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Javier Díaz-Giménez and Josep Pijoan-Mas

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Javier Díaz-Giménez, IESE Business School and CAERP
Josep Pijoan-Mas, CEMFI and CEPR

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Centre for Economic Policy Research
77 Bastwick Street, London EC1V 3PZ, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

Flat Tax Reforms: Investment Expensing and Progressivity*

In this article we quantify the aggregate, distributional and welfare consequences of investment expensing and progressivity in flat-tax reforms of the United States economy. We find that investment expensing as in the Hall and Rabushka type of reform brings about sizable output gains and a non-trivial increase in after-tax income inequality. But we also find that it results in large aggregate welfare gains in steady-state. Two additional flat-tax reforms with full investment expensing and varying degrees of progressivity reveal that the distributional role of the tax-exemption in the labor income tax is limited. But we also find that the progressivity of the reforms matters for welfare: economies with more progressive consumption-based flat-taxes are good for the very poor and are ultimately preferred by a Benthamite social planner because they allow households to do more consumption and leisure smoothing. Our findings suggest that moving towards a progressive consumption-based flat tax scheme could achieve the goals of raising government income, stimulating the economy and providing a safety net for the households that have been hit the hardest by the recession

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Javier Díaz-Giménez
IESE Business School
University of Navarra
Barcelona Campus
Avenida Pearson, 21
08034 Barcelona
SPAIN
Email: jdiaz@iese.edu

Josep Pijoan-Mas
CEMFI
Casado del Alisal 5
28014 Madrid,
SPAIN

Email: pjoan@cemfi.es

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1 Introduction

Many economists today share the perception that the tax codes in modern market economies are becoming very complex, that they are costly to run, that they are full of loop-holes, and that they create many distortions. The debate on fundamental tax reforms is heating up as some countries, mostly in Eastern Europe, have adopted much simplified tax systems, and as other countries need to increase their tax collections to finance their growing public deficits.

One important aspect being debated is whether fundamental tax reforms should tax a broad definition of income, or whether they should tax exclusively consumption expenditures.¹ The key distinction between these two families of tax reforms is the tax treatment of investment expenditures. Income-based taxes tax both consumption and investment. This yields a broader tax base, but it generates distortions in capital accumulation. Consumption-based taxes do not tax investment expenditures and they have a smaller tax base. But they do not distort the capital accumulation decision. The classical optimality results of Chamley (1986) and Judd (1985), prescribe only consumption-based taxes because in the long run the distortions in capital accumulation are more severe than the distortions in the labor choices.²

Against this argument in favor of taxing consumption expenditures exclusively, there are other arguments that defend taxing a broad definition of income. Aiyagari (1995) points out that when labor income is uncertain and uninsurable the aggregate stock of capital may be too large, and some capital income taxation may be needed in order to bring it back to the *modified golden rule* of the textbook representative household growth model. Additionally, taxing capital income instead of labor income is a way to insure households against idiosyncratic labor market risks. This is because poor households own few assets and most of their income comes from labor sources, therefore a shift of taxation from labor to capital makes sour times better for the poor. In a quantitative exercise that supports this argument, Domeij and Heathcote (2004) show that the welfare losses brought about by eliminating capital income taxes can be large. In their model economy, even though eliminating capital income taxation increases output by 10 percent, it reduces aggregate welfare by about 1.5 percent.

In addition, implementing a consumption tax as a sales tax or as the European value added tax rises concerns about fairness because it is believed that it cannot be made a function of income. However this is not entirely true, since there are ways to design fundamental tax

¹See for instance Hubbard (1997) or Lazear and Poterba (2005) for a discussion of this issue.

²Lucas (1990), for instance, uses a representative household model to measure the welfare gains associated to the elimination of the current capital income taxation and he finds them to be large, of the order of a 5% equivalent variation in consumption every period.

reforms that are both consumption-based and progressive. A famous example is the flat-tax reform originally suggested by Hall and Rabushka (1995). These authors propose to abolish the personal income tax and the corporate income tax and to substitute them with a unified flat tax on labor and business income. This tax scheme is equivalent to a consumption tax because it makes investment expenditures deductible from the business income tax base. The average tax rates on labor income are progressive because a fixed amount of labor income is tax-exempt. Simulation results such as those reported by Ventura (1999) or by Altig, Auerbach, Kotlikoff, Smetters, and Walliser (2001) find that a reform along these lines generates large increases in the accumulation of productive capital, and non-trivial increases in income and wealth inequality. However, other authors such as Gentry and Hubbard (1997) argue that this type of tax reforms may not generate more inequality than similar reforms that tax all income.

In this article we contribute to the debate on whether fundamental tax reforms should be income-based or consumption-based, and we find that revenue-neutral consumption-based tax reforms should be preferred because they result in larger welfare gains. We also study the role played by the size of the labor income tax deductions in consumption-based reforms, and we find that the welfare gains are increasing in the progressivity of the reforms.

To do so we calibrate a heterogeneous household, general equilibrium model economy to United States data and we use it to compare the steady-state aggregate, distributional, and welfare consequences of four fundamental tax reforms. Our model economy is an extension of the model economy described in Castañeda, Díaz-Giménez, and Ríos-Rull (2003). In essence it is a variation of the neoclassical growth model with heterogeneous households and uninsurable idiosyncratic risk that combines life-cycle and dynastic features.

We introduce the heterogeneity in labor market opportunities and wealth using an uninsurable process on the endowment of efficiency labor units. We model the life-cycle features using stochastic aging and retirement as in Gertler (1999). We model the dynastic links making households altruistic towards their descendants. Once our model economy is properly calibrated, these features guarantee that households in our model economy save for precautionary reasons, for life-cycle reasons, and for altruistic reasons. This property is important because capital accumulation is one of the main channels through which investment expensing and progressivity affect the economy.

Another important feature of our benchmark model economy is that our households choose their work effort. This is important for two reasons. First, because it allows us to quantify the direct effect of tax distortions on labor supply. Second, because as Pijoan-Mas (2006)

shows, when labor market opportunities are uncertain, the labor supply becomes an important self-insurance mechanism that allows households to reduce their precautionary savings. Given that changes in the progressivity of the tax code will change the uncertainty of after tax income, the interaction of labor and savings decisions can have sizable aggregate, distributional, and welfare consequences.

A final distinguishing feature of our model economy is that it replicates the United States marginal distributions of labor earnings, income, and wealth in very much detail. And, in contrast with the model economies that focus exclusively on life-cycle features, it does a particularly good job in replicating the very top tails of those distributions. This feature is crucial for the quantitative evaluation of tax reforms because the tax burdens and the incentives to work and save that a tax code creates are very different at different points of the earnings and wealth distributions, and their effects are largest on the very income-rich and wealthy. Moreover, as Mirrlees (1971) points out, the distributional details are fundamental in measuring the trade-offs involved in choosing between efficiency and equality of tax reforms, because both the aggregate and the welfare changes depend critically on the number of households of each type that populate the economy.

In this article we start by evaluating a version of the consumption-based flat-tax reform originally proposed by Hall and Rabushka (1995). To do so, we substitute the current personal and corporate income taxes with an integrated 19 percent flat tax on labor income and capital income—which we use as a proxy for business income. We deduct investment expenditures from the capital income tax base and we choose the personal deduction on the labor income tax to make the reform revenue neutral. We find that aggregate output and labor productivity increase by 11.3 and 12.6 percent, and that both after-tax income and wealth inequality increase substantially. Specifically the Gini index of after-tax income increases from 0.51 to 0.55, and the Gini index of wealth increases from 0.82 to 0.84. These results are consistent with most findings in the literature. But, perhaps surprisingly, we also find that the reform yields welfare gains equivalent to an average increase of 5.2 percent of consumption in all periods and in all states. Part of these gains arise from the general equilibrium consequences of the reform: in the reformed economy the capital stock and output are larger, and the increase in the capital stock implies that workers earn higher wages. Another part of the welfare gains is due to the large deduction in the labor income tax, which acts as a boon for the income-poor.

To measure the quantitative importance of not taxing the capital accumulation decision at the margin, we compare the allocations that obtain in the reformed economy with those

that obtain when we simulate an alternative income-based flat-tax reform where the tax rate remains at 19 percent, but where investment is not expendable, and where we adjust the deduction in the labor income tax deduction to make the reform revenue neutral. We find that the aggregate gains brought about by this reform are more modest —output increases by only 4 percent— and that the increases in income and wealth inequality are also smaller —the steady state Gini index of after tax income that obtains after the reform is 0.53. Furthermore, we find that this reform reduces aggregate welfare.

To sum up, our contribution to the consumption versus income-based tax reforms is that consumption-based tax reforms bring about larger output gains, more income and wealth inequality and higher aggregate welfare. Admittedly, they shift the tax burden away from the income rich, but the large output gains that consumption-based tax reforms bring about can be used to provide a larger deduction in the labor income tax. And these deductions result in sizable benefits for the very poor.

Our second contribution to the fundamental tax reform debate is to measure the quantitative importance of the labor income tax exemptions in consumption-based flat-tax reforms. To this purpose, we compare the steady-state allocations that obtain in the 19 percent flat-tax reform with the steady-state allocations of two other reformed economies which differ in the sizes of their flat-tax rates and of their labor income tax exemptions. Specifically, we study a proportional flat-tax reform in which all labor income is taxed at an integrated flat-tax rate of 15.3 percent, and a very progressive flat-tax reform in which we double the labor income tax deduction and in which the integrated flat-tax rate is 24.7 percent. In the model economy with the more progressive flat tax, output, consumption, aggregate hours, and the capital stock are all smaller. These results were to be expected. But more surprisingly, we also find that labor productivity is increasing in the progressivity of the reform and that the inequality of income after-taxes is very similar across the three consumption-based tax reforms.

These novel results are justified by a better allocation of household labor hours. It turns out that in the more progressive reforms, household hours are more correlated with labor market productivity. This is because the more progressive tax code provides more insurance against labor market uncertainty, and this allows households to improve their inter-temporal allocation of labor, and to make better use of their labor market opportunities —essentially the more progressive tax reforms allow the households to work less when the times are bad. Consequently, since more progressive tax reforms increase the correlation of labor hours with the idiosyncratic labor shocks, they make the distribution of labor earnings before taxes more

unequal and the average productivity per hour worked higher. This increased inequality in the distribution of before-tax earnings partly offsets the increased redistribution brought about by the higher labor income tax exemption and the higher flat-tax rate. And they result in similar concentrations of after-tax income.

Finally, we find that the more progressive tax reform brings about the largest welfare gains in spite of the smallest increase in aggregate consumption and the mild decrease in after-tax income inequality. Once again, the reason is that the more progressive flat-tax reform allows households to take better advantage of their labor market opportunities without compromising their ability to smooth consumption. We conclude that progressive consumption-based flat taxes allow households to improve their life-time allocations of consumption and leisure and that a Benthamite social planner would recommend them because they bring about large welfare gains.

The remaining of the paper is organized as follows. In Section 2 we describe the model economy and in Section 3 we discuss how we parameterize it to reproduce the main aggregate and distributional statistics of the US economy. Then, in Section 4 we present the results of the economies with the different tax reforms and compare them to the benchmark economy. Finally, Section 5 concludes.

2 The Model Economy

2.1 Population and endowment dynamics

Our model economy is inhabited by a measure one continuum of heterogeneous dynastic households. The households are endowed with ℓ units of disposable time each period, and they are either workers or retirees. Workers face an uninsured idiosyncratic stochastic process that determines their endowment of efficiency labor units. They also face an exogenous probability of retiring. Retirees are endowed with zero efficiency labor units and they face an exogenous probability of dying. When a retiree dies, it is replaced by a working-age descendant who inherits the retiree's estate and, possibly, some of its earning abilities. We use the one-dimensional shock, s , to denote the household's random age and random endowment of efficiency labor units jointly.

We assume that the process on s is independent and identically distributed across households, and that it follows a finite state Markov chain with conditional transition probabilities given by $\Gamma = \Gamma(s' | s) = Pr\{s_{t+1} = s' | s_t = s\}$, where s and $s' \in S$. We assume that s takes values in one of two possible J -dimensional sets, \mathcal{E} and \mathcal{R} . Therefore the formal

description of set S is $S = \mathcal{E} \cup \mathcal{R} = \{1, 2, \dots, J\} \cup \{J+1, J+2, \dots, 2J\}$. When a household draws shock $s \in \mathcal{E}$, it is a worker and its endowment of efficiency labor units is $e(s) > 0$. When a household draws shock $s \in \mathcal{R}$ it is a retiree, and its endowment of efficiency labor units is $e(s) = 0$. When a household's shock changes from $s \in \mathcal{E}$ to $s' \in \mathcal{R}$, we say that it has retired and when it changes from $s \in \mathcal{R}$ to $s' \in \mathcal{E}$, we say that it has died and has been replaced by a working-age descendant. When a household dies, its estate is liquidated, and its descendant inherits a fraction $1 - \tau_e(z_t)$ of the estate, where z_t denotes the value of the household's stock of wealth at the end of period t . The rest of the estate is taxed away by the government.

This specification of the joint age and endowment process implies that the transition probability matrix, Γ , controls the demographics of the model economy, the life-time persistence of earnings, their life-cycle pattern, and their intergenerational persistence. The demographics is controlled choosing the expected durations of the households' working-lives and retirements. The life-time persistence of earnings, selecting the mobility of households between the states in \mathcal{E} . The life-cycle pattern of earnings, deciding how the endowment of efficiency labor units of new entrants differs from that of senior working-age households. And the intergenerational persistence of earnings, choosing the correlation between the states in \mathcal{E} for consecutive members of the same dynasty.

To specify the process on s we must choose the values of $(2J)^2 + J$ parameters, of which $(2J)^2$ are the conditional transition probabilities and the remaining J are the values of the endowment of efficiency labor units. To reduce this large number of parameters, we impose some additional restrictions on matrix Γ . To understand these restrictions better, it helps to consider the following partition of matrix Γ :

$$\Gamma = \begin{bmatrix} \Gamma_{\mathcal{E}\mathcal{E}} & \Gamma_{\mathcal{E}\mathcal{R}} \\ \Gamma_{\mathcal{R}\mathcal{E}} & \Gamma_{\mathcal{R}\mathcal{R}} \end{bmatrix}$$

Submatrix $\Gamma_{\mathcal{E}\mathcal{E}}$ contains the transition probabilities of working-age households that are still of working-age one period later. Since we impose no restrictions on these transitions, to characterize $\Gamma_{\mathcal{E}\mathcal{E}}$ we must choose the values of J^2 parameters.

Submatrix $\Gamma_{\mathcal{E}\mathcal{R}}$ describes the transitions from the working-age states into the retirement states. The value of this submatrix is $\Gamma_{\mathcal{E}\mathcal{R}} = p_{e\ell}I$, where $p_{e\ell}$ is the probability of retiring and I is the identity matrix. This is because we assume that every working-age household faces the same probability of retiring, and because we use only the last realization of the working-age shock to keep track of the earnings ability of retirees. Consequently, to characterize $\Gamma_{\mathcal{E}\mathcal{R}}$

we must choose the value of only one parameter.

Submatrix $\Gamma_{\mathcal{R}\mathcal{E}}$ describes the transitions from the retirement states into the working-age states that take place when a retiree exits the economy and is replaced by a working-age descendant. The rows of this submatrix contain a two parameter transformation of the stationary distribution of $s \in \mathcal{E}$, which we denote by $\gamma_{\mathcal{E}}^*$. This transformation allows us to control both the life-cycle profile of earnings and its intergenerational correlation. Intuitively, the transformation amounts to shifting the probability mass from $\gamma_{\mathcal{E}}^*$ towards both the first row of $\Gamma_{\mathcal{R}\mathcal{E}}$ and towards its diagonal.³ Consequently, to characterize $\Gamma_{\mathcal{R}\mathcal{E}}$ we must choose the value of the two shift parameters.

Finally, submatrix $\Gamma_{\mathcal{R}\mathcal{R}}$ contains the transition probabilities of retired households that are still retired one period later. The value of this submatrix is $\Gamma_{\mathcal{R}\mathcal{R}} = p_{\rho\rho}I$, where $(1 - p_{\rho\rho})$ is the probability of exiting the economy. This is because the type of retired households never changes, and because we assume that every retired household faces the same probability of exit. Therefore, to identify this submatrix we must choose the value of only one parameter.

To keep the dimension of process $\{s\}$ as small as possible while still being able to achieve our calibration targets, we choose $J = 4$. Therefore, to characterize process $\{s\}$, we must choose the values of $J^2 + J + 4 = 24$ parameters.⁴

2.2 Preferences

We assume that households derive utility from consumption, $c_t \geq 0$, and from non-market uses of their time, and that they care about the utility of their descendants as if it were their own utility. Consequently, the households' preferences can be described by the following standard expected utility function:

$$E \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, \ell - h_t) \mid s_0 \right\},$$

where function u is continuous and strictly concave in both arguments; $0 < \beta < 1$ is the time-discount factor; ℓ is the endowment of productive time; and $0 \leq h_t \leq \ell$ is labor. Consequently, $\ell - h_t$ is the amount of time that the households allocate to non-market activities. Our choice

³The definitions of the two shift parameters can be found in Section A of the Appendix, and a detailed description of our mass shifting procedure can be found in Castañeda, Díaz-Giménez, and Ríos-Rull (2003).

⁴Notice that we have not yet imposed that Γ must be a Markov matrix. When we do this, the number of free parameters is reduced to 20.

for the households' common utility function is

$$u(c, l) = \frac{c^{1-\sigma_1}}{1-\sigma_1} + \chi \frac{(\ell - l)^{1-\sigma_2}}{1-\sigma_2}$$

We make this choice because the households in our model economies face very large changes the market value of their time. And if we had chosen the more standard non-separable specification for preferences, these changes would have resulted in extremely large variations in hours worked.

2.3 Production

We assume that aggregate output, Y_t , depends on aggregate capital, K_t , and on the aggregate labor input, L_t , through a constant returns to scale aggregate production function, $Y_t = f(K_t, L_t)$. We choose a standard Cobb-Douglas aggregate production function with capital share θ .⁵ Aggregate capital is obtained aggregating the wealth of every household, and the aggregate labor input is obtained aggregating the efficiency labor units supplied by every household. We assume that capital depreciates geometrically at a constant rate, δ , and we use r and w to denote the prices of capital and of the efficiency units of labor before all taxes.

2.4 The government sector

The government in our model economies taxes capital income, labor income, consumption, and estates, and it uses the proceeds of taxation to make real transfers to retired households and to finance an exogenous amount of government consumption.

Social security in our model economy takes the form of transfers to retired households, which we denote by $\omega(s)$, and which are financed with a payroll tax. The inclusion of a social security system has important implications for our research questions. First, it reduces the size of the steady-state aggregate stock of capital.⁶ Second, it plays an important role in helping us to replicate the large fraction of households who own very few or zero assets in the United States.⁷ Third, since public pensions are paid as life-time annuities, it insures the

⁵In the U.S. after World War II, the real wage has increased at an approximately constant rate—at least until 1973—and factor income shares have displayed no trend. To replicate this behavior, the elasticity of substitution between capital and labor of the aggregate production function be 1, as is the case in Cobb-Douglas functions.

⁶Samuelson (1975) proves this result in a pure overlapping generations model. Our model economy is a dynastic model, so the pay-as-you-go social security system is isomorphic to a transfer system that reduces uncertainty in income. Therefore, the social security system reduces aggregate capital by reducing the need for precautionary savings.

⁷See Hubbard, Skinner, and Zeldes (1994).

households against the risk of living for too long, and therefore it reduces their incentives to save.

Our calibration procedure allows us to match the size of the average public retirement pension paid in the United States and it ensures that the motives for saving in our model economy are quantitatively realistic. But pensions in our model economy are independent of contributions to social security and this feature qualifies the precision of our analysis in two ways. First, the overall amount of idiosyncratic risk in our model economy diminishes because the labor market history does not condition the retirement benefits. Second, we abstract from a potentially important reason to work, since in real world economies increasing the labor effort entitles the households to receive larger pension benefits.⁸

The capital income taxes in the economy are described by the function:

$$\tau_k(y_k) = a_1 y_k \tag{1}$$

where y_k denotes capital income. Of course, in the U.S. economy different types of capital are taxed at different rates and receive different types of deductions. In order to simplify our model economy we consider just one type of capital good.⁹

Labor income taxes are described by function $\tau_l(y_a)$, where y_a denotes the labor income tax base. This tax is not used in the current United States tax system. But it is part of the flat-tax reforms which we describe in Section 4.

Payroll taxes paid by firms are described by function $\tau_{sf}(y_l)$, where y_l denotes labor income, and payroll taxes paid by households are described by function $\tau_{sh}(y_l)$. Our choice for the payroll tax function is

$$\tau_{sf}(y_l) = \tau_{sh}(y_l) = \begin{cases} a_2 y_l & \text{for } 0 \leq y_l \leq a_3 \\ a_2 a_3 & \text{otherwise} \end{cases} \tag{2}$$

This function approximates the cap on U.S. payroll taxes.¹⁰ To replicate the U.S. Social

⁸We make this assumption for two technical reasons. First, because discriminating between households according to their past contributions to a social security system requires a second asset-type state variable. And second, because in a model where the labor supply decision is endogenous, linking pensions to contributions makes the optimality condition for leisure an intertemporal decision. These two facts make our computational costs unmanageable. (See Section C of the Appendix for the details on our computational algorithm).

⁹To be consistent with this assumption, we calibrate the value of the tax rate on capital, a_1 , as the average tax rate levied on all capital income. By doing this we are implicitly assuming that every household in the economy owns varying amounts of shares of an identical portfolio of assets.

¹⁰In our model economy this cap on payroll taxes creates a non-convexity in the choice set of the households. We discuss this non-convexity in Section B of the Appendix.

Security tax code, we assume that the payroll taxes paid by the model economy households and firms are identical.

Household income taxes are described by the function:

$$\tau_y(y_b) = a_4 [y_b - (y_b^{-a_5} + a_6)^{-1/a_5}] \quad (3)$$

where the definition of the tax base is $y_b = y_k + y_l - \tau_k - \tau_{sf}$. This is the function chosen by Gouveia and Strauss (1994) to model the 1989 U.S. effective federal personal income taxes. Notice that both capital income taxes and payroll taxes paid by firms are excluded from the household income tax base both in the United States personal income taxes and in our model economy household income taxes.

We assume that consumption taxes are proportional and that they are described by the function:

$$\tau_c(c) = a_9 c \quad (4)$$

And finally, we assume that the estate tax function is

$$\tau_e(z) = \begin{cases} 0 & \text{for } z < a_7 \\ a_8(z - a_7) & \text{otherwise} \end{cases} \quad (5)$$

This function replicates the main features of the current effective estate taxes in the United States.¹¹

Therefore, in our model economies, a government policy rule is a specification of $\{\tau_k(y_k), \tau_l(y_l), \tau_{sf}(y_l), \tau_{sh}(y_l), \tau_y(y_b), \tau_c(c), \tau_e(z), \omega(s)\}$ and of a process on government consumption, $\{G_t\}$. Since we also assume that the government balances its budget every period, these policies must satisfy the following restriction: $G_t + Z_t = T_t$, where Z_t and T_t denote aggregate transfers and aggregate tax revenues.

2.5 Market arrangements

We assume that there are no insurance markets for the household-specific shock. Instead, to buffer their streams of consumption against the shocks, the households in our model economy can accumulate wealth in the form of real capital. We assume that these wealth holdings, a_t , belong to a compact set \mathcal{A} . The lower bound of this set can be interpreted as a form of

¹¹See Aaron and Munnell (1992), for example.

liquidity constraints, or as a solvency requirement.¹² The existence of an upper bound for the asset holdings is guaranteed as long as the after-tax rate of return to savings is smaller than the households' common rate of time preference. This condition is always satisfied in equilibrium.¹³

We also assume that firms rent factors of production from households in competitive spot markets. This assumption implies that factor prices are given by the corresponding marginal productivities.

2.6 The households' decision problem

The individual state variables are the realization of the household-specific shock, s , and the value of the stock of assets, a .¹⁴ The Bellman equation of the household decision problem is the following:

$$v(a, s) = \max_{\substack{c \geq 0 \\ z \in \mathcal{A} \\ 0 \leq h \leq \ell}} u(c, \ell - h) + \beta \sum_{s' \in \mathcal{S}} \Gamma_{ss'} v[a'(z), s'], \quad (6)$$

$$s.t. \quad c + z = y - \tau + a, \quad (7)$$

$$y = ar + e(s)hw + \omega(s), \quad (8)$$

$$\tau = \tau_k(y_k) + \tau_l(y_l) + \tau_{sf}(y_l) + \tau_{sh}(y_l) + \tau_y(y_b) + \tau_c(c), \quad (9)$$

$$a'(z) = \begin{cases} z - \tau_e(z) & \text{if } s \in \mathcal{R} \text{ and } s' \in \mathcal{E}, \\ z & \text{otherwise.} \end{cases} \quad (10)$$

where function v is the households' common value function. Notice that household income, which we denote by y , includes three terms: capital income, $y_k = ar$, labor income, $y_l = e(s)hw$, and retirement pensions, $\omega(s)$. Every household can earn capital income. Only workers can earn labor income. And only retirees receive retirement pensions. The household policy that solves this problem is a set of functions that map the individual state into the optimal choices for consumption, end-of-period savings, and labor hours. We denote this policy by $\{c(a, s), z(a, s), h(a, s)\}$.

¹²Given that leisure is an argument in the households' utility function, this borrowing constraint can be interpreted as a solvency constraint that prevents the households from going bankrupt in every state of the world.

¹³Huggett (1993) and Marcet, Obiols-Homs, and Weil (2007) prove this proposition.

¹⁴In our model economy there are no aggregate state variables because we abstract from aggregate uncertainty and we restrict our analysis to the steady states of the economies.

2.7 Equilibrium

Each period the economy-wide state is a probability measure, x_t , defined over an appropriate family of subsets of $S \times \mathcal{A}$ that counts the households of each type, and that we denote by \mathcal{B} . In the steady-state this measure is time invariant, even though the individual state variables and the decisions of the individual households change from one period to the next.¹⁵

Definition 1. *A steady state equilibrium for this economy is a household value function, $v(a, s)$; a household policy, $\{c(a, s), z(a, s), h(a, s)\}$; a government policy, $\{\tau_k(y_k), \tau_l(y_a), \tau_{sf}(y_l), \tau_{sh}(y_l), \tau_y(y_b), \tau_c(c), \tau_e(z), \omega(s), G\}$; a stationary probability measure of households, x ; factor prices, (r, w) ; and macroeconomic aggregates, $\{K, L, T, Z\}$, such that:*

- (i) *Given factor prices and the government policy, the household value function and the household policy solve the households' decision problem described in expressions (6)-(10).*
- (ii) *Firms behave as competitive maximizers. That is, their decisions imply that factor prices are factor marginal productivities $r = f_1(K, L) - \delta$ and $w = f_2(K, L)$.*
- (iii) *Factor inputs, tax revenues, and transfers are obtained aggregating over households:*

$$\begin{aligned}
 K &= \int a \, dx \\
 L &= \int h(a, s) \, e(s) \, dx \\
 T &= \int [\tau_k(y_k) + \tau_l(y_a) + \tau_{sf}(y_l) + \tau_{sh}(y_l) + \tau_y(y_b) + \tau_c(c)] \, dx + \int \mathbf{I}_{s \in \mathcal{R}} \gamma_{s\mathcal{E}} \tau_e(z) \, z(a, s) \, dx \\
 Z &= \int \omega(s) \, dx.
 \end{aligned}$$

where \mathbf{I} denotes the indicator function, the definition of parameter $\gamma_{s\mathcal{E}}$ is $\gamma_{s\mathcal{E}} \equiv \sum_{s' \in \mathcal{E}} \Gamma_{ss'}$ and, consequently, $(\mathbf{I}_{s \in \mathcal{R}} \gamma_{s\mathcal{E}})$ is the probability that a retiree of type s exits the economy. And where every integral in the four definitions above is defined over the state space $S \times \mathcal{A}$.

- (iv) *The goods market clears: $\int [c(a, s) + z(a, s)] \, dx + G = f(K, L) + (1 - \delta) K$.*

- (v) *The government budget constraint is satisfied: $G + Z = T$*

¹⁵See Hopenhayn and Prescott (1992) and Huggett (1993).

(vi) *The measure of households is stationary:*

$$x(B) = \int_B \left\{ \int_{S \times \mathcal{A}} [\mathbf{I}_{z=z(a,s)} \mathbf{I}_{s \notin \mathcal{R} \vee s' \notin \mathcal{E}} + \mathbf{I}_{z=[1-\tau_e(z)]z(a,s)} \mathbf{I}_{s \in \mathcal{R} \wedge s' \in \mathcal{E}}] \Gamma_{ss'} dx \right\} dz ds'$$

for all $B \in \mathcal{B}$, where \vee and \wedge are the logical operators “or” and “and”. This equation counts the households, and the cumbersome indicator functions and logical operators are used to account for estate taxation. We describe the procedure that we use to compute this equilibrium in Section C of the Appendix.

3 Calibration

Our model economy is characterized by 42 parameters. Of these parameters, 5 describe the preferences of the households, 2 the production technology, 11 the government policy, and 24 the joint process on the age of the households and on the endowments of efficiency labor units. To choose the values of these parameters we need 42 calibration targets. Of these targets, 6 are normalization conditions, and the remaining 36 are statistics that describe relevant features of the United States economy. Seven of these calibration conditions uniquely determine the value of 7 of the model economy parameters. To determine the values of the remaining 29 parameters, we solve the system of 29 non-linear equations that results from equating the values of the statistics of the model economy to those of their corresponding United States targets. The details of the procedure that we use to solve this system can be found in Section C of the Appendix.

3.1 Model period

The U.S. tax code defines tax bases in annual terms. Since the income tax, the payroll tax and the estate tax are not proportional taxes, the obvious choice for our model period is one year. Moreover, the Survey of Consumer Finances, which is our main source of micro-data, is also yearly.

3.2 Normalization conditions

The household endowment of disposable time is an arbitrary constant and we choose it to be $\ell = 3.2$. We also normalize the endowment of efficiency labor units of the least productive households to be $e(1) = 1.0$. Finally, since matrix Γ is a Markov matrix, its rows must add

up to one. This property imposes four additional normalization conditions on the rows of $\Gamma_{\mathcal{E}\mathcal{E}}$.¹⁶

3.3 Macroeconomic and demographic targets

Ratios: We target a capital to output ratio, K/Y , of 3.58, a capital income share of 0.376, and an investment to output ratio, I/Y , of 22.5 percent. We obtain our target value for the capital output share dividing \$288,000, which was average household wealth in the United States in 1997 according the 1998 Survey of Consumer Finances, by \$80,376, which was per household Gross Domestic Product according to the Economic Report of the President (2000) for the United States in 1997.¹⁷ Our target for the capital income share is the value that obtains when we use the methods described in Cooley and Prescott (1995) and we exclude the public sector from the computations.¹⁸ To calculate the value of our target for I/Y , we define investment as the sum of gross private fixed domestic investment, change in business inventories, and 75 percent of the private consumption expenditures in consumer durables using data for 1997 from the Economic Report of the President (2000).¹⁹

Allocation of time and consumption: We target a value of $H/\ell = 33$ percent for the average share of disposable time allocated to working in the market.²⁰ For the curvature of consumption we choose a value of $\sigma_1 = 1.5$. This value falls within the range (1–3) that is standard in the literature.²¹ Finally, we want our model economy to replicate the relative cross-sectional variability of U.S. consumption and hours. To this purpose, we target a value of $cv(c)/cv(h) = 3.5$ for the ratio of the cross-sectional coefficients of variation of these two variables.

The age structure of the population: We target the expected durations of working-lives and retirement of the model economy households to be 45 and 18 years. These targets

¹⁶Note that our assumptions about the structure of matrix Γ imply that once submatrix $\Gamma_{\mathcal{E}\mathcal{E}}$ has been appropriately normalized, every row of Γ adds up to one without imposing any further restrictions.

¹⁷We obtained this number dividing the U.S. population quoted for 1997 in Table B-34 of the Economic Report of the President (2000) by the U.S. average household size which was 2.59 according to the 1998 SCF (see Budría, Díaz-Giménez, Quadrini, and Ríos-Rull (2002)).

¹⁸See Castañeda, Díaz-Giménez, and Ríos-Rull (1998) for details about this number.

¹⁹This definition of investment is approximately consistent with the 1998 Survey of Consumer Finances definition of household wealth, which includes the value of vehicles, but does not include the values of other consumer durables.

²⁰See Juster and Stafford (1991) for details about this number.

²¹Recent calibration exercises find very similar values for σ_1 . For example, Heathcote, Storesletten, and Violante (2004) report a value of 1.44 and Pijoan-Mas (2006) reports a value of 1.46 for this parameter.

replicate the average durations of working-lives and retirement in the United States.

The life-cycle profile of earnings: To replicate the life-cycle profile of earnings of the United States in our model economy, we target the ratio of the average earnings of households between ages 60 and 41 to that of households between ages 40 and 21. In the 1972–1991 period the average value of this statistic in the United States was 1.303, according to the Panel Study of Income Dynamics.

The intergenerational transmission of earnings ability: To replicate the intergenerational correlation of earnings of the United States in our model economy, we target the cross-sectional correlation between the average life-time earnings of one generation of households and the average life-time earnings of their immediate descendants. Solon (1992) and Zimmerman (1992) measure this statistic for fathers and sons in the United States, and they report that it is 0.4, approximately.

3.4 Government policy

In Table 1 we report the revenues obtained by the combined Federal, State, and Local Governments in the United States in the 1997 fiscal year. To choose the parameter values of the tax functions in our model economy we must first allocate the United States tax revenues to the tax instruments of our benchmark model economy. We choose the parameters of the model economy household income tax so that they collect the revenues levied by the U.S. personal income tax, the parameters of the model economy capital income tax so that it collects the revenues levied by the U.S. corporate income tax, and with the model economy payroll and estate taxes we do likewise. The remaining sources of government revenues in the United States are sales and gross receipts taxes, property taxes, excise taxes, custom duties and fees, and other taxes. Added together, these tax instruments collected 7.09 percent of U.S. GDP in 1997. In our model economy we allocate these revenues to the consumption tax.²²

To choose the parameters of the expenditure side of the government budget, we do the following: First, since the government of our model economy must balance its budget, we require that the output shares of government consumption and government transfers—the two expenditure items in our model economy—add up to 27.52 percent, which was the GDP

²²Since we also target government transfers and government expenditures (see below), the model economy’s consumption tax rate is determined residually to balance the government budget.

Table 1: Federal, State, and Local Government Receipts

Fiscal Year	1997	
	\$Billion	%GDP
Gross Domestic Product (GDP)	8185.20	100.00
Total Federal, State and Local Gvt Receipts	2252.75	27.52
Individual Income Taxes	896.54	10.95
Social Insurance and Retirement	539.37	6.60
Sales and Gross Receipts Taxes	261.73	3.20
Property Taxes	218.83	2.67
Corporate Profit Taxes	216.11	2.64
Excise Taxes	56.92	0.70
Estate and Gift Taxes	19.85	0.24
Custom Duties and Fees	17.93	0.22
Other Taxes	25.47	0.30

Source: Tables B78, B81, and B86 of the Economic Report of the President 2000.

share of total tax revenues in the United States in 1997. Then we target a value for the transfers to output ratio in the model economy of 5.21 percent. This value corresponds to the share GDP accounted for by Medicare and by two thirds of Social Security transfers in the United States in 1997. We chose this target because transfers in our model economies are lump-sum, and Social Security transfers in the U.S. economy are mildly progressive. This choice leaves us with a residual share for government expenditures to GDP of 22.31(= 27.52 – 5.21) which is our target for the G/Y ratio in our model economy.²³ We discuss the details of our choices for the various model economy tax function parameters in the paragraphs below.

Capital income taxes: We choose a_1 , the capital income tax rate of function (1), so that the revenues collected by this tax in the benchmark model economy match the revenues collected by the corporate profit tax in the U.S. economy.

Payroll taxes: To characterize the payroll tax function described in expression (2), we must choose the values of parameters a_2 and a_3 . In 1997 in the U.S. the payroll tax rate paid by both households and firms was 7.65 percent each and it was levied only on the first \$62,700 of gross labor earnings. This value was approximately equal to 78 percent of the U.S. per household GDP. To replicate these values, in our model economy we make $a_2 = 0.0765$

²³Our target for the G/Y ratio is 4.48 percentage points larger than the 17.89 obtained for the Government Expenditures and Gross Investment entry in the NIPA tables. The difference is essentially accounted for by the sum of net interest payments and the deficit (3.58 percent of GDP).

and $a_3 = 0.78\bar{y}$, where \bar{y} denotes output per household. These choices imply that the payroll tax collections in our model economy are endogenous, and that we can use them as an overidentification restriction.

Household income taxes: To characterize the income tax function described in expression (3), we must choose the values of parameters a_4 , a_5 and a_6 . Since a_4 and a_5 are unit-independent, we use the values reported by Gouveia and Strauss (1994) for these parameters, namely, $a_4 = 0.258$ and $a_5 = 0.768$. To determine the value of a_6 , we equate the tax rate levied on a value of income equal to average output per household in our model economy to the effective tax rate on GDP per household levied in the U.S. economy. Again, these choices imply that the household income tax collections in our model economy are endogenous, and that we can use them as another overidentification restriction.

Estate taxes: To characterize the estate tax function described in expression (5), we must choose the values of parameters a_7 and a_8 . During the 1987–1997 period in the United States the first \$600,000 of the value of estates were tax exempt. This value was approximately equal to ten times the average value of GDP per household.²⁴ In our model economy we make $a_7 = 10\bar{y}$, to replicate this feature of the United States estate tax code. Finally, we choose the value of a_8 so that the estate tax in our model economy collects the same revenues as the estate tax in the United States.

Consumption taxes: We choose the value of parameter a_9 in the consumption tax function described in expression (4) residually, so that the government in our model economy balances its budget. Therefore, the consumption tax collections in our model economy are also endogenous, and they can be interpreted as a third overidentification restriction.

3.5 The distributions of earnings and wealth

The conditions that we have described so far specify a total of 27 targets. To solve our model economy we must choose the values of 42 parameters. Therefore, we need 15 additional targets. These 15 targets are the 2 Gini indexes and 13 additional points from the Lorenz curves of the United States distributions of earnings and wealth which we report in Table 4.²⁵

²⁴See, for example, Aaron and Munnell (1992).

²⁵A detailed discussion of this last, non-standard feature of our calibration procedure can be found in Castañeda, Díaz-Giménez, and Ríos-Rull (2003).

3.6 Calibration results

Our calibration procedure allow us to characterize the stochastic process on the endowment of efficiency labor units. This process is not to be taken literally, since it is a black box that represents everything that we do not know about our model economy. In particular, we cannot compare our process with the panel data estimates of wage processes for prime-age males, such as those reported in Blundell and MaCurdy (1999), or in the more recent Heathcote, Storesletten, and Violante (2004). This is because our process is a measure of household labor market opportunities and not of individual labor market opportunities. In our model economy labor market opportunities result in household labor supply decisions, which include participation decisions of the members of the household.²⁶ Also, panel data sets, such as the Panel Study of Income Dynamics (PSID) or the National Longitudinal Survey of Youth (NLSY), miss the upper tail of the wage distribution, both because of top-coding and because they are not explicitly designed to measure the earnings of the very rich. The upper tail of the earnings distribution is very important if we want our model economy to be consistent with the upper tail of the wealth distribution of the Unites States as reported by the Survey of Consumer Finances which does not have either one of these two problems.

Table 2: The stochastic process for the endowment of efficiency labor units

	$e(s)$	$\gamma_{\mathcal{E}}^*$ (%)	$\Gamma_{\mathcal{E}\mathcal{E}}$ (%) From s To s'			
			$s' = 1$	$s' = 2$	$s' = 3$	$s' = 4$
$s = 1$	1.00	47.78	96.15	1.39	0.23	0.009
$s = 2$	3.17	37.24	1.60	96.00	0.18	0.000
$s = 3$	9.91	14.91	1.19	0.00	96.56	0.028
$s = 4$	634.98	0.0638	6.63	0.45	6.52	84.18

Note: $e(s)$ denotes the relative endowments of efficiency labor units; $\gamma_{\mathcal{E}}^*$ denotes the stationary distribution of working-age households; $\Gamma_{\mathcal{E}\mathcal{E}}$ denotes the transition probabilities of the process on the endowment of efficiency labor units for working-age households that are still workers one period later.

In the second column of Table 2 we report the relative endowments of efficiency labor units, and in the third column the invariant measures of each type of working-age households. The endowments of workers of $s = 2$, $s = 3$, and $s = 4$ are, approximately, 3, 10, and 635. This means that, in our model economy, the luckiest workers are 635 times as lucky as the unluckiest ones. The stationary distribution shows that each period 85 percent of the workers are unlucky and draw states $s = 1$ or $s = 2$, and that only one out of every 1,567 workers is extremely lucky and draws state $s = 4$.

²⁶See Guner, Kaygusuz, and Ventura (2007) for a first attempt to evaluate tax reforms modeling two-member households explicitly.

In the last four columns of Table 2 we also report the transition probabilities between the working-age states. Every row sums up to 97.78 percent plus or minus rounding errors. This is because the probability that a worker retires is 2.22 percent. The first three states are very persistent. Their expected durations are 25.7, 25.3 and 29.4 years. In contrast, state $s = 4$ is relatively transitory and its expected duration is only 7.6 years.

As far as the transitions are concerned, we find that a worker whose current state is $s = 1$ is more likely to move to state $s = 2$ than to any of the other states. Likewise, a worker whose current state is either $s = 2$ or $s = 3$ is most likely to move back to state $s = 1$. Only very rarely workers whose current state is either $s = 1$ or $s = 2$ will make a transition either to state $s = 3$ or to state $s = 4$. Finally, when a worker draws state $s = 4$, it is most likely that she will draw either state $s = 3$ or state $s = 1$ shortly afterwards.

Table 3: Parameter values for the benchmark model economy

<i>Preferences</i>		
Time discount factor	β	0.930
Curvature of consumption	σ_1	1.500
Curvature of leisure	σ_2	1.119
Relative share of consumption and leisure	χ	1.050
Endowment of discretionary time	ℓ	3.200
<i>Technology</i>		
Capital income share	θ	0.376
Capital depreciation rate	δ	0.050
<i>Age and endowment process</i>		
Probability of retiring	p_{ee}	0.022
Probability of dying	$1 - p_{ee}$	0.056
Life cycle earnings profile	ϕ_1	1.000
Intergenerational persistence of earnings	ϕ_2	0.733
<i>Fiscal policy</i>		
Government consumption	G	0.369
Retirement pensions	ω	0.800
Capital income tax function	a_1	0.146
Payroll tax function	a_2	0.076
	a_3	1.262
Household income tax function	a_4	0.258
	a_5	0.768
	a_6	0.456
Estate tax function	a_7	16.179
	a_8	0.246
Consumption tax function	a_9	0.099

We report the values of every other parameter of our model economy in Table 3, and in Table 4 we report the statistics that describe the main aggregate and distributional features

of the United States and the benchmark model economies. These numbers confirm that overall our model economy succeeds in replicating the most relevant features of the United States in very much detail.²⁷ We are particularly encouraged by our model economy’s ability to replicate the U.S. fiscal policy ratios and the U.S. distributions of earnings, income and wealth, since these two sets of targets are the main focus of this article. Recall that in our calibration exercise we have not targeted either the payroll tax collections, the household income tax collections, the consumption tax collections, or the statistics that describe the income distribution, and that all of these statistics can be considered to be overidentification restrictions.

Table 4: The Benchmark Model Economy (E_B) and the United States

<i>Macroeconomic Ratios</i>									
	C/Y	I/Y	G/Y	K/Y	—	—			
<i>U.S.</i>	54.2	22.5	23.3	3.58	—	—			
E_B	59.2	18.0	22.8	3.56	—	—			
<i>Fiscal Policy Ratios</i>									
	G/Y	Z/Y	T/Y	T_y/Y	T_l/Y	T_k/Y	T_s/Y	T_c/Y	T_e/Y
<i>U.S.</i>	22.3	5.2	27.5	11.0	—	2.6	6.6	7.1	0.24
E_B	22.8	4.5	27.3	11.6	—	2.9	5.9	6.5	0.37
<i>The Distributions of Earnings</i>									
	<i>Gini</i>	<i>Quintiles (%)</i>					<i>Top groups (%)</i>		
<i>Economy</i>		1st	2nd	3rd	4th	5th	90–95	95–99	99–100
<i>United States</i>	0.611	–0.4	3.2	12.5	23.3	61.4	12.4	16.4	14.8
E_B	0.613	0.0	4.2	14.4	18.8	62.5	11.8	16.7	15.2
<i>The Distributions of Income (before all taxes and after transfers)</i>									
<i>United States</i>	0.550	2.4	7.2	12.5	20.0	58.0	10.3	15.3	17.5
E_B	0.533	3.7	8.9	10.9	17.2	59.3	10.1	16.6	16.0
<i>The Distributions of Wealth</i>									
<i>United States</i>	0.803	–0.4	1.7	5.7	13.4	79.5	12.6	24.0	29.6
E_B	0.818	0.0	0.3	1.5	15.9	82.2	12.6	19.8	34.7

4 The Flat-Tax Reforms

We study two families of flat tax reforms: the consumption-based flat tax reform originally proposed by Hall and Rabushka (1995) and an income-based flat tax reform. In both cases

²⁷Naturally, there are some exceptions. For instance, our parsimonious modeling of the life cycle does not allow us to match the life-cycle profile of earnings and the intergenerational correlation of earnings simultaneously. Castañeda, Díaz-Giménez, and Ríos-Rull (2003) discuss this issue in detail, and they show that our class of model economies can account for these two statistics one at a time.

we replace the household income tax with a flat tax on all labor income above a tax-exempt level, and the calibrated capital income tax with an integrated flat tax on capital income. The function that describes the labor income tax is

$$\tau_l(y_a) = \begin{cases} 0 & \text{for } y_a < a_{10} \\ a_{11}(y_a - a_{10}) & \text{otherwise} \end{cases} \quad (11)$$

where the tax base is labor income net of social security taxes paid by firms, $y_a = y_l - \tau_{sf}(y_l)$, parameter a_{10} is the tax-exempt level of labor income, and parameter a_{11} is the flat-tax rate. The capital income tax function in the reformed economies is the same as the capital income tax function defined in Expression (1) above. The only difference is that in the consumption-based tax reforms investment expenditures are tax-exempt and, consequently, the capital income tax base is capital net of depreciation income minus savings. Therefore, in these reforms $y_k = r a - (a' - a)$.²⁸ Since in every reform capital and labor income are taxed at the same marginal tax rate, we impose that $a_1 = a_{11}$. Finally, every flat-tax reform is designed to be revenue neutral and none of them changes the composition of public outlays. Therefore, in every flat-tax reform the values of T , G , and Z remain unchanged, and they are equal to their values in the benchmark model economy.

4.1 Investment expensing in flat-tax reforms

In this section we compare the allocations that obtain in the steady-states of two flat-tax model economies that differ only in the fiscal treatment of investment expenditures. In the consumption-based flat tax economy investment expenditures are fully deductible, and in the income-based flat tax economy they are fully taxed.

The consumption-based flat-tax economy, which in this section we call E_C , is the standard flat-tax reform originally proposed by Hall and Rabushka (1995). As these authors suggested, its marginal tax rate on capital and labor income is 19 percent. And we choose the size of its labor income tax deduction to make the reform revenue neutral. This requires a labor income tax deduction $a_{10} = 0.3236$, which corresponds to 20 percent of output per household in the benchmark model economy or \$16,000, approximately.

In the income-based flat-tax economy, which we call E_Y , we keep the 19 percent marginal integrated flat tax rate, but since investment expenditures are not deductible, we change the

²⁸Taxing capital income at the household level is equivalent to the proposed business income tax of Hall and Rabushka (1995), which is applied to firms by taxing business income net of wages, depreciation expenses and net investment.

labor income tax deduction to make the reform revenue neutral. In principle, the direction of this change could go either way. Taxing investment increases the base of the capital income tax. In partial equilibrium this would increase the capital income tax revenues and it would require a larger deduction in the labor income tax to make the reform revenue neutral. But in general equilibrium taxing investment reduces the capital stock. Therefore it also reduces aggregate output and the bases of both the capital and the labor income flat taxes.

It turns out that this second effect dominates. And we find that the value of the labor income tax deduction that makes this reform revenue neutral is $a_{10} = 0.1805$, which is 45 percent smaller than the one that obtains in the consumption-based flat-tax reform, and which corresponds to approximately \$8,924. This is because steady-state output in the income-based flat-tax reform is substantially smaller than in the consumption-based flat-tax reform, as we discuss below.

4.1.1 Macroeconomic aggregates and factor ratios

In Table 5 we report the main macroeconomic aggregates and factor ratios of our model economies. We find that the steady-state aggregate changes brought about by the consumption-based flat tax reform are substantial. Steady state output in model economy E_C is 11 percent larger than in the benchmark economy, E_B . This increase in output is brought about by a very large increase in aggregate capital, of 35 percent. In contrast, the changes brought about in the aggregate labor input are small. Both total labor hours and the total labor input decrease by about 1 percent. We also find that the productivity of labor hours increases by approximately 13 percent, as a result of capital deepening.

Table 5: Production, inputs and input ratios in the model economies

	Y	K	L^a	H^b/ℓ	K/L	L/H	Y/H	K/Y
E_B	1.62	5.76	0.75	33.7	7.64	2.24	4.80	3.56
E_C/E_B (%)	11.3	35.2	-1.0	-1.2	36.5	0.2	12.6	21.4
E_Y/E_B (%)	4.5	11.5	0.4	0.6	11.0	-0.2	3.8	6.7

^aVariable L denotes the aggregate labor input.

^bVariable H denotes the share of the endowment of time allocated to the market.

There are two reasons that justify the increase in the capital stock. First, the consumption-based flat tax reform eliminates the distortion in the intertemporal allocation of consumption, which encourages the households to save and to accumulate capital. Second, as we discuss below, the distribution of after tax income becomes more unequal. This increases the need for precautionary savings and therefore it increases the size of the capital stock even further.

We find that the changes brought about by the income-based flat tax reform are much smaller. As expected, taxing investment has large implications for the capital accumulation decision. Compared to the benchmark model economy, in the income-based flat-tax reform aggregate capital increases by 11.5 percent, which is only one third of the increase that obtains in the consumption-based reform. Still, the increase in the capital stock is not small.

There are two reasons for this increase. First, as in the consumption-based reform, there is an increase in the precautionary motive for savings. Second, the income-based flat-tax reform reduces the marginal capital income tax rates faced by the wealthy, and they increase the marginal capital income tax rates faced by the wealth poor. This is because in our benchmark economy capital income is taxed twice —once by the capital income tax and a second time by the household income tax— and because the rates on capital income of the household income tax are progressive. The aggregate effect of these changes is to increase capital accumulation because wealthy households are more concerned with after tax returns and less concerned with precautionary motives than poor households.

The income-based flat-tax reform also brings about changes in the aggregate labour input that are very small. Consequently, aggregate output in this model economy is only 4 percent larger than in the benchmark economy, while in the consumption-based flat-tax reform it is 11 percent larger.

These findings can be compared with those reported in Altig, Auerbach, Kotlikoff, Smetters, and Walliser (2001), albeit in a somewhat indirect way. Altig, Auerbach, Kotlikoff, Smetters, and Walliser (2001) study a sequence of reforms. First, they look at a purely proportional flat tax on all income. Second, they allow for full expensing of new investment, which makes their income tax equivalent to a consumption tax. And third, they add a labor income tax exemption.²⁹ They find that these three reforms increase aggregate output in the long run. A strictly proportional income tax increases output by 5 percent. Allowing for the expensing of new investment increases output by an additional 4 percent. And adding a fixed deduction to labor income requires a higher marginal tax rate that brings the output increase back to 4.5 percent. Therefore, their results are more modest than ours.

This is partly because our model economy extends Altig, Auerbach, Kotlikoff, Smetters, and Walliser (2001) in several important dimensions. First, we consider uninsurable labor market uncertainty. This brings into the analysis of flat-tax reforms the partial insurance role played by the various tax codes, which is absent from Altig, Auerbach, Kotlikoff, Smetters, and Walliser (2001). Second, we allow for earnings and wealth mobility. This feature

²⁹They consider two additional reforms with different tax breaks for capital holders during the transition.

of our model economy should reduce the welfare consequences of the reforms because our income process is mean reverting, at least at the dynastic level. Third, earnings, income and wealth are more concentrated in our model economy than in Altig, Auerbach, Kotlikoff, Smetters, and Walliser (2001), and they match their counterparts in the data. Finally, our households are altruistic towards their descendants and our model economy displays some of the intergenerational correlation of earnings observed in the United States. We think that this feature is important because the bequest motive is arguably one of the main determinants of wealth accumulation (see De Nardi (2004), for example). Meaningful evaluations of the distributional consequences of tax reforms require realistic wealth distributions, but this realism should be achieved through the appropriate margins.

4.1.2 Expenditure ratios

In Table 6 we report the key expenditure ratios in the benchmark model economy and in the two reformed flat-tax model economies. Since the level of public expenditure, G , is the same in the three model economies, the G/Y ratios fall whenever output increases. Since both reforms bring about sizable increases in aggregate capital, the decreasing marginal returns to capital make the investment to output ratios increase and the consumption to output ratios fall. However, even though the C/Y ratios fall in both flat-tax reforms, it is important to highlight that in both of them aggregate consumption increases (see Column 4 in Table 6). This increase is about 4.5 percent larger in the consumption-based flat-tax reform than in the income-based flat-tax reform.

Table 6: Expenditure ratios in the model economies (%)

	C/Y	I/Y	G/Y	C/Y_B	I/Y_B	G/Y_B
E_B	59.2	18.0	22.8	59.2	18.0	22.8
E_C	57.9	21.6	20.6	64.4	24.0	22.8
E_Y	59.0	19.2	21.8	61.6	20.0	22.8

Note: Columns 1, 2 and 3 report aggregate consumption, investment and government expenditure as a fraction of each economy's output. Columns 4, 5 and 6 report these same magnitudes as a fraction of output in the benchmark economy.

4.1.3 Fiscal policy ratios

In Table 7 we report the main fiscal policy ratios of the model economies. We have already mentioned that in both reformed model economies the government expenditures to output ratios are smaller than in the benchmark model economy. Moreover, the tax revenue to

output ratios and the transfers to output ratios of these model economies are reduced in the same proportion.

Table 7: The Fiscal Policy Ratios in the Model Economies (%)

	G/Y	Z/Y	T/Y	T_y/Y	T_l/Y	T_k/Y	T_s/Y	T_c/Y	T_e/Y
E_B	22.8	4.5	27.3	11.6	–	2.9	5.9	6.5	0.37
E_C	20.6	4.0	24.6	–	9.0	3.0	5.6	6.4	0.57
E_Y	21.8	4.3	26.1	–	9.9	3.5	5.7	6.5	0.47
E_C/Y_B	22.8	4.5	27.3	–	10.0	3.3	6.2	7.1	0.63
E_Y/Y_B	22.8	4.5	27.3	–	10.3	3.7	6.0	6.8	0.49

Note: Rows 1, 2 and 3 report aggregate magnitudes as a fraction of each economy’s output. Rows 4 and 5 report these same magnitudes as a fraction of output in the benchmark economy.

The bottom 2 rows of Table 7 display the tax ratios relative to the output in the benchmark model economy. We observe that, as Hall and Rabushka (1995) had guessed, the labor income tax in the consumption-based flat-tax reform collects less revenues than the personal income tax of the benchmark model economy. This is also the case in the income-based flat-tax reform. The revenue losses are compensated by the higher revenues collected by all the other tax instruments. The lion’s shares of these revenues correspond to consumption taxes in model economy E_C and to capital income taxes in model economy E_Y .

4.1.4 Earnings, income, and wealth inequality

In Table 8 we report the Gini indexes and the Lorenz curves of earnings, after-tax income, and wealth in the benchmark and in the reformed model economies. We find that the effects of the flat tax reforms on earnings inequality are very small, but that both reforms bring about sizable increases in after-tax income inequality and in wealth inequality. The first result is not surprising since the three model economies have identical processes on the endowments of efficiency labor units.

The higher inequality in wealth is easy to understand because the marginal taxes on capital income for the wealthy are lower after the flat-tax reforms. And this gives rich households stronger incentives to accumulate capital. The inequality in after-tax income is larger in the flat-tax economies because of the increase in the inequality in the wealth distribution and because of the lower redistributive power of flat taxes.

When we compare both reforms, we find that consumption-based flat-tax reform creates more inequality than the income-based flat-tax reform. The Gini index of after-tax income in model economy E_C is 0.550, and in model economy E_Y it is only 0.532. These changes

Table 8: The Distributions of Earnings, Income, and Wealth in the Model Economies

<i>The Distributions of Earnings</i>									
<i>Economy</i>	<i>Gini</i>	<i>Quintiles (%)</i>					<i>Top groups (%)</i>		
		1st	2nd	3rd	4th	5th	90–95	95–99	99–100
E_B	0.613	0.0	4.2	14.4	18.8	62.5	11.8	16.7	15.2
E_C	0.612	0.0	4.4	14.2	18.9	62.5	11.9	16.8	14.9
E_Y	0.613	0.0	4.1	14.4	18.9	62.6	12.0	16.6	14.9
<i>The Distributions of Income (after all income taxes and transfers)</i>									
E_B	0.510	4.6	9.3	11.0	17.2	57.9	10.1	16.3	14.9
E_C	0.550	4.3	7.4	10.6	16.2	61.5	10.8	17.3	16.9
E_Y	0.532	4.6	8.6	10.2	16.7	59.9	10.2	16.9	16.6
<i>The Distributions of Wealth</i>									
E_B	0.818	0.0	0.3	1.5	15.9	82.2	12.6	19.8	34.7
E_C	0.841	0.0	0.2	0.9	14.1	84.8	12.3	20.8	38.5
E_Y	0.837	0.0	0.1	1.0	14.6	84.4	12.6	20.4	37.3

are brought about by changes the income shares earned by the households in both tails of the distributions. The shares earned by the households in the bottom two quintiles of model economy E_C are 0.3 and 1.2 percentage points smaller than in model economy E_Y , and the share earned by the households in the top quintile is 1.6 percentage points larger. Finally, we find that the differences between the Gini indexes of wealth of both flat-tax reforms are smaller —0.841 in model economy E_C and 0.837 in model economy E_Y .

Interestingly, in Section 4.3 below we show that, even though there is more after-tax income and wealth inequality in the steady state of model economy E_C , the consumption-based flat-tax reform results in a welfare gain which is equivalent to a 5.24 percent increase in consumption in every period and in every state. In contrast, we find that the income-based flat-tax reform results in a welfare loss which is equivalent to a 0.14 percent reduction in consumption.

4.2 Progressivity in consumption-based flat-tax reforms

In this section we compare the allocations that obtain in the steady-states of three consumption-based flat-tax reforms. The first flat-tax reform is the least progressive of the three. In this model economy all labor income is taxed. Therefore the value of the labor income tax deduction is zero and $a_{10} = 0.0$. The integrated flat-tax rate that makes this reform revenue neutral is 15.3 percent. To keep this in mind we call this economy E_{15} .

The second consumption-based flat-tax reform is the standard flat-tax reform proposed

by Hall and Rabushka (1995) which we have discussed in the previous section. Its marginal tax rate on capital and labor income is 19 percent, and the labor income tax deduction that makes this reform revenue neutral is $a_{10} = 0.3236$, which corresponds to 20 percent of the benchmark model economy output per household, or approximately \$16,000. To be consistent we have relabeled this model economy and we now call it E_{19} .

The third consumption-based flat-tax reform is the most progressive of the three. In this model economy, we double the labor income tax deduction of model economy E_{19} . Therefore, in this model economy $a_{10} = 0.6472$, which corresponds to approximately \$32,000. The value of the integrated flat-tax rate that makes this reform revenue neutral is 24.7 percent, and we call this model economy E_{25} .

4.2.1 Macroeconomic aggregates and factor ratios

In Table 9 we report the main macroeconomic aggregates and factor ratios of our three consumption-based flat-tax reforms. Relative to the benchmark model economy, we find that the three flat-tax reforms are expansionary. We also find that reforms generate large increases in the stock of capital —between 33 and 36 percent— and that the three reforms generate small changes in the labor decision. But while in model economy E_{15} both aggregate hours and the aggregate labor input increase, in model economies E_{19} and E_{25} , these two variables decrease. When we compare the three reformed model economies with each other, we find that every statistic that we report in Table 9 is monotonic in the flat-tax rate. This behavior is consistent with the idea that higher marginal taxes create larger distortions.

Table 9: Production, inputs and input ratios in the model economies

	Y	K	L^a	H^b/ℓ	K/L	L/H	Y/H	K/Y
E_B	1.62	5.76	0.75	33.7	7.64	2.24	4.80	3.56
E_{15}/E_B (%)	12.5	36.2	0.4	1.8	35.7	-1.4	10.6	21.1
E_{19}/E_B (%)	11.3	35.2	-1.0	-1.2	36.5	0.2	12.6	21.3
E_{25}/E_B (%)	9.1	33.0	-3.2	-5.3	37.3	2.3	15.2	21.9

^aVariable L denotes the aggregate labor input.

^bVariable H denotes the share of the endowment of time allocated to the market.

But we find that the increases in the productivity of labor hours are larger in the reformed economies with higher flat-tax rates: 10.6 percent in model economy E_{15} , 12.6 percent in model economy E_{19} , and 15.2 percent in model economy E_{25} . These increases in labor productivity are due to increases both in the labor to hours ratios, L/H , and in the capital to labor ratios, K/L —see the fifth, sixth, seventh, and eighth columns of Table 9.

By definition, the increases in the L/H ratios are the result of household hours being more correlated with the endowment of efficiency labor units. This tells us that as we move towards a more progressive flat-tax system households need to provide less self-insurance. Consequently, they accumulate less precautionary savings —and hence the stock of capital is lower— and they use less precautionary hours —and hence people work less, but labor hours become more correlated with productivity. This makes the allocations more similar to the ones that would obtain under complete markets.³⁰

The increases in the K/L ratios are ultimately due to the same reason: the reduction in hours reduces the aggregate labor input and, therefore, capital per efficiency unit of labor increases. These results lead us to conclude that the fixed deduction in labor income makes labor hours more productive, and that it improves the allocation of the work effort. In economies with higher flat-tax rates, people end up working less on average, but they work more when they are more productive.

Our results are surprisingly consistent with the findings of Ventura (1999). He considers two consumption-based flat-tax reforms with labor income tax exemptions equal to 20 and 40 percent of the income per household of his benchmark economy. The flat-tax rates that make his reforms revenue neutral are 19.1 and 25.2 percent, which are only slightly larger than the 19.0 and 24.7 percent rates of our model economies E_{19} and E_{25} , which have similar deductions. Ventura (1999) finds that both flat-tax reforms bring about large output gains, 17 and 13 percent —these numbers are 11 and 9 percent in our comparable reforms. And he also finds that these gains are mainly the result of large increases in capital accumulation. These results are remarkably similar to ours in spite of some important differences between his model economies and ours. First, to proxy for the effective personal income tax rates, Ventura (1999) uses the statutory tax rates and income brackets. Therefore, in his benchmark model economy the marginal taxes rates of the personal income tax are higher than ours, and his flat-tax reforms bring about efficiency gains that are larger than ours. Second, households in Ventura (1999) lack altruistic motives for saving. And Third, Ventura (1999) largely understates the concentration of the earnings, income and wealth distributions.³¹

³⁰See Pijoan-Mas (2006) for an analysis of the interaction of work effort and savings as self-insurance mechanisms, and for a comparison of capital and labor allocations in complete and incomplete-market economies.

³¹The Gini indexes of earnings and wealth in Ventura (1999) benchmark model economy are 0.47 and 0.60. The corresponding statistics in ours are 0.61 and 0.82. This difference comes from Ventura (1999) largely underestimating the concentrations of income and wealth in the top tails of the distributions.

4.2.2 Expenditure ratios

In Table 10 we report the key expenditure ratios in the benchmark model economy and in the three reformed flat-tax model economies. Since the flat tax reforms are expansionary and the levels of government expenditures do not change, the G/Y ratios in the flat-tax model economies are smaller than in the benchmark model economy. The lower G/Y shares are compensated with large increases in the investment to output ratios, because the consumption to output ratios also decrease. These results are consistent with the large increases in the capital stock which we have discussed in Section 4.2.1 above.

Table 10: Expenditure Ratios in the Model Economies (%)

	C/Y	I/Y	G/Y	C/Y_B	I/Y_B	G/Y_B
E_B	59.2	18.0	22.8	59.2	18.0	22.8
E_{15}	58.2	21.5	20.3	65.5	24.2	22.8
E_{19}	57.9	21.6	20.6	64.4	24.0	22.8
E_{25}	57.5	21.6	20.9	62.7	23.6	22.8

Note: Columns 1, 2 and 3 report aggregate consumption, investment and government expenditure as a fraction of each economy's output. Columns 4, 5 and 6 report these same magnitudes as a fraction of output in the benchmark economy.

We also find that aggregate consumption increases in the three consumption-based flat-tax reforms. The increase is largest in model economy E_{15} and it gets smaller as we increase the progressivity of the flat-tax system. This notwithstanding, we find that the differences in consumption and investment —and in their ratios to output— brought about by differences in the progressivity of the flat-tax reforms are small.

4.2.3 Fiscal policy ratios

Table 11: The Fiscal Policy Ratios in the Model Economies (%)

	G/Y	Z/Y	T/Y	T_y/Y	T_l/Y	T_k/Y	T_s/Y	T_c/Y	T_e/Y
E_B	22.8	4.5	27.3	11.6	–	2.9	5.9	6.5	0.37
E_{15}	20.3	4.0	24.2	–	9.1	2.4	5.6	6.4	0.57
E_{19}	20.6	4.0	24.6	–	9.0	3.0	5.6	6.4	0.57
E_{25}	20.9	4.1	25.0	–	8.5	3.9	5.5	6.3	0.58
E_{15}/Y_B	22.8	4.5	27.3	–	10.2	2.7	6.3	7.2	0.64
E_{19}/Y_B	22.8	4.5	27.3	–	10.0	3.3	6.2	7.1	0.63
E_{25}/Y_B	22.8	4.5	27.3	–	9.3	4.3	6.0	6.9	0.63

Note: Rows 1 to 4 report aggregate magnitudes as a fraction of each economy's output. Rows 5 to 7 report these same magnitudes as a fraction of output in the benchmark economy.

In Table 11 we report the main fiscal policy ratios in the benchmark model economy and in the three reformed flat-tax model economies. In the three reforms total government revenues, T , government consumption, G , and total transfers, Z , do not change, and hence their ratios to output fall.

When we compare the changes in the composition of government revenues, we confirm that in every consumption-based flat-tax reform the labor income tax collects less revenues than the personal income tax of the benchmark model economy (see bottom 3 rows in Table 11). In contrast, payroll and consumption taxes collect more revenues in the three flat-tax model economies, and capital income taxes collect more revenues in model economies E_{19} and E_{25} , but less in the model economy E_{15} . The total revenues from the labor income tax, the payroll tax and the consumption tax are decreasing in the progressivity of the reforms, and the revenues from the capital income taxes are increasing in the progressivity of the reform.

Revenues from the consumption tax decrease with the progressivity of the reform because the consumption tax rate remains unchanged and aggregate consumption—which is the tax base—decreases as the flat-tax rates increase (see Table 10). The same is true for the payroll tax: the tax rate does not change, and the tax base, which is essentially aggregate labor income, decreases with the flat-tax rate.³² The labor income tax revenues fall with the progressivity of the reforms because the tax base is lower for more progressive reforms due to both lower total labor income and higher labor income tax deduction.³³

Finally, the changes in the capital income tax are a bit more involved. Its aggregate tax base is

$$\int y_k dx = (r - \delta)K = \theta Y - \delta K \tag{12}$$

Compared to the benchmark economy the proportional flat-tax reform has less capital tax income. This is because the tax rates are very similar (15.3 against 14.6 percent) and the tax base is lower in the proportional tax flat-tax economy as the increase in aggregate output is much lower than the increase in aggregate capital (see Table 9). When we compare the consumption-based flat-tax reforms with each other, the capital income tax revenues increase with the more progressive reforms because the flat-tax rate is higher and the changes in the

³²Indeed, the tax base of the payroll tax is not exactly the aggregate labor income as labor income above the threshold a_3 is exempt and changes in the distribution of labor earnings make the exact fraction of untaxed labor income different in different economies.

³³This lower tax base is not compensated by the higher tax rate of the labor income tax associated to more progressive reforms. This is because the higher tax rates of the more progressive reforms also affect capital income and hence the labor income tax does not need to collect the whole amount of the labor income tax deduction.

tax base are not too large.

4.2.4 Earnings, income, and wealth inequality

In Table 12 we report the Gini indexes and the Lorenz curves of earnings, income, and wealth of the benchmark model economy and of the consumption-based flat-tax reforms. The Gini indexes of before-tax income and wealth are higher in the three reformed economies and the Gini index of model economy E_{15} is also higher. Moreover earnings, before-tax income, and wealth inequality increase as the flat-tax reforms become more progressive. But the maximum differences between the Gini indexes of earnings, before-tax income, and wealth in the three model economies are small —9, 9, and 5 points per thousand. We conclude that the flat-tax reforms bring about increases in inequality and that, overall, the distributional role played by the progressivity of consumption-based flat taxes is small.

Table 12: The Gini Indexes and the Lorenz Curves in the Model Economies

<i>The Distributions of Earnings</i>									
	<i>Gini</i>	<i>Quintiles (%)</i>					<i>Top groups (%)</i>		
<i>Economy</i>		1st	2nd	3rd	4th	5th	90–95	95–99	99–100
E_B	0.613	0.0	4.2	14.4	18.8	62.5	11.8	16.7	15.2
E_{15}	0.607	0.0	4.4	14.6	19.2	61.8	11.9	16.5	14.7
E_{19}	0.612	0.0	4.4	14.2	18.9	62.5	11.9	16.8	14.9
E_{25}	0.616	0.0	5.2	13.2	18.2	63.5	11.8	17.3	15.4
<i>The Distributions of Income (before all taxes and after transfers)</i>									
E_B	0.533	3.7	8.9	10.9	17.2	59.3	10.1	16.6	16.0
E_{15}	0.537	3.4	8.4	11.6	17.4	59.3	10.4	16.6	15.5
E_{19}	0.541	3.5	8.4	11.2	17.2	59.8	10.5	16.8	15.8
E_{25}	0.546	3.5	8.6	10.5	16.8	60.7	10.5	17.2	16.3
<i>The Distributions of Income (after all income taxes and transfers)</i>									
E_B	0.510	4.6	9.3	11.0	17.2	57.9	10.1	16.3	14.9
E_{15}	0.556	4.2	7.4	10.3	16.2	61.9	10.8	17.5	17.0
E_{19}	0.550	4.3	7.4	10.6	16.2	61.5	10.8	17.3	16.9
E_{25}	0.541	4.4	7.7	11.1	16.2	60.6	10.6	17.0	16.6
<i>The Distributions of Wealth</i>									
E_B	0.818	0.0	0.3	1.5	15.9	82.2	12.6	19.8	34.7
E_{15}	0.841	0.0	0.2	0.9	14.2	84.8	12.7	20.8	38.0
E_{19}	0.841	0.0	0.2	0.9	14.1	84.8	12.3	20.8	38.5
E_{25}	0.846	0.0	0.2	1.0	13.2	85.5	12.0	21.1	39.6

Earnings inequality increases as the flat tax reform becomes more progressive because higher flat-tax rates and higher deductions increase the correlation between wages and work

effort (see the discussion in Section 4.2.1). Since this implies that labor income becomes more volatile, households transfer income between periods using larger buffer stocks of precautionary savings to smooth their consumption profiles. This implies that wealth inequality also increases.

Perhaps more importantly, we find that consumption-based flat-taxes bring about large redistributions of income. So much so that the ranking of the Gini indexes of the after-tax income distributions is reversed. This means that after-tax income inequality is reduced as the flat-tax reforms become more progressive. Interestingly though, the two less progressive consumption-based flat tax reforms exacerbate after-tax income inequality—in model economy E_{15} the Gini index income increases in 19 points per thousand after accounting for all income taxes and transfers, and in model economy E_{19} it increases by 9 percentage points.

Notice, however, that the increases in income and wealth inequality brought about by the flat-tax reforms do not necessarily imply welfare losses to risk averse households because they are silent about their effects on the inequality in consumption and leisure. In Section 4.3 below we show that all three consumption-based flat-tax reforms result in sizable welfare gains, and that these welfare gains increase as the reforms become more progressive.

4.3 Welfare

In the previous section we have reported that the consumption-based flat-tax reform with a 19 percent marginal rate results in larger aggregate output, consumption, and productivity than the model economy with the current U.S. tax code. But after-tax income becomes more unequally distributed. We have also shown that a consumption-based flat-tax reform brings about higher output gains than an income-based flat-tax reform with the same flat-tax rate, but that it also results in higher increases in after-tax income and wealth inequality. Finally, we have also shown that as the consumption-based flat-tax reforms become more progressive, the aggregate gains become smaller but the inequality of after-tax income is also reduced (see Table 13 for a summary of these changes). Therefore, a policymaker who had to choose between any of these reforms would face different versions of the classical trade-off between efficiency and equality. Which one of these reforms results in a steady states with higher aggregate welfare? In this section we use a Benthamite social social welfare function to answer this question.³⁴

³⁴Benthamite social welfare functions give identical weights to every household in the economy. Consequently, when the utility function is concave, equal sharing is the welfare maximizing allocation. Notice also that in this section we compare the welfare of steady-state allocations and we remain conspicuously silent about the transitions between these steady-states.

Table 13: Efficiency, Inequality, and Welfare

	Y	C/Y_B	H/ℓ^a	$G(YD)^b$	$G(E)^c$	$G(W)^d$	Δ^e
E_B	1.62	59.2	33.7	0.510	0.613	0.818	—
E_{15}	1.82	65.5	34.3	0.556	0.607	0.841	3.47
E_{19}	1.80	64.4	33.3	0.550	0.612	0.841	5.24
E_{25}	1.77	62.7	31.9	0.541	0.616	0.846	7.49
E_Y	1.69	61.6	33.9	0.532	0.613	0.837	-0.14

^aVariable H/ℓ denotes the share of the endowment of time allocated to the market.

^bVariable $G(YD)$ denotes the Gini index of income after all taxes and transfers.

^cVariable $G(E)$ denotes the Gini index of labor earnings.

^dVariable $G(W)$ denotes the Gini index of wealth.

^eVariable Δ denotes the aggregate welfare change.

4.3.1 Aggregate welfare changes

To carry out the welfare comparisons, we define $v_B(a, s, \Delta)$ as the equilibrium value function of a household of type (a, s) in the benchmark model economy, whose equilibrium consumption allocation is changed by a fraction Δ every period and whose leisure remains unchanged. Formally,

$$v_B(a, s, \Delta) = u(c_B(a, s)(1 + \Delta), \ell - h_B(a, s)) + \beta \sum_{s' \in S} \Gamma_{ss'} v(z_B(a, s), s', \Delta)$$

where $c_B(a, s)$, $h_B(a, s)$ and $z_B(a, s)$ are the solutions to the household decision problem defined in expressions (6)–(10). Next, we define the welfare gain of living in the steady-state of flat-tax economy E_i , for $i = 15, 19, 25, Y$, as the fraction of additional consumption, Δ_i , that we must give to, or take away from, the households of the benchmark model economy so that the aggregate steady-state welfare in model economy E_i is the same as in economy E_B . Formally, Δ_i is the solution to the equation

$$\int v_B(a, s, \Delta_i) dx_B = \int v_i(a, s) dx_i \tag{13}$$

where v_i and x_i are the equilibrium value function and the equilibrium stationary distribution of households in the flat-tax model economy E_i .

The consumption-based flat-tax reforms. We find that the equivalent variation in consumption for the original Hall and Rabushka reform —model economy E_{19} — is 5.24 percent (see last column in Table 13). This means that, from a Benthamite point of view, the steady state generated by such a reform is largely preferred to the steady state under

the current tax code. Indeed, we would need to increase consumption of every household by 5.24 percent in every period and in every state for the social planner to be indifferent between the steady-state allocation implied by the current U.S. tax code and the steady-state allocation that results from the consumption-based flat-tax system of model economy E_{19} . The consumption-based flat-tax system with no deduction in labor income —model economy E_{15} — generates a welfare gain equivalent to a 3.47 percent increase in consumption. And the consumption-based flat-tax system with a labor income tax exemption that is twice as large as the one in model economy E_{19} , and with a 24.7 flat-tax rate —model economy E_{25} — generates a welfare gain that is equivalent to a 7.5 percent increase in consumption.

Therefore, the three consumption-based flat tax reforms yield steady states that are preferred to the steady state implied by the current U.S. tax code. Moreover, the most progressive consumption-based flat-tax reform is preferred in welfare terms to the two less progressive consumption-based flat-tax reforms, even though it brings about a smaller increase in output and consumption. Our results show that a Benthamite social planner is willing to give up these gains in efficiency to reduce the after-tax income inequality, which in model economy E_{25} is the smallest (see Table 13).

The income-based flat tax-reform. The picture is very different for the income-based flat-tax reform. As we report in Table 13, when compared to the current tax code in the United States, the revenue neutral flat-tax reform with a 19 percent integrated flat-tax rate and full expensing of investment —model economy E_Y — increases aggregate output, consumption, and hours, but by a smaller amount than any of the three consumption-based reforms. The reformed tax code also increases after-tax income inequality. The size of this increase in inequality is such that it more than compensates the efficiency gains and, from the point of view of a Benthamite social planner, its new steady state results in an average welfare loss which is equivalent to a 0.14 percent reduction in consumption.

4.3.2 A decomposition of the aggregate welfare changes

To improve the understanding these results, it is useful to decompose the equivalent variation in consumption discussed above. To this purpose, we define two auxiliary measures of the equivalent variations in consumption for each reform. First, we compute the equivalent variation in consumption that makes the households indifferent between the benchmark model economy E_B and the flat-tax model economy E_i ignoring the changes in the equilibrium

distribution of households. We denote this variation by Δ_i^a , and we define it as follows:

$$\int v_B(a, s, \Delta_i^a; r_B, w_B) dx_B = \int v_i(a, s; r_i, w_i) dx_B$$

Notice that in this expression we calculate the aggregate welfare of the reformed model economies using their equilibrium price vectors, (r_i, w_i) , and the equilibrium stationary distribution of the benchmark model economy.

Second, we compute the equivalent variation in consumption that makes the households indifferent between the benchmark model economy E_B and the reformed model economies E_i ignoring both the changes in the equilibrium distributions of households and the changes in the sizes of the economies. We denote this variation by Δ_i^b , and we define it as follows:

$$\int v_B(a, s, \Delta_i^b; r_B, w_B) dx_B = \int v_i(a, s; r_B, w_B) dx_B$$

Notice that now we calculate the aggregate welfare of the reformed model economies using both the equilibrium stationary distribution and the equilibrium price vector of the benchmark model economy.

These two equivalent variations allow us to decompose the total equivalent variation that we have defined in Expression (13) above as follows:

$$\Delta_i = \Delta_i^b + (\Delta_i^a - \Delta_i^b) + (\Delta_i - \Delta_i^a) \tag{14}$$

The first term of Expression (14) measures the welfare changes that are due to the reshuffling of resources between the households and it ignores both the general equilibrium effects of the reforms and the changes in the distributions of households. The second term measures the welfare changes that are due to the general equilibrium effects of the reforms only. And the third term measures the welfare changes that are due to the changes in the distributions of households only.

Table 14: Decomposing the aggregate welfare changes

<i>Economy</i>	<i>Equivalent variations in consumption (%)</i>			Δ_i
	Δ_i^b	$(\Delta_i^a - \Delta_i^b)$	$(\Delta_i - \Delta_i^a)$	
E_{15}	-1.74	3.42	1.79	3.47
E_{19}	0.39	3.20	1.65	5.24
E_{25}	3.69	2.82	0.98	7.49
E_Y	-1.32	1.21	-0.04	-0.14

The consumption-based flat tax-reforms. In the first three rows of Table 14 we report this decomposition for our three consumption-based flat-tax reforms. We find that, the original Hall and Rabushka (1995) reform —model economy E_{19} — yields welfare gains, even when we disregard the distributional and general equilibrium changes that it brings about. These gains are sizably larger in the more progressive consumption-based flat-tax reform —model economy E_{25} . In contrast, the purely proportional consumption-based reform —model economy E_{15} — results in a welfare loss. These welfare changes are the direct consequence of the redistribution of the tax burden, and of the new individual allocations of consumption and leisure that the flat-tax systems generate. The welfare gains in model economies E_{19} and E_{25} result from the reduction of the tax burden on poor households through the fixed deduction in the labor income tax, which is absent in model economy E_{15} .

When we consider the general equilibrium effects brought about by the change in prices, the sign of the welfare change is positive for the three consumption-based flat-tax reforms. These welfare changes include an aggregate and a distributional component. The aggregate component is the result of the efficiency gains and losses that result from the new aggregate values of consumption and leisure. The distributional component is the result of the change in relative prices: the large increases in the capital to labor ratio increase wages and reduce the interest rates. Consequently they shift income from capital owners to workers. Not surprisingly the general equilibrium effects are larger for the less progressive consumption-based flat-tax reforms that bring about larger gains in output and consumption (see the second and third columns of Table 13).

Finally, the three new equilibrium distributions of the consumption-based flat-tax reforms result in aggregate welfare gains. These welfare gains arise because these reforms put more households in points in the state space that have a higher utility.

The income-based flat tax-reform. Once again, the picture is very different for the income-based flat-tax reform (see the last row of Table 14). We find that the reshuffling of the tax burden towards poorer households brought about by this reform creates a welfare loss that is not compensated by the welfare gains that result from the increases in efficiency. The labor income tax deduction in this reform is small, and a large share of the tax burden goes to households with a high marginal utility of consumption. Moreover, the distributional changes brought about by this reform also result in a small welfare loss.

4.3.3 Welfare changes by household types

Next, we look at the welfare gains and losses for different types of households. We compare the benchmark model economy and model economy E_{15} . We do this for the sake of brevity and because this reform is the one that results in the smallest welfare gains. We conjecture that qualitatively our results would not change in the more progressive consumption-based reforms. And that quantitatively the welfare changes would be larger.

In our model economies there are as many household-types as there are $\{a, s\}$ pairs in the individual state space. To calculate the welfare gains at each point in the state space, $\Delta_{15}(a, s)$, we solve the following equation,

$$v_B(a, s, \Delta_{15}(a, s)) = v_{15}(a, s)$$

In Figure 1 we report the averages of these measures of the individual welfare gains for various groups of households.

The Wealth Sorting. In Panel A of Figure 1 we sort the households by wealth and we average the welfare gains of the households in increments of five percentile points of the wealth distribution. It turns out that the welfare gains are larger for the asset-poor households and that they are decreasing in wealth. This reduction in welfare is not monotonic because as households become wealthier the share of households with different labor market shocks changes. But nonetheless, we find that the welfare changes are positive for the households in the bottom 65 percent of the wealth distribution.

These results may seem puzzling since the marginal tax-rates of the personal income tax in model economy E_B are progressive and both the marginal and the average labor income tax-rates of model economy E_{15} are flat, because in model economy E_{15} there is no tax-exempt level of labor income. But there are two reasons that justify these results. First, the expensing of net savings, that is a part of every consumption-based flat-tax system, benefits the asset poor. When households with low assets get a good labor market opportunity they start accumulating resources, and the consumption-based taxation makes this capital accumulation tax-exempt. Conversely, when asset rich households get a bad labor market shock they start to deplete assets in order to smooth their consumption, and this makes them pay higher taxes because all income obtained from dissaving is taxed. In a sense, the tax exemption of savings contains a strong redistributive component by helping wealth poor households escape from poverty, in exchange of the higher taxes paid by the households who

run down their assets.

The second reason behind the welfare gains of asset-poor households are the general equilibrium effects of this flat-tax reform. Model economy E_{15} has a larger capital stock than the benchmark model economy and, hence, its wage rate is higher and its interest rate is lower. These differences in prices benefit the asset poor, who are intensive in labor income, and harm the asset rich, who are intensive in capital income.

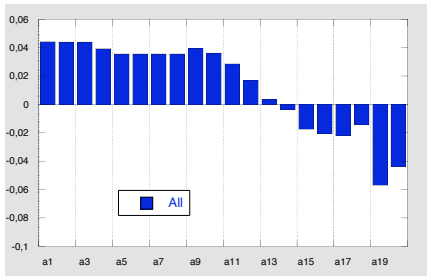
In Panels B and C of Figure 1 we first sort the households according to the realizations of the idiosyncratic shock, s , and then we rank them by wealth. This allows us to rank the households according to their two state variables. In Panel B we plot the welfare changes for the four types of workers, and in Panel C for the four types of retirees. Panel B clearly shows that the welfare gains of the workers are decreasing in wealth and increasing in labor market opportunities. Households with good labor market shocks benefit from the reduced taxes on capital and, since they tend to save more for a given level of assets, they also benefit from the tax-exemption of savings.

The retirees are dis-savers and large shares of their income come from capital sources. These are two important reasons that make them loose with the reform. Recall that the tax-exemption on savings results in a larger tax burden when savings are negative. Moreover, the reduction in the interest rate reduces the income of the asset holders. And as households become wealthier these effects become larger. On the other hand, households in our model economy, and of course the retirees, are altruistic towards their offspring. The average duration of retirement is 17 years, and the average working-life of the working-age household that will replace it is 45 years. Therefore, the welfare gains of the retirees contain part of the welfare gains of the workers, and they may more than compensate for their own welfare losses.

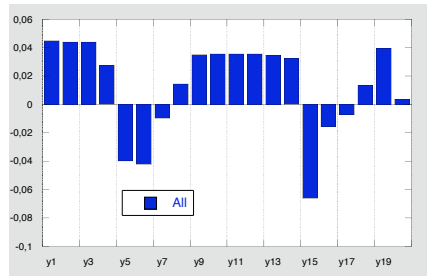
Panel C of Figure 1 shows that for shocks $s = 5, 6, \text{ or } 7$ the welfare gains decrease with wealth. They are positive for asset-poor retirees because they take into account the welfare gains of their asset-poor kids. In contrast, for shock $s = 8$, the welfare gains of retired households increase with wealth. This is because retirees of this type are very wealthy. And for sufficiently high asset holdings, the welfare gains of the reform increase with wealth because wealthy households benefit from not having to pay the large marginal taxes on capital income of the benchmark model economy.

The After-Tax Income Sorting. Next we sort the households according to their after-tax income, and we discuss the welfare gains of the income-rich and of the income-poor. In

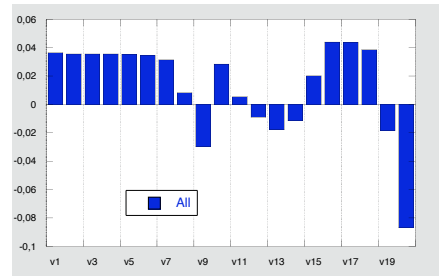
Figure 1: Welfare Gains and Losses



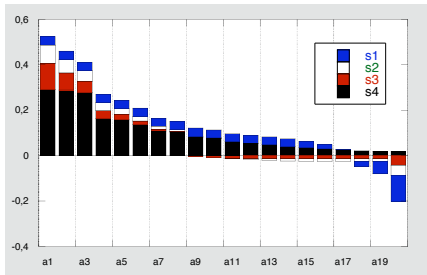
A: All Households Ranked by Wealth



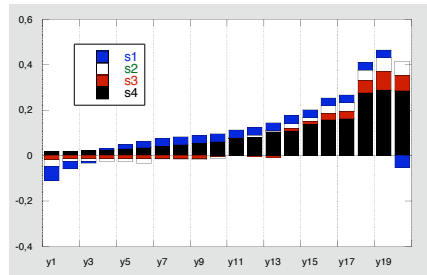
D: All Households Ranked by After-Tax Income



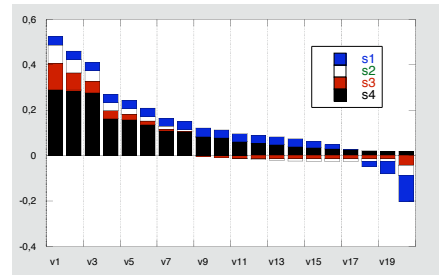
G: All Households Ranked by Value



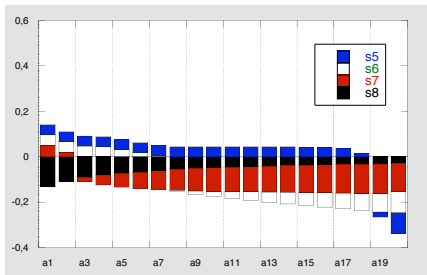
B: Workers Ranked by Wealth



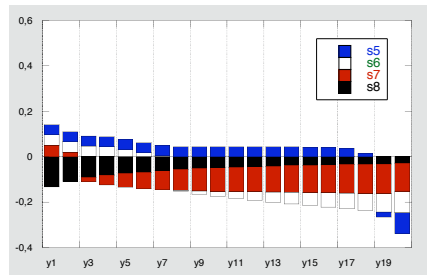
E: Workers Ranked by After-Tax Income



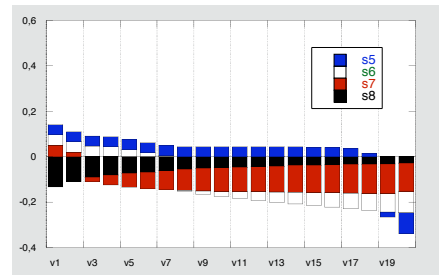
H: Workers Ranked by Value



C: Retirees Ranked by Wealth



F: Retirees Ranked by After-Tax Income



I: Retirees Ranked by Value

Panel D of Figure 1 we show that the households in the bottom 20 percent of the after-tax income distribution benefit from the flat-tax reform. These households are wealth-poor and their labor market productivity is low. The welfare changes for the households in the top eighty percent of the after-tax income distribution are not monotonic. This is because after-tax income increases with both labor income and wealth. In Panels E and F of Figure 1 we sort the households according to their after-tax income but conditioning on the realization of the household-specific shock. In contrast to what we found in the wealth sorting, we find that the welfare gains of workers are increasing in income. This is consistent with the fact that conditional on wealth, welfare gains increase with the labor market shock, and it is a consequence of the large role played by earnings in the sorting of households by disposable income.

The Value Sorting. It is hard to decide whether the tax reform benefits the poor sorting the households according to their wealth or according to their current after-tax income. This is because permanent income is a function both of financial wealth and of human wealth. Since labor market shocks are very persistent in our model economies, the permanent income of asset-poor households who receive a good labor market shock may end up being larger than the permanent income of asset-rich households who receive a bad labor market shock. For this reason, we also want to compare the welfare gains sorting households by the expected value of their individual state, (a, s) .

The value function maps the various states (a, s) to the present value of the flow of utilities that a household in that state expects to receive when it behaves optimally. Hence, the value function contains all the information that we need to rank the households according to their welfare because it takes into account not only the expected present value of income, but also the income fluctuations and the utility cost of having to work to take advantage of good labor market opportunities. In Panel G of Figure 1 we plot the welfare changes brought about by the less progressive flat-tax reform sorting the households according to their values.

We find that the households in the bottom 35 percent of the distribution of values benefit from this reform. This result is similar to the results that we obtained for the wealth and the after-tax income rankings. The welfare changes for the households in the top 65 percent of the distribution of values are non-monotonic, and while some of them experience welfare gains after the reform, others experience welfare losses. The reason that justifies these non-monotonicities is that the reform treats differently the various ways of obtaining utility — through high assets holdings or through good labor market shocks. In Panels H and I of

Figure 1 we plot the welfare gains of the value sorting, conditioning first on the realizations of the shock. The plots are identical to those in Panels B and C because the values turn out to increase monotonically with the assets-holdings, after we condition on the realizations of the shock.

5 Concluding Comments

Hall and Rabushka (1995) claimed that revenue-neutral consumption-based flat-tax reforms would be expansionary and that the tax exemption in their proposed labor income tax could be used to achieve certain distributional targets. Our results confirm that consumption-based flat-tax reforms can indeed generate large gains in output, but that they do so at the expense of increases in the inequality of after-tax income and wealth. These findings are consistent with those reported in the flat-tax literature.

However, we find that the redistributive role of the fixed deduction in the labor income tax results in sizable differences in aggregate welfare. More specifically, we show that a Benthamite social planner prefers a reform with a large labor income tax exemption and a high marginal tax rate. The reason for this is that more progressive flat-taxes allow the households to insure themselves better against the fluctuations in their labor market opportunities. Moreover, they allow them to allocate their labor effort more efficiently, and to do a better job at smoothing their consumption and leisure through time and across the various realizations of the shocks. In other words, part of the increases in income and wealth inequality that arise from the consumption-based tax reforms do not translate into inequality of consumption.

Finally, we find that the differences in the allocations that obtain in consumption-based and in income-based flat-tax reforms can be large. This tells us that the role played by the expensing of investment is important. Indeed, we show that it accounts for two thirds of the output increases brought about by the reforms, and that it would increase the Gini index of the after-tax income distribution well beyond the value that would obtain in a purely income-based flat-tax reform. Overall, we show that a Benthamite social planner favors this deduction because of the large general equilibrium effects that it generates.

In the midst of a long recession that has generated large public deficits, governments are looking for sources of public income that do not depress the economy. Our findings suggest that moving towards a progressive consumption-based flat tax scheme could achieve the goals of raising government income, stimulating the economy and providing a safety net for the households that have been hit the hardest by the recession.

One way to continue with this research project would be to compute the transitions

between the steady states of our model economies. Welfare comparisons between steady states can be misleading because they ignore the welfare changes that take place during these transitions. Since the flat tax reforms result in steady states with sizably larger asset holdings, during the transitions the households must give up some of their consumption in order to accumulate these assets. This will most likely reduce the size of the aggregate welfare gains brought about by the consumption-based reforms, and increase the welfare losses brought about by the income-based reform. Moreover, accounting for the transition also requires being explicit about the possible transition relief schemes designed to compensate the recently-retired households with large asset holdings for their sizable welfare losses. These households have accumulated savings without expensing their savings, and they would be liable to very large tax obligations under the new consumption-based tax reforms when they start running down their asset holdings. But using the algorithms that are currently available, the computational costs of computing the transitions are bound to be very large.

Appendix

A The definition of parameters ϕ_1 and ϕ_2

Let p_{ij} denote the transition probability from $i \in \mathcal{R}$ to $j \in \mathcal{E}$, let γ_i^* be the invariant measure of households that receive shock $i \in \mathcal{E}$, and let ϕ_1 and ϕ_2 be the two parameters that shift the probability mass towards the diagonal and towards the first column of submatrix $\Gamma_{\mathcal{E}\mathcal{E}}$ ³⁵, then the recursive procedure that we use to compute the p_{ij} is the following:

- *Step 1:* First, we use parameter ϕ_1 to shift the probability mass from a matrix with vector $\gamma_{\mathcal{E}}^* = (\gamma_1^*, \gamma_2^*, \gamma_3^*, \gamma_4^*)$ in every row towards its diagonal, as follows:

$$\begin{aligned}
 p_{51} &= \gamma_1^* + \phi_1 \gamma_2^* + \phi_1^2 \gamma_3^* + \phi_1^3 \gamma_4^* \\
 p_{52} &= (1 - \phi_1)[\gamma_2^* + \phi_1 \gamma_3^* + \phi_1^2 \gamma_4^*] \\
 p_{53} &= (1 - \phi_1)[\gamma_3^* + \phi_1 \gamma_4^*] \\
 p_{54} &= (1 - \phi_1)\gamma_4^* \\
 p_{61} &= (1 - \phi_1)\gamma_1^* \\
 p_{62} &= \phi_1 \gamma_1^* + \gamma_2^* + \phi_1 \gamma_3^* + \phi_1^2 \gamma_4^* \\
 p_{63} &= (1 - \phi_1)[\gamma_3^* + \phi_1 \gamma_4^*] \\
 p_{64} &= (1 - \phi_1)\gamma_4^* \\
 p_{71} &= (1 - \phi_1)\gamma_1^* \\
 p_{72} &= (1 - \phi_1)[\phi_1 \gamma_1^* + \gamma_2^*] \\
 p_{73} &= \phi_1^2 \gamma_1^* + \phi_1 \gamma_2^* + \gamma_3^* + \phi_1 \gamma_4^* \\
 p_{74} &= (1 - \phi_1)\gamma_4^* \\
 p_{81} &= (1 - \phi_1)\gamma_1^* \\
 p_{82} &= (1 - \phi_1)[\phi_1 \gamma_1^* + \gamma_2^*] \\
 p_{83} &= (1 - \phi_1)[\phi_1^2 \gamma_1^* + \phi_1 \gamma_2^* + \gamma_3^*] \\
 p_{84} &= \phi_1^3 \gamma_1^* + \phi_1^2 \gamma_2^* + \phi_1 \gamma_3^* + \gamma_4^*
 \end{aligned}$$

³⁵A detailed description of this probability mass shifting procedure can be found in Castañeda, Díaz-Giménez, and Ríos-Rull (2003).

- *Step 2:* Then for $i = 5, 6, 7, 8$ we use parameter ϕ_2 to shift the resulting probability mass towards the first column as follows:

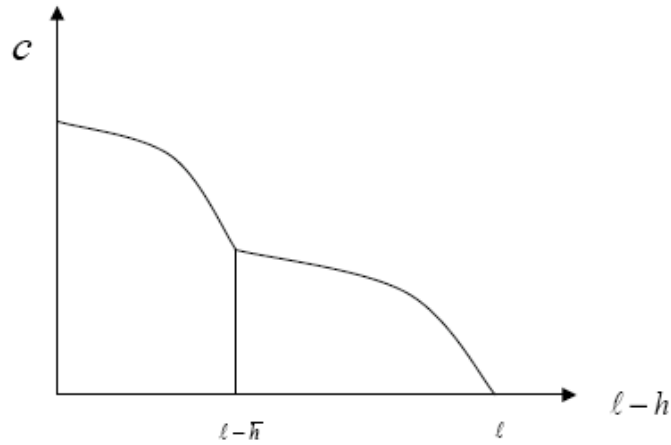
$$\begin{aligned}
 p_{i1} &= p_{i1} + \phi_2 p_{i2} + \phi_2^2 p_{i3} + \phi_2^3 p_{i4} \\
 p_{i2} &= (1 - \phi_2)[p_{i2} + \phi_2 p_{i1} + \phi_2^2 p_{i4}] \\
 p_{i3} &= (1 - \phi_2)[p_{i3} + \phi_2 p_{i4}] \\
 p_{i4} &= (1 - \phi_2)p_{i4}
 \end{aligned}$$

B Non-convexities

Due to the upper cap in payroll taxes, the marginal tax on labor income has a discontinuity at the income level where the cap is reached. This creates a serious problem when we try to find the optimal household policy. Specifically, for a given value of the choice of end-of-period period assets, z , the budget set of the contemporaneous labor decision becomes non-convex. In Figure 2 we illustrate this point. Consider pair of individual state variables (a, s) and a choice of end-of-period assets, z . Then, equations (7), (8) and (9) and the boundary constraints on c and h define the consumption possibilities set for c and $\ell - h$. In Figure 2 we plot an example of this set for $a = 0$. When the household chooses not to work and to enjoy ℓ units of leisure, its consumption is zero. As the household starts to work, its consumption increases albeit at a decreasing rate. This is because of the progressivity of the personal income tax, τ_y , which reduces the after-tax wage of every extra hour of work. Let \bar{h} be the hours of work such that $e(s)\bar{h}w = a_3$. For $h > \bar{h}$ the marginal payroll tax is zero. Therefore the slope of the consumption possibilities set increases discretely at $h = \bar{h}$ and from that point onwards it decreases monotonically as we increase h , again because of the progressivity of τ_y .

This lack of convexity is twice unfortunate. First, because it implies that the first order necessary conditions are no longer sufficient for optimality and, therefore, they do not identify the optimal solution uniquely. In fact there are two points that potentially satisfy the first order conditions, one above and one below the threshold \bar{h} , and only one of these points is the optimal solution. Second, as we change the choice of end-of-period assets, z , the optimal choice of hours becomes discontinuous exactly when we move from a solution on one side of \bar{h} to a solution on the other side of \bar{h} . This is much more troublesome for our computational procedure. And it forces us to solve the household decision problem using discrete value function iterations which are much more computationally intensive, than the Euler equation iterations which can only be used when the choice sets are convex.

Figure 2: Non-convex constraints



C Computation

As we have mentioned in Section 3, to calibrate our model economy we must solve a system of 29 non-linear equations in 29 unknowns. Actually, we solve a smaller system of 25 non-linear equations in 25 unknowns because the value of government expenditures, G , is determined residually from the government budget, and because three of the tax parameters are functions of our guess for aggregate output. This non-linear system is only the outer loop of our computational procedure because we must also find the stationary equilibrium values of the capital labor ratio, K/L , and of aggregate output, Y , for each vector of unknowns. The details of our computational procedure are the following:

- *Step 1:* We choose a vector of weights, one for each of the 25 non-linear equations. These weights measure the relative importance that we attach to each one of our targets.
- *Step 2:* We guess an initial value for the 25 unknowns
- *Step 3:* We guess an initial value for aggregate output, Y_0 (which determines the values of the three tax parameters mentioned above).
- *Step 4:* We guess an initial value for the capital labor ratio $(K/L)_0$
- *Step 5:* We compute the decision rules, the stationary distribution of households and the new value of the capital labor ratio, $(K/L)_1$
- *Step 6:* We iterate on K/L until convergence

- *Step 7:* We compute the new value of aggregate output, Y_1 , that results from the converged value of K/L
- *Step 8:* We iterate on Y until convergence
- *Step 9:* We iterate on the 25-dimensional vector of unknowns until we find an acceptable solution to the system of 25 non-linear equations.

To find the solution of the system of 25 non-linear equations in 25 unknowns, we use a standard non-linear equation solver. Specifically, we use a modification of Powell’s hybrid method that is implemented in subroutine DNSQ from the SLATEC package.

To calculate the decision rules, we discretize the state space and we use a refinement of the discrete value function iteration method. Our refinement exploits the upper bounds of the asset space and the monotonicity of the decision rules to reduce the size of the control space, and it uses Howard’s policy improvement algorithm to reduce the number of the decision rule searches.

Without the first refinement, the size of our state space is $n_k \times n_s = 681 \times 8 = 5,448$ points. The size of our control space is $n_k \times n_n = 681 \times 201 = 136,881$ points for workers and $n_k = 681$ for retirees. Since the numbers of working-age and retirement states are $n_w = n_r = 4$, the total number of search points is $[(n_k \times n_w) \times (n_k \times n_n)] + [(n_k \times n_r) \times n_k] = 374,718,888$ points.

We approximate the stationary distribution, x^* , with a piecewise linearization of its associated distribution function. The grid for this approximation has 80,000 unequally spaced points which are very close to each other near the origin (see Aiyagari (1994), Huggett (1995) or Ríos-Rull (1998) for details).

To compute the model economy’s distributional and aggregate statistics we compute the integrals with respect to the stationary distribution, x^* . We evaluate these integrals directly using our approximation to the distribution function for every statistic except for those that measure mobility, the earnings life cycle, and the intergenerational correlation of earnings. To compute these three statistics, we use a representative sample of 20,000 households drawn from x^* (see Ríos-Rull (1998) for details).

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