,b}$  which is a policy instrument chosen by the government. Hence,  $\mu^b = e^{p,b} \mu^{p,b} + e^{g,b} \mu^{g,b}$ .

The market tightness is  $\theta = \frac{\mu^s}{\mu^b}$  and characterizes the buying and selling intensities  $\beta = A\theta^\alpha$ , and  $\lambda = A\theta^{\alpha-1}$ , respectively. Let  $\lambda^{s-b}$  be the intensity at which a seller of type  $s = p, g$  meets a buyer of type  $b = p, g$ . Similarly, let  $\beta^{s-b}(y)$  be the intensity at which a buyer of type  $b = p, g$  meets a seller of type  $s = p, g$  with an asset of time-to-maturity  $y$ . The matching technology implies that

$$\begin{aligned} \lambda^{p-p} &= \lambda e^{p,s} e^{p,b} \frac{\mu^{p,b}}{\mu^b} & \lambda^{p-g} &= \lambda e^{p,s} e^{g,b} \frac{\mu^{g,b}}{\mu^b} & \lambda^{g-p} &= \lambda e^{g,s} e^{p,b} \frac{\mu^{p,b}}{\mu^b} \\ \beta^{p-p}(y) &= \beta e^{p,s} e^{p,b} \frac{\mu^{l,p}(y)}{\mu^s} & \beta^{g-p}(y) &= \beta e^{g,s} e^{p,b} \frac{\mu^{l,g}(y)}{\mu^s} & \beta^{p-g}(y) &= \beta e^{p,s} e^{g,b} \frac{\mu^{l,p}(y)}{\mu^s} \end{aligned}$$

Finally, we have to assume what happens after a meeting between a government buyer and a government seller. The idea is to interpret the government as a large player and private agents as atomistic. However, for tractability, we assume that all investors can hold either zero or one unit of the asset. To bypass this restriction, assume that a government seller cannot trade with a government buyer, i.e.,  $\lambda^{g-g} = 0$ . Note, that this is a conservative assumption as the cost of the policy is smaller if there are these type of trades. In fact, Section 5.4 solves the model with these type of trades and shows that there are larger effects.

**Prices in secondary markets** There are three types of meetings in secondary markets. Let  $P^{S,s-b}(y)$  be the price when a seller of an asset with time-to-maturity  $y$  of type  $s = p, g$  meets a buyer of type  $b = p, g$ . Upon a meeting between private agents the price is determined by Nash Bargaining in which the seller has bargaining power  $\gamma$  so

$$P^{S,p-p}(y) = D^L(y) + \gamma(D^H(y) - D^L(y)).$$

The prices that involve either a government buyer or seller are determined by the government. In the quantitative solution we restrict prices to be in the following parametric family

$$P^{S,g-p}(y) = D^L(y) + \gamma^{g,s}(D^H(y) - D^L(y)) \quad (20)$$

$$P^{S,p-g}(y) = D^L(y) + \gamma^{g,b}(D^H(y) - D^L(y)) \quad (21)$$

and let the government choose  $\gamma^{g,s}$  and  $\gamma^{g,b}$  in  $[0, 1]$ . Note that prices are similar to those in private meetings but the government can choose a different bargaining power. As we show later, it is optimal to set  $\gamma^{g,s} = 0$  and  $\gamma^{g,b} = 1$ . This implies that the government gives all the bargaining power to the private sector—i.e., the government buys at a *high* price and sell at a *low* price.

Of course, this is an important restriction on government prices but it follows from the objective on finding a lower bound. For example, one can use the model with segmented markets presented in Section 6.4 and allow the government to set different prices according to the maturity. However, we will show that even without this flexibility, the effects of GSIs are quantitatively significant and the extension of targeting prices according to the maturity is likely to improve the results from the lower bound identified in this exercise. Importantly, note that this alternative specification would work through the same channel than the mechanism described in the benchmark policy.

**Private valuations** The value of holding an asset for a high-valuation private agent is equivalent to the benchmark model, Equation (9). However, the value of a low-valuation private agent is different as now he can sell it both to private and government buyers. Under the government prices specified in (20), the price that the government offers is equivalent to that in private meetings but in which the seller has a different bargaining power. Hence, the value for a low-valuation agent is equal to the benchmark model, Equation (10) with an augmented selling intensity:  $\lambda = \lambda^{p-p}\gamma + \lambda^{p-g}\gamma^{g,b}$ .

Let  $\lambda^{GSI}$  and  $\lambda^{EQ}$  be the equilibrium liquidity in the economy with and without GSIs, respectively. If  $\lambda^{GSI} > \lambda^{EQ}$ , Lemma 2 implies that the liquidity spread will be lower in an economy with GSIs. However, borrowers have to pay a distortionary tax to finance the intervention. Hence, ex-ante, we don't know if the policy increases welfare for borrowers.

Private buyers can meet with private and government sellers. The free entry condition is

$$c = \int_0^\tau \beta^{p-p}(y) (D^H(y) - P^{S,p-p}(y)) dy + \int_0^\tau \beta^{g-p}(y) (D^H(y) - P^{S,g-p}(y)) dy.$$

**Cost of GSIs** The government runs a balanced budget. The constraint is

$$\begin{aligned} & \mu^f(\tau)x^c f(\tau) + [\mu^{g,h}(0) + \mu^{g,l}(0)] + \lambda^{g-p} \int_0^\tau \mu^{g,l}(y)P^{S,g-p}(y)y \\ & = \mu^{g,b}c + \mu^{g,b} \int_0^\tau \beta^{p-g}(y)P^{S,p-g}(y)dy + h \int_0^\tau \mu^{g,l}(y)dy. \end{aligned} \quad (22)$$

The left-hand-side of Equation (22) represents the government's income. First, the government charge a proportional corporate tax  $x^c$  to producing firms  $\mu^f$ , where flow profits are  $f(\tau) = z(\tau)$ . Second, some of the securities held by government agents mature. Third, some low-valuation government agents sell the securities to the private sector.

The right-hand side of Equation (22) captures the expenditures. A measure  $\mu^{g,b}$  of agents are searching in secondary markets, and some of them buy a bond. Moreover, some government agents are low-valuation and have to pay the holding cost  $h$ .

**Optimal policy** The objective of the government is to maximize steady state profits of the production sector subject to the equilibrium conditions and the budget constraint (22)<sup>21</sup>

$$\max_{x^c, \mu^{g,b}, \gamma^{g-b}, \gamma^{g-s}} \mu^f(\tau)e^{-(\rho+\delta)\tau} ((1-x^c)f(\tau) - I(\tau)e^{r(\tau)\tau}) \quad \text{s.t. (22) and equilibrium conditions.}$$

GSIs cause a direct and an equilibrium effect. On the one hand, a larger intervention needs a higher taxes which lowers welfare. On the other hand, if the policy increases the equilibrium liquidity, credit spreads for long-term borrowing reduce which benefits borrowers. Therefore, the optimal policy solves the trade-off between these two effects.

## 5.2 Optimal GSIs in the US

First, consider the optimal policy under the calibration for the US. The bargaining power when the government acts as a buyer,  $\gamma^{g,b}$ , directly affects the value of low and high valuation private agents. The optimal policy sets  $\gamma^{g,b} = 1$  so private sellers get more gains from trade when trading with the government. This generates a direct effect on increasing the value of private agents in the financial sector and reduce the financial cost for the production sector.

The bargaining power when the government acts as a seller,  $\gamma^{g,s}$ , has a direct effect on the incentives of private agents to search in the secondary market. The optimal value is  $\gamma^{g,s} = 0$ . Finally, the measure of government agents searching in the secondary markets is optimally chosen to maximize the welfare gains. If  $\mu^{g,b} = 0$  the economy is equivalent to no intervention,

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<sup>21</sup>We consider steady state welfare because the transitions involve manipulating the boundary conditions of the distributions. However, note that this is a conservative assumption because during a transition old generations holding a security issued before the intervention are better off because asset prices increase.

while as  $\mu^{g,b}$  increases, the tax rate also increases to balance the budget. Given the results in this section, the next exercises set  $\gamma^{g,b} = 1$  and  $\gamma^{g,s} = 0$  and let the government chooses  $\mu^{g,b}$  and the tax rate.<sup>22</sup>

### 5.3 GSIs across countries

To evaluate GSIs for economies with higher trading frictions consider variations in the search cost. The lower and higher values of  $c$  correspond to the calibration for the US and Argentina, respectively.

The intervention has non-linear effects across countries. Figure 6 compares economies with and without GSIs for different level of trading frictions. The top left panel shows that the policy is more efficient to improve liquidity of financial markets when search frictions are relatively low. However, the right top figure shows that the flattening of the yield curve due to the policy is more effective when there are larger frictions. In an advanced financial market, the marginal effect of an increase in liquidity is smaller than in a less developed financial system. Hence, even though the improvement in liquidity is lower in emerging markets the consequences might be larger.

The bottom right figure shows that GSIs increase the maturity of corporate debt about five months in the US. Note that this effect is larger in less developed financial markets. For example, in a system similar to Argentina, GSIs increase the maturity of corporate debt by eight months. Finally, the bottom right figure shows that the increase in welfare due to GSIs is 3.84% for the US and 4.94% for Argentina.

### 5.4 Robustness: Alternative policies

The intervention considered so far should be consider as a lower bound on the effects of GSIs. Table 5 explores alternative assumptions that can improve the effects of government interventions for two levels of financial frictions. The first panel consider search costs at the level calibrated for the US while the second panel consider an economy with trading frictions similar to Argentina.

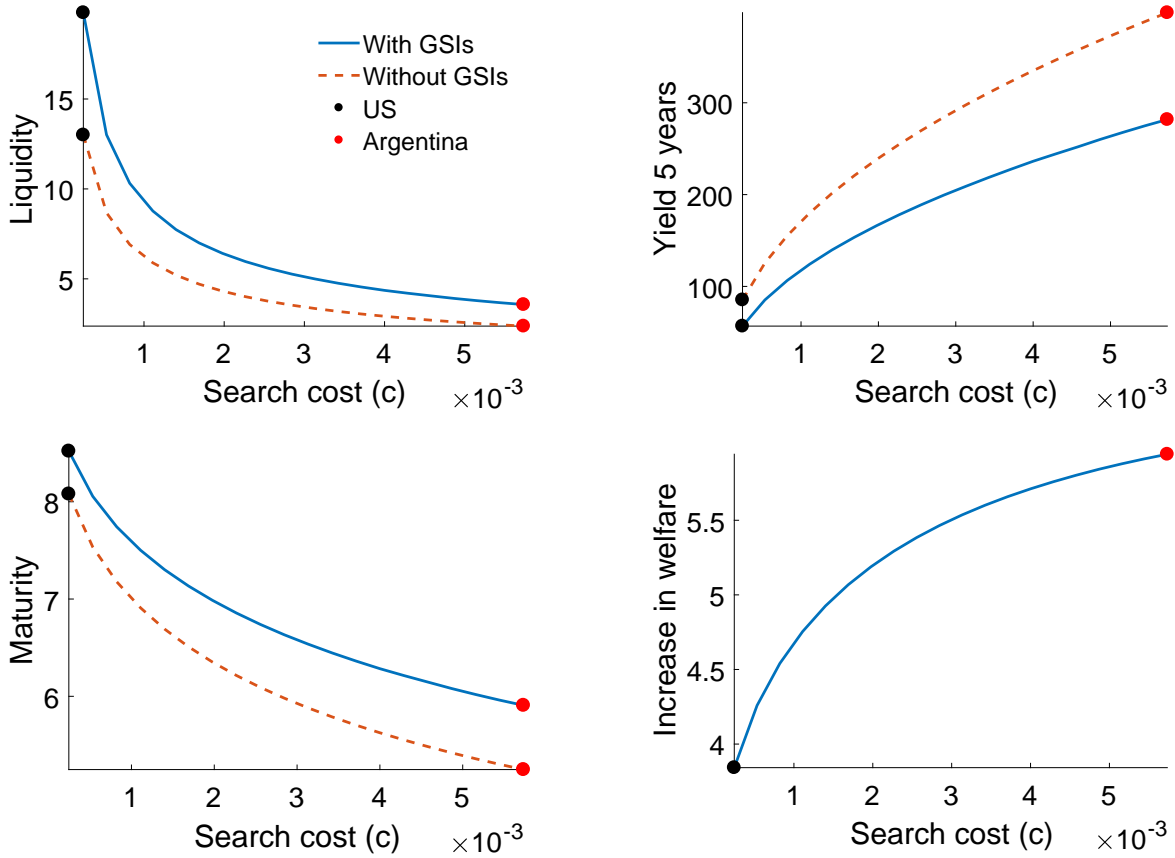
First, consider government agents that are more efficient to search for counterparts. The third and fourth rows of each panel of Table 5 show the result of increasing the search efficiency of government agents by 10% and 50%, respectively (i.e.,  $e^{g,b} = e^{g,s} = 1.1$ , and  $e^{g,b} = e^{g,s} = 1.5$ , respectively). Overall, the results show that as the efficiency of the government increases, the

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<sup>22</sup>In fact, in all exercises we verified that if the government can choose the bargaining they choose this values. However, this restriction simplifies the description of the results without adding additional intuition.



Figure 6: **GSI: Effects for different trading frictions.**



*Note: Results with and without GSIs for different trading frictions.*

intervention becomes more effective in increasing the liquidity of the economy, the yield curve flattens even more and firms issue at longer maturities.

Finally, recall that the benchmark policy assume that  $\lambda^{g-g} = 0$ , i.e., a government seller cannot trade with a government buyer. For a given size  $\mu^{g,b}$ , the cost of GSIs decreases if the government can reallocate securities among its trading agents. The last row of table 5 shows that if government agents can trade among them GSIs are more efficient and the effects on credit spreads, maturity, and welfare improve.

There are legitimate reasons to imagine that government agents might have more flexibility than private agents. Hence, the results of the benchmark policy should be considered as a lower bound on the implications for GSIs. For example, Table 5 shows that the gains from the government intervention can be larger if government agents are more efficient to find counterparts or can trade within each agents.

Table 5: **GSI: Alternative policies.**

	Liquidity	Spread	Maturity	Welfare	Output
		5 years		gains	gains
<i>Low trading frictions (US)</i>					
No GSIs	13.00	85	8.08		
Benchmark policy	19.80	57	8.52	3.84	4.07
Gov. 10% more efficient	20.39	55	8.55	4.07	4.31
Gov. 50% more efficient	21.98	51	8.61	4.62	4.89
Gov. transactions	21.89	51	8.61	4.59	4.86
<i>High trading frictions (Argentina)</i>					
No GSIs	2.37	398	5.25		
Benchmark policy	3.57	282	5.91	5.94	10.38
Gov. 10% more efficient	3.63	278	5.94	6.51	10.87
Gov. 50% more efficient	3.81	267	6.04	8.32	12.40
Gov. transactions	4.09	250	6.23	12.93	15.23

## 6 Extensions

This section extends the model in four dimensions and shows that results do not hinge on many of the assumptions of the theory. The first extension studies how borrowers choose and finance investment projects when they can rollover short-term debt to finance long-term projects. Second, we evaluate how changes in the default probability interact with maturity choices. Third, the effects on the real economy are quantitatively similar with alternative production functions. Finally, the yield curve has the same shape if there is random matching across maturities in the secondary market or buyers direct themselves to markets segmented by maturity.

### 6.1 Rollover

This extension shows that changes in the liquidity of the secondary market have similar effects on borrower choices even if they can rollover short-term debt to finance long-term projects. Consider a firm that chooses the maturity of the project  $\tau$  and issues zero coupon bonds to finance investment costs. A bond of maturity  $y$  has a fixed cost of issuance  $\Phi$  and an interest rate  $r(y)$ .<sup>23</sup> Let  $J$  be the total number of issuances up to age  $\tau$ . It is easy to show that a fixed

<sup>23</sup>In the data, issuance costs include management fee, selling concession, registration fee, underwriter fee, underwriter spread (the difference between the offering price and the guaranteed price to the issuer), underpricing (the difference between the market price and the offering price), and printing, legal and auditing costs. For the Eurobond market, [Melnik and Nissim \(2003\)](#) finds that the total issuance cost is 37 basis points. [Lee et al. \(1996\)](#) finds similar costs and reports evidence of economies of scale, reflecting that a significant fraction is a

issuance cost implies a finite number of issuances.

The firm chooses the maturity of the project,  $\tau$ , the number of issuances  $J$ , the amount borrowed  $B_j$ , and the maturity structure of their liabilities  $y_j$  to solve

$$\begin{aligned} & \max_{\tau, J, y_j, B_j} e^{-(\rho+\delta)\tau} (F(\tau) - B_j) & (23) \\ \text{s.t.} & \quad B_j P(y_j, \lambda) = B_{j-1} + \Phi + I(y_j) \quad \text{for } j = 1, \dots, J \\ & B_0 = 0 \quad \text{and} \quad \sum_{j=1}^J y_j = \tau. \end{aligned}$$

Each issuance borrows to rollover existing debt  $B_{j-1}$ , cover the issuance cost  $\Phi$ , and invest for the next  $y_j$  periods,  $I(y_j)$ .<sup>24</sup> Iterate on  $B_j$  to cast the firm's problem (23) as

$$\max_{\tau} e^{-(\rho+\delta)\tau} F(\tau) - \text{FIN}^{\text{COST}}(\tau)$$

in which the financial cost is

$$\begin{aligned} \text{FIN}^{\text{COST}}(\tau) &= e^{-(\rho+\delta)\tau} \min_{J, \{y_j\}_{j=1}^J} \sum_{i=1}^J (\Phi + I(y_i)) e^{\sum_{s=i}^J r_s y_s} \\ \text{s.t.} & \quad \sum_{j=1}^J y_j = \tau. \end{aligned}$$

**Financing cost** Consider a project of a given maturity  $\tau$ . The financial cost  $\text{FIN}^{\text{COST}}(\tau)$  chooses the number of issuances  $J$  and the maturity structure  $y_j$  to minimize the net present value of issuance costs  $\Phi$  and investment needs  $I(y_j)$ . Both the issuance cost and the liquidity spread affect financial decisions.

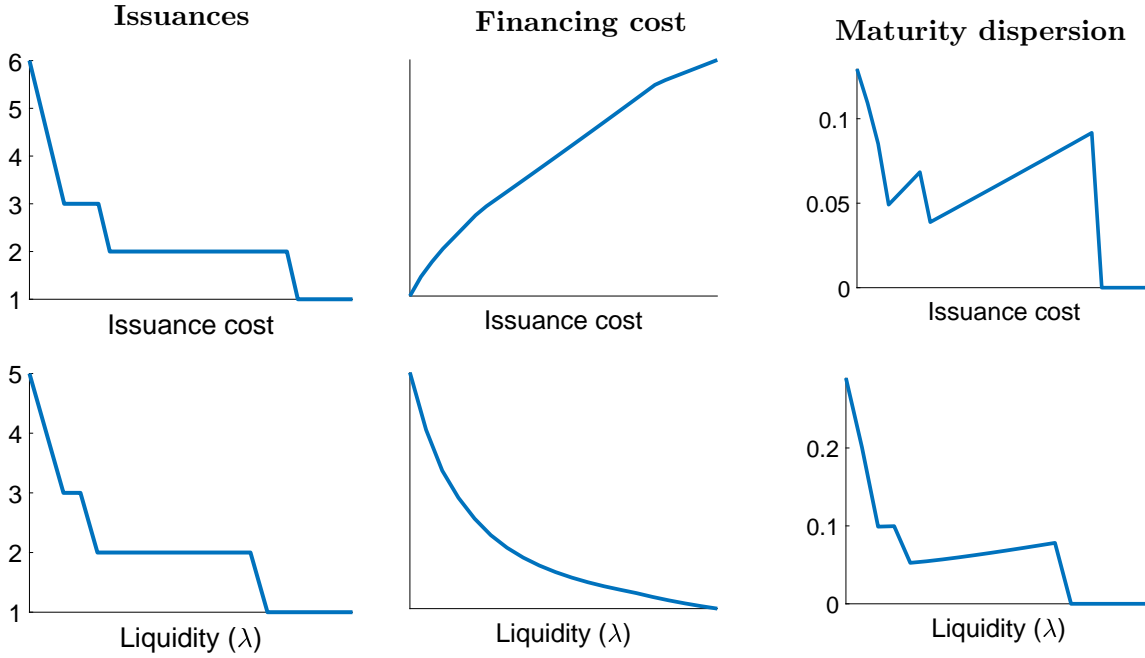
Evaluate the financial choices for different issuance costs  $\Phi$ , given a project  $\tau$ . The top panel of Figure 7 shows the number of issuances, the total financial cost, and the dispersion of maturities for different issuance cost  $\Phi$ . Naturally, as the issuance cost increases, the number of issuances decreases and the total financial cost increases. Note that if  $\Phi$  is sufficiently large, the firm optimally chooses to issue only one time, matching the maturity of the project and the liabilities. In the benchmark model we focus in this particular case with  $J = 1$  and derive a sharper analytical characterization of the effects of secondary markets liquidity on the project's choice.

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fixed cost.

<sup>24</sup>A firm with positive cash holdings will always wait until she runs out of money to issue new debt. Hence, without loss of generality, we only have to consider the choices of the firm when outstanding debt matures which coincides with the moment in which the firm runs out of money.

Figure 7: **Financial cost with rollover.**



*Note: Financial cost for a given maturity  $\tau$  and different issuance cost  $\Phi$  and liquidity  $\lambda$ . Parameter values are discussed in Section 4.*

The bottom panel of Figure 7 evaluates how the liquidity of the secondary market affects financial choices. In markets with lower trading frictions—higher  $\lambda$ —long-term finance is more attractive (Proposition 1) which induces firms to rollover less often, borrow at longer maturities, and reduces the total financial cost. Hence, for a given project  $\tau$ , the financial cost diminishes when  $\lambda$  increases. The next exercise shows that this effect induces firms to invest in projects of longer horizons.

**Maturity structure** The optimal maturity structure solves the trade-off between equalizing credit spreads across different issuances and decreasing future fixed issuance costs. In equilibrium, the maturity structure is decreasing—i.e.,  $y_1 \geq y_2 \geq \dots \geq y_J$ . On the one hand, Figure 7 shows that, conditional on the number of issuances, when the issuance cost increases firms choose to increase the maturity dispersion. Intuitively, as  $\Phi$  increases, firms want to postpone the fixed cost payments of later issuances, and, as a result, they extend the maturity of the first issuances and decrease later ones. On the other hand, the bottom panel shows that for financial markets with larger trading frictions (lower  $\lambda$ ), the dispersion of maturities diminishes to generate similar liquidity spreads across issuances.

**Investment choice** Proposition 3 and the quantitative exercises on Section 4 show that the liquidity of the secondary market is important for investment decisions when firms match the maturity of assets and liabilities. Moving from an OTC secondary market with liquidity as in the US economy ( $\lambda = 26$ ) to a shut-down of the market ( $\lambda = 0$ ) reduces the project's maturity by 5.4 years (from 8.1 to 2.7, first panel of Table 6, rows two and three).

If the firm can rollover short-term debt the effects of trading frictions on project's choice can be substantially weaker because as  $\lambda$  decreases the firm can rollover more often, as suggested by Figure 7. Note that this adjustment should be more important when issuances costs are low. The second panel of Table 6 consider the case of a low issuance cost such that under an OTC secondary market the firm chooses a project of 11.1 years of duration and issue bonds 22 times (i.e., bonds have a maturity of 6 months on average). However, when there is a shut-down of the secondary market the firm chooses to rollover more often (30 times) and shorten the duration of investment to 6.7 years (bonds have a maturity of 2.4 months on average). Hence, the reduction in the duration of the project due to changes in trading frictions is of 4.4 years, similar to the effect when the firm cannot rollover of 5.4 years. The third and fourth panel of Table 6 consider the case of higher issuance costs and find that the effect of trading frictions on maturity choices is similar, around 4 years. These exercises suggest that the results on Section 4 about how trading frictions affect investment decisions do not depend on the assumption of matching the maturity of the project and the bond.

An increase in the issuance cost generate two effects. First, conditional on the duration of the project, firms issue less frequent and increase debt's maturity. Second, the firm reduces the duration of the project. Quantitatively, most of the adjustment is made by the number of issuances while the choice of projects is rather unaffected. This result also confirms that abstracting from rollover decisions in the main analysis of the paper is rather inconsequential for the real side of the economy.

One potential concern about these exercises is that the liquidity of the secondary market and rollover costs might be correlated. In the model, liquidity costs are endogenous while rollover costs are exogenous and fixed, so they do not respond to changes in the liquidity of secondary markets, and a potential Lucas critique may apply. However, we expect that when the secondary market becomes more liquid, both issuance and rollover costs should diminish. Hence, it is conservative to assume fixed issuance costs. As in Table 6, when liquidity of secondary markets improves, the firm adopts a project of longer duration. Similarly, when issuance costs diminish, the firm chooses longer-term projects. If the two effects are present (an increase in liquidity and a reduction of issuance costs), the firm will extend the maturity of the project even more than with fixed issuance costs.

Table 6: Solution under alternative specifications.

Default	Secondary market	Issuance cost	Issuances	Maturity Project	Maturity Bond	Interest rate	Credit spread Default	Credit spread Liquidity
<i>No rollover</i>								
Yes	Centralized		1	9.6	9.6	5.0%	3.0%	0.0%
Yes	OTC		1	8.1	8.1	5.9%	3.0%	0.9%
Yes	Shut down		1	2.7	2.7	15.4%	3.0%	10.4%
No	Centralized		1	30.4	30.4	2.0%	0.0%	0.0%
No	OTC		1	19.1	19.1	3.0%	0.0%	1.0%
No	Shut down		1	3.1	3.1	13.3%	0.0%	11.3%
<i>Rollover with low issuance cost</i>								
Yes	Centralized	0.01	15	12.0	0.8	5.0%	3.0%	0.0%
Yes	OTC	0.01	22	11.1	0.5	5.6%	3.0%	0.6%
Yes	Shut down	0.01	30	6.7	0.2	6.1%	3.0%	1.1%
No	Centralized	0.01	1	30.4	30.4	2.0%	0.0%	0.0%
No	OTC	0.01	36	24.5	0.7	2.6%	0.0%	0.6%
No	Shut down	0.01	30	7.1	0.2	3.1%	0.0%	1.1%
<i>Rollover with medium issuance cost</i>								
Yes	Centralized	0.11	4	11.4	2.9	5.0%	3.0%	0.0%
Yes	OTC	0.11	5	10.5	2.1	5.8%	3.0%	0.8%
Yes	Shut down	0.11	10	6.7	0.7	8.0%	3.0%	3.0%
No	Centralized	0.11	1	30.4	30.4	2.0%	0.0%	0.0%
No	OTC	0.11	6	24.1	4.0	2.8%	0.0%	0.8%
No	Shut down	0.11	10	7.1	0.7	5.2%	0.0%	3.2%
<i>Rollover with high issuance cost</i>								
Yes	Centralized	0.20	3	11.2	3.7	5.0%	3.0%	0.0%
Yes	OTC	0.20	3	10.0	3.3	5.8%	3.0%	0.8%
Yes	Shut down	0.20	7	6.6	0.9	9.1%	3.0%	4.1%
No	Centralized	0.20	1	30.4	30.4	2.0%	0.0%	0.0%
No	OTC	0.20	4	23.6	5.9	2.8%	0.0%	0.8%
No	Shut down	0.20	8	7.1	0.9	5.9%	0.0%	3.9%

## 6.2 Default

Default affects credit spreads and investment decisions. Lemma 2 shows that the liquidity spread increases with the default intensity. Quantitatively, long-term rates react more than short-term rates to changes in default. Figure 8 shows how the default rate changes the liquidity spread at maturities of 1 and 10 years, relative to the benchmark of  $\delta = 0.03$ . The liquidity spread for long maturities reacts more to changes in the default rate. Hence, when  $\delta$  increases the yield curve shifts upwards both due to default and liquidity. As a result, the firm chooses shorter-term projects (see row four to six on Table 6).

Moreover, when there is no default risk and secondary markets are centralized firms have no incentive to issue short-term debt regardless of the issuance cost. However, when default risk is positive, even if secondary markets are centralized firms choose to rollover debt when the issuance cost is not too high. Hence, both default risk and trading frictions shape rollover choices.

## 6.3 Alternative production functions

One might be worry that the quantitative effects on productivity and output depend on assumptions and calibration of the production function. This extension considers alternative production functions and shows that the effects of liquidity on the real economy are quantitatively similar across different models.

Consider a production function that combines labor  $l$  and productivity  $z$  to produce output with technology  $y = z^{1-\sigma}l^\sigma$ . Let  $z$  be the productivity developed in the investment stage as in the benchmark model. To assess the importance of the curvature of the production function  $\sigma$ , we calibrate and do counterfactual analysis related to the liquidity of the financial markets for different values of the labor share,  $\sigma$ .

Static profits are

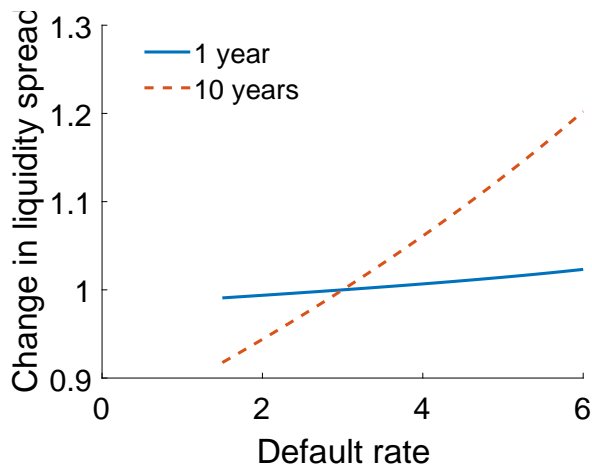
$$\pi(z) = \max_l z^{1-\sigma}l^\sigma - wl,$$

and labor demand is  $l = z \left(\frac{\sigma}{w}\right)^{\frac{1}{1-\sigma}}$ . Note that static profits are linear in  $z$  as in the benchmark model

$$\pi(z) = z \left(\frac{\sigma}{w}\right)^{\frac{\sigma}{1-\sigma}} (1 - \sigma).$$

Assume an exogenous labor supply normalized to one. In steady state there is a measure  $\mu^F = \frac{\epsilon^{-\delta\tau}}{\delta}$  of identical firms with productivity  $z$ . Labor market clearing implies  $\mu^F l = 1$  so

Figure 8: **Liquidity-default interactions.**



*Note: The figure shows how the liquidity spread at 1 and 10 years changes with  $\delta$  with respect to the benchmark of 0.03.*

$w = \sigma (\mu^F z)^{1-\sigma}$ . Finally, aggregate output is

$$Y = \mu^F z^{1-\sigma} l^\sigma = (\mu^F z)^{1-\sigma}.$$

**Calibration** For each  $\sigma \in \{0.2, 0.5, 0.8\}$  calibrate the model to match the same moments as in the benchmark case. In particular, choose search costs  $c$ , holding cost  $h$ , intensity of liquidity shocks  $\eta$ , and productivity growth  $\zeta$  to match the turnover rate, liquidity spread, expected time-to-sell, and maturity. The results of the calibration show the same value for  $c$ ,  $h$  and  $\eta$  than in the benchmark calibration, but a different  $\zeta$  for each  $\sigma$ . This highlights that the moments for the financial sector discipline the parameters related to the trading friction ( $c$ ,  $h$ , and  $\eta$ ), while the production technology parameters ( $\sigma$  and  $\zeta$ ) affect the choice of maturity.

**Counterfactual** For each  $\sigma$  increase the search cost  $c$  to the level in Argentina as in the benchmark quantitative counterfactual exercise. Table 7 shows that as  $\sigma$  increases (i.e., an increase in the labor share) the change in the liquidity spread has a lower effect on maturity and aggregate output. Quantitatively, moving from a labor share of 0.2 to 0.8 the effect on maturity reduces by about six months and the effect on aggregate output reduces by 6 percentage points. Table 4 shows that in the benchmark model the same change on credit spreads generates a decrease in maturity and aggregate output of 2.9 years and 30%, respectively. Hence, we conclude that changes in the liquidity spread are important for maturity choices and the real economy independently of the curvature of the production function.



Table 7: **Alternative production functions.**

Labor share	0.2	0.5	0.8
$\Delta$ Liquidity spread (bps)	317	324	330
$\Delta$ Maturity (years)	-2.56	-2.26	-2.03
$\Delta$ Output (%)	-26	-23	-20

*Note: Model is re-calibrated for each value of  $\sigma$  to match the target moments.*

## 6.4 Segmented markets

In the benchmark model assets of different maturities are traded in a single secondary market. A potential concern could be that assets of short maturities, with small gains from trade, preclude the entry of buyers to the secondary market. This section considers secondary markets segmented by the time-to-maturity of assets. The main takeaway is that even though the market tightness for short-term bonds increases, the tightness for long-term assets remains similar to the case with only one market. Hence, the secondary market in the benchmark model is effectively a market for long-term assets. The intuition for this result is that in equilibrium the single market is not dominated by short-term assets, so there are always sufficient gains from trade.

Intuitively, in long-term markets, there are more gains from trade and therefore more entry of buyers. However, because there is Nash Bargaining over the gains from trade and buyers keeps only a fraction  $(1 - \gamma)$  of the gains, the increase in the entry of buyers in long-term markets is not enough to compensate the increase in the importance of the secondary markets for longer securities. As a result, the yield curve increases in maturity even with segmented markets. We describe the key features of the model in the main text and relegate to Appendix C.2 the full characterization of this extension.

Lets  $\tau$  be the initial maturity and consider the case in which secondary markets are segmented in  $N$  markets. Let  $0 = \tau_1 < \dots < \tau_{N+1} = \tau$  so each market  $j = 1, \dots, N$  trade assets with time-to-maturity  $t \in [\tau_j, \tau_{j+1}]$ .

**Matching and distribution of agents** Let  $\mu^j(y) = [\mu^{H,j}(y), \mu^{L,j}(y)]$  be the measure of high- and low-valuation agents holding an asset with time-to-maturity  $t$  in market  $j$ . Start by market  $N$  and solve for the distribution of agents backwards. The boundary condition is  $\mu^N(\tau) = [\mu^0, 0]$ . Next, iterate towards markets of shorter maturities with boundary conditions  $\mu^j(\tau_{j+1}) = \mu^{j+1}(\tau_{j+1})$  for  $j = 1, \dots, N - 1$ . Lemma 4 characterize the distribution of agents in each market.

**Lemma 4.** *The measure of agents for markets  $j = 1, \dots, N$  is given by the following backward*

recursion

$$\begin{bmatrix} \mu^{H,N+1}(\tau) \\ \mu^{L,N+1}(\tau) \end{bmatrix} = \begin{bmatrix} \mu^0 \\ 0 \end{bmatrix}$$

and

$$\begin{aligned} \mu^{H,j}(y) &= \frac{\eta}{\eta + \lambda^j} \left[ \frac{\lambda^j}{\eta} e^{\delta(y-\tau_{j+1})} (\mu^{H,j+1}(\tau_{j+1}) + \mu^{L,j+1}(\tau_{j+1})) \right. \\ &\quad \left. - e^{(\eta+\lambda^j+\delta)(y-\tau_{j+1})} (-\mu^{H,j+1}(\tau_{j+1}) + \lambda^j \mu^{L,j+1}(\tau_{j+1})) \right] \\ \mu^{L,j}(t) &= \frac{\eta}{\eta + \lambda^j} \left[ e^{\delta(y-\tau_{j+1})} (\mu^{H,j+1}(\tau_{j+1}) + \mu^{L,j+1}(\tau_{j+1})) \right. \\ &\quad \left. + e^{(\eta+\lambda^j+\delta)(y-\tau_{j+1})} (-\mu^{H,j+1}(\tau_{j+1}) + \lambda^j \mu^{L,j+1}(\tau_{j+1})) \right]. \end{aligned}$$

where  $\lambda^j$  is the selling intensity in market  $j = 1, \dots, N$ .

**Valuations** Let  $D^j(y) = [D^{H,j}(y), D^{L,j}(y)]$  be the value for high- and low-valuation agents of holding an asset with time-to-maturity  $y$  in market  $j = 1, \dots, N$ . To solve for the value of holding the asset start with the first market in which the boundary condition is that at maturity the value is equal to one, and then iterate towards longer-term markets. The boundary condition for market  $j = 1$  is  $D^1(\tau_1) = [1, 1]$ . Value matching for market  $j = 2, \dots, N$  implies  $D^j(\tau_j) = D^{j-1}(\tau_j)$  and the Hamilton-Jacobi-Bellman equations are the same as in the benchmark model, Equations (9) and (10).

**Free entry** Free entry in each market implies that

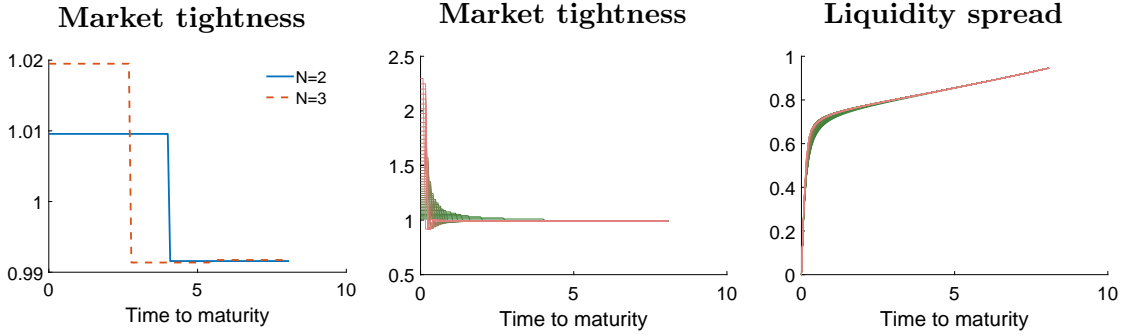
$$c = (1 - \gamma) \int_{\tau_j}^{\tau_{j+1}} \beta^j(y) (D^{H,j}(y) - D^{L,j}(y)) dy$$

where  $\beta^j$  is the intensity at which a buyer finds a seller in market  $j = 1, \dots, N$ . Appendix C.2 provides analytical solutions for the value functions and the free entry condition.

**Results** The first panel of Figure 9 shows the market tightness relative to the case of only one market when  $N = 2$  and  $N = 3$ . With segmentation, markets for short-term assets are tighter (more sellers to buyers) as there are fewer gains from trade. However, for long-term bonds we find a tightness similar to the case of no segmentation. The second panel repeats the exercise under different degrees of segmentation ( $N = 1, \dots, 50$ ). Note that even with 50 different markets, the tightness for markets with maturity above four years is almost identical to the case of no segmentation.

The third panel of Figure 9 shows the effects on the liquidity spread for different models

Figure 9: Segmented markets.



Note: The first (second) figure shows the market tightness relative to no segmentation for  $N = 2, 3$  ( $N = 2, \dots, 50$ ). The third figure shows the liquidity spread for  $N = 1, \dots, 50$ .

with  $N = 1$  to  $N = 50$ . As the market tightness after four years is identical in all these models, the implied liquidity spread is also the same. For short-term assets (maturities up to 4 years), there are some differences in the market tightness. However, they generate small variations in the yield curve. Therefore, we conclude that the secondary market in the benchmark model with  $N = 1$  is effectively a market for long-term assets.

## 7 Conclusion

This paper studies the linkages between the maturity of corporate debt, the liquidity of financial markets and the real economy. Long-term finance is particularly more expensive in economies with severe trading frictions which induce firms to invest at shorter horizons. A calibration of the model suggest that even though it is a stylized and tractable model, the theory reconciles data on maturities, credit spreads, and the real economy. Finally, an intervention like GSIs can improve the liquidity of financial markets, reduce long-term financial cost, and induce firms to borrow and invest at longer horizons. Several extensions suggest that the results of the paper do not hinge on particular modeling assumptions.

Through the paper firms have access to corporate bonds since these assets are already well studied in the literature about trading frictions (e.g., [He and Milbradt, 2014](#)). However, similar frictions also affect other sources of finance such as bank loans, venture capital or private equity funds.<sup>25</sup> The secondary markets for these assets are probably more frictional than for corporate debt, which indicates that aggregate effects can be potentially larger.

<sup>25</sup>There exists a secondary market for bank loans in the US (see [Altman et al., 2010](#); [Drucker and Puri, 2008](#)). However, the maturity of bank loans tend to be shorter than the maturity of corporate bonds ([Cortina Lorente et al., 2016](#)) and financial systems become market-based during the process of economic development ([Demirgüç-Kunt et al., 2013](#)). Moreover, banks need to raise capital to extend long-term loans. Hence, the frictions analyzed in this paper also affect bank's borrowing rates which are likely to affect the loans' rates.

The framework and results developed in this paper transcend the particular application to corporate bonds and can be used to study other markets for long-term finance such as household borrowing for real estate (mortgages) or education (student debt). Interestingly, [Hicks \(1969\)](#) argues that the products manufactured during the first decades of the Industrial Revolution had been invented much earlier. Rather, the critical innovation that ignited growth in England in the 18th century was capital market liquidity such that savers can easily sell their assets if they need their savings while at the same time the capital is committed for longer periods for investment (see [Bencivenga et al., 1995](#)).

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# Appendix

<b>A Proofs</b>	<b>2</b>
A.1 Lenders . . . . .	2
A.2 Borrowers . . . . .	8
A.3 Existence of equilibrium . . . . .	9
<b>B Alternative demands for long-term finance</b>	<b>10</b>
B.1 Quality ladder model . . . . .	10
B.2 Time-to-build capital . . . . .	11
<b>C Extensions</b>	<b>12</b>
C.1 Government-Sponsored Intermediaries . . . . .	12
C.2 Segmented markets . . . . .	12
<b>D Identification</b>	<b>14</b>
<b>E Cross country evidence and quantitative results</b>	<b>14</b>
E.1 Empirical evidence across countries: Maturity . . . . .	14
E.2 Calibration for the US . . . . .	16
E.3 Counterfactual trading frictions . . . . .	17
<b>F Data sources</b>	<b>19</b>

# A Proofs

This section provides the proofs of the main results of the paper.

## A.1 Lenders

### A.1.1 Distribution of financiers

*Proof of Lemma 1.* Let  $\mu(y) = [\mu^H(y), \mu^L(y)]$ . Matching implies  $\mu^B \beta \frac{\mu^L(y)}{\mu^S} = \lambda \mu^L(y)$ . Then, (5)-(6) imply that  $\dot{\mu}(y) = A\mu(y)$  with

$$A = \begin{bmatrix} \delta + \eta & -\lambda \\ -\eta & \delta + \lambda \end{bmatrix}$$

The boundary condition is  $\mu(\tau) = [\mu^0, 0]$ . Note that  $A$  has two real and distinct eigenvalues. Let  $R$  be the vector with the eigenvalues and  $V$  be the matrix with eigenvectors of  $A$ . Define  $B = (V)^{-1} \mu(\tau)$  so

$$V = \begin{bmatrix} -1 & \frac{\lambda}{\eta} \\ 1 & 1 \end{bmatrix} \quad R = \begin{bmatrix} \eta + \lambda + \delta \\ \delta \end{bmatrix} \quad B = \frac{\eta \mu^0}{\eta + \lambda} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

It is standard to show that

$$\mu^H(y) = \sum_{i=1}^2 e^{R_i(y-\tau)} B_i V(1, i) \quad \mu^L(y) = \sum_{i=1}^2 e^{R_i(y-\tau)} B_i V(2, i)$$

Finally, a few lines of algebra deliver

$$\begin{aligned} \mu^H(y) &= \frac{\mu^0 \eta}{\eta + \lambda} \left( e^{\delta(y-\tau)} \frac{\lambda}{\eta} + e^{(\eta+\lambda+\delta)(y-\tau)} \right) \\ \mu^L(y) &= \frac{\mu^0 \eta}{\eta + \lambda} \left( e^{\delta(y-\tau)} - e^{(\eta+\lambda+\delta)(y-\tau)} \right) \end{aligned}$$

□

### A.1.2 Value functions

*Proof of Proposition 1.* Replace the price of the asset in the secondary market  $P^S(y; \lambda)$  in (9)-(10) so

$$\begin{aligned} (\rho + \delta) D^H(y; \lambda) &= -\frac{\partial D^H(y; \lambda)}{\partial y} + \eta (D^L(y; \lambda) - D^H(y; \lambda)) \\ (\rho + \delta) D^L(y; \lambda) &= -h - \frac{\partial D^L(y; \lambda)}{\partial y} + \lambda \gamma (D^H(y; \lambda) - D^L(y; \lambda)) \end{aligned}$$

Let  $H(y; \lambda) = D^H(y; \lambda) - D^L(y; \lambda)$  then

$$(\rho + \delta + \eta + \lambda\gamma) H(y; \lambda) = h - \frac{\partial H(y; \lambda)}{\partial y}$$

with  $H(0; \lambda) = 0$ . It is straight forward to see that  $H(y; \lambda) = h \frac{1 - e^{-c_1 y}}{c_1}$  where  $c_1 = \rho + \delta + \eta + \lambda\gamma$ .

Next, solve for  $D^H(y; \lambda)$  as

$$(\rho + \delta) D^H(y; \lambda) = - \frac{\partial D^H(y; \lambda)}{\partial y} - \eta h \frac{1 - e^{-c_1 y}}{c_1}$$

with boundary  $D^H(0; \lambda) = 1$ . The solution is  $D^H(y; \lambda) = A + B e^{-(\rho + \delta)y} + C e^{-c_1 y}$  with constants

$$A = - \frac{1}{\rho + \delta} \frac{\eta h}{c_1} \quad C = - \frac{1}{\eta + \lambda\gamma} \frac{\eta h}{c_1} \quad B = 1 + \frac{\eta h}{(\eta + \lambda\gamma)(\rho + \delta)}$$

Finally, a few lines of algebra deliver

$$\begin{aligned} D^H(y; \lambda) &= e^{-(\rho + \delta)y} - \mathcal{L}(y, \lambda) \\ \mathcal{L}(y, \lambda) &= \frac{\eta h}{\eta + \lambda\gamma} \left( \frac{1 - e^{-(\rho + \delta)y}}{\rho + \delta} - \frac{1 - e^{-(\rho + \delta + \eta + \lambda\gamma)y}}{\rho + \delta + \eta + \lambda\gamma} \right) \\ \mathcal{L}(\tau, \lambda) &= h \frac{\eta}{\eta + \lambda\gamma} \int_0^\tau e^{-(\rho + \delta)y} \left( 1 - e^{-(\eta + \lambda\gamma)y} \right) dy \end{aligned}$$

The value of a low-valuation agent is  $D^L(y; \lambda) = D^H(y; \lambda) - H(y; \lambda)$ , that is

$$D^L(y, \lambda) = e^{-(\rho + \delta)y} - h \frac{1 - e^{-(\rho + \delta)y}}{\rho + \delta} + \frac{\lambda\gamma}{\eta} \mathcal{L}(y, \lambda).$$

### Properties of the illiquidity cost:

1. **Positive:**  $\mathcal{L}(\tau, \lambda)$  is positive as  $\rho + \delta + \eta + \lambda\gamma \geq \rho + \delta$ .

2. **Sensitivity with respect to maturity  $\tau$ :**

(a) Increasing in  $\tau$ :

$$\frac{\partial \mathcal{L}(\tau, \lambda)}{\partial \tau} = h \frac{\eta}{\eta + \lambda\gamma} e^{-(\rho + \delta)\tau} \left( 1 - e^{-(\eta + \lambda\gamma)\tau} \right) \geq 0$$

(b) Limit:

$$\begin{aligned} \lim_{\tau \rightarrow \infty} \mathcal{L}(\tau, \lambda) &= h \frac{\eta}{\eta + \lambda\gamma} \left( \frac{1}{\rho + \delta} - \frac{1}{\rho + \delta + \eta + \lambda\gamma} \right) \\ &= h \frac{\eta}{(\rho + \delta)(\rho + \delta + \eta + \lambda\gamma)} \end{aligned}$$

### 3. Sensitivity with respect to liquidity shocks $\eta$ :

- (a) If there are no liquidity shocks ( $\eta = 0$ ), then  $\mathcal{L}(\tau, \lambda) = 0$ .  
(b) If  $\eta \rightarrow \infty$  (i.e., always has to pay the cost  $h$ ) then

$$\lim_{\eta \rightarrow \infty} \mathcal{L}(\tau, \lambda) = h \frac{1 - e^{-(\rho+\delta)\tau}}{\rho + \delta}$$

### 4. Sensitivity with respect to liquidity of the secondary market $\lambda$ :

- (a) It is decreasing in  $\lambda$ . Note that the illiquidity cost is

$$\begin{aligned} \mathcal{L}(\tau, \lambda) &= \eta h \left( \frac{1}{(\rho + \delta)(\rho + \delta + \eta + \lambda\gamma)} - \frac{e^{-(\rho+\delta)\tau}}{(\eta + \lambda\gamma)(\rho + \delta)} \right) \\ &\quad + \eta h \frac{e^{-(\rho+\delta+\eta+\lambda\gamma)\tau}}{(\eta + \lambda\gamma)(\rho + \delta + \eta + \lambda\gamma)} \end{aligned}$$

so

$$\begin{aligned} \frac{\partial \mathcal{L}(\tau, \lambda)}{\partial \lambda} &= \eta h \left( -\frac{1}{(\rho + \delta)(\rho + \delta + \eta + \lambda\gamma)^2} + \frac{e^{-(\rho+\delta)\tau}}{(\rho + \delta)(\eta + \lambda\gamma)^2} \right) \\ &\quad - \eta h \left( \frac{\tau e^{-(\rho+\delta+\eta+\lambda\gamma)\tau}}{(\eta + \lambda\gamma)(\rho + \delta + \eta + \lambda\gamma)} + \frac{e^{-(\rho+\delta+\eta+\lambda\gamma)\tau}}{(\eta + \lambda\gamma)^2(\rho + \delta + \eta + \lambda\gamma)} \right) \\ &\quad - \eta h \frac{e^{-(\rho+\delta+\eta+\lambda\gamma)\tau}}{(\eta + \lambda\gamma)(\rho + \lambda\gamma + \eta + \lambda\gamma)^2} \end{aligned}$$

Let  $a = \eta + \lambda\gamma$  and  $b = \rho + \delta$  so

$$\frac{\partial \mathcal{L}(\tau, \lambda)}{\partial \lambda} = \eta h \left( -\frac{1}{b(a+b)^2} + \frac{e^{-b\tau}}{ba^2} \right) - \eta h \frac{e^{-(a+b)\tau}}{a(a+b)} \left( \tau + \frac{2a+b}{a(a+b)} \right)$$

We want to show that

$$\frac{e^{-b\tau}}{ba^2} \leq \frac{1}{b(a+b)^2} + \frac{e^{-(a+b)\tau}}{a(a+b)} \left( \tau + \frac{2a+b}{a(a+b)} \right) \quad (24)$$

Define  $L(\tau)$  and  $R(\tau)$  to the left- and right-hand-side of (24), respectively. Note that  $R(0) = L(0) = \frac{1}{ba^2}$ . Hence, it is sufficient to show that the slope of  $L(\tau)$  is lower than the slope of  $R(\tau)$  for all  $\tau$ . Note that

$$\frac{\partial L(\tau)}{\partial \tau} = -\frac{e^{-b\tau}}{a^2} \quad \frac{\partial R(\tau)}{\partial \tau} = -\frac{e^{-(a+b)\tau}}{a} \left( \tau + \frac{1}{a} \right)$$

Hence, the slope of  $L$  is lower than the slope of  $R$  because  $a\tau \geq \log(a\tau + 1)$ .

- (b) If there are no secondary markets, i.e.,  $\lambda = 0$ , then the illiquidity cost represents the *expected holding costs, i.e.*,

$$\mathcal{L}(\tau, 0) = h \int_0^\tau e^{-(\rho+\lambda^D)y} (1 - e^{-\eta y}) dy$$

- (c) If secondary markets are totally liquid (i.e.,  $\lambda \rightarrow \infty$ ) then  $\mathcal{L}(\tau, \lambda) = 0$ .

5. **Liquidity is more important for long-term assets:** Recall that

$$\begin{aligned} \frac{\partial \mathcal{L}(\tau, \lambda)}{\partial \tau} &= h \frac{\eta}{\eta + \lambda \gamma} e^{-(\rho+\delta)\tau} \left(1 - e^{-(\eta+\lambda\gamma)\tau}\right) \\ \frac{\partial \mathcal{L}(\tau, \lambda)}{\partial \tau} &= \eta h e^{-(\rho+\delta)\tau} \int_0^\tau e^{-(\eta+\lambda\gamma)y} dy \end{aligned}$$

therefore

$$\frac{\partial \mathcal{L}(\tau, \lambda)}{\partial \tau \lambda} = -\eta h e^{-(\rho+\delta)\tau} \int_0^\tau y e^{-(\eta+\lambda\gamma)y} dy \leq 0$$

□

### A.1.3 Liquidity spread

*Proof of Lemma 2.* We show that:

1. **The liquidity spread  $cs^{liq}(\tau, \lambda)$  is increasing in maturity  $\tau$ :**

$$\frac{\partial cs^{liq}(t, \lambda)}{\partial t} = \frac{1}{t^2} \log \left(1 - e^{(\rho+\delta)t} \mathcal{L}(t, \lambda)\right) + \frac{e^{(\rho+\delta)t} (\rho + \delta) \mathcal{L}(t, \lambda) + \frac{\partial \mathcal{L}(t, \lambda)}{\partial t}}{1 - e^{(\rho+\delta)t} \mathcal{L}(t, \lambda)}$$

Recall that  $\log(x) \geq \frac{x-1}{x}$ . Hence

$$\log \left(1 - e^{(\rho+\delta)t} \mathcal{L}(t, \lambda)\right) \geq \frac{-e^{(\rho+\delta)t} \mathcal{L}(t, \lambda)}{1 - e^{(\rho+\delta)t} \mathcal{L}(t, \lambda)}$$

Which implies that

$$\frac{\partial cs^{liq}(t, \lambda)}{\partial t} \geq \frac{1}{t^2} \frac{e^{(\rho+\delta)t} \mathcal{L}(t, \lambda)}{1 - e^{(\rho+\delta)t} \mathcal{L}(t, \lambda)} \left( t(\rho + \delta) + \frac{\partial \mathcal{L}(t, \lambda)}{\partial t} \frac{t}{\mathcal{L}(t, \lambda)} - 1 \right)$$

Let  $\varepsilon_{\mathcal{L}, t} = \frac{\partial \mathcal{L}(t, \lambda)}{\partial t} \frac{t}{\mathcal{L}(t, \lambda)}$  and note that

$$\varepsilon_{\mathcal{L}, t} = t \left[ e^{-(\rho+\delta)\tau} - e^{-(\rho+\delta+\eta+\lambda\gamma)t} \right] \left[ \frac{1 - e^{-(\rho+\delta)t}}{\rho + \delta} - \frac{1 - e^{-(\rho+\delta+\eta+\lambda\gamma)t}}{\rho + \delta + \eta + \lambda\gamma} \right]^{-1}$$

A sufficient condition is  $t(\rho + \delta) + \varepsilon_{\mathcal{L},t} - 1 \geq 0$ . Let  $a = \rho + \delta$  and  $b = \eta + \lambda\gamma$  and define

$$E(t, a, b) = t \left( a + \left[ e^{-at} - e^{-(a+b)t} \right] \left[ \frac{1 - e^{-at}}{a} - \frac{1 - e^{-(a+b)t}}{a+b} \right]^{-1} \right) - 1$$

It is easy to show numerically that  $E(t, a, b) \geq 0$  for all  $t, a, b \geq 0$ . Hence, the liquidity spread is increasing in maturity. Finally, it is straight forward to see that the liquidity spread is decreasing in liquidity  $\lambda$ .

**2. The liquidity spread is increasing in the default intensity  $\delta$ :** Note that

$$\begin{aligned} e^{(\rho+\delta)\tau} \mathcal{L}(\tau, \lambda) &= \frac{\eta}{\eta + \lambda\gamma} \int_0^\tau e^{(\rho+\delta)(\tau-t)} \left( 1 - e^{-(\eta+\lambda\gamma)t} \right) dt \\ \frac{\partial (e^{(\rho+\delta)\tau} \mathcal{L}(\tau, \lambda))}{\partial \delta} &= \frac{\eta}{\eta + \lambda\gamma} \int_0^\tau (\tau - t) e^{(\rho+\delta)(\tau-t)} \left( 1 - e^{-(\eta+\lambda\gamma)t} \right) dt > 0 \end{aligned}$$

□

*Proof of Lemma 3.* The mid-price is

$$\frac{1}{2} (D^H(y; \lambda) + D^L(y; \lambda)) = e^{-(\rho+\delta)y} - \frac{1}{2} \left( h \frac{1 - e^{-(\rho+\delta)y}}{\rho + \delta} + \left( \frac{\eta - \lambda\gamma}{\eta} \right) \mathcal{L}(y, \lambda) \right)$$

where

$$\left( \frac{\eta - \lambda\gamma}{\eta} \right) \mathcal{L}(y, \lambda) = h \frac{\eta - \lambda\gamma}{\lambda^H + \lambda\gamma} \left( \frac{1 - e^{-(\rho+\delta)y}}{\rho + \delta} - \frac{1 - e^{-(\rho+\delta+\lambda^H+\lambda\gamma)y}}{\rho + \delta + \eta + \lambda\gamma} \right)$$

The mid-price is

$$e^{-(\rho+\delta)y} - \frac{h}{\eta + \lambda\gamma} \left( \eta \frac{1 - e^{-(\rho+\delta)y}}{\rho + \delta} - \frac{(\eta - \lambda\gamma)}{2} \frac{1 - e^{-(\rho+\delta+\eta+\lambda\gamma)y}}{\rho + \delta + \eta + \lambda\gamma} \right)$$

Define the gains from trade as

$$GT(y) = h \frac{1 - e^{-(\rho+\delta+\lambda^H+\lambda\gamma)y}}{\rho + \delta + \eta + \lambda\gamma}$$

so

$$BA(y) = GT(y) \left[ e^{-(\rho+\lambda^D)y} - \frac{1}{\eta + \lambda\gamma} \left( h\eta \frac{1 - e^{-(\rho+\delta)y}}{\rho + \delta} - \frac{(\eta - \lambda\gamma)}{2} GT(y) \right) \right]^{-1}$$

$$BA(y) = \left[ \frac{e^{-(\rho+\delta)y}}{GT(y)} - h \frac{\eta}{\eta + \lambda\gamma} \frac{1 - e^{-(\rho+\delta)y}}{\rho + \delta} + \frac{1}{2} \frac{\eta - \lambda\gamma}{\eta + \lambda\gamma} \right]^{-1}$$

Note that  $\frac{e^{-(\rho+\delta)y}}{GT(y)}$  is decreasing in  $y$  as  $e^{-(\rho+\delta)y}$  is decreasing and  $GT(y)$  is increasing in  $y$ . Note that  $\frac{1-e^{-(\rho+\delta)y}}{\frac{\rho+\delta}{GT(y)}}$  is increasing in  $y$  as the discount in  $GT$  is larger than in the numerator. Hence, with the negative sign it is decreasing. Therefore, all the square bracket is decreasing in  $y$ , and as it is to the power of  $-1$ , the  $BA(y)$  is increasing in  $y$ .  $\square$

#### A.1.4 Free entry

*Proof of Proposition 2.* Gains from trade are

$$D^H(y; \lambda) - D^L(y; \lambda) = h \frac{1 - e^{-c_1 y}}{c_1} \quad c_1 = \rho + \delta + \eta + \lambda \gamma$$

The buyer gets  $(1 - \gamma)$  of the gains from trade. Hence, the free entry condition reads

$$c = (1 - \gamma) \int_0^\tau \beta \frac{\mu^L(y)}{\mu^S} h \frac{1 - e^{-c_1 y}}{c_1} dy$$

And  $\theta = \frac{\mu^S}{\mu^B}$ . Also, recall that  $\mu^S = \int_0^\tau \mu^L(y) dy$ . Hence, the free entry condition is

$$\begin{aligned} c &= \frac{(1 - \gamma) h}{c_1} A \theta^\alpha \int_0^\tau \frac{\mu^L(y)}{\mu^S} (1 - e^{-c_1 t}) dy \\ c &= \frac{(1 - \gamma) h}{c_1} A \theta^\alpha \left( 1 - \frac{\int_0^\tau e^{-c_1 y} \mu^L(y) dy}{\int_0^\tau \mu^L(y) dy} \right) \end{aligned}$$

Define  $c_2 = \eta + \delta + \lambda$  and note that

$$\int_0^\tau e^{-c_1 t} \mu^L(y) dy = \mu^0 \frac{\eta}{\eta + \lambda} \left( \frac{e^{-c_1 \tau} - e^{-\delta \tau}}{\delta - c_1} - \frac{e^{-c_1 \tau} - e^{-c_2 \tau}}{c_2 - c_1} \right)$$

As a result, the ratio of integrals in the free entry condition reads

$$\left( \frac{e^{-c_1 \tau} - e^{-\lambda \tau}}{\delta - c_1} - \frac{e^{-c_1 \tau} - e^{-c_2 \tau}}{c_2 - c_1} \right) \left( \frac{1 - e^{-\tau \delta}}{\delta} - \frac{1 - e^{-(\eta + \delta + \lambda) \tau}}{\eta + \delta + \lambda} \right)^{-1} \quad (25)$$

And the free-entry condition boils down to

$$c = \frac{(1 - \gamma) h}{c_1} A \theta^\alpha \left( 1 - \left( \frac{e^{-c_1 \tau} - e^{-\delta \tau}}{\delta - c_1} - \frac{e^{-c_1 \tau} - e^{-c_2 \tau}}{c_2 - c_1} \right) \left( \frac{1 - e^{-\tau \delta}}{\delta} - \frac{1 - e^{-c_2 \tau}}{c_2} \right)^{-1} \right)$$

First, note that it is easy to show that Equation (25) is increasing in  $\tau$ . Next, consider  $\tau = 0$ .



Note that the ratio of integrals in the free entry condition is equal to one as

$$\begin{aligned}
& \lim_{\tau \rightarrow 0} \left( \frac{e^{-c_1\tau} - e^{-\delta\tau}}{\lambda^D - c_1} - \frac{e^{-c_1\tau} - e^{-c_2\tau}}{c_2 - c_1} \right) \left( \frac{1 - e^{-\delta\tau}}{\delta} - \frac{1 - e^{-c_2\tau}}{c_2} \right)^{-1} \\
&= \lim_{\tau \rightarrow 0} \left( \frac{-c_1 e^{-c_1\tau} + \delta e^{-\delta\tau}}{\delta - c_1} - \frac{-c_1 e^{-c_1\tau} + c_2 e^{-c_2\tau}}{c_2 - c_1} \right) \left( \frac{\delta e^{-\tau\delta}}{\delta} - \frac{c_2 e^{-c_2\tau}}{c_2} \right)^{-1} \\
&= \lim_{\tau \rightarrow 0} \left( \frac{(c_1)^2 e^{-c_1\tau} - (\delta)^2 e^{-\delta\tau}}{\delta - c_1} - \frac{(c_1)^2 e^{-c_1\tau} - (c_2)^2 e^{-c_2\tau}}{c_2 - c_1} \right) \left( \frac{-(\delta)^2 e^{-\tau\delta}}{\delta} - \frac{-(c_2)^2 e^{-c_2\tau}}{c_2} \right)^{-1} \\
&= \left( \frac{(c_1)^2 - (\delta)^2}{\delta - c_1} - \frac{(c_1)^2 - (c_2)^2}{c_2 - c_1} \right) \left( \frac{-(\delta)^2}{\lambda^D} - \frac{-(c_2)^2}{c_2} \right)^{-1} \\
&= \left( \frac{(c_1 + \delta)(c_1 - \lambda^D)}{\delta - c_1} - \frac{(c_1 + c_2)(c_1 - c_2)}{c_2 - c_1} \right) (c_2 - \delta)^{-1} \\
&= (-(c_1 + \delta) + (c_1 + c_2))(c_2 - \delta)^{-1} = (c_2 - \lambda^D)(c_2 - \delta)^{-1} = 1
\end{aligned}$$

where we applied L'Hopital's rule in the second and third line. As a result, the free-entry condition is satisfied if and only if  $\lim_{\tau \rightarrow 0} \theta = \infty$ . Hence,  $\lim_{\tau \rightarrow 0} \lambda = 0$ . That is,  $\lambda(0) = 0$ .

Next, consider the case of  $\tau \rightarrow \infty$ . The ratio of integrals in the free entry condition is equal to zero. Hence  $c = \frac{h}{c_1} (1 - \gamma) A \theta^\alpha$ . Recall that  $c_1 = \rho + \delta + \eta + \lambda\gamma$  and  $\lambda = A \theta^{\alpha-1}$ . Hence,

$$\rho + \delta + \eta + \gamma A \theta^{\alpha-1} = \frac{h(1-\gamma)}{c} A \theta^\alpha$$

As  $\alpha \in (0, 1)$  the left hand side is decreasing in  $\theta$  and the right hand side is increasing in  $\theta$ . As a result, there exists a unique  $\theta \in \mathbb{R}_+$ . That is,  $\lim_{\tau \rightarrow \infty} \lambda(\tau) = \bar{\lambda} \in \mathbb{R}_+$ .  $\square$

## A.2 Borrowers

*Proof of Proposition 3.* Let  $J(\tau, \lambda)$  be the value of the firm with maturity  $\tau$  and liquidity  $\lambda$  and let  $A = \frac{\zeta}{\rho + \delta}$ . The first order condition is

$$\begin{aligned}
J_\tau(\tau, \lambda) &= e^{-(\rho + \delta)\tau} A (1 - (\rho + \delta)\tau) \\
&\quad - e^{cs^{liq}(\lambda, \tau)\tau} \left[ \frac{\partial I(\tau)}{\partial \tau} + (\Phi + I(\tau)) cs^{liq}(\lambda, \tau) (1 + \varepsilon_{cs^{liq}, \tau}(\lambda, \tau)) \right]
\end{aligned}$$

Note that  $\tau$  is increasing in  $\lambda$  if the derivative of the first order condition with respect to  $\lambda$  is positive

$$\begin{aligned} J_{\tau\lambda}(\tau, \lambda) &= -e^{cs^{liq}(\lambda, \tau)\tau} \frac{\partial cs^{liq}(\lambda, \tau)}{\partial \lambda} \tau \left[ \frac{\partial I(\tau)}{\partial \tau} + (\Phi + I(\tau)) cs^{liq}(\lambda, \tau) (1 + \varepsilon_{cs^{liq}, \tau}(\lambda, \tau)) \right] \\ &\quad - e^{cs^{liq}(\lambda, \tau)\tau} \frac{\partial cs^{liq}(\lambda, \tau)}{\partial \lambda} (\Phi + I(\tau)) (1 + \varepsilon_{cs^{liq}, \tau}(\lambda, \tau)) \\ &\quad - e^{cs^{liq}(\lambda, \tau)\tau} (\Phi + I(\tau)) cs^{liq}(\lambda, \tau) \frac{\partial \varepsilon_{cs^{liq}, \tau}(\lambda, \tau)}{\partial \lambda} \end{aligned}$$

Recall that  $\frac{\partial cs^{liq}(\lambda, \tau)}{\partial \lambda} \leq 0$ . so the first and second terms are positive. However, the last term involves  $\frac{\partial \varepsilon_{cs^{liq}, \tau}(\lambda, \tau)}{\partial \lambda}$  for which we do not know the sign. We can write  $J_{\tau\lambda}(\tau, \lambda)$  as

$$\begin{aligned} J_{\tau\lambda}(\tau, \lambda) &= -e^{cs^{liq}(\lambda, \tau)\tau} \frac{\partial cs^{liq}(\lambda, \tau)}{\partial \lambda} \frac{\partial I(\tau)}{\partial \tau} \tau - e^{cs^{liq}(\lambda, \tau)\tau} (\Phi + I(\tau)) \\ &\quad \left[ \frac{\partial cs^{liq}(\lambda, \tau)}{\partial \lambda} (1 + \varepsilon_{cs^{liq}, \tau}(\lambda, \tau)) (\tau cs^{liq}(\lambda, \tau) + 1) + cs^{liq}(\lambda, \tau) \frac{\partial \varepsilon_{cs^{liq}, \tau}(\lambda, \tau)}{\partial \lambda} \right] \end{aligned}$$

The first term is positive. A sufficient condition for  $J_{\tau\lambda}(\tau, \lambda) \geq 0$  is that the second term is also positive. This implies

$$\frac{\partial cs^{liq}(\lambda, \tau)}{\partial \lambda} (1 + \varepsilon_{cs^{liq}, \tau}(\lambda, \tau)) (\tau cs^{liq}(\lambda, \tau) + 1) \leq -cs^{liq}(\lambda, \tau) \frac{\partial \varepsilon_{cs^{liq}, \tau}(\lambda, \tau)}{\partial \lambda}$$

This expression depends only on  $cs^{liq}(\lambda, \tau)$ . By Lemma 2 we can approximate the liquidity spread as a linear function increasing in  $\tau$  and decreasing in  $\lambda$ . Let  $cs^{liq}(\lambda, \tau) = c_\tau \tau + c_\lambda \lambda$  with  $c_\tau \geq 0$  and  $c_\lambda \leq 0$ . Then  $\varepsilon_{cs^{liq}, \tau}(\lambda, \tau) = \frac{c_\tau \tau}{c_\tau \tau + c_\lambda \lambda}$  and  $\frac{\partial \varepsilon_{cs^{liq}, \tau}(\lambda, \tau)}{\partial \lambda} = -\frac{c_\tau \tau c_\lambda}{(c_\tau \tau + c_\lambda \lambda)^2}$ . The sufficient condition reads

$$(c_\tau \tau + cs^{liq}(\lambda, \tau)) cs^{liq}(\lambda, \tau) \tau + cs^{liq}(\lambda, \tau) \geq 0$$

which is satisfied. Therefore,  $J_{\tau\lambda}(\tau, \lambda) \geq 0$  and  $\frac{\partial \tau(\lambda)}{\partial \lambda} \geq 0$ . Finally it is straightforward to see that  $\underline{\tau} = \tau(0) \leq \lim_{\lambda \rightarrow \infty} \tau(\lambda) = \tau^* < \infty$ .  $\square$

### A.3 Existence of equilibrium

*Proof of Proposition 4.* First, Proposition 2 define a schedule for the lenders  $\tau^L(\lambda)$ . Note that  $\tau^L(0) = 0$  and there exists  $\bar{\lambda}$  such that  $\tau^L(\bar{\lambda}) = \infty$ .

Second, Proposition 3 define  $\tau^B(\lambda)$  and notice that  $\tau^B(0) = \underline{\tau} > 0$  and  $\tau^B(\lambda) \geq 0$  for all  $\lambda$ .

Finally, define  $F(\lambda) = \tau^L(\lambda) - \tau^B(\lambda)$  and note that  $F(0) = -\underline{\tau} < 0$  and  $F(\bar{\lambda}) = \infty$ . Hence, as  $F$  is continuous, Bolzano's theorem implies that there exists  $\lambda^*$  such that  $F(\lambda^*) = 0$  which defines the equilibrium.  $\square$

## B Alternative demands for long-term finance

In this Appendix we provide alternative microfoundations for the demand of long-term finance.

### B.1 Quality ladder model

We follow the standard quality-ladder model with a continuum of intermediate goods producers with monopolistic power and final goods producers in perfect competition (e.g., [Akcigit and Kerr, 2017](#)). The final good is produced with labor and a continuum of intermediate goods  $j \in [0, 1]$  with the production technology

$$Y = \frac{L^\beta}{1 - \beta} \int_0^1 q_j^\beta k_j^{1-\beta} dj$$

where  $L$  is the labor input,  $k_j$  is the quantity of intermediate good  $j$ , and  $q_j$  is its quality. Without loss of generality normalize the price of the final good  $Y$  to one in every period. The final good is produced competitively with input prices taken as given. The inverse demand of intermediate inputs reads  $p_j = L^\beta q_j^\beta k_j^{-\beta}$  and the labor demand is  $L = \frac{\beta Y}{w}$ .

Each intermediate good  $j$  is produced with a linear technology  $k_j = \bar{q} l_j$  where  $\bar{q} = \int_0^1 q_j dj$  is the average quality, and  $l_j$  is the labor input. Intermediate producers take the inverse demand and maximize static profits

$$\pi(q) = \max_{k_j \geq 0} p_j k_j - \frac{w}{\bar{q}} k_j \quad \text{s.t. } p_j = L^\beta q_j^\beta k_j^{-\beta}$$

There is an exogenous labor supply normalized to one. Labor market clearing implies  $L + \int_0^1 l_j = 1$ . It is easy to show that static profits are linear in quality and aggregate output is linear in the average quality of the economy

$$\pi(q) = \underline{\pi} q_j \quad Y = \underline{Y} \bar{q} \tag{26}$$

where the constants  $\underline{\pi}$  and  $\underline{Y}$  are

$$\underline{\pi} = \beta L \left( \frac{\bar{q}}{w} \right)^{\frac{1-\beta}{\beta}} (1 - \beta)^{\frac{1-\beta}{\beta}} \quad L = \frac{\beta}{(1 - \beta)^2 + \beta} \quad \underline{Y} = \frac{\beta^\beta (1 - \beta)^{(1-2\beta)}}{(1 - \beta)^2 + \beta}.$$

**Life cycle of intermediate goods producers** There are two important empirical facts which motivate assumptions about the back-loaded profile of investment projects. First, the *Arrow's replacement effect* establish that small and young firms are more innovative than large and old firms (e.g., [Arrow, 1962](#); [Itenberg, 2015](#); [Akcigit and Kerr, 2017](#)). Second, small firms are more financially constrained, and in particular for research and development ([Midrigan and Xu, 2014](#); [Itenberg, 2015](#)). Based on these facts, we assume that a newborn firm chooses a project maturity  $\tau$  such that she is *young* for  $t \leq \tau$  and *mature* otherwise.

A young firm invest in research and development to improve the quality of the product. The evolution of quality is given by  $\dot{q}(t) = \delta_1 + \lambda^Q \delta_2$ . The first component,  $\delta_1$ , captures a deterministic growth on quality. We assume  $\delta_1$  is large enough so at maturity the firm wants to repay the debt.<sup>26</sup> Second, as it is standard in the literature, we assume that quality makes jumps of size  $\delta_2$  that arrive at Poisson rate  $\lambda^Q$ . Doing research is costly. The firm pays  $\kappa$  per unit of time doing research. Moreover, the firm is financially constrained when young. She borrows to finance investment which generates a demand for long-term finance.

At age  $\tau$  the firm becomes mature, stop R&D and start production with quality  $q(\tau, N)$ , where  $N$  is a counting process with the number of jumps on quality before  $\tau$ . Static profits are linear in quality  $\pi(q) = \underline{\pi}q$ . Hence, the net present value of a mature firm with quality  $q$  is given by  $F(q) = \frac{\pi(q)}{\rho + \delta}$  where  $\delta$  is the Poisson arrival rate of an exogenous exit shock.

Hence, this model deliver a similar structure to the benchmark model where we can interpret the investments as improvements in the quality of the product rather than in firm's productivity.

## B.2 Time-to-build capital

Rioja and Valev (2004) finds that finance boosts growth in rich countries primarily by speeding-up productivity growth, while finance encourages growth in poorer countries primarily by accelerating capital accumulation. In this section we propose an alternative microfoundation for back-loaded projects based on time-to-build capital.

At every moment a new cohort of identical firms  $\mu^0$  enters the economy and choose a project to implement. There is a continuum of projects differentiated by the time-to-build  $\tau \geq 0$ . For  $t \in [0, \tau)$  the firm is *young* and is investing. For  $t \geq \tau$  the firm is *mature* and is producing.

A young firm starts with no capital,  $K(0) = 0$  and investment is subject to a time-to-build constraint as in Majd and Pindyck (1987) such that  $dK = idt$  and  $i \leq \kappa$ . The investment rate  $i$  per unit of time cannot exceed  $\kappa$ . Given the linearity, it is optimal to build at maximum capacity. Hence, a firm with a project of duration  $\tau$  concludes its investment stage with capital equal to  $K = \tau\kappa$ . This firm is subject to an exogenous exit shock that arrives at Poisson intensity  $\delta$  in which case the project is destroyed and the firm defaults.

A mature firm use the capital to produce and profits are given by  $\pi = zK^\sigma$ , where  $z$  is the productivity and  $\sigma$  are the returns to scale. This firm is subject to an exogenous exit shock that arrives at Poisson intensity  $\delta$ . The net present value for a mature firm with project  $\tau$  is

$$F(\tau) = \frac{zK^\sigma}{\rho + \delta}. \quad (27)$$

Note that the return on the project  $F$  is increasing in the time spent on investment. Hence, time-to-build is an alternative formulation for back-loaded projects.

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<sup>26</sup>If  $\delta_1 = 0$  some firms will prefer to default at maturity. This might be an interesting setup to study corporate default.

## C Extensions

This appendix describe the solution of the model with GSIs and segmented markets.

### C.1 Government-Sponsored Intermediaries

This appendix describe how to solve the distribution of financiers with GSIs. The Total assets with time-to-maturity  $t$  are  $\mu(t) = \mu^0 e^{-\delta t}$ . These assets are hold by 4 type of agents:  $\mu(t) = \mu^{p,h}(t) + \mu^{p,l}(t) + \mu^{g,h}(t) + \mu^{g,l}(t)$ . The law of motions for private sector are

$$\begin{aligned} -\dot{\mu}^{p,h}(t) &= -(\eta + \delta) \mu^{p,h}(t) + (\beta^{p-p}(t) + \beta^{g-p}(t)) \mu^{p,b} \\ -\dot{\mu}^{p,l}(t) &= \eta \mu^{p,h}(t) - (\delta + \lambda^{p-p} + \lambda^{p-g}) \mu^{p,l}(t) \end{aligned}$$

with boundary conditions  $\mu^{p,h}(\tau) = \mu^0$  and  $\mu^{p,l}(\tau) = 0$ . The law of motions for government agents are

$$\begin{aligned} -\dot{\mu}^{g,h}(t) &= -(\eta + \delta) \mu^{g,h}(t) + (\beta^{p-g}(t) + \beta^{g-g}(t)) \mu^{g,b} \\ -\dot{\mu}^{g,l}(t) &= \eta \mu^{g,h}(t) - (\delta + \lambda^{g-p} + \lambda^{g-g}) \mu^{g,l}(t) \end{aligned}$$

with boundary conditions  $\mu^{g,h}(\tau) = \mu^{g,l}(\tau) = 0$ . Matching implies

$$\begin{aligned} \mu^{p,b} \beta^{p-p}(t) &= \mu^{p,l}(t) \lambda^{p-p} \\ \mu^{p,b} \beta^{g-p}(t) &= \mu^{g,l}(t) \lambda^{g-p} \\ \mu^{g,b} \beta^{p-g}(t) &= \mu^{p,l}(t) \lambda^{p-g} \\ \mu^{g,b} \beta^{g,b-g}(t) &= \mu^{g,l}(t) \lambda^{g-g} \end{aligned}$$

Define  $\mu(t) = [\mu^{p,h}(t), \mu^{p,l}(t), \mu^{g,h}(t), \mu^{g,l}(t)]$ . The boundary condition is  $\mu(\tau) = [\mu^0, 0, 0, 0]$  and the system is  $\dot{\mu}(t) = A\mu(t)$  where

$$A = \begin{bmatrix} \eta + \delta & -\lambda^{p-p} & 0 & -\lambda^{g-p} \\ -\eta & \delta + \lambda^{p-p} + \lambda^{p-g} & 0 & 0 \\ 0 & -\lambda^{p-g} & \eta + \delta & -\lambda^{g-g} \\ 0 & 0 & -\eta & \delta + \lambda^{g-p} + \lambda^{g-g} \end{bmatrix}$$

The solution of this system is standard. The only caveat is that we should pay attention to the real and complex eigenvalues of the matrix  $A$ .

### C.2 Segmented markets

This appendix has the results for the model with segmented markets.

## Distributions of financiers

*Proof of Lemma 4.* Total assets are  $\mu^j(t) = e^{(t-\tau)\delta}\mu^0$ . The evolution for high- and low-valuation agents in market  $j$  are

$$\begin{aligned} -\dot{\mu}^{H,j}(t) &= -(\eta + \delta)\mu^{H,j}(t) + \mu^{L,j}\beta^j(t) \\ -\dot{\mu}^{L,j}(t) &= \eta\mu^{H,j}(t) - (\delta + \lambda^j)\mu^{L,j}(\tau_j) \end{aligned}$$

Matching implies that  $\mu^{B,j}\beta^j(t) = \mu^{L,j}(t)\lambda^j$ . Hence the system is

$$\dot{\mu}^j(t) = \begin{bmatrix} \eta + \delta & -\lambda^j \\ -\eta & \delta + \beta^j \end{bmatrix} \begin{bmatrix} \mu^{H,j}(t) \\ \mu^{L,j}(t) \end{bmatrix}$$

with eigenvalues  $\delta$  and  $\eta + \lambda^j + \delta$ . Define  $V^j$  to be the matrix with the eigenvectors and  $R^j$  the diagonal matrix with the eigenvalues and  $B^j = (V^j)^{-1}\mu^{j+1}(\tau_{j+1})$ . Then

$$\begin{aligned} \mu^{H,j}(t) &= \sum_{i=1}^2 e^{R^j(i)(t-\tau_{j+1})} B^j(i) V^j(1, i) \\ \mu^{L,j}(t) &= \sum_{i=1}^2 e^{R^j(i)(t-\tau_{j+1})} B^j(i) V^j(2, i) \end{aligned}$$

For  $j = 1, \dots, N-1$  we have that

$$\begin{aligned} \mu^{H,j}(t) &= \frac{\eta}{\eta + \lambda^j} \left[ \frac{\lambda^j}{\eta} e^{\delta(t-\tau_{j+1})} (\mu^{H,j+1}(\tau_{j+1}) + \mu^{L,j+1}(\tau_{j+1})) \right] \\ &\quad - \frac{\eta}{\eta + \lambda^j} \left[ -e^{(\eta+\lambda^j+\delta)(t-\tau_{j+1})} (-\mu^{H,j+1}(\tau_{j+1}) + \lambda^j \mu^{L,j+1}(\tau_{j+1})) \right] \\ \mu^{L,j}(t) &= \frac{\eta}{\eta + \lambda^j} \left[ e^{\delta(t-\tau_{j+1})} (\mu^{H,j+1}(\tau_{j+1}) + \mu^{L,j+1}(\tau_{j+1})) \right] \\ &\quad + \frac{\eta}{\eta + \lambda^j} \left[ +e^{(\eta+\lambda^j+\delta)(t-\tau_{j+1})} (-\mu^{H,j+1}(\tau_{j+1}) + \lambda^j \mu^{L,j+1}(\tau_{j+1})) \right] \end{aligned}$$

□

**Value functions** Let  $Z^j(t) = D^{H,j}(t) - D^{L,j}(t)$  and  $c_j = \rho + \delta + \eta + \gamma\lambda^j$ , then  $c_j Z^j(t) = h - \dot{Z}^j(t)$ . The solution is  $Z^j(t) = A^{Z,j} e^{-c_j t} + B^{Z,j}$  with  $B^{Z,j} = \frac{h}{c_j}$  and the boundary condition pin downs  $A^{Z,j}$ .

For  $j = 1$  the boundary condition is  $D^{H,1}(\tau_1) = D^{L,1}(\tau_1) = 1$  so  $A^{Z,1} = -\frac{h}{c_1}$

For  $j = 2, \dots, N$  we have that  $Z^j(\tau_j) = Z^{j-1}(\tau_j)$  which implies  $A^{Z,j} = e^{c_j \tau_j} \left( A^{Z,j-1} e^{-c_{j-1} \tau_j} + \frac{h}{c_{j-1}} - \frac{h}{c_j} \right)$

Next, we can solve for the value of high and low valuation agents using  $Z^j$  and the initial conditions.

For high valuation agents

$$(\rho + \delta) D^{H,j}(t) = -\dot{D}^{H,j}(t) - \eta \left( A^{Z,j} e^{-c_j t} + \frac{h}{c_j} \right)$$

The solution is  $D^{H,j}(t) = A^{H,j} + B^{H,j}e^{-(\rho+\lambda^D)t} + C^{H,j}e^{-c_j t}$ , with  $A^{H,j} = -\frac{\eta h}{(\rho+\delta)c_j}$ ,  $C^{H,j} = \frac{\eta A^{Z,j}}{\eta+\gamma\lambda^j}$ , and the boundary condition pin down  $B^{H,j}$ .

For  $j = 1$  we have that  $D^{H,1}(\tau_1) = 1$  and  $\tau_1 = 0$  so  $B^{U,1} = 1 - A^{U,1} - C^{U,1}$ . For  $j = 2, \dots, N$  we have that  $D^{H,j}(\tau_j) = D^{H,j-1}(\tau_j)$  so

$$B^{H,j} = e^{(\rho+\delta)\tau_j} \left( A^{H,j-1} - A^{H,j} + B^{H,j-1}e^{-(\rho+\delta)\tau_j} + C^{H,j-1}e^{-c_{j-1}\tau_j} - C^{H,j}e^{-c_j\tau_j} \right)$$

which defines a recursion in  $B^{H,j}$ .

**Free entry** The free entry condition in each market is

$$c_j = (1 - \gamma) \int_{\tau_j}^{\tau_{j+1}} \beta^j(t) (D^{H,j}(t) - D^{L,j}(t)) dt$$

where  $\beta^j(t) = A(\theta^j)^\alpha \frac{\mu^{C,j}(t)}{\mu^{C,j}}$ , and both the measures and value functions are sum of exponential functions. Hence, it is easy to solve for the integrals on the free entry condition in each market.

## D Identification

Although the model is highly nonlinear, so that (almost) all parameters affect all outcomes, the identification of some parameters relies on some key moments in the data. We keep all parameters fixed at their calibrated value and change only one parameter to study the numerical comparative statics of the moment with respect to the parameter in the neighborhood of the calibrated parameter. First, Figure 10 shows that as the intensity of liquidity shock increases there are more needs for trading and the turnover increases. Second, as the search cost increases, there are more frictions in the secondary market which increases the slope of the liquidity spread yield curve. Third, as the holding cost increases, there is more entry in secondary markets and the expected time to sell decreases. Finally, as the return on investment increases—larger  $\zeta$ —the firm issue at longer maturity.

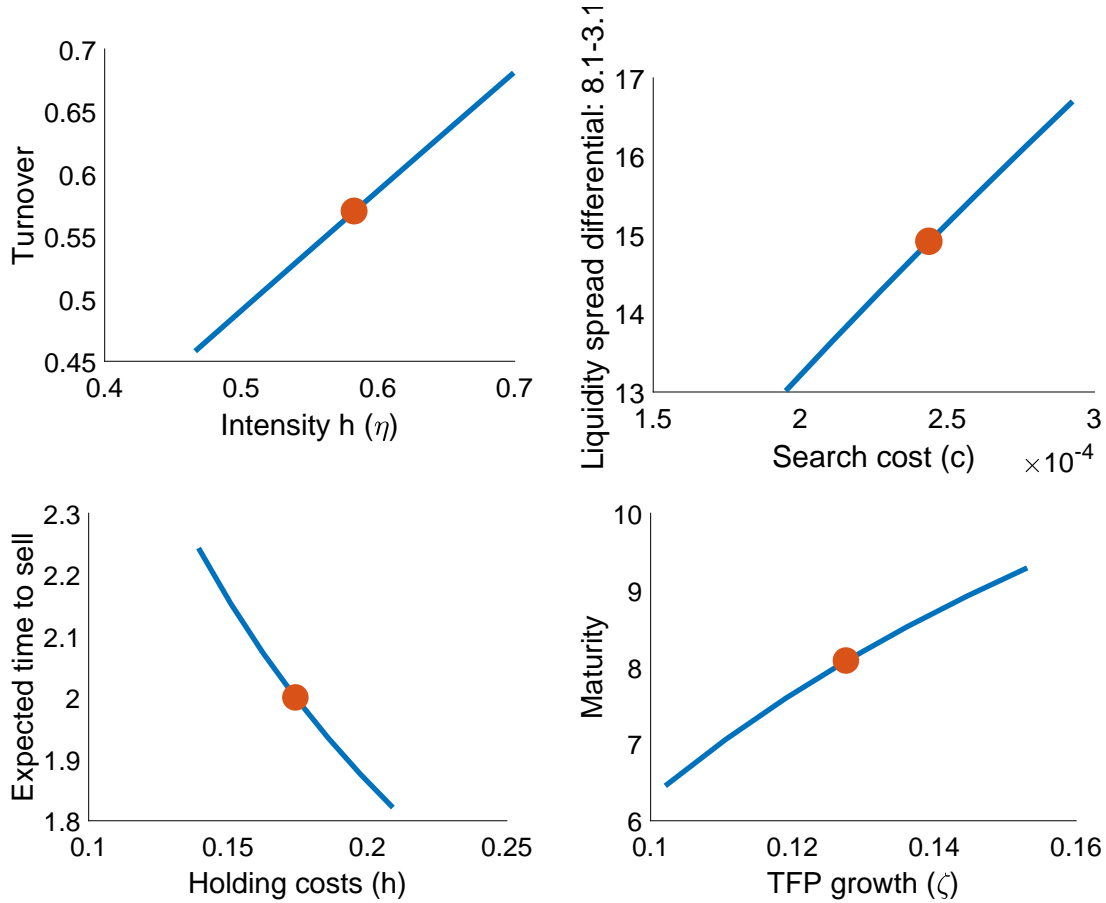
## E Cross country evidence and quantitative results

This appendix summarize the empirical evidence on corporate debt maturity across countries. Next, the cross country evidence disciplines additional quantitative exercises.

### E.1 Empirical evidence across countries: Maturity

There is a vast empirical literature showing that firms in developing countries tend to borrow at shorter maturities than in advanced economies. First, Demirgüç-Kunt and Maksimovic (1998) uses firm-level balance sheet data—i.e., including different securities, not only corporate bonds—and finds that the ratio of long-term debt (defined as maturity greater than one year to total liabilities) is typically

Figure 10: Identification.



*Note: We keep all parameters fixed at their calibrated value and change only one parameter to study the numerical comparative statics of the moment with respect to the parameter in the neighborhood of the calibrated parameter.*

lower in developing countries than in advanced ones, even after controlling for firm characteristics.<sup>27</sup> Second, the [World Bank \(2015\)](#) report on long-term finance uses bank-level balance sheet data and finds similar results. Third, [Cortina Lorente et al. \(2016\)](#) arrives to the same conclusion looking at corporate bonds issuances in domestic markets. As this data set is the closest to the theory we use it for the quantitative exercise.

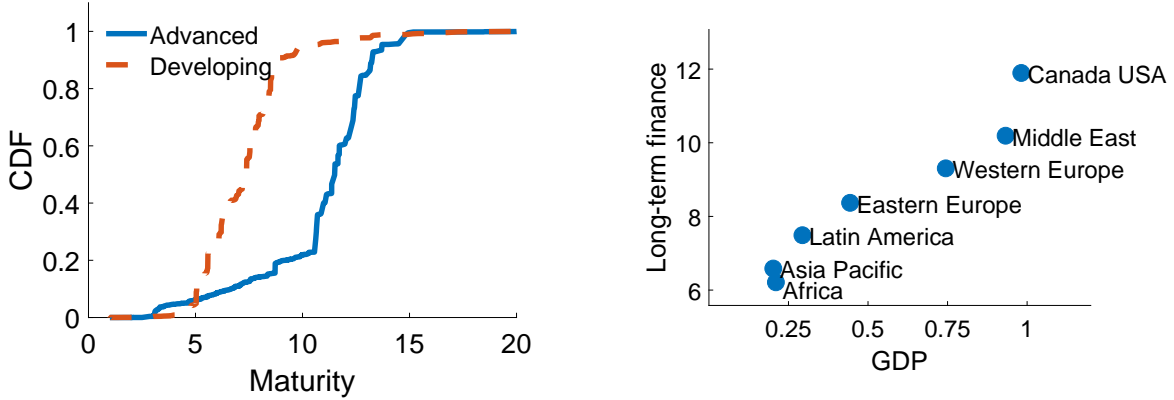
[Cortina Lorente et al. \(2016\)](#) uses Thomson Reuters Security Data Corporation (SDC) Platinum database to compile an extensive dataset of corporate bond issuances in domestic markets for 1991-2014 across 80 countries (41 developed and 39 developing economies).<sup>28</sup> The left panel of Figure 11 shows the empirical distribution of corporate debt maturities for advanced and developing countries. In developing countries, the median maturity is about seven years, while it is about eleven years in

<sup>27</sup>See also [Caprio and Demirgüç-Kunt \(1998\)](#); [Demirgüç-Kunt and Maksimovic \(1999\)](#); [Booth et al. \(2001\)](#); [Fan et al. \(2012\)](#).

<sup>28</sup>I thank the authors for sharing their data, for details see Appendix F.



Figure 11: Maturity of corporate bonds markets across countries.



Note: Maturity is computed as the weighted average maturity (in years) of corporate bonds at issuance in domestic markets. Source: Cortina Lorente et al. (2016). See Appendix F for details.

advanced ones. The right figure shows a positive relationship between corporate debt maturity and output per capita across regions. For example, in the US the average maturity is 12.2 years while in Latin America the average maturity is about 7.6 and GDP per capita is about 30% relative to the US.

## E.2 Calibration for the US

We match moments from the US corporate debt market. Except the liquidity spread, the rest of the calibration is the same as in Section 4.2. To discipline the component of the credit spread driven by liquidity in the data we use the spread between Treasuries and high quality corporate bonds from Section 4.3.<sup>29</sup> The calibration targets this spread only at the equilibrium maturity, i.e., one point of this yield curve, but as a validation the model replicates the data at different maturities. Table 8 shows the calibrated parameters and moments.

**Financial sector** To discipline the search cost  $c$  and the holding costs  $h$  target the liquidity spread at maturity to be equal to 1.6% (from the spread between Treasuries and corporate bonds) and the expected time to sell of 2 weeks (i.e.,  $\lambda = 26$ , see He and Milbradt (2014)). Calibrate the intensity of the liquidity shocks,  $\eta$ , to match an annual turnover rate of 57% (He and Milbradt, 2014; Chen et al., 2017). Note that turnover is equal to  $(\eta^{-1} + \lambda^{-1})^{-1}$ , so we can directly calibrate  $\eta$  without simulating the model.

**Maturity** Normalize the measure of new firms  $\mu^0 = 1$ . For the maturity of corporate bonds target a debt maturity at issuance of 12.2 years (Cortina Lorente et al., 2016). By inspection of equation (17) note that we can normalize  $\kappa = 1$  and internally calibrate  $\zeta$  to match the maturity target.

<sup>29</sup>Recall that high quality corporate bonds are rated above A so we can abstract from the default component.

**Matching** For the secondary market assume a constant-return-to-scale Cobb-Douglas matching function with efficiency parameter normalized to one, and symmetric loadings on sellers and buyers. Further assume symmetric bargaining power,  $\gamma = 0.5$ .

Finally, the discount factor is  $\rho = 0.02$  (He and Milbradt, 2014) and default rate is  $\delta = 3\%$  to match the default rate of speculative grade firms (Moody, 2015).

Table 8: **Parameters and moments.**

Parameter		Value	Target/source	Model	Data
<i>Financial sector</i>					
Intensity of liquidity shocks	$\eta$	0.58	Turnover rate	0.57	0.57
Search cost	$c$	0.0003	Liquidity spread	160	160
Holding cost	$h$	0.25	Expected time to sell	2.00	2.00
<i>Production sector</i>					
$F(\tau) = A\tau$	$A$	5.97	Maturity	12.20	12.20
$I(\tau) = \kappa \frac{1 - \exp^{-\rho\tau}}{\rho}$	$\kappa$	1.00	Normalization		
<i>Matching</i>					
Share of sellers	$\alpha$	0.50	Normalization		
Bargaininw power of sellers	$\gamma$	0.50	Normalization		
<i>Others</i>					
Discount factor	$\rho$	0.02	He Milbradt (2014)		
Default rate	$\delta$	0.03	Moody's (2015)		

### E.3 Counterfactual trading frictions

For sovereign debt markets, Broner et al. (2013) shows that it is more expensive to borrow long-term in emerging countries than in advanced ones. However, there is very little evidence on the yield curve for corporate bonds across countries. A moment commonly use in the literature to compare intermediation costs across countries is the bank's net interest margin which measures the difference between interest income and payments to lenders using banks' balance sheet data from Bankscope (the data is available at The World Bank Global Financial Development Database). For example, Greenwood et al. (2013) attributes these spreads to the intermediation costs related to acquiring information about borrowers. In this paper we explore a different reason for these spreads, namely the illiquidity cost. Select the same countries as in the maturity data from Cortina Lorente et al. (2016) and split them between advanced and emerging countries. According to these measures, credit spreads are 295 basis points higher and maturity is 3.6 years shorter in emerging countries than in advanced ones. The net interest margin is not a perfect measure of the illiquidity cost. However, it is in line with the estimate for US and Argentina of Section 4.1.

Table 9 evaluates the counterfactual economy in which the search cost increases to match a change in the liquidity spread of 295 basis points as in the data. As a result, firms borrow at 3.29 years shorter which is close to the data counterpart of 3.60 years.

The third column of Table 9 considers an alternative model in which the interest rate is exogenous and independent of maturity ( $r = cs^{def} + cs^{liq}$  for all  $\tau$ ). Start with a constant liquidity spread of 160 basis points and increase it by 295 basis points as in the benchmark model. As a result, firms borrow at a shorter maturity but the effect is lower than in the model with endogenous liquidity. In the benchmark model a change in the search cost not only increases the liquidity spread but it also change the slope of the yield curve which induces firms to borrow at shorter maturities. Hence, providing a theory of liquidity spreads is not only important to understand the frictions and study policy implications but also to perform quantitative evaluations.

Table 9: **Credit spreads across countries.**

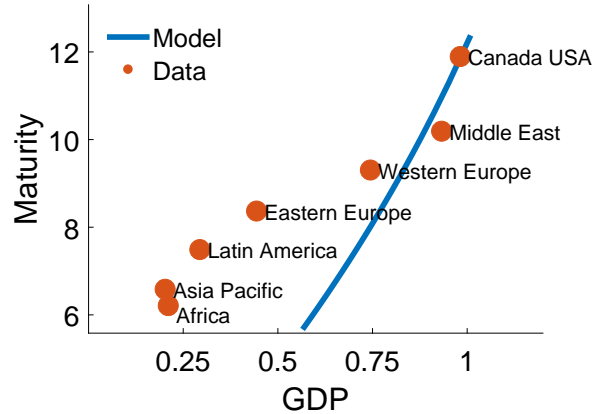
	Data	Model	
		Endogenous	Exogenous
$\Delta$ Liquidity spread (bps)	295	295	295
$\Delta$ Maturity (years)	-3.60	-3.29	-2.96
$\Delta$ Output (%)	-60	-19	-16

*Note: Credit spread data is from World Bank financial structure and economic development database. Maturity data is from Cortina Lorente et al. (2016).*

**Real economy** When the search cost increases long-term finance becomes relatively more expensive and firms tilt their maturity choices toward the short-end. On the real side of the economy, this implies that entrepreneurs invest in shorter-term projects which have lower productivity and, therefore, lower aggregate output. Hence, changes in the real side of the model are driven by changes in the endogenous productivity that is an outcome of the maturity of investment. The third row of Table 9 shows that aggregate output reduces by 19% when the liquidity spread increases by 295 basis points and the firm shorten the maturity by 3.29 years. Trading frictions can account for about one-third of the differences between advanced and developing countries due to changes in the duration of investment and aggregate productivity.

Finally, use the cross-country data on maturity and aggregate output to evaluate the performance of the model. For each country change the search cost to match the data on maturity, and then evaluate the performance on aggregate output. Figure 12 shows that the model predicts a relationship between maturity and output across countries quite similar to the data counterpart. Note that the model has a high explanatory power for developed countries. However, for less-developed economies trading frictions have lower explanation power. Even though we present a stylized and tractable model, the quantitative results suggest that the model captures important features of corporate debt markets and their effect on productivity and output across countries.

Figure 12: **Maturity and the real economy.**



*Note: The blue line shows the equilibrium allocations of maturity and GDP under different frictions in the secondary market and the data is the same as in Figure 11.*

## F Data sources

This appendix describes the data sources.

**Credit spreads in the US** Consider all corporate debt issuances in 2017 from Mergent Fixed Income Securities Database (FISD) and keep issuances of corporate bonds by domestic borrowers, in local currency and with fixed interest rate. The credit spread is the difference between the interest rate and the Treasury of same maturity.

**Credit spreads in Argentina** Consider all the active corporate bonds in August 2017 in the domestic market (MAE) and keep issuances in local currency, with 100% amortization, and interest rate as a spread on the Badlar rate (which is the reference short-term rate in Argentina). These are floating interest rates bonds with a fixed spread so the credit spread is just the spread on the Badlar rate because non-arbitrage implies that agents can swap the variable Badlar rate for a fixed rate.

**Treasury yield curve** For interest rates we use estimates from the US Treasury for the synthetic zero-coupon bonds yield curve. The daily treasury data was retrieved from <https://www.treasury.gov/resource-center/data-chart-center/interest-rates>. Define the treasury yield curve as the daily average for the year 2017.

**High quality corporate bonds yield curve** The corporate yield curve corresponds to the High quality market (bonds with rating above A) and it is available at <https://www.treasury.gov/resource-center/economic-policy/corp-bond-yield/>. Define the corporate yield curve as the monthly average for the year 2017. Tables 10 shows that for the high quality market of corporate debt in the US both default credit losses and default rates are quantitatively small.

Table 10: **Default credit losses.**

Rating	Default credit losses		Default rates	
	Average 1982-2014	Maximum 2008	Average 1920-2014	Maximum 2008
Aaa	0.00%	0.00%	0.000%	0.00%
Aa	0.03%	0.48%	0.061%	0.724%
A	0.03%	0.37%	0.096%	0.547%

*Source: Moodys 2015.*

**Corporate debt maturity across countries** The data for maturities in Figure 11 was shared by Cortina Lorente et al. (2016). The source is Thomson Reuters Security Data Corporation Platinum database and details about the data can be found in Cortina Lorente et al. (2016). In particular, the dataset comprises 80 economies, 41 of them are developed and 39 developing. For GDP we use GDP per capita relative to US for 2014 downloaded from the World Bank database (GDP per capita PPP, constant 2011 international \$, series NY.GDP.PCAP.PP.KD).