Appendix

A Computational Procedures

This appendix explains the computer algorithm used to solve the model. The algorithm is based on the *partial information* approach used by Krusell and Smith (1997).[12] The model solved in this paper requires extending the algorithm in order to include an extra individual state variable, the habit stock h, and a new dimension in the aggregate distribution of households. These two issues justify the need to make explicit what is actually done in this paper.

The general strategy is as follows. I replace the endogenous state μ by its first moments K and H. Using only first moments, we have to replace the equilibrium condition *(vi)* in section 2.3 by

$$K' = f^K(z, K, H) \tag{9}$$

and introduce a new equation to predict aggregate habits

$$H' = f^H(z, K, H) \tag{10}$$

We also need to approximate $R^b(z, \mu)$, which is a direct function of the distribution of agents, so I postulate:

$$R^b = f^{R^b}(z, K, H) \tag{11}$$

Under this approximation, the state space of the household problem is reduced. In order to predict prices, instead of z and μ, consumers only need z, K and H. Of course, the forecasting rules $\left\{f^K, f^H, f^{R^b}\right\}$ are unknown and they are part of the solution. Therefore, solving the household problem implies maximizing equation (1) subject to the constrains (2), (3), (4) and (6) and to the forecasting rules (9), (10) and (11). The problem is that the forecasting rules f^K, f^H and f^{R^b} are not known. I start explaining how to solve the household problem for given forecasting rules and then I discuss how to find the forecasting rules consistent with a rational expectations equilibrium.

A.1 Solving the household's problem

For the household problem the state space is given by the individual vector $j = \{\omega, h, e, \xi\}$ plus the aggregate variables z, K and H. I collapse the three exogenous and stochastic state variables e, ξ and z into one variable ϵ that can take $n_\epsilon = n_z(n_\xi + 1) = 8$ different values. We are therefore left with the two endogenous individual state variables ω and h, the exogenous stochastic shock ϵ and the exogenous (at the household level) aggregate variables K and H. Let's define labor earnings in terms of the newly defined exogenous stochastic process ϵ as $\nu(\epsilon, K)$. Households have to solve the following system formed by

[12] They were already extending previous work by Castañeda, Díaz-Giménez, and Ríos-Rull (1998) and Krusell and Smith (1998). Ríos-Rull (1998) explains it in good detail.

the FOC, the constraints and the forecasting rules:

$$
\begin{aligned}
&0 = u_c(c,h) + \lambda\beta E_{\epsilon'|\epsilon}\left[v_h(\omega',h',\epsilon',K',H')\right] - \beta E_{\epsilon'|\epsilon}\left[v_\omega(\omega',h',\epsilon',K',H')\, R^s\left(\epsilon',K',R^b\right)\right]\\
&0 = u_c(c,h) + \lambda\beta E_{\epsilon'|\epsilon}\left[v_h(\omega',h',\epsilon',K',H')\right] - \beta E_{\epsilon'|\epsilon}\left[v_\omega(\omega',h',\epsilon',K',H')\, R^b\right]\\
&c = \omega - b - s\\
&h' = \lambda c + (1-\lambda)\, h\\
&\omega' = bR^b + sR^s(\epsilon',K') + \nu(\epsilon',K')\\
&K' = f^K(\epsilon,K,H)\\
&H' = f^H(\epsilon,K,H)\\
&R^b = f^{R^b}(\epsilon,K,H)\\
&c \geq 0;\quad b \geq \underline{b};\quad s \geq \underline{s};\quad b+s \geq \underline{\omega}
\end{aligned}
\tag{12}
$$

A standard way to solve this problem would be to use the envelope conditions to substitute out v_ω and v_h from the FOC and then solve for the unknown policy functions $\left\{g^c, g^b, g^s\right\}$. However, doing so with the habits dependence would imply adding variables dated up to the infinity and therefore losing the recursive properties of the problem. What I will do instead is to find a solution for the unknown function $\{v_\omega, v_h\}$ together with the policy functions $\left\{g^c, g^b, g^s\right\}$ as in a value function iteration algorithm.

For a given pair $\{v_\omega^0, v_h^0\}$, the system (12) delivers a set of policy functions $\left\{g^{0,c}, g^{0,b}, g^{0,s}\right\}$. Then, we can substitute them in the right hand side of the envelope conditions to obtain a new pair of derivatives for the value function $\{v_\omega^1, v_h^1\}$,

$$
v_\omega^1(\omega,h,\epsilon,K,H) = u_c(c,h) + \lambda\beta E_{\epsilon'|\epsilon}\left[v_h^0(\omega',h',\epsilon',K',H')\right] \tag{13}
$$

$$
v_h^1(\omega,h,\epsilon,K,H) = u_h(c,h) + (1-\lambda)\,\beta E_{\epsilon'|\epsilon}\left[v_h^0(\omega',h',\epsilon',K',H')\right] \tag{14}
$$

where c, b and s are replaced by $g^{0,c}(\omega,h,\epsilon,K,H)$, $g^{0,b}(\omega,h,\epsilon,K,H)$ and $g^{0,s}(\omega,h,\epsilon,K,H)$. The system of equations (12) together with the envelope conditions (13) and (14) define a mapping T from the cartesian product of the space where v_ω and v_h belong into itself. Solving the household problem amounts to finding a fixed point of this mapping, i.e., a pair such that $\{v_\omega^*, v_h^*\} = T\{v_\omega^*, v_h^*\}$. One problem with this approach is that the space where v_ω and v_h belong to is unknown. I need thus to specify a class of functions that the computer can understand in order to approximate for this space. I will do as follows. For every value of ϵ, I approximate $\{v_\omega, v_h\}$ piece-wise linearly in a four-dimensional grid.[13] Given an initial guess $\{v_\omega^0, v_h^0\}$, I solve the system (12) to get the policy functions $\left\{g^{0,c}, g^{0,b}, g^{0,s}\right\}$. Then, using the envelope conditions (13) and (14), I obtain a new pair $\{v_\omega^1, v_h^1\}$. If the new pair $\{v_\omega^1, v_h^1\}$ is close to $\{v_\omega^0, v_h^0\}$ I have found an approximation to the fixed point of the mapping T and I take $\left\{g^{0,c}, g^{0,b}, g^{0,s}\right\}$ as the solution of the model. If not, I update $\{v_\omega^0, v_h^0\} = \{v_\omega^1, v_h^1\}$ and start again. Notice that there is no contraction theorem for this mapping, which means that there is no guarantee to succeed by using this successive approximations approach. For the iterations to make good progress, it turns out to be very important to select proper initial conditions $\{v_\omega^0, v_h^0\}$.

[13] In the K and H dimension there is not much curvature, so I use fewer points than in the ω and h dimensions. I typically use 6 points for the aggregate variables, 60 for wealth ω and 33 for the habit stock h. This implies solving the system (12) at $570,240$ different points for every pair $\left\{v_\omega^0, v_h^0\right\}$.

A.2 Finding the equilibrium forecasting rules

The nature of the stationary stochastic equilibrium implies keeping track of a distribution μ in order to forecast future prices. The *partial information* approach is based on using just a subset of moments of μ instead. I will use only first moments. We need to find a vector of forecasting functions $f \equiv \left\{f^K, f^H, f^{R^b}\right\} \in \mathcal{F} \equiv \mathcal{F}^K \times \mathcal{F}^H \times \mathcal{F}^{R^b}$ consistent with rational expectations. I.e., given that agents forecast K, H and R^b with certain f, the simulated economy should display this same behavior. Or in other words, the simulated series for K, H and R^b should be *well* predicted by f. The idea is to start with an initial f^0, solve the household's problem defined in section A.1, simulate the economy for a long series of periods and estimate a new f^1 within the same parametric class $\mathcal{F}$. Krusell and Smith (1997) show that one needs to make one correction to this procedure. Precisely, the market for bonds does not clear in every period and state. In order to achieve the bond market clearing in every period and state, I define the following problem:

$$V\left(\omega, h, \epsilon, K, H, R^b\right) = \max_{c,b,s} \left\{u(c,h) + \beta E_{\epsilon'|\epsilon}\left[v\left(\omega', h', \epsilon', K', H'\right)\right]\right\} \tag{15}$$

subject to

$$\begin{aligned}
&c = \omega - b - s \\
&h' = \lambda c + (1-\lambda) h \\
&\omega' = bR^b + sR^s\left(\epsilon', K'\right) + \nu\left(\epsilon', K'\right) \\
&K' = f^K\left(\epsilon, K, H\right) \\
&H' = f^H\left(\epsilon, K, H\right) \\
&R^b = f^{R^b}\left(\epsilon, K, H\right) \\
&c \geq 0; \quad b \geq \underline{b}; \quad s \geq \underline{s}; \quad b + s \geq \underline{\omega}
\end{aligned}$$

This problem differs from the original one in the fact that R^b is a state variable for today, although tomorrow's R^b is perceived to follow the forecasting rule f^{R^b}. I.e., tomorrow's value function is given by problem (12). In this manner one can find an R^b that exactly clears the bond market. The solution to this problem delivers the policy functions $g^c\left(\omega, h, \epsilon, K, H, R^b\right)$, $g^b\left(\omega, h, \epsilon, K, H, R^b\right)$ and $g^s\left(\omega, h, \epsilon, K, H, R^b\right)$. At this stage I can state the algorithm as follows

1. Guess an initial f^0.
2. Solve the household' problem given by the system (12).
3. Simulate the economy,
   (a) set an initial distribution of agents over ω, h and ϵ,
   (b) Look for the R^b that clears the market for bonds. To do so, guess an initial $R^{b,0}$ and solve the problem (15) to find $g^c\left(\omega, h, \epsilon, K, H, R^{b,0}\right)$, $g^b\left(\omega, h, \epsilon, K, H, R^{b,0}\right)$ and $g^s\left(\omega, h, \epsilon, K, H, R^{b,0}\right)$.[14] If there is an excess of lending in the bond market

[14] Solving the problem (15) is quite straightforward since the function v has already been obtained in step 2.

try $R^{b,1} < R^{b,0}$, if there is an excess of borrowing try $R^{b,1} > R^{b,0}$. Go on until finding an $R^{b,*}$ that clears the market,[15,16]

(c) get the next period distribution over ω, h and ϵ by use of $g^c\left(\omega, h, \epsilon, K, H, R^{b,*}\right)$, $g^b\left(\omega, h, \epsilon, K, H, R^{b,*}\right)$ and $g^s\left(\omega, h, \epsilon, K, H, R^{b,*}\right)$ and the law of motion for the shock ϵ,

(d) come back to step (b). Do it for a large number of periods.

4. Drop a number of observations from the beginning such that the remaining time series is clean from the initial conditions. Use the simulated series for K, H and $R^{b,*}$ to estimate new forecasting rules f^1.

5. Compare f^1 and f^0. If they are similar we are done, if not start again by setting $f^0 = f^1$ and going back to point 2.

There is just one last issue to be clarified. Once we have agreed to use only first moments, which is the proper class $\mathcal{F}$ where to define our forecasting rules? In a problem without habit formation Krusell and Smith (1997) show that a log-linear specification on the first moment of the wealth distribution does a good job. I therefore set up the following functional forms for the forecasting rules:

$$\begin{aligned}
\log K' &= \nu_{K0}(z) + \nu_{K1}(z)\log K + \nu_{K2}(z)\log H \\
\log H' &= \nu_{H0}(z) + \nu_{H1}(z)\log K + \nu_{H2}(z)\log H \\
R^b &= \nu_{R0}(z) + \nu_{R1}(z)\log K + \nu_{R2}(z)\log H + \nu_{R3}(z)(\log K)^2 + \nu_{R4}(z)(\log H)^2 + \\
&\quad \nu_{R5}(z)(\log K)(\log H)
\end{aligned}$$

where notice that the coefficients depend on the aggregate shock and therefore estimation of these forecasting rules implies running two different regressions for each. However, as shown in the appendix B.2, my findings are that we do not need so much information. Aggregate habits do not substantially improve the forecasts. This actually means that the aggregate habit stock turns out not to be a state variable of the system. The forecasting rules end up being:

$$\begin{aligned}
\log K' &= \nu_{K0}(z) + \nu_{K1}(z)\log K \\
R^b &= \nu_{R0}(z) + \nu_{R1}(z)\log K + \nu_{R2}(z)(\log K)^2
\end{aligned}$$

and one can drop H from all the formulations of the problem.

B Accuracy of solutions

There are two levels of accuracy in the solutions to the model that we may be worried about. First, for a given vector of forecasting rules f, we may wonder how accurate are

[15] Or until $R^{b,1} \simeq R^{b,0}$

[16] An alternative approach would be to solve the problem generally for a grid of different values of R^b and then interpolate the different guesses $R^{b,0}$, $R^{b,1}$, ... until market clears. The problem with this is its inexactitude. We would need an extremely fine grid on R^b to make the results along different periods of the simulation consistent among them.

the solutions to the household problem. Second, we may want to know how good are the forecasting rules f compared to what the economy actually delivers, that is to say, how well are the households doing in terms of predicting prices. In section B.1 I present accuracy measure for the solutions to the household problem and in section B.2 I discuss the forecasting rules used and their actual predictive power.

B.1 The household problem

The solution method explained in the previous section is based on solving the two Euler equations *exactly* at the given grid points. However, the supports for our state variables (with the exception of the shocks) are continuous. We may be interested in knowing how accurate is the solution outside the grid points; in other words, we may be interested in knowing how far from a zero of the Euler equations we are at any point of the state space.

To convey a meaningful measure of distance from a zero in the Euler equations, I follow Judd (1992) in reporting the relative consumption error. To compute this measure, we need to take the policy functions $\{g^c, g^b, g^s\}$ at any given point of the state space and check which relative change in consumption would yield equality in the Euler equation. If that number is for example 0.01 it means that for every dollar spent in consumption the household makes an error of one cent. I report the base 10 log of this measure. Therefore, if the error measure is -2 we say that the household makes one dollar mistake for every 100 dollars spent or one every 1000 if the error measure is -3.

I report these relative errors for both Euler equations only for those points in the state space where the Euler equations are solved with interior solution. Figures 3 and 4 plot the relative errors for the economy *HA* and figures 5 and 6 for the economy *HAH*. Since we cannot go beyond 3D graphics, I have fixed an arbitrary value for the aggregate state (z, K and H) and plotted the errors for the two equation at all possible points in the wealth, habits and idiosyncratic shock dimensions.

While useful, this graphical information is overwhelming. As a summary of the accuracy of the solutions I have taken the average of these errors over the equilibrium distribution of households in a given period and then taken the base 10 log. For the economy *HA* the average of errors is -3.37 for the bonds equation and -3.38 for the equation for capital (with the maximum errors equal to -2.52 and -2.84 respectively). In the *HAH* economy the equivalent figures are -3.08 and -3.12 (with the maximum errors equal to -2.55 and -2.37 respectively). This means that the error, on average, is less than one dollar for every one thousand dollars spent in consumption.[17]

B.2 The forecasting rules

As discussed in the section A.2, we can try to predict the evolution of the economy by use of a log-linear function of the first moments of the distribution μ. For the non-habits

[17] This errors are computed with the forecasted value of next period aggregate capital. In any case, as shown in section B.2, the actual and predicted values for aggregate capital are almost identical. Indeed, the largest relative discrepancy between the actual and predicted values is 0.067% in the non habits economy and 0.088% in the habits economy.

Figure 3: **Errors in the Euler Equation for bonds. Economy *HA***

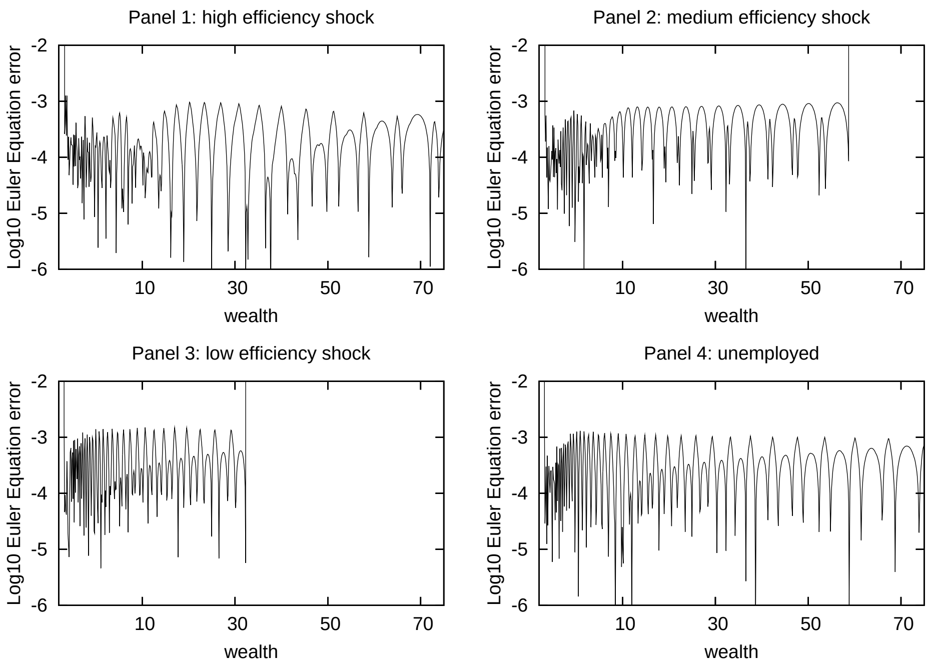


Figure 4: **Errors in the Euler Equation for capital. Economy *HA***

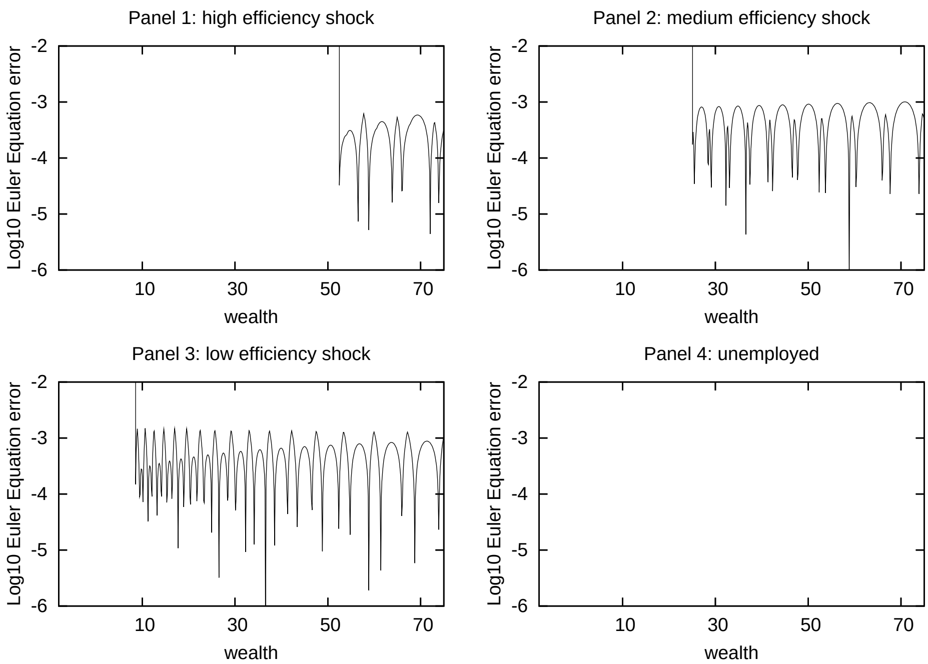


Figure 5: **Errors in the Euler Equation for bonds. Economy *HAH***

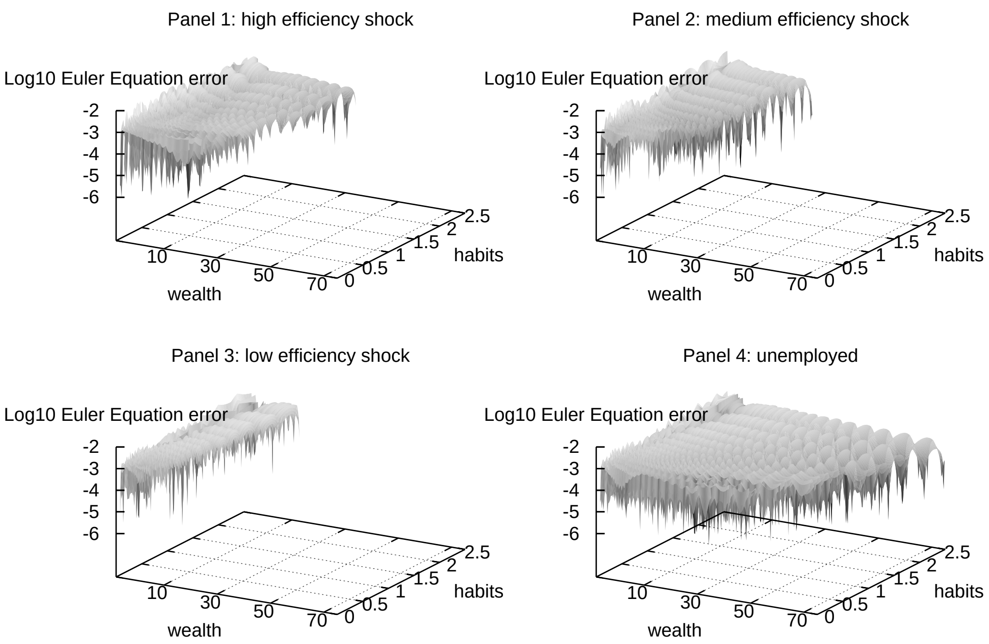
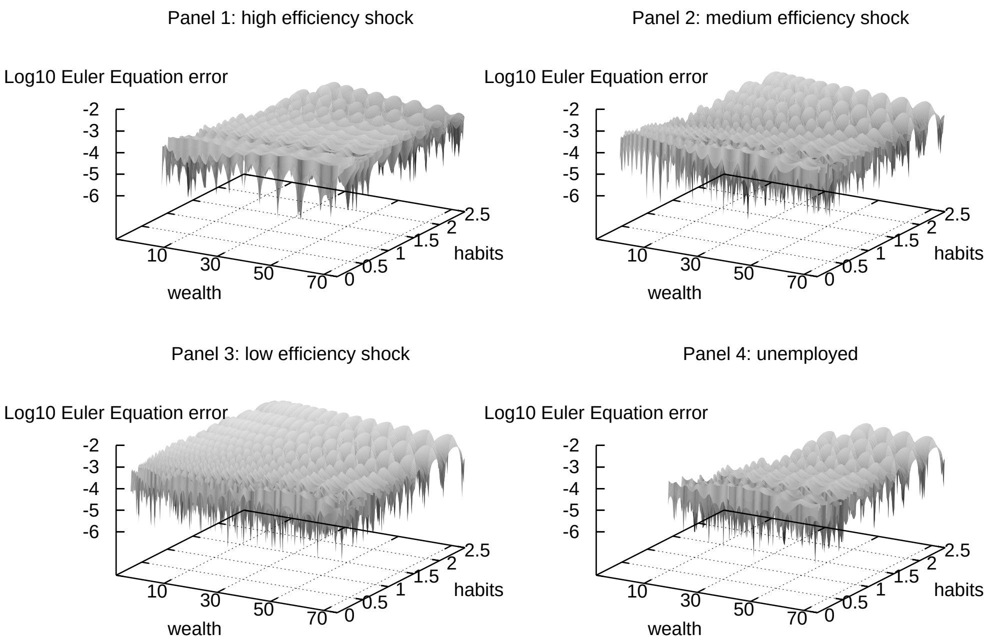


Figure 6: **Errors in the Euler Equation for capital. Economy *HAH***



economy HA the estimated forecasting rules are:

$$\begin{array}{llcll}
\text{if } z = z_g & \log K' & = & 0.083 + 0.973 \log K & \text{with } R^2 = 0.999987 \\
 & R^b & = & 1.105 - 0.052 \log K + 0.006 \left(\log K\right)^2 & \text{with } R^2 = 0.999999
\end{array}$$

$$\begin{array}{llcll}
\text{if } z = z_b & \log K' & = & 0.058 + 0.978 \log K & \text{with } R^2 = 0.999994 \\
 & R^b & = & 1.094 - 0.048 \log K + 0.005 \left(\log K\right)^2 & \text{with } R^2 = 0.999999
\end{array}$$

Notice that the R^2 are very big, larger than 0.9999 in all cases, which tells us that almost all the variation in the time series of $\log K$ and R^b is well predicted by these forecasting rules.

What about the economies with habits? As already anticipated in previous sections, one important finding of this paper is that the first moment of the marginal distribution of agents over habits does not bring any valuable information in predicting tomorrow's state once we are already considering the marginal distribution of assets (or its first moment). For this reason, I have solved the habits economies of this paper with only aggregate capital K as endogenous aggregate state variable. The drop of aggregate habits H does not suppose any change in the results and it dramatically speeds up the computations. Nevertheless, to convince the reader I also present some results for the HAH economy solved with forecasting rules that include the aggregate habit stock.

The estimated forecasting rules for the HAH economy when the aggregate habit stock is included are as follow:

$$\begin{array}{llcll}
\text{if } z = z_g & \log K' & = & +0.035 + 0.988 \log K - 0.028 \log H & \text{with } R^2 = 0.999986 \\
 & \log H' & = & -0.149 + 0.048 \log K + 0.907 \log H & \text{with } R^2 = 0.999992 \\
 & R^b & = & 1.106 - 0.052 \log K + 0.006 \left(\log K\right)^2 & \\
 & & - & 0.001 \log H - 0.001 \left(\log H\right)^2 & \text{with } R^2 = 0.999999
\end{array}$$

$$\begin{array}{llcll}
\text{if } z = z_b & \log K' & = & +0.010 + 0.993 \log K - 0.027 \log H & \text{with } R^2 = 0.999994 \\
 & \log H' & = & -0.161 + 0.051 \log K + 0.903 \log H & \text{with } R^2 = 0.999994 \\
 & R^b & = & 1.095 - 0.048 \log K + 0.005 \left(\log K\right)^2 & \\
 & & + & 0.001 \log H - 0.000 \left(\log H\right)^2 & \text{with } R^2 = 0.999999
\end{array}$$

If we drop the habit stock from the forecasting rules, we obtain a set of forecasting rules very similar to the ones for the HA economy:

$$\begin{array}{llcll}
\text{if } z = z_g & \log K' & = & 0.083 + 0.973 \log K & \text{with } R^2 = 0.999971 \\
 & R^b & = & 1.105 - 0.052 \log K + 0.006 \left(\log K\right)^2 & \text{with } R^2 = 0.999999
\end{array}$$

$$\begin{array}{llcll}
\text{if } z = z_b & \log K' & = & 0.056 + 0.978 \log K & \text{with } R^2 = 0.999977 \\
 & R^b & = & 1.094 - 0.048 \log K + 0.005 \left(\log K\right)^2 & \text{with } R^2 = 0.999999
\end{array}$$

Notice that the loss of predictive power as measured by the R^2 is virtually unnoticeable. Regarding the accuracy measures for the euler equations, the average errors in the euler equations are -3.07 and -3.11 which are almost identical to the ones obtained with the basic HAH economy.

Finally, it needs to be shown that the economy HAH, if solved with the aggregate habit stock as state variable, does not change in any perceptible way from the version without H, which is the one reported in the tables within the main part of the paper. Solving the *HAH* economy with H for the same parameter values as the *HAH* in the main part of the paper delivers a capital to output ratio of 12.56, a standard deviation of aggregate consumption growth of 0.26 and a Sharpe ratio of 0.017. All these three statistics are identical to the ones obtained in the *HAH* economy solved without aggregate habit as a state variable.