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Financial distress, bankruptcy law and the business cycle

Abstract This paper explores the business cycle implications of financial distress and bankruptcy law. We find that due to the presence of financial imperfections the effect of liquidations on the price of capital goods can generate endogenous fluctuations. We show that a law reform that 'softens' bankruptcy law may increase the amplitude of the cycle in the long run. In contrast, a policy of bailing out businesses during the bust or actively managing the interest rate across the cycle could stabilize the economy in the long run. A comprehensive welfare analysis of these policies is provided as well.

Keywords bankruptcy law · business cycles · financial distress · liquidation

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1 Introduction

The interrelations between financial distress, bankruptcy law and macroeconomic fluctuations are capturing growing interest among policy makers and academics alike. For example, [13] in his analysis of the Asian Crisis argues that policy makers failed to understand these interrelations, and as a result implemented policies that exacerbated the crisis. It is implied that macroeconomic effects should be taken into consideration when bankruptcy law is designed, and that bankruptcy and distress should be taken into consideration when macroeconomic policy is implemented.

Several authors have noticed the importance of the general equilibrium implications of financial distress in the context of the ongoing debate on bankruptcy law. This debate has centered on the social desirability of *soft* laws, such as US' Chapter 11, that give borrowers an opportunity to reorganize, or *hard* laws, like the UK Bankruptcy Code, which is essentially a procedure for the enforcement of default-contingent liquidation rights.[1] In particular, [12] argue that once we take into consideration the "general equilibrium aspects of asset-sales ... [particularly when] the shock that causes the seller's distress is industry or economy-wide ... the policy of automatic auctions for the assets of distressed firms, without the possibility of Chapter 11 protection, is not theoretically sound."[2] This suggests that, once the macroeconomic effects are considered, the merits of a soft bankruptcy law would be evident.

In this paper we offer an explicitly dynamic, general equilibrium analysis of bankruptcy law in its relation with financial distress, asset sales, and macroeconomic fluctuations. We compare the possibility of softening bankruptcy law to alternative stabilization policies such as bail-outs or an active interest-rate policy (which one might interpret as monetary policy). To keep things analytically tractable, we model bankruptcy law in a framework similar to [14]. An important characteristic of this framework is that macroeconomic fluctuations

[1] See [6].

[2] Similarly [10] argues that "Immediate cash liquidation of distressed firms' assets via Chapter 7 of the U.S. bankruptcy code could result in suboptimal outcomes" (p. 941).

are generated endogenously, through a (deterministic) mechanism entirely due to agency problems between lenders and borrowers. In the current model, even without any external shock, the dynamics of debt accumulation and asset liquidation can push the economy from boom to bust and *vice-versa*; crucially, the rationality of expectations is maintained throughout.[3]

A central element in the analysis is the contractual relationship between borrowers and lenders. We follow [8] in that debt is enforced under a threat of liquidation, and viable projects may be liquidated as a result of the agency problem between the lender and the borrower. This framework allows two simple formalizations of a softening in bankruptcy law: either an increase in the borrower's bargaining power or an increase in the systematic 'dilution' of lenders' liquidation rights by courts. We analyze both formalizations.

We obtain four main results. First, as noted above, it is possible to construct an equilibrium where the dynamics of debt, financial distress, and asset liquidation are the sole forces behind economic fluctuations. During a boom, high prices of capital goods push new business into high levels of debt and collateral. Those of them which fall into financial distress will have to liquidate assets, which will depress the prices (and production) of new capital goods and push the economy into a bust. However, the low prices of capital goods during the bust will create a favorable environment for new businesses, which will be able to start-up with low levels of debt and collateral, will be less vulnerable to financial distress, and will push the economy back into a boom. Some recent empirical results corroborate the idea that industry busts create opportunities for financially unconstrained firms.[4]

[3] We do not intend to argue that booms and recessions occur with deterministic regularity nor do we deny the importance of uncertainty. Actually we see our mechanism as complementary to the type of propagation mechanisms analyzed by [2] or [9]. However, we think that a setting where financial distress and asset liquidations are solely responsible for the business cycle can help to clarify the interrelations between these important phenomena.

[4] [10] studies the market for second-hand narrow-body aircraft in the US and shows that during industry busts, non-distressed firms with high debt capacity are actively buying aircraft at discount prices (see his Figure 1 and Table V). Similarly, [5] studies the 1989-1991 real-estate bust in the US. He shows that during that period, Real Estate Investment Trusts (henceforth REITs) that were less sensitive to financial distress bought assets from those REITs that were more sensitive to financial distress. Moreover, the former were characterized by better stock

Second, softening bankruptcy law is not a socially desirable policy. The unanticipated enactment of a softer bankruptcy law would produce a temporary debt relief (through renegotiations that would favor the borrowers) and, thus, a temporary reduction in the amount of liquidations, perhaps smoothing the immediate bust if the timing is right. However, in later periods, as lenders would rationally foresee how a softer law erodes their bargaining position or dilutes their nominal liquidation rights, they would demand larger collateralization of their debt, so as to guarantee that their participation constraints are satisfied.[5] In some cases, the new law may lead to even larger liquidations during busts, increasing the amplitude of business fluctuations. Although we do not have an explicit political-economy analysis, these results suggest that soft bankruptcy laws may be enacted by myopic legislators who are willing to use bankruptcy law to accelerate the recovery from economic recession at the expense of long-term stability.[6]

Third, contrary to what the previous finding might suggest, rational expectations do not make all possible policies ineffective. In fact, alternative stabilizing policies, such as bail-outs (during the bust) or an active interest-rate policy (directed to decrease interest charges during busts) may have an endurable stabilizing effect. The crucial difference between these policies and a softening in bankruptcy law is that the former systematically transfer wealth from the less to the more financially constrained, while the latter provokes a purely transitory relief and then leads to contract adjustments that, if anything, make things worse for cyclicality.

Fourth, we show that long-term equilibria are constrained Pareto efficient but, under some stabilizing policies, the gains of winners, who happen to be the most financially constrained, exceed the losses of the losers (where gains and losses are partly due to lower and greater amounts of asset liquidation). Hence, over the cycle, financial frictions can be diminished, at a gain in terms of overall expected

market performance.

[5] See [11] for a corroboration of this mechanism. They show that following the 1978 reform of US bankruptcy law, the cost of secured borrowing increased while credit availability decreased.

[6] As reported by [3], early US legislation often consisted of ad-hoc debt relief. For example, in 1841, following the bank panic of 1837, a law gave some 1% of the adult, white, male population the opportunity to cancel large amounts of debt. The law was repealed in 1843.

income.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 considers the benchmark economy without financial frictions. Section 4 characterizes the equilibrium debt contract. Section 5 discusses the existence of a competitive rational-expectations equilibrium. In Section 6 we analyze the effects of softening bankruptcy law, while in Section 7 we examine the effect of other stabilizing policies. The welfare analysis is in Section 8 and the conclusions in Section 9.

2 The Model

Consider a discrete time ($t = 0, 1, 2, \dots$), small open economy with overlapping cohorts of *entrepreneurs*. There are two commodities: a perishable consumption good which is used as a numeraire and a capital good. The relative price of the capital good in terms of the consumption good in period t is denoted by q_t. The capital good is not tradable across countries, hence its price in the home economy may differ from the world price. In contrast the consumption good can be shipped costlessly from one country to another, which allows a complete integration of the home financial market into the world's market.[7] Hence, the *financiers*, which will play a prominent role in the model below, can be either locals or foreigners, with no material distinction between them. We assume that the default-free interest rate is constant over time and we normalize it to zero. All agents are risk-neutral.[8]

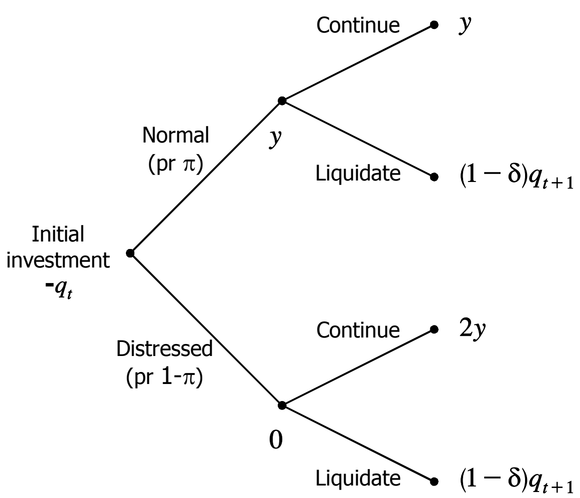
Each period a measure-one continuum of two-period-lived entrepreneurs is born. Entrepreneurs have exclusive access to an investment project each. These projects are the only means to produce the consumption good in the (home) economy. In order to be started-up, the project of a t-born entrepreneur requires the investment of one unit of capital at t (see Figure 1 for details about the

[7] The assumption that consumption goods are tradable and capital goods are not is the simplest – not necessarily the most realistic – way to model a system where capital-good prices fall in response to an increased supply of liquidated capital goods.

[8] Thus utility can be measured in units of the consumption good, as we do below.

timing of the payoffs associated with the project). This capital is purchased at its market price q_t. Projects are subject to some purely idiosyncratic uncertainty that we identify with the risk of financial distress. Projects turn out to be *normal* with probability π and *distressed* with probability $1-\pi$. Both types of projects, if completed, yield the same total output of $2y$ units of the consumption good at $t+1$. However, normal projects yield y units of the consumption good before an interim liquidation deadline and, if completed, y more units after the deadline, while distressed projects, if completed, yield all their output $2y$ after the liquidation deadline.[9] As project-type uncertainty is purely idiosyncratic, the proportion of distressed projects is exactly $1-\pi$ at every period, producing no macroeconomic uncertainty.

Figure 1: Timing of project payoffs



At the liquidation deadline projects can be totally or partially liquidated. When a fraction β of the installed capacity of a project is liquidated, the same proportion of the output which it would produce after the liquidation deadline,

[9]The symmetric distribution of the output of normal projects before and after the liquidation deadline is not important for the argument.

if continued, is lost. However, for each initial unit of capital which is liquidated, $1-\delta$ units can be sold to the entrepreneurs of the next cohort. So the liquidation value of a period t start-up is $(1-\delta)q_{t+1}$, which depends on both the physical depreciation of capital before the liquidation deadline, δ, and the price of capital at $t+1$.

Entrepreneurs are born penniless, so their projects have to be externally financed. We assume that the relationship between financiers and entrepreneurs is burdened by an agency problem: the output of a project is *observable* by both the entrepreneur and her financiers, but it is *not verifiable* by the judge or court on whom the enforcement of the contract depends (see [7]). Hence output-contingent cash flow rights cannot be contracted upon.[10] In contrast, the settlement of payments is verifiable so if a promised repayment is not settled, the relevant judge or court can safely infer that the entrepreneur has defaulted. Also, as commonly assumed in the literature, liquidation and the proceeds from the sale of liquidated assets are verifiable. This allows an *incomplete contract* to be implemented that fixes some repayment to be settled some time before the liquidation deadline and, contingent upon the event of default, gives financiers the right to liquidate all or a part of the project.

Such a contract leaves room for *strategic default*, that occurs when a non-distressed entrepreneur defaults just in order to renegotiate some better terms. The renegotiation will take place after it is already known whether the project's early output is y or 0, but before the liquidation deadline. We model the renegotiation as a take-it-or-leave-it-offer game in which the financier makes the offer with probability λ and the entrepreneur with probability $1-\lambda$. Hence, the lower is λ, the *softer* is the lending relationship for the entrepreneur.

It is quite common to view Chapter 11 of the US bankruptcy code as a court-supervised renegotiation process.[11] Such 'supervision' is rarely neutral: typically,

[10] The "observable but not verifiable" assumption – like others in the model – is made for analytical tractability rather than realism. Observability avoids the need to model bargaining under asymmetric information; non-verifiability introduces an agency problem between debtor and creditor. This assumption could be partially relaxed (e.g. a certain part of the cash could be verifiable) without affecting the results.

[11] See [1].

it tilts the terms of the contract in favor of one of the parties. The literature considers Chapter 11 as tilted towards the entrepreneur, while the British law, that insists on the strict enforcement of contracts, is put at the other end of the bankruptcy-law spectrum.[12]

Formally, one can model the effects of moving from a tougher environment to an American-type bankruptcy law as either a change in the bargaining power in favor of the entrepreneur (i.e. a lower λ), or as a direct dilution of the liquidation rights held by the secured lenders. We consider both formalizations.

In contrast with the consumption good in the above projects, capital can be instantaneously produced and, thus, the industry that produces it bears no agency problem. We assume that this industry is perfectly competitive and has a technology described by a constant-returns-to-scale Cobb-Douglas production function:

$$N_t = AX_t^{\gamma} L_t^{1-\gamma}, \tag{1}$$

where N_t is the period-t production of new capital, X_t and L_t are the inputs of consumption good and labor, and $\gamma \in [0, 1]$ is the elasticity to the consumption good input. For simplicity we assume that the supply of labor is inelastic and equal to one. Moreover, we assume that

$$\alpha \equiv \frac{1}{\gamma} A^{-(1/\gamma)} < y, \tag{2}$$

which guarantees that the price of capital is low enough to make all projects to be continued in the first-best economy (see Section 3).

Lastly, we assume that the economy starts functioning at $t = 0$ with an *initial condition* given by some supply of old capital $O_0 > 0$ coming from the liquidations of the previous generation of entrepreneurs.

[12]The British approach still leaves plenty of room for renegotiations, but they take place out of court, and the law makes no attempt to affect the 'natural' outcome of the bargaining process; see [6].

3 Verifiable cash flows: the first best

To start with, it is useful to analyze, as a benchmark, an economy which is identical to the one just described except because entrepreneurs are not burdened with an agency problem. We show that under our parametric assumptions projects are never liquidated and the economy converges to a (unique) stationary equilibrium after one period.

The supply of new capital can be derived by ordinary marginal-cost pricing considerations. Remembering that the supply of labor is fixed and normalized to one, the industry's supply function becomes:

$$N_t = \left(\frac{q_t}{\alpha}\right)^{\theta}, \tag{3}$$

where

$$\theta \equiv \frac{\gamma}{1-\gamma}. \tag{4}$$

The magnitude of α relative to y is fixed by assumption (2). The price-elasticity is constant and equals θ, which rises from zero to infinity as γ moves from zero to one.

We can now state the main result of this section.

Proposition 1 *If project output is verifiable, the unique equilibrium features $q_t = \alpha$ for all $t \geq 1$. No project is ever liquidated.*

Proof. We start by establishing an upper bound to the market-price of capital: since the demand of new capital never exceeds one, (3) implies $q_t \leq \alpha$.

Next, we show that within this range of feasible prices, no project is ever liquidated, and no project is ever left idle. Note that in the absence of cash-verifiability problems the Modigliani-Miller theorem holds so all investment and liquidation decisions are taken so as to maximize NPV. Consider first the liquidation decision for the t-born cohort. By assumption (2), we have $(1-\delta)q_{t+1} < y$, so a normal project is never discontinued. As distressed projects have even greater continuation value, $2y$, their continuation is even more profitable. Thus no project is ever liquidated in this economy, regardless of financial distress. Hence, when started up, each project's output has an expected present value of $2y$. Since $q_t < 2y$ investments have positive NPV and will be funded. It follows that from $t = 1$ onwards, the demand for new capital is exactly one, so its price is α, by (3).

Things are slightly different at $t = 0$, where some capital O_0 is offered for sale by the previous generation. In this case, the market clearing price of capital is

$$q_0 = \alpha\,(1 - O_0)^{1/\theta} < \alpha. \blacksquare \tag{5}$$

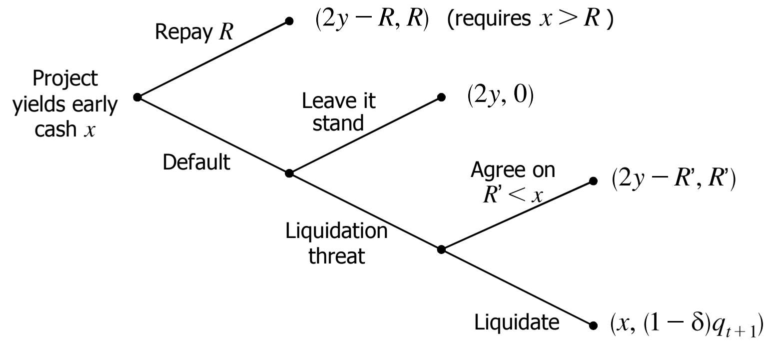
4 Unverifiable cash flows: the contract

Our contract problem is similar to [4] and [8]: unverifiable cash flows can be directly appropriated by the entrepreneur, who can only be induced to repay to his financiers under a liquidation threat. The main difference is that the price of capital has a critical effect on the problem. As we show in this section, the start-up price q_t affects the entrepreneur's financial requirements and thus his financiers' participation constraint, while the liquidation price q_{t+1} affects his incentives to default strategically.

When cash-flows are unverifiable, the first best is no longer attainable. If the contract establishes that the project is never liquidated, as in the first-best case, then, by the liquidation deadline and no matter the project has yielded some output or not, the entrepreneur will claim that his project is distressed and he has nothing with which to repay the financier at that stage. But obviously once the project has yielded all its output the entrepreneur will simply 'take the money and run'. Although by the liquidation deadline the financier may know that the entrepreneur is cheating (remember that output is observable), he cannot prove it in court (since output is in that precise sense unverifiable). Evidently, if financiers foresee this course of action, they will not fund to the entrepreneur in the first place, despite all projects have positive NPV.

The standard incomplete-contract solution in this context is to provide the financier with the right to liquidate all or a fraction of the project in case of default. To illustrate how this will induce the entrepreneur to repay, suppose that an entrepreneur born at t is obliged to repay a certain amount $R_t < y$ in period $t+1$. Suppose that the project is not distressed but the entrepreneur refuses to pay R_t. If the financier has to choose between fully liquidating the project for $(1-\delta)q_{t+1}$ and forgiving the entrepreneur, he will obviously choose the first option.[13] The entrepreneur would foresee the action and avoid (strategic) default. A more refined version of this argument should take into consideration the possibility of renegotiation between the entrepreneur and the financier. Figure 2 depicts the sequence of events, including the possibility of renegotiation, when the project yields some early cash flow x before the liquidation deadline (in a non-distressed project, we have $x = y$).

Figure 2: Events after early cash flow x realizes



If the financier has the chance to make the entrepreneur a take-it-or leave it offer, he will be able to appropriate up to the project's continuation value y. If the entrepreneur has the chance to make the financier a take-it-or-leave-it offer he would push his repayment down to the liquidation value of the capital, $(1-\delta)q_{t+1}$.

[13] Remember that: (i) the financier cannot operate the project by himself, and that (ii) capital will fully depreciate if the project is continued up to $t+1$. Hence, the best the financier can do is to liquidate the project and sell the remaining capital in the second-hand market at $t+1$.

Thus, the entrepreneur will choose not to default as long as the repayment R_t is no larger than the expected renegotiated payment $R'_t = \lambda y + (1-\lambda)(1-\delta)q_{t+1}$.

Unfortunately, this solution involves a deadweight loss. When the project is distressed (that is, if $x = 0$ in Figure 2), the entrepreneur faces a genuine cash shortage and she has no choice but to default. In such a case, it is still in the financier's best interest to fully liquidate the project. Although the project still has a continuation value that exceeds its liquidation value, the entrepreneur does not have the liquidity to buy out the financier's liquidation rights, nor can she credibly commit to pay at $t+1$. In sum, if the lender is given the right to fully liquidate the project, full liquidation will follow in case of distress.

However, the financier's liquidation rights need not be so large. They should be just sufficient to induce repayment when the project is not distressed. In general, only a fraction β_t of the capital will be pledged as collateral so that, in case of default, lenders only have the right to liquidate such a fraction of the project. In this case, the entrepreneur will choose not to default insofar as $R_t \leq \beta_t[\lambda y + (1-\lambda)(1-\delta)q_{t+1}]$.

A contract is thus a pair (R_t, β_t), where R_t is the debt-repayment, β_t is the collateral, and t is the date at which the contract is signed. The contract problem for the entrepreneur born at t has the following form:

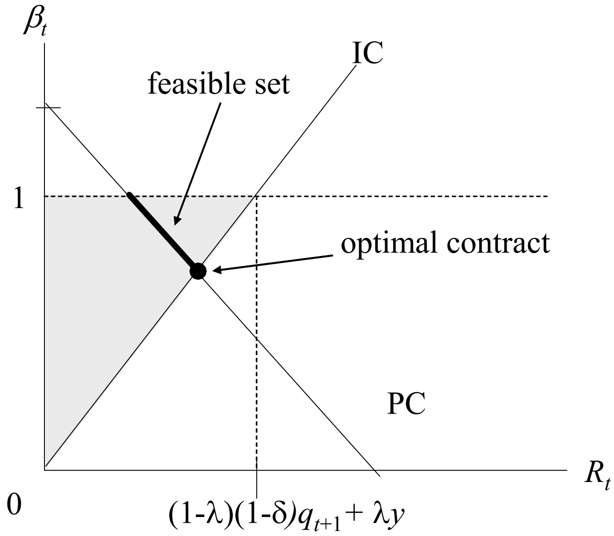
$$\max_{\beta_t, R_t} \quad \pi(2y - R_t) + (1-\pi)(1-\beta_t)2y \qquad (6)$$
s.t.:
$$\pi R_t + (1-\pi)\beta_t(1-\delta)q_{t+1} = q_t \qquad \text{(participation constraint)} \qquad (7)$$
$$R_t \leq \beta_t[\lambda y + (1-\lambda)(1-\delta)q_{t+1}] \qquad \text{(incentive constraint)} \qquad (8)$$
$$\pi(2y - R_t) + (1-\pi)(1-\beta_t)2y \geq 0 \qquad \text{(positive-profit constraint)} \qquad (9)$$
$$R_t \in [0, y], \;\; \beta_t \in [0, 1] \qquad \text{(feasibility constraints)} \qquad (10)$$

The participation constraint (7) holds with equality, reflecting that entrepreneurs have all the bargaining power at the contracting stage.

When solving the program (6)-(10), we focus on β_t; the repayment R_t can always be obtained recursively, but its analysis is of limited interest for our argument. We analyze the problem with the aid of Figure 3, where the downward-sloping PC line represents equation (7), and the shaded area above the upward-sloping IC line represents the set of incentive-compatible contracts as defined in equation (8). The two lines intersect at point

$$\beta\left(q_t, q_{t+1}\right) \equiv \frac{q_t}{\left(1-\delta\right)\left(1-\lambda\pi\right) q_{t+1} + \lambda\pi y}. \tag{11}$$

Figure 3: The contract problem



Proposition 2 *The optimal contract is $\beta_t = \beta\left(q_t, q_{t+1}\right)$, provided that $\beta\left(q_t, q_{t+1}\right) \leq 1$. When $\beta\left(q_t, q_{t+1}\right) > 1$, the entrepreneur is credit-rationed and the project receives no funding.*

Proof. We start by showing that the problem's constraints can be reduced to equations (7), (8), and $\beta_t \leq 1$ only. First notice that under (10), constraint (9) is redundant, while $R_t \leq 0$ and $\beta_t \leq 0$ are clearly incompatible with (7) and (8). Second, substituting $\beta_t = 1$ into equation (7) we get $R_t = \lambda\left(1-\delta\right) q_{t+1} + \left(1-\lambda\right) y$, as represented in Figure 3; but then it follows that the constraint $R_t \leq y$ will never be binding since $q_{t+1} \leq \alpha < y$ by assumption (2) (recall from Proposition 1 that q_{t+1} would reach its maximum value of α if no project started up at t were liquidated).

Hence, the feasible set for the program (6)-(10) is determined by (7), (8) and the feasibility constraint $\beta_t \leq 1$. Graphically, the feasible set is the section of the PC line that belongs to the IC set (shaded) and is below the $\beta_t = 1$ line: the bold segment in Figure 3. Clearly, if the IC and the PC lines intersect at a point with $\beta_t > 1$, then the feasible set is empty and the project is credit-rationed.

Substituting equation (7) into (6), the objective function (6) can be written as

$$v(q_t, q_{t+1}) = 2y - q_t - (1-\pi)\,\beta_t\left[2y - (1-\delta)\,q_{t+1}\right]. \tag{12}$$

It follows that the objective function is maximized when β_t is minimized. Hence, the optimal contract is the point with the lowest β_t within the feasible set. Namely, $\beta\,(q_t, q_{t+1})$, provided that $\beta\,(q_t, q_{t+1}) \leq 1$.■

The economic intuition behind Proposition 2 is best described through equation (12). It decomposes the entrepreneur's profit into the first-best output of the project, minus the purchase-price of the unit of capital invested in it, minus the expected deadweight loss that results from the agency problem. The latter term equals the probability of default, times the fraction of the project that the lender can liquidate in case of default β_t, times the difference between the continuation and liquidation values of the project. The optimal contract should minimize the expected deadweight loss, and that is done by minimizing the size of the collateral. Clearly, the smaller is the collateral, the smaller is the scope for losses due to premature liquidation.

Proposition 1 provides one of the model's basic building blocks, establishes a relationship between market prices and agency problems. Note that

$$\frac{\partial \beta}{\partial q_t} > 0 \quad \text{and} \quad \frac{\partial \beta}{\partial q_{t+1}} < 0. \tag{13}$$

Namely, when the current price of capital increases, entrepreneurs' funding requirements increase and debt repayments must increase as well. Each entrepreneur will have to provide more collateral in order to ensure his financiers that he has no incentive to engage in strategic default. In contrast, when the next-period price of capital increases, strategic default becomes less attractive, permitting repayments to be enforced with less collateral. In short, the underlying incentive problem is more severe when the purchase price of capital is high and its liquidation price is low.

Lastly, notice that the contract that we have just characterized shares many features with real-world debt contracts; particularly, the repayment R_t is not conditional on the project's output and is enforced through a liquidation threat whose effectiveness is tied to the value of the collateralized assets. We shall henceforth refer to the financier as a lender, and to the entrepreneur as a borrower. More importantly, identifying the financial arrangement as 'debt' enables us to interpret what happens after default as 'bankruptcy' and to analyze the effects of bankruptcy law.

5 Competitive non-rationing equilibria

We construct a competitive equilibrium by combining the supply schedule (3) with the solution of the contract problem derived in Section 4. We limit ourselves to non-rationing equilibria. Tractability is the only reason: non-rationing equilibria are governed by a first-order non-linear difference equation (equation (22) below) which is complicated enough. Because every entrepreneur is funded in such an equilibrium and the size of both the population of entrepreneurs and their projects are fixed, investment per cohort is constant over time (and equal to one). In contrast, once the entrepreneurs are credit-rationed (at some date), investment is no longer constant and prices follow a second-order non-linear difference equation, which is much more difficult to analyze.

To facilitate the presentation, let:

$$a \equiv (1-\delta)(1-\lambda\pi), \quad b \equiv \lambda\pi y, \quad c \equiv (1-\delta)(1-\pi), \tag{14}$$

so that

$$\beta(q_t, q_{t+1}) \equiv \frac{q_t}{aq_{t+1}+b}. \tag{15}$$

Formally, the equilibria on which we focus are defined as follows:

Definition 1 *A competitive, rational-expectations, non-rationing equilibrium is a sequence* $\{q_t\}_{t=0}^{\infty}$ *that satisfies*

$$\left(\frac{q_{t+1}}{\alpha}\right)^{\theta} = 1 - c\beta_t, \tag{16}$$

$$\beta_t = \beta\left(q_t, q_{t+1}\right), \tag{17}$$

$$\beta\left(q_t, q_{t+1}\right) \leq 1, \tag{18}$$

and the initial condition (5).

Note the important differences between this economy and the benchmark economy described in Section 3. With verifiable cash flows, no project is ever liquidated, the production of capital equals one in every period, and the price of capital is constant and equal to α. With unverifiable cash flows, a fraction β_t of the capital invested in each project at date t is pledged as collateral, of which a fraction c becomes reusable, through liquidation and sale in the second-hand market, at $t+1$. As reflected in the market clearing condition (16), the production of new capital will typically be lower than one and its price will typically be lower than α.

Our endogenous-cycles results depend crucially on the non-linear nature of price dynamics. This non-linearity complicates the analysis of existence. Solving for β_t in (16) and substituting the resulting expression into equation (17) gives the temporal equilibrium condition

$$g\left(q_{t+1}\right) = \beta\left(q_t, q_{t+1}\right), \tag{19}$$

where

$$g\left(q_{t+1}\right) \equiv \frac{1}{c}\left[1 - \left(\frac{q_{t+1}}{\alpha}\right)^{\theta}\right]. \tag{20}$$

Equation (19) allows us to explicitly solve for q_t, giving rise to a well-defined backward looking difference equation

$$q_t = \phi\left(q_{t+1}\right). \tag{21}$$

However, since our dynamic system is *forward looking* (i.e. with an initial condition given by (5)), we are interested in the difference equation

$$q_{t+1} = f(q_t) \equiv \phi^{-1}(q_t), \tag{22}$$

which will be well-defined if any price q_t prevailing at some period t can be associated with a unique price q_{t+1} at $t+1$ (in other words, if ϕ is monotonic and, hence, invertible over the relevant range). Additionally, for a sequence of prices produced by such difference equation to correspond to a non-rationing equilibrium, we should make sure that all pairs (q_t, q_{t+1}) in the sequence satisfy the non-rationing constraint (18). The following lemma shows that, if we assume

$$\alpha \leq b, \tag{23}$$

then the difference equation is well-defined, the non-rationing constraint is satisfied, and there exists a unique stationary non-rationing price q^*.

Lemma 1 *Under assumption (23), there exists a well-defined sequence of competitive, rational-expectations, non-rationing equilibrium prices, and a unique stationary price q^*.*

Proof. See the Appendix.

In Figure 4 we plot the values of $g(q_{t+1})$ and $\beta(q_t, q_{t+1})$ against the values of q_{t+1}, for a given value of q_t. The graphs of both functions are downward sloping. The graph of g crosses the vertical axis at $1/c$ and the horizontal axis at α, and it is concave for $\theta > 1$ and convex for $\theta < 1$. The graph of β crosses the vertical axis at q_t/b and falls asymptotically towards the horizontal axis; it is always convex and shifts upwards when q_t increases. Assumption (23) guarantees that the intercept of g is above the intercept of β for all $q_t \leq \alpha$, which in turn guarantees that, for any given q_t, the graphs of g and β intersect at least once at some $q_{t+1} < \alpha$. Actually, for $\theta > 1$, the concavity of g directly implies that this intersection is unique and defines the value of $f(q_t)$. For $\theta < 1$, the problem might in principle be more complicated but, as shown in the lemma, it happens that

(23) is also sufficient for uniqueness. Importantly, the fact that the intercept of β is below the intercept of g implies that the graph of β crosses the graph of g from below, so when q_t increases, β shifts upwards, and the intersection occurs at a lower q_{t+1}. In other words, the difference equation has negative slope ($f'(q_t) < 0$).

Figure 4: Analysis of the *f* function

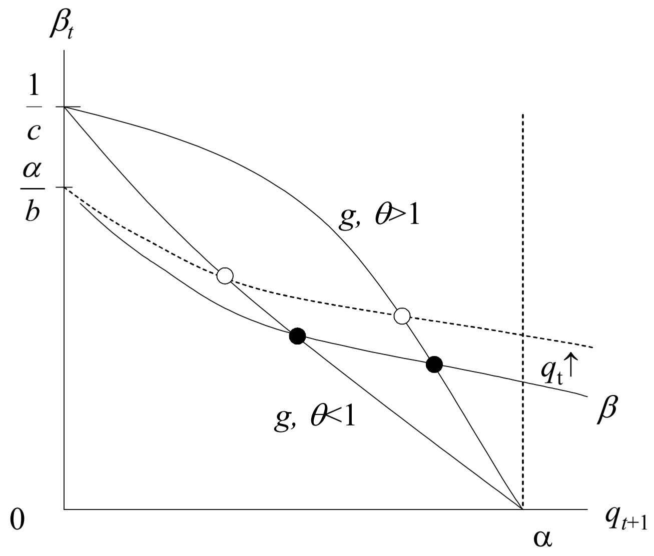
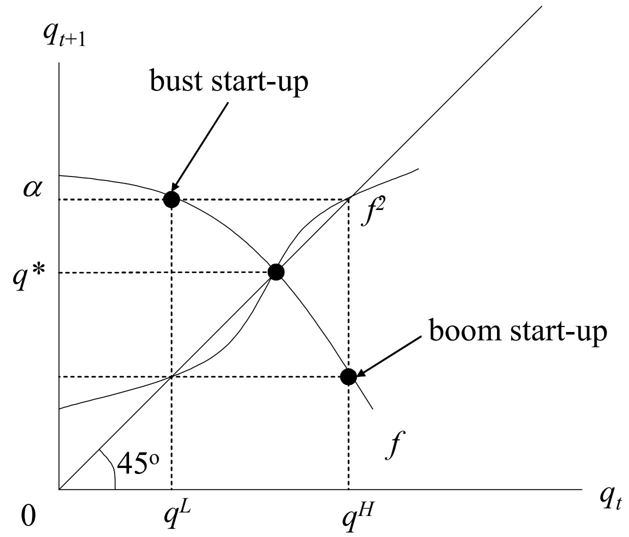


Figure 5 depicts the graph of the difference equation (22). The stationary price q^* corresponds to its intersection with the 45° line. It is well-known that with a non-linear monotonically decreasing difference equation like ours, the equilibrium will consist on a sequence of prices that cyclically converge to either the stationary price q^* (stable equilibrium) or a limit cycle with a periodicity of two (periodic equilibrium). To analyze the latter, we define

$$f^2(q) \equiv f[f(q)]. \tag{24}$$

Figure 5: Limit cycles



Hence,

Proposition 3 *If $f'(q) > -1$, the equilibrium converges to the stationary point. If $f'(q) < -1$, the equilibrium converges to a two-period stable limit cycle.*[14]

Proof. See the Appendix.

Since we will focus on limit cycles, their mechanics is worth some further elaboration.[15] If a limit cycle exists, then long-run equilibrium prices would fluctuate from q^L to q^H and vice versa. A low spot price, q^L, indicates that the demand for new capital is at a relatively low level. That is due to a relatively large amount of capital coming from liquidations and offered for sale in the second-hand market. In such a period, the production of the consumption good is also relatively low due to so much premature liquidation. It seems reasonable to dub

[14] This proposition does not show that the limit cycle is unique. However, simulating the model, we never found more than one limit cycle. In case of several limit cycles, the results above apply to the one next to the stationary point.

[15] We focus on limit cycles just because this is where the results are most dramatic, as all effects survive for the long run. A weaker version of the results can be derived by examining the dynamics of the system around its (stable) stationary point.

such a period a 'bust'. For similar reasons, we dub a period with high capital prices a 'boom'.

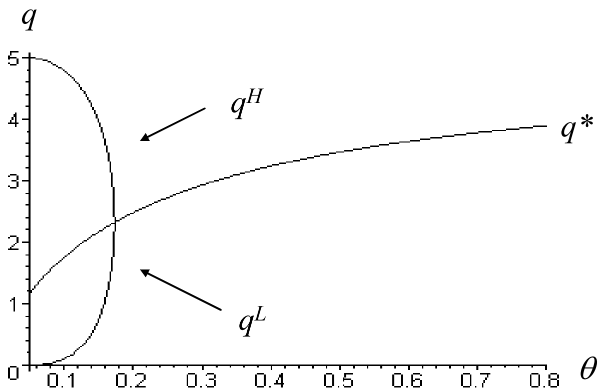
Since each entrepreneur operates for two periods, each goes through both a boom and a bust. However, entrepreneurs differ greatly according to the period at which they start up. Someone starting up in a boom would operate under tighter financial conditions, relative to a bust start-up: she has to borrow a larger amount in order to finance the purchase of expensive capital, and has to repay the debt when capital prices are depressed. Even if she is not distressed, she has a stronger incentive to default strategically; if distressed, her capital will be liquidated at a lower price (see Figure 5). Hence, to get funding, she will have to mortgage a larger fraction of her project (see the partial derivatives in (13)).

Hence, the mechanics of our equilibrium cycle: suppose that period t is a boom. Then period-t start-ups would be forced to pledge a large amount of collateral against the funds they borrow. As lenders foreclose all the collateral of distressed entrepreneurs, the supply of second-hand capital at $t+1$ would increase, depressing its price and pushing the economy into the bust. However, the low price of capital would ease financial conditions for $t+1$ start-ups, which would be able to borrow against less collateral. But then, the supply of second-hand capital at $t+2$ would be relatively small, which would push capital prices and production back into a boom. And so on.

More insight into the mechanics of the cycle can be obtained with the aid of Figure 6, a bifurcation diagram. The model is simulated with the following parameters: $\pi = 0.6$, $\lambda = 0.5$, $\delta = 0.10$, $\alpha = 5$ and $y = 17.5$. One may verify that assumption (23) is satisfied. The stationary price q^* and the two periodic prices, q^L and q^H are plotted (when they exist) against various levels of θ. As one should anticipate from Figure 4, the stationary point increases with θ, as the g function moves outwards. Also consistent with Figure 4 is that as θ increases, the slope of the g curve increases and shifts in the β curve (due to changes in q_t) tend to have smaller effects on q_{t+1}. The f function becomes flatter, excluding periodic equilibria for high θs. Intuitively, when the price-elasticity of the supply of new capital increases, the price of capital becomes less sensitive to changes in the supply of second-hand capital so cyclicality is less pronounced. For low θs, large price changes generate great variability in financial constraints along the cycle, and feed back into the cycle itself.

Figure 6: Bifurcation diagram

π=0.6, λ=0.5, δ=0.1, α=5, y=17.5



It is noteworthy that our model seems to capture the essence of the notion of 'financial instability'. Consider an economy with a stable limit cycle, and an initial condition that is just off the stationary point q^*. Since the stationary point is not stable (see Proposition 3), the economy will start oscillating away from q^*. Initially, the amplitude of these oscillations is very small; we can make them as small as we wish by bringing the initial point closer to q^*. But gradually, the cycle will build up until it converges to the limit cycle. This build-up of 'instability' is wholly endogenous, and will take place without the economy absorbing any exogenous shock. Rather, market prices coordinate lenders and borrowers into collateral positions that amplify the cycle. Moreover, the whole equilibrium is driven by financial factors: we know, from Proposition 1 that without financial frictions in the form of non-verifiable cash-flows, the system would converge up-front to an equilibrium with stable prices and output (and with higher net income).

6 Bankruptcy-law reform

Since the fluctuations in our model are driven by the endogenous dynamics of collateralized borrowing and asset liquidation, it is interesting to examine how court involvement in the resolution of financial distress would affect the business cycle. As noted above, authors such as [12] have argued that once bankruptcy law is analyzed within a proper general-equilibrium framework, the rationale for Chapter 11 becomes evident. Our model seems to provide *prima facie* support to such a claim: since liquidations are driven by financial distress rather than negative continuation values, and since distressed asset-sales have an adverse price effect, which by itself tightens the financial constraints of other business (of the same cohort), a softer bankruptcy law might help to stabilize the economy. However, the analysis below demonstrates the fallacy of this argument in a dynamic equilibrium framework.

Our model offers two ways through which court involvement can soften bankruptcy law. Firstly, the court can dilute the liquidation rights of the secured lenders, disallowing them to exercise their rights on a certain part of the collateral. Thus, the effective collateral, β, decreases below the nominal collateral, β^N. Secondly, the court can establish bargaining procedures that favor the borrower. In the context of our model, decrease λ. We analyze both formalizations.

In line with the observations of [3], we assume that the reform is announced when the economy is already in a bust. We also assume that the reform is not anticipated in advance, so that the current debt contracts were signed under the expectation that the contract would be implemented under the old law. Clearly, in a situation like that contracts might be renegotiated, which previously happened only off the equilibrium path. One might think that in such a case it matters whether the parties renegotiate before or after the uncertainty about financial distress is resolved. As a matter of fact, in our case it makes no difference and we thus assume, without loss of generality, that renegotiations take place ex-post, after the uncertainty is resolved.

6.1 A dilution of liquidation rights

Let β^N be the lender's 'nominal' liquidation rights as written in the debt contract, and assume that the bankruptcy court cancels a fraction ξ while enforcing the contract, so that the 'effective' liquidation rights are

$$\beta = (1 - \xi)\,\beta^N. \tag{25}$$

Proposition 4 *A small dilution factor ξ has no long-run effect. Introducing it in the bust produces a transitory stabilizing effect.*

Proof. To see why the long-run equilibrium is unaltered, note that if the nominal collateral is $\beta^N = \beta / (1 - \xi)$, then after dilution the effective collateral remains the same, and the program (6)-(10), remains the same in terms of effective collateral. Hence, the equilibrium conditions in Definition 1 remain unaltered, and so is the long-run equilibrium. The argument holds for small ξs only because the inflation of nominal collateral may lead to the violation of the feasibility condition $\beta^N \leq 1$.

In the short run, however, before contract terms are fully adjusted, the dilution decreases the effective collateral supporting the existing credit relationships: the incentive constraint (8) no longer holds so that the repayment R will have to be renegotiated downwards.[16] The smaller amount of liquidation will have a positive, short-run effect on prices, capital production, and output. Clearly, if the reform takes place in the bust, the effect is stabilizing.■

In terms of Figure 3, the dilution would move the effective β downwards, away from the optimal-contract point. Being below the IC line, the parties will have to adjust the repayment R, moving horizontally. Note that the renegotiated contract will be below the original participation constraint, which implies that the

[16] Recall that in the absence of unexpected events such as a reform in bankruptcy law, contract renegotiations are an off-the-equilibrium-path phenomenon.

law-reform generates a one-off wealth transfer from lenders to borrowers, which explains the short-run stabilizing effect of the policy. In the long run, both the IC and the PC curves are not affected.

6.2 Giving the borrower more bargaining power

In the rest of the paper, we make extensive use of the following representation of the periodic equilibrium: the periodic prices q^L and q^H solve

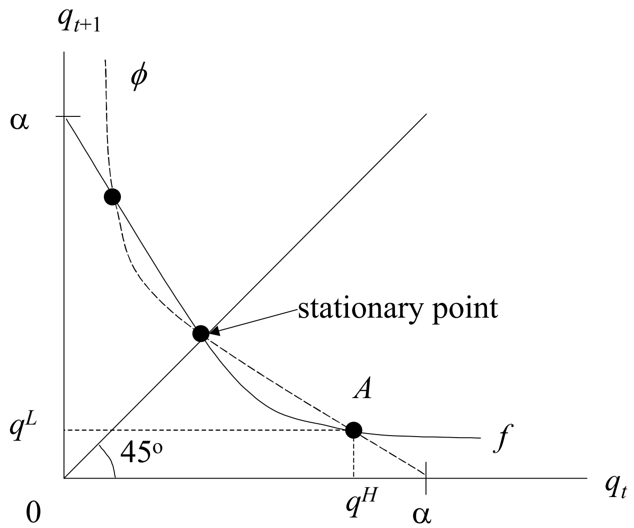
$$q^L = f\left(q^H; \lambda\right) \tag{26}$$

and

$$q^H = f\left(q^L; \lambda\right). \tag{27}$$

In words, (26) maps q^H into q^L and (27) maps q^L back into q^H. These two equations are described in Figure 7. The bold line is the graph of the f function with its argument on the horizontal axis. The dashed line is the graph of the f function with its argument on the vertical axis.

Figure 7: A periodic equilibrium



We can now analyze the effect of a reform that softens bankruptcy law by giving more bargaining power to the borrower:

Proposition 5 *For a 'moderately-cyclical' economy, a reform that gives the borrower more bargaining power will increase the long-run amplitude of the business cycle; this reform will have no short-run effects.*

Proof. See the Appendix.

Intuitively, when λ decreases the IC line of Figure 3 would rotate counterclockwise. Again, repayments will be renegotiated, moving the contracts horizontally and towards the left, at exactly the same β, so that the one-off wealth transfer from lenders to borrowers will have no short-run effect on the amount of liquidation. In the long run (for given market prices), the feasible set will shrink. The borrower will have to pledge a greater fraction of her capital as collateral in order to commit herself not to default strategically, now that she has more bargaining power. With more collateral there will be more liquidations, a stronger effect of financial distress on the price of capital and a greater cyclicality of the economy.

If a real-world softening of bankruptcy law (such as, say, the introduction of Chapter 11 in the US) is a mixture of the two formalizations that we have just explored, then introducing it in the bust will have a short-run stabilizing effect, but it will increase cyclicality in the long run.[17] So it seems that such a policy is a dubious stabilizing policy, specially once its short-run benefits pass and its dynamic general equilibrium implications come into effect.

[17] This result leaves two possible (not necessarily mutually exclusive) interpretations for the observation that US bankruptcy law was softened during busts ([3]). The first is a political-economy one: a Chapter-11 type of law reform is a policy introduced by a myopic government that simply ignores the long-run effects. The second interpretation is a more of a historical-institutional one: initially, bankruptcy legislation was an *ad-hoc* debt relief policy, employed by a government that still lacked more refined instruments such as monetary and fiscal policy. The application of the law created legal precedents and ended up being implemented both in booms and in busts, exacerbating the economy's cyclicality.

7 Other stabilizing policies

The absence of stabilizing long-run effects in bankruptcy law reform is due to the adjustment of contract terms that follows the rational anticipation of the way the contract will be enforced under the new law. However, rational expectations and subsequent adjustments in the equilibrium do not invalidate all possible stabilizing policies. In particular, in this section we show that bail-outs and an active management of interest rates along the cycle (monetary policy?) can have long-run stabilizing effects. The key difference is that these policies involve a sustained transfer of wealth from less to more financially-constrained agents.[18]

7.1 Bail outs

Suppose that the government initiates a policy of recompensing the debt of (some) borrowers in the bust (remember that bust borrowers are those that having started up during the boom, when the price of capital is high, carry over a larger amount of collateralized debt). Suppose that the subsidy is funded by a lump-sum tax on the businesses that mature during the boom (i.e. those started up during the bust, when the price of capital is low). Of course, the government operates under the same informational constraints as the private sector: cash-flows are non-verifiable. Thus the government cannot run a bail-out policy exclusively for the firms that declare default since in such case all firms would declare default. So we assume that the government recompenses the debt of a fraction ψ of all companies, chosen at random. In contrast, the tax can only be levied on the firms serving their debt, whose incentive constraint will be tightened by this additional repayment requirement.

From the description above it should be clear that the tax, T, inserts a wedge between what a borrower pays, say $R' = R + T$, and what its lender gets, R. Substituting R' for R in the incentive constraint (8), working through the contract problem again, substituting the result in the equilibrium condition, and focusing

[18] Our model has no money and thus no monetary policy. Still, it is reasonable to expect that real-world monetary policy might achieve the effect on real interest rate charges that our 'active interest-rate policy' requires.

on a periodic equilibrium, we get

$$\left(\frac{q^H}{\alpha}\right)^\theta = 1 - c\frac{q^L + \pi T}{aq^H + b}. \tag{28}$$

Note that, for given prices of capital, the tax tightens the incentive constraint and thus forces an increase in the collateral taken up by the lender.

For the subsidized cohort of entrepreneurs, the bail-out policy decreases the probability of default from $(1-\pi)$ to $(1-\pi)(1-\psi)$. Substituting the new probabilities of success and failure into the participation constraint (7) and taking the same steps as with equation (28) we get

$$\left(\frac{q^L}{\alpha}\right)^\theta = 1 - (1-\psi)\, c\frac{q^H}{aq^L + b + \lambda\psi(1-\pi)\left[y - (1-\delta)\, q^L\right]}. \tag{29}$$

Bail-outs relax the participation constraint of the entrepreneurs who start producing in the bust and thus (for given prices of capital) allow for a reduction in the amounts of collateral that they offer. Additionally, as less projects are liquidated (notice that parameter c is factored by $1-\psi$), even less capital good gets finally liquidated.

To close the system, we assume that the government balances its budget in every period so

$$\psi R^L = \pi T, \tag{30}$$

where

$$R^L = \frac{q^H\left[\lambda y + (1-\lambda)(1-\delta)\, q^L\right]}{aq^L + b + \lambda\psi(1-\pi)\left[y - (1-\delta)\, q^L\right]} \tag{31}$$

is the debt-repayment of bust entrepreneurs.

Proposition 6 *A small-scale bail-out policy would decrease the long-run amplitude of the business-cycle.*

Proof. See the Appendix.

7.2 Active interest-rate policy

The structure of an active interest-rate policy is even simpler: bust repayments are subsidized, while boom repayments are taxed. Namely, prices are determined by the system

$$q^H = f^H \left(q^L; T\right), \tag{32}$$

$$q^L = f^L \left(q^H; s\right), \tag{33}$$

where T and s must satisfy the government's budget constraint

$$\pi \left(T - s\right) = 0. \tag{34}$$

The functions f^H and f^L are implicitly defined by equations with the same form as (28) (where, in the case of f^L, $-s$ replaces T).

Proposition 7 *A small-scale active interest-rate policy would decrease the amplitude of the business-cycle in the long run.*

Proof. See the Appendix.

Why are the results in this section so different than those of the previous section? The reason is that the fluctuations in this economy are driven by financial constraints. Financial constraints can be relaxed through wealth transfers from the less constrained to the more constrained agents –in our case, from the entrepreneurs who start producing in the boom to those who start producing in the bust. As we have seen, bankruptcy-law reform does not generate such an effect. If anything, it makes the incentive constraint more binding and, thus, exacerbates the problems associated with financial constraints. More "traditional" measures such as interest-rate or bail-out policies perform the stabilizing role more effectively.

8 Welfare analysis

What are the welfare properties of the type of stabilizing policies described above? In answering this question, we must draw a clear line between those welfare-improving mechanisms that could be directly implemented by private agents and those that can only be implemented by the government. This distinction is more difficult to make within incomplete-contracts models, since some of the policies may consist in widening an implicitly restricted set of contracting possibilities.

Indeed, the contract we have considered leaves room for a type of improvement that government subsidies and taxes might achieve but, in principle, private agents might also achieve by themselves. Specifically, consider a subsidy to the initial investment s^I, so that the debt contract becomes $\beta\left(q_t - s^I, q_{t+1}\right)$; since investment always equals one, this subsidy requires a government budget $B = s^I$. For given market prices and a 'small' subsidy, the marginal effect on β of this kind of government expense is

$$\frac{d\beta}{dB} = -\frac{1}{aq_{t+1} + b}. \tag{35}$$

Now, consider an alternative subsidy s^L to liquidation prices under which the debt contract features $\beta\left(q_t, q_{t+1} + s^L\right)$ and the required government budget is $B = c\beta\left(q_t, q_{t+1} + s^L\right) s^L$. Then

$$\frac{d\beta}{dB} = -\frac{a}{c}\frac{1}{aq_{t+1} + b}. \tag{36}$$

Since $a > c$, it follows that a balanced-budget policy combining a tax on the initial investment of a given generation of entrepreneurs with a subsidy that supports the liquidation prices of their projects could reduce the dead-weight losses associated with their debt contracts. Notice, however, that this policy could be replicated by the private agents without the need of government intervention. In essence, entrepreneurs could borrow in excess of the initial cost of investment, keep the difference in a safe account and commit it to indemnifying lenders against credit losses. Without contradicting our initial constraints on contract design, establishing a cash payment contingent on the verifiable event of liquidation should be feasible. This self-provided insurance scheme has pure strategic value: facing a higher liquidation value, the lender would be a tougher bargainer vis-a-vis borrowers who defaulted strategically, but then the amount of collateralized assets necessary to prevent strategic default would diminish and, thus, the amount of capital liquidated in the case of genuine distress.

Interestingly, the above opportunity to improve on the original debt contract vanishes when the lender already has all the bargaining power, $\lambda = 1$, since then $a = c$. It is easy to check that with $\lambda = 1$ interest-payment subsidies such as those analyzed in a previous section are as effective (per unit of government expense) as the two subsidies considered above. Hence we will simplify the discussion on tax/subsidy schemes by focusing on the case with $\lambda = 1$. On the other hand, bailouts are in general less effective than more targeted tax/subsidy schemes as they allocate a significant part of the budget to firms which are not cash-constrained, so we shall not consider them any further. Finally, softer bankruptcy laws will not be considered either as we have shown that their stabilizing effect is, at best, limited to the short-run.

We thus focus on tax/subsidy schemes based on the subsidization of the initial investment of some entrepreneurs and ask whether they bring about a Pareto-improvement in the *laissez-faire* equilibrium (16)-(18). The definition of constrained Pareto optimality that we use deserves some comments. First, we consider the welfare of both entrepreneurs and the workers employed in the capital-good industry. As lenders always receive the (zero) market rate of return, they can be safely ignored. Second, we assume that lump-sum taxes can be imposed on workers, but not on entrepreneurs. The reason is that the government faces the same enforcement problem as the lenders, so it could only extract cash from the entrepreneurs that revealed themselves as not being distressed, that is, by imposing a tax on debt repayments. But taxing debt repayments will undo the effect of the subsidy to the initial investment.

A first question to analyze is whether a subsidy to the initial investment s, financed by lump-sum taxes on the workers T, may increase the price of the capital good and, hence, wages w sufficiently so as to compensate the workers for the tax imposed on them. That would require a strong 'multiplier effect': start-up subsidies would decrease collateral requirements and thus liquidations, this would increase the demand for new capital and thus the price of capital. This, in turn, will improve enforceability (see equation (13)), decrease liquidation even further, and cause a further increase in the price of capital. Although financial imperfections might, in principle, allow for such a multiplier effect to be welfare increasing, this is not the case.

Consider first a stable steady state equilibrium with

$$q = f\left(q - s\right), \qquad w = \left(1 - \gamma\right) q \left(\frac{q}{\alpha}\right)^{\theta} - T, \qquad T = s. \tag{37}$$

It tuns out that

$$\frac{\partial w}{\partial q} = \left(\frac{q}{\alpha}\right)^{\theta} < 1, \tag{38}$$

so a necessary condition for the existence of a Pareto improvement is $\frac{dq}{ds} > 1$. However,

$$1 > \frac{dq}{ds} \mid_{s=0} = -\frac{f'\left(q\right)}{1 - f'\left(q\right)} > 0 \tag{39}$$

since stability requires $-1 < f'(q) < 0$.

A similar, albeit more cumbersome, reasoning applies to a periodic equilibrium. In this case it is convenient to describe the tax-subsidy scheme as consisting of a subsidy τs for the boom cohort and a subsidy $\left(1 - \tau\right) s$ for the bust cohort, with $\tau \in [0, 1]$, so that

$$q^L = f(q^H - \tau s), \quad q^H = f(q^L - \left(1 - \tau\right) s). \tag{40}$$

A Pareto improvement would require that both boom and bust start-ups are better off

$$\frac{dv(q^H - \tau s, q^L)}{ds} > 0, \quad \frac{dv(q^L - \left(1 - \tau\right) s, q^H)}{ds} > 0, \tag{41}$$

where v is defined in (12), and that both boom and bust workers are also better off[19]

$$\left(\frac{q^H}{\alpha}\right)^\theta \frac{dq^H}{ds} + \left(\frac{q^L}{\alpha}\right)^\theta \frac{dq^L}{ds} > 1. \tag{42}$$

We can now prove:

Proposition 8 *With $\lambda = 1$, any long-term equilibrium, periodic or stationary, is constrained Pareto efficient.*

Proof. See the Appendix.

Thus, in what sense might a stabilizing policy be socially desirable? Apart from additional non-modeled considerations (such as imperfect consumer-debt markets that do not allow long-lived workers to smooth consumption over the cycle), one might think that a stabilizing policy that spreads financial constraints more evenly over the boom and the bust might decrease their overall 'average' effect upon the economy. We prove that this is actually the case, at least for the case where f is concave, which is usually the case for periodic equilibria (see Figure 6).

Proposition 9 *Suppose that f is concave. With $\lambda = 1$, a stabilizing interest-rate policy (namely a subsidy to more-constrained start-ups financed by a tax on less-constrained start-ups) will increase the through-the-cycle value of the entrepreneurial sector. Such a policy, however, may make workers worse off.*

Proof. See the Appendix.

9 Conclusions

In this paper we provide an analysis of the interrelations between cyclical movements in financial distress and various policy measures intended to mitigate the consequences of financial distress, including bankruptcy law. The analysis is developed in the context of a dynamic model of entrepreneurial financing where

[19] As workers can be lump-sum taxed, we can abstract from the exact allocation of taxes and welfare across boom and bust participants.

asset liquidations are a second-best implication of the existence of a contract enforcement problem. We show the contribution of this problem to the emergence of cyclical movements in the economy. Our main finding is that softening bankruptcy law produces no reduction in cyclicality, while other, more traditional measures such as like bail-outs, active interest-rate policies or various subsidies to financially constrained firms may have stabilizing effects and increase aggregate net income.

The model is deliberately simple since the joint analysis of financial distress, bankruptcy law, and macroeconomic stability in a dynamic general-equilibrium setup can easily become mathematically intractable. Future research could relax some of our assumptions in order to gain realism. For example, one could consider more realistic distributions of project cash flows, allow for a verifiable component in them, and add states of nature where projects have a negative continuation value so that liquidation is socially efficient. Another challenge is to model projects that last more than two periods; in that case, the relevant investment and liquidation decisions might depend on the entire path of future prices, leading to more complex (and possibly more realistic) dynamics of liquidation and prices. Such a model should surely be analyzed numerically.

Appendix

Proof of Lemma 1. We first want to guarantee that the non-rationing constraint (18) holds for every possible pair (q_t, q_{t+1}) in the price sequence produced by the remaining equilibrium conditions. We know that prices never exceed α. Also $\beta(q_t, q_{t+1})$ is increasing in q_t and decreasing in q_{t+1}, so the price pair that makes credit-rationing most likely is $(\alpha, 0)$. But $\beta(\alpha, 0) \leq 1$ if and only if $\alpha \leq b$.

We next want to prove that $\alpha \leq b$ is a sufficient condition for $g(q_{t+1})$ and $\beta(q_t, q_{t+1})$ to cross at least once for any given $q_t \leq \alpha$. To see this, notice that, the graph of g crosses the vertical axis at $1/c$ and the horizontal axis at α, and it is concave for $\theta > 1$ and convex for $\theta < 1$. The graph of β crosses the vertical axis at q_t/b and falls asymptotically towards the horizontal axis; it is always convex and shifts upwards when q_t increases. Thus, if $1/c > \alpha/b$ then the intercept of g is above the intercept of β for any $q_t \leq \alpha$ and the existence of at least one intersection between the two graphs is guaranteed. But $1/c > \alpha/b$ is implied by $\alpha \leq b$ since $c < 1$.

- When $\theta > 1$, the concavity of g and the convexity of β directly imply that the intersection between the two graphs is unique, so there exist a well defined value of $f(q_t)$ for all $q_t \leq \alpha$.

- When $\theta < 1$, both graphs are concave and several intersections might, in principle exist. This will not be the case, however, if, at any possible intersection, the slope of the g graph is higher than the slope of the β graph, that is,

$$-g'(q_{t+1}) > \frac{\partial \beta}{\partial q_{t+1}} = \frac{a\beta(q_t, q_{t+1})}{(aq_{t+1} + b)^2}. \tag{43}$$

  Now, due to the convexity of g, we have

$$-g'(q_{t+1}) > \frac{g(q_{t+1})}{\alpha - q_{t+1}}. \tag{44}$$

  But in an intersection we will have $g(q_{t+1}) = \beta(q_t, q_{t+1})$ so having

$$\frac{1}{\alpha - q_{t+1}} > \frac{a}{(aq_{t+1} + b)^2} \tag{45}$$

is a sufficient condition for uniqueness. It turns out that the above inequality holds if $\frac{b}{\alpha} > a$, which, in turn, is implied by $\alpha \leq b$ since $a < 1$.

Finally, we want to show the existence of a unique stationary price q^*. Formally, from (19), a stationary price must satisfy

$$g(q^*) = \frac{q^*}{aq^* + b}. \tag{46}$$

We already know that g is decreasing, taking value $1/c$ at $q = 0$ and 0 at $q = \alpha$. The expression in the right hand side is increasing, taking value 0 at $q = 0$ and $\alpha/(a\alpha + b)$ at $q = \alpha$. So a unique intersection exists, which corresponds to the stationary price q^*.■

Proof of Proposition 3. By standard arguments, if $f'(q) > -1$, the equilibrium price sequence converges to the stationary price q^*. If $f'(q) < -1$, it would converge to a limit cycle. To see why, note that prices are strictly positive along the equilibrium path, so $f^2(0) > 0$.

Now,

$$f^{2'}(q^*) = f'[f(q^*)] \cdot f'(q^*) = [f'(q^*)]^2. \tag{47}$$

It follows that if $f'(q^*) < -1$, then $f^{2'}(q^*) > 1$ (see Figure 5), so that f^2 has another fixed point, q^L, within $(0, q^*)$. If $f\left[f\left(q^L\right)\right] = q^L$, then there must exist yet another point, $q^H = f\left(q^L\right)$, within (q^*, α), such that $f(q^H) = q^L$ and thus $f(f(q^L)) = q^L$. Hence the limit cycle.

It follows that at q^L, f^2 must intersect with the diagonal from above (see Figure 5). Hence $f^{2'}\left(q^L\right) < 1$, which is a sufficient condition for the stability of the limit cycle.■

Proof of Proposition 5. We start with the long-run effect. Differentiating the system (26)-(27), one can compute

$$\frac{dq^H}{d\lambda} = -\frac{f_q\left(q^H;\lambda\right)\frac{f_\lambda\left(q^H;\lambda\right)}{f_q\left(q^H;\lambda\right)} + \frac{f_\lambda\left(q^L;\lambda\right)}{f_q\left(q^L;\lambda\right)}}{f_q\left(q^H;\lambda\right) - \frac{1}{f_q\left(q^L;\lambda\right)}}. \tag{48}$$

Note that we have

$$\frac{dq^i}{d\lambda}\mid_{f(q^i;\lambda)=const}= -\frac{f_\lambda\left(q^i;\lambda\right)}{f_q\left(q^i;\lambda\right)}, \tag{49}$$

for $i = L, H$, so (48) can be written as

$$\frac{dq^H}{d\lambda} = \frac{f_q\left(q^H;\lambda\right)\cdot\frac{dq^H}{d\lambda}\mid_{f(q^H;\lambda)=const}+\frac{dq^L}{d\lambda}\mid_{f(q^L;\lambda)=const}}{f_q\left(q^H;\lambda\right)-\frac{1}{f_q(q^L;\lambda)}}. \tag{50}$$

Equation (50) has the following graphical interpretation: the denominator is the difference between the slopes of the f and the ϕ functions at point A of Figure 7, the numerator is the sum of the vertical shifts in f and ϕ due to the change in λ. The denominator of (50) must be positive: by Proposition 3, f is steeper than ϕ at the stationary point, so at point A, ϕ is steeper than f —recall that both derivatives are negative.

Also one can clearly see that

$$\frac{dq^H}{d\lambda}\mid_{f(q^H;\lambda)=const}= \pi q^H\frac{y-(1-\delta)\,q^L}{aq^L+b} > \pi q^L\frac{y-(1-\delta)\,q^H}{aq^H+b} = \frac{dq^L}{d\lambda}\mid_{f(q^L;\lambda)=const} \tag{51}$$

but, in a moderately cyclical economy q^H will be close to the stationary point, implying that $f_q\left(q^H;\lambda\right) < -1$, so the numerator of equation (50) is negative and thus, $\frac{dq^H}{d\lambda} < 0$.

By a parallel reasoning, it also follows that $\frac{dq^L}{d\lambda} > 0$. Namely, the amplitude of the cycle falls when λ increases. Remember that λ measures the lender's power. Hence, a softer system (lower λ) would increase the amplitude of the business cycle.

As for the short-run, note that after an unanticipated fall in λ the incentive constraint (8) no longer holds and repayments, R, will be renegotiated downwards following strategic default. However collateral and, thus, liquidations in case of financial distress will not vary.■

Proof of Proposition 6. The logic of the proof is the same as in Proposition 5. Let the functions f^H and f^L be implicitly defined by (28) and (29) so as to describe situations with bail-outs and taxes, respectively:

$$q^H = f^H\left(q^L;T\right) \quad \text{and} \quad q^L = f^L\left(q^H,\psi\right). \tag{52}$$

Note that with $\psi = 0$, we would have $f^H = f^L = f$. We consider a small-scale bail-out policy as a perturbation of Figure 5, which depicts the $\psi = 0$ case. Similar to the differential (50) we now have

$$\frac{dq^H}{d\psi} = \frac{f_q^L\left(q^H;\psi\right) \cdot \frac{dq^H}{d\psi} \mid_{f^L(q^H;\psi)=const} + \frac{dq^L}{dT} \mid_{f^H(q^L;T)=const} \cdot \frac{dT}{d\psi}}{f_q^L\left(q^H;\psi\right) - \frac{1}{f_q^H(q^L;T)}}, \tag{53}$$

where

$$\frac{dq^H}{d\psi} \mid_{\substack{f^L\left(q^H;\psi\right)=\text{constant} \\ \psi=0}} = \frac{q^H\left(aq^L+b\right) + \lambda\left(1-\pi\right)\left[y-\left(1-\delta\right)q^L\right]}{\left(aq^L+b\right)} > 0. \tag{54}$$

From the government's budget constraint (30), we have

$$\frac{dq^L}{dT} \mid_{\substack{f^H\left(q^L;T\right)=const \\ \psi=0}} \cdot \frac{dT}{d\psi} = -R^L < 0. \tag{55}$$

But we already know that the denominator of (53) is positive so we conclude that

$$\frac{dq^H}{d\psi} \mid_{\psi=0} < 0. \tag{56}$$

The geometrical interpretation of the result is that both curves in Figure 5 move leftward at point A, so unambiguously q^H falls when ψ increases. For similar reasons, q^L increases when ψ increases.■

Proof of Proposition 7. The proof is very similar to the previous one. We have:

$$\frac{dq^H}{dT} = \frac{f_q^L\left(q^H;s\right) \cdot \frac{dq^H}{ds} \cdot \frac{ds}{dT} \mid_{f^L(q^H;s)=const} + \frac{dq^L}{dT} \mid_{f^H(q^L;T)=const}}{f_q^L\left(q^H;s\right) - \frac{1}{f_q^H(q^L;T)}}, \tag{57}$$

which is negative since

$$-\frac{ds}{dT} \mid_{f^L(q^H;s)=const} = \frac{dq^L}{dT} \mid_{f^H(q^L;T)=const} < 0 \quad \text{and} \quad \frac{ds}{dT} = 1. \blacksquare \tag{58}$$

Proof of Proposition 8. The case of a stationary equilibrium is already discussed above. As for a periodic equilibrium, we start by noting that when $\lambda = 1$, the entrepreneur's welfare function can be reduced to

$$v\left(q_t, q_{t+1}\right) = 2y - y\left(2-\pi\right)\beta\left(q_t, q_{t+1}\right). \tag{59}$$

Hence, condition (41) can be rewritten as

$$\frac{d}{ds}\beta\left(q^H - \tau s, q^L\right) < 0, \ \frac{d}{ds}\beta\left(q^L - (1-\tau)s, q^H\right) < 0. \tag{60}$$

Differentiating and evaluating the derivative at $s = 0$, we get

$$\frac{dq^H}{ds} - \tau < \beta\left(q^H, q^L\right)\frac{dq^L}{ds}, \ \frac{dq^L}{ds} - (1-\tau) < \beta\left(q^L, q^H\right)\frac{dq^H}{ds}. \tag{61}$$

By plotting the two conditions in $\left(dq^L/ds, dq^H/ds\right)$ space, it is possible to see that they are satisfied within a cone that opens towards the south-west quadrant; the points $(0, \tau)$ and $(0, -(1-\tau))$ lie on the boundaries of that cone. In the same space, one can plot condition (42) and see that it is satisfied above a downwards-sloping straight line passing through the points

$$\left(0, \frac{1}{1 - c\beta\left(q^L, q^H\right)}\right) \text{ and } \left(\frac{1}{1 - c\beta\left(q^H, q^L\right)}, 0\right). \tag{62}$$

Since $\tau < 1$ and $1/\left[1 - c\beta\left(q^L, q^H\right)\right] > 1$, it follows that if a Pareto improving subsidy exists, then both

$$\frac{dq^H}{ds} > 0 \text{ and } \frac{dq^L}{ds} > 0. \tag{63}$$

In such a case, a necessary condition for (42) is that

$$\frac{dq^H}{ds} + \frac{dq^L}{ds} > 1. \tag{64}$$

We show now that this is never the case for a periodic equilibrium. Differentiating (40) we compute

$$\frac{dq^H}{ds} = \frac{-(1-\tau)f'\left(q^L\right) - \tau f\left(q^H\right)f\left(q^L\right)}{1 - f\left(q^H\right)f\left(q^L\right)}, \tag{65}$$

$$\frac{dq^L}{ds} = \frac{-\tau f'\left(q^H\right) - (1-\tau)f(1-\tau)\left(q^H\right)f\left(q^L\right)}{1 - f\left(q^H\right)f\left(q^L\right)}, \tag{66}$$

so that

$$\frac{dq^H}{ds} + \frac{dq^L}{ds} = \frac{-(1-\tau)f'\left(q^L\right) - \tau f'\left(q^H\right) - f\left(q^H\right)f\left(q^L\right)}{1 - f\left(q^H\right)f\left(q^L\right)}. \tag{67}$$

Note that $0 < f\left(q^H\right) f\left(q^L\right) = \frac{df^2(q^H)}{dq} < 1$ within a (stable) periodic equilibrium so that the denominator in (65) and (66) is positive. Hence, condition (63) can be written as

$$
\begin{aligned}
-(1-\tau) f'\left(q^L\right) &> \tau f\left(q^H\right) f\left(q^L\right) \text{ and} \qquad (68)\\
-\tau f'\left(q^H\right) &> (1-\tau) f\left(q^H\right) f\left(q^L\right), \qquad (69)
\end{aligned}
$$

while condition (64) can be written as

$$
-(1-\tau) f'\left(q^L\right) - \tau f'\left(q^H\right) > 1. \qquad (70)
$$

Obviously, the last condition is violated if both $-f'\left(q^L\right)$ and $-f'\left(q^H\right)$ are smaller than 1. However, within a periodic equilibrium one derivative can exceed 1 (in which case the other must be smaller than 1 due to the stability condition). Suppose, without loss of generality, that $-f'\left(q^L\right) > 1$. Then, we can best satisfy condition (70) by minimizing τ within the limits imposed by conditions (68) and (69). That means setting

$$
\tau = \frac{-f'\left(q^L\right)}{1 - f'\left(q^L\right)}. \qquad (71)
$$

However, substituting this value of τ into (70), we get

$$
\begin{aligned}
-(1-\tau) f'\left(q^L\right) - \tau f'\left(q^H\right) &= \frac{-f'\left(q^L\right)}{1-f'\left(q^L\right)} + \frac{f'\left(q^L\right) f'\left(q^H\right)}{1-f'\left(q^L\right)} \\
&< \frac{-f'\left(q^L\right)}{1-f'\left(q^L\right)} + \frac{1}{1-f'\left(q^L\right)} = 1. \qquad (72)
\end{aligned}
$$

So a Pareto improvement is impossible.■

Proof of Proposition 9. Given

$$
q^L = f\left(q^H - s\right), \quad \text{and} \quad q^H = f\left(q^L + s\right), \qquad (73)
$$

the effect of the tax on prices is

$$
\begin{aligned}
\frac{dq^L}{ds} &= -\frac{f'\left(q^H\right) - f'\left(q^L\right) \cdot f'\left(q^H\right)}{1 - f'\left(q^L\right) \cdot f'\left(q^H\right)} > 0, \qquad (74)\\
\frac{dq^H}{ds} &= \frac{f'\left(q^L\right) - f'\left(q^L\right) \cdot f'\left(q^H\right)}{1 - f'\left(q^L\right) \cdot f'\left(q^H\right)} < 0. \qquad (75)
\end{aligned}
$$

Due to the concavity of f, we have $\left|\frac{dq^H}{ds}\right| > \left|\frac{dq^L}{ds}\right|$. Now compute the effect of s on $\beta^H = \beta\left(q^H - s, q^L\right)$ and $\beta^L = \beta\left(q^L - s, q^H\right)$:

$$\frac{d\beta^H}{ds} = \frac{1}{aq^L + b}\left[\frac{dq^H}{ds} - 1 - a\beta^H \frac{dq^L}{ds}\right] < 0, \tag{76}$$

and

$$\frac{d\beta^L}{ds} = \frac{1}{aq^H + b}\left[\frac{dq^L}{ds} + 1 - a\beta^L \frac{dq^H}{ds}\right] > 0. \tag{77}$$

However

$$\begin{aligned}\frac{d\beta^H}{ds} + \frac{d\beta^L}{ds} &= \left[\frac{1}{aq^H + b} - \frac{1}{aq^L + b}\right] + \frac{dq^H}{ds}\left[\frac{1}{aq^L + b} - \frac{a\beta^L}{aq^H + b}\right] \\ &\quad + \frac{dq^L}{ds}\left[\frac{1}{aq^H + b} - \frac{a\beta^H}{aq^L + b}\right].\end{aligned} \tag{78}$$

The first two terms are unambiguously negative. Even if the third is positive, since $\frac{1}{aq^L+b} - \frac{a\beta^L}{aq^H+b} > \frac{1}{aq^H+b} - \frac{a\beta^H}{aq^L+b}$, it follows that $\frac{d\beta^H}{ds} + \frac{d\beta^L}{ds} < 0$. Using (59) we conclude that the value of the entrepreneurial sector increases.

However, since boom-prices fall by more than bust-prices rise, and since $\left(\frac{q^H}{\alpha}\right)^\theta > \left(\frac{q^L}{\alpha}\right)^\theta$, the effect on wages throughout the entire cycle is negative (see (42)).■

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