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JEL Classification: D83, E24, J41, L11

Keywords: Dual Labor Markets, Unemployment

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Dual Labor Markets and the Equilibrium Distribution of Firms*

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Keywords: Dual Labor Markets, Temporary Contracts, Firm Dynamics, Unemployment.
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1 Introduction

European labor markets are characterized by a two-tier system, with the co-existence of open-ended (OE) contracts, with large termination costs, and fixed-term (FT) contracts, of short duration. In this paper we study the firm-level determinants in the usage of FT contracts in the context of dual labor markets and its consequences for firm dynamics, the equilibrium distribution of firms, and aggregate productivity.

We start by looking at Spanish administrative firm-level data for the period 2004-2019 to document some new facts about the use of temporary contracts across firms.[1] The case of Spain is of particular interest because this country has the largest incidence of FT contracts in Europe (26.3% in 2021 according to OECD data, compared to 11.8% on average across all OECD countries).[2] Moreover, Spanish law has traditionally provided strong employment protection, making the duality very stark. Using our data, we document the following four facts. First, there is a large degree of heterogeneity in the usage of FT contracts across firms, with the distribution being rightly skewed. For instance, while the average FT share across firms is 18.1%, the median firm employs just 2.7% of workers under FT contracts. Second, although the use of FT contracts varies greatly across sectors, regions, and time, variation along these dimensions alone only explains 16% of the overall variation in the share of temporary workers across firms. Therefore, 84% of the variation is due to firm-specific factors that go beyond conventional explanations like seasonality of demand (most likely absorbed by industry and province dummies) or business cycles (absorbed by time dummies). Third, the share of temporary workers increases monotonically with firm size up to firms with 60 workers (95th percentile of firm size distribution). When looking at the share of temporary workers against firm size while controlling for unobserved firm fixed effects, the share of temporary workers increases monotonically with firm size up to firms of 1,000 workers (99.9th percentile). And fourth, most of the variation in the share of temporary workers across firms is due to unobserved firm fixed effects.[3]

To rationalize these facts and understand the macroeconomic effects of dual labor markets, we write a model of firm dynamics with search-and-matching frictions and a two-tier labor market structure. In the model, a set of multi-worker firms subject to idiosyncratic productivity shocks can direct unemployed workers by posting (and committing to) dynamic long-term contracts. Contracts specify state-contingent trajectories for wages and layoff rates for the duration of a worker's tenure in the firm. Firms may simultaneously post two types of dynamic contracts: OE and FT. Workers hired under each contract type potentially differ (ex-post) in terms of their relative productivity within the firm, the costs that the firm must incur to fire them, and the rate at which worker separation shocks occur. For FT contracts, besides choosing wages and layoff policies, firms must also set the rate at

[1] The data contains the quasi-universe of Spanish firms and provides complete balance-sheet information. Among many other items of a firm's balance sheet, we observe the firm's number of employees, their average yearly wage, and the number of them that are hired with a temporary contract.

[2] Other OECD countries with a high share of temporary employment are Poland (26.2%), Portugal (22.0%), France (16.9%), Italy (15.4%) and Germany (12.9%). All numbers are for 2017. The data are available at stats.oecd.org.

[3] In the companion paper Auciello, Pijoan-Mas, Pau-Roldan-Blanco and Tagliati (2023) we document these facts in more detail.

which they promote their FT workers to an OE position. Finally, the two labor markets are also potentially different in terms of aggregate matching efficiency, i.e. the residual of the mapping from a sub-market's tightness (the postings-to-applicants ratio) to the actual number of matches in that market segment. A free-entry condition pins down the aggregate measure of firms in equilibrium.

Unemployed workers are ex-ante indifferent between applying to either contract, as well as between the firms that offer them, because in equilibrium less ex-post attractive offers are posted in tighter markets. Similar to other models of directed search and firm dynamics, the menu of optimal contracts maximizes the joint surplus generated by the firm and its OE and FT workers, and this endogenously determines the rate at which new workers of each type are hired in equilibrium, giving rise to a non-degenerate distribution of firms in the space of idiosyncratic productivity and the number of FT and OE workers.

We calibrate the model to cross-sectional moments related to the distribution of firms and the share of temporary employment from our firm-level Spanish data, as well as to aggregate data on job flows into and out of unemployment by contract type. Moreover, we target the increasing relationship between the temporary share and firm size that we observe in the data. The calibrated model offers a good fit of the data. Crucially, to rationalize these empirical patterns, the calibration delivers that (i) the matching in the FT market is more efficient, with about 3 times more matches per unit of time than in the OE market for given market tightness, and that (ii) workers hired under each type of contract have similar productivity and are highly substitutable.

In the calibrated economy, firms face a trade-off between the lower costs of attracting workers to FT contracts (due to the higher matching efficiency) and the higher turnover of FT positions. This leads to different incentives to hire FT workers for firms of different sizes and productivities. On the one hand, larger firms of similar productivity hire a higher fraction of temporary workers. This is because, due to the presence of decreasing returns to scale, the marginal product of labor is lower for these firms, and so is the opportunity cost of unfilled vacancies. On the other hand, more productive firms target a lower long-run share of temporary workers, as the hiring effort to replace expiring FT contracts is larger for them. In particular, at their long-run target size, firms of different productivity levels have similar opportunity costs of unfilled vacancies (due to decreasing returns to scale and larger firm size), but more productive firms are larger. Because the flow rate of vacancy posting is the same for all firm sizes, for more productive firms to replace their larger number of expiring FT contracts they need to post vacancies in less tight labor markets, which are more expensive. That is, worker turnover is more expensive for them, and hence they turn to OE contracts. All in all, the calibrated economy reproduces the joint distribution of firm sizes and temporary share. Instead, in a counterfactual experiment we show that if both the FT and OE markets had the same match efficiency, more job-seekers would self-select into the OE market, leading to an equilibrium where the share of FT workers is much smaller on average and declining with firm size.

Finally, we use the calibrated model for policy analysis. A recent labor market reform in Spain, approved in December 2021, reduced the duration of FT contracts in an attempt to limit their usage and switch firms into using more OE contracts. To explore this type of policy, we solve for a series of counterfactual economies where we vary the maximum duration of FT contracts, from 1 quarter to 2 years. We find that reducing the duration of FT contracts, from half-a-year under the old regulation to one quarter, does indeed reduce the share of temporary jobs and the unemployment rate. Yet, it does so at the expense of a reduction in aggregate productivity and an overall output decline. The reduction in the share of temporary contracts is intuitive as shorter FT contracts makes them less attractive to firms. The decline in the unemployment rates comes from both (i) the overall reduction in worker separations, due to the increase in the share OE contracts, and (ii) the increase in the job-finding rate in the OE market, where most workers are searching, due to the higher interest of firms for posting contracts in this market. The decline in aggregate productivity happens mainly through the reduction in the number of firms in equilibrium. Instead, if the degree of duality in the labor market were reduced by making FT contracts more similar to OE contracts through an increase in their duration, we find that aggregate productivity would increase and unemployment would decline, leading to an increase in aggregate output.

Related Literature There is a large literature studying the effects of the duality of employment contracts on the labor market outcomes of workers, like the average unemployment rate, the volatility of employment, or the dynamics of labor market flows.[4] Blanchard and Landier (2002), Cahuc and Postel-Vinay (2002), Bentolila, Cahuc, Dolado and Le Barbanchon (2012), and Sala, Silva and Toledo (2012) study the effect of dual labor markets in models with search and matching frictions *à la* Mortensen and Pissarides (1994). In these models search is random and firms do not choose which type of contracts to offer. If they did, they would hire all new workers in FT contracts as in Costain, Jimeno and Thomas (2010) because from the firm side the flexibility of FT contracts dominates OE contracts.[5] Hence, these models are not designed to understand the differential choices of FT vs OE contracts across firms. Furthermore, because firms can only hire one worker, these models cannot be used to link contract choices to firm dynamics.

Several papers provide arguments for the co-existence of FT and OE contracts. In a similar framework as the papers above, Cahuc, Charlot and Malherbet (2016) allow for firms to be heterogeneous in their expected job duration and to choose the type of contract associated to their vacancies. This serves to show that firms prefer to use FT contracts for jobs of short expected duration (in order to save on firing costs) and OE contracts for jobs of long expected duration (to save on vacancy posting costs). In our model expected job duration is driven by the persistence of the stochastic productivity process, but firms are homogeneous in this dimension. In the context of directed search models, Berton and Garibaldi (2012) argue that an advantage of OE over FT contracts for firms is that the vacancy-filling rate will be higher in equilibrium when posting OE contracts as more job-seekers will self-select into the OE market, which offers them higher value jobs. Our model features the same equilibrium logic. However, by properly parameterizing the matching

[4] For a worker-side analysis in the case of Spain, see Dolado, García-Serrano and Jimeno (2002).

[5] Still, the endogenous firing decision in these models allows to make separations contingent on contract type and hence the aggregate share of FT contracts is determined in equilibrium.

function of the OE and FT markets, we can obtain vacancy-filling rates that are larger in the FT than OE markets, a feature that is needed to match the empirical labor market flows. Other explanations for the coexistence of OE and FT contracts are to diminish on-the-job search and hence to retain high-quality workers as in Cao, Shao and Silos (2013) or the presence of financial frictions as in Caggese and Cuñat (2008). Our model with homogeneous workers and perfect financial markets does not allow for these elements.

On the empirical side, several papers have studied the effects of employment protection legislation in the context of dual labor markets. For example, Daruich, Di Addario and Saggio (2020) show that relaxing constraints on FT contracts relative to OE contracts in Italy failed to increase overall employment, and that it increased firms' profits mostly at the expense of young workers. Using data from Portugal, Cahuc, Carry, Malherbet and Martins (2022) show that a policy intended to restrict the use of fixed-term contracts by new establishments of large firms did not increase the number of permanent contracts and ended up decreasing employment in large firms. We complement these studies by showing that, within the context of our calibrated model, changing the duration of fixed-term contracts yields non-monotonic effects on aggregate productivity and employment. Finally, in the context of Spain, Garcia-Louzao, Hospido and Ruggieri (2022) estimate the returns from the accumulation of experience in OE versus FT contracts, finding substantially lower returns from the latter, which the authors attribute to worse human capital accumulation. This justifies our assumption of modelling FT and OE workers as imperfect substitutes.

As we analyze dual labor markets from the firm's side, we also relate to a growing macro literature studying the equilibrium dynamics of multi-worker firms in the context of frictional labor markets. We use a directed search framework with dynamic long-term contracts in the spirit of Kaas and Kircher (2015) and Schaal (2017). We adapt this framework to a continuous-time setting with slow-moving state transitions similar to Roldan-Blanco and Gilbukh (2021) and extend it to incorporate two different labor markets. Similar to these models, the optimal contract is determined by a joint surplus optimization problem. We show, in particular, that the marginal surplus is a function of both total firm size and the share of FT workers in the firm, as firms operate with a decreasing returns-to-scale technology and workers employed under different contracts differ in terms of relative productivity within the firm. An alternative approach is to assume random search and study firm dynamics in the context of decreasing returns and Nash bargaining (as in Elsby and Michaels (2013) or Acemoglu and Hawkins (2014)) or in setting with on-the-job search and various wage-setting protocols (e.g. Moscarini and Postel-Vinay (2013), Coles and Mortensen (2016), Bilal, Engbom, Mongey and Violante (2019), Gouin-Bonenfant (2020), Audoly (2020) and Elsby and Gottfries (2021)). These models have been shown to provide an outstanding quantitative fit of the labor market flows and the hiring and firing decisions of firms in non-dual markets, such as the United States. Our contribution to this literature is to provide a quantitative model for the firm and aggregate labor market dynamics of dual markets, which are more common in Europe.

Table 1: Temporary share: descriptive statistics

Firm size (employment)	share of firms (%)	Distribution of FT contracts mean	p10	p25	p50	p75	p90	p95
Total	100	0.181	0.000	0.000	0.027	0.294	0.591	0.800
1-10	77.65	0.164	0.000	0.000	0.000	0.250	0.541	0.776
11-50	19.04	0.250	0.000	0.031	0.163	0.391	0.677	0.825
51-100	1.78	0.255	0.000	0.034	0.160	0.393	0.701	0.861
101-200	0.99	0.237	0.000	0.029	0.147	0.361	0.645	0.833
201-500	0.30	0.222	0.000	0.026	0.137	0.329	0.589	0.796
501-1,000	0.14	0.229	0.000	0.033	0.142	0.333	0.616	0.841
1,001-5,000	0.10	0.258	0.000	0.048	0.165	0.374	0.746	0.947
5,000+	0.02	0.279	0.000	0.063	0.191	0.393	0.709	0.964

Outline The rest of the paper is organized as follows. Section 2 describes the data and our main empirical findings. Section 3 outlines the model and characterizes its equilibrium conditions. Section 4 discusses the estimation and presents a few counterfactual experiments. Section 5 describes our main policy experiment. Section 6 offers concluding remarks. All proofs and additional results are relegated to the Appendix.

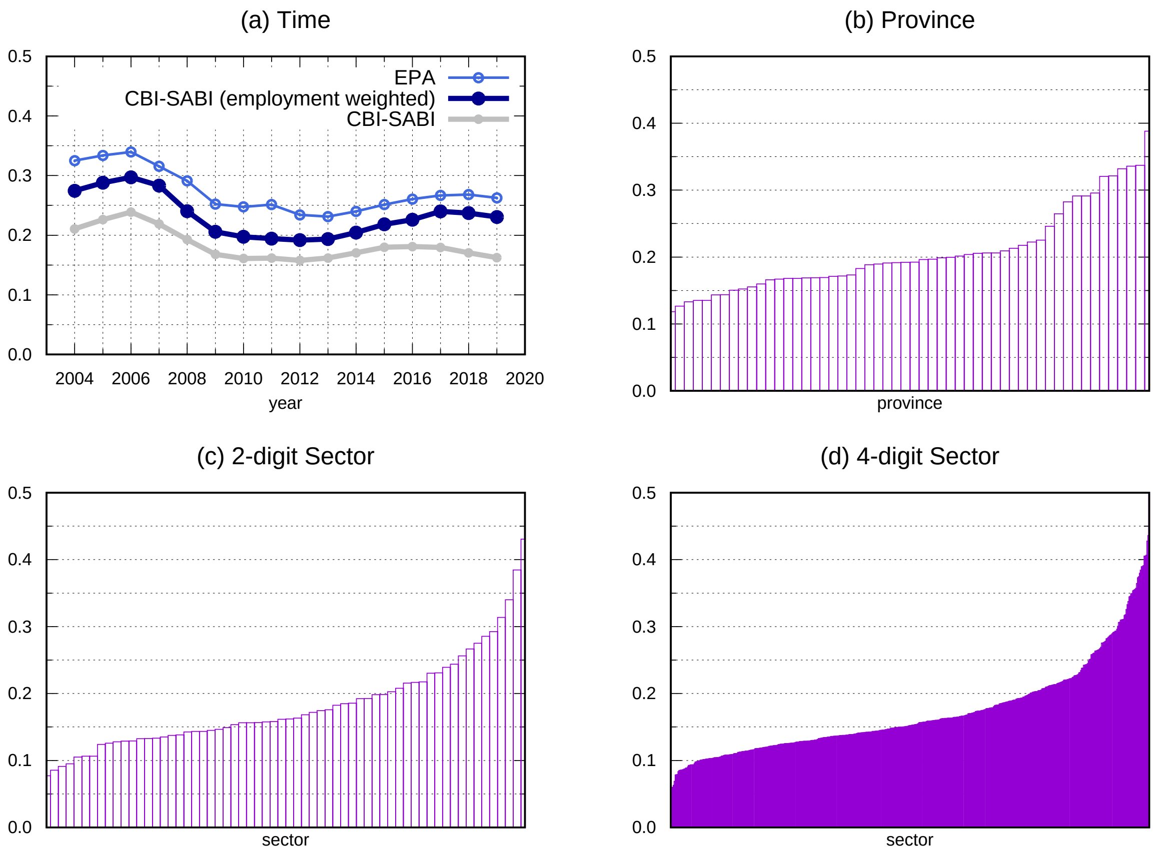
2 Empirical Findings

Data We use annual data for Spain from the *Central de Balances Integrada* (CBI) dataset, a comprehensive and unbalanced panel of confidential firm-level balance-sheet data from the Spanish Commercial Registry (*Registro Mercantil Central*), collected by the *Central de Balances*, a department within the Banco de España.[6] This dataset covers the quasi-universe of Spanish firms, including large and small firms as well as privately held and publicly traded firms. Among many other items from the balance sheet of firms, the data provide information about total employment and type of employment contract. The data has excellent geographical coverage and is representative of all non-financial sectors of economic activity, where sectors are defined up to the 4 digits in the NACE Rev. 2 classification. We use data for the period is 2004-2019.[7] We restrict our sample to firms observed for at least 5 years and whose average employment over the period is at least one worker. After some cleaning, we keep data for 7,153,669 firm-year observations, corresponding to 705,879 different firms.

[6]All Spanish firms are legally obliged to report their end-of-year accounting results to the Spanish Commercial Registry. The *Central de Balances* then obtains these data from the Commercial Registry, and compiles and homogenizes the data into a unique dataset. In order to further expand the coverage of the data, Banco de España merged CBI with the SABI database (Iberian Balance-Sheet Analysis System), owned by Informa-Bureau van Dijk, which is used as Spain's input to the widely-used Amadeus and Orbis datasets. Detailed information about the CBI-SABI data set and its representativeness is provided by Almunia, López-Rodríguez and Moral-Benito (2018).

[7]We start in 2004 because earlier years of the sample do not track well the aggregate share of temporary workers from the labor force survey.

Figure 1: Temporary share: aggregate variation



Notes: Panel (a) reports the average share of temporary workers by year. The "CBI-SABI" line corresponds to the sample average across firms; the "CBI-SABI (employment weighted)" line weights each firm by the employment size, thereby providing the share of temporary workers across workers; the "EPA" line provides the share of temporary workers across workers computed through the labor force survey. Panel (b) reports the average share of temporary workers across firms by province (sorted from smallest to largest). Panels (c) and (d) report the average share of temporary workers across firms by sector (2 and 4 digit respectively, sorted from smallest to largest).

Findings We start by looking at the distribution of the temporary share of workers across firms (Table 1). In our sample, the average share of temporary workers across all years is 18.1%, while the median is 2.7%. This gap reflects a highly right-skewed distribution. Indeed, as is shown in the first row of Table 1, the share of temporary workers at the 25th, 75th and 90th percentiles is 0.0%, 29.4%, and 59.1%, respectively. Thus, a relatively small fraction of firms make a very intensive use of fixed-term contracts. In the next rows of Table 1 we report the same statistics conditional on firm size. We find that the large right-skewness is present across all size bins, and that average and median temporary share tend to increase with firm size (which will be discussed in more detail later). Finally, we note that most of Spanish firms (more than 95%) have 50 workers or less and firms of 500 or more workers only represent 0.25% of the total.

Second, we look at the aggregate determinants of the share of temporary workers across firms. Panel (a) in Figure 1 reports the average temporary share by year (gray line). The temporary share

Table 2: Temporary share regressions

	(1)	(2)	(3)	(4)	(5)	(6)
Size Dummies	No	No	No	No	Yes	Yes
Firm FE	No	No	No	No	No	Yes
Year FE	Yes	No	No	Yes	Yes	Yes
Province FE	No	Yes	No	Yes	Yes	Yes
Industry FE	No	No	Yes	Yes	Yes	Yes
N	6,843,672	6,843,672	6,842,273	6,842,273	6,842,273	6,841,042
R^2	0.009	0.052	0.113	0.162	0.184	0.620

Notes: Each column corresponds to an OLS regression of the share of temporary workers against several controls.

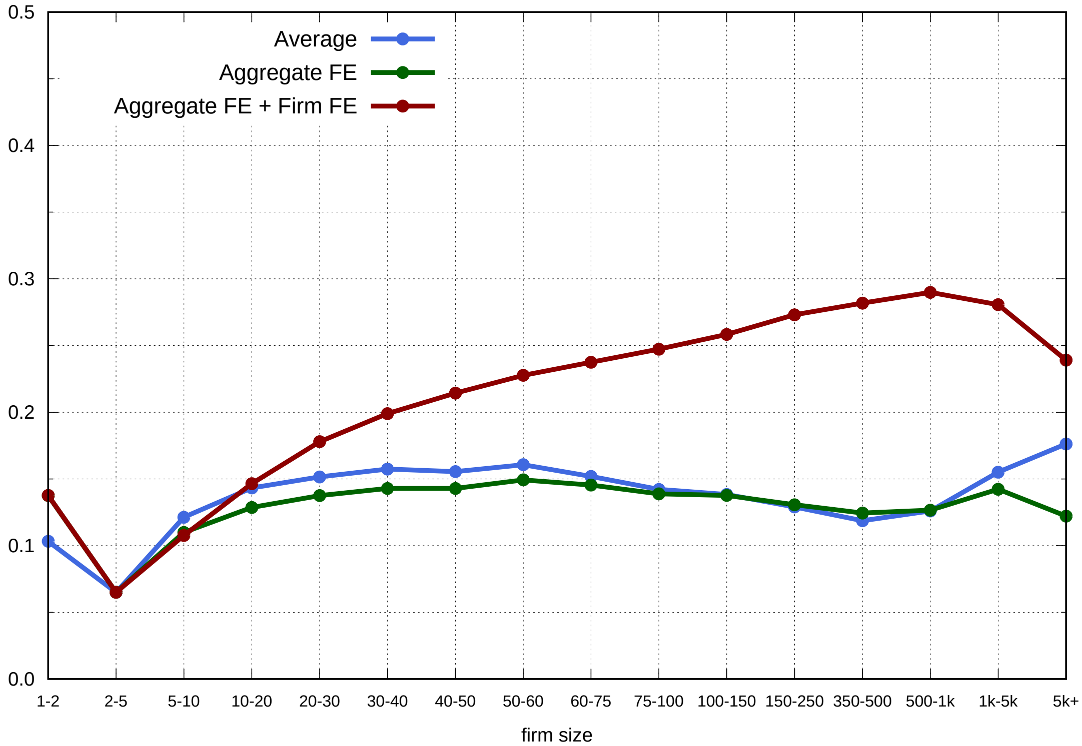
increases between the expansion years of 2004 and 2006, with a peak of 23.6% in 2006. Then, as the recession strikes in 2007, the temporary share declines to 15.7% in 2012. This is consistent with the notion that a large proportion of hiring and firing in economies with a dual labor market structure is done through temporary contracts. In order to compare our data with data from the labor force survey (light blue line), labeled EPA for *Encuesta de Población Activa*, we can compute the average fraction of workers in temporary contracts by weighting each firm by its employment level (dark blue line). We find that the time evolution of the temporary share in our firm-level data tracks the labor force survey well, although there is a small level difference between the two series. Next, panels (b) to (d) in Figure 1 report the average temporary share by province and by 2-digit and 4-digit industries. These figures show substantial heterogeneity in the share of temporary workers along these aggregate dimensions. Across provinces, the lowest 5 values range from 11.5% to 13.5% (Barcelona, Álava, La Rioja, Soria, and Madrid) while the highest 5 values range from 32.1% to 38.8% (Jaén, Córdoba, Cádiz, Almería, and Huelva), which are about three times as large. Across 2-digit sectors, the 4 lowest values range from 7.7% to 9.5% ("*Real estate activities*", "*Legal and accounting activities*", "*Activities of head offices*", and "*Manufacture of basic pharmaceutical products*") while the highest 4 values range from 31.4% to 43.1% ("*Construction of buildings*", "*Services to buildings and landscape activities*", "*Civil engineering*", and "*Employment activities*"), which is more than three times as much. There are several reasons to expect heterogeneity in the temporary share across sectors, like seasonality or uncertainty of demand, or the relative importance of firm-specific human capital. The geographic heterogeneity may be explained by differences in the sectoral composition of provinces, among others.

To quantify the importance of these factors on the overall variation of the temporary share across firms, we regress the temporary share of firms f in year t against year (α_t), province (α_p), and 4-digit industry (α_i) fixed effects:

$$TempSh_{ft} = \alpha_i + \alpha_p + \alpha_t + \varepsilon_{ft}$$

We find that these aggregate factors explain 1%, 5%, and 11% of the overall variation in the temporary share, respectively, when added one by one, and 16% when added together (see columns (1)-(4) in Table 2). Hence, while business cycles and sectoral differences are important determinants of the variation in the use of fixed-term contracts, most of the variation (84%) is not accounted for

Figure 2: Temporary share, by firm size.



Notes: The blue line is the average of the temporary share across firms of different sizes (employment). The green line reports the coefficients of the size dummies of a regression of temporary share that controls for aggregate variables. The red line corresponds to a regression that additionally controls for firm fixed effects. See columns (5) and (6) in Table 2 for these two regressions. The intervals defining the size bins of the X-axis are closed to the left and open to the right.

by them.

Finally, we look at firm-level determinants of the share of temporary employment. To do so, to the previous regression we add dummies for different size bins measured in total employment ($\boldsymbol{X}_{ft}$), as well as firm fixed effects (α_f) capturing unobserved and permanent firm-specific factors:

$$TempSh_{ft} = \boldsymbol{X}_{ft}\boldsymbol{\beta} + \alpha_f + \alpha_i + \alpha_p + \alpha_t + \epsilon_{ft}$$

The estimated coefficients for the size dummies are reported in Figure 2, while the R^2 coefficients are reported in columns (5)-(6) of Table 2. When not controlling for unobserved firm fixed effects, the temporary share is hump-shaped with firm size: it increases for firms up to 60 workers (the 95th percentile of the firm size distribution) and declines afterwards (see green line). This pattern is very similar to the plain average of the temporary share by firm size (see blue line).[8] Instead, when controlling for firm fixed effects, the share of temporary workers increases monotonically for firms between 2 and 1,000 employees (the 99.9th percentile of the firm size distribution), and declines

[8]It is important to note, however, that firms of less than 2 workers display quite a large temporary share that does not square with the hump. This firm size category is mainly made up of 1 worker firms, and the dynamics of the temporary share may obey to different reasons (arithmetically, these firms have temporary shares of either 0% or 100%).

afterwards (see black line). The regression with firm fixed effects can be interpreted in terms of firm dynamics: a firm increases (decreases) its temporary share when increasing (decreasing) in size, which means that temporary contracts are used to adjust firm size. Finally, firm fixed effects are an important determinant of the temporary share: the R^2 of the regression of the temporary share against aggregate fixed effects and size dummies is 0.18 and it increases to 0.62 when adding the firm fixed effects.

Our empirical analysis has shown that most cross-sectional variation in the temporary share is explained by firm-specific factors, with firms increasing their share of temporary workers as they grow in size. Following these findings, we write a firm-dynamics model with a dual labor market structure to study the implications for the aggregate economy.

3 Model

3.1 Environment

Demographics Time is continuous, infinite and indexed by $t \in \mathbb{R}_+$. We consider a stationary environment with no aggregate shocks. The economy is populated by a mass of workers with fixed unit measure and an endogenous measure F of firms. Both types of agents are risk-neutral and infinitely-lived, and discount the future at a common and exogenous rate $\rho > 0$. Firms may either be employing workers and producing, or have no workers.

Workers and firms interact in frictional labor markets. Workers are ex-ante identical. Ex-post, they may be unemployed and receiving a flow unemployment benefit $b > 0$, or employed by a firm under a contract that provides them with a wage. A firm can employ a worker under either one of two types of contract: open-ended (OE) and fixed-term (FT). Workers employed under these two contract types differ in various dimensions, namely firing costs, productivity within the firm, matching efficiency and promotion opportunities, as we describe below.

Production Technology A firm employs $n_i = 0, 1, 2, \ldots$ workers of each type $i \in \mathcal{I} \equiv \{OE, FT\}$. A firm is said to be *active* if $n_{OE} + n_{FT} \geq 1$. We denote the employment vector of an active firm by $\vec{n} \equiv (n_{OE}, n_{FT}) \in \mathcal{N}$, where $\mathcal{N} \equiv \mathbb{Z}_+^2 \backslash \vec{0}$. The production technology is:

$$Y(\vec{n}, z) = \exp(z) y(\vec{n}), \quad \text{with} \quad y(\vec{n}) = \left(\omega n_{OE}^{\alpha} + (1-\omega) n_{FT}^{\alpha} \right)^{\frac{\nu}{\alpha}} \tag{1}$$

where $\alpha < 1$, $\omega \in (0,1)$, and $\nu \in (0,1]$ are parameters, and $z \in \mathcal{Z} \equiv \{z_1 < \cdots < z_k\}$ is idiosyncratic productivity, following a continuous-time Markov chain, where $\lambda(z'|z)$ denotes the intensity rate of a z-to-z' transition.[9] Equation (1) states that workers employed under different contracts are imperfect substitutes within the firm, with the elasticity of substitution being $\frac{1}{1-\alpha}$. The parameter ω

[9]The transition rates satisfy standard properties: for all $z \in \mathcal{Z}$, we have (i) $\lambda(z|z) \leq 0$; (ii) $\lambda(z'|z) \geq 0, \forall z' \neq z$, (iii) $\sum_{z'} \lambda(z'|z) = 0$; and (iv) $\sum_{z'} \lambda(z'|z) < +\infty$.

measures the relative productivity of OE-type workers, while ν measures the degree of diminishing returns to scale in technology. Worker types are complements in production if $\alpha < \nu$.[10]

Worker Flows Firms hire workers of either type i by posting job contracts, as described in detail below. Firms may lose workers for three different reasons: (i) because of an exogenous firm exit shock, with intensity s^F, dissolving the firm and sending all of its workers to unemployment; (ii) because of exogenous worker-specific separation shocks, with intensity s_i^W for each contract type i; or (iii) because the firm endogenously decides to fire some of them. In particular, the firm must choose a per-worker firing rate δ_i for each type-i contract. A firing rate δ_i carries a layoff cost equal to

$$C^F(\delta_i) = \chi_i \delta_i^{\psi_i} \tag{2}$$

with $\chi_i > 0$ and $\psi_i > 1$. The cost $C^F(\delta_i)$ is meant to capture monetary expenses associated to laying off workers (such as administrative expenses and legal costs) but not severance payments.[11]

Firms may also choose to promote their FT workers into an OE contract, with the opposite transition being prohibited by law. In particular, firms choose a promotion rate p for each one of their type-FT workers (if any), which carries a promotion cost:

$$C^P(p) = \chi_p p^{\psi_p} \tag{3}$$

with $\chi_p > 0$ and $\psi_p > 1$. The cost $C^P(p)$ is meant to capture monetary expenses associated to the promotion of workers (e.g. the cost of training programs associated to upgrade workers to a permanent position).

In the event that a firm loses all of its workers, it must exit the market and become a so-called potential entrant. Potential entrants must incur a cost $\kappa > 0$ in order to attract their first worker. Upon successful entry, they draw a productivity $z_e \in \mathcal{Z}$ from some probability mass function $\pi_e(z_e)$.

Contracts Search is directed. Every period, firms publicly announce long-term FT and OE contracts in order to attract new workers. For simplicity, the 1st posting in each of the two markets is free while the rest are infinitely costly, that is, we only allow firms to post one vacancy per instant of time. Unemployed workers can perfectly observe the terms of all posted contracts (there are no information asymmetries). Let $(\vec{n}_t^{t+j}, z_t^{t+j})$ be the full history of possible firm states between t and $t+j$. At any date t and for all $j > 0$, a contract of type $i \in \mathcal{I}$ is a complete state-dependent sequence $c_i(\vec{n}_t^{t+j}, z_t^{t+j})$ of wages $w_i(\vec{n}_t^{t+j}, z_t^{t+j})$, firing rates $\delta_i(\vec{n}_t^{t+j}, z_t^{t+j})$, and for workers employed under FT contracts, intensities $p(\vec{n}_t^{t+j}, z_t^{t+j})$ of promotion from FT to OE contract, conditional on no worker separation.

[10] In this case, the production function is supermodular, i.e. $\dfrac{\partial^2 y(\vec{n})}{\partial n_{OE} \partial n_{FT}} > 0$ if, and only if, $\alpha < \nu$.

[11] A severance payment in this model would be a pure lump-sum transfer from the firm to the worker. As the optimal contract maximizes the joint surplus of the firm and all of its workers (as we will show later), severance payments will not affect any relevant choice.

We assume the following commitment structure. On the worker's side, there is no commitment: workers may forfeit their contract and quit the firm, but they must go back to costly search if they seek to regain employment (we do not allow them to search while on the job). On the firm's side, there is full commitment to the posted contract, and the terms of the contract cannot be revised or renegotiated for the duration of the match. Therefore, contracts must always comply with the firm's initial promises. Moreover, we assume that the firm cannot discriminate between workers with the same contract type.[12]

Matching Technology Given these assumptions, a labor market segment may be summarized by the long-term value that the worker can expect to obtain from it, denoted by $W_i \in [\underline{W}, \overline{W}]$ for each contract type i. Each firm can simultaneously post, and each worker can simultaneously search for, at most one offer W_i. Let $f(W_i)$ be the measure of firms posting value W_i, $u(W_i)$ the number of unemployed workers applying for it, and denote the market's tightness by $\theta(W_i) = f(W_i)/u(W_i)$. The frequency of meetings is determined by a constant-returns-to-scale matching function $\mathcal{M}_i(f, u)$, whose parameters are potentially specific to each labor market i.[13] Given a market tightness $\theta \equiv f/u$, unemployed workers match with a firm at Poisson rate $\mu_i(\theta) \equiv \mathcal{M}_i(\theta, 1)$ for contract type $i \in \mathcal{I}$, while firms find a worker with Poisson intensity $\eta_i(\theta) \equiv \mathcal{M}_i(1, \theta^{-1})$, so that $\mu_i(\theta) = \theta\eta_i(\theta)$, $\forall i \in \mathcal{I}$. We assume μ_i is increasing and concave, η_i is decreasing and convex, and the usual Inada conditions apply, namely (i) $\mu_i(0) = \lim\limits_{\theta\to+\infty} \eta_i(\theta) = 0$, and (ii) $\lim\limits_{\theta\to+\infty} \mu_i(\theta) = \lim\limits_{\theta\to 0} \eta_i(\theta) = +\infty$.

Recursive Formulation Because contracts are seemingly large and complex objects, we focus on *symmetric Markov perfect equilibria*, where the problem can conveniently be made recursive. Within this class of equilibria, contracts are functions of the firm's state, $(\vec{n}, z) \in \mathcal{N} \times \mathcal{Z}$, and the vector of outstanding promises, $\vec{W} = (W_{OE}, W_{FT}) \in \mathbb{R}^2_+$, that the firm promised to deliver to the workers that she is currently employing. Precisely, the recursive contracts (c_{OE}, c_{FT}) are defined by the following arrays:

$$
\begin{aligned}
c_{OE} &\equiv \Big(w_{OE}, \delta_{OE}, \{W'_{OE}(\vec{n}', z')\}\Big) \\
c_{FT} &\equiv \Big(w_{FT}, \delta_{FT}, p, \{W'_{FT}(\vec{n}', z')\}\Big)
\end{aligned}
$$

These contracts include a wage w_i, a per-worker layoff rate δ_i, a promotion rate (for the case of the FT contract) p_i, and a continuation promise $W'_i(\vec{n}', z')$ for each new possible set of states $(\vec{n}', z')$ for

[12]As it is standard in this type of models (e.g. Schaal (2017)), assuming worker discrimination would lead to wage indeterminacy, because as we shall see the distribution of utilities across workers within a type is irrelevant for determining the optimal contract. Assuming no within-contract discrimination is one way to circumvent this issue.

[13]This assumption is meant to capture the idea that the markets for temporary and permanent employment may differ in terms of the ease with which to find matches for reasons exogenous to the firm's decisions.

the firm, where:

$$(\vec{n}', z') \in \left\{ \begin{array}{l} (n_{OE}+1,\ n_{FT},\ z),\ (n_{OE},\ n_{FT}+1,\ z), \\ (n_{OE}-1,\ n_{FT},\ z),\ (n_{OE},\ n_{FT}-1,\ z), \\ (n_{OE}+1,\ n_{FT}-1,\ z), \\ \{(n_{OE},\ n_{FT},\ z') : z' \in \mathcal{Z}\} \end{array} \right\} \tag{4}$$

In words, the firm may hire a new worker (first line in (4)) or fire a worker (second line) of either contract type, she may promote an FT worker into an OE contract (third line), or she may get a shock to her idiosyncratic productivity (fourth line). Since the contract space is complete, prospective worker and firm agree on a wage, a firing rate, a promotion rate and continuation payoffs under each one of these possible contingencies.

We are now ready to characterize the equilibrium of this economy. To proceed, we first pose the value functions of the different agents (Section 3.2). Then, we show that the optimal menu of contracts can be found by solving a joint surplus problem, and we derive the optimality conditions of this problem to characterize the different elements of each contract (Section 3.3).

3.2 Value Functions

3.2.1 Unemployed Worker's Problem

Unemployed workers consume a flow utility $b > 0$ while searching in the labor market. Search is directed toward the sub-market that offers the most profitable expected return for workers. In particular, unemployed workers look for employment in the labor market $i \in \mathcal{I}$ that yields the highest return and, within this market, they apply to the firm that offers the highest payoff. Thus, the total value of unemployment is given by $\boldsymbol{U} = \max\{\boldsymbol{U}_{OE}, \boldsymbol{U}_{FT}\}$, where the value of searching for type-i contracts is defined by:

$$\boldsymbol{U}_i = \max_{W \in [\underline{W}, \overline{W}]} U_i(W) \tag{5}$$

In turn, $U_i(W)$ solves the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\rho U_i(W) = b + \mu_i\big(\theta(W)\big) \max\Big(W - U_i(W), 0\Big) \tag{6}$$

Equation (6) states that the value of search is given by the leisure utility b plus the option value of matching with a firm promising continuation utility W. As workers prefer the most profitable offers, when unemployed they must remain indifferent ex-ante between all of those offers which they decide to apply to. This means that: (i) unemployed workers must be ex-ante indifferent between applying for an OE or an FT contract, and (ii) they must also be ex-ante indifferent between all firms making offers within the same contract type. Formally, condition (i) implies that $\boldsymbol{U}_{OE} = \boldsymbol{U}_{FT} = \boldsymbol{U}$. Requirement (ii), in turn, can be summarized by the following complementary slackness condition:

$$\forall (W, i) \in \mathbb{R}_+ \times \mathcal{I} : \quad U_i(W) \leq \boldsymbol{U}, \text{ with equality if, and only if, } \mu_i\big(\theta(W)\big) > 0$$

This condition states that submarkets either maximize the value of being unemployed, or remain unvisited by workers. Imposing this condition into (6) we find:[14]

$$\theta_i(W) = \mu_i^{-1}\left(\frac{\rho \boldsymbol{U} - b}{W - \boldsymbol{U}}\right) \tag{7}$$

This equation defines the equilibrium market tightness mapping for type-i contracts. This mapping is used by both workers and firms to evaluate payoffs in equilibrium. In particular, note that market tightness is decreasing in W (more attractive contracts for workers ex-post attract more workers per job posting ex-ante), and increasing in $\boldsymbol{U}$ (a better outside option for workers makes job postings relatively less attractive ex-ante).[15] Finally, note that if the matching function is the same in both sub-markets $i \in \mathcal{I}$ (same $\mu_i(\theta)$) so must be $\theta_i(W)$, but if matching is more efficient in one sub-market (larger $\mu_i(\theta)$), then $\theta_i(W)$ will be lower in that sub-market.

3.2.2 Employed Worker's Problem

We now move to the problem of the employed worker. Assume this worker was hired in market $i \in \mathcal{I}$ and is now employed by a firm of type $(\vec{n}, z) \in \mathcal{N} \times \mathcal{Z}$ under contract $c_i = \{w_i, \delta_i, p\mathbf{1}_{[i=FT]}, W_i'(\vec{n}', z')\}$.[16] Let us adopt the convention that $\vec{n} = (n_i, n_{-i})$ and $\mathcal{C} = (c_i, c_{-i})$. Moreover, throughout we will make use of the short-hand notation:

$$\vec{n}_i^+ \equiv (n_i + 1, n_{-i}), \qquad \vec{n}_i^- \equiv (n_i - 1, n_{-i}), \qquad \vec{n}^p \equiv (n_{OE} + 1, n_{FT} - 1).$$

In words, $(\vec{n}_i^+, \vec{n}_i^-, \vec{n}^p)$ denote the employment vector of a firm after hiring or firing a type-i worker, and promoting an existing FT worker into an OE contract, respectively. Then, the worker's value satisfies the HJB equation:

$$\begin{aligned}
\rho \boldsymbol{W}_i(\vec{n}, z; \mathcal{C}) = {} & w_i + (\delta_i + s_i^W + s^F)\Big(\boldsymbol{U} - \boldsymbol{W}_i(\vec{n}, z; \mathcal{C})\Big) \\
& + (n_i - 1)(\delta_i + s_i^W)\Big(W_i'(\vec{n}_i^-, z) - \boldsymbol{W}_i(\vec{n}, z; \mathcal{C})\Big) \\
& + n_{-i}(\delta_{-i} + s_{-i}^W)\Big(W_i'(\vec{n}_{-i}^-, z) - \boldsymbol{W}_i(\vec{n}, z; \mathcal{C})\Big) \\
& + \sum_{j \in \mathcal{I}} \eta_j\Big(W_j'(\vec{n}_j^+, z)\Big)\Big(W_i'(\vec{n}_j^+, z) - \boldsymbol{W}_i(\vec{n}, z; \mathcal{C})\Big) \\
& + n_{FT}\, p\Big(W_i^p(\vec{n}^p, z) - \boldsymbol{W}_i(\vec{n}, z; \mathcal{C})\Big) \\
& + \sum_{z' \in \mathcal{Z}} \lambda(z'|z)\Big(W_i'(\vec{n}, z') - \boldsymbol{W}_i(\vec{n}, z; \mathcal{C})\Big)
\end{aligned} \tag{8}$$

[14] In order to save on notation, throughout we will write $\theta(W)$ when we in fact mean $\theta(W, \boldsymbol{U})$. Nonetheless, the reader should keep in mind that $\boldsymbol{U}$ is an endogenous object.

[15] Note $\theta(W)$ is decreasing in W as long as $W \geq \boldsymbol{U}$, but in equilibrium this condition will always hold thanks to a worker-participation constraint on the firm's problem.

[16] The indicator variable $\mathbf{1}_{[i=FT]}$ is needed because the promotion rate p is only specified for FT contracts.

The right-hand side of this equation incorporates the different sources of value for the employed worker. On the first line, the worker gets a wage w_i, and the option value from separation into unemployment (second additive term), either because she gets laid off (at rate δ_i), her contract breaks off (at rate s_i^W), or because the firm exits (at rate s^F). The second and third lines include the change in value from the separation of a co-worker that was employed under the same contract (second line) or under the other contract type (third line). The fourth line is the change in value due to the firm hiring an additional worker of some type j, where throughout we will use the short-hand notation $\eta_i\big(W(\cdot)\big) \equiv \eta_i\big(\theta_i(W(\cdot))\big)$ for the job-filling rate, with $\theta_i(\cdot)$ defined in equation (7). The fifth line refers to the change in value when a promotion takes place, where we have defined the value after a promotion by:

$$W_i^p(\vec{n}^p, z) \equiv \begin{cases} \dfrac{1}{n_{FT}}\Big(W'_{OE}(\vec{n}^p, z) + (n_{FT}-1)W'_{FT}(\vec{n}^p, z)\Big) & \text{for } i = FT \\ W'_{OE}(\vec{n}^p, z) & \text{for } i = OE \end{cases}$$

The last line of equation (8) includes the change in value do a productivity shock.

3.2.3 Active Firm's Problem

Consider now an active firm in state $(\vec{n}, z)$. This firm must choose a set of contracts $\mathcal{C} = (c_i, c_{-i})$ with $c_i = \{w_i, \delta_i, p\mathbf{1}_{[i=FT]}, W'_i(\vec{n}', z')\}$ for each $i \in \mathcal{I} = \{OE, FT\}$, subject to the outstanding promises $\vec{W} = (W_i, W_{-i})$ that were made to its current workers. Denote by $\boldsymbol{J}(\vec{n}, z, \vec{W})$ the value of this firm. The problem is:

$$\begin{aligned} \rho \boldsymbol{J}(\vec{n}, z, \vec{W}) = \max_{\{w_i, \delta_i, p, W'_i(\vec{n}', z')\}_{i\in\mathcal{I}}} \Bigg\{ & \exp(z) y(\vec{n}) - \chi_p p^{\psi_p} + \sum_{i\in\mathcal{I}} \Bigg[-w_i n_i - \chi_i \delta_i^{\psi_i} \\ & + s^F\Big(\boldsymbol{J}^e - \boldsymbol{J}(\vec{n}, z, \vec{W})\Big) + n_i(\delta_i + s_i^W)\Big(\boldsymbol{J}\big(\vec{n}_i^-, z, \vec{W}'(\vec{n}_i^-, z)\big) - \boldsymbol{J}(\vec{n}, z, \vec{W})\Big) \\ & + \eta_i\Big(W'_i(\vec{n}_i^+, z)\Big)\Big(\boldsymbol{J}\big(\vec{n}_i^+, z, \vec{W}'(\vec{n}_i^+, z)\big) - \boldsymbol{J}(\vec{n}, z, \vec{W})\Big)\Bigg] \\ & + n_{FT} p\Big(\boldsymbol{J}\big(\vec{n}^p, z, \vec{W}'(\vec{n}^p, z)\big) - \boldsymbol{J}(\vec{n}, z, \vec{W})\Big) \\ & + \sum_{z'\in\mathcal{Z}} \lambda(z'|z)\Big(\boldsymbol{J}\big(\vec{n}, z', \vec{W}'(\vec{n}, z')\big) - \boldsymbol{J}(\vec{n}, z, \vec{W})\Big)\Bigg\} \end{aligned} \tag{9}$$

This maximization problem is subject to two constraints:

$$\forall i: \quad \boldsymbol{W}_i(\vec{n}, z; \mathcal{C}) \geq W_i \tag{10a}$$

$$\forall (\vec{n}', z'),\ \forall i: \quad W'_i(\vec{n}', z') \geq \boldsymbol{U} \tag{10b}$$

where $\vec{W}'(\cdot) \equiv \big(W'_i(\cdot), W'_{-i}(\cdot)\big)$. Equation (9) states that the firm chooses the menu of contracts that maximize firm value. Firm value is composed of the following terms: on the first line of equation

(9), the firm makes profits from its current workers, net of wage, firing and promotion costs; the second line includes the changes in value due to firm exit (first term) and worker separation (second term), where $\boldsymbol{J}^e$ is the value of a potential entrant (derived below); the third line includes the event of the firm hiring a new worker; the fourth line is the event of a promotion of FT workers to an OE contract; the last line includes the change in value due to a productivity shock.

Importantly, this maximization problem is subject to two constraints. Constraint (10a) is a *promise-keeping* constraint: in choosing the contract, the firm must deliver an expected value to workers (left-hand side) which is no lower than the outstanding promise (right-hand side). This constraint is in place because of the firm's initial commitment to the posted contracts. Constraint (10b) is a *worker-participation* constraint: for every possible future state $(\vec{n}', z')$, the value that each worker obtains cannot be lower than its outside option. This constraint is in place because workers do not commit, and must therefore be enticed to remain matched.

3.2.4 Potential Entrant's Problem

Finally, we characterize the problem of the potential entrant firm. These are firms with no workers, perceiving a value $\boldsymbol{J}^e$. In order to post a contract, they must incur a flow cost κ and, upon entry, draw an initial productivity z^e from the π_z distribution. Firms enter with one worker, whether it is under an OE or an FT contract. Formally, their problem is:

$$\rho \boldsymbol{J}^e = -\kappa + \sum_{z^e \in \mathcal{Z}} \pi_z(z^e) \sum_{i \in \mathcal{I}} \widetilde{\boldsymbol{J}}_i^e(z^e) \tag{11}$$

where $\widetilde{\boldsymbol{J}}_i^e(z^e)$ is the option value for the firm of entering with one type-i worker and productivity draw z^e, defined by:

$$\widetilde{\boldsymbol{J}}_i^e(z^e) \equiv \max_{W_i^e(z^e)} \left\{ \eta_i\Big(W_i^e(z^e)\Big) \Big(\boldsymbol{J}(\vec{n}_i^e, z^e, \{W_i^e(z^e)\}) - \boldsymbol{J}^e \Big) \right\} \tag{12}$$

where $\vec{n}_i^e \equiv (n_i^e, n_{-i}^e) = (1, 0)$. The usual worker-participation constraint applies:

$$W_i^e(z^e) \geq \boldsymbol{U}, \ \forall (i, z^e) \tag{13}$$

However, as the potential entrant does not yet have workers, there is no promise-keeping constraint. We assume free entry into the labor market, i.e. we allow the aggregate measure of firms F to freely adjust in equilibrium. This implies that, in an equilibrium with positive entry, we must have $\boldsymbol{J}^e = 0$.

3.3 Optimal Contract

We are now ready to characterize the optimal menu of contracts. As we have seen, these contracts maximize firm value subject to providing workers with enough utility (and, in particular, with at least the continuation value they were promised going forward when they first joined the firm). As it is standard in these type of models, in order to derive the optimal contract we can conveniently solve an equivalent and simpler problem, whereby the joint surplus of the match (that is, the sum of the firm's value and that of all of its workers) is maximized.

To show this equivalence, note first that by monotonicity of preferences, the promise-keeping constraint (10a) must always bind with equality in equilibrium. Otherwise, the firm could increase its value by offering a combination of flow and continuation payoffs to the workers that would yield lower value to them and still comply with the firm's initial promises. This means that, for a firm $(\vec{n}, z)$ promising values $\vec{W} = (W_i, W_{-i})$ to each worker type, the value of its type-i workers $\boldsymbol{W}_i(\vec{n}, z; \mathcal{C})$, which we defined in equation (8), must equal W_i, the outstanding promise. In what follows, we will therefore write W_i in place of $\boldsymbol{W}_i(\vec{n}, z; \mathcal{C})$.

3.3.1 Joint Surplus Problem

Let us define the *joint surplus* of a match in firm state $(\vec{n}, z, \vec{W})$, where $\vec{W} = (W_i, W_{-i})$, as the sum of the firm's value and the value of all of its workers:

$$\boldsymbol{\Sigma}(\vec{n}, z) \equiv \boldsymbol{J}(\vec{n}, z, \vec{W}) + \sum_{i \in \mathcal{I}} n_i W_i \tag{14}$$

As discussed in detail below, the joint surplus is independent from promised values. In anticipation of this result, on the left-hand side of equation (14) we have written $\boldsymbol{\Sigma}(\vec{n}, z)$ instead of $\boldsymbol{\Sigma}(\vec{n}, z, \vec{W})$. In Appendix A.1 we then show that $\boldsymbol{\Sigma}(\vec{n}, z)$ solves:

$$\begin{aligned}\boldsymbol{\Sigma}(\vec{n}, z) = \max_{p, \{\delta_i, W_i'(\vec{n}_i^+, z)\}_{i \in \mathcal{I}}} \quad & \frac{1}{\rho + s^F} \Bigg\{ \mathcal{S}(\vec{n}, z) + \sum_{i \in \mathcal{I}} n_i (\delta_i + s_i^W) \Big(\boldsymbol{\Sigma}(\vec{n}_i^-, z) - \boldsymbol{\Sigma}(\vec{n}, z) \Big) \\ & + \sum_{i \in \mathcal{I}} \eta_i \Big(W_i'(\vec{n}_i^+, z) \Big) \Big(\boldsymbol{\Sigma}(\vec{n}_i^+, z) - \boldsymbol{\Sigma}(\vec{n}, z) \Big) + n_{FT} p \Big(\boldsymbol{\Sigma}(\vec{n}^p, z) - \boldsymbol{\Sigma}(\vec{n}, z) \Big) \\ & + \sum_{z'} \lambda(z'|z) \Big(\boldsymbol{\Sigma}(\vec{n}, z') - \boldsymbol{\Sigma}(\vec{n}, z) \Big) \Bigg\}, \quad \text{subject to } W_i'(\vec{n}_i^+, z) \geq \boldsymbol{U} \end{aligned} \tag{15}$$

where we have used the short-hand notation:

$$\mathcal{S}(\vec{n}, z) \equiv \underbrace{\exp(z) y(\vec{n})}_{\text{Firm's profits}} + \underbrace{\sum_{i \in \mathcal{I}} n_i (\delta_i + s_i^W + s^F) \boldsymbol{U}}_{\text{Workers' outside options}} - \underbrace{\chi_p p^{\psi_p}}_{\substack{\text{Promotion} \\ \text{costs}}} - \underbrace{\sum_{i \in \mathcal{I}} \chi_i \delta_i^{\psi_i}}_{\text{Firing costs}} - \underbrace{\sum_{i \in \mathcal{I}} \eta_i \Big(W_i'(\vec{n}_i^+, z) \Big) W_i'(\vec{n}_i^+, z)}_{\text{Commitment costs}} \tag{16}$$

to denote the expected flow surplus.[17] Equation (15) states that the joint surplus is composed of the flow surplus, plus the changes in joint surplus value due to worker separation or firing (first line of (15)), hiring or promotion (second line), and productivity shocks (third line).

[17]The flow surplus is the sum of the firm's flow profits and worker's outside options in case of separation (first line of (16)), net of three types of costs (second line, in this order): promotion costs, firing costs, and commitment costs (namely the costs of having to deliver the promised value in case of hiring, in expected terms).

Importantly, problem (15) is simpler than the firm's problem in equation (9) for two reasons. First, the space of payoff-relevant states has a lower dimension, namely $(\vec{n}, z)$ instead of $(\vec{n}, z, \vec{W})$. Second, the choice set is smaller as well: instead of having to choose a full contract vector $\mathcal{C} = \{w_i, \delta_i, p\mathbf{1}_{[i=FT]}, W_i'(\vec{n}', z')\}_{i\in\mathcal{I}}$, only the subset

$$\mathcal{C}_\Sigma \equiv \left\{\delta_i, p\mathbf{1}_{[i=FT]}, W_i'(\vec{n}_i^+, z)\right\}_{i\in\mathcal{I}} \subset \mathcal{C} \tag{17}$$

matters for the joint surplus. By contrast, the remaining elements of the contract set, i.e. $\widehat{\mathcal{C}} \equiv \mathcal{C}\backslash\mathcal{C}_\Sigma = \{w_i, W_i'(\vec{n}_i^-, z), W_i'(\vec{n}^p, z), \{W_i'(\vec{n}, z')\}_{z'\in\mathcal{Z}}\}_{i\in\mathcal{I}}$, leave the joint surplus unchanged, and are purely redistributive between the firm and its workers. Equipped with this formulation, we arrive at our main equivalence result:

Proposition 1 *The firm's and joint surplus problems are equivalent in the following sense:*

1. *For any set of contracts $\mathcal{C}$ that solves problem (9)-(10a)-(10b), the subset $\mathcal{C}_\Sigma \subset \mathcal{C}$ defined in (17) solves problem (15).*
2. *Conversely, if $\mathcal{C}_\Sigma$ is a solution to problem (15), then there exists a unique set of wages and continuation promises $\widehat{\mathcal{C}} \equiv \{w_i, W_i'(\vec{n}_i^-, z), W_i'(\vec{n}^p, z), \{W_i'(\vec{n}, z')\}_{z'\in\mathcal{Z}}\}_{i\in\mathcal{I}}$ such that $\widehat{\mathcal{C}} \cup \mathcal{C}_\Sigma$ solves problem (9)-(10a)-(10b).*

Proof. See Appendix A.1.

Proposition 1 establishes a useful equivalence result: in order to find the optimal contract, one may solve the joint surplus problem rather than the firm's problem. The proposition also lays a solution procedure, in two steps. In the first stage, we solve problem (15) to find the optimal job-filling rate $\eta_i(W_i'(\vec{n}_i^+, z))$, firing rate δ_i, and promotion rate p. Since the contract space is complete, all agents are risk-neutral, and utilities are fully transferable between them, there always exists a combination of promised utilities $\vec{W} = (W_i, W_{-i})$ and wages $\{w_i\}$ such that, for any future state of the match, rents can be redistributed across firm and workers in a surplus-maximizing way. Therefore, in the second stage, we may find wages and promised utilities residually by ensuring that the promise-keeping constraint (10a) will bind with equality, and the worker-participation constraint (10b) be satisfied at all points of the state space.

As stated above, this two-step solution procedure may appear to lead to an indeterminacy problem between promised values and wages: indeed, from equation (8), there exist different combinations of these two objects that leave $\boldsymbol{W}_i$ unchanged. This indeterminacy is resolved, however, by the free entry condition, which uniquely pins down the subset $\widehat{\mathcal{C}}$ via a simple iterative routine. In particular, the (unique) solution to the entrant's problem provides the outstanding promise of one-worker firms; from the optimal contract of these we may then find the outstanding promises of two-worker firms; and so on.

More precisely, write the free-entry condition in joint-surplus terms:[18]

$$\kappa = \max_{\{W_i^e(z^e)\}} \left\{ \sum_{z^e \in \mathcal{Z}} \pi_z(z^e) \left[\sum_{i \in \mathcal{I}} \eta_i \Big(W_i^e(z^e) \Big) \Big(\mathbf{\Sigma}(\vec{n}_i^e, z^e) - W_i^e(z^e) \Big) \right] \right\} \tag{18}$$

The unique solution to this problem is the outstanding promised value of active firms with a single type-i worker and productivity z^e which, because of commitment, is precisely the state vector of these firms. For them, there is a unique downsizing decision $W_i^-(\vec{n}_i^e, z^e)$, which is found from the free entry problem, and a unique promotion decision (in case $i = FT$), which is found as the solution of the joint-surplus maximization problem of firms with one FT-type worker and no OE workers. Using these, all that remains is to find the wage w_i, but this can be found directly from the promise-keeping constraint (10a) binding with equality, i.e. such that $\boldsymbol{W}_i(\vec{n}_i^e, z^e; \{c_i\}) = W_i^e$. This then gives a unique surplus-maximizing contract for the single-worker firm. The continuation promises in this contract are then taken as the outstanding promises of two-worker firms, i.e. of firms with either $(n_i, n_{-i}) = (1, 1)$ or $(n_i, n_{-i}) = (2, 0)$. Proceeding similarly through the $\vec{n}$ space, this procedure allows us to construct full sequences $\{c_i(\vec{n}, z)\}$ by forward iteration over $(n_i, n_{-i}) \in \{(1,0), (1,1), (1,2), \ldots, (2,0), (2,1), (2,2), (2,3), \ldots\}$, and for all $z \in \mathcal{Z}$.

3.3.2 Equilibrium Hiring, Firing and Promotion Policies

Hiring, firing, and promotion rates can be found by taking first-order conditions of problem (15). For the optimal upsizing choice $W_i^+ \equiv W_i'(\vec{n}_i^+, z)$, we have:

$$\frac{\partial \eta_i(W_i^+)}{\partial W_i^+} W_i^+ + \eta_i(W_i^+) = \Big(\mathbf{\Sigma}(\vec{n}_i^+, z) - \mathbf{\Sigma}(\vec{n}, z) \Big) \frac{\partial \eta_i(W_i^+)}{\partial W_i^+} \tag{19}$$

Intuitively, firms equate the marginal cost of hiring, on the left-hand side, to the marginal benefit, on the right-hand side. Providing an additional util of W_i^+ yields costs both in terms of the new worker as well as the pre-existing ones. On the one hand, the firm must deliver its promise to the new worker in case she is hired, as captured by the first additive term on the left-hand side of (19). On the other hand, by providing this additional utility to the new worker, the promised utility of the pre-existing workers must be modified through the wage, as captured by the second additive term on the left-hand side of equation (19).[19] On the right-hand side, the marginal benefit is given by the increase in joint surplus from hiring, times the change in the probability of a hire.

Equation (19) determines the continuation promise of firms $(\vec{n}, z)$ after hiring an additional worker, $W_i^+ \equiv W_i'(\vec{n}_i^+, z)$, and therefore the starting promises of firms of size $\{\vec{n}_i^+\}_{i \in \mathcal{I}}$. Starting from $(0, 0)$ via the free-entry condition, this allows us to construct the whole sequence of promised

[18] To arrive at this expression, impose $\boldsymbol{J}^e = 0$ into equation (11), and specialize the definition of the joint surplus (equation (14)) to the case $\vec{n} = \vec{n}_i^e = (n_i^e, n_{-i}^e) = (1, 0)$ and $z = z^e$.

[19] One util of promised utility changes wages by the expected value, which is the probability that this util will have to be delivered (i.e. the probability of hiring, $\eta_i(W_i^+)$).

values in the $(\vec{n}, z)$ space. In turn, the objects $W_i^- \equiv W_i'(\vec{n}_i^-, z)$ and $W_i^p \equiv W_i'(\vec{n}^p, z)$ are chosen to be consistent with this sequence.[20]

For the firing and promotion rates, $\delta_i(\vec{n}, z)$ and $p(\vec{n}, z)$, we have, respectively:

$$\delta_i(\vec{n}, z) = \left(n_i \frac{\boldsymbol{U} + \boldsymbol{\Sigma}(\vec{n}_i^-, z) - \boldsymbol{\Sigma}(\vec{n}, z)}{\psi_i \chi_i} \right)^{\frac{1}{\psi_i - 1}} \tag{20}$$

$$p(\vec{n}, z) = \left(n_{FT} \frac{\boldsymbol{\Sigma}(\vec{n}^p, z) - \boldsymbol{\Sigma}(\vec{n}, z)}{\psi_p \chi_p} \right)^{\frac{1}{\psi_p - 1}} \tag{21}$$

Intuitively, firms equate the marginal firing and promotion costs to the marginal gain, given by the corresponding changes in the joint surplus value. Having found the optimal promises $\{W_i'(\vec{n}, z)\}$ and optimal rates $\{\delta_i(\vec{n}, z), p(\vec{n}, z)\}$, we may find wages residually from the promise-keeping constraint (10a), which must bind with equality.

To close the characterization of the equilibrium, we must describe the distribution of firms and workers. The law of motion for the measure of firms $f_t(n_{OE}, n_{FT}, z)$ is characterized by a set of flow equations, which we provide in full in Appendix A.2. Appendix A.3 then shows how to obtain the aggregate measure of active firms, $F \equiv \sum_{n_{OE}} \sum_{n_{FT}} \sum_z f(n_{OE}, n_{FT}, z)$, and inactive firms, F^e, as well as the aggregate unemployment rate, in the stationary equilibrium.

4 Estimation

In this section, we offer a quantification of the model by matching moments from the Spanish micro-level data presented in Section 2. The purpose of this exercise is to study the macroeconomic implications of dual labor markets and conduct policy analysis. To do all this, however, we must first parameterize the model.

4.1 Parameterization

First, we must parameterize the productivity shock z. For this, we must choose values for $k(k-1)$ intensity rates $\{\lambda(z'|z)\}$. As this is a potentially large number (in the numerical implementation, $k = 5$, so $k(k-1) = 20$), to reduce the dimensionality we specialize the productivity process to an Ornstein-Uhlenbeck diffusion:

$$\mathrm{d}\log(z_t) = -\rho_z \log(z_t)\mathrm{d}t + \sigma_z \mathrm{d}B_t \tag{22}$$

[20] That is, for any given n_i and n_{-i}, we have that W_i^- for a firm with employment vector $(n_i + 1, n_{-i})$ must equal W_i^+ for a firm with employment vector $(n_i - 1, n_{-i})$. Similar consistency conditions hold for W_i^p.

where B_t is a Wiener process, and (ρ_z, σ_z) are positive parameters to be calibrated.[21] For the entrant firms' productivity distribution π_z we take the ergodic distribution associated with the (calibrated) Markov chain implied by equation (22).

Second, we must choose a matching function. For each labor market $i \in \{OE, FT\}$, we choose a Cobb-Douglas specification:

$$\mathcal{M}_i(f, u) = A_i f^{\gamma} u^{1-\gamma} \tag{23}$$

where $A_i > 0$ is a market-specific matching efficiency parameter, and $\gamma \in (0, 1)$ is the matching elasticity (assumed to be common across labor markets). This implies meeting rates $\mu_i(\theta) = A_i\theta^{\gamma}$ for the worker, and $\eta_i(\theta) = A_i\theta^{\gamma-1}$ for the firm. Assuming a Cobb-Douglas functional form leads to convenient analytical representations for the promised value and the job-filling rate. Using equation (19), some algebra yields:

$$W_i'(\vec{n}_i^+, z) = \gamma \boldsymbol{U} + (1-\gamma)\Big(\boldsymbol{\Sigma}(\vec{n}_i^+, z) - \boldsymbol{\Sigma}(\vec{n}, z)\Big) \tag{24}$$

This expression is intuitive: the continuation promise $W_i'(\vec{n}_i^+, z)$ for the new worker is a weighted average of the worker's outside option, $\boldsymbol{U}$, and the marginal net joint surplus gain from the hire, $\boldsymbol{\Sigma}(\vec{n}_i^+, z) - \boldsymbol{\Sigma}(\vec{n}, z)$. The parameter $(1-\gamma)$ gives the share of the overall gains (net of the worker's outside option) that accrue to the new worker. On the other hand, and using the definition of joint surplus (equation (14)), the firm obtains the following change in value from a type-i hire:

$$\boldsymbol{J}\Big(\vec{n}_i^+, z, \vec{W}'(\vec{n}_i^+, z)\Big) - \boldsymbol{J}(\vec{n}, z, \vec{W}) = \underbrace{\gamma\Big(\boldsymbol{\Sigma}(\vec{n}_i^+, z) - \boldsymbol{\Sigma}(\vec{n}, z) - \boldsymbol{U}\Big)}_{\substack{\text{New surplus, which is}\\ \text{shared with new hire}}} + \underbrace{\sum_{j \in \mathcal{I}} n_j \Big(W_j'(\vec{n}, z) - W_j'(\vec{n}_i^+, z)\Big)}_{\substack{\text{Transfer of value between}\\ \text{firm and pre-existing workers}}} \tag{25}$$

The firm's marginal gain in value is composed of two terms. On the one hand, the firm absorbs the share γ of the total net gain in joint surplus that is not absorbed by the new hire.[22] On the other hand, since we assume that all workers within the firm that are employed under the same contract must earn the same value, there must be a transfer of value between the firm and all its pre-existing workers (for *both* contract types) after hiring takes place. This is captured by the second additive term. Therefore, in equilibrium, firms must strike a balance between the surplus they extract from new hires and the surplus they extract from their pre-existing workers when a hire takes place.

[21] An Ornstein-Uhlenbeck diffusion process is a type of mean-reverting process which can be thought of as the continuous-time analogue of an AR(1) process in discrete time. For details on how to implement the Ornstein-Uhlenbeck process in our context, and on the mapping between the $\{\lambda\}$ intensity rates and the (ρ_z, σ_z) parameters, see Appendix B.2.

[22] Indeed, by equation (24), this part of firm gains can be written as

$$\gamma\Big(\boldsymbol{\Sigma}(\vec{n}_i^+, z) - \boldsymbol{\Sigma}(\vec{n}, z) - \boldsymbol{U}\Big) = \boldsymbol{\Sigma}(\vec{n}_i^+, z) - \boldsymbol{\Sigma}(\vec{n}, z) - W_i'(\vec{n}_i^+, z),$$

an identity stating that the share of net gains in joint surplus that is absorbed by the firm (left-hand side) must equal the total gains in joint surplus that are left after payments to the new hire (right-hand side).

The optimal job-filling rate can then be written as follows:

$$\eta_i(\vec{n}_i^+, z) = A_i^{\frac{1}{\gamma}} \left[(1-\gamma) \frac{\mathbf{\Sigma}(\vec{n}_i^+, z) - \mathbf{\Sigma}(\vec{n}, z) - \boldsymbol{U}}{\rho \boldsymbol{U} - b} \right]^{\frac{1-\gamma}{\gamma}} \tag{26}$$

Given our choice of $\gamma = 0.5$ (discussed below), equation (26) implies that the job-filling rate is linear (as $\frac{1-\gamma}{\gamma} = 1$) in the ratio of the marginal net gain in joint surplus from a new match that accrues to a worker to the expected value of that worker's search. Therefore, in our calibrated model, firms can expect to find workers at a rate that is proportional to the returns that they offer to them.

Finally, given our choice for the matching function, the free-entry condition (equation (18)) reads:

$$\kappa = \gamma \left(\frac{1-\gamma}{\rho \boldsymbol{U} - b} \right)^{\frac{1-\gamma}{\gamma}} \sum_{z^e \in \mathcal{Z}} \pi_z(z^e) \left\{ \sum_{i \in \mathcal{I}} \left[A_i \Big(\mathbf{\Sigma}(\vec{n}_i^e, z^e) - \boldsymbol{U} \Big) \right]^{\frac{1}{\gamma}} \right\}$$

Notice that the expected value of a single-worker firm (the right-hand side of this equation) is monotonically decreasing in the value of unemployment, because for higher $\boldsymbol{U}$ both the job-filling rate and the ex-post gains in match surplus that accrue to the firm are lower. In the numerical implementation of the model, we exploit this monotonic relationship to find the unique equilibrium value of unemployment, $\boldsymbol{U}$ (see Appendix B.1 for details).

4.2 Calibration Strategy

We calibrate the model in steady state. We separate the parameters into two blocks: those parameters whose values are set externally and those that are calibrated internally via indirect inference by comparing the model's quantitative predictions with various empirical targets.

Data sources We use two main data sources to calibrate the parameters of the model. First, we draw moments from the *Central de Balances* (CBI) firm-level data introduced in Section 2. To avoid a problem of state space dimensionality, we focus on a subsample of firms with at most 50 workers (with no restriction on their temporary share), representing nearly 97% of firms in the total sample (see Table 1). Second, we use aggregate data on worker flows into and out of employment by contract type from the *Encuesta de Población Activa* (EPA), the Spanish labor force survey. The CBI comes at yearly frequency and the EPA comes at the quarterly frequency. We set the model period to one quarter to match the EPA frequency as we only use information on stock variables from the CBI data.

Externally calibrated parameters We set $\rho = 0.0129$, corresponding to an annualized real interest rate of $1 - (1+\rho)^{-4} \approx 5\%$. For the matching elasticity, we choose $\gamma = 0.5$, a standard value in the literature (e.g. Petrongolo and Pissarides (2001)).[23] The productivity parameters (ρ_z, σ_z)

[23]This value is routinely used in models estimated to U.S. data, but has also been used for European labor markets, specifically in models of dual labor markets (e.g. Thomas (2006), Costain *et al.* (2010) and Bentolila *et al.* (2012)).

are calibrated to match a yearly autocorrelation of 0.81 and volatility of 0.34. We borrow these values from Ruiz-García (2021), which estimates an AR(1) process for firm-level TFP using Spanish firm-level balance sheet data from CBI, the same data source that we use in our empirical analysis. As explained in Appendix B.2, these targets imply that $\rho_z = 0.2053$ and $\sigma_z = 0.1700$.

To keep symmetry in the policy functions, we set all cost curvature parameters to $\psi_{OE} = \psi_{FT} = \psi_p = 2$, so that firing and promotion rates are linear in the corresponding net surplus changes. Moreover, following Spanish legislation, we assume that workers hired under an FT contract cannot be fired, which in the model amounts to imposing that $\chi_{FT} = +\infty$.

Finally, the entry cost κ is pinned down by our targets for average firm size and the aggregate employment rate E. To see this, note that the average firm size equals the ratio of the employment rate E to the measure of firms F, so if we have targets on average firm size and the employment rate we know the value for F that we must obtain in equilibrium. Because κ monotonically lowers the firm's expected value upon entry, there is only one κ for which the model matches this target-implied F, a fact that we exploit in the numerical algorithm that solves the model (see Appendix B.1). Thus, the targets for average firm size and employment rate pin down κ uniquely. Our target for average firm size is 6.76 employees, which we obtain from our CBI subsample of firms with 50 or less employees. Our target for the employment rate is uniquely derived from our targets for the labor market flows and the share of workers with an FT contract (17.9%, on average across firms in our CBI sample). We target four labor market flow rates from EPA: the unemployment-to-employment (UE) rates into OE and FT contracts, and the employment-to-unemployment (EU) rates out of OE and FT contracts (see Table 3 for the empirical values, and Appendix A.4 for the description of how these rates are computed in both the data and the model). Together with an average temporary share of 17.9%, these empirical targets imply a steady state unemployment rate of 13.7%, which is roughly in line with the actual average unemployment rate in Spain during this period.[24] Therefore, the measure of active firms in equilibrium must be equal to $F = \frac{\text{EmpRate}}{\text{AvgFirmSize}} = \frac{1-0.137}{6.76} = 0.1276$.

Internally calibrated parameters We have 11 parameters left to identify:

$$\boldsymbol{\theta} \equiv \left(\alpha, \omega, \nu, \chi_{OE}, \chi_p, b, s^F, s^W_{OE}, s^W_{FT}, A_{OE}, A_{FT}\right)$$

We estimate these parameters jointly via an indirect inference procedure that matches a series of model-generated moments to their empirical counterparts. The parameters are chosen to minimize the objective function:

$$\left(M^{\text{data}} - M^{\text{model}}(\boldsymbol{\theta})\right)^{\top} \Omega^{-1} \left(M^{\text{data}} - M^{\text{model}}(\boldsymbol{\theta})\right) \tag{27}$$

[24] To obtain this number for the unemployment rate, we first write down a three-state, discrete-time dynamic system $\vec{s}_{t+1} = \boldsymbol{\Pi}\,\vec{s}_t$, where t is one quarter, $\vec{s}_t = [E_{OE,t}\ E_{FT,t}\ U_t]^{\top}$ contains the shares of workers employed in each type of contract and of unemployed workers, and $\boldsymbol{\Pi}$ is a Markov transition matrix which we populate with the four empirical flow rates ($\pi_{UE_{OE}}$, $\pi_{UE_{FT}}$, $\pi_{E_{FT}U}$ and $\pi_{E_{OE}U}$), assuming no transitions from OE to FT ($\pi_{E_{OE}E_{FT}} = 0$), and leaving the transition rate from FT to OE as an unknown $\pi_{E_{FT}E_{OE}}$. Then, we solve for the ergodic distribution of this system that delivers $\frac{E_{FT}}{E_{FT}+E_{OE}} = 0.179$, our target for the temporary share. This gives $U = 0.137$.

where M^{data} is a vector of moments from the data, $M^{\text{model}}(\boldsymbol{\theta})$ is the model counterpart of these moments, and Ω is a diagonal matrix of weights, containing the squares of the data moments as the diagonal elements. The key to the success of this calibration strategy is that we pick moments that are both informed by the parameters as well as relevant for the economic mechanisms that lie behind our key quantitative counterfactual exercises. Due to the high degree of non-linearities in the model, however, the identification of each parameter with a single specific moment is challenging, in the sense that all moments are sensitive to changes in all parameters to some degree. In what follows, therefore, we offer an economic intuition for how each moment may be informative about each parameter, and in Appendix B.3 we offer a more formal (global) identification test which lends support to these verbal intuitions.

Model Fit Table 3 summarizes the calibrated parameter values and the fit of the model in terms of targeted moments. First, the parameters of the production function (α, ν, ω) are informative about firm size, the labor share, and the average temporary share. For firm size we target 6.76 employees per firm, obtained from our CBI subsample, and for the labor share we target 61.32%, which we obtain from the latest available report of EU KLEMS for Spain.[25] These two moments help identify the degree of decreasing returns to scale ν, which controls the firms' optimal size, and the degree of complementarity between worker types, α. The model predicts that $\nu = 0.7821$ and $\alpha = 0.8980$. Since $\alpha > \nu$, the calibration implies that worker types are *substitutes* in production (in the sense that $\frac{\partial^2 Y(\vec{n},z)}{\partial n_{OE}\partial n_{FT}} < 0$ in equation (1)). The temporary share, which we compute as the ratio of workers employed under an FT contract to the total measure of workers, is primarily identified by ω, which controls the relative productivity of OE workers. The calibration delivers $\omega = 0.4901$, meaning that worker types are essentially equally productive. In the data, the average temporary share in our CBI subsample equals 17.94%, very close to the model's prediction (17.71%).

Second, we target the employment-to-unemployment (EU) and unemployment-to-employment (UE) quarterly flow rates for each type of contract, which we obtain in the data from the Spanish labor force survey (EPA). Appendix A.4 explains how to compute these rates in the model. The UE rates are primarily affected by the matching efficiency parameters (A_{OE}, A_{FT}), as these act as shifters for the number of matches that take place in each market per unit time, given market tightness. In the data, UE transitions are far more frequent among FT workers (18.52%) than among OE workers (2.73%). The model rationalizes this by making the FT labor market a lot more "liquid": for given market tightness, there are $A_{FT}/A_{OE} = 3.438$ matches in the FT market per unit of time for every match that takes place in the OE market. On the other hand, the EU rates are most directly affected by the worker exogenous separation rates, (s^W_{OE}, s^W_{FT}). In particular, in the data, transitions into unemployment are far more common in the FT market (12.97%) than they are in the OE market (1.39%). As firing is not permitted in the FT market because $\chi_{FT} = +\infty$ by assumption, in the model there is only one way (conditional on firm survival) for this transition to take place, which is for the FT worker to exogenously separate. Indeed, the calibration delivers that the separation

[25] The data is publicly available at http://www.euklems.net/.

Table 3: Benchmark Model Parameters and Targeted Moments

A. Externally Calibrated Parameters

Parameter	*Description*	*Equation*	*Value*	*Target/Source*
ρ	Discount rate	.	0.0129	5% annual real interest rate
χ_{FT}	Firing cost scale	(2)	$+\infty$	Spanish LM regulation
(ψ_{EO}, ψ_{FT})	Firing cost curvature	(2)	(2,2)	Normalization
ψ_p	Promotion cost curvature	(3)	2	Normalization
κ	Fixed entry cost	(11)	2,373.05	Measure of active firms
γ	Matching elasticity	(23)	0.5	Petrongolo and Pissarides (2001)
(ρ_z, σ_z)	Productivity parameters	(22)	(0.2053,0.1700)	Ruiz-García (2021)

B. Internally Calibrated Parameters

Parameter	*Description*	*Equation*	*Value*
α	Degree of complementarity between worker types	(1)	0.8980
ω	Relative productivity of OE-type workers	(1)	0.4901
ν	Degree of returns to scale in technology	(1)	0.7821
χ_{OE}	Scale parameters in firing cost function	(2)	2.9653
χ_p	Scale parameter in promotion cost function	(3)	0.0149
b	Worker's opportunity cost of employment	(6)	0.1099
s^F	Exogenous firm exit rate	(8)	0.0091
(s^W_{OE}, s^W_{FT})	Exogenous worker separation rates	(8)	(0.0490,0.5257)
(A_{OE}, A_{FT})	Matching efficiency parameters	(23)	(0.4463,1.5342)

C. Model Fit

Targeted moment	*Model value*	*Data value*	*Data source*	*Helps identify*
Average employment per firm	6.70	6.72	CBI	ν
Labor share	68.63%	61.32%	EU KLEMS	α
Average temporary share	17.71%	18.1%	CBI	ω
UE rate (FT)	19.44%	18.52%	EPA	A_{FT}
UE rate (OE)	1.44%	2.73%	EPA	A_{OE}
EU rate (FT)	13.23%	12.97%	EPA	s^W_{FT}
EU rate (OE)	1.48%	1.39%	EPA	s^W_{OE}
Firm entry rate	0.93%	1.50%	INE	s^F
Value of leisure to output	29.06%	40.00%	Standard	b
Temp share by size bin	See Figure 3	See Figure 3	CBI	.

Notes: The model period is one quarter. UE and EU rates are averages over HP-filtered quarterly series from the EPA over the period 2005Q1-2018Q4 (data before 2005 is unavailable). The firm entry rate is a quarterly figure. Appendix A.4 describes how labor market flows are computed in the data and in the model. Appendix B.3 discusses identification. **Data sources:** *CBI* means our subsample from the Central de Balances Integrada data; *INE* means data from the Instituto Nacional de Estadística; *EPA* means data from the Encuesta de Población Activa; and *EU KLEMS* refers to the EU KLEMS Growth and Productivity Accounts 2017 release, available at http://www.euklems.net/.

rate of FT workers is much higher (equal to 0.5257, i.e. an average duration of about half a year on the job) than that of OE workers (equal to 0.0490, i.e. an average duration of about 5 years on the job). We interpret the exogenous separations in OE contracts as voluntary quits of workers, while the exogenous separations in FT contracts may be driven by voluntary quits but also by regulations on the maximum duration of these contracts. The latter will be our preferred interpretation in our policy analysis of Section 5.

Third, we target the entry rate of firms. In the data, we take this number as reported by the *Instituto Nacional de Estadística* (INE), the Spanish national statistical agency. The firm entry rate fluctuated steadily around 6% annually for the last few years in our sample, about 1.5% in quarterly terms. In the model, we compute the firm entry rate as the ratio of actual entrants (the share of potential entrants F_e that successfully attract their first worker of either contract type) to the total measure of active firms F, or:

$$FirmEntryRate = \frac{F_e}{F}\left(\sum_{z^e\in\mathcal{Z}} \pi_z(z^e) \sum_{i\in\mathcal{I}} \eta_i\big(W_i'(\vec{n}_i^e, z^e)\big)\right)$$

This moment is informed by the exogenous exit rate of firms, s^F, as the exit rate and the entry rate must coincide in a stationary solution.[26] On the other hand, the flow value of unemployment b is picked to match the value of leisure to average output per worker, a standard target for this parameter. We choose the standard value of 40% (e.g. Shimer (2005)). The calibrated model underpredicts this value slightly (29.06%).

Finally, since one of our main goals is to explain how the share of temporary workers differs across firms, we target the average share of FT workers by employment bins. In Figure 2 we showed that, in the full CBI sample, the temporary share is increasing in firm size for most of the firm size distribution, even after controlling for unobserved fixed effects. Figure 3 shows that this remains true in the subsample of CBI that we use to calibrate the model: in the data, firms with 1 to 5 employees have a lower share of FT workers (about 15%) than firms with 26 to 30 employees (about 30%). The model quantitatively tracks this behavior well.

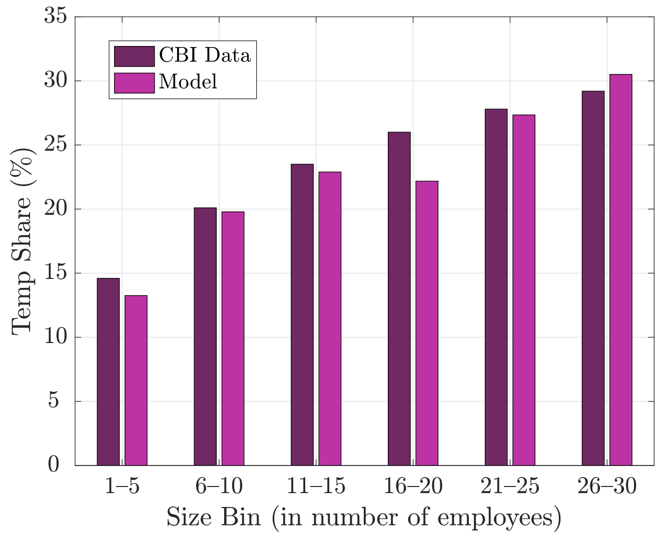
4.3 Policy Functions and the Firm Size Distribution

In the model, firms of different productivity z have different long-run targets of employment and temporary share (or n_{OE} and n_{FT}), but idiosyncratic shocks move them up and down the distribution, and in equilibrium there is dispersion in firm sizes and in temporary shares within each productivity level z. Table 4 reports the long-run targets of employment and temporary share for firms in each productivity bin.[27] We see that more productive firms choose a larger employment

[26] The firm exit rate is equal to $s^F + \sum_{z\in\mathcal{Z}}\sum_{i\in\mathcal{I}}\left(\delta_i(\vec{n}_i^e, z) + s_i^W\right)\frac{f(\vec{n}_i^e, z)}{F}$. Therefore, s^F directly affects the gap between entry and exit rates, and tightly pins down the entry rate.

[27] We obtain them by simulating a large number of firms over a long period, but forcing the path of realized z to be identical over time within each firm and equal to the initial one. We report the average across all firms within each z category.

Figure 3: Temporary share, by firm size: model versus data.



Notes: Each bar plot shows the average temporary share within each employment size bin, in the CBI subsample used to calibrate the model, and in the calibrated model (where model averages use the stationary distribution of firms).

Table 4: Long-run targets of employment and temporary share

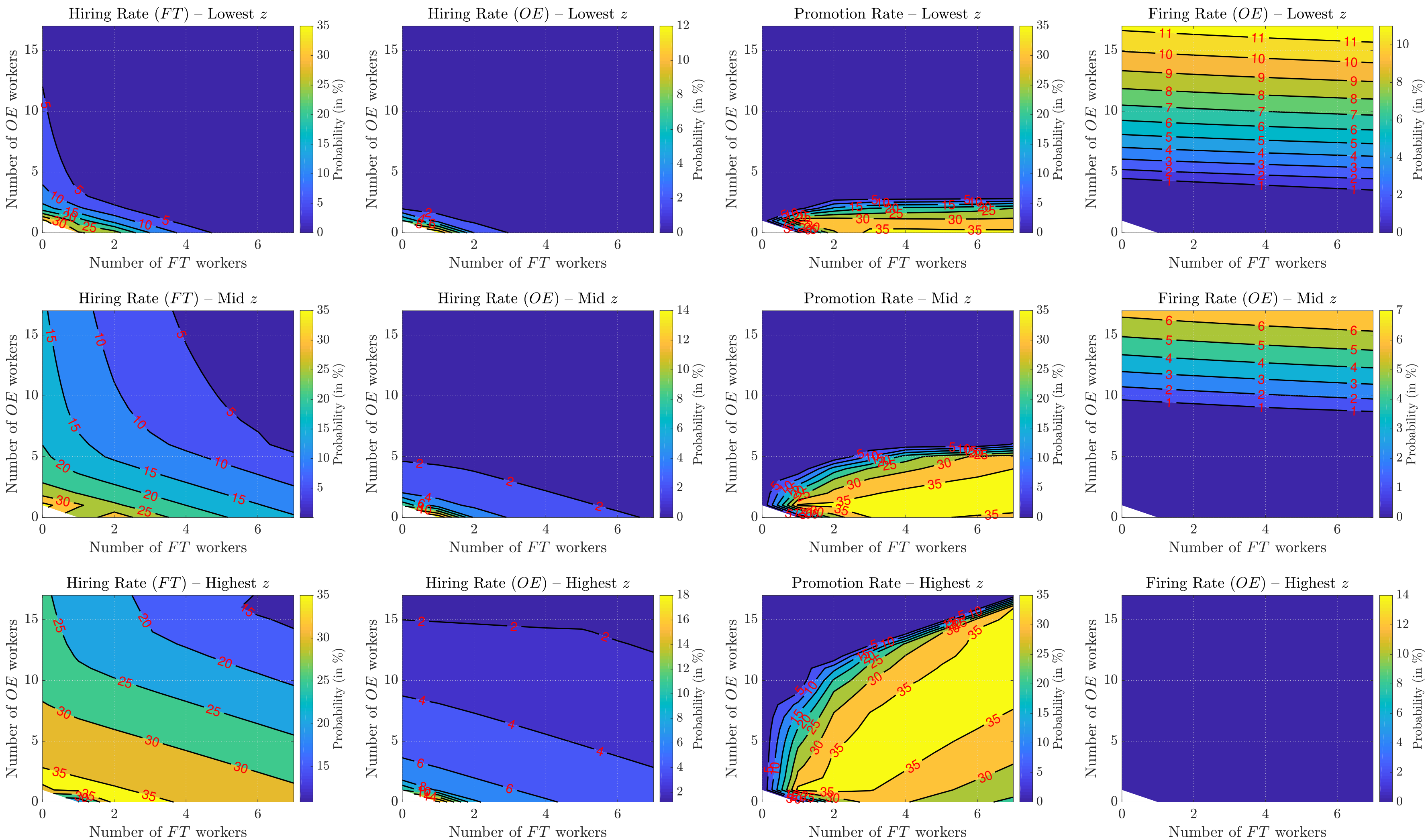
	z_1	z_2	z_3	z_4	z_5
Employment ($n_{OE} + n_{FT}$)	3.237	4.136	6.010	8.978	13.105
Temporary share ($n_{FT}/(n_{OE} + n_{FT})$)	0.298	0.303	0.246	0.191	0.135

Notes: Each column reports the target long-run employment and temporary share for each productivity level, where $z_1 < \cdots < z_5$. See footnote 27 for details

level. For instance, firms with the highest productivity (z_5) target 13.1 employees, while firms with median productivity (z_3) target 6.0 employees. This is a standard result: with decreasing returns to scale in production, firms with higher productivity may achieve a similar marginal product of labor as firms with lower productivity by reaching a larger size.

Table 4 also shows that the long-run target share of temporary workers is lower for more productive firms. For instance, firms with the highest productivity (z_5) target a temporary share of workers of 13.5%, while firms with median productivity (z_3) target a temporary share of 24.6%. This is because, at their long-run target size, firms of different productivity have similar opportunity cost of unfilled vacancies (due to decreasing returns to scale and larger firm size) but more productive firms are larger. This larger size is critical. Because the flow rate of vacancy posting is the same for firms of all sizes, more productive firms (which are larger) need a higher filling rate of vacancies to replace their expiring FT contracts. This requires posting vacancies in less tight but more expensive labor markets. As a result, the higher turnover of FT contracts is relatively less interesting for more productive firms and they turn to OE contracts.

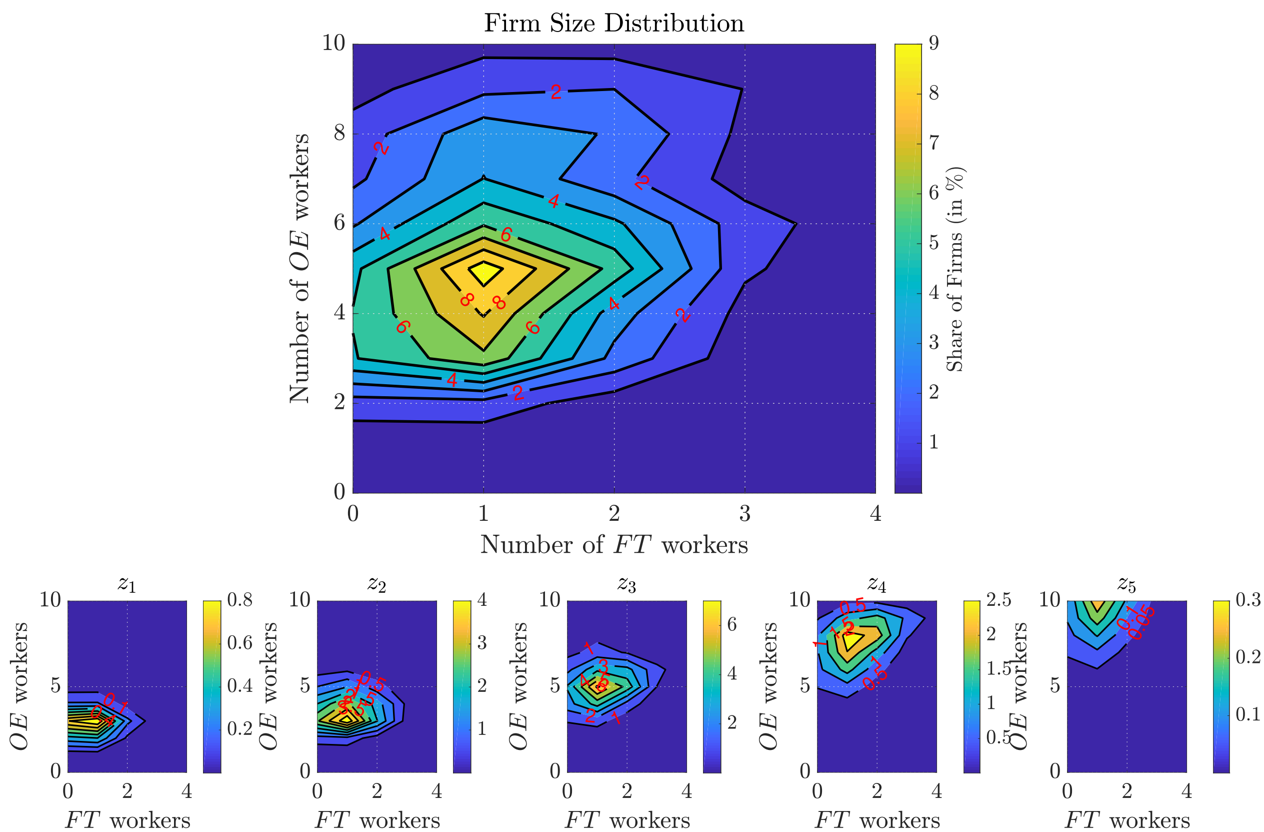
Figure 4: Hiring, promotion and firing policy functions for a single worker in the calibrated model.



Notes: These contour plots show the policy functions for hiring/promoting/firing of a single worker, expressed as quarterly probabilities, in the (n_{FT}, n_{OE}) space, for different levels of productivity. For example, the top-left plot gives 22% for $n_{OE} = n_{FT} = 1$. The way to read this number is: "the probability that, within the next quarter, a firm that has one OE worker and one FT worker hires exactly one more FT worker is equal to 22%."

To see how firms move up and down the distribution, Figure 4 shows the policy functions for hiring under FT and OE contracts, for promotion of FT workers into an OE contract, and for firing of OE workers. Each row in the figure corresponds to a different firm productivity level: lowest (z_1), median (z_3), and highest (z_5).[28] Figure 5, in turn, depicts the equilibrium firm size distribution.

Figure 5: Firm size distribution in the calibrated model.



Notes: *Top panel:* Equilibrium distribution of firms in the (n_{FT}, n_{OE}) space, added across productivity states z. *Bottom panel:* Equilibrium distribution in the (n_{FT}, n_{OE}) space, by z-type, where $z_1 < \cdots < z_5$.

The first thing to observe in these plots is that, (a) for given productivity, both job-filling rates (for both the OE and FT markets) are decreasing in firm size (smaller firms are more likely to hire a new worker within a quarter than larger firms are); and (b) for given size, both job-filling rates are increasing in firm productivity. This is because smaller and more productive firms are further away from their optimal size, the opportunity cost of unfilled vacancies is larger, and hence they are willing to post vacancies in less tight labor markets (within each type of contract) in exchange for offering more value to workers. Second, both hiring probabilities decrease faster in the number of OE workers than in the number of FT workers. This happens because the higher separation rate of FT contracts requires more vacancy posting to keep firm size unchanged. Third, for given productivity z, larger firms are more likely to hire FT workers. As discussed above, larger firms face a lower

[28]For ease of interpretation, we have transformed each Poisson policy rate into the quarterly probability of hiring/promoting/firing exactly one worker.

opportunity cost of unfilled vacancies and hence a lower cost of the higher turnover associated to FT contracts. Fourth, the probability of promotion, which generates within-firm job-to-job transitions from FT to OE, decreases sharply in the number of OE workers, but is fairly constant in the number of FT workers except for the very productive firms. Finally, an interesting feature of the equilibrium is that there is little firing of workers (far right column in Figure 4). Indeed, by assumption firms cannot fire their FT workers. However, few firms in equilibrium fire their OE workers (from Figure 5, note that very few firms exist in the region of the state space in which firing rates are high). In fact, only the less productive firms have non-negligible quarterly probabilities of firing an OE worker. Instead, firms optimally wait for these contracts to expire, i.e. for the separation shock s_{OE}^{W} to hit and destroy the match, which allows them to save on the firing cost.

In sum, in the model, small firms hire either type of worker to start growing. If a new firm does not obtain an OE worker and hires an FT worker instead, the firm promotes the worker into an OE contract with a high probability. Subsequently, the firm continues to accumulate OE workers and to promote their FT workers into OE positions, though this probability tends to decrease as the firm grows in size. Part of the reason why the firm relies on this promotion margin is that FT workers separate very quickly (they last, on average, a little less than two quarters on the job).

These properties help explain the shape of the stationary firm size distribution (see Figure 5), where we see that the highest shares of firms are located in states with relatively few FT workers compared to the number of OE workers. This relative balance between the two types of workers is informed by the average temporary share seen in the data, of around one FT worker for every five employees.

4.4 Counterfactual Exercises: Inspecting the Mechanisms

Our calibrated economy matches the higher share of temporary workers for larger firms (Figure 3). As discussed in the previous Section, this results from the combination of (a) a larger long-run target of the temporary share for more productive (and in the long run larger) firms, and (b) a lower temporary share of larger firms of similar productivity. We now run a series of counterfactual exercises to understand which elements of our model help generate this pattern. In particular, we explore the relationship between firm size and the temporary share along three relevant dimensions: (i) the matching features of each labor market (A_{OE} versus A_{FT}); (ii) differences in firing costs between contract types (χ_{OE} versus χ_{FT}); and (iii) the persistence of productivity shocks. We discuss exercise (i) in this Section, as it is the most relevant one, and relegate (ii) and (iii) to Appendix C.

In the data, UE rates are about one order of magnitude higher for FT than for OE workers. The model rationalizes this fact by making the FT labor market more liquid, i.e. by estimating that the FT market delivers more matches for given market tightness per unit of time ($A_{FT}/A_{OE} > 3$). We now solve for a new economy where both markets have the same matching efficiency, in particular, where we lower A_{FT} to $0.4463 = A_{OE}$. We present the main statistics of this economy, which turns out to be very different from the benchmark, in Table 5.

First, once the hiring advantage of FT contracts disappears, firms hire a much smaller share of FT

Table 5: Results from the counterfactual experiment.

	(A)	(B)
Moment	*Baseline*	*Counterfactual*
Average employment per firm	6.70	13.84
Average temporary share	17.71%	1.15%
UE rate (total)	20.84%	4.86%
... UE rate (FT)	19.44%	2.27%
... UE rate (OE)	1.44%	2.60%
EU rate (total)	3.54%	1.58%
... EU rate (FT)	13.23%	13.36%
... EU rate (OE)	1.48%	1.45%
Promotion rate	5.50%	50.93%
Unemployment rate	14.51%	24.56%
Output per worker	1.000	0.813
... keeping avg. firm size fixed at baseline	.	1.680
... keeping distribution fixed at baseline	.	0.484

Notes: Column (A) corresponds to the baseline calibration; in column (B), we set $A_{FT} = A_{OE} = 0.4463$ and keep all parameters fixed at their calibrated values. The last two rows of the table compute output per worker while keeping either the average firm size or the distribution of firms fixed at the baseline calibration, following equation (28). For the computation of EU and UE rates, see Appendix A.4.

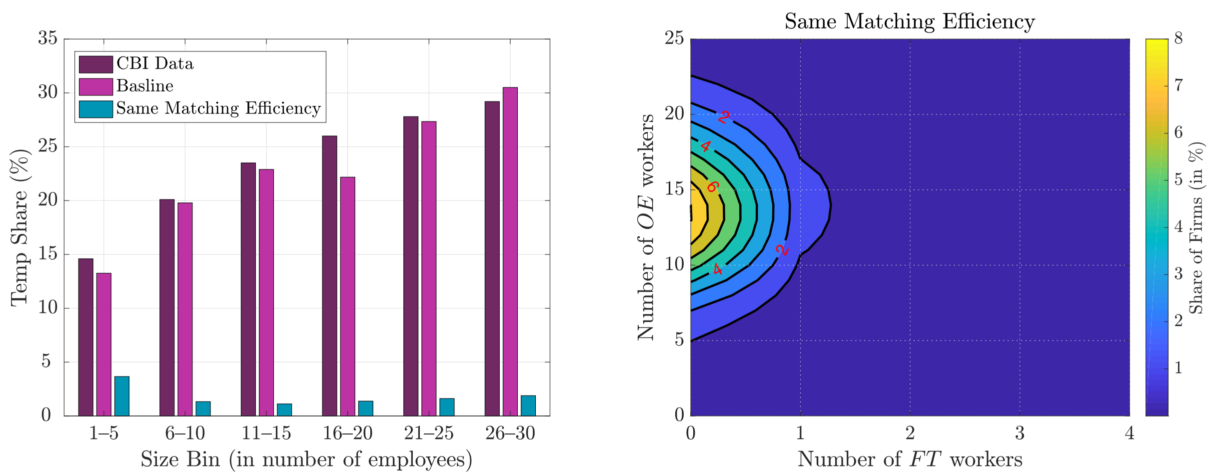
workers (the average temporary share falls from 17.71% to 1.22%) and rely mostly on OE contracts. As a result, UE flows are now more frequent among OE workers (2.59% per quarter vs 1.44% before), less frequent among FT workers (2.27% vs 19.44% before), and less frequent overall (4.86% per quarter vs 20.84% before) due to the loss of the high matching efficiency in the FT market. Second, we see that firms are much larger on average: 13.78 workers, compared to 6.70 in the baseline calibration. This is the result of the larger use of OE contracts, which diminishes worker separations and allows firms to reach larger size without the cost of constantly going to the FT market to hire. Third, we find that the economy is less productive and exhibits higher unemployment: as seen in Table 5, aggregate productivity falls by almost 20%, and the unemployment rate increases from less than 15% to nearly 25%.

The increase in the unemployment rate is the result of the decline in the UE rate discussed above that dominates the decline in the EU rate (from 3.54% to 1.58%) due the fall in the share of temporary workers. To understand the change in productivity, note that aggregate productivity can change for several reasons. To see this formally, aggregate output per workers can be written as

$$\frac{Y}{E} = \sum_{z \in \mathcal{Z}} \sum_{\vec{n} \in \mathcal{N}} \frac{Y(\vec{n}, z) \widetilde{f}(\vec{n}, z)}{E/F} \tag{28}$$

where $Y(\vec{n}, z)$ is firm output defined in equation (1), $\widetilde{f}(\vec{n}, z)$ is the *share* of active firms in state $(\vec{n}, z)$, derived in Appendices A.2, and E/F is average firm size (the ratio of the aggregate employment rate E to the measure of active firms F), derived in Appendix A.3. Thus, three components drive

Figure 6: Temp share on firm size, and firm distribution, for the counterfactual experiment.



Notes: On the left panel, each bar plot shows the average temporary share within each employment size bin, in the CBI subsample used to calibrate the model, in the calibrated model, and in the counterfactual economy with equal matching efficiency. On the right panel, we plot the firm size distribution in the counterfactual economy, adding up across z states.

changes in aggregate productivity: (i) changes in the production function, $Y(.)$; (ii) changes in the distribution of firms, $\widetilde{f}(.)$; and (iii) changes in the average number of employed workers per firm (E/F), which can in turn come from changes in the overall employment rate and in the measure of active firms. Following this result, we compute the change in productivity of the counterfactual economy by isolating the last two of these channels in turn (the production function does not change in the counterfactual exercises). We find that in this counterfactual economy there is a productivity gain due to the change in the equilibrium distribution of firms $\widetilde{f}(\vec{n}, z)$: if we run the counterfactual but keep the average size of firms fixed at 6.70 as in the baseline economy, productivity goes up to 70% (see the penultimate row of Table 5). However, the increase in average firm size E/F decreases productivity: keeping the distribution fixed at the benchmark, productivity in the counterfactual economy decreases by 50% (see the last row of Table 5). This comes from the decreasing returns to scale production, whereby spreading the same number of workers across fewer firms reduces productivity. Overall this latter effect dominates and in the counterfactual economy sees a productivity drop of 20%.

Finally, the share of temporary workers is no longer increasing in firm size, as seen on the left panel of Figure 6.[29] Indeed, the temporary share is largest for the smallest firms (1 to 5 workers) and does not change much with firm size after that. The overall conclusion, therefore, is that differences in matching efficiency between the two labor markets are essential to explain the data, and have quantitatively significant consequences for aggregate output and productivity.

[29]Figure D.1 in the Appendix further breaks this down by productivity level.

Table 6: Policy Results

	(A)	(B)	(C)
Average duration of FT contracts →	*Shorter FT duration* (1 quarter)	*Baseline calibration* (1.9 quarters)	*Longer FT duration* (4 quarters)
Average employment per firm	6.97	6.70	6.62
Average temp share	6.80%	17.71%	36.91%
UE rate (total)	19.78%	20.84%	20.28%
... UE rate (FT)	18.19%	19.44%	19.04%
... UE rate (OE)	1.63%	1.44%	1.28%
EU rate (total)	3.06%	3.54%	3.28%
... EU rate (FT)	25.00%	13.23%	6.41%
... EU rate (OE)	1.48%	1.48%	1.48%
Promotion rate	16.57%	5.50%	1.97%
Unemployment rate	13.40%	14.51%	13.93%
Output per worker	0.968	1.000	1.027
... keeping avg. firm size fixed	1.008	.	1.016
... keeping distribution fixed	0.961	.	1.011

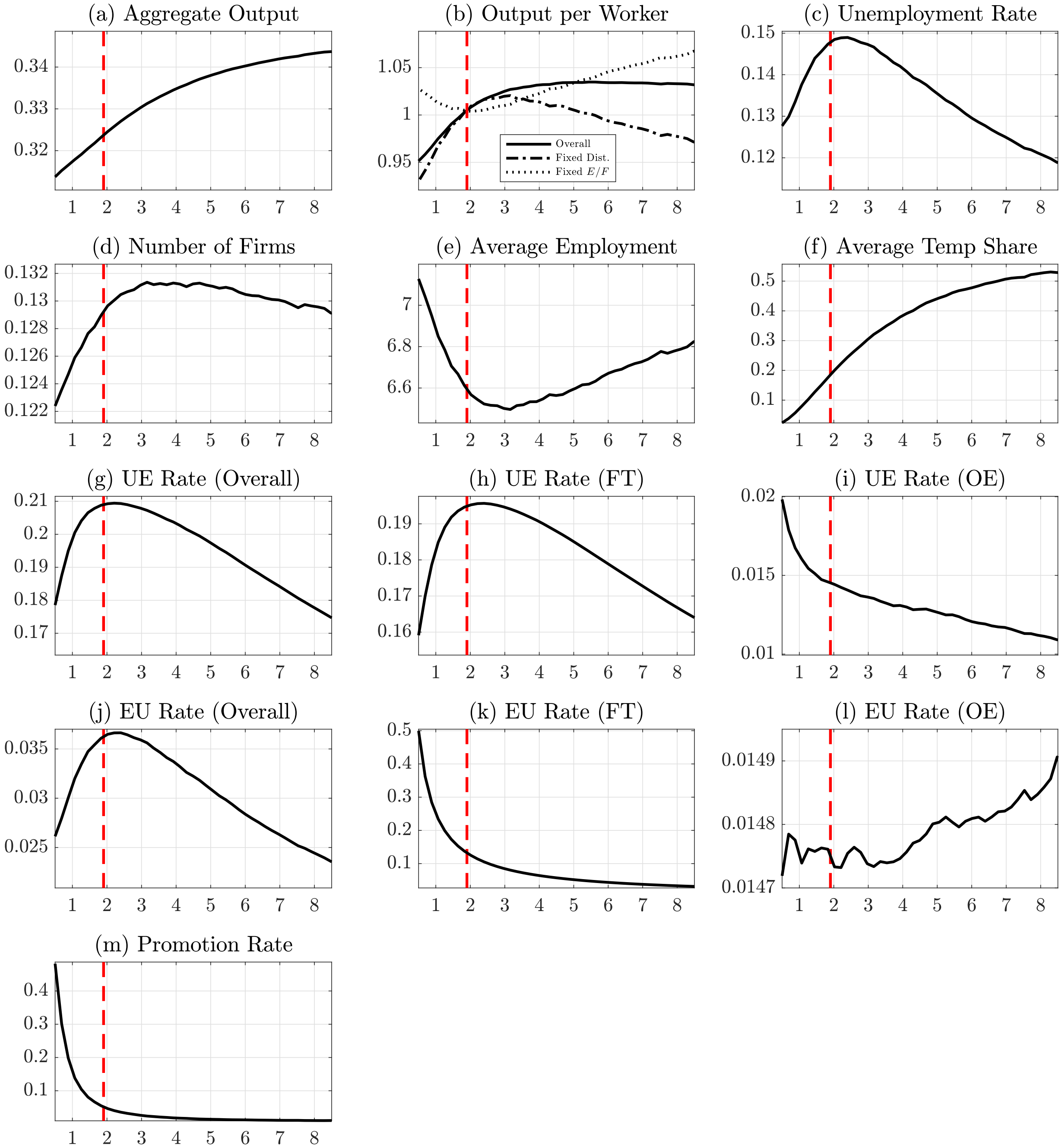
Notes: Column (B) corresponds to the baseline calibration; in column (C), we set $s^W_{FT} = 1/4$ so that FT contracts expire on average after 1 year; in column (A) we set $s^W_{FT} = 1$, so that FT contracts expire on average after 1 quarter. The last two rows of the table compute output per worker while keeping either the average firm size or the distribution of firms fixed at the baseline calibration, following equation (28). For the computation of EU and UE rates, see Appendix A.4.

5 Policy

What are the macroeconomic effects of dual labor markets, and what impact may policies that regulate FT contracts have? In this Section we use our calibrated model to study the effect of changes in the legal duration of FT contracts on the economy. To do so, we solve for a series of economies in which we vary the exogenous separation rate for FT contracts, s^W_{FT}, such that average duration for FT contracts moves from 1 to 10 quarters (the average duration is about 2 quarters in the calibrated economy). We leave all other parameters unchanged at their calibrated values, and compare across steady-state solutions. The results for this series of policy exercises are reported in Figure 7, while Table 6 provides exact numbers for two specific policies (one with shorter and and another one with longer average duration of FT contracts).

In our economy with ex-ante identical risk-neutral workers and firms, aggregate output is a measure of welfare. We find that aggregate output increases monotonically with the duration of FT contracts (see panel (a)). That is, welfare increases as FT contract are made more similar to OE contracts and hence the duality of the labor market is reduced. This increase in aggregate output with the duration of FT contracts is the result of two components. First, output per worker increases monotonically with the duration of FT contracts up to 4 quarters (one year) and remains stable afterwards (solid line in panel (b)). Second, the unemployment rate increases with the duration of

Figure 7: Effects of FT contracts duration on various equilibrium variables.



Notes: For all panels, the horizontal axis represents $1/s^W_{FT}$, and is measured in quarters. The plots shows different stationary solutions of the model, keeping all parameters fixed at their baseline calibration values except for s^W_{FT}. The dashed vertical line shows the expected duration in the baseline calibration. The "output per worker" panel is normalized to one for the baseline calibration. This panel also shows the counterfactual output per worker when the distribution is kept fixed at its baseline calibration value (dashed-dotted line), and the case when the average firm size (the ratio of the employment rate to the measure of firms, E/F) is kept fixed at its calibrated value (dotted line), following the decomposition shown in equation (28). For the computation of EU and UE rates, see Appendix A.4.

FT contracts up to a little above 2 quarters, and declines monotonically afterwards (see panel (c)). The decline of the unemployment rate at longer FT contract duration is what allows total output to increase despite no changes in output per worker. Let us discuss each one of these key variables (unemployment and output per worker) in turn.

The increase in the duration of FT contracts mechanically lowers its EU rate (see panel (k)). This makes FT contracts relatively more attractive to firms, who can now benefit from the higher matching efficiency of this market and suffer a lower turnover rate. As a result, the share of FT workers increases monotonically with the duration of their contracts (see panel (f)). The consequences for worker flows into unemployment and the unemployment rate are non-monotonic. At low levels of FT contract duration, the increase in the share of temporary workers dominates the increased duration of the FT contracts and job separations increase (the overall EU rate increases, see panel (j)), while at higher levels of FT contract duration (above 2 quarters) the increased duration of the FT contracts dominates the increase in the share of temporary workers and the job separations decline (the overall EU rate declines, see panel (j)). Qualitatively, the evolution of the worker flows from unemployment to employment are similar: the UE rates increase with FT contract duration first and decrease at higher levels of FT contract duration (above two quarters). This is the result of firms making a larger (smaller) effort for hiring when they lose more (less) workers by posting offers in more (less) expensive and less tight (tighter) sub-markets. Overall, the changes in the EU rate rate dominate the changes in the UE rate (see panel (g)) and the unemployment rate tracks the EU rate.[30]

As we did in Section 4.4, we can decompose the aggregate productivity gains into the effects due to a change in the equilibrium distribution of firms and the change in the number of firms per worker. In panel (b) we plot this decomposition. We find that the sharp increase in output per worker until FT contract durations of 4 quarters is mainly driven by the increase in the number of firms per worker (or decline in the average firm size, see panel (e)) due to the decreasing returns to scale technology. After this point, the number of firms per worker decline (the average firm size starts increasing), which lowers aggregate productivity. However, in this region the changes in the distribution of firms contribute positively to aggregate productivity, which offsets the fall due to average firm size.

All in all, our results indicate that changing the duration of FT contracts may yield non-monotonic effects on productivity and unemployment. In the context of our calibrated model, though, increasing the flexibility of FT contracts by extending their duration would decrease unemployment and increase firm size, though at the expense of decreasing the overall job-finding rate (as the UE rate decreases with duration in both labor markets) as well as the overall number of firms in the economy.

[30]Figures D.4 and D.5 in the Appendix show the effects of the policy on the relationship between the temporary share, firm size and productivity.

6 Conclusion

Many labor markets are characterized by a dual structure, in which firms offer both open-ended (OE) contracts of long duration with large layoff costs, and fixed-term (FT) contracts of limited duration. Using rich administrative balance-sheet data for Spain (2004-2019), we document that there exists a high degree of heterogeneity in the usage of FT contracts between firms. We find that most cross-sectional variation (around 84%) in the temporary share, i.e. in the ratio of workers employed with an FT to the total number of employees, is due to firm-level factors. In particular, both unconditionally and after controlling for time, region and sectoral fixed effects, smaller firms have on average a lower share of their workers employed under an FT contract than larger firms.

To understand this fact, and to analyze the implications of dual labor markets for aggregate productivity and the distribution of firms, we build and calibrate a firm-dynamics search-and-matching model of multi-worker firms. In the model, firms offer dynamic long-term OE and FT contracts to their new hires, specifying trajectories for wages, layoff and promotion rates. We calibrate the model to our Spanish data and target, among other moments, the employment-to-unemployment and unemployment-to-employment worker transition rates by contract type, as well as the empirically-observed monotonic relationship between the temporary share and firm size. In the calibrated model, the FT market is very liquid, delivering more matches per unit of time and for given market tightness than the OE market. However, separation of FT workers are far more frequent. With decreasing returns to scale, the opportunity cost of unfilled vacancies is lower for larger firms, so these firms hire a higher fraction of temporary workers in equilibrium. In counterfactual policy experiments, we find that changing the duration of FT contract may have important macroeconomic consequences: limiting their duration may decrease the share of temporary employment and the unemployment rate, but at the expense of decreasing aggregate productivity.

Our paper emphasizes that the firm side of dual labor markets may be an important dimension to consider in order to understand the aggregate implications of labor market duality, a phenomenon which is pervasive in both emerging and developed economies which has been traditionally studied only from the worker side. Combining both the worker and the firm side of this phenomenon remains an interesting avenue for future research.

References

ACEMOGLU, D. and HAWKINS, W. B. (2014). Search with multi-worker firms. *Theoretical Economics*, **9** (3), 583 – 628.

ALMUNIA, M., LÓPEZ-RODRÍGUEZ, D. and MORAL-BENITO, E. (2018). Evaluating the Macro-Representativeness of a Firm-Level Database: An Application for the Spanish Economy. *Banco de España Occasional Paper No. 1802*.

AUCIELLO, I., PIJOAN-MAS, J., PAU-ROLDAN-BLANCO and TAGLIATI, F. (2023). Dual Labor Markets in Spain: a Firm-Side Perspective, mimeo.

AUDOLY, R. (2020). Firm Dynamics and Random Search over the Business Cycle. *Working Paper*.

BENTOLILA, S., CAHUC, P., DOLADO, J. J. and LE BARBANCHON, T. (2012). Two-Tier Labour Markets in the Great Recession: France Versus Spain. *The Economic Journal*, **122** (562), F155–F187.

BERTON, F. and GARIBALDI, P. (2012). Workers and Firms Sorting into Temporary Jobs. *The Economic Journal*, **122** (562), F125–F154.

BILAL, A., ENGBOM, N., MONGEY, S. and VIOLANTE, G. L. (2019). Firm and Worker Dynamics in a Frictional Labor Market. *Working Paper*.

BLANCHARD, O. and LANDIER, A. (2002). The Perverse Effects of Partial Labour Market Reform: Fixed-Term Contracts in France. *The Economic Journal*, **112** (480), F214–F244.

CAGGESE, A. and CUÑAT, V. (2008). Financing Constraints and Fixed-term Employment Contracts. *The Economic Journal*, **118** (533), 2013–2046.

CAHUC, P., CARRY, P., MALHERBET, F. and MARTINS, P. (2022). Employment Effects of Restricting Fixed-Term Contracts: Theory and Evidence. *CEPR Working Paper, DP16875*.

—, CHARLOT, O. and MALHERBET, F. (2016). Explaining the Spread of Temporary Jobs and its Impact on Labor Turnover. *International Economic Review*, **57** (2), 522–572.

— and POSTEL-VINAY, F. (2002). Temporary jobs, employment protection and labor market performance. *Labour Economics*, **9** (1), 63–91.

CAO, S., SHAO, E. and SILOS, P. (2013). Match Quality, Labor Market Protection, and Temporary Employment, mimeo.

COLES, M. and MORTENSEN, D. T. (2016). Equilibrium Labor Turnover, Firm Growth, and Unemployment. *Econometrica*, **84** (1), 347–363.

COSTAIN, J. S., JIMENO, J. F. and THOMAS, C. (2010). Employment Fluctuations in a Dual Labor Market. *Working Paper 1013, Banco de España*.

DARUICH, D. (2022). The Macroeconomic Consequences of Early Childhood Development Policies. *Mimeo*.

—, DI ADDARIO, S. and SAGGIO, R. (2020). The Effects of Partial Employment Protection Reforms: Evidence from Italy. *Working Paper*.

DOLADO, J. J., GARCÍA-SERRANO, C. and JIMENO, J. F. (2002). Drawing Lessons from the Boom of Temporary Jobs in Spain. *The Economic Journal*, **112** (480), F270–F295.

ELSBY, M. W. L. and GOTTFRIES, A. (2021). Firm Dynamics, On-The-Job Search and Labor Market Fluctuations. *Working Paper*.

— and MICHAELS, R. (2013). Marginal Jobs, Heterogeneous Firms, and Unemployment Flows. *American Economic Journal: Macroeconomics*, **5** (1), 1–48.

GARCIA-LOUZAO, J., HOSPIDO, L. and RUGGIERI, A. (2022). Dual Returns to Experience. *Working Paper 2211, Banco de España*.

GOUIN-BONENFANT, E. (2020). Productivity Dispersion, Between-Firm Competition, and the Labor Share. *Working Paper*.

KAAS, L. and KIRCHER, P. (2015). Efficient Firm Dynamics in a Frictional Labor Market. *American Economic Review*, **105** (10), 3030 – 3060.

MORTENSEN, D. and PISSARIDES, C. (1994). Job Creation and Job Destruction in the Theory of Unemployment. *Review of Economic Studies*, **61**, 397–415.

MOSCARINI, G. and POSTEL-VINAY, F. (2013). Stochastic Search Equilibrium. *Review of Economic Studies*, **80** (4), 1545 – 1581.

PETRONGOLO, B. and PISSARIDES, C. A. (2001). Looking into the Black Box: A Survey of the Matching Function. *Journal of Economic Literature*, **39** (2), 390–431.

ROLDAN-BLANCO, P. and GILBUKH, S. (2021). Firm Dynamics and Pricing under Customer Capital Accumulation. *Journal of Monetary Economics*, **118**, 99 – 119.

RUIZ-GARCÍA, J. C. (2021). Financial Frictions, Firm Dynamics and the Aggregate Economy: Insights from Richer Productivity Processes. *Working Paper*.

SALA, H., SILVA, J. I. and TOLEDO, M. (2012). Flexibility at the Margin and Labor Market Volatility in OECD Countries. *The Scandinavian Journal of Economics*, **114** (3), 991–1017.

SCHAAL, E. (2017). Uncertainty and Unemployment. *Econometrica*, **85** (6), 1675 – 1721.

SHIMER, R. (2005). The Cyclical Behavior of Equilibrium Unemployment and Vacancies. *American Economic Review*, **95** (1), 25 – 39.

TAUCHEN, G. (1986). Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions. *Economics Letters*, **20** (2), 177 – 181.

THOMAS, C. (2006). Firing costs, labor market search and the business cycle. *Working Paper*.

Dual Labor Markets and the Equilibrium Distribution of Firms

by Josep Pijoan-Mas and Pau Roldan-Blanco

Appendix Materials

A Derivations and Proofs

A.1 Proof of Proposition 1

Proof. Let $\overline{\mathcal{C}} = \{\overline{c}_i\}_{i\in\mathcal{I}}$ be a given policy, where $\overline{c}_i \equiv \left\{\overline{w}_i, \overline{\delta}_i, \overline{p}\mathbf{1}_{[i=FT]}, \overline{W}'_i(\vec{n}', z')\right\}$. Then, we can write the firm's problem (9)-(10a)-(10b) as follows:

$$\boldsymbol{J}(\vec{n}, z, \vec{W}) = \max_{\overline{\mathcal{C}}} \ \widetilde{\boldsymbol{J}}(\vec{n}, z, \vec{W}|\overline{\mathcal{C}}) \quad \text{subject to} \begin{cases} \boldsymbol{W}_i(\vec{n}, z; \overline{\mathcal{C}}) \geq W_i, & \forall i \\ \overline{W}'_i(\vec{n}', z') \geq \boldsymbol{U}, & \forall (\vec{n}', z'), i \end{cases}$$

where:

$$\begin{aligned} \widetilde{\boldsymbol{J}}(\vec{n}, z, \vec{W}|\overline{\mathcal{C}}) \equiv \frac{1}{\overline{\rho}(\vec{n}, z)} \Bigg\{ & \exp(z) y(\vec{n}) - \chi_p \overline{p}^{\psi_p} + \sum_{i\in\mathcal{I}} \Bigg[-\overline{w}_i n_i - \chi_i \overline{\delta}_i^{\psi_i} \\ & + n_i \big(\overline{\delta}_i + s_i^W\big) \boldsymbol{J}\big(\vec{n}_i^-, z, \vec{W}'(\vec{n}_i^-, z)\big) + \eta_i\big(\overline{W}'_i(\vec{n}_i^+, z)\big) \boldsymbol{J}\big(\vec{n}_i^+, z, \vec{W}'(\vec{n}_i^+, z)\big) \Bigg] \\ & + \overline{p} n_{FT} \boldsymbol{J}\big(\vec{n}^p, z, \vec{W}'(\vec{n}^p, z)\big) + \sum_{z'} \lambda(z'|z) \boldsymbol{J}\big(\vec{n}, z', \vec{W}'(\vec{n}, z')\big) \Bigg\} \end{aligned} \tag{A.1}$$

where it is understood that $\vec{W}'(\cdot) = \left(\overline{W}'_i(\cdot), \overline{W}'_{-i}(\cdot)\right)$, and we have defined:

$$\overline{\rho}(\vec{n}, z) \equiv \rho + s^F + \sum_i \Big(n_i \big(\overline{\delta}_i + s_i^W\big) + \eta_i\big(\overline{W}'_i(\vec{n}_i^+, z)\big) \Big)$$

as the effective discount rate of the firm. By monotonicity of preferences, the promise-keeping constraint (10a) holds with equality: $\boldsymbol{W}_i(\vec{n}, z; \overline{\mathcal{C}}) = W_i$, $\forall i \in \mathcal{I}$. Imposing this into equation (8), we can solve for wages to obtain:

$$\begin{aligned} \overline{w}_i = \overline{\rho}(\vec{n}, z) W_i - \Bigg[& \big(\overline{\delta}_i + s_i^W + s^F\big) \boldsymbol{U} + (n_i - 1)\big(\overline{\delta}_i + s_i^W\big) \overline{W}'_i(\vec{n}_i^-, z) + n_{-i}\big(\overline{\delta}_{-i} + s_{-i}^W\big) \overline{W}'_i(\vec{n}_{-i}^-, z) \\ & + n_{FT} \overline{p} \left(\mathbf{1}_{[i=FT]} \frac{\overline{W}'_{OE}(\vec{n}^p, z) + (n_{FT} - 1) \overline{W}'_{FT}(\vec{n}^p, z)}{n_{FT}} + \mathbf{1}_{[i=OE]} \overline{W}'_{OE}(\vec{n}^p, z) \right) \end{aligned}$$

$$+\sum_{j\in\mathcal{I}}\eta_j\Big(\overline{W}'_j(\vec{n}_j^+,z)\Big)\overline{W}'_i(\vec{n}_j^+,z)+\sum_{z'}\lambda(z'|z)\overline{W}'_i(\vec{n},z')\Bigg] \tag{A.2}$$

Define the joint surplus under this given policy as:

$$\widetilde{\mathbf{\Sigma}}\Big(\vec{n},z,\vec{W}\Big|\overline{\mathcal{C}}\Big)\equiv\widetilde{\boldsymbol{J}}\Big(\vec{n},z,\vec{W}\Big|\overline{\mathcal{C}}\Big)+\sum_{i\in\mathcal{I}}n_iW_i$$

Likewise, define the *maximized* joint surplus as:

$$\mathbf{\Sigma}(\vec{n},z,\vec{W})\equiv\max_{\overline{\mathcal{C}}}\ \widetilde{\mathbf{\Sigma}}\Big(\vec{n},z,\vec{W}\Big|\overline{\mathcal{C}}\Big),\ \text{such that } \overline{W}'_i(\vec{n}',z')\geq\boldsymbol{U},\ \forall(\vec{n}',z'),i$$

Plugging (A.2) inside (A.1) and collecting terms:

$$\begin{aligned}
\underbrace{\widetilde{\boldsymbol{J}}(\vec{n},z,\vec{W}|\overline{\mathcal{C}})+\sum_{i\in\mathcal{I}}n_iW_i}_{=\widetilde{\mathbf{\Sigma}}(\vec{n},z,\vec{W}|\overline{\mathcal{C}})} &= \frac{1}{\overline{\rho}(\vec{n},z)}\Bigg\{\exp(z)y(\vec{n})-\chi_p\overline{p}^{\psi_p}+\sum_{i\in\mathcal{I}}\Bigg(n_i(\overline{\delta}_i+s_i^W+s^F)\boldsymbol{U}-\chi_i\overline{\delta}_i^{\psi_i}\Bigg)\\
&+\sum_{i\in\mathcal{I}}n_i(\overline{\delta}_i+s_i^W)\Bigg(\underbrace{\boldsymbol{J}(\vec{n}_i^-,z,\vec{W}'(\vec{n}_i^-,z))+(n_i-1)\overline{W}'_i(\vec{n}_i^-,z)+n_{-i}\overline{W}'_{-i}(\vec{n}_i^-,z)}_{=\mathbf{\Sigma}(\vec{n}_i^-,z,\vec{W}'(\vec{n}_i^-,z))}\Bigg)\\
&+\underbrace{\sum_{i\in\mathcal{I}}\Bigg[n_i\sum_{j\in\mathcal{I}}\eta_j\Big(\overline{W}'_j(\vec{n}_j^+,z)\Big)\overline{W}'_i(\vec{n}_j^+,z)+\eta_i(\overline{W}'_i(\vec{n}_i^+,z))\boldsymbol{J}(\vec{n}_i^+,z,\vec{W}'(\vec{n}_i^+,z))\Bigg]}_{\equiv[A]}\\
&+\ n_{FT}\overline{p}\Bigg(\underbrace{\boldsymbol{J}(\vec{n}^p,z,\vec{W}'(\vec{n}^p,z))+(n_{FT}-1)\overline{W}'_{FT}(\vec{n}^p,z)+(n_{OE}+1)\overline{W}'_{OE}(\vec{n}^p,z)}_{=\mathbf{\Sigma}(\vec{n}^p,z,\vec{W}'(\vec{n}^p,z))}\Bigg)\\
&+\sum_{z'}\lambda(z'|z)\Bigg(\underbrace{\boldsymbol{J}(\vec{n},z',\vec{W}'(\vec{n},z'))+\sum_{i\in\mathcal{I}}n_i\overline{W}'_i(\vec{n},z')}_{=\mathbf{\Sigma}(\vec{n},z',\vec{W}'(\vec{n},z'))}\Bigg)\Bigg\}
\end{aligned}$$

Note the term labeled $[A]$ above can be written as follows:

$$\begin{aligned}
[A] &= \sum_{i\in\mathcal{I}}\eta_i\Big(\overline{W}'_i(\vec{n}_i^+,z)\Big)\Bigg[\boldsymbol{J}(\vec{n}_i^+,z,\vec{W}'(\vec{n}_i^+,z))+n_i\overline{W}'_i(\vec{n}_i^+,z)+n_{-i}\overline{W}'_{-i}(\vec{n}_i^+,z)\Bigg]\\
&= \sum_{i\in\mathcal{I}}\eta_i\Big(\overline{W}'_i(\vec{n}_i^+,z)\Big)\Bigg[\underbrace{\boldsymbol{J}(\vec{n}_i^+,z,\vec{W}'(\vec{n}_i^+,z))+(n_i+1)\overline{W}'_i(\vec{n}_i^+,z)+n_{-i}\overline{W}'_{-i}(\vec{n}_i^+,z)}_{=\mathbf{\Sigma}(\vec{n}_i^+,z,\vec{W}'(\vec{n}_i^+,z))}\Bigg]\\
&\quad -\sum_{i\in\mathcal{I}}\eta_i\Big(\overline{W}'_i(\vec{n}_i^+,z)\Big)\overline{W}'_i(\vec{n}_i^+,z)
\end{aligned}$$

Therefore, putting things together:

$$
\begin{aligned}
\widetilde{\mathbf{\Sigma}}\left(\vec{n}, z, \vec{W}\middle|\overline{\mathcal{C}}\right) = \frac{1}{\overline{\rho}(\vec{n}, z)}\Bigg\{ & \exp(z)y(\vec{n}) - \chi_p \overline{p}^{\psi_p} + \sum_{i\in\mathcal{I}} \Bigg(n_i\left(\overline{\delta}_i + s_i^W + s^F\right)\boldsymbol{U} - \chi_i \overline{\delta}_i^{\psi_i} \\
& - \eta_i\left(\overline{W}'_i(\vec{n}_i^+, z)\right)\overline{W}'_i(\vec{n}_i^+, z) + n_i\left(\overline{\delta}_i + s_i^W\right)\mathbf{\Sigma}\left(\vec{n}_i^-, z, \vec{W}'(\vec{n}_i^-, z)\right) \\
& + \eta_i\left(\overline{W}'_i(\vec{n}_i^+, z)\right)\mathbf{\Sigma}\left(\vec{n}_i^+, z, \vec{W}'(\vec{n}_i^+, z)\right)\Bigg) + n_{FT}\overline{p}\mathbf{\Sigma}(\vec{n}^p, z, \vec{W}'(\vec{n}^p, z)) \\
& + \sum_{z'} \lambda(z'|z)\mathbf{\Sigma}(\vec{n}, z', \vec{W}'(\vec{n}, z'))\Bigg\}
\end{aligned} \tag{A.3}
$$

Note that the right-hand side of (A.3) is independent of $\vec{W}$ and w, so we can omit this dependence from $\mathbf{\Sigma}$ in equation (A.3), and further simplify the equation into:

$$
\begin{aligned}
\widetilde{\mathbf{\Sigma}}\left(\vec{n}, z\middle|\overline{\mathcal{C}}_\Sigma\right) = \frac{1}{\overline{\rho}(\vec{n}, z)}\Bigg\{ & \exp(z)y(\vec{n}) - \chi_p \overline{p}^{\psi_p} + \sum_{i\in\mathcal{I}} \Bigg(n_i\left(\overline{\delta}_i + s_i^W + s^F\right)\boldsymbol{U} - \chi_i \overline{\delta}_i^{\psi_i} \\
& - \eta_i\left(\overline{W}'_i(\vec{n}_i^+, z)\right)\overline{W}'_i(\vec{n}_i^+, z) + n_i\left(\overline{\delta}_i + s_i^W\right)\mathbf{\Sigma}\left(\vec{n}_i^-, z\right) + \eta_i\left(\overline{W}'_i(\vec{n}_i^+, z)\right)\mathbf{\Sigma}\left(\vec{n}_i^+, z\right)\Bigg) \\
& + n_{FT}\overline{p}\mathbf{\Sigma}(\vec{n}^p, z) + \sum_{z'} \lambda(z'|z)\mathbf{\Sigma}(\vec{n}, z')\Bigg\}
\end{aligned}
$$

Thus, out of the full set $\overline{\mathcal{C}} = \left\{\overline{w}_i, \overline{\delta}_i, \overline{p}\mathbf{1}_{[i=FT]}, \overline{W}'_i(\vec{n}', z')\right\}_{i\in\mathcal{I}}$, only the elements

$$
\overline{\mathcal{C}}_\Sigma \equiv \left\{\overline{\delta}_i, \overline{p}\mathbf{1}_{[i=FT]}, \overline{W}'_i(\vec{n}_i^+, z)\right\}_{i\in\mathcal{I}} \subset \overline{\mathcal{C}}
$$

matter for the joint surplus. The optimal contract is then:

$$
\overline{\mathcal{C}}_\Sigma^* = \arg\max_{\overline{\mathcal{C}}_\Sigma} \widetilde{\mathbf{\Sigma}}\left(\vec{n}, z\middle|\overline{\mathcal{C}}_\Sigma\right) \text{ s.t. } W'_i(\vec{n}_i^+, z) \geq \boldsymbol{U},\ \forall i \in \mathcal{I} \tag{A.4}
$$

Wages $\{\overline{w}_i\}_{i\in\mathcal{I}}$ are then given by equation (A.2), while the remaining continuation values are chosen to be consistent with the solution to problem (A.4), as explained in the main text. Summing up: by expressing the firm's problem in terms of continuation promises, we have shown that the optimal contract must maximize the joint surplus. Conversely, for any combination of continuation promises that maximizes the joint surplus, there is a unique wage and set of outstanding promises that maximizes the firm's value subject to the promise-keeping constraint. ■

A.2 Distribution Dynamics

Let $f_t(\vec{n}, z) > 0$ be the measure of firms with $\vec{n} = (n_i, n_{-i}) \in \mathcal{N}$ workers, and productivity $z \in \mathcal{Z}$, at time t. These firms seek to find new workers of type i in market segment $W_i'(\vec{n}_i^+, z)$. We denote the total measure of active firms by $F_t \equiv \sum_{\vec{n}} \sum_z f_t(\vec{n}, z)$, and the measure of potential entrants by $F_t^e > 0$.[31] Both of these aggregate measures are endogenous objects, and are determined in equilibrium to be consistent with the sorting patterns of firms and workers. Let $\theta_i(\vec{n}_i^+, z)$ denote the equilibrium tightness in the market of firms of type $(\vec{n}, z)$ looking to hire an additional worker under contract i, with

$$\theta_i(\vec{n}_i^+, z) = \mu_i^{-1}\left(\frac{\rho \boldsymbol{U} - b}{W_i'(\vec{n}_i^+, z) - \boldsymbol{U}}\right)$$

by equation (7). Market tightness must adjust to guarantee that:

$$u_{it}(\vec{n}_i^+, z)\theta_i(\vec{n}_i^+, z) = f_t(\vec{n}, z) \tag{A.5}$$

for all t, where $u_{it}(\vec{n}_i^+, z)$ is the measure of unemployed workers looking to be hired in a type-i contract by a firm in state $(\vec{n}, z)$. In words, this condition guarantees that, for every state and contract type, the total number of jobs created by firms in that state equals the total number of jobs found by workers in that submarket.[32]

Let $e_{it}(\vec{n}, z)$ be the measure of workers of type i employed by firms of type $(\vec{n}, z)$ at time t. By construction:

$$e_{it}(\vec{n}, z) = n_i f_t(\vec{n}, z) \tag{A.6}$$

The unit measure of workers must be either matched with a firm or unmatched and searching. This gives us a formula for the unemployment rate, $U_t = 1 - E_t$, where $E_t = \sum_{\vec{n}} \sum_z \sum_i n_i f_t(\vec{n}, z)$.

Consider a firm in state $(\vec{n}, z)$. Then, the rate of change in the measure of firms is:

$$\begin{aligned}
\frac{\partial f_t(\vec{n}, z)}{\partial t} &= \sum_{i \in \mathcal{I}} \eta_i\big(W_i'(\vec{n}_i^-, z)\big) f_t(\vec{n}_i^-, z) + \sum_{i \in \mathcal{I}} (n_i + 1)\Big(\delta_i(\vec{n}_i^+, z) + s_i^W\Big) f_t(\vec{n}_i^+, z) \\
&\quad + (n_{FT} + 1) p(\vec{n}_p^-, z) f_t(\vec{n}_p^-, z) + \sum_{\hat{z} \neq z} \lambda(z|\hat{z}) f_t(\vec{n}, \hat{z}) \\
&\quad - \left[s^F + \sum_{i \in \mathcal{I}} \eta_i\big(W_i'(\vec{n}, z)\big) + \sum_{i \in \mathcal{I}} n_i\Big(\delta_i(\vec{n}, z) + s_i^W\Big) + n_{FT} p(\vec{n}, z) + \sum_{\hat{z} \neq z} \lambda(\hat{z}|z)\right] f_t(\vec{n}, z)
\end{aligned} \tag{A.7}$$

[31] The symbol $\sum_{\vec{n}}$ expresses an element-wise sum, i.e. summing over both n_{OE} and n_{FT}.

[32] To see this, multiply both sides of equation (A.5) by $\eta_i(\theta_i(\vec{n}_i^+, z))$ and use the identify $\eta(\theta)\theta = \mu(\theta)$ to write:

$$\underbrace{u_{it}(\vec{n}_i^+, z)\mu_i\big(\theta_i(\vec{n}_i^+, z)\big)}_{\text{Jobs found by workers}} = \underbrace{f_t(\vec{n}, z)\eta_i\big(\theta_i(\vec{n}_i^+, z)\big)}_{\text{Jobs created by firms}}$$

The first two lines on the right-hand side of equation (A.7) give inflows into state $(\vec{n}, z)$. Inflows come from firms with $\vec{n}_i^- \equiv (n_i - 1, n_{-i})$ that hire a type-i worker, firms with $\vec{n}_i^+ \equiv (n_i + 1, n_{-i})$ that fire a type-i worker, firms with $\vec{n}_p^- \equiv (n_{OE} - 1, n_{FT} + 1)$ that promote an FT worker into an OE contract, and firms at $\vec{n} = (n_i, n_{-i})$ that transition into productivity z from some $\hat{z} \neq z$. The last line gives the corresponding outflows.[33] On the other hand, the dynamics of inactive firms are:

$$\frac{\partial F_t^e}{\partial t} = s^F F_t + \sum_{z \in \mathcal{Z}} \sum_{i \in \mathcal{I}} \left(\delta_i(\vec{n}_i^e, z) + s_i^W \right) f_t(\vec{n}_i^e, z) - F_t^e \left(\sum_{z^e \in \mathcal{Z}} \pi_z(z^e) \sum_{i \in \mathcal{I}} \eta_i\big(W_i'(\vec{n}_i^e, z^e)\big) \right) \tag{A.8}$$

where the first term are inflows from exiting firms, the second term are inflows from firms with $\vec{n}_i^e \equiv (n_i, n_{-i}) = (1, 0)$ that lose their last remaining worker, and the third term are outflows from successful entrants.

A.3 Aggregate Measures of Agents

To find the aggregate measures of agents in the stationary solution, first we impose steady state, i.e. $\frac{\partial f_t(\vec{n}, z)}{\partial t} = \frac{\partial F_t^e}{\partial t} = 0$, into equations (A.7)-(A.8). In the numerical implementation, we solve for the *share* (not the measure) $\phi_t(n_i, n_{-i}, z)$ of firms (active or inactive) in each state $(n_i, n_{-i}, z) \in \mathbb{S} \equiv \{0, 1, \ldots, N_i\} \times \{0, 1, \ldots, N_{-i}\} \times \{z_1, \ldots, z_k\}$. This gives us a system of $(N_i + 1)(N_{-i} + 1)k$ equations and the same number of unknowns, $\{\phi(n_i, n_{-i}, z)\} \in [0, 1]^{\mathbb{S}}$. This system of linear equations can be solved with a simple matrix inversion.[34]

Having found the invariant distribution, we then make the following normalization:

$$\widetilde{f}(n_i, n_{-i}, z) \equiv \frac{\phi(n_i, n_{-i}, z)}{1 - \sum_{z=1}^k \phi(0, 0, z)} \qquad \text{and} \qquad \widetilde{F}_e \equiv \sum_{z=1}^k \widetilde{f}(0, 0, z)$$

In words, $\widetilde{f}(n_i, n_{-i}, z)$ is the measure of firms in state (n_i, n_{-i}, z) as a share of *active* firms, i.e. $\widetilde{f} \equiv f/F$, whereas $\widetilde{F}_e$ is the measure of potential entrants as a share of active firms, i.e. $\widetilde{F}_e \equiv F_e/F$.

To proceed, use identity (A.6) to compute:

$$\widetilde{e}_i(\vec{n}, z) = n_i \widetilde{f}(\vec{n}, z)$$

That is, $\widetilde{e}_i \equiv e_i/F$ is the measure of workers of type i employed in firms of type $(\vec{n}, z)$, as a share of the total measure of active firms. From this we can find $\widetilde{E}_i \equiv E_i/F = \sum_{\vec{n}} \sum_z \widetilde{e}_i(\vec{n}, z)$, and $\widetilde{E} \equiv E/F = \sum_{i \in \mathcal{I}} \widetilde{E}_i$.

On the other hand, using (A.5), we know:

[33] Inflows from hiring must be multiplied by $\pi_z(z)$ whenever they come from successful entrants, i.e. if $(n_i, n_{-i}) = (1, 0)$.

[34] In particular, we impose the additional condition $\sum_{n_i=0}^{N_i} \sum_{n_{-i}=0}^{N_{-i}} \sum_{z=z_1}^{z_k} \phi(n_i, n_{-i}, z)$, which guarantees a unique solution.

$$u_{OE}(n_{OE}+1,n_{FT},z) = F\cdot\frac{\widetilde{f}(n_{OE},n_{FT},z)}{\theta_{OE}(n_{OE}+1,n_{FT},z)}$$
$$u_{FT}(n_{OE},n_{FT}+1,z) = F\cdot\frac{\widetilde{f}(n_{OE},n_{FT},z)}{\theta_{FT}(n_{OE},n_{FT}+1,z)}$$

for any $(n_{OE},n_{FT},z)\in\overline{\mathcal{N}}_{OE}\times\overline{\mathcal{N}}_{FT}\times\mathcal{Z}$. In the first equation, add both sides across all $z\in\mathcal{Z}$, and over $n_{OE}=1,\dots,N_{OE}$ (i.e. omitting $n_{OE}=0$) and all $n_{FT}\in\overline{\mathcal{N}}_{FT}=\{0,1,2,\dots,N_{FT}\}$. Similarly, for the second equation, add across all $z\in\mathcal{Z}$, all $n_{OE}\in\overline{\mathcal{N}}_{OE}=\{0,1,2,\dots,N_{OE}\}$ and over $n_{FT}=1,\dots,N_{FT}$ (i.e. omitting $n_{FT}=0$). That is:

$$\sum_{n_{OE}\geq 1}\sum_{n_{FT}\in\overline{\mathcal{N}}_{FT}}\sum_{z}u_{OE}(n_{OE}+1,n_{FT},z) = F\left(\sum_{n_{OE}\geq 1}\sum_{n_{FT}\in\overline{\mathcal{N}}_{FT}}\sum_{z}\frac{\widetilde{f}(n_{OE},n_{FT},z)}{\theta_{OE}(n_{OE}+1,n_{FT},z)}\right) \tag{A.9}$$
$$\sum_{n_{OE}\in\overline{\mathcal{N}}_{OE}}\sum_{n_{FT}\geq 1}\sum_{z}u_{FT}(n_{OE},n_{FT}+1,z) = F\left(\sum_{n_{OE}\in\overline{\mathcal{N}}_{OE}}\sum_{n_{FT}\geq 1}\sum_{z}\frac{\widetilde{f}(n_{OE},n_{FT},z)}{\theta_{FT}(n_{OE},n_{FT}+1,z)}\right) \tag{A.10}$$

Developing the left-hand side of (A.9) and (A.10) yields:

$$\begin{aligned}\sum_{n_{OE}\geq 1}\sum_{n_{FT}\in\overline{\mathcal{N}}_{FT}}\sum_{z}u_{OE}(n_{OE}+1,n_{FT},z) &= U_{OE}-\sum_{n_{FT}\in\overline{\mathcal{N}}_{FT}}\sum_{z}u_{OE}(1,n_{FT},z)\\ &= \underbrace{1-E-U_{FT}}_{=U_{OE}}-\sum_{z}\left(\frac{F^{e}}{\theta_{OE}(1,0,z)}+\sum_{n_{FT}\geq 1}\frac{f(0,n_{FT},z)}{\theta_{OE}(1,n_{FT},z)}\right)\\ &= 1-U_{FT}-F\left[\widetilde{E}+\sum_{z}\left(\frac{\widetilde{F}^{e}}{\theta_{OE}(1,0,z)}+\sum_{n_{FT}\geq 1}\frac{\widetilde{f}(0,n_{FT},z)}{\theta_{OE}(1,n_{FT},z)}\right)\right]\end{aligned}$$

and

$$\begin{aligned}\sum_{n_{OE}\in\overline{\mathcal{N}}_{OE}}\sum_{n_{FT}\geq 1}\sum_{z}u_{FT}(n_{OE},n_{FT}+1,z) &= U_{FT}-\sum_{n_{OE}\in\overline{\mathcal{N}}_{OE}}\sum_{z}u_{FT}(n_{OE},1,z)\\ &= U_{FT}-\sum_{z}\left(\frac{F^{e}}{\theta_{FT}(0,1,z)}+\sum_{n_{OE}\geq 1}\frac{f(n_{OE},0,z)}{\theta_{FT}(n_{OE},1,z)}\right)\\ &= U_{FT}-F\sum_{z}\left(\frac{\widetilde{F}^{e}}{\theta_{FT}(0,1,z)}+\sum_{n_{OE}\geq 1}\frac{\widetilde{f}(n_{OE},0,z)}{\theta_{FT}(n_{OE},1,z)}\right)\end{aligned}$$

respectively. Substituting these back into (A.9) and (A.10) yields:

$$1-U_{FT}-F\left[\widetilde{E}+\sum_{z}\left(\frac{\widetilde{F}^{e}}{\theta_{OE}(1,0,z)}+\sum_{n_{FT}\geq 1}\frac{\widetilde{f}(0,n_{FT},z)}{\theta_{OE}(1,n_{FT},z)}\right)\right] = F\left[\sum_{n_{OE}\geq 1}\sum_{n_{FT}\in\overline{\mathcal{N}}_{FT}}\sum_{z}\frac{\widetilde{f}(n_{OE},n_{FT},z)}{\theta_{OE}(n_{OE}+1,n_{FT},z)}\right]$$

and

$$U_{FT} - F\left[\sum_z \left(\frac{\widetilde{F}^e}{\theta_{FT}(0,1,z)} + \sum_{n_{OE}\geq 1} \frac{\widetilde{f}(n_{OE},0,z)}{\theta_{FT}(n_{OE},1,z)}\right)\right] = F\left[\sum_{n_{OE}\in\overline{\mathcal{N}}_{OE}} \sum_{n_{FT}\geq 1} \sum_z \frac{\widetilde{f}(n_{OE},n_{FT},z)}{\theta_{FT}(n_{OE},n_{FT}+1,z)}\right]$$

Solving for F on each equation yields:

$$F = \frac{1-U_{FT}}{\widetilde{E}+\widetilde{U}_{OE}} \qquad \text{and} \qquad F = \frac{U_{FT}}{\widetilde{U}_{FT}} \tag{A.11}$$

respectively, where we have defined:

$$\widetilde{U}_{OE} \equiv \sum_z \left(\frac{\widetilde{F}^e}{\theta_{OE}(1,0,z)} + \sum_{n_{FT}\geq 1} \frac{\widetilde{f}(0,n_{FT},z)}{\theta_{OE}(1,n_{FT},z)} + \sum_{n_{OE}\geq 1} \sum_{n_{FT}\in\overline{\mathcal{N}}_{FT}} \frac{\widetilde{f}(n_{OE},n_{FT},z)}{\theta_{OE}(n_{OE}+1,n_{FT},z)} \right)$$

$$\widetilde{U}_{FT} \equiv \sum_z \left(\frac{\widetilde{F}^e}{\theta_{FT}(0,1,z)} + \sum_{n_{OE}\geq 1} \frac{\widetilde{f}(n_{OE},0,z)}{\theta_{FT}(n_{OE},1,z)} + \sum_{n_{OE}\in\overline{\mathcal{N}}_{OE}} \sum_{n_{FT}\geq 1} \frac{\widetilde{f}(n_{OE},n_{FT},z)}{\theta_{FT}(n_{OE},n_{FT}+1,z)} \right)$$

Solving for U_{FT} from equation (A.11) gives $U_{FT} = \frac{\widetilde{U}_{FT}}{\widetilde{E}+\widetilde{U}_{OE}+\widetilde{U}_{FT}}$, which finally gives us the aggregate measure of active firms:

$$F = \left(\widetilde{E}+\widetilde{U}_{OE}+\widetilde{U}_{FT}\right)^{-1} \tag{A.12}$$

Therefore, $U_{FT} = \widetilde{U}_{FT}F$ and $U_{OE} = \widetilde{U}_{OE}F$. Once we have F, we can compute all the remaining aggregates: (i) the total measure of potential entrants is $F^e = \widetilde{F}^e F$; (ii) the employment and unemployment rates are given by $E = \widetilde{E}F$ and $U = 1 - E = U_{OE} + U_{FT}$; ans (iii) the aggregate temp-share is given by E_{FT}/E.

A.4 Worker Dynamics

In the data We compute the EU and UE quarterly rates by type of contract using data from the *Encuesta de Población Activa* (EPA), compiled by the *Instituto Nacional de Estadística* (INE), the Spanish national statistical agency. The data come at the quarterly frequency for the period 2006Q1-2019Q4. Denote by $UE^i_{t,t+1}$ the U-to-E flow from quarter t to $t+1$ into a contract of type $i = OE, FT$, and similarly for $EU^i_{t,t+1}$. E-to-E flows from an FT into an OE contract are denoted by $EE^{FtoO}_{t,t+1}$. Labor market rates are defined as follows:

$$\widehat{UE}_i^{\text{data}} \equiv \frac{\sum UE^i_{t,t+1}}{\sum U_t} \qquad \text{and} \qquad \widehat{EU}_i^{\text{data}} \equiv \frac{\sum EU^i_{t,t+1}}{\sum E^i_t}$$

where $\sum$ denotes the sum of sample weights for all observations in that category, $\sum U_t$ is the number of unemployed at time t, and $\sum E^i_t$ is the number of employed in contract type i at time t. Similarly, the promotion rate in the data is computed as follows:

$$\widehat{EE}^{\text{data}}_{FtoO} \equiv \frac{\sum EE^{FtoO}_{t,t+1}}{\sum E^{FT}_t}$$

where E^{FT}_t is the stock of FT workers in quarter t.

In the model We categorize workers into the three employment status that we have available in the data: employed with an OE contract, employed with an FT contract, and unemployed. The following set of equations describes flows between these states through the lens of the model:

$$\begin{aligned}
\frac{\partial E_{OE}}{\partial t} &= \sum_{\vec{n}}\sum_{z}\left\{p(\vec{n},z)e_{FT}(\vec{n},z) + \mu_{OE}(\vec{n}^{+}_{OE},z)u_{OE}(\vec{n}^{+}_{OE},z) - \left(\delta_{OE}(\vec{n},z) + s^W_{OE} + s^F\right)e_{OE}(\vec{n},z)\right\}\\
\frac{\partial E_{FT}}{\partial t} &= \sum_{\vec{n}}\sum_{z}\left\{\mu_{FT}(\vec{n}^{+}_{FT},z)u_{FT}(\vec{n}^{+}_{FT},z) - \left(\delta_{FT}(\vec{n},z) + s^W_{FT} + s^F\right)e_{FT}(\vec{n},z) - p(\vec{n},z)e_{FT}(\vec{n},z)\right\}\\
\frac{\partial U}{\partial t} &= \sum_{\vec{n}}\sum_{z}\left\{\left(\delta_{OE}(\vec{n},z) + s^W_{OE} + s^F\right)e_{OE}(\vec{n},z) + \left(\delta_{FT}(\vec{n},z) + s^W_{FT} + s^F\right)e_{FT}(\vec{n},z)\right.\\
&\qquad\left. - \sum_{i=OE,FT}\mu_i(\vec{n}^{+}_i,z)u_i(\vec{n}^{+}_i,z)\right\}
\end{aligned}$$

A more compact way of writing this dynamical system is:

$$\frac{\partial E_{OE}}{\partial t} = -\lambda_{EU_{OE}}E_{OE} + \lambda_{EE_{FtoO}}E_{FT} + \lambda_{UE_{OE}}U \tag{A.13a}$$

$$\frac{\partial E_{FT}}{\partial t} = -\left(\lambda_{EU_{FT}} + \lambda_{EE_{FtoO}}\right)E_{FT} + \lambda_{UE_{FT}}U \tag{A.13b}$$

$$\frac{\partial U_{OE}}{\partial t} = \lambda_{EU_{OE}}E_{OE} + \lambda_{EU_{FT}}E_{FT} - \left(\lambda_{UE_{OE}} + \lambda_{UE_{FT}}\right)U \tag{A.13c}$$

where we have defined the following average intensities:

$$\lambda_{EU_{OE}} \equiv \frac{EU_{OE}}{E_{OE}} \qquad \lambda_{EE_{FtoO}} \equiv \frac{EE_{FtoO}}{E_{FT}} \qquad \lambda_{UE_{OE}} \equiv \frac{UE_{OE}}{U}$$

$$\lambda_{EU_{FT}} \equiv \frac{EU_{FT}}{E_{FT}} \qquad \lambda_{UE_{FT}} \equiv \frac{UE_{FT}}{U}$$

with

$$EU_{OE} \equiv \sum_{\vec{n}}\sum_{z}\left(\delta_{OE}(\vec{n},z) + s^W_{OE} + s^F\right)e_{OE}(\vec{n},z) \qquad EE_{FtoO} \equiv \sum_{\vec{n}}\sum_{z}p(\vec{n},z)e_{FT}(\vec{n},z)$$

$$UE_{OE} \equiv \sum_{\vec{n}}\sum_{z}\mu_{OE}(\vec{n}^{+}_{OE},z)u_{OE}(\vec{n}^{+}_{OE},z) \qquad EU_{FT} \equiv \sum_{\vec{n}}\sum_{z}\left(\delta_{FT}(\vec{n},z) + s^W_{FT} + s^F\right)e_{FT}(\vec{n},z)$$

$$UE_{FT} \equiv \sum_{\vec{n}}\sum_{z}\mu_{FT}(\vec{n}^{+}_{FT},z)u_{FT}(\vec{n}^{+}_{FT},z)$$

We can write system (A.13a)-(A.13c) in matrix form as follows:

$$\frac{\partial}{\partial t}\begin{bmatrix} E_{OE} \\ E_{FT} \\ U \end{bmatrix} = \begin{pmatrix} -\lambda_{EU_{OE}} & \lambda_{EE_{FtoO}} & \lambda_{UE_{OE}} \\ 0 & -(\lambda_{EU_{FT}} + \lambda_{EE_{FtoO}}) & \lambda_{UE_{FT}} \\ \lambda_{EU_{OE}} & \lambda_{EU_{FT}} & -(\lambda_{UE_{OE}} + \lambda_{EU_{FT}}) \end{pmatrix} \begin{bmatrix} E_{OE} \\ E_{FT} \\ U \end{bmatrix}$$

Setting the right-hand side to the zero vector will give us the stationary measures in the reduced-form model.[35] To have numbers that can be compared to the ones from the quarterly data, we compute for each contract type $i = OE, FT$:

$$\widehat{UE}_i^{\text{model}} = \frac{1 - \exp\left(-UE_i \Delta\right)}{U} \qquad \text{and} \qquad \widehat{EU}_i^{\text{model}} = \frac{1 - \exp\left(-EU_i \Delta\right)}{E_i}$$

where, in the numerator, we have transformed Poisson flow rates into quarterly probabilities by setting $\Delta = 1/4$.[36] For the overall UE and EU rates, we compute:

$$\widehat{UE}_{\text{total}}^{\text{model}} = \frac{1 - \exp\left(-(UE_{OE} + UE_{FT})\Delta\right)}{U} \qquad \text{and} \qquad \widehat{EU}_{\text{total}}^{\text{model}} = \frac{1 - \exp\left(-(EU_{OE} + EU_{FT})\Delta\right)}{E_{OE} + E_{FT}}$$

Similarly, to obtain the promotion rate at the quarterly frequency in the model, we compute:

$$\widehat{EE}_{FtoO}^{\text{model}} = \frac{1 - \exp\left(-EE_{FtoO}\Delta\right)}{E_{FT}}$$

For estimation purposes, we treat $\widehat{UE}_i^{\text{model}}$, $\widehat{EU}_i^{\text{model}}$ and $\widehat{EE}_{FtoO}^{\text{model}}$ as the direct model counterparts of $\widehat{UE}_i^{\text{data}}$, $\widehat{EU}_i^{\text{data}}$ and $\widehat{EE}_{FtoO}^{\text{data}}$, respectively.

B Numerical Appendix

B.1 Stationary Solution Algorithm

The idea of the algorithm is to loop over the fixed firm entry cost κ in order to find the fixed point that obtains the targeted measure of active firms, denoted F^*, as an outcome of the equilibrium. We solve the model on a grid $\overline{\mathcal{N}}_{OE} \times \overline{\mathcal{N}}_{FT} \times \mathcal{Z}$, where $\overline{\mathcal{N}}_i \equiv \{0, 1, 2, \ldots, N_i\}$, $i \in \mathcal{I}$, for a sufficiently large $N_i \in \mathbb{N}$. The solution algorithm is as follows:

Step 0. Set $k = 0$. Choose guesses $\underline{\kappa}^{(0)} \in \mathbb{R}_+$ and $\overline{\kappa}^{(0)} \gg \underline{\kappa}^{(0)}$.

Step 1. At iteration $k \in \mathbb{N}$, set the entry cost to:

$$\kappa^{(k)} = \frac{\underline{\kappa}^{(k)} + \overline{\kappa}^{(k)}}{2}$$

[35] The resulting measures (E_{OE}, E_{FT}, U) coincide exactly, by construction, with the ones derived in Appendix A.3.

[36] In particular, the numerator is the probability that there is *at least* one transition (i.e. one or more transitions), which we compute as the complementary probability of no transitions.

Step 2. Use Value Function Iteration to solve for $\mathbf{\Sigma}^{(k)} \in \overline{\mathcal{N}}_{OE} \times \overline{\mathcal{N}}_{FT} \times \mathcal{Z}$ using:[37]

$$\begin{aligned}\mathbf{\Sigma}^{(k)}(\vec{n},z) = \frac{1}{\rho^{(k)}(\vec{n},z)} \Bigg\{ & \exp(z) y(\vec{n}) - \chi_p \left[p^{(k)}(\vec{n},z)\right]^{\psi_p} + \sum_{i\in\mathcal{I}} \Bigg[n_i \left(\delta_i^{(k)}(\vec{n},z) + s_i^W + s^F\right) \boldsymbol{U}^{(k)} \\ & - \chi_i \left[\delta_i^{(k)}(\vec{n},z)\right]^{\psi_i} + n_i \left(\delta_i^{(k)}(\vec{n},z) + s_i^W\right) \mathbf{\Sigma}^{(k)}\left(\vec{n}_i^-, z\right) \\ & + \eta_i^{(k)}(\vec{n},z) \max\left(\mathbf{\Sigma}^{(k)}\left(\vec{n}_i^+, z\right) - \boldsymbol{U}^{(k)}, \gamma\left(\mathbf{\Sigma}^{(k)}\left(\vec{n}_i^+, z\right) - \boldsymbol{U}^{(k)}\right) + (1-\gamma)\mathbf{\Sigma}^{(k)}\left(\vec{n},z\right)\right)\Bigg] \\ & + p^{(k)}(\vec{n},z) \mathbf{\Sigma}^{(k)}\left(\vec{n}^p, z\right) + \sum_{z'} \lambda(z'|z) \mathbf{\Sigma}^{(k)}\left(\vec{n}, z'\right) \Bigg\}\end{aligned}$$

where:

$$\delta_i^{(k)}(\vec{n},z) = \left[\frac{n_i}{\psi_i \chi_i}\left(\mathbf{\Sigma}^{(k)}\left(\vec{n}_i^-, z\right) - \mathbf{\Sigma}^{(k)}(\vec{n},z) + \boldsymbol{U}^{(k)}\right)\right]^{\frac{1}{\psi_i - 1}}$$

$$p^{(k)}(\vec{n},z) = \left[\frac{n_{FT}}{\psi_p \chi_p}\left(\mathbf{\Sigma}^{(k)}\left(\vec{n}^p, z\right) - \mathbf{\Sigma}^{(k)}(\vec{n},z)\right)\right]^{\frac{1}{\psi_p - 1}}$$

$$\eta_i^{(k)}(\vec{n},z) = A_i^{\frac{1}{\gamma}} \left[\left(\frac{1-\gamma}{\rho \boldsymbol{U}^{(k)} - b}\right)\left(\mathbf{\Sigma}^{(k)}\left(\vec{n}_i^+, z\right) - \mathbf{\Sigma}^{(k)}(\vec{n},z) - \boldsymbol{U}^{(k)}\right)\right]^{\frac{1-\gamma}{\gamma}}$$

$$\rho^{(k)}(\vec{n},z) = \rho + s^F + \sum_{i\in\mathcal{I}} \left(n_i \left(\delta_i^{(k)}(\vec{n},z) + s_i^W\right) + \eta_i^{(k)}(\vec{n},z)\right)$$

and where $\boldsymbol{U}^{(k)}$ is the solution to the free-entry condition:

$$\kappa^{(k)} = \gamma \left(\frac{\rho \boldsymbol{U}^{(k)} - b}{1-\gamma}\right)^{\frac{\gamma-1}{\gamma}} \sum_{z^e \in \mathcal{Z}} \pi_z(z^e) \left\{\sum_{i\in\mathcal{I}} \left[A_i \left(\mathbf{\Sigma}^{(k)}\left(\vec{n}_i^e, z^e\right) - \boldsymbol{U}^{(k)}\right)\right]^{\frac{1}{\gamma}}\right\}$$

where $\vec{n}_i^e = (n_i^e, n_{-i}^e) = (1, 0)$.

Step 3. Compute the invariant distribution of firms and the aggregate measure of active firms $F^{(k)}$, as described in Appendix A.3.

Step 4. Compute $\Psi^{(k)} \equiv F^{(k)}/F^* - 1$. Stop if $\Psi^{(k)} \in [-\varepsilon, \varepsilon]$ for some small tolerance $\varepsilon > 0$. Otherwise,

(a) if $\Psi^{(k)} < -\varepsilon$, set $\underline{\kappa}^{(k+1)} = \underline{\kappa}^{(k)}$ and $\overline{\kappa}^{(k+1)} = \iota \cdot \kappa^{(k)} + (1-\iota) \cdot \overline{\kappa}^{(k)}$, or

(b) if $\Psi^{(k)} > \varepsilon$, set $\underline{\kappa}^{(k+1)} = \iota \cdot \kappa^{(k)} + (1-\iota) \cdot \underline{\kappa}^{(k)}$ and $\overline{\kappa}^{(k+1)} = \overline{\kappa}^{(k)}$,

where $\iota \in (0, 1]$ is a dampening parameter, and go back to Step 1 with $[k] \leftarrow [k+1]$.

[37] To arrive at this expression, we have used results (19) and (26) into equation (A.3).

B.2 Numerical Implementation of the Productivity Process

In the model, idiosyncratic productivity z is governed by a continuous-time Markov chain (CTMC), with associated infinitesimal generator matrix:

$$\mathbf{\Lambda}_z = \begin{pmatrix} -\sum_{j\neq 1}\lambda_{1j} & \lambda_{12} & \dots & \lambda_{1k} \\ \lambda_{21} & -\sum_{j\neq 2}\lambda_{2j} & \dots & \lambda_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{k1} & \lambda_{k2} & \dots & -\sum_{j\neq k}\lambda_{kj} \end{pmatrix}$$

where $\lambda_{ij} > 0$ is short-hand for $\lambda(z_j|z_i)$, $z_i, z_j \in \mathcal{Z}$. For the numerical implementation, we specialize this to an Ornstein-Uhlenbeck (OU) process in logs:

$$\mathrm{d}\log(z_t) = -\rho_z \log(z_t)\mathrm{d}t + \sigma_z \mathrm{d}B_t$$

where B_t is a Wiener process.[38] For the numerical implementation, and the mapping between the (ρ_z, σ_z) and the $\{\lambda_{ij}\}$ parameters, we use the following steps:

1. For a given time interval $[0, T] \subset \mathbb{R}_+$, partition the space into M subintervals of equal length Δ, i.e. $\mathbb{T} \equiv \{0 = t_0 < t_1 < \cdots < t_M = T\}$ with $t_{m+1} - t_m = \Delta$ and $\Delta = T/M$. As the model is calibrated at the quarterly frequency, we choose $\Delta = 1/4$.

2. Approximate the OU process using the *Euler-Maruyama method*:

$$\log(z_k) = \underbrace{(1-\rho_z\Delta)}_{\equiv\tilde{\rho}_z}\log(z_{k-1}) + \underbrace{\sigma_z\sqrt{\Delta}}_{\equiv\tilde{\sigma}_z}\ \varepsilon^z_k, \qquad \varepsilon^z_k \sim iid\ \mathcal{N}(0,1) \tag{B.1}$$

   for each $k \in \mathbb{T}$. This is an AR(1) process on $\mathbb{T}$ with autocorrelation $\tilde{\rho}_z \equiv 1 - \rho_z\Delta$ and volatility $\tilde{\sigma}_z \equiv \sigma_z\sqrt{\Delta}$. Therefore:

   (a) In order to obtain a quarterly autocorrelation of $\tilde{\rho}_z$, we must set:

$$\rho_z = \frac{1-\tilde{\rho}_z}{\Delta}$$

   Since we target a *yearly* autocorrelation of 0.81, we have $\tilde{\rho}_z = (0.81)^{\frac{1}{4}}$, and therefore $\rho_z = 0.2053$.

   (b) In order to obtain a quarterly volatility of $\tilde{\sigma}_z$, we must set:

$$\sigma_z = \frac{\tilde{\sigma}_z}{\sqrt{\Delta}}$$

   Since we target an *annualized* volatility of 0.34, we set $\sigma_z = 0.17$.

[38] In levels, this is a diffusion of the type $\mathrm{d}z_t = \mu(z_t)\mathrm{d}t + \sigma(z_t)\mathrm{d}B_t$, with $\mu(z) = z\left(-\rho_z\log(z) + \frac{\sigma_z^2}{2}\right)$ and $\sigma(z) = \sigma_z z$.

3. Estimate the discrete-time process (B.1) via the Tauchen (1986) method, using a discrete-state Markov chain that defined on the theoretical grid, $\mathcal{Z}$. The outcome of this method are estimates for (ρ_z, σ_z), and a transition probability matrix $\mathbf{\Pi}_z = (\pi_{ij})$, where π_{ij} denotes the probability of a z_i-to-z_j transition in the $\mathbb{T}$ space.

4. For the mapping back into the $\{\lambda_{ij}\}$ rates, we use that any CTMC with generator matrix $\mathbf{\Lambda}_z$ maps into a discrete-time Markov chain with transition matrix $\mathbf{\Pi}_z(t)$ at time $t \in \mathbb{Z}_+$ in which holding times between arrivals are independently and exponentially distributed, so that $\mathbf{\Pi}_z(t) = \exp(\mathbf{\Lambda}_z t)$, from which we can solve for $\{\lambda_{ij}\}$ to obtain:

$$\lambda_{ij} = \begin{cases} -\frac{\pi_{ij}}{1-\pi_{ii}} \frac{\log(\pi_{ii})}{\Delta} & \text{for } i \neq j \\ \frac{\log(\pi_{ii})}{\Delta} & \text{otherwise} \end{cases}$$

where $\{\pi_{ij}\}$ are the estimates found in Step 3.

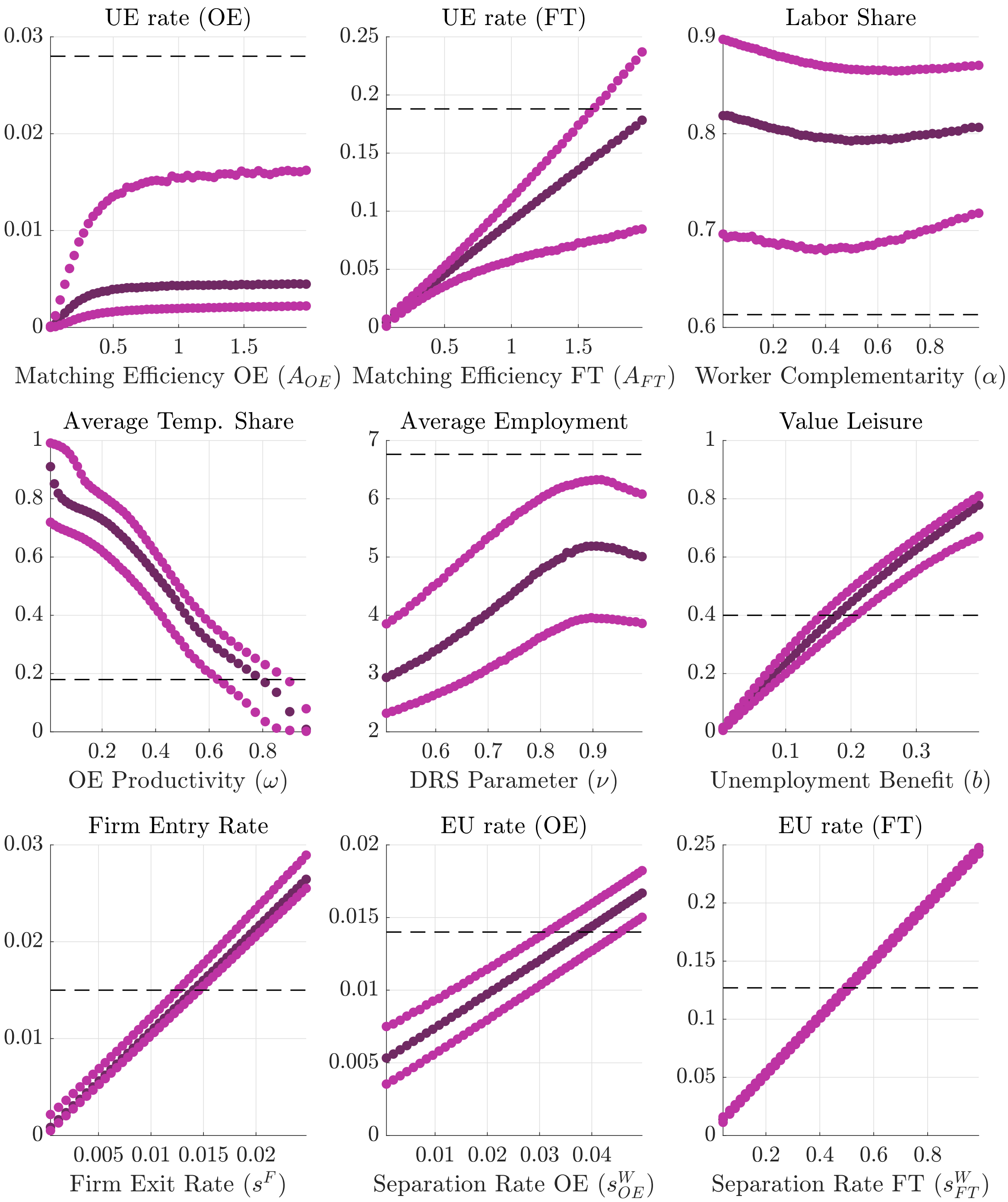
B.3 Estimation Procedure and Global Identification

To estimate the model, we seek to find the vector of parameters $\boldsymbol{\theta}$ that minimizes the distance function $\mathcal{D}(\boldsymbol{\theta})$, defined in equation (27). To do this, we first we create a large P-dimensional hypercube $\mathcal{P}$ in the parameter space, where P is the number of parameters. Then, we pick quasi-random realizations from it using a Sobol sequence, which successively forms finer uniform partitions of the parameter space, thereby efficiently and comprehensively exploring every corner of $\mathcal{P}$. For each parameter-vector draw, we solve the model and store its results in a matrix. For this step, we use a high performance computer (HPC), which allows us to parallelize the procedure and saves us a huge amount of computational time. After drawing N distinct parameter vectors (in practice, $N \approx 1.65$M), we have an $N \times M$ matrix $\boldsymbol{R}$ of results and an $N \times M$ matrix $\mathbf{\Theta} \subset \mathcal{P}$ of the corresponding parameters. The set of internally calibrated parameter values $\widehat{\boldsymbol{\theta}} \in \mathbf{\Theta}$ then satisfies:

$$\mathcal{D}(\widehat{\boldsymbol{\theta}}) \leq \mathcal{D}(\boldsymbol{\theta}), \quad \forall \boldsymbol{\theta} \in \mathbf{\Theta}$$

The advantage of this method over other estimation techniques is that the model-generated data contained in the $(\boldsymbol{R}, \mathbf{\Theta})$ matrices can be exploited to obtain information about identification, following Daruich (2022). First, for each parameter $\theta \in \boldsymbol{\theta}$, we select a target moment m which we think, based on economic intuition, may be particularly sensitive to the parameter. Because of our Sobol routine, for each given value of θ there is a distribution of values for m that results from the underlying random variation in all the remaining $P-1$ parameters. Using this, we divide the support of θ into 50 quantiles, and compute the 25th, 50th and 75th percentiles of this underlying distribution at each quantile. We may then study how sensitive m is to changes in θ by exploring how the distribution of m behaves across different values for θ. We say that θ is well-identified by m when (i) the distribution changes across different quantiles of p, (ii) the rate of this change is high, and (iii) the inter-quartile range of the m distribution is small throughout the support for θ. Criterion

Figure B.1: Global identification results.



Notes: The dark purple dots are the median of the distribution of each targeted moment for given value of the chosen parameter, and the light purple dots are the 25th and 75th percentiles. The dashed horizontal line is the target.

(i) implies that m is *sensitive* to variation in θ, (ii) gives an idea of *how strong* this relationship is, and (iii) implies that *other parameters* are relatively unimportant to explain it. As all the remaining parameters are not fixed throughout this analysis but instead are varying in a random fashion, this method gives us a global view of identification.

Figure B.1 presents the results from the global identification procedure explained above, where we have associated each targeted moment with the parameter that the moment most plausibly identifies (the same pairing as in Table 3 and in our verbal discussion in Section 4). We find that some parameters are well identified by all criteria, particularly $(\omega, b, s^F, s^W_{OE}, s^W_{FT})$. The matching efficiency parameters (A_{OE}, A_{FT}) generate responses in the UE rates, but the efficiency of the OE labor market misses the target. The DRS parameter ν also generates the expected response in the average employment, though the target falls on the upper tail of the distribution. The one parameter that is not well identified is α, which does not generate enough variation in the labor share.

C Additional Counterfactual Exercises

In Section 4.4 we discussed the counterfactual exercise in which we equalize the matching efficiency across the FT and OE markets. In this Appendix we discuss two additional counterfactual exercises, one in which we make firing of OE workers infinitely-costly and another one in which we increase the persistence of productivity shocks.

Table C.1: Results from additional counterfactual experiments.

	(A)	(B)	(C)
Moment	*Baseline*	*Firing costs*	*z persistence*
Average employment	6.70	6.70	6.81
Average temporary share	17.71%	17.89%	18.23%
UE rate (total)	20.84%	20.80%	20.74%
... UE rate (FT)	19.44%	19.43%	19.27%
... UE rate (OE)	1.44%	1.42%	1.54%
EU rate (total)	3.54%	3.53%	3.57%
... EU rate (FT)	13.23%	13.23%	13.23%
... EU rate (OE)	1.48%	1.45%	1.45%
Promotion rate	5.50%	5.31%	5.04%
Unemployment rate	14.51%	14.50%	14.68%
Output per worker	1.000	0.999	0.984
... keeping avg. firm size fixed	.	0.999	1.003
... keeping distribution fixed	.	0.999	0.962

Notes: For each counterfactual experiment, we change one parameter and keep all the remaining ones at their calibrated values. Column (A) corresponds to the baseline calibration; in column (B), we set $\chi_{OE} = \chi_{FT} = +\infty$; in column (C) we increase the persistence of idiosyncratic productivity shocks to a yearly autocorrelation of 98%, and change σ_z accordingly so that the overall variance of shocks remains unchanged. The last two rows of the table compute output per worker while keeping either the average firm size or the distribution of firms fixed at the baseline calibration, following equation (28). For the computation of EU and UE rates, see Appendix A.4.

C.1 Firing Costs

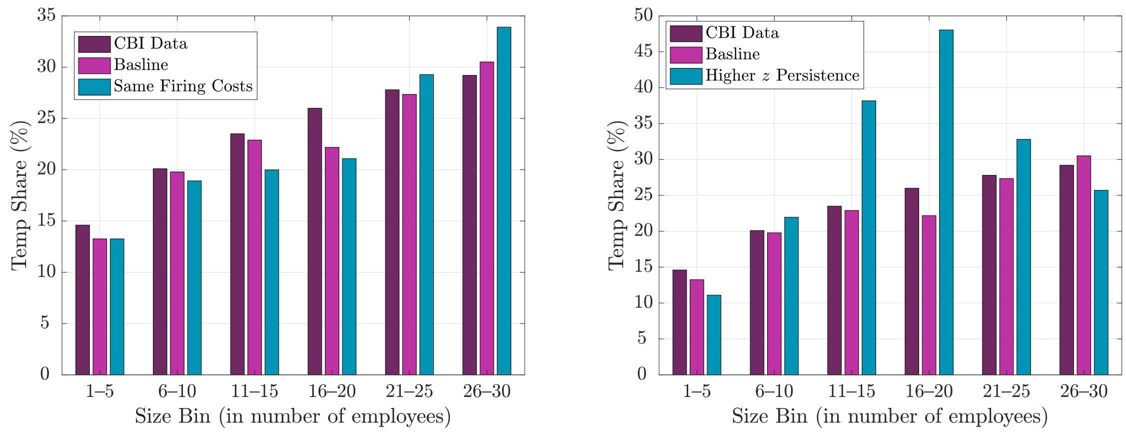
In Section 4.2, we argued that, in the calibrated model, firms do not make extensive use of their firing of OE workers. Instead, they rely on jobs to be destroyed. Therefore, it is not surprising that changes in firing costs have a small quantitative impact on economic aggregates. In particular, in this counterfactual we assume that firms are not allowed to fire their OE workers, which amounts to assuming that $\chi_{OE} = +\infty$. All the other parameters are kept at their calibrated values, including the firing cost of FT workers, which is also $\chi_{FT} = +\infty$.

The left panel of Figure C.1 shows that, in this counterfactual economy, there is still an increasing relationship between the temporary share and firm size, even though the levels are slightly lower for some small firms.[39] The left panel in Figure C.2 shows that the firm size distribution changes very

[39]Figure D.2 further breaks this down into productivity levels.

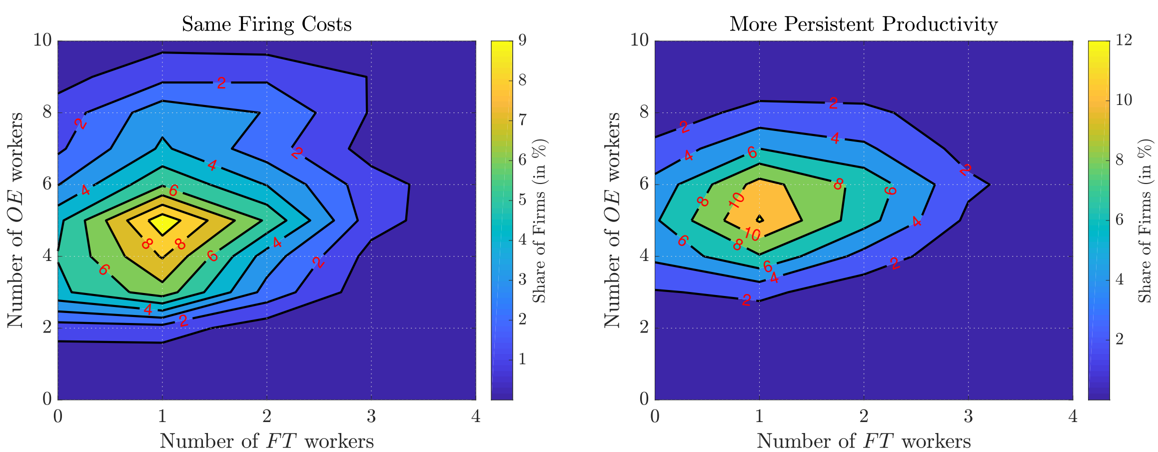
little as well (compare it to Figure 5), and column (B) in Table C.1 shows that economic aggregates are not affected in any significant way: the flow rates are similar and aggregate productivity is unaffected by the experiment. Therefore, this counterfactual suggests that policies intended to change the firing costs of workers employed in specific type of contracts would be ineffective in making the economy more productive.

Figure C.1: Temp share on firm size for different counterfactual scenarios.



Notes: Each bar plot shows the average temporary share within each employment size bin, in the CBI subsample used to calibrate the model, in the calibrated model, and in the model under different counterfactual scenarios.

Figure C.2: Firm size distribution for different counterfactual scenarios.



Notes: Each contour plot shows, under different scenarios, the distribution of firms in the space of (n_{OE}, n_{FT}), adding across all productivity states z.

C.2 Productivity Shocks

Next, we analyze the role of productivity shocks. In particular, we run a counterfactual in which productivity shocks are more persistent. The reason this may change the hiring, promotion and firing decisions of firms is that, if firms expect to retain their productivity for a longer period of time, they may alter their optimal mix of OE and FT workers, as these have different expected durations on the job. In the baseline calibration, we assumed that z had a yearly autocorrelation of 0.81. Keeping all the other parameters at their calibrated values, we now assume that productivity shocks have a yearly autocorrelation of 0.98. Moreover, we change the volatily of the process to ensure that productivity shocks retain the same variance. Using the formulas developed in Appendix B.2, this implies that ρ_z changes from $\rho_z = 0.2053$ to $\rho_z = 0.0202$, and σ_z from $\sigma_z = 0.1700$ to $\sigma_z = 0.0577$.

The right panels of Figures C.1 and C.2, and column (C) in Table C.1, show the results of this experiment.[40] For the temporary share, we now find an inverse-U shaped relationship between the temporary share and firm size. As in the baseline economy, the smallest firms prefer a high share of OE workers because these firms face the highest risk of exiting, and therefore prefer to hire OE workers, who are less likely to separate from the firm. The largest firms, by contrast, are very consolidated and face a very small risk of exiting. However, they now face productivity shocks of higher persistence, and therefore can expect to keep their productivity level for a longer period of time. Given this, these firms may want to have a higher share of OE, because these are workers that they can retain more easily, ensuring that overall firm size is unlikely to change in the future. This intuition is in a similar vain to the ideas developed in Cahuc *et al.* (2016), where firms choose OE contracts when they expect jobs to last for longer. In our setting, job duration and firm size interact with idiosyncratic productivity, rendering more productive firm to choose higher OE shares when faced with more persistent productivity states. In the aggregate, as seen in column (C) of Table C.1, the economy is less productive though with a lower unemployment rate. However, these differences are not quantitatively large.

[40]Figure D.3 further breaks down the temporary share vs firm size relationship into productivity levels.

D Additional Figures

Figure D.1: Temporary share by firm size and by productivity, for the counterfactual where we equate the matching efficiency in both labor markets.

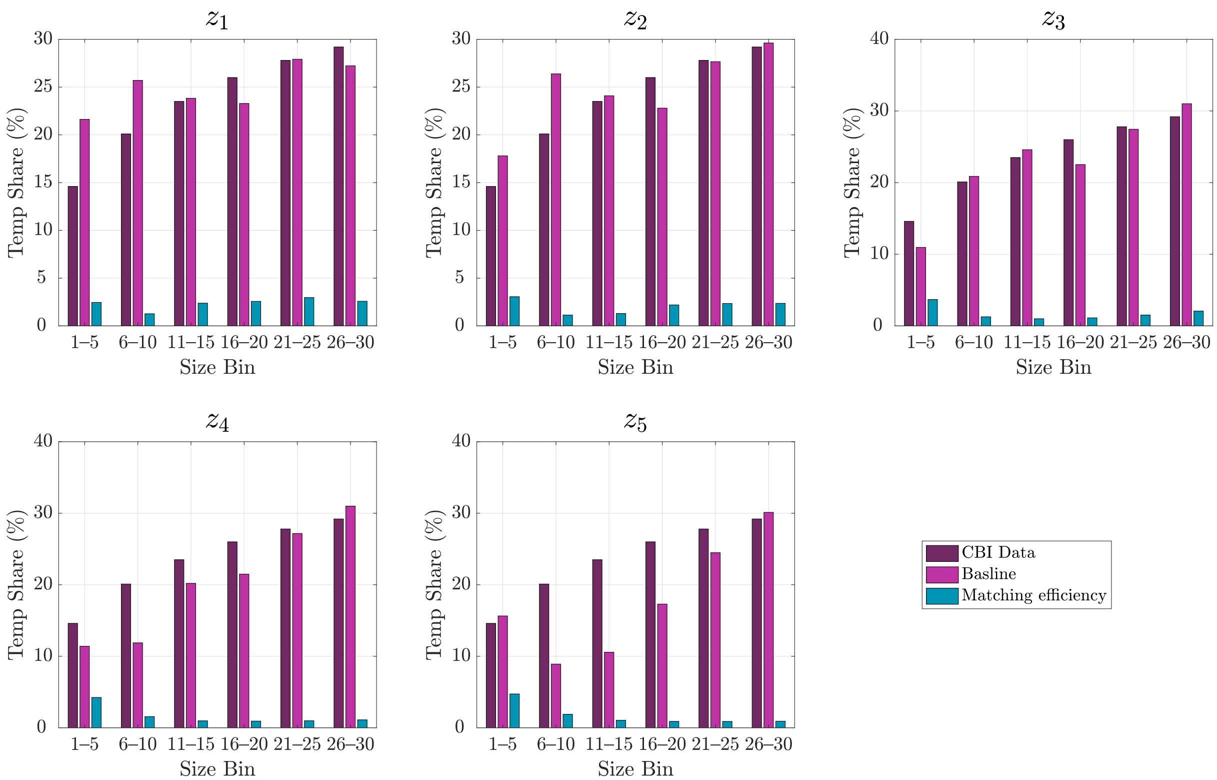


Figure D.2: Temporary share by firm size and by productivity, for the counterfactual where we set firing costs to infinity in both labor markets.

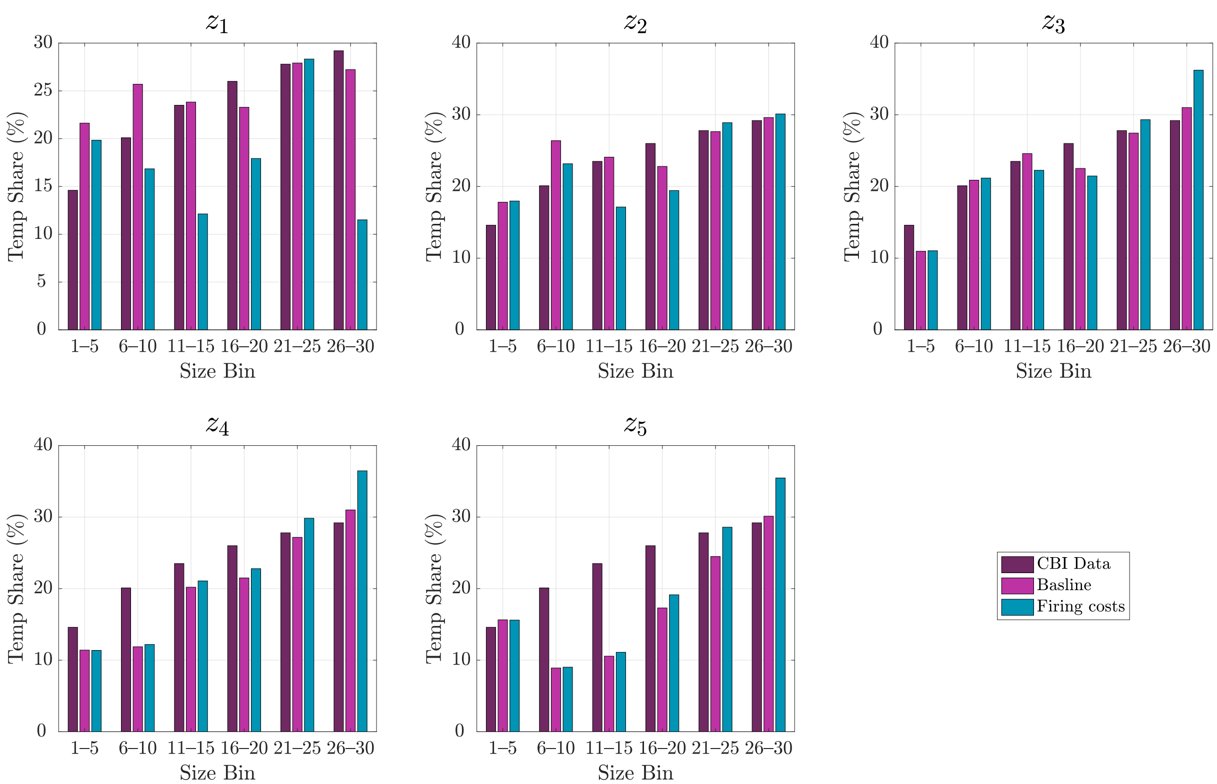


Figure D.3: Temporary share by firm size and by productivity, for the counterfactual where we increase the persistence of idiosyncratic productivity shocks.

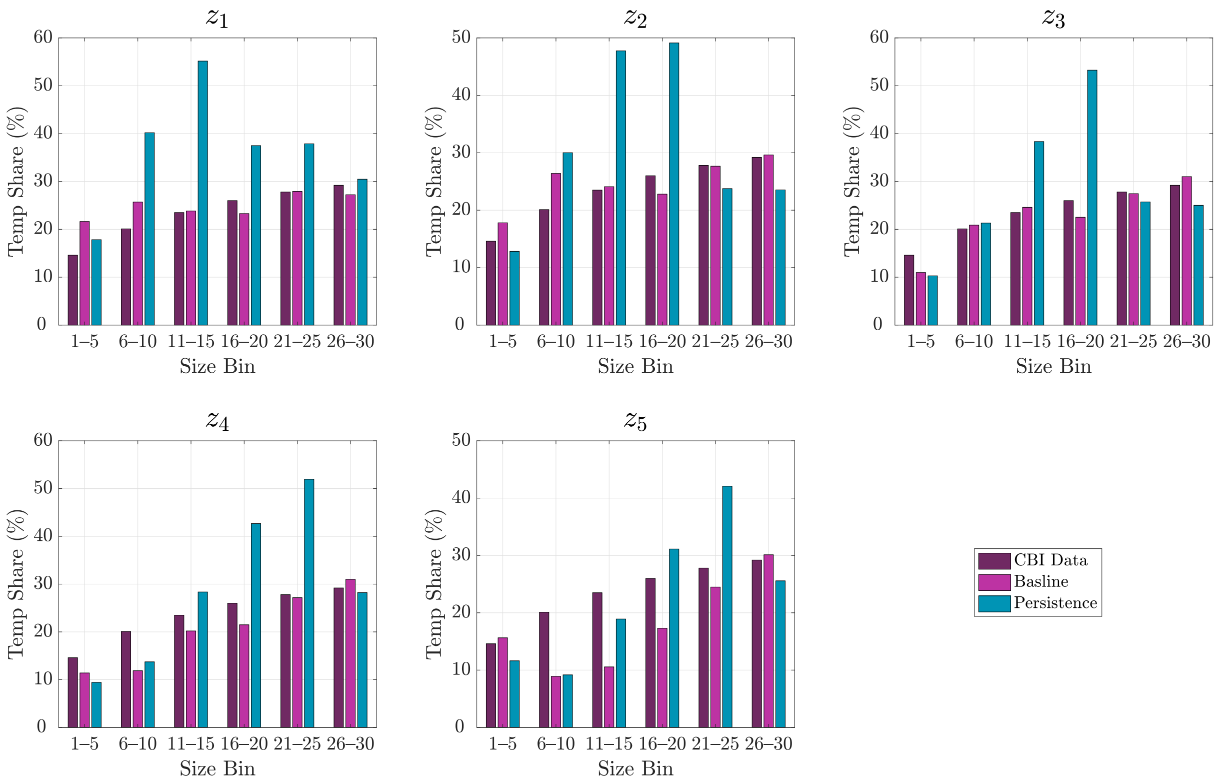


Figure D.4: Temporary share by firm size across different policies.

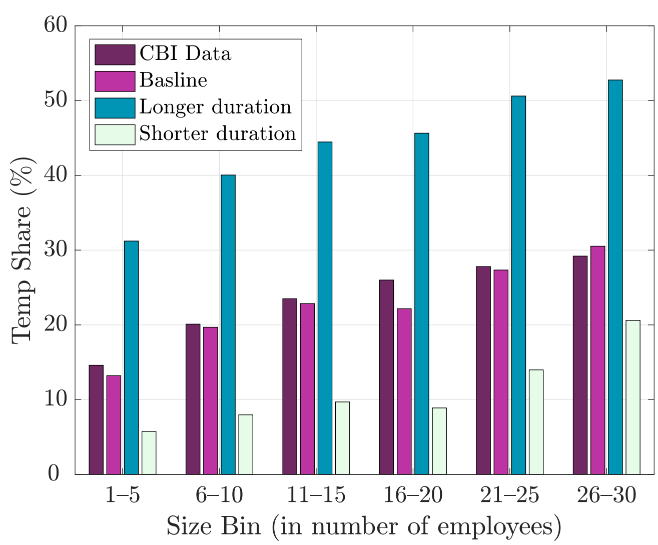


Figure D.5: Temporary share by firm size and by productivity, across different policies.