Quantitative Macroeconomics
(with heterogeneous agents)

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Schedule.

Objective. This course introduces the techniques of modern quantitative macroeconomics to study economies with either (a) household heterogeneity –with a special focus on the life cycle dimension– or (b) firm heterogeneity –with a special focus on firm dynamics. One important aspect of the course is the emphasis on learning how to solve these economies in the computer. To this end, there is sequence of problem sets that will guide you to solve the canonical models of Aiyagari (1994) and Huggett (1996) for the household heterogeneity part, and Hopenhayn and Rogerson (1993) for the firm heterogeneity part.

Requirements. For this course, the main tools you need to understand are: (1) dynamic programming, (2) a little bit of measure theory, and (3) Markov chains. We will cover dynamic programming during the first week. All the measure theory you need to know for this course is contained in some short notes I will provide to you. But if you want more you can check (Stokey et al., 1989, chapter 7) or even better Royden (1988). For Markov chains, a good reference is (Ljungqvist and Sargent, 2004, chapter 2) and I will also give you some notes. A very comprehensive treatment can be found in (Stokey et al., 1989, chapter 8).

Structure. As with all CEMFI courses, we have 30 ninety-minute sessions spread over 10 weeks. My plan is to use 20 sessions for theory, 7 sessions to discuss your homework, and 3 sessions for the reading group.

Homework. There will be two types of problem sets: computer-based (4 of them) and paper-and-pencil (2 of them). Plus a mixed one. Computer-based problem sets have to be solved in (stable) teams of two, but only one copy per team needs to be handed in. Paper-and-pencil problem sets follow the standard rules: you can work in teams, but every student is responsible for submitting her own solutions. All problem sets will be discussed in class on the date of submission.

Computer languages. During the course you will have to do a substantial amount of programming. I do not care which language you use, it is your choice and your responsibility. Students taking a course like this at CEMFI and elsewhere tend to choose Julia or Matlab, but this might as well be a good moment to invest in learning Fortran. There will be a voluntary extra session on Julia during the first week.

Reading group. We reserve 3 sessions to discuss recent papers that relate to the topics covered in the course. Students will have to read the papers in advance, and a few students will be asked to present the papers. The idea is to cover two related papers in each session.

Books and references. There is no basic textbook for this course and most of the material comes from papers and chapters of different books. I list the basic references for each part on the next pages. Regarding numerical methods, easy introductions can be found in (Adda and Cooper, 2003, chaper 3) and (Ljungqvist and Sargent, 2004, chapter 4). In-depth coverage of some very useful methods for economists can be found in Marimon and Scott (1999). Judd (1998) is very comprehensive (encyclopedic) and is a very good reference. Finally, Heer and Maussner (2009) is a more recent textbook also worth looking at.

Teaching assistant. Francesco Chiocchio will work as TA for this course. He will take care of the paper-and-pencil sessions of the problem sets, he will teach the initial Julia session, and he will be available to you if you need help with the computational problem sets.

Evaluation. The final mark will be an average of the final exam (70%), the homework (25%), and the reading group sessions (5%).

More information. This syllabus, exercise lists, and any other supporting material can be found on the intranet (https://intranet.cemfi.es/). I will update its contents throughout the course.

Part I. Dynamic Programming.

Estimated duration: 3 theory sessions

1. Dynamic Programming

   The class lectures follow, loosely, (Ljungqvist and Sargent, 2004, chapter 3). You can also find an alternative coverage in (Adda and Cooper, 2003, chapters 2, 3 and 5). The ultimate reference (all the theorems and some proofs) is Stokey et al. (1989).

2. The neoclassical stochastic growth model: recursive formulation of the competitive equilibrium.

   Brock and Mirman (1972) and (Stokey et al., 1989, chapter 1)

Part II. The Heterogeneous Household Model.

Estimated duration: 3 theory sessions

1. What the RA Agent model cannot do
2. The inter-temporal consumption problem
   - The permanent income hypothesis
   - Uncertainty and the random walk
   - Uncertainty and precautionary savings
3. The heterogeneous agents model in steady state.

   Huggett (1993), Aiyagari (1994)

   For a textbook exposition see (Ljungqvist and Sargent, 2004, chapters 16 and 17).

Part III. Numerical Methods Applied to Heterogeneous Agents Economies.

Estimated duration: 3 theory sessions

1. Solving the household problem
   - Projection methods

     Judd (1992), (Judd, 1998, chapter 11) and McGrattan (1998)
   - A simple application: policy function iteration w/ piecewise linear approximation
2. Finding the steady state equilibrium.

- Finding the stationary distribution: Montecarlo simulation, Young's method

  Young (2010)

- Finding the equilibrium prices

  Aiyagari (1994) and Ríos-Rull (1998)

3. Accuracy

   Judd (1992)

4. Solving non-linear equations

   (Judd, 1998, chapter 5) or (Heer and Maussner, 2009, section 11.5)

Part IV. Some Extensions of the Heterogeneous Households Model.

Estimated duration: 3 theory sessions

1. Life cycle

   Huggett (1996)

2. Endogenous labor

   Pijoan-Mas (2006), Heathcote et al. (2010)

3. Discrete choices with extreme value shocks

4. Outside the Steady State

   Krusell and Smith (1998), Ríos-Rull (1998), Krusell and Smith (2006), Boppart et al. (2018)

Part V. Labour market uncertainty: characterizing labor earnings.

Estimated duration: 1 theory session

1. The standard income process and the evolution of earnings inequality

   Storesletten et al. (2001), Storesletten et al. (2004), Heathcote et al. (2010)

2. Heterogeneous income profiles

   Guvenen (2007), Guvenen (2009), and Guvenen and Smith (2014)

3. Non-linear earnings processes

   Guvenen et al. (2021), Arellano et al. (2017)

4. Endogenous earnings

   Huggett et al. (2011)

Part VI. Wealth Inequality.

Estimated duration: 2 theory sessions

1. Some facts
2. Non-linear earnings

   Castañeda et al. (2003), De Nardi et al. (2020)
3. Heterogenous returns to savings

   Angeletos (2007), Hubmer et al. (2021)

Part VII. Firm Heterogeneity.

Estimated duration: 5 theory sessions

A good survey on the topic can be found in Hopenhayn (2014b)

1. Some data
2. Entrepreneurship

   Lucas (1978), Guner et al. (2008)
3. Firm dynamics

   Hopenhayn (1992), Hopenhayn and Rogerson (1993), Restuccia and Rogerson (2008)
4. Misallocation

   Hopenhayn (2014a), Hsieh and Klenow (2009), Bartelsman et al. (2013)
5. Financial frictions

   Moll (2014), Midrigan and Xu (2014)

Part VIII. Reading Group.

Estimated duration: 3 sessions

1. Consumption responses to income changes

   (Week 7, Thursday)

   - F. Guvenen and A. Smith. Inferring labor income risk and partial insurance from economic choices. *Econometrica*, 82(6):2085–2129, 2014
   - G. Kaplan and G. Violante. A model of the consumption response to fiscal stimulus payments. *Econometrica*, 82(4):1199–1239, 2014

2. Health inequality

   (Week 8, Thursday)

   - J. Ameriks, J. Briggs, A. Caplin, M. Shapiro, and C. Tonetti. Long-term care utility and late in life saving. *Journal of Political Economy*, 128(6):2375–2451, 2020
   - M. De Nardi, S. Pashchenko, and P. Porapakkarm. The lifetime costs of bad health. NBER Working Paper 23963, 2022

3. Financial Frictions and Misallocation

   (Week 10, Thursday)

   - V. Midrigan and D. Xu. Finance and misallocation: Evidence from plant level data. *American Economic Review*, 104(2):422–58, 2014
   - F. Kochen. Finance over the life cycle of firms. Mimeo University of New York, 2023

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